6.E: Regions of Spacetime (Exercises)

PRACTICE

6.1 relations between events

This is a continuation of Sample Problem 6-1. Events 1,2 , and 3 all have the laboratory coordinates $y=z=0$. Their x- and t-coordinates are plotted on the laboratory spacetime diagram.

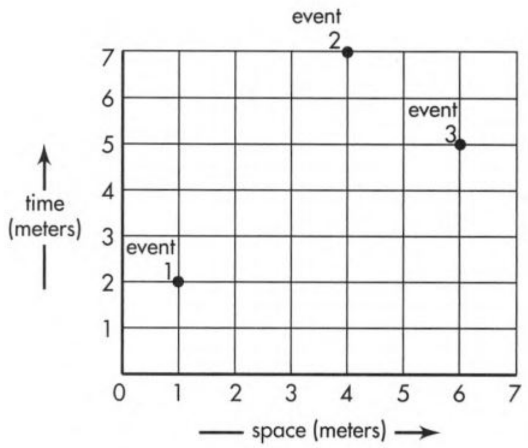


Figure $6.E.1$: Laboratory spacetime map.

a. Answer the following questions three times: once for the timelike pair of events 1 and 2 , once for the spacelike pair of events 1 and 3 , and once for the lightlike pair of events 2 and 3 .
(1) What is the proper time (or proper distance) between the two events?
(2) Is it possible that one of the events caused the other event?
(3) Is it possible to find a rocket frame in which the spatial order of the two events is reversed? That is, is it possible to find a rocket frame in which the event that occurs to the right of the other event in the laboratory frame will occur to the left of the other event in the rocket frame?
(4) Is it possible to find a rocket frame in which the temporal order of the two events is reversed? That is, is it possible to find a rocket frame in which the event that occurs before the other event in the laboratory frame occurs after the other event in the rocket frame?

b. For the timelike pair of events, find the speed and direction of a rocket frame with respect to which the two events occurred at the same place. For the spacelike pair of events, find the speed and direction of a rocket frame with respect to which the two events occurred at the same time.

6-2 timelike, lightlike, or spacelike?

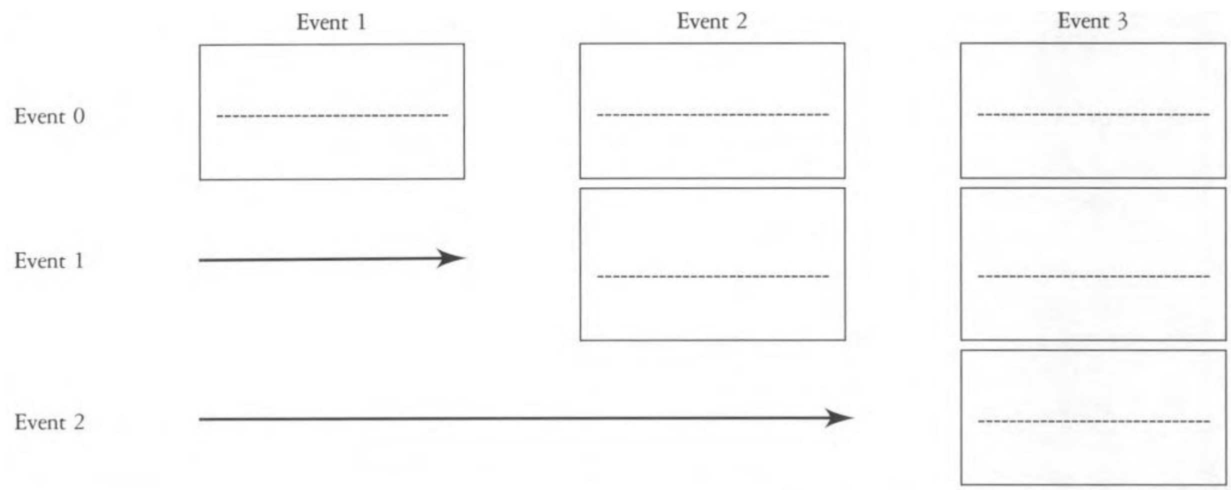
The first table lists the space and time coordinates of three events plus the reference event (event 0) as observed in the laboratory frame.

Laboratory Coordinates of Three Events

	t (years)	x (years)	y (years)
Event 0	0	0	0
Event 1	3	4	0
Event 2	6	5	0
Event 3	8	8	3

a. Copy the second table. In the top half of each box in the second table, write the nature of the interval - timelike, lightlike, or spacelike between the two corresponding events.

b. In the bottom half of each box in the second table, write "yes" if it is possible that one of the events caused the other and "no" if it is not possible. c Find the speed (with respect to the laboratory frame) of a rocket frame in which event 1 and event 2 in the first table occur at the same place.
c. Find the speed (with respect to the laboratory frame) of a rocket frame moving along the x-axis in which event 2 and event 3 in the first table occur at the same time.
d. Find the speed (with respect to the laboratory frame) of a rocket frame moving along the x-axis in which event 2 and event 3 in the first table occur at the same time.



Interval Between Events: Timelike, Lightlike, or Spacelike?

6-3 proper time and proper distance

Note: This exercise uses the Lorentz transformation equations.

a. Two events P and Q have a spacelike separation. Show in general that a rocket frame can be found in which the two events occur at the same time. Also show that in this rocket frame the distance between the two events is equal to the proper distance between them. (One method: assume that such a rocket frame exists and then use the Lorentz transformation equations to show that the relative velocity of this rocket frame is less than the speed of light, thus justifying the assumption made.)
b. Two events P and R have a timelike separation. Show in general that a rocket frame can be found in which the two events occur at the same place. Also show that in this rocket frame the time between the two events is equal to the proper time between them.

PROBLEMS

6-4 autobiography of a photon

A photon emitted by a star on one side of our galaxy is absorbed near a star on the other side of our galaxy, 100,000 light-years away from its point of origin as measured in the frame of the galaxy. How does the photon experience its own birth and death? That is to say, what are the space and time separations between the birth and death of the photon in the frame of the photon?

Discussion: We cannot answer this question, because we cannot move along with the photon. No matter how fast the unpowered rocket in which we ride, we still measure light to move past us with the speed of light! Still, we can try to answer the question as a limiting case in the galaxy frame. Think of extremely energetic PROTONS traveling the same path. As protons of greater and greater energy are emitted by the first star and are absorbed near the second star at the other side of the galaxy, what happens to the distance between these two events in the frame of the proton? What happens to the time between these events in the frame of the proton? Come in this way to a limiting case in which the PROTON is moving arbitrarily close to the speed of light in the galaxy frame. In this limit, what would you expect the distance and time to be between birth and death in the frame of a PHOTON traveling the same path in space?

a. You are the photon. Using the above argument, write the first few sentences of your autobiography.
At the end of the trip, near a star at the fringe of our galaxy, a galaxy-spanning photon travels 10 kilometers vertically through the atmosphere of a planet before it enters a telescope and is absorbed in the eye of an astronomer.

The average index of refraction of the atmosphere of this planet is $n=1.00030$. The speed of the photon in such an atmosphere is $v=v_{\text{conv}}/c= 1/n$. (The speed of light in a vacuum is unity.)

b. What is the proper time for this last leg of the trip-the time in the rest frame of the "slowed-down" photon? How far apart is the top of the atmosphere and the astronomer's eye in the frame of the photon?
c. Complete your photon autobiography with an additional couple of sentences.

Discussion: Relativity is a classical theory - that is, a nonquantum theory - in which photons are postulated to move at light speed in a vacuum and at a speed $v=1/n$ in air, where n is the index of refraction. **Quantum electrodynamics** (QED), the quantum theory of interactions between light and matter, tells us that it is incorrect to talk of a single photon moving through air. Rather, one thinks of an initial photon being absorbed by an atom in the air and a second photon emitted, the second photon then absorbed by another atom, which emits a third photon, and so forth. The classical relativistic analysis is not correct when viewed from the quantum perspective. For more on quantum electrodynamics, read Richard P. Feynman, QED: *The Strange Theory of Light and Matter* (Princeton, Princeton University Press, 1985)

6-5 the detonator paradox

A U-shaped structure made of the strongest steel contains a detonator switch connected by wire to one metric ton (1000 kilograms) of the explosive TNT, as shown in the figure. A T-shaped structure made of the same strong steel fits inside the U, with the long arm of the T not quite long enough to reach the detonator switch when both structures are at rest in the laboratory.

Now the T structure is removed far to the left and accelerated to high speed. It is Lorentz-contracted along its direction of motion. As a result, its long arm is not long enough to reach the detonator switch when the two collide. Therefore there will be no explosion.

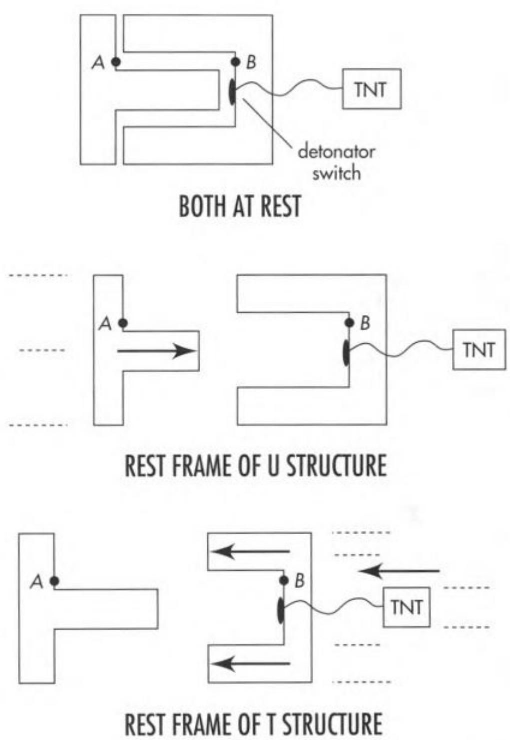


Figure $6.E.2$: **Both at rest:** The leg of the T almost reaches the detonator switch when both the T and the U are at rest. Points A and B are used in parl b of the exercise. **Rest frame of U structure:** The leg of the moving T is Lorentz contracted in the rest frame of the U. Does this mean that the explosion will not take place? **Rest frame of T structure:** The legs of the moving U are Lorentz-contracted in the rest frame of the T. Does this mean explosion will take place?

However, look at the same situation in the rest frame of the T structure. In this frame the arm of the T has its rest length, while the two arms of the U structure are Lorentz-contracted. Therefore the arm of the T will certainly strike the detonator switch and there will be a terrible explosion.

a. Make a decisive prediction: Will there be an explosion or not? Your life depends on it!

b. The wire from the detonator switch to the TNT is restrung through point B on the U structure when both structures are at rest, and a laser is installed at point A on the T structure. Later, when the two structures collide at A, the laser fires a pulse at B that cuts the detonator wire. Does this new apparatus change your prediction about detonation of the TNT?

Acknowledgment: A paper describing this paradox crossed the desk of one of the authors, but the paper and the name of its author have been lost. The laser inhibitor device was devised by Gordon Roesler.

6-6 how fast can you walk?

Webster's Eighth says that to "walk" means to "go on foot without lifting one foot clear of the ground before the other touches the ground." In other words, at least one foot must be on the ground at all times. Use this definition to discover the maximum speed of walking imposed by relativity.

We assume advanced technology here! A walking robot moves its free foot forward at nearly the speed of light. Then one might argue (ambiguously) as follows: While the free foot is moving forward, the planted foot is on the ground, ready to be picked up when [look out!] the free foot comes down in front. Half the time each foot is in motion at nearly light speed and half the time it is at rest. Therefore the average speed of each foot, equal to the maximum possible speed of the walking robot, is half the speed of light.

Why is this argument ambiguous? Because of the relativity of simultaneity. The word when applied to separated events should always unfurl a red flag. The event "front foot down" (label FrontDown) and the event "rear foot up" (label RearUp) occur at different places along the line of motion. Observers in relative motion will disagree about whether or not events FrontDown and RearUp occur at the same time. Therefore they will disagree about whether or not the robot has one foot on the ground at all times in order to satisfy the dictionary definition of walking.

How to remove the ambiguity in the definition of walking? One way is to make the conventional definition frame-independent: One foot must be on the ground at all times as observed in every free-float frame of reference. What limits does this place on the two events FrontDown and RearUp? The rear foot must leave the ground after, or at least simultaneous with, the front foot touching the ground, as observed by all free-float observers. Use the following outline to derive the consequences of this definition for the maximum speed of walking.

a. Consider the three possible relationships between events FrontDown and RearUp: timelike, lightlike, and spacelike. For each of these three relationships, write down answers to the following three questions:
(1) Will the temporal order of the two events be the same for all observers?
(2) Does this relationship adequately satisfy the frame-independent definition of walking?
(3) If so, does this relationship give the maximum possible speed for walking?
Show that you answer "yes" to all three questions only for a lightlike relationship between the two events.
b. A lightlike relationship between events FrontDown and RearUp means that light can just travel from one event to the other with no time left over. Let the distance between these events - the length of one step in the Earth frame-be the unit of distance and time. Show that for the limiting speed in this frame, each foot spends two units of time moving forward, then waits one unit while the light signal propagates to the other foot, then waits three units while the other foot goes through the same process. Summary: Out of six units of time, each foot moves forward at (nearly) the speed of light for two units. What is the average speed of each foot, and therefore the speed of the walker, as measured in the Earth frame?
c. Draw a spacetime diagram for the Earth frame, showing worldlines for each of the robot's feet and worldlines for the connecting light flashes. Add a worldline showing the averaged motion of the torso, always located halfway between the two feet in the Earth frame. Demonstrate that this torso moves at the speed of the walker reckoned above.
d. Paul Horwitz says, "We determined the value of a maximum walking speed by finding a frame independent definition of walking. Therefore this walking robot moves at the same speed as observed in every frame." Is Paul right?

Reference: George B. Rybicki, American Journal of Physics, Volume 59, pages 368-369 (April 1991).

6-7 the flickering bulb paradox: a project

Note: The following is too long for a regular exercise, but it has many insights worth pursuing as a longer activity. Therefore we call it a project.

Two long parallel conducting rails are open at one end but connected electrically at the other end through a lamp and battery, as shown in the figure (rail frame). One of the rails has a square vertical offset 2 meters long. Between the rails moves (without friction) an H-shaped slider, whose vertical legs are conductors but whose horizontal crosspiece is an insulator. (Assume that the vertical legs are not perfect conductors so that, with a sufficiently powerful battery, a voltage is maintained between the rails even when they are connected by the vertical legs of the slider.) If either vertical leg of the slider connects the two rails, the electrical circuit is completed, permitting the lamp to light.

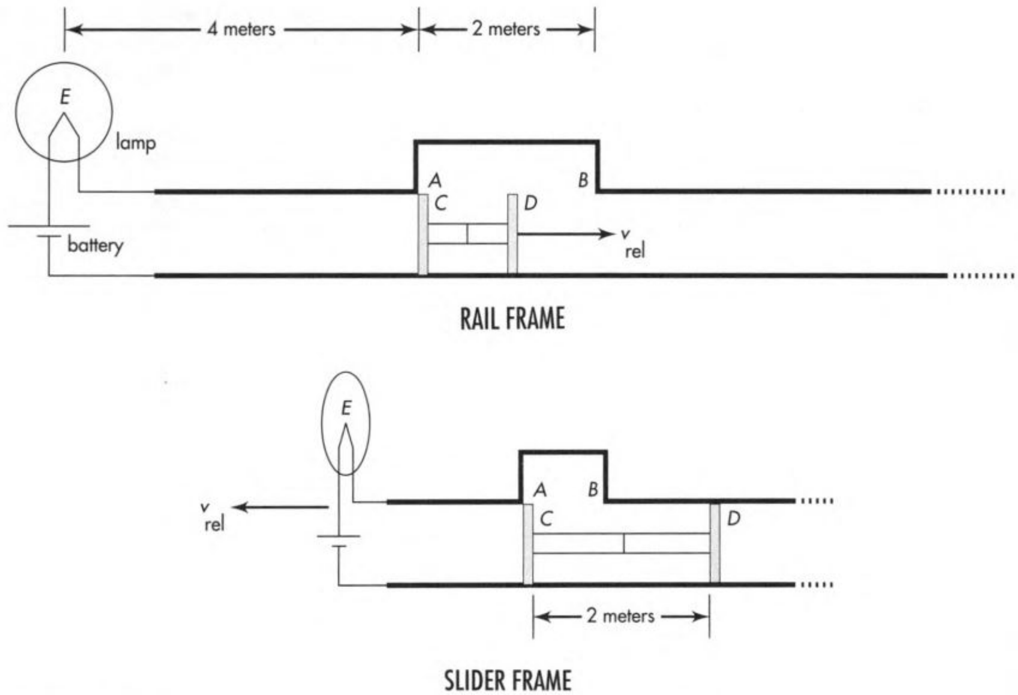


Figure $6.E.3$: **Rail frame:** Configuration at $\mathrm{t}=0$ in the rest frame of the rails. Slider CD moves to the right with speed $\mathrm{v}_{\mathrm{rel}}$ such that the Lorentz-contraction factor equals 2 . The vertical legs of the slider are conductors; the horizontal crosspiece is an insulator. **Slider frame:** Configuration at $\mathrm{t}'=0$ in the rest frame of the slider. The rails and lamp move to the left with speed $\mathrm{v}_{\mathrm{rel}}$ such that the Lorentz-contraction factor is 2.

The rest (proper) length of the slider is also 2 meters, but it moves at such a speed that its Lorentz contracted length is 1 meter in the rail frame. Hence in the rail frame there is a lapse of time during which neither leg of the slider is in contact with the upper rail. Since the circuit is open during this period, the bulb should switch off for a time and then on again - it should flicker.

The figure (slider frame) shows the configuration at $t'=0$ in the slider frame. In this frame the slider is at rest, its length is equal to its rest length, 2 meters, while the rails, the lamp, and the battery all move to the left with a speed such that their lengths along the direction of motion are reduced by a factor of 2 . In particular the offset in the upper rail is Lorentz contracted to a length of one meter. Therefore, in the slider frame, one or the other of the slider conductors always spans the rails, so the circuit is never broken and the bulb should never switch off-it should NOT flicker!

Those trying to disprove relativity shout, "Paradox! In the rest frame of the rails the lamp switches off and then on again - it flickers. In contrast, in the rest frame of the slider the lamp stays on - it does not flicker. Yet all observers must agree: The lamp either flickers or it does not flicker. Relativity must be wrong!"

Analyze the system in sufficient detail either to demonstrate conclusively the correctness of this objection or to pinpoint its error.

Reference: G. P. Sastry, *American Journal of Physics*, Volume 55, pages 943-946 (October 1987).

6-8 the contracting spaceship paradox: a project

Note: The following is too long for a regular exercise, but it has many insights worth pursuing as a longer activity. Therefore we call it a project.

Kerwin Warnick writes in with the following paradox. A spaceship of proper length L_o accelerates from rest. Its front end travels a distance x_F in time t_P to a final speed at which the ship is contracted to half its rest length. In the same time t_F the rear end moves the same distance x_F as the front end plus the distance $L_o/2$ by which the ship has contracted. Distance traveled by the rear end $x_F+(L_o/2)$ in time t_F means an average speed $[x_F+(L_o/2)]/t_F$. Since the proper length L_o can be arbitrarily large, this average speed can be arbitrarily great, even greater than the speed of light. "This disproves relativity!" he exclaims.

Analyze this thought experiment in sufficient detail either to demonstrate conclusively the correctness of Warnick's objection or to pinpoint its error.

Reference: Edwin F. Taylor and A. P. French, *American Journal of Physics*, Volume 51, pages 889-893 (October 1983).