10.3: Microturbulence

In the treatment of microturbulence in a stellar atmosphere, we can suppose that there are many small cells of gas moving in random directions with a maxwellian distribution of speeds. The distinction between microturbulence and macroturbulence is that in microturbulence the size of the turbulent cells is very small compared with the optical depth, so that, in looking down through a stellar atmosphere we are seeing many cells of gas whose distribution of velocity components is gaussian. In macroturbulence the size of the cells is not very small compared with the optical depth, so that , in peering through the haze of an atmosphere, we can see at most only a very few cells.

If the distribution of velocity components of the microturbulent cells is supposed gaussian, then the line profiles will be just like that for thermal broadening, except that, instead of the modal speed $V_{\mathrm{m}} = \sqrt{2kT/m}$ of the atoms we substitute the modal speed ξ_m of the microturbulent cells. Thus the line profile resulting from microturbulence is

$$\frac{I_\nu(\nu)}{I_\nu(\nu_0)} = 1 - d\exp\left[-\frac{c^2}{\xi_m^2}\frac{(\nu-\nu_0)^2}{\nu_0^2}\right]. \tag{10.3.1}$$

The FWHM in frequency units is

$$\frac{\xi_{\mathrm{m}}\nu_0\sqrt{\ln 16}}{c} \tag{10.3.2}$$

or, in wavelength units,

$$\frac{\xi_{\mathrm{m}}\lambda_0\sqrt{\ln 16}}{c}. \tag{10.3.3}$$

If the thermal and microturbulent broadening are comparable in size, we still get a gaussian profile, except that for V_{m} or ξ_{m} we must substitute

$$\sqrt{V_{\mathrm{m}}^2 + \xi_{\mathrm{m}}^2} = \sqrt{2kT/m + \xi_{\mathrm{m}}^2}. \tag{10.3.4}$$

This actually requires formal proof, and this will be given as an exercise in Section 5.

Since either thermal broadening or microturbulence will result in a gaussian profile, one might think that it would not be possible to tell, from a spectrum exhibiting gaussian line profiles, whether the broadening was caused primarily by high temperature or by microturbulence. But a little more thought will show that in principle it *is* possible to distinguish, and to determine separately the kinetic temperature and the modal microturbulent speed. Think about it, and see if you can devise a way.

THINKING

The key is, in purely thermal broadening, the light atoms (such as lithium) move faster than the heavier atoms (such as cadmium), the speeds being inversely proportional to the square roots of their atomic masses. Thus the lines of the light atoms will be broader than the lines of the heavy atoms. In microturbulence all atoms move *en masse* at the same speed and are therefore equally broad. We have seen, beneath Equation 10.3.7, that the FWHM, in frequency units, is

$$w = \frac{\nu_0}{c}\sqrt{\left(2kT/m + \xi_{\mathrm{m}}^2\right)\ln 16}. \tag{10.3.5}$$

If we form the quantity

$$X = \frac{w^2c^2}{\nu_0^2\ln 16} \tag{10.3.6}$$

for a lithium line and for a cadmium line, we will obtain

$$X_{\mathrm{Li}} = \frac{2kT}{m_{\mathrm{Li}}} + \xi_{\mathrm{m}}^2 \quad \text{and } X_{\mathrm{Cd}} = \frac{2kT}{m_{\mathrm{Cd}}} + \xi_{\mathrm{m}}^2, \tag{10.3.7}$$

from which T and ξ_{m} are immediately obtained.

Exercise 10.3.1

A Li line at 670.79 nm has a gaussian FWHM = 9 pm (picometres) and a Cd line at 508.58 nm has a gaussian FWHm = 3 pm. Calculate the kinetic temperature and the modal microturbulent speed.
