3.6: Some kinematic identities

Learning Objectives

- List of various kinematics equations and identities

In addition to the relations

$$D(v)=\sqrt{\frac{1+v}{1-v}} \tag{3.6.1}$$

and

$$v_c=\frac{v_1+v_2}{1+v_1v_2} \tag{3.6.2}$$

the following identities can be handy. If stranded on a desert island you should be able to rederive them from scratch. Don't memorize them.

$$v=\frac{D^2-1}{D^2+1} \tag{3.6.3}$$

$$\gamma=\frac{D^{-1}+D}{2} \tag{3.6.4}$$

$$v\gamma=\frac{D-D^{-1}}{2} \tag{3.6.5}$$

$$D(v)D(-v)=1 \tag{3.6.6}$$

$$\eta=\ln D \tag{3.6.7}$$

$$v=\tanh\eta \tag{3.6.8}$$

$$\gamma=\cosh\eta \tag{3.6.9}$$

$$v\gamma=\sinh\eta \tag{3.6.10}$$

$$\tanh(x+y)=\frac{\tanh x+\tanh y}{1+\tanh x\tanh y} \tag{3.6.11}$$

$$D_c=D_1D_2 \tag{3.6.12}$$

$$\eta_c=\eta_1+\eta_2 \tag{3.6.13}$$

$$v_C\gamma_c=(v_1+v_2)\gamma_1\gamma_2 \tag{3.6.14}$$

The hyperbolic trig functions are defined as follows:

$$\sinh x=\frac{e^x-e^{-x}}{2} \tag{3.6.15}$$

$$\cosh x=\frac{e^x+e^{-x}}{2} \tag{3.6.16}$$

$$\tanh x=\frac{\sinh x}{\cosh x} \tag{3.6.17}$$

Their inverses are built in to some calculators and computer software, but they can also be calculated using the following relations:

$$\sinh^{-1}x=\ln\left(x+\sqrt{x^2+1}\right) \tag{3.6.18}$$

$$\cosh^{-1}x=\ln\left(x+\sqrt{x^2-1}\right) \tag{3.6.19}$$

$$\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right) \tag{3.6.20}$$

Their derivatives are, respectively, $\left(x^2+1\right)^{-1/2}$, $\left(x^2-1\right)^{-1/2}$ and $\left(1-x^2\right)^{-1}$.
