14.7: RLC Series Circuits

Learning Objectives

By the end of this section, you will be able to:

- Determine the angular frequency of oscillation for a resistor, inductor, capacitor (**RLC**) series circuit
- Relate the **RLC** circuit to a damped spring oscillation

When the switch is closed in the **RLC** circuit of Figure $14.7.1a$, the capacitor begins to discharge and electromagnetic energy is dissipated by the resistor at a rate i^2R. With **U** given by Equation 14.4.2, we have

$$\frac{dU}{dt} = \frac{q}{C}\frac{dq}{dt} + Li\frac{di}{dt} = -i^2R$$

where **i** and **q** are time-dependent functions. This reduces to

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0. \tag{14.7.1}$$

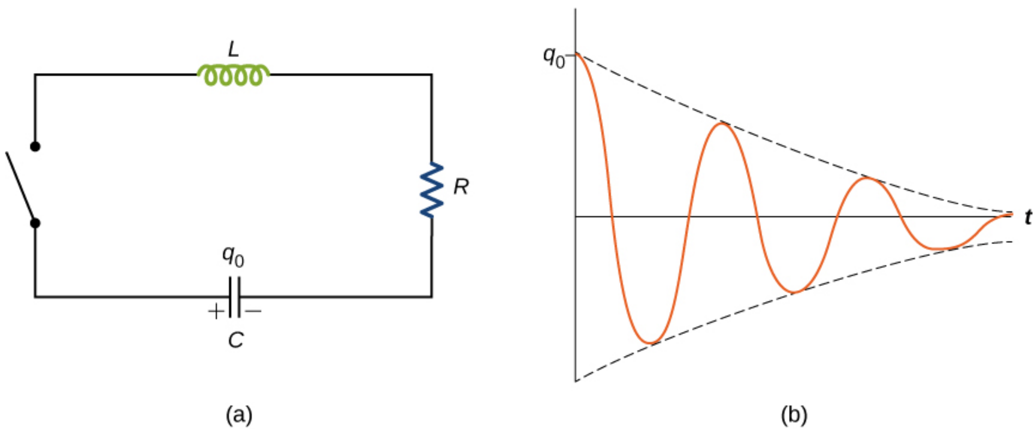

Figure 14.7.1: (a) An **RLC** circuit. Electromagnetic oscillations begin when the switch is closed. The capacitor is fully charged initially. (b) Damped oscillations of the capacitor charge are shown in this curve of charge versus time, or **q** versus **t**. The capacitor contains a charge q_0 before the switch is closed.

This equation is analogous to

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0,$$

which is the equation of motion for a **damped mass-spring system** (you first encountered this equation in Oscillations). As we saw in that chapter, it can be shown that the solution to this differential equation takes three forms, depending on whether the angular frequency of the undamped spring is greater than, equal to, or less than **b**/2**m**. Therefore, the result can be underdamped $(\sqrt{k/m} > b/2m)$, critically damped $(\sqrt{k/m} = b/2m)$, or overdamped $(\sqrt{k/m} < b/2m)$. By analogy, the solution **q(t)** to the **RLC** differential equation has the same feature. Here we look only at the case of under-damping. By replacing **m** by **L**, **b** by **R**, **k** by 1/**C**, and **x** by **q** in Equation 14.7.1, and assuming $\sqrt{1/LC} > R/2L$, we obtain

Note

$$q(t) = q_0 e^{-Rt/2L} cos(\omega' t + \phi) \tag{14.7.2}$$

where the angular frequency of the oscillations is given by

Note

$$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} \tag{14.7.3}$$

This underdamped solution is shown in Figure $14.7.1b$. Notice that the amplitude of the oscillations decreases as energy is dissipated in the resistor. Equation 14.7.2 can be confirmed experimentally by measuring the voltage across the capacitor as a function of time. This voltage, multiplied by the capacitance of the capacitor, then gives **q(t)**.

Note

Try an interactive circuit construction kit that allows you to graph current and voltage as a function of time. You can add inductors and capacitors to work with any combination of **R**, **L**, and **C** circuits with both dc and ac sources.

Note

Try out a circuit-based java applet website that has many problems with both dc and ac sources that will help you practice circuit problems.

Exercise 14.7.1

In an **RLC** circuit, $L = 5.0\, mH$, $C = 6.0\, \mu F$, and $R = 200\, \Omega$. (a) Is the circuit underdamped, critically damped, or overdamped? (b) If the circuit starts oscillating with a charge of $3.0 \times 10^{-3} C$ on the capacitor, how much energy has been dissipated in the resistor by the time the oscillations cease?

Answer

a. overdamped; b. 0.75 J