4.5: Flight Plan

out and back in 40 years to meet our remote descendents

Wide awake now, we face yesterday's question: Shall we go to Canopus, 99 light-years distant, as the Space Agency asks? Yes. And yes, we shall live to return and report.

We take paper and pencil and sketch our plan. The numbers have to be different from those we dreamed about. Trial and error gives us the following plan: After a preliminary run to get up to speed, we will zoom past Earth at $99/101 = 0.9802$light speed. We will continue at that speed all the 99 light-years to Canopus. We will make a loop around it and record in those few minutes, by high-speed camera, the features of that strange star. We will then return at unaltered speed, flashing by our finish line without any letup, and as we do so, we will toss out our bundle of records to colleagues on Earth. Then we will slow down, turn, and descend quietly to Earth, our mission completed.

Round trip: 202 Earth years

The first long run takes 101 Earth years. We have already decided to travel at a speed of $99/101$, or 99 light-years of distance in 101 years of time. Going at that speed for 101 Earth years, we will just cover the 99 light-years to Canopus. The return trip will likewise take 101 Earth years. Thus we will deliver our records to Earth 202 Earth-clock years after the start of our trip.[1]

Even briefer will be the account of our trip as it will be perceived in the free-float rocket frame. Relative to the ship we will not go anywhere, either on the outbound or on the return trip. But time will go on ticking away on our shipboard clock. Moreover our biological clock, by which we age, and all other good clocks carried along will tick away in concord with it. How much time will that rocket clock rack up on the outbound trip? Twenty years. How do we know? We reach this answer in three steps.

Round trip: 40 astronaut years

First, we already know from records in the Earth-linked laboratory frame that the spacetime interval - the proper time - between departure from Earth and arrival at Canopus will equal 20 years:[2]

$$\begin{aligned} (\text{interval})^2 &= (\text{time separation})^2_{\textbf{Laboratory}} - (\text{space separation})^2_{\textbf{Laboratory}} \\ &= (101 \text{ years})^2 - (99 \text{ years})^2 \\ &= 10,201 \text{ years}^2 - 9801 \text{ years}^2 \\ &= 400 \text{ years}^2 \\ &= (20 \text{ years})^2 \end{aligned}$$

Second, as the saying goes, "interval is interval is interval" : The spacetime interval is invariant between frames. The interval as registered in the rocket frame must therefore also have this 20 -year value. Third, in the rocket frame, separation between the two events (departure from Earth and arrival at Canopus) lies all in the time dimension, zero in the space dimension, since we do not leave the rocket. Therefore separation in rocket time itself between these two events is the proper time and must likewise be 20 years:

$$\begin{aligned} (\text{interval})^2 &= (\text{time separation})^2_{\textbf{Rocket}} - (\text{space separation})^2_{\textbf{Rocket}} \\ &= (\text{time separation})^2 - (\text{zero})^2 \\ &= (\text{rocket time})^2 = (\text{proper time})^2 \\ &= (20 \text{ years})^2 \end{aligned}$$

We boil down our flight plan to bare bones and take it to the Space Agency for approval: Speed $99/101 = 0.9802$light speed; distance 99 light-years out, 99 light-years back; time of return to Earth 202 years after start; astronaut's aging during trip, 40 years. The responsible people greet the plan with enthusiasm. They thank us for volunteering for a mission so unprecedented. They ask us

to take our proposal before the Board of Directors for final approval. We agree, not realizing what a hornets' nest we are walking into.

The Board of Directors consists of people from various walks of life, set up by Congress to assure that major projects have support of the public at large. The media have reported widely on our proposal in the weeks before we meet with the board, and many people with strong objections to relativity have written to voice their opinions. A few have met with board members and talked to them at length. We are unaware of this as we enter the paneled board room.

At the request of the chairman we summarize our plan. The majority appear to welcome it. Several of their colleagues, however, object.

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1 Round trip: 202 Earth years

2 Round trip: 40 astronaut years
