4.9: Relativity of Simultaneity

we turn around; our changing colleagues say Earth's clock flies forward

By this time James Fastlane has gotten his second wind. "I am still stuck in this Twin Paradox thing. The time for the outward trip is less as measured in the rocket frame than as measured in the Earth frame. But if relativity is correct, every free-float frame is equivalent. As you sit on the rocket, you feel yourself to be at rest, stationary, motionless; you measure our Earth watch-station clocks to be zipping by you at high velocity. Who cares about labels? For you these Earth clocks are in motion! Therefore the time for the outward trip should be less as measured on the ('moving') Earth clock than as measured on your ('stationary') rocket clock."

Rocket observer: Fewer Earth-clock ticks on outward trip . . .

We nod assent and he continues. "Nothing prevents us from supposing the existence of a series of rocket lookout stations moving along in step with your rocket and strung out at separations of one light-year as measured in your rocket frame, all with clocks synchronized in your rocket frame and running at the same rate as your rocket clock. Now, as Earth passes each of these rocket lookout stations in turn, won't those stations read and record the times on the passing Earth clock to be less than their own times? Otherwise how can relativity be correct?"[1]

"Yes, your prediction is reasonable," we reply.

. . . also fewer Earth-clock ticks on return trip

"And on the return trip will not the same be true: Returning-rocket lookout stations will measure and record time lapses on the passing Earth clock to be less than on their own clocks?"[2]

"That conclusion is inevitable if relativity is consistent."

"Aha!'" exclaims Mr. Fastlane, "Now I've got you! If Earth clock is measured by rocket lookout stations to show smaller time lapses during the outward trip - and also during the return trip - then obviously total Earth time must be less than rocket round-trip time. But you claim just the opposite: that total rocket time is less than Earth time. This is a fundamental contradiction. Your relativity is wrong!" Folding his arms he glowers at us.

There is a long silence. Everyone looks at us except Professor Bright, who has his head down. It is hard to think with all this attention. Yet our mind runs over the trip again. Going out . . coming back ... turning around .... that's it!

"All of us have been thinking the wrong way?" we exclaim. "We have been talking as if there is only a single rocket frame. True, the same vehicle, with its traveler, goes out and returns. True, a single clock makes the round trip with the traveler. But this vehicle *turns around* - reverses its direction of travel - and that changes everything.

Astronaut jumps from outgoing frame to returning frame

"Maybe it's simpler to think of two rockets, each moving without change of velocity. We ride on the first rocket going out and on the second rocket coming back. Each of these two is really a rocket *frame*: each has its own long train of lookout stations with recording clocks synchronized to its reference clock (Figure 4.9.1). The traveler can be thought of as 'jumping trains' at Canopus - from outward-bound rocket frame to inward-bound rocket frame- carrying the calendar clock."[3]

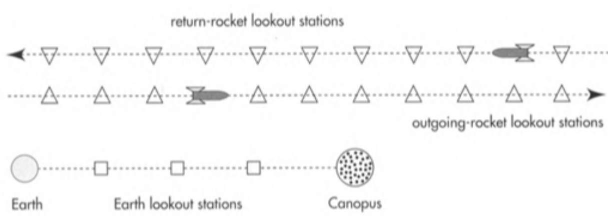

Figure 4.9.1: **Schematic plot in the Earth-linked frame showing the outgoing rocket and the return rocket used in the round trip between Earth and Canopus.** The two rockets meet at Canopus, where the traveler jumps from outgoing rocket to return rocket. Each reference frame has its own string of lookout stations, at rest and synchronized in that frame, shown by small squares, triangles, and inverted triangles. In this figure the outgoing and return rocket lines of motion are displaced vertically for purposes of analysis; tn reality, all motion lies along the single line between Earth and Canopus. The figure is not to scale!

"Now follow Mr. Fastlane's prescription to analyze the trip in the rocket frame, but with this change: make this analysis using *two* rocket frames - one outward bound, the other inward bound."

Outgoing rocket: As it arrives at Canopus, Earth clock reads 3.96 years

"It is 20 years by outward-rocket time when the traveler arrives at Canopus. That is the reading on all lookout station clocks in that outward-rocket frame. One of those lookout stations is passing Earth when this rocket time arrives. Its clock, synchronized to the clock of the outward traveler at Canopus, also reads 20 years. What time does that rocket lookout-station guard read on the passing Earth clock? For the rocket observer Earth clock reads less time by the same factor that rocket clocks read less time (20 years at arrival at Canopus) for Earth observers (who read 101 years on their own clocks). This factor is $20/101$. Hence for the outward-rocket observer the Earth clock must read $20/101$ times 20 years, or 3.96 years."[4]

"What!" explodes Fastlane. "According to your plan, the turnaround at Canopus occurs at 101 years of Earth time. Now you say this time equals less than 4 years on Earth clock."

"No sir, I do *not* say that," we reply, feeling confident at last. "I did say that *at the same time* as the outgoing rocket arrives at Canopus, Earth clock reads 3.96 years *as measured in that outgoing rocket frame*. An equally true statement is that at the same time as the outgoing rocket arrives at Canopus, Earth clock reads 101 years *as measured in the Earthbound frame*. Apparently observers in different reference frames in relative motion do not agree on what events occur *at the same time* when these events occur far apart along the line of relative motion."

Once again Professor Bright supplies the label. "Yes, that is called **relativity of simultaneity**. Events that occur at the same time-simultaneously - judged from one free-float frame but far apart along the line of relative motion do not occur simultaneously as judged from another free-float frame."

"As an example of relativity of simultaneity, consider either chain of lookout stations strung along the line of relative motion. If all clocks in the lookout stations of one frame strike exactly at noon in that frame, these strikes are not simultaneous as measured in another frame in relative motion with respect to the first. This is called **relative synchronization of clocks**."

"Incidentally, most of the so-called 'paradoxes' of relativity, one of which we are considering now, turn on misconceptions about relativity of simultaneity."

Dr. Short breaks in. "What about the returning rocket? What time on the Earth clock will the returning rocket lookout station measure as the traveler starts back?"

Returning rocket: As it leaves Canopus, Earth clock reads 198.04 years

"That shouldn't be too difficult to figure out," we reply. "We know that the clock on the returning rocket reads 40 years when we arrive home on Earth. And the Earth clock reads 202 years on that return. Both of these readings occur at the same place (Earth), so we do not need to worry about relativity of simultaneity of that reading. And during the return trip Earth clock records less elapsed time than rocket clocks' 20 years by the same factor, $20/101$, or a total elapsed time of $20 \times 20/101 = 3.96$ years according to return rocket observations. Therefore at the earlier turnaround, return rocket observers will see Earth clock reading $202 - 3.96 = 198.04$ years."

"Wait a minute!" bellows Fastlane. "First you say that the rocket observer sees the Earth clock reading 3.96 years at turnaround in the outward-bound frame. Now you say that the rocket observer sees the Earth clock read 198.04 years at turnaround in the inward-bound frame. Which one is right?"

"Both are right," we reply. "The two observations are made from two different frames. Each of these frames has a duly synchronized system of lookout-station clocks, as does the Earth-linked frame (Figure $4.9.1$). The so-called Twin Paradox is resolved by noticing that between the Earth-clock reading of 3.96 years, taken from the outward rocket lookout station at turnaround and the Earth-clock reading of 198.04 years, taken by the returning-rocket lookout station at turnaround, there is a difference of 194.08 years."[5]

Forward "jump" in Earth clock results from frame change

"This 'jump' appears on no single clock but is the result of the traveler changing frames at Canopus. Yet this jump, or difference, resolves the paradox: For the traveler, the Earth clock reads small time lapses on the outward leg-and also small time lapses on the return leg - but it jumps *way* ahead at turnaround.[6] This jump accounts for the large value of Earth-aging during the trip: 202 years. In contrast the traveler ages only 40 years during the trip (Table $4.9.1$)."

"And notice that the traveler is unique in the experience of changing frames; only the traveler suffers the terrible jolt of reversing direction of motion. In contrast, the Earth observer stays relaxed and comfortable in the same frame during the astronaut's entire trip. Therefore there is no symmetry between rocket traveler and Earth dweller, so no genuine contradiction in their differing time lapses, and the story of the twins is not a paradox."

Table 4.9.1: Observations of Events on Canopus Trip

		return-rocket lo…	Earth-clock reading observed by	
Event	Time measured in Earth-linked frame	Time measured by traveler	outgoing-rocket lookout stations passing Earth	return-rocket lookout stations passing Earth
Depart Earth	0 years	0 years	0 years	
Arrive Canopus	101 years	20 years	20 years × 20/101 = 3.96 years	
Depart Canopus	101 years	20 years	3.96 years	202 - 3.96 = 198.04 years
Arrive Earth	202 years	40 years		202 years

All observers agree on result, disagree on reason

"In fact, the observer in each of the three frames - Earth-linked, outward-rocket, and inward-rocket - has a perfectly consistent and non-paradoxical interpretation of the sequence of events. However, in accounting for disagreements between his or her readings and those of observers in other free-float frames, each observer infers some misbehavior of measuring devices in these other frames. Each observes less elapsed time on clocks in the other frame than on his or her own clocks (time stretching or time dilation). Each thinks that an object lying along the line of relative motion and at rest in another frame is contracted (Lorentz contraction).[7] Each thinks that lookout-station clocks in other frames are not synchronized with one another (relative synchronization of clocks). As a result, each cannot agree with other observers as to which events far apart along the line of relative motion occur at the same time (relativity of simultaneity)."

"Boy," growls Fastlane, "all these different reference frames sure do complicate the story!"

Spacetime interval is universal language

"Exactly!" we exclaim. "These complications arise because observations from any one frame are limited and parochial. All disagreements can be bypassed by talking only in the invariant language of spacetime interval, proper time, wristwatch time.[8] The proper time from takeoff from Earth to arrival at Canopus equals 20 years, period. The proper time from turnaround at Canopus to re-arrival at Earth equals 20 years, period. The sum equals 40 years as experienced by the astronaut, period. On the Earth clock, the proper time between departure and return is 202 years, period. End of story. Observers in all free-float frames reckon proper times - spacetime intervals between these events - using their differing space and time measurements. However, once the data are translated into the common language of proper time, every observer agrees. Proper times provide a universal language independent of reference frame."

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1 Rocket observer: Fewer Earth-clock ticks on outward trip...

2 ...Rocket observer: Fewer Earth-clock ticks on outward trip

3 Astronaut jumps from outgoing frame to returning frame

4 Outgoing rocket: As it arrives at Canopus, Earth clock reads 3.96 years

5 Returning rocket: As it leaves Canopus, Earth clock reads 198.04 years

6 Forward "jump" in Earth clock results from frame change

7 Forward "jump" in Earth clock results from frame change

8 Spacetime interval is universal language
