3.9: Interpretation of Coordinate Independence

This section discusses some of the issues that arise in the interpretation of coordinate independence. It can be skipped on a first reading.

Is Coordinate Independence Obvious?

One often hears statements like the following from relativists: "Coordinate independence isn't really a physical principle. It's merely an obvious statement about the relationship between mathematics and the physical universe. Obviously the universe doesn't come equipped with coordinates. We impose those coordinates on it, and the way in which we do so can never be dictated by nature." The impressionable reader who is tempted to say, "Ah, yes, that is obvious," should consider that it was far from obvious to Newton ("Absolute, true and mathematical time, of itself, and from its own nature flows equably without regard to anything external . . . "), nor was it obvious to Einstein. Levi-Civita nudged Einstein in the direction of coordinate independence in 1912. Einstein tried hard to make a coordinate-independent theory, but for reasons described in section 3.6, he convinced himself that that was a dead end. In 1914-15 he published theories that were not coordinate independent, which you will hear relativists describe as "obvious" dead ends because they lack any geometrical interpretation. It seems to me that it takes a highly refined intuition to regard as intuitively "obvious" an issue that Einstein struggled with like Jacob wrestling with Elohim.

Is Coordinate Independence Trivial?

It has also been alleged that coordinate independence is trivial. To gauge the justice of this complaint, let's distinguish between two reasons for caring about coordinate independence:

1. Coordinate independence tells us that when we solve problems, we should avoid writing down any equations in notation that isn't manifestly intrinsic, and avoid interpreting those equations as if the coordinates had intrinsic meaning. Violating this advice doesn't guarantee that you've made a mistake, but it makes it much harder to tell whether or not you have.
2. Coordinate independence can be used as a criterion for judging whether a particular theory is likely to be successful.

Nobody questions the first justification. The second is a little trickier. Laying out the general theory systematically in a 1916 paper,[14] Einstein wrote "The general laws of nature are to be expressed by equations which hold good for all the systems of coordinates, that is, are covariant with respect to any substitutions whatever (generally covariant)." In other words, he was explaining why, with hindsight, his 1914-1915 coordinate-dependent theory had to be a dead end.

The only trouble with this is that Einstein's way of posing the criterion didn't quite hit the nail on the head mathematically. As Hilbert famously remarked, "Every boy in the streets of Göttingen understands more about four-dimensional geometry than Einstein. Yet, in spite of that, Einstein did the work and not the mathematicians." What Einstein had in mind was that a theory like Newtonian mechanics not only lacks coordinate independence, but would also be impossible to put into a coordinate-independent form without making it look hopelessly complicated and ugly, like putting lipstick on a pig. But Kretschmann showed in 1917 that any theory could be put in coordinate independent form, and Cartan demonstrated in 1923 that this could be done for Newtonian mechanics in a way that didn't come out particularly ugly. Physicists today are more apt to pose the distinction in terms of "background independence" (meaning that a theory should not be phrased in terms of an assumed geometrical background) or lack of a "prior geometry" (meaning that the curvature of spacetime should come from the solution of field equations rather than being imposed by fiat). But these concepts as well have resisted precise mathematical formulation.[15] My feeling is that this general idea of coordinate independence or background independence is like the equivalence principle: a crucial conceptual principle that doesn't lose its importance just because we can't put it in a mathematical box with a ribbon and a bow. For example, string theorists take it as a serious criticism of their theory that it is not manifestly background independent, and one of their goals is to show that it has a background independence that just isn't obvious on the surface.

Coordinate Independence as a Choice of Gauge

It is instructive to consider coordinate independence from the point of view of a field theory. Newtonian gravity can be described in three equivalent ways: as a gravitational field $\mathbf{g}$, as a gravitational potential ϕ, or as a set of gravitational field lines. The field lines are never incident on one another, and locally the field satisfies Poisson's equation. The electromagnetic field has polarization properties different from those of the gravitational field, so we describe it using either the two fields ($\mathbf{E}, \mathbf{B}$), a pair of potentials,[16] or two sets of field lines. There are similar incidence conditions and local field equations (Maxwell's equations). Gravitational fields in relativity have polarization properties unknown to Newton, but the situation is qualitatively similar to the two foregoing cases. Now consider the analogy between electromagnetism and relativity. In electromagnetism, it is the fields that are directly observable, so we expect the potentials to have some extrinsic properties. We can, for example, redefine our electrical ground, $\Phi \rightarrow \Phi + C$, without any observable consequences. As discussed in more detail in Section 5.6, it is even possible to modify the electromagnetic potentials in an entirely arbitrary and nonlinear way that changes from point to point in spacetime. This is called a gauge transformation. In relativity, the gauge transformations are the smooth coordinate transformations. These gauge transformations distort the field lines without making them cut through one another.

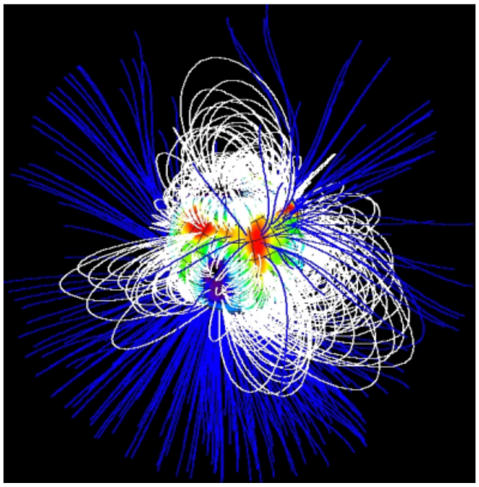

Figure 3.9.1: Since magnetic field lines can never intersect, a magnetic field pattern contains coordinate-independent information in the form of the knotting of the lines. This figure shows the magnetic field pattern of the star SU Aurigae, as measured by Zeeman-Doppler imaging (Petit at al.). White lines represent magnetic field lines that close upon themselves in the immediate vicinity of the star; blue lines are those that extend out into the interstellar medium.

Note

There is the familiar electrical potential ϕ, measured in volts, but also a vector potential $\mathbf{A}$, which you may or may not have encountered. Briefly, the electric field is given not by $-\nabla\phi$ but by $-\nabla\phi - \frac{\partial A}{\partial t}$, while the magnetic field is the curl of $\mathbf{A}$. This is introduced at greater length in section 4.2.

References

[15] Giulini, "Some remarks on the notions of general covariance and background independence," arxiv.org/abs/gr-qc/0603087v1