1.14: Relations between Flux, Intensity, Exitance, Irradiance

In this section I am going to ask, and answer, three questions.

i. (See figure I.3)

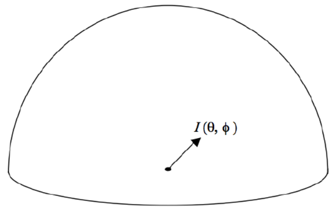

FIGURE I.3

A point source of light has an intensity that varies with direction as $I(\theta, \phi)$. What is the radiant flux radiated into the hemisphere $\theta < \pi/2$? This is easy; we already answered it for a complete sphere in equation 1.6.3. For a hemisphere, the answer is

$$\phi = \int_0^{2\pi} \int_0^{\pi/2} I(\theta, \phi) \sin\theta d\theta d\phi. \tag{1.14.1}$$

ii. At a certain point on an extended plane radiating surface, the radiance is $L(\theta, \phi)$. What is the emergent exitance M at that point?

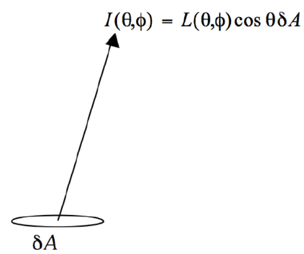

FIGURE I.4

Consider an elemental area δA (see figure I.4). The intensity $I(\theta, \phi)$ radiated in the direction (θ, ϕ) is the radiance times the projected area $\cos\theta\,\delta A$. Therefore the radiant power or flux radiated by the element into the hemisphere is

$$\delta\phi = \int_0^{2\pi} \int_0^{\pi/2} L(\theta, \phi) \cos\theta \sin\theta d\theta d\phi \delta A, \tag{1.14.2}$$

and therefore the exitance is

$$M = \int_0^{2\pi} \int_0^{\pi/2} L(\theta, \phi) \cos\theta \sin\theta d\theta d\phi \tag{1.14.3}$$

iii. A point O is at the centre of the base of a hollow radiating hemisphere whose radiance in the direction (θ, ϕ) is $L(\theta, \phi)$. What is the irradiance at that point O ? (See figure I.5.)

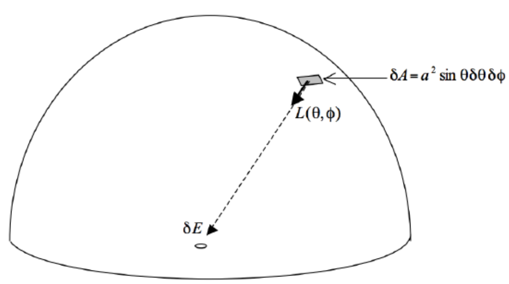

FIGURE I.5

Consider an elemental area $a^2 \ \sin\theta\,\delta\theta\,\delta\phi$ on the inside of the hemisphere at a point where the radiance is $L(\theta, \phi)$ (figure I.5). The intensity radiated towards O is the radiance times the area:

$$\delta I(\theta, \phi) = L(\theta, \phi) a^2 \sin\theta \delta\theta \delta\phi \tag{1.14.4}$$

The irradiance at O from this elemental area is (see equation (1.10.1)

$$\delta E = \frac{\delta I(\theta, \phi)\cos\theta}{a^2} = L(\theta, \phi)\cos\theta \sin\theta \delta\theta \delta\phi, \tag{1.14.5}$$

and so the irradiance at O from the entire hemisphere is

$$E = \int_0^{2\pi} \int_0^{\pi/2} L(\theta, \phi)\cos\theta \sin\theta \delta\theta \delta\phi. \tag{1.14.6}$$

The same would apply for any shape of inverted bowl - or even an infinite plane radiating surface (see figure I.6.)

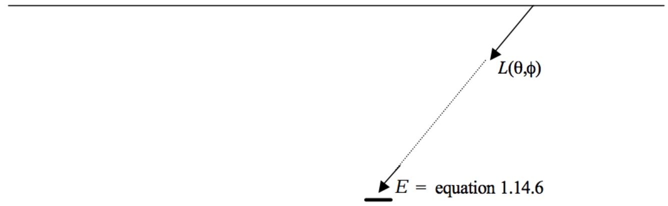

FIGURE I.6