3.8: Invariance of the Interval for All Free-float Frames

super-rocket observer joins the agreement

The interval between two events has the same value for *all possible* relative speeds of overlapping free-float frames. As an example of this claim, consider a third free-float frame moving at a different speed with respect to the laboratory frame-a speed different from that of the rocket frame.

Super-rocket: Same interval between events

We now measure the same events of emission and reception from a "super-rocket frame" moving faster than the rocket (but not faster than light!) along the line between events E and R (Figure 3.7.2, Super-rocket plot). For convenience we arrange that the reference clock of this frame also coincides with reference clocks of the other two frames at event E.[1]

Events E and R occur at the same place in the rocket frame. Between these two events the super-rocket moves to the *right* with respect to the rocket. As a result, the super-rocket observer records event R as occurring to the *left* of the emission event. How far to the left? That depends on the relative speed of the super-rocket frame.

The super-rocket is not super-size; rather it has super-speed. We adjust this super-speed so that the reception occurs 20 meters to the left of the emission for the super-rocket observer. Then the flash of light that rises vertically in the rocket must travel the same 3 meters upward in the super-rocket but also 10 meters to the left as it slants toward the mirror. Hence the distance it travels to the mirror in the super-rocket frame is the length of a hypotenuse, 10.44 meters:

$$\begin{aligned}(3\text{ meters})^2+(10\text{ meters})^2 &= 9\text{ meters}^2+100\text{ meters}^2=109\text{ meters}^2 \\ &= (10.44\text{ meters})^2\end{aligned}$$

It must travel another 10.44 meters as it slants downward and leftward to the event of reception. The total distance traveled equals 20.88 meters. It follows that the total time lapse between E and R equals 20.88 meters of light-travel time for the super-rocket observer.

The speed of the super-rocket is very high. As a result the space separation between emission and reception is very great. But then the time separation is also very great. Moreover, the magnitude of the time separation is perfectly tailored to the size of the space separation. In consequence, the particular quantity equal to the difference of their squares has the value (6 meters)^2, no matter how great the space separation and time separation individually may be. For the super-rocket frame:

$$\begin{aligned}(20.88\text{ meters})^2-(20\text{ meters})^2 &= 436\text{ meters}^2-400\text{ meters}^2=36\text{ meters}^2 \\ &= (6\text{ meters})^2\end{aligned}$$

In spite of the difference in space separation observed in the three frames (0 meters for the rocket, 8 meters for the laboratory, 20 meters for the super-rocket) and the difference in time separation (6 meters for the rocket, 10 meters for the laboratory, 20.88 meters for the super-rocket), the interval between the two events has the same value for all three observers:

In general: $(\text{time separation})^2-(\text{ space separation})^2=(\text{ interval})^2$

Rocket frame: $(6\text{ meters})^2-(0\text{ meters})^2=(6\text{ meters})^2$

Laboratory frame: $(10\text{ meters})^2-(8\text{ meters})^2=(6\text{ meters})^2$

Super-rocket frame: $(20.88\text{ meters})^2-(20\text{ meters})^2=(6\text{ meters})^2$

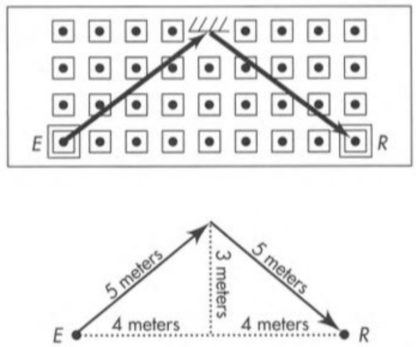

Figure $3.8.1$: (repeated). Laboratory plot of the path of the light flash.

The laboratory observer clocks the time between the flash and its reception as 10 meters, in total disagreement with the 6 meters of timelike interval he figures between those two events. The observer in the super-rocket frame marks an even greater discrepancy, 20.88 meters of her time versus the 6 meters of timelike interval. Only for the rocket observer does clock time agree with interval. Why? Because only she sees reception at the same place as emission.

Invariance of interval from invariance of transverse dimension

The invariance of the interval can be seen at a glance in Figure $3.8.1$. The hypotenuse of the first right triangle has a length equal to half the time separation between E and R. Its base has a length equal to half the space separation. To say that (time separation)^2 - (space separation)^2 has a standard value, and consequently to state that (half the time separation)^2 - (half the space separation)^2 has a standard value, is simply to say that the altitude of this right triangle has a fixed magnitude (3 meters in the diagram) for rocket and all super-rocket frames, no matter how fast they move. And this altitude has a length equal to half the interval between these two events.[2]

Example $3.8.1$: The K$^+$ Meson

A beam of (unstable) K^+mesons, traveling at a speed of $v = 0.868$, passes through two counters 9 meters apart. The particles suffer negligible loss of speed and energy in passing through the counters speed and energy in passing through the counters but give electrical pulses that can be counted. The first counter records 1000 pulses (1000 passing particles); the second records 250 counts (250 passing particles). This decrease arises almost entirely from decay of particles in flight. Determine the half-life of the K^+ meson in its own rest frame.

Solution

Unstable particles of different kinds decay at different rates. By definition, the half-life of unstable particles of a particular species measures the particle wristwatch time during which - on the average - half of the particles decay. Half of the remaining particles decay in an additional time lapse equal to the same half-life, and so forth. In this case, one quarter of the K^+particles remain after passage from counter to counter. Therefore the particles that survive experience the passage of two half-lives between counter and counter. We make the interval between those two passages, those two events, the center of our attention, because it has the same value in the laboratory frame where we do our measuring as it does in the free-float frame of the representative particle.

Basis of invariance of interval: Principle of Relativity

The keystone of the argument establishing the invariance of the interval between two events for all free-float frames? The Principle of Relativity, according to which there is no difference in the laws of physics between one free-float frame and another. This principle showed here in two very different ways. First, it said that distances at right angles to the direction of relative motion are recorded as of equal magnitude in the laboratory frame and the rocket frame (Section 3.6). Otherwise one frame could be distinguished from the other as the one with the shorter perpendicular distances.[3]

Second, the Principle of Relativity demanded that the speed of light be the same in the laboratory frame as in the rocket frame. The speed being the same, the fact that the light-travel path in the laboratory frame (the hypotenuse of two triangles) is longer than the simple round-trip path in the rocket frame (the altitudes of these two triangles: up 3 meters and down again) directly implies a longer time in the laboratory frame than in the rocket frame.

In brief, one elementary triangle in Figure 3.8.1 displays four great ideas that underlie all of special relativity: invariance of perpendicular distance, invariance of the speed of light, dependence of space and time separations upon the frame of reference, and invariance of the interval.

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1 Super-rocket: Same interval between events

2 Invariance of interval from invariance of transverse dimension

3 Basis of invariance of interval: Principle of Relativity
