5.8.7: Solid Cylinder

Refer to figure V.8. The potential from the elemental disc is

$$d\psi = -2\pi G\rho\delta z\left[\left(z^2+a^2\right)^{1/2} - z\right] \tag{5.8.21}$$

and therefore the potential from the entire cylinder is

$$\psi = const. -2\pi G\rho\left[\int_h^{h+l}\left(z^2+a^2\right)^{1/2}dz - \int_h^{h+1} zdz\right]. \tag{5.8.22}$$

I leave it to the reader to carry out this integration and obtain a final expression. One way to deal with the first integral might be to try $z = a\tan\theta$. This may lead to $\int\sec^3\theta d\theta$. From there, you could try something like $\int\sec^3\theta = \int\sec\theta d\tan\theta = \sec\theta\tan\theta - \int\tan\theta d\sec\theta = \sec\theta\tan\theta - \int\sec\theta\tan^2\theta d\theta = \sec\theta\tan\theta - \int\sec^3\theta +$ $\int\sec\theta d\theta$, and so on.