10.11: K- Useful Formulas

Isothermal compressibility:

$$\kappa_T = -\frac{1}{V}\frac{\partial V}{\partial p})_{T,N} = \frac{1}{\rho}\frac{\partial \rho}{\partial p})_T = \frac{1}{\rho^2}\frac{\partial^2 p}{\partial \mu^2})_T$$

Master thermodynamic equation:

$$dF = -SdT - pdV - MdH + \sum_i \mu_i dN_i$$

Gibbs-Helmholtz equation:

$$E(T,V,N) = \frac{\partial (F/T)}{\partial (1/T)})_{V,N} = -\frac{\partial \ln Z}{\partial \beta})_{V,N}$$

Free energy from partition function:

$$F(T,V,N) = -k_B T \ln Z(T,V,N)$$

Classical pure point-particle partition function:

$$Z(T,V,N) = \frac{1}{h^{3N}N!}\int d^{3N}p \int d^{3N}r e^{-H(r,p)/k_BT}$$

$$Z(T,V,N) = \frac{1}{\lambda^{3N}(T)}\int d^{3N}r e^{-U(r)/k_BT} \quad \text{where} \quad \lambda(T) = \frac{h}{\sqrt{2\pi m k_B T}}$$

Quantal ideal gases (+ for fermions, − for bosons):

$$\Xi(\beta,\mu) = \prod_{r=1}^{M}\left[1 \pm e^{-\beta(\epsilon_r-\mu)}\right]^{\pm 1}$$

$$\langle n_r\rangle = \frac{1}{e^{\beta(\epsilon_r-\mu)}\pm 1}$$