6.8: Gauss's Law (Answers)

Check Your Understanding

6.1. Place it so that its unit normal is perpendicular to $\vec{E}$.

6.2. $mab^2/2$

6.3 a. $3.4 \times 10^5 N \cdot m^2/C$;

b. $-3.4 \times 10^5 N \cdot m^2/C$;

c. $3.4 \times 10^5 N \cdot m^2/C$;

d. 0

6.4. In this case, there is only $\vec{E}_{out}$. So,yes.

6.5. $\vec{E} = \frac{\lambda_0}{2\pi\varepsilon_0}\frac{1}{d}\hat{r}$; This agrees with the calculation of Example 5.5 where we found the electric field by integrating over the charged wire. Notice how much simpler the calculation of this electric field is with Gauss's law.

6.6. If there are other charged objects around, then the charges on the surface of the sphere will not necessarily be spherically symmetrical; there will be more in certain direction than in other directions.

Conceptual Questions

1. a. If the planar surface is perpendicular to the electric field vector, the maximum flux would be obtained. b. If the planar surface were parallel to the electric field vector, the minimum flux would be obtained.

3. true

5. Since the electric field vector has a $\frac{1}{r^2}$ dependence, the fluxes are the same since $A = 4\pi r^2$.

7. a. no;

b. zero

9. Both fields vary as $\frac{1}{r^2}$. Because the gravitational constant is so much smaller than $\frac{1}{4\pi\varepsilon_0}$, the gravitational field is orders of magnitude weaker than the electric field.

11. No, it is produced by all charges both inside and outside the Gaussian surface.

13. No, since the situation does not have symmetry, making Gauss's law challenging to simplify.

15. Any shape of the Gaussian surface can be used. The only restriction is that the Gaussian integral must be calculable; therefore, a box or a cylinder are the most convenient geometrical shapes for the Gaussian surface.

17. yes

19. Since the electric field is zero inside a conductor, a charge of $-2.0\mu C$ is induced on the inside surface of the cavity. This will put a charge of $+2.0\mu C$ on the outside surface leaving a net charge of $-3.0\mu C$ on the surface.

Problems

21. $\Phi = \vec{E} \cdot \vec{A} \rightarrow EAcos\theta = 2.2 \times 10^4 N \cdot m^2/C$ electric field in direction of unit normal; $\Phi = \vec{E} \cdot \vec{A} \rightarrow EAcos\theta = -2.2 \times 10^4 N \cdot m^2/C$ electric field opposite to unit normal

23. $\frac{3 \times 10^{-5} N \cdot m^2/C}{(0.05m)^2} = E \Rightarrow \sigma = 2.12 \times 10^{-13} C/m^2$

25. a. $\Phi = 0.17 N \cdot m^2/C$;

b. $\Phi = 0$;

c. $\Phi = EAcos0^\circ = 1.0 \times 10^3 N/C (2.0 \times 10^{-4} m)^2 cos0^\circ = 0.20 N \cdot m^2/C$

27. $\Phi = 3.8 \times 10^4 N \cdot m^2/C$

29. $\vec{E}(z) = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}\hat{k}, \int \vec{E} \cdot \hat{n} dA = \frac{\lambda}{\varepsilon_0} l$

31. a. $\Phi = 3.39 \times 10^3 N \cdot m^2/C$;

b. $\Phi = 0$;

c. $\Phi = -2.25 \times 10^5 N \cdot m^2/C$;

d. $\Phi = 90.4 N \cdot m^2/C$

33. $\Phi = 1.13 \times 10^6 N \cdot m^2/C$

35. Make a cube with **q** at the center, using the cube of side a. This would take four cubes of side a to make one side of the large cube. The shaded side of the small cube would be 1/24th of the total area of the large cube; therefore, the flux through the shaded area would be $\Phi = \frac{1}{24}\frac{q}{\varepsilon_0}$.

37. $q = 3.54 \times 10^{-7} C$

39. zero, also because flux in equals flux out

41. $r > R, E = \frac{Q}{4\pi\varepsilon_0 r^2}; r < R, E = \frac{qr}{4\pi\varepsilon_0 R^3}$

43. $EA = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = 4.50 \times 10^7 N/C$

45. a. 0;

b. 0;

c. $\vec{E} = 6.74 \times 10^6 N/C(-\hat{r})$

47. a. 0;

b. $E = 2.70 \times 10^6 N/C$

49. a. Yes, the length of the rod is much greater than the distance to the point in question.

b. No, The length of the rod is of the same order of magnitude as the distance to the point in question.

c. Yes, the length of the rod is much greater than the distance to the point in question.

d. No. The length of the rod is of the same order of magnitude as the distance to the point in question.

51. a. $\vec{E} = \frac{R\sigma_0}{\varepsilon_0}\frac{1}{r}\hat{r} \Rightarrow \sigma_0 = 5.31 \times 10^{-11} C/m^2, \lambda = 3.33 \times 10^{-12} C/m$;

b. $\Phi = \frac{q_{enc}}{\varepsilon_0} = \frac{3.33 \times 10^{-12} C/m(0.05m)}{\varepsilon + 0} = 0.019 N \cdot m^2/C$

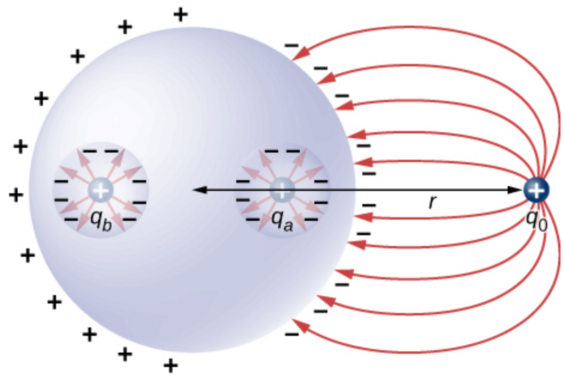
53. $E2\pi rl = \frac{\rho\pi r^2 l}{\varepsilon_0} \Rightarrow E = \frac{\rho r}{2\varepsilon_0}(r \leq R); E2\pi rl = \frac{\rho\pi R^2 l}{\varepsilon_0} \Rightarrow E = \frac{\rho R^2}{2\varepsilon_0 r}(r \geq R)$

55. $\Phi = \frac{q_{enc}}{\varepsilon_0} \Rightarrow q_{enc} = -1.0 \times 10^{-9} C$

57. $q_{enc} = \frac{4}{5}\pi\alpha r^5, E4\pi r^2 = \frac{4\pi\alpha r^5}{5\varepsilon_0} \Rightarrow E = \frac{\alpha r^3}{5\varepsilon_0}(r \leq R), q_{enc} = \frac{4}{5}\pi\alpha R^5, E4\pi r^2 = \frac{4\pi\alpha R^5}{5\varepsilon_0} \Rightarrow E = \frac{\alpha R^5}{5\varepsilon_0 r^2}(r \geq R)$

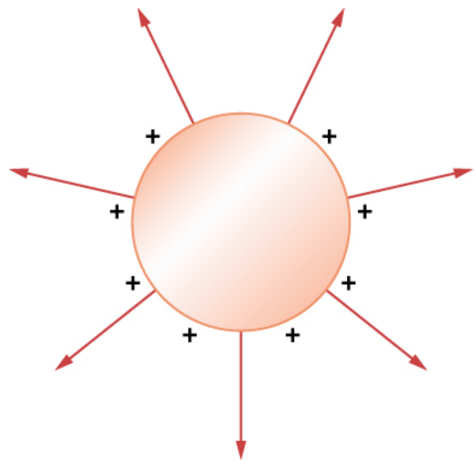
59. integrate by parts: $q_{enc} = 4\pi\rho_0[-e^{-\alpha r}(\frac{(r)^2}{\alpha} + \frac{2r}{\alpha^2} + \frac{2}{\alpha^3}) + \frac{2}{\alpha^3}] \Rightarrow E = \frac{\rho_0}{r^2\varepsilon_0}[-e^{-\alpha r}(\frac{(r)^2}{\alpha} + \frac{2r}{\alpha^2} + \frac{2}{\alpha^3}) + \frac{2}{\alpha^3}]$

61.



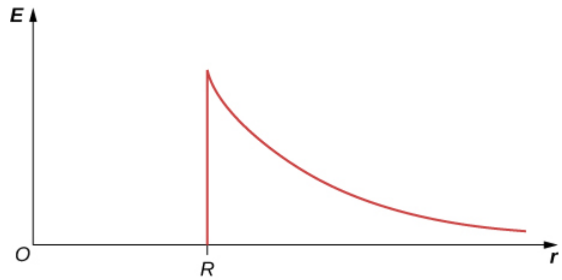
63. a. Outside: $E2\pi rl = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{3.0C/m}{2\pi\varepsilon_0 r}$; Inside $E_{in} = 0$;

b.



65. a. $E2\pi rl = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r} r \geq R$ **E** inside equals 0;

b.



67. $E = 5.65 \times 10^4 N/C$

69. $\lambda = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{a\sigma}{\varepsilon_0 r} r \geq a, E = 0$ inside since q **enclosed**=0

71. a. $E = 0$;

b. $E2\pi rL = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{2\pi\varepsilon_0 rL}$; c. $E = 0$ since **r** would be either inside the second shell or if outside then **q** enclosed equals 0.

Additional Problems

73. $\int \vec{E}\cdot\hat{n}dA = a^4$

75. a. $\int \vec{E}\cdot\hat{n}dA = E_0 r^2\pi$; b. zero, since the flux through the upper half cancels the flux through the lower half of the sphere

77. $\Phi = \frac{q_{enc}}{\varepsilon_0}$; There are two contributions to the surface integral: one at the side of the rectangle at $x=0$ and the other at the side at $x=2.0m$; $-E(0)[1.5m^2]+E(2.0m)[1.5m^2] = \frac{q_{enc}}{\varepsilon_0} = -100Nm2/C$

where the minus sign indicates that at $x=0$, the electric field is along positive **x** and the unit normal is along negative **x**. At $x=2$, the unit normal and the electric field vector are in the same direction: $q_{enc} = \varepsilon_0\Phi = -8.85\times10^{-10}C$

79. didn't keep consistent directions for the area vectors, or the electric fields

81. a. $\sigma = 3.0\times10^{-3}C/m^2, +3\times10^{-3}C/m^2$ on one and $-3\times10^{-3}C/m^2$ on the other;

b. $E = 3.39\times10^8 N/CE = 3.39\times10^8 N/C$

83. Construct a Gaussian cylinder along the **z**-axis with cross-sectional area **A.**

$|z| \geq \frac{a}{2} q_{enc} = \rho Aa, \Phi = \frac{\rho Aa}{\varepsilon_0} \Rightarrow E = \frac{\rho a}{2\varepsilon_0}$,

$|z| \leq \frac{a}{2} q_{enc} = \rho A2z, E(2A) = \frac{\rho A2z}{\varepsilon_0} \Rightarrow E = \frac{\rho z}{\varepsilon_0}$

85. a. $r > b_2\ \ E4\pi r^2 = \frac{\frac{4}{3}\pi[\rho_1(b_1^3-a_1^3)+\rho_2(b_2^3-a_2^3)}{\varepsilon_0} \Rightarrow E = \frac{\rho_1(b_1^3-a_1^3)+\rho_2(b_2^3-a_2^3)}{3\varepsilon_0 r^2}$;

b. $a_2 < r < b_2\ \ E4\pi r^2 = \frac{\frac{4}{3}\pi[\rho_1(b_1^3-a_1^3)+\rho_2(r^3-a_2^3)]}{\varepsilon_0} \Rightarrow E = \frac{\rho_1(b_1^3-a_1^3)+\rho_2(r^3-a_2^3)}{3\varepsilon_0 r^2}$;

c. $b_1 < r < a_2\ \ E4\pi r^2 = \frac{\frac{4}{3}\pi\rho_1(b_1^3-a_1^3)}{\varepsilon_0} \Rightarrow E = \frac{\rho_1(b_1^3-a_1^3)}{3\varepsilon_0 r^2}$;

d. $a_1 < r < b_1\ \ E4\pi r^2 = \frac{\frac{4}{3}\pi\rho_1(r^3_- a_1^3)}{\varepsilon_0} \Rightarrow E = \frac{\rho_1(r^3-a_1^3)}{3\varepsilon_0 r^2}$;

e. 0

87. Electric field due to plate without hole: $E = \frac{\sigma}{2\varepsilon_0}$.

Electric field of just hole filled with \(\displaystyle −σE=\frac{−σ}{2ε_0}(1−\frac{z}{\sqrt{R^2+z^2}}).

Thus, $E_{net} = \frac{\sigma}{2\varepsilon_0}\frac{h}{\sqrt{R^2+h^2}}$

89. a. $E=0$; b. $E = \frac{q_1}{4\pi\varepsilon_0 r^2}$; c. $E = \frac{q_1+q_2}{4\pi\varepsilon_0 r^2}$; d. 0 $q_1 - q_1, q_1+q_2$

Challenge Problems

91. Given the referenced link, using a distance to Vega of $237\times10^{15}m$[4] and a diameter of 2.4 m for the primary mirror,[5] we find that at a wavelength of 555.6 nm, Vega is emitting $2.44\times10^{24}J/s$ at that wavelength. Note that the flux through the mirror is essentially constant.

93. The symmetry of the system forces $\vec{E}$ to be perpendicular to the sheet and constant over any plane parallel to the sheet. To calculate the electric field, we choose the cylindrical Gaussian surface shown. The cross-section area and the height of the cylinder are **A** and **2x**, respectively, and the cylinder is positioned so that it is bisected by the plane sheet. Since **E** is perpendicular to each end and parallel to the side of the cylinder, we have **EA** as the flux through each end and there is no flux through the side. The charge enclosed by the cylinder is σA, so from Gauss's law, $2EA = \frac{\sigma A}{\varepsilon_0}$, and the electric field of an infinite sheet of charge is

$E = \frac{\sigma}{2\varepsilon_0}$, in agreement with the calculation of in the text.

95. There is **Q/2** on each side of the plate since the net charge is $Q : \sigma = \frac{Q}{2A}$,

$$\oint_S \vec{E} \cdot \hat{n} dA = \frac{2\sigma \Delta A}{\varepsilon_0} \Rightarrow E_P = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 2A}$$