11.4.1: An Alternate Formulation

An alternate way to formulate Heisenberg's Uncertainty principle is:

$$\Delta E \Delta t \geq \frac{\hbar}{2} \tag{11.28}$$

The interpretation of this is a little less clear than in the case of position and momentum. Uncertainty in energy seems obvious enough; it's the square root of the variance of all the values of energy that might be measured for a particle in a given quantum state. But what is "uncertainty on time"? Rather than interpreting this as an uncertainty, we shall interpret it as a time interval. In a sense, that's the same thing; an interval of time is qualitatively similar to an uncertainty on what time it is.

What this means, then, is that the uncertainty in the energy of a quantum state is related to how long that state hangs around. If a system is in an energy eigenstate, then it has a definite energy and $\Delta E = 0$. Such a state must be stable then, for Δt has to be infinite. In other words, in the absence of any interactions, a particle in an energy eigenstate will stay, forever, in an energy eigenstate.

For small time intervals, however, there will be a finite uncertainty in the energy of a system. One thing that this means is that it becomes possible to violate the conservation of energy, so long as you do it so fast that nobody can catch you at it! Among other things, this leads to the possibility of quantum tunneling– that is, if a particle is up against a potential barrier it doesn't have enough energy to penetrate, there is some finite probability that the particle may be located inside the barrier. And, the particle may be able to cross the barrier, even though classically it could not.

Later, when we talk about atoms, states other than the ground state (i.e. lowest energy state) of the atom aren't going to be perfectly stable. Over time, they will decay to the ground state, with a characteristic lifetime analogous to the half-life of a radioactive isotope. Although we will describe these excited states as being energy eigenstates, the fact that they decay tells us that they can't exactly be energy eigenstates. It also tells us that there must be some uncertainty as to the exact energy value associated with those states. There will be observational consequences of this, although in practice for real atoms these consequences are extremely difficult to observe.