10.6: F- The Euler-MacLaurin Formula and Asymptotic Series

You know that a sum can be approximated by an integral. How accurate is that approximation? The *Euler-MacLaurin formula* gives the corrections.

$$\sum_{k=0}^{n-1} f(k) \approx \int_0^n f(x) dx$$

$$-\frac{1}{2}[f(n)-f(0)]+\frac{1}{12}[f'(n)-f'(0)]-\frac{1}{720}[f'''(n)-f'''(0)]+\frac{1}{30240}\left[f^{(v)}(n)-f^{(v)}(0)\right]-\frac{1}{1209600}\left[f^{(\mathrm{vii})}(n)\right.$$

This series is asymptotic. If the series is truncated at any point, it can give a highly accurate approximation. But the series may be either convergent or divergent, so adding additional terms to the truncated series might give rise to a poorer approximation. The Stirling approximation is a truncation of an asymptotic series.

References

C.M. Bender and S.A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers*, (McGraw-Hill, New York, 1978).

M. Abramowitz and I. Stegum, *Handbook of Mathematical Functions*, (National Bureau of Standards, Washington, D.C., 1964).

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