1.4: Same Unit for Space and Time- Meter, Second, Minute, or Year

meter for particle accelerators; minute for planets; year for the cosmos

Measure time in meters

The parable of the surveyors cautions us to use the same unit to measure both space and time. So we use meter for both. Time can be measured in meters. Let a flash of light bounce back and forth between parallel mirrors separated by 0.5 meter of distance (Figure $1.4.1$). Such a device is a "clock" that "ticks" each time the light flash arrives back at a given mirror. Between ticks the light flash has traveled a round-trip distance of 1 meter. Therefore we call the stretch of time between ticks **1 meter of light-travel time** or more simply **1 meter of time**.

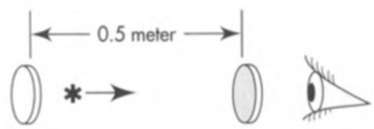

Figure $1.4.1$: This two-mirror "clock" sends to the eye flash after flash, each separated from the next by 1 meter of light-travel time. A light flash (represented by an asterisk) bounces back and forth between parallel mirrors separated from one another by 0.5 meter of distance. The silver coating of the right-band mirror does not reflect perfectly: It lets 1 percent of the light pass through to the eye each time the light pulse bits it. Hence the eye receives a pulse of light every meter of light-travel time.

One meter of light-travel time is quite small compared to typical time lapses in our everyday experience. Light travels nearly 300 million meters per second (300,000,000 meters/second $= 3 \times 10^8$ meters/second, four fifths of the way to Moon in one second). Therefore one second equals 300 million meters of light-travel time. So 1 meter of light-travel time has the small value of one three-hundred-millionth of a second. [How come? Because (1) light goes 300 million meters in one second, and (2) one three-hundred-millionth of that distance (one meter!) is covered in one three-hundred-millionth of that time.] Nevertheless this unit of time is very useful when dealing with light and with high-speed particles. A proton early in its travel through a particle accelerator may be jogging along at "only" one half the speed of light. Then it travels 0.5 meter of distance in 1 meter of light-travel time.

We, our cars, even our jet planes, creep along at the pace of a snail compared with light. We call a deed quick when we've done it in a second. But a second for light means a distance covered of 300 million meters, seven trips around Earth. As we dance around the room to the fastest music, oh, how slow we look to light! Not zooming. Not dancing. Not creeping. Oozing! That long slow ooze racks up an enormous number of meters of light-travel time. That number is so huge that, by the end of one step of our frantic dance, the light that carries the image of the step's beginning is well on its way to Moon.

Meter officially defined using light speed

In 1983 the General Conference on Weights and Measures officially redefined the meter in terms of the speed of light. **The meter is now defined as the distance that light travels in a vacuum in the fraction $\mathbf{1/299,792,458}$ of a second.** (For the definition of the second, see Box 3-2.) Since 1983 the speed of light is, by definition, equal to $c = 299,792,458$ meters/second. This makes official the central position of the speed of light as a conversion factor between time and space.

This official action defines distance (meter) in terms of time (second). Every day we use time to measure distance. "My home is only ten minutes (by car) from work." "The business district is a five-minute walk." Each statement implies a speed - the speed of driving or walking - that converts distance to time. But these speeds can vary - for example, when we get caught in traffic or walk on crutches. In contrast, the speed of light in a vacuum does not vary. It always has the same value when measured over time and the same value as measured by every observer.

Measure distance in light-years

We often describe distances to stars and galaxies using a unit of time. These distances we measure in light-years. One light-year equals the distance that light travels in one year. Along with the light-year of space goes the year of time. Here again space and time are measured in the same units - years. Here again the speed of light is the conversion factor between measures of time and space. From our everyday perspective one light-year of space is quite large, almost 10,000 million million meters: 1 light-year $= 9,460,000,000,000,000$ meters $= 0.946 \times 10^{16}$ meters. Nevertheless it is a convenient unit for measuring distance between stars. For example, the nearest star to our Sun, Proxima Centauri, lies 4.28 light-years away.

Any common unit of space or time may be used as the same unit for both space and time. For example, Table $1.4.1$ gives us another convenient measure of time, seconds, compared with time in meters. We can also measure space in the same units, light-seconds. Our Sun is 499 light-seconds - or, more simply, 499 seconds-of distance from Earth. Seconds are convenient for describing distances and times among events that span the solar system. Alternatively we could use minutes of time and light-minutes of distance: Our Sun is 8.32 light-minutes from Earth. We can also use hours of time and light-hours of distance. In all cases, the speed of light is the conversion factor between units of space and time.

Table 1.4.1: Some Light-travel Times

	Time in seconds of light-travel time	Time in meters
Telephone call one way: New York City to San Francisco via surface microwave link	0.0138	4,139,000
Telephone call one way: New York City to San Francisco via Earth satellite	0.197	59,000,000
Telephone call one way: New York City to San Francisco bounced off Moon	2.51	752,000,000
Flash of light: Emitted by Sun, received on Earth	499.0	149,600,000.000

Use convenient units, the same for space and time

Expressing time and space in the same unit **meter** is convenient for describing motion of high-speed particles in the confines of the laboratory. Time and space in the same unit **second** (or **minute** or **hour**) is convenient for describing relations among events in our solar system. Time and space in the same unit **year** is convenient for describing relations among stars and among galaxies. In all three arenas spacetime is the stage and special relativity is the spotlight that illuminates the inner workings of Nature.

We are not accustomed to measuring time in meters. So as a reminder to ourselves we add a descriptor: meters *of light-travel time*. But the unit of time is still the meter. Similarly, the added words "seconds *of distance*" and "*light-years*" help to remind us that distance is measured in seconds or years, units we usually associate with time. But this unit of distance is really just second or year. The modifying descriptors are for our convenience only. In Nature, space and time form a unity: spacetime!

Question and Answer

The words sound OK. The mathematics appears straightforward. The Sample Problems seem logical. But the ideas are so strange! Why should I believe them? How can invariance of the interval be proved?

Answer

No wonder these ideas seem strange. Particles zooming by at nearly the speed of light - how far this is from our everyday experience! Even the soaring jet plane crawls along at less than one-millionth light speed. Is it so surprising that the world appears different at speeds a million times faster than those at which we ordinarily move with respect to Earth?

The notion of *spacetime interval* distills a wealth of real experience. We begin with interval because it endures: It illuminates observations that range from the core of a nucleus to the center of a black hole. Understand the spacetime interval and you vault, in a single bound, to the heart of spacetime.

Chapter 3 presents a logical proof of the invariance of the interval. Chapter 4 reports a knock-down argument about it. Chapters that follow describe many experiments whose outcomes are totally incomprehensible unless the interval is invariant. Real verification comes daily and hourly in the on-going enterprise of experimental physics.

Example 1.4.1: Proton, Rock, and Starship

a. A proton moving at $3/4$ light speed (with respect to the laboratory) passes through two detectors 2 meters apart. Events 1 and 2 are the transits through the two detectors. What are the laboratory space and time separations between the two events, in meters? What are the space and time separations between the events in the proton frame?

b. A speeding rock from space streaks through Earth's outer atmosphere, creating a short fiery trail (Event 1) and continues on its way to crash into Sun (Event 2) 10 minutes later as observed in the Earth frame. Take Sun to be 1.4960×10^{11} meters from Earth. In the Earth frame, what are space and time separations between Event 1 and Event 2 in minutes? What are space and time separations between the events in the frame of the rock?

c. In the twenty-third century a starship leaves Earth (Event 1) and travels at 95 percent light speed, later arriving at Proxima Centauri (Event 2), which lies 4.3 light-years from Earth. What are space and time separations between Event 1 and Event 2 as measured in the Earth frame, in years? What are space and time separations between these events in the frame of the starship?

Solution

a. The space separation measured in the laboratory equals 2 meters, as given in the problem. A flash of light would take 2 meters of light-travel time to travel between the two detectors. Something moving at $1/4$ light speed would take four times as long: 2 meters$/(1/4) = 8$ meters of light-travel time to travel from one detector to the other. The proton, moving at $3/4$ light speed, takes 2 meters$/(3/4) = 8/3$ meters $= 2.66667$ meters of light-travel time between events as measured in the laboratory.

Event 1 and Event 2 both occur at the position of the proton. Therefore the space separation between the two events equals zero in the proton frame. This means that the spacetime interval - the proper time-equals the time between events in the proton frame.

$$\begin{aligned}
(\text{ proton time })^2-(\text{ proton distance })^2 &=(\text{ interval })^2=(\text{ lab time })^2-(\text{ lab distance })^2 \\
(\text{proton time})^{\,2}-(\text{ zero })^2 &=(2.66667 \text{ meters })^2-(2 \text{ meters })^2 \\
&=(7.1111-4)(\text{ meters })^2 \\
(\text{ proton time })^2 &=3.1111(\text{ meters })^2
\end{aligned}$$

So time between events in the proton frame equals the square root of this, or 1. 764 meters of time.

b. Light travels 60 times as far in one minute as it does in one second. Its speed in meters per minute is therefore:

$$\begin{aligned}
&2.99792458 \times 10^8 \text{ meters/second } \times 60 \text{ seconds / minute} \\
&=1.798754748 \times 10^{10} \text{ meters / minute}
\end{aligned}$$

So the distance from Earth to Sun is

$$\frac{1.4960 \times 10^{11} \text{ meters}}{1.798754748 \times 10^{10} \text{ meters/minute}}=8.3169 \text{ light-minutes}$$

This is the distance between the two events in the Earth frame, measured in light-minutes. The Earth-frame time between the two events is 10 minutes, as stated in the problem.

In the frame traveling with the rock, the two events occur at the same place; the time between the two events in this frame equals the spacetime interval-the proper time - between these events:

$$\begin{aligned}
(\text{ interval })^2 &=(10 \text{ minutes })^2-(8.3169 \text{ minutes })^2 \\
&=(100-69.1708)(\text{ minutes })^2 \\
&=30.8292(\text{ minutes })^2
\end{aligned}$$

The time between events in the rest frame of the rock equals the square root of this, or 5.5524minutes.

c. The distance between departure from Earth and arrival at Proxima Centauri is 4.3 light-years, as given in the problem. The starship moves at 95 percent light speed, or 0.95 light-years/year. Therefore it takes a time 4.3 light-years$/(0.95$ light-years/year$) = 4.53$ years to arrive at Proxima Centauri, as measured in the Earth frame.

Starship time between departure from Earth and arrival at Proxima Centauri equals the interval:

$$\begin{aligned}
(\text{ interval })^2 &=(4.53 \text{ years })^2-(4.3 \text{ years })^2 \\
&=(20.52-18.49)(\text{ years })^2 \\
&=2.03(\text{ years })^2
\end{aligned}$$

The time between events in the rest frame of the starship equals the square root of this, or 1.42 years. Compare with the value 4.53 years as measured in the Earth frame. This example illustrates the famous idea that astronaut wristwatch time - proper time-between two events is less than the time between these events measured by any other observer in relative motion. Travel to stay young! This result comes simply and naturally from the invariance of the interval.

Note

- Measure time in meters
- Meter officially defined using light speed
- Measure distance in light-years
- Use convenient units, the same for space and time