1.9: Hemispheres

Uniform solid hemisphere

Figure I.4 will serve. The argument is exactly the same as for the cone. The volume of the elemental slice is $\pi y^2 \delta x = \pi(a^2 - x^2)\delta x$ and the volume of the hemisphere is $\frac{2\pi a^3}{3}$, so the mass of the slice is

$$M \times \pi(a^2 - x^2)\delta x \div (2\pi a/3) = \frac{3M(a^2-x^2)\delta x}{2a^3}$$

where M is the mass of the hemisphere. The first moment of mass of the elemental slice is x times this, so the position of the centre of mass is

$$\overline{x} = \frac{3}{2a^3}\int_0^a x(a^2 - x^2)dx = \frac{3a}{8}$$

Hollow hemispherical shell.

We may note to begin with that we would expect the centre of mass to be further from the base than for a uniform solid hemisphere.

Again, Figure I.4 will serve. The area of the elemental annulus is $2\pi a\delta x$ (NOT $2\pi y\delta x$!) and the area of the hemisphere is $2\pi a^2$. Therefore the mass of the elemental annulus is

$$M \times 2\pi a\delta x \div (2\pi a^2) = M\delta x/a$$

The first moment of mass of the annulus is *x* times this, so the position of the centre of mass is

$$\overline{x} = \int_0^a \frac{xdx}{a} = \frac{a}{2}$$