18.3: Preliminary Elements from the Velocity Curve

We have seen in the previous section how to calculate the velocity curve given the elements. The more practical problem is the inverse: In this section, we assume that we have obtained a velocity curve observationally, and we want to determine the elements. The assumption that we have obtained a precise radial velocity curve is, of course, rather a large one; but, for the present, let us assume that this has been done and we are trying to determine what we can about the orbit. We limit ourselves in this section to determining from the curve only very rough first estimates of the elements. This will also serve the purpose of establishing what information is *obtainable in principle* from the velocity curve. A later section will deal with refining our estimates and obtaining precise values.

The assumption that we have already obtained the radial velocity curve implies that we already know the period P of the orbit.

The radial velocity curve is given by Equation 18.2.12:

$$V = V_0 + K_1(\cos(\omega + v) + e\cos\omega). \tag{18.3.1}$$

Here $v = v(t, T, e)$. Thus, from the radial velocity curve, we should be able to determine V_0, K_1, e, ω and T. We shall remind ourselves a little later of the meaning of K_1, but in the meantime we can note that the radial velocity varies between a maximum of $V_{max} = V_0 + K_1\,(e \cos \omega + 1)$ and a minimum of $V_{min} = V_0 + K_1\,(e \cos \omega - 1)$. The difference between these two is $2K_1$. Thus K_1 is the *semiamplitude of the radial velocity curve*, regardless of the shape of the curve and the values of ω and e, and so (again assuming that we have a well-determined radial velocity curve) K_1 can be readily determined.

The systemic velocity V_0 is such that the area under the radial velocity curve above it is equal to the area above the radial velocity curve below it. Thus at least a rough preliminary estimate can be made of V_0, regardless of the shape of the curve and of the values of ω and e.

The shape of the radial velocity curve (as distinct from its amplitude and phase) is determined by ω and e. As suggested in the previous section, we can prepare a set of, say, 360 theoretical curves covering 36 values of ω from 0 to 350° and 10 values of e from 0.0 to 0.9. (By making use of symmetries, one need cover ω only from 0 to 90°, but computers are so fast today that one might as well go from 0 to 350°) By comparing the observed curve with these theoretical curves, we get a first estimate of ω and e. We could then I suppose, take advantage of today's fast computers and prepare a set of velocity curves with much finer intervals around one's first estimate. This would not, of course, allow us to calculate definitive precise values of ω and e, but it would give us a pretty good first guess.

I have already pointed out that

$$V_{\max} = V_0 + K_1(e\cos\omega + 1) \tag{18.3.2}$$

and

$$V_{\min} = V_0 + K_1(e\cos\omega - 1) \tag{18.3.3}$$

From these we see that

$$e\cos\omega = \frac{V_{\max} + V_{\min}}{2K_1}. \tag{18.3.4}$$

This allows us to determine $e\cos\omega$ without reference to the slightly uncertain V_0, and we will want to see that our estimates of e and ω from the shape of the curve are consistent with Equation 18.3.3.

The velocity curve also allows us to determine T, the time of periastron passage. For example, the sample theoretical velocity curves I have drawn in figures XIII.3, 4 and 5 all start at periastron at the left hand limit of each curve.

Note that we have been able to determine K_1, which is $\frac{na_1 \sin i}{\sqrt{1-e^2}}$, and we can determine e and n, which is $2\pi/P$. This means that we can determine $a_1 \sin i$, but that is as far as we can go without additional information; we cannot separate a_1 from i.