4.1: Ultrarelativistic particles

Learning Objectives

- Explain the ultrarelativistic particle

A typical 22-caliber rifle shoots a bullet with a mass of about $3\ g$ at a speed of about $400\ m/s$. Now consider the firing of such a rifle as seen through an ultra-powerful telescope by an alien in a distant galaxy. We happen to be firing in the direction away from the alien, who gets a view from over our shoulder. Since the universe is expanding, our two galaxies are receding from each other. In the alien's frame, our own galaxy is the one that is moving — let's say at $^1\ c-(200\ m/s$. If the two velocities simply added, the bullet would be moving at $c+(200\ m/s$. But velocities don't simply add and subtract relativistically, and applying the correct equation for relativistic combination of velocities, we find that in the alien's frame, the bullet flies at only $c-(199.9995\ m/s)$. That is, according to the alien, the energy in the gunpowder only succeeded in accelerating the bullet by $0.0005\ m/s$. If we insisted on believing in $K=\frac{1}{2}mv^2$, this would clearly violate conservation of energy in the alien's frame of reference. It appears that kinetic energy must not only rise faster than v^2 as v approaches c, it must blow up to infinity. This gives a dynamical explanation for why no material object can ever reach or exceed c, as we have already inferred on purely kinematical grounds.

To the alien, both our galaxy and the bullet are ultrarelativistic objects, i.e., objects moving at nearly c. A good way of thinking about an ultrarelativistic particle is that it's a particle with a very small mass. For example, the subatomic particle called the neutrino has a very small mass, thousands of times smaller than that of the electron. Neutrinos are emitted in radioactive decay, and because the neutrino's mass is so small, the amount of energy available in these decays is always enough to accelerate it to very close to c. Nobody has ever succeeded in observing a neutrino that was not ultrarelativistic. When a particle's mass is very small, the mass becomes difficult to measure. For almost 70 years after the neutrino was discovered, its mass was thought to be zero. Similarly, we currently believe that a ray of light has no mass, but it is always possible that its mass will be found to be nonzero at some point in the future. A ray of light can be modeled as an ultrarelativistic particle.

Let's compare ultrarelativistic particles with train cars. A single car with kinetic energy E has different properties than a train of two cars each with kinetic energy $E/2$. The single car has half the mass and a speed that is greater by a factor of $\sqrt{2}$. But the same is not true for ultrarelativistic particles. Since an idealized ultrarelativistic particle has a mass too small to be detectable in any experiment, we can't detect the difference between m and $2m$. Furthermore, ultrarelativistic particles move at close to c, so there is no observable difference in speed. Thus we expect that a single ultrarelativistic particle with energy E compared with two such particles, each with energy $E/2$, should have all the same properties as measured by a mechanical detector.

An idealized zero-mass particle also has no frame in which it can be at rest. It always travels at c, and no matter how fast we chase after it, we can never catch up. We can, however, observe it in different frames of reference, and we will find that its energy is different. For example, distant galaxies are receding from us at substantial fractions of c, and when we observe them through a telescope, they appear very dim not just because they are very far away but also because their light has less energy in our frame than in a frame at rest relative to the source. This effect must be such that changing frames of reference according to a specific Lorentz transformation always changes the energy of the particle by a fixed factor, regardless of the particle's original energy; for if not, then the effect of a Lorentz transformation on a single particle of energy E would be different from its effect on two particles of energy $E/2$.

How does this energy-shift factor depend on the velocity v of the Lorentz transformation? Here it becomes nicer to work in terms of the variable D. Let's write $f(D)$ for the energy-shift factor that results from a given Lorentz transformation. Since a Lorentz transformation D_1 followed by a second transformation D_2 is equivalent to a single transformation by D_1D_2, we must have $f(D_1D_2)=f(D_1)f(D_2)$. This tightly constrains the form of the function f ; it must be something like $f(D)=D^n$, where n is a constant. The interpretation of n is that under a Lorentz transformation corresponding to 1% of c, energies of ultrarelativistic particles change by about $n\%$ (making the approximation that $v=0.1$ gives $(D\simeq 1.01)$. In his original 1905 paper on special relativity, Einstein used Maxwell's equations and the Lorentz transformation to show that for a light wave $n=1$, and we will prove in section 4.3 that this holds for any ultrarelativistic object. He wrote, "*It is remarkable that the energy and the frequency . . . vary with the state of motion of the observer in accordance with the same law.*" He was presumably interested in this fact because 1905 was also the year in which he published his paper on the photoelectric effect, which formed the foundations of quantum mechanics. An axiom of quantum mechanics is that the energy and frequency of any particle are related by $E=hf$, and if E and f hadn't transformed in the same way relativistically, then quantum mechanics would have been incompatible with relativity.

If we assume that certain objects, such as light rays, are truly massless, rather than just having masses too small to be detectable, then their D doesn't have any finite value, but we can still find how the energy differs according to different observers by finding the D of the Lorentz transformation between the two observers' frames of reference.

Example 4.1.1: The astronomical energy shift of the Andromeda Galaxy

For quantum-mechanical reasons, a hydrogen atom can only exist in states with certain specific energies. By conservation of energy, the atom can therefore only absorb or emit light that has an energy equal to the difference between two such atomic energies. The outer atmosphere of a star is mostly made of monoatomic hydrogen, and one of the energies that a hydrogen atom can absorb or emit is $3.0276 \times 10^{-19}\ J$. When we observe light from stars in the Andromeda Galaxy, it has an energy of $3.0306 \times 10^{-19}\ J$. If this is assumed to be due entirely to the motion of the Milky Way and Andromeda Galaxy relative to one another, along the line connecting them, find the direction and magnitude of this velocity.

Solution

The energy is shifted upward, which means that the Andromeda Galaxy is moving toward us. (Galaxies at cosmological distances are always observed to be receding from one another, but this doesn't necessarily hold for galaxies as close as these.) Relating the energy shift to the velocity, we have

$$\frac{E'}{E} = D = \sqrt{\frac{1+v}{1-v}} \tag{4.1.1}$$

Since the shift is only about one part per thousand, the velocity is small compared to c — or small compared to 1 in units where $c = 1$. Therefore we can employ the low-velocity approximation $D \approx 1 + v$, which gives

$$\begin{aligned} v \approx D - 1 &= \frac{E'}{E} - 1 \\ &= -1.0 \times 10^{-3} \end{aligned}$$

The negative sign confirms that the source is approaching rather than receding. This is in units where $c = 1$. Converting to SI units, where $c \neq 1$, we have

$$v = (-1.0 \times 10^{-3})c = -300\ km/s. \tag{4.1.2}$$

Although the Andromeda Galaxy's tangential motion is not accurately known,it is considered likely that it will collide with the Milky Way in a few billion years.

References

[1] In reality when two velocities move at relativistic speeds compared with one another, they are separated by a cosmological distance, and special relativity does not actually allow us to construct frames of reference this large.