1.10: Irradiance E

Suppose that some surface is being irradiated from a point source of radiation of intensity $I\ \mathrm{W\ sr^{-1}}$ at a distance r. The normal flux density ("normal" meaning normal to the direction of propagation), as we have seen, is I/r^2. If the surface being irradiated is inclined so that its normal is inclined at an angle θ to the line joining it to the point source of radiation, the rate at which radiant energy is falling on unit area of the surface will be $I\cos\theta/r^2$.

In any case, the rate at which radiant energy is falling upon unit area of a surface is called the *irradiance* of that surface. It is denoted by the symbol E, and the units are $\mathrm{W\ m^{-2}}$. In the simple geometry that we have described, the relation between the intensity of the source and the irradiance of the surface is

$$E=(I\cos\theta)/r^2 \tag{1.10.1}$$

If we are dealing with visible radiation, the number of lumens falling per unit area on a plane surface is called the *illuminance*, and is expressed in lumens per square metre, or *lux*. Recall that a lumen is the SI unit of luminous flux, and the candela is the unit of luminous intensity, and that an isotropic point source of light radiating with a luminous intensity of I cd (that is, $I\ \mathrm{lm\ sr^{-1}}$) emits a total luminous flux of $4\pi\ \mathrm{lm}$. The relation between the illuminance of a surface and the luminous intensity of a source of light is the same as the relation between irradiance and radiant intensity, namely, equation 1.10.1, or, if the surface is being illuminated normally, equation 1.8.2. If the luminous intensity of a source of light in some direction is one candela, the irradiance of a point on a surface that is closest to the source is $1\mathrm{lm\ m^{-2}}$ if the distance is one metre, $1\mathrm{lm\ cm^{-2}}$ if the distance is one cm, and $1\mathrm{lm\ ft^{-2}}$ if the distance is one foot. A lumen per square metre is a **lux**, and a lumen per square cm is a **phot**. A lumen per square foot is often (usually!) given the extraordinary name of a "foot-candle". This is a most illogical misuse of language, and is mentioned here only because the term is still in frequent use in non-scientific circles. Lumen, candela and lux are, respectively, the SI units of luminous flux, luminous intensity and illuminance. Phot and "foot-candle" are non-SI units of illuminance. The exact definition of the candela will be given in section 1.12; the lumen and lux are derived from the candela. Those who are curious about other strange-sounding units encountered in the quantitative measurement of the visible portion of radiation will also find the definition of "stilb" in section 1.12.

Example 1.10.1

A table is being illuminated by a light bulb fixed at a distance h vertically above the table. The fixture is such that the socket is above the bulb, and the luminous intensity of the bulb varies as

$$I(\theta)=\frac{1}{2}I(0)(1+\cos\theta) \tag{1.10.2}$$

where θ is the angle from the downward vertical from the bulb. Show that the illuminance at a point on the table at an angular distance θ from the sub-bulb point is

$$E(\theta)=\frac{1}{2}E(0)\left(\cos^3\theta+\cos^4\theta\right) \tag{1.10.3}$$

For what angle θ does the irradiance fall to half of the illuminance at the sub-bulb point??

Example 1.10.2

If the table in the above problem is a circular table whose radius subtends an angle α at the bulb, show that the flux that it intercepts is

$$\Phi(\alpha)=\pi I(0)\left(1-\cos\alpha+\frac{1}{2}\sin^2\alpha\right) \tag{1.10.4}$$

What is this if $\alpha\ =0$ and if $\alpha\to\pi/2$? Is this what you would expect? (Compare equation 1.6.7.)