

5.4.7: Solid Cylinder

- We do this not because it has any particular relevance to celestial mechanics, but because it is easy to do. We imagine a solid cylinder, density ρ , radius a , length l . We seek to calculate the field at a point P on the axis, at a distance h from one end of the cylinder (figure V.8).

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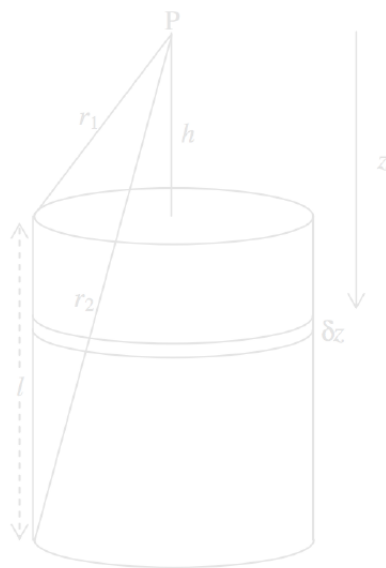


FIGURE V.8

The field at P from an elemental disc of thickness δz a distance z below P is (from Equation 5.4.9)

$$g = G \rho \pi a^2 \omega. \quad (5.4.19)$$

Here ω is the solid angle subtended at P by the disc, which is $\omega = \pi \left[1 - \frac{z}{(z^2 + a^2)^{1/2}} \right]$. Thus the field at P from the entire cylinder is

$$g = 2\pi G \rho \int_h^{l+h} \left[1 - \frac{z}{(z^2 + a^2)^{1/2}} \right] dz, \quad (5.4.20)$$

or

$$g = 2\pi G \rho \left(l - \sqrt{(l+h)^2 + a^2} + \sqrt{h^2 + a^2} \right), \quad (5.4.21)$$

or

$$g = 2\pi G \rho (l - r_2 + r_1). \quad (5.4.22)$$

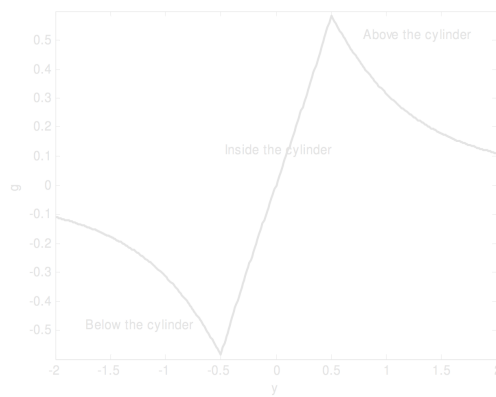
It might also be of interest to express g in terms of the height $y (= \frac{1}{2}l + h)$ of the point P above the mid-point of the cylinder. Instead of Equation 5.4.21, we then have

$$g = 2\pi G \rho \left(l - \sqrt{\left(y + \frac{1}{2}l\right)^2 + a^2} + \sqrt{\left(y - \frac{1}{2}l\right)^2 + a^2} \right). \quad (5.4.23)$$

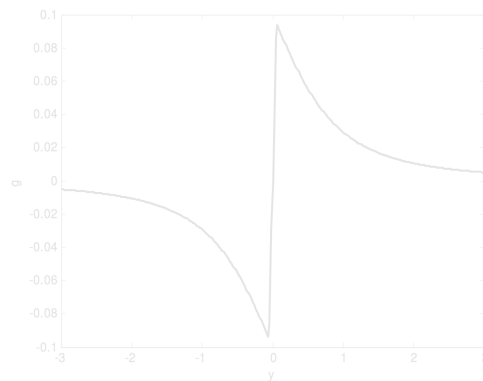
If the point P is *inside* the cylinder, at a distance h below the upper end of the cylinder, the limits of integration in Equation 5.4.20 are h and $l - h$, and the distance y is $\frac{1}{2}l - h$. In terms of y the gravitational field at P is then

$$g = 2\pi G \rho \left(2y - \sqrt{\left(y + \frac{1}{2}l\right)^2 + a^2} + \sqrt{\left(y - \frac{1}{2}l\right)^2 + a^2} \right). \quad (5.4.24)$$

In the graph below I have assumed, by way of example, that l and a are both 1, and I have plotted g in units of $2\pi G \rho$ (counting g as positive when it is directed downwards) from $y = -1$ to $y = +1$. The portion inside the cylinder ($-\frac{1}{2} \leq y \leq \frac{1}{2}l$), represented by Equation 5.4.24, is almost, but not quite, linear. The field at the centre of the cylinder is, of course, zero.



Below, I draw the same graph, but for a thin disc, with $a = 1$ and $l = 0.1$. We see how it is that the field reaches a maximum immediately above or below the disc, but is zero at the centre of the disc.



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