5.5: Exercises

Exercises

Exercise 5.5.1

In Section 5.3, we derived the vector potential operator, in an infinite volume, to be

$$\hat{\mathbf{A}}(\mathbf{r},t)=\int d^3k\sum_{\lambda}\sqrt{\frac{\hbar}{16\pi^3\epsilon_0\omega_{\mathbf{k}}}}\left(\hat{a}_{\mathbf{k}\lambda}\,e^{i(\mathbf{k}\cdot\mathbf{r}-\omega_{\mathbf{k}}t)}+\mathrm{h.\,c.}\right)\mathbf{e}_{\mathbf{k}\lambda}. \tag{5.5.1}$$

Since $[\hat{a}_{\mathbf{k}\lambda},\hat{a}^{\dagger}_{\mathbf{k}'\lambda'}]=\delta^3(\mathbf{k}-\mathbf{k}')\delta_{\lambda\lambda'}$, the creation and annihilation operators each have units of $[x^{3/2}]$. Prove that $\hat{\mathbf{A}}$ has the same units as the classical vector potential.

Exercise 5.5.2

Repeat the spontaneous decay rate calculation from Section 5.4 using the finite-volume versions of the creation/annihilation operators and the vector potential operator (5.4.3). Show that it yields the same result (5.4.16).

Exercise 5.5.3

The density of photon states at energy E is defined as

$$\mathcal{D}(E)=2\int d^3k\;\delta(E-E_{\mathbf{k}}), \tag{5.5.2}$$

where $E_{\mathbf{k}}=\hbar c|\mathbf{k}|$. Note the factor of 2 accounting for the polarizations. Prove that

$$\mathcal{D}(E)=\frac{8\pi E^2}{\hbar^3c^3}, \tag{5.5.3}$$

and show that $\mathcal{D}(E)$ has units of $[E^{-1}V^{-1}]$.

Further Reading

[1] F. J. Dyson, *1951 Lectures on Advanced Quantum Mechanics Second Edition*, arxiv:quant-ph/0608140.

[2] A. Zee, *Quantum Field Theory in a Nutshell* (Princeton University Press, 2010). [cite:zee]

[3] L. L. Foldy and S. A. Wouthuysen, *On the Dirac Theory of Spin $1/2$ Particles and Its Non-Relativistic Limit*, Physical Review **78**, 29 (1950).