5.2: Bouncing Balls

When a ball is dropped to the ground, one of four things may happen:

1. It may rebound with exactly the same speed as the speed at which it hit the ground. This is an *elastic collision*.
2. It may come to a complete rest, for example if it were a ball of soft putty. I shall call this a completely *inelastic collision*.
3. It may bounce back, but with a reduced speed. For want of a better term I shall refer to this as a somewhat *inelastic collision*.
4. If there happens to be a little heap of gunpowder lying on the table where the ball hits it, it may bounce back with a faster speed than it had immediately before collision. That would be a *superelastic collision*.

The ratio

$$\frac{\text{speed after collision}}{\text{speed before collision}}$$

is called the *coefficient of restitution*, for which I shall use the speed before collision symbol e. The coefficient is 1 for an elastic collision, less than 1 for an inelastic collision, zero for a completely inelastic collision, and greater than 1 for a **superelastic** collision. The ratio of kinetic energy (after) to kinetic energy (before) is evidently, *in this situation*, e^2.

If a ball falls on to a table from a height h_0, it will take a time $t_0 = \sqrt{2H_0lg}$ to fall. If the collision is somewhat inelastic it will then rise to a height $h_1 = e^2h_0$ and it will take a time *et* to reach height h_1. Then it will fall again, and bounce again, this time to a lesser height. And, if the coefficient of restitution remains the same, it will continue to do this for an infinite number of bounces. After a billion bounces, there is still an infinite number of bounces yet to come. The total distance travelled is

$$h = h_0 + 2h_0(e^2 + e^4 + e^6 + \dots) \tag{5.2.1}$$

and the time taken is

$$t = t_0 + 2t_0(e + e^2 + e^3 + \dots). \tag{5.2.2}$$

These are geometric series, and their sums are

$$h = h_0\left(\frac{1+e^2}{1-e^2}\right), \tag{5.2.3}$$

which is independent of g (i.e. of the planet on which this experiment is performed), and

$$t = t_0\left(\frac{1+e}{1-e}\right) \tag{5.2.4}$$

For example, suppose h_0 = 1 m, e = 0.5, g = 9.8 m s^{-2}, then the ball comes to rest in 1.36 s after having travelled 1.67 m after an infinite number of bounces.

Discuss

Does the ball ever stop bouncing, given that, after every bounce, there is still an infinite number yet to come; yet after 1.36 seconds it is no longer bouncing...?