9.7: Rotational and Translational Relationships

Learning Objectives

- Use the work-energy theorem to analyze rotation to find the work done on a system when it is rotated about a fixed axis for a finite angular displacement
- Solve for the angular velocity of a rotating rigid body using the work-energy theorem
- Find the power delivered to a rotating rigid body given the applied torque and angular velocity
- Summarize the rotational variables and equations and relate them to their translational counterparts

Thus far in the section, we have extensively addressed kinematics and dynamics for rotating rigid bodies around a fixed axis. In this final subsection, we define work and power within the context of rotation about a fixed axis, which has applications to both physics and engineering. The discussion of work and power makes our treatment of rotational motion almost complete, with the exception of rolling motion and angular momentum, which are discussed in Angular Momentum. We begin this subsection with a treatment of the work-energy theorem for rotation.

9.7.1 Rotational and Translational Relationships Summarized

The rotational quantities and their linear analog are summarized in three tables. Table 10.5 summarizes the rotational variables for circular motion about a fixed axis with their linear analogs and the connecting equation, except for the centripetal acceleration, which stands by itself. Table 10.6 summarizes the rotational and translational kinematic equations. Table 10.7 summarizes the rotational dynamics equations with their linear analogs.

9.7.1.1 Table - Rotational and Translational Variables: Summary

Rotational	Translational	Relationship
θ	x	$\theta=\frac{s}{r}$
ω	v_f	$\omega=\frac{v_t}{r}$
α	a_t	$\alpha=\frac{a_t}{r}$
	a_c	$a_c=\frac{v_t^2}{r}$

9.7.1.2 Table - Rotational and Translational Kinematic Equations: Summary

Rotational	Translational
$\theta_f=\theta_0+\bar{\omega}t$	$x=x_0+\bar{v}t$
$\omega_f=\omega_0+\alpha t$	$v_f=v_0+at$
$\theta_f=\theta_0+\omega_0t+\frac{1}{2}\alpha t^2$	$x_f=x_0+v_0t+\frac{1}{2}at^2$
$\omega_f^2=\omega_0^2+2\alpha(\Delta\theta)$	$v_f^2=v_0^2+2a(\Delta x)$

9.7.1.3 Table - Rotational and Translational Equations: Dynamics

Rotational	Translational
$I=\sum_i m_ir_i^2$	m
$K=\frac{1}{2}I\omega^2$	$K=\frac{1}{2}mv^2$
$\sum_i \overrightarrow{\tau_i}=I\vec{\alpha}$	$\sum_i \vec{F}_i=m\vec{a}$
$W_{AB}=\int_{\theta_A}^{\theta_B}\left(\sum_i \overrightarrow{\tau_i}\right)\cdot d\vec{\theta}$	$W=\int\vec{F}\cdot d\vec{s}$
$P=\vec{\tau}\cdot\vec{\omega}$	$P=\vec{F}\cdot\vec{v}$

- **10.9: Work and Power for Rotational Motion** by OpenStax is licensed CC BY 4.0. Original source: https://openstax.org/details/books/university-physics-volume-1.