11.3: Theory of the Curve of Growth

Let us think again of our homogeneous slab of gas in front of a continuum source. Let $I_\lambda(\mathrm{c})$ be the radiance per unit wavelength interval of the continuum at wavelength λ. Let $\tau(x)$ be the optical thickness in the vicinity of a line and $x=\lambda-\lambda_0$. If the slab is of thickness D, the emergent radiance per unit wavelength as a function of wavelength will be

$$I_\lambda(x)=I_\lambda(\mathrm{c})\exp[-\tau(x)]. \tag{11.3.1}$$

The equivalent width W is given by

$$WI_\lambda(\mathrm{c})=\int_{-\infty}^{\infty}\left(I_\lambda(\mathrm{c})-I_\lambda(x)\right)\,dx, \tag{11.3.2}$$

or, by making use of Equation 11.3.1,

$$W=\int_{-\infty}^{\infty}\left[1-\exp\left\{-\tau(x)\right\}\right]\,dx. \tag{11.3.3}$$

If the line is symmetric, this may be evaluated as

$$W=2\int_{0}^{\infty}\left[1-\exp\left\{-\tau(x)\right\}\right]\,dx. \tag{11.3.4}$$

In former days, gallant efforts were made to find, using various approximations in the different regimes of the curve of growth, algebraic expressions for evaluating this integral. The availability of modern computers enables us to carry out the integration numerically.