15.23: Some Mathematical Results

Before proceeding with the next section, I just want to establish few mathematical results, so that we do not get bogged down in heavy algebra later on when we should be concentrating on understanding physics.

First, if

$$\gamma = \left(1 - \frac{u^2}{c^2}\right), \tag{15.23.1}$$

Then, by trivial differentiation,

$$\frac{d\gamma}{du} = \frac{\gamma^3 u}{c^2}. \tag{15.23.2}$$

$$\dot{\gamma} = \frac{\gamma^3 u \dot{u}}{c^2}. \tag{15.23.3}$$

From this, we quickly find that

$$\frac{\gamma u \dot{u}}{\dot{\gamma}} = c^2 - u^2. \tag{15.23.4}$$

Now for a small result concerning a scalar (dot) product.

Let **A** be a vector such that **A** * **A** = A^2.

Then

$$\tfrac{d}{dt}(A^2) = 2A\dot{A} \text{ and } \tfrac{d}{dt}(\mathbf{A}\cdot\mathbf{A}) = \mathbf{2A}\cdot\dot{\mathbf{A}}$$

$$A \cdot \dot{A} = A\dot{A} \tag{15.23.5}$$

We can now safely proceed to the next section.