3.S: Same Laws for All (Summary)

same laws for all; invariant interval for all

The **Principle of Relativity** says that the laws of physics are the same in every inertial (free-float) reference frame (Section 3.1). This simple principle has important consequences. Specifically:

$$\begin{aligned}
(\text{ interval })^2 &= \begin{pmatrix}\text{separation}\\ \text{in lab}\\ \text{time}\end{pmatrix}^2 - \begin{pmatrix}\text{separation}\\ \text{in lab}\\ \text{position}\end{pmatrix}^2 \\
&= \begin{pmatrix}\text{separation}\\ \text{in moving-}\\ \text{particle time}\end{pmatrix}^2 - \begin{pmatrix}\text{separation}\\ \text{in moving-}\\ \text{particle position}\end{pmatrix}^2 \\
&= \begin{pmatrix}\dfrac{\text{9 meters of distance}}{\begin{matrix}\text{0.868 meters of distance}\\ \text{per meter of time}\end{matrix}}\end{pmatrix}^2 - \begin{pmatrix}\text{9 meters}\\ \text{of distance}\end{pmatrix}^2 \\
&= (\text{2 half-lives })^2 - \begin{pmatrix}\text{zero separation}\\ \text{in space (in}\\ \text{particle frame)}\\ \text{between those}\\ \text{two events}\end{pmatrix}^2 \\
&= \begin{pmatrix}\text{10.368 meters}\\ \text{of light-travel time}\end{pmatrix}^2 - \begin{pmatrix}\text{9 meters}\\ \text{of distance}\end{pmatrix}^2 \\
&= (\text{2 half-lives })^2
\end{aligned}$$

A little arithmetic tells us that two half-lives total 5.15 meters of light-travel time. Consequently the K^+ half-life itself is 2.57 meters of time or (2.57 meters)/(3.00 × 10^8 meters/second) = 8.5 × 10^{-9} second or 8.5 nanoseconds.

1. Two events that lie along the direction of relative motion between two frames cannot be simultaneous as measured in both frames (**relativity of simultaneity**). (Section 3.4)
2. An object in high-speed motion is measured to be shorter along its direction of motion than its proper length, measured in its rest frame (**Lorentz contraction**). (Section 3.5)
3. The dimensions of moving objects transverse to their direction of relative motion are measured to be the same, whatever the relative speed (**invariance of transverse distances**). (Section 3.6)
4. Two events with separation only transverse to the direction of relative motion and simultaneous in either frame are simultaneous in both. (Section 3.6)
5. The spacetime interval between two events is invariant-it has the same value in laboratory and rocket frames (Sections 3.7 and 3.8):

$$\begin{aligned}
(\text{ interval })^2 &= \overset{\textbf{Laboratory}}{\begin{pmatrix}\text{time}\\ \text{separation}\end{pmatrix}^2} - \overset{\textbf{Laboratory}}{\begin{pmatrix}\text{space}\\ \text{separation}\end{pmatrix}^2} \\
&= \overset{\textbf{Rocket}}{\begin{pmatrix}\text{time}\\ \text{separation}\end{pmatrix}^2} - \overset{\textbf{Rocket}}{\begin{pmatrix}\text{space}\\ \text{separation}\end{pmatrix}^2}
\end{aligned}$$

6. In any free-float frame, no object moves with a speed greater than the speed of light (Box 3-3).

Box 3-3: Faster Than Light?

We always want to go faster. Faster than what? Faster than anything has gone before. What is our greatest possible speed, according to the theory of relativity? The speed of light in a vacuum! How do we know that this is the greatest possible speed that we can travel? Many lines of evidence reach this conclusion. Rocket speed greater than the speed of light would lead to the destruction of the essential relation between cause and effect, a result explored in Special Topic: Lorentz Transformation (especially Box L-1) and in Chapter 6. In particular, we could find a frame in which a faster-than-light object arrives before it starts! Moreover, in particle accelerators built over several decades we have spent hundreds of millions of dollars effectively trying to accelerate electrons and protons to the greatest possible speed — which by experiment never exceeds light speed.

The conclusion that no thing can move faster than light arises also from the invariance of the interval. To see this, let a rocket emit two flashes of light a time t' apart as measured in the rocket frame. (Use a prime to distinguish rocket measurements from laboratory measurements.) In the rocket frame the two emissions occur at the same place: the separation x' between them equals zero. Let t and x be the corresponding separations in time and space as measured in the laboratory frame. Then the invariance of the interval tells us that the three quantities t', t, and x are related by the equation

$$(t')^2 - (x')^2 = (t')^2 - (0)^2 = t^2 - x^2$$

whence

$$(t')^2 = t^2 - x^2 \tag{3.S.1}$$

In the laboratory frame the rocket is moving with some speed; give this speed the symbol v. The distance x between emissions is just the distance that the rocket moves in time t in the laboratory frame. The relation between distance, time, and speed is

$$x = vt \tag{3.S.2}$$

Substitute this into equation (3.S.1) to obtain $(t')^2 = t^2 - (vt)^2 = t^2\left[1 - v^2\right]$, or

$$t' = t\left(1 - v^2\right)^{1/2} \tag{3.S.3}$$

Now, v is the speed of the rocket. How large can that speed be? Equation (3.S.3) makes sense for any rocket speed less than the speed of light, or when v has a value less than one.

Suppose we try to force the rocket to move faster than the speed of light. If we should succeed, v would have a value greater than one. Then v^2 also would have a value greater than one. But in this case the expression $1 - v^2$ would have a negative value and its square root would have no physical meaning. In a formal mathematical sense, the rocket time t' would be an imaginary number for the case of rocket speed greater than the speed of light. But clocks do not read imaginary time; they read real time - three hours, for example. Therefore a rocket speed greater than the speed of light leads to an impossible consequence.

Equation (3.S.3) does not forbid a rocket to go as close to the speed of light as we wish, as long as this speed remains less than the speed of light. For v very close to the speed of light, equation (3.S.3) tells us that the rocket time can be very much smaller than the laboratory time. Now suppose that emission of the first flash occurs when the rocket passes Earth on its outward trip to a distant star. Let emission of the second flash occur as the rocket arrives at that distant star. No matter how long the laboratory time t between these two events, we can find a rocket speed, v, such that the rocket time t' is as small as we wish. This means that in principle we can go to any remote star in as short a rocket time as we want. In brief, although our speed is limited to less than the speed of light, the distance we can travel in a lifetime has no limitation. We can go anywhere! This result is explored further in Chapter 4.

Box 3-4: Does a Moving Clock Really "Run Slow"?

Question and Answer

You keep saying, "The time between clock-ticks is shorter as MEASURED in the rest frame of the clock than as MEASURED in a frame in which the clock is moving." I am interested in reality, not someone's measurements. Tell me what really happens!

Answer

What is reality? You will have your own opinion and speculations. Here we pose two related scientific questions whose answers may help you in forming your opinion..

Are differences in clock rates really verified by experiment?
Different values of the time between two events as observed in different frames? Absolutely! Energetic particles slam into solid targets in accelerators all over the world, spraying forward newly created particles, some of which decay in very short times as measured in their rest frames. But these "short-lived" particles survive much longer in the laboratory frame as they streak from target to detector. In consequence, the detector receives a much larger fraction of the undecayed fast-moving particles than would be predicted from their decay times measured at rest. This result has been tested thousands of times with many different kinds of particles. Such experiments carried out over decades lead to dependable, consistent, repeatable results. As far as we can tell, they are correct, true, and reliable and cannot effectively be denied. If that is what you personally mean by "real," then these results are "what really happens."

Does something about a clock really change when it moves, resulting in the observed change in tick rate?
Absolutely not! Here is why: Whether a free-float clock is at rest or in motion in the frame of the observer is controlled by the observer. You want the clock to be at rest? Move along with it! Now do you wont the clock to move? Simply change your own velocity! This is true even when you and the clock are separated by the diameter of the solar system. The magnitude of the clock's steady velocity is entirely under your control. Therefore the time between its ticks as measured in your frame is determined by your actions. How can your change of motion affect the inner mechanism of a distant clock? It cannot and does not.

Every time you change your motion on Earth — and even when you sit down, letting the direction of your velocity change as Earth rotates — you change the rate at which the planets revolve around Sun, as measured in your frame. (You also change the shape of planetary orbits, contracting them along the direction of your motion relative to Sun.) Do you think this change on your velocity really affects the workings of the "clock" we call the solar system? If so, what about a person who sits down on the other side of Earth? That person moves in the opposite direction around the center of Earth, so the results are different from yours. Are each of you having a different effect on the solar system? And are there still different effects — different solar-system clocks — for observers who could in principle be scattered on other planets?

We conclude that free-float motion does not affect the structure or operation of clocks (or rods). If this is what you mean by reality, then there are really no such changes due to uniform motion.

Is there some unity behind these conflicting measurements of time and space? Yes! The interval: the proper time (wristwatch time) between ticks of a clock as measured in a frame in which ticks occur at the same place, in which the clock is at rest. Proper time can also be calculated by all free-float observers, whatever their state of motion, and all agree on its value. Behind the confusing clutter of conflicting measurements stands the simple, consistent, powerful view provided by spacetime.