3.8: Trigonometrical Formulas

I gather here merely for reference a set of commonly-used trigonometric formulas. It is a matter of personal preference whether to commit them to memory. It is probably fair to remark that anyone who is regularly engaged in problems in celestial mechanics or related disciplines will be familiar with most of them, at least from frequent use, whether or not any conscious effort was made to memorize them. At the very least, the reader should be aware of their existence, even if he or she has to look to recall the exact formula.

$$\frac{\sin A}{\cos A} = \tan A \tag{3.8.1}$$

$$\sin^2 A + \cos^2 A = 1 \tag{3.8.2}$$

$$1 + \cot^2 A = \csc^2 A \tag{3.8.3}$$

$$1 + \tan^2 A = \sec^2 A \tag{3.8.4}$$

$$\sec A \csc A = \tan A + \cot A \tag{3.8.5}$$

$$\sec^2 A \csc^2 A = \sec^2 A + \csc^2 A \tag{3.8.6}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \tag{3.8.7}$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \tag{3.8.8}$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \tag{3.8.9}$$

$$\sin 2A = 2 \sin A \cos A \tag{3.8.10}$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A \tag{3.8.11}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \tag{3.8.12}$$

$$\sin \frac{1}{2} A = \sqrt{\frac{1 - \cos A}{2}} \tag{3.8.13}$$

$$\cos \frac{1}{2} A = \sqrt{\frac{1 + \cos A}{2}} \tag{3.8.14}$$

$$\tan \frac{1}{2} A = \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{1 - \cos A}{\sin A} = \frac{\sin A}{A + \cos A} = \csc A - \cot A \tag{3.8.15}$$

$$\sin A + \sin B = 2 \sin \frac{1}{2} S \cos \frac{1}{2} D, \tag{3.8.16}$$

where

$$S = A + B \quad \text{and} \quad D = A - B \tag{3.8.17}$$

$$\sin A - \sin B = 2 \cos \frac{1}{2} S \sin \frac{1}{2} D \tag{3.8.18}$$

$$\cos A + \cos B = 2 \cos \frac{1}{2} S \cos \frac{1}{2} D \tag{3.8.19}$$

$$\cos A - \cos B = -2 \sin \frac{1}{2} S \sin \frac{1}{2} D \tag{3.8.20}$$

$$\sin A \sin B = \frac{1}{2}(\cos D - \cos S) \tag{3.8.21}$$

$$\cos A \cos B = \frac{1}{2}(\cos D + \cos S) \tag{3.8.22}$$

$$\sin A\cos B=\frac{1}{2}(\sin S+\sin D) \tag{3.8.23}$$

$$\sin A=\frac{T}{\sqrt{1+T^2}}=\frac{2T}{1+t^2}, \tag{3.8.24}$$

where

$$T=\tan A \text{ and } t=\tan\frac{1}{2}A \tag{3.8.25}$$

$$\cos A=\frac{1}{\sqrt{1+T^2}}=\frac{1-t^2}{1+t^2} \tag{3.8.26}$$

$$\tan A=T=\frac{2t}{1-t^2} \tag{3.8.27}$$

$$s=\sin A,\quad c=\cos A \tag{3.8.28}$$

$$\begin{array}{ll}
\cos A=c & \sin A=s\\
\cos 2A=2c^2-1 & \sin 2A=2cs\\
\cos 3A=4c^3-3c & \sin 3A=3s-4s^3\\
\cos 4A=8c^4-8c^2+1 & \sin 4A=4c(s-2s^3)\\
\cos 5A=16c^5-20c^3+5c & \sin 5A=5s-20s^3+16s^5\\
\cos 6A=32c^6-48c^4+18c^2-1 & \sin 6A=2c(3s-16s^3+16s^5)\\
\cos 7A=64c^7-112c^5+56c^3-7c & \sin 7A=7s-56s^3+112s^5-64s^7\\
\cos 8A=128c^8-256c^6+160c^4-32c^2+1 & \sin 8A=8c(s-10s^3+24s^5-16s^7)
\end{array} \tag{3.8.29}$$

$$\begin{array}{l}
\cos^2 A=\frac{1}{2}(\cos 2A+1)\\
\cos^3 A=\frac{1}{4}(\cos 3A+3\cos A)\\
\cos^4 A=\frac{1}{8}(\cos 4A+4\cos 2A+3)\\
\cos^5 A=\frac{1}{16}(\cos 5A+5\cos 3A+10\cos A)\\
\cos^6 A=\frac{1}{32}(\cos 6A+6\cos 4A+15\cos 2A+10)\\
\cos^7 A=\frac{1}{64}(\cos 7A+7\cos 5A+21\cos 3A+35\cos A)\\
\cos^8 A=\frac{1}{128}(\cos 8A+8\cos 6A+28\cos 4A+56\cos 2A+35)
\end{array} \tag{3.8.30}$$

$$\begin{array}{l}
\sin^2 A=\frac{1}{2}(1-\cos 2A)\\
\sin^3 A=\frac{1}{4}(3\sin A-\sin 3A)\\
\sin^4 A=\frac{1}{8}(\cos 4A-4\cos 2A+3)\\
\sin^5 A=\frac{1}{16}(\sin 5A-5\sin 3A+10\sin A)\\
\sin^6 A=\frac{1}{32}(10-15\cos 2A+6\cos 4A-\cos 6A)\\
\sin^7 A=\frac{1}{64}(35\sin A-21\sin 3A+7\sin 5A-\sin 7A)\\
\sin^8 A=\frac{1}{128}(\cos 8A-8\cos 6A+28\cos 4A-56\cos 2A+35)
\end{array} \tag{3.8.31}$$

$$\sin A=A-\frac{A^3}{3!}+\frac{A^5}{5!}-\ldots \tag{3.8.32}$$

$$\cos A=1-\frac{A^2}{2!}+\frac{A^4}{4!}-\ldots \tag{3.8.33}$$

$$\int_0^{\pi/2}\sin^m\theta\cos^n\theta d\theta=\frac{(m-1)!!(n-1)!!X}{(m+n)!!},\quad \text{where } X=\pi/2 \text{ if } m \text{ and } n \text{ are both even, and } X=1 \text{ otherwise.} \tag{3.8.34}$$

$e^{ni\theta}=e^{in\theta}$ (de Moivre's theorem - the only one you need know. All others can be deduced from it.)

Plane triangles:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \tag{3.8.35}$$

$$a^2 = b^2 + c^2 - 2bc\cos A \tag{3.8.36}$$

$$a\cos B + b\cos A = c \tag{3.8.37}$$

$$s = \frac{1}{2}(a+b+c) \tag{3.8.38}$$

$$\sin\frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \tag{3.8.39}$$

$$\cos\frac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}} \tag{3.8.40}$$

$$\tan\frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \tag{3.8.41}$$

Spherical triangles

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \tag{3.8.42}$$

$$\cos a = \cos b\cos c + \sin b\sin c\cos A \tag{3.8.43}$$

$$\cos A = -\cos B\cos C + \sin B\sin C\cos a \tag{3.8.44}$$

$$\cos(\mathrm{IS})\cos(\mathrm{IA}) = \sin(\mathrm{IS})\cot(\mathrm{OS}) - \sin(\mathrm{IA})\cot(\mathrm{OA}) \tag{3.8.45}$$
