14.4: Lagrange's Planetary Equations

We now go to Equation 14.2.8 to obtain *Lagrange's Planetary Equations*, which will enable us to calculate the rates of change of the orbital elements if we know the form of the perturbing function:

$$\dot{a} = -\frac{2a^2}{GMm}\frac{\partial R}{\partial T}, \tag{14.4.1}$$

$$\dot{e} = -\frac{a(1-e^2)}{GMme}\frac{\partial R}{\partial T}, \tag{14.4.2}$$

$$i = -\frac{1}{\sqrt{GMm^2a(1-e^2)}\sin i}\frac{\partial R}{\partial \Omega} - \frac{1}{me}\sqrt{\frac{1-e^2}{GMa}}\frac{\partial R}{\partial \omega}, \tag{14.4.3}$$

$$\dot{\omega} = \frac{1}{me}\sqrt{\frac{1-e^2}{GMa}}\frac{\partial R}{\partial e} - \frac{1}{\sqrt{GMm^2a(1-e^2)}\tan i}\frac{\partial R}{\partial i}, \tag{14.4.4}$$

$$\dot{\Omega} = \frac{1}{\sqrt{GMm^2(1-e^2)}\sin i}\frac{\partial R}{\partial i}, \tag{14.4.5}$$

$$\dot{T} = \frac{2a^2}{GMm}\frac{\partial R}{\partial a} + \frac{a(1-e^2)}{GMme}\frac{\partial R}{\partial e}. \tag{14.4.6}$$