4.7: Nonrigid Rotator

The rotational kinetic energy of a body rotating about a principal axis is $\frac{1}{2}I\omega^3$, where I is the moment of inertia about that principal axis, and the angular momentum is L = Iω. (For rotation about a nonprincipal axis, see Section 4.3.) Thus the rotational kinetic energy can be written as

$$KE_{rot} = \frac{L^2}{2I}. \tag{4.7.1}$$

When an asymmetric top is rotating about a nonprincipal axes, the body experiences internal stresses, which, if the body is nonrigid, result in periodic strains which periodically distort the shape of the body. As a result of this, rotational kinetic energy becomes degraded into heat; the rotational kinetic energy of the body gradually decreases. In the absence of external torques, however, the angular momentum is constant. Equation 4.7.1 shows that the kinetic energy is least for a given angular momentum when the moment of inertia is greatest. Thus eventually the body rotates about its principal axis of greatest moment of inertia. After that, it no longer loses kinetic energy to heat, because, when the body is rotating about a principal axis, it is no longer subject to internal stresses.

The time taken (the “relaxation time”) for a body to reach its final state of rotation about its principal axis of greatest moment of inertia depends, among other things, on how fast the body is rotating. A fast rotator will reach its final state relatively soon, whereas it takes a long time for a slow rotator to reach its final state. Thus it is not surprising to find that, among the asteroids, most of the fast rotators are principal axis rotators, whereas many slow rotators are also nonprincipal axis rotators. There are, however, a few fast rotators that are still rotating about a nonprincipal axis. It is assumed that such asteroids may have suffered a collision in the recent past.