3.8: Faster-than-light frames of reference?

Learning Objectives

- Explain the phenomenon of faster-than-light motion in relativity

We recall from Section 3.4 that special relativity doesn't permit the existence of observers who move at c. This is because if two observers differ in velocity by c, then the Lorentz transformation between them is not a one-to-one map, which is physically unacceptable.

But what about a superluminal observer, one who moves faster than c? With charming naivete, the special-effects technicians for Star Trek attempted to show the frame of reference of such an observer in scenes where a field of stars rushed past the Enterprise. (Never mind that the stars, which pass in front of and behind the spaceship, should actually be a million times larger than it.) Actually such an observer would consider her own world-line, which we call spacelike, to be timelike, while the world-line of a star such as our sun, which we consider timelike, would be spacelike in her opinion. Our sun's world-line might, for example, be orthogonal to hers, in which case the sun would not appear to her as an object in motion but rather as a line stretching across space, which would wink into existence and then wink back out. A typical transformation between our frame and the frame of such an observer would be the map S defined by $(t', x') = (x, t)$, simply swapping the time and space coordinates. The "swap" transformation S is one-to-one, and therefore not subject to the objection raised previously to frames moving at c. S happens to be a boost by an infinite velocity, but we can also obtain boosts for velocities $c < v < \infty$ and $-\infty < v < -c$ by combining S with a (subluminal) Lorentz transformation; given a superluminal world-line l, we first transform into a frame in which l is a line of simultaneity, and the we apply S.

But this was all in $1+1$ dimensions. In $3+1$ dimensions, what is the equivalent of S? One possibility is something like $(t', x', y', z') = (x, t, t, t)$, but this isn't one-to-one. We can't squish three dimensions to one or expand one to three without merging points or splitting one point into many.

Another possibility would be a one-to-one transformation such as $(t', x', y', z') = (x, t, y, z)$. The trouble with this version is that it violates the isotropy of spacetime (section 2.3). For example, consider the vector $(1, 0, 1, 0)$ in the unprimed coordinates. This lies on the light cone, and could point along the world-line of a ray of light. After the transformation to the primed coordinates, this vector becomes $(0, 1, 1, 0)$, which points along a line of simultaneity. The primed observer says that the speed of light in this direction is infinite, and yet there are other directions in which it has a finite value. This clearly violates isotropy.

A surprisingly large number of papers, going all the way back to the birth of relativity, have been written by people trying to find a way to extend the Lorentz transformations to superluminal speeds, and these have all turned out to be failures. In fact, there are no-go theorems showing that there can be no such thing as a superluminal observer in our $3+1$-dimensional universe.[1,2]

The nonexistence of FTL frames does not immediately rule out the possibility of FTL motion. (After all, we do have motion at c, but no frames moving at c.) For more about faster-than-light motion in relativity, see section 4.7.

References

[1] Gorini, "Linear Kinematical Groups," Commun. Math. Phys. 21 (1971) 150. Open access via Project Euclid at projecteuclid.org/DPubS?service= UI&version=1.0&verb=Display&handle=euclid.cmp/1103857292.

[2] Andreka et al., "A logic road from special relativity to general relativity," arxiv.org/abs/1005.0960, theorem 2.1.