1.5: Unity of Spacetime

time and space: equal footing but distinct nature

When time and space are measured in the same unit - whether meter or second or year - the expression for the square of the spacetime interval between two events takes on a particularly simple form:

$$\begin{aligned}(\text{interval})^{2} &= (\text{ time separation })^{2}-(\text{ space separation })^{2} \\ &= t^{2}-x^{2} \qquad [\text{same units for time and space}]\end{aligned}$$

Spacetime is a unity

This formula shows forth the unity of space and time. Impressed by this unity, Einstein's teacher Hermann Minkowski (1864-1909) wrote his famous words, "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a union of the two will preserve an independent reality." Today this union of space and time is called spacetime. Spacetime provides the true theater for every event in the lives of stars, atoms, and people. Space is different for different observers. Time is different for different observers. Spacetime is the same for everyone.

Box 1.5.1: Payoff of the Parable from distance in space to interval in spacetime

DISCUSSION	SURVEYING TOWNSHIP	ANALYZING NATURE
Location marker	Steel stake driven in ground	Collision between two particles Emission of flash from atom Spark jumping from antenna to pen
General name for such a location marker	**Point** or **place**	**Event**
Can its location be staked out for all to see, independent of any scheme of measurement, and independent of all numbers?	Yes	Yes
Simple descriptor of separation between two location markers	**Distance**	**Spacetime interval**
Are there ways directly to measure this separation?	Yes	Yes
With enough markers already staked out, how can we tell someone where we want the next one?	Specify **distances** from other points.	Specify **spacetime intervals** from other events.
Instead of boldly staking out the new marker, or instead of positioning it relative to existing markers, how else can we place the new marker?	By locating point relative to a **reference frame**	By locating event relative to a **reference frame**
Nature of this reference frame?	**Surveyor's grid** yields northward and eastward readings of point (Chapter 1).	**Lattice frame** of rods and clocks yields space and time readings of event (Chapter 2).
Is such a reference frame unique?	No	No
How do two such reference frames differ from one another?	**Tilt** of one surveyor's grid relative to the other	**Uniform velocity** of one frame relative to the other
What are names of two such possible reference frames?	**Daytime grid**: oriented to magnetic north **Nighttime grid**: oriented to North- Star north	**Laboratory frame** **Rocket frame**

DISCUSSION	SURVEYING TOWNSHIP	ANALYZING NATURE
What common unit simplifies analysis of the results?	The unit **meter** for both northward and eastward readings	The unit **meter** for both space and time readings
What is the conversion factor from conventional units to meters?	Converting miles to meters: k= 1609.344 meters/mile	Converting seconds to meters using the speed of light: = 299,792,458 meters/second
For convenience, all measurements are referred to what location?	A common **origin** (center of town)	A common **event** (reference spark)
How do readings for a single marker differ between two reference frames?	Individual northward and eastward readings for one point — for one steel stake — do not have the same values respectively for two surveyors' grids that are tilted relative to one another.	Individual space and time readings for one event — for one spark — do not have the same values respectively for two frames that are in motion relative to one another.
When we change from one marker to two, how do we specify the offset between them in reference-frame language?	**Subtract**: Figure the difference between eastward readings of the two points; also the difference in northward readings.	**Subtract**: Figure the difference between space readings of the two events; also the difference in time readings.
language?How to figure from offset readings a measure of separation that has the same value whatever the choice of reference frame?	Figure the distance **between** the two points.	Figure the **spacetime interval** between the two events.
Figure how?	$(\text{ distance })^2=\left(\begin{array}{c}\text{difference in}\\\text{northward readings}\end{array}\right)^2+\left(\begin{array}{c}\text{difference in}\\\text{eastward readings}\end{array}\right)^2$	$(\text{ interval })^2=\left(\begin{array}{c}\text{difference in}\\\text{time readings}\end{array}\right)^2-\left(\begin{array}{c}\text{difference in}\\\text{space readings}\end{array}\right)^2$
Result of this reckoning?	Distance between points as figured from readings using one surveyor's grid is the **same** as figured from readings using a second surveyor's grid tilted with respect to first grid.	Interval between events as figured from readings using one lattice- work frame is the **same** as figured from readings using a second frame in steady straight-line motion relative to first frame.
Phrase to summarize this identity of separation as figured in two reference frames?	**Invariance of the distance** between points	**Invariance of the spacetime interval** between events.
Conclusions from this analysis?	(1) Northward and eastward dimensions are part of a single entity: **space**. (2) **Distance** is the simple measure of separation between two points, natural because invariant: the same for different surveyor grids.	(1) Space and time dimensions are part of a single entity: **spacetime**. (2) **Spacetime interval** is the simple measure of separation between two **events**, natural because invariant: the same for different reference frames.

Minkowski's insight is central to the understanding of the physical world. It focuses attention on those quantities, such as spacetime interval, electrical charge, and particle mass, that are the same for all observers in relative motion. It brings out the merely relative character of quantities such as velocity, momentum, energy, separation in time, and separation in space that depend on relative motion of observers.

Today we have learned not to overstate Minkowski's argument. It is right to say that time and space are inseparable parts of a larger unity. It is wrong to say that time is identical in quality with space.

Question and Answer

Why is it wrong? Is not time measured in meters, just as space is? In relating the positions of two steel stakes driven into the ground, does not the surveyor measure northward and eastward separations, quantities of identical physical character? By analogy, in locating two events is not the observer measuring quantities of the same nature: space and time separations? How else could it be legitimate to treat these quantities on an equal footing, as in the formula for the interval?

Answer

Equal footing, yes; same nature, no. There is a minus sign in the formula for the interval squared = (time separation)^2 — (space separation)^2 that no sleight of hand can ever conjure away. This minus sign distinguishes between space and time. No twisting or turning can ever give the same sign to real space and time separations in the expression for the interval.

The invariance of the spacetime interval evidences the unity of space and time while also preserving — in the formula's minus sign — the distinction between the two.

The principles of special relativity are remarkably simple—simpler than the axioms of Euclidean geometry or the principles of operating an automobile. Yet both Euclid and the automobile have been mastered—perhaps with insufficient surprise — by generations of ordinary people. Some of the best minds of the twentieth century struggled with the concepts of relativity, not because nature is obscure, but because (1) people find it difficult to outgrow established ways of looking at nature, and (2) the world of the very fast described by relativity is so far from common experience that everyday happenings are of limited help in developing an intuition for its descriptions.

By now we have won the battle to put relativity in understandable form. The concepts of relativity can now be expressed simply enough to make it easy to think correctly — "to make the bad difficult and the good easy." This leaves only the second difficulty, that of developing intuition — a practiced way of seeing. We understand distance intuitively from everyday experience. Box 1.1 applies our intuition for **distance in space** to help our intuition for **interval in spacetime**.

To put so much into so little, to subsume all of Einstein's teaching on light and motion in the single word *spacetime*, is to cram a wealth of ideas into a small picnic basket that we shall be unpacking throughout the remainder of this book.