12.3: The Exchange Operator

The exchange operator, notated here by $\hat{P}_{12}$, just exchanges particle 1 for particle 2. In order to satisfy the conditions described in Section 12.1, a state composed of two indistinguishable particles (e.g. two electrons) must be an eigenstate of the exchange operator. Suppose that $|\xi\rangle$ is such a state. This means that

$$\hat{P}_{12}|\xi\rangle = c|\xi\rangle \tag{12.10}$$

where c is the eigenvalue. Suppose that we apply the exchange operator twice. What will happen? We should get back to the original state! We've just swapped the two particles back. Let's apply this twice:

$$\begin{aligned} \hat{P}_{12}\hat{P}_{12}|\xi\rangle &= \hat{P}_{12}(c|\xi\rangle) \\ &= c\hat{P}_{12}|\xi\rangle \\ &= c^2|\xi\rangle \end{aligned} \tag{12.11}$$

If the result of applying this exchange operator twice must be the state we started with, then we must have $c^2 = 1$. This is regular old fashioned squaring, not taking the absolute square. That $c^2 = 1$ means that there are only two possibilities for the eigenvalue of the exchange operator working on a state of two indistinguishable particles: $c = 1$ or $c = -1$.