12.5: The Pauli Exclusion Principle

The Pauli Exclusion Principle states that no two fermions may occupy the same quantum state. This principle is absolutely crucial to life as we know it; without it, we would not have the Periodic Table of chemistry, nor would we have a lot of the rest of the structure of matter. This doesn't mean, however, that only one electron in the Universe is allowed to have positive z spin! Obviously, we have many more than two electrons in the Universe. However, if you have a quantum state, such as an energy level in an atom, where you can put electrons, you can only put two electrons into that energy level. Why two, and not one? Because of electron spin; as long as the two electrons have opposite spin (or, more precisely, are in a combined spin state with spin angular momentum zero such that they are antisymmetric under exchange), then you can put two electrons into the same state. It is possible to have two electrons with the same spin, so long as something else is different about their quantum states. So, for example, you could have two electrons with the same spin if they were in different orbitals in an atom.

Why can't you put more than one fermion in the same state? Because it's impossible to construct an antisymmetric state vector two fermions in the same state. Suppose you have a state $|\psi\rangle$, and you want to put two fermions into it. We know that the state:

$$|\psi_1\rangle\,|\psi_2\rangle \tag{12.17}$$

won't work, because the exchange operator working on it just produces the same state back, not the negative of the same state:

$$\hat{P}_{12}\,|\psi_1\rangle\,|\psi_2\rangle = |\psi_2\rangle\,|\psi_1\rangle = |\psi_1\rangle\,|\psi_2\rangle \tag{12.18}$$

This is an eigenvalue of the exchange operator, which is good, but the eigenvalue is +1. This would work for bosons; indeed, because of this, you can put as many bosons as you want all into the same state. However, for fermions, the eigenvalue of the exchange operator working on the two-particle state needs to be −1. If we try to construct an antisymmetric wave vector with both of these electrons in the same state:

$$\frac{1}{\sqrt{2}}|\psi_1\rangle\,|\psi_2\rangle - \frac{1}{\sqrt{2}}|\psi_2\rangle\,|\psi_1\rangle \tag{12.19}$$

we just end up with 0, which isn't a state at all. Thus, if you have two indistinguishable fermions, there must be something different about their states; you can't put more than one fermion into a single quantum state.