

## 7.18: Hyperfine Structure

The levels and lines of many atoms have a hyperfine structure that is detectable only with high resolution, which may require not only interferometry but also a low temperature and low pressure source so that the intrinsic line width is small. Part of this very fine structure is due to the existence of several isotopes, and is not technically what is ordinarily meant by hyperfine structure, but is better referred to as isotope effects, which are dealt with in section 7.19. Hyperfine structure proper arises from the existence of *nuclear spin*, and it is this aspect that is dealt with in this section.

Protons and neutrons, the constituents of an atomic nucleus, collectively known as "nucleons", have, like the electron, a spin of  $1/2$ . That is to say, they possess an angular momentum

$$\sqrt{\frac{1}{2}\left(\frac{1}{2} + 1\right)}\hbar = \frac{1}{2}\sqrt{3}\hbar \quad (7.18.1)$$

oriented such that the component in some direction can have only one of the two values  $\pm\frac{1}{2}\hbar$ . Consequently a nucleus with an even number of nucleons must have an integral spin (which might be zero) while a nucleus with an odd number of nucleons must have an integral-plus-half spin, which cannot be zero. The spin quantum number of a nucleus is denoted by the symbol  $I$ . The magnitude of the nuclear angular momentum is  $\sqrt{I(I+1)}\hbar$ . It should be noted that different isotopes of a given element in general have different nuclear spins and consequently different hyperfine structure.

Whether the electrons in an atom are coupled by  $LS$  or  $jj$  or intermediate coupling, the coupling between the electrons is much stronger than the weak coupling between electrons and nucleus. Thus, in considering the coupling between the electrons and the nucleus,  $J$  can usually be regarded as a "good quantum number". To determine the total angular momentum of an atom, we have to add (vectorially, and by the rules of quantum mechanics) the nuclear angular momentum  $\mathbf{I}$  to the electronic angular momentum  $\mathbf{J}$ . This forms the total angular momentum of the atom, including nuclear spin, denoted by the symbol  $\mathbf{F}$ :

$$\mathbf{F} = \mathbf{J} + \mathbf{I}. \quad (7.18.1)$$

This equation is very similar to equation 7.13.3, except that, in equation 7.13.3, although  $S$  can be either integral or integral-plus-one-half,  $L$  is integral; whereas, in equation 7.18.1 both  $J$  and  $I$  have the possibility of being either integral or integral-plus-one-half. In any case, there are  $2\min\{I, J\} + 1$  values of  $F$ , going from  $|J - I|$  to  $J + I$ . If  $J + I$  is integral, all values of  $F$  are integral; and if  $J + I$  is integral-plus-one-half, so are all values of  $F$ . The magnitude of  $\mathbf{F}$  is  $\sqrt{F(F+1)}\hbar$ .

The nature of the interaction between  $\mathbf{J}$  and  $\mathbf{I}$  is the same as that between  $\mathbf{L}$  and  $\mathbf{S}$  in  $LS$ -coupling, and consequently the spacing of the term values of the hyperfine levels is similar to that described by equation 7.17.1 for the spacing of the levels within a term for  $LS$ -coupling namely:

$$T = \frac{1}{2}b[F(F+1) - J(J+1) - I(I+1)], \quad (7.18.2)$$

except that  $b \ll a$ . Landé's interval rule is obeyed; that is to say, the separation of two hyperfine levels within a level is proportional to the larger of the two  $F$ -values involved. There are similar selection rules for transitions between the hyperfine levels of one level and those of another, namely  $\Delta F$  and  $\Delta J = 0, \pm 1$  ( $0 \leftrightarrow 0$ ) forbidden, and (naturally!)  $\Delta I = 0$ . Calculating the spacings and intensities in the hyperfine structure of a line is precisely like calculating the spacings and intensities of the lines within a multiplet in  $LS$ -coupling.

For nuclei with zero spin, the quantum number  $M$  was associated with the vector  $\mathbf{J}$ , which was oriented such that its  $z$ -component was  $M\hbar$ , where  $M$  could have any of the  $2J + 1$  values from  $-J$  to  $+J$ . (We are here describing the situation in the quasi-mechanical descriptive vector model, rather than in terms of the possible eigenvalues of the quantum-mechanical operators, which supplies the real reason for the restricted values of the quantum numbers.) With nuclear spin, however, the quantum number  $M$  is associated with the vector  $\mathbf{F}$ , which is oriented such that its  $z$ -component is  $M\hbar$ , where  $M$  can have any of the  $2F + 1$  values from  $-F$  to  $+F$ , these values being integral or integral-plus-one-half according to whether  $F$  is integral or integral-plus-one-half. Thus each level is split into  $2\min\{J, I\} + 1$  hyperfine levels, and each hyperfine level is  $(2F + 1)$ -fold degenerate. Thus the statistical weight of a level is  $(2I + 1)(2J + 1)$ . (For a derivation of this, recall the *Exercise* in Section 7.14 concerning the statistical weight of a term in  $LS$ -coupling.)

If the nuclear spin is zero, the statistical weight of a level is the same as its degeneracy, namely just  $2J + 1$ . For nonzero nuclear spin, the correct expression for the statistical weight is  $(2I + 1)(2J + 1)$ . Nevertheless, the statistical weight is often treated as

though it were merely  $2J + 1$ , and indeed there are many contexts in which this can be safely done. We shall return to this in Chapter 9 when discussing the Boltzmann and Saha equations.

Hyperfine structure of spectrum lines is not often evident in the visible spectrum of stars. Generally the resolution is too poor and the lines are so broadened by high temperature as to mask any hyperfine structure. However, the nuclear spin of, for example, the  ${}^5\text{V}$  (vanadium) atom is  $I = 7/2$ , and the hyperfine structure of the lines, even if not fully resolved, is sufficient to make the lines noticeably broad. In the radio region, the most famous line of all is the 21-cm line of atomic hydrogen, and this involves hyperfine structure. The ground term of  $\text{H I}$  is  $1s\,{}^2S$ , which consists of the single level  ${}^2S_{1/2}$ . This level has  $J = \frac{1}{2}$ . The nuclear spin (spin of a proton) is  $I = \frac{1}{2}$ , so the level is split into two hyperfine levels with  $F = 0$  and 1. The 21-cm line is the transition between these two hyperfine levels. The transition is forbidden to electric dipole radiation (there is no parity change) and so it involves magnetic dipole radiation. It is therefore intrinsically a very weak line, but there is an awful lot of space out there with an awful lot of hydrogen in it.

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