7.16: Return to the Hydrogen Atom

We have until now talked about the "levels" $n = 1, 2, 3\ldots$ of hydrogen, and the Lyman and Balmer "lines" connecting them. This nomenclature is now suspect.

In its lowest configuration the sole electron in hydrogen is in the K-shell and has $n = 1$. The only possible l-value is zero, and hence the lowest configuration in hydrogen is $1s$. This single electron has $l = 0$ and $s = 1/2$, and hence for the atom $L = 0$ and $S = 1/2$. Thus the only term in the K-shell is ^{2}S, and this has but a single level, $^2S_{1/2}$. (We are not considering here the very small contribution made by the spin of the nucleus; this is unimportant for most contexts in optical spectroscopy, but is important - VERY important - in the microwave region.)

In the L-shell, $n = 2$, and hence the electron can be in an s-orbital or a p-orbital. (See section 7.12 to recall the phrase "the electron is in a p-orbital".) That is, the configuration can be $2s$ or $2p$. In either case, of course, $s = 1/2$, so that the two possible terms in the L-shell are ^{2}S (with one level, $^2\mathrm{S}_{1/2}$) and ^{2}P (with two levels, $^2\mathrm{P}_{1/2}$ and $^2\mathrm{P}_{3/2}$).

I leave it up to the honour and integrity of the reader to convince him/herself that the possible configurations for $n = 3$ are $3s$, $3p$ or $3d$. There are three terms: ^{2}S, ^{2}P and ^{2}D, 5 levels (write down the J-levels of each) and 18 states.

In brief, although we have hitherto talked about the $n = 1,\ 2,\ 3$ levels, this is all right for $n = 1$, but $n = 2$ is three levels in two terms, and $n = 3$ is five levels in three terms.

The term values of the nine levels included in $n = 1, 2$ and 3 are as follows, in $\mu\mathrm{m}^{-1}$. I have also written out the statistical weight $2J + 1$ of each level.

$$\begin{array}{lll}
3d\ ^2\mathrm{D}_{\frac{5}{2}} & 9.7492342 & 6 \\
3d\ ^2\mathrm{D}_{\frac{3}{2}} & 9.7492306 & 4 \\
3p\ ^2\mathrm{P}^{\mathrm{o}}_{\frac{3}{2}} & 9.7492306 & 4 \\
3s\ ^2\mathrm{S}_{\frac{1}{2}} & 9.7492208 & 2 \\
3p\ ^2\mathrm{P}^{\mathrm{o}}_{\frac{1}{2}} & 9.7492198 & 2 \\
 & & \\
2p\ ^2\mathrm{P}^{\mathrm{o}}_{\frac{3}{2}} & 8.2259272 & 4 \\
2s\ ^2\mathrm{S}_{\frac{1}{2}} & 8.2258942 & 2 \\
2p\ ^2\mathrm{P}^{\mathrm{o}}_{\frac{1}{2}} & 8.2258907 & 2 \\
 & & \\
1s\ ^2\mathrm{S}_{\frac{1}{2}} & 0.0000000 & 2
\end{array} \tag{7.16.1}$$

Notice that the statistical weights of the K, L and M shells ($n = 1, 2, 3$) are, respectively, 2, 8 and 18 - i.e $2n^2$, as explained in section 7.11. In section 7.9 (see especially immediately following equation 7.9.2), before the introduction of electron spin, we had deduced that the statistical weight of each shell was just n^2; the introduction of electron spin has doubled that.

So just how many transitions *are* there in the "line" Hα? We cannot answer that until we have familiarized ourselves with the *selection rules*, but I make it that Hα comprises three transition arrays, three multiplets, seven lines, and I'm not sure just how many components! We'll come back to this in section 7.24..