1.E: Geometric Theory of Spacetime (Exercises)

Problems

1. In classical mechanics, one hears the term "the acceleration of gravity," which doesn't literally make sense, since it is *objects* that accelerate. Explain why this term's usefulness is dependent on the equivalence principle.
2. The New Horizons space probe communicates with the earth using microwaves with a frequency of about 10 GHz. Estimate the sizes of the following frequency shifts in this signal, when the probe flies by Pluto in 2015, at a velocity of $\sim$ 10 A.U./year:
   a. the Doppler shift due to the probe's velocity;
   b. the Doppler shift due to the Earth's orbital velocity;
   c. the gravitational Doppler shift.
3. Euclid's axioms E1-E5 do not suffice to prove that there are an infinite number of points in the plane, and therefore they need to be supplemented by an extra axiom that states this (unless one finds the nonstandard realizations with finitely many points to be interesting enough to study for their own sake). Prove that the axioms of ordered geometry O1-O4 do not have this problem.
4. In the science fiction novel *Have Spacesuit — Will Travel*, by Robert Heinlein, Kip, a high school student, answers a radio distress call, encounters a flying saucer, and is knocked out and kidnapped by aliens. When he wakes up, he finds himself in a locked cell with a young girl named Peewee. Peewee claims they're aboard an accelerating spaceship. "If this was a spaceship," Kip thinks. "The floor felt as solid as concrete and motionless."
   The equivalence principle can be stated in a variety of ways. Earlier, it was stated as (1) gravitational and inertial mass are always proportional to one another. An alternative formulation is (2) that Kip has no way, by experiments or observations inside his sealed prison cell, to determine whether he's in an accelerating spaceship or on the surface of a planet, experiencing its gravitational field.
   a. Show that any violation of statement 1 also leads to a violation of statement 2.
   b. If we'd intended to construct a geometrical theory of gravity roughly along the lines of axioms O1-O4, which axiom is violated in this scenario?
5. Clock A sits on a desk. Clock B is tossed up in the air from the same height as the desk and then comes back down. Compare the elapsed times.
6. (a) Find the difference in rate between a clock at the center of the earth and a clock at the south pole. (b) When an antenna on earth receives a radio signal from a space probe that is in a hyperbolic orbit in the outer solar system, the signal will show both a kinematic red-shift and a gravitational blueshift. Compare the orders of magnitude of these two effects.
7. Consider the following physical situations: (1) a charged object lies on a desk on the planet earth; (2) a charged object orbits the earth; (3) a charged object is released above the earth's surface and dropped straight down; (4) a charged object is subjected to a constant acceleration by a rocket engine in outer space. In each case, we want to know whether the charge radiates. Analyze the physics in each case (a) based on conservation of energy; (b) by determining whether the object's motion is inertial in the sense intended by Isaac Newton; (c) using the most straightforward interpretation of the equivalence principle (i.e., not worrying about the issues discussed on p. that surround the ambiguous definition of locality).
8. Consider the physical situation depicted in Figure 1.5.12. Let ag be the gravitational acceleration and a_r the acceleration of the charged particle due to radiation. Then $\frac{a_r}{a_g}$ measures the violation of the equivalence principle. The goal of this problem is to make an order-of-magnitude estimate of this ratio in the case of a neutron and a proton in low earth orbit.
   a. Let m the mass of each particle, and q the charge of the charged particle. Without doing a full calculation like the ones by the DeWitts and Grøn and Næss, use general ideas about the frequency-scaling of radiation (see section 9.2) to find the proportionality that gives the dependence of $\frac{a_r}{a_g}$ on q, m, and any convenient parameters of the orbit.
   b. Based on considerations of units, insert the necessary universal constants into your answer from part a.
   c. The result from part b will still be off by some unitless factor, but we expect this to be of order unity. Under this assumption, make an order-of-magnitude estimate of the violation of the equivalence principle in the case of a neutron and a proton in low earth orbit.