3.2: Moment of Force

First, let's look at a familiar two-dimensional situation. In Figure III.1 I draw a force $\mathbf{F}$ and a point O. The moment of the force with respect to O can be defined as

Force times perpendicular distance from O to the line of action of $\mathbf{F}$.

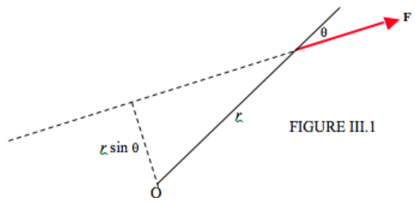

FIGURE III.1

Alternatively, (Figure III.2) the moment can be defined equally well by

Transverse component of force times distance from O to the point of application of the force.

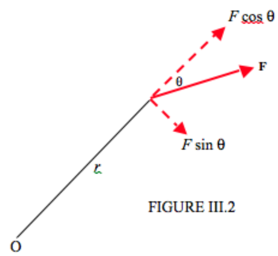

FIGURE III.2

Either way, the magnitude of the moment of the force, also known as the *torque*, is $rF\sin\theta$ We can regard it as a vector, $\boldsymbol{\tau}$, perpendicular to the plane of the paper:

$$\boldsymbol{\tau}=\mathbf{r}\times\mathbf{F} \tag{3.2.1}$$

Now let me ask a question. Is it correct to say the moment of a force with respect to (or "about") a *point* or with respect to (or "about") an *axis*?

In the above two-dimensional example, it does not matter, but now let me move on to three dimensions, and I shall try to clarify.

In Figure III.3, I draw a set of rectangular axes, and a force $\mathbf{F}$, whose position vector with respect to the origin is $\mathbf{r}$.

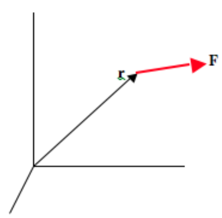

FIGURE III.3

The moment, or *torque*, of $\mathbf{F}$ *with respect to the origin* is the vector

$$\boldsymbol{\tau}=\mathbf{r}\times\mathbf{F} \tag{3.2.2}$$

The $x-, y-$ and z *-components* of $\boldsymbol{\tau}$ are the moments of $\mathbf{F}$ *with respect to the* $x-, y-$ *and z-axes.* You can easily find the components of $\boldsymbol{\tau}$ by expanding the cross product 3.2.2:

$$\boldsymbol{\tau}=\hat{\mathbf{x}}(yF_z-zF_y)+\hat{\mathbf{y}}(yF_x-xF_z)+\hat{\mathbf{z}}(xF_y-yF_x) \tag{3.2.3}$$

where $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ are the unit vectors along the x, y, z axes. In Figure III.4, we are looking down the x-axis, and I have drawn the components F_y and F_z, and you can see that, indeed, $\tau_x=yF_z-zF_y$.

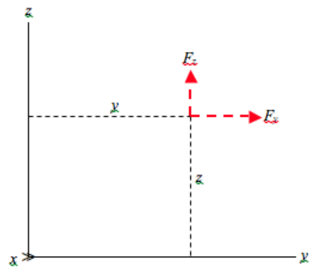

FIGURE III.4

The dimensions of moment of a force, or torque, are ML^2T^{-2}, and the SI units are N m. (It is best to leave the units as N m rather than to express torque in joules.)