6.4: Gravitational Wave Sources

In general, gravitational waves are produced by moving masses. This can include the collapse of stars or objects in orbit. One caveat, though, is that the motion can't be spherically symmetric. A perfectly spherical star, then, would not produce gravitational waves when it collapses or explodes, but an asymmetric star would. While perfectly spherical stars don't actually exist, the degree to which a typical star deviates from perfect spherical symmetry is small enough that wouldn't expect very strong gravitational waves to come from it. For this reason, the primary source that we can expect to detect gravitational waves from is objects in orbit. While many objects in space orbit around something, only the most massive objects have any hope of producing gravitational waves large enough for us to detect them. Furthermore, the orbit has to be fast enough to produce gravitational waves with a frequency on the order of 100 Hz. This vastly reduces the possible orbital scenarios that we can detect. So far, for example, we have only been able to detect binary systems (that is, two objects orbiting around each other) that consist of black holes and/or neutron stars.

Let's analyze a simple binary system consisting of objects with mass M_1 and M_2 separated by a distance r. As long as r is much greater than the Schwarszchild radius and the orbital speed is non-relativistic, the total energy can be approximated using Newtonian mechanics. The result (see Box 6.4.1), in natural units, is

$$E=-\frac{M_1 M_2}{2r}. \tag{6.4.1}$$

The gravitational waves, meanwhile, carry energy away, which makes the total energy of the binary system *more negative*.

Exercise 6.4.1

Use Equation 6.4.1 to argue that, as the binary system loses energy, the distance between the orbiting objects decreases.

Answer

Fig. 6.4.1 shows a plot of the total energy as a function of r.

Figure 6.4.1: Total energy of a binary system as a function of r. (Copyright; author via source)

You can see from the graph that energy loss corresponds to smaller values of r.

It can be shown (though we will not do so here), that the rate of energy loss is

$$\frac{dE}{dt} = -\frac{32}{5r^5}\left(M_1 M_2\right)^2 \left(M_1 + M_2\right). \tag{6.4.2}$$

Equation 6.4.2 can then be used to determine the rate at which the distance between them decreases. The result (see Box 6.4.2) is

$$\frac{dr}{dt} = -\frac{64}{5r^3} M_1 M_2 \left(M_1 + M_2\right). \tag{6.4.3}$$

This result shows two important things. First, $\frac{dr}{dt}$ is negative, which confirms that the distance between the orbiting objects decreases with time. Second, the rate at which the distance decreases gets larger as the orbiting objects get closer together.

Finally, the orbital speed increases as the distance between them decreases (see Box 6.4.3), which increases the frequency of the emitted gravitational waves. The result is a **chirp**. That is, there is a very rapid increase in the frequency of the gravitational wave in the moments just before the orbiting objects merge. Fig. 6.4.2 shows what this looks like for an actual gravitational wave event at three different detectors (Virgo is another detector that came online after the original detection by Hanford and Livingston).

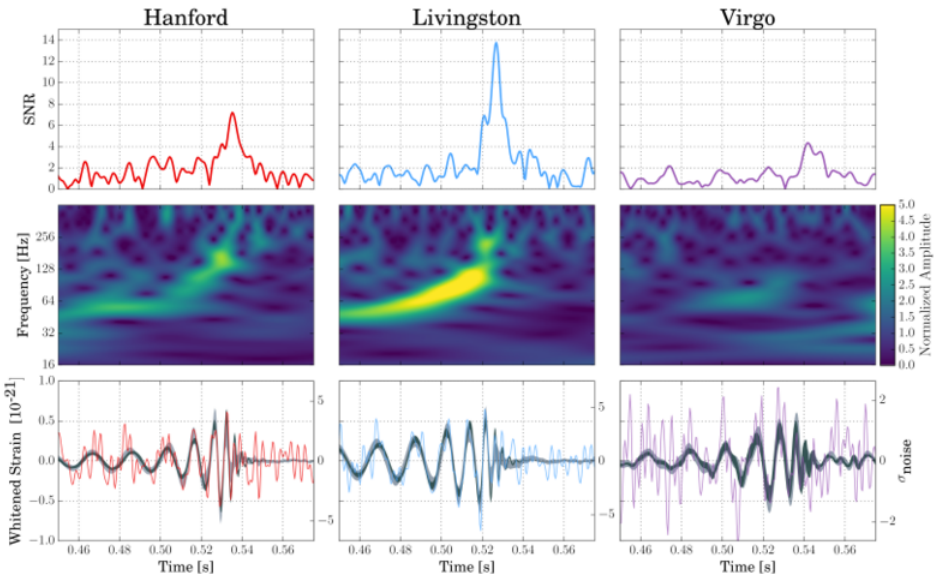

Figure 6.4.2: The top row shows the signal-to-noise ratio for a gravitational wave event at three different detectors. The middle row shows a spectrogram, which depicts both frequency and amplitude as a function of time. The bottom row shows the gravitational wave strain as a function of time. Both the middle row and the bottom row show that the frequency rapidly increases just before the merger. (Image credit: Wikimedia commons)

The point of this is that we have a theoretical expectation for what a gravitational wave signal should look like, and the properties of that signal depend on the masses of the orbiting objects, the distance between them, and the distance from us to the binary system. Gravitational wave detectors continuously look for signals with this characteristic shape. When they do find one, they perform a parameter fit, which gives us information about the binary system. Then by looking at the time delay between the signal at the different detectors, we can determine where on the sky on the signal came from.

Box 6.4.1

The total energy in Equation 6.4.1 is a combination of potential energy and kinetic energy. The potential energy, in natural units, is

$$PE = -\frac{M_1 M_2}{r}$$

while the kinetic energy is

$$KE = \frac{1}{2}\mu v^2,$$

where $\mu = \frac{M_1 M_2}{M_1 + M_2}$ is called the **reduced mass** (reduced mass is used when two objects revolve around a point between them rather than one object orbiting around another stationary object). Meanwhile, the gravitational force of attraction is given by

$$F = \frac{M_1 M_2}{r^2}.$$

Finally, the centripretal force is

$$F = \mu \frac{v^2}{r}.$$

Use these to derive Equation 6.4.1.

Box 6.4.2

Take the derivative of Equation 6.4.1 with respect to time, then equate it to Equation 6.4.2 in order to prove Equation 6.4.3. (Hint: the r in Equation 6.4.1 is a function of time, so you must use the chain rule when taking the derivative.)

Box 6.4.3

Use information from Box 6.4.1 to argue that orbital speed increases as the distance between the orbiting objects decreases.