14.1: The Schrödinger Equation

The one-dimensional Coulomb potential was presented in section 13.6. Of course, the real Hydrogen atom is three-dimensional. The Schr¨odinger equation that results is almost identical:

$$\hat{K}\psi(\vec{r}) + V(r)\psi(\vec{r}) = E\psi(\vec{r}) \tag{14.1}$$

First, remember that when we say $\psi(\vec{r})$, that's a shorthand for $\psi(x, y, z)$. On the left, we've broken the Hamiltonian $\hat{H}$ into the kinetic energy part $(\hat{K})$ and the potential. Notice that in here, we don't have $V(x, y, z)$, but only $V(r)$. When we say r without the little vector arrow, we mean the distance from the origin, that is, $r = \sqrt{x^2 + y^2 + z^2}$. The potential here is spherically symmetric. Because it depends only on the origin, if you rotated the whole system through any angle, the potential would be no different.

If we substitute in the correct expression for the potential $V(r)$, this equation becomes:

$$\hat{K}\psi(\vec{r}) - \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}\psi(\vec{r}) = E\psi(\vec{r}) \tag{14.2}$$

As noted in the previous chapter, the potential is always negative. As r gets very large (i.e. the electron is very far away from the proton), the potential approaches zero. Again, this is just a convention; we could add any constant we wanted to the potential without changing the physics of what's happening. We've chosen this because it's convenient not to have to worry about the nuclei of atoms that are far away. The solution to this equation will be individual functions $\psi(\vec{r})$, each corresponding to a different allowed state, each with a corresponding energy level. The solutions that represent an atom— where the electron is bound to the proton— have $E < 0$. Classically, if a particle moves in this potential, that will set a maximum distance away from the origin that the particle could reach.