

15.19: The Transverse and Oblique Doppler Effects

I pointed out in Section 15.18 that the observed Doppler effect, when the transmitted signal is electromagnetic radiation and observer or source or both are travelling at speed comparable to that of light, is a combination of two effects – the “true” Doppler effect, caused by the changing distance between source and observer, and the effect of time dilation. This raises the following questions.

Transverse Doppler effect

If a source of light is moving at right angles to (transverse to) the line joining observer to source, will the observer see a change in frequency or wavelength, even though the distance between observer and source at that instant is not changing? The answer is yes, certainly, and the effect is sometimes called the “transverse Doppler effect”, although it is the effect of relativistic time dilation rather than of a true Doppler effect.

Thus let us suppose that a source is moving transverse to the line of sight at a speed described by its parameter β or γ , and that the period of the radiation referred to the reference frame in which the source is at rest is T_0 and the frequency is ν_0 . The time interval between emission of consecutive wavecrests when referred to the frame in which the observer is at rest is longer by the gamma-factor, and the frequency is correspondingly less. That is, the frequency, referred to the observer’s reference frame, is

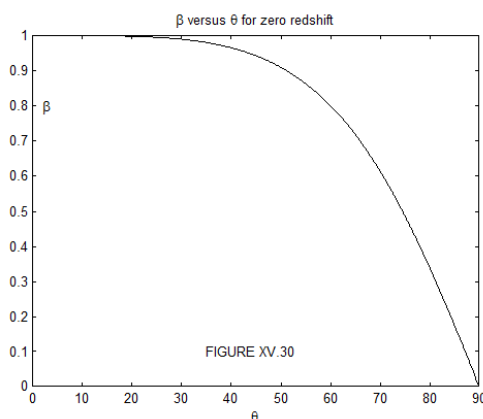
$$\nu = \frac{\nu_0}{\gamma} = \nu_0 \sqrt{1 - \beta^2} \quad (15.19.1)$$

The light from the source is therefore seen by the observer to be redshifted, even though there is no radial velocity component.

Oblique Doppler Effect

This raises a further question. Suppose a source is moving almost but not quite at right angles to the line of sight, so that it has a large transverse velocity component, and a small velocity component towards the observer. In that case, its “redshift” resulting from the time dilation might be appreciable, while its “blueshift” resulting from “true” Doppler effect (the decreasing distance between source and observer) is still very small. Therefore, even though the distance between source and observer is slightly decreasing, there is a net redshift of the spectrum. This is in fact correct, and is the “oblique Doppler effect”.

In Figure XV.30, a source S is moving at speed β times the speed of light in a direction that makes an angle θ with the line of sight. It is emitting a signal of frequency ν_0 in S. (I am here using the frame “in S” as earlier in the chapter to mean “referred to a reference frame in which S is at rest.”) The signal arrives at the observer O at a slightly greater frequency as a result of the decreasing distance of S from O, and at a slightly lesser frequency as a result of the time dilation, the two effects opposing each other.



The frequency of the received signal at O, in O, is

$$\nu = \frac{\nu_0 \sqrt{1 - \beta^2}}{1 - \beta \cos \theta}. \quad (15.19.2)$$

For a given angle θ the redshift is zero for a speed of

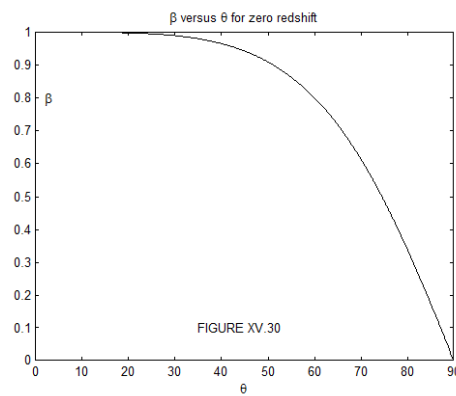
$$\beta = \frac{2 \cos \theta}{1 + \cos^2 \theta} \quad (15.19.3)$$

or, for a given speed, the direction of motion resulting in a zero redshift is given by

$$\beta \cos^2 \theta - 2 \cos \theta + \beta = 0. \quad (15.19.4)$$

This relation is shown in Figure XV.30. (Although Equation 15.19.4 is quadratic in $\cos \theta$ there is only one real solution θ for β between 0 and 1. Prove this assertion.) It might be noted that if the speed of the source is 99.99% of the speed of light the observer will see a redshift unless the direction of motion of S is no further than $9^\circ 36'$ from the line from S to O. That is worth repeating: S is moving very close to the speed of light in a direction that is close to being directly towards the observer; the observer will see a redshift.

Equation 15.19.2 which gives ν as a function of θ for a given β , will readily be recognized at the equation of an ellipse of eccentricity β , semi minor axis ν_0 and semi major axis $\gamma\nu_0$. This relation is shown in Figure XV.31 for several β . The curves are red where there is a redshift and blue where there is a blueshift. There is no redshift or blueshift for $\beta = 0$, and the ellipse for that case is a circle and is drawn in black.



An alternative and perhaps more useful way of looking at Equation 15.19.2 is to regard it as an equation that gives β as a function of θ for a given Doppler ratio $\frac{\nu}{\nu_0}$. For example, if the Doppler ratio of a galaxy is observed to be 0.75, the velocity vector of the galaxy could be any arrow starting at the black dot and ending on the curve marked 0.75. The curves are ellipses with semi major axis equal to $\frac{1}{\sqrt{1-(\frac{\nu}{\nu_0})^2}}$ and semi minor axis $\frac{1}{(1-(\frac{\nu}{\nu_0})^2)}$.

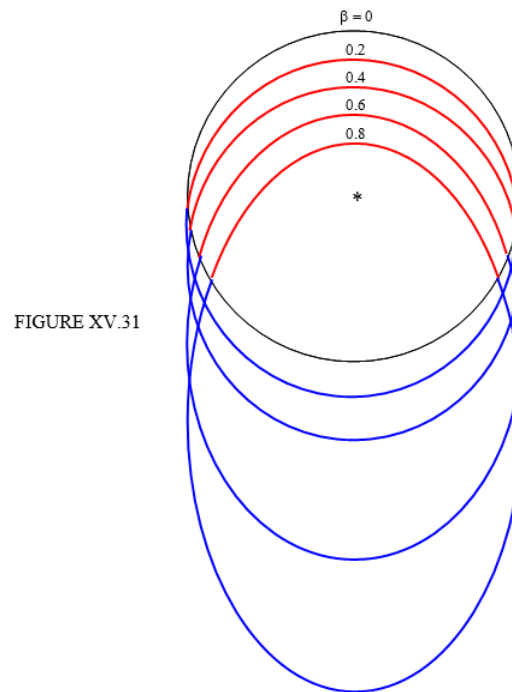


FIGURE XV.31

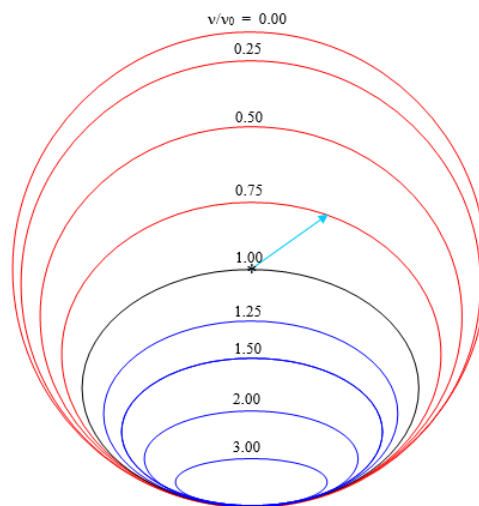


FIGURE XV.32

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