1.5: "Per unit"

We have so far on three occasions used the phrase "per unit", as in flux per unit frequency interval, per unit wavelength interval, and per unit solid angle. It may not be out of place to reflect briefly on the meaning of "per unit".

The word *density* in physics is usually defined as "mass per unit volume" and is expressed in kilograms per cubic metre. But do we really mean the mass contained within a volume of a cubic metre? A cubic metre is, after all, a rather large volume, and the density of a substance may well vary greatly from point to point within that volume. Density, in the language of thermodynamics, is an *intensive* quantity, and it is defined *at a point*. What we really mean is the following. If the mass within a volume δV is δm, the *average density* in that volume is $\delta m/\delta V$. The density *at a point* is

$$\lim_{\delta V\to 0}\frac{\delta m}{\delta V}, \text{i.e.} \frac{dm}{dV}. \tag{1.5.1}$$

Perhaps the short phrase "per unit mass" does not describe this concept with precision, but it is difficult to find an equally short phrase that does so, and the somewhat loose usage does not usually lead to serious misunderstanding.

Likewise, Φ_λ is described as the flux "per unit wavelength interval", expressed in $\mathrm{W\ m^{-1}}$. But does it really mean the flux radiated in the absurdly large wavelength interval of a metre? Let $\delta\Phi$ be the flux radiated in a wavelength interval $\delta\lambda$. Then

$$\Phi_\lambda = \lim_{\delta\lambda\to 0}\frac{\delta\Phi}{\delta\lambda}; \text{i.e.} \frac{d\Phi}{d\lambda}. \tag{1.5.2}$$

Intensity is flux "per unit solid solid angle", expressed in watts per steradian. Again a steradian is a very large angle. What is actually meant is the following. If $\delta\Phi$ is the flux radiated into an elemental solid angle $\delta\omega$ (which, in spherical coordinates, is $\sin\theta\,\delta\theta\,\delta\phi$) then the *average* intensity over the solid angle $\delta\omega$ is $\delta\Phi/\delta\omega$. The intensity *in a particular direction* (θ, Φ) is

$$\lim_{\delta\omega\to 0}\frac{\delta\Phi}{\delta\omega}. \tag{1.5.3}$$

That is,

$$I = \frac{d\Phi}{d\omega}. \tag{1.5.4}$$