

1.3: 1.3 The Logarithm Function

Since the exponential is a one-to-one function, its inverse is a well-defined function. We call this the **natural logarithm**:

$$\ln(x) \equiv y \text{ such that } \exp(y) = x. \quad (1.3.1)$$

For brevity, we will henceforth use “logarithm” to refer to the natural logarithm, unless otherwise stated (the “non-natural” logarithms are not our concern in this course). The domain of the logarithm is $y \in \mathbb{R}^+$, and its range is \mathbb{R} . Its graph is shown below:

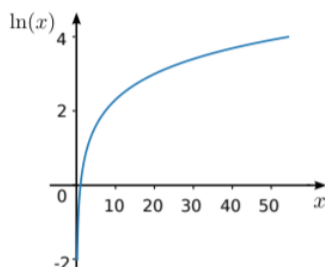


Figure 1.3.1

Observe that the graph increases extremely slowly with x , precisely the opposite of the exponential's behavior.

Using Eq. (1.2.3), we can prove that the logarithm satisfies the product and quotient rules

$$\ln(xy) = \ln(x) + \ln(y) \quad (1.3.2)$$

$$\ln(x/y) = \ln(x) - \ln(y). \quad (1.3.3)$$

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