9.6: Mass and Energy

Learning Objectives

- You will be able to calculate rest energy and mass.
- You will understand that mass and energy are different aspects of the same underlying quantity, and that is the total of mass and energy that is truly conserved.
- You will know that a tiny amount of matter can release a great deal of energy.
- You will understand that it is possible to produce a pair of charged particles from an energetic photon, and that the annihilation of a particle-antiparticle pair releases energy.
- You will know that the total energy is greater than the rest energy if a particle is moving

What Do You Think: Mass and Energy

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The last major topic in special relativity that we will discuss is that of mass–energy equivalence. This most famous of equations in physics was not even included in Einstein's original 1905 paper on special relativity. He did derive an expression for the increase of the mass of a particle as its velocity increased, suggesting that the mass of a particle should be multiplied by the relativistic gamma factor to obtain its true mass. Only later did Einstein derive his famous $E = mc^2$. But, what does this equation mean?

Quite simply, the mass–energy equivalence relation means just what it says: Mass and energy are equivalent and can be converted one into the other. Do not be distracted by the factor of c^2 in the equation; it is just a constant that depends on the system of units we are using. There is nothing special about a system of units; we just have to be consistent and use just one set of units when carrying out a calculation. The essence of the equation is that energy (E) is proportional to mass (m), and that the two can be interconverted, one to the other.

However, the amount of energy equivalent to given mass can be understood in terms of the units. The speed of light is a large number in standard SI units. When we square it we get an even larger number: $(3 \times 10^8 \text{ m/s})^2 = 9 \times 10^{16} \text{ m}^2/\text{s}^2$. So, converting a single kilogram of mass would release an enormous amount of energy, almost 10^{17} joules. That is enough energy to power a typical house, which uses a kilowatt or so, for several million years.

Generally, the actual reactions that happen in nature are not 100% efficient at converting mass to energy, so only a small fraction of mass is actually converted. For instance, in nuclear reactions like the ones converting hydrogen to helium in main sequence stars, just under 1% of the mass is converted to energy. Typical chemical reactions, like burning a match, are about a million times less efficient than that. Only when a particle meets its antiparticle, like when a positron meets an electron, is all of the mass converted to energy.

Of course, the mass–energy equivalence does not only allow for mass to be converted into energy. It also means that energy can be converted into mass. This is what happens in particle accelerators like the one at CERN near Geneva, Switzerland (Figure 9.18). In those machines, protons and antiprotons are accelerated to extremely high energies and speeds very close to the speed of light. When the particles collide inside the machines, their enormous energies are converted into the showers of particles that are then detected. It is a common misconception that the physicists are somehow "smashing atoms" so that they can see what is inside of them. The atoms in the collisions are certainly destroyed, but the particles that come out are not made of the insides of the destroyed atoms. They are made of the energy contained in the *motion* of those atoms (and from the particles' masses that have been converted into some of the energy). The next few activities will help to illustrate this concept.

Figure 9.18: Aerial view of CERN, near Geneva, Switzerland. The large yellow ring outlines the location of the Large Hadron Collider (LHC). Credit: CERN/Maximilien Brice

REST ENERGY OF PROTONS AND NEUTRONS

Worked Example:

1. The mass of a proton is about 1.673×10^{-27} kg. Assuming the proton is at rest, how much energy does this correspond to in SI units (joules)?

The energy contained in this mass is given by Einstein's famous equation. We will use a subscript zero to remind ourselves that the particle is assumed to be at rest.

- Given: $m = 1.673 \times 10^{-27}\ \mathrm{kg}, \mathrm{c} = 3 \times 10^{8}\ \mathrm{m/s}$
- Find: E_0
- Concept: $E_0 = mc^2$
- Solution: $E_0 = (1.673 \times 10^{-27}\ \mathrm{kg})(3 \times 10^{8}\ \mathrm{m/s})^2 = 1.506 \times 10^{-10}\ \mathrm{J}$

This seems like quite a small amount of energy. However, typical objects contain many, many protons. For instance, a typical human body has roughly 10^{28} protons in it (as well as about the same number of neutrons and electrons!). So, converting a typical object into pure energy would release tremendous amounts of energy. Converting all of the protons in a human body into energy would release $(10^{28})(1.506 \times 10^{-10}\ \mathrm{J}) = 1.506 \times 10^{18}\ \mathrm{J}!$

Questions:

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In the activity above, we used the subscript zero for the energy when a particle is assumed to be at rest. This is called its rest energy because it is the amount of energy contained in the mass of a particle observed to be at rest. If the particle is not at rest, then its energy will be different. We explore this case in the next activity.

Total Energy of a Proton in the LHC

In this activity, we will explore how the rest energy of a proton in the Large Hadron Collider (LHC) at CERN compares to its total energy.

Protons and antiprotons are circulated at extremely high speeds in the LHC. The total energies of each of the protons and antiprotons accelerated are currently (in 2021) about 10^{-6} J.

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From these two activities, we can see that most of the energy of the protons in the LHC is not in their rest energy (due to their mass), but must instead be in their motion. We will explore this idea in more detail.

For moving particles, we have to include the relativistic gamma factor when computing their energy. If an object is moving, its total energy is:

$$E = \gamma m c^2$$

or

$$E = \gamma E_0$$

It is this gamma factor that boosts the total energy above and beyond the rest energy value.

ENERGIES, VELOCITIES, AND THE GAMMA FACTOR

Worked Examples:

1. The ratio of an object's total energy to its rest energy is its gamma factor. (In other words, $E/E_0 = \gamma$, as follows from the discussion above.) If a proton were measured to have a total energy of 3 × 10^{-10} J, what would its gamma factor be?

- Given: $E = 3 \times 10^{-10}$ J, $E_0 = 1.506 \times 10^{-10}$ J
- Find γ
- Concept: $\gamma = E/E_0$
- Solution: \(\gamma = (3\times 10^{-10}\, {\rm J}) / (1.506\times 10^{-10}\, {\rm J}) = 1.99

2. What would the speed of this proton be in terms of the speed of light?

We can use the clickable gamma vs. v/c graph to see that v/c = 0.86. To see the math worked out using the equation for gamma, see Math Exploration 9.3.

USE GRAPH

To see the math worked out using the equation for gamma, see Math Exploration 9.3.

Math Exploration 9.3

Questions:

The most energetic cosmic rays ever detected have total energies around 10^{22} eV.

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From the activity, you should have noticed that the velocities of the protons in the LHC and of the cosmic rays are nearly the speed of light. Notice how, at this point, increasing gamma cannot increase the speed of the particles significantly. What's more, no matter how big gamma gets, the speed will never exceed *c*. However, if the speed increases even by a tiny amount toward the speed of light, then gamma increases by a large amount, and therefore, so does the energy of the particles. It is the increase in gamma, not the increase in velocity directly, that is responsible for the gain in energy. To put this more directly, when a particle is moving at near the speed of light, a minuscule increase in speed results in an enormous increase in gamma, and therefore a correspondingly enormous increase in energy.

You have probably heard that special relativity prohibits particles from moving at or above the speed of light. The last examples and activities give part of the reason why. As a particle with mass approaches the speed of light, its gamma factor, and thus its energy, increase without bound. The particle energy, mostly due to its motion, must come from somewhere. In the LHC, the energy is provided by electricity that is used to create strong fields that bump the particles up in energy as they race around their circular track. We do not have access to boundless energy to put into these particles, we have only whatever power can be delivered by the power plants in Europe. Yet, as the speed of the particles nears the speed of light, their energies increase without limit, and for a massive particle to attain the speed of light we would require an infinite amount of energy to put into it. Clearly, this is more energy than we can ever generate, so there is never enough energy available to accelerate a particle to the speed of light. As a result, massive particles are forbidden from attaining this speed.

On the other hand, some particles have no mass. Photons are one example. The only reason photons have any energy at all is by virtue of the fact that they move. For massless particles like photons, they must *always* travel at the speed of light for any observer in any frame of reference, but that is what we assumed as a postulate of special relativity.

So, particles with nonzero mass cannot travel at or above the speed of light because they would need infinite energy to pass through the point where $v = c$. Could it be possible for particles to travel faster than light if they *always* traveled at such high speeds? In that case, they would never actually have the speed of light, where they would have infinite energy. If you look at the expression for gamma, you will see that if *v* is larger than *c*, then the argument of the radical in the denominator will be less than zero. This will cause gamma to be an imaginary number. So, particles that travel faster than light must have an imaginary mass and imaginary energy, whatever that means. Such particles have been suggested, and they are commonly called tachyons. No evidence for tachyons has ever been found, but they are one of the strange ideas that emerge from the theory of special relativity.
