6.5: Interpreting the Inner Product

So far, all we know about the inner product is that for a properly normalized quantum state, the inner product of that state with itself is 1, and that the inner product between two different states corresponding to definite states of the same observable must be zero. But what about the inner product between two arbitrary states? Consider:

$$\langle\phi \mid \psi\rangle$$

The interpretation of this is that it is the **amplitude for a particle in state $|\psi\rangle$ to subsequently be measured in state $|\phi\rangle$**. As an example, suppose that we have an electron in the following state:

$$|\psi\rangle=\frac{3}{5}|+z\rangle+\frac{4 i}{5}|-z\rangle$$

Suppose we send this electron through an SGz machine. If this state is properly normalized (is it?), then we could work out the amplitude for it to be measured in the $|-z\rangle$ state (i.e. the amplitude for measuring its z-spin to be $-\hbar/2$) as follows:

$$\begin{aligned}
\langle-z \mid \psi\rangle &=\langle-z|\left(\frac{3}{5}|+z\rangle+\frac{4 i}{5}|-z\rangle\right) \\
&=\frac{3}{5}\langle-z \mid+z\rangle+\frac{4 i}{5}\langle-z \mid-z\rangle \\
&=\frac{3}{5}(0)+\frac{4 i}{5}(1) \\
&=\frac{4 i}{5}
\end{aligned}$$

This tells us the amplitude for the electron to be found in the $|-z\rangle$ state. Remember that the probability, what we really care about, is the absolute square of the amplitude. That probability is:

$$\begin{aligned}
|\langle-z \mid \psi\rangle|^{2} &=\langle-z \mid \psi\rangle^{*}\langle-z \mid \psi\rangle \\
&=\left(\frac{-4 i}{5}\right)\left(\frac{4 i}{5}\right) \\
&=\left(\frac{-16 i^{2}}{25}\right) \\
&=\frac{16}{25}=0.64
\end{aligned}$$

If the electron was in the state $|\psi\rangle$ defined above upon entering a SGz machine, there's an 64% chance it will come out the $-z$ output of the machine, being measured with a z-spin of $-\hbar/2$.