2.6: Wien's Law

The wavelengths or frequencies at which these functions reach a maximum, and what these maximum values are, can be found by differentiation of these functions. They do not all come to a maximum at the same wavelength. For the four Planck functions discussed in Section 2.6 (Equations 2.6.1- 2.6.4), the wavelengths or frequencies at which the maxima occur are given by:

For Equation 2.6.1:

$$\lambda = W_1/T \tag{2.6.1}$$

For Equation 2.6.2:

$$\lambda = W_2/T \tag{2.6.2}$$

For Equation 2.6.3:

$$\nu = W_3T \tag{2.6.3}$$

For Equation 2.6.4:

$$\nu = W_4T \tag{2.6.4}$$

Any of these equations (but more usually the first one) may be referred to as *Wien's law*.

The constants are

$$W_n = \frac{hc}{kx_n}, \quad (n = 1, 2) \tag{2.6.5}$$

$$W_n = \frac{kx_n}{h}, \quad (n = 3, 4) \tag{2.6.6}$$

where the x_n are the solutions of

$$x_n = (6-n)\left(1 - e^{-x_n}\right) \tag{2.6.7}$$

and have the values

$$x_1 = 4.965114 \tag{2.6.8}$$

$$x_2 = 3.920690 \tag{2.6.9}$$

$$x_3 = 2.821439 \tag{2.6.10}$$

$$x_4 = 1.593624 \tag{2.6.11}$$

The Wien constants then have the values

$$W_1 = 2.8978 \times 10^{-3} \text{ m K} \tag{2.6.12}$$

$$W_2 = 3.6697 \times 10^{-3} \text{ m K} \tag{2.6.13}$$

$$W_3 = 5.8790 \times 10^{10} \text{ Hz K}^{-1} \tag{2.6.14}$$

$$W_4 = 3.3206 \times 10^{10} \text{ Hz K}^{-1} \tag{2.6.15}$$

The maximum ordinates of the functions are given by

$$M_\lambda(\max) = A_1T^5 \tag{2.6.16}$$

$$N_\lambda(\max) = A_2T^4 \tag{2.6.17}$$

$$M_\nu(\max) = A_3T^3 \tag{2.6.18}$$

$$N_\nu(\max) = A_4T^2 \tag{2.6.19}$$

The constants A_n are given by

$$A_n = \frac{2\pi k^{6-n} y_n}{h^4 c^3}, \quad (n = 1, 2) \tag{2.6.20}$$

$$A_n = \frac{2\pi k^{6-n} y_n}{h^2 c^2}, \quad (n = 3, 4) \tag{2.6.21}$$

where the y_n are dimensionless numbers defined by

$$y_n = \frac{x_n^{6-n}}{e^{x_n} - 1} \tag{2.6.22}$$

That is,

$$y_1 = 21.20144 \tag{2.6.23}$$

$$y_2 = 4.779841 \tag{2.6.24}$$

$$y_3 = 1.421435 \tag{2.6.25}$$

$$y_4 = 0.6476102 \tag{2.6.26}$$

The constants A_n therefore have the values

$$A_1 = 1.2867 \times 10^{-5} \quad \mathrm{W\ m^{-2} K^{-5} m^{-1}} \tag{2.6.27}$$

$$A_2 = 2.1011 \times 10^{17} \quad \mathrm{ph\, s^{-1} m^{-2} K^{-4} m^{-1}} \tag{2.6.28}$$

$$A_3 = 5.9568 \times 10^{-19} \quad \mathrm{W\ m^{-2} K^{-3} Hz^{-1}} \tag{2.6.29}$$

$$A_4 = 1.9657 \times 10^{4} \quad \mathrm{ph\, s^{-1} m^{-2} K^{-2} Hz^{-1}} \tag{2.6.30}$$
