CHAPTER OVERVIEW

3: Integrals

If we have a function $f(x)$ which is well-defined for some $a \le x \le b$, its integral over those two values is defined as

$$\int_a^b dx\ f(x) \;=\; \lim_{N\to\infty} \sum_{n=0}^{N} \Delta x\ f(x_n) \quad \text{where}\ \ x_n = a + n\Delta x,\ \ \Delta x \equiv \left(\frac{b-a}{N}\right). \tag{3.1}$$

This is called a **definite integral**, and represents the area under the graph of $f(x)$ in the region between $x = a$ and $x = b$, as shown in the figure below. The function $f(x)$ is called the **integrand**, and the two points a and b are called the **bounds** of the integral. The interval between the two bounds is divided into N segments, of length $(b-a)/N$ each. Each term in the sum represents the area of a rectangle, and as $N \to \infty$, the sum converges to the area under the curve.

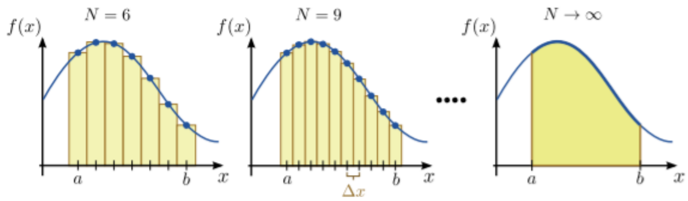

Figure 3.1

A **multiple integral** involves integration over more than one variable. For instance, when we have a function $f(x_1, x_2)$ that depends on two independent variables, x_1 and x_2, we can perform a double integral by integrating over one variable first, then the other variable:

$$\int_{a_1}^{b_1} dx_1 \int_{a_2}^{b_2} dx_2\ f(x_1, x_2) \equiv \int_{a_1}^{b_1} dx_1 F(x_1) \quad \text{where}\ \ F(x_1) \equiv \int_{a_2}^{b_2} dx_2\ f(x_1, x_2). \tag{3.2}$$