21.1: Introduction to Central Forces

When a particle is in orbit around a point under the influence of a central attractive force (i.e. a force $F(r)$ which is directed towards a central point, with no transverse component) it experiences, *when referred to an inertial reference frame*, a centripetal acceleration. If, however, the system is described *with respect to a co-rotating reference frame*, there is no centripetal acceleration; rather, it appears as though an additional force, the centrifugal force, is pushing it away from the centre of attraction. In the co-rotating frame, this force depends only on the distance of the particle from the centre of attraction, and it is therefore a conservative force – and, like any conservative force, it can be described by the negative of the derivative of a potential energy function. When describing the motion with respect to the co-rotating frame, we must add this potential to any additional "real" potentials (such as originate from the gravitational fields of other bodies), to form an equivalent potential which constrains the motion of the particle. An excellent example of this method is the analysis of the restricted three-body problem given in some detail in Chapter 16 of the Celestial Mechanics notes. But I deal first, by way of example, with some simpler problems involving central forces, in which we shall be able, by simple arguments, to deduce some basic characteristics of the motion.