# 13.4: Kepler's Second Law

In section 13.3 we made use of Kepler's second law, namely that the radius vector sweeps out equal areas in equal times. Explicitly,

$$\dot{B} = \frac{1}{2}h = \frac{1}{2}\sqrt{GMl}. \tag{13.4.1}$$

We are treating this as a two-body problem and therefore ignoring planetary perturbations. It is nevertheless worth reminding ourselves – from section 9.5 of chapter 9, especially Equations 9.5.17, 9.4.3, 9.5.19, 9.5.20 and 9.5.21, of the precise meanings of the symbols in Equation 13.4.1. The symbol $h$ is the angular momentum per unit mass of the orbiting body, and $l$ is the semi latus rectum of the orbit. If we are referring to the centre of mass of the two-body system as origin, then $h$ and $l$ are the angular momentum per unit mass of the orbiting body and the semi latus rectum relative to the centre of mass of the system, and $M$ is the *mass function* $M^3/(M+m)^2$ of the system, $M$ and $m$ being the masses of Sun and planet respectively. In chapter 9 we used the symbol $\mathfrak{M}$ for the mass function. If we are referring to the centre of the Sun as origin, then $h$ and $l$ are the angular momentum per unit mass of the planet and the semi latus rectum of the planet's orbit relative to that origin, and $M$ is the sum of the masses of Sun and planet, for which we used the symbol $\mathbf{M}$ in chapter 9. In any case, for all but perhaps the most massive asteroids, we are probably safe in regarding the mass of the orbiting body as being negligible compared with the mass of the Sun. In that case there is no distinction between the centre of the Sun and the centre of mass of the two-body system, and the $M$ in Equation 13.4.1 is then merely the mass of the Sun. (Note that I have n*ot* said that the barycentre of the entire solar system coincides with the centre of the Sun. The mass of Jupiter, for example, is nearly one thousandth of the mass of the Sun, and that is by no means negligible.)

The symbol $G$, of course, stands for the universal gravitational constant. Its numerical value is not known to any very high precision, and consequently the mass of the Sun is not known to any higher precision than $G$ is. Approximate values for them are $G = 6.672 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$ and $M = 1.989 \times 10^{30}\ \mathrm{kg}$. The *product* $GM$, is known to considerable precision; it is $1.327\,124\,38 \times 10^{20}\ \mathrm{m^3\ s^{-2}}$.

*Definition*: Until June 2012 the *astronomical unit of distance* (au) was defined as the radius of a circular orbit in which a body of negligible mass will, in the absence of planetary perturbations, move around the Sun at an angular speed of exactly $0.017\,202\,098\,95$radians per mean solar day, or $1.990\,983\,675 \times 10^{-7}\ \mathrm{rad\,s^{-1}}$, or $0.985\,607\,668\,6$degrees per mean solar day. This angular speed is sometimes called the *gaussian constant* and is given the symbol $k$. With this definition, the value of the astronomical unit is approximately $1.495\,978\,70 \times 10^{11}\ \mathrm{m}$.

However, in June 2012 the International Astronomical Union re-defined the astronomical unit as $149\,597\,870\,700$metres exactly. This means that a body of negligible mass moving around the Sun in a circular orbit will, in the absence of planetary perturbations, move at an angular speed of approximately $0.017\,202\,098\,95$ radians per mean solar day, This angular speed is the *gaussian constant* $k$ - but, with the new definition of the au, it is no longer regarded as one of the fundamental astronomical constants. The $\mathrm{IAU}$ also recommended that the official abbreviation for the astronomical unit should be au.

If we equate the centripetal acceleration of the hypothetical body moving in a circular orbit of radius $1\ \mathrm{au}$ at angular speed $k$ to the gravitational force on it per unit mass, we see that $ak^2 = GM/a^2$, so that

$$GM = k^2a^3, \tag{13.4.2}$$

where $a$ is the length of the astronomical unit and $k$ is the gaussian constant.