

3.3: The Einstein-Podolsky-Rosen "Paradox"

In 1935, Einstein, Podolsky, and Rosen (EPR) formulated a thought experiment, now known as the **EPR paradox**, that highlights the counter-intuitive features of quantum entanglement. They tried to use this thought experiment to argue that quantum theory cannot serve as a fundamental description of reality. Subsequently, however, it was shown that the EPR paradox is not an actual paradox; physical systems *really do* have the strange behavior that the thought experiment highlighted.

Consider an entangled state, like the following “singlet state” of two spin-1/2 particles:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|+\rangle|-\rangle - |-\rangle|+\rangle \right). \quad (3.3.1)$$

As before, let the two particles be labeled A and B . Measuring S_z on A collapses the system into a two-particle state that is unentangled, where each particle has a definite spin. If the measurement outcome is $+\hbar/2$, the new state is $|+\rangle|-\rangle$, whereas if the outcome is $-\hbar/2$, the new state is $|-\rangle|+\rangle$.

The postulates of quantum theory seem to indicate that the state collapse happens instantaneously, regardless of the distance separating the particles. Imagine that we prepare the two-particle state in a laboratory on Earth. Particle A is then transported to the laboratory of Alice, in the Alpha Centauri star system, and particle B is transported to the laboratory of Bob, in the Betelgeuse system, separated by ~ 640 light years. In principle, this can be done carefully enough to avoid disturbing the two-particle quantum state.

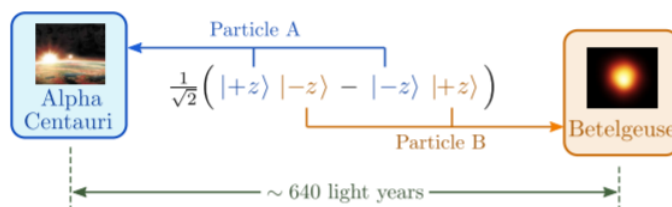


Figure 3.3.1

Once ready, Alice measures \hat{S}_z on particle A , which induces an instantaneous collapse of the two-particle state. Immediately afterwards, Bob measures \hat{S}_z on particle B , and obtains—with 100% certainty—the opposite spin. During the time interval between these two measurements, no classical signal could have traveled between the two star systems, not even at the speed of light. Yet the state collapse induced by Alice’s measurement has a definite effect on the result of Bob’s measurement.

There are three noteworthy aspects of this phenomenon:

First, it dispels some commonsensical but mistaken “explanations” for quantum state collapse in terms of perturbative effects. For instance, it is sometimes explained that if we want to measure a particle’s position, we need to shine a light beam on it, or disturb it in some way, and this disturbance generates an uncertainty in the particle’s momentum. The EPR paradox shows that such stories don’t capture the full weirdness of quantum state collapse, for we can collapse the state of a particle by doing a measurement on *another* particle far away!

Second, our experimentalists have a certain amount of control over the state collapse, due to the choice of what measurement to perform. So far, we have considered S_z measurements performed by Alice on particle A . But Alice can choose to measure the spin of A along another axis, say S_x . In the basis of spin-up and spin-down states, the operator \hat{S}_x has matrix representation

$$\hat{S}_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (3.3.2)$$

The eigenvalues and eigenvectors are

$$\begin{aligned} s_x = \frac{\hbar}{2}, \quad |+\rangle &= \frac{1}{\sqrt{2}} (|+\rangle + |-\rangle) \\ s_x = -\frac{\hbar}{2}, \quad |-\rangle &= \frac{1}{\sqrt{2}} (|+\rangle - |-\rangle). \end{aligned} \quad (3.3.3)$$

Conversely, we can write the \hat{S}_z eigenstates in the $\{|+\rangle, |-\rangle\}$ basis:

$$\begin{aligned} | +z \rangle &= \frac{1}{\sqrt{2}} (| +x \rangle + | -x \rangle) \\ | -z \rangle &= \frac{1}{\sqrt{2}} (| +x \rangle - | -x \rangle). \end{aligned} \quad (3.3.4)$$

This allows us to write the two-particle entangled state in the \hat{S}_x basis:

$$|\psi\rangle = \frac{1}{\sqrt{2}} (| -x \rangle | +x \rangle - | +x \rangle | -x \rangle). \quad (3.3.5)$$

Alice's measurement still collapses the particles into definite spin states with opposite spins—but now spin states of S_x rather than S_z .

Third, this ability to choose the measurement axis *does not* allow for superluminal communication. Alice can choose whether to (i) measure S_z or (ii) measure S_x , and this choice instantaneously affects the quantum state of particle B . If Bob can find a way to distinguish between the cases (i) and (ii), even statistically, this would serve as a method for instantaneous communication, violating the theory of relativity! Yet this turns out to be impossible. The key problem is that quantum states themselves cannot be measured; only observables can be measured. Suppose Alice's measurement is \hat{S}_z , which collapses B to either $| +z \rangle$ or $| -z \rangle$, each with probability $1/2$. Bob must now choose which measurement to perform. If he measures S_z , the outcome is $+\hbar/2$ or $-\hbar/2$ with equal probabilities. If he measures S_x , the probabilities are:

$$\begin{aligned} P(S_x = +\hbar/2) &= \frac{1}{2} |\langle +x | +z \rangle|^2 + \frac{1}{2} |\langle +x | -z \rangle|^2 = \frac{1}{2} \\ P(S_x = -\hbar/2) &= \frac{1}{2} |\langle -x | +z \rangle|^2 + \frac{1}{2} |\langle -x | -z \rangle|^2 = \frac{1}{2}. \end{aligned} \quad (3.3.6)$$

The probabilities are still equal! Repeating this analysis for any other choice of spin axis, we find that the two possible outcomes always have equal probability. Thus, Bob's measurement does not yield any information about Alice's choice of measurement axis.

Since quantum state collapse does not allow for superluminal communication, it is consistent *in practice* with the theory of relativity. However, state collapse is still **nonlocal**, in the sense that unobservable ingredients of the theory (quantum states) can change faster than light can travel between two points. For this reason, EPR argued that quantum theory is *philosophically* inconsistent with relativity.

EPR suggested an alternative: maybe quantum mechanics is an approximation of some deeper theory, whose details are currently unknown, but which is deterministic and local. Such a “**hidden variable theory**” may give the appearance of quantum state collapse in the following way. Suppose each particle has a definite but “hidden” value of S_z , either $S_z = +\hbar/2$ or $S_z = -\hbar/2$; let us denote these as $[+]$ or $[-]$. We can hypothesize that the two-particle quantum state $|\psi\rangle$ is not an actual description of reality; rather, it corresponds to a *statistical* distribution of “hidden variable” states, denoted by $[+; -]$ (i.e., $S_z = +\hbar/2$ for particle A and $S_z = -\hbar/2$ for particle B), and $[-; +]$ (the other way around).

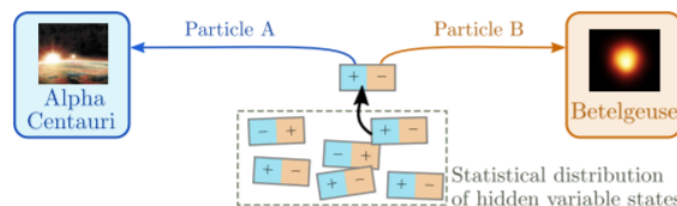


Figure 3.3.2

When Alice measures S_z , the value of the hidden variable is revealed. A result of $+z$ implies $[+; -]$, whereas $-z$ implies $[-; +]$. When Bob subsequently measures S_z , the result obtained is the opposite of Alice's result. But those were simply the values all along—there is no instantaneous physical influence traveling between their two laboratories.

Clearly, there are many missing details in this hypothetical description. Any actual hidden variable theory would also need to replicate the huge list of successful predictions made by quantum theory. Trying to come up with a suitable theory of this sort seems difficult, but with enough hard work, one might imagine that it is doable.

This page titled 3.3: The Einstein-Podolsky-Rosen "Paradox" is shared under a [CC BY-SA 4.0](https://creativecommons.org/licenses/by-sa/4.0/) license and was authored, remixed, and/or curated by [Y. D. Chong](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.