CHAPTER OVERVIEW

15: N2) 1 Dimensional Kinematics

If the last chapter, on Newton's laws, can be considered the study of "why" objects move (because of the forces that act on them), we can consider this chapter to focus on the question of "how" objects move. This is the study of **kinematics**. In particular, we are going to focus on the motion of points (rather then extended objects), moving in 1 dimension, and moving under constant acceleration. To start with let's recall the definitions of velocity and acceleration, both the average and instantaneous:

$$v_{x,ave} = \frac{\Delta x}{\Delta t}, \quad v_x = \frac{dx}{dt}, \quad a_{x,ave} = \frac{\Delta v}{\Delta t}, \quad a_x = \frac{dv_x}{dt}. \tag{15.1}$$

Now, under the condition of constant acceleration in 1 dimension, there are two independent equations of motion - one of which describes the position as a function of time, and one of which describes the velocity as a function of time. We'll present them here, and derive them in this chapter:

$$x(t) = \frac{1}{2}a_x t^2 + v_{0x}t + x_0, \tag{15.2}$$

$$v_x(t) = a_x t + v_{0x}. \tag{15.3}$$

These are quite a jumble of symbols, so let's discuss them carefully. The left hand side of the first equation, $x(t)$, is the position as a function of time along the x-direction. That's relatively simple to understand, but keep in mind the functional notation here - for each value of t, we get a position $x(t)$ by evaluating the right hand side of this equation. On the right hand side, we have the acceleration in the x-direction a_x, the initial velocity in the x-direction v_{0x}, and the initial position in the x-direction x_0. Notice the "physicist" convention of labeling these initial values with a zero, 0 - they are pronounced "-naught", like "x-naught". The last variable on this side is the time t.

The second equation is actually simplier after you know what's going on in the first. It's the velocity in the x-direction (again in the functional notation), with the acceleration, time and initial position on the right hand side. The first two things to notice about these two equations:

* They share the exact same set of symbols, with on their the left hand side being unique to each. They are also tied together through the time parameter t.

* The second one is the derivative, with respect to time, of the first. This comes from the calculus behind how these equations work, but we can check it pretty easily:

$$\frac{d}{dt}x(t) = v_x(t), \tag{15.4}$$

$$\frac{d}{dt}\left(\frac{1}{2}a_x t^2 + v_{0x}t + x_0\right) = \frac{d}{dt}\left(\frac{1}{2}a_x t^2\right) + \frac{d}{dt}(v_{0x}t) + \frac{d}{dt}(x_0) = a_x t + v_0. \tag{15.5}$$

The first is just the definition of velocity, whereas the second is carrying out a few pretty easy differentiations.

Using these equations is pretty straightforward, and is usually just a matter of identifying the "knowns and unknowns" in your problem. For example, let's do a runner, who starts at a position of 5 m (from some origin), and accelerates from rest at 2 m/s^2. We can find the position and velocity of this runner at *any* point in the future, so let's say after 5 seconds. The first job is to identify all the variables in the two equations above: x_0=5 m, a_x=2 m/s^2, and v_{0x}=0 (since we said "at rest"). Since we were asked for $t=5$ seconds, we can just plug in our values to find our two unknowns:

$$x(5\ s) = \frac{1}{2}(2\ m/s^2)(5\ s)^2 + (0)(5\ s) + (5\ m) = 30\ m, \tag{15.6}$$

$$v_x(5\ s) = (2\ m/s^2)(5\ s) + (0) = 10\ m/s. \tag{15.7}$$

It's important to realize that we did not label these positions "initial" and "final" - there is a reason for that. When we studied the conservation laws (momentum and energy), we were frequently concerned with very specific points - the final height of the ball, after a collision, etc. In the kinematic equations, we can consider *any value of time we like*, not only initial and final but any time in-between...not only 5 s, but also 5.1 s, and 4.9 s, or 4.99 s - you get the point. That is what is going on with the left side of these equations: $x(t)$ is "the position at a time t" - you have to know what time t you are specifically referring to in order to use these equations. Another way to think about that is the variable t is something you plug into these expressions - an input that needs to be specified to evaluate them. In this way, you might write these equations with that input more explicit:

$$x(\square) = \frac{1}{2}a_x(\square)^2 + v_{0x}(\square) + x_0, \tag{15.8}$$

$$v_x(\square) = a_x(\square) + v_{0x}. \tag{15.9}$$

Plug something into the boxes, and you are good to go!