

## 9.6: Gravitation as Curvature of Spacetime

### spacetime curvature accounts for tidal accelerations of objects

#### ***Spacetime curvature demonstrated by change in separation of two originally parallel worldlines***

Einstein smiles, too, as he hears gravitation described as action at a distance. Curvature of spacetime and nothing more, he tells us, is all that is required to describe the millimeter or two change in separation in 8 seconds of two ball bearings, originally 20 meters apart in space above Earth, and endowed at the start with zero relative velocity. Moreover, this curvature completely accounts for gravitation.

#### ***Acceleration toward Earth: Totalized effect of relative accelerations, each particle toward its neighbor, in a chain of test particles that girdles globe***

What a preposterous claim!" is one's first reaction. "How can such minor - and slow - changes in the distance between one tiny ball and another offer any kind of understanding of the enormous velocity with which a falling mass hits Earth?" The answer is simple: Many local reference frames, fitted together, make up the global structure of spacetime. Each local Lorentz frame can be regarded as having one of the ball bearings at its center. The ball bearings all simultaneously approach their neighbors (curvature). Then the large-scale structure of spacetime bends and pulls nearer to Earth (Figure 9.6.1). In this way many local manifestations of curvature add up to give the appearance of long-range gravitation originating from Earth as a whole.

In brief, the geometry used to describe motion in any local free-float frame is the flat-spacetime geometry of Lorentz (special relativity). Relative to such a local freefloat frame, every nearby electrically neutral test particle moves in a straight line with constant velocity. Slightly more remote particles are detected as slowly changing their velocities, or the directions of their worldlines in spacetime. These changes are described as tidal effects of gravitation. They are understood as originating in the local curvature of spacetime.

From the point of view of the student of local physics, gravitation shows itself not at all in the motion of one test particle but only in the change of separation of two or more nearby test particles. "Rather than have one global frame with gravitational forces we have many local frames without gravitational forces." However, these local dimension changes add up to an effect on the global spacetime structure that one interprets as "gravitation" in its everyday manifestations.

In contrast, Newton supposed the existence of one ideal overall reference frame. For him, "Absolute space, in its own nature, without relation to anything external, remains always similar and immovable." The ball bearing or spaceship is regarded by Newton as actually accelerated with respect to this ideal frame. The "gravitational force" that accelerates it acts mysteriously across space and is produced by distant objects. That the man in the spaceship finds no evidence either of the acceleration or the force is an accident of nature, according to the Newtonian view. Pundits used to interpret this accident of nature as the fortuitous equality of "gravitational mass" and "inertial mass" or in other "learned" ways.

In conversations with one of the authors of this book at various times over the years, Einstein emphasized his great respect for Newton and, in particular, his admiration for Newton's courage and judgment. He stressed that Newton was even better aware than his seventeenth-century critics of the difficulties with the ideas of absolute space and time. To postulate those ideas was nevertheless the only practical way to get on with the task of describing motion in Newton's century. In effect, Newton chopped the problem of motion into two parts: (1) space and time and their meaning: ideas that were puzzling but usable and that were destined to be clarified only 230 years later and (2) the laws of acceleration with respect to that idealized spacetime: laws that Newton gave the world.

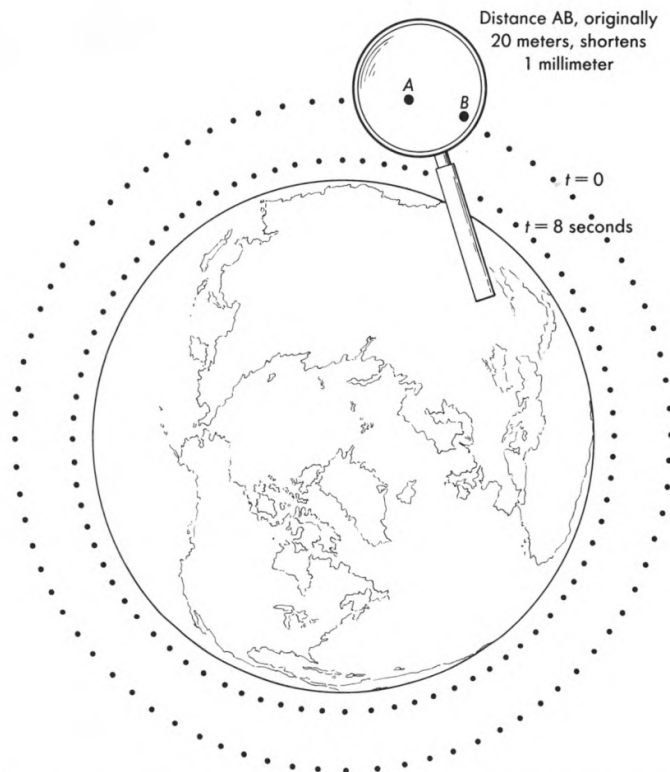


Figure 9.6.1: Local curvature adding up to the appearance of long-range gravitation. The shortening of distance between any one pair, A and B, of ball bearings is small when the distance itself is small. However, small separation between each ball bearing and its partner demands many pairs to encompass Earth. The totalized shortening of the circumference in any given time - the shortening of one separation times the number of separations - is independent of the fineness of the subdivision. That totalized pulling in of the circumference carries the whole necklace of masses inward. This is free fall, this is gravity, this is a large scale motion interpreted as a consequence of local curvature. Example:

Original separation between A and B-and every other pair: 20 meters

Time of observation: 8 seconds

Shortening of separation in that time: 1 millimeter

Fractional shortening: 1 millimeter/20 meters =  $1/20,000$

Circumference of Earth (length of airy necklace of ball bearings):  $4.0030 \times 10^7$  meters

Shrinkage of this circumference in 8 seconds:  $1/20,000 \times 4.0030 \times 10^7$  meters = 2001.5 meters

Decrease in the distance from the center of Earth (drops by the same factor  $1/20,000$ ):

$1/20,000 \times 6.371 \times 10^6$  meters = 315 meters.

This apparently large-scale effect is caused - in Einstein's picture - by the addition of a multitude of small-scale effects: the changes in the local dimensions associated with the curvature of geometry ( failure of B to remain at rest as observed in the free-float frame associated with A).

What is the source of the curvature of spacetime? Momenergy is the source. In Chapter 8 we saw the primacy of momenergy in governing interactions between particles. Crash of mass on mass, no matter how elastic or how destructive, leaves the total momenergy of the system quite unaltered. By what miracle does this come about? Education of momenergy from birth onward to good behavior? Goodness of heart?

#### Box 9.1 HOW SPACETIME CURVATURE CARRIES INFLUENCE FROM ONE MASS TO ANOTHER

The necklace of ball bearings (Figure 9.6.1) as they approach Earth, examined more closely, reveals a remarkable feature of spacetime curvature outside a great, essentially uniform, essentially isolated sphere of mass. The curvature in its character is totally "tideproducing," totally " noncontractile."

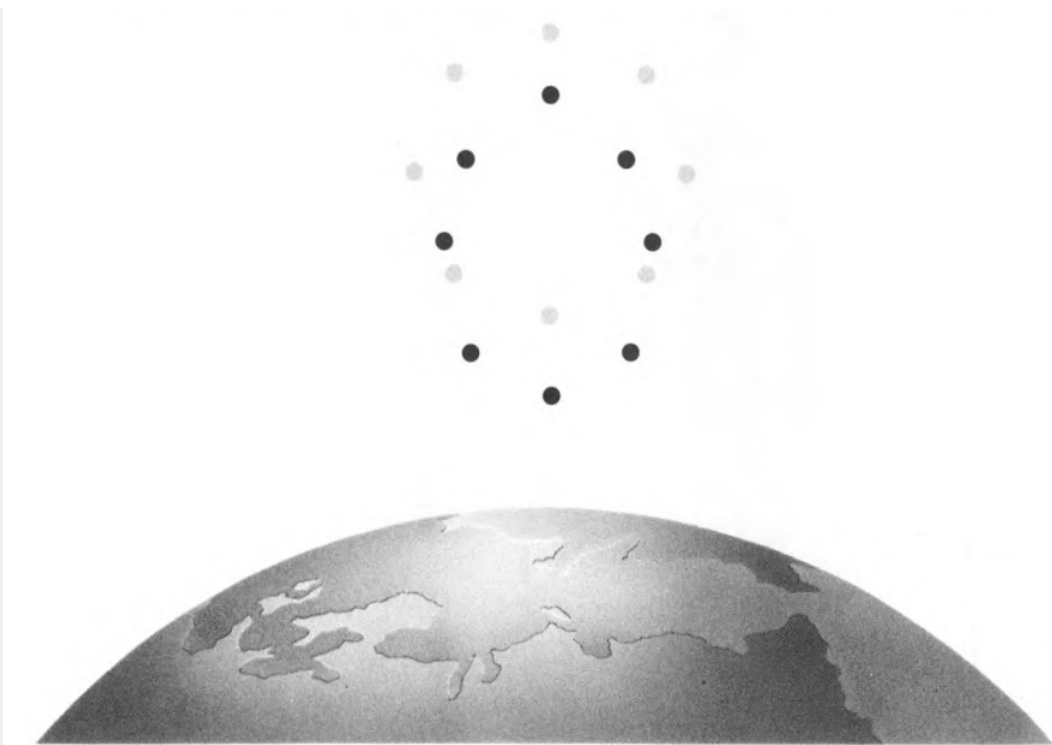


Figure 9.6.2: An array of test masses covering the surface of a hollow sphere freely floating above the Earth's surface will shrink in two dimensions and lengthen in one. The volume remains constant; only the shape changes. This change is evidence of the noncontractile, tide-driving spacetime curvature outside Earth.

What do these descriptive terms mean, and how do we verify that they apply? We look at a cluster of ball bearings dotted here and there over the surface of an imaginary small sphere, all momentarily at rest relative to each other and relative to Earth. That shape, however, as the seconds tick by, changes from sphere to ellipsoid. How come? First let's look at the two dimensions of the sphere that lie perpendicular to each other but parallel to Earth's surface. Both these dimensions of the sphere shrink as the ball bearings converge toward Earth's center. The up-down dimension of the pattern, however, lengthens, and twice as much. Why? Newton says because of the greater gravitational acceleration of the one nearer Earth. Einstein says because two-percent stretch in that dimension compensates one-percent shrinkage in the other two dimensions and keeps the volume of the pattern unchanged. Spacetime curvature, yes; but a totally noncontractile curvature. Einstein's famous equation, stated in simple terms, tells us how spacetime curvature responds to mass:

$$\left( \begin{array}{c} \text{appropriate measure of} \\ \text{spacetime contractile curvature} \\ \text{at any place, any time,} \\ \text{in any Lorentz frame} \end{array} \right) = \left( \begin{array}{c} \text{a universal} \\ \text{constant} \end{array} \right) \times \left( \begin{array}{c} \text{density of energy} \\ \text{at that locale} \\ \text{perceived in that} \\ \text{Lorentz frame} \end{array} \right)$$

Outside, no mass, no energy, a spacetime curvature that is totally noncontractile. Inside Earth, however, there is mass, therefore there is energy - or in a moving frame, energy plus energy flow-and therefore spacetime curvature there has a contractile character. The ball bearings - when shafts are drilled for them so that not one of them encounters any obstacle to freefloat motion - start to converge vertically as well as horizontally. The volume shrinks. That, overlooking details, is what we mean when we say that "mass grips spacetime, telling it how to curve."

There is no Earth mass out at Moon's orbit. How then does Einstein's spacetime geometry account for Moon's motion? Answer: Earth's mass imposes on spacetime a contractile curvature throughout Earth's interior, as a jumper's feet impose a contractile curvature on a trampoline. That contractile curvature, where the feet push, forces on the surrounding nontear fabric

a corresponding lateral stretch. That effect transmits itself in ever more dilute measure to the ever more remote regions of the trampoline.



Figure 9.6.3: The deformation of the nontear trampoline fabric under the jumper's feet and elsewhere is analogous to the nontear curvature of spacetime geometry inside Earth and elsewhere.

Likewise spacetime does not tear. Its fabric just above Earth's surface experiences the same lateral contractility as it does just below the surface. Not so with the curvature in the two-dimensional domain defined by time and by direction perpendicular to Earth's surface. In that one plane, curvature within Earth is contractile but suddenly jumps just above Earth's surface to the opposite character. Hence the tide-producing character of spacetime curvature outside Earth. A point twice as far from Earth's center lies on an imaginary Earthcentered sphere that encompasses eight times the volume. There the tide-producing curvature experiences eight times the dilution and has one eighth the strength. Despite this rapid dilution of tide-producing power with distance, it has strength enough at Moon, 60 Earth radii away from Earth's center, to deform Moon from sphere to ellipsoid, 1738.35 kilometers in radius along the Earth-Moon direction, 1738.15 kilometers in radius for each of the other two perpendicular directions.

Easy as it is to regard Earth as running the whole show, Moon too has its part. Like an infant standing on the trampoline some distance from its mother, it imposes its own small curvature on top of the curvature evoked by Earth. That additional curvature, contractile in Moon's interior, has tide-driving character outside. Were the Earth an ideal sphere covered by an ideal ocean of uniform depth, then Moon would draw that ocean's surface 35.6 centimeters higher than the average in two domains, one directly facing Moon, one directly opposite to it - simultaneously lowering those waters 17.8 centimeters below the average on the circle of points midway between the two. (These low figures show how important are funneling and resonant sloshing in determining heights of actual ocean tides on Earth.)

The local contractile curvature of spacetime at Moon's location added up along Moon's path yields the appearance of long-range gravitation, similar to that illustrated in Figure 9-6. Box 2-1 tells a little of the many influences that have to be taken into account in any fuller treatment of the tides.

### ***Momenergy tells spacetime how to curve***

Spacetime controls momenergy. Obedience to the eyes of a corps of bookkeepers? No, Einstein taught us. The enforcing agency does not lie far away. It's close at hand. It's the geometry of spacetime, right where the crash takes place. Not only does spacetime grip isolated mass, telling it how to move. In addition, in a crash it sees to it that the participants neither gain nor lose momenergy. But there is more! Spacetime, in so acting, cannot maintain the perfection assumed in textbooks of old. To every action there is a corresponding reaction. *Spacetime acts on momenergy, telling it how to move; momenergy reacts back on spacetime, telling it how to curve.* This "handshake" between momenergy and spacetime is the origin of momenergy conservation - and the source of spacetime curvature that leads to gravitation (Box 9-1).

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Quotation: "Rather than have one global frame with gravitational forces we have many local frames without gravitational forces." Steven Schutz, in January 1966 final examination in course in relativity, Princeton University.

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