10.E: Electromagnetism (Exercises)

Q1

a. A parallel-plate capacitor has charge per unit area $\pm\sigma$ on its two plates. Use Gauss's law to find the field between the plates.
b. In the style of Example 10.4.1, transform the field to a frame moving perpendicular to the plates, and verify that the result makes sense in terms of the sources that are present.
c. Repeat the analysis for a frame moving parallel to the plates.

Q2

We've seen examples such as figure 10.1.1 in which a purely magnetic field in one frame becomes a mixture of magnetic and electric fields in another, and also cases like Example 10.4.1 in which a purely electric field transforms to a mixture. Can we have a case in which a purely electric field in one frame transforms to a purely magnetic one in another? The easy way to do this problem is by using invariants.

Q3

a. Starting from Equation 10.3.5 for $\mathcal{F}^{\mu\nu}$, lower an index to find $\mathcal{F}^{\mu}{}_{\nu}$. Assume Minkowski coordinates and metric signature $+---$.
b. Let $v=\gamma(1,u_x,u_y,u_z)$, where (u_x,u_y,u_z) is the velocity threevector. Write out the matrix multiplication $F^{\mu}=q\mathcal{F}^{\mu}{}_{\nu}\,v^{\nu}$, and show that the result is the Lorentz force law.

Q4

In section 10.6, I presented a list of properties of the electromagnetic stress tensor, followed by an argument in which the tensor is constructed with three unknown constants a, b, and c, to be determined from those properties. The values of a and b are derived in the text, and the purpose of this problem is to finish up by proving that $c=-1$. The idea is to take the field of a point charge, which we know satisfies Maxwell's equations, and then apply property 8, which requires that the energy-conservation condition $\partial T^{ab}/\partial x^a=0$ hold. This works out nicely if you apply this property to the x column of T, at a point that lies in the positive x direction relative to the charge.

Q5

Show that the number of independent conditions contained in equations 10.7.8 and 10.7.9 agrees with the number found in Maxwell's equations.

Q6

Show that

$$\frac{\partial\mathcal{F}^{\mu\nu}}{\partial x^{\lambda}}+\frac{\partial\mathcal{F}^{\nu\lambda}}{\partial x^{\mu}}+\frac{\partial\mathcal{F}^{\lambda\mu}}{\partial x^{\nu}}=0 \tag{10.E.1}$$

implies that the magnetic field has zero divergence.

Q7

Write down the fields of an electromagnetic plane wave propagating in the z direction, choosing some polarization. Do not assume a sinusoidal wave. Show that this is a solution of

$$\frac{\partial\mathcal{F}^{\mu\nu}}{\partial x^{\nu}}=0 \tag{10.E.2}$$