8.4: Energy Without Mass- Photon

light moves with zero aging. photons move with zero mass.

A striking example of the primacy of momenergy over mass is furnished by a quantum of light colliding with an electron.

Quantum? A quantum of luminous energy of a given color or, in more technical terms, light of a given wavelength or frequency of vibration. Max Planck discovered in 1900 that light of a given color comes only in quanta - "hunks" - of energy of a standard amount, an amount completely determined by the color (Table 8-2). We can have one quantum, one hunk, one photon, of green light, or two, or fifteen, but never two and a half.

Compton demonstrates quantum of radiation - photon!

Nothing did more to raise the light quantum, the hunk of luminous energy, the photon, to the status of a particle than experiments carried out by 28-year-old Arthur Holly Compton at Washington University, St. Louis, in 1920. Shining X-rays of known wavelength (and hence of known frequency and known quantum energy) on a variety of different substances, he measured the wavelength (and hence the quantum energy) of the emergent "scattered" X-rays. He got identical changes in wavelength at identical angles of observation from many kinds of materials. There was no way he could explain this result except to say that the scattering object was in every case the same, an electron, whatever the atom in which the electron happened to reside.

But why did the change of wavelength have a unique value, the same for all materials at a given angle of scattering? Every idea of classical physics failed to fit, Compton found. "Compton arrived at his revolutionary quantum theory for the scattering process rather suddenly in late 1922," a biographer tells us. "He now treated the interaction as a simple collision between [an X-ray quantum] and a free electron... [He] found that [this hypothesis gave results] which agreed perfectly with his data ... When Compton reported his discovery at meetings of the American Physical Society, it aroused great interest and strong opposition . .." By 1927, however, his finding was generally accepted and in that year won him the Nobel Prize.

What does it mean to treat a photon on the same footing as a particle? It means this: attribution to the photon of an energy and a momentum, in other words momenergy.

TABLE 8.4.1: MOMENTUM AND ENERGY CARRIED BY ONE PHOTON, ONE QUANTUM, ONE HUNK OF LUMINOUS ENERGY OF VARIOUS "COLORS" (Unit of energy used in this table: electron-volt or eV, the amount of energy given to an electron by accelerating it through an electrical potential difference of one volt)

	Momentum (and energy) of a single quantum	Frequency in vibrations per second	Wavelength in meters
KDKA, Pittsburgh: world's first radio broadcast station	$4.22 \times 10^{-9} \mathrm{eV}$	1.02×10^{6}	294
A sample infrared beam Yellow radiation from a sodium arc lamp	$1.24 \times 10^{-2} \mathrm{eV}$	3×10^{12}	10^{-4}
Ultraviolet light from a mercury arc lamp Ultraviolet star radiation of just barely sufficient quantum	$2.11 \mathrm{eV}$	5.09×10^{14}	5.90×10^{-7}
energy to strip a hydrogen atom of its electron	$4.89 \mathrm{eV}$	1.18×10^{15}	2.54×10^{-7}
Each of two gamma rays given off in the mutual annihilation of a slow positron and a slow electron	$5.11 \times 10^{5} \mathrm{eV}$	1.23×10^{20}	2.43×10^{-12}
Each of two gamma rays given out when a neutral pi meson, at rest, decays	$6.75 \times 10^{7} \mathrm{eV}$	1.63×10^{22}	1.84×10^{-14}

	Momentum (and energy) of a single quantum	Frequency in vibrations per second	Wavelength in meters
Each of two gamma rays given off in the mutual annihilation of a slow proton and a slow antiproton	$0.938 \times 10^{9} \mathrm{eV}$	2.27×10^{23}	1.32×10^{-15}

Photon momenergy points in lightlike direction

In what direction in spacetime does the photon's arrow of momenergy point? In a lightlike direction, because the photon - a quantum of light - travels with light speed!

When we turn from spacetime to a particular free-float frame of reference and observe a pulse of light at one event along its worldline and then observe it at a second event (Figure $8.4.1$), we know in advance something important about the interval between the two events: It equals zero.

$$\begin{aligned} (\text{ interval })^{2} &= (\text{ distance between two events })^{2}-(\text{ time between two events })^{2} \\ &= (\text{ difference between two quantities of identical magnitude }) \\ &= 0 \end{aligned}$$

Photon momenergy: magnitude zero (photon mass $= \mathbf{0}$)

A photon in a pulse of light has a momenergy arrow with a tip and a tail, like the momenergy vector for any other particle. Between the tip and tail there is a magnitude. The magnitude for the photon, however, has the value zero-zero because this arrow points in the same direction in spacetime as the worldline of the light pulse (Figure $8.4.1$). For that reason its space component (momentum) and its time component (energy) are equal. And, of course, we express the square of this magnitude as we express the square of any interval, as a difference between the squared timelike and spacelike separations between the two ends of the arrow:

$$\begin{aligned} (\text{magnitude of momenergy arrow of photon})^{2} &= (\text{ photon energy })^{2}-(\text{ photon momentum })^{2} \\ &= (\text{ photon mass })^{2}=0 \end{aligned}$$

In brief, the lightlike character of the arrow of photon momenergy tells us that (1) photon mass equals zero and (2) the magnitude of momentum, or punch-delivering power, of the photon is identical in value with the energy of the photon:

$$(\text{photon energy}) = (\text{magnitude of photon momentum})$$

and

$$(\text{ photon mass })=0$$

ENERGY WITHOUT MASS: PHOTON

Figure $8.4.1$ summarizes these features of the elementary quanta of visible light and other electromagnetic radiations. For a "handle" on the momenergy 4-vector of a photon - representative of its magnitude - we choose a stylized zero, 0.

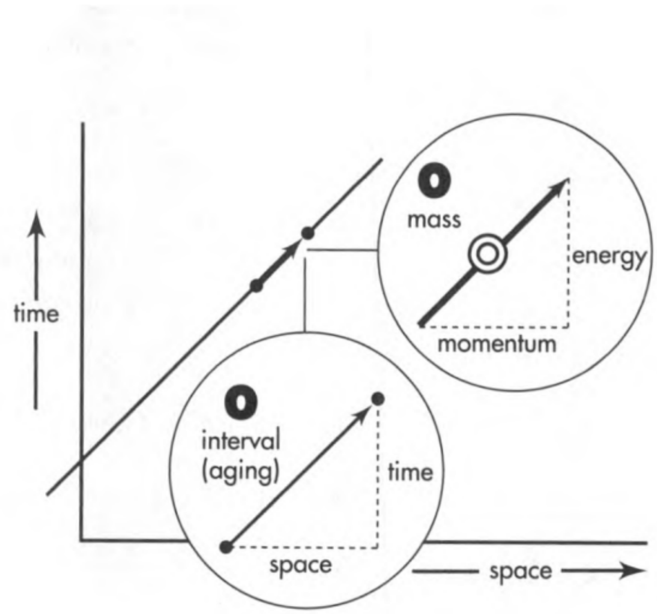

Figure 8.4.1: Worldline o f a photon. Note its “unit slope in spacetime. ” Insets; Unit slope of worldline means equal space and time separations between events on this worldline, hence zero interval between them — and zero aging for the photon. Momenergy of the same photon, also with unit slope, symbolizing three properties of the photon: it has zero mass (hence the bigzero as an invariant “handle"), it travels with light speed, and it has a momentum identical in magnitude with its energy.

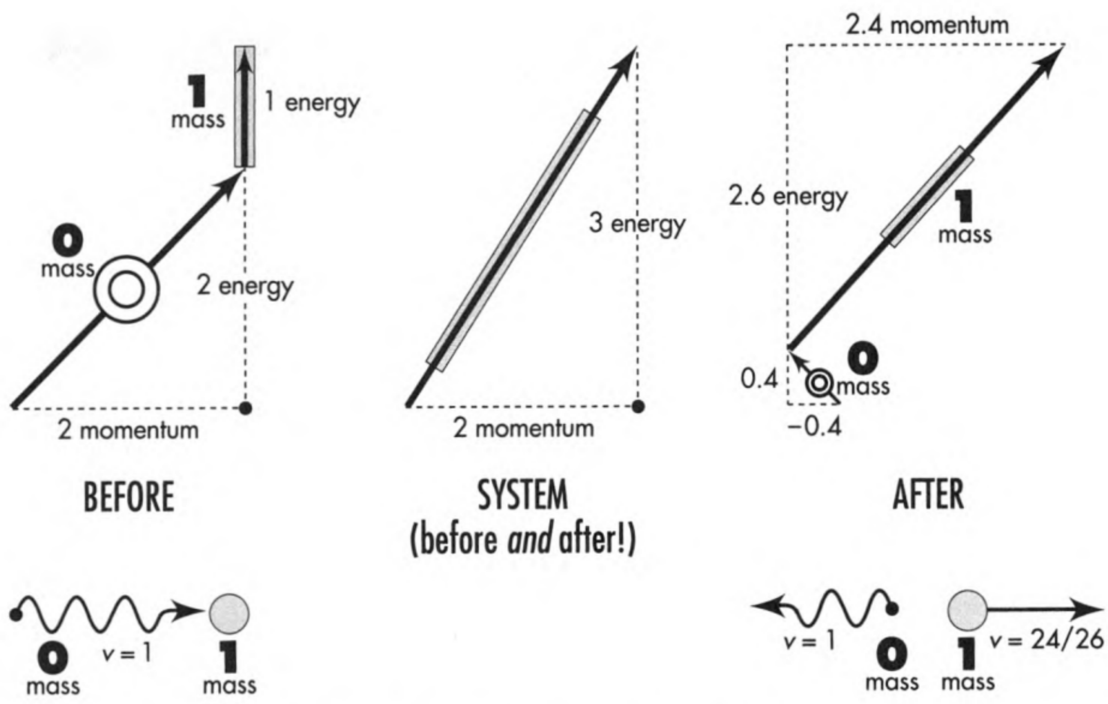

Figure 8.4.2: Backscattering of a photon by a free electron. The wiggly arrow symbol represents a photon. Energy, momentum, and mass of all particles are expressed in units of electron mass. Before: The electron at rest bas an energy equal to its mass (vertical arrow); the photon bas an energy (and a momentum) of 2 electron masses (angled arrow). System: Arrow of total momenergy. (What is the mass of the system?) After: Arrows of momenergy of knocked-on electron (labeled 1) and backscattered photon (labeled 0) after the encounter. Arrow of total momenergy of the system remains the same (is conserved!) during this process

Compton collision analyzed

Nothing shows these revolutionary features of light to better advantage than the very collision process studied by Arthur Compton: the encounter between a single photon and a single electron. We take the electron, loosely bound though it may be in one or another outer orbit of an atom, as essentially free and essentially at rest - at rest compared to the swift motion in which it finds itself after the high-energy photon hits it (Figure $8.4.2$).

To simplify all numbers, we pick for the photon energy a value typical of gamma rays, considerably greater than that of the X-rays with which Compton worked but easily available today from various sources of radioactivity: $1.022 \mathrm{MeV}$ (million electron-volts). We pick this number because we want to express all energies in units of electron mass, 9.11×10^{-31} kilograms or $0.511 \mathrm{MeV}$. Our choice of photon energy equals exactly two electron masses. Convenient!

Incoming photons of this energy, encountering an electron, are scattered by the electron sometimes in one direction, sometimes in another, and sometimes straight backward. In that most extreme of encounters-backward scattering-an interchange of momentum takes place that nevertheless preserves total momentum and also total energy, as illustrated in Figure $8.4.2$. The electron is kicked forward with a momentum of $2.4=12/5$ times the electron mass, and the photon bounces backward with a momentum (and energy) of $0.4=2/5$ times the electron mass, much less than the two-electron masses of momentum (and energy) with which it approached. 232

Example $8.4.1$ MASS OF A SYSTEM THAT INCLUDES PHOTONS\)

A photon has no rest energy - that is, no mass of its own. However, a photon can contribute energy and momentum to a system of objects. Hence the presence of one or more photons in a system can increase the mass of that system. More: A system consisting entirely of zero-mass photons can itself have nonzero mass!

Find system mass M_{system} for each of the following systems. The particles that make up these systems do not interact with one another. Express the system mass in terms of the unit mass m (or the unit energy E in the photons-only systems). Use only energy and mass in your answers: no momenta or velocities.

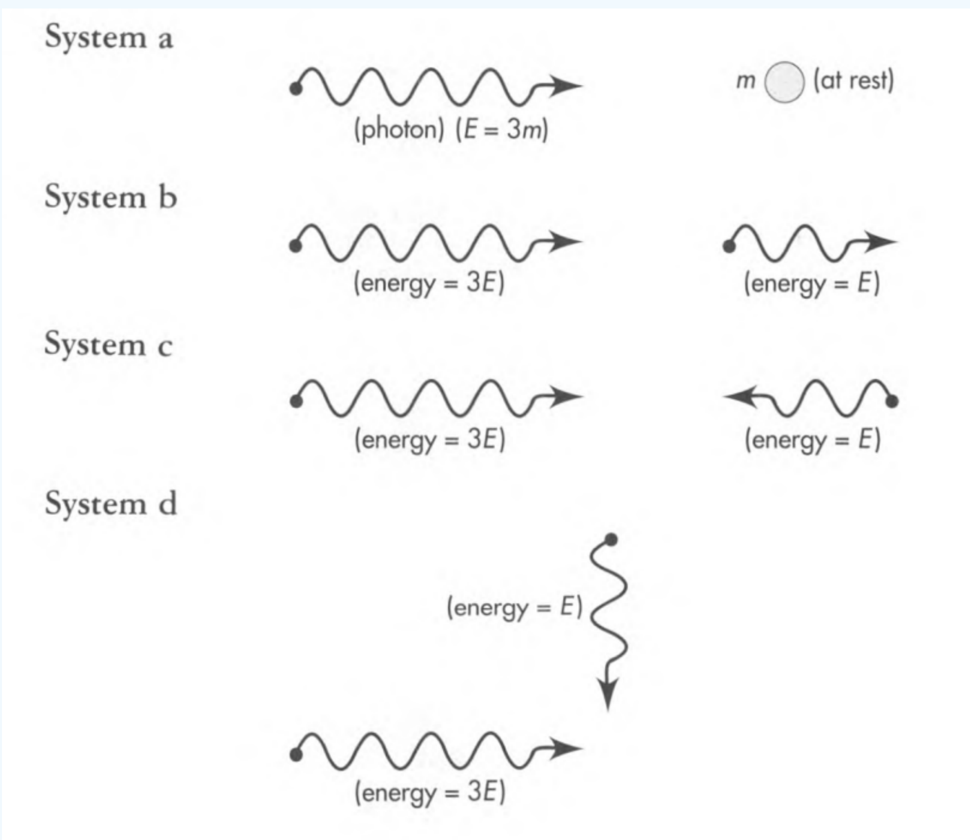

Figure $8.4.3$:

Solution

System a: System energy equals the rest energy m of the material particle plus the energy $E=3m$ of the photon: $E_{\text{system}}=m+3m=4m$. The momentum of the system is equal to the momentum of the photon, which is equal to its energy: $p_{\text{system}}=3m$. The mass of the system is reckoned from the difference of the square of energy and momentum:

$$\begin{aligned} M_{\text{system}} &= \left[E_{\text{system}}^2 - p_{\text{system}}^2\right]^{1/2} = \left[(4m)^2 - (3m)^2\right]^{1/2} = \left[16m^2 - 9m^2\right]^{1/2} \\ &= [7]^{1/2} m = 2.646m \end{aligned}$$

System b: System energy equals the sum of the energies of the two photons: $E_{\text{system}} = 3E + E = 4E$. System momentum equals sum of momenta of the two photons - which in this case also equals the sum of the energies of the two photons: $p_{\text{system}} = 3E + E = 4E$. Therefore system mass equals zero:

$$M_{\text{system}} = \left[E_{\text{system}}^2 - p_{\text{system}}{}^2\right]^{1/2} = \left[(4E)^2 - (4E)^2\right]^{1/2} = 0n$$

We could have predicted this result immediately. Two photons moving along in step are, as regards momentum and energy, completely equivalent to a single photon of energy equal to the sum of energies of the separate photons. And a single photon has, of course, zero mass.

System c: Total energy $=$ system energy $= E_{\text{system}} = 3E + E = 4E$. System momentum equals the difference between the rightward momentum of the first particle and the leftward momentum of the second particle: $p_{\text{ssstem}} = 3E - E = 2E$. Hence the system mass is

$$M_{\text{system}} = \left[16E^2 - 4E^2\right]^{1/2} = [12]^{1/2} E = 3.464En$$

Why can't we simply make a single photon by adding the energies of the two photons, as in system b? Because energies add as scalars, and momenta add as 3 -vectors. In this case the total energy is $4E$ and the total momentum is $2E$. No way to make a single photon out of this; for a photon, energy and momentum must have equal magnitudes!

System d: This part serves as an additional reminder that momentum is a 3 -vector. The system energy equals $E_{\text{system}} = E + 3E = 4E$. The squared momentum of the system equals the sum of squares of the momenta of the separate particles, since they move in perpendicular directions in this frame: $p_{\text{system}}^2 = E^2 + (3E)^2 = 10E^2$.

Hence system mass is:

$$M_{\text{system}} = \left[16E^2 - 10E^2\right]^{1/2} = [6]^{1/2} E = 2.449E$$