10.4: Appendix C

Euclidean Geometry

E1 Two points determine a line.

E2 Line segments can be extended.

E3 A unique circle can be constructed given any point as its center and any line segment as its radius.

E4 All right angles are equal to one another.

E5 *Parallel postulate*: Given a line and a point not on the line, exactly one line can be drawn through the point and parallel to the given line.[1]

> **Note**
>
> This is a form known as Playfair's axiom, rather than the version of the postulate originally given by Euclid.

Ordered Geometry

O1 Two events determine a line.

O2 Line segments can be extended: given A and B, there is at least one event such that [ABC] is true.

O3 Lines don't wrap around: if [ABC] is true, then [BCA] is false.

O4 Betweenness: For any three distinct events A, B, and C lying on the same line, we can determine whether or not B is between A and C (and by statement 3, this ordering is unique except for a possible over-all reversal to form [CBA]).

Affine Geometry

In addition to O1-O4, postulate the following axioms:

A1 Constructibility of parallelograms: Given any P, Q, and R, there exists S such that [PQRS], and if P, Q, and R are distinct then S is unique.

A2 Symmetric treatment of the sides of a parallelogram: If [PQRS], then [QRSP], [QPSR], and [PRQS].

A3 Lines parallel to the same line are parallel to one another: If [ABCD] and [ABEF], then [CDEF].

Experimentally Motivated Statements about Lorentzian Geometry

L1 *Spacetime is homogeneous and isotropic.* No point has special properties that make it distinguishable from other points, nor is one direction distinguishable from another.

L2 *Inertial frames of reference exist.* These are frames in which particles move at constant velocity if not subject to any forces. We can construct such a frame by using a particular particle, which is not subject to any forces, as a reference point.

L3 *Equivalence of inertial frames*: If a frame is in constant-velocity translational motion relative to an inertial frame, then it is also an inertial frame. No experiment can distinguish one inertial frame from another.

L4 *Causality*: There exist events 1 and 2 such that $t_1 < t_2$ in all frames.

L5 *Relativity of time*: There exist events 1 and 2 and frames of reference (t, x) and (t', x') such that $t_1 < t_2$, but $t'_1 > t'_2$.

Statements of the Equivalence Principle

Accelerations and gravitational fields are equivalent. There is no experiment that can distinguish one from the other (section 1.5).

It is always possible to define a local Lorentz frame in a particular neighborhood of spacetime (section 1.5).

There is no way to associate a preferred tensor field with spacetime (section 4.4).

Vectors

Coordinates cannot in general be added on a manifold, so they don't form a vector space, but infinitesimal coordinate differences can and do. The vector space in which the coordinate differences exist is a different space at every point, referred to as the tangent space at that point (see section 7.1).

Vectors are written in abstract index notation with upper indices, x^a, and are represented by column vectors, arrows, or birdtracks with incoming arrows, → x.

Dual vectors, also known as covectors or 1-forms, are written in abstract index notation with lower indices, xa, and are represented by row vectors, ordered pairs of parallel lines (see section 2.1), or birdtracks with outgoing arrows, ← x.

In concrete-index notation, the x^μ are a list of numbers, referred to as the vector's contravariant components, while x_μ would be the covariant components of a dual vector.

Fundamentally the distinction between the two types of vectors is defined by the tensor transformation laws, section 4.3. For example, an odometer reading is contravariant because converting it from kilometers to meters increases it. A temperature gradient is covariant because converting it from degrees/km to degrees/m decreases it.

In the absence of a metric, every physical quantity has a definite vector or dual vector character. Infinitesimal coordinate differences dx a and velocities dx a/ dτ are vectors, while momentum pa and force Fa are dual (see section 4.3). Many ordinary and interesting real-world systems lack a metric (see section 2.1). When a metric is present, we can raise and lower indices at will. There is a perfect duality symmetry between the two types of vectors, but this symmetry is broken by the convention that a measurement with a ruler is a Δx^a, not a Δx_a.

For consistency with the transformation laws, differentiation with respect to a quantity flips the index, e.g., $\partial_\mu = \frac{\partial}{\partial x^\mu}$. The operators $\partial\mu$ are often used as basis vectors for the tangent plane. In general, expressing vectors in a basis using the Einstein notation convention results in an ugly notational clash described in section 7.1.

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