13.3: Holonomic Constraints

The complete description of a system of *[Math Processing Error]* unconstrained particles requires *[Math Processing Error]* coordinates. You can think of the state of the system at any time as being represented by a single point in *[Math Processing Error]*-dimensional space. If the system consists of molecules in a gas, or a cluster of stars, or a swarm of bees, the coordinates will be continually changing, and the point that describes the system will be moving, perhaps completely unconstrained, in its *[Math Processing Error]*-dimensional space.

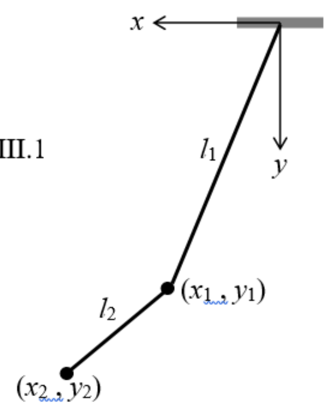
However, in many systems, the particles may not be free to wander anywhere at will; they may be subject to various *constraints*. A constraint that can be described by an equation relating the coordinates (and perhaps also the time) is called a *holonomic constraint*, and the equation that describes the constraint is a *holonomic equation*. If a system of *[Math Processing Error]* particles is subject to *[Math Processing Error]* holonomic constraints, the point in *[Math Processing Error]*-dimensional space that describes the system at any time is not free to move anywhere in *[Math Processing Error]*-dimensional space, but it is constrained to move over a surface of dimension *[Math Processing Error]*. In effect only *[Math Processing Error]* coordinates are needed to describe the system, given that the coordinates are connected by *[Math Processing Error]* holonomic equations.

Incidentally, I looked up the word "holonomic" in *The Oxford English Dictionary* and it said that the word was from the Greek ὅ*[Math Processing Error]*, meaning "whole" or "entire" and *[Math Processing Error]*, meaning "law". It also said "applied to a constrained system in which the equations defining the constraints are integrable or already free of differentials, so that each equation effectively reduces the number of coordinates by one; also applied to the constraints themselves."

As an example, consider a bar of wet soap slithering around in a hemispherical basin of radius *[Math Processing Error]*. You can describe its position in the basin by means of the usual two spherical angles *[Math Processing Error]*; the motion is otherwise constrained by its remaining in contact with the basin; that is to say it is subject to the holonomic constraint *[Math Processing Error]*. Thus instead of needing three coordinates to describe the position of a totally unconstrained particle, we need only two coordinates.

Or again, consider the double pendulum shown in Figure XIII.1, and suppose that the pendulum is constrained to swing only in the plane of the paper – or of the screen of your computer monitor.

FIGURE XIII.1



Two unconstrained particles would require six coordinates to specify their positions but this system is subject to four holonomic constraints. The holonomic equations *[Math Processing Error]* and *[Math Processing Error]* constrain the particles to be moving in a plane, and, if the strings are kept taut, we have the additional holonomic constraints *[Math Processing Error]* and *[Math Processing Error]*. Thus only two coordinates are needed to describe the system, and they could conveniently be the angles that the two strings make with the vertical.