

6.4: Waves in 3D Space

The wave equation can be generalized to three spatial dimensions by replacing $f(x, t)$ with a wavefunction that depends on three spatial coordinates, $f(x, y, z, t)$. The second-order derivative in x is then replaced by second-order derivatives in each spatial direction:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right) f(x, y, z, t) = 0. \quad (6.4.1)$$

This PDE supports complex plane wave solutions of the form

$$f(x, y, z, t) = A e^{i(\vec{k} \cdot \vec{r} - \omega t)}, \quad (6.4.2)$$

where

$$\vec{k} = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}, \quad \vec{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \frac{\omega}{\sqrt{k_x^2 + k_y^2 + k_z^2}} = v. \quad (6.4.3)$$

Again, we can verify that this is a solution by direct substitution. We call \vec{k} the **wave-vector**, which generalizes the wavenumber k . The direction of the wave-vector specifies the spatial direction in which the wave travels.

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