

2.E: Floating Free (Exercises)

PRACTICE

2.1 human cannonball

A person rides in an elevator that is shot upward out of a cannon. Think of the elevator after it leaves the cannon and is moving freely in the gravitational field of Earth. Neglect air resistance.

- a. While the elevator is still on the way up, the person inside jumps from the "floor" of the elevator. Will the person
 - (1) fall back to the "floor" of the elevator?
 - (2) hit the "ceiling" of the elevator?
 - (3) do something else? If so, what?
- b. The person waits to jump until after the elevator has passed the top of its trajectory and is falling back toward Earth. Will your answers to part a be different in this case?
- c. How can the person riding in the elevator tell when the elevator reaches the top of its trajectory?

2.2 free-float bounce

Test your skill as an acrobat and contortionist! Fasten a weight-measuring bathroom scale under your feet and bounce up and down on a trampoline while reading the scale. Describe readings on the scale at different times during the bounces. During what part of each jump will the scale have zero reading? Neglecting air resistance, what is the longest part of the cycle during which you might consider yourself to be in a free-float frame?

2.3 practical synchronization of clocks

You are an observer in the laboratory frame stationed near a clock with spatial coordinates $x = 6$ lightseconds, $y = 8$ lightseconds, and $z = 0$ light-seconds. You wish to synchronize your clock with the one at the origin. Describe in detail and with numbers how to proceed.

2.4 synchronization by a traveling clock

Mr. Engelsberg does not approve of our method of synchronizing clocks by light flashes (Section 2.6).

- a. "I can synchronize my clocks in any way I choose!" he exclaims. Is he right?

Mr. Engelsberg wishes to synchronize two identical clocks, named Big Ben and Little Ben, which are relatively at rest and separated by one million kilometers, which is 10^9 meters or approximately three times the distance between Earth and Moon. He uses a third clock, identical in construction with the first two, that travels with constant velocity between them. As his moving clock passes Big Ben, it is set to read the same time as Big Ben. When the moving clock passes Little Ben, that outpost clock is set to read the same time as the traveling clock.
- b. "Now Big Ben and Little Ben are synchronized," says Mr. Engelsberg. Is he right?
- c. How much out of synchronism are Big Ben and Little Ben as measured by a latticework of clocks - at rest relative to them both - that has been synchronized in the conventional manner using light flashes? Evaluate this lack of synchronism in milliseconds when the traveling clock that Mr. Engelsberg uses moves at 360,000 kilometers/hour, or 10^5 meters/ second.
- d. Evaluate the lack of synchronism when the traveling clock moves 100 times as fast.
- e. Is there any earthly reason - aside from matters of personal preference - why we all should not adopt the method of synchronization used by Mr. Engelsberg?

2.5 Earth's surface as a free-float frame

Many experiments involving fast-moving particles and light itself are observed in earthbound laboratories. Typically these laboratories are not in free fall! Nevertheless, under many circumstances laboratories fixed to the surface of Earth can satisfy the conditions required to be called free-float frames. An example:

a. In an earthbound laboratory, an elementary particle with speed $v = 0.96$ passes from side to side through a cubical spark chamber one meter wide. For what length of laboratory time is this particle in transit through the spark chamber? Therefore for how long a time is the experiment "in progress"? How far will a separate test particle, released from rest, fall in this time? [Distance of fall from rest $= (1/2)gt_{\text{sec}}^2$, where g = acceleration of gravity ≈ 10 meters/second² and t_{sec} is the time of free fall in seconds.] Compare your answer with the diameter of an atomic nucleus (a few answer with the diameter of an atomic nucleus (a few times 10^{-15} meter).

b. How wide *can* the spark chamber be and still be considered a free-float frame for this experiment? Suppose that by using sensitive optical equipment (an **interferometer**) you can detect a test particle change of position as small as one wavelength of visible light, say 500 nanometers $= 5 \times 10^{-7}$ meter. How long will it take the test particle to fall this distance from rest? How far does the fast elementary particle of part a move in that time? Therefore how long can an earthbound spark chamber be and still be considered free-float for this sensitivity of detection?

2.6 horizontal extent of free-float frame near Earth

Consider two ball bearings near the surface of Earth and originally separated horizontally by 20 meters (Section 2.3). Demonstrate that when released from rest (relative to Earth) the particles move closer together by 1 millimeter as they fall 315 meters, using the following method of similar triangles or some other method.

Each particle falls from rest toward the center of Earth, as indicated by arrows in the figure. Solve the problem using the ratio of sides of similar triangles abc and $a'b'c'$. These triangles are upside down with respect to each other. However, they are similar because their respective sides are parallel: Sides ac and $a'c'$ are parallel to each other, as are sides bc and $b'c'$ and sides ab and $a'b'$. We know the lengths of some of these sides. Side $a'c' = 315$ meters is the height of fall (greatly exaggerated in the diagram); side ac is effectively equal to the radius of Earth, 6,371,000 meters. Side $ab = (1/2)(20 \text{ meters})$ equals half the original separation of the particles. Side $a'b'$ equals HALF their CHANGE in separation as they fall onto Earth's surface. Use the ratio of sides of similar triangles to find this "half-change" and therefore the entire change in separation as two particles initially 20 meters apart horizontally fall from rest 315 meters onto the surface of Earth.

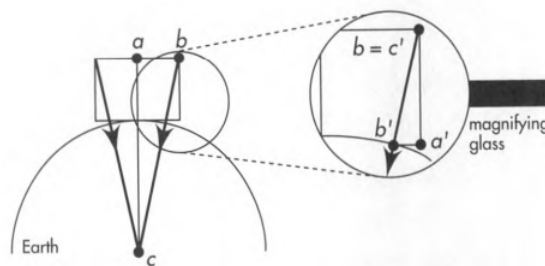


Figure 2.E. 1: Schematic diagram of two ball bearings falling onto Earth's surface. Not to scale.

2.7 limit on free-float frame near Earth's Moon

Release two ball bearings from rest a horizontal distance 20 meters apart near the surface of Earth's Moon. By how much does the separation between them decrease as they fall 315 meters? How many seconds elapse during this 315-meter fall? Assume that an initial vertical separation of 20 meters is increased by twice the change in horizontal separation in a fall through the same height. State clearly and completely the dimensions of the region of spacetime in which such a freely falling frame constitutes an inertial frame (to the given accuracy). Moon radius equals 1738 kilometers. Gravitational acceleration at Moon's surface: $g = 1.62$ meters/second².

2.8 vertical extent of free-float frame near Earth

Note: This exercise makes use of elementary calculus and the Newtonian theory of gravitation.

A paragraph in Section 2.3 says:

As another example, drop the same antique [20-meterlong] railway coach from rest in a vertical orientation, with the lower end of the coach initially 315 meters from the surface of Earth (Figure 2.3.1, right). Again release two tiny ball bearings from rest at opposite ends of the coach. In this case, during the time of fall [8 seconds], the ball bearings move apart by a distance of two millimeters because of the greater gravitational acceleration of the one nearer Earth, as Newton would put it. This is twice the change that occurs for horizontal separation.

Demonstrate this 2-millimeter increase in separation. The following outline may be useful. Take the gravitational acceleration at the surface of Earth to be $g_0 = 9.8 \text{ meters/second}^2$ and the radius of Earth to be $r_0 = 6.37 \times 10^6 \text{ meters}$. More generally, the gravitational acceleration g of a particle of mass m a distance r from the center of Earth (mass M) is given by the expression

$$g = \frac{F}{m} = \frac{GM}{r^2} = \frac{GM}{r_0^2} \frac{r_0^2}{r^2} = \frac{g_0 r_0^2}{r^2}$$

- Take the differential of this equation for g to obtain an approximate algebraic expression for Δg , the change in g , for a small change Δr in height.
- Now use $\Delta y = \frac{1}{2} \Delta g t^2$ to find an algebraic expression for increase in distance Δy between ball bearings in a fall that lasts for time t .
- Substitute numbers given in the quotation above to verify the 2 -millimeter change in separation during fall.

2.9 the rising railway coach

You are launched upward inside a railway coach in a horizontal position with respect to the surface of Earth, as shown in the figure. After the launch, but while the coach is still rising, you release two ball bearings at opposite ends of the train and at rest with respect to the train.

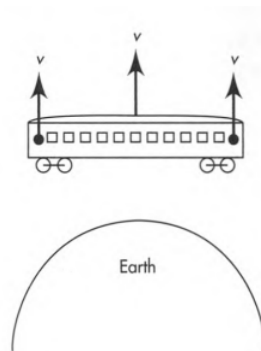


Figure 2.E. 2: Free-float railway coach rising from Earth's surface, as observed in Earth frame. Two ball bearings were just released from rest with respect to the coach. What will be their subsequent motion as observed from inside the coach? Figure not to scale.

- Riding inside the coach, will you observe the distance between the ball bearings to increase or decrease with time?
- Now you ride in a second railway coach launched upward in a *vertical* position with respect to the surface of Earth (not shown). Again you release two ball bearings at opposite ends of the coach and at rest with respect to the coach. Will you observe these ball bearings to move together or apart?
- In either of the cases described above, can you, the rider in the railway coach, distinguish whether the coach is rising or falling with respect to the surface of Earth solely by observing the ball bearings from inside the coach? What do you observe at the moment the coach stops rising with respect to Earth and begins to fall?

2.10 test particle?

- Verify the statement in Section 2.5 that a candidate test particle of mass 10 kilograms placed 0.1 meter from a less massive particle (initially stationary with respect to it), draws the second toward it by 1 millimeter in less than 3 minutes. If this relative motion is detectable by equipment in use at the test site, the result disqualifies the 10 -kilogram particle as a "test particle." Assume that both particles are spherically symmetric. Use Newton's Law of Gravitation:

$$F = \frac{GMm}{r^2}$$

where the gravitation constant G has the value $G = 6.673 \times 10^{-11} \text{ meter}^3 / (\text{kilogram-second}^2)$. Assume that this force does not change appreciably as the particles decrease separation by one millimeter.

- Section 2.3 describes two ball bearings released 20 meters apart horizontally in a freely falling railway coach. They move 1 millimeter closer together during 8 seconds of free fall, showing the limitations on this inertial frame. Verify that these ball bearings

qualify as test particles by estimating the distance that one will move from rest in 8 seconds under the gravitational attraction of the other, if both were initially at rest in interstellar space far from Earth. Make your own estimate of the mass of each ball bearing.

PROBLEMS

2.11 communications storm!

Sun emits a tremendous burst of particles that travels toward Earth. An astronomer on Earth sees the emission through a solar telescope and issues a warning. The astronomer knows that when the particles arrive, they will wreak havoc with broadcast radio transmission. Communications systems require three minutes to switch from broadcast to underground cable. What is the maximum speed of the particle pulse emitted by Sun such that the switch can occur in time, between warning and arrival of the pulse? Take Sun to be 500 light-seconds from Earth.

2.12 the Dicke experiment

a. The Leaning Tower of Pisa is about 55 meters high. Galileo says, "The variation of speed in air between balls of gold, lead, copper, porphyry, and other heavy materials is so slight that in a fall of 100 cubits [about 46 meters] a ball of gold would surely not outstrip one of copper by as much as four fingers. Having observed this I came to the conclusion that in a medium totally devoid of resistance all bodies would fall with the same speed.

Taking four fingers to be equal to 7 centimeters, find the maximum fractional difference in the acceleration of gravity $\Delta g/g$ between balls of gold and copper that would be consistent with Galileo's experimental result.

b. The result of the more modern Dicke experiment is that the fraction $\Delta g/g$ is not greater than 3×10^{-11} . Assume that the fraction has this more recently determined maximum value. Reckon how far behind the first ball the second one will be when the first reaches the ground if they are dropped simultaneously from the top of a 46-meter vacuum chamber. Under these same circumstances, how far would balls of different materials have to fall in a vacuum in a uniform gravitational field of 10 meters /second/second for one ball to lag behind the other one by a distance of 1 millimeter? Compare this distance with the Earth-Moon separation (3.8×10^8 meters). Clearly the Dicke experiment was not carried out using falling balls!

c. A plumb bob of mass m hangs on the end of a long line from the ceiling of a closed room, as shown in the first figure (left). A very massive sphere at one side of the closed room exerts a horizontal gravitational force mg_s on the plumb bob, where $g_s = GM/R^2$, M is the mass of the large sphere, and R the distance between plumb bob and the center of the sphere. This horizontal force causes a static deflection of the plumb line from the vertical by the small angle ε . (Similar practical example: In northern India the mass of the Himalaya Mountains results in a slight sideways deflection of plumb lines, causing difficulties in precise surveying.) The sphere is now rolled around to a corresponding position on the other side of the room (right), causing a static deflection of the plumb by an angle ε of the same magnitude but in the opposite direction.

Now the angle ε is very small. (Deflection due to the Himalayas is about 5 seconds of arc, which equals 0.0014degrees.) However, as the sphere is rolled around and around outside the closed room, an observer inside the room can measure the gravitational field g_s due to the sphere by measuring with greater and greater precision the total deflection angle $2\varepsilon \approx 2 \sin \varepsilon$ of the plumb line, where ε is measured in radians. Derive the equation that we will need in the calculation of g_s .

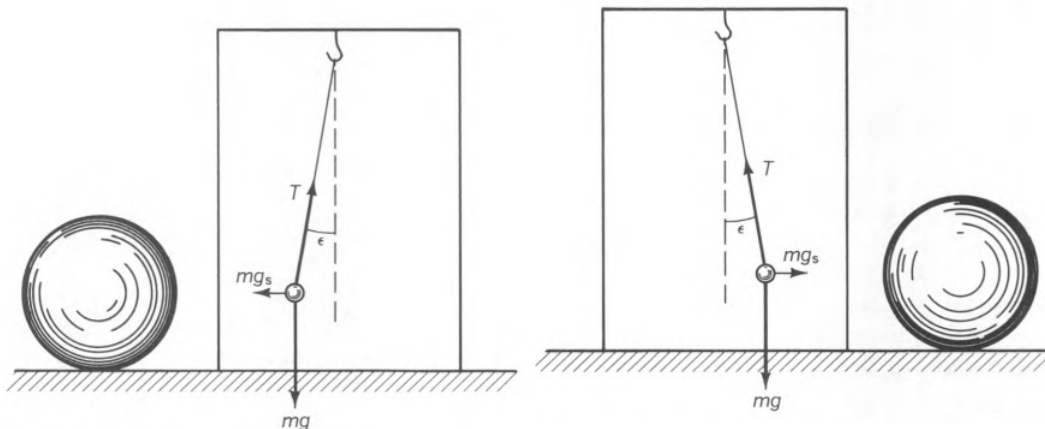


Figure 2.E. 3: **Left:** Nearby massive sphere results in static deflection of plumb line from vertical

Right: Rolling the sphere to the other side results in static deflection of plumb line in the opposite direction.

d. We on Earth have a large sphere effectively rolling around us once every day. It is the most massive sphere in the solar system: Sun itself? What is the value of the gravitational acceleration $g_s = GM/R^2$ due to Sun at the position of Earth? (Some constants useful in this calculation appear inside the back cover of this book.)

e. One additional acceleration must be considered that, however, will not enter our final comparison of gravitational acceleration g_s for different materials. This additional acceleration is the centrifugal acceleration due to the motion of Earth around Sun. When you round a corner in a car you are pressed against the side of the car on the outward side of the turn. This outward force-called the centrifugal pseudoforce or the centrifugal inertial force - is due to the acceleration of your reference frame (the car) toward the center of the circular turn. This centrifugal inertial force has the value mv_{conv}^2/r , where v_{conv} is the speed of the car in conventional units and r is the radius of the turn. Now Earth moves around Sun in a path that is nearly circular. Sun's gravitational force mg_s acts on a plumb bob in a direction toward Sun; the centrifugal inertial force mv_{conv}^2/R acts in a direction away from Sun. Compare the "centrifugal acceleration" v_{conv}^2/R at the position of Earth with the oppositely directed gravitational acceleration g_s calculated in part *d*. What is the net acceleration toward or away from Sun of a particle riding on Earth as observed in the (accelerated) frame of Earth?

f. Of what use is the discussion thus far? A plumb bob hung near the surface of Earth experiences a gravitational acceleration g_s toward Sun - and an equal but opposite centrifugal acceleration mv_{conv}^2/R away from Sun. Therefore - in the accelerating reference frame of Earth - the bob experiences no net force at all due to the presence of Sun. Indeed this is the method by which we constructed an inertial frame in the first place (Section 2.2): Let the frame be in free fall about the center of gravitational attraction. A particle at rest on Earth's surface is in free fall about Sun and therefore experiences no net force due to Sun. What then does all this have to do with measuring the equality of gravitational acceleration for particles made of different substances - the subject of the Dicke experiment? Answer: Our purpose is to detect the difference - if any - in the gravitational acceleration g_s toward Sun for different materials. The centrifugal acceleration v^2/R away from Sun is presumably the same for all materials and therefore need not enter any comparison of different materials.

Consider a torsion pendulum suspended from its center by a thin quartz fiber (second figure). A light rod of length L supports at its ends two bobs of equal mass made of different materials - say aluminum and gold. Suppose that the gravitational acceleration g_1 of the gold due to Sun is slightly greater than the acceleration g_2 of the aluminum due to Sun. Then there will be a slight net torque on the torsion pendulum due to Sun. For the position of Sun shown at left in the figure, show that the net torque is counterclockwise when viewed from above. Show also that the magnitude of this net torque is given by the expression

$$\begin{aligned}\text{torque} &= mg_1 L/2 - mg_2 L/2 = m(g_1 - g_2) L/2 \\ &= mg_s (\Delta g/g_s) L/2\end{aligned}$$

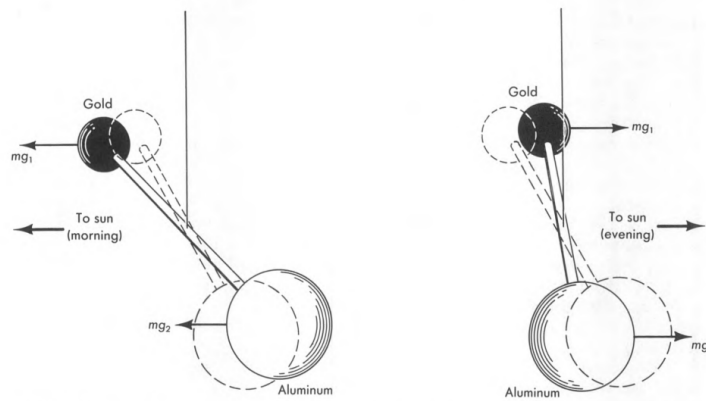


Figure 2.E.4: **second figure.** Schematic diagram of the Dicke experiment. **Left:** Hypothetical effect: morning. **Right:** Hypothetical effect: evening. Any difference in the gravitational acceleration of Sun for gold and aluminum should result in opposite sense of net torque on torsion pendulum in the evening compared with the morning. The large aluminum ball has the same mass as the small high-density gold ball.

g. Suppose that the fraction $(\Delta g/g_s)$ has the maximum value 3×10^{-11} consistent with the results of the final experiment, that L has the value 0.06 meters, and that each bob has a mass of 0.03 kilograms. What is the magnitude of the net torque? Compare this to the torque provided by the added weight of a bacterium of mass 10^{-15} kilogram placed on the end of a meter stick balanced at its center in the gravitational field of Earth.

h. Sun moves around the heavens as seen from Earth. Twelve hours later Sun is located as shown at right in the second figure. Show that under these changed circumstances the net torque will have the same magnitude as that calculated in part *g* but now will be clockwise as viewed from above - in a sense opposite to that of part *g*. This change in the sense of the torque every twelve hours allows a small difference $\Delta g = g_1 - g_2$ in the acceleration of gold and aluminum to be detected using the torsion pendulum. As the torsion pendulum jiggles on its fiber because of random motion, passing trucks, Earth tremors and so forth, one needs to consider only those deflections that keep step with the changing position of Sun.

i. A torque on the rod causes an angular rotation of the quartz fiber of θ radians given by the formula

$$\text{torque} = k\theta$$

where k is called the **torsion constant** of the fiber. Show that the maximum angular rotation of the torsion pendulum from one side to the other during one rotation of Earth is given by the expression

$$\theta_{\text{tot}} = \frac{mg_s L}{k} \left(\frac{\Delta g}{g_s} \right)$$

j. In practice Dicke's torsion balance can be thought of as consisting of 0.030-kilogram gold and aluminum bobs mounted on the ends of a beam 6×10^{-2} meter in length suspended in a vacuum on a quartz fiber of torsion constant 2×10^{-8} newton meter/radian. A statistical analysis of the angular displacements of this torsion pendulum over long periods of time leads to the conclusion that the fraction $\Delta g/g$ for gold and aluminum is less than 3×10^{-11} . To what mean maximum angle of rotation from side to side during one rotation of Earth does this correspond? Random motions of the torsion pendulum - noise! - are of much greater amplitude than this; hence the need for the statistical analysis of the results.

References: R. H. Dicke, "The Eötvös Experiment," *Scientific American*, Volume 205, pages 84-94 (December, 1961). See also P. G. Roll, R. Krotkov, and R. H. Dicke, *Annals of Physics*, Volume 26, pages 442 – 517 (1964). The first of these articles is a popular exposition written early in the course of the Dicke experiment. The second article reports the final results of the experiment and takes on added interest because of its account of the elaborate precautions required to insure that no influence that might affect the experiment was disregarded. Galileo quote from Galileo Galilei, *Dialogues Concerning Two New Sciences*, translated by Henry Crew and Alfonso de Salvio (Northwestern University Press, Evanston, Illinois, 1950).

2.13 deflection of starlight by Sun

Estimate the deflection of starlight by Sun using an elementary analysis. Discussion: Consider first a simpler example of a similar phenomenon. An elevator car of width L is released from rest near the surface of Earth. At the instant of release a flash of light is

fired horizontally from one wall of the car toward the other wall. After release the elevator car is an inertial frame. Therefore the light flash crosses the car in a straight line with respect to the car. With respect to Earth, however, the flash of light is falling - because the elevator is falling. Therefore a light flash is deflected in a gravitation field, as Newton would phrase it. (How would Einstein phrase it? See Chapter 9.) As another example, a ray of starlight in its passage tangentially across Earth's surface receives a gravitational deflection (over and above any refraction by Earth's atmosphere). However, the time to cross Earth is so short, and in consequence the deflection so slight, that this effect has not yet been detected on Earth. At the surface of Sun, however, the acceleration of gravity has the much greater value of 275 meters/ second/second. Moreover, the time of passage across the surface is much increased because Sun has a greater diameter, 1.4×10^9 meters. In the following, assume that the light just grazes the surface of Sun in passing.

- a. Determine an "effective time of fall" from the diameter of Sun and the speed of light. From this time of fall deduce the net velocity of fall toward Sun produced by the end of the whole period of gravitational interaction. (The maximum acceleration acting for this "effective time" produces the same net effect [calculus proof] produced by the actual acceleration - changing in magnitude and direction along the path - in the entire passage of the ray through Sun's field of force.)
- b. Comparing the lateral velocity of the light with its forward velocity, deduce the angle of deflection. The accurate analysis of special relativity gives the same result. However, Einstein's 1915 general relativity predicted a previously neglected effect, associated with the change of lengths in a gravitational field, that produces something like a supplementary refraction of the ray of light and doubles the predicted deflection. [Deflection observed in 1947 eclipse of Sun: $(9.8 \pm 1.3) \times 10^{-6}$ radian; in the 1952 eclipse: $(8.2 \pm 0.5) \times 10^{-6}$ radian..]

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