4.4: The Paraboloid

The Equation $x^2 = 4qz = 2lz$ is a parabola in the xz-plane. The distance between vertex and focus is q, and the length of the semi latus rectum $l = 2q$. The Equation can also be written

$$\frac{x^2}{a^2} = \frac{z}{h} \tag{4.4.1}$$

Here a and h are distances such that $x = a$ when $z = h$, and the length of the semi latus rectum is $l = a^2/(2h)$.

If this parabola is rotated through 360° about the z-axis, the figure swept out is a *paraboloid of revolution*, or *circular paraboloid*. Many telescope mirrors are of this shape. The Equation to the circular paraboloid is

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = \frac{z}{h}. \tag{4.4.2}$$

The cross-section at $z = h$ is a circle of radius a.

The Equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{h}, \tag{4.4.3}$$

in which we shall choose the x- and y-axes such that $a > b$, is an elliptic paraboloid and, if $a \neq b$, is not formed by rotation of a parabola. At $z = h$, the cross section is an ellipse of semi major and minor axes equal to a and b respectively. The section in the plane $y = 0$ is a parabola of semi latus rectum $a^2/(2h)$. The section in the plane $x = 0$ is a parabola of semi latus rectum $b^2/(2h)$. The elliptic paraboloid lies entirely above the xy-plane.

The Equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{z}{h} \tag{4.4.4}$$

is a hyperbolic paraboloid, and its shape is not quite so easily visualized. Unlike the elliptic paraboloid, it extends above and below the plane. It is a saddle-shaped surface, with the saddle point at the origin. The section in the plane $y = 0$ is the "nose down" parabola $x^2 = a^2z/h$ extending above the xy-plane. The section in the plane $x = 0$ is the "nose up" parabola $y^2 = -b^2z/h$ extending below the xy-plane. The section in the plane $z = h$ is the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1. \tag{4.4.5}$$

The section with the plane $z = -h$ is the conjugate hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1. \tag{4.4.6}$$

The section with the plane $z = 0$ is the asymptotes

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0. \tag{4.4.7}$$

The surface for $a = 3$, $b = 2$, $h = 1$ is drawn in figure $\mathrm{IV}.4$.

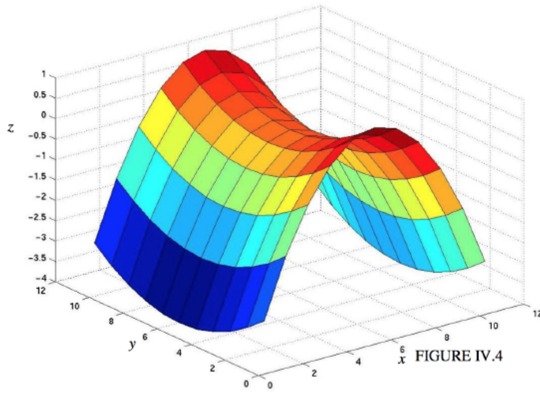

FIGURE IV.4