# 0.4: Basics of Scientific Measurement

## Things You Should Already Know

As a final section for the "Preliminaries" chapter, we will review a few things you should have encountered somewhere in a science class prior to enrolling in the university. Some things that fall into this category are covered by course prerequisites. For example, everyone entering Physics 9A should already have a solid working knowledge of trigonometry and basic calculus (differentiation and integration). This section will discuss other basics that are not explicitly in the course prerequisites, but will nevertheless be assumed to be understood by students entering the course.

## Physical Dimensions and Units

When describing a physical quantity in the subject of mechanics (covered in Physics 9A), it can be broken down into a combination of three distinct *physical dimensions*. These dimensions are *distance*, *time*, and *mass*. Physical quantities can be measured in many different scales called *units* (distance can be measured in centimeters, yards, or light years, time measured in seconds, hours, or fortnights, and so on), but while two measurements may use different units, their physical dimensions are the same. For example, two speeds may be measured in meters per second and miles per hour, respectively, but they have in common the dimensions of "length per unit time."

### Dimensional Analysis

When solving physics problems, it is often useful to check one's work at the end (or even while in progress), as it is possible to have made an algebraic error in the calculation. One way to make a quick check is to see if the dimensions of a quantity work out right. For example, in this class we will talk about a quantity called energy. It turns out that energy has units of: $mass \times \frac{length^2}{time^2}$ . Suppose a problem asks for a computation of velocity, after some algebra you get:

$$v=\sqrt{\frac{2E}{m}}$$

While it is not a definitive check of whether the answer is right, it is possible to gain confidence in the answer or catch an error if it is wrong by plugging in the dimensions to see if they work out in the equality. The dimensions of velocity we know to be length-per-time, so if we plug in the dimensions for energy $E$ and mass $m$, we can check to see if things work out. Note that the number 2 has no physical dimensions, so it can be ignored (the brackets around the variable $v$ mean "dimensions of"):

$$[v]=\sqrt{\frac{mass \times \frac{length^2}{time^2}}{mass}}=\frac{length}{time}$$

The mass dimensions divide out, and the quare root of the squared length and time results in a confirmation of the dimensions.

### Unit Conversion

While the dimensions of physical quantities are always the same, they may be measured differently. Occasionally it is desirable to convert a numerical value from one system of units to another. One reason might be that a problem is given where different quantities are measured in different systems of units, and the final answer should not contain both (say) inches and meters. There is a simple procedure for making this conversions...

It starts with knowing what a measurement of a dimensions in one system of units is in another system. For example, 1 inch is a length equal to 2.54 centimeters. Given that this is true, then the ratio of these two values (with either one in the numerator) must be equal to one:

$$\frac{2.54cm}{1in}=1=\frac{1in}{2.54cm}$$

This is useful to know, because a quantity can always be multiplied by 1 without affecting its value. So suppose we want to know how many centimeters there are in 3.5 inches. All we have to do is multiply 3.5 inches by 1, using the fraction with inches in the denominator so that the inches unit cancels, and the number of centimeters is left behind:

$$3.5in = (3.5in) \cdot 1 = \left(3.5\ \cancel{in}\right)\left(\frac{2.54cm}{1\ \cancel{in}}\right) = 8.89cm$$

If the quantity is more complicated than just a single length, the same procedure can be followed for each separate unit. For example, suppose we wish to know how fast a car is traveling in miles per hour when we are given that it is moving at a speed of 40 meters per second. Now we need to convert meters to miles, and seconds to hours. We can even use intermediate units like kilometers and minutes along the way. That is, suppose we know the conversion between kilometers and miles is $1.60km = 1mi$. Of course we know that there are 1000 meters in 1 kilometer, 60 seconds in a minute, and 60 minutes in an hour, so we form the following ratios with a value of 1 for the purposes of our conversion:

$$1 = \frac{1.61km}{1mi} = \frac{1mi}{1.61km}, \quad 1 = \frac{1000m}{1km} = \frac{1km}{1000m}, \quad 1 = \frac{60s}{1min} = \frac{1min}{60s}, \quad 1 = \frac{60min}{1hr} = \frac{1hr}{60min}$$

Now we take the original value and start multiplying it by as many 1's as we need to in order to replace the units as we want them:

$$40\frac{m}{s} = \left(40\frac{\cancel{m}}{\cancel{s}}\right)\left(\frac{1\ \cancel{km}}{1000\ \cancel{m}}\right)\left(\frac{1mi}{1.61\ \cancel{km}}\right)\left(\frac{60\ \cancel{s}}{1\ \cancel{min}}\right)\left(\frac{60\ \cancel{min}}{1hr}\right) = 89.4\frac{mi}{hr}$$

## Significant Figures

Most (but certainly not all) physics problems that you will encounter in this course provide you with some numeric values associated with a physical system. The presumption is that those numbers were determined by a measuring device of some kind. Measuring devices have varying levels of precision. For example, if one measures a distance with a meter stick, one can expect the measurement to be accurate to within about a millimeter. If a measurement is taken using a microscope, the measurement may be accurate to within a micron (one thousandth of a millimeter). In all scientific disciplines, it is understood that the measuring device's precision is reflected in the number itself. So for example, a measurement with a meter stick (in meters) will show no more than three decimal places, because the fourth decimal place signifies fractions of millimeters. Conversely, whenever we see a number given in a problem that describes a length to three decimal places, we assume that the measuring device could only do that well (so maybe it was a meter stick with millimeter subdivisions). Note that if the result is a round number, then the number provided can still provide information about the true level of precision by including trailing zeros. So if a number $0.100m$ is given, then the implication is that it is accurate down to millimeters.

For larger numbers this can get a little weird. For example, suppose we want to express the number above in microns. Clearly the value is $10,000\mu m$, but now it seems that we have lost the information about precision (which is the same if we measured with the same device). The solution here is to use scientific notation, and only keep trailing zeros to the place where the precision ends. So the above number with the same precision would be expressed as: $1.00 \times 10^5 \mu m$.

Understanding the precision of given numbers is one thing, but ultimately these numbers are used in calculations to get new numbers, so the question becomes, "How do we express the precision of our calculated numbers?" The simple answer is that the calculated number's level of precision is only as good as the least precise number in the calculation. For example, suppose we want to know how far something travels when we are given its speed and the time it moves at this constant speed. The distance is obviously computed by multiplying these numbers. Suppose the speed is measured fairly crudely, no more precisely than 1 meter per second. Let's say the speed is $35\frac{m}{s}$. Let's also assume that the time is measured *very* precisely – with an atomic clock, and the time comes out to be $3.8933274s$ Simply multiplying these numbers together gives a distance traveled equal to $136.266459m$ But if we claim this is the answer, then someone reading it will assume that this distance value is accurate down to $10^{-6}m$ (microns). But suppose the actual speed value was $35.3\frac{m}{s}$ (this extra decimal place was not caught by our measuring device). Then the correct distance value would be over $137m$, and the accuracy implied by our 9-digit answer would be misleading. So rather than keep all 9 digits generated by the exact calculation, we round off that answer to the number of digits ("significant figures") in the less-precise number. The speed has only two digits, so we would round-off our final answer to only two significant figures, changing the answer from $136.266459m$ to $140m$.

There is one other thing to mention here regarding significant figures. If a measured value is multiplied by an exact number, then the exact number is not taken into the significant figure calculation. For example, a physics formula might include a factor of one-half. This number is not a measured value – it is exact – so even though you might think it is 0.5 and has only one significant figure, this is not correct. In fact it is 0.50000000000..., which means that it will not limit the number of significant figures of the final answer at all – only the measured value(s) in the calculation will. Another interesting example is the calculation of the circumference of a circle. Suppose the diameter is given to be $1.9cm$, then the circumference is equal to $1.9\pi cm$. To express this as a decimal, you might think that the two significant figures for the diameter means that you only keep two significant figures for $\pi$. But $\pi$ is an exact number (even if we can't express it completely as a decimal), so you need to keep more decimal places of that number. Using $\pi \approx 3.1$ gives an answer for the circumference (to two significant figures) of $5.9cm$, while using more decimal places for pi raises the answer for the circumference to $6.0cm$.

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