5.A: Relativity (Answers)

Check Your Understanding

5.1. Special relativity applies only to objects moving at constant velocity, whereas general relativity applies to objects that undergo acceleration.

5.2. $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.650c)^2}{c^2}}} = 1.32$

5.3. a. $\Delta t = \frac{\Delta\tau}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2.10\times10^{-8}s}{\sqrt{1-\frac{(1.90\times10^8 m/s)^2}{(3.00\times10^8 m/s)^2}}} = 2.71\times10^{-8}s$.

b. Only the relative speed of the two spacecraft matters because there is no absolute motion through space. The signal is emitted from a fixed location in the frame of reference of A, so the proper time interval of its emission is $\tau = 1.00s$. The duration of the signal measured from frame of reference B is then

$$\Delta t = \frac{\Delta\tau}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1.00s}{\sqrt{1-\frac{(4.00\times10^7 m/s)^2}{(3.00\times10^8 m/s)^2}}} = 1.01s\ .$$

5.4. $L = L_0\sqrt{1-\frac{v^2}{c^2}} = (2.50km)\sqrt{1-\frac{(0.750c)^2}{c^2}} = 1.65km$

5.5. Start with the definition of the proper time increment:

$$d\tau = \sqrt{-(ds)^2/c^2} = \sqrt{dt^2-(dx^2+dx^2+dx^2)/c^2}\ .$$

where (dx, dy, dx, cdt) are measured in the inertial frame of an observer who does not necessarily see that particle at rest. This therefore becomes

$$\begin{aligned} d_\tau &= \sqrt{-(ds)^2/c^2} = \sqrt{dt^2-[(dx)^2+(dy)^2+(dz)^2]/c^2} \\ &= dt\sqrt{1-[(\frac{dx}{dt})^2+(\frac{dy}{dt})^2+(\frac{dz}{dt})^2]/c^2} \\ &= dt\sqrt{1-v^2/c^2} \end{aligned}$$

$$dt = \gamma d\tau.$$

5.6. Although displacements perpendicular to the relative motion are the same in both frames of reference, the time interval between events differ, and differences in **dt** and dt' lead to different velocities seen from the two frames.

5.7. We can substitute the data directly into the equation for relativistic Doppler frequency:

$$f_{obs} = f_s\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} = (1.50GHz)\sqrt{\frac{1-\frac{0.350c}{c}}{1+\frac{0.350c}{c}}} = 1.04GHz.$$

5.8. Substitute the data into the given equation:

$$p = \gamma mu = \frac{mu}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{(9.11\times10^{-31}kg)(0.985)(3.00\times10^8 m/s)}{\sqrt{1-\frac{(0.985c)^2}{c^2}}} = 1.56\times10^{-21}kg-m/s\ .$$

5.9.

$$K_{rel} = (\gamma-1)mc^2 = (\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}mc^2 = (\frac{1}{\sqrt{1-\frac{(0.992c)^2}{c^2}}-1}(9.11\times10^{-31}kg)(3.00\times10^8 m/s)^2 = 5.67\times10^{-13}J$$

Conceptual Questions

1. the second postulate, involving the speed of light; classical physics already included the idea that the laws of mechanics, at least, were the same in all inertial frames, but the velocity of a light pulse was different in different frames moving with respect to each other

3. yes, provided the plane is flying at constant velocity relative to the Earth; in that case, an object with no force acting on it within the plane has no change in velocity relative to the plane and no change in velocity relative to the Earth; both the plane and the ground are inertial frames for describing the motion of the object

5. The observer moving with the process sees its interval of proper time, which is the shortest seen by any observer.

7. The length of an object is greatest to an observer who is moving with the object, and therefore measures its proper length.

9. a. No, not within the astronaut's own frame of reference.

b. He sees Earth clocks to be in their rest frame moving by him, and therefore sees them slowed.

c. No, not within the astronaut's own frame of reference.

d. Yes, he measures the distance between the two stars to be shorter.

e. The two observers agree on their relative speed.

11. There is no measured change in wavelength or frequency in this case. The relativistic Doppler effect depends only on the relative velocity of the source and the observer, not any speed relative to a medium for the light waves.

13. It shows that the stars are getting more distant from Earth, that the universe is expanding, and doing so at an accelerating rate, with greater velocity for more distant stars.]

15. Yes. This can happen if the external force is balanced by other externally applied forces, so that the net external force is zero.

17. Because it loses thermal energy, which is the kinetic energy of the random motion of its constituent particles, its mass decreases by an extremely small amount, as described by energy-mass equivalence.

19. Yes, in principle there would be a similar effect on mass for any decrease in energy, but the change would be so small for the energy changes in a chemical reaction that it would be undetectable in practice.

21. Not according to special relativity. Nothing with mass can attain the speed of light.

Problems

23. a. 1.0328;

b. 1.15

25. $5.96 \times 10^{-8} s$

27. 0.800c

29. 0.140c

31. 48.6 m

33. Using the values given in Example 5.3:

a. 1.39 km;

b. 0.433 km;

c. 0.433 km

35. a. 10.0c;

b. The resulting speed of the canister is greater than c, an impossibility.

c. It is unreasonable to assume that the canister will move toward the earth at 1.20**c.**

37. The angle α approaches 45°, and the $t' -$ and $x' - axes$ rotate toward the edge of the light cone.

39. 15 m/s east

41. 32 m/s

43. a. The second ball approaches with velocity $-v$ and comes to rest while the other ball continues with velocity $-v$;

b. This conserves momentum.

45. a. $t_1' = 0; x_1' = 0$;

$t_2' = \tau; x_2' = 0;$

b. $t_1' = 0; x_1' = 0$;

$t_2' = \dfrac{\tau}{\sqrt{1-v^2/c^2}}; x_2' = \dfrac{-v\tau}{\sqrt{1-v^2/c^2}}$

47. 0.615c

49. 0.696c

51. (Proof)

53. $4.09 \times 10^{-19} kg \cdot m/s$

55. a. $3.000000015 \times 10^{13} kg \cdot m/s$;

b. 1.000000005

57. $2.988 \times 10^8 m/s$

59. 0.512 MeV according to the number of significant figures stated. The exact value is closer to 0.511 MeV.

61. $2.3 \times 10^{-30} kg$; to two digits because the difference in rest mass energies is found to two digits

63. a. $1.11 \times 10^{27} kg$;

b. $5.56 \times 10 - 5$

65. a. $7.1 \times 10^{-3} kg$;

b. $7.1 \times 10^{-3} = 7.1 \times 10^{-3}$;

c. $\dfrac{\Delta m}{m}$ is greater for hydrogen

67. a. 208;

b. 0.999988c; six digits used to show difference from c

69. a. $6.92 \times 10^5 J$;

b. 1.54

71. a. 0.914c;

b. The rest mass energy of an electron is 0.511 MeV, so the kinetic energy is approximately 150% of the rest mass energy. The electron should be traveling close to the speed of light.

Additional Problems

73. a. 0.866c;

b. 0.995c

75. a. 4.303 y to four digits to show any effect;

b. 0.1434 y;

c. $1/\sqrt{(1-v^2/c^2)} = 29.88$..

77. a. 4.00;

b. $v = 0.867c$

79. a. A sends a radio pulse at each heartbeat to B, who knows their relative velocity and uses the time dilation formula to calculate the proper time interval between heartbeats from the observed signal.

b. $(66 beats/min)\sqrt{1-v^2/c^2} = 57.1$ beats/min

81. a. first photon: $(0, 0, 0)$ at $t = t'$; second photon:

$$t' = \frac{-vx/c^2}{\sqrt{1-v^2/c^2}} = \frac{-(c/2)(1.00m)/c^2}{\sqrt{0.75}} = \frac{0.577m}{c} = 1.93 \times 10^{-9} s$$

$$x' = \frac{x}{\sqrt{1-v^2/c^2}} = \frac{1.00m}{\sqrt{0.75}}$$ =1.15m\)

b. simultaneous in A, not simultaneous in B

83. $$t' = \frac{t-vx/c^2}{\sqrt{1-v^2/c^2}} = \frac{(4.5 \times 10^{-4} \text{ s}) - (0.6c)\left(\frac{150\times10^3 \text{ m}}{c^2}\right)}{\sqrt{1-(0.6)^2}}$$

$= 1.88 \times 10^{-4}\ s$

$$x' = \frac{x-vt}{\sqrt{1-v^2/c^2}} = \frac{150 \times 10^3 \text{ m} - (0.60)\left(3.00 \times 10^8 \text{ m/s}\right)\left(4.5 \times 10^{-4} \text{ s}\right)}{\sqrt{1-(0.6)^2}}$$

$= 8.6 \times 10^4\ m = 86\ km$

$y = y' = 15\ km$

$z = z' = 1\ km$

85. $$\Delta t = \frac{\Delta t' + v\Delta x'/c^2}{\sqrt{1-v^2/c^2}}$$

$$0 = \frac{\Delta t' + v(500m)/c^2}{\sqrt{1-v^2/c^2}};$$

since $v \ll c$, we can ignore the term v^2/c^2 and find

$$\Delta t' = -\frac{(50m/s)(500m)}{(3.00 \times 10^8 m/s)^2} = -2.78 \times 10^{-13} s$$

The breakdown of Newtonian simultaneity is negligibly small, but not exactly zero, at realistic train speeds of 50 m/s.

87. $$\Delta t' = \frac{\Delta t - v\Delta x/c^2}{\sqrt{1-v^2/c^2}}$$

$$0 = \frac{(0.30s) - \frac{(v)(2.0\times10^9 m)}{(3.00\times10^8 m/s)^2}}{\sqrt{1-v^2/c^2}}$$

$$v = \frac{(0.30s)}{(2.0 \times 10^9 m)}(3.00 \times 10^8 m/s)^2$$

$v = 1.35 \times 10^7 m/s$

89. Note that all answers to this problem are reported to five significant figures, to distinguish the results.

a. 0.99947c;

b. $1.2064 \times 10^{11} y$;

c. $1.2058 \times 10^{11} y$

91. a. –0.400c;

b. –0.909c

93. a. 1.65 km/s;

b. Yes, if the speed of light were this small, speeds that we can achieve in everyday life would be larger than 1% of the speed of light and we could observe relativistic effects much more often.

95. 775 MHz

97. a. $1.12 \times 10^{-8} m/s$;

b. The small speed tells us that the mass of a protein is substantially smaller than that of even a tiny amount of macroscopic matter.

99. a. $F = \frac{dp}{dt} = \frac{d}{dt}(\frac{mu}{\sqrt{1-u^2/c^2}}) = \frac{du}{dt}(\frac{m}{\sqrt{1-u^2/c^2}}) - \frac{1}{2}\frac{mu^2}{(1-u^2/c^2)^{3/2}} 2\frac{du}{dt} = \frac{m}{(1-u^2/c^2)^{3/2}}\frac{du}{dt}$;

b. $F = \frac{m}{(1-u^2/c^2)^{3/2}}\frac{du}{dt} = \frac{1kg}{(1-(\frac{1}{2})^2)^{3/2}}(1m/s^2) = 1.53N$

101. 90.0 MeV

103. a. $\gamma^2 - 1$;

b. yes

105. 1.07×10^3

107. a. $6.56 \times 10^{-8} kg$;

b. $m = (200L)(1m^3/1000L)(750kg/m^3) = 150kg$ therefore, $\frac{\Delta m}{m} = 4.37 \times 10^{-10}$

109. a. 0.314c;

b. 0.99995c (Five digits used to show difference from c)

111. a. 1.00 kg;

b. This much mass would be measurable, but probably not observable just by looking because it is 0.01% of the total mass.

113. a. $6.06 \times 10^{11} kg/s$;

b. $4.67 \times 10^{10} y$;

c. $4.27 \times 10^9 kg$;

d. 0.32