5.3: The Pauli Exclusion Principle

Imagine five neutrons and four protons trapped in an atomic nucleus. Model the nucleus as an infinite potential well of length 10 fm. Find the total kinetic energy of the nucleons. Ignore the mass difference between protons and neutrons.

The allowed energy levels in this infinite square well are:

$$\begin{aligned} E &= n^2 \frac{(hc)^2}{8mc^2L^2} \\ E &= n^2 \frac{(1240 \text{ MeV fm})^2}{8(938 \text{ MeV})(10 \text{ fm})^2} \\ E &= n^2 (2.35 \text{ MeV}) \end{aligned} \tag{5.3.1}$$

Although all of the particles would love to occupy the lowest energy state, the Pauli Exclusion Principle states that no two identical fermions can occupy the exact same quantum state. Thus, only two neutrons (and two protons) can occupy the lowest energy state, one with spin “up” and one with spin “down”. In this way, the well is filled from the bottom up as indicted below:

n	E (2.05 MeV)	# neutrons	# protons
1	1	2	2
2	4	2	2
3	9	1	

(5.3.2)

Therefore, the total kinetic energy of the nucleons is:

$$KE = [4(1) + 4(4) + 1(9)]2.05 \text{ MeV} = 59.5 \text{ MeV} \tag{5.3.3}$$