

2.1: Introduction to Geometry of Flat Spacetime

The geometrical treatment of space, time, and gravity only requires as its basis the equivalence of inertial and gravitational mass. Given this assumption, we can describe the trajectory of any free-falling test particle as a geodesic. Equivalence of inertial and gravitational mass holds for Newtonian gravity, so it is indeed possible to redo Newtonian gravity as a theory of curved spacetime. This project was carried out by the French mathematician Cartan. The geometry of the local reference frames is very simple. The three space dimensions have an approximately Euclidean geometry, and the time dimension is entirely separate from them. This is referred to as a Euclidean spacetime with 3+1 dimensions. Although the outlook is radically different from Newton's, all of the predictions of experimental results are the same.

The experiments in [Section 1.2](#) show, however, that there are real, experimentally verifiable violations of Newton's laws. In Newtonian physics, time is supposed to flow at the same rate everywhere, which we have found to be false. The flow of time is actually dependent on the observer's state of motion through space, which shows that the space and time dimensions are intertwined somehow. The geometry of the local frames in relativity therefore must not be as simple as Euclidean 3+1. Their actual geometry was implicit in Einstein's 1905 paper on special relativity, and had already been developed mathematically, without the full physical interpretation, by Hendrik Lorentz. Lorentz's and Einstein's work were explicitly connected by Minkowski in 1907, so a Lorentz frame is often referred to as a Minkowski frame.



Figure 2.0.1 - Hendrik Antoon Lorentz (1853-1928)

To describe this Lorentz geometry, we need to add more structure on top of the axioms O1-O4 of ordered geometry, but it will not be the additional Euclidean structure of E3-E4, it will be something different. To see how to proceed, let's start by thinking about what bare minimum of geometrical machinery is needed in order to set up frames of reference.

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