# 1.8: Dynamics of the Expansion

## Introduction to the Dynamics chapters, 10-16.

In the following set of chapters we will derive the dynamical equations that relate the matter content in a homogeneous and isotropic universe to the evolution of the scale factor over time. There are fundamentally two independent dynamical equations. The first of these is the Friedmann equation:

$$H^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} \tag{1.8.1}$$

where $H \equiv \dot{a}/a$, $G$ is Newton's gravitational constant, $k$ is the curvature constant we have already seen in the FRW invariant distance rule. For simplicity here we have assumed a single type of matter fills the universe; $\rho$ is its mass density.

**Note**

The generalization could not be more straightforward. For $i=1$ to $N$ components, replace $\rho$ with $\Sigma_{i=1}^{N}\rho_i$ where $\rho_i$ is the mass/energy density of the $i^{\text{th}}$ component.

The second equation tells us how $\rho$ evolves with time (or scale factor) and can be taken to be:

$$a\frac{d\rho}{da} = -3\left(\rho + P/c^2\right) \tag{1.8.2}$$

where $P$ is the pressure.

One might think we need general relativity to derive these equations, and one would be correct. However, we can get surprisingly close with a Newtonian analysis. Here we will follow a Newtonian analysis, and clarify where we are taking something from GR without proof.

We know that a Newtonian analysis of gravitational dynamics will lead to highly accurate predictions when relative speeds are slow and gravitational fields are weak; Newtonian theory follows from general relativity in these limits. Our Newtonian analysis takes advantage of this fact. We can study the dynamics over as small a distance as we like, over which relative speeds are as small as we want, and gravitational potential differences are arbitrarily small. As we will see, the results we achieve by doing so are *independent* of the size we assume. If our results are describing one small region of the universe accurately, by the assumed homogeneity of the spacetime they must be describing all other small regions of the universe accurately, and hence the whole universe accurately.

Our approach risks confusion because we will be moving back and forth between Newtonian descriptions of spacetime and relativistic descriptions. So consider yourself duly warned. Be vigilant and seek clarification when something seems amiss.