22.5: Dimensionless Quantities

Dimensionless Quantities are used extensively in fluid dynamics. For example, if a body of some difficult shape, such as an aircraft, is moving through a fluid at speed V, it will experience all sorts of forces, external and internal. The ratio of the internal forces to the external forces will depend upon its speed, and the viscosity of the fluid, and the size of the body. By “size” of a body of “difficult” shape we could take the distance between two defined points on the body, such as its top and bottom, or its front and back, or its greatest width, or whatever. Call that distance l. But the ratio of the internal to the viscous forces is dimensionless, so it must depend on some combination of the viscosity, speed V and linear size l that is dimensionless. Since V and l do not contain M in their dimensions, the viscosity concerned must be the *kinematic* viscosity ν, which is the ratio of dynamic viscosity to density and does not have M in its dimensions. So, what combination of ν, V and l is dimensionless?

It is easy to see that $\frac{VI}{V}$ - or any power of it, positive, negative, zero, integral, nonintegral - is dimensionless. $\frac{VI}{V}$ is called the Reynolds number, and is usually given the symbol Re. It is supposed that if you make a small model of the aircraft (or whatever the body is) and move it through some fluid and some speed, the ratio of internal to viscous forces in the model will be the same as in the real thing provided that the Reynolds numbers in the model and in the real thing are the same.

There are oodles of similar dimensionless numbers used in fluid dynamics, such as **Froude’s number** and **Mach number**, but this example of Reynolds number should give the general idea.