2.3: Force and Impulse

In the previous section we looked at the quantity $m\Delta v$, but notice that if mass is now changing, this is the same as the change in momentum of the object itself, Δp. That makes Equation (2.2.4) look like $\Delta p_1 = -\Delta p_2$, which is the statement that if one object gains $\Delta\vec{p}$, the object that it's interacting with must have lost that same amount of momentum. This phenomena known as **a conservation law**.

But of course we know that this is not the only way to describe interacting objects - in fact, perhaps the most intuitive way to describe two objects interacting is actually using a **force between them**. We are going to talk a lot more about forces in the second half of the this book, but right now we just want to acknowledge that if interactions can be described with either forces or momentum transfer, there must be some relationship between these two quantities. In fact, the relationship can be made concrete once you know the **time period Δt** over which the interaction occurs.

Specifically, if you have a change in momentum $\Delta\vec{p}$ resulting from an interaction that happens over a time period Δt, we can associate a force with this interaction via

$$\boxed{\vec{F}_{ave} = \frac{\Delta\vec{p}}{\Delta t}} \tag{2.3.1}$$

Notice that this definition only really works for an *average* force $\vec{F}_{ave}$, since we are talking about the changes in time being possibly relatively large. If the change in time is very small (in the sense of an infinitesimal dt from calculus), we can actually write

$$\vec{F} = \frac{d\vec{p}}{dt} \tag{2.3.2}$$

for the *instantaneous* force, which is the usual notion of force we are used to (this relationship can also be derived from Newton's second law).

Finally, we want to mention one more definition that comes up in this topic, **the impulse $\vec{J}$,** which is the amount of momentum delivered by a particular interaction. In many ways, this is simply a renaming of the change in momentum $\Delta\vec{p}$, and in fact the concrete mathematical definition of the impulse demonstrates this:

$$\boxed{\vec{J} = \Delta\vec{p}}. \tag{2.3.3}$$

All of the above is a perfectly fine description of a force that does not depend on time, which is often what we are dealing with. However, if a force does depend on time (think of throwing a baseball, or a "real" car crash - the force over the time period of those interactions may not be constant at all), we can still find the impulse delivered by performing an integration,

$$\vec{J} = \int \vec{F} dt. \tag{2.3.4}$$