6.3: Light Cone- Partition in Spacetime

invariance of the interval preserves cause and effect

Thus far in dealing with the interval between two events, A and B, we have considered primarily the situation in which these events lie along a single direction in space-on the reference line where the laboratory and rocket reference clocks are located. In contrast, the surveyors in our imaginary kingdom made use of two space dimensions - northward and eastward. We know, however, that Euclidean space is truly three dimensional. A surveyor measuring hilly terrain soon appreciates the need for a third dimension: the direction vertically upward! The measure of distance in three dimensions requires a simple extension of the expression for distance in two dimensions: The square of the distance becomes the sum of the squares of three mutually perpendicular separations:[1]

Interval generalized to three space dimensions

$$(\text { distance })^2=(\text { north separation })^2+(\text { east separation })^2+(\text { up separation })^2$$

Euclidean space requires three dimensions. In contrast, spacetime, which includes the time dimension, demands four. The expression for the square of a timelike interval now has four terms: a positive term (the square of the time separation) and three negative terms (the squares of the separations in three space dimensions).

$$(\text { interval })^2=(\text { time separation })^2-(\text { north separation })^2-(\text { east separation })^2-(\text { up separation })^2$$

The three space terms can be represented by the single distance term in the equation above, yielding

\[(\text { timelike interval })^2=(\text { time separation })^2-(\text { distance })^2 \\ (\text { spacelike interval })^2=(\text { distance })^2-(\text { time separation })^2 \\(\text { lightlike interval })^2=0=(\text { time separation })^2-(\text { distance })^2 \nonumber \]

or, for the lightlike interval,

$$\text{magnitude of (separation in time)} = \text{(distance in space)} \quad \text{[lightlike interval]} \quad \text{(6-3)} \tag{6.3.1}$$

For pairs of events with lightlike separation, the interval equals zero. The zero interval is a unique feature of Lorentz geometry, new and quite different from Interval generalized to three space dimensions anything in Euclidean geometry. In Euclidean geometry it is never possible for distance AG between two points to be zero unless all three of the separations (northward, eastward, and upward) equal zero. In contrast, interval AG between two events can vanish even when separation in space and separation in time are individually quite large. Equation (6-3) describes the separation between lightlike events, but now separation in space may show up in two or three space dimensions as well as one time dimension. The distance in space is always positive.

Example 6.3.1 EXPLETIVE DELETED

At 12:00 noon Greenwich Mean Time (GMT) an circuit (event D) temporarily disables the receiving astronaut on Moon drops a wrench on his toe and amplifier at Mission Control on Earth. Take Earth shouts "Damn!" into his helmet microphone and Moon to be 3.84×10^8 meters apart in the (event A), carried by a radio signal toward Earth. Earth frame and assume zero relative motion.

At one second after 12:00 noon GMT a short

a. Does Mission Control on Earth hear the astronaut’s expletive?

b. Could the astronaut’s strong language have caused the short circuit on Earth?

c. Classify the spacetime separation between events A and D : timelike, spacelike, or lightlike.

d. Find the proper distance or proper time between events A and D.

e. For all possible rocket frames passing between Earth and Moon, find the shortest possible distance between events A and D. In the rocket frame for which this distance is shortest, determine the time between the two events.

Solution

a. In one second, electromagnetic radiation (light and radio waves) travels $3.0 X 10^8$ meters in a vacuum. Therefore the radio signal does not have time to travel the 3.84×10^8 meters between Moon and Earth in the one second available between the events A and D as measured in the Earth frame. So Mission Control does not hear the exclamation.

b. No signal travels faster than light. So the astronaut's strong language cannot have caused the short circuit.

c. The space part of the separation between events $\left(3.84 \times 10^{8}\right.$ meters $)$ dominates the time part (one second $=3.0 \times 10^{8}$ meters). Therefore the separation is spacelike.

d. The square of the proper distance s comes from the expression

$$\begin{aligned} s^{2} & =(\text { space separation })^{2}-(\text { time separation })^{2} \\ & =\left(3.84 \times 10^{8} \text { meters }\right)^{2}-\left(3.00 \times 10^{8} \text { meters }\right)^{2} \\ & =(14.75-9.00) \times 10^{16}(\text { meters })^{2} \\ & =5.75 \times 10^{16} \text { (meters) }^{2} \end{aligned}$$

The proper distance equals the square root of this value: $s=2.40 \times 10^{8}$ meters

e. The proper distance equals the shortest distance between two spacelike events as measured in any rocket frame moving between them (Figure 6-2, laboratory map). Hence 2.40×10^{8} meters equals the shortest possible distance between events A and D. In the particular rocket frame for which the distance is shortest, the time between the two events has the value zero-events A and D are simultaneous in this frame.

Example 6.3.2 SUNSPOT

Bradley grabs his sister's wand and waves it, be eruption of the sunspot at the surface of Sun shouting "Sunspot"' At that very instant his faitself. The Earth-Sun distance equals approxither, Lloyd, who is operating a home solar obsermately 1.5×10^{11} meters. Neglect relative motion vatory, sees a spot appear on the face of Sun. Let between Earth and Sun.

event E be Bradley waving the wand and event A

a. Is it possible that Bradley's wand waving caused the sunspot to erupt on Sun?

b. Is it possible that the sunspot erupting on Sun caused Bradley to wave his wand?

c. Classify the spacetime separation between events A and E : timelike, spacelike, or lightlike.

d. Find the value of proper distance or proper time between events A and E.

e. For all possible rocket frames passing between Earth and Sun, find the shortest possible distance or the shortest possible time between events A and E.

Solution

a. Light travels 1 meter of distance in 1 meter of time-or 1.5×10^{11} meters of distance in 1.5×10^{11} meters of time. Hence in the Earth-Sun frame, eruption of the sunspot (event A) occurred 1.5×10^{11} meters of time before Bradley waved the wand (event E). So Bradley's wand waving could not have caused the eruption on Sun.

b. On the other hand, it is possible that eruption of the sunspot caused Bradley to wave his wand: He raises the wand in the air, looks over his father's shoulder, and waves the wand as the spot appears on the projection screen. (We neglect his reaction time.)

c. Events A and E are connected by one light pulse; their space and time separations both have the value 1.5×10^{11} meters in the Earth frame. Therefore the spacetime separation between them is lightlike.

d. Space and time separations between events A and E are equal. Therefore the interval between them has value zero. Hence proper time between them - equal to proper distance between them - also has value zero.

e. The interval is invariant. Therefore all possible free-float rocket frames passing between Earth and Sun reckon zero interval between events A and E. This means each of them measures space separation between events A and E equal to the time separation between these events. The common value of the space and time separations are not the same for all rocket frames, but they are equal to one another in every individual rocket frame. We are asked to find the shortest possible value for this time.

Think of a rocket just passing Sun as the sunspot erupts, the rocket headed toward Earth at nearly light speed with respect to Earth. Rocket lattice clocks record the light flash from the sunspot moving away from the rocket at standard speed unity. However, these clocks record that Earth lies very close to Sun (Lorentz contraction of distance) and that Earth rushes toward the rocket at nearly light speed. Therefore light does not travel far to get to Earth in this rocket frame; neither does it take much time. For a rocket moving arbitrarily close to light speed, this distance between A and E approaches zero, and so does the time between A and E. Hence the shortest possible distance between A and E— equal to the shortest possible time between A and E — has the value zero. But this constitutes a limiting case, since rocket speed may approach but cannot equal the speed of light in any free-float frame.

Light flash traces out light cone in spacetime diagram

It is interesting to plot on an appropriate map locations of all events, $G, G_1, G_2, G_3, \ldots$, that can be connected with one given event A by a single spreading pulse of light. Every such future event has a distance in space from A identical to its delay in time after A. Only so can it satisfy the requirement $(6.3.1)$ for a null interval. For it:

$$\text{((future time with respect to } A) = +(\text{ distance in space from } A) \text{) [lightlike interval]} \tag{6.3.2}$$

It is equally interesting to display - and on the same diagram - all the events $\mathrm{H}, \mathrm{H}_1, \mathrm{H}_2, \mathrm{H}_3, \ldots$. that can send a light pulse to A. Every such event fulfills the condition

$$\text{((past time relative to } A) = -(\text{ distance in space from } A) \text{) [for lightlike interval]} \tag{6.3.3}$$

Both of these equations satisfy the magnitude equation $(6-3)$.

In Figure 6-4 we suppress display of a third space dimension in the interest of simplicity. We limit attention to future events $G, G_1, G_2, \ldots$ and past events $H, H_1, H_2, \ldots$ that lie on a north - south/east-west plane in space. A flash emitted from event A expands as a circle on this space plane. As it spreads out from event A, this circle of light traces out a cone opening upward in the spacetime map of Figure $6-4$. This is called the future light cone of event A. The cone opening downward traces the history of an in-coming circular pulse of radiation so perfectly focused that it converges toward event A, collapsing exactly at event A at time zero. This downwardopening cone has the name past light cone of event A. All the events $G, G_1, G_2, \ldots$. lie on the future light cone of event A, all events $H, H_1, H_2, \ldots$ on its past light cone.

Numerous as the events may be that lie on the light cone, typically there are many more that don't! Look, for example, at all the events that occur 7 meters of time later than the zero time of event A. On the spacetime map, these events define a plane 7 meters above the $t=0$ plane in which event A lies, and parallel to that plane. The light cone intersects this plane in a circle (circle in the present map; a sphere in a full spacetime map with three space dimensions). An event on the plane falls into one or another of three categories, relative to event A, according as it lies inside the circle (as does B in Figure 6-4), on it (as does G), or outside it (as does D).

The light cone is unique to Lorentz geometry. It gives nature a structure beyond any power of Euclidean geometry. The light cone does more than divide events on a single plane into categories. It classifies every event, everywhere in spacetime, into one or another of five distinct categories according to the causal relation that event bears to the chosen event, A :

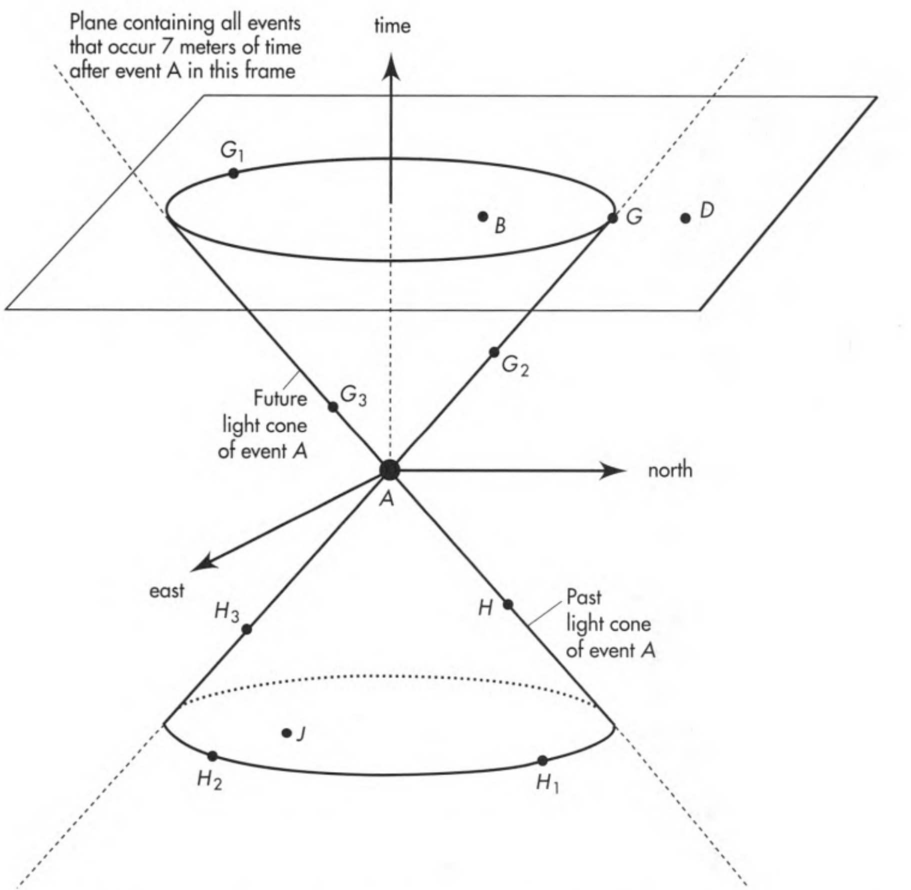

Figure 6.3.1: Light cone as partition in spacetime; perspective three-dimensional spacetime map showing eastward, northward, and time locations of events occurring on a flat plane in space. Events $\mathrm{G}, \mathrm{G}_1, \mathrm{G}_2$, and G_3 are on the future light cone of event A; events $\mathrm{H}, \mathrm{H}_1, \mathrm{H}_2$, and H_3 are on its past light cone. See also Figure 6.3.2

1. Can a material particle emitted at A affect what is going to happen at B ? If so, B lies inside the future light cone of A and forms a timelike pair with event A.

Cause and effect preserved by light cone

2. Can a light ray emitted at A affect - with no time to spare - what is going to happen at G ?

Cause and effect preserved by

If so, G lies on the future light cone of A and forms a lightlike pair with event

light cone A.

4. Can no effect whatever produced at A affect what happens at D ?

If so, D lies outside the future and past light cones of A and forms a spacelike pair with event A. It lies in the absolute elsewhere of A.

5. Can a material particle emitted at J affect what is happening at A ?

If so, J lies inside the past light cone of A and forms a timelike pair with event A.

6. Can a light ray emitted at H affect - with no time to spare-what is happening at A ?

If so, H lies on the past light cone of A and forms a lightlike pair with event A.

Nature reveals a cause-and-effect structure beyond the vision of Euclidean geometry. The causal relation between an event B and another event A falls into one or the other of five categories picked out by the light cone of A. That light cone and those categories have an existence in spacetime quite apart from any space and time measurements that may be used to describe them. Zero interval between events in one free-float frame means zero interval between the same events in every overlapping free-float frame. The light cone is the light cone is the light cone!

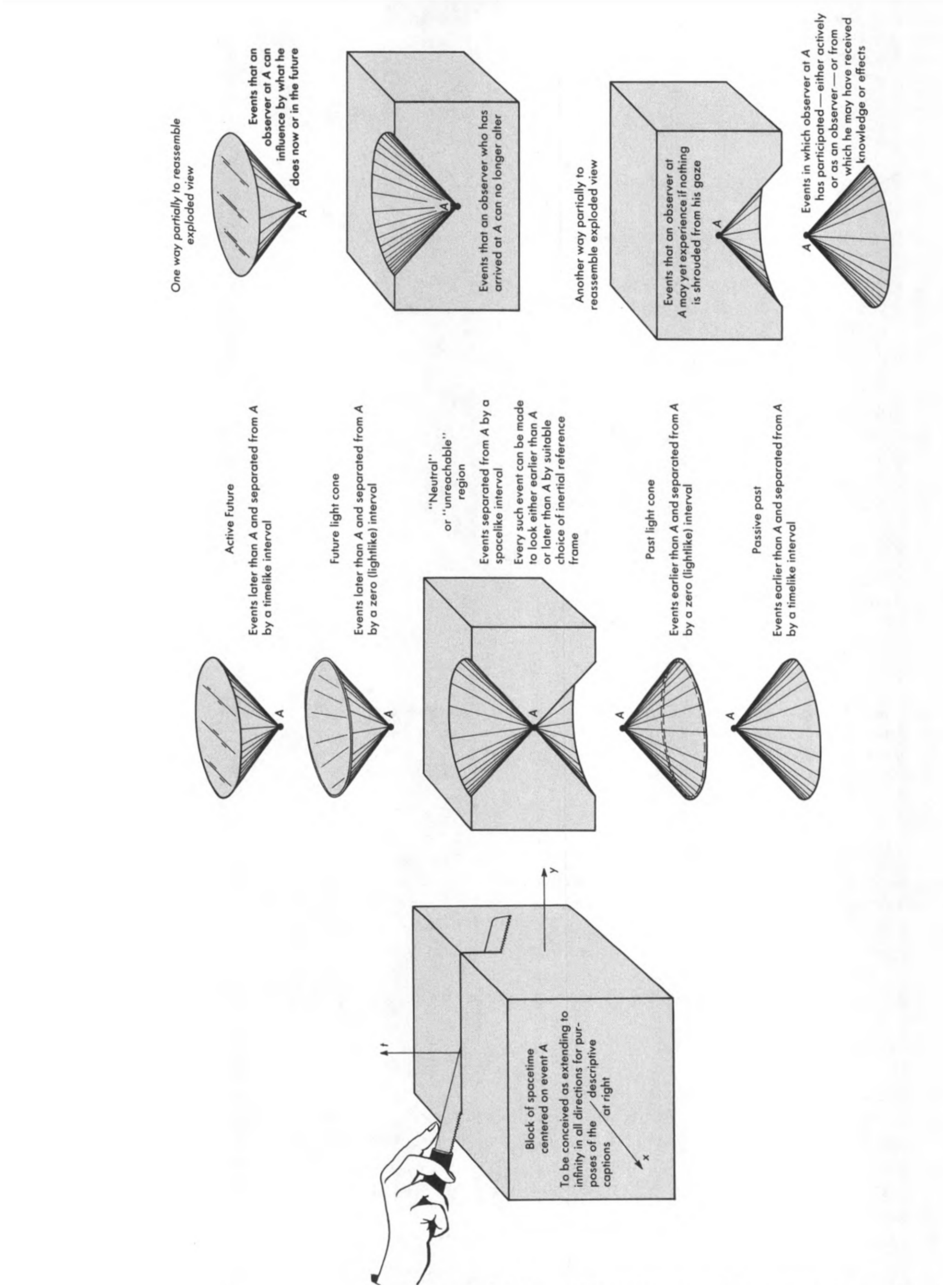

Figure 6.3.1: Exploded view of the regions into which the events of spacetime fall apart when classified with respect to a selected event A. other of five categories picked out by the light cone of A. That light cone and those categories have an existence in spacetime quite apart from any space and time measurements that may be used to describe them. Zero interval between events in

one free-float frame means zero interval between the same events in every overlapping free-float frame. The light cone is the light cone is the light cone!

Question and Answer

Event A appears at the origin of every spacetime map in this chapter. What's so special about event A ?

Answer

Nothing whatever is special about event A ! ' On the contrary, we have not captured the full story of the causal structure of spacetime until for every event $A\left(A_1, A_2, A_3,, \therefore\right.$ We have classified every other event $B\left(B_1, B_2, B_3, \ldots\right)$ into the appropriate category - timelike! lightlike! spacelike! - with respect to that event.

Figure $6.3.2$ summarizes the relations between a selected event A and all other events of spacetime.

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1 Interval generalized to three space dimensions

2 Light flash traces out light cone in spacetime diagram

3 Cause and effect preserved by light cone
