13.13: Resuming the Numerical Example

Let us start with our previous iteration

$$\begin{array}{ll}\Delta_1 = 2.65825 & r_1 = 3.41952\\ \Delta_2 = 2.61558 & r_2 = 3.41673\\ \Delta_3 = 2.54579 & r_3 = 3.41082\end{array}$$

- or rather with the more precise values that will at this stage presumably be stored in our computer.

These are the values that we had reached when we last left the numerical example.

I promised to say how we know f_3. We defined $2f_3$ as $v_2 - v_1$, and this is the angle between the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$. Thus

$$\cos 2f_3 = \frac{\xi_1\xi_2 + \eta_1\eta_2 + \zeta_1\zeta_2}{r_1r_2}. \tag{13.13.1}$$

The heliocentric coordinates can be obtained from Equations 13.5.1, 2 and 3. For example,

$$\xi_1 = l_1\,\Delta_1 - \mathfrak{x}_{01}, \tag{13.13.2}$$

and of course

$$r_1 = \sqrt{\xi_1^2 + \eta_1^2 + \zeta_1^2}\,. \tag{13.13.3}$$

We know how to find the components $(\xi,\ \eta,\ \zeta)$ of the heliocentric radius vector (see Equations 13.7.8 and 9), and so we can now find f_3. I obtain

$$\cos 2f_3 = 0.999\ 929\ 1, \quad \cos f_3 = 0.999\ 982\ 3\ .$$

This means that the true anomaly is advancing at about $0^\circ.68$ in five days. It is interesting to see whether we are on the right track. According to the MPC, Pallas has a period of 4.62 years, which means that, on average, it will move through $1^\circ.067$ in five days. But Pallas has a rather eccentric orbit (according to the MPC, $e = 0.23$). The semi major axis of the orbit must be $P^{2/3} = 2.77$ AU (which agrees with the MPC), and therefore its aphelion distance $a(1+e)$ is about 3.41 AU. Thus Pallas must be close to aphelion in July 2002. By conservation of angular momentum, its angular motion at aphelion must be less than its mean motion by a factor of $(1+e)^2$ so the increase in the true anomaly in five days should be about $1^\circ.067/1.23^2$ or $0^\circ.71$. Thus we do seem to be on the right track.

We can now calculate M_3 and N_3 from Equations 13.12.27 and 28:

$$M_3^2 = 0.000\ 046\ 313\ 0$$

$$N_3 = 1.000\ 018$$

and so we have the following Equations 13.12.25 and 26 for the sector-triangle ratios:

$$R_3^2 = \frac{0.000\ 046\ 313\ 0}{1.000\ 018 - \cos g_3}$$

$$\text{and } R_3^3 - R_3^2 = \frac{0.000\ 046\ 313\ 0(g_3 - \sin g_3 \cos g_3)}{\sin^3 g_3}.$$

Since we discussed how to solve these Equations in section 1.9 of chapter 1, I merely give the solutions here. The one useful hint worth giving is that you can make the first guess for the iteration for g_3 equal to f_3, which we know ($\cos f_3 = .0999\ 9823$), and $R_3 = 1$.

$$\cos g_3 = 0.999\ 972, \quad R_3 = 1.000\ 031$$

We can proceed similarly with R_1 and R_2.

Here is a summary:

subscript	$\cos f$	M^2	N	$\cos g$	R
1	0.999 928 7	$1.859\ 91 \times 10^{-4}$	1.000 072	0.999 886	1.000 124
2	0.999 839 9	$4.180\ 80 \times 10^{-4}$	1.000 161	0.999 743	1.000 279
3	0.999 982 3	$4.631\ 30 \times 10^{-5}$	1.000 018	0.999 972	1.000 031

(13.13.1)

Our new triangle ratios will be

$$a_1 = \frac{R_2}{R_1} b_1 = \frac{1.000\,279}{1.000\,124} \times \frac{2}{3} = 0.666\,770$$

$$\text{and } a_3 = \frac{R_2}{R_3} b_3 = \frac{1.000\,279}{1.000\,031} \times \frac{1}{2} = 0.333\,416.$$

We can now go back to Equations 13.7.4,5 and 6, and calculate the geocentric and heliocentric distances anew. Skip sections 13.8, 13.9 and 13.10, and calculate new sector- triangle ratios and hence new triangle ratios, and repeat until convergence is obtained. After three iterations, I obtained convergence to six significant figures and after seven iterations I obtained convergence to 11 significant figures. The results to six significant figures are as follows:

$$\begin{array}{ll} \Delta_1 = 2.65403 & r_1 = 3.41539 \\ \Delta_2 = 2.61144 & r_2 = 3.41268 \\ \Delta_3 = 2.54172 & r_3 = 3.40681 \end{array}$$

This is not to be expected to agree exactly with the published MPC values, which are based on all available Pallas observations, whereas we arbitrarily chose three approximate ephemeris positions, but, based on these three positions, we have now broken the back of the problem and have found the geocentric and heliocentric distances.

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