2.3: Local Character of Free-float Frame

tidal effects intrude in larger domains

First to strike us about the concept free float has been its paradoxical character. As a first step to explaining gravity Einstein got rid of gravity. There is no evidence of gravity in the freely falling house.

Earth's pull nonuniform: Large spaceship not a free-float frame

Well, *almost* no evidence. The second feature of free float is its local character. Riding in a very small spaceship (Figure $2.3.1$, left) we find no evidence of gravity.[1] But the enclosure in which we ride-falling near Earth or plunging through Earth cannot be too large or fall for too long a time without some unavoidable relative changes in motion being detected between particles in the enclosure. Why? Because widely separated particles within a large enclosed space are differently affected by the nonuniform gravitational field of Earth, to use the Newtonian way of speaking. For example, two particles released side by side are both attracted toward the center of Earth, so they move closer together as measured inside a falling long narrow *horizontal* railway coach (Figure $2.3.1$, center). This has nothing to do with "gravitational attraction" between the particles, which is entirely negligible.

As another example, think of two particles released far apart vertically but directly above one another in a long narrow *vertical* falling railway coach (Figure $2.3.1$, right). This time their gravitational accelerations toward Earth are in the same direction, according to the Newtonian analysis. However, the particle nearer Earth is more strongly attracted to Earth and slowly leaves the other behind: the two particles move farther apart as the coach falls. Conclusion: the large enclosure is not a free-float frame.

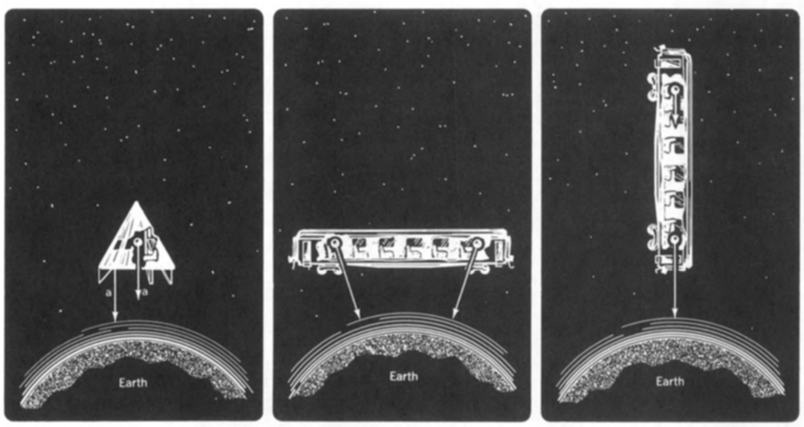


Figure $2.3.1$: Three vehicles in free fall near Earth: small space capsule, Einstein's old-fashioned railway coach in free fall in a horizontal orientation, and another railway coach in vertical orientation.

Free-float frame is local

Even a small room fails to qualify as free-float when we sample it over a long enough time. In the 42 minutes it takes our small room to fall through the tunnel from North Pole to South Pole, we notice relative motion between test particles released initially from rest at opposite sides of the room.[2]

Now, we want the laws of motion to look simple in our floating room. Therefore we want to eliminate all relative accelerations produced by external causes. "Eliminate" means to reduce these accelerations below the limit of detection so that they do not interfere with more important accelerations we wish to study, such as those produced when two particles collide. We eliminate the problem by choosing a room that is sufficiently small. Smaller room? Smaller relative accelerations of objects at different points in the room!

Let someone have instruments for detection of relative accelerations with any given degree of sensitivity. No matter how fine that sensitivity, the room can always be made so small that these perturbing relative accelerations are too small to be detectable. Within these limits of sensitivity our room is a free-float frame. "Official" names for such a frame are the **inertial reference frame** and the **Lorentz reference frame**. Here, however, we often use the name **free-float frame**, which we find more descriptive. These are all names for the same thing.

Free-float (inertial) frame formally defined

A reference frame is said to be an "inertial" or "free-float" or "Lorentz" reference frame in a certain region of space and time when, throughout that region of spacetime-and within some specified accuracy-every free test particle initially at rest with respect to that frame remains at rest, and every free test particle initially in motion with respect to that frame continues its motion without change in speed or in direction.[3]

Wonder of wonders! This test can be carried out entirely within the free-float frame The observer need not look out of the room or refer to any measurements made external to the room. A free-float frame is "local" in the sense that it is limited in space and time - and also "local" in the sense that its free-float character can be determined from within, locally.

Sir Isaac Newton stated his First Law of Motion this way: "Every body perseveres in its state of rest, or of uniform motion in a right [straight] line, unless it is compelled to change that state by forces impressed upon it." For Newton, **inertia** was a property of objects that described their tendency to maintain their state of motion, whether of rest or constant velocity. For him, objects obeyed the "Law of Inertia." Here we have turned the "Law of Inertia" around: Before we certify a reference frame to be inertial, we *require* observers in that frame to demonstrate that every free particle maintains its initial state of motion or rest. Then Newton's First Law of Motion *defines* a reference frame-an arena or playing field - in which one can study the motion of objects and draft the laws of their motion.

Question and Answer

When is the room, the spaceship, or any other vehicle small enough to be called a local free-float frame? Or when is the relative acceleration of two free particles placed at opposite ends of the vehicle too slight to be detected?

Answer

"Local" is a tricky word. For example, drop the old-fashioned 20-meter-long railway coach in a horizontal orientation from rest at a height of 315 meters onto the surface of Earth (Figure $2.3.1$, center). Time from release to impact equals 8 seconds, or 2400 million meters of light-travel time. At the same instant you drop the coach, release tiny ball bearings from rest — and in midair — at opposite ends of the coach.

Box 2-1

THE TIDE-DRIVING POWER OF MOON AND SUN

Note: Neither astronomers nor newspapers say "the Venus" or "the Mars." All say simply "Venus" or "Mars." Astronomers follow the same snappy practice for Earth, Moon, and Sun. More and more of the rest of the world now follows — as do we in this book — the recommendations of the International Astronomical Union.

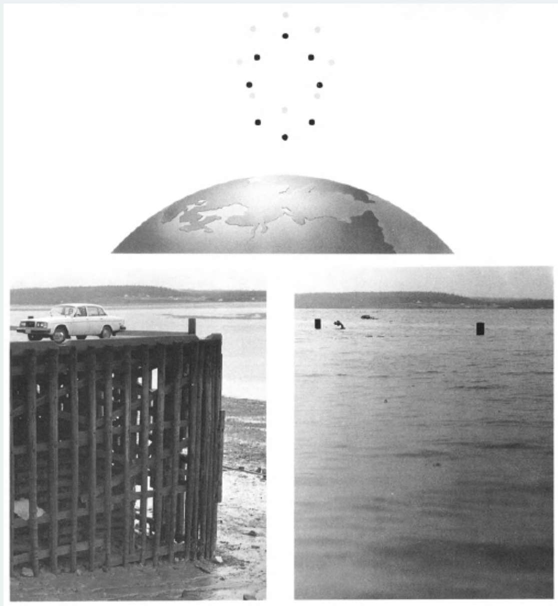
The ocean's rise and fall in a never-ending rhythmic cycle bears witness to the tide-driving power of Moon and Sun. In principle those influences are no different from those that cause relative motion of free particles in the vicinity of Earth. In a free-float frame near Earth, particles separated vertically increase their separation with time; particles separated horizontally decrease their separation with time (Figure $2.3.1$). More generally, a thin spattering of free-float test masses, spherical in pattern, gradually becomes egg-shaped, with the long axis vertical. Test masses nearer Earth, more strongly attracted than the average, move downward to form the lower bulge. Similarly, test masses farther from Earth, less strongly attracted than the average, lag behind to form the upper bulge.

By like action Moon, acting on the waters of Earth — floating free in space — would draw them out into an egg-shaped pattern if there were water everywhere, water of uniform depth. There isn't. The narrow Straits of Gibraltar almost cut off the Mediterranean from the open ocean, and almost kill all tides in it. Therefore it is no wonder that Galileo Galilei, although a great pioneer in the study of gravity, did not take the tides as seriously as the more widely traveled Johannes Kepler, an expert on the motion of Moon and the planets. Of Kepler, Galileo even said, "More than other people he was a person of independent genius . . . [but he] later pricked up his ears and became interested in the action of the moon on the water, and in other occult phenomena, and similar childishness.''

Foolishness indeed, it must have seemed, to assign to the tiny tides of the Mediterranean an explanation so cosmic as Moon. But mariners in northern waters face destruction unless they track the tides. For good reason they remember that Moon reaches its summit overhead an average 50.47 minutes later each day. Their own bitter experience tells them that, of the two high tides a day — two because there are two projections on an egg — each also comes about 50 minutes later than it did the day before.

Geography makes Mediterranean tides minuscule. Geography also makes tides in the Gulf of Maine and Bay of Fundy the highest in the world. How come? Resonance! The Bay of Fundy and the Gulf of Maine make together a great bathtub in which water sloshes back and forth with a natural period of 13 hours, near to the 12.4-hour timing of Moon's tide-driving power — and to the 12-hour timing of Sun's influence. Build a big power- producing dam in the upper reaches of the Bay of Fundy? Shorten the length of the bathtub? Decrease the slosh time from 13 hours to exact resonance with Moon? Then get one-foot higher tides along the Maine coast!

Want to see the highest tides in the Bay of Fundy? Then choose your visit according to these rules: (1) Come in summer, when this northern body of water tilts most strongly toward Moon. (2) Come when Moon, in its elliptic orbit, is closest to Earth — roughly 10 percent closer than its most distant point, yielding roughly 35 percent greater tide-producing power. (3) Take into account the tide-producing power of Sun, about 45 percent as great as that of Moon. Sun's effect reinforces Moon's influence when Moon is dark, dark because interposed, or almost interposed, between Earth and Sun, so Sun and Moon pull from the same side. But an egg has two projections, so Sun and Moon also assist each other in producing tides when they are on opposite sides of Earth; in this case we see a full Moon.



The result? Burncoat Head in the Minas Basin, Nova Scotia, has the greatest mean range of 14.5 meters (47.5 feet) between low and high tide when Sun and Moon line up. At nearby Leaf Basin, a unique value of 16.6 meters (54.5 feet) was recorded in 1953.

High and low tides witness to the relative accelerations of portions of the ocean separated by the diameter of Earth. High tides show the "stretching" relative acceleration at different radial distances from Moon or Sun. Low tides witness to the "squeezing" relative accelerations at the same radial distance from Moon or Sun but at opposite sides of Earth.

During the time of fall, they move toward each other a distance of 1 millimeter-a thousandth of a meter, the thickness of 16 pages of this book. Why do they move toward one another? Not because of the gravitational attraction between the ball bearings; this is far too minute to bring about any "coming together." Rather, according to Newton's nonlocal view, they are both attracted toward the center of Earth. Their relative motion results from the difference in direction of Earth's gravitational pull on them, says Newton.

As another example, drop the same antique railway coach from rest in a *vertical* orientation, with the lower end of the coach initially 315 meters from the surface of Earth (Figure 2.3.1, right). Again release tiny ball bearings from rest at opposite ends of the coach. In this case, during the time of fall, the ball bearings move *apart* by a distance of 2 millimeters because of the greater gravitational acceleration of the one nearer Earth, as Newton would put it. This is twice the change that occurs for horizontal separation.

In either of these examples let the measuring equipment in use in the coach be just short of the sensitivity required to detect this relative motion of the ball bearings. Then, with a limited time of observation of 8 seconds, the railway coach - or, to use the earlier example, the freely falling room - serves as a free-float frame.

When the sensitivity of measuring equipment is increased, the railway coach may no longer serve as a local free-float frame unless we make additional changes. Either shorten the 20 -meter domain in which observations are made, or decrease the time given to the observations. Or better, cut down some appropriate combination of space and time dimensions of the region under observation. Or as a final alternative, shoot the whole apparatus by rocket up to a region of space where one cannot detect locally the "differential gravitational acceleration" between one side of the coach and another - to use Newton's way of speaking. In another way of speaking, relative accelerations of particles in different parts of the coach must be too small to perceive. Only when these relative accelerations are too small to detect do we have a reference frame with respect to which laws of motion are simple. That's why "local" is a tricky word!

Question and Answer

Hold on? You just finished saying that the idea of local gravity is unnecessary. Yet here you use the "differential gravitational acceleration" to account for relative accelerations of test particles and ocean tides near Earth. Is local gravity necessary or not?

Answer

Near Earth, two explanations of projectile paths or ocean flow give essentially the same numerical results. Newton says there is a force of gravity, to be treated like any other force in analyzing motion. Einstein says gravity differs from all other forces: Get rid of gravity locally by climbing into a free-float frame. Near the surface of Earth both explanations accurately predict relative accelerations of falling particles toward or away from one another and motions of the tides. In this chapter we use the more familiar Newtonian analysis to predict relative accelerations.

When tests of gravity are very sensitive, or when gravitational effects are large, such as near white dwarfs or neutron stars, then Einstein's predictions are not the same as Newton's. In such cases Einstein's battle-tested 1915 theory of gravity (general relativity) predicts results that are observed; Newton's theory makes incorrect predictions. This justifies Einstein's insistence on getting rid of gravity locally using free-float frames. All that remains of gravity is the relative accelerations of nearby particles - tidal accelerations.

---

1 Earth's pull nonuniform: Large spaceship not a free-float frame

2 Free-float frame is local

3 Free-float (inertial) frame formally defined
