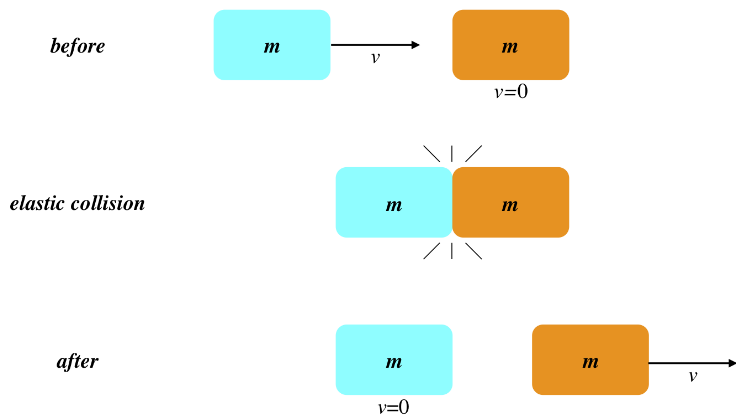
4.5: More Collisions

Elastic Collisions

In the previous section, we focused on inelastic collisions. Here we will look at elastic collisions, where the kinetic energy of the system remains unchanged, meaning none of the rest frame kinetic energy is converted into thermal energy. This kind of collision is standard between particles, but between macroscopic objects, this is really only an approximation. When we are told that a given collision is elastic (or at least can be approximated as such), then that gives us an additional condition that we can use to solve the problem. We'll go through a few examples of elastic collisions in one dimension below. In each case, the diagram will show the experimental result, which we will then show mathematically using the combination of momentum and kinetic energy conservation.

Figure 4.5.1 – Elastic Collision of Equal Masses, Target Stationary

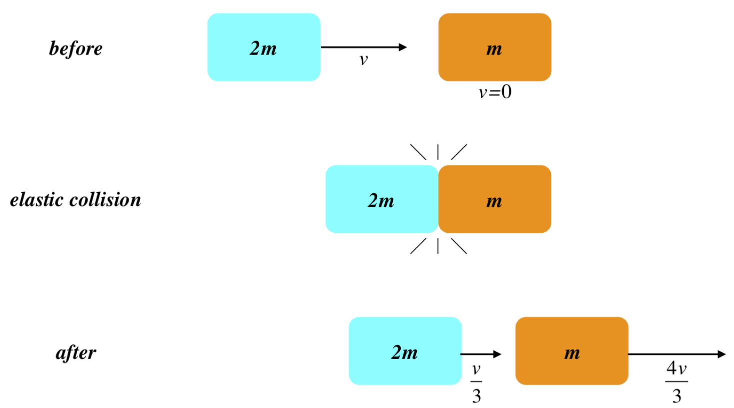


We see that the incoming cart stops completely and the target cart moves off with the same velocity as the original cart (note that the center of mass continues moving at a constant speed, as it should). We now show this mathematically... Dropping the vector arrows, since the motion is in one dimension, and choosing to the right as the (+) direction, we have:

$$\left.\begin{array}{llll} p_o = p_f: & mv + 0 = mv_1 + mv_2 & \Rightarrow & v = v_1 + v_2 \\ KE_o = KE_f: & \frac{1}{2}mv^2 + 0 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 & \Rightarrow & v^2 = v_1^2 + v_2^2 \end{array}\right\} \Rightarrow \quad v_1 = 0, \; v_2 = v \quad or \quad v_1 = v, \; v_2 = 0 \tag{4.5.1}$$

Wait, why do we get two solutions? That is, why can *either* velocity equal zero? Well, if the incoming cart were to *miss the target cart*, then that too is an elastic "collision," inasmuch as the momentum and kinetic are both conserved, so the math takes into account that as a possibility.

Figure 4.5.2 – Elastic Collision of Unequal Masses, Target Lighter and Stationary



The algebra is only a little tougher this time:

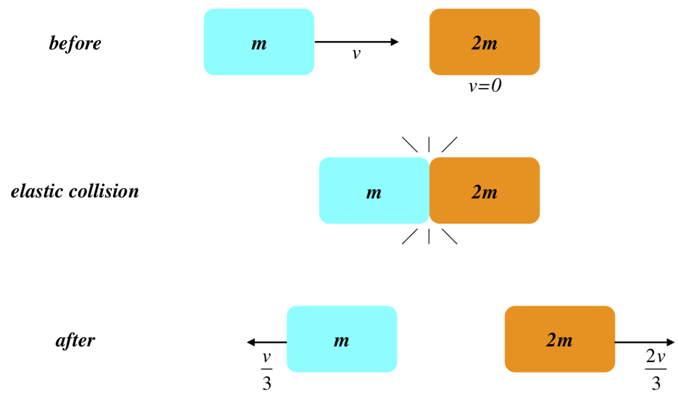
$$\left.\begin{array}{llll} p_o = p_f: & 2mv + 0 = 2mv_1 + mv_2 & \Rightarrow & 2v = 2v_1 + v_2 \\ KE_o = KE_f: & \frac{1}{2}2mv^2 + 0 = \frac{1}{2}2mv_1^2 + \frac{1}{2}mv_2^2 & \Rightarrow & 2v^2 = 2v_1^2 + v_2^2 \end{array}\right\} \Rightarrow \quad 4v_1 = v_2 \quad \Rightarrow \quad v_1 = \frac{v}{3}, \; v_2 = \frac{4v}{3} \tag{4.5.2}$$

Both carts continue forward, the lighter one at 4 times the speed of the heavier one. Note that once again $v_1 = v,\ \ v_2 = 0$ is a solution (the incoming cart misses the target).

Let's consider an application of this in the real world. Suppose we are passengers in one of two vehicles involved in a head-on collision. Which vehicle would we rather be in, the lighter one or the heavier one? Intuitively we know we would rather be in the heavier vehicle, but why? Well, we would want to experience as little force as possible (force is what breaks bones). The force that our dashboard or steering column exerts on us is going to equal our mass times our acceleration (as it constitutes our net horizontal force), and we are constrained to experience the same acceleration as our car. So compare the accelerations of the two carts here. The heavier cart goes from a speed v down to a speed of v/3, for a change of 2v/3. The lighter cart's velocity changes from 0 to 4v/3 in the same period of time, which means it experiences twice the acceleration. More acceleration for our car means more acceleration for us, which means more force on us, which is bad.

Lastly, we look at the lighter object bouncing off the heavier one:

Figure 4.5.3 – Elastic Collision of Unequal Masses, Target Heavier and Stationary



The math:

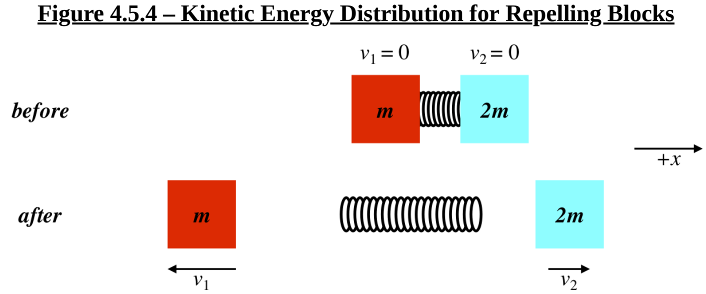
$$\left.\begin{array}{lll} p_o = p_f: & mv + 0 = mv_1 + 2mv_2 & \Rightarrow \quad v = v_1 + 2v_2 \\ KE_o = KE_f & \frac{1}{2}mv^2 + 0 = \frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2 & \Rightarrow \quad v^2 = v_1^2 + 2v_2^2 \end{array}\right\} \Rightarrow \quad 2v_1 = -v_2 \quad \Rightarrow \quad v_1 = -\frac{v}{3},\ v_2 = \frac{2v}{3} \tag{4.5.3}$$

The lighter cart bounces off the heavier one at half the speed that the heavier one continues forward (or the incoming cart misses the target). There is actually a clever way we could have solved this case more quickly by using the solution of the previous case and what we know about relative motion. If we move along with the incoming block and declare ourselves to be "stationary," then we see the heavier mass coming toward us at a speed v, which is exactly the same physical situation as we had above. After the collision, we will see the heavier mass continuing in the same direction at a speed of $v/3$, while the target block moves in the same direction at a speed of $4v/3$. That is what *we see*. Going back to the original frame, these two speeds change by v, which means the heavy object is not going left at $v/3$ – it is going *right* at $v - v/3 = 2v/3$, while the smaller block is moving left at a speed of $4v/3 - v = v/3$.

Kinetic Energy Distribution Within a System

Let's return once again to an example we looked at in the previous section (Figure 4.3.1), and ask a new question about it (the example has been simplified slightly by giving one block exactly twice the mass of the second block).

Figure 4.5.4 – Kinetic Energy Distribution for Repelling Blocks



The spring stored some potential energy when it was compressed, and it gave this energy to the kinetic energy of the two blocks. What fraction of this energy is given to each of the blocks? One might be inclined to believe that since the spring exerts equal forces on both blocks, they both get equal amounts of kinetic energy. But by now we know better! They only get the same amount of energy if the spring does the same amount of work on both, and it's clear here that the lighter mass is pushed a longer distance before losing contact with the spring than the heavier mass, so with equal forces acting on each, more work is done on the lighter mass. Specifically, the lighter mass is accelerated twice as much by the equal force, so it displaces twice as far, and therefore gets twice as much energy as the heavier block.

Another way to see it is to note that both blocks must have the same magnitude of momentum after the spring expands (since the momenta must cancel to equal zero and remain conserved), so using Equation 4.1.6 we can compare their kinetic energies:

$$KE_1 = \frac{p_1^2}{2m_1} = \frac{p^2}{2m} \quad \Rightarrow \quad KE_2 = \frac{p_2^2}{2m_2} = \frac{p^2}{2\,(2m)} = \frac{1}{2}KE_1 \tag{4.5.4}$$

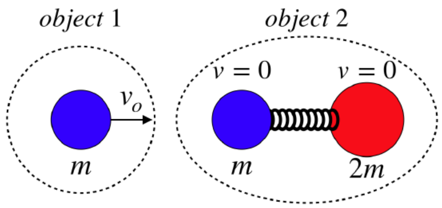
This confirms what we reasoned above.

Now we can see more clearly why we are able to refer to the gravitational potential energy of the system of a small stone and the earth as simply the gravitational potential energy "of the stone," ignoring the fact that the earth is also involved. This is because when the potential energy is converted to kinetic energy, virtually all of the kinetic energy goes to the stone, and none of it to the earth (imagine the heavier block above being *much* heavier).

Source of Inelasticity

We have said more than once that all collisions between particles are elastic, while collisions between objects are not. But objects are *made* of particles, which means that when they collide, it involves the collision of particles. How is this not a contradiction? Let's see if we can sort this out with the simplest possible example imaginable. Let's look at a collision between object made up of a *single* particle, and another object made up of two particles. The latter object we will model as two particles of different masses, bound together by a spring. We will further simplify things by assuming that the two particles are separated by the equilibrium length of the spring, and they are not vibrating (i.e. this two-particle object has zero internal energy). A diagram of the collision is shown in the figure below.

Figure 4.5.5 – Microscopic View of Two Object Collision



When the blue particles collide, they will do so elastically. As we saw in the example above, when a projectile collides head-on elastically with a stationary target of equal mass, the incoming particle stops, and the target particle continues forward at the same speed that the incoming particle had. This means that object 1 will stop, and one of the particles in object 2 will start moving toward the other particle. This will compress the spring, which will cause the other particle to also move to the right. That is, object 2 as a whole will start moving to the right. Its particles will also vibrate back-and-forth within the object – the object will have internal energy. This is precisely the recipe for an inelastic collision. In the case of bigger objects with trillions upon trillions of particles, this internal energy is spread throughout the particles, and their motions are randomly-distributed, which is to say that this internal energy is thermal. The simplicity of this example, by contrast, allows us to precisely track this energy. Let's do that.

The motion of object 2 after the collision is measured by its center of mass velocity, which the spring will not affect, as it provides only an internal force within the collection of particles. Therefore the center of mass velocity can simply be computed using the initial condition when the blue particle is moving and the red particle is not:

$$v_{cm} = \frac{mv_o + 0}{m + 2m} = \frac{1}{3}v_o \tag{4.5.5}$$

The total energy of the two-object system is unchanged, and initially it was just the kinetic energy of the incoming particle, so it is:

$$E_{system} = \frac{1}{2}mv_o^2 \tag{4.5.6}$$

The kinetic energies of the two objects after the collision are: Zero for object 1 (which stops), and for object 2:

$$KE_2 = \frac{1}{2}Mv_{cm}^2 = \frac{1}{2}(3m)\left(\frac{1}{3}v_o\right)^2 = \frac{1}{3}\left(\frac{1}{2}mv_o^2\right) = \frac{1}{3}E_{system} \tag{4.5.7}$$

So we see that this collision between two objects is inelastic, because only one-third of the original kinetic energy remains in the objects. The remaining two thirds is stored in the internal energy of object 2 as its particles vibrate.

Exercise

Suppose the same two objects as above collide again, but this time the incoming object strikes the other side. They are the same two objects, so would you expect the result to be the same? Confirm or refute your intuition mathematically.

Solution

Repeating the process above, we have the same total system energy as before. The elastic collision between the incoming blue particle and the larger red particle gives a different result, however. We calculated earlier what happens when an object collides elastically with another object twice as heavy. The incoming object bounces-back with one-third its incoming speed, and the heavier one moves forward with two thirds the incoming speed.

The center of mass velocity of the two-particle object can be computed from the red particle's new speed:

$$v_{cm}=\frac{2m\left(\frac{2}{3}v_o\right)+0}{2m+m}=\frac{4}{9}v_o$$

Now we can use the reflected speed of the incoming object and the center of mass speed of the target object to determine the kinetic energy in the system after the collision:

$$KE_{sys}=KE_1+KE_2=\frac{1}{2}m\left(\frac{1}{3}v_o\right)^2+\frac{1}{2}(3m)\left(\frac{4}{9}v_o\right)^2=\frac{19}{27}\left(\frac{1}{2}mv_o^2\right)=\frac{19}{27}E_{system}$$

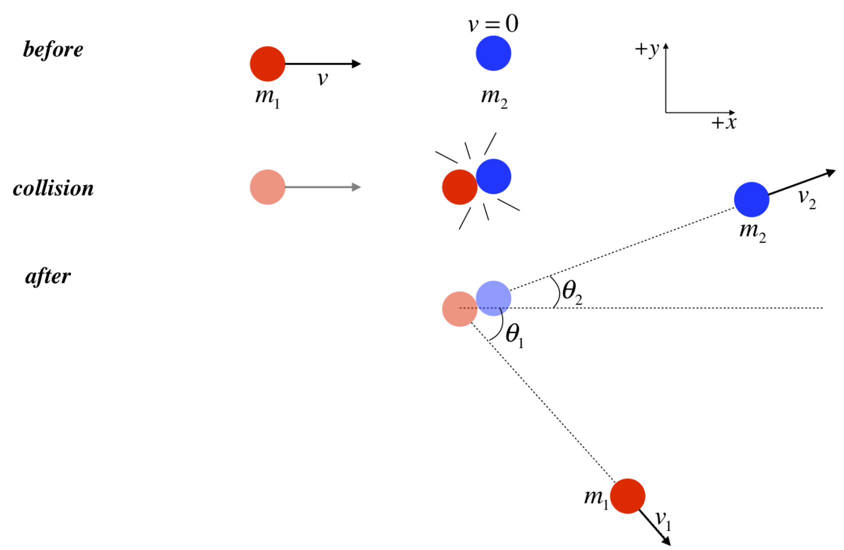
So it appears that considerably less energy goes into the internal energy of the two-particle object in this case than when the smaller particles collide.

General Two-Dimensional Collisions

We have been saying for awhile now that one of the big differences between momentum conservation and energy conservation is the fact that momentum is a vector while energy is not. This means that there are actually three momentum quantities that are equal before and after (if the full momentum vector is conserved). Here we will look at what this entails.

Let's look at a standard two-dimensional collision. In this example, we will have a stationary ball struck by another. The two balls have different masses, and they collide off-center, so that they emerge from the collision in directions angled off the original direction of motion. We'll set up the geometry and label all the known and unknown variables with a diagram, and then do the physics:

Figure 4.5.6 – General Two-Dimensional Collision in the Target Frame



Now we need to apply momentum conservation. Since momentum is a conserved vector, each of its components are individually conserved, which means that momentum conservation provides us two separate equations to work with. In the "before" case, we have an x-component of momentum that is simply the incoming mass times the incoming velocity (m_1v), while the y-component of momentum is zero. in the "after" case, we need to resolve the momenta into components. Setting before equal to after gives:

$$\begin{aligned} x-direction: &\quad m_1v=m_1v_1\cos\theta_1+m_2v_2\cos\theta_2 \\ y-direction: &\quad 0=-m_1v_1\sin\theta_1+m_2v_2\sin\theta_2 \end{aligned} \tag{4.5.8}$$

You'll note the minus sign for the component in the $-y$-direction. This is not strictly necessary, as this negative sign could be absorbed into θ_1, but it is generally less confusing to put the signs in explicitly, and let all the angle values be positive.

Let's consider what would be required to solve a problem that looks like this. We have two equations, and seven distinct variables. If this is all we know about the collision, then to completely unravel this physical situation, we need to know five of these quantities. So for example, we could be given the two masses, the incoming speed, and the outgoing speed and direction of one of the balls, and we can solve for the outgoing speed and direction of the other ball. If we also provided the target ball a starting velocity, or a y-component to the incoming ball's velocity, then there would be even more unknowns. But we can quickly reduce this problem back to the one above, by first rotating our coordinate system so that the incoming velocity is once again in the x direction, and then changing the reference frame to the rest frame of the target ball. It is also sometimes useful to change to the center of mass reference frame.

Notice that once such a problem is solved, once can then check to see if the collision is elastic, by comparing the kinetic energy before and after the collision:

$$KE_{before}=\frac{1}{2}m_1v^2 \qquad KE_{after}=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2 \tag{4.5.9}$$

This comparison could be a difference (determining how much kinetic energy is lost), or a fraction (determining the percentage of kinetic energy remaining or the percentage lost). Note that a collision *can* result in an increase of kinetic energy, but this can only happen if there is some potential energy stored within the colliding objects that is unleashed by the collision. This is such an uncommon occurrence (the circumstances need to be quite contrived), that it is safe to assume that a collision is either elastic (conserves kinetic energy) or is inelastic such that kinetic energy is lost.

Not all problems are posed with five of the seven variables given. The energy condition can be given instead, which provides a third equation, requiring only four of the seven variables in the statement of the problem. Needless to say, these problems can require a lot of tedious algebra, but getting the equations set up using momentum conservation and the fate of the system's kinetic energy is where the physics is.

Elastic Two-Dimensional Collisions

As daunting as the full-blown problem shown above can be, there are cases where shortcuts or simplifications exist. We look first at the case of elastic collisions. If we want to know all the information shown above, we have no choice but to go through the algebra involved. But we can achieve an interesting result without recourse to the coordinate system at all. Namely, it turns out that the ratios of the masses of the colliding objects and their outgoing speeds completely determine the angle *between* the outgoing velocity vectors, $\theta_1+\theta_2$. To get this result, we will use Equation 4.1.5 extensively...

Let's call the incoming momentum $\overrightarrow{p}$ and the mass of the incoming object m_1. Then the kinetic energy of the system (in the frame where the target is stationary) is:

$$KE_{before}=\frac{1}{2m_1}\overrightarrow{p}\cdot\overrightarrow{p} \tag{4.5.10}$$

Now let's define the outgoing momenta of the two objects as $\overrightarrow{p}_1$ and $\overrightarrow{p}_2$, with the latter being for the target object after collision. The kinetic energy after the collision is therefore:

$$KE_{after}=\frac{1}{2m_1}\overrightarrow{p}_1\cdot\overrightarrow{p}_1+\frac{1}{2m_2}\overrightarrow{p}_2\cdot\overrightarrow{p}_2 \tag{4.5.11}$$

Now we apply momentum conservation:

$$\overrightarrow{p}=\overrightarrow{p}_1+\overrightarrow{p}_2 \quad\Rightarrow\quad KE_{before}=\frac{1}{2m_1}\left(\overrightarrow{p}_1+\overrightarrow{p}_2\right)\cdot\left(\overrightarrow{p}_1+\overrightarrow{p}_2\right)=\frac{1}{2m_1}\overrightarrow{p}_1\cdot\overrightarrow{p}_1+\frac{1}{2m_1}\overrightarrow{p}_2\cdot\overrightarrow{p}_2+\frac{1}{m_1}\overrightarrow{p}_1\cdot\overrightarrow{p}_2 \tag{4.5.12}$$

Applying kinetic energy conservation (remember, we are assuming an elastic collision):

$$KE_{before}=KE_{after} \quad\Rightarrow\quad \cancel{\frac{1}{2m_1}\overrightarrow{p}_1\cdot\overrightarrow{p}_1}+\frac{1}{2m_1}\overrightarrow{p}_2\cdot\overrightarrow{p}_2+\frac{1}{m_1}\overrightarrow{p}_1\cdot\overrightarrow{p}_2=\cancel{\frac{1}{2m_1}\overrightarrow{p}_1\cdot\overrightarrow{p}_1}+\frac{1}{2m_2}\overrightarrow{p}_2\cdot\overrightarrow{p}_2 \tag{4.5.13}$$

Now multiply through by m_1 and rearrange things a bit to get:

$$\overrightarrow{p}_1\cdot\overrightarrow{p}_2=\frac{1}{2}\left(\frac{m_1}{m_2}-1\right)\overrightarrow{p}_2\cdot\overrightarrow{p}_2 \tag{4.5.14}$$

Now write the dot products in terms of the magnitudes of the vectors and the angles between them:

$$p_1 p_2 \cos\theta = \frac{1}{2}\left(\frac{m_1}{m_2} - 1\right) p_2^2 \tag{4.5.15}$$

The angle θ is of course the angle between the two outgoing velocity vectors (which point the same direction as the momentum vectors). The $p_2 = 0$ solution to this corresponds to the case of the incoming object missing the target entirely (because the target remains stationary), so assuming the target is not missed, we can divide both sides by p_2 and if we also plug in $p_1 = m_1 v_1$ and $p_2 = m_2 v_2$, we get the promised relationship of the *scattering angle* in terms of the masses and outgoing speeds:

$$\theta = \cos^{-1}\left[\frac{1}{2}\left(1 - \frac{m_2}{m_1}\right)\frac{v_2}{v_1}\right] \tag{4.5.16}$$

We can extract some interesting information from this result:

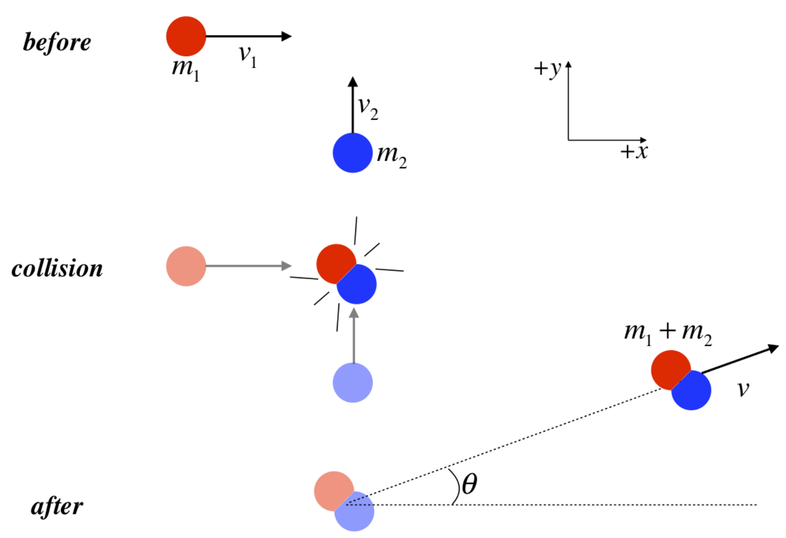
- We see that if the masses are equal, then the scattering angle is precisely 90^o, since the cosine of this angle vanishes. In this case, the scattering angle doesn't depend at all on how off-center the collision is (except that a direct head-on hit naturally leads to an angle of 0^o or 180^o). The degree of how off-center the collision is (which is measured by a quantity known as the *impact parameter*) *does* effect the angles θ_1 and θ_2 in Figure 4.5.6, but not the sum of those angles. If the masses are not equal, then the impact parameter does play a role in the scattering angle, because it has a say in the ratio of the outgoing speeds.
- If $m_2 > m_1$, the argument of the inverse cosine is negative, so the angle must be greater than 90^o. This makes sense, because if the target mass is greater than the incoming mass, the incoming mass "bounces back," rather than "plowing through" (a result we found for the one-dimensional elastic collisions we examined above), and since the target mass has a forward component to its final velocity, the angle is greater than 90^o.
- The argument of the inverse cosine can never be larger than +1 or smaller than –1, which places limits on the outgoing speeds given the masses. For example, if the incoming mass m_1 is twice the target mass m_2, then the largest possible ratio of the two outgoing velocities is 4. This ratio occurs when $\theta = 0$, and indeed we have seen this result already above (Equation 4.5.2).

It should be noted that this result could also be achieved using the formulas resulting from Figure 4.5.5, but it would require an unnatural desire to slog through trigonometric identities.

Perfectly Inelastic Two-Dimensional Collisions

As much as we were able to do with elastic collisions, perfectly inelastic collisions are even easier to handle. This is because the outgoing motions of the two objects are constrained to be the same (i.e. they stick together and have the same final speed and direction). This constraint means that if we are given all of the incoming conditions (the masses of the two objects, and their incoming velocity vectors), we can determine the result completely. That is, the amount of energy lost in the collision does not need to be given – it is unique and can in fact be computed. The figure below is a diagram for an example of a perfectly inelastic collision. [*This is somewhat simplified by having the incoming objects approach each other at right angles, but not as simple as the case of looking at it from the target frame, which makes the collision one-dimensional!*]

Figure 4.5.7 – A Perfectly Inelastic Two-Dimensional Collision



We follow the same procedure as we did for Figure 4.5.6, this time with the simplification that we have a single outgoing momentum:

$$\begin{aligned} x-direction: \quad & m_1 v_1 = (m_1 + m_2)\, v\cos\theta \\ y-direction: \quad & m_2 v_2 = (m_1 + m_2)\, v\sin\theta \end{aligned} \tag{4.5.17}$$

The amount of energy converted to thermal from this collision equals the loss of kinetic energy from the system, and as we saw in the one-dimensional case, this amount doesn't depend upon the details of the internal non-conservative force. It only matters that eventually (after the two objects end their tumultuous collision) settle into moving off together with a common velocity. The amount of energy converted is:

$$\Delta E_{thermal} = KE_{before} - KE_{after} = \left[\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right] - \left[\frac{1}{2}(m_1+m_2)\,v^2\right] \tag{4.5.18}$$

For the case above where the two incoming objects have velocities are right angles to each other, we can turn this into an equation that includes only the masses and incoming speeds. Sparing the reader the algebra, the result is:

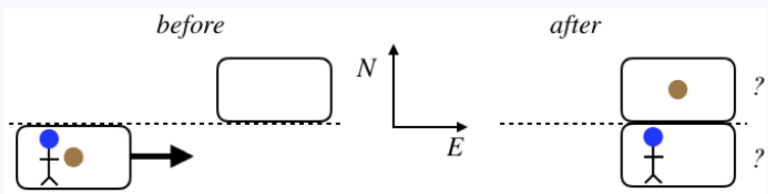
$$E_{thermal} = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)\left(v_1^2+v_2^2\right) \tag{4.5.19}$$

Notice that since the two velocities are perpendicular, the sum of their squares is actually the square of their *relative* velocity. This is not a surprising result, and in fact will translate into collisions at any angle (though the equation will look different), because we would not expect the post-collision blob to be any hotter when the collision is viewed in one frame as opposed to another. As mentioned above, we can always view this collision from the target frame, making the collision one-dimensional, and the total kinetic energy of the system before the collision is a function of the relative velocity. In that case, we can use Equation 4.4.3 to compute the energy converted to thermal.

So suppose we drop a ball of clay to the ground. Viewing this from the earth's rest frame, the earth becomes the stationary target with mass m_2, and essentially all of the clay's incoming kinetic energy is converted to thermal (because $m_2 \approx m_1 + m_2$), and the clay's (and earth's) temperature goes up a bit. If we view it from the clay's rest frame, then the kinetic energy of the earth is enormous (same relative speed, much larger mass), and after the collision we might therefore expect the temperatures to go up a lot, but making the clay the stationary target now makes the target mass m_2 very small compared to $m_1 + m_2$, which makes the fraction multiplying the earth's kinetic energy very small – exactly small enough to give the same energy change as before.

Analyze This

A cart slides along a frictionless surface in an easterly direction. The cart contains a person and a medicine ball. The cart slides past an identical (but empty) stationary cart, also on the frictionless surface. When the carts are side-by-side, the person throws the medicine ball into the other cart by pushing the ball in the north direction.



Your goal in the analysis is to extract everything you can from what has been given. At a minimum, every analysis should include these items:

- *what we are given (perhaps translated from English to mathematics)*
- *what we can infer, if anything*
- *quantities we can compute (or almost compute!), if anything*

Analysis

This situation is a bit more complicated than a simple two-object collision, but the principles behind it are the same. Treating both carts, the person, and the medicine ball together as a single system, there are not external forces on the system, and the total momentum vector is conserved – the x and y components of momentum are each separately conserved.

Let's call the magnitude of the initial easterly velocity of the cart v_o. This gives us the initial momentum (which will remain unchanged):

$$p_x = (m_{cart} + m_{person} + m_{ball})\,v_o$$
$$p_y = 0$$

Next we need to think about the "after" situation. Now we have two separate components to deal with, as the objects involved will have some y components to their motion. Calling the incoming cart #1, and the other cart #2, we have:

$$\begin{array}{ll} x\text{-direction:} & p_x = (m_{cart} + m_{person})\,v_{1x} + (m_{cart} + m_{ball})\,v_{2x} \\ y\text{-direction:} & p_y = (m_{cart} + m_{person})\,v_{1y} + (m_{cart} + m_{ball})\,v_{2y} \end{array}$$

Invoking momentum conservation gives:

$$\begin{array}{lll} x\text{-direction:} & \left(m_{cart}+m_{person}+m_{ball}\right)v_o & =\left(m_{cart}+m_{person}\right)v_{1x}+\left(m_{cart}+m_{ball}\right)v_{2x} \\ y\text{-direction:} & 0 & =\left(m_{cart}+m_{person}\right)v_{1y}+\left(m_{cart}+m_{ball}\right)v_{2y} \end{array}$$

With the push being in the north direction, the x-component of person + cart 1 is not changed, which means we can put in $v_{1x}=v_o$, giving:

$$m_{ball}v_o=\left(m_{cart}+m_{ball}\right)v_{2x}$$

This is all fine if we are given the velocities of the carts after the ball has been transferred, but it seems clear that in such a problem the details of the ball transfer itself might be given. If this is the case, then really we have two momentum problems to solve – the problem of the first cart ejecting the ball, and the second of the second car receiving the ball.

*We in fact already know that the ball is thrown northward by the person. Does this mean we can write the momentum of the ball as being purely in the y-direction? No! The person throws the ball northward, but it **was already moving eastward** when it was thrown, so its momentum actually has both x and y components. So for the ball-is-thrown half of the problem, the ball, person, and cart #1 will not change their speeds in the x-direction at all.*

*How to compute the effects on the y-components of the velocities of the ball and cart #1 depends upon what is given about the medicine ball's motion. If the northward component of its velocity relative to the ground is given, then things are pretty straightforward. Just use this quantity and the mass of the ball to compute its y-component of momentum, and that same amount of momentum is what cart #1 + person must have in the southerly direction, to conserve the momentum in the y-direction (which was initially zero). But if the velocity of the medicine ball **as seen by the person** is given, then it gets considerably trickier, because this is the speed of the medicine ball north **relative to** the person + cart #1, which will recoil south.*

Calling the northward component of the ball's velocity relative to the person v_{bp}, the ball relative to earth v_b, and the southward component of the person + cart #1 after the ball is released v_p (which has a negative value), we can use the y-components of the usual relative motion formula (Equation 1.8.3) to get:

$$v_b=v_{bp}+v_p$$

We then just need to use the quantities v_b and v_p in the y-component parts of our momentum conservation equations. We already noted that the x-component for the throwing part is not very interesting, so the y-component part looks like:

$$0=m_{ball}v_b+\left(m_{cart}+m_{person}\right)v_p$$

For the second part, where cart #2 receives the ball, both the x and y components of the ball's motion are important, and the momentum conservation equations are:

$$\begin{array}{ll} x\text{-direction:} & m_{ball}v_o+0=\left(m_{ball}+m_{cart}\right)v_{2x} \\ y\text{-direction:} & m_{ball}v_b+0=\left(m_{ball}+m_{cart}\right)v_{2y} \end{array}$$