9.E: Gravitational Waves (Exercises)

1. (a) Starting in section 1.5, we have associated geodesics with the world-lines of low-mass objects (test particles). Use the HulseTaylor pulsar as an example to show that the assumption of low mass was a necessary one. How is this similar to the issues encountered in chapter 1 problems involving charged particles?
(b) Show that if low-mass, uncharged particles did not follow geodesics (in a spacetime with no ambient electromagnetic fields), it would violate Lorentz invariance. Make sure that your argument explicitly invokes the low mass and the lack of charge, because otherwise your argument is wrong.
2. Show that the metric $ds^2 = dt^2 - A\,dx^2 - B\,dy^2 - dz^2$ with

$$\begin{aligned} A &= 1 - f + \frac{3}{8}f^2 - \frac{25}{416}f^3 + \frac{15211}{10729472}f^5 \\ B &= 1 + f + \frac{3}{8}f^2 + \frac{25}{416}f^3 - \frac{15211}{10729472}f^5 \\ f &= Ae^{k(t-z)} \end{aligned}$$

   is an approximate solution to the vacuum field equations, provided that k is real — which prevents this from being a physically realistic, oscillating wave. Find the next nonvanishing term in each series.
3. Verify the claims made in example 2. Characterize the (somewhat complex) behavior of the function q obtained when p(u) = 1 + A cos u.
4. Verify the claims made in example 3 using Maxima. Although the result holds for any function f, you may find it more convenient to use some specific form of f, such as a sine wave, so that Maxima will be able to simplify the result to zero at the end. Note that when the metric is expressed in terms of the line element, there is a factor of 2 in the 2h dz dt term, but when expressing it as a matrix, the 2 is not present in the matrix elements, because there are two elements in the matrix that each contribute an equal amount.