15.2: Orbital elements and the position and velocity vector

The six elements used to describe the orbit of an asteroid are the familiar

$$a, e, i, \Omega, \omega, T$$

Because of the precession and nutation of Earth, the angular elements must, of course, be referred to a particular equinox and equator, usually chosen to be that of the standard epoch J2000.0, which means 12h 00m TT on 2000 January 01. (The “J” stands for “Julian Year”.)

The element T is the instant of perihelion passage. If the orbit is nearly circular, the instant of perihelion passage is ill-defined, and if the orbit is exactly circular, it is not defined at all. In such cases, instead of T, we may give either the *mean anomaly* M_0 or the *mean longitude* L_0 at a specified epoch (see Chapter 10). This epoch need not be (and usually is not) the same as the standard epoch referred to in the previous paragraph.

Suppose that, at some instant of time (to be known, for reasons to be explained later, as the epoch of osculation), the heliocentric ecliptic coordinates of an asteroid or comet in an elliptic orbit are (X, Y, Z) and the components of the velocity vector are ($\dot{X}, \dot{Y}, \dot{Z}$). We have shown in Chapter 10, Section 10.10) how to calculate, from these, the six elements $a, e, i, \Omega, \omega, T$ of the orbit at that instant. Conversely, given the orbital elements, we could reverse the calculation and calculate the components of the position and velocity vectors. Thus an orbit may equally well be described by the six numbers

$$X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}$$

That is to say the components, at some specified instant of time, of the position and velocity vector in heliocentric ecliptic coordinates.

We could equally well give the components, at some instant of time, of the position and velocity vectors in heliocentric equatorial coordinates:

$$\xi, \eta, \zeta, \dot{\xi}, \dot{\eta}, \dot{\zeta}$$

We saw in Section 10.9 that yet another set of six numbers,

$$P_x, Q_x, P_y, Q_y, P_z, Q_z$$

will also suffice to describe an orbit.

It is assumed here that the reader is familiar with all four of these alternative sets of elements, and can convert between them. Indeed, before reading on, it may be a useful exercise to prepare a computer program that will convert instantly between them. This may not be a trivial task, but I strongly recommend doing so before reading further. The facility to convert instantly between one set and another is an enormous help. To convert between ecliptic and equatorial coordinates, you will need, of course, the obliquity of the ecliptic at that instant - it varies, of course, with time.) The reader will have noticed the frequent occurrence of the phrase “at that instant” in the previous paragraphs. If the asteroid were not subject to perturbations from the other planets, it would retain its orbital elements forever. However, because of the planetary perturbations, the elements a, e i,, Ω,ω,T computed from $X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}$ or from $\xi, \eta, \zeta, \dot{\xi}, \dot{\eta}, \dot{\zeta}$ at a particular instant of time are valid only for that instant. The elements will change with time. Therefore in quoting the elements of an asteroidal orbit, it is entirely necessary to state clearly and without ambiguity the instant of time to which these elements are referred. The unperturbed orbit, and the real perturbed orbit, will coincide in position and velocity at that instant. The real and unperturbed orbits will “kiss” or osculate at that instant, which is therefore known as the epoch of osculation.

The elements $a, e, i, \Omega, \omega, T$ calculated for a particular epoch of osculation may suffice for the computation of an ephemeris for weeks to come. But after months the observed position of the object will start to deviate from its calculated ephemeris position. It is then necessary to calculate a new set of elements for a later epoch of osculation. Depending on circumstances, orbital elements may be recalculated every year, or every 200 days or every 40 days or every 10 days, or at some other convenient interval. It will be the purpose in what follows to do the following. Given that at some instant (i.e. at some epoch of osculation) the elements are $a, e, i, \Omega, \omega, T$ (or the position and velocity vectors are $\xi, \eta, \zeta, \dot{\xi}, \dot{\eta}, \dot{\zeta}$, how do we calculate the elements at some subsequent epoch, taking into account planetary perturbations?

As pointed out at the end of Section 15.1, we shall need to know the positions and distances of the major planets as a function of time. We suppose that we have subroutines in our program that we can call upon to calculate these data at any date. As mentioned above, the Equations of motion can be written in equatorial or ecliptic coordinates, though it is more likely that, for the positions of the major planets, we shall have available their positions in *equatorial coordinates*.