6.1: Frequency

Learning Objectives

- Explain time and clock

This chapter and the preceding one have good, solid physical titles. Inertia. Waves. But underlying the physical content is a thread of mathematics designed to teach you a language for describing spacetime. Without this language, the complications of relativity rapidly build up and become unmanageable. In section 5.2, we saw that there are physically compelling reasons for switching back and forth between different coordinate systems — different ways of attaching names to the events that make up spacetime. A toddler in a bilingual family gets a payoff for switching back and forth between asking Mamá in Spanish for *dulces* and alerting Daddy in English that Barbie needs to be rescued from falling off the couch. She may bounce back and forth between the two languages in a single sentence — a habit that linguists call "code switching." In relativity, we need to build fluency in a language that lets us talk about actual phenomena without getting hung up on the naming system.

Is time's flow constant?

The simplest naming task is in $0+1$ dimensions: a time-line like the ones in history class. If we name the points in time $A, B, C, \ldots$ or $1, 2, 3, \ldots$, or Bush, Clinton, Bush, ..., how do we know that we're marking off equal time intervals? Does it make sense to imagine that time itself might speed up and slow down, or even start and stop? The second law of thermodynamics encourages us to think that it could. If the universe had existed for an infinite time, then entropy would have maximized itself — a long time ago, presumably — and we would not exist, because the heat death of the universe would already have happened.

Clock-comparison experiments

But what would it actually mean empirically for time's rate of flow to vary? Unless we can tie this to the results of experiments, it's nothing but cut-rate metaphysics. In a Hollywood movie where time could stop, the scriptwriters would show us the stopping through the eyes of an observer, who would stroll past frozen waterfalls and snapshotted bullets in mid-flight. The observer's brain is a kind of clock, and so is the waterfall. We're left with what's known as a clock-comparison experiment. To date, all clock-comparison experiments have given null results. Matsakis *et al.*[1] found that pulsars match the rates of atomic clocks with a drift of less than about 10^{-6} seconds over 10 years. Guéna *et al.*[2] observed that atomic clocks using atoms of different isotopes drifted relative to one another by no more than about 10^{-16} per year. Any non-null result would have caused serious problems for relativity. One of the expectations in an Aristotelian description of spacetime is that the motion of material objects on earth would naturally slow down relative to celestial phenomena such as the rising and setting of the sun. The relativistic interpretation of time dilation as an effect on time itself also depends crucially on the null results of these experiments.

Birdtracks notation

As a simple example of clock comparison, let's imagine using the hourly emergence of a mechanical bird from a pendulum-driven cuckoo clock to measure the rate at which the earth spins. There is clearly a kind of symmetry here, since we could equally well take our planet's rotation as the standard and use it to measure the frequency with which the bird pops out of the door. Schematically, let's represent this measurement process with the following notation, which is part of a system called called birdtracks:[3]

$$c \rightarrow e = 24 \tag{6.1.1}$$

Here c represents the cuckoo clock and e the rotation of the earth. Although the measurement relationship is nearly symmetric, the arrow has a direction, because, for example, the measurement of the earth's rotational period in terms of the clock's frequency is

$$c \rightarrow e = (1\ hr^{-1})(24\ hr) = 24 \tag{6.1.2}$$

but the clock's period in terms of the earth's frequency is

$$e \rightarrow c = \frac{1}{24} \tag{6.1.3}$$

We say that the relationship is not symmetric but "dual." By the way, it doesn't matter how we arrange these diagrams on the page. The notations $c \rightarrow e$ and $e \rightarrow c$ mean exactly the same thing, and expressions like this can even be drawn vertically.

Suppose that e is a displacement along some one-dimensional line of time, and we want to think of it as the thing being measured. Then we expect that the measurement process represented by c produces a real-valued result and is a linear function of e. Since the relationship between c and e is dual, we expect that c also belongs to some vector space. For example, vector spaces allow multiplication by a scalar: we could double the frequency of the cuckoo clock by making the bird come out on the half hour as well as on the hour, forming $2c$. Measurement should be a linear function of both vectors; we say it is "bilinear."

Duality

The two vectors c and e have different units, hr^{-1} and hr, and inhabit two different one-dimensional vector spaces. The "flavor" of the vector is represented by whether the arrow goes into it or comes out. Just as we used notation like $\overrightarrow{v}$ in freshman physics to tell vectors apart from scalars, we can employ arrows in the birdtracks notation as part of the notation for the vector, so that instead of writing the two vectors as c and e, we can notate them as $c \rightarrow$ and $\rightarrow e$. Performing a measurement is like plumbing. We join the two "pipes" in $c \rightarrow \rightarrow e$ and simplify to $c \rightarrow e$.

A confusing and nonstandardized jungle of notation and terminology has grown up around these concepts. For now, let's refer to a vector such as $\rightarrow e$, with the arrow coming in, simply as a "vector," and the type like $c \rightarrow$ as a "covector." In the one-dimensional example of the earth and the cuckoo clock, the roles played by the two things were completely equivalent, and it didn't matter which one we expressed as a vector and which as a covector.

References

[1] Astronomy and Astrophysics 326 (1997) 924, adsabs.harvard.edu/full/1997A&26A...326..924M

[2] arxiv.org/abs/1205.4235

[3] The system used in this book follows the one defined by Cvitanovi´c, which was based closely on a graphical notation due to Penrose. For a more complete exposition, see the Wikipedia article "Penrose graphical notation" and Cvitanovi´c's online book at birdtracks.eu.