11.4: Form of the Lorentz Transformation

any event can be reference event? then transformation is linear

What general form does the Lorentz transformation have? It has the form that mathematicians call a linear transformation. This means that laboratory coordinates x and t are related to linear (first) power of rocket coordinates x' and t' by equations of the form

Lorentz transformation: linear equations

$$\begin{aligned} t &= Bx' + Dt' \\ x &= Gx' + Ht' \end{aligned} \tag{11.4.1}$$

where our task is to find expressions for the coefficients $B, D, G,$ and H that do not depend on either the laboratory or the rocket coordinates of a particular event, though they do depend on the relative speed v_{rel} .

Why must these transformations be linear? Because we are free to choose any event as our reference event, the common origin $x = y = z = t = 0$ in all reference frames. Let our rocket sparkplug emit the flashes at $t' = 1$ and 2 and 3 meters. These are equally spaced in rocket time. According to equation (L-3) these three events occur at laboratory times $t = 1\gamma$ and 2γ and 3γ meters of time. These are equally spaced in laboratory time. Moving the reference event to the first of these events still leaves them equally spaced in time for both observers: $t' = 0$ and 1 and 2 meters in the rocket and $t = 0$ and 1γ and 2γ in the laboratory.

Arbitrary event as reference event? Then Lorentz transformation must be linear

In contrast, suppose that equation (L-3) were not linear, reading instead $t = Kt'^2$, where K is some constant. Rocket times $t' = 1$ and 2 and 3 meters result in laboratory times $t = 1K$ and $4K$ and $9K$ meters. These are not equally spaced in time for the laboratory observer. Moving the reference event to the first event would result in rocket times $t' = 0$ and 1 and 2 meters as before, but in this case laboratory times $t = 0$ and $1K$ and $4K$ meters, with a completely different spacing. But the choice of reference event is arbitrary: Any event is as qualified to be reference event as any other. A clock that runs steadily as observed in one frame must run steadily in the other, independent of the choice of reference event. We conclude that the relation between t and t' must be a linear one. A similar argument requires that events equally separated in space in the rocket must also be equally separated in space as measured in the laboratory. Hence the Lorentz transformation must be linear in both space and time coordinates.