3.6: Force and Rate of Change of Momentum

Theorem:

The rate of change of the total momentum of a system of particles is equal to the sum of the external forces on the system.

Thus, consider a single particle. By Newton's second law of motion, the rate of change of momentum of the particle is equal to the sum of the forces acting upon it:

$$\dot{\mathbf{P}}_i = \mathbf{F}_i + \sum_i \mathbf{F}_{ij} \qquad (j \neq i) \tag{3.6.1}$$

Now sum over all the particles:

$$\begin{gathered}\dot{\mathbf{P}}_i = \sum_i \mathbf{F}_i + \sum_i \sum_j \mathbf{F}_{ij} \qquad (j \neq i) \\ \mathbf{F} + \frac{1}{2}\sum_i \sum_j \mathbf{F}_{ij} + \frac{1}{2}\sum_j \sum_i \mathbf{F}_{ij} \\ \mathbf{F} + \frac{1}{2}\sum_i \sum_j \mathbf{F}_{ji} + \mathbf{F}_{ij}\end{gathered} \tag{3.6.2}$$

But, by Newton's third law of motion, $\mathbf{F}_{ji} + \mathbf{F}_{ij} = 0$, so the theorem is proved.

Corollary:

If the sum of the external forces on a system is zero, the linear momentum is constant. (Law of Conservation of Linear Momentum.)