

## 13.2: Stating the Equation

As was briefly mentioned in Section 8.5, the energy operator is an operator so important to quantum mechanics that it gets its own proper name, the Hamiltonian, and the eigenvalue equation for it also gets its own name, the Schrödinger equation. It is this equation that allows us to figure out the energy states of a system, and it could be argued that energy states are the most important states in quantum mechanics. It is energy levels in atoms that provides all of the structure that gives us the Periodic Table of the Elements, and it is transitions between those energy levels that we observe in a number of both terrestrial and astrophysical contexts.

At the most base level, the Schrödinger equation is just the energy operator eigenvalue equation:

$$\hat{H}|\psi\rangle = E|\psi\rangle \quad (13.2)$$

$\hat{H}$  is the Hamiltonian; it's the operator that corresponds to energy as an observable. Solutions  $|\psi\rangle$  to this equation are the eigenstates of energy. The value  $E$  that goes with a given solution  $|\psi\rangle$  is the energy associated with that state. Technically, this equation is called the Time-Independent Schrödinger Equation. (There is also a full Schrödinger equation that describes how quantum states evolve in time.)

A full investigation of the Hamiltonian requires differential calculus, so we won't fully present it here. However, you can break the Hamiltonian into two parts. In doing so, we're going to go to a wave function representation of the state vector  $|\psi\rangle$ . Whereas we have used column vectors to represent spin states, it is more traditional (and more useful) to represent energy states as functions of position  $\psi(x, y, z)$ . As with a regular function,  $\psi(x, y, z)$  is just something into which you can plug a position (i.e. values of  $x$ ,  $y$ , and  $z$ ) and get a number—although in this case that number can be a complex number. Dividing the Hamiltonian into two parts and writing the state as a wave function yields this form of the Schrödinger equation:

$$\hat{K}\psi(x, y, z) + V(x, y, z)\psi(x, y, z) = E\psi(x, y, z) \quad (13.3)$$

The Hamiltonian here has been divided into the kinetic energy operator  $\hat{K}$ , and the potential energy  $V(x, y, z)$ . Note that the action of the potential energy operator is just multiplying the wave function by the potential energy! The aforementioned differential calculus is buried inside  $\hat{K}$ . In fact, there isn't a single Schrödinger equation. Rather, there's a different one for each form of the potential  $V(x, y, z)$ . This also means that the solutions  $\psi(x, y, z)$  will be different for each potential.

Although the full time-independent Schrödinger equation is in fact a function in the full 3-D space that we live in, for much of what we do below we will simplify it and consider only one dimensional systems. This makes dealing with it conceptually simple, but does not obscure any of the essential physical results. Such systems can in fact be realistic. For example, if you consider a mass moving on a spring attached to a wall, that is essentially a one-dimensional system, as the mass moves only forwards and backwards along the direction the spring is oriented.

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