5.9: Work Required to Assemble a Uniform Sphere

Let us imagine a uniform solid sphere of mass M, *density* ρ and radius a. In this section we ask ourselves, how much work was done in order to assemble together all the atoms that make up the sphere if the atoms were initially all separated from each other by an infinite distance? Well, since massive bodies (such as atoms) attract each other by gravitational forces, they will naturally eventually congregate together, so in fact you would have to do work in dis-assembling the sphere and removing all the atoms to an infinite separation. To bring the atoms together from an infinite separation, the amount of work that you do is *negative*.

Let us suppose that we are part way through the process of building our sphere and that, at present, it is of radius r and of mass $M_r = \frac{4}{3}\pi r^3 \rho$. The potential at its surface is

$$-\frac{GM_r}{r} = -\frac{G}{r} \cdot \frac{4\pi r^3 \rho}{3} = -\frac{4}{3}\pi G \rho r^2. \tag{5.9.1}$$

The amount of work required to add a layer of thickness δr and mass $4\pi\rho r^2 \delta r$ to this is

$$-\frac{4}{3}\pi G\rho r^2 \times 4\pi r^2 \rho \delta r = -\frac{16}{3}\pi^2 G \rho^2 r^4 \delta r. \tag{5.9.2}$$

The work done in assembling the entire sphere is the integral of this from $r = 0$ to a, which is

$$-\frac{16\pi^2 G\rho^2 a^5}{15} = -\frac{3GM^2}{5a}. \tag{5.9.1}$$