# 10.7: Calculating the Position of a Comet or Asteroid

We suppose that we are given the orbital elements of an asteroid or comet. Our task is to be able to predict, from these, the right ascension and declination of the object in the sky at some specified future (or past) date. If we can do it for one date, we can do it for many dates - e.g. every day for a year if need be. In other words, we will have constructed an ephemeris. Nowadays, of course, we can obtain ephemeris-generating programs and ephemerides with a few deft clicks on the Web, without knowing so much as the difference between a sine and a cosine; but that way of doing it is not the purpose of this section.

For example, according to the Minor Planet Center, the osculating elements for the minor planet (1) Ceres for the epoch of osculation $t_0 = 2002 \text{ May } 6.0 \text{ TT}$ are as follows:

$a = 2.766\ 412\ 2 \text{ AU} \quad \Omega = 80^\circ.486\ 32$
$e = 0.079\ 115\ 8 \quad \omega = 73^\circ.984\ 40$
$i = 10^\circ.583\ 47 \quad M_0 = 189^\circ.275\ 00$

$i$, $\Omega$ and $\omega$ are referred to the equinox and equator of $\text{J}2000.0$

Calculate the right ascension and declination (referred to $\text{J}2000.0$) at 2002 July $15.0 \text{ TT}$.

We have already learned how to achieve much of our aim from Chapter 9. Thus, from the elements $a$, $e$, $\omega$ and $T$ for an elliptic orbit (or the corresponding elements for a parabolic or hyperbolic orbit) we can already compute the *true anomaly* $v$ and the heliocentric distance r as a function of time. These are the heliocentric polar coordinates of the body (henceforth “asteroid”). In order to find the right ascension and declination (i.e. geocentric coordinates with the celestial equator as $xy$-plane) all we have to do is to find the coordinates relative to the ecliptic, rotate the coordinate system from ecliptic to equatorial, and shift the origin of coordinates from Sun to Earth,. We just have to do some straightforward geometry, and no further dynamics.

Let’s start by doing what we already know how to do from Chapter 9, namely, we’ll calculate the true anomaly and the heliocentric distance.

- Mean anomaly at the epoch ($t_0 = \text{May } 6.0$) is $M_0 = 189^\circ.275\ 00$.
- Mean anomaly at time $t$ ($= \text{July } 15$. which is 70 days later) is given by

$$M - M_0 = \frac{2\pi}{P}(t - t_0). \tag{10.7.1}$$

The quantity $2\pi/P$ is called the mean motion (actually the average orbital angular speed of the planet), usually given the symbol $n$. We can calculate $P$ in sidereal years from $P^2 = a^3$, and, given that a sidereal year is $365^{\text{d}}.25636$ and that $2\pi$ radians is 360 degrees, we can calculate the mean motion in its usual units of degrees per day. We find that $n = 0.214\ 205$ degrees per day. In fact the Minor Planet Center, as well as giving the orbital elements, also lists, for our convenience, the mean motion, and they give $n = 0.214\ 204\ 57$ degrees per day. The small discrepancy between the $n$ given by the Minor Planet Center and the value that we have calculated from the published value of a presumably arises because the published values of the elements have been rounded off for publication, and the Minor Planet Center presumably carries all digits in its calculations. I would recommend using the value of $n$ published by the Minor Planet Center, and I do so here. By July 15, then, Equation 10.7.1 tells us that the mean anomaly is $M = 204^\circ.269342$. (I’m carrying six decimal places, even though $M_0$ is given only to five, just to be sure that I’m not accumulating rounding-off errors in the intermediate calculations. I’ll round off properly when I reach the final result.)

We now have to find the eccentric anomaly from Kepler’s Equation $M = E - e \sin E$ . Easy. (See chapter 9 if you’ve forgotten how.) We find $E = 202^\circ.5322784$ and, from Equations 2.3.16 and 17, we obtain the true anomaly $v = 200^\circ.8540289$. The polar Equation to an ellipse is , $r = \frac{a(1-e^2)}{1+e\cos v}$. so we find that the heliocentric distance is $r = 2.968\ 5716 \text{ au}$ (The Minor Planet Centre gives $r$, to four significant figures, as $2.969 \text{ au}$) So much we could already do from Chapter 9. Note also that $\omega + v$, known as the argument of latitude and often given the symbol $\theta$, is $274^\circ.838\ 429$.

We are going to have to make use of three heliocentric coordinate systems and one geocentric coordinate system.

1. *Heliocentric plane-of-orbit*. $\odot xyz$ with the ?x axis directed towards perihelion. The polar coordinates in the plane of the orbit are the heliocentric distance r and the true anomaly v. The z-component of the asteroid is necessarily zero, and $x = r\cos v$ and $y = r\sin v$.
2. Heliocentric ecliptic. $\odot XYZ$ with the $\odot X$ axis directed towards the *First Point of Aries*, where Earth, as seen from the Sun, will be situated on or near September 22. The spherical coordinates in this system are the heliocentric distance r, the

ecliptic longitude $\lambda$, and the ecliptic latitude $\beta$, such that $X = r\cos\beta\cos\lambda$, $Y = r\cos\beta\sin\lambda$ and $Z = r\sin\beta$.

INSERT FIGURE HERE

FIGURE X.2

3. *Heliocentric equatorial coordinates.* $\odot\xi\eta\zeta$ with the $\odot\xi$ axis directed towards the First Point of Aries and therefore coincident with the X axis . The angle between the Z axis and the ζ axis is ε, the obliquity of the ecliptic. This is also the angle between the XY-plane (plane of the ecliptic, or of Earth's orbit) and the ξη-plane (plane of Earth's equator). See figure X.4.

4. *Geocentric equatorial coordinates.* ⊕xyz with the ⊕x axis directed towards the First Point of Aries. The spherical coordinates in this system are the geocentric distance $\Delta$, the right ascension $\alpha$ and the declination $\delta$, such that $x = \Delta\cos\delta\cos\alpha$, $y = \Delta\cos\delta\sin\alpha$ and $z = \Delta\sin\delta$.

In figure X.2, the arc ϒN is the heliocentric ecliptic longitude $\lambda$ of the asteroid, and so $N$N is $\lambda - \Omega$. The arc NX is the heliocentric ecliptic latitude $\beta$. By two applications of Equation 3.5.5 we find

$$\cos(\lambda-\Omega)\cos i = \sin(\lambda-\Omega)\cot(\omega+v) - \sin i \cot 90^\circ \tag{10.7.2}$$

and

$$\cos(\lambda-\Omega)\cos 90^\circ = \sin(\lambda-\Omega)\cot\beta - \sin 90^\circ \cot i. \tag{10.7.3}$$

These reduce to

$$\tan(\lambda-\Omega) = \cos i \tan(\omega+v) \tag{10.7.4}$$

and

$$\tan\beta = \sin(\lambda-\Omega)\tan i. \tag{10.7.5}$$

In our particular example, we obtain (if we are careful to watch the quadrants),

$$\lambda - \Omega = 274^\circ.921\,7550, \quad \lambda = 355^\circ.408\,0750, \quad \beta = -10^\circ.545\,3234$$

Now, we'll take the X-axis for the heliocentric ecliptic coordinates through ϒ and the $Y$-axis 90° east of this. Then, by the usual formulas for converting between spherical and rectangular coordinates, that is, $X = r\cos\beta\cos\lambda$, $Y = r\cos\beta\sin\lambda$ and $Z = r\sin\beta$, we obtain

$$X = +2.909\,0661, \quad Y = -0.233\,6453, \quad Z = -0.543\,2880 \quad \text{au.}$$

(Check: $X^2 + Y^2 + Z^2 = r^2$ . )

**Exercise 10.7.1**

Show, by elimination of $\lambda$ and $\beta$, or otherwise, that:

$$X = r(\cos\Omega\cos\theta - \sin\Omega\sin\theta\cos i) \tag{10.7.6}$$

$$Y = r(\sin\Omega\cos\theta + \cos\Omega\sin\theta\cos i) \tag{10.7.8}$$

$$Z = r\sin\theta\sin i. \tag{10.7.9}$$

This will provide a more convenient way of calculating the coordinates. Verify that these give the same numerical result as before. Here are some suggestions for doing it "otherwise"

Refer to Figure $X.3$, in which K is the pole of the ecliptic, and X is the asteroid. The radius of the celestial sphere can be taken as equal to $r$, the heliocentric distance of the asteroid. The rectangular heliocentric ecliptic coordinates are

$X = r\cos \Upsilon\odot\mathrm{X} \quad Y = r\cos \mathrm{R}\odot\mathrm{X} \quad Z = r\cos \mathrm{K}\odot\mathrm{X}$