7.3: Normal Modes for a One-dimensional Chain

The matrix A is all zeros except for 2 on the diagonal and −2 on the superdiagonal. But this doesn't really help us solve the problem. The solution comes from physical insight, not mathematical trickery!

Dispersion relation:

$$\omega(k) = 2\sqrt{\frac{K}{m}}\left|\sin\left(\frac{1}{2}ka\right)\right| \tag{7.3.1}$$

Meaning of term "dispersion relation":

> Start with an arbitrary wave packet, break it up into Fourier components.
>
> Each such component moves at a particular speed.
>
> After some time, find how all the components have moved, then sew them back together.
>
> The wave packet will have changed shape (usually broadened. . . dispersed).

Remember that we haven't done any statistical mechanics in this section, nor even quantum mechanics. This has been classical mechanics!