15.20: Acceleration

Figure XV.33 shows two references frames, Σ and Σ', the latter moving at speed v with respect to the former.

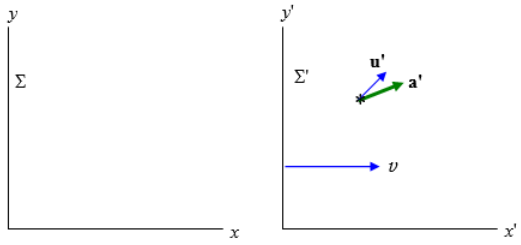

FIGURE XV.33

A particle is moving with acceleration $\mathbf{a}'$ in Σ'. (" in Σ' " = "referred to the reference frame Σ' ".) The velocity is not necessarily, of course, in the same direction as the acceleration, and we'll suppose that its velocity in Σ' is $\mathbf{u}'$. The acceleration and velocity components in Σ' are $a'_{x'}, a'_{y'}, u'_{x'}, u'_{y'}$.

What is the acceleration of the particle in Σ? We shall start with the x-component.

The x-component of its acceleration in Σ is given by

$$a_x = \frac{du_x}{dt}, \tag{15.20.1}$$

where

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}} \tag{15.20.2}$$

and

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right) \tag{15.5.19}$$

Equations 15.16.2 and 15.5.19 give us

$$du_x = \frac{du_x}{du'_x} du'_x = \frac{du'_x}{\gamma^2(1 + \frac{u'_x v}{c^2})^2} \tag{15.20.3}$$

and

$$dt = \frac{\partial t}{\partial t'} dt' + \frac{\partial t}{\partial x'} dx' = \gamma dt' + \frac{\gamma v}{c^2} dx' \tag{15.20.4}$$

On substitution of these into Equation 15.20.1 and a very little algebra, we obtain

$$a_x = \frac{a'}{\gamma^3(1 + \frac{u'_x v}{c^2})^3} \tag{15.20.5}$$

The y-component of its acceleration in Σ is given by

$$a_y = \frac{du_y}{dt}, \tag{15.20.6}$$

We have already worked out the denominator dt (Equation 15.20.4). We know that

$$u_y = \frac{u'_{y'}}{\gamma(1 + \frac{u'_{x'} v}{c^2})} \tag{15.16.3}$$

from which

$$du_y = \frac{\partial u_y}{\partial u'_{x'}} + \frac{\partial u_y}{\partial u'_{y'}} \partial u'_{y'} = \frac{1}{\gamma} \left(- \frac{v u'_{y'}}{c^2 \left(1 + \frac{v u'_{x'}}{c^2} \right)^2} du'_{x'} + \frac{1}{1 + \frac{v u'_{x'}}{c^2}} du'_{y'} \right). \tag{15.20.7}$$

Divide Equation 15.20.7 by Equation 15.20.4 to obtain

$$a_y = \frac{1}{\gamma^2} \left(- \frac{v u'_{y'}}{c^2 \left(1 + \frac{v u'_{x'}}{c^2} \right)^2} a'_{x'} + \frac{1}{1 + \frac{v u'_{x'}}{c^2}} a'_{y'} \right). \tag{15.20.8}$$

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