

9.2: Kepler's Second Law from Conservation of Angular Momentum

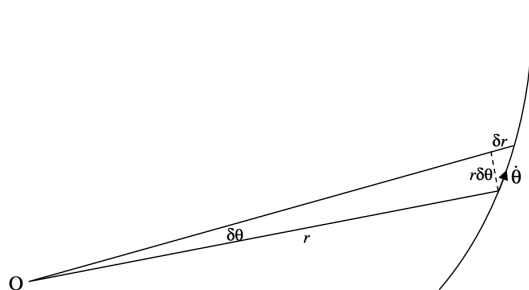


FIGURE IX.1

In figure IX.1, a particle of mass m is moving in some sort of trajectory in which the only force on it is directed towards or away from the point O . At some time, its polar coordinates are (r, θ) . At a time δt later these coordinates have increased by δr and $\delta \theta$.

Using the formula one half base times height for the area of a triangle, we see that the area swept out by the radius vector is approximately

$$\delta A = \frac{1}{2} r^2 \delta \theta + \frac{1}{2} r \delta \theta \delta r. \quad (9.3.1)$$

On dividing both sides by δt and taking the limit as $\delta t \rightarrow 0$, we see that the rate at which the radius vector sweeps out area is

$$\dot{A} = \frac{1}{2} r^2 \dot{\theta}. \quad (9.3.2)$$

But the angular momentum is $m r^2 \dot{\theta}$ and since this is constant, the areal speed is also constant. The areal speed, in fact, is half the angular momentum per unit mass.

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