# CHAPTER OVERVIEW

## 4: C4) Systems and The Center of Mass



There are two basic concepts we are going to cover in this chapter: what a *system* is, and *the center of mass*.

A system is a very generic word ("the system of the Earth and the Moon", "a fridge is a closed system","my computer system") that we are going to make precise so that we can use it to describe our abstraction of the real world. Simply put, a **system is a collection of objects**. When we turn our objects into points, we are going to describe specific collections of those points as the systems. For a given problem, there may be multiple possible systems. For a simple example, consider the collision of two blocks sliding along the ground. We replace the blocks with points, and say "the system is the two blocks." Well that's a fine system if we're going to ignore friction, but if the blocks are experiencing friction between themselves and the ground, we had better include the ground in our system as well. So maybe system A is the blocks, but system B is the blocks *and* the ground, and if we want to include the effects of friction, we have to use system B. A simple idea, but really important for us as we start to think about how we model interactions, and which interactions are present in any given physical problem.

The center of mass, in comparison, is a definite mathematical concept. It's the answer to "when we replace our objects with points, what point do we actually use?" We use the center of mass because it's the balance point of the system, and therefore satisfies some important properties that random other points in our system don't satisfy. For example, consider the motion of a runner. If we replaced the runner with a point, and used the tip of their fingers as the point, we would have a very hard time confirming any of our calculations, because a real runners fingers are flying back and forth as they run! However, if we described their motion using their center of mass (a point inside their chest, at the center of their body), their motion would essentially be just a straight line - much easier to understand and use in our calculations.

The specific mathematical definition of the center of mass is

$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum m_i\vec{r}_i}{\sum m_i} \tag{4.1}$$

In this formula, each $m_i$ is the mass of an object, and each $\vec{r}_i$ is the position of the mass. If we were going to use this definition to find the center of mass of the runner, we would break up their body into a bunch of little masses and find all their vector positions - it would take a long time[1]! In this course, we will generally do things more like "find the center of mass between two objects of mass $m_1$ and $m_2$, separated by a distance $d$". So what are $\vec{r}_1$ and $\vec{r}_2$ in this situation? They are position vectors, so you first have to decide where the origin of your system is - where you are measuring the position relative to. It's often easy to pick one of the masses as the origin, say the first one, so that $r_1 = 0$, $r_2 = d$, and the center of mass is

$$r_{CM} = \frac{m_1 \cdot 0 + m_2 \cdot d}{m_1 + m_2}. \tag{4.2}$$

Notice that I got rid of the vector signs in this example - we will generally be more careful and keep track of both x- and y-directions separately, but here it's easy to see that the center of mass will be in a line between the two masses. If our numbers were $m_1 = 1$ kg, $m_2 = 10$ kg, and $d = 1$ m, the answer is $r_{CM} \simeq 0.91$ m. This makes sense as "the balance point", because it's much closer to the larger mass.

[1] Of course, if we really wanted to do that calculation, we would probably use an integral!