

3.6: Some kinematic identities

Learning Objectives

- List of various kinematics equations and identities

In addition to the relations

$$D(v) = \sqrt{\frac{1+v}{1-v}} \quad (3.6.1)$$

and

$$v_c = \frac{v_1 + v_2}{1 + v_1 v_2} \quad (3.6.2)$$

the following identities can be handy. If stranded on a desert island you should be able to rederive them from scratch. Don't memorize them.

$$v = \frac{D^2 - 1}{D^2 + 1} \quad (3.6.3)$$

$$\gamma = \frac{D^{-1} + D}{2} \quad (3.6.4)$$

$$v\gamma = \frac{D - D^{-1}}{2} \quad (3.6.5)$$

$$D(v)D(-v) = 1 \quad (3.6.6)$$

$$\eta = \ln D \quad (3.6.7)$$

$$v = \tanh \eta \quad (3.6.8)$$

$$\gamma = \cosh \eta \quad (3.6.9)$$

$$v\gamma = \sinh \eta \quad (3.6.10)$$

$$\tanh(x+y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y} \quad (3.6.11)$$

$$D_c = D_1 D_2 \quad (3.6.12)$$

$$\eta_c = \eta_1 + \eta_2 \quad (3.6.13)$$

$$v_C \gamma_c = (v_1 + v_2) \gamma_1 \gamma_2 \quad (3.6.14)$$

The **hyperbolic trig** functions are defined as follows:

$$\sinh x = \frac{e^x - e^{-x}}{2} \quad (3.6.15)$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \quad (3.6.16)$$

$$\tanh x = \frac{\sinh x}{\cosh x} \quad (3.6.17)$$

Their inverses are built in to some calculators and computer software, but they can also be calculated using the following relations:

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}) \quad (3.6.18)$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}) \quad (3.6.19)$$

$$\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad (3.6.20)$$

Their derivatives are, respectively, $(x^2 + 1)^{-1/2}$, $(x^2 - 1)^{-1/2}$ and $(1 - x^2)^{-1}$.

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