3.2: Integrals as Antiderivatives

Since the value of a definite integral depends on the values of the upper and lower bounds, we can ask what happens to the value of the definite integral when either bound is varied. Using the defintion of the derivative from the previous chapter, we can show that

$$\frac{d}{db}\left[\int_a^b dx\; f(x)\right] = f(b), \tag{3.2.1}$$

$$\frac{d}{da}\left[\int_a^b dx\; f(x)\right] = -f(a). \tag{3.2.2}$$

To prove the first equation, observe that increasing the upper bound from b to $b+\delta b$ increases the area under the curve by $f(b)\delta b$ (to lowest order in δb). Hence, the definite integral's rate of change with b is $f(b)$. Likewise, increasing the lower bound from a to δa *decreases* the area under the curve, leading to a rate of change of $-f(a)$.

From the above result, we define the concept of an **indefinite integral**, or **antiderivative**, as the inverse of a derivative operation:

$$\int^x dx' f(x') \equiv F(x) \;\; \text{such that} \;\; \frac{d}{dx}F(x) = f(x). \tag{3.2.3}$$

Since derivatives are not one-to-one (i.e., two different functions can have the same derivative), an antiderivative does not have a unique, well-specified value. Rather, its value is only defined up to an additive constant, called an **integration constant**. A definite integral, by contrast, always has a well-defined value.

Finding antiderivatives is much harder than differentiation. Once you know how to differentiate a few special functions, differentiating some combination of those functions usually involves a straightforward (if tedious) application of composition rules. By contrast, there is no general systematic procedure for symbolic integration. Integration often requires creative steps, like guessing a solution and checking if its derivative yields the desired integrand.

Some common techniques are summarized in the following sections; others will be introduced later in this course.