11.2: A Review of Some Terms

Before continuing, a review of some terms such as *absorption coefficient, absorptance, central depth* and *optical thickness* may be of some use.

Imagine a thin slice of absorbing gas of thickness δx. At one side of the slice, suppose that the specific intensity (radiance) per unit wavelength interval wavelength λ is $I_\lambda(\lambda)$ and that, after passage of the radiation through the slice, the specific intensity is now only $I_\lambda + \delta I_\lambda$. (δI_λ is negative.) The fractional decrease in the specific intensity is proportional to δx:

$$-\frac{\delta I_\lambda(\lambda)}{I_\lambda(\lambda)} = \alpha(\lambda)\delta x. \tag{11.2.1}$$

$\alpha(\lambda)$ is the *linear absorption coefficient*, and is of dimension L^{-1}. Now let imagine that, rather than an infinitesimally thin slice of gas, we have a slab of gas of finite thickness D and that it is sitting in front of a continuum source of specific intensity (radiance) per unit wavelength interval $I_\lambda(\mathrm{c})$, where "c" indicates "continuum". The radiance after passage through the slice is given by integrating Equation 11.2.1:

$$I_\lambda(\lambda) = I_\lambda(\mathrm{c})\exp\left[-\int_0^D \alpha(\lambda)\,dx\right], \tag{11.2.2}$$

The quantity

$$\tau(\lambda) = \int_0^D \alpha(\lambda)\,dx \tag{11.2.3}$$

is the *optical thickness* of the slab. It is dimensionless. If the slab of gas is *homogeneous*, in the sense that $\alpha(\lambda)$ is the same throughout the slab and is not a function of x, this becomes simply

$$\tau(\lambda) = D\alpha(\lambda). \tag{11.2.4}$$

Let us suppose that, after passage through the slab of gas, the specific intensity (radiance) of the gas at wavelength λ, which was initially $I_\lambda(\mathrm{c})$, is now $I_\lambda(\lambda)$, The fraction of the radiance that has been absorbed is the *absorptance* at wavelength λ, $a(\lambda)$:

$$a(\lambda) = \frac{I_\lambda(\mathrm{c}) - I_\lambda(\lambda)}{I_\lambda(\mathrm{c})} \tag{11.2.5}$$

It is dimensionless. The relation between absorptance and optical thickness is evidently

$$a(\lambda) = 1 - e^{-\tau(\lambda)}, \quad \tau(\lambda) = -\ln(1 - a(\lambda)). \tag{11.2.6 a,b}$$

For a gas of very small optical thickness, in which only a tiny fraction of the radiation has been absorbed (which will not in general be the case in this chapter), Maclaurin expansion of either of these Equations will show that

$$a(\lambda) \approx \tau(\lambda). \tag{11.2.7}$$

If, in addition, the slab of gas is homogeneous,

$$a(\lambda) \approx D\alpha(\lambda). \tag{11.2.8}$$

The *absorptance at the line centre* is

$$a(\lambda_0) = \frac{I_\lambda(\mathrm{c}) - I_\lambda(\lambda_0)}{I_\lambda(\mathrm{c})}, \tag{11.2.9}$$

and we have also called this, in Chapter 10, the *central depth* d.

The *absorption coefficient* of a gas at the wavelength of an absorption line is proportional to n_1, the number of atoms per unit volume in the initial (lower) level of the transition. This is so whether the slab of gas is optically thin or not; we are concerned here with the absorption coefficient and the number density *at a point within the gas*, not with the slab as a whole. The optical thickness of a slab of gas is proportional to $\mathcal{N}_1$, the number of atoms per unit area in the line of sight (column density) in the initial level. This is so whether or not the slab is homogeneous. In the sense in which the words *intensive* and *extensive* are used in thermodynamics, it could be said that absorption coefficient is an intensive quantity and optical thickness and absorptance are extensive quantities. While the optical thickness is proportional to $\mathcal{N}_1$, the radiance per unit wavelength interval and the equivalent width are not linearly proportional to $\mathcal{N}_1$ unless the gas is optically thin.

For the terms such as atomic and mass absorption coefficient, opacity, and the distinction between absorption, scattering and extinction, see Chapter 5.
