5.3: Common Forces - Normal (or Perpendicular) Force

Learning Objectives

- Define normal forces

Weight (also called the force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in Figure $5.3.1$(a). But how do inanimate objects like a table support the weight of a mass placed on them, such as shown in Figure $5.3.1$(b)? When the bag of dog food is placed on the table, the table sags slightly under the load. This would be noticeable if the load were placed on a card table, but even a sturdy oak table deforms when a force is applied to it. Unless an object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or a trampoline or diving board). The greater the deformation, the greater the restoring force. Thus, when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point, the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly and the sag is slight, so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.

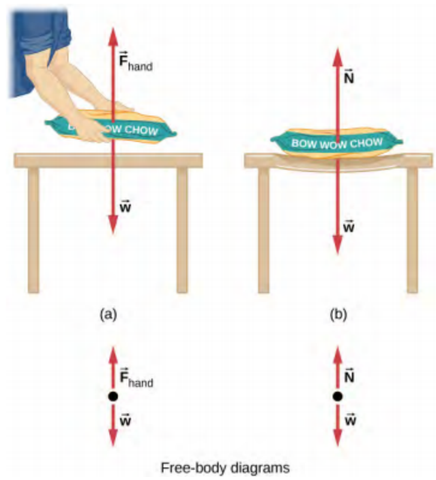

Figure $5.3.1$: (a) The person holding the bag of dog food must supply an upward force $\vec{F}$ hand equal in magnitude and opposite in direction to the weight of the food $\vec{w}$ so that it doesn't drop to the ground. (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force $\vec{N}$ equal in magnitude and opposite in direction to the weight of the load.

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. If the force supporting the weight of an object, or a load, is perpendicular to the surface of contact between the load and its support, this force is defined as a **normal force** and here is given by the symbol $\vec{N}$. (This is not the newton unit for force, or N.) The word **normal** means perpendicular to a surface. This means that the normal force experienced by an object resting on a horizontal surface can be expressed in vector form as follows:

$$\vec{F}_N = -m\vec{g}.$$

In scalar form, this becomes

$$F_N = mg.$$

The normal force can be less than the object's weight if the object is on an incline.

Example 5.12: Weight on an Incline

Consider the skier on the slope in Figure $5.3.2$. Her mass including equipment is 60.0 kg. (a) What is her acceleration if friction is negligible? (b) What is her acceleration if friction is 45.0 N?

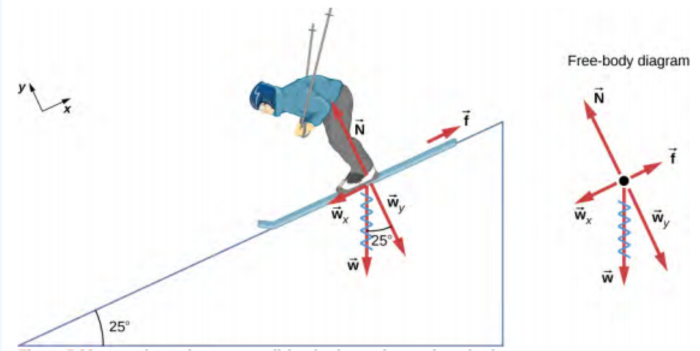

Figure 5.3.2: Since the acceleration is parallel to the slope and acting down the slope, it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular to it (axes shown to the left of the skier). $\vec{F}_N$ is perpendicular to the slope and $\vec{F}_f$ is parallel to the slope, but $\vec{F}_g$ has components along both axes, namely, w_y and w_x. Here, $\vec{F}_g$ has a squiggly line to show that it has been replaced by these components. The force $\vec{F}_N$ is equal in magnitude to w_y, so there is no acceleration perpendicular to the slope, but f is less than w_x, so there is a downslope acceleration (along the axis parallel to the slope).

Strategy

This is a two-dimensional problem, since not all forces on the skier (the system of interest) are parallel. The approach we have used in two-dimensional kinematics also works well here. Choose a convenient coordinate system and project the vectors onto its axes, creating two one-dimensional problems to solve. The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. (Motions along mutually perpendicular axes are independent.) We use x and y for the parallel and perpendicular directions, respectively. This choice of axes simplifies this type of problem, because there is no motion perpendicular to the slope and the acceleration is downslope. Regarding the forces, friction is drawn in opposition to motion (friction always opposes forward motion) and is always parallel to the slope, w_x is drawn parallel to the slope and downslope (it causes the motion of the skier down the slope), and w_y is drawn as the component of weight perpendicular to the slope. Then, we can consider the separate problems of forces parallel to the slope and forces perpendicular to the slope.

Solution

The magnitude of the component of weight parallel to the slope is

$$w_x = w\sin 25^o = mg\sin 25^o,$$

and the magnitude of the component of the weight perpendicular to the slope is

$$w_y = w\cos 25^o = mg\cos 25^o.$$

a. Neglect friction. Since the acceleration is parallel to the slope, we need only consider forces parallel to the slope. (Forces perpendicular to the slope add to zero, since there is no acceleration in that direction.) The forces parallel to the slope are the component of the skier's weight parallel to slope w_x and friction f. Using Newton's second law, with subscripts to denote quantities parallel to the slope, $a_x = \frac{F_{net\ x}}{m}$ where $F_{net\ x} = w_x$ - mg sin 25°, assuming no friction for this part. Therefore, $a_x = \frac{F_{net\ x}}{m} = \frac{mg\sin 25^o}{m} = g\sin 25^o \quad (9.80\ m/s^2)(0.4226) = 4.14\ m/s^2$ is the acceleration.
b. Include friction. We have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is $F_{net\ x} = w_x - f$. Substituting this into Newton's second law, $a_x = \frac{F_{net\ x}}{m}$, gives $a_x = \frac{F_{net\ x}}{m} = \frac{w_x - f}{m} = \frac{mg\sin 25^o - f}{m}$. We substitute known values to obtain $a_x = \frac{(60.0\ kg)(9.80\ m/s^2)(0.4226) - 45.0\ N}{60.0\ kg}$. This give us $a_x = 3.39\ m/s^2$, which is the acceleration parallel to the incline when there is 45.0 N of opposing friction.

Significance

Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is none. It is a general result that if friction on an incline is negligible, then the acceleration down the incline is a =

g sin θ, regardless of mass. As discussed previously, all objects fall with the same acceleration in the absence of air resistance. Similarly, all objects, regardless of mass, slide down a frictionless incline with the same acceleration (if the angle is the same).

When an object rests on an incline that makes an angle θ with the horizontal, the force of gravity acting on the object is divided into two components: a force acting perpendicular to the plane, wy , and a force acting parallel to the plane, wx (Figure $5.3.3$). The normal force $\vec{N}$ is typically equal in magnitude and opposite in direction to the perpendicular component of the weight w_y. The force acting parallel to the plane, w_x, causes the object to accelerate down the incline.

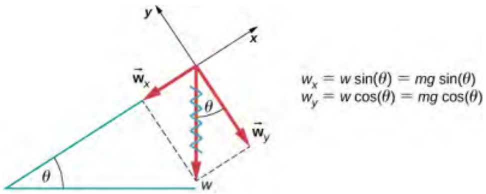

Figure $5.3.3$: An object rests on an incline that makes an angle θ with the horizontal.

Be careful when resolving the weight of the object into components. If the incline is at an angle θ to the horizontal, then the magnitudes of the weight components are

$$w_x = w\sin\theta = mg\sin\theta$$

and

$$w_y = w\cos\theta = mg\cos\theta$$

We use the second equation to write the normal force experienced by an object resting on an inclined plane:

$$N = mg\cos\theta.$$

Instead of memorizing these equations, it is helpful to be able to determine them from reason. To do this, we draw the right angle formed by the three weight vectors. The angle θ of the incline is the same as the angle formed between w and w_y. Knowing this property, we can use trigonometry to determine the magnitude of the weight components:

$$\cos\theta = \frac{w_y}{w}, \quad w_y = w\cos\theta = mg\cos\theta$$
$$\sin\theta = \frac{w_x}{w}, \quad w_x = w\sin\theta = mg\sin\theta$$