5.6: Wristwatch Time Along a Worldline

straight worldline has longest proper time between two given events in spacetime A curved path in Euclidean space is determined by laying down a flexible tape

Measure proper time along curved worldline with wristwatch . . .

A curved path in Euclidean space is determined by laying down a flexible tape measure and recording distance along the path's length. A curved worldline in Lorentz spacetime is measured by carrying a wristwatch along the worldline and recording what it shows for the elapsed time. The summed spacetime interval — the proper time read directly on the wristwatch — measures the worldline in Lorentz geometry in the same way that distance measures path length in Euclidean geometry.[1]

A particle moves along the worldline in Figure $5.6.1$. This particle carries a wristwatch and a sparkplug; the sparkplug fires every meter of time $(1, 2, 3, 4, \ldots.)$ as read off the particle's wristwatch. The laboratory observer notes which of his clocks the traveling particle is near every time the sparkplug fires. He plots that location and that lattice clock time on his spacetime map, tracing out the worldline of the particle. He numbers spark points sequentially on the resulting worldline, as shown in Figure $5.6.1$, knowing that these numbers register meters of time recorded on the moving wristwatch.

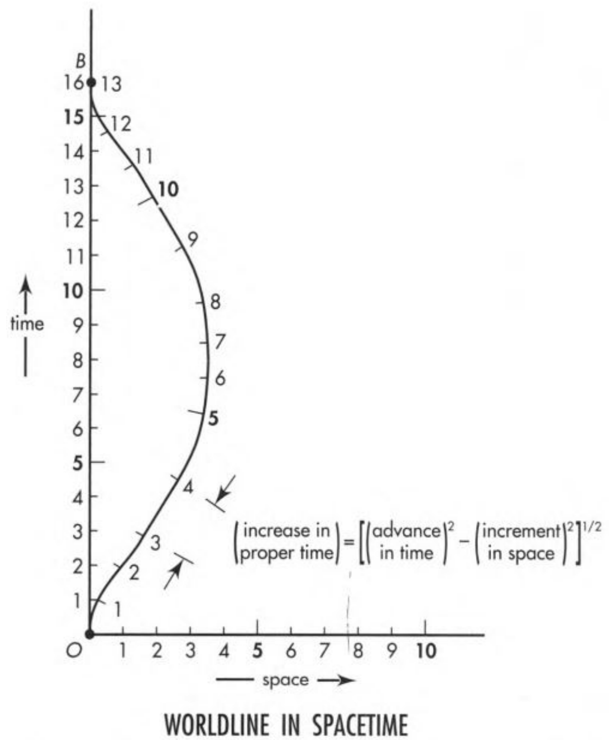

Figure $5.6.1$: **Proper time along a curved worldline.** Notice that the total proper time along the curved worldline from event O to event B is smaller than the proper time along the straight line from O to B.

. . . or as sum of intervals between adjacent events

Consider the spacetime interval between two sequential numbered flashes of the sparkplug, for instance those marked 3 and 4 in the figure. In the laboratory frame these two sparks are separated by a difference in position and also by a difference in time (the time between them). The squared interval— the proper time squared — between the sparks is given by the familiar spacetime relation:[2]

$$(\text{ proper time })^2=(\text{ difference in time })^2-(\text{ difference in position })^2$$

What about the proper time between sparks 3 and 4 calculated from measurements made in the sparkplug frame? In this frame, both sparks occur at the same place, namely at the position of the sparkplug. The difference in position between the sparks equals zero in this frame. As a result, the time difference in the sparkplug frame — the "wristwatch time" — is equal to the proper time between these two events:

$$(\text{ proper time })^2 = (1 \text{ meter })^2 - (\text{ zero })^2 = (1 \text{ meter })^2 \qquad [\text{recorded on traveling wristwatch}]$$

This analysis assumes that sparks are close together in both space and time. For sparks close enough together, the velocity of the emitting particle does not change much from one spark to the next; the particle velocity is effectively constant between sparks; the piece of curved worldline can be replaced with a short straight segment. Along this straight segment the particle acts like a free-float rocket. The proper time is invariant in free-float rocket and free-float laboratory frames. Thus the laboratory observer can compute the value of the proper time between events 3 and 4 and predict the time lapse — one meter— on the traveling wristwatch, which measures the proper time directly.

Elsewhere along the worldline the particle moves with a different speed. Nevertheless the proper time between each consecutive pair of sparks must also be independent of the free-float frame in which that interval is reckoned. For sparks close enough together, this proper time equals the time read directly on the wristwatch.

All observers agree on proper time along worldline

All observers agree on the proper time between every sequential pair of sparks emitted by the sparkplug. Therefore the sum of of all individual proper times has the same value for all observers. This sum equals the value of the total proper time, on which all free-float observers agree. And this total proper time is just the wristwatch time measured by the traveling sparkplug.[3]

In brief, proper time is the time registered in a rocket by its own clock, or by a person through her own wristwatch or her own aging. Like aging, proper time is cumulative. To obtain total proper time racked up along a worldline between some marked starting event and a designated final event, we first divide up the worldline into segments so short that each is essentially "straight" or "free-float." For each segment we determine the interval, that is, the lapse of proper time, the measurement of aging experienced on that segment. Then we add up the aging, the proper time for each segment, to get total aging, total wristwatch time, total lapse of proper time.

An automobile may travel the most complicated route over an entire continent, but the odometer adds it all up and gives a well-understood number. The traveler through the greater world of spacetime, no matter how many changes of speed or direction she undergoes, has the equivalent of the odometer with her on her journey. It is her wristwatch and her body - her aging. Your own wristwatch and your biological clock automatically add up the bits of proper time traced out on all successive segments of your worldline.

Straight worldline has longest proper time

It is possible to proceed from event O to event B along quite another worldline - for example, along the straight worldline OB in Figures $5.6.1$ and $5.6.2$ (bottom). The proper time from O to B along this new worldline can be measured directly by a flashing clock that follows this new worldline. It can also be calculated from records of flashes emitted by the clock as recorded in any laboratory or rocket frame.[4]

Total proper time along this alternative worldline has a different value than total proper time along the original worldline. In Lorentz geometry a curved worldline between two specified events is shorter than the direct worldline between them- shorter in terms of total proper time, total wristwatch time, total aging.

Total proper time, the aging along any given worldline, straight or curved, is an invariant: it has the same value as reckoned by observers in all overlapping free-float frames. This value correctly predicts elapsed time recorded directly on the wristwatch of the particle that travels this worldline. It correctly predicts the aging of a person or a mouse that travels this worldline. A different worldline between the same two events typically leads to a different value of aging-a new value also agreed on by all free-float observers: Aging is maximal along the straight worldline between two events. This uniqueness of the straight worldline is also a matter of complete agreement among all free-float observers. All agree also on this: The straight worldline is the one actually followed by a free particle. Conclusion: Between two fixed events, a free particle follows the worldline of maximal aging.

Principle of Maximal Aging predicts motion of free particle

This more general prediction of the worldline of a free particle is called the **Principle of Maximal Aging**.[5] It is true not only for "straight" particle worldlines in the limited regions of spacetime described by special relativity but also, with minor modification, for the motion of free particles in wider spacetime regions in the vicinity of gravitating mass. The Principle of Maximal Aging provides one bridge between special relativity and general relativity.

Stark contrast between Euclidean and Lorentz geometries

The stark contrast between Euclidean geometry and Lorentz geometry is shown in Figure $5.6.2$. In Euclidean geometry distance between nearby points along a curved path is always equal to or greater than the northward separation between those two points. In contrast, proper time between nearby events along a curved worldline is always equal to or less than the corresponding time along the direct worldline as measured in that frame.[6]

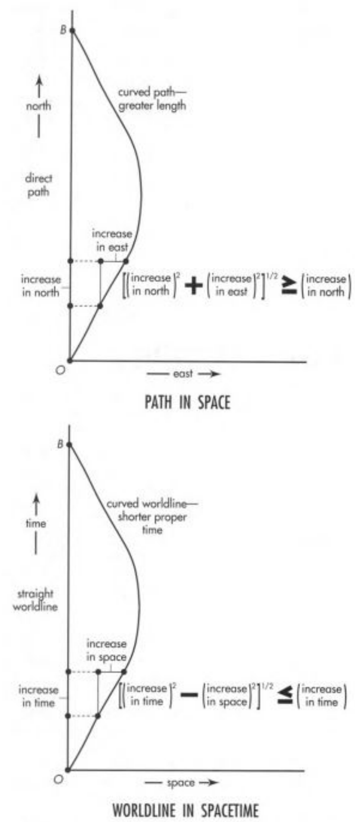

Figure $5.6.2$: **Path in space:** In Euclidean geometry the curved path bas greater length. **Worldline in spacetime:** In Lorentz geometry the curved worldline is traversed in shorter proper time.

The difference of proper time between two alternative worldlines in spacetime violates no law, just as the difference of length between two alternative paths in space violates no law. There is nothing wrong with a wristwatch that reads different proper times when carried along different worldlines between events O and B in spacetime, just as there is nothing wrong with a tape measure that records different lengths for different paths between points O and B in space. In both cases the measuring device is simply giving evidence of the appropriate geometry: Euclidean geometry for space, Lorentz geometry for spacetime.

In brief, the determination of cumulative interval, proper time, wristwatch time, aging along a worldline between two events is a fundamental method of comparing different worldlines that connect the same two events.[7]

Among all possible worldlines between two events, the straight worldline is unique. All observers agree that this worldline is straight and has the longest proper time - greatest aging - of any possible worldline connecting these events.

---

1 Measure proper time along curved worldline with wristwatch...

2 ...or as sum of intervals between adjacent events

3 All observers agree on proper time along worldline

4 Straight worldline has longest proper time

5 Principle of Maximal Aging predicts motion of free particle

6 Stark contrast between Euclidean and Lorentz geometries

7 Proper times compare worldlines
