CHAPTER OVERVIEW

11: C11) Rotational Energy

In this chapter we are going to continue our study of conservation laws by moving into a new kind of energy conservation - rotational energy. Just like we first studied conservation of linear momentum ($\Delta\vec{p}=0$) before moving onto rotational momentum ($\Delta\vec{L}=0$), we are going to move from linear kinetic energy ($1/2mv^2$) to learn how to include rotational energy into our energy conservation law, $\Delta E=0$. It turns out this is going to be easy, because rotational motion is just another kind of kinetic motion, so rotational energy is just another kind of kinetic energy.

First we should quickly recall what we already know about rotational motion (Chapter 6 and Chapter 7). A rotating object has an angular velocity $\vec{\omega}$, and the "rotational inertia" of the object is the moment of inertia, I. Moments of inertia are different for each object, but generally look like $I=\alpha mr^2$, where α is 1 for a point or a hollow cylinder, 1/2 for a disk, etc. We might not easily remember this coefficient, but we can always get a sense of the moment of inertia by going back to the original definition of $I=\sum m_i r_i^2$, which tells us that "the more masses m_i that are closer to the axis, the smaller the moment of inertia will be".

So how do we use this for rotational energy? Using the correspondance principle between linear and rotational quantities (remember, that's how we got from $\vec{p}=m\vec{v}$ to $\vec{L}=I\vec{\omega}$), we get from linear ("center-of-mass") kinetic energy $K_{cm}=\frac{1}{2}mv^2$ to rotational kinetic energy,

$$K_{rot}=\frac{1}{2}I\omega^2. \tag{11.1}$$

Note that this equation satisfies a lot of the same conceptual framework that $\frac{1}{2}mv^2$ does - the larger moment of inertia or angular speed, the more rotational energy is being stored in the system. It's also worthwhile here to note what kinds of objects have large and small moments of inertia - objects with lots of mass near the axis of rotation are easy to rotate, and therefore have small moments of inertia, and objects with lots of mass far away from the axis of rotation are hard to rotate, and have large moments of inertia. So while you could store the same amount of energy in two different objects, the object with the smaller I will be spinning faster (have larger ω), to keep K_{rot} constant.

We are going to use this in the exact same way that we use other sources of energy - but unlike linear vs rotational momentum, which are separately conserved, we only have one conservation of energy law. So, if our system has rotational energy, we are just going to write:

$$\Delta E=E_f-E_i=(K_{cm,f}+K_{rot,f}+U_f)-(K_{cm,i}+K_{rot,i}+U_i)=0. \tag{11.2}$$

(Naturally, we can have more than one source of potential energy U_f and U_i as well.)