3.2: Tangent Vectors

It's not immediately clear what a vector means in the context of curved spacetime. The freshman physics notion of a vector carries all kinds of baggage, including ideas like rotation of vectors and a magnitude that is positive for nonzero vectors. We also used to assume the ability to represent vectors as arrows, i.e., geometrical figures of finite size that could be transported to other places — but in a curved geometry, it is not in general possible to transport a figure to another location without distorting its shape, so there is no notion of congruence. For this reason, it's better to visualize vectors as tangents to the underlying space, as in Figure $3.2.1$. Intuitively, we want to think of these vectors as arrows that are infinitesimally small, so that they fit on the curved surface without having to be bent. In the pictures, we simply scale them up to make them visible without an infinitely powerful microscope, and this scaling only makes them *appear* to rise out of the space in which they live.

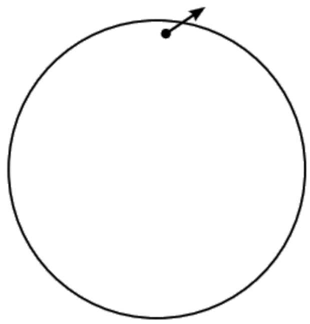

Figure $3.2.1$: A vector can be thought of as lying in the plane tangent to a certain point.

A more formal definition of the notion of a tangent vector is given later.