13.5: The Simple Harmonic Oscillator

One important potential energy function is the Simple Harmonic Oscillator, or SHO. This is the potential energy of a spring (so long as you don't stretch of squish the spring too much). It also turns out to be a decent approximation, at least for lower energy levels, for a number of quantum systems. One such system is the vibrational energy states of a Hydrogen molecule H_2. The form of this potential, in one dimension, is:

$$V(x) = \frac{1}{2} m\omega^2 x^2 \tag{13.9}$$

Here, m is the mass of the particle moving in the potential. ω is the "natural frequency of oscillation" for the potential; for a classical spring, it would correspond to $2\pi/T$, where T is the period of oscillations. (Of course, for a classical spring, the system could also have any energy!)

The solution to the one dimensional Schrödinger equation for this potential gives the following energies for the energy eigenstates:

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \tag{13.10}$$

where n is an integer 0, 1, 2, ... As written, this potential is an infinitely high potential $(V(x)$ just keeps going up as x gets farther and farther from 0.) As such, there are an infinite number of allowed energy levels. Of course, as an approximation to a real physical system, usually the approximation will get worse and worse as x gets farther and farther from 0, which means that the solutions less and less of a good approximation to the real energy system for higher and higher energy levels.