2.2: Momentum

Inspired by Equation (2.1.2), let's define an objects momentum as

$$p=mv, \tag{2.2.1}$$

with mass m and velocity v, moving in one dimension (the choice of the letter p for momentum is apparently related to the Latin word "im*p*etus").

We can think of momentum as a sort of extension of the concept of inertia, from an object at rest to an object in motion. When we speak of an object's inertia, we typically think about what it may take to get it moving; when we speak of its momentum, we typically think of that it may take to stop it (or perhaps deflect it). So, both the inertial mass m and the velocity v are involved in the definition.

We may also observe that what looks like inertia in some reference frame may look like momentum in another. For instance, if you are driving in a car towing a trailer behind you, the trailer has only a large amount of inertia, but no momentum, relative to you, because its velocity relative to you is zero; however, the trailer definitely has a large amount of momentum (by virtue of both its inertial mass and its velocity) relative to somebody standing by the side of the road.

Conservation of Momentum; Isolated Systems

For a system of objects, we treat the momentum as an *additive* quantity. So, if two colliding objects, of masses m_1 and m_2, have initial velocities v_{1i} and v_{2i}, we say that the total initial momentum of the system is $p_i=m_1v_{1i}+m_2v_{2i}$, and similarly if the final velocities are v_{1f} and v_{2f}, the total final momentum will be $p_f=m_1v_{1f}+m_2v_{2f}$.

We then assert that *the total momentum of the system is not changed by the collision*. Mathematically, this means

$$p_i=p_f \tag{2.2.2}$$

or

$$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}. \tag{2.2.3}$$

But this last equation, in fact, follows directly from Equation (2.1.2): to see this, move all the quantities in Equation (2.2.3) having to do with object 1 to one side of the equal sign, and those having to do with object 2 to the other side. You then get

$$\begin{aligned} m_1\left(v_{1i}-v_{1f}\right) &= m_2\left(v_{2f}-v_{2i}\right) \\ -m_1\Delta v_1 &= m_2\Delta v_2 \end{aligned} \tag{2.2.4}$$

which is just another way to write Equation (2.1.2). Hence, the result (2.1.2) ensures the conservation of the total momentum of a system of any two interacting objects ("particles"), regardless of the form the interaction takes, as long as there are no external forces acting on them.

Momentum conservation is one of the most important principles in all of physics, so let's take a little time to explain how we got here and elaborate on this result. First, as just mentioned, we have been more or less implicitly assuming that the two interacting objects form an *isolated* system, by which we mean that, throughout, they interact with nothing other than each other. (Equivalently, there are no external forces acting on them.)

It is pretty much impossible to set up a system so that it is *really* isolated in this strict sense; instead, in practice, we settle for making sure that the external forces on the two objects *cancel out*. This is what happens in the car collisions we have been considering so far: gravity is acting on the carts, but that force is balanced out by the upwards push of the track. A system on which there is no *net* external force is as good as isolated for practical purposes, and we will refer to it as such. (It is harder, of course, to completely eliminate friction and drag forces, so we just have to settle for approximately isolated systems in practice.)

Secondly, we have assumed so far that the motion of the two objects is restricted to a straight line—one dimension. In fact, momentum is a *vector* quantity (just like velocity is), so in general we should write

$$\vec{p}=m\vec{v}$$

and conservation of momentum, in general, holds as a vector equation for any isolated system in three dimensions:

$$\vec{p}_i=\vec{p}_f. \tag{2.2.5}$$

What this means, in turn, is that each separate component (x, y and z) of the momentum will be separately conserved (so Equation (2.2.5) is equivalent to three scalar equations, in three dimensions). When we get to study the vector nature of forces, we will see an interesting implication of this, namely, that it is possible for one component of the momentum vector to be conserved, but not another—depending on whether there is or there isn't a net external force in that direction or not. For example, anticipating things a bit, when you throw an object horizontally, as long as you can ignore air drag, there is no horizontal force acting on it, and so that component of the momentum vector is conserved, but the vertical component is changing all the time because of the (vertical) force of gravity.

Thirdly, although this may not be immediately obvious, for an isolated system of two colliding objects the momentum is truly conserved throughout the whole collision process. It is not just a matter of comparing the initial and final velocities: at any of the times shown in Figures 2.1.1, 2.1.2, or 2.1.3, if we were to measure v_1 and v_2 and compute $m_1 v_1 + m_2 v_2$, we would obtain the same result. In other words, the total momentum of an isolated system is *constant*: it has the same value at all times.

Finally, all these examples have involved interactions between only two particles. Can we really generalize this to conclude that the total momentum of an isolated system of any number of particles is constant, even when all the particles may be interacting with each other simultaneously? Here, again, the experimental evidence is overwhelmingly in favor of this hypothesis[4], but much of our confidence on its validity comes in fact from a consideration of the nature of the internal interactions themselves. It is a mathematical fact that all of the interactions so far known to physics have the property of conserving momentum, whether acting individually or simultaneously. No experiments have ever suggested the existence of an interaction that does not have this property.

[4]For an important piece of indirect evidence, just consider that any extended object is in reality a collection of interacting particles, and the experiments establishing conservation of momentum almost always involve such extended objects. See the following section for further thoughts on this matter.