12.E: Quantum Mechanics (Exercise)

Conceptual Questions

13.1: Blackbody Radiation

1. Give an example of a physical entity that is quantized. State specifically what the entity is and what the limits are on its values.

2. Give an example of a physical entity that is not quantized, in that it is continuous and may have a continuous range of values.

3. What aspect of the blackbody spectrum forced Planck to propose quantization of energy levels in the thermal oscillators?

4. Why don't we notice quantization in everyday events?

13.2: The Photoelectric Effect

5. Is visible light the only type of EM radiation that can cause the photoelectric effect?

6. Which aspects of the photoelectric effect cannot be explained without photons? Which can be explained without photons? Are the latter inconsistent with the existence of photons?

7. Is the photoelectric effect a direct consequence of the wave character of EM radiation or of the particle character of EM radiation? Explain briefly.

8. Insulators (nonmetals) have a higher BE than metals, and it is more difficult for photons to eject electrons from insulators. Discuss how this relates to the free charges in metals that make them good conductors.

9. If you pick up and shake a piece of metal that has electrons in it free to move as a current, no electrons fall out. Yet if you heat the metal, electrons can be boiled off. Explain both of these facts as they relate to the amount and distribution of energy involved with shaking the object as compared with heating it.

13.3: The Wave Nature of Matter

10. How does the interference of water waves differ from the interference of electrons? How are they analogous?

11. Describe one type of evidence for the wave nature of matter.

12. Describe one type of evidence for the particle nature of EM radiation.

13.4: Uncertainty Principle

13. What is the Heisenberg uncertainty principle? Does it place limits on what can be known?

13.5: Discovery of the Atomic Nucleus

14. What two pieces of evidence allowed the first calculation of m_e, the mass of the electron?

(a) The ratios q_e/m_e and q_p/m_p.

(b) The values of q_e and E_B.

(c) The ratio q_e/m_e and q_e.

Justify your response.

15. How do the allowed orbits for electrons in atoms differ from the allowed orbits for planets around the sun? Explain how the correspondence principle applies here.

13.6: Bohr's Theory of the Hydrogen Atom

16. How do the allowed orbits for electrons in atoms differ from the allowed orbits for planets around the sun? Explain how the correspondence principle applies here.

17. Explain how Bohr's rule for the quantization of electron orbital angular momentum differs from the actual rule.

18. What is a hydrogen-like atom, and how are the energies and radii of its electron orbits related to those in hydrogen?

13.7: The Wave Nature of Matter Causes Quantization

19. How is the de Broglie wavelength of electrons related to the quantization of their orbits in atoms and molecules?

Problems & Exercises

13.1: Blackbody Radiation

20. A LiBr molecule oscillates with a frequency of 1.7×10^{13} Hz

(a) What is the difference in energy in eV between allowed oscillator states?

(b) What is the approximate value of n for a state having an energy of 1.0 eV?

Solution

(a) 0.070 eV

(b) 14

21. The difference in energy between allowed oscillator states in HBr molecules is 0.330 eV. What is the oscillation frequency of this molecule?

22. A physicist is watching a 15-kg orangutan at a zoo swing lazily in a tire at the end of a rope. He (the physicist) notices that each oscillation takes 3.00 s and hypothesizes that the energy is quantized.

(a) What is the difference in energy in joules between allowed oscillator states?

(b) What is the value of n for a state where the energy is 5.00 J?

(c) Can the quantization be observed?

13.2: The Photoelectric Effect

23. What is the longest-wavelength EM radiation that can eject a photoelectron from silver, given that the binding energy is 4.73 eV? Is this in the visible range?

Solution

263 nm

24. Find the longest-wavelength photon that can eject an electron from potassium, given that the binding energy is 2.24 eV. Is this visible EM radiation?

25. What is the binding energy in eV of electrons in magnesium, if the longest-wavelength photon that can eject electrons is 337 nm?

Solution

3.69 eV

26. Calculate the binding energy in eV of electrons in aluminum, if the longest-wavelength photon that can eject them is 304 nm.

27. What is the maximum kinetic energy in eV of electrons ejected from sodium metal by 450-nm EM radiation, given that the binding energy is 2.28 eV?

Solution

0.483 eV

28. UV radiation having a wavelength of 120 nm falls on gold metal, to which electrons are bound by 4.82 eV. What is the maximum kinetic energy of the ejected photoelectrons?

29. Violet light of wavelength 400 nm ejects electrons with a maximum kinetic energy of 0.860 eV from sodium metal. What is the binding energy of electrons to sodium metal?

Solution

2.25 eV

30. UV radiation having a 300-nm wavelength falls on uranium metal, ejecting 0.500-eV electrons. What is the binding energy of electrons to uranium metal?

31. What is the wavelength of EM radiation that ejects 2.00-eV electrons from calcium metal, given that the binding energy is 2.71 eV? What type of EM radiation is this?

Solution

(a) 264 nm

(b) Ultraviolet

32. Find the wavelength of photons that eject 0.100-eV electrons from potassium, given that the binding energy is 2.24 eV. Are these photons visible?

33. A laser with a power output of 2.00 mW at a wavelength of 400 nm is projected onto calcium metal.

(a) How many electrons per second are ejected?

(b) What power is carried away by the electrons, given that the binding energy is 2.71 eV?

Solution

(a) $4.02 \times 10^{15} / \mathrm{s}$

(b) 0.256 mW

34. (a) Calculate the number of photoelectrons per second ejected from a 1.00-mm ^{2} area of sodium metal by 500-nm EM radiation having an intensity of $1.30\ \mathrm{kW/m^2}$ (the intensity of sunlight above the Earth's atmosphere).

(b) Given that the binding energy is 2.28 eV, what power is carried away by the electrons? (c) The electrons carry away less power than brought in by the photons. Where does the other power go? How can it be recovered?

Unreasonable Results

35. Red light having a wavelength of 700 nm is projected onto magnesium metal to which electrons are bound by 3.68 eV.

(a) Use $\mathrm{KE}_e = hf - \mathrm{BE}$ to calculate the kinetic energy of the ejected electrons.

(b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?

Solution

(a) $-1.90\ \mathrm{eV}$

(b) Negative kinetic energy

(c) That the electrons would be knocked free.

Unreasonable Results

36. (a) What is the binding energy of electrons to a material from which 4.00-eV electrons are ejected by 400-nm EM radiation?

(b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?

13.3: The Wave Nature of Matter

37. At what velocity will an electron have a wavelength of 1.00 m?

Solution

$7.28 \times 10^{-4}\ \mathrm{m}$

38. What is the wavelength of an electron moving at 3.00% of the speed of light?

39. At what velocity does a proton have a 6.00-fm wavelength (about the size of a nucleus)? Assume the proton is nonrelativistic. (1 femtometer = $10^{-15}\ \mathrm{m}$)

Solution

$6.62 \times 10^7 \ \mathrm{m/s}$

40. What is the velocity of a 0.400-kg billiard ball if its wavelength is 7.50 cm (large enough for it to interfere with other billiard balls)?

41. Find the wavelength of a proton moving at 1.00% of the speed of light.

Solution

$1.32 \times 10^{-13} \ \mathrm{m}$

42. Experiments are performed with ultracold neutrons having velocities as small as 1.00 m/s. (a) What is the wavelength of such a neutron? (b) What is its kinetic energy in eV?

43. (a) Find the velocity of a neutron that has a 6.00-fm wavelength (about the size of a nucleus). Assume the neutron is nonrelativistic.

(b) What is the neutron's kinetic energy in MeV?

Solution

(a) $6.62 \times 10^7 \ \mathrm{m/s}$

(b) $22.9 \ \mathrm{MeV}$

44. What is the wavelength of an electron accelerated through a 30.0-kV potential, as in a TV tube?

45. What is the kinetic energy of an electron in a TEM having a 0.0100-nm wavelength?

Solution

15.1 keV

46. (a) Calculate the velocity of an electron that has a wavelength of $1.00 \ \mu\mathrm{m}$

(b) Through what voltage must the electron be accelerated to have this velocity?

47. The velocity of a proton emerging from a Van de Graaff accelerator is 25.0% of the speed of light.

(a) What is the proton's wavelength?

(b) What is its kinetic energy, assuming it is nonrelativistic?

(c) What was the equivalent voltage through which it was accelerated?

Solution

(a) 5.29 fm

(b) $4.70 \times 10^{-12} \ \mathrm{J}$

(c) 29.4 MV

48. The kinetic energy of an electron accelerated in an X-ray tube is 100 keV. Assuming it is nonrelativistic, what is its wavelength?

Unreasonable Results

49. (a) Assuming it is nonrelativistic, calculate the velocity of an electron with a 0.100-fm wavelength (small enough to detect details of a nucleus).

(b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?

Solution

(a) $7.28 \times 10^{12} \ \mathrm{m/s}$

(b) This is thousands of times the speed of light (an impossibility).

(c) The assumption that the electron is non-relativistic is unreasonable at this wavelength.

13.4: Uncertainty Principle

50. (a) If the position of an electron in a membrane is measured to an accuracy of $1.00\ \mu\text{m}$, what is the electron's minimum uncertainty in velocity?

(b) If the electron has this velocity, what is its kinetic energy in eV?

(c) What are the implications of this energy, comparing it to typical molecular binding energies?

Solution

(a) 57.9 m/s

(b) $9.55 \times 10^{-9}\ \text{eV}$

(c) Typical molecular binding energies range from about 1eV to 10 eV, therefore the result in part (b) is approximately 9 orders of magnitude smaller than typical molecular binding energies.

51. (a) If the position of a chlorine ion in a membrane is measured to an accuracy of $1.00\ \mu\text{m}$, what is its minimum uncertainty in velocity, given its mass is $5.86 \times 10^{-26}\ \text{kg}$?

(b) If the ion has this velocity, what is its kinetic energy in eV, and how does this compare with typical molecular binding energies?

52. Suppose the velocity of an electron in an atom is known to an accuracy of $2.0 \times 10^{3}\ \text{m/s}$ (reasonably accurate compared with orbital velocities). What is the electron's minimum uncertainty in position, and how does this compare with the approximate 0.1-nm size of the atom?

Solution

29 nm,

290 times greater

53. The velocity of a proton in an accelerator is known to an accuracy of 0.250% of the speed of light. (This could be small compared with its velocity.) What is the smallest possible uncertainty in its position?

54. A relatively long-lived excited state of an atom has a lifetime of 3.00 ms. What is the minimum uncertainty in its energy?

Solution

$1.10 \times 10^{-13}\ \text{eV}$

55. (a) The lifetime of a highly unstable nucleus is $10^{-20}\ \text{s}$. What is the smallest uncertainty in its decay energy?

(b) Compare this with the rest energy of an electron.

56. The decay energy of a short-lived particle has an uncertainty of 1.0 MeV due to its short lifetime. What is the smallest lifetime it can have?

Solution

$3.3 \times 10^{-22}\ \text{s}$

57. The decay energy of a short-lived nuclear excited state has an uncertainty of 2.0 eV due to its short lifetime. What is the smallest lifetime it can have?

58. What is the approximate uncertainty in the mass of a muon, as determined from its decay lifetime?

Solution

$2.66 \times 10^{-46}\ \text{kg}$

59. Derive the approximate form of Heisenberg's uncertainty principle for energy and time, $\Delta E \Delta t \approx h$, using the following arguments: Since the position of a particle is uncertain by $\Delta x \approx \lambda$, where λ is the wavelength of the photon used to examine it, there is an uncertainty in the time the photon takes to traverse Δx. Furthermore, the photon has an energy related to its wavelength, and it can transfer some or all of this energy to the object being examined. Thus the uncertainty in the energy of the object is also related to λ. Find Δt and ΔE; then multiply them to give the approximate uncertainty principle.

13.5: Discovery of the Atomic Nucleus

60. Rutherford found the size of the nucleus to be about $10^{-15}\ \mathrm{m}$. This implied a huge density. What would this density be for gold?

Solution

$6\times 10^{20}\ \mathrm{kg/m^3}$

61. In Millikan's oil-drop experiment, one looks at a small oil drop held motionless between two plates. Take the voltage between the plates to be 2033 V, and the plate separation to be 2.00 cm. The oil drop (of density $0.81\ \mathrm{g/cm^3}$) has a diameter of $4.0\times 10^{-6}\ \mathrm{m}$. Find the charge on the drop, in terms of electron units.

62. (a) An aspiring physicist wants to build a scale model of a hydrogen atom for her science fair project. If the atom is 1.00 m in diameter, how big should she try to make the nucleus?

(b) How easy will this be to do?

Solution

(a) $10.0\ \mu\mathrm{m}$

(b) It isn't hard to make one of approximately this size. It would be harder to make it exactly $10.0\ \mu\mathrm{m}$.

13.6: Bohr's Theory of the Hydrogen Atom

63. By calculating its wavelength, show that the first line in the Lyman series is UV radiation.

Solution

$\frac{1}{\lambda}=R\left(\frac{1}{n_\mathrm{f}^2}-\frac{1}{n_\mathrm{i}^2}\right)\Rightarrow\lambda=\frac{1}{R}\left[\frac{(n_\mathrm{i}\cdot n_\mathrm{f})^2}{n_\mathrm{i}^2-n_\mathrm{f}^2}\right]$; $n_\mathrm{i}=2,\, n_\mathrm{f}=1$ so that

$\lambda=\left(\frac{\mathrm{m}}{1.097\times 10^7}\right)\left[\frac{(2\times 1)^2}{2^2-1^2}\right]=1.22\times 10^{-7}\ \mathrm{m}=122\ \mathrm{nm}$, which is UV radiation.

64. Find the wavelength of the third line in the Lyman series, and identify the type of EM radiation.

65. Look up the values of the quantities in $a_\mathrm{B}=\frac{h^2}{4\pi^2 m_e k q_e^2}$, and verify that the Bohr radius a_B is $0.529\times 10^{-10}\ \mathrm{m}$.

Solution

$$a_\mathrm{B}=\frac{h^2}{4\pi^2 m_e kZq_e^2}=\frac{\left(6.626\times 10^{-34}\ \mathrm{J\cdot s}\right)^2}{4\pi^2\left(9.109\times 10^{-31}\ \mathrm{kg}\right)\left(8.988\times 10^9\ \mathrm{N\cdot m^2/C^2}\right)(1)\left(1.602\times 10^{-19}\ \mathrm{C}\right)^2}=0.529\times 10^{-10}\ \mathrm{m}$$

66. Verify that the ground state energy E_0 is 13.6 eV by using $E_0=\frac{2\pi^2 q_e^4 m_e k^2}{h^2}$.

67. If a hydrogen atom has its electron in the $n=4$ state, how much energy in eV is needed to ionize it?

Solution

0.850 eV

68. A hydrogen atom in an excited state can be ionized with less energy than when it is in its ground state. What is n for a hydrogen atom if 0.850 eV of energy can ionize it?

69. Find the radius of a hydrogen atom in the $n=2$ state according to Bohr's theory.

Solution

$2.12\times 10^{-10}\ \mathrm{m}$

70. Show that $(13.6\mathrm{eV})/hc=1.097\times 10^7\ \mathrm{m}=R$ (Rydberg's constant), as discussed in the text.

71. What is the smallest-wavelength line in the Balmer series? Is it in the visible part of the spectrum?

Solution

365 nm

It is in the ultraviolet.

72. Show that the entire Paschen series is in the infrared part of the spectrum. To do this, you only need to calculate the shortest wavelength in the series.

73. Do the Balmer and Lyman series overlap? To answer this, calculate the shortest-wavelength Balmer line and the longest-wavelength Lyman line.

Solution

No overlap

365 nm

122 nm

74. (a) Which line in the Balmer series is the first one in the UV part of the spectrum?

(b) How many Balmer series lines are in the visible part of the spectrum?

(c) How many are in the UV?

75. A wavelength of $4.653\ \mu\mathrm{m}$ is observed in a hydrogen spectrum for a transition that ends in the $n_\mathrm{f} = 5$ level. What was n_i for the initial level of the electron?

Solution

7

76. A singly ionized helium ion has only one electron and is denoted He^+. What is the ion's radius in the ground state compared to the Bohr radius of hydrogen atom?

77. A beryllium ion with a single electron (denoted Be^{3+}) is in an excited state with radius the same as that of the ground state of hydrogen.

(a) What is n for the Be^{3+} ion?

(b) How much energy in eV is needed to ionize the ion from this excited state?

Solution

(a) 2

(b) 54.4 eV

78. Atoms can be ionized by thermal collisions, such as at the high temperatures found in the solar corona. One such ion is C^{+5}, a carbon atom with only a single electron.

(a) By what factor are the energies of its hydrogen-like levels greater than those of hydrogen?

(b) What is the wavelength of the first line in this ion's Paschen series?

(c) What type of EM radiation is this?

79. Verify Equations $r_n = \frac{n^2}{Z} a_\mathrm{B}$ and $a_\mathrm{B} = \frac{h^2}{4\pi^2 m_e k q_e^2} = 0.529 \times 10^{-10}\ \mathrm{m}$ using the approach stated in the text. That is, equate the Coulomb and centripetal forces and then insert an expression for velocity from the condition for angular momentum quantization.

Solution

$\frac{kZq_e^2}{r_n^2} = \frac{m_e V^2}{r_n}$, so that $r_n = \frac{kZq_e^2}{m_e V^2} = \frac{kZq_e^2}{m_e} \frac{1}{V^2}$. From the equation $m_e v r_n = n \frac{h}{2\pi}$, we can substitute for the velocity, giving: $r_n = \frac{kZq_e^2}{m_e}$ so that $r_n = \frac{n^2}{Z} \frac{h^2}{4\pi^2 m_e k q_e^2} = \frac{n^2}{Z} a_\mathrm{B}$, where $a_\mathrm{B} = \frac{h^2}{4\pi^2 m_e k q_e^2}$.

80. The wavelength of the four Balmer series lines for hydrogen are found to be 410.3, 434.2, 486.3, and 656.5 nm. What average percentage difference is found between these wavelength numbers and those predicted by $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_1^2}\right)$? It is amazing how well a simple formula (disconnected originally from theory) could duplicate this phenomenon.