13.9: Iterating

We can now use Equations 13.8.35a,b and get a better estimate of the triangle ratios. The numerical data are

$$b_1 = 2/3, \quad b_3 = 1/3, \quad r_2 = 3.481\,33,$$

$\tau_1 = t_3 - t_2 = 10$ mean solar days and $\tau_3 = t_2 - t_1 = 5$ mean solar days, but recall that we are expressing time intervals in units of $1/k$, which is $58.132\,440\,87$mean solar days, and therefore

$$\tau_1 = 0.172\,021 \quad \text{and} \quad \tau_3 = 0.086\,010.$$

Equations 13.8.35 then result in

$$a_1 = 0.666\,764, a_3 = 0.333\,411$$

Now we can go back to Equation 13.7.4 and start again with our new values for the triangle ratios – und so weiter − until we obtain new values for Δ_1, Δ_2, Δ_3 and r_2. I show below in the first two columns the first crude estimates (already given above), in the 16 second two columns the results of the first iteration, and, in the last two columns, the values given in the published IAU ephemeris.

	First crude estimates		First iteration		MPC	
	Δ	r	Δ	r	Δ	r
1	2.72571	3.48532	2.65825	3.41952	2.644	3.406
2	2.68160	3.48133	2.61558	3.41673	2.603	3.404
3	2.61073	3.47471	2.54579	3.41082	2.536	3.401

(13.9.1)

We see that we have made a substantial improvement, but we are not there yet. We can now calculate new values of a_1 and a_3 from Equations 13.8.35a,b to get

$$a_1 = 0.666\,770 \quad a_3 = 0.333\,416$$

We *could* (if we so wished) now go back to Equations 13.7.4,5,6, and iterate again. However, this will result in only small changes to a_1, a_3, Δ and r, and we have to bear in mind that Equations 13.8.35a,b are only approximations (to order τ^3). Therefore, even if successive iterations converge, they will still not give precise correct answers for Δ and r.

To anticipate, eventually we shall arrive at some exact Equations (Equations 13.12.25 and 13.12.26) that will allow us to solve the problem. But these Equations will not be easy to solve. They have to be solved by iteration using a reasonably good first guess. It is our present aim to obtain a reasonably good first guess for a_1, a_3, Δ and r, in order to prepare for the solution of the exact Equations 13.12.25 and 13.12.26. Our current values of a_1 and a_3, while not exact, will enable us to solve Equations 13.12.25 and 13.12.26 exactly, so we should now, rather than going back again to Equations 13.7.4,5,6, proceed straight to Sections 13.11, 13.12 and 13.13.

Nevertheless, in the following section, we provide (in Equations 13.10.9 and 13.10.10), after considerable effort, higher-order expansions for a_1 and a_3. These may be useful, but for reasons explained in the previous paragraph, it may be easier to skip Section 13.10 entirely.