4.6: Twin Paradox

> *a kink in the path explains the difference*

"Your whole plan depends on relativity," stresses James Fastlane, "but relativity is a swindle. You can see for yourself that it is self-contradictory. It says that the laws of physics are identical in all free-float frames. Very well, here's your rocket frame and here's Earth frame. You tell me that identical clocks, started near Earth at identical times, each in one of these free-float frames, will read very different time lapses. You go away and return only 40 years older, while we and our descendants age 202 years. But if there's any justice, if relativity makes any sense at all, it should be equally possible to regard *you* as the stay-at-home.[1] Relative to you, *we* speed away in the opposite direction and return. Hence we should be younger than you when we meet again. In contrast, you say you will be younger than we are. This is a flat contradiction Nothing could show more conclusively that neither result can be right. Aging is aging It is impossible to live long enough to cover a distance of 99 light-years twice - going and coming. Forget the whole idea.

Which twin travels?

"Jim," we reply, "your description is the basis for the famous Twin Paradox, in which one twin stays on Earth while the other takes the kind of round trip we have been describing. Which twin is older when they come together again? I would like to leave this question for a minute and consider a similar trip across the United States.

"We all know, Jim, that every July you drive straight north on Interstate Highway 35 from Laredo, Texas, on the Mexican border, to Duluth, Minnesota, near the Canadian border. Your tires roll along a length of roadway equal to 2000 kilometers and the odometer on your car shows it.

Curved path in space is a longer path

"I too drive from Laredo to Duluth, but last year I had to make a stop in Cincinnati Ohio, on the way. I drove northeast as straight as I could from Laredo to Cincinnati 1400 kilometers, and northwest as straight as I could from Cincinnati to Duluth another 1400 kilometers. Altogether, my tires rolled out 2800 kilometers. When we left Laredo you could have said that my route was deviating from yours, and I could have said with equal justice that yours was deviating from mine. The great difference between our travels is this, that my course has a sharp turn in it. That's why my kilometerage is greater than yours in the ratio of 2800 to 2000."[2]

Fastlane interrupts: "Are you telling me that the turn in the rocket trajectory at Canopus explains the *smaller* aging of the rocket traveler? The turn in your trip to Duluth made your travel distance longer, not shorter." Which twin travels? Curved path in space is a *longer* path Astronaut who turns around ages less ...

Astronaut who turns around ages less . . .

"That is the difference between path length in Euclidean space geometry and wristwatch time in Lorentz spacetime geometry," we reply. "In Euclidean geometry the *shortest path length* between two points is achieved by the traveler who does not change direction. All indirect paths are longer than this minimum. In spacetime the *greatest aging* between two events is experienced by the traveler who does not change direction. For all travelers who change direction, the total proper time, the total wristwatch time, the total aging is *less* than this maximum.[3]

"The distinction between distance in Euclidean geometry and aging in spacetime comes directly from the contrast between *plus* sign in the expression for distance between two locations and *minus* sign in the expression for interval between two events. In going to Duluth by way of Cincinnati I use the *plus* sign:

$$\begin{pmatrix}\text{distance:}\\ \text{Laredo to}\\ \text{Cincinnati}\end{pmatrix}^2 = \begin{pmatrix}\text{northward}\\ \text{separation:}\\ \text{Laredo to}\\ \text{Cincinnati}\end{pmatrix}^2 + \begin{pmatrix}\text{eastward}\\ \text{separation:}\\ \text{Laredo to}\\ \text{Cincinnati}\end{pmatrix}^2$$

"Contrast this with motion in spacetime. In analyzing my trip to Canopus, I use the minus sign:[4]

$$\begin{pmatrix}\text{Proper time:}\\ \text{Earth to}\\ \text{Canopus}\end{pmatrix}^2 = \begin{pmatrix}\text{rocket time:}\\ \text{Earth to}\\ \text{Canopus}\end{pmatrix}^2 = \begin{pmatrix}\text{Earth time:}\\ \text{Earth to}\\ \text{Canopus}\end{pmatrix}^2 - \begin{pmatrix}\text{Earth distance:}\\ \text{Earth to}\\ \text{Canopus}\end{pmatrix}^2$$

"The contrast between a plus sign and a minus sign: This is the distinction between distance covered during travel in space and time elapsed - aging - during travel in spacetime."
