6.2: Relation Between Events- Timelike, Spacelike, or Lightlike

minus sign yields three possible relations between pairs of events

squared distance: Positive or zero

Using Euclidean geometry, a surveyor reckons the distance between two steel stakes from the sum of the squares of the northward and eastward separations of these stakes:[1]

$$(\text{ distance })^2=(\text{ northward separation })^2+(\text{ eastward separation })^2$$

In consequence, in Euclidean geometry a distance-or its square-always has a positive value or zero.

Squared interval: Positive, zero, or negative

In contrast, the spacetime interval between events in Lorentz geometry arises from the difference of squares of time and space separations:[2]

$$(\text{ interval })^2=(\text{ separation in time })^2-(\text{ separation in space })^2$$

In consequence of the minus sign, this equation yields a number that may be positive, negative, or zero, depending on whether the time or the space separation predominates. Moreover, whichever of these three descriptions characterizes the interval in one free-float frame also characterizes the interval in any other free-float frame. Why? Because the spacetime interval between two events has the same value in all overlapping free-float frames. In the threefold possibilities for an interval, nature reveals the causal relation between events.

An interval between two events earns the name **timelike** or **spacelike** or **lightlike** depending on whether the time part predominates, the space part predominates, or the time and space parts are equal, respectively, as shown in Table $6.2.1$. For convenience, the minus sign is placed so that the resulting squared interval is greater than or equal to zero.

Timelike interval: Time part dominates

Timelike Interval: We picture the sequence of sparks emitted by a moving sparkplug.[3] Points representing these sparks on the spacetime map trace out the worldline of the particle (Chapter 5). No material particle has ever been measured to travel faster than light. Every material particle always travels less than one meter of distance in one meter of light-travel time. The sparks emitted by the particle have a greater time separation than their separation in space. In other words, the worldline of a particle consists of events that have a timelike relation with one another and with the initial event. We say that a material particle follows a **timelike worldline**.

The interval τ between two timelike events reveals itself to the observer in any free-float frame:

$$(\text{timelike interval})^{2}=\tau^{2}=(\text{ time separation })^2-(\text{ space separation })^2 \tag{6.2.1}$$

Table $6.2.1$: Classification of the Relation Between Two Events

Description	Squared interval is named and reckoned
Time part of interval dominates space part	$(\text{ timelike interval })^2=\tau^2=(\text{ time })^2-(\text{ distance })^2$
Space part of interval dominates time part	$(\text{ spacelike interval })^2=s^2=(\text{ distance })^2-(\text{ time })^2$
Time part of interval equals space part	$(\text{ lightlike interval })^2=0=(\text{ time })^2-(\text{ distance })^2$

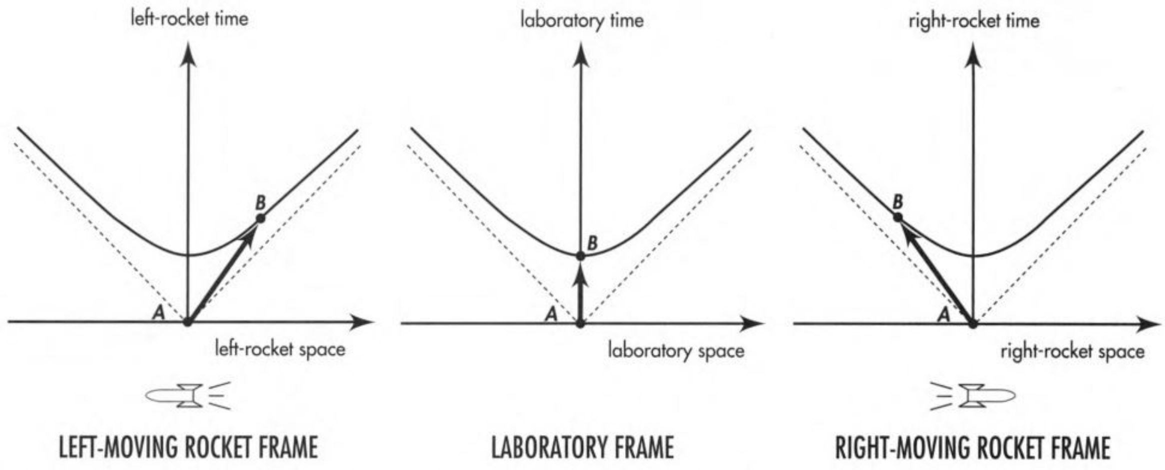


Figure 6.2.1: **Events A and B form a timelike pair (with event A arbitrarily chosen as reference event), here recorded in the spacetime maps of three free-float frames, Point B lies on a hyperbola opening along the time axis in each frame.** The shortest time between events A and B is recorded in the laboratory frame, the frame in which the two events occur at the same place.

Same two sparks registered in different frames? Different records for the separation in time between those sparks. Different records for the separation in space. Same figure for the timelike interval between them!

Timelike interval: Invariant hyperbola opens along time axis

Nobody can keep us from tracing out on one and the same diagram (Figure $6.2.1$) the very different records for the separation AB that observers get in different free-float frames. One frame? One point on the diagram. Another frame? Another point on the diagram. And so on. These many records for the same pair of events AB trace out a hyperbola. This hyperbola opens out in the time direction.[4]

The two sparks, A and B - definite locations though they occupy in spacetime nevertheless register in different frames of reference as having different separations in reference-frame time. Among the many conceivable frames, which one records this separation in time as smallest? Answer: The frame in which spark B occurs at the same place as spark A. In other words, the frame that happens to move along in sync with the sparkplug, even if only briefly. In that frame the clock records a separation in time between A and B identical with the timelike interval AB.

As seen in the left-moving rocket frame in Figure $6.2.1$, spark B lies to the right of spark A. In contrast, spark B occurs to the left of spark A in the right-moving rocket. The position of B relative to A depends on the reference frame from which it is measured. For a pair of events separated by a timelike interval, labels "right" and "left" have no invariant meaning: they are frame-dependent.

Spacelike interval: Space part dominates

Spacelike Interval: The interval between two events A and D is spacelike when the space part predominates over the time part. Such was the case for a possible explosion of Sun (event A) and Meredith's wand waving (event D), simultaneous with A as recorded in the Earth frame (Section 6.1). Events A and D, if they occurred, would be separated in the Earth - Sun frame by a distance of 150,000 million meters and separated by a time of zero meters. Clearly the space part predominates over the time part! Whenever the space part predominates, we call the relation between the two events spacelike.[5]

The interval s (sometimes called by the Greek letter sigma, σ) between two spacelike events reveals itself to the observer in any free-float frame:

$$(\text{ spacelike interval })^2=s^2=(\text{ space separation })^2-(\text{ time separation })^2 \tag{6.2.2}$$

Events A and D registered in different frames? Then different records for the separation in time between those events. Also different records for the separation in space. Same numerical value for the spacelike interval between them!

Spacelike interval: Invariant hyperbola opens along space axis

We plot on another spacetime diagram (Figure $6.2.2$) all of the very different records for the separation AD that observers get in different free-float frames. One frame? One point on the diagram. Another frame? Another point on the diagram. And so on. These many records for the same pair of events AD trace out a hyperbola. This hyperbola opens out in the space direction.[6]

The two events, A and $D-$ definite locations though they occupy in space time nevertheless register in different frames of reference as having different separations in reference-frame space. Among the many conceivable frames, which one records this separation in space as smallest? Answer: The frame in which spark D occurs at the same time as spark A. In that frame a long stick records a separation in space between A and D identical with the spacelike interval, AD. This is called the **proper distance** between the two spacelike events.

In the Earth - laboratory frame in Figure $6.2.2$, Meredith waves her wand (event D) at the same time as Sun explodes (event A). In the right-moving rocket frame Sun explodes after Meredith waves her wand. In the left-moving rocket frame Sun explodes before the wand wave. For a pair of events separated by a spacelike interval, labels "before" and " after" have no invariant meaning: they are frame-dependent. To allow the wand to control Sun would be to scramble cause and effect!

No particle - not even a flash of light - can move between two events connected by a spacelike interval. To do so would require it to cover a distance greater than the time available to cover this distance (space separation greater than time separation). In brief, it would have to travel faster than light. This is alternative evidence that two events separated by a spacelike interval cannot be causally connected: one of them cannot "get at" the other one by any possible signal.

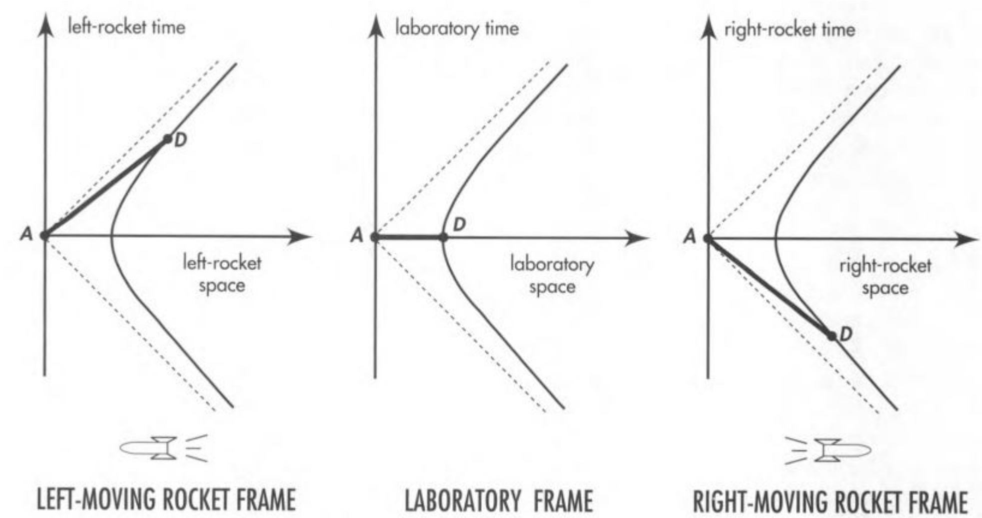


Figure $6.2.2$: The spacelike pair of events A and D {with event A arbitrarily chosen as reference event) as recorded in the spacetime maps of three free-float frames. Point D lies on a hyperbola opening along the space axis in every rocket and laboratory frame. The shortest distance between these events is recorded in the laboratory frame, the frame in which the two events occur at the same time. A heavy line represents the spacetime separation AD. No particle can travel along this line; the speed would be greater than light speed— and would be infinitely great as measured in the laboratory frame, since the particle would have to cover the distance from A to D in zero time!

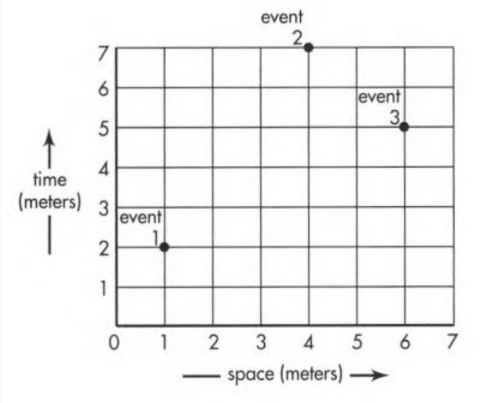
Example $6.2.1$: Relations Between Events

Events 1,2, and 3 all have laboratory locations $y=z=0$. Their x and t measurements are plotted on the laboratory spacetime map.

a. Classify the interval between events 1 and 2: timelike, spacelike, or lightlike.

b. Classify the interval between events 1 and 3.

c. Classify the interval between events 2 and 3.



Solution

a. For event $1, t=2$ meters and $x=1$ meter. For event $2, t=7$ meters and $x=4$ meters. The squared interval between them: $(\text{interval})^{2}=(7-2)^{2}-(4-1)^{2}= \quad 5^{2}-3^{2}=25-9=16(\text{ meters })^{2}$. The time part is greater than the space part, so the interval between these two events is timelike: $\tau=4$ meters.
b. For event $1, t=2$ meters and $x=1$ meter. For event $3, t=5$ meters and $x=6$ meters. The squared interval between them: $(\text{interval})^{2}=(5-2)^{2}-(1-6)^{2}= \quad 3^{2}-5^{2}=9-25=-16(\text{ meters })^{2}$. The space part is greater than the time part, so the interval is spacelike: $s=4$ meters. (For spacelike intervals, we subtract the squared time part from the squared space part before taking the square root.)
c. For event $2, t=7$ meters and $x=4$ meters. For event $3, t=5$ meters and $x=6$ meters. The squared interval between them: $(\text{interval})^{2}=(7-5)^{2}-(4-6)^{2}= \quad 2^{2}-2^{2}=4-4=0 \quad (\text{meters})^{2}$. The time part equals the space part, so the interval is lightlike: it is a null interval.

Lightlike interval; Time separation equal space separation

Lightlike Interval (Null Interval): Two events stand in a lightlike relation when the interval between them is zero:[7]

$$(\text{time separation})^{2}-(\text{ space separation })^{2}=0$$

or

$$\text{magnitude of (separation in time) } =(\text{ distance in space}) \qquad [\text{for lightlike interval}]$$

Lightlike interval: Plotted along ±45 degree lines

An interval that is lightlike? A separation in time between two events, A and G, identical to the distance in space between them? What does this condition mean? This: A pulse of light can fly directly from event A and arrive with perfect timing at event G.[8] How come? Distance in meters between the two locations measures the meters of time required for light to fly from one place to the other. Separation in time between the two events represents the time available for the trip. Time available equals time needed? Guarantee that the pulse from A arrives in coincidence with event G! More generally, whenever the influence of one event, spreading out at the speed of light, can directly affect a second event, then the interval between those two events rates as lightlike, zero, null.

Only light ("photons"), neutrinos, and gravitons can move directly between two events connected by a lightlike interval. Only by means of one of these light-speed particles can the one event in a lightlike pair cause the other.

The spherical out-going pulse of light from an event, A, may trigger two widely separated events, E and G (Figure $6.2.3$). Does this common genesis imply that E and G occur at the same time? Yes and no! Yes, there's always a free-float reference frame in which the two daughter events appear as simultaneous. That frame-for no good reason - we call the laboratory frame in Figure $6.2.3$. In other frames of reference - for example, the left-moving rocket frame in Figure $6.2.3$ - the clocks show that E occurs before G. There are still other frames - the right-moving rocket frame is one-in which the clocks register E and G in the opposite order of time. But no frame shows either E or G in the past of A.

> **Question and Answer**
>
> *Hold it! Aren't spacelike separations impossible? I understand timelike and lightlike separations between two events, because a particle - or at least a light flash - can travel between them. Not even a light flash, however, can travel from one event to a second event separated from the first by an interval that is spacelike. The first event cannot possibly cause the second event in the spacelike case. Therefore a spacelike interval cannot arise in nature. So why talk about it?*
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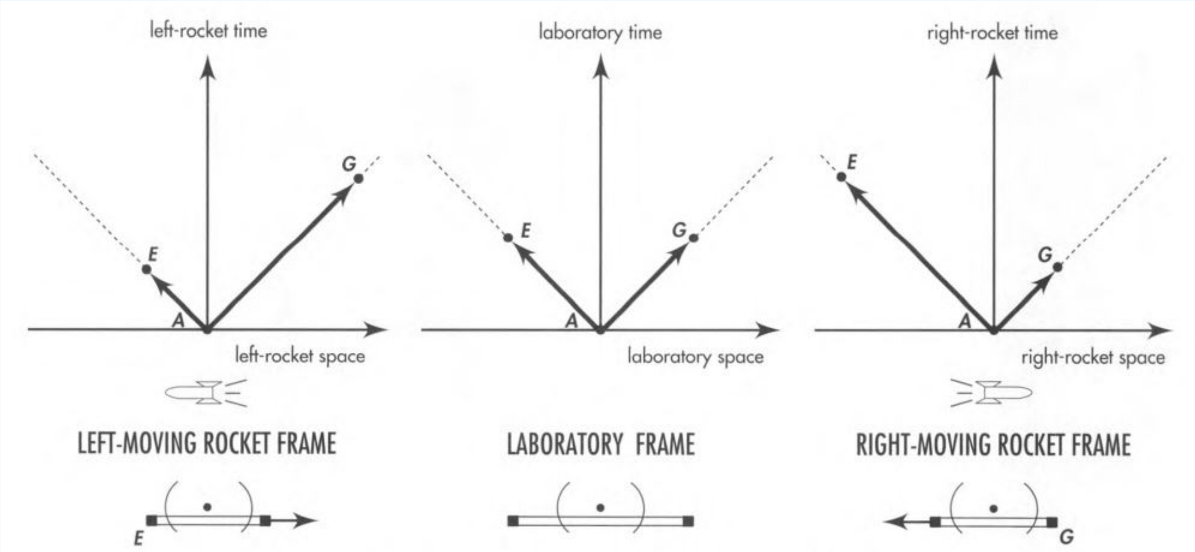
> 
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> Figure 6.2.3: Two lightlike pairs of events AE and AG (with event A arbitrarily chosen as reference event) as recorded in spacetime maps of three free-float frames. A flash originates at A and spreads outward from the center of a rod at rest in the laboratory frame. Events E and G are receptions of this flash at the two ends of the rod as recorded by different observers. In the laboratory frame, reception events E and G occur at the same time. In the right-moving rocket frame, the rod moves to the left, so event G occurs sooner than event E. In the left-moving rocket frame, the rod moves to the right, so event E occurs sooner than event G.
>
> **Answer**
>
> Oops! A spacelike interval between two events certainly can and does arise in nature.
>
> Signals from the supernova labeled 1987A reported that event to us in 1987, which was 150,000 years after the explosion occurred. Yet occur it did! No astronomer of Babylonian, Egyptian, or Greek days reported it, nor could they even know of it. Yet it had already happened for them. That event separated itself from each of them by a spacelike interval. Only the advance of time to the year 1987 brought down the interval between that explosion and Earthbound observers from spacelike to lightlike. In that year a light pulse carried the earliest possible report of that explosion to our eyes. And look today? See no explosion at that location in the sky. The light from it has passed us by. Our present relation to that event? Timelike!

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1 squared distance: Positive or zero

2 Squared interval: Positive, zero, or negative

3 Timelike interval: Time part dominates

4 Timelike interval: Invariant hyperbola opens along time axis

5 Spacelike interval: Space part dominates

6 Spacelike interval: Invariant hyperbola opens along space axis

7 Lightlike interval: Time separation equation, space separation

8 Lightlike interval: Plotted along ±45 degree lines
