9.15: Rotational Kinetic Energy - Work and Energy

Learning Objectives

By the end of this section, you will be able to:

- Derive the equation for rotational work.
- Calculate rotational kinetic energy.
- Demonstrate the Law of Conservation of Energy.

In this module, we will learn about work and energy associated with rotational motion. Figure $9.15.1$ shows a worker using an electric grindstone propelled by a motor. Sparks are flying, and noise and vibration are created as layers of steel are pared from the pole. The stone continues to turn even after the motor is turned off, but it is eventually brought to a stop by friction. Clearly, the motor had to work to get the stone spinning. This work went into heat, light, sound, vibration, and considerable rotational kinetic energy.

Figure $9.15.1$: The motor works in spinning the grindstone, giving it rotational kinetic energy. That energy is then converted to heat, light, sound, and vibration. (credit: U.S. Navy photo by Mass Communication Specialist Seaman Zachary David Bell)

Work must be done to rotate objects such as grindstones or merry-go-rounds. Work was defined in Uniform Circular Motion and Gravitation for translational motion, and we can build on that knowledge when considering work done in rotational motion. The simplest rotational situation is one in which the net force is exerted perpendicular to the radius of a disk (as shown in Figure $9.15.2$) and remains perpendicular as the disk starts to rotate. The force is parallel to the displacement, and so the net work done is the product of the force times the arc length traveled:

$$net\ W = (net\ F)\Delta s. \tag{9.15.1}$$

To get torque and other rotational quantities into the equation, we multiply and divide the right-hand side of the equation by r, and gather terms:

$$net\ W = (r\ net\ F)\frac{\Delta s}{r}. \tag{9.15.2}$$

We recognize that $r\ net\ F = net\ \tau$ and $\Delta s/r = \theta$, so that

$$net\ W = (net\ \tau)\theta. \tag{9.15.3}$$

This equation is the expression for rotational work. It is very similar to the familiar definition of translational work as force multiplied by distance. Here, torque is analogous to force, and angle is analogous to distance. Equation $9.15.3$ is valid in general, even though it was derived for a special case. To get an expression for rotational kinetic energy, we must again perform some algebraic manipulations. The first step is to note that

$$net\ W = I\alpha\theta \tag{9.15.4}$$

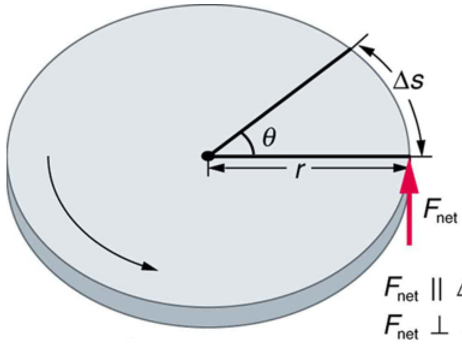

Figure 9.15.2: The net force on this disk is kept perpendicular to its radius as the force causes the disk to rotate. The net work done is thus $(net\,F)\Delta s$. The net work goes into rotational kinetic energy.

> Work and energy in rotational motion are completely analogous to work and energy in translational motion, first presented in Uniform Circular Motion and Gravitation.

Now, we solve one of the rotational kinematics equations for $\alpha\theta$. We start with the equation

$$\omega^2 = \omega_0^2 + 2\alpha\theta. \tag{9.15.5}$$

Next, we solve for $\alpha\theta$:

$$\alpha\theta = \frac{\omega^2 - \omega_0^2}{2}. \tag{9.15.6}$$

Substituting this into the equation for net W and gathering terms yields

$$net\,W = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2. \tag{9.15.7}$$

This equation is the work-energy theorem for rotational motion only. As you may recall, net work changes the kinetic energy of a system. Through an analogy with translational motion, we define the term $\left(\frac{1}{2}\right)\omega^2$ to be rotational kinetic energy KE_{rot} for an object with a moment of inertia I and an angular velocity ω:

$$KE_{rot} = \frac{1}{2}I\omega^2. \tag{9.15.8}$$

The expression for rotational kinetic energy is exactly analogous to translational kinetic energy, with I being analogous to m and ω to v. Rotational kinetic energy has important effects. Flywheels, for example, can be used to store large amounts of rotational kinetic energy in a vehicle, as seen in Figure $9.15.3$.

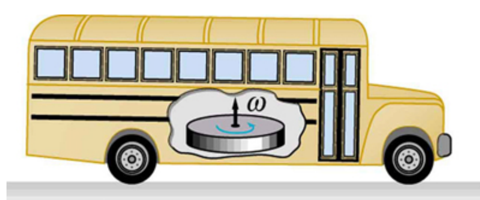

Figure 9.15.3: Experimental vehicles, such as this bus, have been constructed in which rotational kinetic energy is stored in a large flywheel. When the bus goes down a hill, its transmission converts its gravitational potential energy into KE_{rot}. It can also convert translational kinetic energy, when the bus stops, into KE_{rot}. The flywheel's energy can then be used to accelerate, to go up another hill, or to keep the bus from slowing down due to friction.

✔ Example $9.15.1$: Calculating the Work and Energy for Spinning a Grindstone

Consider a person who spins a large grindstone by placing her hand on its edge and exerting a force through part of a revolution as shown in Figure. In this example, we verify that the work done by the torque she exerts equals the change in rotational energy.

a. How much work is done if she exerts a force of 200 N through a rotation of $1.00\,rad (57.3^o)$? The force is kept perpendicular to the grindstone's 0.320-m radius at the point of application, and the effects of friction are negligible.

b. What is the final angular velocity if the grindstone has a mass of 85.0 kg?
c. What is the final rotational kinetic energy? (It should equal the work.)

Strategy

To find the work, we can use Equation 9.15.3. We have enough information to calculate the torque and are given the rotation angle. In the second part, we can find the final angular velocity using one of the kinematic relationships. In the last part, we can calculate the rotational kinetic energy from its expression in $KE_{rot} = \frac{1}{2}I\omega^2$.

Solution for (a)

The net work is expressed in the equation

$$net\,W = (net\,\tau)\theta, \tag{9.15.9}$$

where net τ is the applied force multiplied by the radius (rF) because there is no retarding friction, and the force is perpendicular to r. The angle θ is given. Substituting the given values in the equation above yields

$$\begin{aligned} net\,W &= rF\theta \\ &= (0.320\,m)(200\,N)(1.00\,rad) \\ &= 64.0\,N\cdot m. \end{aligned}$$

Noting that $1\,N\cdot m = 1\,J.$,

$$net\,W = 64.0\,J. \tag{9.15.10}$$

Figure 9.15.4 *A large grindstone is given a spin by a person grasping its outer edge.*

Solution for (b)

To find ω from the given information requires more than one step. We start with the kinematic relationship in the equation

$$\omega^2 = \omega_0^2 + 2\alpha\theta. \tag{9.15.11}$$

Note that $\omega_0 = 0$ because we start from rest. Taking the square root of the resulting equation gives

$$\omega = (2\alpha\theta)^{1/2}. \tag{9.15.12}$$

Now we need to find α. One possibility is

$$\alpha = \frac{net\,\tau}{I}, \tag{9.15.13}$$

where the torque is

$$net\,\tau = rF = (0.320\,m)(200\,N) = 64.0\,N\cdot m. \tag{9.15.14}$$

The formula for the moment of inertia for a disk is found in [link]:

$$I = \frac{1}{2}MR^2 = 0.5(85.0\,kg)(0.320\,m)^2 = 4.352\,kg\cdot m^2. \tag{9.15.15}$$

Substituting the values of torque and moment of inertia into the expression for α, we obtain

$$\alpha = \frac{64.0\,N\cdot m}{4.352\,kg\cdot m^2} = 14.7\frac{rad}{s^2}. \tag{9.15.16}$$

Now, substitute this value and the given value for θ into the above expression for ω:

$$\omega = (2\alpha\theta)^{1/2} = [2(14.7\frac{rad}{s^2})(1.00\,rad)]^{1/2} = 5.42\frac{rad}{s}. \tag{9.15.17}$$

Solution for (c)

The final rotational kinetic energy is

$$KE_{rot} = \frac{1}{2}I\omega^2. \tag{9.15.18}$$

Both I and ω were found above. Thus,

\[KE_{rot} = (0.5)(4.352 \, kg \cdot m^2)(5.42 \, rad/s)^2 = 64.0 \, J.

Discussion

The final rotational kinetic energy equals the work done by the torque, which confirms that the work done went into rotational kinetic energy. We could, in fact, have used an expression for energy instead of a kinematic relation to solve part (b). We will do this in later examples.

Helicopter pilots are quite familiar with rotational kinetic energy. They know, for example, that a point of no return will be reached if they allow their blades to slow below a critical angular velocity during flight. The blades lose lift, and it is impossible to immediately get the blades spinning fast enough to regain it. Rotational kinetic energy must be supplied to the blades to get them to rotate faster, and enough energy cannot be supplied in time to avoid a crash. Because of weight limitations, helicopter engines are too small to supply both the energy needed for lift and to replenish the rotational kinetic energy of the blades once they have slowed down. The rotational kinetic energy is put into them before takeoff and must not be allowed to drop below this crucial level. One possible way to avoid a crash is to use the gravitational potential energy of the helicopter to replenish the rotational kinetic energy of the blades by losing altitude and aligning the blades so that the helicopter is spun up in the descent. Of course, if the helicopter's altitude is too low, then there is insufficient time for the blade to regain lift before reaching the ground.

Problem-Solving Strategy for Rotational Energy

1. Determine that energy or work is involved in the rotation.
2. Determine the system of interest. A sketch usually helps.
3. Analyze the situation to determine the types of work and energy involved.
4. For closed systems, mechanical energy is conserved. That is, $KE_i + PE_f = KE_f + PE_f$. Note that KE_i and KE_f may each include translational and rotational contributions.For open systems, mechanical energy may not be conserved, and other forms of energy (referred to previously as OE),
5. such as heat transfer, may enter or leave the system. Determine what they are, and calculate them as necessary.
6. Eliminate terms wherever possible to simplify the algebra.
7. Check the answer to see if it is reasonable.

Example $9.15.2$: Calculating Helicopter Energies

A typical small rescue helicopter, similar to the one in Figure $9.15.5$, has four blades, each is 4.00 m long and has a mass of 50.0 kg. The blades can be approximated as thin rods that rotate about one end of an axis perpendicular to their length. The helicopter has a total loaded mass of 1000 kg.

a. Calculate the rotational kinetic energy in the blades when they rotate at 300 rpm.
b. Calculate the translational kinetic energy of the helicopter when it flies at 20.0 m/s, and compare it with the rotational energy in the blades
c. To what height could the helicopter be raised if all of the rotational kinetic energy could be used to lift it?

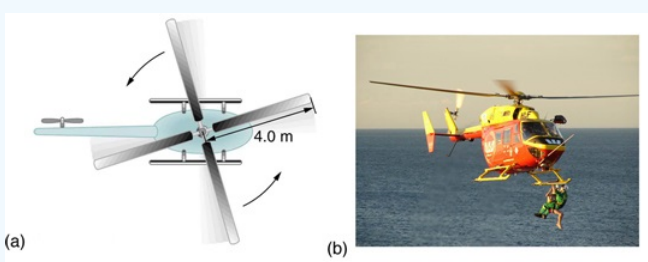

Figure $9.15.5$: The first image shows how helicopters store large amounts of rotational kinetic energy in their blades. This energy must be put into the blades before takeoff and maintained until the end of the flight. The engines do not have enough power to simultaneously provide lift and put significant rotational energy into the blades. The second image shows a helicopter from the Auckland Westpac Rescue Helicopter Service. Over 50,000 lives have been saved since its operations beginning in 1973. Here, a water rescue operation is shown. (credit: 111 Emergency, Flickr)

Strategy

https://phys.libretexts.org/@go/page/81621

Rotational and translational kinetic energies can be calculated from their definitions. The last part of the problem relates to the idea that energy can change form, in this case from rotational kinetic energy to gravitational potential energy.

Solution for (a)

The rotational kinetic energy is

$$KE_{rot} = \frac{1}{2} I\omega^2$$

We must convert the angular velocity to radians per second and calculate the moment of inertia before we can find KE_{rot}, the angular velocity ω is

$$\omega = \frac{300\,rev}{1.00\,min} \cdot \frac{2\pi\,rad}{1\,rev} \cdot \frac{1.00\,min}{60.0\,s} = 31.4\frac{rad}{s}. \tag{9.15.19}$$

The moment of inertia of one blade will be that of a thin rod rotated about its end, found in [link]. The total I is four times this moment of inertia, because there are four blades. Thus,

$$I = 4\frac{Ml^2}{3} = 4 \times \frac{(50.0\,kg)(4.00\,kg)^2}{3} = 1067\,kg \cdot m^2. \tag{9.15.20}$$

Entering ω and I into the expression for rotational kinetic energy gives

$$KE_{rot} = 0.5(1067\,kg \cdot m^2)(31.4\,rad/s)^2 \tag{9.15.21}$$

$$= 5.26 \times 10^5\ J \tag{9.15.22}$$

Solution for (b)

Translational kinetic energy was defined in Uniform Circular Motion and Gravitation. Entering the given values of mass and velocity, we obtain

$$KE_{trans} = \frac{1}{2}mv^2 = (0.5)(1000\,kg)(20.0\,m/s)^2 = 2.00 \times 10^5\ J. \tag{9.15.23}$$

To compare kinetic energies, we take the ratio of translational kinetic energy to rotational kinetic energy. This ratio is

$$\frac{2.00 \times 10^5\ J}{5.26 \times 10^5\ J} = 0.380. \tag{9.15.24}$$

Solution for (c)

At the maximum height, all rotational kinetic energy will have been converted to gravitational energy. To find this height, we equate those two energies:

$$KE_{rot} = PE_{grav} \tag{9.15.25}$$

or

$$\frac{1}{2}I\omega^2 = mgh. \tag{9.15.26}$$

We now solve for h and substitute known values into the resulting equation

$$h = \frac{\frac{1}{2}I\omega^2}{mg} = \frac{5.26 \times 10^5\ J}{(1000\,kg)(9.80\,m/s^2)} = 53.7\,m \tag{9.15.27}$$

Discussion

The ratio of translational energy to rotational kinetic energy is only 0.380. This ratio tells us that most of the kinetic energy of the helicopter is in its spinning blades—something you probably would not suspect. The 53.7 m height to which the helicopter could be raised with the rotational kinetic energy is also impressive, again emphasizing the amount of rotational kinetic energy in the blades.

Conservation of energy includes rotational motion, because rotational kinetic energy is another form of KE. Uniform Circular Motion and Gravitation has a detailed treatment of conservation of energy.

How Thick Is the Soup? Or Why Don't All Objects Roll Downhill at the Same Rate?

One of the quality controls in a tomato soup factory consists of rolling filled cans down a ramp. If they roll too fast, the soup is too thin. Why should cans of identical size and mass roll down an incline at different rates? And why should the thickest soup roll the slowest?

The easiest way to answer these questions is to consider energy. Suppose each can starts down the ramp from rest. Each can starting from rest means each starts with the same gravitational potential energy PE_{grav}, which is converted entirely to KE, provided each rolls without slipping. KE however, can take the form of KE_{trans} or KE_{rot}, and total KE is the sum of the two. If a can rolls down a ramp, it puts part of its energy into rotation, leaving less for translation. Thus, the can goes slower than it would if it slid down. Furthermore, the thin soup does not rotate, whereas the thick soup does, because it sticks to the can. The thick soup thus puts more of the can's original gravitational potential energy into rotation than the thin soup, and the can rolls more slowly, as seen in Figure $9.15.6$.

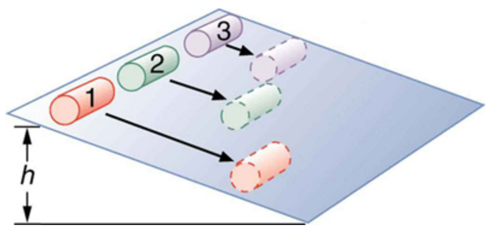

Figure $9.15.6$: Three cans of soup with identical masses race down an incline. The first can has a low friction coating and does not roll but just slides down the incline. It wins because it converts its entire PE into translational KE. The second and third cans both roll down the incline without slipping. The second can contains thin soup and comes in second because part of its initial PE goes into rotating the can (but not the thin soup). The third can contains thick soup. It comes in third because the soup rotates along with the can, taking even more of the initial PE for rotational KE, leaving less for translational KE.

Assuming no losses due to friction, there is only one force doing work—gravity. Therefore the total work done is the change in kinetic energy. As the cans start moving, the potential energy is changing into kinetic energy. Conservation of energy gives

$$PE_i = KE_f. \tag{9.15.28}$$

More specifically,

$$PE_{grav} = KE_{trans} + KE_{rot} \tag{9.15.29}$$

or

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2. \tag{9.15.30}$$

So, the initial mgh is divided between translational kinetic energy and rotational kinetic energy; and the greater I is, the less energy goes into translation. If the can slides down without friction, then $\omega = 0$ and all the energy goes into translation; thus, the can goes faster.

TAKE-HOME EXPERIMENT

Locate several cans each containing different types of food. First, predict which can will win the race down an inclined plane and explain why. See if your prediction is correct. You could also do this experiment by collecting several empty cylindrical containers of the same size and filling them with different materials such as wet or dry sand.

Example $9.15.3$: Calculating the Speed of a Cylinder Rolling Down an Incline

Calculate the final speed of a solid cylinder that rolls down a 2.00-m-high incline. The cylinder starts from rest, has a mass of 0.750 kg, and has a radius of 4.00 cm.

Strategy

We can solve for the final velocity using conservation of energy, but we must first express rotational quantities in terms of translational quantities to end up with as the only unknown.

Solution

Conservation of energy for this situation is written as described above:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2. \tag{9.15.31}$$

Before we can solve for v, we must get an expression for I from [link]. Because v and $\omega$are related (note here that the cylinder is rolling without slipping), we must also substitute the relationship $\omega = v/R$ into the expression. These substitutions yield

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right). \tag{9.15.32}$$

Interestingly, the cylinder's radius R and mass m cancel, yielding

$$gh = \frac{1}{2}v^2 + \frac{1}{4}v^2 = \frac{3}{4}v^2. \tag{9.15.33}$$

Solving algebraically, the equation for the final velocity v gives

$$v = \left(\frac{4gh}{3}\right)^{1/2}. \tag{9.15.34}$$

Substituting known values into the resulting expression yields

$$v = \left[\frac{4(9.80\,m/s^2)(2.00\,m)}{3}\right]^{1/2} = 5.11\,m/s. \tag{9.15.35}$$

Discussion

Because m and R cancel, the result $v = (\frac{4}{3}gh)^{1/2}$ is valid for any solid cylinder, implying that all solid cylinders will roll down an incline at the same rate independent of their masses and sizes. (Rolling cylinders down inclines is what Galileo actually did to show that objects fall at the same rate independent of mass.) Note that if the cylinder slid without friction down the incline without rolling, then the entire gravitational potential energy would go into translational kinetic energy. Thus, $\frac{1}{2}mv^2 = mgh$ and $v = (2gh)^{1/2}, which is 22$. That is, the cylinder would go faster at the bottom.

? Exercise 9.15.1: Analogy of Rotational and Translational Kinetic Energy

Is rotational kinetic energy completely analogous to translational kinetic energy? What, if any, are their differences? Give an example of each type of kinetic energy.

Solution

Yes, rotational and translational kinetic energy are exact analogs. They both are the energy of motion involved with the coordinated (non-random) movement of mass relative to some reference frame. The only difference between rotational and translational kinetic energy is that translational is straight line motion while rotational is not. An example of both kinetic and translational kinetic energy is found in a bike tire while being ridden down a bike path. The rotational motion of the tire means it has rotational kinetic energy while the movement of the bike along the path means the tire also has translational kinetic energy. If you were to lift the front wheel of the bike and spin it while the bike is stationary, then the wheel would have only rotational kinetic energy relative to the Earth.

Moment of Inertia and Rotational Kinetic Energy

the kinetic energy due to the rotation of an object. This is part of its total kinetic energy

Any moving object has kinetic energy. We know how to calculate this for a body undergoing translational motion, but how about for a rigid body undergoing rotation? This might seem complicated because each point on the rigid body has a different velocity. However, we can make use of angular velocity—which is the same for the entire rigid body—to express the kinetic energy for a rotating object. Figure $9.15.1$ shows an example of a very energetic rotating body: an electric grindstone propelled by a motor. Sparks are flying, and noise and vibration are generated as the grindstone does its work. This system has considerable energy, some of it in the form of heat, light, sound, and vibration. However, most of this energy is in the form of **rotational kinetic energy**.

Figure $9.15.1$: The rotational kinetic energy of the grindstone is converted to heat, light, sound, and vibration. (credit: Zachary David Bell, US Navy)

Energy in rotational motion is not a new form of energy; rather, it is the energy associated with rotational motion, the same as kinetic energy in translational motion. However, because kinetic energy is given by $K=\frac{1}{2}mv^2$, and velocity is a quantity that is different for every point on a rotating body about an axis, it makes sense to find a way to write kinetic energy in terms of the variable ω, which is the same for all points on a rigid rotating body. For a single particle rotating around a fixed axis, this is straightforward to calculate. We can relate the angular velocity to the magnitude of the translational velocity using the relation $v_t=\omega r$, where r is the distance of the particle from the axis of rotation and v_t is its tangential speed. Substituting into the equation for kinetic energy, we find

$$K=\frac{1}{2}mv_t^2=\frac{1}{2}m(\omega r)^2=\frac{1}{2}\left(mr^2\right)\omega^2.$$

In the case of a rigid rotating body, we can divide up any body into a large number of smaller masses, each with a mass m_j and distance to the axis of rotation r_j, such that the total mass of the body is equal to the sum of the individual masses: $M=\sum_j m_j$. Each smaller mass has tangential speed v_j, where we have dropped the subscript t for the moment. The total kinetic energy of the rigid rotating body is

$$K=\sum_j \frac{1}{2}m_jv_j^2=\sum_j \frac{1}{2}m_j(r_j\omega_j)^2$$

and since $\omega_j=\omega$ for all masses,

$$K=\frac{1}{2}\left(\sum_j m_jr_j^2\right)\omega^2. \tag{9.15.36}$$

The units of Equation $9.15.36$ are joules (J). The equation in this form is complete, but awkward; we need to find a way to generalize it.

Moment of Inertia

If we compare Equation $9.15.36$ to the way we wrote kinetic energy in Work and Kinetic Energy, ($\frac{1}{2}mv^2$), this suggests we have a new rotational variable to add to our list of our relations between rotational and translational variables. The quantity $\sum_j m_jr_j^2$ is the counterpart for mass in the equation for rotational kinetic energy. This is an important new term for rotational motion. This quantity is called the **moment of inertia** I, with units of kg·m^2:

$$I=\sum_j m_jr_j^2. \tag{9.15.37}$$

For now, we leave the expression in summation form, representing the moment of inertia of a system of point particles rotating about a fixed axis. We note that the moment of inertia of a single point particle about a fixed axis is simply mr^2, with r being the distance from the point particle to the axis of rotation. In the next section, we explore the integral form of this equation, which can be used to calculate the moment of inertia of some regular-shaped rigid bodies.

The moment of inertia is the quantitative measure of rotational inertia, just as in translational motion, and mass is the quantitative measure of linear inertia—that is, the more massive an object is, the more inertia it has, and the greater is its resistance to change in linear velocity. Similarly, the greater the moment of inertia of a rigid body or system of particles, the greater is its resistance to change in angular velocity about a fixed axis of rotation. It is interesting to see how the moment of inertia varies with *r*, the distance to the axis of rotation of the mass particles in Equation 9.15.37. Rigid bodies and systems of particles with more mass concentrated at a greater distance from the axis of rotation have greater moments of inertia than bodies and systems of the same mass, but concentrated near the axis of rotation. In this way, we can see that a hollow cylinder has more rotational inertia than a solid cylinder of the same mass when rotating about an axis through the center. Substituting Equation 9.15.37 into Equation 9.15.36, the expression for the kinetic energy of a rotating rigid body becomes

$$K = \frac{1}{2} I \omega^2. \tag{9.15.38}$$

We see from this equation that the kinetic energy of a rotating rigid body is directly proportional to the moment of inertia and the square of the angular velocity. This is exploited in flywheel energy-storage devices, which are designed to store large amounts of rotational kinetic energy. Many carmakers are now testing flywheel energy storage devices in their automobiles, such as the flywheel, or kinetic energy recovery system, shown in Figure 9.15.2.

Figure 9.15.2: A KERS (kinetic energy recovery system) flywheel used in cars. (credit: “cmonville”/Flickr)

The rotational and translational quantities for kinetic energy and inertia are summarized in Table 10.4. The relationship column is not included because a constant doesn’t exist by which we could multiply the rotational quantity to get the translational quantity, as can be done for the variables in Table 10.3.

Table 10.4: Rotational and Translational Kinetic Energies and Inertia

Rotational	Translational
$I = \sum_j m_j r_j^2$	m
$K = \frac{1}{2} I \omega^2$	$K = \frac{1}{2} m v^2$

Example 9.15.1: Moment of Inertia of a system of particles

Six small washers are spaced 10 cm apart on a rod of negligible mass and 0.5 m in length. The mass of each washer is 20 g. The rod rotates about an axis located at 25 cm, as shown in Figure 9.15.3. (a) What is the moment of inertia of the system? (b) If the two washers closest to the axis are removed, what is the moment of inertia of the remaining four washers? (c) If the system with six washers rotates at 5 rev/s, what is its rotational kinetic energy?

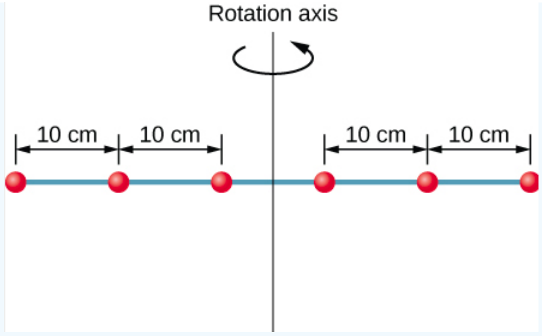

Figure $9.15.3$: Six washers are spaced 10 cm apart on a rod of negligible mass and rotating about a vertical axis.

Strategy

a. We use the definition for moment of inertia for a system of particles and perform the summation to evaluate this quantity. The masses are all the same so we can pull that quantity in front of the summation symbol.
b. We do a similar calculation.
c. We insert the result from (a) into the expression for rotational kinetic energy.

Solution

a. $I = \sum m_j r_j^2 = (0.02 \text{ kg}) \left(2 \times (0.25 \text{ m})^2 + 2 \times (0.15 \text{ m})^2 + 2 \times (0.05 \text{ m})^2\right) = 0.0035 \text{ kg} \cdot \text{m}^2$
b. $I = \sum_j m_j r_j^2 = (0.02 \text{ kg}) \left(2 \times (0.25 \text{ m})^2 + 2 \times (0.15 \text{ m})^2\right) = 0.0034 \text{ kg} \cdot \text{m}^2$
c. $K = \frac{1}{2} I \omega^2 = \frac{1}{2} \left(0.0035 \text{ kg} \cdot \text{m}^2\right) (5.0 \times 2\pi \text{ rad/s})^2 = 1.73 \text{ J}$

Significance

We can see the individual contributions to the moment of inertia. The masses close to the axis of rotation have a very small contribution. When we removed them, it had a very small effect on the moment of inertia.

In the next section, we generalize the summation equation for point particles and develop a method to calculate moments of inertia for rigid bodies. For now, though, Figure $9.15.4$ gives values of rotational inertia for common object shapes around specified axes.

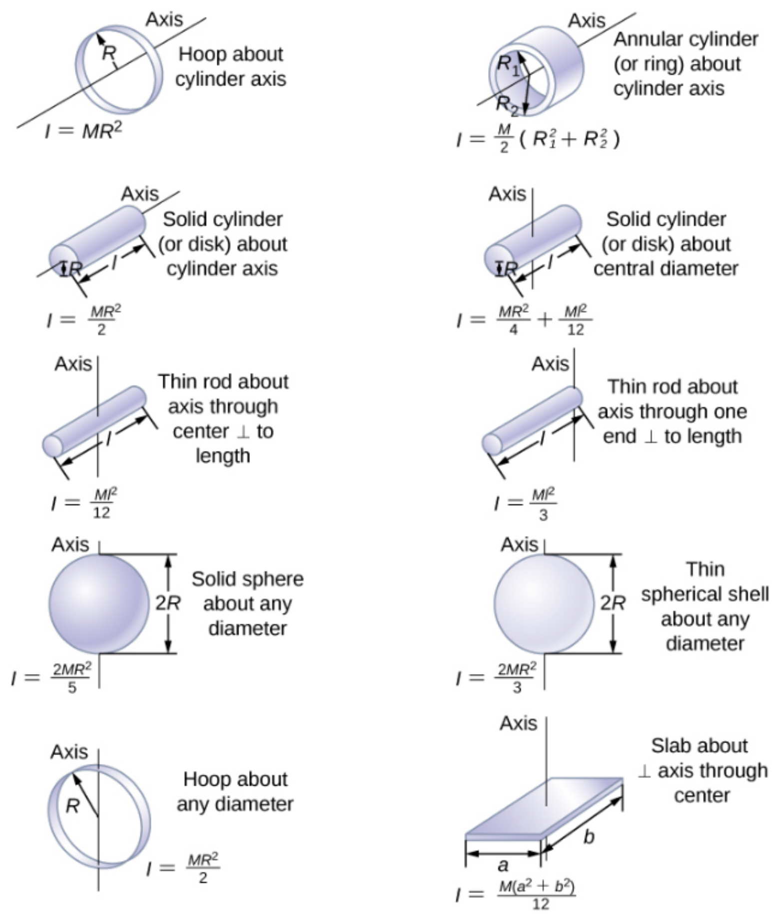

Figure 9.15.4: Values of rotational inertia for common shapes of objects.

Applying Rotational Kinetic Energy

Now let's apply the ideas of rotational kinetic energy and the moment of inertia table to get a feeling for the energy associated with a few rotating objects. The following examples will also help get you comfortable using these equations. First, let's look at a general problem-solving strategy for rotational energy.

? PROBLEM-SOLVING STRATEGY: ROTATIONAL ENERGY

1. Determine that energy or work is involved in the rotation.
2. Determine the system of interest. A sketch usually helps.
3. Analyze the situation to determine the types of work and energy involved.
4. If there are no losses of energy due to friction and other nonconservative forces, mechanical energy is conserved, that is, $K_i + U_i = K_f + U_f$.
5. If nonconservative forces are present, mechanical energy is not conserved, and other forms of energy, such as heat and light, may enter or leave the system. Determine what they are and calculate them as necessary.
6. Eliminate terms wherever possible to simplify the algebra.
7. Evaluate the numerical solution to see if it makes sense in the physical situation presented in the wording of the problem.

✓ Example 9.15.2: Calculating helicopter energies

A typical small rescue helicopter has four blades: Each is 4.00 m long and has a mass of 50.0 kg (Figure 9.15.5). The blades can be approximated as thin rods that rotate about one end of an axis perpendicular to their length. The helicopter has a total loaded mass of 1000 kg. (a) Calculate the rotational kinetic energy in the blades when they rotate at 300 rpm. (b) Calculate the translational kinetic energy of the helicopter when it flies at 20.0 m/s, and compare it with the rotational energy in the blades.

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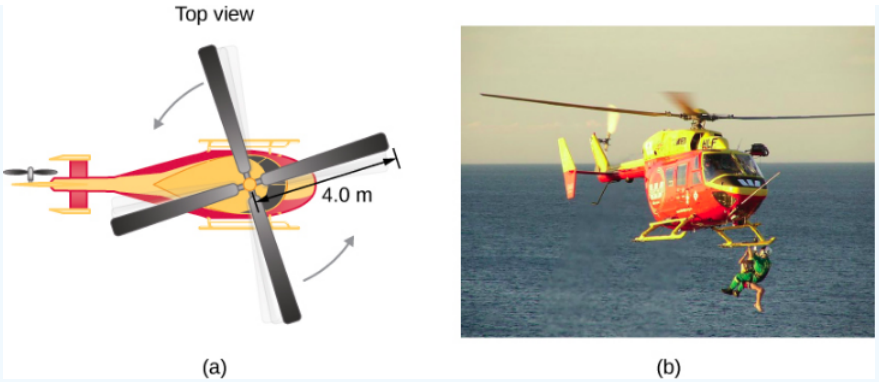

Figure 9.15.5: (a) Sketch of a four-blade helicopter. (b) A water rescue operation featuring a helicopter from the Auckland Westpac Rescue Helicopter Service. (credit b: modification of work by “111 Emergency”/Flickr)

Strategy

Rotational and translational kinetic energies can be calculated from their definitions. The wording of the problem gives all the necessary constants to evaluate the expressions for the rotational and translational kinetic energies.

Solution

a. The rotational kinetic energy is

$$K = \frac{1}{2} I \omega^2$$

We must convert the angular velocity to radians per second and calculate the moment of inertia before we can find K. The angular velocity ω is

$$\omega = \frac{300 \text{ rev}}{1.00 \text{ min}} \frac{2\pi \text{ rad}}{1 \text{ rev}} \frac{1.00 \text{ min}}{60.0 \text{ s}} = 31.4 \frac{\text{rad}}{\text{s}}.$$

The moment of inertia of one blade is that of a thin rod rotated about its end, listed in Figure $9.15.4$ The total I is four times this moment of inertia because there are four blades. Thus,

$$I = 4\frac{Ml^2}{3} = 4 \times \frac{(50.0 \text{ kg})(4.00 \text{ m})^2}{3} = 1067.0 \text{ kg} \cdot \text{m}^2.$$

Entering ω and I into the expression for rotational kinetic energy gives

$$K = 0.5 \left(1067 \text{ kg} \cdot \text{m}^2\right) (31.4 \text{ rad/s})^2 = 5.26 \times 10^5 \text{ J}.$$

b. Entering the given values into the equation for translational kinetic energy, we obtain

$$K = \frac{1}{2} m v^2 = (0.5)(1000.0 \text{ kg})(20.0 \text{ m/s})^2 = 2.00 \times 10^5 \text{ J}.$$

To compare kinetic energies, we take the ratio of translational kinetic energy to rotational kinetic energy. This ratio is

$$\frac{2.00 \times 10^5 \text{ J}}{5.26 \times 10^5 \text{ J}} = 0.380.$$

Significance

The ratio of translational energy to rotational kinetic energy is only 0.380. This ratio tells us that most of the kinetic energy of the helicopter is in its spinning blades.

Example 9.15.3: Energy in a boomerang

A person hurls a boomerang into the air with a velocity of 30.0 m/s at an angle of 40.0° with respect to the horizontal (Figure $9.15.6$). It has a mass of 1.0 kg and is rotating at 10.0 rev/s. The moment of inertia of the boomerang is given as $I = \frac{1}{12} mL^2$

where L = 0.7 m. (a) What is the total energy of the boomerang when it leaves the hand? (b) How high does the boomerang go from the elevation of the hand, neglecting air resistance?

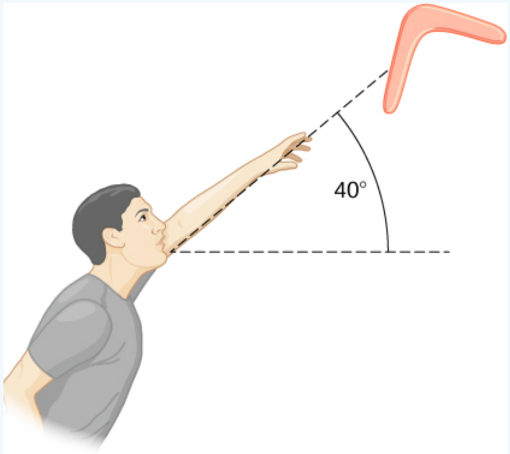

Figure 9.15.6: A boomerang is hurled into the air at an initial angle of 40°.

Strategy

We use the definitions of rotational and linear kinetic energy to find the total energy of the system. The problem states to neglect air resistance, so we don't have to worry about energy loss. In part (b), we use conservation of mechanical energy to find the maximum height of the boomerang.

Solution

a. Moment of inertia: $I = \frac{1}{12}mL^2 = \frac{1}{12}(1.0\text{ kg})(0.7\text{ m})^2 = 0.041\text{ kg}\cdot\text{m}^2$.

Angular Velocity: $\omega = (10.0\text{ rev/s})(2\pi) = 62.83\text{ rad/s}$

The rotational kinetic energy is therefore

$$K_R = \frac{1}{2}\left(0.041\text{ kg}\cdot\text{m}^2\right)(62.83\text{ rad/s})^2 = 80.93\text{ J}$$

The translational kinetic energy is

$$K_\text{T} = \frac{1}{2}mv^2 = \frac{1}{2}(1.0\text{ kg})(30.0\text{ m/s})^2 = 450.0\text{ J}$$

Thus, the total energy in the boomerang is

$$K_\text{Total} = K_R + K_T = 80.93 + 450.0 = 530.93\text{ J}.$$

b. We use conservation of mechanical energy. Since the boomerang is launched at an angle, we need to write the total energies of the system in terms of its linear kinetic energies using the velocity in the *x*- and *y*-directions. The total energy when the boomerang leaves the hand is

$$E_\text{Before} = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}I\omega^2$$

The total energy at maximum height is

$$E_\text{Final} = \frac{1}{2}mv_x^2 + \frac{1}{2}I\omega^2 + mgh$$

By conservation of mechanical energy, $E_{Before} = E_{Final}$ so we have, after canceling like terms,

$$\frac{1}{2}mv_y^2 = mgh.$$

Since v_y = 30.0 m/s $(\sin 40°)$ = 19.28 m/s, we find

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$$h=\frac{(19.28\ \mathrm{m/s})^2}{2\,(9.8\ \mathrm{m/s^2})}=18.97\ \mathrm{m}$$

Significance

In part (b), the solution demonstrates how energy conservation is an alternative method to solve a problem that normally would be solved using kinematics. In the absence of air resistance, the rotational kinetic energy was not a factor in the solution for the maximum height.

? Exercise

A nuclear submarine propeller has a moment of inertia of 800.0 kg • m^2. If the submerged propeller has a rotation rate of 4.0 rev/s when the engine is cut, what is the rotation rate of the propeller after 5.0 s when water resistance has taken 50,000 J out of the system?

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