22.6: Different Fundamental Quantities

We stated at the beginning of this chapter that any mechanical quantity could be expressed in terms of three fundamental quantities, *mass*, *length* and *time*. But there is nothing particularly magic about these quantities. For example, we might decide that we could express any mechanical quantity in terms of, say, *energy* E, *speed* V and *angular momentum* J. We might then say that the dimensions of area could be expressed as $E^{-2}V^2J^2$.(Verify this!)

While agreeing that such a system might be possible, you might feel that it would be totally absurd and there is no point in reading further.

But stop! Such a system is not only possible, but it is *normally and routinely used* in the field of *high-energy particle physics.* That, perhaps, is a surprise, but, if you are thinking of taking an interest in particle physics, read on.

The *units* generally used in particle physics to express the fundamental quantities energy, speed and angular momentum are GeV (or MeV, or TeV , etc) for energy, the speed of light c for speed, and the modified Planck constant $\hbar$ for angular momentum. There are often referred to as “natural” units, the speed of light being a “natural” unit of speed and ħ being a “natural” unit for angular momentum, whereas metre, kilogram and second are not so “natural” in this sense as they are “man-made”. It is true that a GeV is not particularly “natural”, but at least a system with GeV, c and $\hbar$ as fundamental quantities is certainly more “natural” than metre-kilogram-second.

In any case, the dimensions of *mass* in this system are EV^{-2}. (You can see this immediately, for example from Einstein’s famous equation $E = mc^2$.) The units used in this system are GeV/c^2. Thus the rest mass of a proton is 0.9383 GeV/c^2, and the rest mass of an electron is 0.5110 MeV/c^2. One way to interpret this, if you like, is to say that the rest-mass energy of a proton (i.e. its m_0c^2) is 0.9383 GeV.

Likewise the dimensions of linear momentum are EV^{-1}, and units in which it is expressed are GeV/c. (You can see this, for example, if you look at the energy and momentum of a photon: E = hν, p = h/γ, from which $\frac{P}{E} = \frac{1}{v\gamma} = \frac{1}{c}$)

Torque (which has the same dimensions as energy) is equal to rate of change of angular momentum, from which we see that time has dimensions E−1J and could be expressed in units of ħ/GeV. Alternatively you can see that [time] = ħ/GeV immediately from Planck’s equation . $E = \hbar\omega$ And speed is distance over time, so that we see that distance, or length, has dimensions $E^{-1}VJ$, and hence units ħc/GeV.

Using data from the 2010 Particle Physics Booklet, I calculate as follows.

Mass:	1 GeV/c^2 = 1.782 661 76 ~ 10^{-27} kg
Length:	1 $\hbar c$/GeV = 1.973 269 63 ~ 10^{-16} m
Time:	1 $\hbar$/GeV = 6.582 118 99 ~ 10^{-26} s
Energy:	1 GeV = 1.602 176 49 ~ 10^{-10} J
Linear Momentum:	1 GeV/c = 5.344 285 50 ~ 10^{-19} kg m s^{-1}

I give here a table of the dimensions (in terms of EVJ) of the same quantities as in the table of page 2. I dare say some of them are never likely to be needed, but some certainly will be needed, and, rather than predict which will be useful and which not, I might as well give them all. The dynamic viscosity of water at room temperature is about 10^{-3} kg m^{-1} s^{-1}, or 10^{-3} dekapoise. I cannot imagine anyone needing to know that the dynamic viscosity of water at room temperature is about 7.3 ~ 10^{-18} $(GeV)^3/(c^3\hbar^2)$, or that its surface tension is so many $(GeV)^3/(c\hbar)^2$ − but you never know

Mass	EV^{-2}	GeV/c^2
Length	$\mathrm{E}^{-1}\mathrm{VJ}$	$\hbar c/\mathrm{GeV}$
Time	$\mathrm{E}^{-1}\mathrm{J}$	$\hbar/\mathrm{GeV}$
Density	$\mathrm{E}^2\mathrm{V}^{-5}\mathrm{J}^{-3}$	$(\mathrm{GeV})^2/(c^5\hbar^3)$
Speed	V	c
Acceleration	EVJ^{-1}	$\mathrm{GeV}\ c/\hbar$
Force	$\mathrm{E}^2\mathrm{V}^{-1}\mathrm{J}^{-1}$	$(\mathrm{GeV})^2/(c\hbar)$
Work, Energy, Torque	E	GeV
Action	J	$\hbar$
Rotational inertia	$\mathrm{E}^{-1}\mathrm{J}^2$	$\hbar^2/\mathrm{GeV}$
Angular speed	EJ	$\mathrm{GeV}\ \hbar$
Angular acceleration	$\mathrm{E}^2\mathrm{J}^2$	$(\mathrm{GeV})^2\hbar^2$
Angular momentum	J	$\hbar$
Pressure, elastic modulus	$\mathrm{E}^4\mathrm{V}^{-3}\mathrm{J}^{-3}$	$(\mathrm{GeV})^4/(c\hbar)^3$
Gravitational constant	$\mathrm{E}^{-2}\mathrm{V}^5\mathrm{J}$	$c^5\hbar/(\mathrm{GeV})^2$
Dynamic viscosity	$\mathrm{E}^3\mathrm{V}^{-3}\mathrm{J}^{-2}$	$(\mathrm{GeV})^3/(c^3\hbar^2)$
Kinematic viscosity	$\mathrm{E}^{-1}\mathrm{V}^2\mathrm{J}$	$c^2\hbar/(\mathrm{GeV})$
Force constant	$\mathrm{E}^3\mathrm{V}^{-2}\mathrm{J}^{-2}$	$(\mathrm{GeV})^3/(c\hbar)^2$
Torsion constant	E	GeV
Surface tension	$\mathrm{E}^3\mathrm{V}^{-2}\mathrm{J}^{-2}$	$(\mathrm{GeV})^3/(c\hbar)^2$
Schrödinger wavefunction Ψ	$\mathrm{E}^2\mathrm{V}^{-3/2}\mathrm{J}^{-2}$	$(\mathrm{GeV})^2/(c^{3/2}\hbar^2)$
Schrödinger wavefunction ψ	$\mathrm{E}^{3/2}\mathrm{V}^{-3/2}\mathrm{J}^{-3/2}$	$(\mathrm{GeV})^{3/2}/(c^{3/2}\hbar^{3/2})$