2.7.2: Geometrical Shapes

✓ Learning Objectives

- know what a polygon is
- know what perimeter is and how to find it
- know what the circumference, diameter, and radius of a circle is and how to find each one
- know the meaning of the symbol ππ and its approximating value
- know what a formula is and four versions of the circumference formula of a circle
- know the meaning and notation for area
- know the area formulas for some common geometric figures
- be able to find the areas of some common geometric figures
- know the meaning and notation for volume
- know the volume formulas for some common geometric objects
- be able to find the volume of some common geometric objects

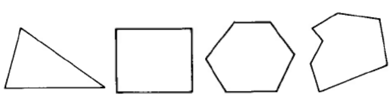
Polygons

We can make use of conversion skills with denominate numbers to make measurements of geometric figures such as rectangles, triangles, and circles. To make these measurements we need to be familiar with several definitions.

Definition: Polygon

A **polygon** is a closed plane (flat) figure whose sides are line segments (portions of straight lines).

Polygons



Not polygons

Perimeter

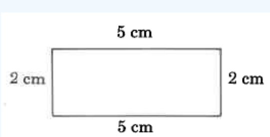
Definition: Perimeter

The **perimeter** of a polygon is the distance around the polygon.

To find the perimeter of a polygon, we simply add up the lengths of all the sides.

✓ Sample Set A

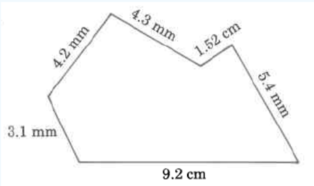
Find the perimeter of each polygon.



Solution

$$\begin{aligned} \text{Perimeter} &= 2 \text{ cm} + 5 \text{ cm} + 2 \text{ cm} + 5 \text{ cm} \\ &= 14 \text{ cm} \end{aligned}$$

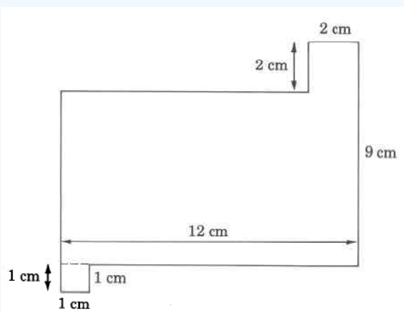
Sample Set A



Solution

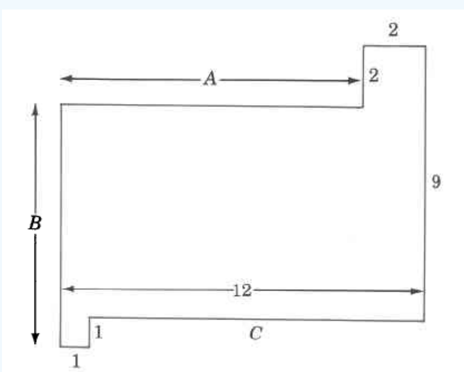
$$\begin{aligned} \text{Perimeter} = \quad & 3.1 \text{ mm} \\ & 4.2 \text{ mm} \\ & 4.3 \text{ mm} \\ & 1.52 \text{ mm} \\ & 5.4 \text{ mm} \\ & \underline{+\, 9.2 \text{ mm}} \\ & 27.72 \text{ mm} \end{aligned}$$

Sample Set A



Solution

Our first observation is that three of the dimensions are missing. However, we can determine the missing measurements using the following process. Let A, B, and C represent the missing measurements. Visualize

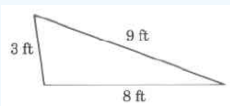


A = 12m - 2m = 10m
B = 9m + 1m - 2m = 8m
C = 12m - 1m = 11m

$$\begin{array}{lll} \text{Perimeter} & = & 8 \text{ m} \\ & & 10 \text{ m} \\ & & 2 \text{ m} \\ & & 2 \text{ m} \\ & & 9 \text{ m} \\ & & 11 \text{ m} \\ & & 1 \text{ m} \\ & & \underline{+1 \text{ m}} \\ & & 44 \text{ m} \end{array}$$

Practice Set A

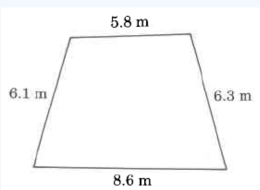
Find the perimeter of each polygon.



Answer

20 ft

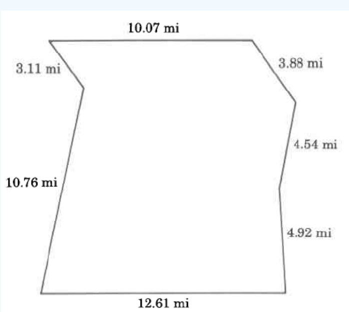
Practice Set A



Answer

26.8 m

Practice Set A



Answer

49.89 mi

Circumference/Diameter/Radius

Diameter (d)

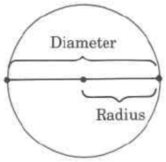
A **diameter** of a circle is any line segment that passes through the center of the circle and has its endpoints on the circle.

Radius (r)

A **radius** of a circle is any line segment having as its endpoints the center of the circle and a point on the circle. The radius is one half the diameter.

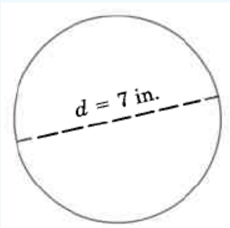
Circumference (C)

The **circumference** of a circle is the distance around the circle. It is given by $C = \pi d = 2\pi r$



Sample Set B

Find the circumference of the circle.



Solution

Use the formula $C = \pi d$.

$C = \pi \cdot 7\ in.$

By commutativity of multiplication,

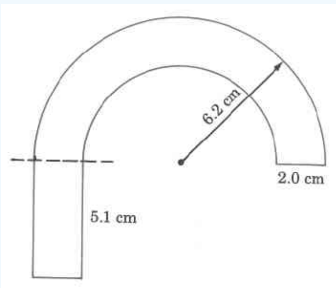
$C = 7\ in. \cdot \pi$

$C = 7\pi in.$, exactly

This result is exact since π has not been approximated.

Sample Set B

Find the perimeter of the figure.



Solution

We notice that we have two semicircles (half circles).

The larger radius is 6.2 cm.

The smaller radius is 6.2 cm - 2.0 cm $= 4.2$ cm.

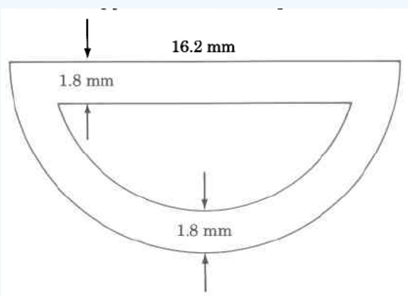
The width of the bottom part of the rectangle is 2.0 cm.

$$\begin{array}{rcll} \text{Perimeter} & = & 2.0 \text{ cm} & \\ & & 5.1 \text{ cm} & \\ & & 2.0 \text{ cm} & \\ & & 5.1 \text{ cm} & \\ & & (0.5)\cdot(2)\cdot(3.14)\cdot(6.2 \text{ com}) & \text{Circumference of outer semicircle.} \\ & + & \underline{(0.5)\cdot(2)\cdot(3.14)\cdot(4.2 \text{ com})} & \text{Circumference of inner semicircle.} \\ & & & 6.2 \text{ cm - } 2.0 \text{ cm} = 4.2 \text{ cm} \\ & & & \text{The 0.5 appears because we want the} \\ & & & \text{perimeter of only half a circle.} \end{array}$$

$$\begin{array}{rcr} \text{Perimeter} & \approx & 2.0 \text{ cm} \\ & & 5.1 \text{ cm} \\ & & 2.0 \text{ cm} \\ & & 5.1 \text{ cm} \\ & & 19.468 \text{ cm} \\ & & \underline{+13.188 \text{ cm}} \\ & & 48.856 \text{ cm} \end{array}$$

Practice Set B

Find the outside perimeter of



Answer

41.634 mm

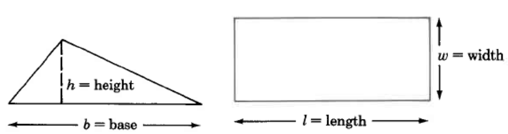
The Meaning and Notation for Area

The product $(\text{length unit}) \cdot (\text{length unit}) = (\text{length unit})^2$, or, square length unit (sq length unit), can be interpreted physically as the *area* of a surface.

Area
The **area** of a surface is the amount of square length units contained in the surface.

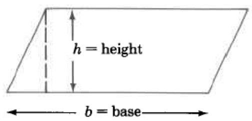
For example, 3 sq in. means that 3 squares, 1 inch on each side, can be placed precisely on some surface. (The squares may have to be cut and rearranged so they match the shape of the surface.)

We will examine the area of the following geometric figures.

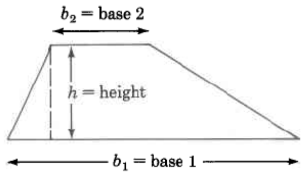


Triangles

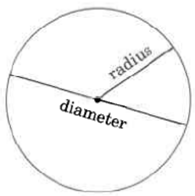
Rectangles



Parallelograms



Trapezoids

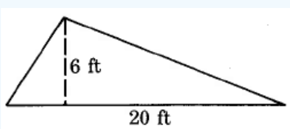


Circles

Finding Areas of Some Common Geometric Figures

✔ Sample Set A

Find the area of the triangle.



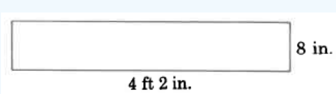
Solution

$$\begin{aligned} A_T &= \frac{1}{2} \cdot b \cdot h \\ &= \frac{1}{2} \cdot 20 \cdot 5 \text{ sq ft} \\ &= 10 \cdot 6 \text{ sq ft} \\ &= 60 \text{ sq ft} \\ &= 60 \text{ ft}^2 \end{aligned}$$

The area of this triangle is 60 sq ft, which is often written as 60 ft^2.

✔ Sample Set A

Find the area of the rectangle.



Solution

Let's first convert 4 ft 2 in. to inches. Since we wish to convert to inches, we'll use the unit fraction $\frac{12 \text{ in.}}{1 \text{ ft}}$ since it has inches in the numerator. Then,

$$\begin{aligned} 4 \text{ ft} &= \frac{4 \text{ ft}}{1} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} \\ &= \frac{4 \cancel{\text{ft}}}{1} \cdot \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} \\ &= 48 \text{ in.} \end{aligned}$$

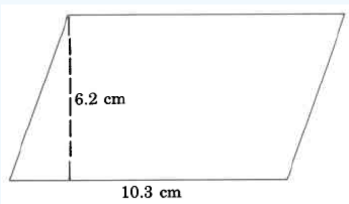
Thus, 4 ft 2 in. $= 48$ in. $+ 2$ in. $= 50$ in.

$$\begin{aligned} A_R &= l \cdot w \\ &= 50 \text{ in.} \cdot 8 \text{ in.} \\ &= 400 \text{ sq in.} \end{aligned}$$

The area of this rectangle is 400 sq in.

Sample Set A

Find the area of the parallelogram.



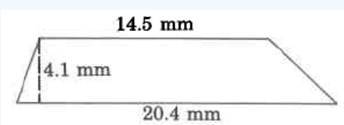
Solution

$$\begin{aligned} A_P &= b \cdot h \\ &= 10.3 \text{ cm} \cdot 6.2 \text{ cm} \\ &= 63.86 \text{ sq cm} \end{aligned}$$

The area of this parallelogram is 63.86 sq cm.

Sample Set A

Find the area of the trapezoid.



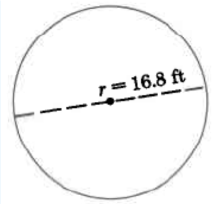
Solution

$$\begin{aligned} A_{Trap} &= \frac{1}{2} \cdot (b_1 + b_2) \cdot h \\ &= \frac{1}{2} \cdot (14.5 \text{ mm} + 20.4 \text{ mm}) \cdot (4.1 \text{ mm}) \\ &= \frac{1}{2} \cdot (34.9 \text{ mm}) \cdot (4.1 \text{ mm}) \\ &= \frac{1}{2} \cdot (143.09 \text{ sq mm}) \\ &= 71.545 \text{ sq mm} \end{aligned}$$

The area of this trapezoid is 71.545 sq mm.

Sample Set A

Find the approximate area of the circle.



Solution

$$\begin{aligned} A_c &= \pi \cdot r^2 \\ &\approx (3.14) \cdot (16.8 \text{ ft})^2 \\ &\approx (3.14) \cdot (282.24 \text{ sq ft}) \\ &\approx 888.23 \text{ sq ft} \end{aligned}$$

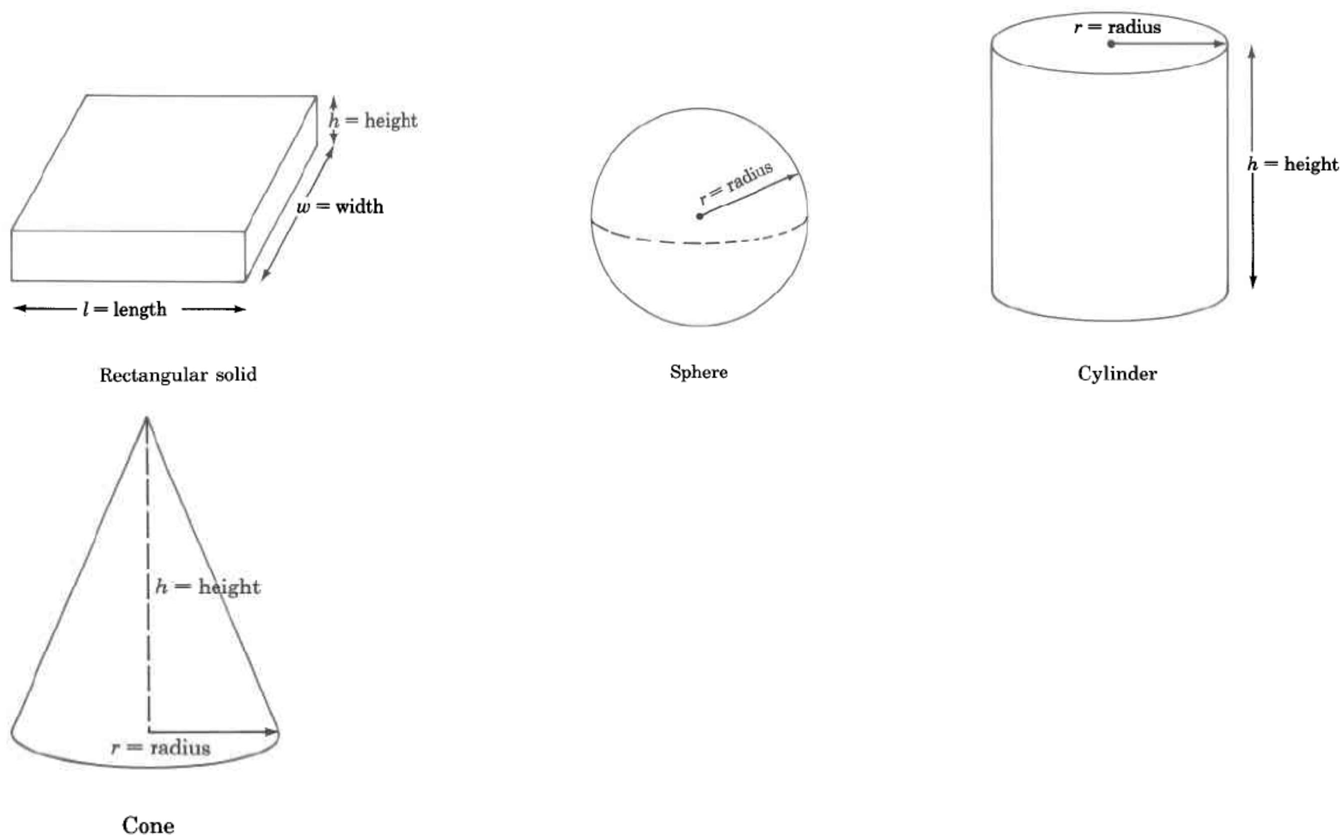
The area of this circle is approximately 886.23 sq ft.

The Meaning and Notation for Volume

The product $(\text{length unit})(\text{length unit})(\text{length unit}) = (\text{length unit})^3$, or cubic length unit (cu length unit), can be interpreted physically as the *volume* of a three-dimensional object.

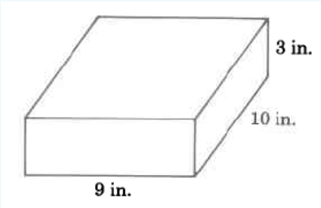
Volume
The **volume** of an object is the amount of cubic length units contained in the object.

For example, 4 cu mm means that 4 cubes, 1 mm on each side, would precisely fill some three-dimensional object. (The cubes may have to be cut and rearranged so they match the shape of the object.)



✓ Sample Set B

Find the volume of the rectangular solid.



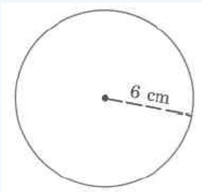
Solution

$$\begin{aligned} V_R &= l \cdot w \cdot h \\ &= 9 \text{ in.} \cdot 10 \text{ in.} \cdot 3 \text{ in.} \\ &= 270 \text{ cu in.} \\ &= 270 \text{ in.}^3 \end{aligned}$$

The volume of this rectangular solid is 270 cu in.

✓ Sample Set B

Find the approximate volume of the sphere.



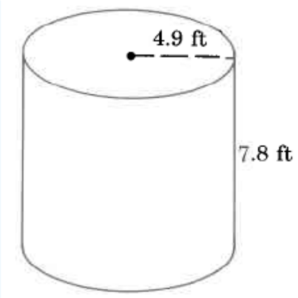
Solution

$$\begin{aligned} V_S &= \frac{4}{3} \cdot \pi \cdot r^3 \\ &\approx (\frac{4}{3}) \cdot (3.14) \cdot (6 \text{ cm})^3 \\ &\approx (\frac{4}{3}) \cdot (3.14) \cdot (216 \text{ cu cm}) \\ &\approx 904.32 \text{ cu cm} \end{aligned}$$

The approximate volume of this sphere is 904.32 cu cm, which is often written as 904.32 cm^3.

✓ Sample Set B

Find the approximate volume of the cylinder.



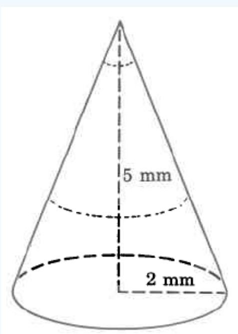
Solution

$$\begin{aligned} V_{Cyl} &= \pi \cdot r^2 \cdot h \\ &\approx (3.14) \cdot (4.9 \text{ ft})^2 \cdot (7.8 \text{ ft}) \\ &\approx (3.14) \cdot (24.01 \text{ sq ft}) \cdot (7.8 \text{ ft}) \\ &\approx (3.14) \cdot (187.278 \text{ cu ft}) \\ &\approx 588.05292 \text{ cu ft} \end{aligned}$$

The volume of this cylinder is approximately 588.05292 cu ft. The volume is approximate because we approximated π with 3.14.

Sample Set B

Find the approximate volume of the cone. Round to two decimal places.



Solution

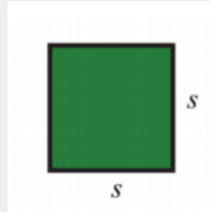
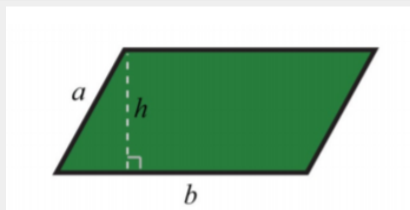
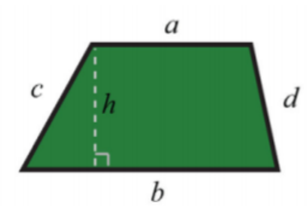
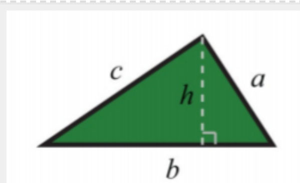
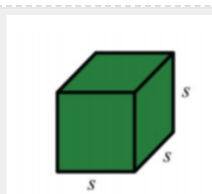
$$\begin{aligned} V_c &= \frac{1}{3} \cdot \pi \cdot r^2 \cdot h \\ &\approx (\frac{1}{3}) \cdot (3.14) \cdot (2 \text{ mm})^2 \cdot (5 \text{ mm}) \\ &\approx (\frac{1}{3}) \cdot (3.14) \cdot (4 \text{ sq mm}) \cdot (5 \text{ mm}) \\ &\approx (\frac{1}{3}) \cdot (3.14) \cdot (20 \text{ cu mm}) \\ &\approx 20.9\overline{3} \text{ cu mm} \\ &\approx 20.93 \text{ cu mm} \end{aligned}$$

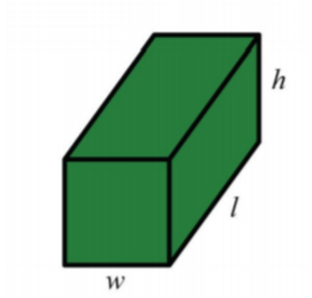
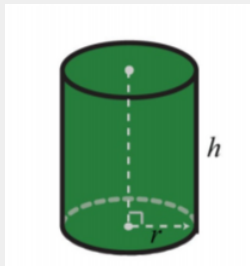
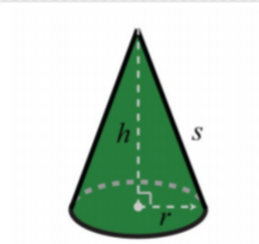
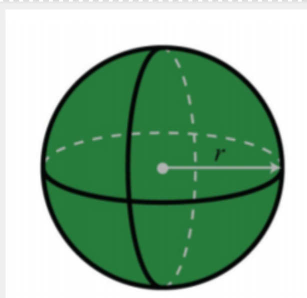
The volume of this cone is approximately 20.93 cu mm. The volume is approximate because we approximated π with 3.14.

Table of Formulas for Common Shapes

	Figure	Formulas

https://phys.libretexts.org/@go/page/72531

Square	 Figure 2.7.2.1	$P = 4s \quad (2.7.2.1)$ $A = s^2 \quad (2.7.2.2)$
Rectangle	 Figure 2.7.2.2	$P = 2l + 2w \quad (2.7.2.3)$ $A = lw \quad (2.7.2.4)$
Parallelogram	 Figure 2.7.2.3	$P = 2a + 2b \quad (2.7.2.5)$ $A = bh \quad (2.7.2.6)$
Trapezoid	 Figure 2.7.2.4	$P = a + b + c + d \quad (2.7.2.7)$ $A = \frac{1}{2}h(a + b) \quad (2.7.2.8)$
Triangle	 Figure 2.7.2.5	$P = a + b + c \quad (2.7.2.9)$ $A = \frac{1}{2}bh \quad (2.7.2.10)$
Circle	 Figure 2.7.2.6	$C = 2\pi \quad (2.7.2.11)$ $r = \pi r^2 \quad (2.7.2.12)$ Volume (***V***) is measured in cubic units and surface area (***SA***) is measured in square units.
Cube	 Figure 2.7.2.1	$SA = 6s^2 \quad (2.7.2.13)$ $V = s^3 \quad (2.7.2.14)$

Rectangular Solid	 Figure 2.7.2.2	$$SA = 2lw + 2lh + 2wh \tag{2.7.2.15}$$ $$V = lwh \tag{2.7.2.16}$$
Right Circular Cylinder	 Figure 2.7.2.3	$$SA = 2\pi r^2 + 2\pi rh \tag{2.7.2.17}$$ $$V = \pi r^2 h \tag{2.7.2.18}$$
Right Circular Cone	 Figure 2.7.2.4	$$SA = \pi r^2 + \pi rs \tag{2.7.2.19}$$ $$V = \frac{1}{3}\pi r^2 h \tag{2.7.2.20}$$
Sphere	 Figure 2.7.2.5	$$SA = 4\pi r^2 \tag{2.7.2.21}$$ $$V = \frac{4}{3}\pi r^3 \tag{2.7.2.22}$$

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