7.7: The Wave Nature of the Electron

In 1906 Barkla had shown that when "soft" (relatively long wavelength, low-energy) x-rays were scattered by carbon, they exhibited polarization phenomena in just the way one would expect if they were transverse waves. In 1913 the father-and-son team of W.H and W.L. Bragg had performed their experiments on the diffraction and interference of x-rays by crystals, and they showed that x-rays behaved just as one would expect for short wavelength electromagnetic waves. In 1919, Compton carried out his famous experiments on the scattering of x-rays by light atoms, including carbon, though he used higher energy ("harder") x-rays than in Barkla's experiments. Some of the x-rays were scattered without change of wavelength, as expected from classical Thomson scattering theory, and the wave nature of x-rays appeared to be very firmly established. Yet not all of the x-rays were scattered thus. Indeed when the scattering was from a light element such as carbon most of the scattered x-rays were found to have a longer wavelength than the incident x-rays. Most readers will be familiar with at least the broad outline of these experiments, and how Compton showed that the phenomenon could most easily be explained if, instead of being treated as waves of wavelength λ, they were treated as though they were particles of momentum h/λ. Thus x-rays appeared to have a "wave-particle duality", their behaviour in some circumstances being better treated as a wave phenomenon, and in others as though they were particles.

In 1924 de Broglie speculated that perhaps the electron, hitherto regarded, following the initial experiments in 1897 of J. J. Thomson, as a particle, might also exhibit "wave-particle duality" and in some circumstances, rather than be treated as a particle of momentum p might better be described as a wave of wavelength h/p. During the 1920s Davisson and Germer indeed did scattering experiments, in which electrons were scattered from a nickel crystal, and they found that electrons were selectively scattered in a particular direction just as they would if they were waves, and in a rather similar manner to the Bragg scattering of x-rays. Thus indeed it seemed that electrons, while in some circumstances behaving like particles, in others behaved like waves.

De Broglie went further and suggested that, if electrons could be described as waves, then perhaps in a Bohr circular orbit, the electron waves formed a standing wave system with an integral number of waves around the orbit:

$$n\lambda = 2\pi r. \tag{7.7.1}$$

Then, if $\lambda = h/(mv)$, we have

$$mvr = nh/(2\pi), \tag{7.7.2}$$

thus neatly "explaining" Bohr's otherwise *ad hoc* assumption, described by equation 7.4.1. From a modern point of view this "explanation" may look somewhat quaint, and little less "*ad hoc*" than Bohr's original assumption. One might also think that perhaps there should have been an integral number of *half*-wavelengths in a Bohr orbit. Yet it portended more interesting things to come. For example, one is reminded of the spherical harmonics in the solution to the vibrating sphere described in section 7.6, in which there is an integral number of antinodes between $\phi = 0$ and $\phi = 2\pi$.

For those who appreciate the difference between the *phase velocity* and the *group velocity* of a wave, we mention here that an electron moves with the group velocity of the wave that describes its behaviour.

We should also mention that the wave description of a particle is not, of course, restricted to an electron, but it can be used to describe the behaviour of any particle.

Finally, the equation $p = h/\lambda$ can also conveniently be written

$$p = \hbar k, \tag{7.7.3}$$

where $\hbar = h/(2\pi)$ and k is the propagation constant $2\pi/\lambda$ The propagation constant can also be written in the form $k = \omega/v$, since $\omega = 2\pi\nu = 2\pi v/\lambda = kv$.