

5.1: Composite Systems

Suppose we have two systems, described by Hilbert spaces \mathcal{H}_1 and \mathcal{H}_2 , respectively. We can choose orthonormal bases for each system:

$$\mathcal{H}_1 : \{|\phi_1\rangle, |\phi_2\rangle, \dots, |\phi_N\rangle\} \quad \text{and} \quad \mathcal{H}_2 : \{|\psi_1\rangle, |\psi_2\rangle, \dots, |\psi_M\rangle\} \quad (5.1)$$

The respective dimensions of \mathcal{H}_1 and \mathcal{H}_2 are N and M . We can construct $N \times M$ basis states for the composite system via $|\phi_j\rangle$ and $|\psi_k\rangle$. This implies that the total Hilbert space of the composite system can be spanned by the tensor product

$$\{|\phi_j\rangle \otimes |\psi_k\rangle\}_{jk} \quad \text{on} \quad \mathcal{H}_{1+2} = \mathcal{H}_1 \otimes \mathcal{H}_2 \quad (5.2)$$

An arbitrary pure state on \mathcal{H}_{1+2} can be written as

$$|\Psi\rangle = \sum_{jk} c_{jk} |\phi_j\rangle \otimes |\psi_k\rangle \equiv \sum_{jk} c_{jk} |\phi_j, \psi_k\rangle \quad (5.3)$$

For example, the system of two qubits can be written on the basis $\{|0, 0\rangle, |0, 1\rangle, |1, 0\rangle, |1, 1\rangle\}$. If system 1 is in state $|\phi\rangle$ and system 2 is in state $|\psi\rangle$, the partial trace over system 2 yields

$$\text{Tr}_2(|\phi, \psi\rangle\langle\phi, \psi|) = \text{Tr}_2(|\phi\rangle\langle\phi| \otimes |\psi\rangle\langle\psi|) = |\phi\rangle\langle\phi| \text{Tr}(|\psi\rangle\langle\psi|) = |\phi\rangle\langle\phi|, \quad (5.4)$$

since the trace over any density operator is 1. We have now lost system 2 from our description! Therefore, taking the partial trace without inserting any other operators is the mathematical version of forgetting about it. This is a very useful feature: you often do not want to deal with every possible system you are interested in. For example, if system 1 is a qubit, and system two is a very large environment the partial trace allows you to “trace out the environment”.

However, tracing out the environment will not always leave you with a pure state as in Eq. (5.4). If the system has interacted with the environment, taking the partial trace generally leaves you with a mixed state. This is due to entanglement between the system and its environment.

This page titled [5.1: Composite Systems](#) is shared under a [CC BY-NC-SA 4.0](#) license and was authored, remixed, and/or curated by [Pieter Kok](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.