CHAPTER OVERVIEW

9: C9) Potential Energy- Graphs and Springs

This chapter is a continuation of our study of energy. Here we want to introduce another important source of energy - the potential energy of a spring. Although it's "just for springs", it turns out that many systems in the world can be modeled with springs (molecules, for instance, and nearly anything that exhibits oscillatory behavior). In fact, it will be the last of the three kinds of potential energy we are going to study in this book (the other two being due to the gravitational interaction, U_g and U_G). So to round out this chapter, we are also going to introduce some graphs of more complicated potential energy functions, and how we can extract information from them.

A spring is an example of something that interacts *elastically*, meaning that it stores energy if you either stretch it *or* compress it. How much energy it stores depends on the spring, so each spring has "a spring constant" k, that describes how much energy is stored in it when you compress it a given distance. The actual formula is

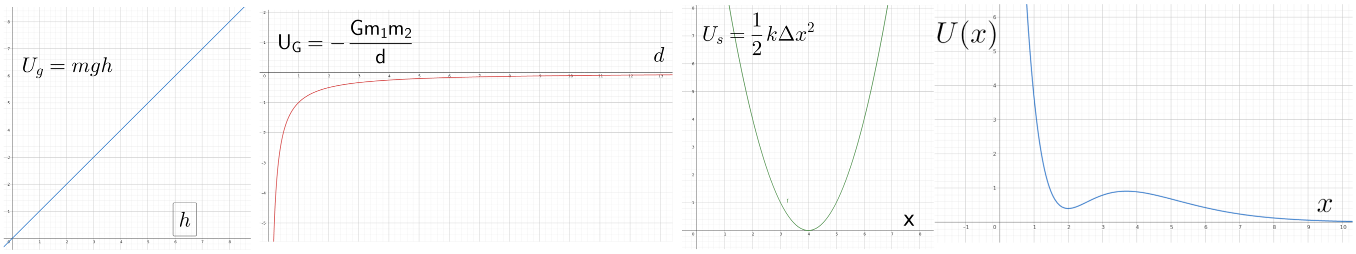
$$U_s = \frac{1}{2}k\Delta x^2, \tag{9.1}$$

where Δx is the amount the end of the spring is moved from it's equilibrium position - we will often write this as $\Delta x = x - x_0$, where x_0 is the equilibrium position of the spring. The fact that it is squared is what tells us that it doesn't matter what direction the change in length is; both compressing and stretching store energy.

To start talking about graphs, let's start with graphs of the three interactions we have seen in this book so far (below). Now think about how objects in each of these interactions behaves. For example, consider two asteroids separated by a distance d, using the potential energy for gravity (second graph). These two asteroids move towards each other right? What happens to the energy in the graph as they do? You can see that the potential energy decreases when that happens. In fact, that's true for the other form of gravitational potential as well. For the spring, something similar happens, but what direction the motion happens in depends on where you start - if you start on the right side of the equilibrium, decreasing energy means moving to smaller positions (closer to the equilibrium). If you're on the left side of the equilibrium, decreasing energy means increasing the x-coordinate (again, back to equilibrium).

What does all that mean? Well, it turns that nature wants to decrease the potential energy in a system! Unless something is actively preventing the motion, the potential will decrease. This means moving to the origin for the gravitational interaction, and towards the equilibrium point for the spring. We actually have special names for interactions that pull things together (**attractive**) as compared to push things apart (**repulsive**). So, just based on their plots, we can see that *gravity is an attractive interaction*. The *elastic interaction (springs) is either attractive or repulsive*, depending on which side of the equilibrium you are on.

Let's consider just one more interaction, one that's more complicated (last figure on the right). Playing the same game (nature wants to lower the energy!), we can see that if you start inside of the dip, you experience a spring-like interaction, either attractive or repulsive depending on where you start. If you start outside of the dip, the lowest energy is at higher and higher x-coordinate, so that interaction is repulsive out there. We can actually describe the qualitative motion of this system without ever looking at any equations, just with these simple considerations of "energy flow". This is only the first half of the story about potential energy graphs, which we will take up in more detail in the second half of this chapter.


$U_g = mgh$
h
$U_G = -\frac{Gm_1m_2}{d}$
d
$U_s = \frac{1}{2}k\Delta x^2$
x
$U(x)$
x