5.S: Trekking through Spacetime (Summary)

straighter worldline? greater aging!

Events? Yes. Each event endowed with its own location in that great fabric we call spacetime? Yes. But time? No point in all that fabric displays any trace of anything we can identify with any such thing as the "time" of that event. Label that event with a "time" anyway? Sure. No one can stop us. Moreover, such labeling often proves quite useful. But it is our labeling! A different reference frame, a different wristwatch brought to that event along a different worldline yields a different time label for that event.

For our own convenience, then, we plot events on a **spacetime map (spacetime diagram)** for a particular free-float frame and its latticework of rods and clocks. This map can be printed on the page of a book if events are limited to one line in space. Distance along this line is plotted horizontally on the spacetime map, with time of the event plotted vertically (Section 5.1). The time and space values of an event are measured with respect to a common **reference event**, plotted at the origin of the spacetime map. The invariance of the interval: $(\text{interval})^2 = (\text{ time })^2 - (\text{ distance })^2$ between an event and the reference event corresponds to the equation of a **hyperbola**, the same hyperbola as plotted on the spacetime map of every overlapping free-float frame. The event point lies somewhere on the same **invariant hyperbola** as plotted on every one of these spacetime maps (Sections 5.2 and 5.3).

Billions of events sparkle like sand grains scattered over the spacetime map. A given event is unconnected to most other events on the map. Here we pay attention to particular strings of events that are connected. The **worldline** of a particle connects in sequence events that occur at the particle (Section 5.4). The "length" of a worldline between an initial and a final event is the elapsed time measured on a clock carried along the worldline between the two events (Section 5.6). This is called the proper time, wristwatch time, or aging along this worldline. The lapse of proper time is given the symbol τ, in contrast to the symbol t for the frame time read on the latticework clocks in a given free-float frame.

Carry a wristwatch (or grow old!) along a worldline: This is one way to measure the total proper time along it from some initial event (such as the birth of a person or a particle) to some final event (such as death of a person or annihilation of a particle). This method is direct, experimental, simple. A second method? Calculate the interval between each pair of adjacent events that make up the worldline, and then add up all these intervals, assuming that each tiny segment is short enough to be considered straight. This method seems more bothersome and detailed, but it can be carried out by the observer in any free-float frame. All such observers will agree with one another - and with the clock-carrier - on the value of the total proper time from the initial event to the final event on the worldline (Section 5.6).

Among all possible worldlines between two given events, the straight line is the worldline of **maximal aging**. This is the actual worldline followed by a free particle that travels from one of these two events to the other (Section 5.6).

As measured in a given free-float frame, the **stretch factor** $=1/\left(1-v^2\right)^{1/2}$ equals the ratio of elapsed frame time t to elapsed proper time τ along a segment of worldline in which the particle moves with speed v in that frame. The stretch factor is also the Lorentz contraction factor (Section 5.8): Locate, at the same time, the front and back ends of an object moving in a given free-float frame. These end locations will be $\left(1-v^2\right)^{1/2}$ as far apart in that frame as they are in a frame in which the object is at rest.

Worldlines connect events. Like events, they exist independent of any reference frame. In principle, worldlines allow us to relate events to one another - to do science - without using reference frames at all (Section 5.9).