18.5: Refinement of the Orbital Elements

By finding the best fit of the observational values of radial velocity to a set of theoretical radial velocity curves, we have by now determined, if only graphically, a preliminary estimate of the orbital elements. We now have to refine these estimates in order to obtain the best set of elements that we can from the data.

Let us remind ourselves of the theoretical Equation (Equation 18.2.12) that we developed for the radial velocity:

$$V = V_0 + K_1(\cos(\omega + V) + e\cos\omega). \tag{18.5.1}$$

Here

$$K_1 = \frac{na_1 \sin i}{\sqrt{1-e^2}} \tag{18.5.2}$$

and

$$n = 2\pi/P. \tag{18.5.3}$$

Also v is a function of the time and the elements T and e, through Equations 9.6.4, 9.6.5 and 2.3.16 cited in Section 18.2. Thus Equation 18.5.1 expresses the radial velocity as a function of the time (hence true anomaly) and of the orbital elements V_0, K_1, ω, e, n and T:

$$V = V\left(t; V_0, K_1, \omega, e, n, T\right). \tag{18.5.4}$$

For each observation (i.e for each time t), we can use our preliminary elements to calculate what the radial velocity should be at that time, and compare it with the observed radial velocity at that time. Our aim is going to be to adjust the orbital elements so that the sum of the squares of the differences V_{obs} - V_{calc} is least.

If we were to change each of the elements of Equation 18.4.4 by a little, the corresponding change in V would be, to first order,

$$\delta V = \frac{\partial V}{\partial V_0}\delta V_0 + \frac{\partial V}{\partial K_1}\delta K_1 + \frac{\partial V}{\partial \omega}\delta\omega + \frac{\partial V}{\partial e}\delta e + \frac{\partial V}{\partial n}\delta n + \frac{\partial V}{\partial T}\delta T. \tag{18.5.5}$$

When the differentiations have been performed, this becomes

$$\begin{array}{l} \delta V = \delta V_0 + (\cos(V+\omega) + e\cos\omega)\delta K_1 - K_1(\sin(V+\omega) + e\sin\omega)\delta\omega \\ +K_1\left(\cos\omega - \frac{(2+e\cos v)\sin(V+\omega)\sin v}{1-e^2}\right)\delta e \\ -\frac{\sin(v+\omega)(1+e\cos v)^2 K_1(t-T)}{(1-e^2)^{3/2}}\delta n + \frac{K_1 n\sin(v+\omega)(1+e\cos v)^2}{(1-e^2)^{3/2}}\delta T \end{array}. \tag{18.5.6}$$

In this Equation, δV is V_{obs} - V_{calc}. There will be one such Equation for each observation, and hence, if there are N (> 6) observations there will be N *Equations of condition*. From these, six *normal Equations* will be formed in the manner described in Section 1.8 and solved for the increments in the orbital elements. These are then subtracted from the preliminary elements to form an improved set of elements, and the process can be repeated until there is no significant change.

This process can be highly automated by computer, but in practice the calculation is best overseen by an experienced human orbit computer. While a computer may produce a formal solution, there are a number of situations that may result in a solution that is unrealistic or even quite wrong. Much depends on the distribution of the observations, and on whether the observational errors are normally distributed. Also, if the system has been observed for a long time over many orbital periods, the period may be known to great precision, and the investigator may prefer to keep P (hence n) as a fixed, known constant during the calculation. Or again, if the period is short, the investigator may wish (perhaps on the basis of additional knowledge) to suppose that the two stars are close together and that the orbits of the components are circular, and hence fix e = 0 throughout the calculation. I am always a little uneasy about making an assumption that some element has some desired value; it seems to me that, once one starts this, one might as well assume values for *all* of the elements. This would have the advantage that one need not make any observations or do any calculations and can just assume all the results according to personal taste. Whether an assumption that P or e can be held as fixed and known, or whether one should let the computer do the entire calculation without any intervention, is something that requires the experience of someone who has been calculating orbits for years.