2.A: Geometric Optics and Image Formation (Answers)

Check Your Understanding

Conceptual Questions

1. Virtual image cannot be projected on a screen. You cannot distinguish a real image from a virtual image simply by judging from the image perceived with your eye.

3. Yes, you can photograph a virtual image. For example, if you photograph your reflection from a plane mirror, you get a photograph of a virtual image. The camera focuses the light that enters its lens to form an image; whether the source of the light is a real object or a reflection from mirror (i.e., a virtual image) does not matter.

5. No, you can see the real image the same way you can see the virtual image. The retina of your eye effectively serves as a screen.

7. The mirror should be half your size and its top edge should be at the level of your eyes. The size does not depend on your distance from the mirror.

9. when the object is at infinity; see the mirror equation

11. Yes, negative magnification simply means that the image is upside down; this does not prevent the image from being larger than the object. For instance, for a concave mirror, if distance to the object is larger than one focal distance but smaller than two focal distances the image will be inverted and magnified.

13. answers may vary

15. The focal length of the lens is fixed, so the image distance changes as a function of object distance.

17. Yes, the focal length will change. The lens maker's equation shows that the focal length depends on the index of refraction of the medium surrounding the lens. Because the index of refraction of water differs from that of air, the focal length of the lens will change when submerged in water.

19. A relaxed, normal-vision eye will focus parallel rays of light onto the retina.

21. A person with an internal lens will need glasses to read because their muscles cannot distort the lens as they do with biological lenses, so they cannot focus on near objects. To correct nearsightedness, the power of the intraocular lens must be less than that of the removed lens.

23. Microscopes create images of macroscopic size, so geometric optics applies.

25. The eyepiece would be moved slightly farther from the objective so that the image formed by the objective falls just beyond the focal length of the eyepiece.

Problems

27.

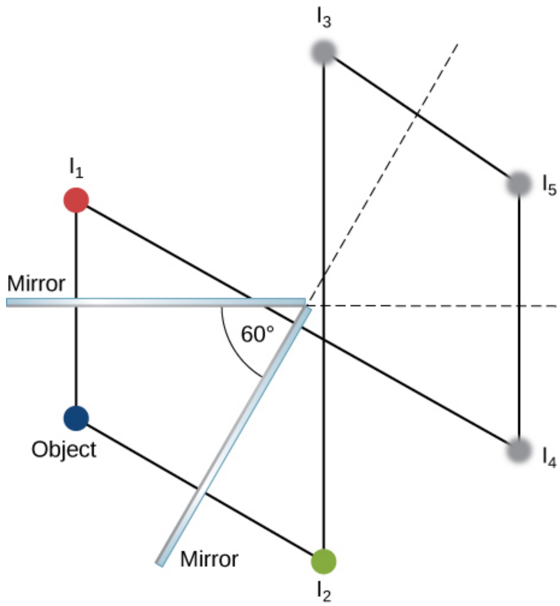

Figure shows cross sections of two mirrors placed at an angle of 60 degrees to each other. Six small circles labeled object, I_1, I_2, I_3, I_4 and I_5 are shown. The object is on the bisector between the mirrors. Line 1 intersects mirror 1 perpendicularly connecting the object to I_1 on the other side of the mirror. Line 2 intersects the mirror 2 perpendicularly connecting the object to I_2 on the other side of the mirror. Lines parallel to these respectively connect I_2 to I_3 and I_1 to I_4. Lines parallel to these respectively connect I_4 to I_5 and I_3 to I_5.

29. It is in the focal point of the big mirror and at the center of curvature of the small mirror.

31. $f = \frac{R}{2} \Rightarrow R = +1.60m$

33. $d_o = 27.3cm$

35. Step 1: Image formation by a mirror is involved.

Step 2: Draw the problem set up when possible.

Step 3: Use thin-lens equations to solve this problem.

Step 4: Find f.

Step 5: Given: $m = 1.50, d_o = 0.120m$.

Step 6: No ray tracing is needed.

Step 7: Using $m = \frac{d_i}{d_o}$, $d_i = -0.180m$. Then, $f = 0.360m$.

Step 8: The image is virtual because the image distance is negative. The focal length is positive, so the mirror is concave.

37. a. for a convex mirror $d_i < 0 \Rightarrow m > 0. m = +0.111$;

b. $d_i = -0.334cm$ (behind the cornea);

c. $f = -0.376cm$, so that $R = -0.752cm$

39. $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = -\frac{-d_o}{d_o} = \frac{d_o}{d_o} = 1 \Rightarrow h_i = h_o$

41. $m = -11.0 \ A' = 0.110m^2 \ I = 6.82kW/m^2$

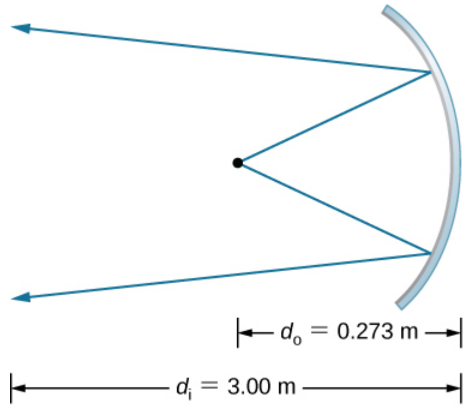

Figure shows the cross section of a concave mirror. Two rays originating from a point strike the mirror and are reflected. The distance of the point from the mirror is labeled $d_o = 0.273m$ and $d_i = 3.00m$.

43. $x_{2m} = -x_{2m-1}, (m = 1, 2, 3, \ldots),$

$x_{2m+1} = b - x_{2m}, (m = 0, 1, 2, \ldots),$ with $x_0 = a.$

45. $d_i = -55cm; m = +1.8$

47. $d_i = -41cm, m = 1.4$

49. proof

51. a. $\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f} \Rightarrow d_i = 3.43m$;

b. $m = -33.33$, so that $(2.40 \times 10^{-2} m)(33.33) = 80.0cm$, and

$(3.60 \times 10^{-2} m)(33.33) = 1.20m \Rightarrow 0.800m \times 1.20m$ or $80.0cm \times 120cm$

53. a. $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ $d_i = 5.08cm;$

b. $m = -1.695 \times 10^{-2}$, so the maximum height is $\frac{0.036m}{1.695 \times 10^{-2}} = 2.12m \Rightarrow 100;$

c. This seems quite reasonable, since at 3.00 m it is possible to get a full length picture of a person.

55. a. $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow d_o = 2.55m$;

b. $\frac{h_i}{h_o} = -\frac{d_i}{d_o} \Rightarrow h_o = 1.00m$

57. a. Using $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$, $d_i = -56.67cm$. Then we can determine the magnification, $m = 6.67$.

b. $d_i = -190cm$ and $m = +20.0$;

c. The magnification m increases rapidly as you increase the object distance toward the focal length.

59. $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$

$d_I = \frac{1}{(1/f) - (1/d_o)}$

$\frac{d_i}{d_o} = 6.667 \times 10^{-13} = \frac{h_i}{h_o}$

$h_i = -0.933mm$

61. $d_i = -6.7cm$

$h_i = 4.0cm$

63. 83 cm to the right of the converging lens, $m = -2.3, h_i = 6.9cm$

65. $P = 52.0D$

67. $\frac{h_i}{h_o} = -\frac{d_i}{d_o} \Rightarrow h_i = -h_o(\frac{d_i}{d_o}) = -(3.50mm)(\frac{2.00cm}{30.0cm}) = -0.233mm$

69. a. $P = +62.5D$;

b. $\frac{h_i}{h_o} = -\frac{d_i}{d_o} \Rightarrow h_i = -0.250mm$;

c. $h_i = -0.0800mm$

71. $P = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow d_o = 28.6cm$

73. Originally, the close vision was 51.0 D. Therefore, $P = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow d_o = 1.00m$

75. originally, $P = 70.0D$; because the power for normal distant vision is 50.0 D, the power should be decreased by 20.0 D

77. $P = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow d_o = 0.333m$

79. a. $P = 52.0D$;

b. $P' = 56.16D\ \frac{1}{d_o} + \frac{1}{d_i} = P \Rightarrow d_o = 16.2cm$

81. We need $d_i = -18.5cm$ when $d_o = \infty$, so $P = -5.41D$

83. Let x = far point $\Rightarrow P = \frac{1}{-(x - 0.0175m)} + \frac{1}{\infty} \Rightarrow -xP + (0.0175m)P = 1 \Rightarrow x = 26.8cm$

85. $M = 6\times$

87. $M = (\frac{25cm}{L})(1 + \frac{L-\ell}{f})\ \ L - \ell = d_o\ \ d_o = 13cm$

89. $M = 2.5\times$

91. $M = -2.1\times$

93. $M = \frac{25cm}{f}\ \ M_{max} = 5$

95. $M_{max}^{young} = 1 + \frac{18cm}{f} \Rightarrow f = \frac{18cm}{M_{max}^{young} - 1}$

$M_{max}^{old} = 9.8\times$

97. a. $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow d_i = 4.65cm \Rightarrow m = -30.01$;

b. $M_{net} = -240$

99. a. $\frac{1}{d_o^{obj}} + \frac{1}{d_i^{obj}} = \frac{1}{f^{obj}} \Rightarrow d_i^{obj} = 18.3cm$ behind the objective lens;

b. $m^{obj} = -60.0$;

c. $d_o^{eye} = 1.70cm$

$d_i^{eye} = -11.3cm$;

d. $M^{eye} = 13.5$;

e. $M_{net} = -810$

101. $M = -40.0$

103. $f^{obj} = \frac{R}{2}, M = -1.67$

105. $M = -\frac{f^{obj}}{f^{eye}}, f^{eye} = +10.0cm$

107. Answers will vary.

109. 12 cm to the left of the mirror, $m = 3/5$

111. 27 cm in front of the mirror, $m = 0.6, h_i = 1.76cm$, orientation upright

113. The following figure shows three successive images beginning with the image Q_1 in mirror M_1. Q_1 is the image in mirror M_1, whose image in mirror M_2 is Q_{12} whose image in mirror M_1 is the real image Q_{121}.

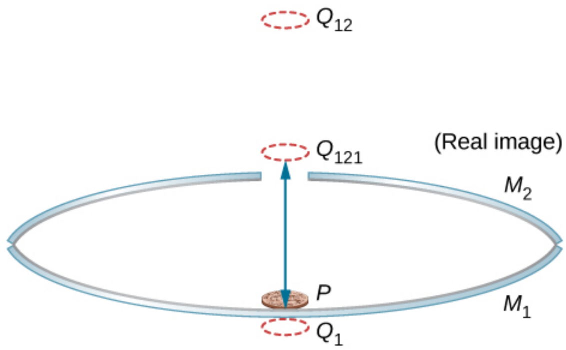

Figure shows the side view of two concave mirrors, M_1 and M_2 placed one on top of the other, facing each other. The top, M_2, one has a small hole in the middle. A penny is placed on the bottom mirror. An image of the penny labeled Q_1 is shown below M_1. Another image of the penny, labeled Q_{121} is shown above the top mirror. This is labeled real image.

115. 5.4 cm from the axis

117. Let the vertex of the concave mirror be the origin of the coordinate system. Image 1 is at −10/3 cm (−3.3 cm), image 2 is at −40/11 cm (−3.6 cm). These serve as objects for subsequent images, which are at −310/83 cm (−3.7 cm), −9340/2501 cm (−3.7 cm), −140,720/37,681 cm (−3.7 cm). All remaining images are at approximately −3.7 cm.

119.

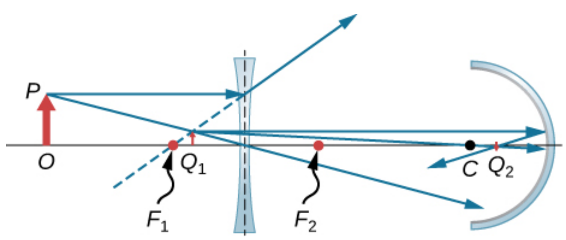

Figure shows two prisms with their bases parallel to each other at an angle of 45 degrees to the horizontal. To the right of this is a bi-convex lens. A ray along the optical axis enters this set up from the left, deviates between the two prisms and travels parallel to the optical axis, slightly below it. It enters the lens and deviates to pass through its focal point on the other side.

121. Figure shows from left to right: an object with base O on the axis and tip P. A bi-concave lens with focal point F1 and F2 on the left and right respectively and a concave mirror with center of curvature C. Two rays originate from P and diverge through the bi-concave lens. Their back extensions converge between F1 and the lens to form image Q1. Two rays originating from the tip of Q1 strike the mirror, are reflected and converge at Q2 between C and the mirror.

123. −5 D

125. 11

Additional Problems

127. a.

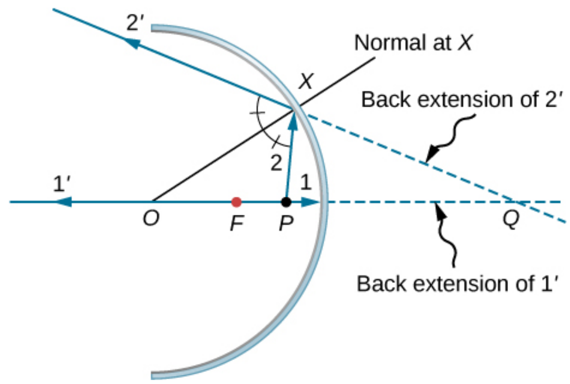

Figure shows the cross section of a concave mirror with centre of curvature O and focal point F. Point P lies on the axis between point F and the mirror. Ray 1 originates from point P, travels along the axis and hits the mirror. The reflected ray 1 prime travels back along the axis. Ray 2 originates from P and hits the mirror at point X. The reflected ray is labeled 2 prime. Line OX, labeled normal at X, bisects the angle formed by PX and ray 2 prime. The back extensions of 1 prime and 2 prime intersect at point Q.

b.

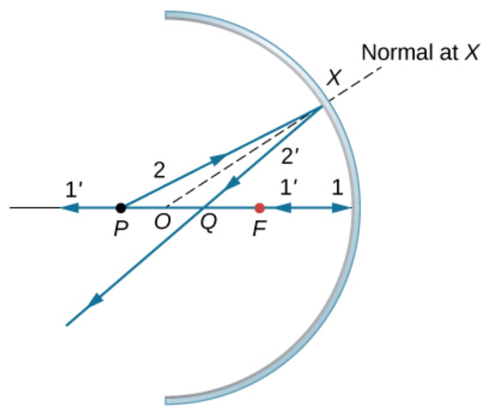

Figure shows the cross section of a concave mirror with points P, O, Q and F lying on the optical axis. Point P is furthest from the mirror. Ray 1 originates from P, travels along the axis and hits the mirror. The reflected ray 1 prime travels back along the axis. Ray 2 originates from P and hits the mirror at point X. The reflected ray 2 prime intersects the axis at point Q, which lies between points P and F. OX, labeled normal at X, bisects the angle PXQ.

c.

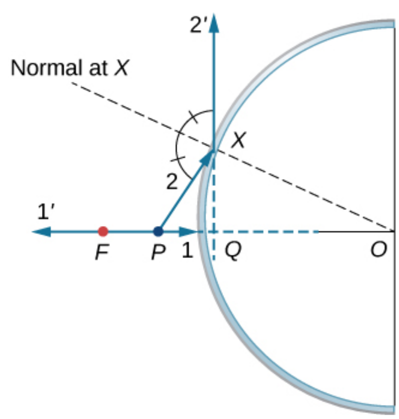

Figure shows a convex mirror with point P lying between point F and the mirror on the optical axis. Ray 1 originates from P, travels along the axis and hits the mirror. The reflected ray 1 prime travels back along the axis. Ray 2 originates from P and hits the mirror at point X. The angle formed by reflected ray 2 prime and PX is bisected by OX, the normal at X. The back extensions of 1 prime and 2 prime intersect at point Q, just behind the mirror.

d. similar to the previous picture but with point P outside the focal length;

e. Repeat (a)–(d) for a point object off the axis. For a point object placed off axis in front of a concave mirror corresponding to parts (a) and (b), the case for convex mirror left as exercises.

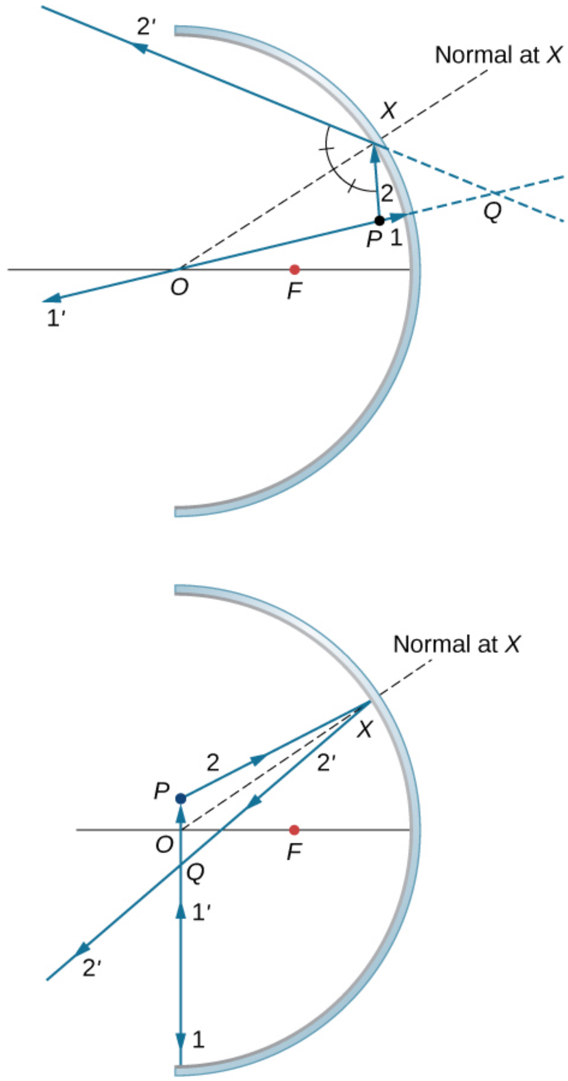

Figure a shows the cross section of a concave mirror. Point P lies above the axis, closer to the mirror than focal point F. Ray 1 originates from P and hits the mirror. Reflected ray 1 prime travels back along the same line as ray 1 and intersects the optical axis at point O. Ray 2 originates from point P and hits the mirror at point X. The reflected ray is labeled 2 prime. The back extensions of 1 prime and 2 prime intersect at point Q behind the mirror. The angle formed by rays 2 and 2 prime is bisected by OX, the normal at X. Figure b shows the cross section of a concave mirror. Point P lies above the axis, further away from the mirror than point F. Ray 1 originates from P and hits the mirror. Reflected ray 1 prime travels back along the same line as ray 1 and intersects the optical axis at point O. Ray 2 originates from point P and hits the mirror at point X. The reflected ray is labeled 2 prime. Rays 1 prime and 2 prime intersect at point Q in front of the mirror. The angle formed by rays 2 and 2 prime is bisected by OX, the normal at X.

129. $d_i = -10/3cm, h_i = 2cm$, upright

131. proof

133.

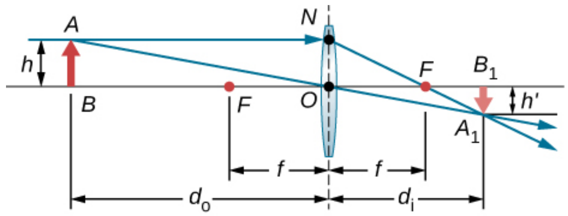

Figure shows a bi-convex lens, an object placed at point A on the optical axis and an inverted image formed at point B_1 on the axis behind the lens. The top of the object is a distance h from the origin. Three rays originate from the top of the object, strike the lens and converge on the other side at the top of the inverted image. It passes the focal point in front of the lens and is parallel to the optical axis behind the lens.

Triangles **BAO** and B_1A_1O are similar triangles. Thus, $\frac{A_1B_1}{AB}=\frac{d_i}{d_o}$. Triangles **NOF** and B_1A_1F are similar triangles. Thus, $\frac{NO}{f}=\frac{A_1B_1}{d_i-f}$. Noting that $NO=AB$ gives $\frac{AB}{f}=\frac{A_1B_1}{d_i-f}$ or $\frac{AB}{A_1B_1}=\frac{f}{d_i-f}$. Inverting this gives $\frac{A_1B_1}{AB}=\frac{d_i-f}{f}$. Equating the two expressions for the ratio $\frac{A_1B_1}{AB}$ gives $\frac{d_i}{d_o}=\frac{d_i-f}{f}$. Dividing through by d_i gives $\frac{1}{d_o}=\frac{1}{f}-\frac{1}{d_i}$ or $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$.

135. 70 cm

137. The plane mirror has an infinite focal point, so that $d_i=-d_o$. The total apparent distance of the man in the mirror will be his actual distance, plus the apparent image distance, or $d_o+(-d_i)=2d_o$. If this distance must be less than 20 cm, he should stand at $d_o=10cm$.

139. Here we want $d_o=25cm-2.20cm=0.228m$. If $x=$near point, $d_i=-(x-0.0220m)$. Thus, $P=\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{0.228m}+\frac{1}{x-0.0220m}$. Using $P=0.75D$ gives $x=0.253m$, so the near point is 25.3 cm.

141. Assuming a lens at 2.00 cm from the boy's eye, the image distance must be $d_i=-(500cm-2.00cm)=-498cm$. For an infinite-distance object, the required power is $P=\frac{1}{d_i}=-0.200D$. Therefore, the $-4.00D$ lens will correct the nearsightedness.

143. $87\mu m$

145. Use, $M_{net}=-\frac{d_i^{obj}(f^{eye}+25cm)}{f^{obj}f^{eye}}$. The image distance for the objective is $d_i^{obj}=-\frac{M_{net}f^{obj}f^{eye}}{f^{eye}+25\ cm}$. Using $f^{obj}=3.0cm$, $f^{eye}=10cm$, and $M=-10$ gives $d_i^{obj}=8.6cm$. We want this image to be at the focal point of the eyepiece so that the eyepiece forms an image at infinity for comfortable viewing. Thus, the distance d between the lenses should be $d=f^{eye}+d_i^{obj}=10cm+8.6cm=19cm$

147. a. focal length of the corrective lens $f_c=-80cm$;

b. −1.25 D

149. $2\times10^{16}km$

151. $105m$