

15.23: Some Mathematical Results

Before proceeding with the next section, I just want to establish few mathematical results, so that we do not get bogged down in heavy algebra later on when we should be concentrating on understanding physics.

First, if

$$\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}, \quad (15.23.1)$$

Then, by trivial differentiation,

$$\frac{d\gamma}{du} = \frac{\gamma^3 u}{c^2}. \quad (15.23.2)$$

$$\dot{\gamma} = \frac{\gamma^3 u \dot{u}}{c^2}. \quad (15.23.3)$$

From this, we quickly find that

$$\frac{\gamma u \dot{u}}{\dot{\gamma}} = c^2 - u^2. \quad (15.23.4)$$

Now for a small result concerning a scalar (dot) product.

Let \mathbf{A} be a vector such that $\mathbf{A} \cdot \mathbf{A} = A^2$.

Then

$$\begin{aligned} \frac{d}{dt}(A^2) &= 2A\dot{A} \text{ and } \frac{d}{dt}(\mathbf{A} \cdot \mathbf{A}) = 2\mathbf{A} \cdot \dot{\mathbf{A}} \\ A \cdot \dot{A} &= A\dot{A} \end{aligned} \quad (15.23.5)$$

We can now safely proceed to the next section.

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