# 13.7: Interpretation of the Wave Function ψ(x)

In general, it is best to view $\psi(x)$ the same way that you view $|\psi\rangle$. It's an abstract mathematical object that represents the state of the system. Quantum mechanics is then a theory, a mathematical model of reality that includes rules for manipulating $\psi(x)$ (or other representations of $|\psi\rangle$) in order to make predictions about the results of experiments, such as probabilities for observing particles in certain states, or expectation values for certain values.

It turns out that there is one particularly simple rule that can be applied to $\psi(x)$ in order to learn something about the state of the system. if $\psi(x)$ is a properly normalized single-particle wave function, then the construction $\psi^*(x)\psi(x)dx$ is the probability of finding that particle between position $x$ and position $x+dx$, where $dx$ is a small range of $x$. (By "small", we mean small enough that $\psi(x)$ does not appreciably change over the range.) As an example, consider the free particle wave function:

$$\psi(x)=A[\cos(2\pi x/\lambda)\pm i\sin(2\pi x/\lambda)] \tag{13.14}$$

If we want to find the probability for finding a particle at a given position, we multiply this function by its complex conjugate:

$$\begin{aligned}\psi^*(x)\psi(x)dx &= A^*A\left[\cos\left(\frac{2\pi x}{\lambda}\right)\mp i\sin\left(\frac{2\pi x}{\lambda}\right)\right)\left(\cos\left(\frac{2\pi x}{\lambda}\right)\pm i\sin\left(\frac{2\pi x}{\lambda}\right)\right]dx \\ &= A^*A\left[\cos^2\left(\frac{2\pi x}{\lambda}\right)+\sin^2\left(\frac{2\pi x}{\lambda}\right)\right. \\ &\qquad \left.\pm i\cos\left(\frac{2\pi x}{\lambda}\right)\sin\left(\frac{2\pi x}{\lambda}\right)\mp i\cos\left(\frac{2\pi x}{\lambda}\right)\sin\left(\frac{2\pi x}{\lambda}\right)\right]dx \\ &= A^*A\left[\cos^2\left(\frac{2\pi x}{\lambda}\right)+\sin^2\left(\frac{2\pi x}{\lambda}\right)\right]dx\end{aligned} \tag{13.15}$$

To simplify this further, we can use the trigonometric identity $\sin^2\phi+\cos^2\phi=1$ (this applies for all $\phi$). Thus, we are left with:

$$\psi^*(x)\psi(x)dx=|A|^2dx \tag{13.16}$$

That is, the probability of finding a free particle at any $x$ within a given range $dx$ is always the same. This correspoinds to an infinite uncertainty in position $x$, which is what we need given that this state has a definite momentum $p=h/\lambda$.

The construction $\psi^*(x)\psi(x)dx$ works for any one-dimensional wave function for calculating the probability of finding the particle at a given position. Using the three dimensional version of this construction on the solutions to the Hydrogen atom is what gives us the "electron cloud" diagrams you may have seen for electron orbitals. More about that in the next chapter.