3.6: Invariance of Transverse Dimension

"faster" does not mean "thinner" or "fatter"

A rocket ship makes many trips past the laboratory observer, each at successively higher speed. For each new and greater speed of the rocket, the laboratory observer measures its length to be shorter than it was on the trip before. This observed contraction is **longitudinal** — along its direction of motion. Does the laboratory observer also measure contraction in the **transverse** dimension, perpendicular to the direction of relative motion? In brief, is the rocket measured to get thinner as well as shorter as it moves faster and faster?

Transverse dimension same for laboratory and rocket observers

The answer is No. This is confirmed experimentally by observing the width of electron and proton beams traveling in high-energy accelerators. It is also easily demonstrated by simple thought experiments.[1]

Speeding-Train Thought Experiment: Return to Einstein's high-speed railroad train seen end-on (Figure $3.6.1$). Suppose the Earthbound observer measures the train to get thinner as it moves faster. Then for the Earth observer the right and left wheels of the train would come closer and closer together as the train speeds up, finally slipping off *between the tracks* to cause a terrible wreck. In contrast, the train observer regards herself as at rest and the tracks as speeding by in the opposite direction. If she measures the speeding tracks to get closer together as they move faster and faster, the train wheels will slip off outside the tracks, also resulting in a wreck. But this is absurd: the wheels cannot end up between the tracks and *outside the tracks* under the same circumstances. Conclusion: High speed leads to no measured change in transverse dimensions - no observed thinning or fattening of fast objects. We are left with the conclusion that high relative speed affects the measured values of longitudinal dimensions but not transverse dimension: a welcome simplification!

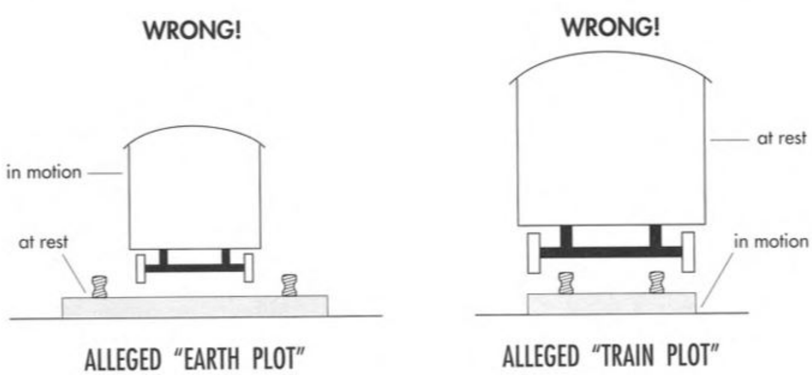

Figure $3.6.1$: **Two possible alternatives (both wrong!) if the moving train is measured to shrink transverse to its direction of motion.** The **"Earth plot"** assumes the speeding train to be measured as getting thinner with increasing speed. The train's wheels would slip off between the tracks. The **"train plot"** of the same circumstance assumes the speeding rails to be measured as getting closer together. In this case the wheels would slip off outside the tracks. But this is a contradiction. Therefore the wheel separation - and the transverse dimensions of train and track - must be invariant, the same for all free-float observers moving along the track. (If you think that the actual transverse contraction might be too small to cause a wreck for the train shown, assume that both the wheels and the track are knife edges; the same argument still applies.)

Thought experiments demonstrate invariance of transverse dimension

Speeding-Pipes Thought Experiment: Start with a long straight pipe. Paint one end with a checkerboard pattern and the other end with stripes. Cut out and discard the middle of the pipe, leaving only the painted ends. Now hurl the ends toward each other, with their cylindrical axes lying along a common line parallel to the direction of relative motion (Figure $3.6.2$). Suppose that a moving object is measured to be thinner. Then someone riding on the checkerboard pipe will observe the striped pipe to pass inside her cylinder. All observers - everyone looking from the side - will see a checkerboard pattern. In contrast, someone riding on the striped pipe will observe the checkerboard pipe to pass inside his cylinder. In this case, all observers will see a striped pattern. Again, this is absurd: *All* observers must see stripes, or all must see checkerboard. The only tenable conclusion is that speed has no measurable effect on transverse dimensions and the pipe segments will collide squarely edge on.[2]

A simple question leads to an even more fundamental argument against the difference of transverse dimensions of a speeding object as observed by different free-float observers in relative motion: *About what axis* does the contraction take place?

We try to define an "axis of shrinkage" parallel to the direction of relative motion. Can we claim that a speeding pipe gets thinner by shrinking uniformly toward an "axis of shrinkage" lying along its center? Then what happens when two pipe segments move along their lengths, side by side as a pair? Does each pipe shrink separately, causing the clear space between them to *increase*? Or does the combination of both pipes contract toward the line midway between them, causing the clear space between them to *decrease*? Is the answer different if one pipe is made of lead and the other one of paper? Or if one pipe is entirely in our imagination?

There is no logically consistent way to define an "axis of shrinkage." Given the direction of relative motion of two objects, we cannot select uniquely an "axis of shrinkage" from the infinite number of lines that lie parallel in this direction. For each different choice of axis a different pattern of distortions results. But this is logically intolerable. The only way out is to conclude that there is no transverse shrinkage at all (and, by a similar argument, no transverse expansion).

The above analysis leads to conclusions about events as well as about objects. A set of explosions occurs around the perimeter of the checkerboard pipe. More: These explosions occur simultaneously in this checkerboard frame. Then these events are simultaneous also in the striped frame. How do we know? By symmetry! For suppose the explosions were *not* simultaneous in the striped frame. Then which one of these events would occur first in the striped frame? The one on the right side of the pipe or the one on the left side of the pipe? But "left" and "right" cannot be distinguished by means of any physical effect: Each pipe is cylindrically symmetric. Moreover, space is the same in all directions - space is **isotropic**, the same to right as to left.[3] So neither the event on the right side nor the event on the left side can be first. They must be simultaneous. The same argument can be made for events at the "top" and "bottom" events separated only transverse to relative motion of the pipe, and for every other pair of events on opposite sides of the pipe. Conclusion: If the explosions are simultaneous in the checkerboard frame, they must also be simultaneous in the striped frame.

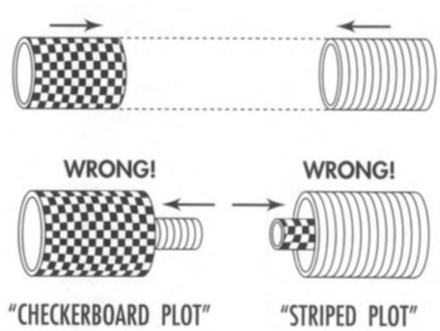

Figure 3.6.2: Two identical-size pipe segments hurtle toward each other along a common centerline. What will happen when they meet? Here are two possible alternatives (both wrong') if a moving object is observed to shrink transverse to direction of motion. Which pipe passes inside the other? The impossibility of a consistent answer to this question leads to the conclusion that neither pipe can be measured to change transverse dimension.

We make the following summary conclusions about dimensions transverse to the direction of relative motion:

Dimensions of moving objects transverse to the direction of relative motion are measured to be the same in laboratory and rocket frames (invariance of transverse distance).

Two events with separation only transverse to the direction of relative motion and simultaneous in either laboratory or rocket frame are simultaneous in both.

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1 Transverse dimension same for laboratory and rocket observers

2 Thought experiments demonstrate invariance of transverse dimension

3 "Same time" agreed on for events separated only transverse to relative motion

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