

## 1.6: Failure of the Newton-Raphson Method

This section is written reluctantly, for fear it may give the impression that the Newton-Raphson method frequently fails and is of limited usefulness. This is not the case; in nearly all cases encountered in practice it is very rapid and does not require a particularly good first guess. Nevertheless for completeness it should be pointed out that there are rare occasions when the method either fails or converges rather slowly.

One example is the quintic Equation that we have just encountered:

$$205 + 111x + 4x^2 - 31x^3 - 10x^4 + 5x^5 = 0 \quad (1.6.1)$$

When we chose  $x = 0$  as our first guess, we reached a solution fairly quickly. If we had chosen  $x = 1$ , we would not have been so lucky, for the first iteration would have taken us to  $-281$ , a very long way from any of the real solutions. Repeated iteration will eventually take us to the correct solution, but only after many iterations. This is not a typical situation, and usually almost any guess will do.

Another example of an Equation that gives some difficulty is

$$x = \tan x, \quad (1.6.2)$$

an Equation that occurs in the theory of single-slit diffraction.

We have

$$f(x) = x - \tan x = 0 \quad (1.6.3)$$

and

$$f'(x) = 1 - \sec^2 x = -\tan^2 x. \quad (1.6.4)$$

The Newton-Raphson process takes the form

$$x = x + \frac{x - \tan x}{\tan^2 x}. \quad (1.6.5)$$

The solution is  $x = 4.493\,409$ , but in order to achieve this the first guess must be between 4.3 and 4.7. This again is unusual, and in most cases almost any reasonable first guess results in rapid convergence.

The Equation

$$1 - 4x + 6x^2 - 4x^3 + x^4 = 0 \quad (1.6.6)$$

is an obvious candidate for difficulties. The four identical solutions are  $x = 1$ , but at  $x = 1$  not only is  $f(x)$  zero, but so is  $f'(x)$ . As the solution  $x = 1$  is approached, convergence becomes very slow, but eventually the computer or calculator will record an error message as it attempts to divide by the nearly zero  $f'(x)$ .

I mention just one last example very briefly. When discussing orbits, we shall encounter an Equation known as Kepler's Equation. The Newton-Raphson process almost always solves Kepler's Equation with spectacular speed, even with a very poor first guess. However, there are some very rare occasions, almost never encountered in practice, where the method fails. We shall discuss this Equation in Chapter 9.

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