6.3: Bra Vectors and the Inner Product

For each ket vector $|\psi\rangle$, there is a corresponding bra vector $\langle\psi|$. We haven't yet looked into any specific representations of ket vectors beyond just the ket vector itself, so at the moment that's all you need to know. However, when we do get into specific representations, the rules for converting ket vectors to bra vectors are generally very easy. You always take the complex conjugate of any numbers in the representation going from the ket vector to the bra vector. (You may also turn a column vector into a row vector, if you're using column vectors to represent ket vectors; much more about that later.) $\langle\psi|$ hψ| is something like the complex conjugat $|\psi\rangle$, although that's not really right. However, just as a number and its complex conjugate are associated with each other, each ket vector $|\psi\rangle$ is uniquely associated with a bra vetor $\langle\psi|$.

With the introduction of bra vectors, it becomes possible to define a new operation you can do on these things. You can always stick a bra vector on to a ket vector. The notation is meant to help suggest this; where there is a straight side, you can stick two of them together. The result is called the inner product. The specific rules for how you calculate the inner product again depend on the detailed representation of the ket vector, so for now we'll keep them abstract. As an example, suppose you have two different quantum states represented by the ket vector $|\psi\rangle$ and the ket vector $|\phi\rangle$. The bra vector corresponding to the latter is $\langle\phi|$, and the inner product of that bra vector with the ket vector $|\psi\rangle$ is:

$$\langle\phi \mid \psi\rangle$$

When you see a bra-ket pair combined like that, the result is a **scalar!** It may well be a complex number, but it is just a number. At that point, you can manipulate it in algebraic equations the way you would manipulate any other complex number.

The inner product of a bra and a ket is the first way we've seen to multiply two of these state vectors together. We've talked about multiplying the state vectors by a scalar, but before we didn't know how to multiply them together. Notice, however, that this is a different sort of multiplication than multiplying two scalars. When you multiply two scalars, you get another scalar out— the same sort of thing as the things you multiplied together. However, when you take the inner product of two state vectors, you get a scalar out, something different from the two things that went into the inner product.

Note that you can only take the inner product between two quantum states if they are the same sort of state. That is, they must be the same kind of state for the same particle or system. For instance, you could take the inner product between two angular momentum states for the same electron, but you couldn't take the inner product between an angular momentum state and a position state.