12.3: Electrical Analogue

Suppose that an alternating potential difference $E = \hat{E} \sin \omega t$ is applied across an LCR circuit. We refer to Equation 11.6.3, and we see that the equation that governs the charge on the capacitor is

$$L\ddot{Q} + R\dot{Q} + \frac{Q}{C} = \hat{E} \sin \omega t. \tag{12.3.1}$$

We can differentiate both sides with respect to time, and divide by L, and hence see that the current is given by

$$\ddot{I} + \frac{R}{L}\dot{I} + \frac{1}{LC} I = \frac{\hat{E}\omega}{L} \cos \omega t. \tag{12.3.2}$$

We can compare this directly with Equation 12.2.2, so that we have

$$\gamma = \frac{R}{L}, \quad \omega_0^2 = \frac{1}{\sqrt{LC}}, \quad \hat{f} = \frac{\hat{E}\omega}{L}. \tag{12.3.3}$$

Then, by comparison with Equation 12.2.5, we see that I will lag behind E by α, where

$$\tan \alpha = \frac{\frac{R\omega}{L}}{\frac{1}{LC} - \omega^2} = \frac{R}{\frac{1}{C\omega} - L\omega}. \tag{12.3.4}$$

This is just what we obtain from the more familiar complex number approach to alternating current circuits.