9.12: Conservation of Energy

Learning Objectives

- Formulate the principle of conservation of mechanical energy, with or without the presence of non-conservative forces
- Use the conservation of mechanical energy to calculate various properties of simple systems

In this section, we elaborate and extend the result we derived in Potential Energy of a System, where we re-wrote the work-energy theorem in terms of the change in the kinetic and potential energies of a particle. This will lead us to a discussion of the important principle of the conservation of mechanical energy. As you continue to examine other topics in physics, in later chapters of this book, you will see how this conservation law is generalized to encompass other types of energy and energy transfers. The last section of this chapter provides a preview.

The terms 'conserved quantity' and 'conservation law' have specific, scientific meanings in physics, which are different from the everyday meanings associated with the use of these words. (The same comment is also true about the scientific and everyday uses of the word 'work.') In everyday usage, you could conserve water by not using it, or by using less of it, or by re-using it. Water is composed of molecules consisting of two atoms of hydrogen and one of oxygen. Bring these atoms together to form a molecule and you create water; dissociate the atoms in such a molecule and you destroy water. However, in scientific usage, a **conserved quantity** for a system stays constant, changes by a definite amount that is transferred to other systems, and/or is converted into other forms of that quantity. A conserved quantity, in the scientific sense, can be transformed, but not strictly created or destroyed. Thus, there is no physical law of conservation of water.

9.12.1 Systems with a Single Particle or Object

We first consider a system with a single particle or object. Returning to our development of Equation 8.2.2, recall that we first separated all the forces acting on a particle into conservative and non-conservative types, and wrote the work done by each type of force as a separate term in the work-energy theorem. We then replaced the work done by the conservative forces by the change in the potential energy of the particle, combining it with the change in the particle's kinetic energy to get Equation 8.2.2. Now, we write this equation without the middle step and define the sum of the kinetic and potential energies, K + U = E; to be the **mechanical energy** of the particle.

Conservation of Energy

The mechanical energy E of a particle stays constant unless forces outside the system or non-conservative forces do work on it, in which case, the change in the mechanical energy is equal to the work done by the non-conservative forces:

$$W_{nc,\ AB} = \Delta(K+U)_{AB} = \Delta E_{AB}. \tag{9.12.1}$$

This statement expresses the concept of **energy conservation** for a classical particle as long as there is no non-conservative work. Recall that a classical particle is just a point mass, is nonrelativistic, and obeys Newton's laws of motion. In Relativity, we will see that conservation of energy still applies to a non-classical particle, but for that to happen, we have to make a slight adjustment to the definition of energy.

It is sometimes convenient to separate the case where the work done by non-conservative forces is zero, either because no such forces are assumed present, or, like the normal force, they do zero work when the motion is parallel to the surface. Then

$$0 = W_{nc,\ AB} = \Delta(K+U)_{AB} = \Delta E_{AB}. \tag{9.12.2}$$

In this case, the conservation of mechanical energy can be expressed as follows: The mechanical energy of a particle does not change if all the non-conservative forces that may act on it do no work. Understanding the concept of energy conservation is the important thing, not the particular equation you use to express it.

Problem-Solving Strategy: Conservation of Energy

1. Identify the body or bodies to be studied (the system). Often, in applications of the principle of mechanical energy conservation, we study more than one body at the same time.
2. Identify all forces acting on the body or bodies.

3. Determine whether each force that does work is conservative. If a non-conservative force (e.g., friction) is doing work, then mechanical energy is not conserved. The system must then be analyzed with non-conservative work, Equation 9.12.2.
4. For every force that does work, choose a reference point and determine the potential energy function for the force. The reference points for the various potential energies do not have to be at the same location.
5. Apply the principle of mechanical energy conservation by setting the sum of the kinetic energies and potential energies equal at every point of interest.

Example 9.12.1: Simple pendulum

A particle of mass m is hung from the ceiling by a massless string of length 1.0 m, as shown in Figure 9.12.1. The particle is released from rest, when the angle between the string and the downward vertical direction is 30°. What is its speed when it reaches the lowest point of its arc?

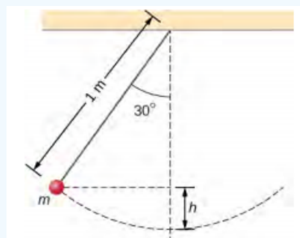

Figure 9.12.1: A particle hung from a string constitutes a simple pendulum. It is shown when released from rest, along with some distances used in analyzing the motion.

Strategy

Using our problem-solving strategy, the first step is to define that we are interested in the particle-Earth system. Second, only the gravitational force is acting on the particle, which is conservative (step 3). We neglect air resistance in the problem, and no work is done by the string tension, which is perpendicular to the arc of the motion. Therefore, the mechanical energy of the system is conserved, as represented by Equation 9.12.2, 0 = Δ(K + U). Because the particle starts from rest, the increase in the kinetic energy is just the kinetic energy at the lowest point. This increase in kinetic energy equals the decrease in the gravitational potential energy, which we can calculate from the geometry. In step 4, we choose a reference point for zero gravitational potential energy to be at the lowest vertical point the particle achieves, which is mid-swing. Lastly, in step 5, we set the sum of energies at the highest point (initial) of the swing to the lowest point (final) of the swing to ultimately solve for the final speed.

Solution

We are neglecting non-conservative forces, so we write the energy conservation formula relating the particle at the highest point (initial) and the lowest point in the swing (final) as

$$K_i + U_i = K_f + U_f. \tag{9.12.3}$$

Since the particle is released from rest, the initial kinetic energy is zero. At the lowest point, we define the gravitational potential energy to be zero. Therefore our conservation of energy formula reduces to

$$\begin{aligned} 0 + mgh &= \frac{1}{2}mv^2 + 0 \\ v &= \sqrt{2gh}. \end{aligned}$$

The vertical height of the particle is not given directly in the problem. This can be solved for by using trigonometry and two givens: the length of the pendulum and the angle through which the particle is vertically pulled up. Looking at the diagram, the vertical dashed line is the length of the pendulum string. The vertical height is labeled h. The other partial length of the vertical string can be calculated with trigonometry. That piece is solved for by

$$\cos\theta = \frac{x}{L} = L\cos\theta. \tag{9.12.4}$$

Therefore, by looking at the two parts of the string, we can solve for the height h,

$$
\begin{aligned}
x + h &= L \\
L\cos\theta + h &= L \\
h &= L - L\cos\theta \\
&= L(1-\cos\theta).
\end{aligned}
$$

We substitute this height into the previous expression solved for speed to calculate our result:

$$v = \sqrt{2gL(1-\cos\theta)} = \sqrt{2(9.8\ m/s^2)(1\ m)(1-\cos 30^o)} = 1.62\ m/s. \tag{9.12.5}$$

Significance

We found the speed directly from the conservation of mechanical energy, without having to solve the differential equation for the motion of a pendulum (see Oscillations). We can approach this problem in terms of bar graphs of total energy. Initially, the particle has all potential energy, being at the highest point, and no kinetic energy. When the particle crosses the lowest point at the bottom of the swing, the energy moves from the potential energy column to the kinetic energy column. Therefore, we can imagine a progression of this transfer as the particle moves between its highest point, lowest point of the swing, and back to the highest point (Figure $9.12.2$). As the particle travels from the lowest point in the swing to the highest point on the far right hand side of the diagram, the energy bars go in reverse order from (c) to (b) to (a).

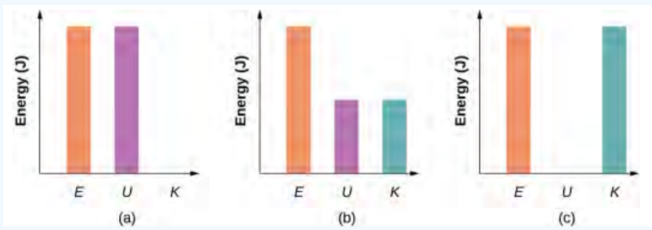

Figure $9.12.2$: Bar graphs representing the total energy (E), potential energy (U), and kinetic energy (K) of the particle in different positions. (a) The total energy of the system equals the potential energy and the kinetic energy is zero, which is found at the highest point the particle reaches. (b) The particle is midway between the highest and lowest point, so the kinetic energy plus potential energy bar graphs equal the total energy. (c) The particle is at the lowest point of the swing, so the kinetic energy bar graph is the highest and equal to the total energy of the system.

Exercise 8.7

How high above the bottom of its arc is the particle in the simple pendulum above, when its speed is 0.81 m/s?

Example 9.12.1: Air resistance on a falling object

A helicopter is hovering at an altitude of 1 km when a panel from its underside breaks loose and plummets to the ground (Figure $9.12.3$). The mass of the panel is 15 kg, and it hits the ground with a speed of 45 m/s. How much mechanical energy was dissipated by air resistance during the panel's descent?

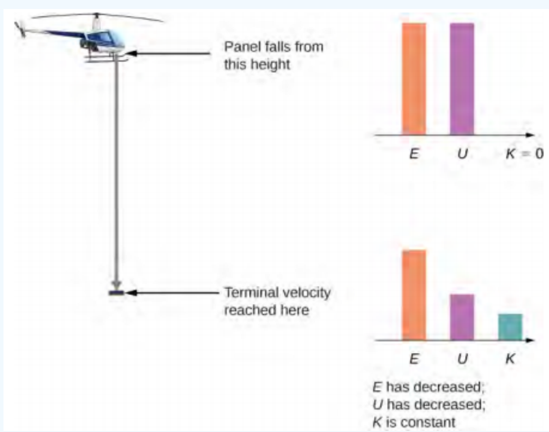

Figure $9.12.3$: A helicopter loses a panel that falls until it reaches terminal velocity of 45 m/s. How much did air resistance contribute to the dissipation of energy in this problem?

Strategy

Step 1: Here only one body is being investigated.

Step 2: Gravitational force is acting on the panel, as well as air resistance, which is stated in the problem.

Step 3: Gravitational force is conservative; however, the non-conservative force of air resistance does negative work on the falling panel, so we can use the conservation of mechanical energy, in the form expressed by Equation 9.12.1, to find the energy dissipated. This energy is the magnitude of the work:

$$\Delta E_{diss} = |W_{nc,if}| = |\Delta(K+U)_{if}|. \tag{9.12.6}$$

Step 4: The initial kinetic energy, at yi = 1 km, is zero. We set the gravitational potential energy to zero at ground level out of convenience.

Step 5: The non-conservative work is set equal to the energies to solve for the work dissipated by air resistance.

Solution

The mechanical energy dissipated by air resistance is the algebraic sum of the gain in the kinetic energy and loss in potential energy. Therefore the calculation of this energy is

$$\begin{aligned} \Delta E_{diss} &= |K_f - K_i 9U_f - U_i| \\ &= \left|\frac{1}{2}(15\ kg)(45\ m/s)^2 - 0 + 0 - (15\ kg)(9.8\ m/s^2)(1000\ m)\right| \\ &= 130\ kJ. \end{aligned}$$

Significance

Most of the initial mechanical energy of the panel (U_i), 147 kJ, was lost to air resistance. Notice that we were able to calculate the energy dissipated without knowing what the force of air resistance was, only that it was dissipative.

? Exercise 8.8

You probably recall that, neglecting air resistance, if you throw a projectile straight up, the time it takes to reach its maximum height equals the time it takes to fall from the maximum height back to the starting height. Suppose you cannot neglect air resistance, as in Example 8.8. Is the time the projectile takes to go up (a) greater than, (b) less than, or (c) equal to the time it takes to come back down? Explain.

In these examples, we were able to use conservation of energy to calculate the speed of a particle just at particular points in its motion. But the method of analyzing particle motion, starting from energy conservation, is more powerful than that. More advanced treatments of the theory of mechanics allow you to calculate the full time dependence of a particle's motion, for a given potential energy. In fact, it is often the case that a better model for particle motion is provided by the form of its kinetic and potential energies, rather than an equation for force acting on it. (This is especially true for the quantum mechanical description of particles like electrons or atoms.)

We can illustrate some of the simplest features of this energy-based approach by considering a particle in one-dimensional motion, with potential energy U(x) and no non-conservative interactions present. Equation 9.12.1 and the definition of velocity require

$$K = \frac{1}{2}mv^2 = E - U(x) \tag{9.12.7}$$

$$v = \frac{dx}{dt} = \sqrt{\frac{2(E-U(x))}{m}}. \tag{9.12.8}$$

Separate the variables x and t and integrate, from an initial time t = 0 to an arbitrary time, to get

$$t = \int_0^t dt = \int_{x_0}^x \frac{dx}{\sqrt{\frac{2(E-U(x))}{m}}}. \tag{9.12.9}$$

If you can do the integral in Equation 9.12.9, then you can solve for x as a function of t.

Example 8.9: Constant Acceleration

Use the potential energy $U(x) = -E\left(\frac{x}{x_0}\right)$, for $E > 0$, in Equation 9.12.9 to find the position x of a particle as a function of time t.

Strategy

Since we know how the potential energy changes as a function of x, we can substitute for U(x) in Equation 9.12.9, integrate, and then solve for x. This results in an expression of x as a function of time with constants of energy E, mass m, and the initial position x_0.

Solution

Following the first two suggested steps in the above strategy,

$$t=\int_{x_0}^{x}\frac{dx}{\sqrt{\left(\frac{2E}{mx_0}\right)(x_0-x)}}=\frac{1}{\sqrt{\left(\frac{2E}{mx_0}\right)}}\left|-2\sqrt{(x_0-x)}\right|_{x_0}^{x}=\frac{-2\sqrt{(x_0-x)}}{\sqrt{\left(\frac{2E}{mx_0}\right)}}. \tag{9.12.10}$$

Solving for the position, we obtain

$$x(t)=x_0-\frac{1}{2}\left(\frac{E}{mx_0}\right)t^2. \tag{9.12.11}$$

Significance

The position as a function of time, for this potential, represents one-dimensional motion with constant acceleration, $a = \left(\frac{E}{mx_0}\right)$, starting at rest from position x_0. This is not so surprising, since this is a potential energy for a constant force, $F = -\frac{dU}{dx} = \frac{E}{x_0}$, and $a = \frac{F}{m}$.

? Exercise 8.9

What potential energy U(x) can you substitute in Equation 9.12.2 that will result in motion with constant velocity of 2 m/s for a particle of mass 1 kg and mechanical energy 1 J?

We will look at another more physically appropriate example of the use of Equation 9.12.2 after we have explored some further implications that can be drawn from the functional form of a particle's potential energy.

9.12.2 More about Mechanical Energy and Conservation of Energy

Recall the Work-Energy Theorem, which relates the net work done on an object to its change in kinetic energy, along a path from point A to point B:

$$W^{net}=\Delta K=K_B-K_A$$

where K_A is the object's initial kinetic energy and K_B is its final kinetic energy. Generally, the net work done is the sum of the work done by conservative forces, W^C, and the work done by none conservative forces, W^{NC}:

$$W^{net}=W^C+W^{NC}$$

The work done by conservative forces can be expressed in terms of changes in potential energy functions. For example, suppose that two conservative forces, $\vec{F}_1$ and $\vec{F}_2$, are exerted on the object. The work done by those two forces is given by:

$$\begin{aligned} W_1 &= -\Delta U_1 \\ W_2 &= -\Delta U_2 \end{aligned}$$

where U_1 and U_2 are the changes in potential energy associated with forces $\vec{F}_1$ and $\vec{F}_2$, respectively. We can re-arrange the Work-Energy Theorem as follows[1]:

$$\begin{aligned} W^{net} = W^C + W^{NC} &= -\Delta U_1 - \Delta U_2 + W^{NC} &= \Delta K \\ \therefore W^{NC} &= \Delta U_1 + \Delta U_2 + \Delta K \end{aligned}$$

That is, the work done by non-conservative forces is equal to the sum of the changes in potential and kinetic energies. In general, we can use ΔU to represent the change in the total potential energy of the object. The total potential energy is the sum of the potential energies associated with each of the conservative forces acting on the object ($\Delta U = \Delta U_1 + \Delta U_2$ above). The above expression can thus be written in a more general form:

$$W^{NC} = \Delta U + \Delta K \tag{9.12.12}$$

In particular, note that if there are no non-conservative forces doing work on the object:

$$\Delta K + \Delta U = 0 \text{ if no non-conservative forces} \tag{9.12.13}$$

$$-\Delta U = \Delta K$$

That is, the sum of the changes in potential and kinetic energies of the object is always zero. This means that if the potential energy of the object increases, then the kinetic energy of the object must decrease by the same amount.

We can introduce the "mechanical energy", E, of an object as the sum of the potential and kinetic energies of the object:

$$E = U + K \tag{9.12.14}$$

If the object started at position A, with potential energy U_A and kinetic energy K_A, and ended up at position B with potential energy U_B and kinetic energy K_B, then we can write the mechanical energy at both positions and its change ΔE, as:

$$\begin{aligned} E_A &= U_A + K_A \\ E_B &= U_B + K_B \\ \Delta E &= E_B - E_A \\ &= U_B + K_B - U_A - K_A \\ \therefore \Delta E &= \Delta U + \Delta K \end{aligned}$$

Thus, the change in mechanical energy of the object is equal to the work done by non-conservative forces:

$$W^{NC} = \Delta U + \Delta K = \Delta E$$

and if there is no work done by non-conservative forces on the object, then the mechanical energy of the object does not change:

$$\begin{aligned} \Delta E &= 0 \quad \text{if no non-conservative forces} \\ \therefore E &= \text{constant} \end{aligned}$$

This is what we generally call the "conservation of mechanical energy". If there are no non-conservative forces doing work on an object, its mechanical energy is conserved (i.e. constant).

The introduction of mechanical energy gives us a completely different way to think about mechanics. We can now think of an object as having "energy" (potential and/or kinetic), and we can think of forces as changing the energy of the object.

? Exercise 9.12.1

Is the value of an an object's mechanical energy meaningful, or is it only the difference in mechanical energy that is meaningful?

A. Yes, the value of the mechanical energy is meaningful. At any given time, an object will have a quantifiable amount of mechanical energy.
B. No, the value is not meaningful because the value of potential energy is arbitrary. Only differences in mechanical energy are meaningful.
C. No, the value is not meaningful because both the potential and kinetic energies are arbitrary. Their values will change depending on where you set the energy to be zero.
D. It depends on which conservative forces act on the object (and therefore what "kind" of potential energy the object has).

Answer

We can also think of the work done by non-conservative forces as a type of change in energy. For example, the work done by friction can be thought of as a change in thermal energy (feel the burn as you rub your hand vigorously on a table!). If we can model the work done by non-conservative forces as a type of "other" energy, $-W^{NC} = \Delta E^{other}$, then we can state that:

$$\Delta E^{other} + \Delta U + \Delta K = 0$$

which is what we usually refer to as "conservation of energy". That is, the total energy in a system, including kinetic, potential and any other form (e.g. thermal, electrical, etc.) is constant unless some external agent is acting on the system.

We can always include that external agent in the system so that the total energy of the system is constant. The largest system that we can have is the Universe itself. Thus, the total energy in the Universe is constant and can only transform from one type into another, but no energy can ever be added or removed from the Universe.

Olivia's Thoughts

Here's an example that may help you understand the concept of external agents and energy conservation. Say we have a mass that hangs from a spring, so that the mass oscillates up and down like a yo-yo. If we define our system to include the spring, the mass, and gravity, energy will be conserved (the energy is transformed from potential energy to kinetic energy and back again).

Now, what if someone is holding the end of the spring and they start walking so that the whole system accelerates? Energy is not conserved because the system is gaining kinetic energy, seemingly out of nowhere. The system is being acted on by an *external agent* (the person). If we expand our system so that it includes the spring, the mass, gravity, *and the person*, energy is conserved. Instead of the kinetic energy "coming out of nowhere", we can see that it is actually coming from the person converting chemical energy in their body in order to move their muscles.

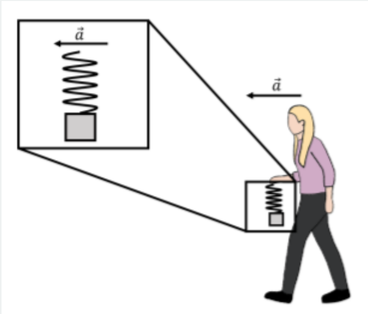

Figure 9.12.1: A person accelerates a mass and spring by walking. If the system does not include the person, energy is not conserved. If it does include the person, energy is conserved.

But what if there's an external agent acting on our new system? We can keep "zooming out" to include more and more external sources in the definition of our system. If you kept zooming out, eventually you would reach the point where the whole Universe was included in your system. At this point, you can't zoom out any more. This means that, if the Universe is your system, energy must always be conserved because there can't be any external agents acting on the system.

Example 9.12.1

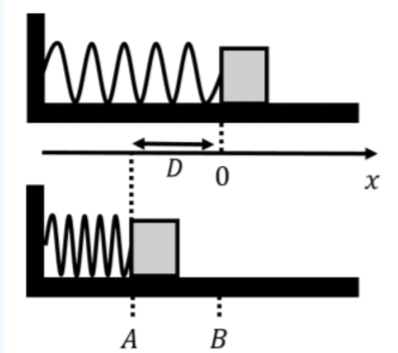

Figure 9.12.2: A block is launched along a frictionless surface by compressing a spring by a distance D. The top panel shows the spring when at rest, and the bottom panel shows the spring compressed by a distance D just before releasing the block.

A block of mass m can slide along a horizontal frictionless surface. A horizontal spring, with spring constant, k, is attached to a wall on one end, while the other end can move, as shown in Figure 8.3.2. A coordinate system is defined such that the x axis is horizontal and the free end of the spring is at $x=0$ when the spring is at rest. The block is pushed against the spring so that the spring is compressed by a distance D. The block is then released. What speed will the block have when it leaves the spring?

Solution:

This is again the same example that we saw in Chapters 6 and 7. We will show here that it is solved very easily using conservation of energy. The forces acting on the block are:

1. Weight, which does no work since it is perpendicular to the block's displacement.
2. The normal force, which does no work since it is perpendicular to the block's displacement.
3. The force from the spring, which is conservative and can be modelled with a potential energy $U(x)=\frac{1}{2}kx^2$, where x is the position of the end of the spring.

The block starts at rest at position A $(x=-D)$, where the spring is compressed by a distance D, and leaves the spring at position B $(x=0)$, where the spring is at its rest position.

At position A, the kinetic energy of the block is $K_A=0$ since the block is at rest, and the potential energy from the spring force of the block is $U_A=\frac{1}{2}kD^2$. The mechanical energy of the block at position A is thus:

$$\begin{aligned} K_A &= 0 \\ U_A &= \frac{1}{2}kD^2 \\ \therefore E_A &= U_A+K_A=\frac{1}{2}kD^2 \end{aligned}$$

At position B, the spring potential energy of the block is zero (since the spring is at rest), and all of the energy is kinetic:

$$\begin{aligned} K_B &= \frac{1}{2}mv_B^2 \\ U_B &= 0 \\ \therefore E_B &= U_B+K_B=\frac{1}{2}mv_B^2 \end{aligned}$$

Since there are no non-conservative forces doing work on the block, the mechanical energies at A and B are the same:

$$\begin{aligned} W^{NC} &= \Delta E=E_B-E_A=0 \\ \therefore E_B &= E_A \\ \frac{1}{2}mv_B^2 &= \frac{1}{2}kD^2 \\ v_B &= \sqrt{\frac{kD^2}{m}} \end{aligned}$$

as we found previously.

> **Example 9.12.2**

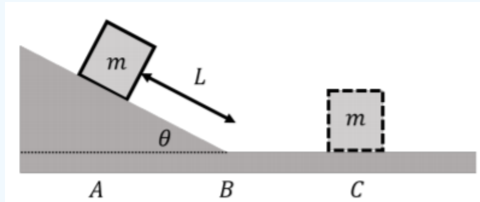

Figure 9.12.3: A block slides down an incline before sliding on a flat surface and stopping.

A block of mass m is placed at rest on an incline that makes an angle θ with respect to the horizontal, as shown in Figure 9.12.3. The block is nudged slightly so that the force of static friction is overcome and the block starts to accelerate down the incline. At the bottom of the incline, the block slides on a horizontal surface. The coefficient of kinetic friction between the block and the incline is μ_{k1}, and the coefficient of kinetic friction between the block and horizontal surface is μ_{k2}. If one assumes that the block started at rest a distance L from the bottom of the incline, how far along the horizontal surface will the block slide before stopping?

Solution

This is the same problem we solved in *Example 6.2.1*. In that case, we solved for the acceleration of the block using Newton's Second Law and then used kinematics to find how far the block went. We can solve this problem in a much easier way using conservation of energy.

It is still a good idea to think about what forces are applied on the object in order to determine if there are non-conservative forces doing work. In this case, the forces on the block are:

1. The normal force, which does no work, as it is always perpendicular to the motion.
2. Weight, which does work when the height of the object changes, which we can model with a potential energy function.
3. Friction, which is a non-conservative force, whose work we must determine.

Let us divide the motion into two segments: (1) a segment along the incline (positions A to B in Figure 9.12.3), where gravitational potential energy changes, and (2), the horizontal segment from positions B to position C on the figure. We can then apply conservation of energy for each segment.

Starting with the first segment, we can choose the gravitational potential energy to be zero when the block is at the bottom of the incline. The block starts at a height $h=L\sin\theta$ above the bottom of the incline. The gravitational potential energy for the beginning and end of the first segment are thus:

$$\begin{aligned}U_A&=mgL\sin\theta\\U_B&=0\end{aligned}$$

Since the block starts at rest, its kinetic energy is zero at position A, and if the speed of the box is v_B at position B, we can write its kinetic energy at both positions as:

$$\begin{aligned}K_A&=0\\K_B&=\frac{1}{2}mv_B^2\end{aligned}$$

The mechanical energy of the object at positions A and B is thus:

$$\begin{aligned}E_A&=U_A+K_A=mgL\sin\theta\\E_B&=U_B+K_B=\frac{1}{2}mv_B^2\\\Delta E&=E_B-E_A=\frac{1}{2}mv_B^2-mgL\sin\theta\end{aligned}$$

Finally, since we have a non-conservative force, the force of kinetic friction, acting on the first segment, we need to calculate the work done by that force. We found in *Example 6.2.1* that the force of friction had magnitude $f_k=\mu_{k1}N=\mu_{k1}mg\cos\theta$. Since the force of friction is anti-parallel to the displacement vector, which points down the incline and has length L, the work done by friction is:

$$W^{NC} = W_f = -f_k L = -\mu_{k1} mg\cos\theta L$$

Applying conservation of energy along the first segment, we have:

$$\begin{aligned} W^{NC} &= \Delta E \\ -\mu_{k1} mg\cos\theta L &= \frac{1}{2}mv_B^2 - mgL\sin\theta \\ \therefore \frac{1}{2}mv_B^2 &= mgL\sin\theta - \mu_{k1} mg\cos\theta L \end{aligned}$$

Note that the above equation, in words, could be read as, "the change in kinetic energy ($\frac{1}{2}mv_B^2$) is equal to the negative change in potential energy ($mgL\sin\theta$) minus the work done by friction ($\mu_{k1} mg\cos\theta L$)". In other words, the block had potential energy, which was converted into kinetic energy and heat (the work done by friction can be thought of as thermal energy).

We now proceed in an analogous way for the second segment, from position B to position C. The only force that can do work along this segment (of length x) is the force of kinetic friction, since both the weight and normal force are perpendicular to the displacement. There are no conservative forces doing work, so there is no change in potential energy. The initial kinetic energy is K_B (from above), and the final kinetic energy, K_C, is zero. The change in mechanical energy is thus:

$$\begin{aligned} \Delta E &= E_C - E_B = K_C - K_B = -K_B \\ &= -\frac{1}{2}mv_B^2 \\ &= -mgL\sin\theta + \mu_{k1} mg\cos\theta L \end{aligned}$$

where, in the last line, we used the result from the first segment. The work done by the force of friction along the horizontal segment of (undetermined) length x is:

$$W^{NC} = W_f = -f_k x = -\mu_{k2} N x = -\mu_{k2} mgx$$

Finally, we can find x by setting the work done by non-conservative forces equal to the change in mechanical energy:

$$\begin{aligned} W^{NC} &= \Delta E \\ -\mu_{k2} mgx &= -mgL\sin\theta + \mu_{k1} mg\cos\theta L \\ \therefore x &= L\frac{1}{\mu_{k2}}(\sin\theta - \mu_{k1}\cos\theta) \end{aligned}$$

which is the same result that we obtained in *Example 6.2.1*.

Discussion

By using conservation of energy, we were able to model the motion of the block down the incline in a way that was much easier than what was done in *Example 6.2.1*. Furthermore, although we modeled friction as a non-conservative force doing work, we gained some insight into the idea that this could be thought of as an energy loss. In terms of energy, we would say that the block initially had gravitational potential energy, which was then converted into kinetic energy as well as thermal energy (in the heat generated by friction).

9.12.3 Footnotes

1. This is why we defined potential energy as negative of the work; it becomes a positive term when we move it to the same side of the equation as the kinetic energy!

9.12.4 Systems with Several Particles or Objects

Systems generally consist of more than one particle or object. However, the conservation of mechanical energy, in one of the forms in Equation 9.12.1 or Equation 9.12.2, is a fundamental law of physics and applies to any system. You just have to include the kinetic and potential energies of all the particles, and the work done by all the non-conservative forces acting on them. Until you learn more about the dynamics of systems composed of many particles, in Linear Momentum and Collisions, Fixed-Axis Rotation, and Angular Momentum, it is better to postpone discussing the application of energy conservation to then.

- **8.4: Conservation of Energy** by OpenStax is licensed CC BY 4.0. Original source: https://openstax.org/details/books/university-physics-volume-1.
- **8.3: Mechanical Energy and Conservation of Energy** by Ryan D. Martin, Emma Neary, Joshua Rinaldo, and Olivia Woodman is licensed CC BY-SA 4.0. Original source: https://github.com/OSTP/PhysicsArtofModelling/blob/master/README.md.