5.4.4: Infinite Plane Laminas

For the gravitational field due to a uniform infinite plane lamina, all one has to do is to put $\alpha = \pi/2$ in Equation 5.4.7 or $\omega = 2\pi$ in Equation 5.4.9 to find that the gravitational field is

$$g = 2\pi G\sigma. \tag{5.4.13}$$

This is, as might be expected, independent of distance from the infinite plane. The lines of gravitational field are uniform and parallel all the way from the surface of the lamina to infinity.

Suppose that the surface density of the infinite plane is not uniform, but varies with distance in the plane from some point in the plane as $\sigma(r)$, we have to calculate

$$g = 2\pi Gz \int_0^\infty \frac{\sigma(r) r dr}{(z^2 + r^2)^{3/2}}. \tag{5.4.14}$$

Try it, for example, with $\sigma(r)$ being one of the following:

$$\sigma_0 e^{-kr}, \quad \sigma_0 e^{-k^2 r^2}, \quad \frac{\sigma_0}{1 + k^2 r^2}. \tag{5.4.4.1}$$