9.E: Flux (Exercises)

Q1

Rewrite the stress-energy tensor of a perfect fluid in SI units. For air at sea level, compare the sizes of its components.

Q2

Prove by direct computation that if a rank-2 tensor is symmetric when expressed in one Minkowski frame, the symmetry is preserved under a boost.

Q3

Consider the following change of coordinates:

$$t' = -t \tag{9.E.1}$$

$$x' = x \tag{9.E.2}$$

$$y' = y \tag{9.E.3}$$

$$z' = z \tag{9.E.4}$$

This is called a time reversal. As in Example 9.2.3, find the effect on the stress-energy tensor.

Q4

Show that in Minkowski coordinates in flat spacetime, all Christoffel symbols vanish.

Q5

Show that if the differential equation for geodesics is satisfied for one affine parameter λ, then it is also satisfied for any other affine parameter $\lambda' = a\lambda + b$, where a and b are constants.

Q6

This problem investigates a notational conflict in the description of the metric tensor using index notation. Suppose that we have two different metrics, $g_{\mu\nu}$ and $g'_{\mu\nu}$. The difference of two rank-2 tensors is also a rank-2 tensor, so we would like the quantity $\partial g_{\mu\nu} = g'_{\mu\nu} - g_{\mu\nu}$ to be a well-behaved tensor both in its transformation properties and in its behavior when we manipulate its indices. Now we also have $g_{\mu\nu}$ and $g'_{\mu\nu}$, which are defined as the matrix inverses of their lower-index counterparts; this is a special property of the metric, not of rank-2 tensors in general. We can then define $\partial g^{\mu\nu} = g'^{\mu\nu} - g^{\mu\nu}$.

a. Use a simple example to show that $\partial g_{\mu\nu}$ and $\partial g^{\mu\nu}$ cannot be computed from one another in the usual way by raising and lowering indices.
b. Find the general relationship between $\partial g_{\mu\nu}$ and $\partial g^{\mu\nu}$.

Q7

In section 9.5, we analyzed the Bell spaceship paradox using the expansion scalar and the Herglotz-Noether theorem. Suppose that we carry out a similar analysis, but with the congruence defined by $x^2 - t^2 = a^{-2}$. The motivation for considering this congruence is that its world-lines have constant proper acceleration a, and each such world-line has a constant value of the coordinate X in the system of accelerated coordinates (Rindler coordinates) described in section 7.1. Show that the expansion tensor vanishes. The interpretation is that it is possible to apply a carefully planned set of external forces to a straight rod so that it accelerates along its own length without any stress, i.e., while remaining Born-rigid.