4.1: Introduction

Various geometrical figures in three-dimensional space can be described relative to a set of mutually orthogonal axes Ox, Oy, Oz, and a point can be represented by a set of rectangular coordinates (x, y, z). The point can also be represented by cylindrical coordinates (ρ, ϕ, z) or spherical coordinates (r, θ, ϕ), which were described in Chapter 3. In this chapter, we are concerned mostly with (x, y, z). The rectangular axes are usually chosen so that when you look down the z-axis towards the xy-plane, the y-axis is 90° counterclockwise from the x-axis. Such a set is called a right-handed set. A left-handed set is possible, and may be useful under some circumstances, but, unless stated otherwise, it is assumed that the axes chosen in this chapter are right-handed.

An Equation connecting x, y and z, such as

$$f(x, y, z) = 0 \tag{4.1.1}$$

or

$$z = z(x, y) \tag{4.1.2}$$

describes a two-dimensional surface in three-dimensional space. A line (which need be neither straight nor two-dimensional) can be described as the intersection of two surfaces, and hence a line or curve in three-dimensional coordinate geometry is described by two Equations, such as

$$f(x, y, z) = 0 \tag{4.1.3}$$

and

$$g(x, y, z) = 0. \tag{4.1.4}$$

In two-dimensional geometry, a single Equation describes some sort of a plane curve. For example,

$$y^2 = 4qx \tag{4.1.5}$$

describes a parabola. But a plane curve can also be described in parametric form by two Equations. Thus, a parabola can also be described by

$$x = qt^2 \tag{4.1.6}$$

and

$$y = 2qt \tag{4.1.7}$$

Similarly, in three-dimensional geometry, a line or curve can be described by three Equations in parametric form. For example, the three Equations

$$x = a\cos t \tag{4.1.8}$$

$$y = a\sin t \tag{4.1.9}$$

$$z = ct \tag{4.1.10}$$

describe a curve in three-space. Think of the parameter t as time, and see if you can imagine what sort of a curve this is.

We shall be concerned in this chapter mainly with six types of surface: the plane, the ellipsoid, the paraboloid, the hyperboloid, the cylinder and the cone.