

14.3: The Poisson Brackets for the Orbital Elements

A worked example is in order. From Equations 14.2.7 and 14.2.8, we see that the Poisson brackets are defined by

$$\{A_i, A_k\}_{\alpha_j, \beta_j} = \sum_j \left(\frac{\partial A_i}{\partial \alpha_j} \frac{\partial A_k}{\partial \beta_j} - \frac{\partial A_i}{\partial \beta_j} \frac{\partial A_k}{\partial \alpha_j} \right). \quad (14.3.1)$$

The A_i are the orbital elements.

For our example, we shall calculate $\{\Omega, i\}$ and we write out the sum in full:

$$\{\Omega, i\} = \sum_j \left(\frac{\partial \Omega}{\partial \alpha_j} \frac{\partial i}{\partial \beta_j} - \frac{\partial \Omega}{\partial \beta_j} \frac{\partial i}{\partial \alpha_j} \right) \quad (14.3.2)$$

$$= \frac{\partial \Omega}{\partial \alpha_1} \frac{\partial i}{\partial \beta_1} + \frac{\partial \Omega}{\partial \alpha_2} \frac{\partial i}{\partial \beta_2} + \frac{\partial \Omega}{\partial \alpha_3} \frac{\partial i}{\partial \beta_3} - \frac{\partial \Omega}{\partial \beta_1} \frac{\partial i}{\partial \alpha_1} - \frac{\partial \Omega}{\partial \beta_2} \frac{\partial i}{\partial \alpha_2} - \frac{\partial \Omega}{\partial \beta_3} \frac{\partial i}{\partial \alpha_3}. \quad (14.3.3)$$

Refer now to Equations 10.11.27 and 29, and we find

$$\{\Omega, i\} = 0 + 0 + 0 - 0 + \frac{1}{\alpha_3 \sqrt{1 - \alpha_2^2 / \alpha_3^2}} - 0. \quad (14.3.4)$$

Finally, referring to Equations 10.11.20 and 21, we obtain

$$\{\Omega, i\} = \frac{1}{\sqrt{GMm^2 a(1 - e^2)} \cdot \sin i}. \quad (14.3.5)$$

Proceeding in a similar manner for the others, we obtain

$$\{a, T\} = -\frac{2a^2}{GMm}, \quad (14.3.6)$$

$$\{e, T\} = -\frac{a(1 - e^2)}{GMme}, \quad (14.3.7)$$

$$\{i, \omega\} = \frac{1}{\sqrt{GMm^2 a(1 - e^2)} \cdot \tan i}. \quad (14.3.8)$$

$$\{e, \omega\} = -\frac{\sqrt{1 - e^2}}{em\sqrt{GMa}}, \quad (14.3.9)$$

In addition, we have, of course,

$$\{i, \Omega\} = -\{\Omega, i\}, \{T, a\} = -\{a, T\}, \{T, e\} = -\{e, T\} \text{ and } \{\omega, i\} = -\{i, \omega\}. \quad (14.3.10)$$

All other pairs are zero.

This page titled [14.3: The Poisson Brackets for the Orbital Elements](#) is shared under a [CC BY-NC 4.0](#) license and was authored, remixed, and/or curated by [Jeremy Tatum](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.