7.3: Internal and External Forces

Learning Objectives

- Demonstrate why only external forces affect the motion of a system of objects

We have been avoiding an important issue up to now: When we say that an object moves (more correctly, accelerates) in a way that obeys Newton's second law, we have been ignoring the fact that all objects are actually made of many constituent particles. A car has an engine, steering wheel, seats, passengers; a football is leather and rubber surrounding air; a brick is made of atoms. There are many different types of particles, and they are generally not distributed uniformly in the object. How do we include these facts into our calculations?

Then too, an extended object might change shape as it moves, such as a water balloon or a cat falling (Figure $7.3.1$). This implies that the constituent particles are applying internal forces on each other, in addition to the external force that is acting on the object as a whole. We want to be able to handle this, as well.

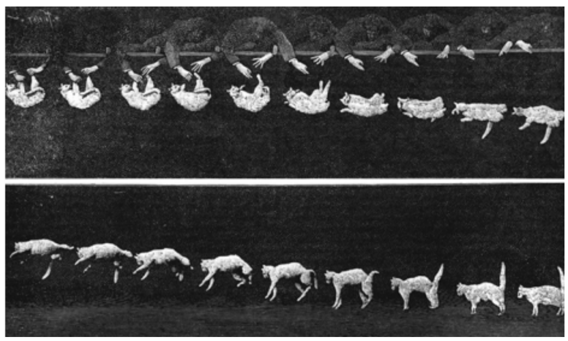

Figure $7.3.1$: As the cat falls, its body performs complicated motions so it can land on its feet, but one point in the system moves with the simple uniform acceleration of gravity.

The problem before us, then, is to determine what part of an extended object is obeying Newton's second law when an external force is applied and to determine how the motion of the object as a whole is affected by both the internal and external forces.

Internal and External Forces

Suppose we have an extended object of mass M, made of N interacting particles. Let's label their masses as m_j, where j = 1, 2, 3, …, N. Note that

$$M=\sum_{j=1}^{N} m_j. \tag{7.3.1}$$

If we apply some net **external force** $\vec{F}_{ext}$ on the object, every particle experiences some "share" or some fraction of that external force. Let:

$\vec{f}_j^{ext}$ = the fraction of the external force that the jth particle experiences

Notice that these fractions of the total force are not necessarily equal; indeed, they virtually never are. (They **can** be, but they usually aren't.) In general, therefore,

$$\vec{f}_1^{ext} \neq \vec{f}_2^{ext} \neq \cdots \neq \vec{f}_N^{ext}. \tag{7.3.2}$$

Next, we assume that each of the particles making up our object can interact (apply forces on) every other particle of the object. We won't try to guess what kind of forces they are; but since these forces are the result of particles of the object acting on other particles of the same object, we refer to them as **internal forces** $\vec{f}_j^{int}$; thus:

$\vec{f}_j^{int}$ = the net internal force that the jth particle experiences from all the other particles that make up the object.

Now, the **net** force, internal plus external, on the jth particle is the vector sum of these:

$$\vec{f}_j = \vec{f}_j^{int} + \vec{f}_j^{ext}. \tag{7.3.3}$$

where again, this is for all N particles; j = 1, 2, 3, … , N. As a result of this fractional force, each particle accelerates:

$$\begin{aligned} \vec{f}_j &= m_j \vec{a}_j \\ \vec{f}_j^{int} + \vec{f}_j^{ext} &= m_j \vec{a}_j. \end{aligned}$$

The net force $\vec{F}$ on the object is the vector sum of these forces:

$$\begin{aligned} \vec{F}_{net} &= \sum_{j=1}^{N} (\vec{f}_j^{int} + \vec{f}_j^{ext}) \\ &= \sum_{j=1}^{N} \vec{f}_j^{int} + \sum_{j=1}^{N} \vec{f}_j^{ext}. \end{aligned}$$

This net force accelerates the object as a whole, and must be the vector sum of all the individual net forces of all of the particles:

$$\vec{F}_{net} = \sum_{j=1}^{N} m_j \vec{a}_j. \tag{7.3.4}$$

Combining these equations gives

$$\sum_{j=1}^{N} \vec{f}_j^{int} + \sum_{j=1}^{N} \vec{f}_j^{ext} = \sum_{j=1}^{N} m_j \vec{a}_j. \tag{7.3.5}$$

Let's now think about these summations. First consider the internal forces term; remember that each $\vec{f}_j^{int}$ is the force on the jth particle from the other particles in the object. But by Newton's third law, for every one of these forces, there must be another force that has the same magnitude, but the opposite sign (points in the opposite direction). These forces do not cancel; however, that's not what we're doing in the summation. Rather, we're simply **mathematically adding up** all the internal force vectors. That is, in general, the internal forces for any individual part of the object won't cancel, but when all the internal forces are added up, the internal forces must cancel in pairs. It follows, therefore, that the sum of all the internal forces must be zero:

$$\sum_{j=1}^{N} \vec{f}_j^{ext} = 0. \tag{7.3.6}$$

This argument is subtle, but crucial.

For the external forces, this summation is simply the total external force that was applied to the whole object:

$$\sum_{j=1}^{N} \vec{f}_j^{ext} = \vec{F}_{ext}. \tag{7.3.7}$$

As a result,

$$\vec{F}_{ext} = \sum_{j=1}^{N} m_j \vec{a}_j. \tag{7.3.8}$$

This is an important result. Equation 7.3.8 tells us that the acceleration of the entire object (all N particles) is due only to the external forces; the internal forces do not change the acceleration of the object as a whole. This is why you can't lift yourself in the air by standing in a basket and pulling up on the handles: For the system of you + basket, your upward pulling force is an internal force. This is why it is not the engine that propels a car forward, it is the static friction with the road.