3.7: Angular Momentum

Notation:

- $\mathbf{L}_C$= angular momentum of system with respect to centre of mass C.
- $\mathbf{L}$ = angular momentum of system relative to some other origin O.
- $\overline{\mathbf{r}}$ = position vector of C with respect to O.
- $\mathbf{P}$ = linear momentum of system with respect to O.
- (The linear momentum with respect to C is, of course, zero.)

Theorem:

$$\mathbf{L}=\mathbf{L}_C+\overline{\mathbf{r}}\times\mathbf{P} \tag{3.7.1}$$

Thus:

$$\begin{aligned}\mathbf{L} &=\sum\mathbf{r}_i\times\mathbf{p}_i=\sum m_i(\mathbf{r}_i\times\mathbf{v}_i)=\sum m_i(\overline{\mathbf{r}}+\mathbf{r}_i')\times(\overline{\mathbf{v}}+\mathbf{v}_i') \\ &=(\overline{\mathbf{r}}\times\overline{\mathbf{v}})\sum m_i+\overline{\mathbf{r}}\times\sum m_i\mathbf{v}_i'+(\sum m_i\mathbf{r}_i')\times\overline{\mathbf{v}}+\sum\mathbf{r}_i'\times\mathbf{p}_i' \\ &=M(\overline{\mathbf{r}}\times\overline{\mathbf{v}})+\overline{\mathbf{r}}\times 0+0\times\overline{\mathbf{v}}+\mathbf{L}_C\end{aligned}$$

therefore

$$\mathbf{L}=\mathbf{L}_C+\overline{\mathbf{r}}\times\mathbf{P}$$

Example 3.7.1

A hoop of radius *a* rolling along the ground (Figure III.6):

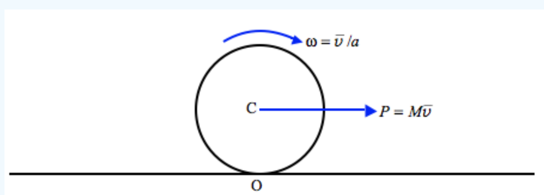

FIGURE III.6

The angular momentum with respect to C is L_C = $I_{C\omega}$ where I_C is the rotational inertia about C. The angular momentum about O is therefore

$$I=I_C\omega+M\overline{v}a=I_C\omega+Ma^2\omega=(I_C+Ma^2)=I\omega$$

where

$$I=I_C+Ma^2$$

is the *rotational inertia* about O.