5.S: Relativity (Summary)

Key Terms

classical (Galilean) velocity addition	method of adding velocities when $v << c$; velocities add like regular numbers in one-dimensional motion: $u = v + u'$, where **v** is the velocity between two observers, **u** is the velocity of an object relative to one observer, and u' is the velocity relative to the other observer
event	occurrence in space and time specified by its position and time coordinates (x, y, z, t) measured relative to a frame of reference
first postulate of special relativity	laws of physics are the same in all inertial frames of reference
Galilean relativity	if an observer measures a velocity in one frame of reference, and that frame of reference is moving with a velocity past a second reference frame, an observer in the second frame measures the original velocity as the vector sum of these velocities
Galilean transformation	relation between position and time coordinates of the same events as seen in different reference frames, according to classical mechanics
inertial frame of reference	reference frame in which a body at rest remains at rest and a body in motion moves at a constant speed in a straight line unless acted on by an outside force
length contraction	decrease in observed length of an object from its proper length L_0 to length **L** when its length is observed in a reference frame where it is traveling at speed **v**
Lorentz transformation	relation between position and time coordinates of the same events as seen in different reference frames, according to the special theory of relativity
Michelson-Morley experiment	investigation performed in 1887 that showed that the speed of light in a vacuum is the same in all frames of reference from which it is viewed
proper length	L_0; the distance between two points measured by an observer who is at rest relative to both of the points; for example, earthbound observers measure proper length when measuring the distance between two points that are stationary relative to Earth
proper time	$\Delta\tau$ is the time interval measured by an observer who sees the beginning and end of the process that the time interval measures occur at the same location
relativistic kinetic energy	kinetic energy of an object moving at relativistic speeds
relativistic momentum	$\vec{p}$, the momentum of an object moving at relativistic velocity; $\vec{p} = \gamma m\vec{u}$
lativistic velocity addition	method of adding velocities of an object moving at a relativistic speeds
rest energy	energy stored in an object at rest: $E_0 = mc^2$
rest frame	frame of reference in which the observer is at rest

rest mass	mass of an object as measured by an observer at rest relative to the object
second postulate of special relativity	light travels in a vacuum with the same speed **c** in any direction in all inertial frames
special theory of relativity	theory that Albert Einstein proposed in 1905 that assumes all the laws of physics have the same form in every inertial frame of reference, and that the speed of light is the same within all inertial frames
speed of light	ultimate speed limit for any particle having mass
time dilation	lengthening of the time interval between two events when seen in a moving inertial frame rather than the rest frame of the events (in which the events occur at the same location)
total energy	sum of all energies for a particle, including rest energy and kinetic energy, given for a particle of mass m and speed **u** by $E=\gamma mc^2$, where $\gamma=\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$
world line	path through space-time

Key Equations

Time dilation	$\Delta t=\frac{\Delta\tau}{\sqrt{1-\frac{v^2}{c^2}}}=\gamma\tau$
Lorentz factor	$\gamma=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$
Length contraction	$L=L_0\sqrt{1-\frac{v^2}{c^2}}=\frac{L_0}{\gamma}$
Galilean transformation	$x=x'+vt, y=y', z=z', t=t'$
Lorentz transformation	$t=\frac{t'+vx'/c^2}{\sqrt{1-v^2/c^2}}$ $x=\frac{x'+vt'}{\sqrt{1-v^2/c^2}}$ $y=y'$ $z=z'$
Inverse Lorentz transformation	$t'=\frac{t-vx/c^2}{\sqrt{1-v^2/c^2}}$ $x'=\frac{x-vt}{\sqrt{1-v^2/c^2}}$ $y'=y$ $z'=z$
Space-time invariants	$(\Delta s)^2=(\Delta x)^2+(\Delta y)^2+(\Delta z)^2-c^2(\Delta t)^2$ $(\Delta\tau)^2=-(\Delta s)^2/c^2=(\Delta t)^2-\frac{[(\Delta x)^2+(\Delta y)^2+(\Delta z)^2]}{c^2}$
Relativistic velocity addition	$u_x=(\frac{u'_x+v}{1+vu'_x/c^2}), u_y=(\frac{u'_y/\gamma}{1+vu'_x/c^2}), u_z=(\frac{u'_z/\gamma}{1+vu'_x/c^2})$

Relativistic Doppler effect for wavelength	$\lambda_{obs} = \lambda_s \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$
Relativistic Doppler effect for frequency	$f_{obs} = f_s \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$
Relativistic momentum	$\vec{p} = \gamma m\vec{u} = \frac{m\vec{u}}{\sqrt{1-\frac{u^2}{c^2}}}$
Relativistic total energy	$E = \gamma mc^2$,where $\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$
Relativistic kinetic energy	$K_{rel} = (\gamma - 1)mc^2$, where $\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$

Summary

5.1 Invariance of Physical Laws

- Relativity is the study of how observers in different reference frames measure the same event.
- Modern relativity is divided into two parts. Special relativity deals with observers in uniform (unaccelerated) motion, whereas general relativity includes accelerated relative motion and gravity. Modern relativity is consistent with all empirical evidence thus far and, in the limit of low velocity and weak gravitation, gives close agreement with the predictions of classical (Galilean) relativity.
- An inertial frame of reference is a reference frame in which a body at rest remains at rest and a body in motion moves at a constant speed in a straight line unless acted upon by an outside force.
- Modern relativity is based on Einstein's two postulates. The first postulate of special relativity is that the laws of physics are the same in all inertial frames of reference. The second postulate of special relativity is that the speed of light c is the same in all inertial frames of reference, independent of the relative motion of the observer and the light source.
- The Michelson-Morley experiment demonstrated that the speed of light in a vacuum is independent of the motion of Earth about the sun.

5.2 Relativity of Simultaneity

- Two events are defined to be simultaneous if an observer measures them as occurring at the same time (such as by receiving light from the events).
- Two events at locations a distance apart that are simultaneous for an observer at rest in one frame of reference are not necessarily simultaneous for an observer at rest in a different frame of reference.

5.3 Time Dilation

- Two events are defined to be simultaneous if an observer measures them as occurring at the same time. They are not necessarily simultaneous to all observers—simultaneity is not absolute.
- Time dilation is the lengthening of the time interval between two events when seen in a moving inertial frame rather than the rest frame of the events (in which the events occur at the same location).
- Observers moving at a relative velocity **v** do not measure the same elapsed time between two events. Proper time $\Delta\tau$ is the time measured in the reference frame where the start and end of the time interval occur at the same location. The time interval Δt measured by an observer who sees the frame of events moving at speed **v** is related to the proper time interval $\Delta\tau$ of the events by the equation:

$$\Delta t = \frac{\Delta\tau}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma\Delta\tau,$$

where

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}.$$

- The premise of the twin paradox is faulty because the traveling twin is accelerating. The journey is not symmetrical for the two twins.
- Time dilation is usually negligible at low relative velocities, but it does occur, and it has been verified by experiment.
- The proper time is the shortest measure of any time interval. Any observer who is moving relative to the system being observed measures a time interval longer than the proper time.

5.4 Length Contraction

- All observers agree upon relative speed.
- Distance depends on an observer's motion. Proper length L_0 is the distance between two points measured by an observer who is at rest relative to both of the points.
- Length contraction is the decrease in observed length of an object from its proper length L_0 to length L when its length is observed in a reference frame where it is traveling at speed **v**.
- The proper length is the longest measurement of any length interval. Any observer who is moving relative to the system being observed measures a length shorter than the proper length.

5.5 The Lorentz Transformation

- The Galilean transformation equations describe how, in classical nonrelativistic mechanics, the position, velocity, and accelerations measured in one frame appear in another. Lengths remain unchanged and a single universal time scale is assumed to apply to all inertial frames.
- Newton's laws of mechanics obey the principle of having the same form in all inertial frames under a Galilean transformation, given by

$$x = x' + vt, y = y', z = z', t = t' \; .$$

The concept that times and distances are the same in all inertial frames in the Galilean transformation, however, is inconsistent with the postulates of special relativity.

- The relativistically correct Lorentz transformation equations are

Lorentz transformation	Inverse Lorentz transformation
$t = \frac{t' + vx'/c^2}{\sqrt{1 - v^2/c^2}}$ $x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$ √ $y = y'$ $z = z'$	$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$ $x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$ $y' = y$ $z' = z$

We can obtain these equations by requiring an expanding spherical light signal to have the same shape and speed of growth, c, in both reference frames.

- Relativistic phenomena can be explained in terms of the geometrical properties of four-dimensional space-time, in which Lorentz transformations correspond to rotations of axes.
- The Lorentz transformation corresponds to a space-time axis rotation, similar in some ways to a rotation of space axes, but in which the invariant spatial separation is given by Δs rather than distances Δr, and that the Lorentz transformation involving the time axis does not preserve perpendicularity of axes or the scales along the axes.
- The analysis of relativistic phenomena in terms of space-time diagrams supports the conclusion that these phenomena result from properties of space and time itself, rather than from the laws of electromagnetism.

5.6 Relativistic Velocity Transformation

- With classical velocity addition, velocities add like regular numbers in one-dimensional motion: $u = v + u'$, where **v** is the velocity between two observers, **u** is the velocity of an object relative to one observer, and u'u′ is the velocity relative to the other observer.
- Velocities cannot add to be greater than the speed of light.
- Relativistic velocity addition describes the velocities of an object moving at a relativistic velocity.

5.7 Doppler Effect for Light

- An observer of electromagnetic radiation sees relativistic Doppler effects if the source of the radiation is moving relative to the observer. The wavelength of the radiation is longer (called a red shift) than that emitted by the source when the source moves away from the observer and shorter (called a blue shift) when the source moves toward the observer. The shifted wavelength is described by the equation:

$$\lambda_{obs} = \lambda_s \sqrt{1+\frac{v}{c}}\,1-\frac{v}{c}\ .$$

where λ_{obs} is the observed wavelength, λ_s is the source wavelength, and **v** is the relative velocity of the source to the observer.

5.8 Relativistic Momentum

- The law of conservation of momentum is valid for relativistic momentum whenever the net external force is zero. The relativistic momentum is $p = \gamma m u$, where m is the rest mass of the object, **u** is its velocity relative to an observer, and the relativistic factor is $\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$.
- At low velocities, relativistic momentum is equivalent to classical momentum.
- Relativistic momentum approaches infinity as **u** approaches **c**. This implies that an object with mass cannot reach the speed of light.

5.9 Relativistic Energy

- The relativistic work-energy theorem is $W_{net} = E - E_0 = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$.
- Relativistically, $W_{net} = K_{rel}$ where K_{rel} is the relativistic kinetic energy.
- An object of mass m at velocity u has kinetic energy $K_{rel} = (\gamma - 1)mc^2$, where $\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$.
- At low velocities, relativistic kinetic energy reduces to classical kinetic energy.
- No object with mass can attain the speed of light, because an infinite amount of work and an infinite amount of energy input is required to accelerate a mass to the speed of light.
- Relativistic energy is conserved as long as we define it to include the possibility of mass changing to energy.
- The total energy of a particle with mass m traveling at speed u is defined as $E = \gamma mc^2$, where $\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$ and **u** denotes the velocity of the particle.
- The rest energy of an object of mass m is $E_0 = mc^2$, meaning that mass is a form of energy. If energy is stored in an object, its mass increases. Mass can be destroyed to release energy.
- We do not ordinarily notice the increase or decrease in mass of an object because the change in mass is so small for a large increase in energy. The equation $E^2 = (pc)^2 + (mc^2)^2$ relates the relativistic total energy **E** and the relativistic momentum p. At extremely high velocities, the rest energy mc^2 becomes negligible, and $E = pc$.