

1.1: Scattering Experiments on Quantum Particles

Quantum particles exhibit a feature known as **wave-particle duality**, which can be summarized in the **quantum double-slit thought experiment**. As shown in the figure below, a source emits electrons with energy E , which travel towards a screen with a pair of slits. A detector is positioned on the other side of the screen. By moving the detector around, we can measure the rate at which electrons are detected at different positions.

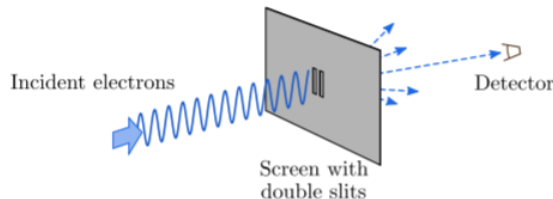


Figure 1.1.1

According to quantum theory, the experiment reveals the following: (i) the electrons arrive in discrete units—one at a time, like classical particles; (ii) when we move the detector around to measure how the detection events are *statistically* distributed in space, the resulting distribution matches an interference pattern formed by a classical wave diffracted by the slits. The wavelength λ is related to the electron energy E by

$$\lambda = \frac{2\pi}{k}, \quad E = \frac{\hbar^2 k^2}{2m}, \quad (1.1.1)$$

where $\hbar = h/2\pi$ is Dirac's constant, and m is the electron mass. (This also implies that we can deduce the spacing of the slits from the diffraction pattern, if E is known.)

Wave-particle duality arises from quantum theory's distinction between a particle's state and the outcomes of measurements performed on it. The state is described by a wavefunction $\psi(\mathbf{r})$, which can undergo diffraction like a classical wave. Measurement outcomes, however, depend *probabilistically* on the wavefunction. In a position measurement, the probability of locating a particle in a volume dV around position \mathbf{r} is $|\psi(\mathbf{r})|^2 dV$.

In this chapter, we will study a generalization of the double-slit experiment called a **scattering experiment**. The idea is to take an object called a **scatterer**, shoot quantum particles at it, and measure the resulting particle distribution. Just as the double-slit interference pattern can be used to deduce the slit spacing, a scattering experiment can be used to deduce various facts about the scatterer. Scattering experiments constitute a large proportion of the methods used to probe the quantum world—from electron- and photon-based laboratory experiments for measuring the properties of materials, to huge accelerator experiments that study high-energy phenomena like the Higgs boson.

We will focus on a simple scenario with a single non-relativistic quantum particle and a classical scatterer. Consider a continuous and unbounded d -dimensional space, describable by coordinates \mathbf{r} . Somewhere around the origin, $\mathbf{r} = 0$, is a finite-sized scatterer. An incoming quantum particle, with energy E , is governed by the Hamiltonian

$$\hat{H} = \hat{H}_0 + V(\hat{\mathbf{r}}), \quad \hat{H}_0 = \frac{\hat{\mathbf{p}}^2}{2m}. \quad (1.1.2)$$

Here, \hat{H}_0 describes the particle's kinetic energy, m is the particle's mass, $\hat{\mathbf{r}}$ and $\hat{\mathbf{p}}$ are position and momentum operators, and V is a **scattering potential** describing how the scatterer affects the quantum particle. We assume that $V(\mathbf{r}) \rightarrow 0$ as $|\mathbf{r}| \rightarrow \infty$, i.e., the scattering potential becomes negligible far from the origin.

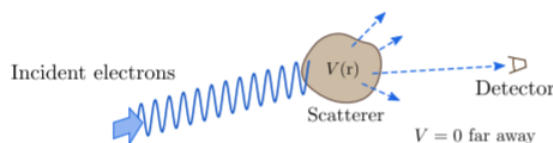


Figure 1.1.2

We prepare an incoming particle state with energy E , and want to see how the particle is scattered by the potential. However, converting these words into a well-defined mathematical problem is a bit tricky! We will give the formulation first, before

discussing its meaning:

1. The particle state $|\psi\rangle$ obeys the time-independent Schrödinger equation

$$\hat{H}|\psi\rangle = E|\psi\rangle, \quad (1.1.3)$$

where E is the incoming particle energy.

2. This state can be decomposed into two terms,

$$|\psi\rangle = |\psi_i\rangle + |\psi_s\rangle, \quad (1.1.4)$$

where $|\psi_i\rangle$ is called the **incident state** and $|\psi_s\rangle$ is called the **scattered state**.

3. The incident state is described by a plane wave—a simultaneous eigenstate of \hat{H}_0 (with energy E) and $\hat{\mathbf{p}}$ (with momentum \mathbf{p}_i):

$$\hat{H}_0|\psi_i\rangle = E|\psi_i\rangle \text{ and } \hat{\mathbf{p}}|\psi_i\rangle = \mathbf{p}_i|\psi_i\rangle, \text{ where } E = \frac{|\mathbf{p}_i|^2}{2m}. \quad (1.1.5)$$

4. We require the scattered state to be an “outgoing” state. This is the most subtle of the conditions, and we will describe what it means later.

The first condition says that the scattering process is elastic. Since the scatterer takes the form of a potential $V(\mathbf{r})$, its interaction with the particle is conservative (i.e., the total energy E is fixed). The second condition says that the particle’s wavefunction, $\psi(\mathbf{r}) = \langle \mathbf{r} | \psi \rangle$, consists of a superposition of an incoming wave and a scattered wave. The third condition defines the incoming wave as a plane wave whose wavelength is determined by the chosen energy E . The final condition says that the scattered wave moves out toward infinity.

Note that *this is not an eigenproblem*! Usually, when we use the time-independent Schrödinger equation, we treat it as an eigenproblem and solve for the energy eigenvalues and eigenstates. But in this case, E is an input to the calculation, describing the energy assigned to the incoming quantum particle. Given $|\psi_i\rangle$, E , and $V(\mathbf{r})$, we want to find $|\psi_s\rangle$.

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