# 11.4: The Heisenberg Uncertainty Principle

As previously stated, quantifying the uncertainty on a given observable for a given quantum state is more interesting when the observable we're talking about has a large number (or even a continuum) of different values it might take on. If you consider two different observables whose operators do not commute, then a system cannot be in a definite state for both of those observables at the same time. The Heisenberg Uncertainty Principle takes this observation, makes it stronger, and quantifies it.

Consider a quantum particle that can move along one direction. Its position is then $x$, and its momentum along that direction is $p_x$. The Heisenberg Uncertainty Principle states that:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \tag{11.26}$$

We've gotten used to thinking of $\hbar$ as an angular momentum unit, because that's where it's shown up before. However, here, it's not really an angular momentum, though it still does of course have the same units (position times position over time). Instead, it represents the fundamental limit in quantum mechanics on how well you can know two different observables, position and momentum. If you know one of them perfectly, e.g. $\Delta x = 0$, then the uncertainty in the other one must be infinite. Although in more advanced quantum mechanics we use such states as they're a good approximation for a lot of things, they're not really physical. In reality, most quantum systems have a small amount of uncertainty in both position and momentum. That is, a particle doesn't have a definite position or a definite momentum, but the range of positions for which it has an appreciable amplitude is confined to a small space, and the range of momenta for which it has an appreciable amplitude is confined to a small range.

As a concrete example, let's consider an electron. For a non-relativistic electron, its momentum is just $p = mv$, where $m$ is its mass and $v$ is its speed. Therefore, $\Delta p = m\Delta v$, as the mass is well known and there is no uncertainty in it. What is a good uncertainty in speed to consider? For practical purposes, let's suppose that we're doing an experiment with an electron that requires it to be localized for 1 second. We don't want the uncertainty in the speed of the electron to cause our uncertainty in the position after one second to be greater than the uncertainty in the position was in the first place. So, we shall choose $\Delta v = \Delta x/t$, where we'll put in $t = 1$ second. If we then put this into the uncertainty principle

$$\begin{aligned} \Delta x \Delta p &\geq \frac{\hbar}{2} \\ \Delta x m_e \frac{\Delta x}{t} &\geq \frac{\hbar}{2} \\ \Delta x &\geq \sqrt{\frac{\hbar t}{2m_e}} \end{aligned} \tag{11.27}$$

If you put in the numbers, you find that the uncertainty on the position of this electron is 0.01 m, or one centimeter. For an electron, that's a lot! (One could argue about whether or not 1 second is a reasonable timescale. When we get to talking about atoms, we'll think more carefully about what a reasonable timescale is.)

Notice, however, that the uncertainty in the position goes down as the mass goes up. Imagine that you stood still your entire life. If you want to balance the uncertainty in your starting position with the uncertainty in your position resulting from the uncertainty in your velocity over your entire life, then you'd put in your age for $t$. Let us assume, optimistically, that you will live 100 years ($3 \times 10^9$ seconds), and that your mass is 80 kg. If you put those numbers in to the equation above, you find out that the uncertainty on your position is $4 \times 10^{-14}$ m. In other words, even though quantum uncertainty can be pretty important for an electron, on everyday scales for macroscopic objects the effect of quantum uncertainty is utterly negligible.