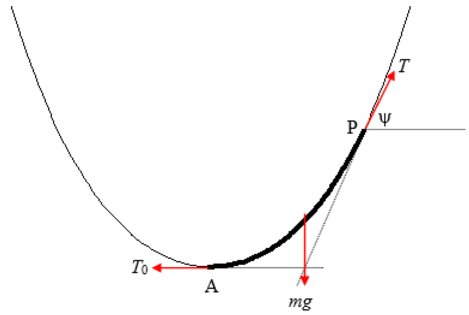
18.2: The Intrinsic Equation to the Catenary

We consider the equilibrium of the portion AP of the chain, A being the lowest point of the chain (Figure XVIII.1).

FIGURE XVIII.1



It is in equilibrium under the action of three forces: The horizontal tension T_0 at A; the tension T at P, which makes an angle ψ with the horizontal; and the weight of the portion AP. If the mass per unit length of the chain is μ and the length of the portion AP is s, the weight is μsg. It may be noted than these three forces act through a single point.

Clearly,

$$T_0 = T\cos\psi \tag{18.2.1}$$

and

$$\mu s\text{g} = T\sin\psi, \tag{18.2.2}$$

from which

$$(\mu s\text{g})^2 + T_0^2 = T^2 \tag{18.2.3}$$

and

$$\tan\psi = \frac{\mu\text{g}s}{T_0} \tag{18.2.4}$$

Introduce a constant a having the dimensions of length defined by

$$a = \frac{T_0}{\mu\text{g}}. \tag{18.2.5}$$

Then Equations 18.2.3 and 18.2.4 become

$$T = \mu\text{g}\sqrt{s^2 + a^2} \tag{18.2.6}$$

and

$$s = a\tan\psi. \tag{18.2.7}$$

Equation 18.2.7 is the *intrinsic equation* of the catenary.