14.8: Inductance (Summary)

Key Terms

henry (H)	unit of inductance, $1H = 1\Omega \cdot s$; it is also expressed as a volt second per ampere
inductance	property of a device that tells how effectively it induces an emf in another device
inductive time constant	denoted by τ, the characteristic time given by quantity L/R of a particular series RL circuit
inductor	part of an electrical circuit to provide self-inductance, which is symbolized by a coil of wire
LC circuit	circuit composed of an ac source, inductor, and capacitor
magnetic energy density	energy stored per volume in a magnetic field
mutual inductance	geometric quantity that expresses how effective two devices are at inducing emfs in one another
RLC circuit	circuit with an ac source, resistor, inductor, and capacitor all in series.
self-inductance	effect of the device inducing emf in itself

Key Equations

Mutual inductance by flux	$M = \frac{N_2\Phi_2}{1_I} = \frac{N_1\Phi_{12}}{I_2}$
Mutual inductance in circuits	$\varepsilon_1 = -M\frac{dI_2}{dt}$
Self-inductance in terms of magnetic flux	$N\Phi_m = LI$
Self-inductance in terms of emf	$\varepsilon = -L\frac{dI}{dt}$
Self-inductance of a solenoid	$L_{solenoid} = \frac{\mu_0 N^2 A}{l}$
Self-inductance of a toroid	$L_{toroid} = \frac{\mu_0 N^2 h}{2\pi} ln\frac{R_2}{R_1}.$
Energy stored in an inductor	$U = \frac{1}{2}LI^2$
Current as a function of time for a **RL** circuit	$I(t) = \frac{\varepsilon}{R}(1 - e^{-t/\tau_L})$
Time constant for a **RL** circuitτ	$\tau_L = L/R$
Charge oscillation in **LC** circuits	$q(t) = q_0 cos(\omega t + \phi)$
Angular frequency in **LC** circuits	$\omega = \sqrt{\frac{1}{LC}}$
Current oscillations in **LC** circuits	$i(t) = -\omega q_0 sin(\omega t + \phi)$
Charge as a function of time in **RLC** circuit	$q(t) = q_0 e^{-Rt/2L} cos(\omega' t + \phi)$
Angular frequency in **RLC** circuit	$\omega' = \sqrt{\frac{1}{LC} - (\frac{R}{2L})^2}$

Summary

14.2 Mutual Inductance

- Inductance is the property of a device that expresses how effectively it induces an emf in another device.
- Mutual inductance is the effect of two devices inducing emfs in each other.
- A change in current dI_1/dt in one circuit induces an emf (ε_2) in the second:

$$\varepsilon_2 = -M\frac{dI_1}{dt},$$

where **M** is defined to be the mutual inductance between the two circuits and the minus sign is due to Lenz's law.

- Symmetrically, a change in current dI_2/dt through the second circuit induces an emf (ε_1) in the first:

$$\varepsilon_1 = -M\frac{dI_2}{dt},$$

where **M** is the same mutual inductance as in the reverse process.

14.3 Self-Inductance and Inductors

- Current changes in a device induce an emf in the device itself, called self-inductance,

$$\varepsilon = -L\frac{dI}{dt},$$

where **L** is the self-inductance of the inductor and dI/dt is the rate of change of current through it. The minus sign indicates that emf opposes the change in current, as required by Lenz's law. The unit of self-inductance and inductance is the henry (H), where $1H = 1\Omega\cdot s$.

- The self-inductance of a solenoid is

$$L = \frac{\mu_0 N^2 A}{l},$$

where **N** is its number of turns in the solenoid, **A** is its cross-sectional area, **l** is its length, and $\mu_0 = 4\pi\times10^{-7}T\cdot m/A$ is the permeability of free space.

- The self-inductance of a toroid is

$$L = \frac{\mu_0 N^2 h}{2\pi}ln\frac{R_2}{R_1},$$

where **N** is its number of turns in the toroid, R_1 and R_2 are the inner and outer radii of the toroid, **h** is the height of the toroid, and $\mu_0 = 4\pi\times10^{-7}T\cdot m/A$ is the permeability of free space.

14.4 Energy in a Magnetic Field

- The energy stored in an inductor **U** is

$$U = \frac{1}{2}LI^2.$$

- The self-inductance per unit length of coaxial cable is

$$\frac{L}{l} = \frac{\mu_0}{2\pi}ln\frac{R_2}{R_1}.$$

14.5 RL Circuits

- When a series connection of a resistor and an inductor—an **RL** circuit—is connected to a voltage source, the time variation of the current is

$$I(t) = \frac{\varepsilon}{R}(1-e^{-Rt/L}) = \frac{\varepsilon}{R}(1-e^{-t/\tau_L}) \text{ (turning on)},$$

where the initial current is $I_0 = \varepsilon/R$..

- The characteristic time constant τ is $\tau_L = L/R$, where **L** is the inductance and **R** is the resistance.

- In the first time constant τ, the current rises from zero to $0.632 I_0$, and to 0.632 of the remainder in every subsequent time interval τ.
- When the inductor is shorted through a resistor, current decreases as

  $I(t) = \frac{\varepsilon}{R} e^{-t/\tau_L}$ (turning off).

  Current falls to $0.368 I_0$ in the first time interval τ, and to 0.368 of the remainder toward zero in each subsequent time τ.

14.6 Oscillations in an LC Circuit

- The energy transferred in an oscillatory manner between the capacitor and inductor in an **LC** circuit occurs at an angular frequency $\omega = \sqrt{\frac{1}{LC}}$.
- The charge and current in the circuit are given by

$$q(t) = q_0 cos(\omega t + \phi)\ ,$$

$$i(t) = -\omega q_0 sin(\omega t + \phi)\ .$$

14.7 RLC Series Circuits

- The underdamped solution for the capacitor charge in an **RLC** circuit is

$$q(t) = q_0 e^{-Rt/2L} cos(\omega' t + \phi).$$

- The angular frequency given in the underdamped solution for the **RLC** circuit is

$$\omega' = \sqrt{\frac{1}{LC} - (\frac{R}{2L})^2}\ .$$