5.8.1: Potential Near a Point Mass

We shall define the potential to be zero at infinity. If we are in the vicinity of a point mass, we shall always have to *do work* in moving a test particle *away from* the mass. We shan't reach zero potential until we are an infinite distance away. It follows that the potential at any finite distance from a point mass is *negative*. The potential at a point is the work required to move unit mass *from infinity to the point*; i.e., it is negative.

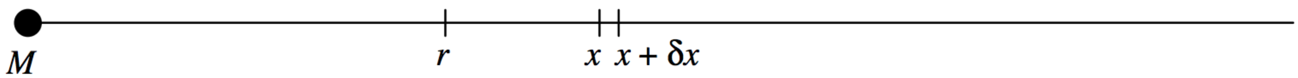

FIGURE V.23

The magnitude of the field at a distance x from a point mass M (figure V.23) is GM/x^2, and the force on a mass m placed there would be GMm/x^2. The work required to move m from x to $x+\delta x$ is $GMm\delta x/x^2$. The work required to move it from r to infinity is

$$GMm\int_r^\infty \frac{dx}{x^2} = \frac{GMm}{r}. \tag{5.8.1.1}$$

The work required to move *unit* mass *from* ∞ to r, which is the potential at r is

$$\psi = -\frac{GM}{r}. \tag{5.8.1}$$

The *mutual potential energy of two point masses a distance r apart*, which is the work required to bring them to a distance r from an infinite initial separation, is

$$V = -\frac{GMm}{r}. \tag{5.8.2}$$

I here summarize a number of similar-looking formulas, although there is, of course, not the slightest possibility of confusing them. Here goes:

Force between two masses:

$$F = \frac{GMm}{r^2}. \quad \mathrm{N} \tag{5.8.3}$$

Field near a point mass:

$$g = \frac{GM}{r^2}, \quad \mathrm{N\,kg^{-1}}\ \mathit{or}\ \mathrm{m\,s^{-2}} \tag{5.8.4}$$

which can be written in vector form as:

$$\mathbf{g} = -\frac{GM}{r^2}\hat{\mathbf{r}} \quad \mathrm{N\,kg^{-1}}\ \mathit{or}\ \mathrm{m\,s^{-2}} \tag{5.8.5}$$

or as:

$$\mathbf{g} = -\frac{GM}{r^3}\mathbf{r}. \quad \mathrm{N\,kg^{-1}}\ \mathit{or}\ \mathrm{m\,s^{-2}} \tag{5.8.6}$$

Mutual potential energy of two masses:

$$V = -\frac{GMm}{r}. \quad \mathrm{J} \tag{5.8.7}$$

Potential near a point mass:

$$\psi = -\frac{GM}{r}. \quad \mathrm{J\,kg^{-1}} \tag{5.8.8}$$

I hope that's crystal clear.