7.1: Momenergy- Total Conserved in a Collision

> *Every physical quantity is represented by a geometric object.*
>
> *Theme of Herman weyl*

momentum conserved, energy conserved. momenergy conserved!

Paradoxically, few examples of motion are more complicated than a collision, and few are simpler. The complication shows nowhere more clearly than in the slow-motion videotape of the smashup of two automobiles. Millisecond by millisecond the fender

Smashup complicated?

Smashup complicated? of one colliding car deforms another fraction of a centimeter. Millisecond by millisecond the radiator grille of the other car bends inward a litcle more on the way to total collapse: steel against steel, force against force, crumpling surface against crumpling surface. What could be more complex?

Smashup is simple!

For the drivers of the colliding cars the experience is shattering. They are hardly aware of noise and complicated damage. A single impression overpowers their senses: the inevitability of the crash. Call it what we will - inertia, momentum, the grip of spacetime on mass - something is at work that drives the two vehicles together as the frantic drivers jam their brake pedals down, locking the wheels as the cars slither over the glassy ice, crash into one another, then slide apart.

Momentum conservation simplifies

Does mass lose its inertia during the collision? No. Inertia does its best to keep each car going as it was, to keep its momentum constant in magnitude and direction. Momentum: we can think of it loosely as an object's will to hold its course, to resist deflection from its appointed way. The higher the object's momentum, the more description violently it hits whatever stands in its way. But the momentum of a single object is not all-powerful. The two vehicles exchange momentum. But spacetime insists and demands that whatever momentum one car gains the other car must lose. Regardless of all complications of detail and regardless of how much the momentum of any one object may change, the combined momentum of the two objects remains constant: the total is unchanged in the collision.

Energy too is conserved

A like statement applies to energy, despite a conversion of energy of motion into heat energy and fender crumpling.

Momenergy is conserved!

A collision thus manifests a wonderful simplicity: the combination of the motiondescriptive quantities (momentum and energy) of the two colliding bodies does not change. That combination is identical before and after the collision. In a word, it is conserved. This conserved combination we call m om entum - energy or, more briefly, m om energy (defined more carefully in Section 7.2). We will use the two terms interchangeably in this book.

A collision cannot be elevated from mere talk to numbers without adopting, directly or indirectly, the principle of conservation of momentum and energy. In the enterprise of identifying the right numbers, using them, and understanding them, no concept is more powerful than what relativity smilingly holds forth: momenergy.

Question and Answer

Wait a minute. Apparently you are going to find new expressions for momentum and energy, then combine them in some way to form a unity: momenergy. But I bave three complaints. (1) What is wrong with what good old-fashioned secondary school physics textbooks give us, the Newtonian expressions for momentum - $p_{\text{Newton}} = mv_{\text{conv}}$ *- and kinetic energy* $K_{\text{Newton}} = 1/2mv_{\text{conv}}^2$ *- where* v_{conv} *is expressed in conventional units, say meters/second? (2) Momentum and energy do not even have the same units, as these formulas make clear. How can you combine auantities with different units? (3) formulas make clear. How can you combine quantities with different units? (3) Momentum and energy are different things entirely; why try to combine them at all*

Answer

Take your questions in order.

1. Newtonian Expressions: Only for slow-moving particles do we get correct results when we use Newtonian expressions for momentum and energy. For particle speeds approaching that of light, however, total energy and momentum of an isolated system, as Newton defined momentum and energy, are not conserved in a collision. In contrast, when momentum and energy are defined relativistically, then total momentum and total energy of particles in an isolated system are conserved, no matter what their observed speeds.

2. Units: It is easy to adopt identical units for momentum and energy. As a start we adopt identical units for space and time. Then the speed of a particle is expressed in unit-free form, v, in meters of distance per meter of light-travel time (Section 2.8). This choice of units, which we have already accepted earlier in this book, gives even Newtonian expressions for momentum - $p_{\text{Newton }}=mv-a_{\text{and kinetic }}$ energy $-K_{\text{Newton }}=1/2mv^{2}$ - the same unit: mass. These are not relativistic expressions, but they do agree in their units, and agree in units with the correct relativistic expressions.

3. Momentum and Energy Different: Yes, of course, momentum and energy are different. Space and time are different too, but their combination, spacetime, provides a powerful unification of physics. Space and time are put on an equal footing, but their separate identities are maintained. Same for momenergy: We will see that its "space part" is momentum, its "time part" energy. We will also discover that its magnitude is the mass of the particle, reckoned using the good ol', ever-lovin', familiar minus sign: $m^{2}=E^{2}-p^{2}$.

Thus relativity offers us a wonderful unity. Instead of three separate motion-descriptive quantities - momentum, energy, and mass - we have a single quantity: momenergy.