7.3: The Hydrogen Spectrum

In 1885, J. J. Balmer, a lecturer in a ladies' college in Switzerland, devised a simple formula relating the wavelengths of the lines in the visible region of the atomic hydrogen spectrum to the natural numbers, and these lines have since been referred to as the Balmer series and have been denoted by Hα, Hβ, Hγ,...,starting at the long wavelength end. The standard air wavelengths in nm and the vacuum wavenumbers in μm^{-1} are as follows:

$$\begin{array}{lcc} & \lambda & \sigma_0 \\ & \mathrm{nm} & \mu\mathrm{m}^{-1} \\ \mathrm{H}\alpha & 656.28 & 1.5233 \\ \mathrm{H}\beta & 486.13 & 2.0565 \\ \mathrm{H}\gamma & 434.05 & 2.3032 \\ \mathrm{H}\delta & 410.17 & 2.4373 \\ \mathrm{H}\epsilon & 397.01 & 2.5181 \end{array} \tag{7.3.1}$$

The series eventually converges to a series limit, the *Balmer limit*, at a standard air wavelength of $364.60\ \mathrm{nm}$ or a vacuum wavenumber of $2.7420\ \mu\mathrm{m}^{-1}$. In the way in which Balmer's formula is usually written today, the vacuum wavenumbers of the lines in the Balmer series are given by

$$\sigma_0 = R\left(\frac{1}{4} - \frac{1}{n^2}\right),\ n = 3, 4, 5\ldots \tag{7.3.2}$$

n being 3, 4, 5, etc., for $\mathrm{H}\alpha$, $\mathrm{H}\beta$, $\mathrm{H}\gamma$, etc. The number R is called the *Rydberg constant for hydrogen*, and has the value $10.9679\ \mu\mathrm{m}^{-1}$.

Later, a similar series, to be named the *Lyman series*, was discovered in the ultraviolet, and several similar series were found in the infrared, named after Paschen, Brackett, Pfund, Humphreys, Hansen and Strong, and successively less famous people. Indeed in the radio region of the spectrum there are series named just for numbers; thus we may talk about the 109α line.

A single formula can be used to generate the wavenumbers of the lines in each of these series:

$$\sigma_0 = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right),\ n_2 = n_1 + 1, n_1 + 2, \ldots \tag{7.3.3}$$

Here $n_1 = 1, 2, 3, 4, 5, 6\ldots$for the Lyman, Balmer, Paschen, Brackett, Pfund, Humphreys... series.

Similar (not identical) spectra are observed for other hydrogen-like atoms, such as He^+, Li^{++}, Be^{+++}, etc., the Rydberg constants for these atoms being different from the Rydberg constant for hydrogen. Deuterium and tritium have very similar spectra and their Rydberg constants are very close to that of the $^1\mathrm{H}$ atom.

Each "line" of the hydrogen spectrum, in fact, has fine structure, which is not easily seen and usually needs carefully designed experiments to observe it. This fine structure need not trouble us at present, but we shall later be obliged to consider it. An interesting historical story connected with the fine structure of hydrogen is that the quantity $e^2/(4\pi\epsilon_0\hbar c)$ plays a prominent role in the theory that describes it. This quantity, which is a dimensionless pure number, is called the *fine structure constant* α, and the reciprocal of its value is close to the prime number 137. Sir Arthur Eddington, one of the greatest figures in astrophysics in the early twentieth century, had an interest in possible connections between the fundamental constants of physics and the natural numbers, and became almost obsessed with the notion that the reciprocal of the fine structure constant should be *exactly* 137, even insisting on hanging his hat on a conference hall coatpeg number 137.