13.5: Coordinates

We need to make use of several coordinate systems, and I reproduce here the descriptions of them from section 10.7 of chapter 10. You may wish to refer back to that chapter as a further reminder.

1. *Heliocentric plane-of-orbit*. $\odot xyz$ with the $\odot x$ axis directed towards perihelion. The polar coordinates in the plane of the orbit are the heliocentric distance r and the true anomaly v. The z-component of the asteroid is necessarily zero, and $x = r\cos v$ and $y = r\sin v$.
2. *Heliocentric ecliptic*. $\odot XYZ$ with the $\odot X$ axis directed towards the First Point of Aries Υ, where Earth, as seen from the Sun, will be situated on or near September 22. The spherical coordinates in this system are the heliocentric distance r, the ecliptic longitude λ, and the ecliptic latitude β, such that $X = r\cos\beta\cos\lambda$, $Y = r\cos\beta\sin\lambda$ and $Z = r\sin\beta$.
3. *Heliocentric equatorial coordinates*. $\odot\xi\eta\zeta$ with the $\odot\xi$ axis directed towards the First Point of Aries and therefore coincident with the $\odot X$ axis . The angle between the $\odot Z$ axis and the $\odot\zeta$ axis is ε, the obliquity of the ecliptic. This is also the angle between the XY-plane (plane of the ecliptic, or of Earth's orbit) and the $\xi\eta$-plane (plane of Earth's equator). See figure $\mathrm{X}.4$.
4. *Geocentric equatorial coordinates*. $\oplus\mathfrak{x}\mathfrak{y}\mathfrak{z}$ with the $\oplus\mathfrak{x}$ axis directed towards the First Point of Aries. The spherical coordinates in this system are the geocentric distance Δ, the right ascension α and the declination δ, such that $\mathfrak{x} = \Delta\cos\delta\cos\alpha$, $\mathfrak{y} = \Delta\cos\delta\sin\alpha$ and $\mathfrak{z} = \Delta\sin\delta$.

A summary of the relations between them is as follows

$$\mathfrak{x} = \Delta\cos\alpha\cos\delta = l\Delta = \mathfrak{x}_o + \xi, \tag{13.5.1}$$

$$\mathfrak{y} = \Delta\sin\alpha\cos\delta = m\Delta = \mathfrak{y}_o + \eta, \tag{13.5.2}$$

$$\mathfrak{z} = \Delta\sin\delta = n\Delta = \mathfrak{z}_o + \zeta. \tag{13.5.3}$$

Here, $(l,\ m,\ n)$ are the direction cosines of the planet's geocentric radius vector. They offer an alternative way to $(\alpha,\ \delta)$ for expressing the direction to the planet as seen from Earth. They are not independent but are related by

$$l^2 + m^2 + n^2 = 1. \tag{13.5.4}$$

The symbols $\mathfrak{x}_o$, $\mathfrak{y}_o$ and $\mathfrak{z}_o$ are the geocentric equatorial coordinates of the Sun.