

## 9: Elements of Coherent atom field interactions

### Objectives

In this chapter we shall consider the dynamics of an atom under the action of a monochromatic or quasimonochromatic field. We start from first principles i.e. from the Schrödinger equation. In the second part of the chapter we consider the incoherent limit, which is important mainly in understanding the laser.

### Prerequisites

The Schrödinger equation from Chapter 2. The energy of a dipole in an external electric field. Solution of differential equations. Dielectric susceptibility and its connection with the index of refraction and absorption.

### Introduction

In this chapter we shall consider the dynamics of an atom under the influence of a given external, linearly polarized harmonic electric field of circular frequency  $\omega$  and amplitude  $E_0$ :

$$E(t) = E_0 \cos \omega t \quad (9.1)$$

We know, of course that an electromagnetic wave has a space dependence, as well, but as the size of an atom is about  $10^{-31}$ – $3$  times smaller than the wavelength of the field in the visible and even in the far UV domain, we can neglect the space dependence. We assume here that the field induces transition between **two resonant** energy levels, which means that there are two atomic stationary states with energies  $\varepsilon_1 < \varepsilon_2$ , such that their difference  $\varepsilon_2 - \varepsilon_1 = \hbar\omega_0$  is **close to**  $\hbar\omega$ . This condition can be expressed as:

$$\omega_0 - \omega =: \Delta \ll \omega_0 \quad (9.2)$$

In the language of quantum mechanics the two levels belong to two stationary eigenstates  $u_1$  and  $u_2$  of the Hamiltonian  $H_0$  of the isolated atom. We shall assume here that these levels are nondegenerate, i.e. there is only one stationary state belonging to them:

$$\begin{aligned} H_0 u_1 &= \varepsilon_1 u_1 \\ H_0 u_2 &= \varepsilon_2 u_2 \end{aligned} \quad (9.3)$$

$u_1$  and  $u_2$  can be for instance the  $1s$  and  $2p_z$  ( $m=0$ ) states of a hydrogen atom, assumed to be time independent. This is the **two-level and resonant approximation**.

If we include the interaction with the external electromagnetic wave, then the Hamilton operator must be modified:

$$H = H_0 + K \quad (9.4)$$

where  $K$  describes the effect of the external field. From this point of view the atom is represented by a dipole moment operator  $D$  which couples the atom to the oscillating external field, and the interaction energy is

$$K = -DE = -DE_0 \cos \omega t \quad (9.5)$$

We assume here that the polarization, the direction of the electric field strength, points to the  $x$  direction, and accordingly  $D$  is the  $x$  component of the dipole moment. Besides being a natural assumption, this form of the interaction energy can be shown to be a very good approximation by more elaborate arguments.

### Solution of the optical Rabi problem

Our task is to solve the time dependent Schrödinger equation

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi \quad (9.6)$$

in this two dimensional problem. According to the two level resonant approximation, we shall look for the solutions as the time dependent linear combination of the two stationary states

$$\Psi(\mathbf{r}, t) = b_1(t)e^{-i\frac{\varepsilon_1}{\hbar}t}u_1(\mathbf{r}) + b_2(t)e^{-i\frac{\varepsilon_2}{\hbar}t}u_2(\mathbf{r}) \quad (9.7)$$

In (9.7) we have only two coefficients. The explicit time dependent factors  $e^{-i\frac{\epsilon_j}{\hbar}t}$  have been separated out to make the calculations more easy. In this section we assume that the amplitude of the linearly polarized electric field is constant at the position of the atom, which is assumed to be in rest.. We substitute the wave function (9.7) into (9.6) and find the equation:

$$\sum_{j=1,2} \left( i\hbar \dot{b}_j + \epsilon_j b_j(t) \right) e^{-i\frac{\epsilon_j}{\hbar}t} u_j = (H_0 - DE_0 \cos \omega t) \sum_{j=1,2} b_j(t) e^{-i\frac{\epsilon_j}{\hbar}t} u_j \quad (9.8)$$

We can now multiply this equation by  $u_i^*(\mathbf{r})$  ( $i = 1, 2$ ), integrate with respect to the spatial coordinates of the eigenfunctions, and make use of the orthogonality condition of the stationary states, which are assumed to be normalized, as well:  $\int u_i^*(\mathbf{r}) u_j(\mathbf{r}) d^3\mathbf{r} = \delta_{ij}$ .

Then we obtain two coupled ordinary differential equations:

$$i\dot{b}_1 = -\frac{dE_0}{2\hbar} (e^{i\omega t} + e^{-i\omega t}) e^{-i\omega_0 t} b_2 \quad (9.9)$$

$$i\dot{b}_2 = -\frac{d^*E_0}{2\hbar} (e^{i\omega t} + e^{-i\omega t}) e^{i\omega_0 t} b_1 \quad (9.10)$$

where

$$d = \int u_1^*(\mathbf{r}) D u_2(\mathbf{r}) d^3\mathbf{r} = q \int u_1^*(\mathbf{r}) x u_2(\mathbf{r}) d^3\mathbf{r} \quad (9.11)$$

is the so called **transition matrix element** of the  $x$  component of the dipole moment operator, or transition dipole moment for short. As the extensions of atomic wave functions are of the order of the Bohr radius,  $a_0$  and the charge in question is the charge of the optically active electron,  $q_0$ , the elementary charge,  $d$  must be of the order of  $q_0 a_0$ . As an example we quote the value of  $d$  for the 589.0nm  $D_2$  line of sodium, which is  $|d| = 3.5 q_0 a_0 = 3 \times 10^{-29} \text{ Cm}$ . We shall assume that

$$d = d^* \quad (9.12)$$

is a real quantity, which is very often the case. We also have taken into account here that the integrals

$$\int u_j^*(\mathbf{r}) x u_j(\mathbf{r}) d^3\mathbf{r} = \int x |u_j(\mathbf{r})|^2 d^3\mathbf{r} = 0 \quad j = 1, 2 \quad (9.13)$$

vanish. The reason is that for atoms the functions  $|u_j(\mathbf{r})|^2$  are always even, and therefore their products with  $x$ , which is odd, is also an odd function. That is why atoms do not possess a permanent electric dipole moment, because these integrals are the expectation values of the dipole moment in the stationary states  $u_j(\mathbf{r})$ .

Close to resonance on the right hand side of Eqs. (9.9), (9.10) we have two kind of exponentials. One of them which contains the difference  $\omega_0 - \omega = \Delta \ll \omega$ , is a slowly varying function of time if compared to the other oscillating with frequency  $\omega_0 + \omega \approx 2\omega$ . We can omit the latter ones, as on integration the terms  $e^{\pm i2\omega t} / 2\omega$  are small and rapidly oscillating relative to those with  $e^{\pm i\Delta t} / \Delta$ . This is called the **rotating wave approximation** abbreviated as RWA.

Thus with the RWA we have:

$$\dot{b}_1 = i \frac{dE_0}{2\hbar} e^{i\omega t} e^{-i\omega_0 t} b_2 = i \frac{dE_0}{2\hbar} e^{-i\Delta t} b_2 \quad (9.14)$$

$$\dot{b}_2 = i \frac{d^*E_0}{2\hbar} e^{-i\omega t} e^{i\omega_0 t} b_1 = i \frac{dE_0}{2\hbar} e^{i\Delta t} b_1 \quad (9.15)$$

It is worth to note that in the case of circularly polarized external field, when  $\hat{\mathbf{E}} = E_0(\hat{\mathbf{x}} \cos \omega t + \hat{\mathbf{y}} \sin \omega t)$  we do not need the RWA, we obtain exactly the above equations.

Now we introduce the notation

$$\frac{dE_0}{\hbar} = \Omega_0 \quad (9.16)$$

which has the dimension of frequency, and describes the strength of the interaction between the atom of dipole moment  $d$ , and field, as given by  $E_0$ .

With this notation

$$\dot{b}_1 = i \frac{\Omega_0}{2} e^{-i\Delta t} b_2 \quad (9.17)$$

$$\dot{b}_2 = i \frac{\Omega_0}{2} e^{i\Delta t} b_1 \quad (9.18)$$

It is easily seen that the system conserves  $|b_1|^2 + |b_2|^2$ , which is set to be

$$|b_1|^2 + |b_2|^2 = 1 \quad (9.19)$$

as by assumption, we have only these two levels populated.

### Problem 9.1

Prove that (9.19) is valid at all times  $|b_1(t)|^2 + |b_2(t)|^2$ , meaning that the equations conserve the probability) if this is valid at  $t = 0$ , i.e.  $|b_1(0)|^2 + |b_2(0)|^2$ .

We shall restrict ourselves to the solution of the system of equations (9.17) and (9.18) above only if  $\Delta = 0$ , which is called the resonant case.

$$\begin{cases} \dot{b}_1 = i \frac{\Omega_0}{2} b_2 \\ \dot{b}_2 = i \frac{\Omega_0}{2} b_1 \end{cases} \quad (9.20)$$

The solution is the following:

$$\begin{cases} b_1(t) = b_1(0) \cos \frac{\Omega_0}{2} t + i b_2(0) \sin \frac{\Omega_0}{2} t \\ b_2(t) = i b_1(0) \sin \frac{\Omega_0}{2} t + b_2(0) \cos \frac{\Omega_0}{2} t \end{cases} \quad (9.23)$$

Assume now that the system is initially in its lower stationary state  $u_1$ . Then we can set  $b_1(0) = 1$ , and  $b_2(0) = 0$ . The probability of finding the system in the upper state is then:

$$|b_2|^2 = \sin^2 \frac{\Omega_0 t}{2} = \frac{1}{2} (1 - \cos \Omega_0 t) \quad (9.26)$$

We see that this probability oscillates with the frequency  $\Omega_0$  between 0 and 1.  $\Omega_0$  is called the **resonant optical Rabi frequency** after I. I. Rabi, who first investigated the very similar phenomenon of magnetic resonance in 1938, and used it to determine experimentally the magnetic moments of atomic nuclei. We note that the Rabi frequency is usually 5 orders of magnitude smaller than the transition (Bohr) frequency.

We can see that the system reaches the upper state exactly when  $\Omega_0 t = \pi$ . Therefore a pulse with constant amplitude and duration of  $t = \pi/\Omega_0$  is called a  $\pi$  pulse. On the other hand if  $t = 2\pi/\Omega_0$ , then the system returns to its initial ground state: this is a  $2\pi$  pulse. A pulse for which  $t = \frac{\pi}{2}/\Omega_0$  brings the system into an equal weight superposition of the ground and excited states: this is a  $\frac{\pi}{2}$  pulse. We note that some texts use the terms for these processes as absorption till  $t = \pi/\Omega_0$ , and induced emission, between  $t = \pi/\Omega_0$  and  $t = 2\pi/\Omega_0$ , but this is not really a good nomenclature. Absorption and induced emission, as they were originally introduced by Einstein are **incoherent** processes, they manifest themselves only on longer time scales, where relaxation processes play an important role, and can be considered a kind of a stationary limit of the process considered here. We shall come to this point in the next section.

This shows that on exact resonance we **can excite an atom or a system exactly to its upper state** if this excitation is so fast, that the atom can be described with the Schrödinger equation for all times. This is called a coherent excitation. It means that the atom does not interact with degrees of freedom, other than the oscillating electric field. Below we shall consider circumstances when this is not valid any more, and will complete the description with relaxation, as well as with pumping terms.

The system (9.17), (9.18) can be solved in a closed form also when  $\omega_0 - \omega = \Delta \neq 0$ , i.e. if the system is not exactly resonant. It can be shown, that in such cases the system initially in its ground state cannot be inverted in this way any more, the probability of finding it in the state  $u_2$  is

$$|b_2(t)|^2 = \frac{\Omega_0^2}{\Omega_0^2 + \Delta^2} \sin^2 \sqrt{\Omega_0^2 + \Delta^2} \frac{t}{2} \quad (9.27)$$

So if  $\Delta \neq 0$  then the oscillation frequency is changed from  $\Omega_0$  to  $\sqrt{\Omega_0^2 + \Delta^2}$ , and more importantly the maximal value of  $|b_2(t)|^2$  is diminishing from 1 to  $\frac{\Omega_0^2}{\Omega_0^2 + \Delta^2}$  with increasing  $\Delta$ , i.e. with detuning the external field from the resonance condition.

As we have mentioned, the quantum theory of the resonant interaction has been derived and used first by Rabi in his molecular beam method. In that case the levels in question were magnetic spin states in nuclei, and the transition was induced by time varying magnetic fields.

In case of electronic levels in the atomic shell the transition comes from the electric dipole interaction with a time dependent electric field, as we have seen in this section. In the infrared domain the validity of these results has been proved experimentally first by G. Hocker and C. Tang in 1969, while in the visible domain by H. Gibbs with Rb atoms in 1973.

### The time dependent polarization of the atom

An important quantity that can be calculated from the dynamics of the atoms is the expectation value of their dipole moment. According to quantum mechanics this can be found:

$$p(t) = \int \Psi^*(\mathbf{r}, t) D \Psi(\mathbf{r}, t) d^3\mathbf{r} \quad (9.28)$$

in the time dependent state  $\Psi(\mathbf{r}, t)$  of (9.7) as

$$\begin{aligned} p(t) &= b_1^* b_2 e^{-i\omega_0 t} \int u_1^*(\mathbf{r}) D u_2(\mathbf{r}) d^3\mathbf{r} + b_1 b_2^* e^{+i\omega_0 t} \int u_2^*(\mathbf{r}) D u_1(\mathbf{r}) d^3\mathbf{r} = \\ &= d (b_1^* b_2 e^{-i\omega_0 t} + b_1 b_2^* e^{+i\omega_0 t}) =: d (\varrho_{21} e^{-i\omega t} + \varrho_{12} e^{i\omega t}) \end{aligned} \quad (9.29)$$

Here we exploited (9.11) and that  $(\varepsilon_2 - \varepsilon_1)/\hbar = \omega_0$  is the Bohr frequency of the transition. We have also introduced the definition

$$\varrho_{21} = \varrho_{12}^* = b_2 b_1^* e^{-i\Delta t} \quad (9.30)$$

which are the slowly varying factors (compared to  $e^{i\omega t}$ ) in the atomic dipole moment. We shall give more explanation about  $\varrho_{21}$  in the next subsection. We have also assumed that the dipole matrix element (transition dipole momentum) is real, see (9.12), and we have exploited once again that the static dipole moments vanish see (9.13). Note the difference between  $d$  and  $p(t)$ :  $d$  the dipole matrix element is a constant, while  $p(t)$  is the time dependent dipole moment of the atom.

If the atoms constitute a gas, the macroscopic polarization density  $P$  which is an important quantity in phenomenological electrodynamics can be calculated by multiplying the expectation value of the atomic dipole moment with  $\mathcal{N}$ , the latter being the number of atoms per unit volume:

$$P = \mathcal{N} p(t) \quad (9.31)$$

### Populations and coherences, a different formulation

Let us observe at this point that the physically interesting quantities can be expressed as products of the quantum mechanical amplitudes  $b_i$  of the levels. The probabilities of finding the atom on level  $i$  is  $|b_i|^2$ , and we shall introduce the notation for these quantities:

$$w_i := |b_i|^2 \quad (9.32)$$

The number of atoms in unit volume on level  $i$  is then obtained by  $\mathcal{N} w_i$  which is called the population of level  $i$ , but frequently the probabilities  $w_i$  are also called as populations. In case of the two-level approximation above, we have always

$$w_1 + w_2 = 1 \quad (9.33)$$

therefore only one of the  $w_i$ -s is an independent variable. One usually chooses the variable

$$w := w_2 - w_1 \quad (9.34)$$

which then determines  $w_1 = (1 - w)/2$  and  $w_2 = (1 + w)/2$ .  $w$  is called the population inversion, or simply inversion variable varying between  $-1$  and  $1$ .

The other important quantity is  $p(t)$  which is determined by the products  $b_i b_j^*$ , or by the quantities:  $\varrho_{ij} = \varrho_{ji}^*$ . These are often called as **coherences between the stationary levels**. They are non-zero only if the system is in state 1, as well as in state 2 simultaneously with nonvanishing amplitudes. The  $\varrho_{ij}$ -s are responsible for the time varying polarization of the atom, and that is why an atom can radiate while it is in a superposition of its stationary states, see (9.29). Because of their direct physical meaning it is straightforward to seek the equations that determine the dynamics of  $w(t)$  and  $\varrho_{ij}(t)$ , instead of those for the  $b_i$  amplitudes.

Using (9.17) and (9.18) this can be simply done and we present the result leaving the calculations as a problem.

$$\dot{w} = -i\Omega_0 (\varrho_{21} - \varrho_{21}^*) \quad (9.35)$$

$$\dot{\varrho}_{21} = -i\Delta \varrho_{21} - i\Omega_0 w/2 \quad (9.36)$$

#### Problem 9.2

Derive these equations from (9.17) and (9.18).

We note that  $w$  is always a real number, and on exact resonance, when  $\Delta = 0$ ,  $\varrho_{21}$  is purely imaginary, which is written customarily as  $\varrho_{21} = -i\nu/2$  with real  $\nu$ . Then the system (9.35), (9.36) takes the simple form

$$\dot{w} = -\Omega_0 v, \quad \dot{v} = \Omega_0 w \quad (9.37)$$

An important constant of motion emerging from these equations is:

$$w^2 + v^2 = \text{const} = 1 \quad (9.38)$$

The value of the constant is 1, because it turns out to be equal to  $|b_2|^2 + |b_1|^2$ .

### Problem 9.3

Show the validity of (9.38).

The above equality implies the following substitution

$$w = -\cos \Theta, \quad v = -\sin \Theta \quad (9.39)$$

the minus sign being a convention. The system (9.37) can be integrated even if the external field **amplitude** is time dependent  $\Omega_0(t) = \frac{d}{dt} E_0(t)$ , as from the first of the equations we have, yielding, when substituting into the second. The integral

$$\Theta(t) = \int_{-\infty}^t \Omega_0(t') dt' = \frac{d}{dt} \int_{-\infty}^t E_0(t') dt' \quad (9.40)$$

is called the **pulse area**. Let us note that we implicitly assumed that the time dependence of the amplitude  $E_0(t)$  – called the **envelope** – is much slower than that of the rapidly oscillating phase factor  $\cos \omega t$ , the **carrier**, i.e.  $|\dot{E}_0(t)| \ll \omega |E_0(t)|$ , otherwise the separation (9.1) would not make sense.

The theory outlined above forms the basis of coherent atom-field interactions, and it is very important in a large class of spectroscopic studies.

## Rate equations

### Relaxation terms

In the coherent interaction between atom and field considered in the previous section, we have assumed that the atom interacts exclusively with the external field. This is valid, however only for times that are short in comparison with relaxation times. The latter describe the time scale how fast (or slow) is the interaction of an atom with other degrees of freedom, which are always present as an environment, even in a perfect vacuum. So we come to the point where the relaxation and possibly pumping processes, mentioned in the introduction of this chapter must be taken into account.

The fastest of all these relaxation processes is the so called **transversal** relaxation, or phase relaxation which effects  $\varrho_{21}$  because it destroys the coherence between the levels. This is introduced here phenomenologically as an exponential decay, the time constant of which is usually denoted by  $T_2$ . This means that we complete the right hand side of Eq. (9.36) with an additional term  $-\varrho_{21}/T_2$ :

$$\dot{\varrho}_{21} = -\frac{\varrho_{21}}{T_2} - i\Delta\varrho_{21} - i\frac{\Omega_0}{2}w \quad (9.41)$$

$T_2$  is characteristic of the level pair in question. In the absence of the external driving field  $\varrho_{21}$  decays exponentially due to the term  $-\varrho_{21}/T_2$ . The new term describes the **loss of the phase memory** between the amplitudes of the levels. For an isolated atom in vacuum this comes from the unavoidable quantum fluctuations of the electromagnetic vacuum causing spontaneous emission, and it is of the order of nanoseconds. In a laser material it is due to collisions with other atoms in a gas, or with phonons in a solid etc., and it is in the ps range, thus much faster than for an isolated atom.

If we consider only two levels and the absence of any other external agent than the coherent excitation given by  $E(t)$ , then we have to consider also the loss of the energy of the atoms which is transferred to its environment. This is done again by adding an additional damping term to the right hand side of (9.35) resulting also in exponential decay to the value  $w = -1$ , corresponding to the lower state

$$\dot{w} = -i\Omega_0(\varrho_{21} - \varrho_{21}^*) - \frac{w+1}{T_1} \quad (9.42)$$

This relaxation which is usually much slower than that of  $\varrho_{21}$  is called **longitudinal** relaxation, and

$$T_2 \ll T_1 \quad (9.43)$$

### Stationary solution

The difference between the transversal and longitudinal relaxation times allows one to find a stationary solution for the  $\rho_{21}$  variable. We can assume that due to the large value of  $1/T_2$  the time derivative of  $\rho_{21}$  can be set to zero, yielding

$$\rho_{21} = -\frac{i}{1/T_2 + i\Delta} \frac{\Omega_0}{2} w \quad (9.44)$$

Let us insert now this value in (9.42) and obtain

$$\dot{w} = -\frac{1}{T_2} \frac{1}{\Delta^2 + T_2^{-2}} \Omega_0^2 w - \frac{w+1}{T_1} \quad (9.45)$$

We may introduce a dimensionless intensity

$$\mathfrak{F} = T_1 T_2 \Omega_0^2 = T_1 T_2 \left( \frac{dE_0}{\hbar} \right)^2 \quad (9.46)$$

and the Lorentzian factor:

$$\mathfrak{L}(\Delta) = \frac{1}{1 + (T_2 \Delta)^2} \quad (9.47)$$

so the equation takes the form:

$$\dot{w} = -\frac{\mathfrak{F}}{T_1} \mathfrak{L}(\Delta) w - \frac{w+1}{T_1} \quad (9.48)$$

This equation describes the change of the population under the effect of the field of amplitude  $E_0$ , or intensity  $\mathfrak{F} \sim E_0^2$ . It is not difficult to solve this equation, but instead of giving its explicit solution, we note only that the value of  $w(t)$  will go exponentially to its equilibrium value  $w_{\infty}$ , where  $\dot{w} = 0$ . This can be easily seen to be

$$w_{\infty} = -\frac{1}{\mathfrak{F} \mathfrak{L}(\Delta) + 1} \quad (9.49)$$

which is always negative, so by incoherent excitation one cannot achieve inversion, in contrast to short time coherent excitation as we have seen it in the case of Rabi oscillations.

The incoherent process described in this section can be considered as absorption, or as induced emission, when starting from  $w = +1$ , in the sense Einstein introduced them originally.

### Linear and stationary limit

Let us consider now the stationary solution for weak fields, where the intensity is not too strong, and the system remains close to its ground state  $w = -1$  during all the process. Then the steady state solution (9.44) gives

$$\rho_{21} = i \frac{\Omega_0}{2} \frac{1}{i\Delta + \frac{1}{T_2}} \quad (9.50)$$

while the  $\Delta$  dependent macroscopic polarization is according to (9.29) and (9.31)

$$P(\Delta) = \mathcal{N} d (\rho_{21} e^{-i\omega t} + \rho_{12} e^{i\omega t}) = 2 \operatorname{Re} \frac{T_2}{2} \frac{d^2}{\hbar} \mathcal{N} \frac{i + T_2 \Delta}{1 + (T_2 \Delta)^2} E_0 e^{-i\omega t} \quad (9.51)$$

In this limiting case we have a linear relation between  $P$  and  $E$ , which we may compare with the phenomenological macroscopic connection between  $P$  and  $E$ :

$$P = \chi \epsilon_0 E \quad (9.52)$$

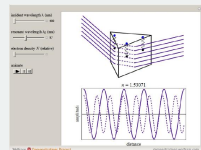
the expression

$$\chi(\Delta) = \frac{1}{\epsilon_0} \frac{d^2}{\hbar} T_2 \mathcal{N} \frac{i + T_2 \Delta}{1 + (T_2 \Delta)^2} = \chi' + i\chi'' \quad (9.53)$$

can be identified with the frequency dependent susceptibility close to resonance. We see that it has a real part,  $\chi'$  which is known to be connected with the index of refraction via  $n^2 = \epsilon' = 1 + \chi'$ , while its imaginary part  $\chi''$  is known to be connected to the linear absorption coefficient.

Essentially the same result can be obtained from the so called oscillator or Lorentz model of the atom, which is just the linear variant of our more precise approach. In the Lorentz model the atom cannot be inverted, which corresponds to  $w = -1$  for all times.

## Animation



This animation shows the relevant parameters of the Lorentz model. The index of refraction is often described as an apparent decrease in the speed of light as it passes through a dielectric medium. In fact, light photons do not actually slow down, but the effect is simulated by a retarding phase shift in the emerging electromagnetic waves. This is caused by superposition of the incident wave with a retarded wave produced by radiation from the electrons in the medium.

<http://demonstrations.wolfram.com/LorentzOscillatorModelForRefractiveIndex/>

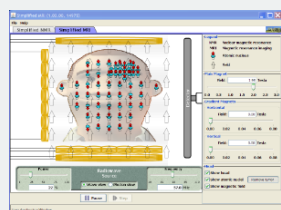
## Conclusions and outlook

We presented the action of a monochromatic or (in the case of time dependent  $E_0$ ) a quasimonochromatic field on atoms, that were described starting from the Schrödinger equation, i.e. from first principles. In the second part we considered the incoherent limit, which is important mainly in understanding the laser, where one has to include the space dependence of the variables, as well, which is a further complication.

The pioneer of the coherent quantum mechanical theory was [I. Rabi](#), who applied it to magnetic resonance, and received the Nobel prize in 1944. Interestingly, after him many other physicists obtained the Nobel for different extensions and applications of this relatively simple two-level model. Among them [F. Bloch](#) and [E. Purcell](#) (1951) for the Nuclear Magnetic Resonance (NMR) in solids. It is worth noting here that the pair of equations (9.41), (9.42) are often called optical Bloch equations. Other Nobel prizes went to [C. Townes](#), [N. Basov](#) and [A. Prokhorov](#) (1964) for the laser principle, [A. Kastler](#) (1966), [A. Schawlow](#) and [N. Bloembergen](#) (1981) for coherent spectroscopy, [N. Ramsey](#) (1989) who was a co-worker and student of Rabi for extending the Rabi method, [C. Cohen-Tannoudji](#), [W. Phillips](#) and [S. Chu](#) (1997) for laser cooling also based on this model, to be presented to some extent in the next section. More recently [S. Haroche](#) and [D. Wineland](#) (2012) got the prize who did experiments with single resonant two-level atoms.

The two-level resonant atomic model plays an important role in many other physical phenomena. As we have mentioned, it gives the basis of the so called semiclassical laser theory. [Atomic clocks](#), magnetic resonance experiments, and their wide applications in [chemical analysis](#), [medical picture processing](#) and many more are based on coherent atom field interactions.

## Animation



Is it a tumor? Magnetic Resonance Imaging (MRI) can tell. Your head is full of tiny radio transmitters (the nuclear spins of the hydrogen nuclei of your water molecules). In an MRI unit, these little radios can be made to broadcast their positions, giving a detailed picture of the inside of your head.

<http://phet.colorado.edu/en/simulation/mri>

More recently this model got another emphasis with the emergence of the science of quantum informatics, where bits are replaced by qubits. Their most important characteristic feature is that two atomic states are not mutually exclusive, but their superpositions can also be used for information processing, as we underscored it in this chapter.

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