15.26: Energy and Mass

The nonrelativistic expression for kinetic energy $T = \frac{1}{2}mu^2$ has just one term in it, a term which depends on the speed. The relativistic expression which approximates to the nonrelativistic expression at low speeds) can be written $T = mc^2 - m_0c^2$ that is, a speed-dependent term minus a constant term. The kinetic energy can be thought of as the excess over the energy over the constant term m_0c^2. The expression m_0c^2 is known as the *rest-mass energy*. The sum of the kinetic energy and the rest-mass energy is the “total energy”, or just the “energy” E:

$$E = T + m_0c^2 = mc^2 \tag{15.26.1}$$

This means that, if the kinetic energy of a particle is zero, the total energy of the particle is not zero – it still has its rest-mass energy m_0c^2.

Of course, giving the name “rest-mass energy” to the constant term m_0c^2, and calling the speed-dependent term mc^2 the “total energy” and writing the famous equation $E = mc^2$, does not by itself immediately and directly tell us that “matter” can be converted to “energy” or the other way round. Whether such conversion can in fact take place is a matter for experiment and observation to determine. The equation by itself merely tells us how much mass is held by a given quantity of energy, or how much energy is held by a given quantity of mass. That entities that we traditionally think of as “matter” can be converted into entities that we traditionally think of as “energy” is well established with, for example, the “annihilation” of an electron and a positron (“matter” and “antimatter”) to form photons (“energy”) as is the inverse process of pair production (production of an electron-positron pair from a gamma ray in the presence of a third body).

It is unfortunate that the main (almost the only) example of application of the equation $E = mc^2$ persistently presented to the nonscientific public is the atom bomb, whose operation actually has nothing at all to do with the equation $E = mc^2$, nor, contrary to the popular mind, is any “matter” converted to energy.

I have heard it said that you can find out on the Web how to build an atom bomb, so here goes – here is how an atom bomb works. A uranium-235 nucleus is held together by strong attractive forces between the nucleons, which, at short femtometre ranges are much stronger than the Coulomb repulsive forces between the protons. When the nucleus absorbs an additional neutron, the resulting ^{236}U nucleus is unstable and breaks up into two intermediate-mass nuclei plus two or three neutrons. The two intermediate-mass nuclei are generally not of exactly equal mass; one is usually a bit less than half of the uranium nucleus and the other a bit more than half, but that’s a detail. The potential energy required to bind the nucleons together in the uranium nucleus is rather greater than the binding energy of the two resulting intermediate-mass nuclei; the difference is of order 200 MeV, and that potential energy is converted into kinetic energy of the two resulting nuclei and, to a lesser extent, the two or three neutrons released. That is all. It is merely the familiar conversion of potential binding energy (admittedly a great deal of energy) into kinetic energy. No matter, no protons, no neutrons, are “destroyed” or “converted into energy”, and $E = mc^2$ simply does not enter into it anywhere! The rest-mass energy of a proton or a neutron is about 1 GeV, and that much energy would be released if a proton were miraculously and for no cause converted into energy. Let us hope that no one invents a bomb that will do that – though we may rest assured that that is rather unlikely.

Where the equation $E = mc^2$ does come in is in the familiar observation that the mass of any nucleus other than hydrogen is a little less than the sum of the masses of the constituent nucleons. It is for that reason that nuclear masses, even for pure isotopes, are not integral. The mass of a nucleus is equal to the sum of the masses of the constituent nuclei plus the mass of the binding energy, the latter being a negative quantity since the inter-nucleon forces are attractive forces. The equation $E = mc^2$ tells us that energy (such as, for example, the binding energy between nucleons) has mass.