1.5: The Scalar and Vector Wave Equation

We consider a homogeneous insulator (i.e. E is independent of position and σ=0) in which there are no external sources:

$$J_{ext}\,,\rho_{ext}=0.$$

In optics the external source, e.g. a laser, is normally spatially separated from objects of interest with which the light interacts. Hence the assumption that the external source vanishes in the region of interest is often justified. Take the curl of (1.12) and the time derivative of (1.13) and add the equations obtained. This gives

$$\bigtriangledown\times\bigtriangledown\times\varepsilon+E\mu_0\frac{\partial^2\varepsilon}{\partial t^2}=0.$$

Now for any vector field **A** there holds:

$$\bigtriangledown\times\bigtriangledown\times A=-\bigtriangledown^2 A+\bigtriangledown\bigtriangledown\cdot A.$$

where ∇^2A is the vector:

$$\bigtriangledown^2 A=\bigtriangledown^2 A_x\hat{x}+\bigtriangledown^2 A_y\hat{y}+\bigtriangledown^2 A_z\hat{z},$$

with

$$\bigtriangledown^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}.$$

Because Gauss's law (1.3.14) with ρ_{ext} = 0 and E constant implies that $\nabla \cdot$ E = 0, (1.5.3) applied to E yields

$$\bigtriangledown\times\bigtriangledown\times\varepsilon=-\bigtriangledown^2\varepsilon.$$

Hence, (1.5.2) becomes

$$\bigtriangledown^2\varepsilon-E\mu_0\frac{\partial^2\varepsilon}{\partial t^2}=0.$$

By a similar derivation it is found that also **H** satisfies (1.5.7). Hence in a homogeneous dielectric without external sources, every component of the electromagnetic field satisfies the scalar wave equation:

$$\bigtriangledown^2 U-E\mu_0\frac{\partial^2 U}{\partial t^2}=0.$$

The **refractive index** is the dimensionless quantity defined by

$$n=(\frac{E}{E_0})^{1/2}.$$

The scalar wave equation can then be written as

$$\bigtriangledown^2 U-n^2E_0\mu_0\frac{\partial^2 U}{\partial t^2}=0.$$

The speed of light in matter is

$$\frac{c}{n}=\frac{1}{(E\mu_0)^{1/2}}.$$