CHAPTER OVERVIEW

4: Complex Numbers

The **imaginary unit**, denoted i, is defined as a solution to the quadratic equation

$$z^2 + 1 = 0. \tag{4.1}$$

In other words, $i = \sqrt{-1}$. As we know, the above equation lacks any real number solutions. For this concept to make sense, we must extend our pre-established notions about what numbers are.

We will let the imaginary unit take part in the usual arithmetic operations of addition and multiplication, treating it as an algebraic quantity that can participate on the same footing as real numbers. It is one of the most profound discoveries of mathematics that this seemingly arbitrary idea gives rise to powerful computational methods with applications in numerous fields.