3.E: Same Laws for All (Exercises)

PRACTICE

3-1 relativity and swimming

The idea here is to illustrate how remarkable is the invariance of the speed of light (light speed same in all free-float frames) by contrasting it with the case of a swimmer making her way through water.

Light goes through space at 3×10^8 meters/second, and the swimmer goes through the water at 1 meter/second. "But how can there otherwise be any difference?" one at first asks oneself.

For a light flash to go down the length of a 30 meter spaceship and back again takes

$$\begin{aligned} \text{time } &= (\text{ distance })/(\text{ speed }) \\ &= 2 \times (30 \text{ meters })/\left(3 \times 10^8 \text{ meters / second }\right) \\ &= 2 \times 10^{-7} \text{ second} \end{aligned}$$

as measured in the spaceship, regardless of whether the ship is stationary at the spaceport or is zooming past it at high speed.

Check how very different the story is for the swimmer plowing along at 1 meter/second with respect to the water.

a. How long does it take her to swim down the length of a 30 -meter pool and back again?
b. How long does it take her to swim from float A to float B and back again when the two floats, A and B, are still 30 meters apart, but now are being towed through a lake at $1/3$ meter/second?
   Discussion: When the swimmer is swimming in the same direction in which the floats are being towed, what is her speed relative to the floats? And how great is the distance she has to travel expressed in the "frame of reference' of the floats? So how long does it take to travel that leg of her trip? Then consider the same three questions for the return trip.
c. Is it true that the total time from A to B and back again is independent of the reference system ("stationary" pool ends vs. moving floats)?
d. Express in the cleanest, clearest, sharpest one-sentence formulation you can the difference between what happens for the swimmer and what happens for a light flash.

3-2 Einstein puzzler

When Albert Einstein was a boy of 16, he mulled over the following puzzler: A runner looks at herself in a mirror that she holds at arm's length in front of her. If she runs with nearly the speed of light, will she be able to see herself in the mirror? Analyze this question using the Principle of Relativity.

3-3 construction of clocks

For the measurement of time, we have made no distinction among spring clocks, quartz crystal clocks, biological clocks (aging), atomic clocks, radioactive clocks, and a clock in which the ticking element is a pulse of light bouncing back and forth between two mirrors (see figure). Let all these clocks be adjusted by the laboratory observer to run at the same rate when at rest in the laboratory. Now let the clocks all be accelerated gently to a high speed in a rocket, which then turns off its engines. Make a simple but powerful argument that the free-float rocket observer will also measure these different clocks all to run at the same rate as one another. Does it follow that the (common) clock rate of these clocks measured by the rocket observer is the same as their (common) rate measured by the laboratory observer as they pass by in the rocket?

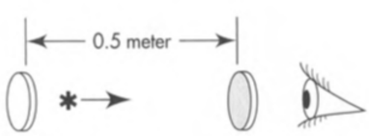

Figure $3.E.1$: This two-mirror "clock" sends to the eye flash after flash, each separated from the next by 1 meter of light-travel time. A light flash (represented by an asterisk) bounces back and forth between parallel mirrors separated from one another by 0.5 meter of distance. The silver coating of the right-band mirror does not reflect perfectly: It lets 1 percent of the light pass through to the eye each time the light pulse bits it. Hence the eye receives a pulse of light every meter of light-travel time.

3-4 the Principle of Relativity

Two overlapping free-float frames are in uniform relative motion. On the following list, mark with a "yes" the quantities that must *necessarily* be the same as measured in the two frames. Mark with a "no" the quantities that are *not* necessarily the same as measured in the two frames.

a. time it takes for light to go one meter of distance in a vacuum
b. spacetime interval between two events
c. kinetic energy of an electron
d. value of the mass of the electron
e. value of the magnetic field at a given point
f. distance between two events
g. structure of the DNA molecule
h. time rate of change of momentum of a neutron

3-5 many unpowered rockets

In the laboratory frame, event 1 occurs at $x=0$ light-years, $t=0$ years. Event 2 occurs at $x=6$ light-years, $t=10$ years. In all rocket frames, event 1 also occurs at the position 0 light-years and the time 0 years. The y- and z-coordinates of both events are zero in both frames.

a. In rocket frame A, event 2 occurs at time $t'=14$ years. At what position x' will event 2 occur in this frame?
b. In rocket frame B, event 2 occurs at position $x''=5$ light-years. At what time t'' will event 2 occur in this frame?
c. How fast must rocket frame C move if events 1 and 2 occur at the same place in this rocket frame?
d. What is the time between events 1 and 2 in rocket frame C of part c ?

3-6 down with relativity!

Mr. Van Dam is an intelligent and reasonable man with a knowledge of high school physics. He has the following objections to the theory of relativity. Answer each of Mr. Van Dam's objections decisively without criticizing him. If you wish, you may present a single connected account of how and why one is driven to relativity, in which these objections are all answered.

a. "Observer A says that B's clock goes slow, and observer B says that A's clock goes slow. This is a logical contradiction. Therefore relativity should be abandoned."
b. "Observer A says that B's meter sticks are contracted along their direction of relative motion, and observer B says that A's meter sticks are contracted. This is a logical contradiction. Therefore relativity should be abandoned."
c. "Relativity does not even have a unique way to define space and time coordinates for the instantaneous position of an object. Laboratory and rocket observers typically record different coordinates for this position and time. Therefore anything relativity says about the velocity of the object (and hence about its motion) is without meaning."
d. "Relativity postulates that light travels with a standard speed regardless of the free-float frame from which its progress is measured. This postulate is certainly wrong. Anybody with common sense knows that travel at high speed in the direction of a receding light pulse will decrease the speed with which the pulse recedes. Hence a flash of light cannot have the same speed for observers in relative motion. With this disproof of the basic postulate, all of relativity collapses."
e. "There isn't a single experimental test of the results of special relativity."
f. "Relativity offers no way to describe an event without coordinates - and no way to speak about coordinates without referring to one or another particular reference frame. However, physical events have an existence independent of all choice of coordinates and all choice of reference frame. Hence relativity - with its coordinates and reference frames - cannot provide a valid description of these events."
g. "Relativity is preoccupied with how we observe things, not what is really happening. Hence it is not a scientific theory, since science deals with reality."

PROBLEMS

3-7 Space War

Two rockets of equal rest length are passing ‘‘head on" at relativistic speeds, as shown in the figure (left). Observer o has a gun in the tail of her rocket pointing perpendicular to the direction of relative motion (center). She fires the gun when points a and a' coincide. In her frame the other rocket ship is Lorentz contracted. Therefore o expects her bullet to miss the other rocket. But in the frame of the other observer o' it is the rocket ship of o that is measured to be Lorentz contracted (right). Therefore when points a and a' coincide, observer o' should observe a hit.

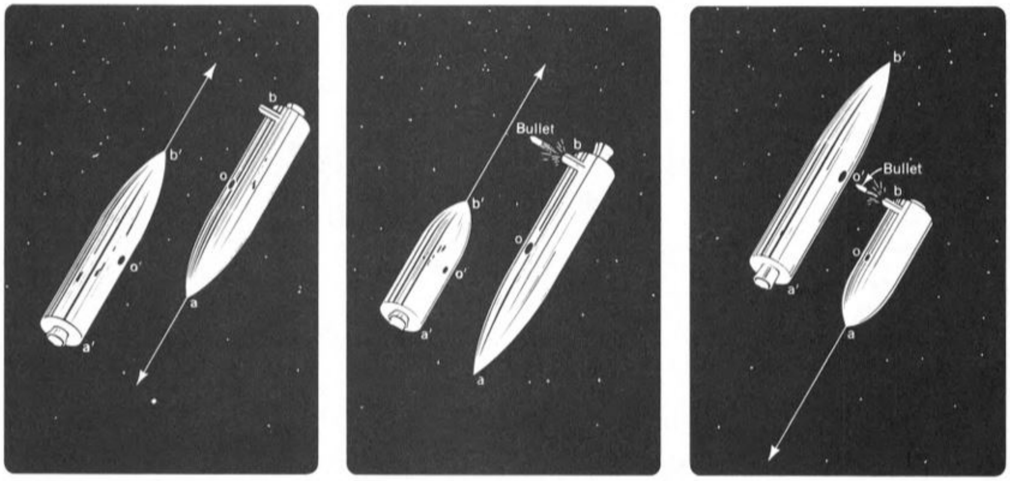

Figure $3.E.2$: **Left**: Two rocket ships passing at high speed. **Center**: In the frame of o one expects a bullet fired when a coincides with a' to miss the other ship. **Right**: In the frame of o' one expects a bullet fired when a coincides with a' to hit the other ship.

Does the bullet actually hit or miss? Pinpoint the looseness of the language used to state the problem and the error in one figure. Show that your argument is consistent with the results of the Train Paradox (Section 3.4).

3-8 $\breve{C}$erenkov radiation

No particle has been observed to travel faster than the speed of light in a *vacuum*. However particles have been observed that travel in a material medium faster than the speed of light *in that medium*. When a charged particle moves through a medium faster than light moves in that medium, it radiates coherent light in a cone whose axis lies along the path of the particle. (Note the rough similarity to waves created by a motorboat speeding across calm water and the more exact similarity to the "cone of sonic boom" created by a supersonic aircraft.) This is called $\breve{C}$erenkov radiation (Russian $\breve{C}$ is pronounced as "ch"). Let v be the speed of the particle in the medium and v_{light} be the speed of light in the medium.

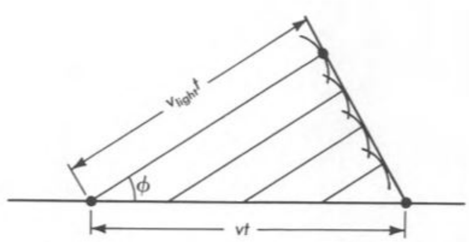

Figure $3.E.3$: **first figure**. Calculation of $\breve{C}$erenkov angle ϕ.

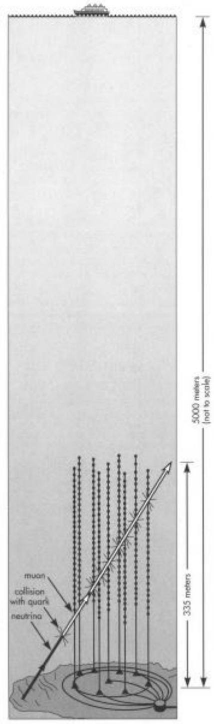

Figure $3.E.4$: Use of $\breve{C}$erenkov radiation for indirect detection of neutrinos in the Deep Underwater Muon and Neutrino Detector (DUMAND) 30 kilometers off Keahole Point on the island of Hawaii. Neutrinos have no electric charge and their mass, if any, has so far escaped detection (Box 8-1). Neutrinos interact extremely weakly with matter, passing through Earth with almost no collisions. Indeed, the DUMAND detector array selects for analysis only neutrinos that come upward through Earth. In this way Earth itself acts as a shield to eliminate all other cosmic-ray particles.

What are possible sources for these neutrinos? Theory predicts the emission of very high-energy {greater than 10^{12} electron-volt) neutrinos from matter plunging toward a black hole. Black holes may be the energy sources for extra-bright galactic nuclei and for quasars —small, distant, enigmatic objects shining with the light of hundreds of galaxies (Section 9.8). Information about conditions deep within these astronomical structures may be carried by neutrinos as they pierce Earth and travel upward through the DUMAND detector array.

In a rare event, a neutrino moving through the ocean slams into one of the quarks that make up a proton or a neutron in, say, an oxygen nucleus in the water, creating a burst of particles. All of these particles are quickly absorbed by the surrounding water except a stable negatively charged muon, 201 times the mass of the electron (thus sometimes called a "fat electron"). This muon streaks through the water in the same direction as the neutrino that created it and at a speed greater than that of light in water, thus emitting $\breve{C}$erenkov radiation. The $\breve{C}$erenkov radiation is detected by photomultiplier tubes in an array anchored to the ocean floor.

Photomultipliers are strung along 9 vertical cables, 8 cables spaced around a circle 100 meters in diameter on the ocean floor, the ninth cable rising from the center of the circle. Each cable is 335 meters long and holds 24 glass spheres positioned 10 meters apart on the top 230 meters of its length. There are no detectors on the bottom no meters, in order to avoid any cloud of sediments from the bottom. Above the bottom, the water is so clear and modem photo detectors so sensitive that $\breve{C}$erenkov radiation can he detected from a muon that passes within 40 meters of a detector.

Photomultipliers in the glass spheres detect $\breve{C}$erenkov radiation from the passing muons, transmitting this signal through underwater optical fibers to computers on the nearby island of Hawaii. The computers select for examination only those events in which (1) several optical sensors detect bursts that are (2) within 40 meters or so of a straight line, (3) spaced in time to show that the particle is moving at essentially the speed of light in a vacuum, and (4) from a particle moving upward through the water. A system of sonar beacons and hydrophones tracks the locations of the photomultipliers as the strings sway with the slow ocean currents. As a result, the direction of motion of the original neutrino can be recorded to an accuracy of one degree.

The DUMAND facility is designed to create a new sky map of neutrino sources to supplement our knowledge of the heavens, so far obtained primarily from the electromagnetic spectrum (radio, infrared, optical, ultraviolet. X-ray, gamma ray).

a. From this information use the first figure to show that the half-angle ϕ, of the light cone is given by the expression

$$\cos \phi = v_{\text{light}} / v$$

b. Consider the plastic with the trade name Lucite, for which $v_{\text{light}} = 2/3$. What is the minimum velocity that a charged particle can have if it is to produce $\breve{C}$erenkov radiation in Lucite? What is the maximum angle ϕ at which $\breve{C}$erenkov radiation can be produced in Lucite? Measurement of the angle provides a good way to measure the velocity of the particle.
c. In water the speed of light is approximately $v_{\text{light}} = 0.75$. Answer the questions of part **b** for the case of water. See the second figure for an application of $\breve{C}$erenkov radiation in water.

3-9 aberration of starlight

A star lies in a direction generally perpendicular to Earth's direction of motion around Sun. Because of Earth's motion, the star appears to an Earth observer to lie in a slightly different direction than it would appear to an observer at rest relative to Sun. This effect is called **aberration**. Using the diagram, find this apparent difference of direction.

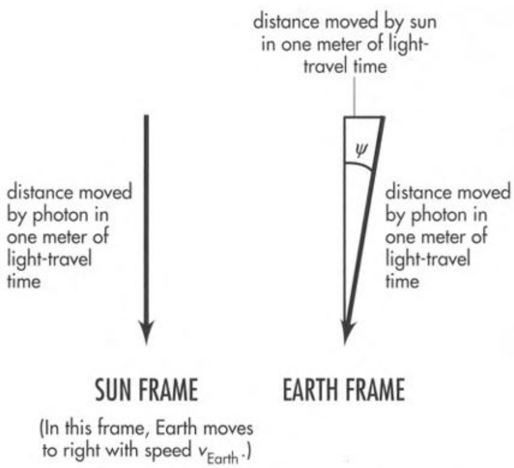

Figure $3.E.5$: Aberration of starlight. Not to scale.

a. Find a trigonometric expression for the aberration angle ψ shown in the figure.
b. Evaluate your expression using the speed of Earth around Sun, $v_{\text{Earth conv}}$ = 30 kilometers/second.Find the answer in radians and in seconds of arc. (One degree equals 60 minutes of arc; one minute equals 60 seconds of arc.) This change in apparent position can be detected with sensitive equipment.
c. The nonrelativistic answer to this problem — the answer using nonrelativistic physics—is tan $\psi = v_{\text{Earth}}$ meters/meter). Do you think that the experimental difference between relativistic and nonrelativistic answers for stellar aberration observed from Earth can be the basis of a crucial experiment to decide between the correctness of the two theories?
Discussion: Of course we cannot climb off Earth and view the star from the Sun frame. But Earth reverses direction every six months (with respect to what?), so light from a "transverse star" viewed in, say, July will appear to be shifted through twice the aberration angle calculated in part **b** compared with the light from the same star in January. New question: Since the background of stars behind the one under observation also shifts due to aberration, how can the effect be measured at all?
d. A rocket in orbit around Earth suddenly changes its velocity from a very small fraction of the speed of light to $v = 0.5$ with respect to Sun, moving in the same direction as Earth is moving around Sun. In what direction will the rocket astronaut now see the star of parts **a** and **b**?

3-10 the expanding universe

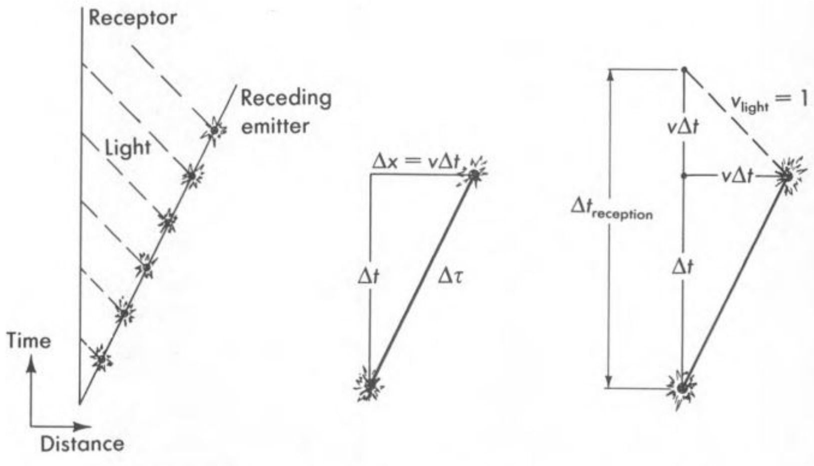

Figure $3.E.6$: Calculation of the time $\Delta t_{\text{reception}}$ between arrival at observer of consecutive flashes from receding emitter. Light moves one meter of distance in one meter of time, so lines showing motion of light are tilted at $\pm 45^{\circ}$ from the vertical.

a. A giant bomb explodes in otherwise empty space. What is the nature of the motion of one fragment relative to another? And how can this relative motion be detected?

   Discussion: Imagine each fragment equipped with a beacon that gives off flashes of light at regular, known intervals $\Delta\tau$ of time as measured in its own frame of reference (proper time!). Knowing this interval between flashes, what method of detection can an observer on one fragment employ to determine the velocity v - relative to her - of any other fragment? Assume that she uses, in making this determination, (1) the known proper time $\Delta\tau$ between flashes and (2) the time $\Delta t_{\text{reception}}$ between the arrival of consecutive flashes at her position. (This is not equal to the time Δt in her frame between the emission of the two flashes from the receding emitter; see the figure.) Derive a formula for v in terms of proper time lapse $\Delta\tau$ and $\Delta t_{\text{reception.}}$ How will the measured recession velocity depend on the distance from one's own fragment to the fragment at which one is looking? Hint: In any given time in any given frame, fragments evidently travel distances in that frame from the point of explosion that are in direct proportion to their velocities in that frame.

b. How can observation of the light from stars be used to verify that the universe is expanding? Discussion: Atoms in hot stars give off light of different frequencies characteristic of these atoms ("spectral lines"). The observed period of the light in each spectral line from starlight can be measured on Earth. From the pattern of spectral lines the kind of atom emitting the light can be identified. The same kind of atom can then be excited in the laboratory to emit light while at rest and the proper period of the light in any spectral line can be measured. Use the results of part a to describe how the observed period of light in one spectral line from starlight can be compared to the proper period of light in the same spectral line from atoms at rest in the laboratory to give the velocity of recession of the star that emits the light. This observed change in period due to the velocity of the source is called the Doppler shift. (For a more detailed treatment of Doppler shift, see the exercises for Chapters 5 and 8.) If the universe began in a gigantic explosion, how must the observed velocities of recession of different stars at different distances compare with one another? Slowing down during expansion - by gravitational attraction or otherwise - is to be neglected here but is considered in more complete treatments.

c. The brightest steadily shining objects in the heavens are called quasars, which stands for "quasistellar objects." A single quasar emits more than 100 times the light of our entire galaxy. One possible source of quasar energy is the gravitational energy released as material falls into a black hole (Section 9.8). Because they are so bright, quasars can be observed at great distances. As of 1991, the greatest observed quasar red shift $\Delta t_{\text{reception}} / \Delta\tau$ has the value 5.9. According to the theory of this exercise, what is the velocity of recession of this quasar, as a fraction of the speed of light?

3-11 law of addition of velocities

In a spacebus a bullet shoots forward with speed $3/4$ that of light as measured by travelers in the bus. The spacebus moves forward with speed $3/4$ light speed as measured by Earth observers. How fast does the bullet move as measured by Earth observers: $3/4+3/4=6/4=1.5$ times the speed of light? No! Why not? Because (1) special relativity predicts that nothing can travel faster than light, and (2) hundreds of millions of dollars have been spent accelerating particles ("bullets") to the fastest possible speed without anyone detecting a single particle that moves faster than light in a vacuum. Then where is the flaw in our addition of velocities? And what is the correct law of addition of velocities? These questions are answered in this exercise.

a. First use Earth observers to record the motions of the spacebus (length L measured in the Earth frame, speed v_{rel}) and the streaking bullet (speed v_{bullet}). The bullet starts at the back of the bus. To give it some competition, let a light flash (speed $=1$) race the bullet from the back of the bus toward the front. The light flash wins, of course, reaching the front of the bus in time t_{forward}. And t_{forward} is also equal to the distance that the light travels in this time. Show that this distance (measured in the Earth frame) equals the length of the bus plus the distance the bus travels in the same time:

$$t_{\text{forward}}=L+v_{\text{rel}}\,t_{\text{forward}} \text{ or } t_{\text{forward}}=\frac{L}{1-v_{\text{rel}}} \tag{3.E.1}$$

b. In order to rub in its advantage over the bullet, the light flash reflects from the front of the bus and moves backward until, after an additional time t_{backward}, it rejoins the forward-plodding bullet. This meeting takes place next to the seat occupied by Fred, who sits a distance fL behind the front of the bus, where f is a fraction of the bus length L. Show that for this leg of the trip the Earth-measured distance t_{backward} traveled by the light flash can also be expressed as

$$\begin{aligned} t_{\text{backward}} &= fL-v_{\text{rel}}\,t_{\text{backward}} \qquad \text{or} \\ t_{\text{backward}} &= \frac{fL}{1+v_{\text{rel}}} \end{aligned} \tag{3.E.2}$$

c. The light flash has moved forward and then backward with respect to Earth. What is the *net* forward distance covered by the light flash at the instant it rejoins the bullet? Equate this with the forward distance moved by the bullet (at speed v_{bullet}) to obtain the equation

$$v_{\text{bullet}}\left(t_{\text{forward}}+t_{\text{backward}}\right)=t_{\text{forward}}-t_{\text{backward}}$$

or

$$\left(1+v_{\text{bullet}}\right) t_{\text{backward}}=\left(1-v_{\text{bullet}}\right) t_{\text{forward}} \tag{3.E.3}$$

d. What are we after? We want a relation between the bullet speed v_{bullet} as measured in the Earth frame and the bullet speed, call it v'_{bullet} (with a prime), as measured in the spacebus frame. The times given in parts **a**, **b**, and **c** are of no use to this end. Worse, we already know that times between events are typically different as measured in the spacebus frame than times between the same events measured in the Earth frame. So get rid of these times! Moreover, the Lorentz-contracted length L of the spacebus itself as measured in the Earth frame will be different from its rest length measured in the bus frame (Section 3.5). So get rid of L as well. Equations (3.E.1), (3.E.2), and (3.E.3) can be treated as three equations in the three unknowns t_{forward}, t_{backward}, and L. Substitute equations for the times (3.E.1) and (3.E.2) into equation (3.E.3). Lucky us: The symbol L cancels out of the result. Show that this result can be written

$$f=\frac{\left(1-v_{\text{bullet}}\right)}{\left(1+v_{\text{bullet}}\right)} \frac{\left(1+v_{\text{rel}}\right)}{\left(1-v_{\text{rel}}\right)} \tag{3.E.4}$$

e. Now repeat the development of parts **a** through **d** for the spacebus frame, with respect to which the spacebus has its rest length L' and the bullet has speed v'_{bullet} (both with primes). Show that the result is:

$$f=\frac{\left(1-v'_{\text{bullet}}\right)}{\left(1+v'_{\text{bullet}}\right)} \tag{3.E.5}$$

Discussion: Instead of working hard, work smart! Why not use the old equations (3.E.1) through (3.E.4) for the spacebus frame? Because there is no relative velocity v_{rel} in the spacebus frame; the spacebus is at rest in its own frame! No problem: Set $v_{\text{rel}}=0$ in equation (3.E.4), replace v_{bullet} by v'_{bullet} and obtain equation (3.E.5) directly from equation (3.E.4). If this is too big a step, carry out the derivation from the beginning in the spacebus frame.

f. Do the two fractions f in equations (3.E.4) and (3.E.5) have the same value? In equation (3.E.4) the number f locates Fred's seat in the bus as a fraction of the total length of the bus in the Earth frame. In equation (3.E.5) the number f locates Fred's seat in the bus as a fraction of the total length of the bus in the bus frame. But this fraction must be the same: Fred cannot be halfway back in the Earth frame and, say, three quarters of the way back in the spacebus frame. Equate the two expressions for f given in equations (3.E.4) and (3.E.5) and solve for v_{bullet} to obtain the Law of Addition of Velocities:

$$v_{\text{bullet}} = \frac{v'_{\text{bullet}} + v_{\text{rel}}}{1 + v'_{\text{bullet}} v_{\text{rel}}} \tag{3.E.6}$$

g. Explore some consequences of the Law of Addition of Velocities.

1. An express bus on Earth moves at 108 kilometers/hour (approximately 67 miles/ hour or 30 meters per second). A bullet moves forward with speed 600 meters/second with respect to the bus. What are the values of v_{rel} and v'_{bullet} in meters/meter? What is the value of their product in the denominator of equation (3.E.6)? Does this product of speeds increase the value of the denominator significantly over the value unity? Therefore what approximate form does equation (3.E.6) take for everyday speeds? Is this the form you would expect from your experience?
2. Analyze the example that began this exercise: Speed of bullet with respect to spacebus $v'_{\text{bullet}} = 3/4$; speed of spacebus with respect to Earth $v_{\text{rel}} = 3/4$. What is the speed of the bullet measured by Earth observers?
3. $v'_{\text{bullet}} = 1$. For $v_{\text{rel}} = 3/4$, with what speed does this light flash move as measured in the Earth frame? Is this what you expect from the Principle of Relativity?
4. Suppose a light flash is launched from the front of the bus directed toward the back $\left(v'_{\text{bullet}} = -1\right)$. What is the velocity of this light flash measured in the Earth frame? Is this what you expect from the Principle of Relativity?

Reference: N. David Mermin, American Journal of Physics, Volume 51 , pages 1130-1131 (1983).

3-12 Michelson-Morley experiment

a. An airplane moves with air speed c (not the speed of light) from point A to point B on Earth. A stiff wind of speed v is blowing from B toward A. (In this exercise only, the symbol v stands for velocity in conventional units, for example meters/second.) Show that the time for a round trip from A to B and back to A under these circumstances is greater by a factor $1/\left(1 - v^2/c^2\right)$ than the corresponding round trip time in still air. Paradox: The wind helps on one leg of the flight as well as hinders on the other. Why, therefore, is the round-trip time not the same in the presence of wind as in still air? Give a simple physical reason for this difference. What happens when the wind speed is nearly equal to the speed of the airplane?

b. The same airplane now makes a round trip between A and C. The distance between A and C is the same as the distance from A to B, but the line from A to C is perpendicular to the line from A to B, so that in moving between A and C the plane flies across the wind. Show that the round-trip time between A and C under these circumstances is greater by a factor $1/\left(1 - v^2/\tau^2\right)^{1/2}$ than the corresponding round-trip time in still air.

c. Two airplanes with the same air speed c start from A at the same time. One travels from A to B and back to A, flying first against and then with the wind (wind speed v). The other travels from A to C and back to A, flying across the wind. Which one will arrive home first, and what will be the difference in their arrival times? Using the first two terms of the binomial theorem,

$$(1+z)^n \approx 1 + nz \quad \text{for } |z| << 1$$

show that if $v << c$, then an approximate expression for this time difference is $\Delta t \approx (L/2c)(v/c)^2$, where L is the round-trip distance between A and B (and between A and C).

d. The South Pole Air Station is the supply depot for research huts on a circle of 300 -kilometer radius centered on the air station. Every Monday many supply planes start simultaneously from the station and fly radially in all directions at the same altitude. Each plane drops supplies and mail to one of the research huts and flies directly home. A Fussbudget with a stopwatch stands on the hill overlooking the air station. She notices that the planes do not all return at the same time. This discrepancy perplexes her because she knows from careful measurement that
(1) the distance from the air station to every research hut is the same,
(2) every plane flies with the same air speed as every other plane - 300 kilometers/hour-and
(3) every plane travels in a straight line over the ground from station to hut and back.
The Fussbudget finally decides that the discrepancy is due to the wind at the high altitude at which the planes fly. With her stopwatch she measures the time from the return of the first plane to the return of the last plane to be 4 seconds. What is the wind speed at the altitude where the planes fly? What can the Fussbudget say about the direction of this wind?

e. In their famous experiment Michelson and Morley attempted to detect the so-called ether drift - the motion of Earth through the "ether," with respect to which light was supposed to have the velocity c. They compared the round-trip times for light to travel equal distances parallel and perpendicular to the direction of motion of Earth around Sun. They reflected the light back and forth between nearly parallel mirrors. (This would correspond to part c if each airplane made repeated round trips.) By this means they were able to use a total round-trip length of 22 meters for each path. If the "ether" is at rest with respect to Sun, and if Earth moves at 30×10^3 meters/second in its path around Sun, what is the approximate difference in time of return between light flashes that are emitted simultaneously and travel along the two perpendicular paths? Even with the instruments of today, the difference predicted by the ether-drift hypothesis would be too small to measure directly, and the following method was used instead.

f. The original Michelson-Morley interferometer is diagrammed in the figure. Nearly monochromatic light (light of a single frequency) enters through the lens at a. Some of the light is reflected by the half-silvered mirror at b and the rest of the light continues toward d. Both beams are reflected back and forth until they reach mirrors e and e_1 respectively, where each beam is reflected back on itself and re- traces its path to mirror b. At mirror b parts of each beam combine to enter telescope f together. The transparent piece of glass at c, of the same dimensions as the half-silvered mirror b, is inserted so that both beams pass the same number of times (three times) through this thickness of glass on their way to telescope f. Suppose that the perpendicular path lengths are exactly equal and the instrument is at rest with respect to the ether. Then monochromatic light from the two paths that leave mirror b in some relative phase will return to mirror b in the same phase. Under these circumstances the waves entering telescope f will add crest to crest and the image in this telescope will be bright. On the other hand, if one of the beams has been delayed a time corresponding to one half period of the light, then it will arrive at mirror b one half period later and the waves entering the telescope will cancel (crest to trough), so the image in the telescope will be dark. If one beam is retarded a time corresponding to one whole period, the telescope image will be bright, and so forth. What time corresponds to one period of the light? Michelson and Morley used sodium light of wavelength 589 nanometers (one nanometer is equal to 10^{-9} meter). Use the equations $f\lambda = c$ and $f = 1/T$ that relate frequency f, period T, wavelength λ, and speed c of an electromagnetic wave. Show that one period of sodium light corresponds to about 2×10^{-15} seconds.
Now there is no way to "turn off' the alleged ether drift, adjust the apparatus, and then turn the alleged ether drift on again. Instead of this, Michelson and Morley floated their interferometer in a pool of mercury and rotated it slowly about its center like a phonograph record while observing the image in the telescope (see the figure). In this way if light is delayed on either path when the instrument is oriented in a certain direction, light on the other path will be delayed by the same amount of time when the instrument has rotated 90 degrees. Hence the total change in delay time between the two paths observed as the interferometer rotates should be twice the difference calculated using the expression derived in part c. By refinements of this method Michelson and Morley were able to show that the time change between the two paths as the instrument rotated corresponded to less than one one-hundredth of the shift from one dark image in the telescope to the next dark image. Show that this result implies that the motion of the ether at the surface of Earth - if it exists at all - is less than one sixth of the speed of Earth in its orbit. In order to eliminate the possibility that the ether was flowing past Sun at the same rate as Earth was moving its orbit, they repeated the experiment at intervals of three months, always with negative results.

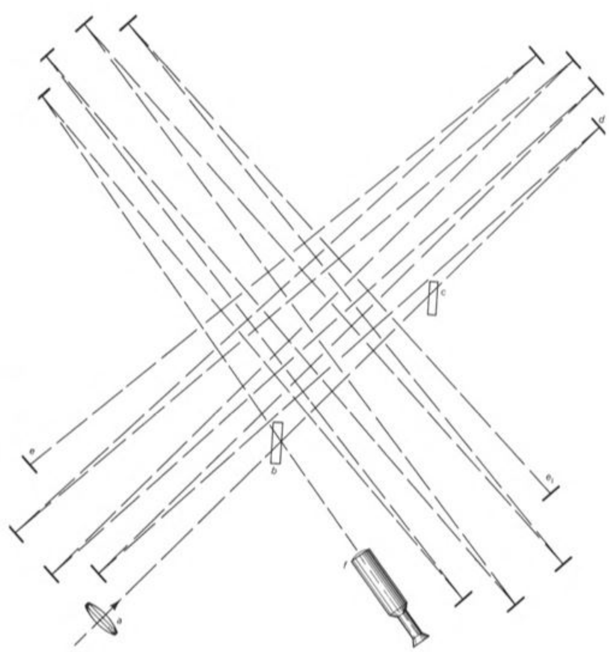

Figure $3.E.7$: Michelson-Morley interferometer mounted on a rotating marble slab.

g. **Discussion question**: Does the Michelson Morley experiment, by itself, disprove the theory that light is propagated through an ether? Can the ether theory be modified to agree with the results of this experiment? How? What further experiment can be used to test the modified theory?

Reference: A. A. Michelson and E. W. Morley, American Journal of Science, Volume 134, pages 333-345 (1887).

3-13 the Kennedy-Thorndike experiment

Note: Part **d** of this exercise uses elementary calculus.

The Michelson - Morley experiment was designed to detect any motion of Earth relative to a hypothetical fluid - the ether - a medium in which light was supposed to move with characteristic speed c. No such relative motion of earth and ether was detected. Partly as a result of this experiment the concept of ether has since been discarded. In the modern view, light requires no medium for its transmission. What significance does the negative result of the Michelson-Morley experiment have for us who do not believe in the ether theory of light propagation? Simply this:

(1) The round-trip speed of light measured on earth is the same in every direction - the speed of light is isotropic.
(2) The speed of light is isotropic not only when Earth moves in one direction around Sun in, say, January (call Earth with this motion the "laboratory frame"), but also when Earth moves in the opposite direction around Sun six months later, in July (call Earth with this motion the "rocket frame").
(3) The generalization of this result to any pair of inertial frames in relative motion is contained in the statement: The round-trip speed of light is isotropic both in the laboratory frame and in the rocket frame.

This result leaves an important question unanswered: Does the round-trip speed of light - which is isotropic in both laboratory and rocket frames - also have the same numerical value in laboratory and rocket frames? The assumption that this speed has the same numerical value in both frames played a central role in demonstrating the invariance of the interval (Section 3.7). But is this assumption valid?

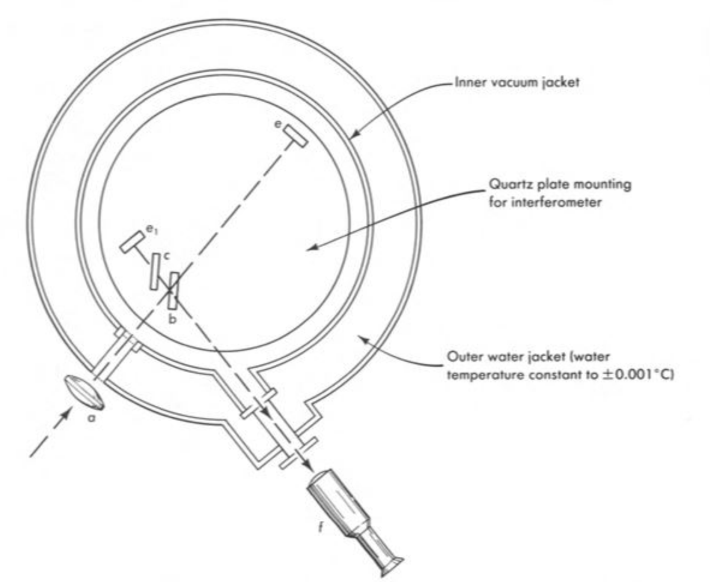

Figure $3.E.8$: Schematic diagram of apparatus used for the Kennedy- Thorndike experiment. Parts of the interferometer have been labeled with letters corresponding to those used in describing the Michelson-Morley interferometer (Exercise 3-12). The experi menters went to great lengths to insure the optical and mechanical stability of their apparatus. The interferometer is mounted on a plate of quartz, which changes dimension very little when tempera ture changes. The interferometer is enclosed in a vacuum jacket so that changes in atmospheric pressure will not alter the effective optical path length of the interferometer arms (slightly different speed of light at different atmospheric pressure). The inner vacuum jacket is surrounded by an outer water jacket in which the water is kept at a temperature that varies less than ± 0.001 degrees Celsius. The entire apparatus shown in the figure is enclosed in a small darkroom (not shown) maintained at a temperature constant within a few hundredths of a degree. The small darkroom is in turn enclosed in a larger darkroom whose temperature is constant within a few tenths of a degree. The overall size of the apparatus can he judged from the fact that the difference in length of the two arms of the interferometer (length eb compared with length e_1b) is 16 centimeters.

a. An experiment to test the assumption of the equality of the round-trip speed of light in two inertial frames in relative motion was conducted in 1932 by Roy J. Kennedy and Edward M. Thorndike. The experiment uses an interferometer with arms of unequal length (see the figure). Assume that one arm of the interferometer is Δl longer than the other arm. Show that a flash of light entering the apparatus will take a time $2 \Delta l / c$ longer to complete the round trip along the longer arm than along the shorter arm. The difference in length Δl used by Kennedy and Thorndike was approximately 16 centimeters. What is the approximate difference in time for the round trip of a light flash along the alternative paths?

b. Instead of a pulse of light, Kennedy and Thorndike used continuous monochromatic light of period $T=1.820 \times 10^{-15}$ seconds ($\lambda=546.1$ nanometers $=546.1 \times 10^{-9}$ meters) from a mercury source. Light that traverses the longer arm of the interferometer will return approximately how many periods n later than light that traverses the shorter arm? If in the actual experiment the number of periods is an integer, the reunited light from the two arms will add (crest-to-crest) and the field of view seen through the telescope will be bright. In contrast, if in the actual experiment the number of periods is a half-integer, the reunited light from the two arms will cancel (crest-to-trough) and the field of view of the telescope will be dark.

c. Earth continues on its path around Sun. Six months later Earth has reversed the direction of its velocity relative to the fixed stars. In this new frame of reference will the round-trip speed of light have the same numerical value c as in the original frame of reference? One can rewrite the answer to part **b** for the original frame of reference in the form

$$c=(2 / n)(\Delta l / T)$$

where Δl is the difference in length between the two interferometer arms, T is the time for one period of the atomic light source, and n is the number of periods that elapse between the return of the light on the shorter path and the return of the light on the longer path. Suppose that as Earth orbits Sun no shift is observed in the telescope field of view from, say, light toward dark. This means that n is observed to be constant. What would this hypothetical result tell about the numerical value c of the speed of light? Point out the standards of distance and time used in determining this result, as they appear in the equation. Quartz has the greatest stability of dimension of any known material. Atomic time standards have proved to be the most dependable earth-bound time keeping mechanisms.

d. In order to carry our the experiment outlined in the preceding paragraphs, Kennedy and Thorndike would have had to keep their interferometer operating perfectly for half a year while continuously ob serving the field of view through the telescope. Uninterrupted operation for so long a time was not feasible. The actual durations of their observations varied from eight days to a month. There were several such periods of observation at three-month time separations. From the data obtained in these periods, Kennedy and Thorndike were able to estimate that over a single six-month observation the number of periods n of relative delay would vary by less than the fraction $3 / 1000$of one period. Take the differential of the equation in part **c** to find the largest fractional change $d c / c$ of the round-trip speed of light between the two frames consistent with this estimated change in n (frame 1 - the "laboratory" frame-and frame 2 - the "rocket" frame - being in the present analysis Earth itself at two different times of year, with a relative velocity twice the speed of Earth in its orbit: 2×30 kilometers/second).

Historical note: At the time of the Michelson Morley experiment in 1887 , no one was ready for the idea that physics - including the speed of light - is the same in every inertial frame of reference. According to today's standard Einstein interpretation it seems obvious that both the Michelson-Morley and the Kennedy-Thorndike experiments should give null results. However, when Kennedy and Thorndike made their measurements in 1932 , two alternatives to the Einstein theory were open to consideration (designated here as theory A and theory B). Both A and B assumed the old idea of an absolute space, or "ether," in which light has the speed c. Both A and B explained the zero fringe shift in the Michelson Morley experiment by saying that all matter that moves at a velocity v (expressed as a fraction of light speed) relative to "absolute space" undergoes a shrinkage of its space dimensions in the direction of motion to a new length equal to $\left(1-v^2\right)^{1 / 2}$ times the old length ("Lorentz-Fitz Gerald contraction hypothesis"). The two theories differed as to the effect of "motion through absolute space" on the running rate of a clock. Theory A said, No effect. Theory B said that a standard seconds clock moving through absolute space at velocity v has a time between ticks of $\left(1-v^2\right)^{1 / 2}$ seconds. In theory B the ratio $\Delta l / T$ in the equation in part b will not be affected by the velocity of the clock, and the Kennedy-Thorndike experiment will give a null result, as observed ("complicated explanation for simple effect'). In theory A the ratio $\Delta l / T$ in the equation will be multiplied by the factor $\left(1-v_1^2\right)^{1 / 2}$ at a time of year when the "velocity of Earth relative to absolute space" is v_1 and multiplied by $\left(1-v_2^2\right)^{1 / 2}$ at a time of year when this velocity is v_2. Thus the fringes should shift from one time of year $\left(v_1=v_{\text {orbital }}+v_{\text {Sun }}\right)$ to another time of year $\left(v_2=v_{\text {orbital }}-v_{\text {Sun }}\right)$ unless by accident Sun happened to have "zero velocity relative to absolute space" - an accident judged so unlikely as not to provide an acceptable explanation of the observed null effect. Thus the Kennedy-Thorndike experiment ruled out theory A (length contraction alone) but allowed theory B (length contraction plus time contraction) - and also allowed the much simpler Einstein theory of equivalence of all inertial reference frames.

The "sensitivity" of the Kennedy-Thorndike experiment depends on the theory under consideration. In the context of theory A the observations set an upper limit of about 15 kilometers/second to the "speed of Sun through absolute space" (sensitivity reported in the Kennedy-Thorndike paper). In the context of Einstein's theory the observations say that the round-trip speed of light has the same numerical magnitude-within an error of about 3 meters/ second - in inertial frames of reference having a relative velocity of 60 kilometers/second.

Reference: R. J. Kennedy and E. M. Thorndike, Physical Review, Volume 42, pages 400-418 (1932).

3-14 things that move faster than light

Can "things" or "messages" move faster than light? Does relativity really say "No" to this possibility? Explore these questions further using the following examples.

a. **The Scissors Paradox**. A very long straight rod, inclined at an angle θ to the x-axis, moves downward with uniform speed v_{rod} as shown in the figure. Find the speed v_A of the point of intersection A of the lower edge of the stick with the x-axis. Can this speed be greater than the speed of light? If so, for what values of the angle θ and v_{rod} does this occur? Can the motion of intersection point A be used to transmit a message faster than light from someone at the origin to someone far out on the x-axis?

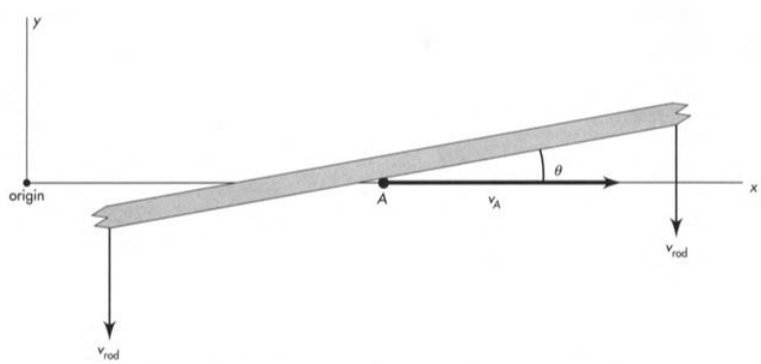

Figure $3.E.9$: Can the point of intersection A move with a speed v_A greater than the speed of light?

b. **Transmission of a Hammer Pulse**. Suppose the same rod is initially at rest in the laboratory with the point of intersection initially at the origin. The region of the rod centered at the origin is struck sharply with the downward blow of a hammer. The point of intersection moves to the right. Can this motion of the point of intersection be used to transmit a message faster than the speed of light?

c. **Searchlight Messenger?** A very powerful searchlight is rotated rapidly in such a way that its beam sweeps out a flat plane. Observers A and B are at rest on the plane and each the same distance from the searchlight but not near each other. How far from the searchlight must A and B be in order that the searchlight beam will sweep from A to B faster than a light signal could travel from A to B ? Before they took their positions, the two observers were given the following instruction:
To A : "When you see the searchlight beam, fire a bullet at B."
To B : "When you see the searchlight beam, duck because A has fired a bullet at you."
Under these circumstances, has a warning message traveled from A to B with a speed faster than that of light?

d. **Oscilloscope Writing Speed.** The manufacturer of an oscilloscope claims a writing speed (the speed with which the bright spot moves across the screen) in excess of the speed of light. Is this possible?

3-15 four times the speed of light?

We look westward across the United States and see the rocket approaching us at four times the speed of light.

> **Note**
>
> How can this be, since nothing moves faster than light?
>
> We did not say the rocket moves faster than light; we said only that we see it moving faster than light.

Here is what happens: The rocket streaks under the Golden Gate Bridge in San Francisco, emitting a flash of light that illuminates the rocket, the bridge, and the surroundings. At time Δt later the rocket threads the Gateway Arch in St. Louis that commemorates the starting point for covered wagons. The arch and the Mississippi riverfront are flooded by a second flash of light. The top figure is a visual summary of measurements from our continent-spanning latticework of clocks taken at this moment.

Now the rocket continues toward us as we stand in New York City. The center figure summarizes data taken as the first flash is about to enter our eye. Flash 1 shows us the rocket passing under the Golden Gate Bridge. An instant later flash 2 shows us the rocket passing through the Gateway Arch.

a. Answer the following questions using symbols from the first two figures. The images carried by the two flashes show the rocket how far apart in space? What is the time lapse between our reception of these two images? Therefore, what is the apparent speed of the approaching rocket we see? For what speed v of the rocket does the apparent speed of approach equal four times the speed of light? For what rocket speed do we see the approaching rocket to be moving at 99 times the speed of light?
b. Our friend in San Francisco is deeply disappointed. Looking eastward, she sees the retreating rocket traveling at less than half the speed of light (bottom figure). She wails, "Which one of us is wrong?" "Neither one." we reply. "No matter how high the speed v of the rocket, you will never see it moving directly away from you at a speed greater than half the speed of light." Use the bottom figure to derive an expression for the apparent speed of recession of the rocket. When we in New York see the rocket approaching at four times the speed of light, with what speed does our San Francisco friend see it moving away from her? When we see a faster rocket approaching at 99 times the speed of light, what speed of recession does she behold?

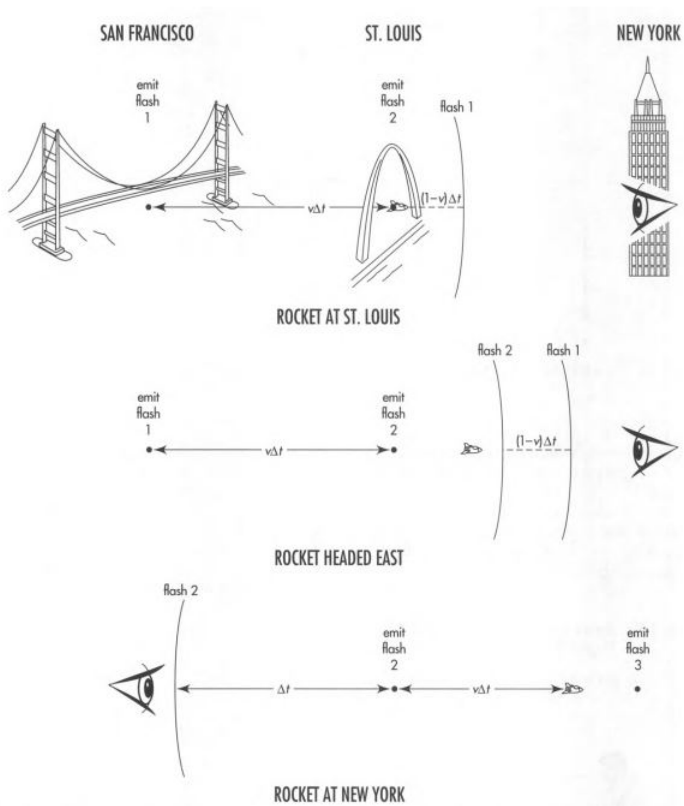

Figure $3.E.10$: **Top**: Rocket headed east, shown at the instant it passes under the Gateway Arch in St. Louis and emits flash 2. The rocket is chasing flash 1, emitted earlier as it passed under the Golden Gate Bridge in San Francisco. **Center**: The two image carrying flashes are close together, so they enter the eye in rapid succession. This gives the viewer the visual impression that the rocket moved from San Francisco to St. Louis in a very short time. **Bottom**: Rocket headed east, shown at the instant it approaches the Empire State Building in New York City and emits flash 3. When the rocket moves away from the viewer, the distance of rocket travel is added to the separation between flashes. This increases the ap parent time between flashes, giving the viewer the impression that the rocket moved from St. Louis to New York at less than one half light-speed.

3-16 superluminal expansion off quasar 3C273?

The most powerful sources of energy we know or conceive or see in all the universe are so-called quasi- stellar objects, or **quasars**, starlike sources of light located billions of light-years away. Despite being far smaller than any galaxy, the typical quasar manages to put out more than 100 times as much energy as our own Milky Way, with its hundred billion stars. Quasars, unsurpassed in brilliance and remoteness, we count today as lighthouses of the heavens.

One of the major problems associated with quasars is that some are composed of two or more components that appear to be separating from each other with relative velocity greater than the speed of light ("superluminal" velocity). One theory that helps explain this effect pictures the quasar as a core that ejects a jet of plasma at relativistic speed. Disturbances or instabilities in such a jet appear as discrete "knots" of plasma. The motion and light emission from a knot may account for its apparent greater-than-light speed, as shown using the first figure.

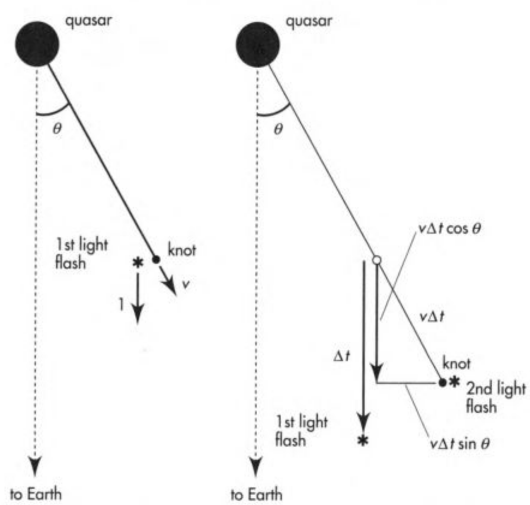

Figure $3.E.11$: **first figure. Left**: Bright "knot" of plasma ejected from a quasar at high speed v emits a first flash of light toward Earth. **Right**: The knot emits a second light flash toward Earth a time Δt later. This time Δt is measured locally near the knot using the Earth-linked latticework of rods and clocks (bar! bar!).

a. The first figure shows two Earth-directed light flashes emitted from the streaking knot. The time between emissions is Δt as measured locally near the knot using the Earth-linked latticework of rods and clocks. Of course the clock readings on this portion of the Earth-linked latticework are not available to us on Earth; therefore we cannot measure Δt directly. Rather, we see the time separation between the arrivals of the two flashes at Earth. From the figure, show that this Earth-seen time separation Δt_{seen} is given by the expression

$$\Delta t_{\text{seen}} = \Delta t(1 - v\cos\theta)$$

b. We have another disability in viewing the knot from Earth. We do not see the motion of the knot toward us, only the apparent motion of the knot across our field of view. Find an expression for this transverse motion (call it Δx_{seen}) between emissions of the two light flashes in terms of Δt.

c. Now calculate the speed v^x_{seen} of the rightward motion of the knot as seen on Earth. Show that the result is

$$v^x_{\text{seen}} = \frac{\Delta x_{\text{seen}}}{\Delta t_{\text{seen}}} = \frac{v\sin\theta}{1 - v\cos\theta}$$

d. What is the value of v^x_{seen} when the knot is emitted in the direction exactly toward Earth? when it is emitted perpendicular to this direction? Find an expression that gives the range of angles θ for which v^x_{seen} is greater than the speed of light. For $\theta = 45$ degrees, what is the range of knot speeds v such that v^x_{seen} is greater than the speed of light?

e. If you know calculus, find an expression for the angle $\theta_{\max}$ at which v^x_{seen} has its maximum value for a given knot speed v. Show that this angle satisfies the equation $\cos\theta_{\max} = v$. Whether or not you derive this result, use it to show that the maximum apparent transverse speed is seen as

$$v^x_{\text{seen, max}} = \frac{v}{(1 - v^2)^{1/2}}$$

f. What is this maximum transverse speed seen on Earth when $v = 0.99$?

g. The second figure shows the pattern of radio emission from the quasar $3C273$. The decreased period of radiation from this source (Exercise 3-10) shows that it is approximately 2.6×10^9 light-years from Earth. A secondary source is apparently moving away from the central quasar. Take your own measurements on the figure. Combine this with data from the figure caption to show that the apparent speed of separation is greater than 9 times the speed of light.

Note: As of 1990 , apparent greater-than-light-speed ("superluminal") motion has been observed in approximately 25 different sources.

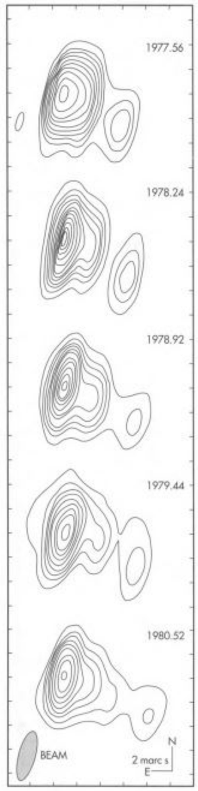

Figure $3.E.12$: **second figure**. Contour lines of radio emission from the quasar 3C273 showing a bright "knot" of plasma apparently moving away from it at a speed greater than the speed of light. The time of each image is given as calendar year and decimal fraction. Horizontal scale divisions are in units of 2 milli arc-seconds. (1 milli arc-second $=10^{-3}/3600$ degree $=4.85\times10^{-9}$ radian)

References: Analysis and first figure adapted from Denise C. Gabuzda, American Journal of Physics, Volume 55, pages 214-215 (1987). Second figure and data taken from T. J. Pearson, S. C. Unwin, M. H. Cohen, R. P. Linfield, A. C. S. Readhead, G. A. Seielstad, R. S. Simon, and R. C. Walker, Nature, Volume 290, pages 365-368 (2 April 1981).

3-17 Contraction or rotation?

A cube at rest in the rocket frame has an edge of length 1 meter in that frame. In the laboratory frame the cube is Lorentz contracted in the direction of motion, as shown in the figure. Determine this Lorentz contraction, for example, from locations of four clocks at rest and synchronized in the laboratory lattice with which the four corners of the cube, E, F, G, H, coincide when all four clocks read the same time. This latticework measurement eliminates time lags in the travel of light from different corners of the cube.

Now for a different observing procedure! Stand in the laboratory frame and look at the cube with one eye as the cube passes overhead. What one sees at any time is light that enters the eye at that time, even if it left the different corners of the cube at different times. Hence, what one sees visually may not be the same as what one observes using a latticework of clocks. If the cube is viewed from the bottom then the distance GO is equal to the distance HO, so light that leaves G and H simultaneously will arrive at O simultaneously. Hence, when one sees the cube to be overhead one will see the Lorentz contraction of the bottom edge.

a. Light from E that arrives at O simultaneously with light from G will have to leave E earlier than light from G left G. How much earlier? How far has the cube moved in this time? What is the value of the distance x in the right top figure?
b. Suppose the eye interprets the projection in the figures as a rotation of a cube that is not Lorentz contracted. Find an expression for the angle of apparent rotation ϕ of this uncontracted cube. Interpret this expression for the two limiting cases of cube speed in the laboratory frame: $v\rightarrow 0$ and $v\rightarrow 1$.
c. **Discussion question**: Is the word "really" an appropriate word in the following quotations?
(1) An observer using the rocket latticework of clocks says, "The stationary cube is really neither rotated nor contracted."

(2) Someone riding in the rocket who looks at the stationary cube agrees, "The cube is really neither rotated nor contracted."
(3) An observer using the laboratory latticework of clocks says, "The passing cube is really Lorentz contracted but not rotated."
(4) Someone standing in the laboratory frame looking at the passing cube says, "The cube is really rotated but not Lorentz contracted."
What can one rightfully say - in a sentence or two - to make each observer think it reasonable that the other observers should come to different conclusions?

d. The analysis of parts **b** and **c** assumes that the visual observer looks with one eye and has no depth perception. How will the cube passing overhead be perceived by the viewer with accurate depth perception?

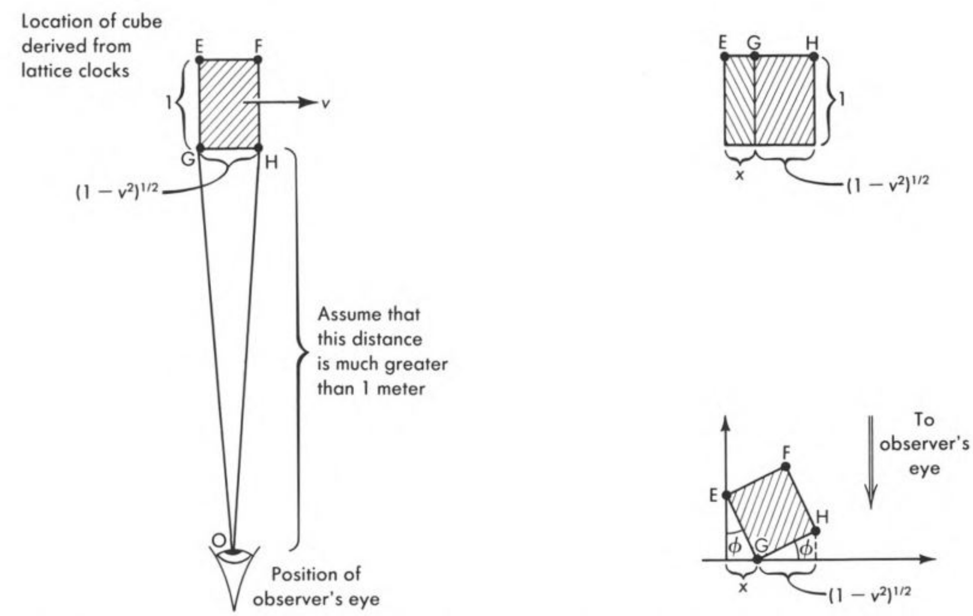

Figure $3.E.13$: **Left**: Position of eye of visual observer watching cube pass overhead. **Right top**: What the visual observer sees as she looks up from below. **Right bottom**: How the visual observer can interpret the projection of the second figure.

Reference: For a more complete treatment of this topic, see Edwin F. Taylor, Introductory Mechanics (John Wiley and Sons, New York, 1963), pages 346-360.