12.1: More on Differential Equations

In Section 11.4 we argued that the most general solution of the differential equation

$$ay'' + by' + cy = 0 \tag{11.4.1}$$

is of the form

$$y = Af(x) + Bg(x). \tag{11.4.2}$$

In this chapter we shall be looking at equations of the form

$$ay'' + by' + cy = h(x). \tag{12.1.1}$$

If you look back at the arguments that led to the conclusion that Equation 11.4.2 is the most general solution of Equation 11.4.1, you will be able to conclude that 11.4.2 is still a solution of Equation 11.4.1, but it is not the only solution. There is another function that is a solution, so that the most general solution to equation 12.1.1 is of the form

$$y = Af(x) + Bg(x) + H(x). \tag{12.1.2}$$

The solution $H(x)$ is called the *particular integral*, while the part $Af(x) + Bg(x)$ is the *complementary function.* I shall be dealing in this chapter mainly with the particular integral, though we shall not entirely forget the complementary function.

This is a book on classical mechanics rather than on differential equations, so I am not going into how to obtain the particular integral $H(x)$ for a given $h(x)$. There are several ways of doing it; for those who know what they are and are in practice with them, Laplace transforms are among the more attractive methods. Some readers will already know how to do it. They will doubtless want to go back to Equation 11.6.3 in the previous chapter and try their hand at finding the particular integral for that. Those who do not may be happy and content to take my word for the particular integral in the sections that follow, or perhaps at least to differentiate it to verify that it is indeed a solution.