9.4: Spacetime Curvature

not one but two particles witness to gravitation

Splendid! And also simple! But isn't Einstein's view of motion too simple? We started out interested in the motion of a spaceship around Earth and in "gravitation." We seem to have ended up talking only about the motion of the satellite - or the proof mass - relative to a strictly local inertial reference frame, a trivially simple straightline motion. Where is there any evidence of "gravitation" to be seen in that? Nowhere.

This is the great lesson of Einstein: Spacetime is always and everywhere locally Lorentzian. No evidence of gravitation whatsoever is to be seen by following the motion of a single particle in a free-float frame.

One has to observe the relative acceleration of two particles slightly separated from each other to have any proper measure of a gravitational effect. Separated by how much? That depends on the region of spacetime and the sensitivity of the measuring equipment. Two ball bearings with a horizontal separation of 20 meters, dropped from a height of 315 meters above Earth's surface with 0 initial relative velocity, hit the ground 8 seconds later (24×10^{8} meters of light-travel time later) with a separation that has been reduced by 10^{-3} meter (Section 2.3). Two ball bearings with a vertical separation of 20 meters, dropped from a height of 315 meters with 0 initial relative velocity, in the same 8 seconds increase their separation by 2×10^{-3} meter. To measuring equipment unable to detect such small relative displacements the ball bearings count as moving in one and the same free-float reference frame. No evidence for gravitation is to be seen. More sensitive apparatus detects the tide-producing action of gravity - the accelerated shortening of horizontal separations parallel to Earth's surface, the accelerated lengthening of vertical separations. Each tiny ball bearing still moves in a straight line in its own local free-float reference frame. But now - with the new precision - the region of validity of the one free-float reference frame does not reach out far enough to give a proper account of the motion of the other steel ball. The millimeter or two discrepancy is the way "gravity" manifests itself.

Tidal acceleration displays gravity as a local phenomenon. No mention here of the distance of the steel balls from the center of Earth! No mention here of acceleration relative to that center! The only accelerations that come into consideration are those of nearby particles relative to each other, the tidal accelerations described in the preceding paragraph.

These relative accelerations double when the separations are doubled. The true measure of the tide-producing effect has therefore the character of an acceleration per unit of separation. Let the acceleration be measured in meters of distance per meter of light-travel time per meter of light-travel time; that is, in units meters/meter ^{2} or $1/$ meter $\left[x=(1/2) a t^{2}\right.$, so $\left.a=2 x / t^{2}\right]$. Then the measure of the tide-producing effect (different for different directions) has the units (acceleration/distance) or $\left(1 / \text{meter}^{2}\right)$. In the example, in the two horizontal directions this quantity has the value $\left\{2(-0.001 \text { meter }) /\left(24 \times 10^{8} \text { meter }\right)^{2}\right] / 20$ meter $=-17.36 \times 10^{-24}$ meter $^{-2}$ and in the vertical direction twice the value and the opposite sign: $+34.72 \times 10^{-24}$ meter $^{-2}$. The tide-producing effect is small but it is real and it is observable. Further, it is a locally defined quantity. And Einstein tells us that we must focus our attention on locally defined quantities if we want a simple description of nature.

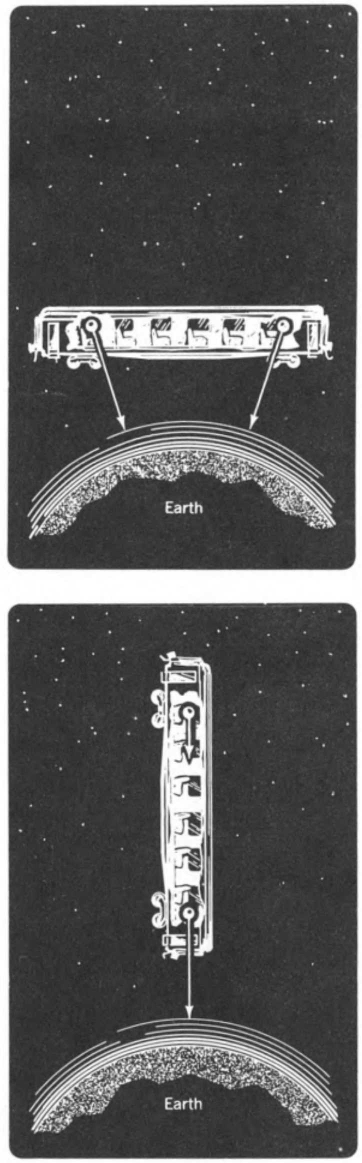

Figure 9.4.1: Einstein's railway coach in free fall.

Einstein says more: This tide-producing effect does not require for its explanation some mysterious force of gravitation, propagated through spacetime and additional to the structure of spacetime. Instead, it can and should be described in terms of the geometry of spacetime itself as the curvature of spacetime.

Though Einstein speaks of four-dimensional spacetime, his concepts of curvature can be illustrated in terms of two-dimensional geometry on the surface of a sphere.