

### 3.4: Change of Variables

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Another useful technique for solving integrals is to change variables. Consider the integral

$$\int_0^{\infty} \frac{dx}{x^2 + 1}. \quad (3.4.1)$$

We can solve this by making a change of variables  $x = \tan(u)$ . This involves (i) replacing all occurrences of  $x$  in the integrand with  $\tan(u)$ , (ii) replacing the integral limits, and (iii) replacing  $dx$  with  $(dx/du) du = 1/[\cos(u)]^2 du$

$$\int_0^{\infty} \frac{dx}{x^2 + 1} = \int_0^{\pi/2} \frac{1}{[\tan(u)]^2 + 1} \cdot \frac{1}{[\cos(u)]^2} du \quad (3.4.2)$$

$$= \int_0^{\pi/2} \frac{1}{[\sin(u)]^2 + [\cos(u)]^2} du. \quad (3.4.3)$$

Due to the Pythagorean theorem, the integrand reduces to 1, so

$$\int_0^{\infty} \frac{dx}{x^2 + 1} = \int_0^{\pi/2} du = \frac{\pi}{2}. \quad (3.4.4)$$

Clearly, this technique often requires some cleverness and/or trial-and-error in choosing the right change of variables.

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