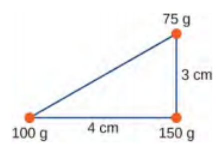
7.9: Practice

7.9.1 Conceptual Questions

1. Suppose a fireworks shell explodes, breaking into three large pieces for which air resistance is negligible. How does the explosion affect the motion of the center of mass? How would it be affected if the pieces experienced significantly more air resistance than the intact shell?
2. What if another planet the same size as Earth were put into orbit around the Sun along with Earth. Would the moment of inertia of the system increase, decrease, or stay the same?
3. A discus thrower rotates with a discus in his hand before letting it go. (a) How does his moment of inertia change after releasing the discus? (b) What would be a good approximation to use in calculating the moment of inertia of the discus thrower and discus?
4. Does increasing the number of blades on a propeller increase or decrease its moment of inertia, and why?
5. The moment of inertia of a long rod spun around an axis through one end perpendicular to its length is $\frac{mL^2}{3}$. Why is this moment of inertia greater than it would be if you spun a point mass m at the location of the center of mass of the rod (at $\frac{L}{2}$) (that would be $\frac{mL^2}{4}$)
6. Why is the moment of inertia of a hoop that has a mass M and a radius R greater than the moment of inertia of a disk that has the same mass and radius?

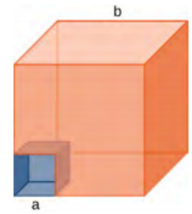
7.9.1.1 Problems

7. Three point masses are placed at the corners of a triangle as shown in the figure below. Find the center of mass of the three-mass system.

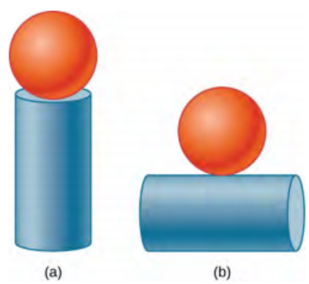


8. Two particles of masses m_1 and m_2 separated by a horizontal distance D are released from the same height h at the same time. Find the vertical position of the center of mass of these two particles at a time before the two particles strike the ground. Assume no air resistance.
9. Two particles of masses m_1 and m_2 separated by a horizontal distance D are let go from the same height h at different times. Particle 1 starts at t = 0 , and particle 2 is let go at t = T. Find the vertical position of the center of mass at a time before the first particle strikes the ground. Assume no air resistance.
10. Two particles of masses m_1 and m_2 move uniformly in different circles of radii R_1 and R_2 about origin in the x,y-plane. The x- and y-coordinates of the center of mass and that of particle 1 are given as follows (where length is in meters and t in seconds): $x_1(t) = 4\cos(2t)$, $y_1(t) = 4\sin(2t)$ and: $x_{CM}(t) = 3\cos(2t)$, $y_{CM}(t) = 3\sin(2t)$. (a) Find the radius of the circle in which particle 1 moves. (b) Find the x- and y-coordinates of particle 2 and the radius of the circle this particle moves.
11. Two particles of masses m_1 and m_2 move uniformly in different circles of radii R_1 and R_2 about the origin in the x, y-plane. The coordinates of the two particles in meters are given as follows (z = 0 for both). Here t is in seconds: $x_1(t) = 4\cos(2t)$, $y_1(t) = 4\sin(2t)$, $x_2(t) = 2\cos\left(3t - \frac{\pi}{2}\right)$, $y_2(t) = 2\sin\left(3t - \frac{\pi}{2}\right)$ (a) Find the radii of the circles of motion of both particles. (b) Find the x- and y-coordinates of the center of mass. (c) Decide if the center of mass moves in a circle by plotting its trajectory.
12. Find the center of mass of a one-meter long rod, made of 50 cm of iron (density 8 g/cm^3) and 50 cm of aluminum (density 2.7 g/cm^3).
13. Find the center of mass of a rod of length L whose mass density changes from one end to the other quadratically. That is, if the rod is laid out along the x-axis with one end at the origin and the other end at x = L, the density is given by $\rho(x) = \rho_0 + (\rho_1 - \rho_0)\left(\frac{x}{L}\right)^2$, where ρ_0 and ρ_1 are constant values.
14. Find the center of mass of a rectangular block of length a and width b that has a nonuniform density such that when the rectangle is placed in the x,y-plane with one corner at the origin and the block placed in the first quadrant with the two edges along the x- and y-axes, the density is given by $\rho(x, y) = \rho_0 x$, where ρ_0 is a constant.
15. Find the center of mass of a rectangular material of length a and width b made up of a material of nonuniform density. The density is such that when the rectangle is placed in the xy-plane, the density is given by $\rho(x, y) = \rho_0 xy$.

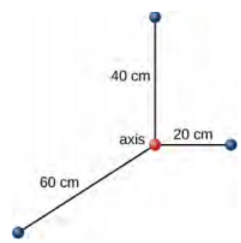
16. A cube of side a is cut out of another cube of side b as shown in the figure below. Find the location of the center of mass of the structure. (**Hint**: Think of the missing part as a negative mass overlapping a positive mass.)



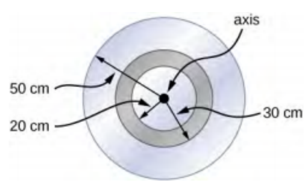
17. Find the center of mass of cone of uniform density that has a radius R at the base, height h, and mass M. Let the origin be at the center of the base of the cone and have +z going through the cone vertex.
18. Find the center of mass of a thin wire of mass m and length L bent in a semicircular shape. Let the origin be at the center of the semicircle and have the wire arc from the +x axis, cross the +y axis, and terminate at the −x axis.
19. Find the center of mass of a uniform thin semicircular plate of radius R. Let the origin be at the center of the semicircle, the plate arc from the +x axis to the −x axis, and the z axis be perpendicular to the plate.
20. Find the center of mass of a sphere of mass M and radius R and a cylinder of mass m, radius r, and height h arranged as shown below. Express your answers in a coordinate system that has the origin at the center of the cylinder.



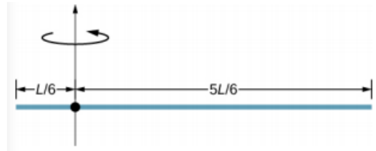
21. A system of point particles is shown in the following figure. Each particle has mass 0.3 kg and they all lie in the same plane. (a) What is the moment of inertia of the system about the given axis?



22. A system consists of a disk of mass 2.0 kg and radius 50 cm upon which is mounted an annular cylinder of mass 1.0 kg with inner radius 20 cm and outer radius 30 cm (see below). The system rotates about an axis through the center of the disk and annular cylinder at 10 rev/s. (a) What is the moment of inertia of the system?



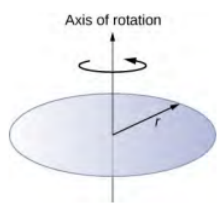
23. Using the parallel axis theorem, what is the moment of inertia of the rod of mass m about the axis shown below?



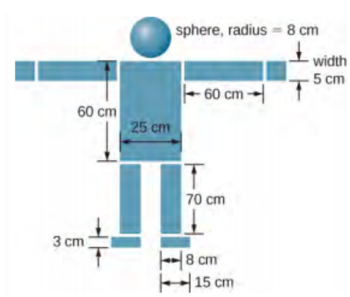
24. Find the moment of inertia of the rod in the previous problem by direct integration.
25. Calculate the moment of inertia by direct integration of a thin rod of mass M and length L about an axis through the rod at L/3, as shown below. Check your answer with the parallel-axis theorem.

7.9.2 Additional Problems

26. Where is the center of mass of a semicircular wire of radius R that is centered on the origin, begins and ends on the x axis, and lies in the x,y plane?
27. Where is the center of mass of a slice of pizza that was cut into eight equal slices? Assume the origin is at the apex of the slice and measure angles with respect to an edge of the slice. The radius of the pizza is R.
28. If the entire population of Earth were transferred to the Moon, how far would the center of mass of the Earth-Moon-population system move? Assume the population is 7 billion, the average human has a mass of 65 kg, and that the population is evenly distributed over both the Earth and the Moon. The mass of the Earth is 5.97 x 10^{24} kg and that of the Moon is 7.34 x 10^{22} kg. The radius of the Moon's orbit is about 3.84 x 10^5 m.
29. A system of point particles is rotating about a fixed axis at 4 rev/s. The particles are fixed with respect to each other. The masses and distances to the axis of the point particles are m_1 = 0.1 kg, r_1 = 0.2 m, m_2 = 0.05 kg, r_2 = 0.4 m, m_3 = 0.5 kg, r_3 = 0.01 m. (a) What is the moment of inertia of the system?
30. Calculate the moment of inertia of a skater given the following information. (a) The 60.0-kg skater is approximated as a cylinder that has a 0.110-m radius. (b) The skater with arms extended is approximated by a cylinder that is 52.5 kg, has a 0.110-m radius, and has two 0.900-m-long arms which are 3.75 kg each and extend straight out from the cylinder like rods rotated about their ends.
31. A disk of mass m, radius R, and area A has a surface mass density $\sigma = \frac{mr}{AR}$ (see the following figure). What is the moment of inertia of the disk about an axis through the center?



32. Find the center of mass of the structure given in the figure below. Assume a uniform thickness of 20 cm, and a uniform density of 1 g/cm^3.



7.9.3 Contributors and Attributions