

13.E: Special Relativity (Exercise)

Conceptual Questions

28.1: Einstein's Postulates

1. Which of Einstein's postulates of special relativity includes a concept that does not fit with the ideas of classical physics? Explain.
2. Is Earth an inertial frame of reference? Is the Sun? Justify your response.
3. When you are flying in a commercial jet, it may appear to you that the airplane is stationary and the Earth is moving beneath you. Is this point of view valid? Discuss briefly.

28.2: Simultaneity and Time Dilation

4. Does motion affect the rate of a clock as measured by an observer moving with it? Does motion affect how an observer moving relative to a clock measures its rate?
5. To whom does the elapsed time for a process seem to be longer, an observer moving relative to the process or an observer moving with the process? Which observer measures proper time?
6. How could you travel far into the future without aging significantly? Could this method also allow you to travel into the past?

28.3: Length Contraction

7. To whom does an object seem greater in length, an observer moving with the object or an observer moving relative to the object? Which observer measures the object's proper length?
8. Relativistic effects such as time dilation and length contraction are present for cars and airplanes. Why do these effects seem strange to us?
9. Suppose an astronaut is moving relative to the Earth at a significant fraction of the speed of light.
 - (a) Does he observe the rate of his clocks to have slowed?
 - (b) What change in the rate of Earth-bound clocks does he see?
 - (c) Does his ship seem to him to shorten?
 - (d) What about the distance between stars that lie on lines parallel to his motion?
 - (e) Do he and an Earth-bound observer agree on his velocity relative to the Earth?

28.4: Relativistic Addition of Velocities

10. Explain the meaning of the terms "red shift" and "blue shift" as they relate to the relativistic Doppler effect.
11. What happens to the relativistic Doppler effect when relative velocity is zero? Is this the expected result?
12. Is the relativistic Doppler effect consistent with the classical Doppler effect in the respect that λ_{obs} is larger for motion away?
13. All galaxies farther away than about $50 \times 10^6 ly$ exhibit a red shift in their emitted light that is proportional to distance, with those farther and farther away having progressively greater red shifts. What does this imply, assuming that the only source of red shift is relative motion? (Hint: At these large distances, it is space itself that is expanding, but the effect on light is the same.)

28.5: Relativistic Momentum

14. How does modern relativity modify the law of conservation of momentum?
15. Is it possible for an external force to be acting on a system and relativistic momentum to be conserved? Explain.

28.6: Relativistic Energy

16. How are the classical laws of conservation of energy and conservation of mass modified by modern relativity?
17. What happens to the mass of water in a pot when it cools, assuming no molecules escape or are added? Is this observable in practice? Explain.
18. Consider a thought experiment. You place an expanded balloon of air on weighing scales outside in the early morning. The balloon stays on the scales and you are able to measure changes in its mass. Does the mass of the balloon change as the day progresses? Discuss the difficulties in carrying out this experiment.
19. The mass of the fuel in a nuclear reactor decreases by an observable amount as it puts out energy. Is the same true for the coal and oxygen combined in a conventional power plant? If so, is this observable in practice for the coal and oxygen? Explain.
20. We know that the velocity of an object with mass has an upper limit of c . Is there an upper limit on its momentum? Its energy? Explain.
21. Given the fact that light travels at c , can it have mass? Explain.
22. If you use an Earth-based telescope to project a laser beam onto the Moon, you can move the spot across the Moon's surface at a velocity greater than the speed of light. Does this violate modern relativity? (Note that light is being sent from the Earth to the Moon, not across the surface of the Moon.)

Problems & Exercises

28.2: Simultaneity and Time Dilation

23. (a) What is γ if $v = 0.250c$?

(b) If $v = 0.500c$?

Solution

(a) 1.0328

(b) 1.15

24. (a) What is γ if $v = 0.100c$?

(b) If $v = 0.900c$?

25. Particles called π -mesons are produced by accelerator beams. If these particles travel at $2.70 \times 10^8 \text{ m/s}$ and live $2.60 \times 10^{-8} \text{ s}$ when at rest relative to an observer, how long do they live as viewed in the laboratory?

Solution

$5.96 \times 10^{-8} \text{ s}$

26. Suppose a particle called a kaon is created by cosmic radiation striking the atmosphere. It moves by you at $0.980c$ and it lives $1.24 \times 10^{-8} \text{ s}$ when at rest relative to an observer. How long does it live as you observe it?

27. A neutral π -meson is a particle that can be created by accelerator beams. If one such particle lives $1.40 \times 10^{-16} \text{ s}$ as measured in the laboratory, and $0.840 \times 10^{-16} \text{ s}$ when at rest relative to an observer, what is its velocity relative to the laboratory?

Solution

$0.800c$

28. A neutron lives 900 s when at rest relative to an observer. How fast is the neutron moving relative to an observer who measures its life span to be 2065 s?

29. If relativistic effects are to be less than 1%, then γ must be less than 1.01. At what relative velocity is $\gamma = 1.01$?

Solution

$0.140c$

30. If relativistic effects are to be less than 3%, then γ must be less than 1.03. At what relative velocity is $\gamma = 1.03$?

31. (a) At what relative velocity is $\gamma = 1.50$?

(b) At what relative velocity is $\gamma = 100$?

Solution

(a) $0.745c$

(b) $0.99995c$ (to five digits to show effect)

32. (a) At what relative velocity is $\gamma = 2.00$?

(b) At what relative velocity is $\gamma = 10.0$?

33. *Unreasonable Results*

(a) Find the value of γ for the following situation. An Earth-bound observer measures 23.9 h to have passed while signals from a high-velocity space probe indicate that 24.0 h have passed on board.

(b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?

Solution

(a) 0.996

(b) γ cannot be less than 1.

(c) Assumption that time is longer in moving ship is unreasonable.

28.3: Length Contraction

34. A spaceship, 200 m long as seen on board, moves by the Earth at $0.970c$. What is its length as measured by an Earth-bound observer?

Solution

48.6 m

35. How fast would a 6.0 m-long sports car have to be going past you in order for it to appear only 5.5 m long?

36. (a) How far does the muon in [link] travel according to the Earth-bound observer?

(b) How far does it travel as viewed by an observer moving with it? Base your calculation on its velocity relative to the Earth and the time it lives (proper time).

(c) Verify that these two distances are related through length contraction $\gamma = 3.20$.

Solution

(a) 1.387 km = 1.39 km

(b) 0.433 km

$$(c) L = \frac{L_0}{\gamma} = \frac{1.387 \times 10^3 \text{ m}}{3.20} = 433.4 \text{ m} = 0.433 \text{ km}$$

Thus, the distances in parts (a) and (b) are related when $\gamma = 3.20$.

37. (a) How long would the muon in [link] have lived as observed on the Earth if its velocity was $0.0500c$?

(b) How far would it have traveled as observed on the Earth? (c) What distance is this in the muon's frame?

38. (a) How long does it take the astronaut in Example to travel 4.30 ly at $0.99944c$ (as measured by the Earth-bound observer)?

(b) How long does it take according to the astronaut?

(c) Verify that these two times are related through time dilation with $\gamma = 30.00$ as given.

Solution

(a) 4.303 y (to four digits to show any effect)

(b) 0.1434 y

$$(c) \Delta t = \gamma \Delta t_0 \Rightarrow \gamma = \frac{\Delta t}{\Delta t_0} = \frac{4.303 \text{ y}}{0.1434 \text{ y}} = 30.0$$

Thus, the two times are related when $\gamma = 30.00$.

39. (a) How fast would an athlete need to be running for a 100-m race to look 100 yd long?

(b) Is the answer consistent with the fact that relativistic effects are difficult to observe in ordinary circumstances? Explain.

40. Unreasonable Results

(a) Find the value of γ for the following situation. An astronaut measures the length of her spaceship to be 25.0 m, while an Earth-bound observer measures it to be 100 m.

(b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?

Solution

(a) 0.250

(b) γ must be ≥ 1

(c) The Earth-bound observer must measure a shorter length, so it is unreasonable to assume a longer length.

41. Unreasonable Results

A spaceship is heading directly toward the Earth at a velocity of $0.800c$. The astronaut on board claims that he can send a canister toward the Earth at $1.20c$ relative to the Earth.

(a) Calculate the velocity the canister must have relative to the spaceship.

(b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?

28.4: Relativistic Addition of Velocities

42. Suppose a spaceship heading straight towards the Earth at $0.750c$ can shoot a canister at $0.500c$ relative to the ship.

(a) What is the velocity of the canister relative to the Earth, if it is shot directly at the Earth?

(b) If it is shot directly away from the Earth?

Solution

(a) $0.909c$

(b) $0.400c$

43. Repeat the previous problem with the ship heading directly away from the Earth.

44. If a spaceship is approaching the Earth at $0.100c$ and a message capsule is sent toward it at $0.100c$ relative to the Earth, what is the speed of the capsule relative to the ship?

Solution

$0.198c$

45. (a) Suppose the speed of light were only 3000 m/s . A jet fighter moving toward a target on the ground at 800 m/s shoots bullets, each having a muzzle velocity of 1000 m/s . What are the bullets' velocity relative to the target?

(b) If the speed of light was this small, would you observe relativistic effects in everyday life? Discuss.

46. If a galaxy moving away from the Earth has a speed of 1000 km/s and emits 656 nm light characteristic of hydrogen (the most common element in the universe). (a) What wavelength would we observe on the Earth?

(b) What type of electromagnetic radiation is this?

(c) Why is the speed of the Earth in its orbit negligible here?

Solution

a) 658 nm

b) red

c) $v/c = 9.92 \times 10^{-5}$ (negligible)

47. A space probe speeding towards the nearest star moves at $0.250c$ and sends radio information at a broadcast frequency of 1.00 GHz . What frequency is received on the Earth?

48. If two spaceships are heading directly towards each other at $0.800c$, at what speed must a canister be shot from the first ship to approach the other at $0.999c$ as seen by the second ship?

Solution
 $0.991c$

49. Two planets are on a collision course, heading directly towards each other at $0.250c$. A spaceship sent from one planet approaches the second at $0.750c$ as seen by the second planet. What is the velocity of the ship relative to the first planet?

50. When a missile is shot from one spaceship towards another, it leaves the first at $0.950c$ and approaches the other at $0.750c$. What is the relative velocity of the two ships?

Solution
 $-0.696c$

51. What is the relative velocity of two spaceships if one fires a missile at the other at $0.750c$ and the other observes it to approach at $0.950c$?

52. Near the center of our galaxy, hydrogen gas is moving directly away from us in its orbit about a black hole. We receive 1900 nm electromagnetic radiation and know that it was 1875 nm when emitted by the hydrogen gas. What is the speed of the gas?

Solution
 $0.01324c$

53. A highway patrol officer uses a device that measures the speed of vehicles by bouncing radar off them and measuring the Doppler shift. The outgoing radar has a frequency of 100 GHz and the returning echo has a frequency 15.0 kHz higher. What is the velocity of the vehicle? Note that there are two Doppler shifts in echoes. Be certain not to round off until the end of the problem, because the effect is small.

54. Prove that for any relative velocity v between two observers, a beam of light sent from one to the other will approach at speed c (provided that v is less than c , of course).

Solution
 $u' = c$, so

$$u = \frac{v + u'}{1 + (vu'/c^2)} = \frac{v + c}{1 + (vc/c^2)} = \frac{v + c}{1 + (v/c)} = \frac{c(v + c)}{c + v} = c$$

55. Show that for any relative velocity v between two observers, a beam of light projected by one directly away from the other will move away at the speed of light (provided that v is less than c , of course).

56. (a) All but the closest galaxies are receding from our own Milky Way Galaxy. If a galaxy $12.0 \times 10^9\text{ ly}$ away is receding from us at $0.0900c$, at what velocity relative to us must we send an exploratory probe to approach the other galaxy at $0.990c$, as measured from that galaxy?

(b) How long will it take the probe to reach the other galaxy as measured from the Earth? You may assume that the velocity of the other galaxy remains constant.

(c) How long will it then take for a radio signal to be beamed back? (All of this is possible in principle, but not practical.)

Solution
 a) $0.99947c$
 b) $1.2064 \times 10^{11}\text{ y}$
 c) $1.2058 \times 10^{12}\text{ y}$ (all to sufficient digits to show effects)

28.5: Relativistic Momentum

57. Find the momentum of a helium nucleus having a mass of $6.68 \times 10^{-27}\text{ kg}$ that is moving at $0.200c$.

Solution
 $4.09 \times 10^{-19}\text{ kg} \cdot \text{m/s}$

58. What is the momentum of an electron traveling at $0.980c$?

59. (a) Find the momentum of a $1.00 \times 10^9 \text{ kg}$ asteroid heading towards the Earth at 30.0 km/s.
 (b) Find the ratio of this momentum to the classical momentum. (Hint: Use the approximation that $\gamma = 1 + (1/2)v^2/c^2$ at low velocities.)

Solution

- (a) $3.000000015 \times 10^{13} \text{ kg} \cdot \text{m/s}$.
 (b) Ratio of relativistic to classical momenta equals 1.000000005 (extra digits to show small effects)
60. (a) What is the momentum of a 2000 kg satellite orbiting at 4.00 km/s?
 (b) Find the ratio of this momentum to the classical momentum. (Hint: Use the approximation that $\gamma = 1 + (1/2)v^2/c^2$ at low velocities.)
61. What is the velocity of an electron that has a momentum of $3.04 \times 10^{-21} \text{ kg} \cdot \text{m/s}$? Note that you must calculate the velocity to at least four digits to see the difference from c.

Solution

- $2.9957 \times 10^8 \text{ m/s}$
62. Find the velocity of a proton that has a momentum of $4.48 \times 10^{-19} \text{ kg} \cdot \text{m/s}$.
63. (a) Calculate the speed of a $1.00 - \mu\text{g}$ particle of dust that has the same momentum as a proton moving at $0.999c$.
 (b) What does the small speed tell us about the mass of a proton compared to even a tiny amount of macroscopic matter?
- Solution**
- (a) $1.121 \times 10^{-8} \text{ m/s}$
 (b) The small speed tells us that the mass of a proton is substantially smaller than that of even a tiny amount of macroscopic matter!
64. (a) Calculate γ for a proton that has a momentum of $1.00 \text{ kg} \cdot \text{m/s}$.
 (b) What is its speed? Such protons form a rare component of cosmic radiation with uncertain origins.

28.6: Relativistic Energy

65. What is the rest energy of an electron, given its mass is $9.11 \times 10^{-31} \text{ kg}$? Give your answer in joules and MeV.
- Solution**
- $8.20 \times 10^{-14} \text{ J}$
 0.512 MeV
66. Find the rest energy in joules and MeV of a proton, given its mass is $1.67 \times 10^{-27} \text{ kg}$.
67. If the rest energies of a proton and a neutron (the two constituents of nuclei) are 938.3 and 939.6 MeV respectively, what is the difference in their masses in kilograms?

Solution

- $2.3 \times 10^{-30} \text{ kg}$
68. The Big Bang that began the universe is estimated to have released 10^{68} J of energy. How many stars could half this energy create, assuming the average star's mass is $4.00 \times 10^{30} \text{ kg}$?
69. A supernova explosion of a $2.00 \times 10^{31} \text{ kg}$ star produces $1.00 \times 10^{44} \text{ J}$ of energy.
 (a) How many kilograms of mass are converted to energy in the explosion?
 (b) What is the ratio $\Delta m/m$ of mass destroyed to the original mass of the star?
- Solution**
- (a) $1.11 \times 10^{27} \text{ kg}$
 (b) 5.56×10^{-5}

70. (a) Using data from [link], calculate the mass converted to energy by the fission of 1.00 kg of uranium.

(b) What is the ratio of mass destroyed to the original mass, $\Delta m/m$?

71. (a) Using data from [link], calculate the amount of mass converted to energy by the fusion of 1.00 kg of hydrogen.

(b) What is the ratio of mass destroyed to the original mass, $\Delta m/m$?

(c) How does this compare with $\Delta m/m$ for the fission of 1.00 kg of uranium?

Solution

$$7.1 \times 10^{-3} \text{ kg}$$

$$7.1 \times 10^{-3}$$

The ratio is greater for hydrogen.

72. There is approximately 10^{34} J of energy available from fusion of hydrogen in the world's oceans.

(a) If 10^{33} J of this energy were utilized, what would be the decrease in mass of the oceans? Assume that 0.08% of the mass of a water molecule is converted to energy during the fusion of hydrogen.

(b) How great a volume of water does this correspond to?

(c) Comment on whether this is a significant fraction of the total mass of the oceans.

73. A muon has a rest mass energy of 105.7 MeV, and it decays into an electron and a massless particle.

(a) If all the lost mass is converted into the electron's kinetic energy, find γ for the electron.

(b) What is the electron's velocity?

Solution

$$208$$

$$0.999988c$$

74. A π -meson is a particle that decays into a muon and a massless particle. The π -meson has a rest mass energy of 139.6 MeV, and the muon has a rest mass energy of 105.7 MeV. Suppose the π -meson is at rest and all of the missing mass goes into the muon's kinetic energy. How fast will the muon move?

75. (a) Calculate the relativistic kinetic energy of a 1000-kg car moving at 30.0 m/s if the speed of light were only 45.0 m/s.

(b) Find the ratio of the relativistic kinetic energy to classical.

Solution

$$6.92 \times 10^5 \text{ J}$$

$$1.54$$

76. Alpha decay is nuclear decay in which a helium nucleus is emitted. If the helium nucleus has a mass of $6.80 \times 10^{-27} \text{ kg}$ and is given 5.00 MeV of kinetic energy, what is its velocity?

77. (a) Beta decay is nuclear decay in which an electron is emitted. If the electron is given 0.750 MeV of kinetic energy, what is its velocity?

(b) Comment on how the high velocity is consistent with the kinetic energy as it compares to the rest mass energy of the electron.

Solution

$$(a) 0.914c$$

(b) The rest mass energy of an electron is 0.511 MeV, so the kinetic energy is approximately 150% of the rest mass energy. The electron should be traveling close to the speed of light.

78. A positron is an antimatter version of the electron, having exactly the same mass. When a positron and an electron meet, they annihilate, converting all of their mass into energy.

(a) Find the energy released, assuming negligible kinetic energy before the annihilation.

(b) If this energy is given to a proton in the form of kinetic energy, what is its velocity?

(c) If this energy is given to another electron in the form of kinetic energy, what is its velocity?

79. What is the kinetic energy in MeV of a π -meson that lives $1.40 \times 10^{-16} \text{ s}$ as measured in the laboratory, and $0.840 \times 10^{-16} \text{ s}$ when at rest relative to an observer, given that its rest energy is 135 MeV?

Solution

90.0 MeV

80. Find the kinetic energy in MeV of a neutron with a measured life span of 2065 s, given its rest energy is 939.6 MeV, and rest life span is 900s.

81. (a) Show that $(pc)^2 / (mc^2)^2 = \gamma^2 - 1$. This means that at large velocities $pc \gg mc^2$.

(b) Is $E \approx pc$ when $\gamma = 30.0$, as for the astronaut discussed in the twin paradox?

Solution

(a) $E^2 = p^2 c^2 + m^2 c^4 = \gamma^2 m^2 c^4$, so that $p^2 c^2 = (\gamma^2 - 1) m^2 c^4$, and therefore $\frac{(pc)^2}{(mc^2)^2} = \gamma^2 - 1$

(b) yes

82. One cosmic ray neutron has a velocity of $0.250c$ relative to the Earth.

(a) What is the neutron's total energy in MeV?

(b) Find its momentum.

(c) Is $E \approx pc$ in this situation? Discuss in terms of the equation given in part (a) of the previous problem.

83. What is γ for a proton having a mass energy of 938.3 MeV accelerated through an effective potential of 1.0 TV (teravolt) at Fermilab outside Chicago?

Solution

1.07×10^3

84. (a) What is the effective accelerating potential for electrons at the Stanford Linear Accelerator, if $\gamma = 1.00 \times 10^5$ for them?

(b) What is their total energy (nearly the same as kinetic in this case) in GeV?

85. (a) Using data from [link], find the mass destroyed when the energy in a barrel of crude oil is released.

(b) Given these barrels contain 200 liters and assuming the density of crude oil is 750 kg/m^3 , what is the ratio of mass destroyed to original mass, $\Delta m / m$?

Solution

$6.56 \times 10^{-8} \text{ kg}$

4.37×10^{-10}

86. (a) Calculate the energy released by the destruction of 1.00 kg of mass.

(b) How many kilograms could be lifted to a 10.0 km height by this amount of energy?

87. A Van de Graaff accelerator utilizes a 50.0 MV potential difference to accelerate charged particles such as protons.

(a) What is the velocity of a proton accelerated by such a potential?

(b) An electron?

Solution

$0.314c$

$0.99995c$

88. Suppose you use an average of $500 \text{ kW} \cdot \text{h}$ of electric energy per month in your home.

(a) How long would 1.00 g of mass converted to electric energy with an efficiency of 38.0% last you?

(b) How many homes could be supplied at the $500 \text{ kW} \cdot \text{h}$ per month rate for one year by the energy from the described mass conversion?

89. (a) A nuclear power plant converts energy from nuclear fission into electricity with an efficiency of 35.0%. How much mass is destroyed in one year to produce a continuous 1000 MW of electric power?

(b) Do you think it would be possible to observe this mass loss if the total mass of the fuel is 10^4 kg ?

Solution

(a) 1.00 kg

(b) This much mass would be measurable, but probably not observable just by looking because it is 0.01% of the total mass.

90. Nuclear-powered rockets were researched for some years before safety concerns became paramount.

(a) What fraction of a rocket's mass would have to be destroyed to get it into a low Earth orbit, neglecting the decrease in gravity? (Assume an orbital altitude of 250 km, and calculate both the kinetic energy (classical) and the gravitational potential energy needed.)

(b) If the ship has a mass of $1.00 \times 10^5 \text{ kg}$ (100 tons), what total yield nuclear explosion in tons of TNT is needed?

91. The Sun produces energy at a rate of $4.00 \times 10^{26} \text{ W}$ by the fusion of hydrogen.

(a) How many kilograms of hydrogen undergo fusion each second?

(b) If the Sun is 90.0% hydrogen and half of this can undergo fusion before the Sun changes character, how long could it produce energy at its current rate?

(c) How many kilograms of mass is the Sun losing per second?

(d) What fraction of its mass will it have lost in the time found in part (b)?

Solution

(a) $6.3 \times 10^{11} \text{ kg/s}$

(b) $4.5 \times 10^{10} \text{ y}$

(c) $4.44 \times 10^9 \text{ kg}$

(d) 0.32%

92. *Unreasonable Results*

A proton has a mass of $1.67 \times 10^{-27} \text{ kg}$. A physicist measures the proton's total energy to be 50.0 MeV.

(a) What is the proton's kinetic energy?

(b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?

93. *Construct Your Own Problem*

Consider a highly relativistic particle. Discuss what is meant by the term "highly relativistic." (Note that, in part, it means that the particle cannot be massless.) Construct a problem in which you calculate the wavelength of such a particle and show that it is very nearly the same as the wavelength of a massless particle, such as a photon, with the same energy. Among the things to be considered are the rest energy of the particle (it should be a known particle) and its total energy, which should be large compared to its rest energy.

94. *Construct Your Own Problem*

Consider an astronaut traveling to another star at a relativistic velocity. Construct a problem in which you calculate the time for the trip as observed on the Earth and as observed by the astronaut. Also calculate the amount of mass that must be converted to energy to get the astronaut and ship to the velocity travelled. Among the things to be considered are the distance to the star, the velocity, and the mass of the astronaut and ship. Unless your instructor directs you otherwise, do not include any energy given to other masses, such as rocket propellants.

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