

Resources: Formulae, Constants, and Conversions

Formulae	Relationship	Initial Chapter
$\lambda = c/f$	wavelength, frequency, and speed	Chapter 2
$E = hf$	energy and frequency	
$E_{\text{photon}} = \Delta E_{\text{electron}}$	energy of photon emitted/absorbed by atom	
$E_n = -13.6/n^2$	energy levels of atom	
$T = 2.9 \times 10^{-3}/\lambda$	temperature and peak wavelength	
$F = \sigma T^4$	brightness and temperature	Chapter 3
$A = \pi r^2$	area and radius	
$\theta = 1.22 \lambda / D$	resolution	Chapter 4
$v = d/t$	speed, distance, and time	
$z = \Delta\lambda/\lambda$	redshift	
$v = cz$	velocity and redshift	Chapter 5
$t = E/L$	stellar lifetime	
$E = mc^2$	energy and mass	
$t \sim m^{-2}$	stellar lifetime and mass	
$L \sim m^3$	stellar mass and luminosity	Chapter 6
$d = 1/a$	parallax angle and distance	
$d = S/\theta$	small angle formula	
$F = L/4\pi d^2$	inverse square law	Chapter 7
$F = ma$	Newton's second law	
$F_g = mg$	Weight and mass	
$a_c = v^2/r$	Centripetal acceleration	
$F_g = Gm_1m_2/r^2$	Newton's law of gravity	
$PE = mgh$	Potential energy (non-relativistic)	
$KE = \frac{1}{2}mv^2$	Kinetic energy (non-relativistic)	
$E_{\text{final}} = E_{\text{initial}}$	Conservation of energy	

Formulae	Relationship	Initial Chapter
$v_{\text{escape}} = (2GM/R)^{1/2}$	Escape velocity	Chapter 8
$v \propto r$	Rotation of rigid disk	
$v \propto 1/r$	Rotation of water around drain	
$v \propto \text{constant}$	Rotation of cars on roundabout	
$v \propto 1/r^{1/2}$	Keplerian rotation (planets)	
$v^2 = GM/r$	Relationship of enclosed mass to velocity and distance	
$M = \rho V$	Mass, density, and volume	Chapter 9
$\gamma = 1/\sqrt{1-v^2/c^2}$	gamma factor	
$\Delta t' = \gamma \Delta t$	time dilation	
$L' = L/\gamma$	length contraction	
$d^2 = \Delta x^2 + \Delta y^2$	Pythagorean Theorem	
$s^2 = \Delta x^2 - c(\Delta t)^2$	spacetime interval	
$E = mc^2$	mass and rest energy	Chapter 10
$E = \gamma E_0$	total energy and rest energy	
$g = \frac{GM}{R^2}$	Surface gravity	
$d = v_0 t + \frac{1}{2} a t^2$	Distance and acceleration	
$v = at$	Velocity and acceleration	
$t = \frac{t_0}{\left(1 - \frac{gH}{c^2}\right)}$	Time dilation (weak field approximation)	
$f = f_0 \left(1 - \frac{gH}{c^2}\right)$	Gravitational redshift (weak field approximation, frequency, photon traveling upward)	

Formulae	Relationship	Initial Chapter
$\lambda = \frac{\lambda_0}{\left(1 - \frac{gH}{c^2}\right)}$	Gravitational redshift (weak field approximation, wavelength)	
$f = f_0 \sqrt{1 - \frac{2GM}{Rc^2}}$	Gravitational redshift (full expression, frequency)	
$\lambda = \frac{\lambda_0}{\sqrt{1 - \frac{2GM}{Rc^2}}}$	Gravitational redshift (full expression, wavelength)	
$d^2 = (\Delta x)^2 + (\Delta y)^2$	Pythagorean theorem	
$d^2 = (R\Delta\theta)^2 + \cos^2\theta(R\Delta\alpha)^2$	Distance on a sphere	
$d^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$	Pythagorean Theorem in 3-D	
$s^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2$	Spacetime interval in flat space	
$s^2 = \left(1 - \frac{2GM}{rc^2}\right)^{-1} [(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2] - \left(1 - \frac{2GM}{rc^2}\right) (c\Delta t)^2$	Spacetime interval in spherically curved space	
$\theta = \frac{2GM}{bc^2}$	Angle of deflection of light	
$P_{gw} = \frac{2}{5} \left(\frac{GM}{Rc^2}\right)^5 \left(\frac{m}{M}\right)^2 \left(\frac{c^5}{G}\right)$	Power emitted by gravitational waves	
$\mathbf{G} = 8\pi G \mathbf{T}/c^4$	Einstein equation	

Formulae	Relationship	Initial Chapter
$R_S = \frac{2GM}{c^2}$	Schwarzschild radius	Chapter 11
$s^2 = \left(1 - \frac{R_S}{d}\right)^{-1} (\Delta d)^2 - \left(1 - \frac{R_S}{d}\right) (c\Delta t)^2$	Spacetime interval in a spherically symmetric space (Schwarzschild interval)	
$T_{bh} = \frac{1.23 \times 10^{23}}{M}$	Temperature of a black hole	
$\Delta E \Delta t \geq \frac{h}{4\pi}$	Uncertainty principle	
$L = \frac{3.56 \times 10^{32}}{M^2}$	Luminosity of a blackhole	
$t \approx 2.5 \times 10^{-16} M^3$	Evaporation time	
$\frac{\Delta m}{\Delta t} = -\frac{2L}{c^2}$	Accretion rate	Chapter 12
$L_{edd} = 6.3 M_{BH}$	Eddington luminosity	
$\alpha = \frac{4GM}{bc^2}$	Deflection angle (full)	
$\theta_E = \sqrt{\left(\frac{4GM(b)}{c^2}\right) \left(\frac{D_{LS}}{D_{LO}D_{SO}}\right)}$	Einstein radius	
$\theta^2 - x\theta - \theta_E^2 = 0$	Lens equation	

Formulae	Relationship	Initial Chapter
$m = \frac{1}{\left[1 - \left(\frac{\theta_E}{\theta}\right)^4\right]}$	Magnification for a point-mass lens	Chapter 13
$v = H_0 d$	Hubble law	
$v = cz$	Cosmological redshift	
$d_{\text{physical}}(t) = d_{\text{comoving}}(t)S(t)$	Comoving coordinates	
$1 + z = \frac{S(t_{\text{observed}})}{S(t_{\text{emitted}})}$	Ratio of scale factors	
$t = \frac{1}{H_0}$	Hubble time (age)	
$H^2 - \frac{8\pi G\rho}{3} = -\frac{kc^2}{S^2}$	Friedman equation	Chapter 14
$d = cz/H_0$	distance and redshift	
$T_e / T_o = 1 + z = S_o / S_e$	Temperature, redshift, and scale factor	Chapter 15
$E \sim kT$	energy and temperature	
$T \sim mc^2/k$	temperature of Universe and mass of particle in reaction	Chapter 16
Constants	Name	
$c = 3 \times 10^8 \text{ m/s} = 3 \times 10^5 \text{ km/s}$	speed of light	
$h = 6.63 \times 10^{-34} \text{ J s} = 4.136 \times 10^{-15} \text{ eV s}$	Planck's constant	
$G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$	Universal gravitational constant	
$k_B = 1.38 \times 10^{-23} \text{ J/K}$	Boltzmann's constant	
$t_{\text{planck}} \sim 10^{-43} \text{ s}$	Planck time	
$t_{\text{planck}} \sim 4 \times 10^{-35} \text{ m}$	Planck length	

Formulae	Relationship	Initial Chapter
$m_e = 9.1 \times 10^{-31} \text{ kg}$	mass of an electron	
Conversions	Units	
1 km = 1000 m	km and meters	
1 km = 0.6 mi	km and miles	
1 AU = $1.5 \times 10^{11} \text{ m} = 1.5 \times 10^8 \text{ km}$	AU and meters and km	
1 ly = $9.5 \times 10^{15} \text{ m} = 9.5 \times 10^{12} \text{ km}$	light-years and meters and km	
1 ly = $6.3 \times 10^4 \text{ AU}$	light-years and AU	
1 eV = $1.6 \times 10^{-19} \text{ J}$	eV and joules	
1 degree = 60 arcmin	degrees and arcminutes	
1 arcmin = 60 arcsec	arcminutes and arcseconds	
1 angstrom (\AA) = $1 \times 10^{-10} \text{ meters}$	angstroms and meters	
1 pc = 3.26 ly	parsecs and light-years	
1 N = 0.2248 pounds	newtons and pounds	
1 kpc = $3.086 \times 10^{19} \text{ m}$	kiloparsecs and meters	
1 solar mass = $2 \times 10^{30} \text{ kg}$	solar masses and kg	
1 Mpc = $3.09 \times 10^{22} \text{ m}$	megaparsecs and meters	
1 radian = $2.06 \times 10^5 \text{ arcsecond}$	radians and arcsec	
1 Mpc = $3.09 \times 10^{19} \text{ km}$	megaparsecs and km	
Units (abbreviation)	Type of quantity	
meters (m)	length (SI)	
kilograms (kg)	mass (SI)	
second (s)	time (SI)	
meters per second (m/s)	speed (SI)	
kelvin (K)	temperature (SI)	
miles (mi)	length	
astronomical unit (AU)	length	
year (yr)	time	
light-year (ly)	length	
light-minutes	length	
light-seconds	length	
g/cm^3	density	

Formulae	Relationship	Initial Chapter
solar masses	mass	
hertz (Hz) = cycles/s = $1/s = s^{-1}$	frequency (SI)	
joules (J)	energy (SI)	
electron volts (eV)	energy	
watts (W) = J/s	power	
radians	angle	
degrees	angle	
arcmin	angle	
arcsec	angle	
angstrom (Å)	length	
parsec (pc)	length	
m/s^2	acceleration (SI)	
newton (N) = $kg\ m/s^2$	force (SI)	
joules (J) = N m	energy (SI)	
μK micro Kelvin = 10^{-6} K	temperature	

Prefix	Meaning (in USA)	Exponent	Symbol
Tera	trillion	10^{12}	T
Giga	billion	10^9	G
Mega	million	10^6	M
kilo	thousand	10^3	k
centi	one-hundredth	10^{-2}	c
milli	one-thousandth	10^{-3}	m
micro	one-millionth	10^{-6}	μ
nano	one-billionth	10^{-9}	n
pico	one-trillionth	10^{-12}	p