7.5: Inertia and Rates of Change

Learning Objectives

- How to get around a curvilinear coordinate system

Suppose that we describe a flying bullet in polar coordinates. We neglect the vertical dimension, so the bullet's motion is linear. If the bullet has a displacement of $(\Delta r_1, \Delta\theta_1)$ in an short time interval Δt, then clearly at a later point in its motion, during an equal interval, it will have a displacement $(\Delta r_2, \Delta\theta_2)$ with two different numbers inside the parentheses. This isn't because its velocity or momentum really changed. It's because the coordinate system is curvilinear. There are three ways to get around this:

1. Use only Minkowski coordinates.
2. Instead of characterizing inertial motion as motion with constant velocity components, we can instead characterize it as motion that maximizes the proper time (section 2.4).
3. Define a correction term to be added when taking the derivative of a vector or covector expressed in non-Minkowski coordinates.

These issues become more acute in general relativity, where curvature of spacetime can make option 1 impossible. Option 3, called the covariant derivative, is discussed in optional section 9.4. If you aren't going to read that section, just keep in mind that in non-Minkowski coordinates, you cannot naively use changes in the components of a vector as a measure of a change in the vector itself.