2.5: Planck's Equation

The importance of Planck's equation in the early birth of quantum theory is well known. Its theoretical derivation is dealt with in courses on statistical mechanics. In this section I merely give the relevant equations for reference.

Planck's equation can be given in various ways, and here I present four. All will be given in terms of exitance. The radiance is the exitance divided by π.(Equation 1.15.2.). The four forms are as follows, in which I have made use of equations 1.3.1 and the expression $h\nu = hc/\lambda$ for the energy of a single photon.

The rate of emission of energy per unit area per unit time (i.e. the exitance) per unit wavelength interval:

$$M_\lambda = \frac{C_1}{\lambda^5 \left(e^{K_1/\lambda T} - 1\right)} \tag{2.6.1}$$

The rate of emission of photons per unit area per unit time per unit wavelength interval:

$$N_\lambda = \frac{C_2}{\lambda^4 \left(e^{K_1/\lambda T} - 1\right)} \tag{2.6.2}$$

The rate of emission of energy per unit area per unit time (i.e. the exitance) per unit frequency interval:

$$M_\nu = \frac{C_3 \nu^3}{e^{K_2 \nu/T} - 1} \tag{2.6.3}$$

The rate of emission of photons per unit area per unit time per unit frequency interval:

$$N_\nu = \frac{C_4 \nu^2}{e^{K_2 \nu/T} - 1} \tag{2.6.4}$$

The constants are:

$$\begin{array}{lllll} C_1 & = & 2\pi hc^2 & = & 3.7418 \times 10^{-16}\,\mathrm{W\ m^2} & (2.6.5) \\ C_2 & = & 2\pi c & = & 1.8837 \times 10^{9}\,\mathrm{m\ s^{-1}} & (2.6.6) \\ C_3 & = & 2\pi h/c^2 & = & 4.6323 \times 10^{-50}\,\mathrm{kg\ s} & (2.6.7) \\ C_4 & = & 2\pi/c^2 & = & 6.9910 \times 10^{-17}\,\mathrm{m^{-2} s^2} & (2.6.8) \\ K_1 & = & hc/k & = & 1.4388 \times 10^{-2}\,\mathrm{m\,K} & (2.6.9) \\ K_2 & = & h/k & = & 4.7992 \times 10^{-11}\,\mathrm{s\,K} & (2.6.10) \end{array} \tag{2.5.1}$$

Symbols:

$$\begin{aligned} h &= \text{Planck's constant} \\ k &= \text{Boltzmann's constant} \\ c &= \text{speed of light} \\ T &= \text{temperature} \\ \lambda &= \text{wavelength} \\ \nu &= \text{frequency} \end{aligned} \tag{2.5.2}$$