1.18: Radiation Pressure (P)

Photons carry momentum h/λ and hence exert pressure. Pressure is rate of change of momentum (i.e. force) per unit area.

The pressure P exerted by radiation (in $\mathrm{N\,m^{-2}}$, or Pa) is related to the energy density u of radiation (in $\mathrm{J\,m^{-3}}$) by

$$P=2u \tag{1.18.1}$$

$$P=u \tag{1.18.2}$$

$$P=u/3 \tag{1.18.3}$$

or

$$P=u/6 \tag{1.18.4}$$

depending on the circumstances!

First, we may imagine a parallel beam of photons that have come a long way from their original source. For example, they might be photons that have arrived at a comet from the Sun, and they are about to push material out from the comet to form the tail of the comet. Each of them is travelling with speed c. We suppose that there are n of them per unit volume, and therefore the number of them per unit area arriving per unit time is nc. Each of them carries momentum h/λ. [As in section 1.17 they need not all carry the same momentum. The total momentum is the sum of each.] The rate of arrival of momentum per unit area is $nhc/\lambda=nh\nu$. But $h\nu$ is the energy of each photon, so the rate of arrival of momentum per unit area is equal to the energy density. (Verify that these are dimensionally similar.) If all the photons stick (i.e. if they are absorbed), the rate of change of momentum per unit area (i.e. the pressure) is just equal to the energy density (Equation 1.18.2); but if they are reflected elastically, the rate of change of momentum per unit area is twice the energy density (Equation 1.18.1).

If the radiation is isotropic, the situation is different. The radiation may be approximately isotropic deep in the atmosphere of a star, though I fancy not completely isotropic, because there is sure to be a temperature gradient in the atmosphere. I suppose for the radiation to be truly isotropic, you'd have to go to the very center of the star.

We'll start from Equation 1.17.4, which gives the rate at which photons arrive at a point per unit area. ("at a point per unit area"? This makes sense only if you bear in mind the meaning of "per unit"!) If the energy of each photon is E, the momentum of each is E/c. (This is the relation, from special relativity, between the energy and momentum of a particle of zero rest mass.) However, it is the normal component of the momentum which contributes to the pressure, and the normal component of each photon is $(\mathrm{E}\cos\theta)/c$. The rate at which this normal component of momentum arrives per unit area is found by multiplying the integrand in Equation 1.17.4 by this. Bearing in mind that $n\mathrm{E}$ is the energy density u, we obtain

$$\frac{u}{4\pi}\int_0^{2\pi}\int_0^{\pi/2}\sin\theta\cos^2\theta\,d\theta d\phi. \tag{1.18.5}$$

The pressure on the surface is the rate at which the normal component of this momentum is changing. If the photons stick, this is

$$\frac{u}{4\pi}\int_0^{2\pi}\int_0^{\pi/2}\sin\theta\cos^2\theta\,d\theta d\phi=u/6. \tag{1.18.6}$$

But if they bounce, it is twice this, or $u/3$.