# 11.6: Electrical Analogues

A charged capacitor of capacitance $C$ is connected in series with a switch and an inductor of inductance $L$. The switch is closed, and charge flows out of the capacitor and hence a current flows through the inductor. Thus while the electric field in the capacitor diminishes, the magnetic field in the inductor grows, and a back electromotive force (EMF) is induced in the inductor. Let $Q$ be the charge in the capacitor at some time. The current $I$ flowing from the positive plate is equal to $-\dot{Q}$. The potential difference across the capacitor is $\frac{Q}{C}$ and the back EMF across the inductor is $L\dot{I} = -L\ddot{Q}$. The potential drop around the whole circuit is zero, so that $\frac{Q}{C} = -L\ddot{Q}$. The charge on the capacitor is therefore governed by the differential equation

$$\ddot{Q} = -\frac{Q}{LC}, \tag{11.6.1}$$

which is simple harmonic motion with $\omega_0 = \frac{1}{\sqrt{LC}}$. You should verify that this has dimensions $T^{-1}$.

If there is a resistor of resistance $R$ in the circuit, while a current flows through the resistor there is a potential drop $RI = -R\dot{Q}$ across it, and the differential equation governing the charge on the capacitor is then

$$LC\ddot{Q} + RC\dot{Q} + Q = 0. \tag{11.6.2}$$

This is damped oscillatory motion, the condition for critical damping being

$$R^2 = \frac{4L}{C}.$$

In fact, it is not necessary actually to have a physical resistor in the circuit. Even if the capacitor and inductor were connected by superconducting wires of zero resistance, while the charge in the circuit is slopping around between the capacitor and the inductor, it will be radiating electromagnetic energy into space and hence losing energy. The effect is just as if a resistance were in the circuit.

If a battery of EMF $E$ were in the circuit, the differential equation for $Q$ would be

$$LC\ddot{Q} + RC\dot{Q} + Q = EC. \tag{11.6.3}$$

This is not quite an equation of the form 11.4.1, and I shan't spend time on it here. However, if we are interested in the current as a function of time, we just differentiate Equation 11.6.3 with respect to time:

$$LC\ddot{I} + RC\dot{I} + I = 0. \tag{11.6.4}$$