8.1: Introduction

As it goes about its business, a particle may experience many different sorts of forces. In Chapter 7, we looked at the effect of forces that depend only on the *speed* of the particle. In a later chapter we shall look at forces that depend only on the *position* of the particle. (Such forces will be called *conservative* forces.) In this chapter we shall look at the effect of forces that vary with *time*. Of course, it is quite possible that an unfortunate particle may be buffeted by forces that depend on its speed, on its position, and on the time - but, as far as this chapter is concerned, we shall be looking at forces that depend only on the time.

Everyone knows that Newton's second law of motion states that when a force acts on a body, the momentum of the body changes, and the rate of change of momentum is equal to the applied force. That is, $F = \frac{dp}{dt}$. If a force that varies with time, $F(t)$, acts on a body for a time T, the integral of the force over the time, $\int_0^T F(t)dt$ is called the *impulse* of the force, and it results in a *change of momentum* ΔP which is equal to the impulse. I shall use the symbol J to represent impulse, or the time integral of a force. Its SI units would be N s, and its dimensions MLT^{-1}, which is the same as the dimensions of momentum.

Thus, Newton's second law of motion is

$$F = \dot{p}.$$

When integrated over time, this becomes

$$J = \Delta p.$$

Likewise, in rotational motion, the *angular momentum* L of a body changes when a *torque* τ acts on it, such that the rate of change of angular momentum is equal to the applied torque:

$$\tau = \dot{L}.$$

If the torque, which may vary with time, acts over a time T, the integral of the torque over the time, $\int_0^T \tau dt$ is the *angular impulse*, which I shall denote by the symbol K, and it results in a change of the angular momentum:

$$K = \Delta L.$$

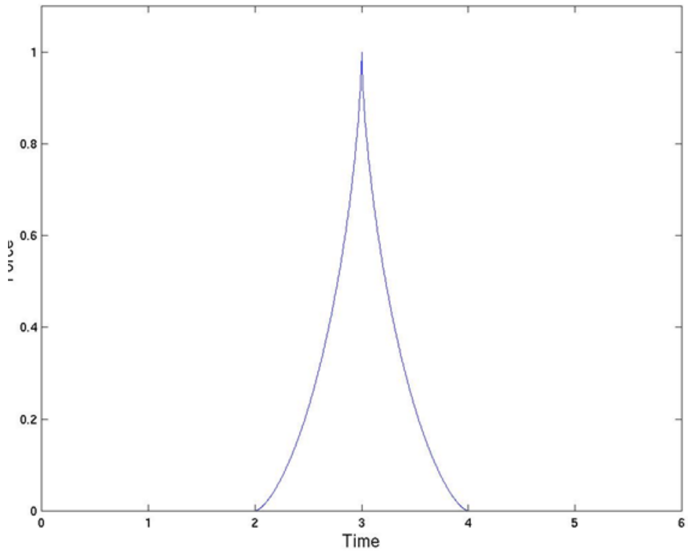
The SI units of angular impulse are N m s, and the dimensions are ML^{2}T^{-1}, which are the same as those of angular momentum.

For example, suppose that a golf ball is struck by a force varies with time as

$$F = \hat{F}\left[1 - \left(\frac{|t - t_0|}{\tau}\right)^{\frac{2}{3}}\right]^{\frac{3}{2}}$$

This may look like a highly-contrived and unlikely function, but in Figure VIII.1 I have drawn it for $\hat{F} = 1$, $t_0 = 3$, $\tau = 1$ and you will see that it is a reasonably plausible function. The club is in contact with the ball from time $t_0 - \tau$ to $t_0 + \tau$.

FIGURE VIII.1



If the ball, of mass m, starts from rest, what will be its speed V immediately after it leaves the club? The answer is that its new momentum, mV, will equal the impulse (or the time integral) of the above force:

$$mV = \hat{F}\int_{t_0-\tau}^{t_0+\tau}\left[1-\left(\frac{|t-t_0|}{\tau}\right)^{\frac{2}{3}}\right]^{\frac{3}{2}}dt.$$

This is very easy to understand; if there is any difficulty it might be in the mechanics of working out this integral. It is good integration practice, but, if you can't do it after a reasonable effort, and you want a hint, ask me (jtatum@uvic.ca) and I'll see what I can do. You should get

$$mV = \frac{3\pi}{16}\hat{F}\tau = 0.589\hat{F}\tau$$

Example 8.1.1

Here is a very similar example, except that the integration is rather easier. A ball of mass 500 g, initially at rest, is struck with a force that varies with time as

$$F = \hat{F}\left[1-\left(\frac{t-t_0}{\tau}\right)^2\right]^{\frac{1}{2}},$$

where $\hat{F}$ = 4000N, t_0 = 10 ms, t = 3 ms. Draw (accurately, by computer) a graph of F versus time (it doesn't look quite like Figure VIII.1). How fast is the ball moving immediately after impact?

(I make it 37.7 m s^{-1}.)