

3.5: The Gaussian Integral

Here's a famous integral:

$$\int_{-\infty}^{\infty} e^{-\gamma x^2} dx. \quad (3.5.1)$$

The integrand is called a **Gaussian**, or **bell curve**, and is plotted below. The larger the value of γ , the more narrowly-peaked the curve.

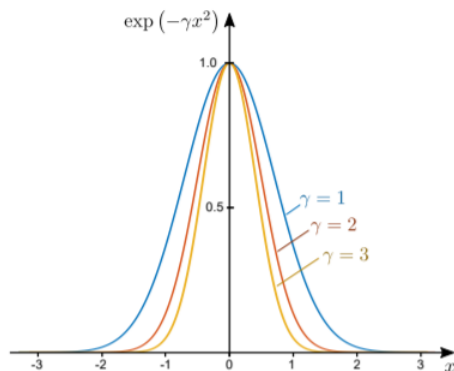


Figure 3.5.1

The integral was solved by [Gauss](#) in a brilliant way. Let $I(\gamma)$ denote the value of the integral. Then I^2 is just two independent copies of the integral, multiplied together:

$$I^2(\gamma) = \left[\int_{-\infty}^{\infty} dx e^{-\gamma x^2} \right] \times \left[\int_{-\infty}^{\infty} dy e^{-\gamma y^2} \right]. \quad (3.5.2)$$

Note that in the second copy of the integral, we have changed the “dummy” label x (the integration variable) into y , to avoid ambiguity. Now, this becomes a two-dimensional integral, taken over the entire 2D plane:

$$I^2(\gamma) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy e^{-\gamma(x^2+y^2)}. \quad (3.5.3)$$

Next, change from Cartesian to polar coordinates:

$$I^2(\gamma) = \int_0^{\infty} dr r \int_0^{2\pi} d\phi e^{-\gamma r^2} = \left[\int_0^{\infty} dr r e^{-\gamma r^2} \right] \times \left[\int_0^{2\pi} d\phi \right] = \frac{1}{2\gamma} \cdot 2\pi. \quad (3.5.4)$$

By taking the square root, we arrive at the result

$$I(\gamma) = \int_{-\infty}^{\infty} dx e^{-\gamma x^2} = \sqrt{\frac{\pi}{\gamma}}. \quad (3.5.5)$$

This page titled [3.5: The Gaussian Integral](#) is shared under a [CC BY-SA 4.0](#) license and was authored, remixed, and/or curated by [Y. D. Chong](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.