15.11: The Twins Paradox

During the late 1950s and early 1960s there was great controversy over a problem known as the “Twins Paradox”. The controversy was not confined to within scientific circles, but was argued, by scientists and others, in the newspapers, magazines and many serious journals. It goes something like this:

There are two 20-year-old twins, Albert and Betty. Albert is a sedentary type who likes nothing better than to stay at home tending the family vineyards. His twin sister Betty is a more adventurous type, and has trained to become an astronaut. On their twentieth birthday, Betty waves a cheery *au revoir* to her brother and takes off on what she intends to be a brief spaceflight, at which she travels at 99.98 % of the speed of light (γ = 50). After six months by her calendar she turns back and on her 21st birthday she arrives back home to greet her brother, only to find that he is now old and sere and has laboured, by his calendar for 50 years and is now an aged man of 71 years. If we accept what we have derived in the previous section about the dilation of time, there would seem to be no particular problem with that. It has even been argued that travel between the stars may not be an impossibility. Whereas to an Earthbound observer it may take many decades for a spacecraft to travel to a star and back, for the astronauts on board much less time has elapsed.

And yet a paradox was pointed out. According to the principles of the relativity of motion, it was argued, one could refer everything to Betty’s reference frame, and from that point of view one could regard Betty as being the stationary twin and Albert as the one who travelled off into the distance and returned later. Thus, it could be argued, it would be Albert who had aged only one year, while Betty would have aged fifty years. Thus we have a *paradox*, which is a problem which apparently gives rise to opposite conclusions depending on how it is argued. And the only way that the paradox could be resolved was to suppose that both twins were the same age when they were re-united.

A second argument in favour of this interpretation that the twins were the same age when re-united points out that dilation of time arises because two events that may occur in the same place when referred to one reference frame do not occur in the same place when referred to another. But in this case, the two events (Betty’s departure and re-arrival) occur at the same place when referred to both reference frames.

The argument over this point raged quite furiously for some years, and a particularly plausible tool that was used was something referred to as the "k-calculus” – an argument that is, however, fatally flawed because the “rules” of the k-calculus inherently incorporate the desired conclusion. Two of the principal leaders of the very public scientific debate were Professors Fred Hoyle and Herbert Dingle, and this inspired the following letter to a weekly magazine, *The Listener*, in 1960:

Sir:

The ears of a Hoyle may tingle;
The blood of a Hoyle may boil
When Hoyle pours hot oil upon Dingle,
And Dingle cold water on Hoyle.

But the dust of the wrangle will settle.
Old stars will look down on new soil.
The pot will lie down with the kettle,
And Dingle will mingle with Hoyle.

So what are you, the reader, expected to believe? Let us say this: If you are a student who has examinations to pass, or if you are an untenured professor who has to hold on to a job, be in no doubt whatever: The original conclusion is the canonically-accepted correct conclusion, namely that Albert has aged 50 years while his astronaut sister has aged but one. This is now firmly accepted truth. Indeed it has even been claimed that it has been “proved” experimentally by a scientist who took a clock on commercial airline flights around the world, and compared it on his return with a stay-at-home clock. For myself I have neither examinations to pass nor, alas, a job to hold on to, so I am not bound to believe one thing or the other, and I elect to hold my peace.

I do say this, however – that what anyone “believes” is not an essential point. It is not a matter of what Albert or Betty or Hoyle or Dingle or your professor or your employer “believes”. The real question is this: What is it that is predicted by the special theory of relativity? From this point of view it does not matter whether the theory of relativity is “true” or not, or whether it represents a correct description of the real physical world. Starting from the basic precepts of relativity, whether “true” or not, it must be only a matter of algebra (and simple algebra at that) to decide what is predicted by relativity.

A difficulty with this is that it is not, strictly speaking, a problem in special relativity, for special relativity deals with transformations between reference frames that are in uniform motion relative to one another. It is pointed out that Albert and Betty are not in uniform motion relative to one another, since one or the other of them has to change the direction of motion – i.e. has to accelerate. It could still be argued that, since motion is relative, one can regard either Albert or Betty as the one who accelerates – but the response to this is that only *uniform* motion is relative. Thus there is no symmetry between Albert and Betty. Betty either accelerates or experiences a gravitational field (depending on whether her experience is referred to Albert's or her own reference frame). And, since there is no symmetry, there is no paradox. This argument, however, admits that the age difference between Albert and Betty on Betty's return is not an effect of special relativity, but of general relativity, and is an effect caused by the acceleration (or gravitational field) experienced by Betty.

If this is so, there are some severe difficulties is describing the effect under general relativity. For example, whether the general theory allows for an instantaneous change in direction by Betty (and infinite deceleration), or whether the final result depends on how she decelerates – at what rate and for how long – must be determined by those who would tackle this problem. Further, the alleged age difference is supposed to depend upon the time during which Betty has been travelling and the length of her journey – yet the portion of her journey during which she is accelerating or decelerating can be made arbitrarily short compared with the time during which she is travelling at constant speed.

If the effect were to occur solely during the time when she was accelerating or decelerating, then the total length and duration of the constant speed part of her journey should not affect the age difference at all.

Since this chapter deals only with special relativity, and this is evidently not a problem restricted to special relativity, I leave the problem, as originally stated, here, without resolution, for readers to argue over as they will

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