2.4: Other Axiomatizations

Learning Objectives

- Einstein's postulates
- Laurent's postulates

Einstein's postulates

Einstein used a different axiomatization in his 1905 paper on special relativity:[1]

E1: Principle of relativity

The laws of electrodynamics and optics are valid for all frames of reference for which the equations of mechanics hold good.

E2: Propagation of light

Light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body.

These should be supplemented with our **P2** and **P3**. Einstein's approach has been slavishly followed in many later textbook presentations, even though the special role it assigns to light is not consistent with how modern physicists think about the fundamental structure of the laws of physics. (In 1905 there was no other phenomenon known to travel at c.) Einstein did not explicitly state anything like our **P2** (flatness), since he had not yet developed the theory of general relativity or the idea of representing gravity in relativity as spacetime curvature. When he did publish the general theory, he described the distinction between special and general relativity as a generalization of the class of acceptable frames of reference to include accelerated as well as inertial frames. This description has not stood the test of time, and today relativists use flatness as the distinguishing criterion. In particular, it is not true, as one sometimes still hears claimed, that special relativity is incompatible with accelerated frames of reference.

Maximal time

Another approach, presented, e.g., by Laurent,[2] combines our **P2** with the following:

t1: Metric

An inner product exists. Proper time is measured by the square root of the inner product of a world-line with itself.

T2: Maximum proper time

Inertial motion gives a world-line along which the proper time is at a maximum with respect to small changes in the world-line. Inertial motion is modeled by vectors and parallelism, and this vector-space apparatus has the usual algebraic properties in relation to the inner product referred to in **T1**, e.g.,

$$a \cdot (b+c) = a \cdot b + a \cdot c \tag{2.4.1}$$

Conceptually, **T2** is similar to defining a line as the shortest path between two points, except that we define a geodesic as being the *longest* one (four our $+---$ signature).

Comparison of the Systems

It is useful to compare the axiomatizations P, E, and T from sections 2.1 - 2.4 with each other in order to gain insight into how much "wiggle room" there is in constructing theories of spacetime. Since they are logically equivalent, any statement occurring in one axiomatization can be proved as a theorem in any one of the others. For example, we might wonder whether it is possible to equip Galilean spacetime with a metric. The answer is no, since a system with a metric would satisfy the axioms of system T, which are logically equivalent to our system P. The underlying reason for this is that in Galilean spacetime there is no natural way to compare the scales of distance and time.

Or we could ask whether it is possible to compose variations on the theme of special relativity, alternative theories whose properties differ in some way. System P shows that this would be unlikely to succeed without violating the symmetry of spacetime. Another interesting example is Amelino-Camelia's doubly-special relativity,[3] in which we have both an invariant speed c and an p invariant length L, which is assumed to be the Planck length:

$$L=\sqrt{\frac{\hbar G}{c^3}}. \tag{2.4.2}$$

The invariance of this length contradicts the existence of length contraction. In order to make his theory work, Amelino-Camelia is obliged to assume that energy-momentum vectors (Section 4.3) have their own special inner product that violates the algebraic properties referred to in **T2**.

References

1. Paraphrased from the translation by W. Perrett and G.B. Jeffery.
2. Bertel Laurent, Introduction to Spacetime: A First Course on Relativity
3. Relativity in space-times with short-distance structure governed by an observer-independent (Planckian) length scale

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