5.S: Photons and Matter Waves (Summary)

Key Terms

absorber	any object that absorbs radiation
absorption spectrum	wavelengths of absorbed radiation by atoms and molecules
Balmer formula	describes the emission spectrum of a hydrogen atom in the visible-light range
Balmer series	spectral lines corresponding to electron transitions to/from the $n=2$ state of the hydrogen atom, described by the Balmer formula
blackbody	perfect absorber/emitter
blackbody radiation	radiation emitted by a blackbody
Bohr radius of hydrogen	radius of the first Bohr's orbit
Bohr's model of the hydrogen atom	first quantum model to explain emission spectra of hydrogen
Brackett series	spectral lines corresponding to electron transitions to/from the $n=4$ state
Compton effect	the change in wavelength when an X-ray is scattered by its interaction with some materials
Compton shift	difference between the wavelengths of the incident X-ray and the scattered X-ray
Compton wavelength	physical constant with the value $\lambda_c = 2.43pm$
cut-off frequency	frequency of incident light below which the photoelectric effect does not occur
cut-off wavelength	wavelength of incident light that corresponds to cut-off frequency
Davisson–Germer experiment	historically first electron-diffraction experiment that revealed electron waves
de Broglie wave	matter wave associated with any object that has mass and momentum
de Broglie's hypothesis of matter waves	particles of matter can behave like waves
double-slit interference experiment	Young's double-slit experiment, which shows the interference of waves
electron microscopy	microscopy that uses electron waves to "see" fine details of nano-size objects
emission spectrum	wavelengths of emitted radiation by atoms and molecules
emitter	any object that emits radiation
energy of a photon	quantum of radiant energy, depends only on a photon's frequency
energy spectrum of hydrogen	set of allowed discrete energies of an electron in a hydrogen atom
excited energy states of the H atom	energy state other than the ground state
Fraunhofer lines	dark absorption lines in the continuum solar emission spectrum
ground state energy of the hydrogen atom	energy of an electron in the first Bohr orbit of the hydrogen atom

group velocity	velocity of a wave, energy travels with the group velocity
Heisenberg uncertainty principle	sets the limits on precision in simultaneous measurements of momentum and position of a particle
Humphreys series	spectral lines corresponding to electron transitions to/from the $n = 6$ state
hydrogen-like atom	ionized atom with one electron remaining and nucleus with charge $+Ze$
inelastic scattering	scattering effect where kinetic energy is not conserved but the total energy is conserved
ionization energy	energy needed to remove an electron from an atom
ionization limit of the hydrogen atom	ionization energy needed to remove an electron from the first Bohr orbit
Lyman series	spectral lines corresponding to electron transitions to/from the ground state
nuclear model of the atom	heavy positively charged nucleus at the center is surrounded by electrons, proposed by Rutherford
Paschen series	spectral lines corresponding to electron transitions to/from the $n = 3$ state
Pfund series	spectral lines corresponding to electron transitions to/from the $n = 5$ state
photocurrent	in a circuit, current that flows when a photoelectrode is illuminated
photoelectric effect	emission of electrons from a metal surface exposed to electromagnetic radiation of the proper frequency
photoelectrode	in a circuit, an electrode that emits photoelectrons
photoelectron	electron emitted from a metal surface in the presence of incident radiation
photon	particle of light
Planck’s hypothesis of energy quanta	energy exchanges between the radiation and the walls take place only in the form of discrete energy quanta
postulates of Bohr’s model	three assumptions that set a frame for Bohr’s model
power intensity	energy that passes through a unit surface per unit time
propagation vector	vector with magnitude $2\pi/\lambda$ that has the direction of the photon’s linear momentum
quantized energies	discrete energies; not continuous
quantum number	index that enumerates energy levels
quantum phenomenon	in interaction with matter, photon transfers either all its energy or nothing
quantum state of a Planck’s oscillator	any mode of vibration of Planck’s oscillator, enumerated by quantum number
reduced Planck’s constant	Planck’s constant divided by 2π
Rutherford’s gold foil experiment	first experiment to demonstrate the existence of the atomic nucleus

Rydberg constant for hydrogen	physical constant in the Balmer formula
Rydberg formula	experimentally found positions of spectral lines of hydrogen atom
scattering angle	angle between the direction of the scattered beam and the direction of the incident beam
Stefan–Boltzmann constant	physical constant in Stefan's law
stopping potential	in a circuit, potential difference that stops photocurrent
wave number	magnitude of the propagation vector
wave quantum mechanics	theory that explains the physics of atoms and subatomic particles
wave-particle duality	particles can behave as waves and radiation can behave as particles
work function	energy needed to detach photoelectron from the metal surface
α-particle	doubly ionized helium atom
α-ray	beam of α-particles (alpha-particles)
β-ray	beam of electrons
γ-ray	beam of highly energetic photons

Key Equations

Wien's displacement law	$\lambda_{max}T = 2.898 \times 10^{-3} m \cdot K$
Stefan's law	$P(T) = \sigma AT^4$
Planck's constant	$h = 6.626 \times 10-^{34} J \cdot s = 4.136 \times 10^{-15} eV \cdot s$
Energy quantum of radiation	$\Delta E = hf$
Planck's blackbody radiation law	$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_B^T} - 1}$
Maximum kinetic energy of a photoelectron	$K_{max} = e\Delta V_s$
Energy of a photon	$E_f = hf$
Energy balance for photoelectron	$K_{max} = hf - \phi$
Cut-off frequency	$f_c = \frac{\phi}{h}$
Relativistic invariant energy equation	$E^2 = p^2c^2 + m_0^2c^4$
Energy-momentum relation for photon	$p_f = \frac{E_f}{c}$
Energy of a photon	$E_f = hf = \frac{hc}{\lambda}$
Magnitude of photon's momentum	$p_f = \frac{h}{\lambda}$
Photon's linear momentum vector	$\vec{p_f} = \hbar\vec{k}$
The Compton wavelength of an electron	$\lambda_c = \frac{h}{m_0c} = 0.00243nm$
The Compton shift	$\Delta\lambda = \lambda_c(1 - cos\theta)$
The Balmer formula	$\frac{1}{\lambda} = R_H(\frac{1}{2^2} - \frac{1}{n^2})$

The Rydberg formula	$\frac{1}{\lambda} = R_H(\frac{1}{n_f^2} - \frac{1}{n_i^2}), n_i = n_f + 1, n_f + 2, \ldots$
Bohr’s first quantization condition	$L_n = n\hbar, n = 1, 2, \ldots$
Bohr’s second quantization condition	$h_f = \lvert E_n - E_m \rvert$
Bohr’s radius of hydrogen	$a_0 = 4\pi\varepsilon 0 \frac{\hbar^2}{m_e e^2} = 0.529\text{Å}$
Bohr’s radius of the *n*th orbit	$r_n = a_0 n^2$
Ground-state energy value, ionization limit	$E_0 = \frac{1}{8\varepsilon_0^2}\frac{m_e e^4}{h^2} = 13.6eV$
Electron’s energy in the *n*th orbit	$E_n = -E_0 \frac{1}{n^2}$
Ground state energy of hydrogen	$E_1 = -E_0 = -13.6eV$
The *n*th orbit of hydrogen-like ion	$r_n = \frac{a_0}{Z} n^2$
The *n*th energy of hydrogen-like ion	$E_n = -Z^2 E_0 \frac{1}{n^2}$
Energy of a matter wave	$E = hf$
The de Broglie wavelength	$\lambda = \frac{h}{p}$
The frequency-wavelength relation for matter waves	$\lambda f = \frac{c}{\beta}$
Heisenberg’s uncertainty principle	$\Delta x \Delta p \geq \frac{1}{2}\hbar$

Summary

6.1 Blackbody Radiation

- All bodies radiate energy. The amount of radiation a body emits depends on its temperature. The experimental Wien’s displacement law states that the hotter the body, the shorter the wavelength corresponding to the emission peak in the radiation curve. The experimental Stefan’s law states that the total power of radiation emitted across the entire spectrum of wavelengths at a given temperature is proportional to the fourth power of the Kelvin temperature of the radiating body.
- Absorption and emission of radiation are studied within the model of a blackbody. In the classical approach, the exchange of energy between radiation and cavity walls is continuous. The classical approach does not explain the blackbody radiation curve.
- To explain the blackbody radiation curve, Planck assumed that the exchange of energy between radiation and cavity walls takes place only in discrete quanta of energy. Planck’s hypothesis of energy quanta led to the theoretical Planck’s radiation law, which agrees with the experimental blackbody radiation curve; it also explains Wien’s and Stefan’s laws.

6.2 Photoelectric Effect

- The photoelectric effect occurs when photoelectrons are ejected from a metal surface in response to monochromatic radiation incident on the surface. It has three characteristics: (1) it is instantaneous, (2) it occurs only when the radiation is above a cut-off frequency, and (3) kinetic energies of photoelectrons at the surface do not depend of the intensity of radiation. The photoelectric effect cannot be explained by classical theory.
- We can explain the photoelectric effect by assuming that radiation consists of photons (particles of light). Each photon carries a quantum of energy. The energy of a photon depends only on its frequency, which is the frequency of the radiation. At the surface, the entire energy of a photon is transferred to one photoelectron.
- The maximum kinetic energy of a photoelectron at the metal surface is the difference between the energy of the incident photon and the work function of the metal. The work function is the binding energy of electrons to the metal surface. Each metal has its own characteristic work function.

6.3 The Compton Effect

- In the Compton effect, X-rays scattered off some materials have different wavelengths than the wavelength of the incident X-rays. This phenomenon does not have a classical explanation.
- The Compton effect is explained by assuming that radiation consists of photons that collide with weakly bound electrons in the target material. Both electron and photon are treated as relativistic particles. Conservation laws of the total energy and of momentum are obeyed in collisions.
- Treating the photon as a particle with momentum that can be transferred to an electron leads to a theoretical Compton shift that agrees with the wavelength shift measured in the experiment. This provides evidence that radiation consists of photons.
- Compton scattering is an inelastic scattering, in which scattered radiation has a longer wavelength than that of incident radiation.

6.4 Bohr's Model of the Hydrogen Atom

- Positions of absorption and emission lines in the spectrum of atomic hydrogen are given by the experimental Rydberg formula. Classical physics cannot explain the spectrum of atomic hydrogen.
- The Bohr model of hydrogen was the first model of atomic structure to correctly explain the radiation spectra of atomic hydrogen. It was preceded by the Rutherford nuclear model of the atom. In Rutherford's model, an atom consists of a positively charged point-like nucleus that contains almost the entire mass of the atom and of negative electrons that are located far away from the nucleus.
- Bohr's model of the hydrogen atom is based on three postulates: (1) an electron moves around the nucleus in a circular orbit, (2) an electron's angular momentum in the orbit is quantized, and (3) the change in an electron's energy as it makes a quantum jump from one orbit to another is always accompanied by the emission or absorption of a photon. Bohr's model is semi-classical because it combines the classical concept of electron orbit (postulate 1) with the new concept of quantization (postulates 2 and 3).
- Bohr's model of the hydrogen atom explains the emission and absorption spectra of atomic hydrogen and hydrogen-like ions with low atomic numbers. It was the first model to introduce the concept of a quantum number to describe atomic states and to postulate quantization of electron orbits in the atom. Bohr's model is an important step in the development of quantum mechanics, which deals with many-electron atoms.

6.5 De Broglie's Matter Waves

- De Broglie's hypothesis of matter waves postulates that any particle of matter that has linear momentum is also a wave. The wavelength of a matter wave associated with a particle is inversely proportional to the magnitude of the particle's linear momentum. The speed of the matter wave is the speed of the particle.
- De Broglie's concept of the electron matter wave provides a rationale for the quantization of the electron's angular momentum in Bohr's model of the hydrogen atom.
- In the Davisson–Germer experiment, electrons are scattered off a crystalline nickel surface. Diffraction patterns of electron matter waves are observed. They are the evidence for the existence of matter waves. Matter waves are observed in diffraction experiments with various particles.

6.6 Wave-Particle Duality

- Wave-particle duality exists in nature: Under some experimental conditions, a particle acts as a particle; under other experimental conditions, a particle acts as a wave. Conversely, under some physical circumstances, electromagnetic radiation acts as a wave, and under other physical circumstances, radiation acts as a beam of photons.
- Modern-era double-slit experiments with electrons demonstrated conclusively that electron-diffraction images are formed because of the wave nature of electrons.
- The wave-particle dual nature of particles and of radiation has no classical explanation.
- Quantum theory takes the wave property to be the fundamental property of all particles. A particle is seen as a moving wave packet. The wave nature of particles imposes a limitation on the simultaneous measurement of the particle's position and momentum. Heisenberg's uncertainty principle sets the limits on precision in such simultaneous measurements.
- Wave-particle duality is exploited in many devices, such as charge-couple devices (used in digital cameras) or in the electron microscopy of the scanning electron microscope (SEM) and the transmission electron microscope (TEM).