5.3: The Ergoregion

When studying the Schwarzschild metric, we saw that $r = 2M$ was interesting for two reasons. First, the coefficient of dt^2 (g_{tt}) in the metric became zero. We can call this an **infinite redshift surface** because the time between two consecutive crests of a light wave coming from just outside the event horizon is infinitely big for a faraway observer (shifted to a lower frequency). Second, we saw that the signs of g_{tt} and g_{rr} switched at $r = 2M$, which we interpreted as meaning that $r = 2M$ is the "point of no return" since the inevitable end was $r = 0$. We called this the **event horizon**. We could also call it the **static limit surface** because it is only possible to stay stationary outside. For a Schwarzschild black hole, the infinite redshift surface, the static limit surface, and the event horizon happen to have the same r-value, but that isn't necessarily the case for other black holes. For a spinning black hole, the infinite redshift surface and the static limit surface are still the same, but they are different from the event horizon. The region in between the infinite redshift surface and the event horizon is called the **ergoregion**.

Definition: Infinite Redshift Surface

The infinite redshift surface of a black hole is the set of all r-values for which $g_{tt} = 0$. A light wave emitted outward from this surface will be shifted to a lower frequency, eventually reaching a frequency of zero at an r-value of infinity.

Definition: Static Limit Surface

The static limit surface of a black hole is the set of all r-values below which a test particle cannot stay stationary.

Definition: Ergoregion

The ergoregion is the region between the infinite redshift surface and the event horizon.

By setting $g_{tt} = 0$, we can show that the infinite redshift surface r_{IR} is given by (see Box 5.3.1)

$$r_{\text{IR}} = M + \sqrt{M^2 - a^2\cos^2\theta}. \tag{5.3.1}$$

Note

Setting $g_{tt} = 0$ in the Kerr metric and solving for r actually yields *two* solutions, but one of those solutions falls within the event horizon. Since light from just outside that surface can't escape to the outside world, we can ignore that solution.

It turns out that the infinite redshift surface is also the static limit surface (see Box 5.3.2), but *not* because particles within the ergoregion must fall to smaller values of r. Instead, particles within the ergoregion must move in the direction of the black hole's spin.

Finally, by setting $\frac{1}{g_{rr}} = 0$ in the Kerr metric, we can show that the event horizon is given by (see Box 5.3.3)

$$r_{\text{EH}} = M + \sqrt{M^2 - a^2}. \tag{5.3.2}$$

Note

Once again, you actually get two solutions when you set $g_{rr} = 0$, but we ignore the smaller one. If the point of defining an event horizon is to identify the point of no return, then the smaller one is rendered irrelevant.

Fig. 5.3.1 shows a polar plot of the infinite redshift surface and the event horizon.

Figure 5.3.1: A polar plot of the infinite redshift surface (green) and the event horizon (purple) for $a = 0.95M$. This shows a cross-section, with the rotation axis in the center and vertical. Click "edit graph on desmos" to see how changing the spin parameter changes each. (Copyright; author via source)

Exercise 5.3.1

Does a faster spin cause the ergoregion to become bigger or smaller?

Answer

If you click on "edit graph on desmos" in Fig. 5.3.1, you can choose the value of a using a slider. You should find that the ergoregion shrinks the closer a is to zero. This makes sense since $a = 0$ is the Schwarzschild limit, where there is no ergoregion.

Exercise 5.3.2

What is the maximum spin for which there is an event horizon?

Answer

According to Equation 5.3.2, the r-value of the event horizon is only a real number if $M^2 - a^2 \geq 0$, which means that there is only an event horizon if $a \leq M$.

Exercise 5.3.3

How is the ergoregion different from the region in the interior of the event horizon?

Answer

It is impossible to stay stationary in both the ergoregion and the interior of the event horizon. The difference is that a particle is able to maintain a constant r-value inside the ergoregion while it is forced to move to $r = 0$ inside the event horizon.

Box 5.3.1

Set $g_{tt} = 0$ in the Kerr metric to prove Equation 5.3.1.

Box 5.3.2

Argue that a particle can only stay stationary ($dr = d\theta = d\phi = 0$) outside of a Kerr black hole if $g_{tt} > 0$. (Remember that $g_{tt} = 0$ defines the infinite redshift surface, so this proves that the static limit surface is the same as the infinite redshift surface.)

Box 5.3.3

Set $\frac{1}{g_{rr}} = 0$ in the Kerr metric to prove Equation 5.3.2.
