5.A: Solving the Finite Well (Project)

Imagine a particle trapped in a one-dimensional well of length 2L. Inside the well there is no potential energy while the region outside the well has a finite potential energy. This potential energy function is referred to as the finite square well.

I. General Solution

Since the potential energy is a piece-wise function, Schrödinger's equation must be solved in the three regions separately.

A. Region I: $x < -L$
1. Apply and solve Schrödinger's equation in region I. Your solution should have two arbitrary constants. Make use of the definition:

2. Clearly explain why one of your two constants must equal zero, and then record your simplified solution below.

B. Region II: $-L < x < L$
1. Apply and solve Schrödinger's equation in region II. Your solution should have two arbitrary constants. Make use of the definition:

C. Region III: $x > L$
1. Apply and solve Schrödinger's equation in region III. Your solution should have two arbitrary constants. Again make use of the definition:

2. Clearly explain why one of your two constants must equal zero, and then record your simplified solution below.

D. Sketching Solutions
1. Sketch the wavefunctions of the four lowest energy states on the diagram below.

Your sketched solutions should fall into one of two categories. Symmetric solutions ($n = 1, 3, \ldots$) are those that are symmetric about the y-axis. Antisymmetric solutions ($n = 2, 4, \ldots$) are, you guessed it, antisymmetric when reflected about the y-axis. We will solve for these two types of solutions separately.

E. Symmetric Solutions
1. Clearly explain why one of the two constants in region II must equal zero for symmetric solutions.

2. What is the relationship between the constant in region I and the constant in region III for symmetric solutions?

3. Record your simplified solutions in the three regions below.
Region I:

Region II:

Region III:

The wave function, and the derivative of the wave function, must be continuous across the boundary at $x = L$. This will lead to two simultaneous equations that can be solved for the energy levels of symmetric solutions.
4. Force the wavefunction to be continuous across the $x = L$ boundary and simplify the resulting expression.

5. Force the derivative of the wavefunction to be continuous across the $x = L$ boundary and simplify the resulting expression.

6. Solve the two equations above for an expression involving and k. Simplify this expression into a transcendental equation for the energy levels of symmetric solutions.

F. Antisymmetric Solutions
1. Clearly explain why one of the two constants in region II must equal zero for antisymmetric solutions.

2. What is the relationship between the constant in region I and the constant in region III for antisymmetric solutions?

3. Record your simplified solutions in the three regions below.
Region I:

Region II:

Region III:

4. Force the wavefunction to be continuous across the x = L boundary and simplify the resulting expression.

5. Force the derivative of the wavefunction to be continuous across the x = L boundary and simplify the resulting expression.

6. Solve the two equations above for an expression involving and k. Simplify this expression into a transcendental equation for the energy levels of antisymmetric solutions.

II. Specific Solution

Find all the allowed energy levels for an electron trapped in a finite square well of total width 2.00 nm and depth 1.00 eV.