

CHAPTER OVERVIEW

11: Discrete Fourier Transforms

The **Discrete Fourier Transform** (DFT) is a discretized version of the [Fourier transform](#), which is widely used in numerical simulation and analysis. Given a set of N numbers $\{f_0, f_1, \dots, f_{N-1}\}$, the DFT produces another set of N numbers $\{F_0, F_1, \dots, F_{N-1}\}$, defined as follows:

$$\text{DFT} \{f_0, f_1, \dots, f_{N-1}\} = \{F_0, F_1, \dots, F_{N-1}\} \quad \text{where} \quad F_n = \sum_{m=0}^{N-1} e^{-2\pi i \frac{mn}{N}} f_m. \quad (11.1)$$

The inverse of this transformation is the Inverse Discrete Fourier Transform (IDFT):

$$\text{IDFT} \{F_0, F_1, \dots, F_{N-1}\} = \{f_0, f_1, \dots, f_{N-1}\} \quad \text{where} \quad f_m = \frac{1}{N} \sum_{n=0}^{N-1} e^{2\pi i \frac{mn}{N}} F_n. \quad (11.2)$$

The inverse relationship between the DFT and the IDFT is straightforward to prove, by using the identity

$$\sum_{m=0}^{N-1} e^{\pm 2\pi i \frac{m(n-n')}{N}} = N \delta_{nn'}, \quad (11.3)$$

where $\delta_{nn'}$ denotes the Kronecker delta. This identity is derived from the geometric series formula.

[11.1: Conversion of Continuous Fourier Transform to DFT](#)

[11.2: Spectral Resolution and Range](#)

[11.3: The Split-Step Fourier Method](#)

This page titled [11: Discrete Fourier Transforms](#) is shared under a [CC BY-SA 4.0](#) license and was authored, remixed, and/or curated by [Y. D. Chong](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.