10.13: Fixed-Axis Rotation Summary

Key Terms

angular acceleration	time rate of change of angular velocity
angular position	angle a body has rotated through in a fixed coordinate system
angular velocity	time rate of change of angular position
instantaneous angular acceleration	derivative of angular velocity with respect to time
instantaneous angular velocity	derivative of angular position with respect to time
kinematics of rotational motion	describes the relationships among rotation angle, angular velocity, angular acceleration, and time
lever arm	perpendicular distance from the line that the force vector lies on to a given axis
linear mass density	the mass per unit length λ of a one dimensional object
moment of inertia	rotational mass of rigid bodies that relates to how easy or hard it will be to change the angular velocity of the rotating rigid body
Newton’s second law for rotation	sum of the torques on a rotating system equals its moment of inertia times its angular acceleration
parallel axis	axis of rotation that is parallel to an axis about which the moment of inertia of an object is known
parallel-axis theorem	if the moment of inertia is known for a given axis, it can be found for any axis parallel to it
rotational dynamics	analysis of rotational motion using the net torque and moment of inertia to find the angular acceleration
surface mass density	mass per unit area σ of a two dimensional object
torque	cross product of a force and a lever arm to a given axis
total linear acceleration	vector sum of the centripetal acceleration vector and the tangential acceleration vector

Key Equations

Angular position	$\theta = \frac{s}{r}$
Angular velocity	$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = \frac{d \theta}{dt}$
Tangential speed	$v_{t} = r \omega$
Angular acceleration	$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t} = \frac{d \omega}{dt} = \frac{d^{2} \theta}{dt^{2}}$
Tangential acceleration	$a_{t} = r \alpha$
Average angular velocity	$\bar{\omega} = \frac{\omega_{0} + \omega_{f}}{2}$
Angular displacement	$\theta_{f} = \theta_{0} + \bar{\omega} t$
Angular velocity from constant angular acceleration	$\omega_{f} = \omega_{0} + \alpha t$
Angular velocity from displacement and constant angular acceleration	$\theta_{f} = \theta_{0} + \omega_{0} t + \frac{1}{2} \alpha t^{2}$
Change in angular velocity	$\omega_{f}^{2} = \omega_{0}^{2} + 2a (\Delta \theta)$
Total acceleration	$\vec{a} = \vec{a}_{c} + \vec{a}_{t}$
Moment of inertia	$I = \sum_{j} m_{j} r_{j}^{2}$
Moment of inertia of a continuous object	$I = \int r^{2} dm$

Parallel-axis theorem	$I_{parallel-axis} = I_{initial} + md^{2}$
Moment of inertia of a compound object	$I_{total} = \sum_{i} I_{i}$
Torque vector	$\vec{\tau} = \vec{r} \times \vec{F}$
Magnitude of torque	$\|\vec{\tau}\| = r_{\perp} F$
Total torque	$\vec{\tau}_{net} = \sum_{i} \|\vec{\tau}_{i}\|$
Newton’s second law for rotation	$\sum_{i} \tau_{i} = I \alpha$

Summary

Rotational Variables

- The angular position θ of a rotating body is the angle the body has rotated through in a fixed coordinate system, which serves as a frame of reference.
- The angular velocity of a rotating body about a fixed axis is defined as ω(rad/s), the rotational rate of the body in radians per second. The instantaneous angular velocity of a rotating body $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$ is the derivative with respect to time of the angular position θ, found by taking the limit Δt → 0 in the average angular velocity $\bar{\omega} = \frac{\Delta\theta}{\Delta t}$. The angular velocity relates v_t to the tangential speed of a point on the rotating body through the relation $v_t = r\omega$, where r is the radius to the point and v_t is the tangential speed at the given point.
- The angular velocity $\vec{\omega}$ is found using the right-hand rule. If the fingers curl in the direction of rotation about a fixed axis, the thumb points in the direction of $\vec{\omega}$ (see Figure 10.5).
- If the system’s angular velocity is not constant, then the system has an angular acceleration. The average angular acceleration over a given time interval is the change in angular velocity over this time interval, $\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$. The instantaneous angular acceleration is the time derivative of angular velocity, $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$. The angular acceleration $\vec{\alpha}$ is found by locating the angular velocity. If a rotation rate of a rotating body is decreasing, the angular acceleration is in the opposite direction to $\vec{\omega}$. If the rotation rate is increasing, the angular acceleration is in the same direction as $\vec{\omega}$.
- The tangential acceleration of a point at a radius from the axis of rotation is the angular acceleration times the radius to the point.

Rotation with Constant Angular Acceleration">10.2 Rotation with Constant Angular Acceleration

- The kinematics of rotational motion describes the relationships among rotation angle (angular position), angular velocity, angular acceleration, and time.
- For a constant angular acceleration, the angular velocity varies linearly. Therefore, the average angular velocity is 1/2 the initial plus final angular velocity over a given time period: $\bar{\omega} = \frac{\omega_{0} + \omega_{f}}{2} \ldotp$
- We used a graphical analysis to find solutions to fixed-axis rotation with constant angular acceleration. From the relation $\omega = \frac{d\theta}{dt}$, we found that the area under an angular velocity-vs.-time curve gives the angular displacement, $\theta_f - \theta_0 = \Delta\theta = \int_{t_0}^{t} \omega(t)dt$. The results of the graphical analysis were verified using the kinematic equations for constant angular acceleration. Similarly, since $\alpha = \frac{d\omega}{dt}$, the area under an angular acceleration-vs.-time graph gives the change in angular velocity: $\omega f - \omega 0 = \Delta\omega = \int_{t_0}^{t} \alpha(t)dt$.

Relating Angular and Translational Quantities">10.3 Relating Angular and Translational Quantities

- The linear kinematic equations have their rotational counterparts such that there is a mapping x → θ, v → ω, a → α.
- A system undergoing uniform circular motion has a constant angular velocity, but points at a distance r from the rotation axis have a linear centripetal acceleration.
- A system undergoing nonuniform circular motion has an angular acceleration and therefore has both a linear centripetal and linear tangential acceleration at a point a distance r from the axis of rotation.
- The total linear acceleration is the vector sum of the centripetal acceleration vector and the tangential acceleration vector. Since the centripetal and tangential acceleration vectors are perpendicular to each other for circular motion, the magnitude of the total linear acceleration is $|\vec{a}| = \sqrt{a_c^2 + a_t^2}$.

Calculating Moments of Inertia

- Moments of inertia can be found by summing or integrating over every 'piece of mass' that makes up an object, multiplied by the square of the distance of each 'piece of mass' to the axis. In integral form the moment of inertia is $I = \int r^2 dm$.
- Moment of inertia is larger when an object's mass is farther from the axis of rotation.
- It is possible to find the moment of inertia of an object about a new axis of rotation once it is known for a parallel axis. This is called the parallel axis theorem given by $I_{\text{parallel-axis}} = I_{\text{center of mass}} + md^2$, where d is the distance from the initial axis to the parallel axis.
- Moment of inertia for a compound object is simply the sum of the moments of inertia for each individual object that makes up the compound object.

Torque

- The magnitude of a torque about a fixed axis is calculated by finding the lever arm to the point where the force is applied and using the relation $|\vec{\tau}| = r_{\perp}F$, where $r_{\perp}$ is the perpendicular distance from the axis to the line upon which the force vector lies.
- The sign of the torque is found using the right hand rule. If the page is the plane containing $\vec{r}$ and $\vec{F}$, then $\vec{r} \times \vec{F}$ is out of the page for positive torques and into the page for negative torques.
- The net torque can be found from summing the individual torques about a given axis.

Newton's Second Law for Rotation">10.6 Newton's Second Law for Rotation

- Newton's second law for rotation, $\sum_i \tau_i = I\alpha$, says that the sum of the torques on a rotating system about a fixed axis equals the product of the moment of inertia and the angular acceleration. This is the rotational analog to Newton's second law of linear motion.
- In the vector form of Newton's second law for rotation, the torque vector $\vec{\tau}$ is in the same direction as the angular acceleration $\vec{\alpha}$. If the angular acceleration of a rotating system is positive, the torque on the system is also positive, and if the angular acceleration is negative, the torque is negative.

Contributors and Attributions