# 5.4: Worldline

## the moving particle traces out a line - its worldline - on the spacetime diagram

***String of event pearls: Worldline!***

We describe the world by listing events and showing how they relate to one another. Until now we have focused on pairs of events and spacetime intervals between them. Now we turn to a whole chain of events, events that track the passage of a particle through spacetime. Think of a speeding sparkplug that emits a spark every meter of time read on its own wristwatch. Each spark is an event; the collection of spark events forms a chain that threads through spacetime, like pearls. String the pearls together. The thread connecting the pearl events, tracing out the path of a particle through spacetime, has a wonderfully evocative name: **worldline**.[1] The sparkplug travels through spacetime trailing its worldline behind it.

The speeding sparkplug is only an example. Every particle has a worldline that connects events along its spacetime path, events such as collisions or near-collisions (close calls) with other particles.

Events - pearls in spacetime - exist independent of any reference frame we may choose to describe them. A worldline strings these event pearls together. The worldline, too, exists independent of any reference frame. A particle traverses spacetime follows a worldline - totally oblivious to our poor efforts to describe its motion using one or another free-float frame. Yet we are accustomed to using a free-float frame and its associated latticework of rods and clocks. One clock after another records its encounter with the particle. The worldline of the particle connects this chain of encounter events.

***Worldline versus line on spacetime map***

We can draw this worldline of a particle on the spacetime map for this reference frame. Such worldlines are shown in Figure $5.4.2$ and in later figures of this chapter. Strictly speaking, the line drawn on the spacetime map is not the worldline itself. It is spacetime map an image of the worldline - a strand of ink printed on a piece of paper. When we use a highway map, we often refer to a line drawn on the paper as "the highway." "Yet is not the highway itself, but an image. Ordinarily this causes no confusion; no one tries to drive a car across a highway map! Similarly, we loosely refer to the line drawn on the spacetime map as the worldline, even though the worldline in spacetime stands above and beyond all our images of it.[2]

***Examples of worldlines***

The worldline is seen in no way more clearly than through example.[3] Particle 1 starts at the laboratory reference clock at zero time and moves to the right with constant speed (Figure $5.4.1$ ). As particle 1 zooms along a line of laboratory latticework of clocks, each clock it encounters records the time at which the particle passes. Each clock record shows where the clock is located and the time at which particle 1 coincides with the clock. "Where and when" determines an event, the event of coincidence of particle and recording clock. Afterwards the chief observer travels throughout the lattice of clocks, collecting the records of these coincidence events. She plots these events as points on her spacetime map. She then draws a line through event points in sequence - the worldline of particle 1 (Figure $5.4.2$ ).

Particle 1 moves with constant speed along a single direction in space. The distance it covers is equal for each tick of the laboratory clocks. The worldline of particle 1 shows equal changes in space during equal lapses of time by being straight on the spacetime map.

Particle 2 moves to the right faster than particle 1 and so covers a greater distance in the same time lapse (Figure $5.4.1$ ). Lattice clocks record their events of coincidence with particle 2, and the observer collects these records and plots the worldline of particle 2 on the same spacetime map (worldline shown in Figure $5.4.2$).

And so it goes: Particle 3 is a light flash and moves to the right in space (Figure $5.4.1$ ) with maximum speed: one meter of distance per meter of time. With horizontal and vertical axes calibrated in meters, the light-flash worldline rises at an angle of 45 degrees (Figure $5.4.2$).

Particle 4 does not move at all in laboratory space; it rests quietly next to the laboratory reference clock. Like you sitting in your chair, it moves only along the time dimension; in the laboratory spacetime map its worldline is vertical (Figure $5.4.2$).

Particle 5 moves not to the right but to the left in space according to the laboratory observer (Figure $5.4.1$ ), so its worldline angles up and leftward in the laboratory spacetime map (Figure $5.4.2$).

Each of these particles moves with constant speed, so each traces out a straight worldline. After 3 meters of time as measured in the laboratory frame, different particles have moved different distances from the starting point (Figure 5.4.1 ). In the laboratory spacetime map their space positions after 3 meters of time lie along the upper horizontal **line of simultaneity**, shown dashed in Figure 5.4.2.

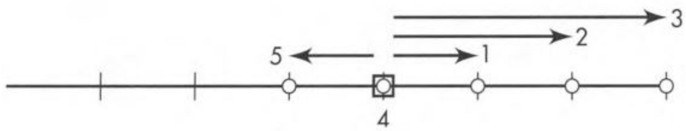


Figure 5.4.1: **Trajectories in space (not in spacetime!) of particles 1 through 5 during 3 meters of time.** Each particle starts at the reference clock (the square) at zero of time and moves with a constant velocity.

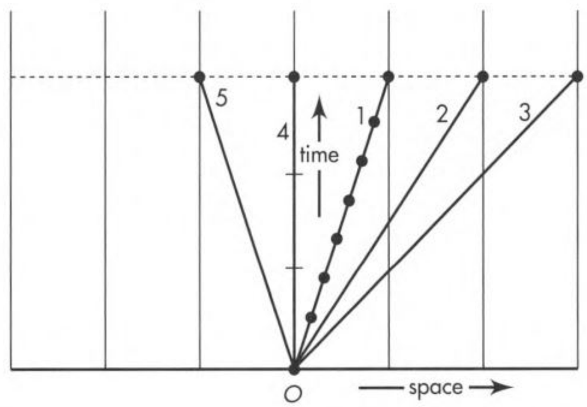


Figure 5.4.2: Worldlines in spacetime of the particles shown in Figure 5.4.1, plotted for the laboratory frame. Only the worldline for particle 1 includes a sample set of event points that are connected to make up the worldline.

Particle 4 is not the only object stationary in space. Every laboratory clock lies at rest in the laboratory frame; it moves neither right nor left as time passes. Nevertheless each laboratory clock moves forward in time, tracing out its own vertical worldline in the laboratory spacetime map. The background vertical lines in Figure 5.4.2 are worldlines of the row of laboratory clocks.

**Exercise 5.4.1**

What is the difference between a "path in space" and a "worldline in spacetime"?

**Note**

The transcontinental airplane leaves a jet trail in still air. That trail is the plane's path in space. Take a picture of that trail and you have a space map of the motion. From that space map alone you cannot tell how fast the jet is moving at this or that different point on its path. The space map is an incomplete record of the motion.[4]

The plane moves not only in space but also in time. Its beacon flashes. Plot those emissions as events on a spacetime map. This spacetime map has not only a horizontal space axis but also a vertical time axis. Now connect those event points with a worldline. The worldline gives a complete description of the motion of the jet as the plane at every event along its path.

Worldline gives spacetime map of the journey of the jet. Likewise a worldline drawn on a spacetime map images the journey of any particle through spacetime. A worldline is not a physical path, not a trajectory, not a line in space. An object at rest in your frame has, for you, no path at all through space; it stays always at one space point. Yet this stationary particle traces out a "vertical" worldline in your spacetime map (such as line 4 in Figure 5.4.2). A particle always has a worldline in spacetime. As you sit quietly in your chair reading this book, you glide through spacetime on your own unique worldline. Every stationary object lying near you also traces out a worldline, parallel to your own on your spacetime map.

Not all particles move with constant speed. When a particle changes speed with respect to a free-float frame, we know why: A force acts on it. Think of a train moving on a straight stretch of track. A force applied by the locomotive speeds up all the cars. Small speed: small distance covered in a given time lapse; worldline inclined slightly to the vertical in the spacetime map. Great speed: great distance covered in the same stretch of time; worldline inclined at a greater angle to the vertical in the spacetime map.

Changing speed: changing distances covered in equal time periods; worldline that changes inclination as it ascends on the spacetime map— a curved worldline![5]

**Question and Answer**

*Wait a minute! The train moves along a straight track. Yet you say its worldline is curved. Straight or curved? Make up your mind!*

**Answer**

Straight in space does not necessarily mean straight in spacetime. Place your finger on the straight edge of a table near you. Now move your finger rapidly back and forth along this edge. Clearly this motion lies along a straight line. As your fingertip changes speed and direction, however, it travels different spans of distance in equal time periods. During a spell in which it is at rest on the table edge, your fingertip traces out a vertical portion of its worldline on the spacetime map. When it moves slowly to the right on the table, it traces out a worldline inclined slightly to the right of vertical on the map. When it moves rapidly to the left, your fingertip leaves a spacetime trail inclined significantly to the left on the map. Changing inclination of the worldline from point to point results in a curved worldline. Your finger moves straight in space but follows a curved worldline in spacetime!

Figure $5.4.3$ shows a curved worldline, not for a locomotive, but for a particle constrained to travel down the straight track of a linear accelerator. The particle starts at the reference clock at the time of the reference event ($O$ on the map). Initially the particle moves slowly to the right along the track. As time passes - advancing upward on the spacetime map - the particle speed increases to a large fraction of the speed of light. Then the particle slows down again, comes to rest at event $Z$, with a vertical tangent to its worldline at that event. Thereafter the particle accelerates to the left in space until it arrives at event $P$.

***Limit on worldline slope: speed of light***

What possible worldlines are available to the particle that has arrived at event $P$? A Limit on worldline slope: speed of light material particle must move at less than the speed of light.[6] In other words, it travels less than one meter of distance in one meter of time. Its future worldline makes an "angle with the vertical" somewhere between plus 45 degrees and minus 45 degrees when space and time are measured in the same units and plotted to the same scale along horizontal and vertical axes on the graph. These limits of slope - which apply to every point on a particle worldline-are shown as dashed lines emerging from event $P$ in Figure $5.4.3$ (and also from event $O$).

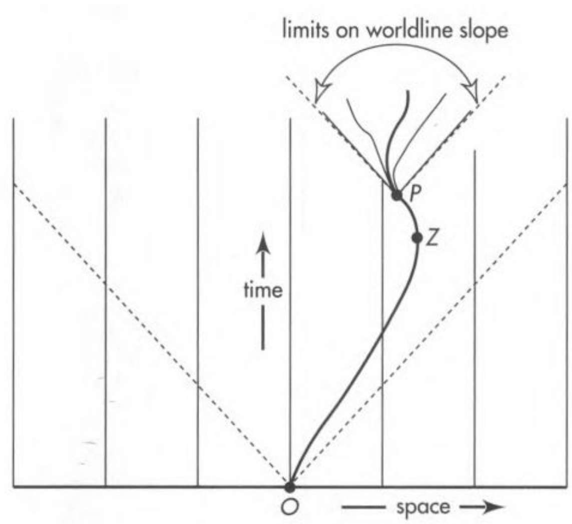


Figure $5.4.3$: Curved laboratory worldline of a particle that changes speed as it moves back a n d forth along a straight line in space. Some possible worldlines available to the particle after event P.

The worldline gives a complete description of particle motion in spacetime. As drawn in the spacetime map for any frame, the worldline tells position and velocity of the particle at every event along its trail. In contrast, the trajectory or orbit or path shape of a particle in space does not give a complete description of the motion. To complete the description we need to know when the particle occupies each location on that trajectory. A worldline in a spacetime map automatically displays all of this information.

*Spacetime map displays only already detected events*

The spacetime map provides a tool for retrospective study of events that have already taken place and have been reported to the free-float observer who plots them.[7] Once she plots these event points, this analyst can trace already plotted worldlines backward in time. She can examine at a single glance event points that may have occurred light-years apart in space. These features of the spacetime map do not violate our experience that time moves only forward or that nothing moves faster than light. Everything plotted on a spacetime map is history; it can be scanned rapidly back and forth in the space dimension or the time dimension or both. The spacetime map supplies a comprehensive tool for recognizing patterns of events and teasing out laws of nature, but it is useless for influencing the events it represents.

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2 Worldline versus line on spacetime map

3 Examples of worldlines

4 Path in space versus worldline in spacetime

5 Changing speed means curving worldline

6 Limit on worldline slope: speed of light

7 Spacetime map displays only already detected events

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