3.5: Lorentz Contraction of Length

space separation between two length-measuring events? disagreement!

How do we measure the length of a moving rod — the distance between one end and the other end? One way is to use our latticework of clocks to mark the location of the two ends at the same time. But when the rod lies along the direction of relative motion, someone riding with the rod does not agree that our marking of the positions of the two ends occurs at the same time (Section 3.4).[1] The relativity of simultaneity tells us that rocket and laboratory observers disagree about the simultaneity of two events (firecrackers exploding at the two ends of the rod) that occur at different locations along the direction of relative motion. Therefore the two observers disagree about whether or not a valid measurement of length has taken place.

Disagree about simultaneity? Then disagree about length.

Go back to the Train Paradox. For the observer standing on the ground, the two lightning bolts strike the front and back of the train at the same time. Therefore for him the distance between the char marks on the track constitutes a valid measure of the length of the train. In contrast, the observer riding on the train measures the front lightning bolt to strike first, the rear bolt later. The rider on the train exclaims to her Earth-based colleague, "See here! Your front mark was made before the back mark — since the flash from the front reached me (at the middle of the train) before the flash from the back reached me. Of course the train moved during the time lapse between these two lightning strikes. By the time the stroke fell at the back of the train, the front of the train had moved well past the front char mark on the track. Therefore your measurement of the length of the train is too small. The train is really longer than you measured."[2]

There are other ways to measure the length of a moving rod. Many of these methods lead to the same result: the space separation between the ends of the rod is less as measured in a frame in which the rod is moving than as measured in a frame in which the rod is at rest. This effect is called Lorentz contraction. Section 5.8 examines the Lorentz contraction quantitatively.

Suppose we agree to measure the length of a rod by determining the position of its two ends at the same time. Then an observer for whom the rod is at rest measures the rod to be longer than does any other observer. This "rest length" of the rod is often called its proper length.

Question and Answer

You keep using the word "measure." Occasionally you say "observe." You never talk about that most delicate, sensitive, and refined of our five senses: sight. Why not just look and see these remarkable relativistic effects?

Answer

We have been careful to say that the relativity of simultaneity and the Lorentz contraction are *measured*, not *seen* with the eye. *Measurement* employs the latticework of rods and clocks that constitutes a free-float frame. As mentioned in Chapter 2, seeing with the eye leads to confused images due to the finite speed of light. Stand in an open field in the southern hemisphere as Sun sets in the west and full Moon rises in the east: You see Moon as it was 1.3 seconds ago, Sun as it was eight minutes ago, the star Alpha Centauri (nearest star visible to the naked eye) as it was 4.34 years ago, the Andromeda nebula as it was *2 million years ago* — you see them all *now*. Similarly, light from the two separated ends of a speeding rod typically takes different times to reach your eye. This relative time delay results in visual distortion that is avoided when the location of each end is recorded locally, with zero or minimal delay, by the nearest lattice clock. Visual appearance of rapidly moving objects is itself an interesting study, but for most scientific work it is an unnecessary distraction. To avoid this kind of confusion we set up the free-float latticework of synchronized recording clocks and insist on its use — at least in principle!

Question and Answer

Aha! Then I have caught you in a contradiction. Figure 3.4.1 shows lightning flashes and trains. Is this not a picture of what we would see with our eyes?

Answer

No. Strictly speaking, each of the three "pictures" in Figure 3.4.1 summarizes where parts of the train are as recorded by the Earth latticework of clocks at a given instant of Earth time. The position of each light flash at this instant is also recorded by the clocks in the lattice. The summary of data is then given to a draftsman, who draws the picture for that Earth time. To distinguish such a drafted picture from the visual view, we will often refer to it as a **plot**. For example. Figure 3.4.1 (top) is the Earth plot at the time when lightning bolts strike the two ends of the train.

Actually, all three plots in Figure 3.4.1 show approximately what you see through a telescope when you are very far from the scene in a direction perpendicular to the direction of motion of the train and at a position centered on the action. At such a remote location, light from all parts of the scene takes approximately equal times to reach your eye, so you would see events and objects at approximately the same time according to Earth clocks. Of course, you receive this information later than it actually occurs because of the time it takes light to reach you.

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1 Length of a rod = separation between simultaneous sparks at its two ends

2 Disagree about simultaneity? Then disagree about length.
