11.8: Summary

Lorentz transformation deals with coordinates, not invariant quanfities

Given the space and time coordinates of an event with respect to the reference event in one free-float frame, the Lorentz coordinate transformation equations tell us the coordinates of the same event in an overlapping free-float frame in relative motion with respect to the first. The equations that transform rocket coordinates (primed coordinates) to laboratory coordinates (unprimed coordinates) have the form

$$\begin{aligned} t &= \frac{v_{\text{rel}}x' + t'}{\left(1 - v_{\text{rel}}^2\right)^{1/2}} \\ x &= \frac{x' + v_{\text{rel}}t'}{\left(1 - v_{\text{rel}}^2\right)^{1/2}} \\ y &= y' \quad \text{and} \quad z = z' \end{aligned} \tag{11.5.5}$$

where v_{rel} stands for relative speed of the two frames (rocket moving in the positive x-direction in the laboratory). The inverse Lorentz transformation equations transform laboratory coordinates to rocket coordinates:

$$\begin{aligned} t' &= \frac{-v_{\text{rel}}x + t}{\left(1 - v_{\text{rel}}^2\right)^{1/2}} \\ x' &= \frac{x - v_{\text{rel}}t}{\left(1 - v_{\text{rel}}^2\right)^{1/2}} \\ y' &= y \quad \text{and} \quad z' = z \end{aligned} \tag{11.6.1}$$

in which v_{rel} is treated as a positive quantity. In both these sets of equations, coordinates of events are measured with respect to a reference event. It is really only the difference in coordinates between events that matter, for example $x_2 - x_1 = \Delta x$ for any two events 1 and 2 , not the coordinates themselves. This is important in deriving the Law of Addition of Velocities.

The Law of Addition of Velocities or Law of Combination of Velocities in one dimension follows from the Lorentz transformation equations. This law tells us the velocity v of a particle in the laboratory frame if we know its velocity v' with respect to the rocket and relative speed v_{rel} between rocket and laboratory, \

[v=\frac{v^{\prime}+v_{\mathrm{rel}}}{1+v^{\prime} v_{\mathrm{rel}}}\]

REFERENCE

Sample Problem 11.7.3, The Firing Meson, was adapted from A. P. French, Special Relativity (W.W. Norton, New York, 1968), page 159.