6.3C: Body thrown vertically upwards with initial speed v₀

If we measure y upwards from the ground, the equation of motion is

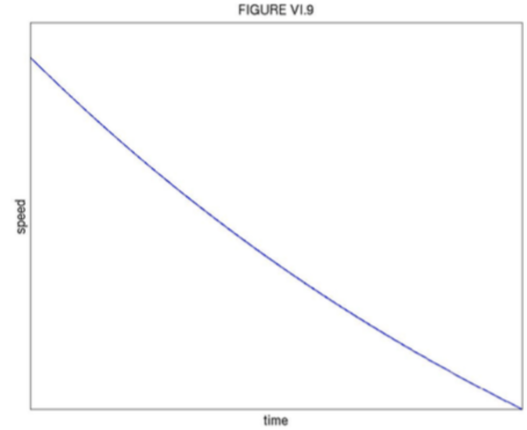
$$\ddot{y} = -g - \gamma v = -\gamma(\hat{v} + v). \tag{6.3.24}$$

The *first time integral* is

$$v = -\hat{v} + (v_0 + \hat{v})e^{-\gamma t} \tag{6.3.25}$$

and this is shown in Figure VI.9.

FIGURE VI.9



It reaches a maximum height after time T, when $v = 0$ (at which time the acceleration is just $-g$):

$$t + \frac{1}{\gamma}\ln(1 + \frac{v_0}{\hat{v}}). \tag{6.3.26}$$

The *second time integral* (obtained by writing v as $\frac{dy}{dt}$ in Equation 6.3.25) and the *space integral* (obtained by writing $\ddot{y}$ as $v\frac{dv}{dy}$ in the equation of motion) require some patience, but the results are

$$y = \frac{(v_0 + \hat{v}}{\gamma}(1 - e^{-\gamma t} - \hat{v}t, \tag{6.3.27}$$

$$v = v_0 - \gamma y - \hat{v}\ln\left(\frac{\hat{v} + v_0}{\hat{v} + v}\right). \tag{6.3.28}$$

These are illustrated in Figures VI.10 and VI.11.

FIGURE VI.10

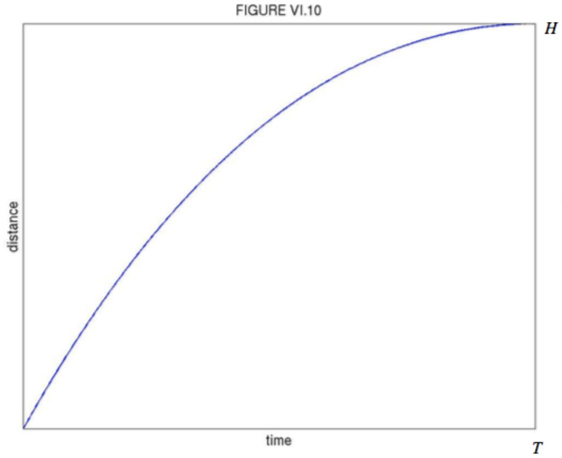


FIGURE VI.11