5.11: Interference and polarisation

In the study of interference we have so far ignored the vectorial nature of light (i.e. its polarisation) by assuming that all the fields have the same polarisation. Suppose now that we have two real vector fields $\mathcal{E}_1, \mathcal{E}_2$. The (instantaneous) intensity of each field is (apart from a constant factor) given by

$$\mathcal{E}_1 \cdot \mathcal{E}_1, \quad \mathcal{E}_2 \cdot \mathcal{E}_2$$

If the two fields interfere, the instantaneous intensity is given by

$$(\mathcal{E}_1 + \mathcal{E}_2) \cdot (\mathcal{E}_1 + \mathcal{E}_2) = \mathcal{E}_1 \cdot \mathcal{E}_1 + \mathcal{E}_2 \cdot \mathcal{E}_2 + 2\mathcal{E}_1 \cdot \mathcal{E}_2$$

where $2\mathcal{E}_1 \cdot \mathcal{E}_2$ is the interference term. Suppose the polarisation of $\mathcal{E}_1$ is orthogonal to the polarisation of $\mathcal{E}_2$, e.g.

$$\mathcal{E}_1 = \begin{pmatrix} \mathcal{E}_{1x} \\ 0 \\ 0 \end{pmatrix}, \quad \mathcal{E}_2 = \begin{pmatrix} 0 \\ \mathcal{E}_{2y} \\ 0 \end{pmatrix}$$

Then $\mathcal{E}_1 \cdot \mathcal{E}_2 = 0$, which means the two fields can not interfere. This observation is the

First Fresnel-Arago Law: fields with orthogonal polarisation cannot interfere.

Next we write the fields in terms of orthogonal components

$$\mathcal{E}_1 = \begin{pmatrix} \mathcal{E}_{1\perp} \\ \mathcal{E}_{1\|} \end{pmatrix} \quad \mathcal{E}_2 = \begin{pmatrix} \mathcal{E}_{2\perp} \\ \mathcal{E}_{2\|} \end{pmatrix}$$

This is always possible, whether the fields are polarised or randomly polarised. Then (5.11.2) becomes

$$\mathcal{E}_1 \cdot \mathcal{E}_1 + \mathcal{E}_2 \cdot \mathcal{E}_2 + 2\mathcal{E}_1 \cdot \mathcal{E}_2 = \mathcal{E}_{1\perp}^2 + \mathcal{E}_{2\perp}^2 + 2\mathcal{E}_{1\perp}\mathcal{E}_{2\perp} + \mathcal{E}_{1\|}^2 + \mathcal{E}_{2\|}^2 + 2\mathcal{E}_{1\|}\mathcal{E}_{2\|}$$

If the fields are randomly polarised, the time average of the $\perp$-part will equal the average of the $|$-part, so the time-averaged intensity becomes

$$\begin{aligned} I &= 2\left\langle \mathcal{E}_{1\perp}^2 + \mathcal{E}_{2\perp}^2 + 2\mathcal{E}_{1\perp}\mathcal{E}_{2\perp} \right\rangle \\ &= 2\left\langle \mathcal{E}_{1\|}^2 + \mathcal{E}_{2\|}^2 + 2\mathcal{E}_{1\|}\mathcal{E}_{2\|} \right\rangle \end{aligned}$$

This is qualitatively the same as what we would get if the fields had parallel polarisation, e.g.

$$\mathcal{E}_1 = \begin{pmatrix} \mathcal{E}_{1\perp} \\ 0 \end{pmatrix}, \quad \mathcal{E}_2 = \begin{pmatrix} \mathcal{E}_{2\perp} \\ 0 \end{pmatrix}$$

This leads to the

Second Fresnel-Arago Law: two fields with parallel polarisation interfere the same way as two fields that are randomly polarised.

This indicates that our initial assumption in the previous sections that all our fields have parallel polarisation is not as limiting as it may have appeared at first.

Suppose now that we have some field

$$\mathcal{E} = \begin{pmatrix} \mathcal{E}_{\perp} \\ \mathcal{E}_{\|} \end{pmatrix}$$

which is **randomly polarised**. Suppose we separate the two polarisations, and rotate one so that the two resulting fields are aligned, e.g.

$$\mathcal{E}_1 = \begin{pmatrix} \mathcal{E}_{\perp} \\ 0 \end{pmatrix}, \quad \mathcal{E}_2 = \begin{pmatrix} \mathcal{E}_{\|} \\ 0 \end{pmatrix}$$

These fields can not interfere because $\mathcal{E}_{\perp}$ and $\mathcal{E}_{\|}$ are incoherent. This leads to

The third Fresnel-Arago Law: the two constituent orthogonal linearly polarised states of natural light cannot interfere to form a readily observable interference pattern, even if rotated into alignment.

External sources in recommended order

1. Veritasium - The original double-slit experiment, starting at 2:15 - Demonstration of an interference pattern obtained with sunlight.
2. MIT OCW - Two-beam Interference - Collimated Beams: Interference of laser light in a Michelson interferometer.
3. MIT OCW - Fringe Contrast - Path Difference: Demonstration of how fringe contrast varies with propagation distance.
4. MIT OCW - Coherence Length and Source Spectrum: Demonstration of how the coherence length depends on the spectrum of the laser light.
5. Lecture - 18 Coherence: Lecture Series on Physics - I: Oscillations and Waves by Prof. S. Bharadwaj, Department of Physics and Meteorology, IIT Kharagpur.
6. Lecture - 19 Coherence: Lecture Series on Physics - I: Oscillations and Waves by Prof. S. Bharadwaj, Department of Physics and Meteorology, IIT Kharagpur.