

7.6: Conservation of Momenergy and Its Consequences

total momenergy of an isolated system of particles is conserved

Momenergy puts us at the heart of mechanics. The relativity concept momenergy gives us the indispensable tool for mastering every interaction and transformation of particles.

What does it mean in practice to say in this language of momenergy components that the punch given to particle *A* by particle *B* in a collision is exactly equal in magnitude and opposite in spacetime direction to the punch given to *B* by *A*? That gain in momenergy of *A* is identical to loss of momenergy by *B*? That the sum of separate momenergies of *A* and *B*-this sum itself regarded as an arrow in spacetime, the arrow of total momenergy (Figure 7-6) - has the same magnitude and direction after the encounter that it had before?

Momenergy of a system of particles

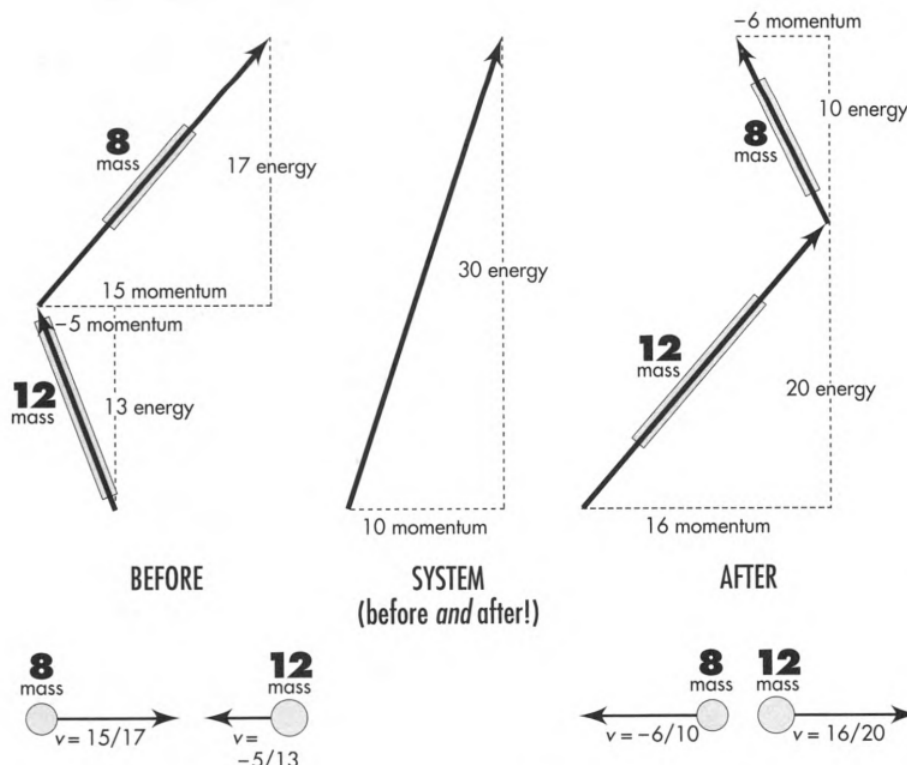


Figure 7.6.1: Conservation of total momenergy in a collision. Before: The lighter 8-unit mass, moving right with 15/17 light speed, collides with the slower and heavier 12-unit mass moving left (with 5 units of momentum to the left and 13 units of energy). System: Arrow of total momenergy of the system of two particles. Combined momentum of the colliding particles has value $-5 + 15 = 10$ units rightward. Combined energy of the two equals $13 + 17 = 30$ units. The total system momenergy is conserved. After: One of many possible outcomes of this collision: The 8-unit mass bounces back leftward after collision, but the punch that it provided has reversed the direction of motion and increased the speed of the heavier 12-unit mass. The handle of the momenergy arrow of each particle gives the true magnitude of that momenergy, figured in the Lorentz geometry of the real physical world, as contrasted to the length of that 4-vector as it appears in the Euclidean-and therefore misleading-geometry of this sheet of paper. The scale of magnitudes in this figure is different from that of Figure 7-3.

Or, in brief, how does the principle of conservation of momenergy translate itself into the language of components in a given free-float frame? Answer: Each component of the momenergy vector, when added together for particles *A* and *B*, has the same value after the collision as before the collision. In other words,

Energy of system conserved

$$\begin{aligned}
 \begin{pmatrix} \text{energy of } A \\ \text{before the} \\ \text{encounter} \end{pmatrix} + \begin{pmatrix} \text{energy of } B \\ \text{before the} \\ \text{encounter} \end{pmatrix} &= \begin{pmatrix} \text{total energy} \\ \text{before the} \\ \text{encounter} \end{pmatrix} \\
 &= \begin{pmatrix} \text{total energy} \\ \text{after the} \\ \text{encounter} \end{pmatrix} \\
 &= \begin{pmatrix} \text{energy of } A \\ \text{after the} \\ \text{encounter} \end{pmatrix} + \begin{pmatrix} \text{energy of } B \\ \text{after the} \\ \text{encounter} \end{pmatrix}
 \end{aligned}$$

called conservation of the time part of momenergy.

Momentum of system conserved

Add to this three statements about the three space components of momenergy, of which the first one reads,

$$\begin{aligned}
 \begin{pmatrix} \text{eastward component} \\ \text{of momentum of } A \\ \text{before the encounter} \end{pmatrix} + \begin{pmatrix} \text{eastward component} \\ \text{of momentum of } B \\ \text{before the encounter} \end{pmatrix} \\
 &= \begin{pmatrix} \text{eastward component} \\ \text{of total momentum} \\ \text{before the encounter} \end{pmatrix} \\
 &= \begin{pmatrix} \text{eastward component} \\ \text{of total momentum} \\ \text{after the encounter} \end{pmatrix} \\
 &= \begin{pmatrix} \text{eastward component} \\ \text{of momentum of } A \\ \text{after the encounter} \end{pmatrix} + \begin{pmatrix} \text{eastward component} \\ \text{of momentum of } B \\ \text{after the encounter} \end{pmatrix}
 \end{aligned}$$

called conservation of the space part of momenergy. Figure 7.6.1 illustrates the conservation of momenergy in a recoil collision between two particles. Momentum is laid out.

Box 7-3

INVARIANT? CONSERVED? CONSTANT?

Is the speed of light a constant? An invariant? Is mass conserved in a collision? Is it an invariant? A constant? Many terms from everyday speech are taken over by science and applied to circumstances far beyond the everyday. The three useful adjectives invariant, conserved, and constant have distinct meanings in relativity.

Invariant

In relativity a quantity is invariant if it has the same value when measured by observers in different free-float frames-frames in relative motion. First among relativistic invariants is the speed of light: It has the same value when reckoned using data from the laboratory latticework of recording clocks as when figured using data from the rocket latticework. A second central invariant is the interval between two events: All inertial observers agree on the interval (proper time or proper distance). A third mighty invariant is the mass of a particle. There are many other invariants, every one with its special usefulness.

Some very important quantities do not qualify as invariants. The time between two events is not an invariant. It differs as measured by observers in relative motion. Neither is the distance between events an invariant. It too differs from one frame to another. Neither the energy nor the momentum of a particle is an invariant.

Conserved

A quantity is conserved if it has the same value before and after some encounter or does not change during some interaction. The total momenergy of an isolated system of particles is conserved in an interaction among the particles. In a given free-float frame this means that the total energy is conserved. So is each component of total momentum. The magnitude of total

momenergy of a system - the mass of that system - is also conserved in an interaction. On the other hand, the sum of the individual masses of the Momentum of system conserved constituent particles of a system ordinarily is not conserved in a relativistic interaction. (For examples, see Chapter 8.)

Constant

Something that is constant does not change with time. The speed of the Great Pyramid with respect to the rock plateau of Giza is constant - equal to zero, or at least less than one millimeter per millennium. This speed may be constant, but it is not an invariant: As observed from a passing rocket, the Great Pyramid moves with blinding speed! Is the speed of the Great Pyramid conserved? Conserved during what encounter? There is no before or after to which the term "conserved" can refer. The term "conserved" simply does not apply to the speed of the Great Pyramid.

It is true that the speed of light in a vacuum is constant - it does not change with time. It is also true, but an entirely different statement, that the speed of light is an invariant - has the same value measured by different observers in uniform relative motion. It is true that total momenergy of an isolated system is constant - does not change with time. It is also true, but an entirely different statement, that total momenergy of an isolated system is conserved in a collision or interaction among particles in that system.

When anyone hears the word invariant, conserved, or constant, she is well-advised to listen for the added phrase with respect to, which should always be expressed or implied. Usually (but not always) constant means with respect to the passage of time. Conserved usually (but not always) means with respect to a collision or interaction. Invariant can have at least as many meanings as there are geometries to describe Nature: In Euclidean geometry, distance is invariant as measured with respect to relatively rotated coordinate axes. In Lorentz geometry, interval and mass are invariants as measured with respect to free-float frames in relative motion. The full meaning of the word invariant or conserved or constant depends on the condition under which this property is invoked.

In Figure 7.6.1:right and left on the page; energy is marked off vertically. The left diagram shows two particles before collision and their momentum-energy vectors. The right diagram shows the corresponding display after the collision.

The center diagram shows total momenergy of the system of two particles. The momenergy vectors of the two particles before the collision add up to this total; the momenergy vectors of the two particles after the collision add up to the same total. Total momenergy of the system has the same value after as before: it is conserved in the collision.

✓ Question and Answer

Well, you've done it again: You've given us a powerful tool that seems impossible to visualize. How can one think about this momenergy 4-vector, anyway? Can you personally picture it in your mind's eye?

Answer

We can almost visualize the momenergy arrow, by looking at Figure 7-6 for example. There momentum and energy components of a given momenergy vector have their correct relative values. And the direction of the momenergy arrow in spacetime is correctly represented in the diagram.

However, the magnitude of this arrow - mass of the particle - does not correspond to its length in the momenergy diagram. This is because mass is reckoned from the difference of squares of energy and momentum, whereas length of a line on the Euclidean page of a book is computed from the sum of squares of horizontal and vertical dimensions. The handle or thickened region on the typical arrow and the big, boldface number for mass remind us of the failure - the lie-that results from trying to represent momenergy on such a page.

To observe a given momenergy 4-vector first from one free-float frame, then from another, and then from another (Figure 7-3) is to see the apparent direction of the arrow changing. The change in frame brings with it changes in the energy and momentum components. However, magnitude does not change. Mass does not change. To examine the momenergy 4-vector of a particle in different frames is to gain improved perspective on what momenergy is and does.

See if this analogy helps: The momentum-energy 4-vector is like a tree. The tree has a location for its base and for its tip whether or not we choose this, that, or the other way to measure it. The shadow the tree casts on the ground, however, depends upon the tilt of the tree and the location of Sun in the sky.

Likewise, momenergy of a particle as it passes through a given event on its worldline has a magnitude and direction, a fixity in spacetime, independent of any choice we make of free-float frame from which to observe and measure it. No means of reporting momenergy is more convenient for everyday purposes than separate specification of momentum and energy of the object in question in some chosen free-float frame. Those two quantities separately, however, are like the shadow of the tree on the ground. As Sun rises the shadow shortens. Similarly the momentum of a car or spaceship depends on the frame in which we see it. In one frame, terrifying. In another frame, tame. In a comoving frame, zero momentum, as the tree's shadow disappears when Sun lies in exactly that part of the sky to which the tilted tree points. In such a special frame of reference, the time component of an object's momenergy - that is, its energy - takes on its minimum possible value, which is equal to the mass itself of that object. However, in whatever free-float frame we observe it, the arrow of momenergy clings to the same course in spacetime, maintains the same length, manifests the same mas

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