11.3: First Steps

invariance off the interval gets us started

Recall that the coordinates y and z transverse to the direction of relative motion between rocket and laboratory have the same values in both frames (Section 3.6):

$$\begin{aligned} y &= y' \\ z &= z' \end{aligned} \tag{11.3.1}$$

where primes denote rocket coordinates. A second step makes use of the difference in observed clock rates when the clock is at rest or in motion (Section 1.3 and Box 3-3). Think of a sparkplug at rest at the origin of a rocket frame that moves with speed v_{rel} relative to the laboratory. The sparkplug emits a spark at time t' as measured in the rocket frame. The sparkplug is at the rocket origin, so the spark occurs at $x' = 0$.

Derive difference in clock rates

Where and when $(x \text{ and } t)$ does this spark occur in the laboratory? That depends on how fast, v_{rel}, the rocket moves with respect to the laboratory. The spark must occur at the location of the sparkplug, whose position in the laboratory frame is given by

$$x = v_{\text{rel}} t$$

Now the invariance of the interval gives us a relation between t and t',

$$\left(t'\right)^2 - \left(x'\right)^2 = \left(t'\right)^2 - (0)^2 = t^2 - x^2 = t^2 - \left(v_{\text{rel}} t\right)^2 = t^2\left(1 - v_{\text{rel}}^2\right)$$

from which

$$t' = t\left(1 - v_{\text{re}}^2\right)^{1/2}$$

or

$$t = \frac{t'}{\left(1 - v_{\text{rel}}^2\right)^{1/2}} \qquad [\text{when x}' = 0\,] \tag{11.3.2}$$

The awkward expression $1/\left(1 - v_{\text{rel}}^2\right)^{1/2}$ occurs often in what follows. For simplicity, this expression is given the symbol Greek lower-case gamma: γ.

$$\gamma \equiv \frac{1}{\left(1 - v_{\text{re}}^2\right)^{1/2}}$$

Time stretch factor defined

Because it gives the ratio of observed clock rates, γ is sometimes called the time stretch factor (Section 5.8). Strictly speaking, we should use the symbol γ_{rel}, since the value of γ is determined by v_{rel}. For simplicity, however, we omit the subscript in the hope that this will cause no confusion. With this substitution, equation $11.3.2$ becomes

$$t = \gamma t' \qquad [\text{when x}' = 0\,] \tag{11.3.3}$$

Substitute this into the equation $x = v_{\text{rel}}\, t$ above to find laboratory position in terms of rocket measurements:

$$t = \gamma t' \qquad [\text{when x}' = 0\,] \tag{11.3.4}$$

Equations $11.3.1$, $11.3.3$ and $11.3.5$ give the first answer to the question, "If we know the space and time coordinates of an event in one free-float frame, what are its space and time coordinates in some other overlapping free-float frame?" These equations are limited, however, since they apply only to a particular situation: one in which both events occur at the same place $(x' = 0)$ in the rocket platform.