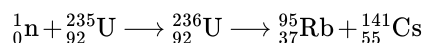


8.7: Converting Mass to Usable Energy- Fission, Fusion, Annihilation

fission and fusion both slide down the energy hill toward the minimum, iron, electron and positron annihilate to yield two energetic photons.

For a final perspective on the evanescence of mass and the preservation of momentum, turn from processes where mass is created to three processes in which mass is destroyed: fission, fusion, and annihilation.

Anyone who first hears about the splitting of a nucleus (fission) as a source of energy, and the joining of two nuclei (fusion) also as a source of energy might gain the mistaken impression that a perpetual motion machine has been invented. Could we split and join the same nucleus over and over again, each time releasing energy? No. Here's why. Fission occurs in the splitting of uranium, for instance when a neutron strikes a uranium nucleus:

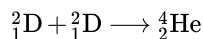


Fission and fusion: Both go from looser to tighter binding

In this equation the lower-left subscript tells the number of protons in the given nucleus and the upper-left superscript shows number of protons plus neutrons in the nucleus. The process described by this equation rearranges the 236 nucleons, that is, 92 protons plus 144 neutrons, into a configuration that comes a bit closer to that most stable of all available nuclear configurations, the iron nucleus:



But fusion too, for example the process of uniting two rather light nuclei such as "heavy hydrogen" or deuterons to form a helium nucleus,



can also be regarded as one step along the way toward rearranging nucleons (protons and neutrons) to achieve the iron configuration or something like it.

In brief, we can get energy out of nucleon rearrangement processes that move from looser binding of both heavier and lighter nuclei toward tighter binding of the (intermediate-mass) iron nucleus (Figure 8-10). In neither fission nor fusion, however, is the fraction of mass converted into energy as great as one percent. (For an example of fusion reaction in Sun, see Sample Problem 8-5, especially c.)

Annihilation converts 100% of matter into radiation

Annihilation is interesting because it can convert 100 percent of matter into radiation. Annihilation is interesting, too, because it has been demonstrated on the microscopic scale. A slow positive electron, a positron, joining up by chance to orbit with an everyday negative electron, eventually unites with it to annihilate them both and produce sometimes two, sometimes three light quanta (photons-called gamma rays in the case of these high energies):

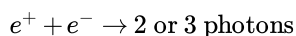


Figure 8.7.2 displays the balance of energy and momentum in the two-quantum annihilation process.

Note

The ideal gas law is easy to remember and apply in solving problems, as long as you get the **proper values a**

Figure 8.7.1: Both the conversion of deuterium to the more massive helium in fusion and the conversion of uranium to lighter nuclei in fission decrease the mass per nucleon, both toward the most stable of nuclei, iron.

? Exercise 8.7.1

Why 2 or 3 photons? Why can't just a single photon be emitted in this process? '

Note

Brief answer: Conservation of momentum. Fuller answer: Look at Figure 8.7.2 Before annihilation, the system has zero total momentum. A single photon remaining after the annihilation could not have zero momentum, no matter in which direction it moved! The presence of a single photon after the collision could not satisfy conservation of momentum. So annihilation never does and never can end up giving only a single photon.

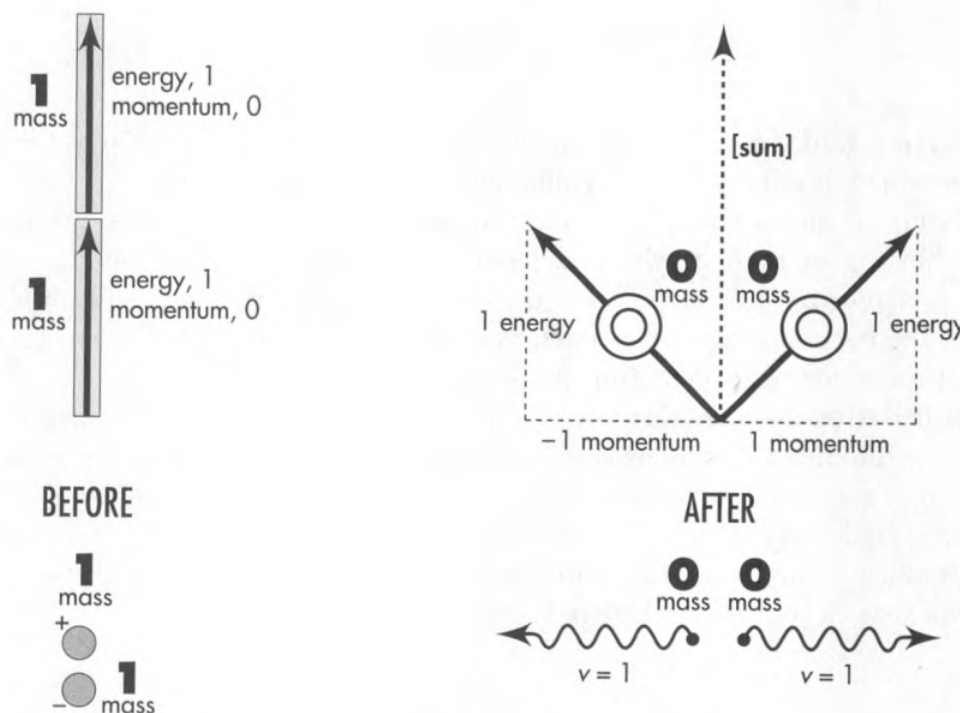


Figure 8.7.2: Momenergy conservation in the two-photon electron-positron annihilation process. Before: Before annihilation each oppositely charged particle has rest energy and no momentum. After: The two particles have annihilated, creating two high-energy photons (gamma rays). The two photons fly apart in opposite directions; total momentum remains zero.

Box 8.2 ANALYZING A PARTICLE ENCOUNTER

Conservation of total momenergy! In any given free-float frame that means conservation of total energy and conservation of each of three components of total momentum. In no way does the power and scope of this principle make itself felt more memorably than the analysis of simple encounters of this, that, and the other kind in an isolated system of particles. "Analyzing an encounter" means using conservation laws and other relations to find unknown masses, energies, and momenta of particles in terms of known quantities. Sometimes a complete analysis is not possible; the information provided may be insufficient. Here are suggested steps in analyzing an encounter. Sample Problems 8.7.1 and 8.7.2 illustrate these methods.

1. Draw a diagram of particles before and particles after the interaction. Label particles entering with numbers or letters and particles leaving with different numbers or letters (even if they are the same particles). Use arrows to show particle directions of motion and label with symbols their masses, energies, and momenta, whether initially known or unknown.
2. Write down algebraically the conservation of total energy. Do not forget to include the rest energy - the mass m - of any particle not moving in the chosen free-float frame.
3. Write down algebraically the conservation of total momentum. Do not forget that momentum is a vector. In general this means demanding conservation of each of three components of total momentum.
4. Try to solve for unknowns in terms of knowns, still using symbols.
 - a. Make liberal use of the relation $m^2 = E^2 - p^2$, where $p^2 = p_x^2 + p_y^2 + p_z^2$. For a photon or neutrino, mass equals zero and $E = p$ (in magnitude: Pay attention to the direction of the momentum vector p - or its sign if motion is in one space

dimension).

b. Do NOT use speed v of a particle unless forced to by requirements of the problem. Relativistic particles typically move with speeds very close to light speed, so speed proves to be a poor measure of significance. Increase by one percent the speed of a particle moving at $v = 0.99$ and you increase its energy by a factor of almost 10.

c. Substitute numerical values into resulting equations as late as possible. Before substituting numerical values, check that all values are expressed in concordant units.

5. Check your result. Check units of the solution. Is the order of magnitude of numerical results reasonable? Substitute limiting values, for example letting energy of an incoming particle become very large (and very small). Is the limiting-case result reasonable?

Is there any general conclusion you can draw from your specific solution? Does this exercise illustrate a deep principle or lead to an even more interesting application of conservation laws? 240 CHAPTER 8 COLLIDE. CREATE. ANNIHILATE.

✓ Example 8.7.1

SYMMETRIC ELASTIC COLLISION

A proton of mass m and kinetic energy K in the laboratory frame strikes a proton initially at rest in that frame. The two protons undergo a symmetric elastic collision: the outgoing protons move in directions that make equal and opposite angles $\theta/2$ with the line of motion of the original incoming particle. Find energy and momentum of each outgoing particle and angle θ between their outgoing directions of motion for this symmetric case. Historical note: When impact speed is small compared to the speed of light, this separation of directions, θ , is 90 degrees, according to Newtonian mechanics. Early cloud-chamber tracks sometimes showed symmetric collisions with angles of separation substantially less than 90 degrees, thereby giving evidence for relativistic mechanics and providing the first reliable measurements of impact energy.

Solution

following steps in Box 8-2

1. **Draw a diagram** and label all four particles with letters:

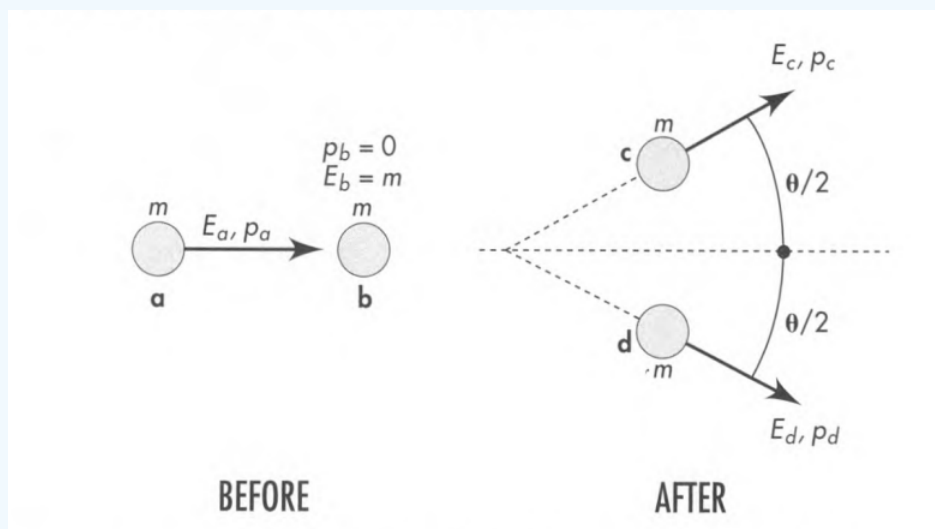


Figure 8.7.3

2. **Conservation of energy:** Energy of each particle equals mass plus kinetic energy. And the masses don't change in this reaction. Therefore total kinetic energy after the encounter (divided equally between the two particles) equals the (known) total kinetic energy before the encounter, all localized on one particle. In brief: $K_c = K_d = K_a/2 = K/2$. Simple answer to one of the three questions we were asked!

3. **Conservation of momentum:** By symmetry, the vertical components of momenta of the outgoing particles cancel. Horizontal components add, leading to the relation

$$p_{\text{tot}} = p_a = p_c \cos(\theta/2) + p_d \cos(\theta/2) = 2p_d \cos(\theta/2)$$

or, in brief,

$$p_a = 2p_d \cos(\theta/2) \quad [\text{conservation of momentum}]$$

4. Solve for the unknown angle θ : Along the way find the other requested quantity, the magnitude $p_c = p_d$ of the momenta after the collision. To that end, first find the momentum p_a before the collision, using the general formula for the momentum of an individual particle:

$$\begin{aligned} p &= [E^2 - m^2]^{1/2} = [(K + m)^2 - m^2]^{1/2} = (K^2 + 2mK + m^2 - m^2)^{1/2} \\ &= (K^2 + 2mK)^{1/2} \end{aligned}$$

Therefore

$$p_a = (K^2 + 2mK)^{1/2}$$

From conservation of energy, $K_c = K_d = K/2$. Therefore

$$p_d = [(K/2)^2 + 2m(K/2)]^{1/2}$$

Substitute these expressions for p_a and p_d into the equation for conservation of momentum:

$$(K^2 + 2mK)^{1/2} = 2[(K/2)^2 + 2m(K/2)]^{1/2} \cos(\theta/2)$$

Square both sides and solve for $\cos^2(\theta/2)$ to obtain

$$\cos^2(\theta/2) = \frac{K + 2m}{K + 4m}$$

Now apply to this result the trigonometric identity

$$\cos^2(\theta/2) = \frac{(\cos \theta + 1)}{2}$$

After some manipulation, obtain the desired result:

$$\cos \theta = \frac{(K/m)}{(K/m) + 4}$$

Here K is the kinetic energy of the incoming particle, m the mass of either particle, and θ the angle between outgoing particles. This result assumes (1) an elastic collision (kinetic energy conserved), (2) one particle initially at rest, (3) equal masses of the two particles, and (4) the symmetry of outgoing paths shown in the diagram.

5a. Limiting case: Low energy. In the case of low energy (Newtonian limit), the incoming particle has a kinetic energy K very much less than its rest energy m , so the ratio K/m approaches zero. In the limit, $\cos \theta$ becomes zero and $\theta = 90^\circ$ degrees. This is the accepted Newtonian result for low velocities (except for an exactly head-on collision, in which case the incoming particle stops dead and the struck particle moves forward with the same speed and direction as the original incoming particle).

5b. Limiting case: High energy. For extremely high-energy elastic collisions, the incident particle has a kinetic energy very much greater than its rest energy, so the ratio K/m increases without limit. In this case the quantity 4 in the denominator becomes negligible compared with K/m , so numerator and denominator both approach the value K/m , with the result $\cos \theta \rightarrow 1$ and $\theta \rightarrow 0$. This means that in the special symmetric case discussed here both resulting particles go forward in the same direction as the incoming particle, sharing equally the kinetic energy of the incoming particle.

For an incoming particle of very high energy, the elastic collision described here is only one of several possible outcomes. Alternative processes include creation of new particles.

✓ Example 8.7.2

ANNIHILATION

A positron of mass m and kinetic energy equal to its mass strikes an electron at rest. They annihilate, creating two high-energy photons. One photon enters a detector placed at an angle of 90 degrees with respect to the direction of the incident positron. What are the energies of both photons (in units of mass of the electron) and direction of motion of the second photon?

Solution

following steps in Box 8-2

1. Draw a diagram and label the particles with letters.

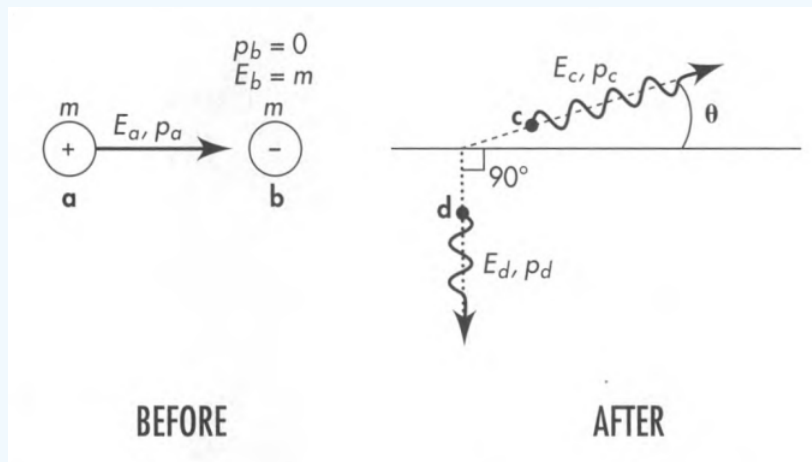


Figure 8.7.4

2. **Conservation of energy** expressed in the symbols of the diagram, and including the rest energy of the initial stationary particle:

$$E_{\text{tot}} = E_a + m = E_c + E_d$$

3. **Conservation of each component of total momentum:**

$$p_{x \text{ tot}} = p_a = p_c \cos \theta$$

$$p_{y \text{ tot}} = 0 = p_c \sin \theta - p_d$$

[horizontal momentum]

[vertical momentum]

4. **Solve:** First of all, the problem states that the kinetic energy K of the incoming positron equals its rest energy m . Therefore its total energy $E_a = m + K = m + m = 2m$. Second, the outgoing particles are photons, for which $p_c = E_c$ and $p_d = E_d$ in magnitude, respectively. With these substitutions, the three conservation equations become

$$E_a + m = 2m + m = 3m = E_c + E_d$$

$$p_a = E_c \cos \theta$$

$$E_d = E_c \sin \theta$$

These are three equations in three unknowns E_c and E_d and θ . Square both sides of the second and third equations, add them, and use a trigonometric identity to get rid of the angle θ :

$$p_a^2 + E_d^2 = E_c^2 (\cos^2 \theta + \sin^2 \theta) = E_c^2$$

Substitute $p_a^2 = E_a^2 - m^2$ on the left side of this equation and again use $E_a = 2m$ to obtain a first expression for E_c^2 :

$$E_c^2 = E_a^2 - m^2 + E_d^2 = 4m^2 - m^2 + E_d^2 = 3m^2 + E_d^2$$

Now solve the equation of conservation of energy for E_c and square it to obtain a second expression for E_c^2 :

$$E_c^2 = (3m - E_d)^2 = 9m^2 - 6mE_d + E_d^2$$

Equate these two expressions for E_c^2 and subtract E_d^2 from both sides to obtain

$$3m^2 = 9m^2 - 6mE_d$$

Solve for unknown E_d :

$$E_d = \frac{9m^2 - 3m^2}{6m} = \frac{6m^2}{6m} = m$$

This yields our first unknown. Use this result and conservation of energy to find an expression for E_c :

$$E_c = 3m - E_d = 3m - m = 2m$$

Finally, angle θ comes from conservation of vertical momentum. For a photon $p = E$, so

$$\sin \theta = \frac{p_d}{p_c} = \frac{E_d}{E_c} = \frac{m}{2m} = \frac{1}{2}$$

from which $\theta = 30$ degrees. We have now solved for all unknowns: $E_c = 2m$, $E_d = m$, and $\theta = 30$ degrees.

5. Limiting cases: There is no limiting case here, since the energy of the incoming positron is specified fully in terms of the mass m common to electron and positron.

✓ Example 8.7.3

CONVERSION OF MASS TO ENERGY IN SUN

Luminous energy from Sun pours down on the outer atmosphere of Earth at a rate of 1372 watts per square meter of area that lies perpendicular to the direction of this radiation. The figure 1372 watts per square meter has the name solar constant. The radius of Earth equals approximately 6.4×10^6 meters and the Earth-Sun distance equals 1.5×10^{11} meters. The mass of Sun is approximately 2.0×10^{30} kilograms.

- How much mass is converted to energy every second in Sun to supply the luminous energy that falls on Earth?
- What total mass is converted to energy every second in Sun to supply luminous energy?
- Most of Sun's energy comes from burning hydrogen nuclei (mostly protons) into helium nuclei (mostly a two-proton-two-neutron combination). Mass of the proton equals 1.67262×10^{-27} kilogram, while the mass of a helium nucleus of this kind equals 6.64648×10^{-27} kilogram. How many metric tons of hydrogen must Sun convert to helium every second to supply its luminous output? (One metric ton is equal to 1000 kilograms, or 2200 pounds.)
- Estimate how long Sun will continue to warm Earth, neglecting all other processes in Sun and emissions from Sun.

Solution

a. One watt equals one joule per second = one kilogram meter ²/ second ³. We want to measure energy in units of mass-in kilograms. Do this by dividing the number of joules by the square of the speed of light (Section 7.5 and Table 7-1):

$$\begin{aligned} \frac{1372 \text{ joules}}{c^2} &= \frac{1.372 \times 10^3 \text{ kilogram meters}^2 / \text{second}^2}{9.00 \times 10^{16} \text{ meters}^2 / \text{second}^2} \\ &= 1.524 \times 10^{-14} \text{ kilograms} \end{aligned}$$

Thus every second 1.524×10^{-14} kilogram of luminous energy falls on each square meter perpendicular to Sun's rays. The following calculations are based on a simplified model of Sun (see last paragraph of this solution). Therefore we use the approximate value 1.5×10^{-14} kilogram per second and two-digit accuracy.

What total luminous energy falls on Earth per second? It equals the solar constant (in kilograms per square meter per second) times some area (in square meters). But what area? Think of a huge movie screen lying behind Earth and perpendicular to Sun's rays (see the figure). The shadow of Earth on this screen forms a circle of radius equal to the radius of Earth. This shadow represents the zone of radiation removed from that flowing outward from Sun. Call the area of this circle the cross-sectional area A of Earth. Earth's radius $r = 6.4 \times 10^6$ meters, so the cross-sectional area A seen by incoming Sunlight equals

$A = \pi r^2 = 1.3 \times 10^{14}$ meters². Hence a total luminous energy equal to $(1.5 \times 10^{-14}$ kilograms/ meter²) $\times (1.3 \times 10^{14}$ meters²) = 2.0 kilograms fall on Earth every second. This equals the mass converted every second in Sun to supply the light incident on Earth.

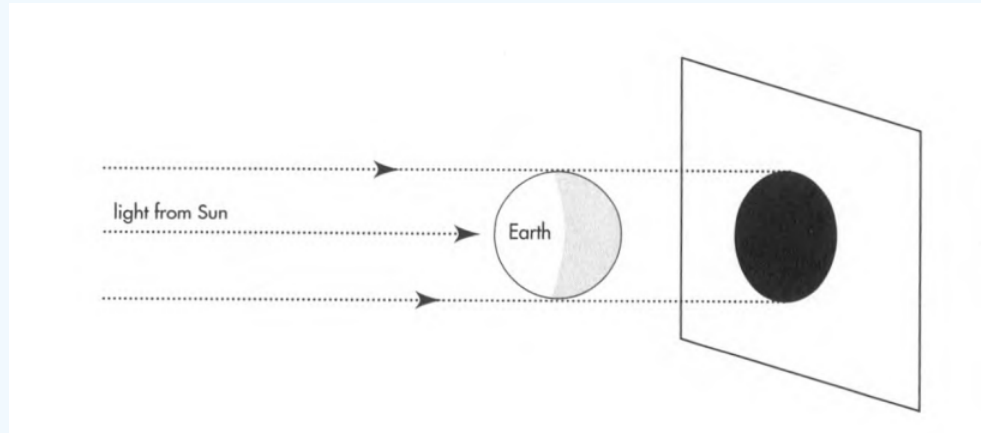


Figure 8.7.5: Copy and Paste Caption here. (Copyright; author via source)

b. Assume that Sun delivers sunlight at the same "solar-constant rate" to every part of a sphere surrounding Sun of radius equal to the Earth-Sun distance. The area of this large sphere has the value $4\pi R^2$ where $R = 1.5 \times 10^{11}$ meters, the average distance of Earth from Sun. This area equals 2.8×10^{23} meters². Therefore Sun converts a total of 2.8×10^{23} meters² $\times 1.5 \times 10^{-14}$ kilograms/meter² (from a) = 4.2×10^9 kilograms of mass into luminous energy every second, or about 4 million metric tons per second.

c. Through a series of nuclear processes not described here, four protons transform into a helium nucleus consisting of two protons and two neutrons. The four original protons have a mass $4 \times 1.67262 \times 10^{-27} = 6.69048 \times 10^{-27}$ kilogram. The helium nucleus has a mass 6.64648×10^{-27} kilogram. The difference, 0.04400×10^{-27} kilogram, comes out mostly as light. (We cannot use two-digit accuracy here, because the important result is a difference between nearly equal numbers.)

The ratio of hydrogen burned to mass converted equals $6.69048/0.04400 = 150$ (back to two-digit accuracy!). So for each kilogram of mass converted to electromagnetic radiation, 150 kilograms of hydrogen burn to helium. In other words, about 0.7 percent of the rest energy (mass) of the original hydrogen is converted into radiation. Hence in order to convert 4.2×10^9 kilograms per second into radiation, Sun burns $150 \times 4.2 \times 10^9$ kilograms per second = 6.3×10^{11} kilograms of hydrogen into helium per second - about 630 million metric tons each second.

d. We can reckon Sun's mass by figuring how much Sun gravity it takes to guide our planet around in an orbit of 8 light-minute radius and one year time of circuit. Result: about 2.0×10^{30} kilograms. If Sun were all hydrogen, then the process of burning to helium at the present rate of 6.3×10^{11} kilograms every second would take $(2.0 \times 10^{30} \text{ kilograms}) / (6.3 \times 10^{11} \text{ kilograms / second}) = 3.2 \times 10^{18}$ seconds. At 32 million seconds per year, this would last about 10^{11} years, or 100 billion years.

Of course the evolution of a star is more complicated than the simple conversion of hydrogen into helium-plus-radiation. Other nuclear reactions fuse helium into more massive nuclei on the way to the most stable nucleus, iron-56 (Section 8.7). These other reactions occur at higher temperatures and typically proceed at faster rates than the hydrogen-to-helium process. Sun emits a flood of neutrinos (invisible; detected with elaborate apparatus; amount presently uncertain by a factor of 2, carry away less than 1 percent of Sun's output). Sun also loses mass as particles blown away from the surface, called the solar wind. And stars do not convert all their hydrogen to helium and other nuclei - or live for 100 billion years. According to current theory, the lifetime of a star like Sun equals approximately 10 billion years (10^{10} years). We believe Sun to be 4 to 5 billion years old. The remaining 6 billion years (6×10^9 years) or so should be sufficient time for our descendants to place themselves in the warmth of nearby stars.

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