

1.3: Straight-Line Motion

Start Simple

There is nothing more fundamental in the study of physics than motion. We will bring a lot of mathematics to bear on this subject (including the vectors we just learned about), but we are going to start as simply as possible – with motion of a single particle that remains on a straight line. This simplifies our task in a couple ways:

1. By restricting ourselves to single particles, we don't have to worry about the complicated motions of systems of particles, where each of the particles can move differently than the others.
2. By keeping the motion along a straight line, there are only two directions involved, and these directions can be characterized simply as "positive" and "negative" – there is no need for unit vectors.

Throughout this course, for new topics we will take this approach of starting as simply as we can in these two ways (single particle in one dimension), and only extend to more general results once the simpler case is understood. It should also be noted that it is possible to treat a system of particles as if it is a single particle when all of the particles follow the same motion. This is assured when the system of particles is a rigid object that does not rotate. Many of the examples we do for the time being will be of this variety, even if it is not explicitly stated that the object has these two properties of rigidity and zero rotation.

Displacement

In order for motion to occur for an object, obviously its **position** must change from one instant in time to another. We will refer to the coordinate position of the straight line on which the object moves as $x(t)$. A change in this position we call the **displacement**, and refer to it as a change in position:

$$\text{displacement} = \Delta x \equiv x_f - x_o \quad (1.3.1)$$

Alert

It's a good idea to get used to this now, as you will use it throughout the Physics 9-series: When referring to a time-dependent quantity, the "delta" (Δ) means "after minus before," or "final minus initial."

Notice that if the final position is a smaller number than the initial position, then the object has a negative displacement. Eventually we will treat displacement as a vector, but for our straight-line motion, the sign of the value provides all the information we need about the direction. In this text you will receive several warnings about the precise use of physics language, which is frequently at odds with how the same words are used in casual conversation. Here is the first example:

Alert

*"Displacement" sounds a lot like "distance covered." Walking a mile to the store and back again is a two mile walk, but the displacement in this case is **not** two miles. Displacement is a **vector** whose magnitude is the distance between the starting and ending points, and whose direction points from the starting point to the ending point.*

Average Velocity

Of course, there is more to motion than just displacement. We will generally also be interested in how fast that displacement occurs. We therefore define a rate called the **average velocity** thus:

$$\text{average velocity} = v_{ave} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_o}{t_f - t_o} \quad (1.3.2)$$

Since we know displacement is a vector (of course in our current simple 1-dimensional model it can only have two directions), then average velocity must be a vector as well.

Instantaneous Velocity

Just talking about the before and after gets pretty boring, so what do we do about during? That is, how do we define a velocity at a single moment in time – the instantaneous velocity? Well, we know the answer to this from calculus. We start with our idea of average velocity, and just shrink the time span down very small, until it vanishes:

$$\text{instantaneous velocity} = v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad (1.3.3)$$

Average and Instantaneous Acceleration

We take our discussion of motion to one level more – we consider that things might speed up or slow down. Just as we defined average velocity in terms of before and after positions, we also define *average acceleration* in terms of before and after (instantaneous) velocities:

$$\text{average acceleration} = a_{ave} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o} \quad (1.3.4)$$

And, as before, we use calculus to extend this notion of average acceleration to instantaneous acceleration, which we describe as the amount that our object is speeding up or slowing down at a single moment in time:

$$\text{instantaneous acceleration} = a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \quad (1.3.5)$$

$$= \frac{d^2x}{dt^2} \quad (1.3.6)$$

Alert

Another language warning – In standard English parlance, we are used to reserving the word “acceleration” to mean only “speeding up.” In physics it means specifically the rate of change of velocity, which for straight line motion includes both speeding up and slowing down (for multi-dimensional motion it gets even trickier).

Conceptual Question

If a moving object is slowing down, is it possible that the magnitude of its acceleration is increasing? If an object is speeding up, is it possible that the magnitude of its acceleration is decreasing? In either of these cases, can the magnitude of the acceleration be zero? Explain.

Solution

If an object is slowing down, then it is experiencing an acceleration in the direction opposite to its motion. If this acceleration increases in magnitude, then it slows down faster. So naturally it can be slowing down as the acceleration magnitude increases. Similarly, as an object is speeding up, it is experiencing an acceleration in the direction of its motion. If the magnitude of this acceleration decreases, then the rate at which it speeds up decreases, but it is still speeding up. If the object is either slowing down or speeding up, then its velocity must be changing, and the acceleration cannot be zero.

Motion Diagrams

A motion diagram starts as merely a series of collinear dots that represent the position of an object at different equally-spaced intervals of time. You can think of it as a time-lapse photograph using a strobe light. There is one other piece of information that goes with this starting diagram: the direction that the object is moving. An example of this starting point might be this:

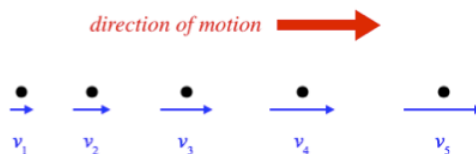
Figure 1.3.1a – Creating a Motion Diagram



From this we need to somehow extract the instantaneous velocity (magnitude and direction, which may be changing) at each position, and the acceleration (magnitude and direction, assumed to be constant throughout) of the object. At this point we are only working qualitatively, so our goal is to sketch onto the diagram velocity vectors at each dot that have magnitudes and directions that approximately represent the velocities of the object at those points (v_1 , v_2 , etc.), keeping in mind that the time intervals between dots are all the same, and the acceleration is constant throughout. You can do this intuitively (it must be going faster if it

covers more distance in the same time), or you can figure it out from [Equation 1.3.2](#). Adding the instantaneous velocity vectors to the above diagram makes it look like this:

Figure 1.3.1b – Creating a Motion Diagram

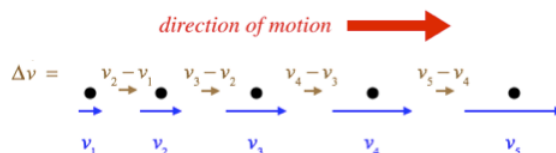


Now for acceleration. Since we are assuming constant acceleration (at least for the five-data-point interval we are considering), the average acceleration equals the instantaneous acceleration. With each dot being separated by the same time interval, the acceleration between dots is proportional to the velocity changes (magnitude and direction), and in this case of constant acceleration, is the same between every pair of dots:

$$a = \text{constant} = a_{\text{ave}} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{v_3 - v_2}{t_3 - t_2} = \dots \quad (1.3.7)$$

Putting this into the diagram gives:

Figure 1.3.1c – Creating a Motion Diagram



Note that Δv is determined using the usual tail-to-head vector addition, which in one dimension just consists of keeping the signs straight.

If we didn't know whether or not the acceleration was constant, we could make a good guess by comparing the Δv 's. Notice that we need two dots to determine the average velocity for a single time interval, since two dots gives us a displacement. But if we want to know how the speed is changing (i.e. the acceleration), we need three dots. If dots #1 and #2 are closer together than dots #2 and #3, we know the object has sped up, and if the first two dots are farther apart, then the object is slowing down. So when we label our motion diagram, we can arbitrarily draw in the first velocity vector on the first dot, but we can't add the velocity vector to the second dot if there is no third dot present to show us if the object is going faster, slower, or the same speed at the second dot. The more changes we want to consider (like if we want to know about a changing acceleration), the more dots we need.

This is in fact the nature of calculus – the change of a change of a change, etc, requires an additional measurement of position for each additional change computed. So the motion diagram only needs three dots if the acceleration is known (or assumed) to be constant, but to confirm that it is constant requires four dots.

Conceptual Question

Suppose we are given a motion diagram like the figure above, except that each velocity vector arrow is twice as long as the one before it. Does this diagram depict a constant acceleration? Explain.

Solution

Since every velocity vector is shown after the same time interval, the acceleration is only constant if the change in the velocity vector is the same each time. Let's call the magnitude of the first velocity vector "1 unit." This means that the second velocity vector has a magnitude of 2 units (since it doubled in length), and the one after that has a magnitude of 4 units. But this means that the change for the first time interval is 1 unit (from 1 to 2), and the change for the second interval is 2 units (from 2 to 4). These changes are not equal – the object is speeding up more in the second interval than it did in the first – so this acceleration is not constant.

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