

5.8: Increase of Spatial Coherence by Propagation

When light propagates, the mutual coherence at transversely separated points in general increases with propagation distance. For example, the sun consists of an incredibly large number of emitters which emit at random times. Therefore, the fields at different points at the surface of the sun are completely uncorrelated, i.e. the field at the surface of the sun is spatially fully incoherent. But after propagation to the earth, the sunlight has become partially coherent, with coherence length of roughly $50\mu\text{m}$ around the wavelength of 510 nm . The further away the sun or a star is, the more spatially coherent the field in two transversely separated points becomes. How can we understand this?

One way of looking at it is that at the star's surface, the field is determined by the closest point sources. Since all point sources emit wavetrains making thousands of uncorrelated phase jumps during an integration time of any detector, the fields in two points at the surface of the star are completely incoherent. But the field at two transversely separated points at a great distance from the star both consist of contributions from all point sources in the star, i.e. the fields of the different point sources mix spatially. Provided the **difference** in distances between each of the transverse points and the point sources of the star is smaller than the coherence length, these contributions can interfere. Interesting, when the distance of the transversely separated points to the star increases, the difference between the distances of the points to the star decreases. Therefore, the field becomes more spatially coherent when the distance to the start increases.

To quantify this phenomenon, we consider two mutually incoherent point sources S_1 and S_2 in the $z = 0$ plane. We assume that their mutual coherence function is given by:

$$\begin{aligned}\Gamma_{S_1 S_2}(\tau) &= 0, \text{ for all } \tau, \\ \Gamma_{S_1 S_1}(\tau) &= \Gamma_{S_2 S_2}(\tau) = \Gamma_0(\tau),\end{aligned}$$

where Γ_0 is the self-coherence which we assume here to be the same for both point sources, $\Gamma_0(\tau)$ is a decreasing function of the delay time τ (although it is not necessarily monotonically decreasing). Using that the long-time average does not depend on the origin of time (which was based on the assumption that the source is stationary), we find:

$$\Gamma_0(-\tau) = \langle U(S_1, t) U(S_1, t + \tau)^* \rangle = \langle U(S_1, t - \tau) U(S_1, t)^* \rangle = \Gamma_0(\tau)^*.$$

Furthermore, for $\tau = 0$: $\Gamma_0(0) = I_0$, which is the intensity of either source.

Consider two points P_1, P_2 at large distance z from the two point sources as seen in Figure 5.8.1. We will compute the mutual coherence $\Gamma_{P_1 P_2}(0)$ of these points for zero time delay $\tau = 0$ (we can also compute the mutual coherence for more general time delay τ , i.e. $\Gamma_{P_1 P_2}(\tau)$, but it will suffice for our purpose to take $\tau = 0$). The field in P_1 is the sum of the fields emitted by S_1 and S_2 :

$$U(P_1, t) \propto \frac{U(S_1, t - |S_1 P_1|/c)}{|S_1 P_1|} + \frac{U(S_2, t - |S_2 P_1|/c)}{|S_2 P_1|},$$

where we used (5.6.3). Similarly,

$$U(P_2, t) \propto \frac{U(S_1, t - |S_1 P_2|/c)}{|S_1 P_2|} + \frac{U(S_2, t - |S_2 P_2|/c)}{|S_2 P_2|}.$$

Let us assume that z is so large that all distances $|S_i P_j|$ in the denominators can be replaced by z . Then these equal distances can be omitted. It can then be shown by substitution of (5.8.4) and (5.8.5) that the mutual coherence in P_1 and P_2 for zero time delay $\tau = 0$ becomes:

$$\begin{aligned}\Gamma_{P_1 P_2}(0) &= \langle (U(P_1, t) U(P_2, t))^* \rangle \\ &= \Gamma_{S_1 S_1} \left(\frac{|S_1 P_2| - |S_1 P_1|}{c} \right) + \Gamma_{S_1 S_2} \left(\frac{|S_2 P_2| - |S_1 P_1|}{c} \right) \\ &\quad + \Gamma_{S_2 S_1} \left(\frac{|S_1 P_2| - |S_2 P_1|}{c} \right) + \Gamma_{S_2 S_2} \left(\frac{|S_2 P_2| - |S_2 P_1|}{c} \right).\end{aligned}$$

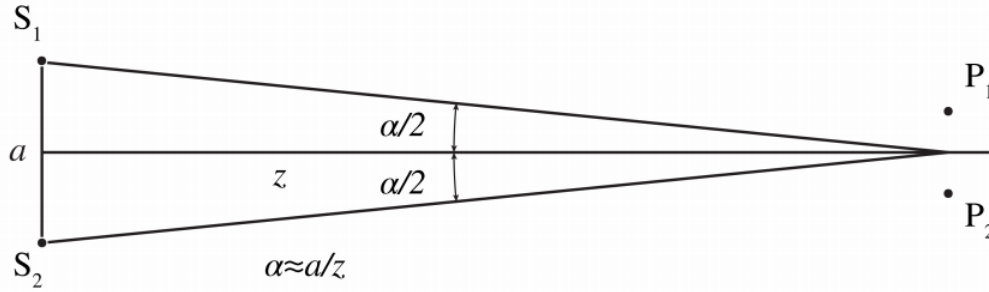


Figure 5.8.1: Two incoherent point sources S_1, S_2 and two points P_1, P_2 in a plane at large distance z from the point sources. The degree of mutual coherence at P_1 and P_2 increases to 1 in the limit $\alpha \rightarrow 0$.

Now we use (5.8.1) and (5.8.2) and get

$$\Gamma_{P_1 P_2}(0) = \Gamma_0 \left(\frac{|S_1 P_2| - |S_1 P_1|}{c} \right) + \Gamma_0 \left(\frac{|S_2 P_2| - |S_2 P_1|}{c} \right).$$

Similarly,

$$\Gamma_{P_1 P_1}(0) = \Gamma_{P_2 P_2}(0) = 2\Gamma_0(0) = 2I_0.$$

The result (5.8.7) confirms what was already remarked in Section 5.5, that the mutual coherence $\Gamma_{P_1 P_2}(\tau = 0)$ depends on the **difference** in distances of source point S_1 to points P_1 and P_2 and on the **difference** in distance from S_2 to P_1 and P_2 .

Remark. The derivation given here is valid for any two points P_1, P_2 sufficiently far away from the points sources S_1 and S_2 and hence applies not only to transversally but also to longitudinally separated points.

Suppose that $S_1 = (a/2, 0, 0)$, $S_2 = (-a/2, 0, 0)$ and let P_1, P_2 be given by: $P_j = (x_j, 0, z)$ for $j = 1, 2$. If z is very large, so that $S_1 P_1$ and $S_1 P_2$ are almost parallel, then with Figure 5.8.2 we find

$$|S_1 P_2| - |S_1 P_1| \approx |Q P_2| = \frac{\alpha}{2}(x_1 - x_2).$$

Similarly,

$$|S_2 P_2| - |S_2 P_1| \approx -\frac{\alpha}{2}(x_1 - x_2).$$

Hence, with (5.8.3):

$$\Gamma_{P_1 P_2}(0) = 2 \operatorname{Re} \Gamma_0 \left(\frac{\alpha}{2} \frac{(x_1 - x_2)}{c} \right).$$

It is thus seen that the degree of mutual coherence depends on the angle $\alpha \approx a/z$ subtended by the two point sources at the midpoint $(0, 0, z)$ on the mask. The smaller this angle, the higher the degree of spatial coherence. The reason is that for smaller α , i.e. for a smaller size of the source and/or a greater distance to the point sources, the fringes due to the point sources become almost overlapping and enforce each other, whereas for larger α the fringes are more displaced with respect to each other and hence the sum of the fringes of the two point sources has less pronounced fringe visibility.

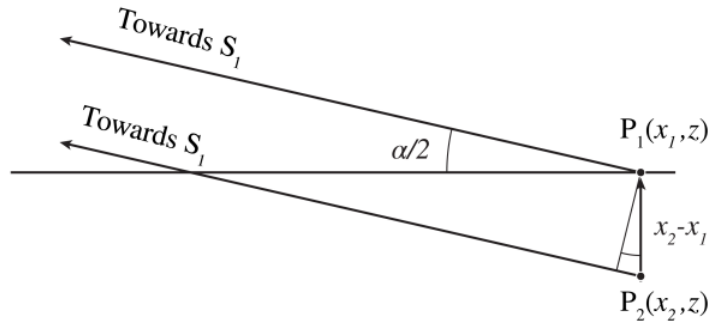


Figure 5.8.2: For z very large, S_1P_1 and S_1P_2 are almost parallel and $|S_1P_2| - |S_1P_1| \approx |QP_2| = (x_1 - x_2) \alpha/2$.

As example consider quasi-monochromatic light for which (see (5.4.10):

$$\Gamma_0(\tau) = I_0 e^{-i\bar{\omega}\tau}, \text{ for all } \tau$$

where $\bar{\omega}$ is the centre frequency. Then

$$\Gamma_{P_1P_2}(0) = 2I_0 \cos \left[\frac{\alpha \bar{\omega} (x_1 - x_2)}{c} \right],$$

and hence the degree of mutual coherence is:

$$\begin{aligned} \gamma_{P_1P_2}(0) &= \frac{\Gamma_{P_1P_2}(0)}{\sqrt{\Gamma_{P_1P_1}(0)} \sqrt{\Gamma_{P_2P_2}(0)}} \\ &= \cos \left[\frac{\alpha \bar{\omega} (x_1 - x_2)}{2} \right]. \end{aligned}$$

We see that when

$$x_1 - x_2 = \bar{\lambda}/(2\alpha),$$

there is no interference. Hence we can say that coherence between points in the mask only occurs when there distance is smaller than $\bar{\lambda}/(2\alpha)$.

We see from (5.8.14) that by keeping P_1 fixed, we can retrieve the angle α by measuring $\text{Re } \gamma_{P_1P_2}(0)$ for a number of different positions of P_2 .

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