1.10: Exercises

Exercises

Exercise 1.10.1

In Sec. 1.2, we derived the eigenstates of a particle in an empty infinite space by considering a box of length L on each side, applying periodic boundary conditions, and taking $L \to \infty$. Suppose we instead use Dirichlet boundary conditions (i.e., the wavefunction vanishes on the walls of the box). Show that this gives rise to the same set of momentum eigenstates in the $L \to \infty$ limit.

Exercise 1.10.2

Using the results for the 1D delta-function scattering problem described in Section 1.3, calculate the probability current

$$J(x) = \frac{\hbar}{2mi}\left(\psi^* \frac{d\psi}{dx} - \psi \frac{d\psi^*}{dx}\right), \tag{1.10.1}$$

where $\psi(x)$ is the *total* (incident + scattered) wavefunction. Explain the relationship between the values of J on the left and right side of the scatterer.

Exercise 1.10.3

Derive the Green's function for a free particle in 1D space:

$$\langle x|\hat{G}_0|x'\rangle = \frac{2m}{\hbar^2} \cdot \frac{1}{2ik_i} \exp(ik_i|x-x'|). \tag{1.10.2}$$

Exercise 1.10.4

In Section 1.8, the scattering amplitude $f(\mathrm{k} \to \mathrm{k}')$ for the 3D scattering problem was derived using the Born series. Derive the corresponding expressions for 1D and 2D.

Further Reading

[1] Bransden & Joachain, §13.1—13.3 and §13.5—13.6.

[2] Sakurai, §7.1–7.3, 7.5–7.6