18.6: Finding the Period

The first five sections of this chapter have dealt with calculating the relations between the orbital elements and the radial velocity curve, and that really completes what is necessary in a book whose primary focus is on celestial mechanics. In practice, the celestial mechanics part is the least of the difficulties. The Equations may look forbidding at first sight, but at least the Equations are unambiguous and clear cut. There are lots of problems of one sort of another that in practice occupy much more of the investigator's time than merely the computation of the orbit, which nowadays is done in the blink of an eye. I mention a few of these only briefly in the remaining sections, partly because they are not particularly concerned with celestial mechanics, and partly because my personal practical experience with them is limited.

If you were able to measure the radial velocity every five minutes throughout a complete period, there would be no difficulty in obtaining a nice velocity curve. In practice, however, you measure a radial velocity "every so often" – with perhaps many orbital periods between consecutive observations. Finding the period, then, is obviously a bit of a problem. (That there is an initial difficulty in finding the period is ultimately compensated for in that, once a preliminary value for the period is found, it can often be calculated to great precision, if the star has been observed over many decades.)

If you have a large number of observations spread out over a long time, it may be possible to identify several observations in which the radial velocity is a maximum, and you might then assume that the least time between consecutive maxima is an integral number of orbital periods. Of course you don't know what this integral number is, but you might be able to do a little better. For example, you might find that there are 100 days between two consecutive maxima, so that there are an integral number of periods in 100 days. You might also find that two other maxima are separated by 110 days. You now know that there are an integral number of periods in 10 days – which is a great improvement.

A difficulty arises if you observe the star at regular and equal intervals. While there is an obvious answer to this – *i.e.* don't do it – it may not in practice be so easy to avoid. For example: if you always observe the star when it is highest in the sky, on the meridian, then you are always observing it at an integral number of sidereal days. You then get a stroboscopic effect. Thus, if you have a piece of machinery that is cycling many times per second, you can illuminate it stroboscopically with a light that flashes periodically, and you can then see the machinery moving apparently much more slowly than it really is. The same thing happens if you observe a spectroscopic binary star at precisely regular intervals – it will appear to have a much longer period that is really the case.

It is easier to understand the effect if we work in terms of frequency (reciprocal of the period) rather than period. Thus let n (= $1/P$) be the orbital frequency of the star and let n' (= $1/T$) be the frequency of observation (the frequency of the stroboscope flash, to recall the analogy). Then the apparent orbital frequency ν' of the star is given by

$$|\nu' - \nu| = mn \tag{18.6.1}$$

where m is an integer. Returning to periods, this means that you can be deceived into deducing a spurious period P' given by

$$\frac{1}{P'} = \frac{1}{P} \pm \frac{1}{mn}. \tag{18.6.2}$$

You don't have to make an observation every single sidereal day to experience this stroboscopic effect. If your stroboscope is defective and it misses a few flashes, the machinery will still appear to slow down. Likewise, if you miss a few observations, you may still get a spurious period.

Once you have overcome these difficulties and have determined the period, in order to construct a radial velocity curve you will have to subtract an integral number of periods from the time of each observation in order to bring all observations on to a single velocity curve covering just one period.