

## 10.11: APPENDIX C- Optical Thinness, Homogeneity and Thermodynamic Equilibrium

It has also occurred to me while preparing these chapters that some of the equations are valid only under certain conditions, such as that the gas is *optically thin*, or is *homogeneous* or is in *thermodynamic equilibrium*, or some combination of these, or none of them. It would be tedious to spell out all of the conditions after each equation. Yet it is important to know under what conditions each is valid. In this Appendix I try to give some guidance. For example, most of the equations in this Chapter deal with line profiles in an optically thin gas, whereas in the next Chapter the gas is no longer optically thin. In the end, however, the only way of being sure of what conditions apply to each equation is to understand the basic physics behind each rather than attempting to memorize which conditions apply to which equations.

The linear absorption coefficient  $\alpha$  at a point within a gas is proportional to the local number density  $n_1$  of absorbers. (The subscript 1 refers to “atoms in the lower level of the line concerned”.) The optical thickness of a slab of gas of thickness  $D$  is related to the absorption coefficient (which may or may not vary throughout the slab) by  $\tau = \int_0^D \alpha(x) dx$ . This is so whether or not the gas is optically thin or whether it is homogeneous. Likewise, the column density  $N_1$  of absorbers is related to the number density by  $N_1 = \int_0^D n_1(x) dx$ . If the gas is homogeneous in the sense that  $n_1$  is not a function of  $x$ , and consequently  $\alpha$  is not a function of  $x$  either, then these equations become simply  $\tau = \alpha D$  and  $N_1 = n_1 D$ , and this is so whether or not the gas is optically thin.

Whether optically thin or thick, and whether homogeneous or not, the optical thickness is proportional to the column density  $N$ , just as the absorption coefficient is proportional to  $n_1$ .

If a layer of gas of thickness  $D$  is not homogeneous, the optical thickness is related to the absorption coefficient and the thickness of the gas by  $\tau = \int_0^D \alpha(x) dx$ . If the gas is homogeneous so that  $\alpha$  is independent of  $x$ , then the relation is merely  $\tau = \alpha D$ . Neither of these equations requires the gas to be optically thin. That is, they are valid whether the gas is optically thin or thick. The absorption coefficient at a point within the gas is proportional to the local density (number of absorbers per unit volume there.) The optical thickness is proportional to the column density of absorbers along the line of sight, whether or not the gas is optically thin and whether or not it is homogeneous.

However, the *equivalent width* and *central depth* of an absorption line, or the *intensity* or *radiance*, or *central intensity* or *radiance per unit wavelength interval* of an emission line, are proportional to the column density of atoms only if the gas is optically thin. Indeed this simple proportionality can serve as a good definition of what is meant by being optically thin.

The equivalent width of an absorption line is given by  $W = \int [1 - \exp\{-\tau(\lambda)\}] d\lambda$ . If the gas is homogeneous, this becomes  $W = \int [1 - \exp\{-D\alpha(\lambda)\}] d\lambda$ . If, in addition, the gas is optically thin at all wavelengths within the line, this becomes (by Maclaurin expansion), merely  $W = D \int \alpha(\lambda) d\lambda$ . Note that, if  $\lambda$  and  $D$  are expressed in m and if  $\alpha$  is expressed in  $\text{m}^{-1}$ , the equivalent width will be in m. If, however, you choose to express wavelengths in angstroms and the thickness of a cloud in parsecs, that is your problem, and you are on your own.

Any equations in which we have gone from  $n$ , the total number of atoms per unit volume in all levels to  $n_1$  via Boltzmann's equation, implies an assumption of thermodynamic equilibrium. An example would be going from equation 9.2.4 (which does not imply thermodynamic equilibrium) to equations 9.2.6-10 (which do imply thermodynamic equilibrium). If a gas is truly in thermodynamic equilibrium, this implies that the gas will be at a single, homogenous temperature – otherwise there will be heat flow and no equilibrium. It is doubtful if anything in the Universe is truly in thermodynamical equilibrium in the very strictest use of the term. However, even in an atmosphere in which the temperature is different from point to point, we may still have *local* thermodynamic equilibrium (LTE), in the sense that, at any point, it is all right to calculate the distribution of atoms among their energy levels by Boltzmann's equation, or the degree of ionization by Saha's equation, or the atomic speeds by the Maxwell-Boltzmann equation, or the radiation energy density by Planck's equation – and you may even be able to use the same temperature for each. This may be all right within a small volume of an atmosphere; only when considered over large ranges of space and time will it be evident that the atmosphere is not in true thermodynamic equilibrium.

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