5.8.9: Solid Sphere

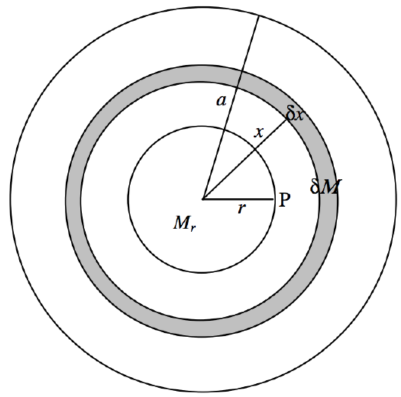

FIGURE V.24A

The potential *outside* a solid sphere is just the same as if all the mass were concentrated at a point in the centre. This is so, even if the density is not uniform, and long as it is spherically distributed. We are going to find the potential at a point P inside a uniform sphere of radius a, mass M, density ρ, at a distance r from the centre ($r < a$). We can do this in two parts. First, there is the potential from that part of the sphere "below" P. This is $-GM_r/r$, where $M_r = \frac{r^3 M}{a^3}$ is the mass within radius r. Now we need to deal with the material "above" P. Consider a spherical shell of radii x, $x + \delta x$. Its mass is $\delta M = \frac{4\pi x^2 \delta x}{\frac{4}{3}\pi a^3} \cdot M = \frac{3Mx^2\delta x}{a^3}$. The potential from this shell is $-G\delta M/x = -\frac{3GMx\delta x}{a^3}$. This is to be integrated from $x = 0$ to a, and we must then add the contribution from the material "below" P. The final result is

$$\psi = -\frac{GM}{2a^3}\left(3a^2 - r^2\right). \tag{5.8.23}$$

Figure $\mathrm{V}.25$ shows the potential both inside and outside a uniform solid sphere. The potential is in units of $-GM/r$, and distance is in units of a, the radius of the sphere.

FIGURE V.25