# 6.1: The Wave Equation

A wave can be described by a function $f(x,t)$, called a **wavefunction**, which specifies the value of a measurable physical quantity at each position $x$ and time $t$. For simplicity, we will assume that space is one-dimensional, so $x$ is a single real number. We will also assume that $f(x,t)$ is a number, rather than a more complicated object such as a vector. For instance, a sound wave can be described by a wavefunction $f(x,t)$ representing the air pressure at each point of space and time.

The evolution of the wavefunction is described by a partial differential equation (PDE) called the **time-dependent wave equation**:

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}, \quad v \in \mathbb{R}^+. \tag{6.1.1}$$

The parameter $v$, which we take to be a positive real constant, is called the **wave speed**, for reasons that will shortly become clear.

Sometimes, we re-arrange the wave equation into the following form, consisting of a linear differential operator acting on $f(x,t)$:

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right) f(x,t) = 0. \tag{6.1.2}$$

This way of writing the wave equation emphasizes that it is a linear PDE, meaning that any linear superposition of solutions is likewise a solution.