7.2: Force, Mass, and Weight

Mass and Weight

To understand gravity, we will have to learn a little bit more about what causes changes in the motion of objects. Clearly, gravity causes such changes, but it is not the only effect that can do so. For instance, we can push on a cart, or pull on it with a cord. In either case, the cart's motion can change. Such a pull or push is called a force by physicists. In physics, force has a very precise mathematical definition:

$$\vec{F}_{tot} = m\vec{a}$$

In this expression, F is force, m is mass, and a is acceleration, and the equation says that total force is the product of mass and acceleration.

There are several things that are important to understand about this equation. The first is the little arrows above the $\vec{F}$ and the $\vec{a}$. They are there to remind us that both force and acceleration have a direction associated with them. Pushing on the back bumper of a car has very different results than pushing on the front bumper - one makes the car accelerate forward, the other makes it accelerate backwards. Quantities that have a size and a direction are called vectors , and the arrows remind us that both force and acceleration are vectors. We will often write the equation without the arrows:

$$F = m\,a.$$

In this case, it means we are not concerned with the direction of the force and acceleration, or sometimes it will mean that their direction is obvious from the context of the problem we are considering. Since the only two vectors in this equation, the F and the a, are on opposite sides of the equals sign, they must point in the same direction.

The second thing we have to consider in this equation is the "tot" subscript on the F. That is there to remind us that we are referring to the total, or net force acting on an object. So, for example, if we push equally hard on the back bumper and on the front bumper of a car in such a way that the two forces are the same size but pointed in opposite directions, then the total force on the car will be zero. In this case, where the total force is zero, the equation says that the acceleration must be zero as well. That means that the car's velocity will remain constant. This is because acceleration is defined to be the change of an object's velocity with respect to time.

On the other hand, if both forces are pointed in the same direction, the total force will be the sum of the two. Then the acceleration will be larger than if only one force was acting because the two forces together, acting in the same direction, are larger than either one alone (Figure 7.2).

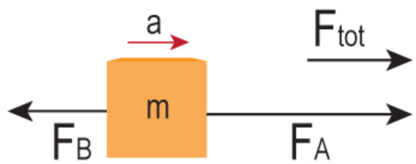

Figure 7.2: Net force and acceleration. The acceleration of an object points in the same direction as the total, or net, force (F_{tot}) acting on that object. The strength of the force, proportional here to the length of the arrow, is equal to the object's mass multiplied by its acceleration. Note that in this image, the net force points in the direction of F_A because F_A is larger than the oppositely directed F_B. Therefore, the acceleration also points in the direction of F_A. Credit: NASA/SSU/Aurore Simonnet

The SI unit of force is the newton (N), named for Isaac Newton, whose discoveries are the subject of this chapter. In SI units, 1 newton = 1 kg m/s^2. The more familiar unit of force in the USA is the pound (lb); 1 N = 0.2248 lb. However, we will use SI units in these modules.

Net Force on an Object

In this activity, you will use what scientists call a **freebody diagram** to see how forces acting on an object in different directions can add together to create a total force on the object.

Play Activity

Worked Example:

1. Consider how two forces add together in one dimension. Assume the first force we are dealing with pushes an object to the right with a force of 3 N. The second force pushes the object to the left with a force of 4 N. What is the total force on the object?

- The results of using the Freebody Diagram tool for this situation are shown in Figure A.7.1.
- The first force pushes 3 N to the right, or 3 N in the positive x direction. There is no push in the y direction. To enter the first force, type 3 in the "x value" box, and 0 in the "y value" box. Now click the "Show" button to enter this force. Notice that the force now appears on the diagram to the right.
- For the second force, there is a push of 4 in the negative x direction, and no push in the y direction. To enter this force, type -4 in the "x value" box, and 0 in the "y value box". Now click the "Show" button to enter this force.
- To add the two forces, click the "Add" button. Toward the bottom of the Input Panel, you should now see the Total Force listed in green. On the right, you can see the two input forces (in red) and the total force (in green) on the diagram.
- The total force is -1 (x-direction) + 0 (y-direction) N. You can see that the total force points toward the left.
- Once you are ready, click "Clear" before going on to the next example.

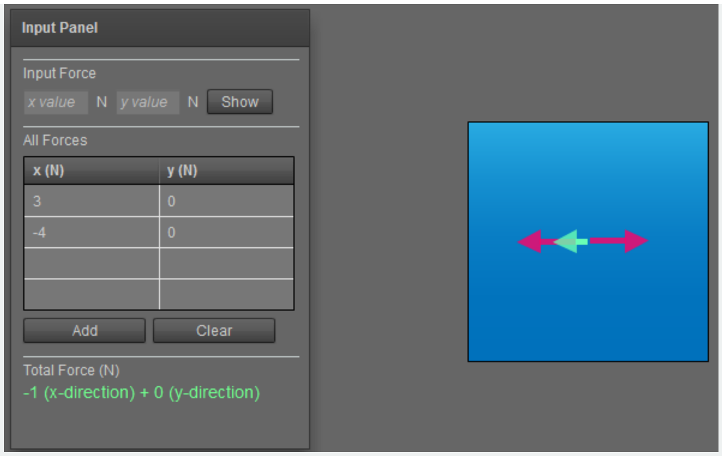

Figure A.7.1: Net force for 2 forces in one dimension. Output from the *Freebody Diagram* tool for 3(x-direction)N + -4(x-direction) N. The red arrows show the individual forces, while the green arrow shows the sum of the two forces. Credit: NASA/SSU

Questions

Now do a few examples on your own, either numerically, or using the interactive *Freebody Diagram tool*.

Another important thing to understand in the equation is the meaning of the symbol *m*, the mass. Mass is a tricky concept to understand at first, but hopefully by the end of this chapter you will understand it. Mass, as defined by the equation above, is a measure of how resistant an object is to changes in its motion. First, we will do a couple of examples to demonstrate how this works. The next thing to be clear about is that mass is not the same as weight. The difference will be addressed at the end of this section.

MASS AND ACCELERATION

Worked Examples:

1. Assume that we want to accelerate a 5 kg object at a rate of 2 m/s^2. We can use the force equation to compute the force we must apply to the object.

- Given: m = 5 kg, a = 2 m/s^2

- Find: F
- Concept: F = ma
- Solution: F = (5 kg) (2 m/s^2) = 10 N

We would therefore have to exert a 10 N force on the object.

2. We can also use the force relation to deduce the mass of an object. If we imagine that when we apply a 100 N force to an object it accelerates at 2 m/s^2, we can calculate its mass:

- Given: F = 100 N, a = 2 m/s^2
- Find: m
- Concept: F = ma, or, rearranging, m = F/a
- Solution: m = F/a = (100 N) / (2 m/s^2) = 50 kg

This defines the mass of an object using its inertia, or its resistance to change in motion. This definition of mass is sometimes referred to as inertial mass. We can also define mass in terms of gravity, as we do below. While philosophically different, the two definitions give the same numerical answer in all cases studied to the best of our ability to measure.

Questions

So the more massive an object is, the more difficult it is to change its velocity. This includes both its direction of motion or its speed. We are all familiar with this from our experiences in the world: It is much less dangerous to run into a person than a freight train. While freight trains do tend to move faster than people, that is not the most vital detail to consider. Even if a train is moving only at walking speed you would probably prefer colliding with a person than with a train. This is because a train has a lot more mass - and thus inertia - than a person has.

ACCELERATION AND MASS

We can use the *Freebody Diagram* again, this time with mass as well as force, to determine both the direction and strength of an object's acceleration.

Play Activity

Worked Examples:

Imagine you and your friend are fighting over a textbook that you both really, really want to read. The textbook has a mass of 5 kg. You pull on the book with a force of 6 N in the x direction, and your friend pulls on the book with a force of 5 N in the negative x direction.

Using the Freebody Diagram tool that allows you to enter the mass, you should get something that looks like Figure A.7.2 below.

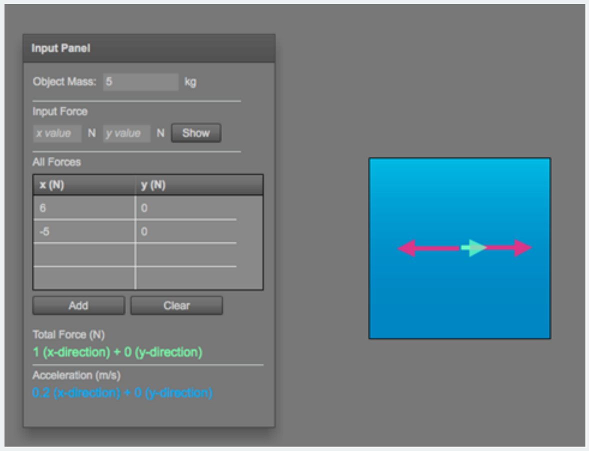

Figure A.7.2: Net force

and acceleration in one dimension. Output from Freebody Diagram for forces of 6 N (x-direction) and -5 N (x-direction) and a mass of 5 kg. The red arrows show the individual forces, while the green arrow shows the net force. The direction of the acceleration is the same as the direction of the net force. Credit: NASA/SSU

1. What is the total force on the textbook (strength and direction)?

We see from the tool that the net force is +6 N – 5 N = +1 N in the x-direction (to the right).

2. What is the total acceleration of the textbook (strength and direction)?

We see from the tool that the acceleration of the textbook is 0.2 m/s^2 in the +x direction. Notice that the direction of the acceleration is the same direction as the net force. The tool calculates the strength of the acceleration from the equation F = ma, or a = F/m. In this case, a = 1 N / 5 kg = 0.2 m/s^2.

3. Will the textbook accelerate toward you or your friend?

It will move toward me.

Questions

Once you decide the matter of the textbook, you and your friend realize that there is a study guide full of answers to all the homework questions in the text, and you begin to fight over that, too! The study guide has a mass of 2 kg. Again, you pull on the guide with a force of 6 newtons in the x direction. Your friend pulls on the study guide with a force of 5 newtons in the negative x direction.

In the activity above, you can see that when two friends are pulling on a fairly massive textbook, the book does not accelerate much. However, when the friends pull with the same force on a less massive study guide, the study guide accelerates more. This demonstrates that under the influence of a force, an object's acceleration is *inversely proportional* to its mass. In other words, an object with a *large* mass will experience a *smaller* acceleration due to a given net force than an object with a *smaller* mass. The object with a *smaller* mass will experience a *larger* acceleration due to a particular net force than the object with the *larger* mass.

Remember that one of the definitions of mass we discussed above refers to the *inertial mass* because it is based on the inertia of an object, or in other words, on its tendency to resist changes in its motion. The equation linking force, mass, and acceleration was first deduced by Isaac Newton, and it is called Newton's second law of motion. Along with his first and third laws of motion, it constitutes the basis of classical mechanics, a branch of physics that describes the motion of bodies through space. We have already encountered Newton's first law, borrowed from Galileo:

Definition: Newton's first law of motion

Objects in motion remain at a constant velocity (straight line, constant speed) unless acted upon by a net force.

We have now encountered Newton's second law in our discussion of force, mass, and acceleration:

Definition: Newton's second law of motion

The acceleration of an object is directly proportional to the total force on the object, and inversely proportional to its mass. In physics, we write this using the equation $F = ma$, which we introduced and have worked with in its full vector form already.

For more about Newton's laws see Going Further 7.1: Newton's Third Law.

Going Further 7.1: Newton's Third Law

Our daily experiences might lead us to think that forces are always applied by one object on another; for example, a horse pulls a carriage, a person pushes a grocery cart, or a hammer hits a nail. It took Sir Isaac Newton to realize that things are not so simple, and not so one-sided. True, if a hammer strikes a nail, the hammer exerts a force on the nail (thereby driving it into a board). Yet, the nail must also exert a force on the hammer since the hammer's state of motion is changed, and according to the first law, this requires a net (outside) force. This is the essence of Newton's third law: For every action there is an equal and opposite reaction. However, it is important to understand that the action and the reaction are acting on different objects.

Try this: Press the side of your hand against the edge of a table. Notice how your hand becomes distorted. Clearly, a force is being exerted on it. You can see the edge of the table pressing into your hand and feel the table exerting a force on your hand. Now press harder. The harder you press, the harder the table pushes back on your hand. Remember this important point: You can only feel the forces being exerted on you, not the forces you exert on something else. So, it is the force the table is exerting on you that you see and feel in your hand.

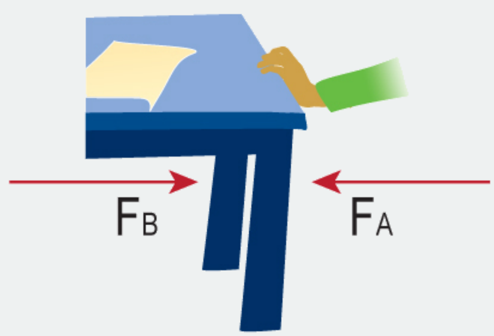

Figure B.7.1. Action–reaction pair illustrating Newton's third law. Newton's third law is illustrated in this figure, which shows that the action of the hand pushing on the table is equal in strength and opposite in direction to the reaction of the table pushing on the hand. Credit: NASA/SSU/Aurore Simonnet

Action–reaction pairs like the force of the hand on the table and the force of the table on hand are all around us. If you are reading this while sitting in a chair, you can feel the force the chair is exerting upward on you while you exert a force downward on it. And if you are reading on a computer monitor that sits on the table in front of you, then there is a balanced action–reaction pair between the monitor and the table, as well as between the table and the floor, and so on for all the objects you see around you. Each of these is an action–reaction pair. Such action–reaction pairs are the domain of Newton's third law.

Not every equal and opposite pair of forces is a third law pair, however. In the third law, two objects are involved with the two forces. For example, a hand and a table, a table and a hand. We can have two forces of equal strength and opposite direction due to Newton's second law, but these forces act on a single object. For example, if you hold a ball still in your hand, there is an upward force due to your hand on the ball that is exactly equal to the downward force of gravity from Earth on the ball. That is why the ball does not move. These two forces are acting on a single object, the ball, whose net force and acceleration are zero by Newton's second law.

Newton's third law is also related to the concept of conservation of momentum. Momentum is defined as an object's mass times its velocity; because velocity is a vector, momentum is also a vector. Momentum is also related to force: the change in momentum over time is equal to the net force.

So, for two objects that are a third law pair, the change in momentum of one object will be equal in strength and opposite in direction to the change in momentum of the other. This principle is important in collisions. For example, if two billiard balls collide, they will bounce off of each other and travel in opposite directions. It is also important for propulsion. For example, a squid is a sea creature that takes in water; when it wants to move, it squirts water out of its body in one direction, and it moves in the opposite direction. As another example, if you are standing on a frozen pond having a snowball fight in the winter, if you throw a snowball, the snowball will move forward, but you will also slide backward a little. The velocity of the snowball will be greater than your velocity because you have a greater mass (unless you make a really big snowball).

We will use Newton's laws in our study of objects moving under the influence of gravity. Combined, they provide a basic framework for understanding how objects move, why bridges are able to stand, how water flows in a stream and many other physical phenomena that fill our everyday experience.

What does all of this have to do with gravity? Well, as was noted in Section 7.1, gravity causes an acceleration, g, and this acceleration is constant for all objects on Earth, regardless of their mass. If mass and acceleration are involved, then according to Newton's second law, there must be a total, or net, force acting.

In fact, we can use the second law to measure the force exerted by gravity on objects of different masses. We have a special name for the force of gravity acting on a mass; we call it the weight of the object. Substituting the acceleration due to gravity into Newton's second law, we get an expression for the force of gravity on Earth, or the weight of an object:

$$F_g = mg$$

Again, F_g is the gravitational force, or weight, m is the mass, and g is the acceleration due to gravity, 9.8 m/s^2. Now the relationship between mass and weight becomes clear: weight is a force, whereas mass is how much "stuff" there is. The weight and the mass are related by a factor of g, but are not the same.

For more information on why the value of the mass is the same, whether the force is gravity or some other force, see Going Further 7.2: Gravity and Inertia.

7.2.1: GOING FURTHER: GRAVITY AND INERTIA

Gravitational Force on a Mass

Worked Example:

1. If we consider a 5 kg mass, we can use Newton's second law to find the force that gravity exerts upon it.

- Given: m = 5 kg, g = 9.8 m/s^2
- Find: F_g
- Concept: F_g = mg
- Solution: F_g = mg = (5 kg) (9.8 m/s^2) = 49 N

This is equivalent to just over two pounds.

Questions

ADAPT 7.2.1

In the previous example we used Newton's second law to compute the force acting on an object falling in Earth's gravity. This force causes objects to accelerate at one g (9.8 m/s^2). But what if an object is not falling? Does a force still act on it? Yes! But in that case the acceleration is zero, so how can there be a force? Here we have to think carefully about what the second law says: it says that acceleration is caused by a total, or net force. A zero acceleration means only that there is no *net* force acting on the body, not that no forces at all act on it. A picture might help clarify this (Figure 7.3).

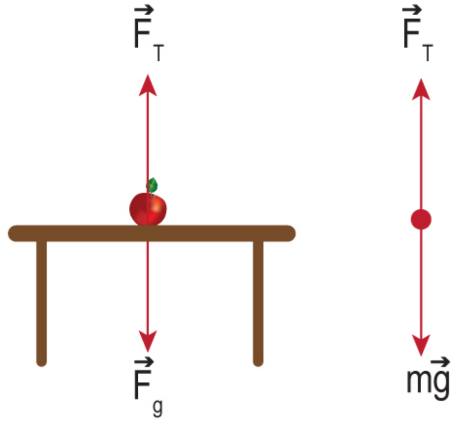

Figure 7.3: Apple sitting on a table. An apple sitting on a table feels the force of gravity, Fg, pulling it downward, but it is not accelerating. If the acceleration of the apple is zero, the net force must be zero. Therefore, there must be a force of equal strength but opposite direction to the gravitational force. This opposing force that is pushing up on the apple is F_T, from the table to the apple, which keeps the apple from falling. The net force on the apple is zero and the apple remains stationary. Credit: NASA/SSU/Aurore Simonnet

Imagine that we have the situation shown in Figure 7.3: An apple sits on a table. In the numerical activity, we saw that the force of gravity is acting on the apple, which should cause it to accelerate downward. However, in this case the apple is not accelerating. Why not? Because the table is in the way. If we apply Newton's second law to this case, we can deduce that the table must be applying an upward force equal in magnitude to the force of gravity. How do we know that the net force is zero? Since the apple does not accelerate, we know the net force is zero. We can draw the apple and the forces acting on it, and since only the apple's weight and the upward force from the table, F_T, are present, they must be exactly equal in strength and opposite in direction.

Let's repeat that again for emphasis: The apple does not remain stationary because there are no forces acting on it. In fact, we have just identified two forces that are acting on the apple. However, one force is from gravity and points downward. The other force is from the table and points upward. The forces are equal in strength and point in opposite directions, so they cancel each other out. The total force (or net force) on the apple is zero.

Lunar Lander

We can put what we have learned so far about Newton's laws and gravity into practice by using the Lunar Lander to simulate landing a spacecraft on the surface of the Moon. To successfully land the spacecraft, you will have to slow it down so that both its horizontal and vertical velocities are very small. You will also have to land the spacecraft upright. Read the Lunar Lander instructions to learn which keys on your keyboard you can use to fire the lander's engine and to rotate the lander. Watch your fuel level!

Some things to think about as you try this activity:

- When you fire the lander's engine, you are giving the lander a "push", or applying a force to it. The direction of the force depends on which direction the lander is pointing. When you stop firing the engine, you stop applying a force to the lander.
- The lander starts with a horizontal velocity . You will have to slow the lander down in the horizontal direction.
- The Moon exerts a gravitational force on the lander, causing it to accelerate downward. You will have to slow the lander down in the vertical direction, too.

Play Activity