

## 13.2: Generalized Coordinates and Generalized Forces

In two-dimensions the positions of a point can be specified either by its *rectangular coordinates*  $(x, y)$  or by its *polar coordinates*. There are other possibilities such as confocal conical coordinates that might be less familiar. In three dimensions there are the options of *rectangular coordinates*  $(x, y, z)$ , or *cylindrical coordinates*  $\rho, \phi, z$  or *spherical coordinates*  $r, \omega, \phi$  – or again there may be others that may be of use for specialized purposes (inclined coordinates in crystallography, for example, come to mind). The state of a molecule might be described by a number of parameters, such as the bond lengths and the angles between the bonds, and these may be varying periodically with time as the molecule vibrates and twists, and these bonds lengths and bond angles constitute a set of *coordinates* which describe the molecule. We are not going to think about any particular sort of coordinate system or set of coordinates. Rather, we are going to think about *generalized coordinates*, which may be lengths or angles or various combinations of them. We shall call these coordinates  $(q_1, q_2, q_3, \dots)$ . If we are thinking of a single particle in three-dimensional space, there will be three of them, which could be rectangular, or cylindrical, or spherical. If there were  $N$  particles, we would need  $3N$  coordinates to describe the system – unless there were some constraints on the system.

With each generalized coordinate  $q_j$  is associated a *generalized force*  $P_j$ , which is defined as follows. If the work required to increase the coordinate  $q_j$  by  $\delta q_j$  is  $P_j \delta q_j$ , then  $P_j$  is the generalized force associated with the coordinate  $q_j$ .

A generalized force need not always be dimensionally equivalent to a force. For example, if a generalized coordinate is an angle, the corresponding generalized force will be a torque.

One of the things that we shall want to do is to identify the generalized force associated with a given generalized coordinate.

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