

## 5.5: The 2D Infinite Square Well

Twelve electrons are trapped in a two-dimensional infinite potential well of x-length 0.40 nm and y-width 0.20 nm. Find the total kinetic energy of the system.

Since the x- and y-directions in space are independent, Schrödinger's equation can be separated into an x-equation and a y-equation. The solutions to these equations are identical to the one-dimensional infinite square well. Thus, the allowed energy states of a particle of mass m trapped in a two-dimensional infinite potential well can be written as:

$$E = n_x^2 \frac{(hc)^2}{8mc^2 L_x^2} + n_y^2 \frac{(hc)^2}{8mc^2 L_y^2} \quad (5.5.1)$$

$$E = \left( \frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} \right) \frac{(hc)^2}{8mc^2}$$

with wavefunction:

$$\Psi(x, y) = A \sin\left(\frac{n_x \pi x}{L_x}\right) \sin\left(\frac{n_y \pi y}{L_y}\right) \quad (5.5.2)$$

Therefore, the allowed energy levels are given by

$$E = \left( \frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} \right) \frac{(hc)^2}{8mc^2} \quad (5.5.3)$$

$$E = \left( \frac{n_x^2}{4^2} + \frac{n_y^2}{0.2^2} \right) \frac{(1240 \text{ eV nm})^2}{8(5111000 \text{ eV})^2 (0.1 \text{ nm})^2}$$

$$E = \left( \frac{n_x^2}{16} + \frac{n_y^2}{4} \right) 37.6 \text{ eV}$$

Rather than deal with fractions, multiply and divide by 16:

$$E = (n_x^2 + 4n_y^2) 2.35 \text{ eV} \quad (5.5.4)$$

To help calculate the total kinetic energy of the system, list the first few lowest allowed energy states:

| Level | $n_x$ | $n_y$ | E (2.35 eV) | # electrons |
|-------|-------|-------|-------------|-------------|
| 1     | 1     | 1     | 5           | 2           |
| 2     | 2     | 1     | 8           | 2           |
| 3     | 3     | 1     | 13          | 2           |
| 4     | 1     | 2     | 17          | 2           |
| 5     | 2     | 2     | 20          | 4           |
|       | 4     | 1     |             |             |
| 6     | 3     | 2     | 25          | 0           |

(5.5.5)

The states  $(n_x, n_y) = (2, 2)$  and  $(4, 1)$  are termed degenerate because two completely different wavefunctions have the same energy. The state  $(2, 2)$  looks like this:

while the state  $(4, 1)$  looks like this:

Since these are different wavefunctions, two electrons (spin up and spin down) can occupy each state. Thus, four electrons have the same energy. Thus, the twelve electrons will have total kinetic energy of

$$KE = [2(5) + 2(8) + 2(13) + 2(17) + 4(20)]2.35 \text{ eV} = 390 \text{ eV} \quad (5.5.6)$$

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