13.2: Triangles

I shall start with a geometric theorem involving triangles, which will be useful as we progress towards our aim of computing orbital elements.

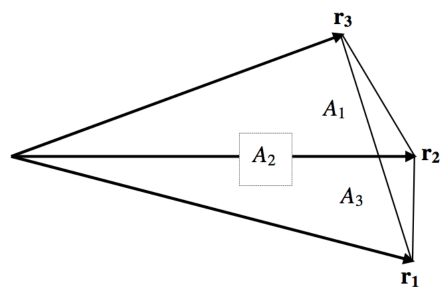

FIGURE XIII.1

Figure $\mathrm{XIII}.1$ shows three coplanar vectors. It is clearly possible to express $\mathbf{r}_2$ as a linear combination of the other two. That is to say, it should be possible to find coefficients such that

$$\mathbf{r}_2 = a_1\mathbf{r}_1 + a_3\mathbf{r}_3. \tag{13.2.1}$$

The notation I am going to use is as follows:

- The area of the triangle formed by joining the tips of $\mathbf{r}_2$ and $\mathbf{r}_3$ is A_1.
- The area of the triangle formed by joining the tips of $\mathbf{r}_3$ and $\mathbf{r}_1$ is A_2.
- The area of the triangle formed by joining the tips of $\mathbf{r}_1$ and $\mathbf{r}_2$ is A_3.

To find the coefficients in Equation 13.2.1, multiply both sides by $\mathbf{r}_1\times$:

$$\mathbf{r}_1 \times \mathbf{r}_2 = a_3\mathbf{r}_1 \times \mathbf{r}_3. \tag{13.2.2}$$

The two vector products are parallel vectors (they are each perpendicular to the plane of the paper), of magnitudes $2A_3$ and $2A_2$ respectively. ($2A_3$ is the area of the *parallelogram* of which the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ form two sides.)

$$\therefore a_3 = A_3/A_2. \tag{13.2.3}$$

Similarly by multiplying both sides of Equation 13.2.1 by $\mathbf{r}_3\times$ it will be found that

$$a_1 = A_1/A_2. \tag{13.2.4}$$

Hence we find that

$$A_2\mathbf{r}_2 = A_1\mathbf{r}_1 + A_3\mathbf{r}_3. \tag{13.2.5}$$