6.4: Waves in 3D Space

The wave equation can be generalized to three spatial dimensions by replacing $f(x,t)$ with a wavefunction that depends on three spatial coordinates, $f(x,y,z,t)$. The second-order derivative in x is then replaced by second-order derivatives in each spatial direction:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\; f(x,y,z,t)=0. \tag{6.4.1}$$

This PDE supports complex plane wave solutions of the form

$$f(x,y,z,t)=A\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}, \tag{6.4.2}$$

where

$$\vec{k}=\begin{bmatrix}k_x\\k_y\\k_z\end{bmatrix},\quad \vec{r}=\begin{bmatrix}x\\y\\z\end{bmatrix},\quad \frac{\omega}{\sqrt{k_x^2+k_y^2+k_z^2}}=v. \tag{6.4.3}$$

Again, we can verify that this is a solution by direct substitution. We call $\vec{k}$ the **wave-vector**, which generalizes the wavenumber k. The direction of the wave-vector specifies the spatial direction in which the wave travels.