15.27: Energy and Momentum

A moving particle has energy arising from its momentum and also from its rest mass, and we need to find an expression relating energy to rest mass and momentum. It is fairly easy and it goes like this:

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[Math Processing Error]

Thus we obtain for the energy in terms of rest mass and momentum

[Math Processing Error]

If the speed (and hence momentum) is zero, the energy is merely *[Math Processing Error]*. If the rest mass is zero (as, for example, a photon) and the energy is not zero, then *[Math Processing Error]*. But also *[Math Processing Error]*, so that, if the rest mass of a particle is zero and the energy is not, the particle must be moving at the speed of light. This could be regarded as the reason why photons, which have zero rest mass, travel at the speed of photons. If neutrinos have zero rest mass, they, too, will travel at the speed of light; if they are not massless, they won't.

In addition to Equation *[Math Processing Error]*, which relates the energy to the magnitude of the momentum, it will be of interest to see how the *components* of momentum transform between reference frames. As usual, we are considering frame *[Math Processing Error]* to be moving with respect to *[Math Processing Error]* at a speed *[Math Processing Error]* with respect to *[Math Processing Error]*. There is no difficulty with the *[Math Processing Error]*- and *[Math Processing Error]*- components. We have merely *[Math Processing Error]* and *[Math Processing Error]*. However:

[Math Processing Error] and *[Math Processing Error]*.

Also *[Math Processing Error]*, from which *[Math Processing Error]*.

After a little algebra, we obtain

[Math Processing Error]

And this is

[Math Processing Error]

The inverse is found in the usual way:

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If we eliminate *[Math Processing Error]* from Equations *[Math Processing Error]* and *[Math Processing Error]*, we'll find *[Math Processing Error]* in terms of *[Math Processing Error]* and *[Math Processing Error]*:

[Math Processing Error]

Thus the transformations between energy and the three spatial components of momentum is similar to the transformation between time and the three space coordinates, and are described by a similar 4-vector:

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The reader should multiply this out to verify that it does reproduce Equations *[Math Processing Error]* and *[Math Processing Error]*.