6.5: Quantum Mechanics of Free Particles

"Particle in a box." Periodic boundary conditions. *k*-space. In the thermodynamic limit, the dots in *k*-space become densely packed, and it seems appropriate to replace sums over levels with integrals over *k*-space volumes. (In fact, there is at least one situation (see equation 6.83) in which this replacement is *not* correct.)

Density of levels in *k*-space:

$$\frac{V}{8\pi^3} \quad (\text{ worth memorizing. }) \tag{6.5.1}$$

Energy density of levels:

$$\text{number of one-body levels with } \epsilon_r \text{ from } \mathcal{E} \text{ to } \mathcal{E}+d\mathcal{E} \equiv G(\mathcal{E})d\mathcal{E} = V\frac{\sqrt{2}m^3}{2\pi^2\hbar^3}\sqrt{\mathcal{E}}d\mathcal{E} \tag{6.5.2}$$

How to use energy density of levels:

$$\sum_r f\left(\epsilon_r\right) \approx \int_0^\infty G(\mathcal{E})f(\mathcal{E})d\mathcal{E} \tag{6.5.3}$$

and this approximation (usually) becomes exact in the thermodynamic limit.

6.5.1 Problems

6.14 Free particles in a box

We argued that, for a big box, periodic boundary conditions would give the same results as "clamped boundary conditions". Demonstrate this by finding the density of levels for three-dimensional particle in a box problem.

6.15 Density of levels for bound particles

What is the density of levels $G(\mathcal{E})$ for a one-dimensional harmonic oscillator with spring constant *K*? For a three-dimensional isotropic harmonic oscillator?

6.16 Density of levels in d dimensions

What is the density of levels $G(\mathcal{E})$ for free particles subject to periodic boundary conditions in a world of d dimensions?