12.1: Indistinguishable Particles

Every electron is exactly the same as every other electron. Thus, all electrons are indistinguishable. This means that if you have a state with two electrons, you can swap the two electrons and it cannot change anything physically observable from that state.

To make this concrete, suppose the state $|\psi\rangle$ is a state with two electrons. Let's define $|\psi'\rangle$ as the state with those two electrons swapped. Then, the expectation value of any operator must be the same for these two different states:

$$\langle\psi|\hat{\mathcal{O}}|\psi\rangle = \left\langle\psi'|\hat{\mathcal{O}}|\psi'\right\rangle \tag{12.1}$$

Also, the probability for any measurement of any observable to be made must be the same for the two states. That is, if $\langle\phi|$ is an eigenstate of a given observable, then

$$|\langle\phi \mid \psi\rangle|^2 = |\langle\phi \mid \psi'\rangle|^2 \tag{12.2}$$

If you think about it, however, this does not mean that the two states must be identical! However, they must be close enough such that anything physically observable from the state must be identical. Below, we will introduce the exchange operator as a way of quantifying the effect of identical particles on quantum states.