4.7: Lorentz Contraction

go a shorter distance in a shorter time

As James Fastlane ponders this response, Dr. Joanne Short breaks in. "The Twin Paradox is not the only one you have to explain in order to convince us of the correctness of your analysis. Look at the outward trip as observed by you yourself, the rocket traveler. You reach Canopus after just 20 years of your time. Yet we know that Canopus lies 99 light-years distant. How can you possibly cover 99 light-years in 20 years?"

"That is exactly what I dreamed about, Joanne!" we reply. "First of all, it is confusing to combine distances measured in one reference frame with time measured in another reference frame. The 99-light-year distance to Canopus is measured with respect to the Earth-linked frame, while the 20 years recorded on the outward traveler's clock refers to the rocket frame. No wonder the result appears to imply a rate of travel faster than light. Why not take what I paid for fuel for my car last week and divide it by the number of gallons you bought today for your car, to figure the cost of a gallon of fuel? A crazy, mixed-up, wrong way to work out cost - but no crazier than that way to figure speed!

Canopus much closer for astronaut

"But your question about time brings up a similar question about distance: distance between Earth and Canopus measured in the frame in which they are at rest does not agree with the distance between them measured from a rocket that moves along the line connecting them.[1]

"Any free-float frame is as good as any other for analyzing motion - that is the Principle of Relativity! So think of the entire outward trip in terms of rocket measurements. At the starting gun (or firecracker) Earth is rushing past the rocket at speed $99/101$. Twenty years later Canopus arrives at the rocket, Canopus also traveling at that speed, $99/101$ in that rocket frame. This means that for the rocket traveler the Earth-Canopus distance is only about 20 light-years. In fact it is just the fraction $(99/101)$ of 20 light-years, so that at speed $99/101$ this distance is covered in exactly 20 years."

Lorentz contraction

"Of course. We are dealing with **Lorentz contraction**," huffs Professor Bright, who thinks any objection to relativity is a waste of time. He has no head for politics, so does not appreciate how important it is for the public to accept the expenditures proposed for this project.

He continues, "Think of a very long stick lying with one end at Earth, the other end at Canopus. Each observer, with the help of colleagues, measures the position of the two ends of this stick *at the same time* in his or her frame. By this means the outward rocket traveler measures a shorter length of the stick-a smaller Earth-Canopus distance - than does an observer in the Earth-linked frame in which the stick lies at rest.

"The factor by which the stick appears contracted in the rocket frame is just the same as the ratio of rocket time to Earth time for the outward trip. This ratio is $(20$ years$)/(101$ years$)$. Hence the rocket observer measures the Earth - Canopus distance to be $(99$ light-years $)(20/101)=19.6$ light-years - just a bit less than 20 light-years, as you said.

"Everybody has a satisfactory picture: The astronaut can get to Canopus in 20 years of rocket time because the astronaut's measurements show Canopus to be slightly less than 20 light-years distant. We on Earth agree that the time lapse on the rocket clock is 20 years, but our 'explanation' rests on the invariance of the interval between the events of departure from Earth and arrival at Canopus." Professor Bright pounds the table: "Why are you giving this poor astronaut such a hard time, when relativity is so utterly simple?" He is surprised by the outburst of laughter from other board members and the audience in the room.

1 Canopus much closer for astronaut