# 5.3: Invariant Hyperbola

## all observers agree: "event point lies somewhere on this hyperbole"

Different reception points marked $R$ in different spacetime maps all refer to the same event. What do these different separations of the same event from the reference event have in common? They all satisfy invariance of the interval, reflected in the equation

$$(\text{ time separation })^2-(\text{ space separation })^2=(\text{ interval })^2=\text{ constant}$$

**Question and Answer**

*Constant? Constant with respect to what?*

**Answer**

With respect to free-float frame. Record different space and time measurements in different frames, but figure out from them always the same interval.

***Invariant hyperbola; Locus of same event in all rocket frames***

Curves drawn on the three maps conform to this equation. This kind of curve, in which the difference of two squares equals a constant, is called a **hyperbola**. Somewhere on this hyperbola is recorded the time and position of one and the same reception event as measured in every possible rocket and super-rocket frame. Same reception event, different frames, all summarized in one hyperbola, the **invariant hyperbola**.

Spacetime arrows in all three maps connect the same pair of events. They imply the identical invariant interval. They embody the same spacetime reality. In a deep sense these three arrows on the page represent the same arrow in spacetime. Spacetime maps of different observers show different projections-different perspectives-of the same arrow in spacetime.

**Question and Answer**

*The same arrow? The same magnitude for the spacetime arrow pictured in all three maps of Figure 5.2.2? Then why do the three arrows have obviously different lengths in the three maps?*

**Answer**

Because the paper picture of spacetime is a lie! The length of an arrow on a piece of paper is Euclidean, related to the sum of squares of the space separations of the endpoints in two perpendicular directions. Euclidean geometry works fine if what is being represented is flat space, for example the map of a township. But Euclidean geometry is the wrong geometry and betrays us when we try to lay out time along one direction on the page. Instead we need to use Lorentz geometry of spacetime. In Lorentz geometry, time must be combined with space through a difference of squares to find the correct magnitude of the resulting spacetime vector - the interval. That is why the arrows in the different spacetime maps of Figure 5.2.2 seem to be of different lengths. The reality that these lengths represent, however - the value of the interval between two events-is the same in all three spacetime maps.

1 Invariant hyperbola; Locus of same event in all rocket frames