2.S: Geometric Optics and Image Formation (Summary)

Key Terms

aberration	distortion in an image caused by departures from the small-angle approximation
accommodation	use of the ciliary muscles to adjust the shape of the eye lens for focusing on near or far objects
angular magnification	ratio of the angle subtended by an object observed with a magnifier to that observed by the naked eye
apparent depth	depth at which an object is perceived to be located with respect to an interface between two media
Cassegrain design	arrangement of an objective and eyepiece such that the light-gathering concave mirror has a hole in the middle, and light then is incident on an eyepiece lens
charge-coupled device (CCD)	semiconductor chip that converts a light image into tiny pixels that can be converted into electronic signals of color and intensity
coma	similar to spherical aberration, but arises when the incoming rays are not parallel to the optical axis
compound microscope	microscope constructed from two convex lenses, the first serving as the eyepiece and the second serving as the objective lens
concave mirror	spherical mirror with its reflecting surface on the inner side of the sphere; the mirror forms a “cave”
converging (or convex) lens	lens in which light rays that enter it parallel converge into a single point on the opposite side
convex mirror	spherical mirror with its reflecting surface on the outer side of the sphere
curved mirror	mirror formed by a curved surface, such as spherical, elliptical, or parabolic
diverging (or concave) lens	lens that causes light rays to bend away from its optical axis
eyepiece	lens or combination of lenses in an optical instrument nearest to the eye of the observer
far point	furthest point an eye can see in focus
farsightedness (or hyperopia)	visual defect in which near objects appear blurred because their images are focused behind the retina rather than on the retina; a farsighted person can see far objects clearly but near objects appear blurred
first focus or object focus	object located at this point will result in an image created at infinity on the opposite side of a spherical interface between two media
focal length	distance along the optical axis from the focal point to the optical element that focuses the light rays
focal plane	plane that contains the focal point and is perpendicular to the optical axis

focal point	for a converging lens or mirror, the point at which converging light rays cross; for a diverging lens or mirror, the point from which diverging light rays appear to originate
image distance	distance of the image from the central axis of the optical element that produces the image
linear magnification	ratio of image height to object height
magnification	ratio of image size to object size
near point	closest point an eye can see in focus
nearsightedness (or myopia)	visual defect in which far objects appear blurred because their images are focused in front of the retina rather than on the retina; a nearsighted person can see near objects clearly but far objects appear blurred
net magnification	(MnetMnet) of the compound microscope is the product of the linear magnification of the objective and the angular magnification of the eyepiece
Newtonian design	arrangement of an objective and eyepiece such that the focused light from the concave mirror was reflected to one side of the tube into an eyepiece
object distance	distance of the object from the central axis of the optical element that produces its image
objective	lens nearest to the object being examined.
optical axis	axis about which the mirror is rotationally symmetric; you can rotate the mirror about this axis without changing anything
optical power	(**P**) inverse of the focal length of a lens, with the focal length expressed in meters. The optical power P of a lens is expressed in units of diopters D; that is, $1D=1/m=1m^{-1}$
plane mirror	plane (flat) reflecting surface
ray tracing	technique that uses geometric constructions to find and characterize the image formed by an optical system
real image	image that can be projected onto a screen because the rays physically go through the image
second focus or image focus	for a converging interface, the point where a bundle of parallel rays refracting at a spherical interface; for a diverging interface, the point at which the backward continuation of the refracted rays will converge between two media will focus
simple magnifier (or magnifying glass)	converging lens that produces a virtual image of an object that is within the focal length of the lens
small-angle approximation	approximation that is valid when the size of a spherical mirror is significantly smaller than the mirror's radius; in this approximation, spherical aberration is negligible and the mirror has a well-defined focal point
spherical aberration	distortion in the image formed by a spherical mirror when rays are not all focused at the same point
thin-lens approximation	assumption that the lens is very thin compared to the first image distance

vertex	point where the mirror’s surface intersects with the optical axis
virtual image	image that cannot be projected on a screen because the rays do not physically go through the image, they only appear to originate from the image

Key Equations

Image distance in a plane mirror	$d_o = -d_i$
Focal length for a spherical mirror	$f = \frac{R}{2}$
Mirror equation	$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$
Magnification of a spherical mirror	$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$
Sign convention for mirrors	
Focal length f	+for concave mirror −for conve x mirror
Object distance d_o	+for real object −for virtual object
Image distance d_i	+for real image −for virtual image
Magnification m	+for upright image −for inverted image
Apparent depth equation	$h_i = (\frac{n_2}{n_1})h_o$
Spherical interface equation	$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R}$
The thin-lens equation	$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$
The lens maker’s equation	$\frac{1}{f} = (\frac{n_2}{n_1} - 1)(\frac{1}{R_1} - \frac{1}{R_2})$
The magnification m of an object	$m \equiv \frac{h_i}{h_o} = -\frac{d_i}{d_o}$
Optical power	$P = \frac{1}{f}$
Optical power of thin, closely spaced lenses	$P_{total} = P_{lens1} + P_{lens2} + P_{lens3} + \cdots$
Angular magnification **M** of a simple magnifier	$M = \frac{\theta_{image}}{\theta_{object}}$
Angular magnification of an object a distance **L** from the eye for a convex lens of focal length **f** held a distance ℓ from the eye	$M = (\frac{25cm}{L})(1 + \frac{L - \ell}{f})$
Range of angular magnification for a given lens for a person with a near point of 25 cm	$\frac{25cm}{f} \leq M \leq 1 + \frac{25cm}{f}$
Net magnification of compound microscope	$M_{net} = m^{obj} M^{eye} = -\frac{d_i^{obj}(f^{eye} + 25cm)}{f^{obj} f^{eye}}$

Summary

2.1 Images Formed by Plane Mirrors

- A plane mirror always forms a virtual image (behind the mirror).
- The image and object are the same distance from a flat mirror, the image size is the same as the object size, and the image is upright.

2.2 Spherical Mirrors

- Spherical mirrors may be concave (converging) or convex (diverging).
- The focal length of a spherical mirror is one-half of its radius of curvature: $f = R/2$.
- The mirror equation and ray tracing allow you to give a complete description of an image formed by a spherical mirror.
- Spherical aberration occurs for spherical mirrors but not parabolic mirrors; comatic aberration occurs for both types of mirrors.

2.3 Images Formed by Refraction

This section explains how a single refracting interface forms images.

- When an object is observed through a plane interface between two media, then it appears at an apparent distance h_i that differs from the actual distance $h_o : h_i = (n_2/n_1)h_o$.
- An image is formed by the refraction of light at a spherical interface between two media of indices of refraction n_1 and n_2.
- Image distance depends on the radius of curvature of the interface, location of the object, and the indices of refraction of the media.

2.4 Thin Lenses

- Two types of lenses are possible: converging and diverging. A lens that causes light rays to bend toward (away from) its optical axis is a converging (diverging) lens.
- For a converging lens, the focal point is where the converging light rays cross; for a diverging lens, the focal point is the point from which the diverging light rays appear to originate.
- The distance from the center of a thin lens to its focal point is called the focal length f.
- Ray tracing is a geometric technique to determine the paths taken by light rays through thin lenses.
- A real image can be projected onto a screen.
- A virtual image cannot be projected onto a screen.
- A converging lens forms either real or virtual images, depending on the object location; a diverging lens forms only virtual images.

2.5 The Eye

- Image formation by the eye is adequately described by the thin-lens equation.
- The eye produces a real image on the retina by adjusting its focal length in a process called accommodation.
- Nearsightedness, or myopia, is the inability to see far objects and is corrected with a diverging lens to reduce the optical power of the eye.
- Farsightedness, or hyperopia, is the inability to see near objects and is corrected with a converging lens to increase the optical power of the eye.
- In myopia and hyperopia, the corrective lenses produce images at distances that fall between the person's near and far points so that images can be seen clearly.

2.6 The Camera

- Cameras use combinations of lenses to create an image for recording.
- Digital photography is based on charge-coupled devices (CCDs) that break an image into tiny "pixels" that can be converted into electronic signals.

2.7 The Simple Magnifier

- A simple magnifier is a converging lens and produces a magnified virtual image of an object located within the focal length of the lens.
- Angular magnification accounts for magnification of an image created by a magnifier. It is equal to the ratio of the angle subtended by the image to that subtended by the object when the object is observed by the unaided eye.

- Angular magnification is greater for magnifying lenses with smaller focal lengths.
- Simple magnifiers can produce as great as tenfold (**10×**) magnification.

2.8 Microscopes and Telescopes

- Many optical devices contain more than a single lens or mirror. These are analyzed by considering each element sequentially. The image formed by the first is the object for the second, and so on. The same ray-tracing and thin-lens techniques developed in the previous sections apply to each lens element.
- The overall magnification of a multiple-element system is the product of the linear magnifications of its individual elements times the angular magnification of the eyepiece. For a two-element system with an objective and an eyepiece, this is

$$M = m^{obj} M^{eye}. \quad (2.41)$$

where m^{obj} is the linear magnification of the objective and M^{eye} is the angular magnification of the eyepiece.

- The microscope is a multiple-element system that contains more than a single lens or mirror. It allows us to see detail that we could not to see with the unaided eye. Both the eyepiece and objective contribute to the magnification. The magnification of a compound microscope with the image at infinity is

$$M_{net} = -\frac{(16cm)(25cm)}{f^{obj} f^{eye}}. \quad (2.42)$$

In this equation, 16 cm is the standardized distance between the image-side focal point of the objective lens and the object-side focal point of the eyepiece, 25 cm is the normal near point distance, f^{obj} and f^{eye} are the focal distances for the objective lens and the eyepiece, respectively.

- Simple telescopes can be made with two lenses. They are used for viewing objects at large distances.
- The angular magnification M for a telescope is given by

$$M = -\frac{f^{obj}}{f^{eye}}, \quad (2.43)$$

where f^{obj} and f^{eye} are the focal lengths of the objective lens and the eyepiece, respectively.