17.2: Determination of the Apparent Orbit

The apparent orbit may be said to be determined if we can determine the size of the apparent ellipse (i.e. its semi major axis), its shape (i.e. its eccentricity), its orientation (i.e. the position angle of its major axis) and the two coordinates of the centre of the ellipse with respect to the primary star. Thus there are five parameters to determine.

The general Equation to a conic section (see Section 2.7 of Chapter 2) is of the form

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + 1 = 0, \tag{17.2.1}$$

so that we can equally say that the apparent orbit has been determined if we have determined the five coefficients $a, \ h, \ b, \ g, \ f$. Sections 2.8 and 2.9 described how to determine these coefficients if the positions of five or more points were given, and section 2.7 dealt with how to determine the semi major axis, the eccentricity, the orientation and the centre given $a, \ h, \ b, \ g$ and f.

We may conclude, therefore, that in order to determine the apparent ellipse all that need be done is to obtain five or more observations of (ρ, θ) or of (x, y), and then just apply the methods of section 2.8 and 2.9 to fit the apparent ellipse. Of course, although five is the minimum number of observations that are essential, in practice we need many, many more (see section 2.9), and in order to get a good ellipse we really need to wait until observations have been obtained to cover a whole period. But merely to fit the best ellipse to a set of (x, y) points is not by any means making the best use of the data. The reason is that an observation consists not only of (ρ, θ) or of (x, y), but also the time, t. In fact the separation and position angle are quite difficult to measure and will have quite considerable errors, while the *time* of each observation is known with great precision. We have so far completely ignored the one measurement that we know for certain!

We need to make sure that the apparent ellipse that we obtain *obeys* Kepler's *second law*. Indeed it is more important to ensure this than blindly to fit a least-squares ellipse to n points.

If I were doing this, I would probably plot two separate graphs – one of ρ (or perhaps ρ^2) against time, and one of θ against time. One thing that this would immediately achieve would be to identify any obviously bad measurements, which we could then reject. I would draw a smooth curve for each graph. Then, for equal time intervals I would determine from the graphs the values of ρ and dθ/dt and I would then calculate $\rho^2 \ d\theta/dt$. According to Kepler's second law, this should be constant and independent of time. I would then adjust my preliminary attempt at the apparent orbit until Kepler's second law was obeyed and $\rho^2 \ d\theta/dt$ was constant. A good question now, is, which should be adjusted, ρ or θ ? There may be no hard and fast invariable answer to this, but, generally speaking, the measurement of the separation is more uncertain than the measurement of the position angle, so that it would usually be best to adjust ρ.

If we are eventually satisfied that we have the best apparent ellipse that satisfies as best as possible not only the positions of the points, but also their times, and that the apparent ellipse satisfies Kepler's law of areas, our next task will be to determine the elements of the true ellipse.