5.7: Kinked Worldline

kink in the worldline decreases aging along that worldline

Acceleration-proof clocks

The change in slope of the worldline from event to event in Figures 5.6.1 and 5.6.2 (bottom) means that the clock being carried along this worldline changes velocity: It accelerates. Different clocks behave differently when accelerated. Typically a clock can withstand a great acceleration only when it is small and compact. A pendulum clock is not an accurate timepiece when carried by car through stop-and-go traffic; a wristwatch is fine. A wristwatch is destroyed by being slammed against a wall; a radioactive nucleus is fine. Typically, the smaller the clock, the more acceleration it can withstand and still register properly, and the sharper can be the curves and kinks on its worldline.[1] In all figures like Figures 5.6.1 and 5.6.2 (bottom), we assume the ideal limit of small (acceleration-proof) clocks.

Simplify: Worldlines with straight segments

We are now free to analyze a motion in which particle and clock are subject to a great acceleration. In particular, consider the simple special case of the worldline of Figure 5.6.1. That worldline gradually changes slope as the particle speeds up and slows down. Now make the period of speeding up shorter and shorter (great driving force!); also make the period of slowing down shorter and shorter.[2] In this way come eventually to the limiting case in which episodes of acceleration and deceleration-curved portions of the worldline - are too short even to show up on the scale of the spacetime map (worldline OQB in Figure $5.7.1$). In this simple limiting case the whole history of motion is specified by (1) initial event O, (2) final event B, and (3) turnaround event Q, halfway in time between O and B. In this case it is particularly easy to see how the lapse of proper time between O and B depends on the location of the halfway event - and thus to compare three worldlines, OPB, OQB, and ORB.

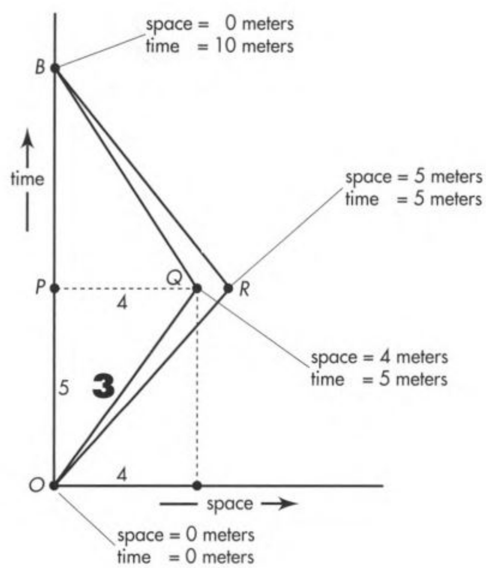


Figure $5.7.1$: **Three alternative worldlines connecting events $\mathbf{O}$ and $\mathbf{B}$.** The sharp changes of velocity at events Q and R bave been drawn for the ideal limit of small clocks that tolerate great acceleration. The bold-face number 3 is the proper time along the segment OQ, reckoned from the difference between the squared time separation and the squared space separation: $3^2=5^2-4^2$.

Zero proper time for light

Path OPB is the worldline of a particle that does not move in space; it stays next to the reference-frame clock. Proper time from O to B by way of P is evidently equal to time as measured in the free-float frame of this reference clock:

$$(\text{total proper time along } OPB) = 10 \text{ meters of time}$$

In contrast, on the way from O to B via R, for each segment the space separation equals the time separation, so the proper time has the value zero:[3]

$$
\begin{aligned}
(\text{ proper time along leg } OR)^2 &= (\text{ time })^2-(\text{ space })^2 \\
&= (5 \text{ meters })^2-(5 \text{ meters })^2 \\
&= 0 \\
(\text{ total proper time along } ORB) &= 2 \times(\text{ proper time along } OR) \\
&= 0
\end{aligned}
$$

As far as we know, only three things can travel 5 meters of distance in 5 meters of time: light (photons), neutrinos, and gravitons (see Box 8-1). No material clock can travel at light speed. Therefore the worldline ORB is not actually attainable by a material particle. However, it can be approached arbitrarily closely. One can find a speed sufficiently close to light speed - and yet less than light speed - so that a trip with this speed first one way then the other will bring an ideal clock back to the reference clock with a lapse of proper time that is as short as one pleases. In the same way we can, in principle, go to the star Canopus and back in as short a round-trip rocket time as we choose (Section 4.8).

Reduced proper time along kinked worldline

As distinguished from the limiting case ORB, worldline OQB demands an amount of proper time that is greater than zero but still less than the 10 meters of proper time along the direct worldline OPB:[4]

$$
\begin{aligned}
(\text{ proper time along leg } OQ)^2 &= (5 \text{ meters })^2-(4 \text{ meters })^2 \\
&= 25 \text{ (meters) }^2-16 \text{ (meters) }^2 \\
&= 9 \text{ (meters) }^2 \\
&= (3 \text{ meters })^2
\end{aligned}
$$

so

$$
(\text{ proper time along leg } OQ)=3 \text{ meters}
$$

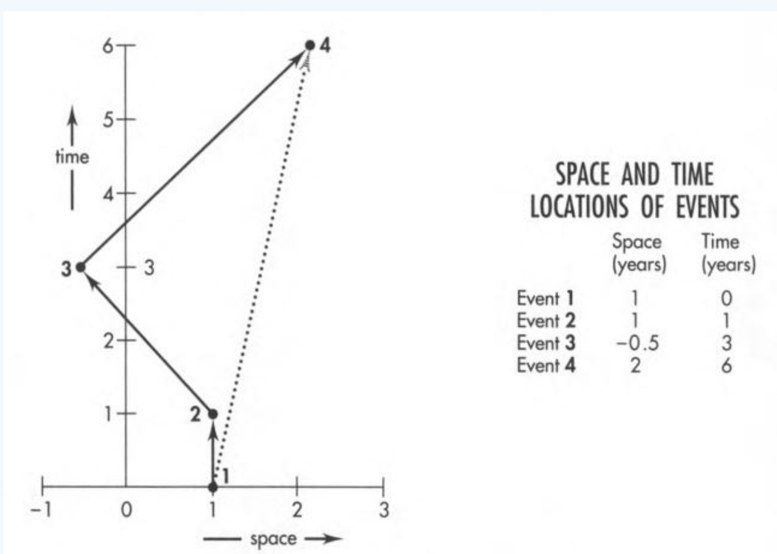
and

$$
\begin{aligned}
(\text{ total proper time along both legs } OQB) &= 2 \times(\text{ proper time along } OQ) \\
&= 6 \text{ meters}
\end{aligned}
$$

This is less proper time than (proper time along OPB) $=10$ meters that characterized the "direct" worldline OPB. Our trip to Canopus and back described in Chapter 4 follows a worldline similar to OQB.

Example 5.7.1: More is Less

In the spacetime map shown, time and space are measured in years. A table shows space and time locations of numbered events in this frame.



Two alternative worldlines between events 1 and 4

a. One traveler moves along the solid straight worldline segments from event 1 to events 2,3, and 4 . Calculate the time increase on her clock between event 1 and event 2; between event 2 and event 3; between event 3 and event 4. Calculate total proper time-her aging-along worldline 1, 2, 3, 4.
b. Another traveler, her twin brother, moves along the straight dotted worldline from event 1 directly to event 4. Calculate the time increase on his clock along the direct worldline 1, 4.
c. Which twin (solid-line traveler or dotted-line traveler) is younger when they rejoin at event 4?

Solution

a. From the table next to the map, space separation between events 1 and 2 equals 0. Time separation equals 1 year. Therefore the interval is reckoned from (interval)$^2 = 1^2 - 0^2 = 1^2$. Thus the proper time lapse on a clock carried between events 1 and 2 equals 1 year.
Space separation between event 2 and event 3 equals $1-(-0.5)=1.5$ light-years. Time separation equals 2 years. Therefore the square of the interval is $2^2-(1.5)^2=4-2.25=1.75$ (years)^2 and the advance of proper time equals the square root of this, or 1.32 years.
Between event 3 and event 4 space separation equals 2.5 light-years and time separation 3 years. The square of the interval has the value $3^2-(2.5)^2=9-\ 6.25=2.75$ (years) ^{2} and proper time between these two events equals the square root of this, or 1.66 years. Total proper time - aging - along worldline 1, 2, 3, 4 equals the sum of proper times along individual segments: $1+1.32+1.66=3.98$ years.
b. Space separation between events 1 and 4 equals 1 light-year. Time separation is 6 years. The squared interval between them equals $6^2-1^2=36-1=35$ (years)^2. A traveler who moves along the direct worldline from event 1 to event 4 records a span of proper time equal to the square root of this value, or 5.92 years.
c. The brother who moves along straight worldline 1,4 ages 5.92 years during the trip. The sister who moves along segmented worldline $1,2,3,4$ ages less: 3.98 years. As always in Lorentz geometry, the direct worldline (shown dotted) is longer - that is, it has more elapsed proper time, greater aging - than the indirect worldline (shown solid).

1 Acceleration-proof clocks

2 Simplify: Worldlines with straight segments

3 Zero proper time for light

4 Reduced proper time along kinked worldline