7.6: Moment of Inertia/Rotational Inertia

Learning Objectives

- Define the physical concept of moment of inertia in terms of the mass distribution from the rotational axis

Figure 7.6.1: This photo shows Yuko Kawaguti, in the 2010 Cup of Russia, free skating competition, pulling in her arms, to reduce her moment of inertia, so that she can rotate faster. (image in the public domain: https://w.wiki/7rrn)

Have you wandered why would a skater pull in her arms to spin faster? When objects rotate, it is not only mass that can affect motion, both mass and how mass is distributed about the axis of rotation matter. This suggests that there is a need for a new rotational variable to add to our list of our relations between rotational and translational variables. The quantity that takes into account both mass and the way mass is distributed about the axis of rotation is called the **moment of inertia** I, with units of kg·m^2. This is an important new term for rotational motion.

The moment of inertia is the quantitative measure of rotational inertia, just as in translational motion, and mass is the quantitative measure of linear inertia—that is, the more massive an object is, the more inertia it has, and the greater is its resistance to change in linear velocity. The moment of inertia of a single point particle of mass m about a fixed axis a distance r from the point particle is simply:

$$I = mr^2. \tag{7.6.1}$$

For a system of point particles rotating about a fixed axis the **moment of inertia** is given by:

$$I = \sum_j m_j r_j^2. \tag{7.6.2}$$

The greater the moment of inertia of a rigid body or system of particles, the greater is its resistance to change in angular velocity about a fixed axis of rotation. It is interesting to see how the moment of inertia varies with r, the distance to the axis of rotation of the mass particles in Equation 7.6.2. Rigid bodies and systems of particles with more mass concentrated at a greater distance from the axis of rotation have greater moments of inertia than bodies and systems of the same mass, but concentrated near the axis of rotation. In this way, we can see that a hollow cylinder has more rotational inertia than a solid cylinder of the same mass when rotating about an axis through the center.

In the next section, we explore the integral form of this equation, which can be used to calculate the moment of inertia of some regular-shaped rigid bodies.

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