

7.7: Momenergy (Summary)

momenergy of an object unifies energy, momentum, and mass

The momenergy 4-vector of a particle equals its mass multiplied by the ratio of its spacetime displacement to proper time - wristwatch time - for that displacement (Section 7.2):

$$\left(\begin{array}{c} \text{momenergy} \\ \text{4-vector} \end{array} \right) = (\text{mass}) \left(\begin{array}{c} \text{spacetime} \\ \text{displacement} \\ \frac{\text{4-vector}}{\text{proper time}} \\ \text{for that} \\ \text{displacement} \end{array} \right) \quad (7.7.1)$$

Momenergy of a particle is a 4-vector. It possesses magnitude equal to the particle's mass. The momenergy at any given event in the motion of the particle points in the direction of the worldline at that event (Section 7.2).

The momenergy of a particle has an existence independent of any frame of reference.

The terms momenergy, momentum, and energy, as we deal with them in this book, all have a common unit: mass. In older times mass, momentum, and energy were all conceived of as different in nature and therefore were expressed in different units. The conventional units are compared with mass units in Table 7-1.

The magnitude of the momenergy 4-vector of a particle is reckoned from the difference of the squares of energy and momentum components in any given frame (Section 7.3):

$$m^2 = E^2 - (p_x)^2 - (p_y)^2 - (p_z)^2$$

or, more simply,

$$m^2 = E^2 - p^2 = (E')^2 - (p')^2 \quad (7.7.2)$$

Mass m of the particle is an invariant, has the same numerical value when computed using energy and momentum components in the laboratory frame (unprimed components) as in any rocket frame (primed components).

In a given inertial frame, the momenergy 4-vector of a particle has four components. Three space components describe the momentum of the particle in that frame (Sections 7.3 and 7.4):

$$\begin{aligned} p_x &= m \frac{dx}{d\tau} \\ p_y &= m \frac{dy}{d\tau} \\ p_z &= m \frac{dz}{d\tau} \end{aligned}$$

The magnitude of the momentum can be expressed as the factor $1/(1-v^2)^{1/2}$ times the Newtonian expression for momentum mv . The result is

$$p = mv/(1-v^2)^{1/2} \quad (7.7.4)$$

The "time part" of the momenergy 4-vector in a given inertial frame equals energy of the particle in that frame (Sections 7.3 and 7.5):

$$E = m \frac{dt}{d\tau} = \frac{m}{(1-v^2)^{1/2}} \quad (7.7.5)$$

For a particle at rest, the energy of the particle has a value equal to its mass:

$$E_{\text{rest}} = m \quad (7.7.6)$$

For a moving particle, the energy combines two parts: rest energy - equal to mass of the particle - plus the additional kinetic energy K that the particle has by virtue of its motion:

$$E = E_{\text{rest}} + K = m + K \quad (7.7.7)$$

From these equations comes an expression for kinetic energy:

$$K = E - m = m \left[\frac{1}{(1 - v^2)^{1/2}} - 1 \right] \quad (7.7.8)$$

The momenergy 4-vector derives from conservation its power to analyze particle interactions. Conservation states that the total momenergy 4-vector of an isolated system of particles is conserved, no matter how particles in the system interact with one another or transform themselves. This conservation law holds independent of choice of the free-float frame in which we employ it (Section 7.6).

In any given inertial frame, conservation of total momenergy of an isolated system breaks apart into four conservation laws:

1. Total energy of the system before an interaction equals total energy of the system after the interaction.

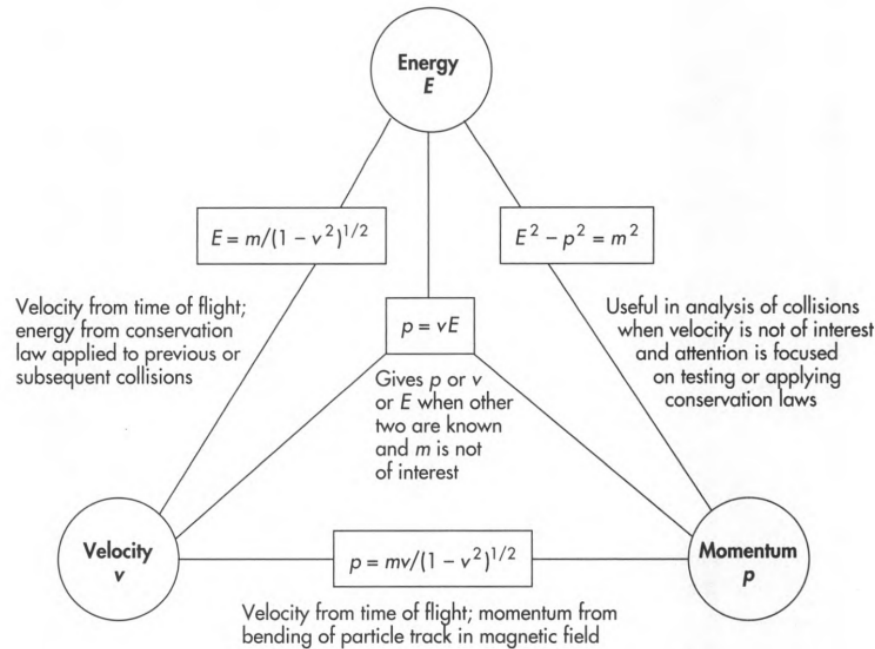


Figure 7.7.1: 7. Formulas that relate momentum, energy, mass, and velocity of a n object, and notes about their uses in analyzing experiments. In this diagram, p is the magnitude of the momentum.

| | In units of mass (for example, E and p both in kilograms; X, y, z, t, T in meters) | Reference equations | In conventional units (for example, E_{conv} in joules, p_{conv} in kilogram meters/second; t_{conv} in seconds) |
|---------------------|--|---------------------|---|
| Energy | $E = m \frac{dt}{d\tau} = \frac{m}{(1-v^2)^{1/2}}$ | (7-2, 5, 11, 17) | $E_{\text{conv}} = \frac{mc^2}{[1-(v_{\text{conv}}/c)^2]^{1/2}}$ |
| Rest energy | $E_{\text{rest}} = m$ | (7-13, 18) | $E_{\text{rest}} = mc^2$ |
| Kinetic energy | $K = m \left(\frac{1}{(1-v^2)^{1/2}} - 1 \right)$ | (7-15, 19) | $K_{\text{conv}} = mc^2 \left(\frac{1}{[1-(v_{\text{conv}}/c)^2]^{1/2}} - 1 \right)$ |
| Momentum | $p = \frac{mv}{(1-v^2)^{1/2}}$ | (7-8, 10) | $p_{\text{conv}} = \frac{mv_{\text{conv}}}{[1-(v_{\text{conv}}/c)^2]^{1/2}}$ |
| Momentum components | $p_x = m \frac{dx}{d\tau} = \frac{mv_x}{(1-v^2)^{1/2}}$ | (7-2, 5) | $p_{x \text{ conv}} = \frac{mv_{x \text{ conv}}}{[1-(v_{\text{conv}}/c)^2]^{1/2}}$ |
| | $p_y = m \frac{dy}{d\tau} = \frac{mv_y}{(1-v^2)^{1/2}}$ | (7-2, 5) | $p_{y \text{ conv}} = \frac{mv_{y \text{ conv}}}{[1-(v_{\text{conv}}/c)^2]^{1/2}}$ |

| | | | |
|---------------------------------------|---|------------|---|
| | $p_z = m \frac{dz}{d\tau} = \frac{mv_z}{(1-v^2)^{1/2}}$ | (7-2, 5) | $p_{z \text{ conv}} = \frac{mv_{z \text{ conv}}}{\left(1-(v_{\text{conv}}/c)^2\right)^{1/2}}$ |
| Mass | $m^2 = E^2 - p^2$ | (7-3) | $m^2 c^4 = E_{\text{conv}}^2 - p_{\text{conv}}^2 c^2$ |
| Particle speed | $v = \frac{p}{E}$ | (7-16) | $v_{\text{conv}} = \frac{p_{\text{conv}} c^2}{E_{\text{conv}}}$ |
| Newtonian low-speed Kinetic energy | it $K_{\text{Newton}} = \frac{1}{2}mv^2$ | (7-12, 20) | $K_{\text{conv Newton}} = \frac{1}{2}mv_{\text{conv}}^2$ |
| Momentum | $p_{\text{Newton}} = mv$ | (7-7, 9) | $p_{\text{conv Newton}} = mv_{\text{conv}}$ |
| Momentum components | $p_{x \text{ Newton}} = mv_x$ | | $p_{x \text{ conv Newton}} = mv_{x \text{ conv}}$ |
| | $p_{y \text{ Newton}} = mv_y$ | | $p_{y \text{ conv Newton}} = mv_{y \text{ conv}}$ |
| | $p_{z \text{ Newton}} = mv_z$ | | $p_{z \text{ conv Newton}} = mv_{z \text{ conv}}$ |

2. Total x -momentum of the system is the same before and after the interaction.
3. Total y -momentum of the system is the same before and after the interaction.
4. Total z -momentum of the system is the same before and after the interaction.

In this chapter we have developed expressions that relate energy, momentum, mass, and velocity. Which of these expressions is useful depends upon circumstances and the system we are trying to analyze. Figure 7-7 summarizes these equations and circumstances under which they may be useful. Table 7 – 1 compares energy and momentum in units of mass and in conventional units.

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