15.8: Timelike and Spacelike 4-Vectors

I am going to refer some events to a coordinate system whose origin is here and now and which is moving at the same velocity as you happen to be moving. In other words, you are sitting at the origin of the coordinate system, and you are stationary with respect to it. Let us suppose that an event A occurs at the following coordinates referred to this reference frame, in which the distances $x_1,\ y_1,\ z_1$ are expressed in light-years (lyr) the time t_1 is expressed in years (yr).

$$x_1 \ = 2 \quad y_1 \ = 3 \quad z_1 \ = 7 \quad t_1 \ = \ -1$$

A "light-year" is a unit of distance used when describing astronomical distances to the layperson, and it is also useful in describing some aspects of relativity theory. It is the distance travelled by light in a year, and is approximately 9.46 % 10^{15} m or 0.307 parsec (pc). Event A, then, occurred a year ago at a distance of $\sqrt{62} \ = \ 7.87$ lyr, when referred to this reference frame. Note that, if referred to a reference frame that coincides with this one at $t = 0$, but is moving with respect to it, all four coordinates might be different, and the distance $\sqrt{x^2+y^2+z^2}$ and the time of occurrence would be different, but, according to the way in which we have defined space and time by the Lorentz transformation, the quantity $\sqrt{x^2+y^2+z^2-c^2t^2}$ would be the same.

Imagine now a second event, B, which occurs at the following coordinates:

$$x_2 \ = 5 \quad y_2 \ = 8 \quad z_2 \ = 10 \quad t_2 \ = \ +2$$

That is to say, when referred to the same reference frame, it will occur in two years' time at a distance of $\sqrt{189} = 13.75$ lyr.

The 4-vector **s** = **B** - **A** connects these two events, and the magnitude s of **s** is the *interval* between the two events. Note that the *distance* between the two events, *when referred to our reference frame*, is $\sqrt{(5-2)^2+(8-3)^2+(10-7)^2} = 6.56$ lyr. The *interval* between the two events is $\sqrt{(5-2)^2+(8-3)^2+(10-7)^2-(2+1)^2} = 5.83$ lyr, and this is independent of the velocity of the reference frame. That is, if we "rotate" the reference frame, it obviously makes no difference to the *interval* between the two events, which is *invariant*.

As another example, consider two events A and B whose coordinates are

$$x_1 \ = 2 \quad y_1 \ = 5 \quad z_1 \ = 3 \quad t_1 \ = \ -2$$

$$x_2 \ = 3 \quad y_2 \ = 7 \quad z_2 \ = 4 \quad t_2 \ = \ +6$$

with distances, as before, expressed in lyr, and times in yr. Calculate the interval between these two events – i.e. the magnitude of the 4-vector connecting them. If you carry out this calculation, you will find that s^2 = -58, so that the interval s is *imaginary* and equal to 7.62i.

So we see that some pairs of events are connected by a 4-vector whose magnitude is real, and other pairs are connected by a 4-vector whose magnitude is imaginary. There are differences in character between real and imaginary intervals, but, in order to strip away distractions, I am going to consider events for which $y \quad = \quad z \quad = 0$. We can now concentrate on the essentials without being distracted by unimportant details.

Let us therefore consider two events A and B whose coordinates are

$$x_1 \ = \ 2 \ \text{lyr} \ t_1 = -2 \ \text{yr}$$

$$x_2 \ = \ 3 \ \text{lyr} \ t_2 = +6 \ \text{yr}$$

These events and the 4-vector connecting them are shown in Figure XV.7.Event A happened two years ago (referred to our reference frame); event B will occur (also referred to our reference frame) in six years' time. The square of the *interval* between the two events (which is invariant) is -63 lyr^2, and the interval is imaginary. If someone wanted to experience both events, he would have to travel only 1 lyr (referred to our reference frame), and he could take his time, for he would have eight years (referred to our reference frame) in which to make the journey to get to event B in time. He couldn't totally dawdle, however; he would have to travel at a speed of at least $\frac{1}{8}$ times the speed of light, but that's not extremely fast for anyone well versed in relativity.

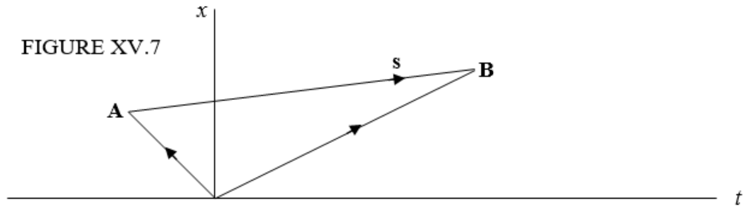

FIGURE XV.7

Let's look at it another way. Let's suppose that event A is the *cause* of event B. This means that some agent must be capable of conveying some information from A to B at a speed at least equal to $\frac{1}{8}$ times the speed of light. That may present some technical problems, but it presents no problems to our imagination.

You'll notice that, in this case, the interval between the two events – i.e. the magnitude of the 4-vector connecting them - is *imaginary*. A 4-vector whose magnitude is *imaginary* is called a *timelike* 4-vector.There is quite a long time between events A and B, but not much distance.

Now consider two events A and B whose coordinates are

$$x_1 = 2 \text{ lyr } t_1 = -1 \text{ yr}$$

$$x_2 = 7 \text{ lyr } t_2 = +3 \text{ yr}$$

The square of the magnitude of the interval between these two events is +9 lyr^2, and the interval is real. A 4-vector whose magnitude is *real* is called a *spacelike* 4-vector. It is shown in Figure XV.8.

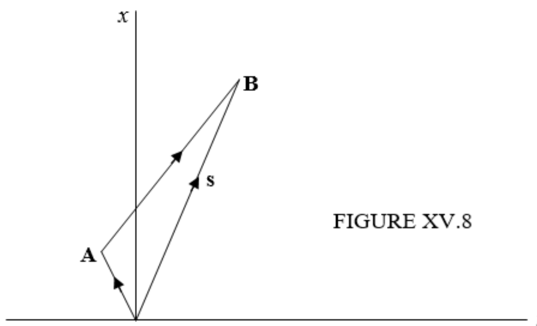

FIGURE XV.8

Perhaps I could now ask how fast you would have to travel if you wanted to experience both events. They are quite a long way apart, and you haven't much time to get from one to the other. Or, if event A is the *cause* of event B, how fast would an information-carrying agent have to move to convey the necessary information from A in order to instigate event B? Maybe you have already worked it out, but I'm not going to ask the question, because in a later section we'll find that *two events A and B cannot be mutually causally connected if the interval between them is real.* Note that I have said "mutually"; this means that A cannot cause B, and B cannot cause A. A and B must be quite independent events; there simply is too much space in the interval between them for one to be the cause of the other. It does not mean that the two events cannot have a common cause. Thus, Figure XV.9 shows two events A and B with a spacelike interval between them (very steep) and a third event C such the intervals CA and CB (very shallow) are timelike. C could easily be the cause of both A and B; that is, A and B could have a common cause.But there can be no *mutual* causal connection between A and B. (It might be noted parenthetically that Charles Dickens temporarily nodded when he chose the title of his novel *Our Mutual Friend*. He really meant our common friend. C was a friend common to A and to B. A and B were friends mutually to each other.)

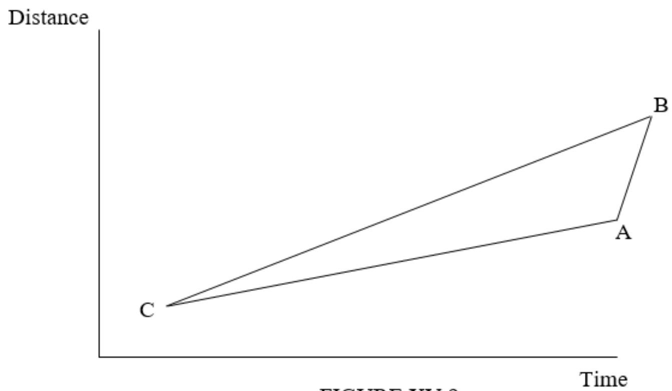

FIGURE XV.9

? Exercise 15.8.1

The distance of the Sun from Earth is 1.496 % 10^{11} m. The speed of light is 2.998 % 10^{8} m s^{-1}.

How long does it take for a photon to reach Earth from the Sun?

Event A: A photon leaves the Sun on its way to Earth. Event B: The photon arrives at Earth.

What is the interval (i.e. *s* in 4-space) between these two events?