

6.7: Gauss's Law (Exercises)

Conceptual Questions

6.2 Electric Flux

1. Discuss how would orient a planar surface of area A in a uniform electric field of magnitude E_0 to obtain
 - (a) the maximum flux and
 - (b) the minimum flux through the area.
2. What are the maximum and minimum values of the flux in the preceding question?
3. The net electric flux crossing a closed surface is always zero. True or false?
4. The net electric flux crossing an open surface is never zero. True or false?

6.3 Explaining Gauss's Law

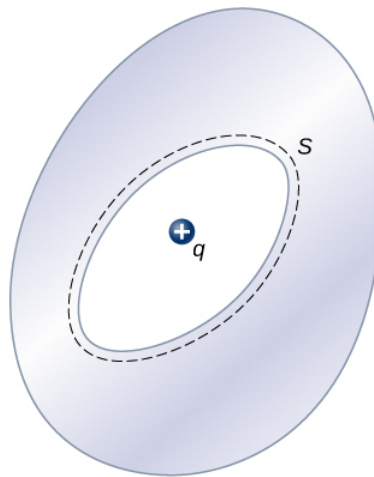
5. Two concentric spherical surfaces enclose a point charge q . The radius of the outer sphere is twice that of the inner one. Compare the electric fluxes crossing the two surfaces.
6. Compare the electric flux through the surface of a cube of side length a that has a charge q at its center to the flux through a spherical surface of radius a with a charge q at its center.
7. (a) If the electric flux through a closed surface is zero, is the electric field necessarily zero at all points on the surface?
(b) What is the net charge inside the surface?
8. Discuss how Gauss's law would be affected if the electric field of a point charge did not vary as $1/r^2$.
9. Discuss the similarities and differences between the gravitational field of a point mass m and the electric field of a point charge q .
10. Discuss whether Gauss's law can be applied to other forces, and if so, which ones.
11. Is the term \vec{E} in Gauss's law the electric field produced by just the charge inside the Gaussian surface?
12. Reformulate Gauss's law by choosing the unit normal of the Gaussian surface to be the one directed inward.

6.4 Applying Gauss's Law

13. Would Gauss's law be helpful for determining the electric field of two equal but opposite charges a fixed distance apart?
14. Discuss the role that symmetry plays in the application of Gauss's law. Give examples of continuous charge distributions in which Gauss's law is useful and not useful in determining the electric field.
15. Discuss the restrictions on the Gaussian surface used to discuss planar symmetry. For example, is its length important? Does the cross-section have to be square? Must the end faces be on opposite sides of the sheet?

6.5 Conductors in Electrostatic Equilibrium

16. Is the electric field inside a metal always zero?
17. Under electrostatic conditions, the excess charge on a conductor resides on its surface. Does this mean that all the conduction electrons in a conductor are on the surface?
18. A charge q is placed in the cavity of a conductor as shown below. Will a charge outside the conductor experience an electric field due to the presence of q ?



19. The conductor in the preceding figure has an excess charge of $-5.0\mu\text{C}$. If a $2.0 - \mu\text{C}$ point charge is placed in the cavity, what is the net charge on the surface of the cavity and on the outer surface of the conductor?

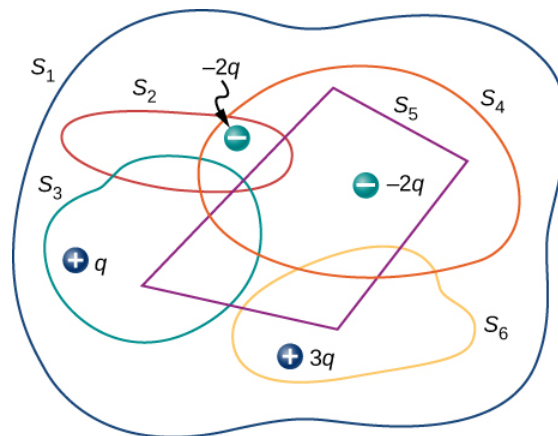
Problems

6.2 Electric Flux

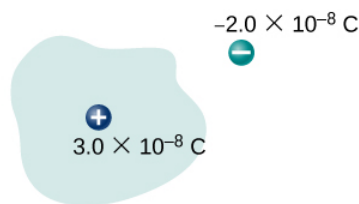
20. A uniform electric field of magnitude $1.1 \times 10^4 \text{ N/C}$ is perpendicular to a square sheet with sides 2.0 m long. What is the electric flux through the sheet?
21. Calculate the flux through the sheet of the previous problem if the plane of the sheet is at an angle of 60° to the field. Find the flux for both directions of the unit normal to the sheet.
22. Find the electric flux through a rectangular area $3\text{cm} \times 2\text{cm}$ between two parallel plates where there is a constant electric field of 30 N/C for the following orientations of the area: (a) parallel to the plates, (b) perpendicular to the plates, and (c) the normal to the area making a 30° angle with the direction of the electric field. Note that this angle can also be given as $180^\circ + 30^\circ$.
23. The electric flux through a square-shaped area of side 5 cm near a large charged sheet is found to be $3 \times 10^{-5} \text{ N} \cdot \text{m}^2 / \text{C}$. when the area is parallel to the plate. Find the charge density on the sheet.
24. Two large rectangular aluminum plates of area 150cm^2 face each other with a separation of 3 mm between them. The plates are charged with equal amount of opposite charges, $\pm 20\mu\text{C}$. The charges on the plates face each other. Find the flux through a circle of radius 3 cm between the plates when the normal to the circle makes an angle of 5° with a line perpendicular to the plates. Note that this angle can also be given as $180^\circ + 5^\circ$.
25. A square surface of area 2cm^2 is in a space of uniform electric field of magnitude 10^3 N/C . The amount of flux through it depends on how the square is oriented relative to the direction of the electric field. Find the electric flux through the square, when the normal to it makes the following angles with electric field: (a) 30° , (b) 90° , and (c) 0° . Note that these angles can also be given as $180^\circ + \theta$.
26. A vector field is pointed along the z -axis, $\vec{v} = \frac{\alpha}{x^2 + y^2} \hat{z}$.
 - (a) Find the flux of the vector field through a rectangle in the xy -plane between $a < x < b$ and $c < y < d$.
 - (b) Do the same through a rectangle in the yz -plane between $a < z < b$ and $c < y < d$. (Leave your answer as an integral.)
27. Consider the uniform electric field $\vec{E} = (4.0\hat{j} + 3.0\hat{k}) \times 10^3 \text{ N/C}$. What is its electric flux through a circular area of radius 2.0 m that lies in the xy -plane?
28. Repeat the previous problem, given that the circular area is (a) in the yz -plane and (b) 45° above the xy -plane.
29. An infinite charged wire with charge per unit length λ lies along the central axis of a cylindrical surface of radius r and length l . What is the flux through the surface due to the electric field of the charged wire?

6.3 Explaining Gauss's Law

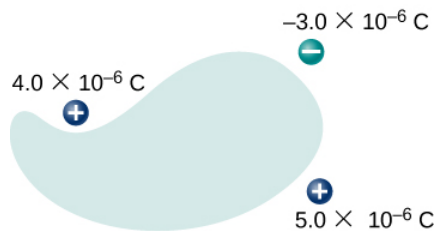
30. Determine the electric flux through each closed surface whose cross-section inside the surface is shown below.



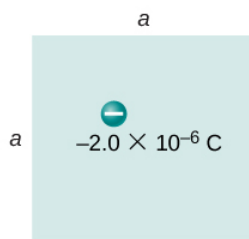
31. Find the electric flux through the closed surface whose cross-sections are shown below.



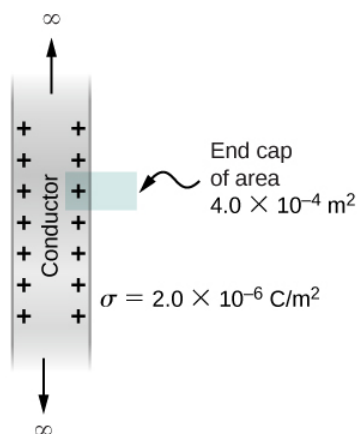
(a)



(b)



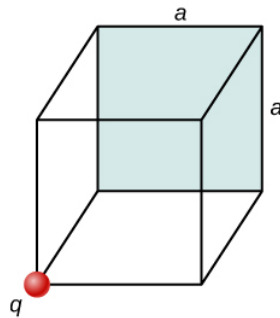
(c)



(d)

32. A point charge q is located at the center of a cube whose sides are of length a . If there are no other charges in this system, what is the electric flux through one face of the cube?
33. A point charge of $10\mu\text{C}$ is at an unspecified location inside a cube of side 2 cm. Find the net electric flux through the surfaces of the cube.
34. A net flux of $1.0 \times 10^4 \text{ N} \cdot \text{m}^2/\text{C}$ passes inward through the surface of a sphere of radius 5 cm.
- How much charge is inside the sphere?
 - How precisely can we determine the location of the charge from this information?

35. A charge q is placed at one of the corners of a cube of side a , as shown below. Find the magnitude of the electric flux through the shaded face due to q . Assume $q > 0$.



36. The electric flux through a cubical box 8.0 cm on a side is $1.2 \times 10^3 \text{ N} \cdot \text{m}^2 / \text{C}$. What is the total charge enclosed by the box?
37. The electric flux through a spherical surface is $4.0 \times 10^4 \text{ N} \cdot \text{m}^2 / \text{C}$. What is the net charge enclosed by the surface?
38. A cube whose sides are of length d is placed in a uniform electric field of magnitude $E = 4.0 \times 10^3 \text{ N/C}$ so that the field is perpendicular to two opposite faces of the cube. What is the net flux through the cube?
39. Repeat the previous problem, assuming that the electric field is directed along a body diagonal of the cube.
40. A total charge $5.0 \times 10^{-6} \text{ C}$ is distributed uniformly throughout a cubical volume whose edges are 8.0 cm long.
- What is the charge density in the cube?
 - What is the electric flux through a cube with 12.0-cm edges that is concentric with the charge distribution?
 - Do the same calculation for cubes whose edges are 10.0 cm long and 5.0 cm long.
 - What is the electric flux through a spherical surface of radius 3.0 cm that is also concentric with the charge distribution?

6.4 Applying Gauss's Law

41. Recall that in the example of a uniform charged sphere, $\rho_0 = Q / (\frac{4}{3}\pi R^3)$. Rewrite the answers in terms of the total charge Q on the sphere.
42. Suppose that the charge density of the spherical charge distribution shown in Figure 6.23 is $\rho(r) = \rho_0 r / R$ for $r \leq R$ and zero for $r > R$. Obtain expressions for the electric field both inside and outside the distribution.
43. A very long, thin wire has a uniform linear charge density of $50 \mu\text{C}/\text{m}$. What is the electric field at a distance 2.0 cm from the wire?
44. A charge of $-30 \mu\text{C}$ is distributed uniformly throughout a spherical volume of radius 10.0 cm. Determine the electric field due to this charge at a distance of
- 2.0 cm,
 - 5.0 cm, and
 - 20.0 cm from the center of the sphere.
45. Repeat your calculations for the preceding problem, given that the charge is distributed uniformly over the surface of a spherical conductor of radius 10.0 cm.
46. A total charge Q is distributed uniformly throughout a spherical shell of inner and outer radii r_1 and r_2 , respectively. Show that the electric field due to the charge is

$$\vec{E} = \vec{0} \quad (r \leq r_1);$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3} \right) \hat{r} \quad (r_1 \leq r \leq r_2);$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r} \quad (r \geq r_2).$$

47. When a charge is placed on a metal sphere, it ends up in equilibrium at the outer surface. Use this information to determine the electric field of $+3.0\mu\text{C}$ charge put on a 5.0-cm aluminum spherical ball at the following two points in space:

- (a) a point 1.0 cm from the center of the ball (an inside point) and
- (b) a point 10 cm from the center of the ball (an outside point).

48. A large sheet of charge has a uniform charge density of $10\mu\text{C}/\text{m}^2$. What is the electric field due to this charge at a point just above the surface of the sheet?

49. Determine if approximate cylindrical symmetry holds for the following situations. State why or why not.

- (a) A 300-cm long copper rod of radius 1 cm is charged with $+500\text{ nC}$ of charge and we seek electric field at a point 5 cm from the center of the rod.
- (b) A 10-cm long copper rod of radius 1 cm is charged with $+500\text{ nC}$ of charge and we seek electric field at a point 5 cm from the center of the rod.
- (c) A 150-cm wooden rod is glued to a 150-cm plastic rod to make a 300-cm long rod, which is then painted with a charged paint so that one obtains a uniform charge density. The radius of each rod is 1 cm, and we seek an electric field at a point that is 4 cm from the center of the rod.
- (d) Same rod as (c), but we seek electric field at a point that is 500 cm from the center of the rod.

50. A long silver rod of radius 3 cm has a charge of $-5\mu\text{C}/\text{cm}$ on its surface.

- (a) Find the electric field at a point 5 cm from the center of the rod (an outside point).
- (b) Find the electric field at a point 2 cm from the center of the rod (an inside point).

51. The electric field at 2 cm from the center of long copper rod of radius 1 cm has a magnitude 3 N/C and directed outward from the axis of the rod.

- (a) How much charge per unit length exists on the copper rod?
- (b) What would be the electric flux through a cube of side 5 cm situated such that the rod passes through opposite sides of the cube perpendicularly?

52. A long copper cylindrical shell of inner radius 2 cm and outer radius 3 cm surrounds concentrically a charged long aluminum rod of radius 1 cm with a charge density of 4 pC/m. All charges on the aluminum rod reside at its surface. The inner surface of the copper shell has exactly opposite charge to that of the aluminum rod while the outer surface of the copper shell has the same charge as the aluminum rod. Find the magnitude and direction of the electric field at points that are at the following distances from the center of the aluminum rod:

- (a) 0.5 cm, (b) 1.5 cm, (c) 2.5 cm, (d) 3.5 cm, and (e) 7 cm.

53. Charge is distributed uniformly with a density ρ throughout an infinitely long cylindrical volume of radius R . Show that the field of this charge distribution is directed radially with respect to the cylinder and that

$$E = \frac{\rho r}{2\epsilon_0} \quad (r \leq R);$$

$$E = \frac{\rho R^2}{2\epsilon_0 r} \quad (r \geq R)$$

54. Charge is distributed throughout a very long cylindrical volume of radius R such that the charge density increases with the distance r from the central axis of the cylinder according to $\rho = \alpha r$, where α is a constant. Show that the field of this charge distribution is directed radially with respect to the cylinder and that

$$E = \frac{\alpha r^2}{3\epsilon_0} \quad (r \leq R);$$

$$E = \frac{\alpha R^3}{3\epsilon_0 r} \quad (r \geq R).$$

55. The electric field 10.0 cm from the surface of a copper ball of radius 5.0 cm is directed toward the ball's center and has magnitude $4.0 \times 10^2 \text{ N/C}$. How much charge is on the surface of the ball?
56. Charge is distributed throughout a spherical shell of inner radius r_1 and outer radius r_2 with a volume density given by $\rho = \rho_0 r_1/r$, where ρ_0 is a constant. Determine the electric field due to this charge as a function of r , the distance from the center of the shell.
57. Charge is distributed throughout a spherical volume of radius R with a density $\rho = \alpha r^2$, where α is a constant. Determine the electric field due to the charge at points both inside and outside the sphere.
58. Consider a uranium nucleus to be sphere of radius $R = 7.4 \times 10^{-15} \text{ m}$ with a charge of $92e$ distributed uniformly throughout its volume. (a) What is the electric force exerted on an electron when it is $3.0 \times 10^{-15} \text{ m}$ from the center of the nucleus? (b) What is the acceleration of the electron at this point?
59. The volume charge density of a spherical charge distribution is given by $\rho(r) = \rho_0 e^{-\alpha r}$, where ρ_0 and α are constants. What is the electric field produced by this charge distribution?

6.5 Conductors in Electrostatic Equilibrium

60. An uncharged conductor with an internal cavity is shown in the following figure. Use the closed surface S along with Gauss' law to show that when a charge q is placed in the cavity a total charge $-q$ is induced on the inner surface of the conductor. What is the charge on the outer surface of the conductor?

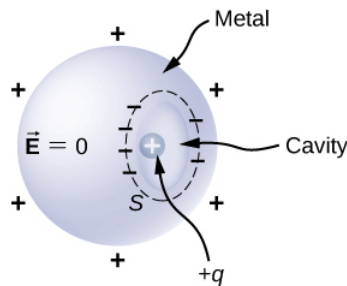
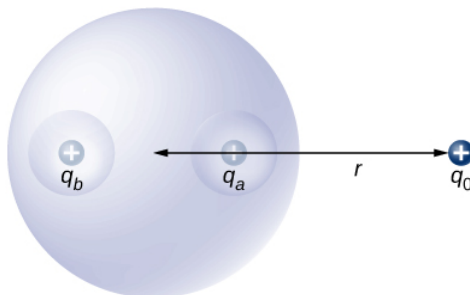
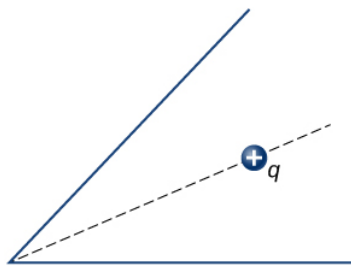


Figure 6.46: A charge inside a cavity of a metal. Charges at the outer surface do not depend on how the charges are distributed at the inner surface since E field inside the body of the metal is zero.

61. An uncharged spherical conductor S of radius R has two spherical cavities A and B of radii a and b , respectively as shown below. Two point charges $+q_a$ and $+q_b$ are placed at the center of the two cavities by using non-conducting supports. In addition, a point charge $+q_0$ is placed outside at a distance r from the center of the sphere.
- (a) Draw approximate charge distributions in the metal although metal sphere has no net charge.
- (b) Draw electric field lines. Draw enough lines to represent all distinctly different places.



62. A positive point charge is placed at the angle bisector of two uncharged plane conductors that make an angle of 45° . See below. Draw the electric field lines.



63. A long cylinder of copper of radius 3 cm is charged so that it has a uniform charge per unit length on its surface of 3 C/m. (a) Find the electric field inside and outside the cylinder. (b) Draw electric field lines in a plane perpendicular to the rod.

64. An aluminum spherical ball of radius 4 cm is charged with $5\mu\text{C}$ of charge. A copper spherical shell of inner radius 6 cm and outer radius 8 cm surrounds it. A total charge of $-8\mu\text{C}$ is put on the copper shell.

(a) Find the electric field at all points in space, including points inside the aluminum and copper shell when copper shell and aluminum sphere are concentric.

(b) Find the electric field at all points in space, including points inside the aluminum and copper shell when the centers of copper shell and aluminum sphere are 1 cm apart.

65. A long cylinder of aluminum of radius R meters is charged so that it has a uniform charge per unit length on its surface of λ . (a) Find the electric field inside and outside the cylinder. (b) Plot electric field as a function of distance from the center of the rod.

66. At the surface of any conductor in electrostatic equilibrium, $E = \sigma/\epsilon_0$. Show that this equation is consistent with the fact that $E = kq/r^2$ at the surface of a spherical conductor.

67. Two parallel plates 10 cm on a side are given equal and opposite charges of magnitude $5.0 \times 10^{-9} \text{ C}$. The plates are 1.5 mm apart. What is the electric field at the center of the region between the plates?

68. Two parallel conducting plates, each of cross-sectional area 400 cm^2 , are 2.0 cm apart and uncharged. If 1.0×10^{12} electrons are transferred from one plate to the other, what are (a) the charge density on each plate? (b) The electric field between the plates?

69. The surface charge density on a long straight metallic pipe is σ . What is the electric field outside and inside the pipe? Assume the pipe has a diameter of 2a.



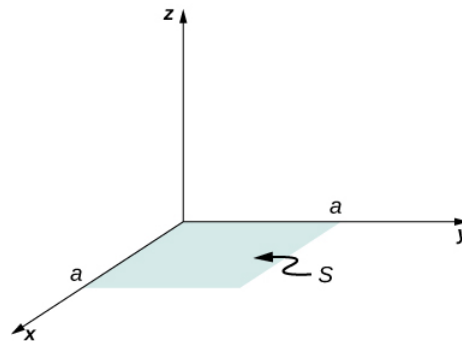
70. A point charge $q = -5.0 \times 10^{-12} \text{ C}$ is placed at the center of a spherical conducting shell of inner radius 3.5 cm and outer radius 4.0 cm. The electric field just above the surface of the conductor is directed radially outward and has magnitude 8.0 N/C.

- What is the charge density on the inner surface of the shell?
- What is the charge density on the outer surface of the shell?
- What is the net charge on the conductor?

71. A solid cylindrical conductor of radius a is surrounded by a concentric cylindrical shell of inner radius b . The solid cylinder and the shell carry charges $+Q$ and $-Q$, respectively. Assuming that the length L of both conductors is much greater than a or b , determine the electric field as a function of r , the distance from the common central axis of the cylinders, for (a) $r < a$; (b) $a < r < b$; and (c) $r > b$.

Additional Problems

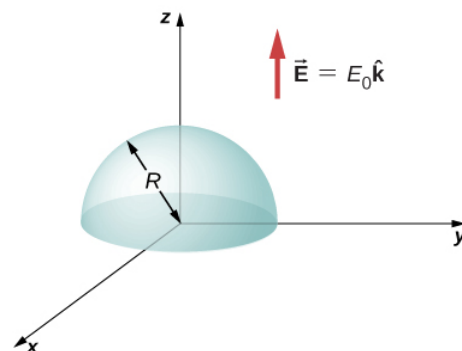
72. A vector field \vec{E} (not necessarily an electric field; note units) is given by $\vec{E} = 3x^2 \hat{k}$. Calculate $\int_S \vec{E} \cdot \hat{n} da$, where S is the area shown below. Assume that $\hat{n} = \hat{k}$.



73. Repeat the preceding problem, with $\vec{E} = 2x \hat{i} + 3x^2 \hat{k}$.

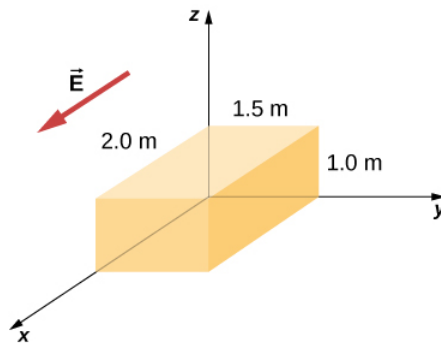
74. A circular area S is concentric with the origin, has radius a , and lies in the yz -plane. Calculate $\int_S \vec{E} \cdot \hat{n} dA$ for $\vec{E} = 3z^2 \hat{i}$.

- Calculate the electric flux through the open hemispherical surface due to the electric field $\vec{E} = E_0 \hat{k}$ (see below).
- If the hemisphere is rotated by 90° around the x -axis, what is the flux through it?

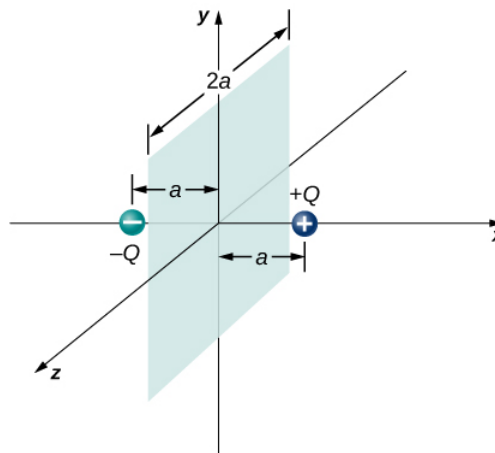


76. Suppose that the electric field of an isolated point charge were proportional to $1/r^{2+\sigma}$ rather than $1/r^2$. Determine the flux that passes through the surface of a sphere of radius R centered at the charge. Would Gauss's law remain valid?

77. The electric field in a region is given by $\vec{E} = a/(b + cx) \hat{i}$, where $a = 200 \text{ N} \cdot \text{m}/\text{C}$, $b = 2.0 \text{ m}$, and $c = 2.0$. What is the net charge enclosed by the shaded volume shown below?



78. Two equal and opposite charges of magnitude Q are located on the x -axis at the points $+a$ and $-a$, as shown below. What is the net flux due to these charges through a square surface of side $2a$ that lies in the yz -plane and is centered at the origin? (Hint: Determine the flux due to each charge separately, then use the principle of superposition. You may be able to make a symmetry argument.)



79. A fellow student calculated the flux through the square for the system in the preceding problem and got 0. What went wrong?

80. A $10\text{cm} \times 10\text{cm}$ piece of aluminum foil of 0.1 mm thickness has a charge of $20\mu\text{C}$ that spreads on both wide side surfaces evenly. You may ignore the charges on the thin sides of the edges.

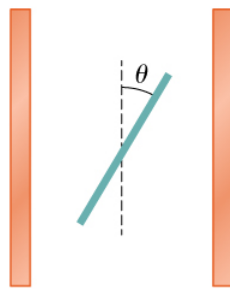
- Find the charge density.
- Find the electric field 1 cm from the center, assuming approximate planar symmetry.

81. Two $10\text{cm} \times 10\text{cm}$ pieces of aluminum foil of thickness 0.1 mm face each other with a separation of 5 mm. One of the foils has a charge of $+30\mu\text{C}$ and the other has $-30\mu\text{C}$.

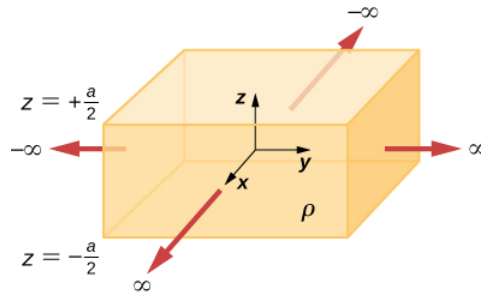
- Find the charge density at all surfaces, i.e., on those facing each other and those facing away.
- Find the electric field between the plates near the center assuming planar symmetry.

82. Two large copper plates facing each other have charge densities $\pm 4.0\text{C}/\text{m}^2$ on the surface facing the other plate, and zero in between the plates. Find the electric flux through a $3\text{cm} \times 4\text{cm}$ rectangular area between the plates, as shown below, for the following orientations of the area.

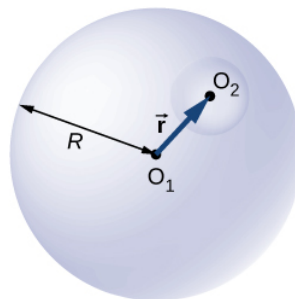
- If the area is parallel to the plates, and
- if the area is tilted $\theta = 30^\circ$ from the parallel direction. Note, this angle can also be $\theta = 180^\circ + 30^\circ$.



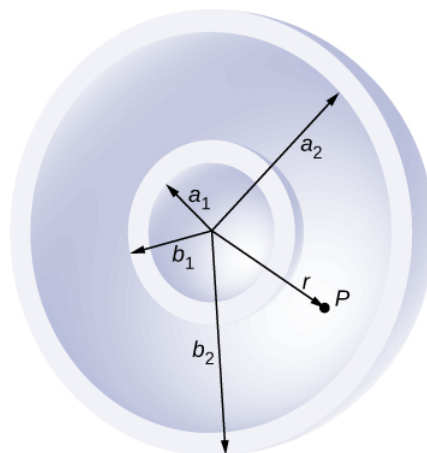
83. The infinite slab between the planes defined by $z = -a/2$ and $z = a/2$ contains a uniform volume charge density ρ (see below). What is the electric field produced by this charge distribution, both inside and outside the distribution?



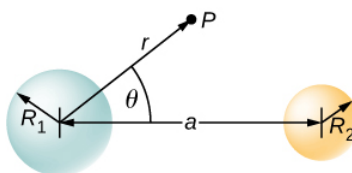
84. A total charge Q is distributed uniformly throughout a spherical volume that is centered at O_1 and has a radius R . Without disturbing the charge remaining, charge is removed from the spherical volume that is centered at O_2 (see below). Show that the electric field everywhere in the empty region is given by $\vec{E} = \frac{Q\vec{r}}{4\pi\epsilon_0 R^3}$, where \vec{r} is the displacement vector directed from O_1 to O_2 .



85. A non-conducting spherical shell of inner radius a_1 and outer radius b_1 is uniformly charged with charge density ρ_1 inside another non-conducting spherical shell of inner radius a_2 and outer radius b_2 that is also uniformly charged with charge density ρ_2 . See below. Find the electric field at space point P at a distance r from the common center such that (a) $r > b_2$, (b) $a_2 < r < b_2$, (c) $b_1 < r < a_2$, (d) $a_1 < r < b_1$, and (e) $r < a_1$.

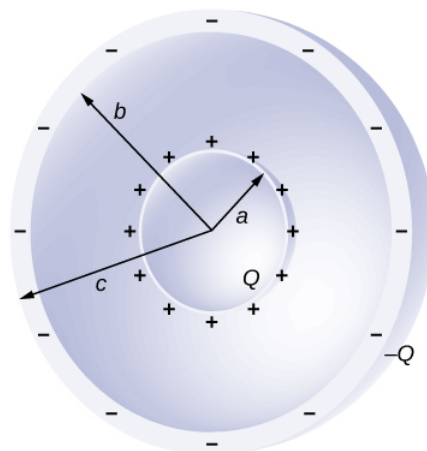


86. Two non-conducting spheres of radii R_1 and R_2 are uniformly charged with charge densities ρ_1 and ρ_2 , respectively. They are separated at center-to-center distance a (see below). Find the electric field at point P located at a distance r from the center of sphere 1 and is in the direction θ from the line joining the two spheres assuming their charge densities are not affected by the presence of the other sphere. (**Hint:** Work one sphere at a time and use the superposition principle.)

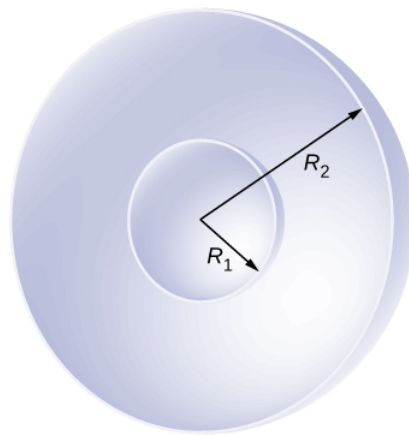


87. A disk of radius R is cut in a non-conducting large plate that is uniformly charged with charge density σ (coulomb per square meter). See below. Find the electric field at a height h above the center of the disk. ($h \gg R, h \ll l$ or w). (**Hint:** Fill the hole with $\pm\sigma$.)

88. Concentric conducting spherical shells carry charges Q and $-Q$, respectively (see below). The inner shell has negligible thickness. Determine the electric field for (a) $r < a$; (b) $a < r < b$; (c) $b < r < c$; and (d) $r > c$.



89. Shown below are two concentric conducting spherical shells of radii R_1 and R_2 , each of finite thickness much less than either radius. The inner and outer shell carry net charges q_1 and q_2 , respectively, where both q_1 and q_2 are positive. What is the electric field for (a) $r < R_1$; (b) $R_1 < r < R_2$; and (c) $r > R_2$? (d) What is the net charge on the inner surface of the inner shell, the outer surface of the inner shell, the inner surface of the outer shell, and the outer surface of the outer shell?



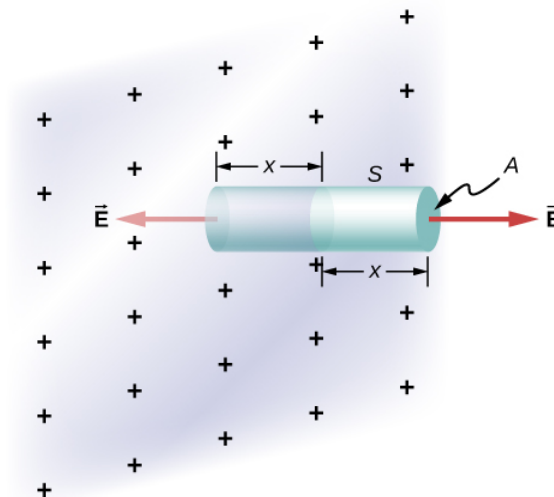
90. A point charge of $q=5.0 \times 10^{-8} \text{ C}$ is placed at the center of an uncharged spherical conducting shell of inner radius 6.0 cm and outer radius 9.0 cm. Find the electric field at (a) $r=4.0 \text{ cm}$, (b) $r=8.0 \text{ cm}$, and (c) $r=12.0 \text{ cm}$. (d) What are the charges induced on the inner and outer surfaces of the shell?

Challenge Problems

91. The Hubble Space Telescope can measure the energy flux from distant objects such as supernovae and stars. Scientists then use this data to calculate the energy emitted by that object. Choose an interstellar object which scientists have observed the flux at the Hubble with (for example, *Vega*), find the distance to that object and the size of Hubble's primary mirror, and calculate the total energy flux. (Hint: The Hubble intercepts only a small part of the total flux.)

92. Re-derive Gauss's law for the gravitational field, with \vec{g} directed positively outward.

93. An infinite plate sheet of charge of surface charge density σ is shown below. What is the electric field at a distance x from the sheet? Compare the result of this calculation with that of worked out in the text.



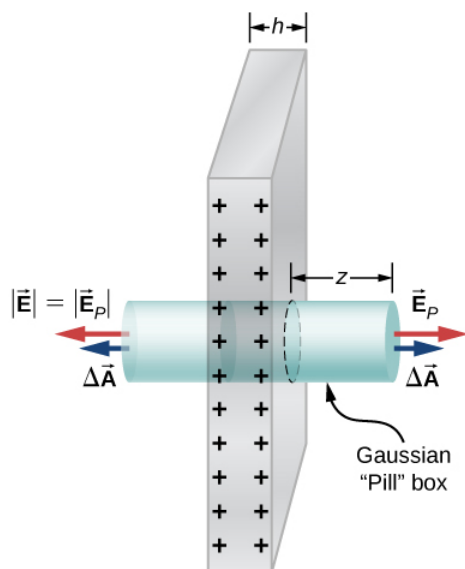
94. A spherical rubber balloon carries a total charge Q distributed uniformly over its surface. At $t = 0$, the radius of the balloon is R . The balloon is then slowly inflated until its radius reaches $2R$ at the time t_0 . Determine the electric field due to this charge as a function of time

(a) at the surface of the balloon,

(b) at the surface of radius R , and

(c) at the surface of radius $2R$. Ignore any effect on the electric field due to the material of the balloon and assume that the radius increases uniformly with time.

95. Find the electric field of a large conducting plate containing a net charge q . Let A be area of one side of the plate and h the thickness of the plate (see below). The charge on the metal plate will distribute mostly on the two planar sides and very little on the edges if the plate is thin.



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