4.E: Trip to Canopus (Exercises)

Note: The following exercises are related to the story line of this chapter. Additional exercises may be selected from Chapter 3 or the Special Topic on the Lorentz Transformation following Chapter 3.

4.1 practical space travel

In 2200 A.D. the fastest available interstellar rocket moves at $v = 0.75$ of the speed of light. James Abbott is sent in this rocket at full speed to Sirius, the Dog Star (the brightest star in the heavens as seen from Earth), a distance $D = 8.7$ light-years as measured in the Earth frame. James stays there for a time $T = 7$ years as recorded on his clock and then returns to Earth with the same speed $v = 0.75$. Assume Sirius is at rest relative to Earth. Let the departure from Earth be the reference event (the zero of time and space for all observers).

According to Earth-linked observers:

a. At what time does the rocket arrive at Sirius?

b. At what time does the rocket leave Sirius?

c. At what time does the rocket arrive back at Farth

According to James's observations:

d. At what time does he arrive at Sirius?

e. At what time does he leave Sirius?

f. At what time does he arrive back at Earth?

g. As he moves toward Sirius, James is accompanied by a string of outgoing lookout stations along his direction of motion, each one with a clock synchronized to his own. What is the spatial distance between Earth and Sirius, according to observations made with this outgoing string of lookout stations?

h. One of James's outgoing lookout stations, call it Q, passes Earth at the same time (in James's outgoing frame) that James reaches Sirius. What time does Q 's clock read at this event of passing? What time does the clock on Earth read at this same event?

i. As he moves back toward Earth, James is accompanied by a string of incoming lookout stations along his direction of motion, each one with a clock synchronized to his own. One of these incoming lookout stations, call it Z, passes Earth at the same time (in James's incoming frame) that James leaves Sirius to return home. What time does Z 's clock read at this event of passing? What time does the clock on Earth read at this same event?

To *really* understand the contents of Chapter 4, repeat this exercise many times with new values of v, D, and T that you choose yourself.

4.2 one-way twin paradox?

A worried student writes, "I still cannot believe your solution to the Twin Paradox. During the outward trip to Canopus, each twin can regard the other as moving away from him; so how can we say which twin is younger? The answer is that the twin in the rocket makes a turn, and in Lorentz spacetime geometry, the greatest aging is experienced by the person who does not turn. This argument is extremely unsatisfying. It forces me to ask: What if the rocket breaks down when I get to Canopus, so that I stop there but cannot turn around? Does this mean that it is no longer possible to say that I have aged less than my Earthbound twin? But if not, then I would never have gotten to Canopus alive." Write a half-page response to this student, answering the questions politely and decisively.

4.3 a relativistic oscillator

In order to test the laws of relativity, an engineer decides to construct an oscillator with a very light oscillating bob that can move back and forth very fast. The lightest bob known with a mass greater than zero is the electron. The engineer uses a cubical metal box, whose edge measures one meter, that is warmed slightly so that a few electrons "boil off" from its surfaces (see the figure). A vacuum pump removes air from the box so that electrons may move freely inside without colliding with air molecules. Across the middle of the box - and electrically insulated from it is a metal screen charged to a high positive voltage by a power supply. A voltage-control knob on the power supply can be turned to change the DC voltage $\mathrm{V_o}$ between box and screen. Let an electron boiled off from the inner wall of the box have very small velocity initially (assume that the initial velocity is zero). The electron is attracted to the positive screen, increases speed toward the screen, passes through a hole in the screen, slows down as it moves away from the attracting screen, stops just short of the opposite wall of the box, is pulled back toward the screen; and in this way oscillates back and forth between the walls of the box.

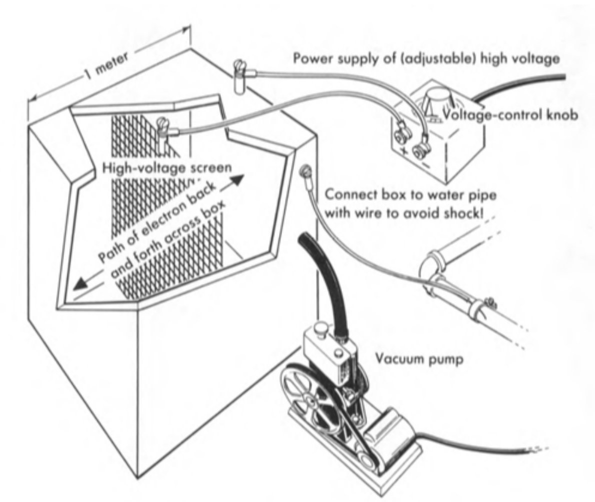

Figure $4.E.1$: Relativistic oscillator with electron as oscillating bob.

a. In how short a time T can the electron be made to oscillate back and forth on one round trip between the walls? The engineer who designed the equipment claims that by turning the voltage control knob high enough he can obtain as high a frequency of oscillation $f=1/T$ as desired. Is he right?

b. For sufficiently low voltages the electron will be nonrelativistic-and one can use Newtonian mechanics to analyze its motion. For this case the frequency of oscillation of the electron is increased by what factor when the voltage on the screen is doubled? Discussion: At corresponding points of the electron's path before and after voltage doubling, how does the Newtonian kinetic energy of the electron compare in the two cases? How does its velocity compare in the two cases?

c. What is a definite formula for frequency f as a function of voltage in the nonrelativistic case? Wait as late as possible to substitute numbers for mass of electron, charge of electron, and so forth.

d. What is the frequency in the extreme relativistic case in which over most of its course the electron is moving ... (rest of sentence suppressed!) ... ? Call this frequency $f_{\max}$.

e. On the same graph, plot two curves of the dimensionless quantity $f/f_{\max}$ as functions of the dimensionless quantity $q\,\mathrm{V}_{\mathrm{o}}/\left(2mc^2\right)$, where q is the charge on the electron and m is its mass. First curve: the nonrelativistic curve from part **c** to be drawn heavily in the region where it is reliable and indicated by dashes elsewhere. Second curve: the extreme relativistic value from part d, also with dashed lines where not reliable. From the resulting graph estimate quantitatively the voltage of transition from the nonrelativistic to the relativistic region. If possible give a simple argument explaining why your result does or does not make sense as regards order of magnitude (that is, overlooking factors of 2, π, etc.).

f. Now think of the round-trip "proper period" of oscillation τ experienced by the electron and logged by its recording wristwatch as it moves back and forth across the box. At low electron speeds how does this proper period compare with the laboratory period recorded by the engineer? What happens at higher electron speeds? At extreme relativistic speeds? How is this reflected in the "proper frequency" of oscillation f_{proper} experienced by the electron? On the graph of part e draw a rough curve in a different color or shading showing qualitatively the dimensionless quantity $f_{\text{proper}}/f_{\max}$ as a function of $q\,\mathrm{V}_{\mathrm{o}}/\left(2mc^2\right)$.