5.8.3: Plane Discs

Refer to figure $\mathrm{V.2A}$. The potential at P from the elemental disc is

$$d\psi = -\frac{G\delta M}{\left(r^2+z^2\right)^{1/2}} = -\frac{2\pi G\sigma r\delta r}{\left(r^2+z^2\right)^{1/2}}. \tag{5.8.10}$$

The potential from the whole disc is therefore

$$\psi = -2\pi G\sigma \int_0^a \frac{rdr}{\left(r^2+z^2\right)^{1/2}}. \tag{5.8.11}$$

The integral is trivial after a brilliant substitution such as $X = r^2+z^2$ or $r = z\tan\theta$, and we arrive at

$$\psi = -2\pi G\sigma\left(\sqrt{z^2+a^2}-z\right). \tag{5.8.12}$$

This increases to zero as $z\to\infty$. We can also write this as

$$\psi = -\frac{2\pi Gm}{\pi a^2}\cdot\left[z\left(1+\frac{a^2}{z^2}\right)^{1/2}-z\right], \tag{5.8.13}$$

and, if you expand this binomially, you see that for large z it becomes, as expected, $-Gm/z$.

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