4.2: Decoherence

The density operator allows us to consider the phenomenon of **decoherence**. Consider the pure state $|+\rangle$. In matrix notation with respect to the basis $\{|+\rangle, |-\rangle\}$, this can be written as

$$\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{4.8}$$

The trace is 1, and one of the eigenvalues is 1, as required for a pure state. We can also write the density operator in the basis $\{|0\rangle, |1\rangle\}$:

$$\rho = |+\rangle\langle+| = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \times \frac{1}{\sqrt{2}}(1,1) = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \tag{4.9}$$

Notice how the outer product (as opposed to the inner product) of two vectors creates a matrix representation of the corresponding projection operator.

Let the time evolution of $|+\rangle$ be given by

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \to \frac{|0\rangle + e^{i\omega t}|1\rangle}{\sqrt{2}}. \tag{4.10}$$

The corresponding density operator becomes

$$\rho(t) = \frac{1}{2}\begin{pmatrix} 1 & e^{i\omega t} \\ e^{-i\omega t} & 1 \end{pmatrix}. \tag{4.11}$$

The "population" in the state $|+\rangle$ is given by the expectation value

$$\langle+|\rho(t)|+\rangle = \frac{1}{2} + \frac{1}{2}\cos(\omega t). \tag{4.12}$$

This oscillation is due to the off-diagonal elements of $\rho(t)$, and it is called the coherence of the system (see Fig. 2).

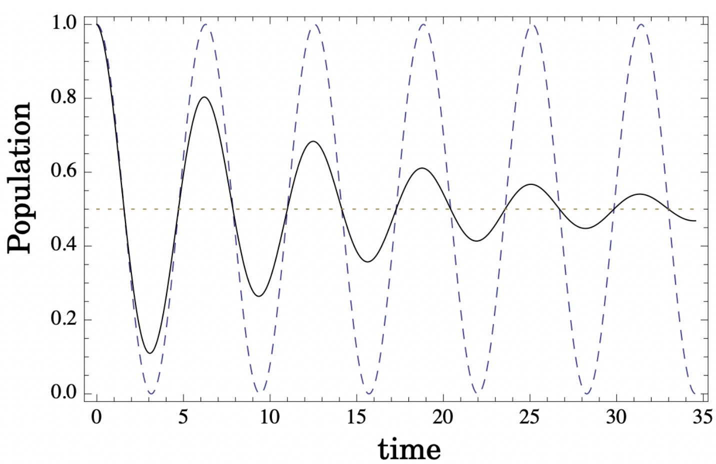

Figure 2: Population in the state $|+\rangle$ with decoherence (solid curve) and without (dashed curve).

The state is pure at any time t. In real physical systems the coherence often decays exponentially at a rate γ, and the density matrix can be written as

$$\rho(t) = \frac{1}{2}\begin{pmatrix} 1 & e^{i\omega t-\gamma t} \\ e^{-i\omega t-\gamma t} & 1 \end{pmatrix}. \tag{4.13}$$

The population in the state $|+\rangle$ decays accordingly as

$$\langle+|\rho(t)|+\rangle = \frac{1}{2} + \frac{e^{-\gamma t}\cos(\omega t)}{2}. \tag{4.14}$$

This is called decoherence of the system, and the value of γ depends on the physical mechanism that leads to the decoherence.

The decoherence described above is just one particular type, and is called **dephasing**. Another important decoherence mechanism is relaxation to the ground state. If the state $|1\rangle$ has a larger energy than $|0\rangle$ there may be processes such as spontaneous emission that drive the system to the ground state. Combining these two decay processes, we can write the density operator as

$$\rho(t)=\frac{1}{2}\begin{pmatrix} 2-e^{-\gamma_1 t} & e^{i\omega t-\gamma_2 t} \\ e^{-i\omega t-\gamma_2 t} & e^{-\gamma_1 t} \end{pmatrix}. \tag{4.15}$$

The study of decoherence is currently one of the most important research areas in quantum physics.