5.6: Common Forces - Tension

Learning Objectives

- Define tension forces

A **tension** is a force along the length of a medium; in particular, it is a pulling force that acts along a stretched flexible connector, such as a rope or cable. The word “tension” comes from a Latin word meaning “to stretch.” Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called tendons. Any flexible connector, such as a string, rope, chain, wire, or cable, can only exert a pull parallel to its length; thus, a force carried by a flexible connector is a tension with a direction parallel to the connector. Tension is a pull in a connector. Consider the phrase: “You can’t push a rope.” Instead, tension force pulls outward along the two ends of a rope. Consider a person holding a mass on a rope, as shown in Figure $5.6.4$. If the 5.00-kg mass in the figure is stationary, then its acceleration is zero and the net force is zero. The only external forces acting on the mass are its weight and the tension supplied by the rope. Thus,

$$F_{net} = T - w = 0,$$

where T and w are the magnitudes of the tension and weight, respectively, and their signs indicate direction, with up being positive. As we proved using Newton’s second law, the tension equals the weight of the supported mass:

$$T = w = mg.$$

Thus, for a 5.00-kg mass (neglecting the mass of the rope), we see that

$$T = mg = (5.00\ kg)(9.80\ m/s^2) = 49.0\ N.$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

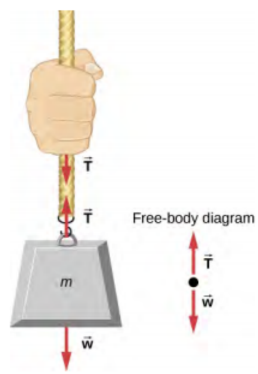

Figure $5.6.4$: When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force $\vec{T}$, that force must be parallel to the length of the rope, as shown. By Newton’s third law, the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). The rope is the medium that carries the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a tendon, or a bicycle brake cable. If there is no friction, the tension transmission is undiminished; only its direction changes, and it is always parallel to the flexible connector, as shown in Figure $5.6.5$.

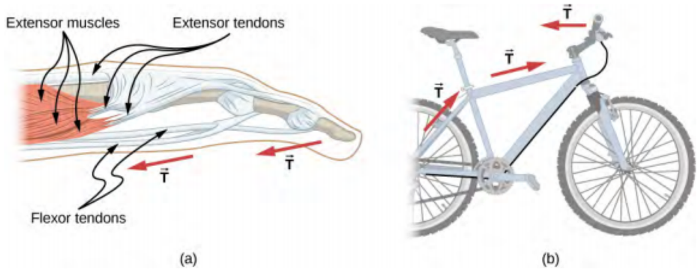

Figure 5.6.5: (a) Tendons in the finger carry force T from the muscles to other parts of the finger, usually changing the force's direction but not its magnitude (the tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension T from the brake lever on the handlebars to the brake mechanism. Again, the direction but not the magnitude of T is changed.

: What is the Tension in a Tightrope?

Calculate the tension in the wire supporting the 70.0-kg tightrope walker shown in Figure 5.6.6.

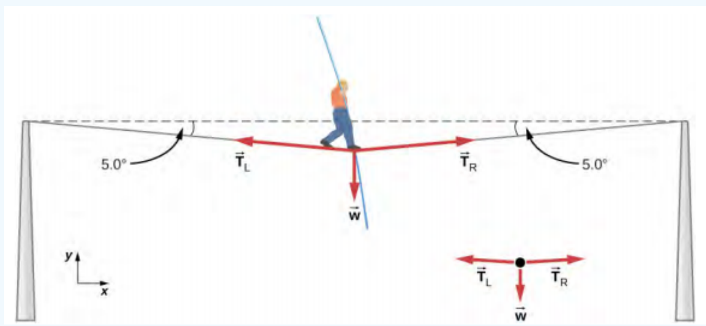

Figure 5.6.6: The weight of a tightrope walker causes a wire to sag by 5.0°. The system of interest is the point in the wire at which the tightrope walker is standing.

Strategy

As you can see in Figure 5.6.6, the wire is bent under the person's weight. Thus, the tension on either side of the person has an upward component that can support his weight. As usual, forces are vectors represented pictorially by arrows that have the same direction as the forces and lengths proportional to their magnitudes. The system is the tightrope walker, and the only external forces acting on him are his weight $\vec{w}$ and the two tensions $\vec{T}_L$ (left tension) and $\vec{T}_R$ (right tension). It is reasonable to neglect the weight of the wire. The net external force is zero, because the system is static. We can use trigonometry to find the tensions. One conclusion is possible at the outset—we can see from Figure 5.6.6(b) that the magnitudes of the tensions T_L and T_R must be equal. We know this because there is no horizontal acceleration in the rope and the only forces acting to the left and right are T_L and T_R. Thus, the magnitude of those horizontal components of the forces must be equal so that they cancel each other out.

Whenever we have two-dimensional vector problems in which no two vectors are parallel, the easiest method of solution is to pick a convenient coordinate system and project the vectors onto its axes. In this case, the best coordinate system has one horizontal axis (x) and one vertical axis (y).

Solution

First, we need to resolve the tension vectors into their horizontal and vertical components. It helps to look at a new free-body diagram showing all horizontal and vertical components of each force acting on the system (Figure 5.6.7).

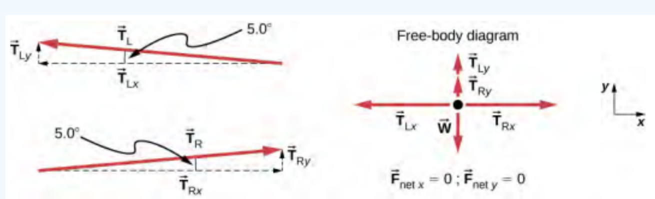

Figure 5.6.7: When the vectors are projected onto vertical and horizontal axes, their components along these axes must add to zero, since the tightrope walker is stationary. The small angle results in T being much greater than w.

Consider the horizontal components of the forces (denoted with a subscript x):

$$F_{netx} = T_{Rx} - T_{Lx}.$$

The net external horizontal force $F_{net\ x} = 0$, since the person is stationary. Thus,

$$F_{netx} = 0 = T_{Rx} - T_{Lx}.$$

$$T_{Lx} = T_{Rx}.$$

Now observe Figure $5.6.7$ You can use trigonometry to determine the magnitude of T_L and T_R:

$$\cos 5.0^o = \frac{T_{Lx}}{T_L}, \quad T_{Lx} = T_L \cos 5.0^o$$

$$\cos 5.0^o = \frac{T_{Rx}}{T_R}, \quad T_{Rx} = T_R \cos 5.0^o.$$

Equating T_{Lx} and T_{Rx}:

$$T_L \cos 5.0^o = T_R \cos 5.0^o.$$

Thus,

$$T_L = T_R = T,$$

as predicted. Now, considering the vertical components (denoted by a subscript y), we can solve for T. Again, since the person is stationary, Newton's second law implies that $F_{net\ y} = 0$. Thus, as illustrated in the free-body diagram,

$$F_{nety} = T_{Ly} + T_{Ry} - w = 0.$$

We can use trigonometry to determine the relationships among T_{Ly}, T_{Ry}, and T. As we determined from the analysis in the horizontal direction, $T_L = T_R = T$:

$$\sin 5.0^o = \frac{T_{Ly}}{T_L}, \quad T_{Ly} = T_L \sin 5.0^o = T \sin 5.0^o$$

$$\sin 5.0^o = \frac{T_{Ry}}{T_R}, \quad T_{Ry} = T_R \sin 5.0^o = T \sin 5.0^o.$$

Now we can substitute the vales for T_{Ly} and T_{Ry}, into the net force equation in the vertical direction:

$$F_{nety} = T_{Ly} + T_{Ry} - w = 0$$

$$F_{nety} = 0 = T \sin 5.0^o + T \sin 5.0^o - w = 0$$

$$2T \sin 5.0^o - w = 0$$

$$2T \sin 5.0^o = w$$

and

$$T = \frac{w}{2 \sin 5.0^o} = \frac{mg}{2 \sin 5.0^o},$$

so

$$T = \frac{(70.0\ kg)(9.80\ m/s^2)}{2(0.0872)},$$

and the tension is

$$T = 3930\ N.$$

Significance

The vertical tension in the wire acts as a force that supports the weight of the tightrope walker. The tension is almost six times the 686-N weight of the tightrope walker. Since the wire is nearly horizontal, the vertical component of its tension is only a fraction of the tension in the wire. The large horizontal components are in opposite directions and cancel, so most of the tension in the wire is not used to support the weight of the tightrope walker.

If we wish to create a large tension, all we have to do is exert a force perpendicular to a taut flexible connector, as illustrated in Figure 5.6.6. As we saw in Example 5.13, the weight of the tightrope walker acts as a force perpendicular to the rope. We saw that the tension in the rope is related to the weight of the tightrope walker in the following way:

$$T = \frac{w}{2\sin\theta}.$$

We can extend this expression to describe the tension T created when a perpendicular force ($F_{\perp}$) is exerted at the middle of a flexible connector:

$$T = \frac{F_{\perp}}{2\sin\theta}.$$

The angle between the horizontal and the bent connector is represented by θ. In this case, T becomes large as θ approaches zero. Even the relatively small weight of any flexible connector will cause it to sag, since an infinite tension would result if it were horizontal (i.e., θ = 0 and sin θ = 0). For example, Figure 5.6.8 shows a situation where we wish to pull a car out of the mud when no tow truck is available. Each time the car moves forward, the chain is tightened to keep it as straight as possible. The tension in the chain is given by T = $\frac{F_{\perp}}{2\sin\theta}$, and since θ is small, T is large. This situation is analogous to the tightrope walker, except that the tensions shown here are those transmitted to the car and the tree rather than those acting at the point where $F_{\perp}$ is applied.

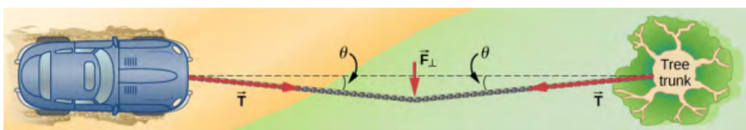

Figure 5.6.8: We can create a large tension in the chain—and potentially a big mess—by pushing on it perpendicular to its length, as shown.