8.E: Atomic Structure (Exercises)

Conceptual Questions

8.1 The Hydrogen Atom

1. Identify the physical significance of each of the quantum numbers of the hydrogen atom.

2. Describe the ground state of hydrogen in terms of wave function, probability density, and atomic orbitals.

3. Distinguish between Bohr's and Schrödinger's model of the hydrogen atom. In particular, compare the energy and orbital angular momentum of the ground states.

8.2 Orbital Magnetic Dipole Moment of the Electron

4. Explain why spectral lines of the hydrogen atom are split by an external magnetic field. What determines the number and spacing of these lines?

5. A hydrogen atom is placed in a magnetic field. Which of the following quantities are affected?

(a) total energy;

(b) angular momentum;

(c) z-component of angular momentum;

(d) polar angle.

6. On what factors does the orbital magnetic dipole moment of an electron depend?

8.3 Electron Spin

7. Explain how a hydrogen atom in the ground state $(l = 0)$ can interact magnetically with an external magnetic field.

8. Compare orbital angular momentum with spin angular momentum of an electron in the hydrogen atom.

9. List all the possible values of **s** and m_s for an electron. Are there particles for which these values are different?

10. Are the angular momentum vectors $\vec{L}$ and $\vec{S}$ necessarily aligned?

11. What is spin-orbit coupling?

8.4 The Exclusion Principle and the Periodic Table

12. What is Pauli's exclusion principle? Explain the importance of this principle for the understanding of atomic structure and molecular bonding.

13. Compare the electron configurations of the elements in the same column of the periodic table.

14. Compare the electron configurations of the elements that belong in the same row of the periodic table of elements.

8.5 Atomic Spectra and X-rays

15. Atomic and molecular spectra are discrete. What does discrete mean, and how are discrete spectra related to the quantization of energy and electron orbits in atoms and molecules?

16. Discuss the process of the absorption of light by matter in terms of the atomic structure of the absorbing medium.

17. NGC1763 is an emission nebula in the Large Magellanic Cloud just outside our Milky Way Galaxy. Ultraviolet light from hot stars ionize the hydrogen atoms in the nebula. As protons and electrons recombine, light in the visible range is emitted. Compare the energies of the photons involved in these two transitions.

18. Why are X-rays emitted only for electron transitions to inner shells? What type of photon is emitted for transitions between outer shells?

19. How do the allowed orbits for electrons in atoms differ from the allowed orbits for planets around the sun?

8.6 Lasers

20. Distinguish between coherent and monochromatic light.

21. Why is a metastable state necessary for the production of laser light?

22. How does light from an incandescent light bulb differ from laser light?

23. How is a Blu-Ray player able to read more information that a CD player?

24. What are the similarities and differences between a CD player and a Blu-Ray player?

Problems

8.1 The Hydrogen Atom

25. The wave function is evaluated at rectangular coordinates (x, y, z)=(2, 1, 1) in arbitrary units. What are the spherical coordinates of this position?

26. If an atom has an electron in the $n = 5$ state with $m = 3$, what are the possible values of **l**?

27. What are the possible values of **m** for an electron in the $n = 4$ state?

28. What, if any, constraints does a value of $m = 1$ place on the other quantum numbers for an electron in an atom?

29. How many possible states are there for the **l** = 4 state?

30. (a) How many angles can **L** make with the z-axis for an $l = 2$ electron?

(b) Calculate the value of the smallest angle.

31. The force on an electron is "negative the gradient of the potential energy function." Use this knowledge and Equation 8.1 to show that the force on the electron in a hydrogen atom is given by Coulomb's force law.

32. What is the total number of states with orbital angular momentum $l = 0$? (Ignore electron spin.)

33. The wave function is evaluated at spherical coordinates $(r, \theta, \phi) = (\sqrt{3}, 45^\circ, 45^\circ)$, where the value of the radial coordinate is given in arbitrary units. What are the rectangular coordinates of this position?

34. Coulomb's force law states that the force between two charged particles is: $F = k\dfrac{Qq}{r^2}$. Use this expression to determine the potential energy function.

35. Write an expression for the total number of states with orbital angular momentum **l.**

36. Consider hydrogen in the ground state, ψ_{100}.

(a) Use the derivative to determine the radial position for which the probability density, $P(r)$, is a maximum.

(b) Use the integral concept to determine the average radial position. (This is called the expectation value of the electron's radial position.) Express your answers into terms of the Bohr radius, a_o. **Hint:** The expectation value is the just average value.

(c) Why are these values different?

37. What is the probability that the 1s electron of a hydrogen atom is found outside the Bohr radius?

38. How many polar angles are possible for an electron in the $l = 5$ state?

39. What is the maximum number of orbital angular momentum electron states in the $n = 2$ shell of a hydrogen atom? (Ignore electron spin.)

40. What is the maximum number of orbital angular momentum electron states in the $n = 3$ shell of a hydrogen atom? (Ignore electron spin.)

8.2 Orbital Magnetic Dipole Moment of the Electron

41. Find the magnitude of the orbital magnetic dipole moment of the electron in in the **3p** state. (Express your answer in terms of μ_B)

42. A current of $I = 2A$ flows through a square-shaped wire with 2-cm side lengths. What is the magnetic moment of the wire?

43. Estimate the ratio of the electron magnetic moment to the muon magnetic moment for the same state of orbital angular momentum. (**Hint:** $m_\mu = 105.7 MeV/c^2$)

44. Find the magnitude of the orbital magnetic dipole moment of the electron in in the **4d** state. (Express your answer in terms of μB.μB.)

45. For a **3d** electron in an external magnetic field of $2.50 \times 10^{-3} T$, find (a) the current associated with the orbital angular momentum, and (b) the maximum torque.

46. An electron in a hydrogen atom is in the $n = 5, l = 4$ state. Find the smallest angle the magnetic moment makes with the **z**-axis. (Express your answer in terms of μ_B.)

47. Find the minimum torque magnitude $|\vec{\tau}|$ that acts on the orbital magnetic dipole of a 3**p** electron in an external magnetic field of $2.50 \times 10^{-3} T$.

48. An electron in a hydrogen atom is in 3**p** state. Find the smallest angle the magnetic moment makes with the **z**-axis. (Express your answer in terms of μ_B.)

49. Show that $U = -\vec{\mu} \cdot \vec{B}$. (**Hint:** An infinitesimal amount of work is done to align the magnetic moment with the external field. This work rotates the magnetic moment vector through an angle $-d\theta$ (toward the positive z-direction), where $d\theta$ is a positive angle change.)

8.3 Electron Spin

50. What is the magnitude of the spin momentum of an electron? (Express you answer in terms of $\hbar$.)

51. What are the possible polar orientations of the spin momentum vector for an electron?

52. For $n = 1$, write all the possible sets of quantum numbers (n, l, m, m_s).

53. A hydrogen atom is placed in an external uniform magnetic field ($B = 200T$). Calculate the wavelength of light produced in a transition from a spin up to spin down state.

54. If the magnetic field in the preceding problem is quadrupled, what happens to the wavelength of light produced in a transition from a spin up to spin down state?

55. If the magnetic moment in the preceding problem is doubled, what happens to the frequency of light produced in a transition from a spin-up to spin-down state?

56. For $n = 2$, write all the possible sets of quantum numbers (n, l, m, m_s).

8.4 The Exclusion Principle and the Periodic Table

57. (a) How many electrons can be in the $n = 4$ shell?

(b) What are its subshells, and how many electrons can be in each?

58. (a) What is the minimum value of **l** for a subshell that contains 11 electrons?

(b) If this subshell is in the $n = 5$ shell, what is the spectroscopic notation for this atom?

59. Unreasonable result. Which of the following spectroscopic notations are not allowed?

(a) $5s^1$

(b) $1d^1$

(c) $4s^3$

(d) $3p^7$

(e) $5g^{15}$

State which rule is violated for each notation that is not allowed.

60. Write the electron configuration for potassium.

61. Write the electron configuration for iron.

62. The valence electron of potassium is excited to a 5d state.

(a) What is the magnitude of the electron's orbital angular momentum?

(b) How many states are possible along a chosen direction?

63. (a) If one subshell of an atom has nine electrons in it, what is the minimum value of **l**?

(b) What is the spectroscopic notation for this atom, if this subshell is part of the $n = 3$ shell?

64. Write the electron configuration for magnesium.

65. Write the electron configuration for carbon.

66. The magnitudes of the resultant spins of the electrons of the elements B through Ne when in the ground state are: $\sqrt{3}\hbar/2, \sqrt{2}\hbar, \sqrt{15}\hbar/2, \sqrt{2}\hbar, \sqrt{3}\hbar/2,$ and 0, respectively. Argue that these spins are consistent with Hund's rule.

8.5 Atomic Spectra and X-rays

67. What is the minimum frequency of a photon required to ionize:

(a) a He^+ ion in its ground state?

(b) A Li^{2+} ion in its first excited state?

68. The ion Li^{2+} makes an atomic transition from an $n = 4$ state to an $n = 2$ state.

(a) What is the energy of the photon emitted during the transition?

(b) What is the wavelength of the photon?

69. The red light emitted by a ruby laser has a wavelength of 694.3 nm. What is the difference in energy between the initial state and final state corresponding to the emission of the light?

70. The yellow light from a sodium-vapor street lamp is produced by a transition of sodium atoms from a 3**p** state to a 3**s** state. If the difference in energies of those two states is 2.10 eV, what is the wavelength of the yellow light?

71. Estimate the wavelength of the K_α X-ray from calcium.

72. Estimate the frequency of the K_α X-ray from cesium.

73. X-rays are produced by striking a target with a beam of electrons. Prior to striking the target, the electrons are accelerated by an electric field through a potential energy difference: $\Delta U = -e\Delta V$, where **e** is the charge of an electron and ΔV is the voltage difference. If $\Delta V = 15,000$ volts, what is the minimum wavelength of the emitted radiation?

74. For the preceding problem, what happens to the minimum wavelength if the voltage across the X-ray tube is doubled?

75. Suppose the experiment in the preceding problem is conducted with muons. What happens to the minimum wavelength?

76. An X-ray tube accelerates an electron with an applied voltage of 50 kV toward a metal target.

(a) What is the shortest-wavelength X-ray radiation generated at the target?

(b) Calculate the photon energy in eV.

(c) Explain the relationship of the photon energy to the applied voltage.

77. A color television tube generates some X-rays when its electron beam strikes the screen. What is the shortest wavelength of these X-rays, if a 30.0-kV potential is used to accelerate the electrons? (Note that TVs have shielding to prevent these X-rays from exposing viewers.)

78. An X-ray tube has an applied voltage of 100 kV.

(a) What is the most energetic X-ray photon it can produce? Express your answer in electron volts and joules.

(b) Find the wavelength of such an X-ray.

79. The maximum characteristic X-ray photon energy comes from the capture of a free electron into a **K** shell vacancy. What is this photon energy in keV for tungsten, assuming that the free electron has no initial kinetic energy?

80. What are the approximate energies of the K_α and K_β X-rays for copper?

81. Compare the X-ray photon wavelengths for copper and gold.

82. The approximate energies of the K_α and K_β X-rays for copper are $E_{K_\alpha} = 8.00keV$ and $E_{K_\beta} = 9.48keV$, respectively. Determine the ratio of X-ray frequencies of gold to copper, then use this value to estimate the corresponding energies of K_α and K_β X-rays for gold.

8.6 Lasers

83. A carbon dioxide laser used in surgery emits infrared radiation with a wavelength of $10.6\mu m$. In 1.00 ms, this laser raised the temperature of $1.00cm^3$ of flesh to $100^\circ C$ and evaporated it.

(a) How many photons were required? You may assume that flesh has the same heat of vaporization as water.

(b) What was the minimum power output during the flash?

84. An excimer laser used for vision correction emits UV radiation with a wavelength of 193 nm.

(a) Calculate the photon energy in eV.

(b) These photons are used to evaporate corneal tissue, which is very similar to water in its properties. Calculate the amount of energy needed per molecule of water to make the phase change from liquid to gas. That is, divide the heat of vaporization in kJ/kg by the number of water molecules in a kilogram.

(c) Convert this to eV and compare to the photon energy. Discuss the implications.

Additional Problems

85. For a hydrogen atom in an excited state with principal quantum number n, show that the smallest angle that the orbital angular momentum vector can make with respect to the **z**-axis is $\theta = cos^{-1}(\sqrt{\frac{n-1}{n}})$.

86. What is the probability that the 1**s** electron of a hydrogen atom is found between $r = 0$ and $r = \infty$?

87. Sketch the potential energy function of an electron in a hydrogen atom.

(a) What is the value of this function at $r = 0$? in the limit that $r = \infty$?

(b) What is unreasonable or inconsistent with the former result?

88. Find the value of *l*, the orbital angular momentum quantum number, for the Moon around Earth.

89. Show that the maximum number of orbital angular momentum electron states in the nth shell of an atom is n^2. (Ignore electron spin.) (**Hint:** Make a table of the total number of orbital angular momentum states for each shell and find the pattern.)

90. What is the magnitude of an electron magnetic moment?

91. What is the maximum number of electron states in the $n = 5$ shell?

92. A ground-state hydrogen atom is placed in a uniform magnetic field, and a photon is emitted in the transition from a spin-up to spin-down state. The wavelength of the photon is $168\mu m$. What is the strength of the magnetic field?

93. Show that the maximum number of electron states in the nth shell of an atom is $2n^2$.

94. The valence electron of chlorine is excited to a 3**p** state.

(a) What is the magnitude of the electron's orbital angular momentum?

(b) What are possible values for the **z**-component of angular measurement?

95. Which of the following notations are allowed (that is, which violate none of the rules regarding values of quantum numbers)?

(a) $1s^1$;

(b) $1d^3$;

(c) $4s^2$;

(d) $3p^7$;

(e) $6h^{20}$

96. The ion Be^{3+} makes an atomic transition from an $n = 3$ state to an $n = 2$ state.

(a) What is the energy of the photon emitted during the transition?

(b) What is the wavelength of the photon?

97. The maximum characteristic X-ray photon energy comes from the capture of a free electron into a **K** shell vacancy. What is this photon frequency for tungsten, assuming that the free electron has no initial kinetic energy?

98. Derive an expression for the ratio of X-ray photon frequency for two elements with atomic numbers Z_1 and Z_2.

99. Compare the X-ray photon wavelengths for copper and silver.

100. (a) What voltage must be applied to an X-ray tube to obtain 0.0100-fm-wavelength X-rays for use in exploring the details of nuclei?

(b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?

101. A student in a physics laboratory observes a hydrogen spectrum with a diffraction grating for the purpose of measuring the wavelengths of the emitted radiation. In the spectrum, she observes a yellow line and finds its wavelength to be 589 nm.

(a) Assuming that this is part of the Balmer series, determine n_i, the principal quantum number of the initial state.

(b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?