5.3: The Stress-energy Tensor

In general, the curvature of spacetime will contain contributions from both tidal forces and local sources, superimposed on one another. To develop the right formulation for the Einstein field equations, we need to eliminate the tidal part. Roughly speaking, we will do this by averaging the sectional curvature over all three of the planes $t-x$, $t-y$, and $t-z$, giving a measure of curvature called the *Ricci curvature*. The "roughly speaking" is because such a prescription would treat the time and space coordinates in an extremely asymmetric manner, which would violate local Lorentz invariance.

To get an idea of how this would work, let's compare with the Newtonian case, where there really is an asymmetry between the treatment of time and space. In the Cartan curved-spacetime theory of Newtonian gravity (Chapter 2), the field equation has a kind of scalar Ricci curvature on one side, and on the other side is the density of mass, which is also a scalar. In relativity, however, the source term in the equation clearly cannot be the scalar mass density. We know that mass and energy are equivalent in relativity, so for example the curvature of spacetime around the earth depends not just on the mass of its atoms but also on all the other forms of energy it contains, such as thermal energy and electromagnetic and nuclear binding energy. Can the source term in the Einstein field equations therefore be the mass-energy E? No, because E is merely the timelike component of a particle's momentum four-vector. To single it out would violate Lorentz invariance just as much as an asymmetric treatment of time and space in constructing a Ricci measure of curvature. To get a properly Lorentz invariant theory, we need to find a way to formulate everything in terms of tensor equations that make no explicit reference to coordinates. The proper generalization of the Newtonian mass density in relativity is the stress-energy tensor T^{ij}, whose 16 elements measure the local density of mass-energy and momentum, and also the rate of transport of these quantities in various directions. If we happen to be able to find a frame of reference in which the local matter is all at rest, then T^{tt} represents the mass density. The reason for the word "stress" in the name is that, for example, the flux of x-momentum in the x direction is a measure of pressure.

For the purposes of the present discussion, it's not necessary to introduce the explicit definition of T; the point is merely that we should expect the Einstein field equations to be tensor equations, which tells us that the definition of curvature we're seeking clearly has to be a rank-2 tensor, not a scalar. The implications in four-dimensional spacetime are fairly complex. We'll end up with a rank-4 tensor that measures the sectional curvature, and a rank-2 Ricci tensor derived from it that averages away the tidal effects. The Einstein field equations then relate the Ricci tensor to the energy-momentum tensor in a certain way. The stress-energy tensor is discussed further in Section 8.1.