3.4: Change of Variables

Another useful technique for solving integrals is to change variables. Consider the integral

$$\int_0^\infty \frac{dx}{x^2+1}. \tag{3.4.1}$$

We can solve this by making a change of variables $x = \tan(u)$. This involves (i) replacing all occurrences of x in the integrand with $\tan(u)$, (ii) replacing the integral limits, and (iii) replacing dx with $(dx/du)\,du = 1/[\cos(u)]^2 du$:

$$\int_0^\infty \frac{dx}{x^2+1} = \int_0^{\pi/2} \frac{1}{[\tan(u)]^2+1} \cdot \frac{1}{[\cos(u)]^2}\, du \tag{3.4.2}$$

$$= \int_0^{\pi/2} \frac{1}{[\sin(u)]^2+[\cos(u)]^2}\, du. \tag{3.4.3}$$

Due to the Pythagorean theorem, the integrand reduces to 1, so

$$\int_0^\infty \frac{dx}{x^2+1} = \int_0^{\pi/2} du = \frac{\pi}{2}. \tag{3.4.4}$$

Clearly, this technique often requires some cleverness and/or trial-and-error in choosing the right change of variables.