2.3: Principle of Fermat

The starting point of the treatment of geometrical optics is the very powerful.

> **Principle of Fermat (1657)**
>
> The path followed by a light ray between two points is the one that takes the least amount of time.

The speed of light in a material with refractive index n, is c/n, where c = 3 × 10^8 m/s is the speed of light in vacuum. At the time of Fermat, the conviction was that the speed of light must be finite, but nobody could suspect how incredibly large it actually is. In 1676 the Danish astronomer Ole Römer computed the speed from inspecting the eclipses of a moon of Jupiter and arrived at an estimate that was only 30% too low.

Let $\mathbf{r}(s)$, be a ray with s the length parameter. The ray links two points S and P. Suppose that the refractive index varies with position: $n(\mathbf{r})$. Over the infinitesimal distance from s to $s+ds$, the speed of the light is

$$\frac{c}{n(\mathbf{r}(s))}.$$

Hence the time it takes for light to go from $\mathbf{r}(s)$ to $\mathbf{r}(s+ds)$ is:

$$dt = \frac{n(\mathbf{r}(s))}{c} ds,$$

and the total total time to go from S to P is:

$$t_{S\rightarrow P} = \int_0^{s_P} \frac{n(\mathbf{r}(s))}{c}\, ds,$$

where s_P is the distance along the ray from S to P. The **optical path length** [m] of the ray between S and P is defined by:

$$OPL = \int_0^{s_P} n(r(s))\, ds,$$

So the OPL is the distance weighted by the refractive index.

Fermat's principle is thus equivalent to the statement that a ray follows the path with shortest OPL.

> **Remark**
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> Actually, Fermat's principle as formulated above is not complete. There are circumstances that a ray can take two paths between two points that have different travel times. Each of these paths then corresponds to a minimum travel time compared to nearby paths, so the travel time is in general a *local minimum*. An example is the reflection by a mirror discussed in the following section.