4.5: Decompose Elliptical Polarisation into Linear and Circular States

Any elliptical polarisation state can be written as the sum of two perpendicular linear polarised states:

$$J=\begin{pmatrix} c\mathcal{A}_x e^{i\varphi_x} \\ \mathcal{A}_y e^{i\varphi_y} \end{pmatrix}=\mathcal{A}_x e^{i\varphi_x}\begin{pmatrix}1\\0\end{pmatrix}+\mathcal{A}_y e^{i\varphi_y}\begin{pmatrix}0\\1\end{pmatrix}$$

Alternatively, any elliptical polarisation state can be written as the sum of two circular polarisation states, one right- and the other left-circular polarised:

$$J=\begin{pmatrix} \mathcal{A}_x e^{i\varphi_x} \\ \mathcal{A}_y e^{i\varphi_y} \end{pmatrix}=\frac{1}{2}\left(\mathcal{A}_x e^{i\varphi_x}-i\mathcal{A}_y e^{i\varphi_y}\right)\begin{pmatrix}1\\i\end{pmatrix}+\frac{1}{2}\left(\mathcal{A}_x e^{i\varphi_x}+i\mathcal{A}_y e^{i\varphi}\right)\begin{pmatrix}1\\-i\end{pmatrix}$$

We conclude that to study what happens to elliptic polarisation, it suffices to consider two orthogonal linear polarisations, or, if that is more convenient, left- and right-circular polarised light. In a birefringent material two linear polarisations, namely the one parallel to the o-axis and the one parallel to the e-axis, each propagate with their own refractive index. To predict what happens to an arbitrary linear polarisation state which is not aligned to either of these axes, or more generally what happens to an elliptical polarisation state, we write this polarisation as a linear combination of o- and e-polarisation states, i.e. we expand the field on the o- and e-basis.

In sugar, it are the left- and right-circular polarisation states which each propagate with their own refractive index. Therefore sugars are said to be **circular birefringent**. To see what happens to an arbitrary elliptical polarisation state in such a material, the incident light is best be written as linear combination of left-and right-circular polarisations.

External sources in recommended order

1. Double Vision - Sixty Symbols: Demonstration of double refraction by a calcite crystal due to birefringence.
2. MIT OCW - Linear Polarizer: Demonstration of linear polarizers and linear polarisation.
3. MIT OCW - Polarization Rotation Using Polarizers: Demonstration of polarisation rotation using linear polarisers.
4. Demonstration of a QuarterWavePlate by Andrew Berger.
5. MIT OCW - Quarter-wave Plate: Demonstration of the quarter-wave plate to create elliptical (in particular circular) polarisation.
6. Demonstration of a HalfWavePlate by Andrew Berger.
7. MIT OCW - Half-wave Plate: Demonstration of the half-wave plate.