9.3: Length Contraction

Learning Objectives

- You will know that moving rulers are shorter.
- You will be able to correctly use the length contraction formula to compare lengths in different reference frames.

What Do You Think: Distances and Time

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You might be puzzled by an apparent contradiction from the example of the muon in the last section. It makes a certain amount of sense that a muon would reach the ground as measured by a ground-based observer. After all, the muon's clock is seen to tick more slowly, and so its lifetime as seen by the ground-based observer appears to be longer. But an observer moving along with the muon would see a clock sitting on the ground moving more slowly than the clock riding along with the muon, so should the muon decay even more rapidly in that frame? The answer is "no," of course, but we have to consider another aspect of relativity for this example to make sense.

It is not just clocks that read differently for different observers moving relative to one another. Meter sticks also give a different measure. We should not be surprised by this fact. After all, velocity is a ratio of distance to time, and if the velocity of light is to be constant between two frames in relative motion, while time is not, then the lengths they measure must not be the same, either. If it was, the ratio of distance traveled by a photon over a time interval could not be the same for both. However, not all lengths are affected, just the ones along the direction of relative motion.

This effect is known as length contraction. If we consider its effect on a ruler of length L, it can be expressed mathematically as below.

$$L' = \frac{L}{\gamma}$$

For this expression, L' is the length in the frame moving with speed v, L is the length measured by an observer at rest with respect to the ruler, and γ is the gamma factor, as defined in the previous section. The ruler is taken to be aligned with the direction of motion, since that is the only direction for which length contraction happens. Since gamma is always bigger than one, L' is always *less than* L, so *moving rulers are shorter* than they are at rest.

LENGTH CONTRACTION

USE GRAPH

Worked Examples:

1. An astronaut whose height on Earth is 1.7 m is flying in a spacecraft at 0.8c. What is her height as measured by her fellow astronauts in the ship?

From the perspective (frame) of the ship, the astronaut is at rest. Therefore, her height is her usual height of 1.7 m

2. What is the height of the astronaut as measured by an observer at mission control?

From the perspective (frame) of mission control, the astronaut is moving, so she will be shorter in height:

- Given: L = 1.7 m, v/c = 0.8
- Find: L'
- Concept: L' = L/ γ
- Solution: from the clickable graph, the γ corresponding to v/c = 0.8 is γ = 1.67, so the astronaut will be L' = 1.7 m / 1.67 = 1.02 m tall, which is indeed shorter.

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Muons From Cosmic Rays, as Seen by an Observer Riding With the Muon

Worked Examples:

1. We will look at the case of the muon again, this time from the reference frame of the muon instead of from the reference frame of the ground.

As the muon travels toward the ground at 0.99c, from its point of view, the distance from the muon to the ground is not as far away as it appears to be when measured by an observer on the ground. If the height of the muon is *L* as measured from the ground, then it will be shorter in the muon's frame (*L'*) by the gamma

factor.

First, calculate the distance to the ground. In the muon's frame:

- Given: L = 10 km, γ = 7.1
- Find: L'
- Concept: *L'* = *L*/γ
- Solution: L' = (10 km)/(7.1) = 1.4 km

Next, calculate the time (t_{muon}) required to travel this distance:

- Given: v = speed of the muon = 0.99c, L' = 1.4 km
- Find: t_{muon}
- Concept: v = L'/t_{muon}
- Solution: t_{muon} = (1.4 km) / [(0.99) × (3 × 10^5 km/s)] = 4.7×10^{-6} s

Comparing this number to the muon's lifetime in its own reference frame, which is 2.2 micorseconds $(2.2 \times 10^{-6}\ \mathrm{s})$, we see that in terms of half-lives, it is $4.7 \times 10^{-6}\ \mathrm{s}/(2.2 \times 10^{-6}\ \mathrm{s})$ = 2.1 half-lives, just as we found before.

So, the results observed in the two frames are entirely consistent. The same numbers of muons reach the ground. The different observers only disagree about why the muons can make the journey successfully. The observer on the ground thinks it is because the muons' clocks tick more slowly. The observer moving with the muons says it is because the ground is not very distant from where the muons are produced.

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LENGTH CONTRACTION OF SPACESHIPS

In this activity, you will use an example of a rocket ship and a space station flying past each other to explore the idea of length contraction.

You can use the slider bar to adjust the relative speed of the ship and station.

The changes in length are shown accurately, but the speeds of the objects moving across the screen are not relativistic (otherwise, the motion would be too fast to see).

Play Activity

USE GRAPH

Worked Examples:

A space station and a rocket ship are both 100 m long at rest. Imagine that they are flying past each other at a velocity of 0.5c (50% of the speed of light).

1. From the point of view of an astronaut on the space station, what is the length of the rocket ship as it flies by?

- To figure this out, slide the speed control to 0.5c, and play the animation.
- Notice that from the frame of reference of the space station, the rocket ship appears to be shorter.
- Using the clickable graph for v = 0.5c, you should find that gamma = 1.15, so the rocket ship is about $L' = L/\gamma = 100$ m/1.15 or about 87 m long.

2. From the point of view of an astronaut on the rocket ship, what is the length of the space station?

- Play the animation again, this time noticing the perspective from the frame of reference of the rocket ship.
- From the rocket ship, the space station flies by and appears shorter, again only about 87 m long.

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