11.7: Observational Curve of Growth

Equation 11.6.2 and figure XI.6 show how the equivalent width of a line will grow as the optical thickness is increased, which can be achieved either by increasing the geometrical thickness of the gas under study or by increasing the number density of the absorbers (atoms). Of course, if you are looking at the spectrum of a stellar atmosphere, you cannot do either of these things, but you might be able to construct a curve of growth by looking at many lines from the same element (such as an element from the iron group, which has lots of lines of a wide range of equivalent widths.) Then, by comparing the form of the observed curve of growth with one of the theoretical curves, you could deduce the gaussian and lorentzian HWHm, g and l, of your lines (and hence possibly the temperature and pressure) even if your resolution were not sufficient to determine the individual line profiles with any precision. You will notice the word "possibly" – because it has to be remembered that g includes both a thermal and a microturbulent component, and l includes both a radiation damping and pressure broadening component. Provided that the gas under study is homogeneous and of a uniform temperature throughout (I suppose this rules out a stellar atmosphere!) g should be the same for all lines of a given element. (The microturbulent component of g should be the same for all elements.) The radiation damping component of l will vary from line to line, but in practice, at least in the atmosphere of a main sequence star (but not necessarily of a giant, where the atmospheric pressure is much lower), the pressure broadening component of l will be much greater than the radiation damping component, and hence l will be the same for all lines of a given element. The theories of thermal, microturbulent, radiation damping and pressure broadening are all covered in Chapter 10.

It is all very well to say plot log W' versus log $\tau(0)$ for many lines of an iron-group element, but we immediately discover that we don't know either. The dimensionless quantity W' is given by

$$W' = W\sqrt{\ln 2}/g, \tag{11.7.1}$$

and we don't know g – indeed one of our aims is to find it. The optical depth at the line centre is given by

$$\tau(0) = \frac{\mathcal{N}_1 f_{12} e^2}{mc\epsilon_0 \Gamma}, \tag{11.7.2}$$

and we don't know Γ − again, one of our aims is to find it. However, these equations can be written in the form

$$\log W' = \log W + \text{ constant} \tag{11.7.3}$$

and

$$\log \tau(0) = \log \mathcal{N}_1 f_{12} + \text{ constant.} \tag{11.7.4}$$

Also, from Boltzmann's equation (where applicable!), we have

$$\ln \mathcal{N}_1 = \ln(\mathcal{N}/u) - E_1/(kT) + \ln \varpi_1, \tag{11.7.5}$$

from which

$$\log \mathcal{N}_1 = \log \varpi_1 + C(E_1) \tag{11.7.6}$$

where the "constant" is a function of E_1, the excitation energy of the lower level of the line. From this, we obtain

$$\log \tau(0) = \log \varpi f + C(E_1). \tag{11.7.7}$$

Thus, provided we take a set of lines all having the same lower excitation level (or at least the same lower term, provided that departures from LS-coupling are not so severe as to scatter the levels widely), we can construct a partial curve of growth by plotting log W versus log ϖf for these lines. These will be displaced both vertically and horizontally from the theoretical curves of figure XI.6 by arbitrary amounts, which will not affect the shape of the curve.

We can then take another set of lines, having a different common lower excitation level (or term), and plot another partial curve of growth. It will be displaced horizontally (but not vertically) from the first fragment, and it must be slid horizontally until it meshes in with the first fragment. And so we continue, building up partial curves of growth from sets of lines with a common lower term, sliding them horizontally until they all mesh with each other in a single continuous curve, which we can then compare with the shapes of the theoretical curves to obtain g and l and hence the temperature and pressure.