1.2: Plane Triangular Lamina

Definition: A median of a triangle is a line from a vertex to the midpoint of the opposite side.

Theorem I. The three medians of a triangle are concurrent (meet at a single, unique point) at a point that is two-thirds of the distance from a vertex to the midpoint of the opposite side.

Theorem II. The centre of mass of a uniform triangular lamina (or the centroid of a triangle) is at the meet of the medians.

The proof of I can be done with a nice vector argument (Figure I.1):

Let $\mathbf{A}$, $\mathbf{B}$ be the vectors OA , OB . Then $\mathbf{A+B}$ is the diagonal of the parallelogram of which OA and OB are two sides, and the position vector of the point C_1 is $\frac{1}{3}(\mathbf{A+B})$.

To get C_2, we see that

$$\mathbf{C}_2 = \mathbf{A} + \frac{2}{3}(\mathrm{AM}_2) = \mathbf{A} + \frac{2}{3}(\mathbf{M_2} - \mathbf{A}) = \mathbf{A} + \frac{2}{3}(\frac{1}{2}\mathbf{B} - \mathbf{A}) = \frac{1}{3}(\mathbf{A} + \mathbf{B})$$

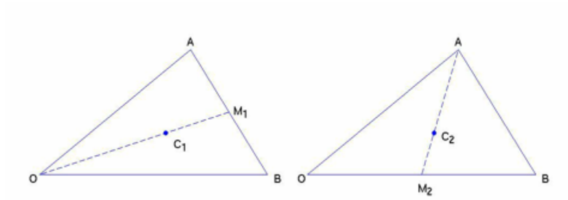

FIGURE I.1

Thus the points C_1 and C_2 are identical, and the same would be true for the third median, so Theorem I is proved.

Now consider an elemental slice as in Figure I.2. The centre of mass of the slice is at its mid-point. The same is true of any similar slices parallel to it. Therefore the centre of mass is on the locus of the mid-points - i.e. on a median. Similarly, it is on each of the other medians, and Theorem II is proved.

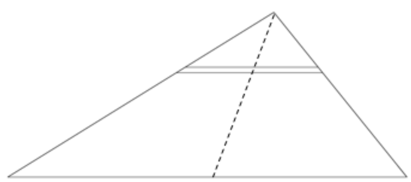
FIGURE 1.2

That needed only some vector geometry. We now move on to some calculus.