1.12: Radiance L

The concept of exitance does nothing to describe a situation in which the brightness of an extended radiating (or reflecting) surface appears to vary with the direction from which it is viewed. For example, the centre of the solar disc is brighter than the limb (which is viewed at an oblique angle), particularly at shorter wavelengths, and the Moon is much brighter at full phase than at first or last quarter.

There are two concepts we can use to describe the directional properties of an extended radiating surface. I shall call them *radiance* L, and "surface brightness" B. I first define them, and then I determine the relationship between them. Please keep in mind the meaning of "per unit", or, as it is written in the next sentence, "from unit".

The radiance L of an extended source is the irradiance of an observer *from* unit solid angle of the extended source. It is an intrinsic property of the source and is independent of the distance of any observer. This is because, while irradiance of an observer falls off inversely as the square of the distance, the area included in unit solid angle increases as the square of the distance of the observer. While the radiance does not depend on the *distance* of the observer, it may well depend on the *direction* (θ,ϕ) from which the observer views the surface.

The s*urface brightness* B of an extended source is the intensity (i.e. flux emitted *into* unit solid angle) from unit projected area of the source. "Projected" here means projected on a plane that is normal to the line joining the observer to a point on the surface. The solid angle referred to here is subtended at a point on the surface. Like radiance, surface brightness is a property intrinsic to the source and is independent of the distance (but not the direction) to the observer.

These concepts may become clearer as I try to explain the relationship between them. This I shall do by supposing that the surface brightness of a point on the surface is B in some direction; and I shall calculate the irradiance of an observer in that direction from unit solid angle around the point.

In figure I.1 I draw an elemental area and the vector $\delta\mathbf{A}$ representing that area. In some direction making an angle with the normal to $\delta\mathbf{A}$, the area projected on a plane at right angles to that direction is $\delta A\ \cos\theta$. We suppose the surface brightness to be B, and, since surface brightness is defined to be intensity per unit projected area, the intensity in the direction of interest is $B\delta A\ \cos\theta$. The irradiance of an observer at a distance r from the elemental area is $\delta E = \delta I/r^2 = B\delta A\ \cos\theta/r^2$. But $\delta A\ \cos\theta/r^2$ is the solid angle $\delta\omega$ subtended by the elemental area at the observer. Therefore, by definition, $\delta E/\delta\omega$ is L, the radiance. Thus $L = B$. We see, then, that radiance L and surface brightness B are one and the same thing. Henceforth we can use the one term *radiance* and the one symbol L for either, and either definition will suffice to define radiance.

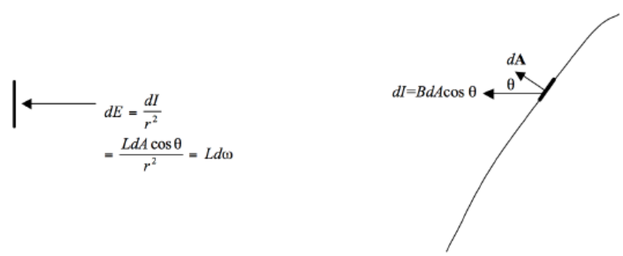

FIGURE I.1

In the figure the surface brightness at some point on a surface in a direction that makes an angle θ with the normal is B. The intensity radiated in that direction by an element of area dA is $dI = BdA\cos\theta$. The irradiance of a surface at a distance r away is $dE = dI/r^2 = BdA\cos\theta/r^2$. But $dA\cos\theta/r^2 = d\omega$, the solid angle subtended by dA. But the radiance L of a point on the right hand surface is the irradiance of the point in the left hand surface from unit solid angle of the former. Thus $L = B$, and we see that the two definitions, namely surface brightness and radiance, are equivalent, and will henceforth be called just radiance.

Radio astronomers usually use the term "surface brightness". In the literature of stellar atmospheres, however, the term used for radiance is often "specific intensity" or even just "intensity" and the symbol used is I. This is clearly a quite different usage of the word intensity and the symbol I that we have used hitherto. The use of the adjective "specific" does little to help, since in most contexts in physics, the adjective "specific" is understood to mean "per unit mass". It is obviously of great importance, in both reading and writing on the subject of stellar atmospheres, to be very clear as to the meaning intended by such terms as "intensity".

The radiance per unit frequency interval *of a black body* is often given the symbol B_ν, and the radiance per unit wavelength interval is given the symbol B_λ. We shall see later that these are related to the blackbody exitance functions (see equation 1.11.2

for M_λ) by $M_\nu = \pi B_\nu$ and $M_\lambda = \pi B_\lambda$. Likewise the integrated (over all wavelengths) radiance of a black body is sometimes written in the form $B = aT^4$. Here $a = \sigma/\pi$, σ being the Stefan-Boltzmann constant used in equation 1.11.1. (But see also section 1.17.)

Summary so far: The concepts "radiance" and "surface brightness", for which we started by using separate symbols, L and B, are identical, and the single name radiance and the single symbol L suffice, as also will either definition. The symbol B can now be reserved specifically for the radiance of a black body.

Although perhaps not of immediate interest to astronomers other than those concerned with light pollution, I now discuss the corresponding terms used when dealing with visible light. Instead of the terms radiant flux, radiant intensity, irradiance and radiance, the terms used are luminous flux, luminous intensity, illuminance and *luminance*. (This is the origin of the symbol L used for luminance and for radiance.) Luminous flux is expressed in *lumens*. Luminous intensity is expressed in lumens per steradian or *candela*. Illuminance is expressed in lumens per square metre, or *lux*. Luminance is expressed in $\mathrm{lm\,m^{-2} sr^{-1}}$, or $\mathrm{lux\,sr^{-1}}$, or $\mathrm{cd\,m^{-2}}$ or *nit*. The standard of luminous intensity was at one time the intensity of light from a candle of specified design burning at a specified rate. It has long been replaced by the candela, whose intensity is indeed roughly that of the former standard candle. The candela, when first introduced, was intended to be a unit of luminous intensity, equal approximately in magnitude to that of the former "standard candle", but making no reference to an actual real candle; it was defined such that the luminance of a black body at the temperature of melting platinum ($2042\ \mathrm{K}$) was exactly $600,000\ \mathrm{cd\,m^{-2}}$. Since 1979 we have gone one step further, recognizing that obtaining and measuring the radiation from a black body at the temperature of melting platinum is a matter of some practical difficulty, and the current definition of the candela makes no mention of platinum or of a black body, and the candela is defined in such a manner that if a source of monochromatic radiation of frequency $5.4 \times 10^{14} \mathrm{Hz}$ has a radiant intensity of $1/683\ \mathrm{W\ sr^{-1}}$ in that direction, then the luminous intensity is one candela. The reader may well ask what if the source in not monochromatic, or what if it is monochromatic but of a different frequency? Although it is not the intention here to treat this topic thoroughly, the answer, roughly, is that scientists involved in the field have prepared a table of a standard "photopic" relative sensitivity of a "standard" photopic human eye, normalized to unity at its maximum sensitivity at $5.4 \times 10^{14} \mathrm{Hz}$ (about $555\ \mathrm{nm}$). For the conversion between watts and lumens for monochromatic light of wavelength other than $555\ \mathrm{nm}$, one must multiply the conversion at $555\ \mathrm{nm}$ by the tabular value of the sensitivity at the wavelength in question. To calculate the luminance of a heterochromatic source, it is necessary to integrate over all wavelengths the product of the radiance per unit wavelength interval times the tabular value of the photopic sensitivity curve.

We have mentioned the word "photopic". The retina of the eye has two types of receptor cells, known, presumably from their shape, as "rods" and "cones". At high levels of illuminance, the cones predominate, but at low levels, the cones are quite insensitive, and the rods predominate. The sensitivity curve of the cones is called the "photopic" sensitivity, and that of the rods (which peaks at a shorter wavelengths than the cones) is the "scotopic" sensitivity. It is the standard photopic curve that defines the conversion between radiance and luminance.

I make one last remark on this topic. Namely , together with the metre, kilogram, second, kelvin, ampère and mole, the *candela* is one of the fundamental base units of the Système International des Unités. The *lumen*, *lux* and *nit* are also SI units, but the phot is not. The SI unit of luminance is the *nit*, although in practice this word is rarely heard ($\mathrm{lux\ sr^{-1}}$ or $\mathrm{cd\ m^{-2}}$ serve). The non-SI unit known as the *stilb* is a luminance of one candela per square centimetre.