19.4: Variations

In Sections 19.1,2,3, we imagined that the cycloid was generated by a circle that was rolling counterclockwise along the line $y = 2a$. We can also imagine variations such as the circle rolling clockwise along $y = 0$, or we can start with P at the top of the circle rather than at the bottom. I summarise in this section four variations. The distinction between ψ and θ is as follows. The angle that the tangent to the cycloid makes with the positively-directed x-axis is ψ; that is to say, $dx/dy = tan\psi$.The circle rolls through an angle 2θ. There is a simple relation between ψ and θ, which is different for each case.

In each figure, x and y are plotted in units of a. The vertical height between vertices and cusps is $2a$, the horizontal distance between a cusp and the next vertex is πa, and the arc length between a cusp and the next vertex is $4a$.

I. Circle rolls counterclockwise along $y = 2a$. P starts at the bottom. The cusps are up. A vertex is at the origin.

$$x = a(2\theta + \sin 2\theta)$$

$$y = 2a \sin^2 \theta \tag{19.4.2}$$

$$s = 4a \sin\theta \tag{19.4.3}$$

$$^2 = 8ay \tag{19.4.4}$$

$$\psi = \theta. \tag{19.4.5}$$

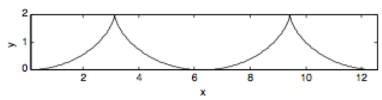

FIGURE XIX.2

II. Circle rolls clockwise along $y = 0$. P starts at the bottom. The cusps are down. A cusp is at the origin.

$$x = a(2\theta - sin2\theta) \tag{19.4.6}$$

$$y = 2asin^2\theta \tag{19.4.7}$$

$$s = 4a(1 - cos\theta) \tag{19.4.8}$$

$$s^2 = 8a(y - s) \tag{19.4.9}$$

$$\psi = 90\circ - \theta. \tag{19.4.10}$$

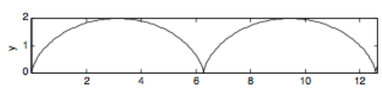

FIGURE XIX.3

III. Circle rolls clockwise along $y = 0$. P starts at the top. The cusps are down. A vertex is at $x = 0$.

$$x = a(2\theta + \sin 2\theta \tag{19.4.11}$$

$$y = 2a \cos^2 \theta \tag{19.4.12}$$

$$s = 4a \sin\theta \tag{19.4.13}$$

$$s^2 = 8a(2a - y) \tag{19.4.14}$$

$$\psi = 180\circ - \theta. \tag{19.4.15}$$

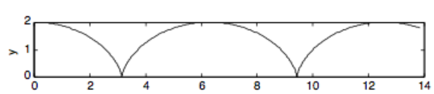

FIGURE XIX.4

IV. Circle rolls counterclockwise along $y = 2a$. P starts at the top. The cusps are up. A cusp is at $x = 0$.

$$x = a(2\theta - \sin 2\theta) \tag{19.4.16}$$

$$y = 2a\cos^2\theta \tag{19.4.17}$$

$$s = 4a(1 - \cos\theta) \tag{19.4.18}$$

$$s^2 - 8as + 8a(2a - y) = 0 \tag{19.4.19}$$

$$\psi = 90\circ + \theta \tag{19.4.20}$$

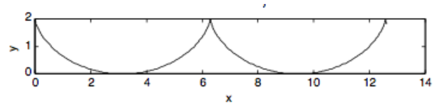

FIGURE XIX.5