18.4: Masses

In Section 18.3 we saw that we could obtain approximate values of P, V_0, K_1, e, ω and T. But, apart from its being the semi-amplitude of the velocity curve, we have forgotten the meaning of K_1. We remind ourselves. It was defined just after Equation 18.2.10 as

$$K_1 = \frac{na_1 \sin i}{\sqrt{1-e^2}}. \tag{18.4.1}$$

Here n is the mean motion 2π/P. Thus, since we know P (hence n), e and K_1, we can determine a_1 sin i – but we cannot determine a_1 or i separately.

Now the mean motion n is given just before Equation 18.2.1 as

$$n^2 a_1^3 = GM, \tag{18.4.2}$$

where

$$M = m_2^3/(m_1+m_2)^2. \tag{18.4.3}$$

(A reminder: The subscript 1 refers, for a single-lined binary, to the star whose spectrum we can observe, and the subscript 2 refers to the star that we cannot observe.) All of this put together amounts to

$$K_1 = \frac{G}{(1-e^2)\, a_1 \sin i} \times \frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}. \tag{18.4.4}$$

Thus we can determine the *mass function* $\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$. We cannot determine the separate masses, or their ratio or sum, or the inclination.

In recent years, it has become possible to measure very small radial velocities of the order of a few metres per second, and a number of single-lined binary stars have been detected with very small values of K_1; that is to say, very small radial velocity amplitudes. These could, of course, refer to stars with small orbital inclinations, so that the plane of the orbit is almost perpendicular to the line of sight. It has been held, however, (on grounds that are not entirely clear to me) that many of these single-lined binary stars with small radial velocity variations are actually single stars with a planet (or planets) in orbit around them. The mass of the star that we can observe (m_1) is very much larger than the mass of the planet, which we cannot observe (m_2). To emphasize this, I shall use the symbol M instead of m_1 for the star, and m instead of m_2 for the planet. The mass function that can be determined is, then

$$\frac{m^3 \sin^3 i}{(M+m)^2}.$$

If m (the mass of the unseen body – the supposed planet) is very much smaller than the star (of mass M) whose radial velocity curve has been determined, then the mass function (which we can determine) is just

$$\frac{m^3 \sin^3 i}{M^2}.$$

And if, further, we have a reasonable idea of the mass M of the star (we know its spectral type and luminosity class from its spectrum, and we can suppose that it obeys the well-established relation between mass and luminosity of main-sequence stars), then we can determine m^3 sin^3 i and hence, of course m sin$i$. It is generally recognized that we cannot determine i for a spectroscopic binary star, and so it is conceded that the mass of the unseen body (the supposed planet) is uncertain by the unknown factor sin i.

However, the entire argument, it seems to me, is fundamentally and rather blatantly unsound, since, in order to arrive at m sin i and to hence to claim that m is of typically planetary rather than stellar mass, *the assumption that m is small and i isn't has already been made in approximating the mass function by* $\frac{m^3 \sin^3 i}{M^2}$. Unless there is *additional evidence* of a *different kind*, the observation of a velocity curve of small amplitude is not sufficient to indicate the presence of an unseen companion of planetary mass. Equally well (without additional evidence) the unseen companion could be of stellar mass and the orbital inclination could be small.

If the system is a *double-lined* spectroscopic binary system, we can determine the mass function for each component. That is, we can determine $\frac{m_1^3 \sin^3 i}{(m_1+m_2)^2}$ and $\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$. The reader should now convince him- or herself that, since we now know these two mass functions, we can determine the *mass ratio* and we can also determine $m_1 \sin^3 i$ and $m_2 \sin^3 i$ separately. But we cannot determine m_1, m_2 or i.