CHAPTER OVERVIEW

7: Complex Derivatives

We have studied functions that take real inputs and give complex outputs (e.g., complex solutions to the damped harmonic oscillator, which are complex functions of time). For such functions, the derivative with respect to its real input is much like the derivative of a real function of real inputs. It is equivalent to taking the derivatives of the real and imaginary parts, separately:

$$\frac{d\psi}{dx} = \frac{d\mathrm{Re}(\psi)}{dx} + i\,\frac{d\mathrm{Im}(\psi)}{dx}. \tag{7.1}$$

Now consider the more complicated case of a function of a *complex* variable:

$$f(z) \in \mathbb{C}, \quad \text{where} \;\; z \in \mathbb{C}. \tag{7.2}$$

At one level, we could just treat this as a function of two independent real inputs: $f(x,y)$, where $z = x + iy$. However, in doing so we would be disregarding the mathematical structure of the complex input—the fact that z is not merely a collection of two real numbers, but a complex *number* that can participate in algebraic operations. This structure has important implications for the differential calculus of complex functions.