# 3.3: Kinetic Energy and the Work-Energy Theorem

Learning Objectives

By the end of this section, you will be able to:

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

## Work Transfers Energy

What happens to the work done on a system? Energy is transferred into the system, but in what form? Does it remain in the system or move on? The answers depend on the situation. For example, if a lawn mower is pushed just hard enough to keep it going at a constant speed, then energy put into the mower by the person is removed continuously by friction, and eventually leaves the system in the form of heat transfer. In contrast, work done on the briefcase by the person carrying it up stairs in is stored in the briefcase-Earth system and can be recovered at any time by allowing the briefcase to fall back down to the ground floor. In fact, the building of the pyramids in ancient Egypt is an example of storing energy in a system by doing work on the system. Some of the energy imparted to the stone blocks in lifting them during construction of the pyramids remains in the stone-Earth system and has the potential to do work.

In this section we begin the study of various types of work and forms of energy. We will find that some types of work leave the energy of a system constant, for example, whereas others change the system in some way, such as making it move. We will also develop definitions of important forms of energy, such as the energy of motion.

## Net Work and the Work-Energy Theorem

We know from the study of Newton's laws in Dynamics: Force and Newton's Laws of Motion that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

To begin with, let us consider a situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown below.

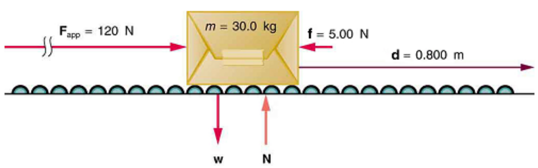


Figure $3.3.2$: A package on a roller belt is pushed horizontally through a distance $d$.

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force $F_{app}$ and the horizontal friction force $f$. Thus, as expected, the net force is parallel to the displacement, and the net work is given by

$$W_{net} = F_{net}d. \tag{3.3.1}$$

The effect of the net force $F_{net}$ is to accelerate the package from $v_0$ to $v$ The kinetic energy of the package increases, indicating that the net work done on the system is positive. (See Example.) By using Newton's second law, and doing some algebra, we can reach an interesting conclusion. Substituting $F = ma$ from Newton's second law gives

$$W_{net} = mad. \tag{3.3.2}$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take $d = x - x_0$ . It is possible to show that if the acceleration has a constant value $a$ for an object traveling a distance $d$, then the final velocity $v$ and initial velocity $v_0$ of the object are related by the equation $v^2 = v_0^2 + 2ad$ . Solving for acceleration gives $a = \frac{v^2 - v_0^2}{2d}$. When $a$ is substituted into the preceding expression for $W_{net}$ we obtain

$$W_{net} = m\left(\frac{v^2 - v_0^2}{2d}\right)d. \tag{3.3.3}$$

The $d$ cancels, and we rearrange this to obtain

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2. \tag{3.3.4}$$

This expression is called the **work-energy theorem,** and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity $\frac{1}{2}mv^2$. This quantity is our first example of a form of energy.

## Work-Energy Theorem

The net work on a system equals the change in the quantity $\frac{1}{2}mv^2$.

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2. \tag{3.3.5}$$

The quantity $\frac{1}{2}mv^2$ in the work-energy theorem is defined to be the translational **kinetic energy** (KE) of a mass $m$ moving at a speed $v$. (*Translational* kinetic energy is distinct from *rotational* kinetic energy, which is considered later.) In equation form, the translational kinetic energy,

$$KE = \frac{1}{2}mv^2, \tag{3.3.6}$$

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

We are aware that it takes energy to get an object, like a car or the package in Figure, up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 100 km/h has four times the kinetic energy it has at 50 km/h, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.

## Example 3.3.1: Calculating the Kinetic Energy of a Package

Suppose a 30.0-kg package on the roller belt conveyor system in Figure 7.03.2 is moving at 0.500 m/s. What is its kinetic energy?

**Strategy**

Because the mass $m$ and the speed $v$ are given, the kinetic energy can be calculated from its definition as given in the equation $KE = \frac{1}{2}mv^2$ .

**Solution**

The kinetic energy is given by

$$KE = \frac{1}{2}mv^2. \tag{3.3.7}$$

Entering known values gives

$$KE = 0.5(30.0\,kg)(0.500\,m/s)^2, \tag{3.3.8}$$

which yields

$$KE = 3.75\,kg\cdot m^2/s^2 = 3.75\,J \tag{3.3.9}$$

**Discussion**

Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

Example 3.3.3: Determining Speed from Work and Energy

Find the speed of the package in Figure 7.03.2. at the end of the push, using work and energy concepts.

**Strategy**

Here the work-energy theorem can be used, because we have just calculated the net work $W_{net}$ and the initial kinetic energy, $\frac{1}{2}mv_0^2$ These calculations allow us to find the final kinetic energy, $\frac{1}{2}mv^2$ and thus the final speed $v$.

**Solution**

The work-energy theorem in equation form is

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2. \tag{3.3.10}$$

Solving for $\frac{1}{2}mv^2$ gives

$$\frac{1}{2}mv^2 = W_{net} + \frac{1}{2}mv_0^2 \tag{3.3.11}$$

Thus,

$$\frac{1}{2}mv^2 = 92.0\,J + 3.75\,J = 95.75\,J. \tag{3.3.12}$$

Solving for the final speed as requested and entering known values gives

$$v = \sqrt{\frac{2(95.75\,J)}{m}} = \sqrt{\frac{191.5\,kg\cdot m^2/s^2}{30.0\,kg}} \tag{3.3.13}$$

$$= 2.53\,m/s \tag{3.3.14}$$

**Discussion**

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work done on the package. This means that the work indeed adds to the energy of the package.

## Summary

- The net work $W_{net}$ is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass $m$ moving at speed $v$ is $KE = \frac{1}{2}mv^2$ .
- The work-energy theorem states that the net work $W_{net}$ on a system changes its kinetic energy, $W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$ .

## Glossary

**net work**

work done by the net force, or vector sum of all the forces, acting on an object

**work-energy theorem**

the result, based on Newton’s laws, that the net work done on an object is equal to its change in kinetic energy

**kinetic energy**

the energy an object has by reason of its motion, equal to $\frac{1}{2}mv^2$ for the translational (i.e., non-rotational) motion of an object of mass $m$ moving at speed $v$

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