7.5: Static and Stationary Spacetimes (Part 2)

No-hair Theorems

Birkhoff's theorem is similar to a set of theorems called no-hair theorems describing black holes. The most general no-hair theorem states that a black hole is completely characterized by its mass, charge, and angular momentum. Other than these three numbers, nobody on the outside can recover any information that was possessed by the matter and energy that were sucked into the black hole.

It has been proposed[11] that the no-hair theorem for nonzero angular momentum and zero charge could be tested empirically by observations of Sagittarius A*. If the observations are consistent with the no-hair theorem, it would be taken as supporting the validity of general relativity and the interpretation of this object as a supermassive black hole. If not, then there are various possibilities, including a failure of general relativity to be the correct theory of strong gravitational fields, or a failure of one of the theorem's other assumptions, such as the nonexistence of closed timelike curves in the surrounding universe.

The no-hair theorems say that relativity only has a small repertoire of types of black-holes, defined as regions of space that are causally disconnected from the universe, in the sense that future light-cones of points in the region do not extend to infinity.[12] That is, a black hole is defined as a region hidden behind an event horizon, and since the definition of an event horizon is dependent on the observer, we specify an observer infinitely far away. Birkhoff's theorem has a somewhat different structure than those of the no-hair theorems, since it assumes a symmetry and proves the existence of an event horizon (if the vacuum region is extended to small enough radii), whereas the no-hair theorems assume an event horizon and prove the form of the metric, including its symmetries.

null infinity

For a more formal statement of this, see Hawking and Ellis, "The Large Scale Structure of Space-Time," p. 315. Essentially, the region must be a connected region on a spacelike three-surface, and there must be no lightlike world-lines that connect points in that region to null infinity. Null infinity was introduced briefly in section 7.3 is defined formally using conformal techniques, but basically refers to points that are infinitely far away in both space and time, and have the two infinities *equal* in a certain sense, so that a free light ray could end up there. The definition is based on the assumption that the surrounding spacetime is asymptotically flat, since otherwise null infinity can't be defined. It is not actually necessary to assume a singularity as part of the definition; the no-hair theorems guarantee that one exists.

The no-hair theorems cannot classify naked singularities, i.e., those not hidden behind horizons. The role of naked singularities in relativity is the subject of the cosmic censorship hypothesis, which is an open problem. The theorems do not rule out the Big Bang singularity, because we cannot define the notion of an observer infinitely far from the Big Bang. We can also see that Birkhoff's theorem does not prohibit the Big Bang, because cosmological models are not vacuum solutions with $\Lambda = 0$. Black string solutions are not ruled out by Birkhoff's theorem because they would lack spherical symmetry, so we need the arguments given in section 6.3 to show that they don't exist.

We saw on example 4 and section 7.3 that there is no clearly defined way to treat a singularity as a geometrical object, and that this ambiguity extends even to such seemingly straightforward questions as how many dimensions it has. Geometrically, as Gertrude Stein said about Oakland, there's "no there there." We could also ask whether a black hole singularity has any *physical properties*. If so, then the no-hair theorems would limit the list of such properties to at most three. But we cannot ascribe these properties to the singularity itself. Rather, they are properties of some large region of the spacetime, measurable by an observer at asymptotic infinity. Such an observer cannot say whether a black hole's mass is a property of the singularity; she cannot even say whether the singularity exists "now." In this sense a black hole singularity is not an "it." Asking about "its" properties is like asking what time it is when the tip of the minute hand is at the center of the clock. The dial only exists around the circumference of the circle, not at its center.

The Gravitational Potential

When Pound and Rebka made the first observation of gravitational redshifts, these shifts were interpreted as evidence of gravitational time dilation, i.e., a mismatch in the rates of clocks. We are accustomed to connecting these two ideas by using the expression $e^{-\Delta\Phi}$ for the ratio of the rates of two clocks (example 11), where Φ is a function of the spatial coordinates, and this is in fact the most general possible definition of a gravitational potential Φ in relativity. Since a stationary field allows us to compare rates of clocks, it seems that we should be able to define a gravitational potential for any stationary field. There is a problem, however, because when we talk about a potential, we normally have in mind something that has encoded within it all there is to know about the field. We would therefore expect to be able to find the metric from the potential. But the example of the rotating earth shows that this need not be the case for a general stationary field. In that example, there are effects like frame-dragging that clearly cannot be deduced from Φ; for by symmetry, Φ is independent of azimuthal angle, and therefore it cannot distinguish between the direction of rotation and the contrary direction. In a static spacetime, these rotational effects don't exist; a static vacuum spacetime can be described completely in terms of a single scalar potential plus information about the spatial curvature.

There are two main reasons why relativity does not offer a gravitational potential with the same general utility as its Newtonian counterpart.

The Einstein field equations are nonlinear. Therefore one cannot, in general, find the field created by a given set of sources by adding up the potentials. At best this is a possible weak-field approximation. In particular, although Birkhoff's theorem is in some ways analogous to the Newtonian shell theorem, it cannot be used to find the metric of an arbitrary spherically symmetric mass distribution by breaking it up into spherical shells.

It is also not meaningful to talk about any kind of gravitational potential for spacetimes that aren't static or stationary. For example, consider a cosmological model describing our expanding universe. Such models are usually constructed according to the Copernican principle that no position in the universe occupies a privileged place. In other words, they are homogeneous in the sense that they have Killing vectors describing arbitrary translations and rotations. Because of this high degree of symmetry, a gravitational potential for such a model would have to be independent of position, and then it clearly could not encode any information about the spatial part of the metric. Even if we were willing to make the potential a function of time, $\Phi(t)$, the results would still be nonsense. The gravitational potential is defined in terms of rate-matching of clocks, so a potential that was purely a function of time would describe a situation in which all clocks, everywhere in the universe, were changing their rates in a uniform way. But this is clearly just equivalent to a redefi- nition of the time coordinate, which has no observable consequences because general relativity is coordinate-invariant. A corollary is that in a cosmological spacetime, it is not possible to give a natural prescription for deciding whether a particular redshift is gravitational (measured by Φ) or kinematic, or some combination of the two (see also section 8.2).

References

[11] Johannsen and Psaltis, http://arxiv.org/abs/1008.3902v1