5.4.9: Solid Sphere

A solid sphere is just lots of hollow spheres nested together. Therefore, the field at an external point is just the same as if all the mass were concentrated at the centre, and the field at an internal point P is the same is if all the mass *interior* to P, namely M_r, were concentrated at the centre, the mass *exterior* to P not contributing at all to the field at P. This is true not only for a sphere of uniform density, but of any sphere in which the density depends only of the distance from the centre – i.e., any spherically symmetric distribution of matter.

If the sphere is uniform, we have $\frac{M_r}{M}=\frac{r^3}{a^3}$, so the field inside is

$$g=\frac{GM_r}{r^2}=\frac{GMr}{a^3}. \tag{5.4.24}$$

Thus, inside a uniform solid sphere, the field increases linearly from zero at the centre to GM/a^2 at the surface, and thereafter it falls off as GM/r^2.

If a uniform solid sphere has a narrow hole bored through it, and a small particle of mass m is allowed to drop through the hole, the particle will experience a force towards the centre of $GMmr/a^3$, and will consequently oscillate with period P given by

$$P^2=\frac{4\pi^2}{GM}a^3. \tag{5.4.25}$$