

## 5.13: Pressure at the Centre of a Uniform Sphere

What is the pressure at the centre of a sphere of radius  $a$  and of uniform density  $\rho$ ?

(Preliminary thought: Show by dimensional analysis that it must be something times  $G\rho^2 a^2$ .)

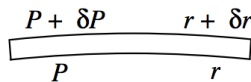


FIGURE V.27

Consider a portion of the sphere between radii  $r$  and  $r + \delta r$  and cross-sectional area  $A$ . Its volume is  $A\delta r$  and its mass is  $\rho A\delta r$ . (Were the density not uniform throughout the sphere, we would here have to write  $\rho(r)A\delta r$ .) Its weight is  $\rho g A\delta r$ , where  $g = GM_r/r^2 = \frac{4}{3}\pi G\rho r$ . We suppose that the pressure at radius  $r$  is  $P$  and the pressure at radius  $r + \delta r$  is  $P + \delta P$ . ( $\delta P$  is negative.) Equating the downward forces to the upward force, we have

$$A(P + \delta P) + \frac{4}{3}\pi A G \rho^2 r \delta r = AP. \quad (5.13.1)$$

That is:

$$\delta P = -\frac{4}{3}\pi G \rho^2 r \delta r. \quad (5.13.2)$$

Integrate from the centre to the surface:

$$\int_{P_0}^0 dP = -\frac{4}{3}\pi G \rho^2 \int_0^a r dr. \quad (5.13.3)$$

Thus:

$$P = \frac{2}{3}\pi G \rho^2 a^2. \quad (5.13.4)$$

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