

### 3.11: Torque and Rate of Change of Angular Momentum

#### Theorem:

The rate of change of the total angular momentum of a system of particles is equal to the sum of the external torques on the system.

Thus:

$$L = \sum_i \mathbf{r}_i \times \mathbf{p}_i \quad (3.11.1)$$

$$\therefore \quad \dot{\mathbf{L}} = \sum_i \dot{\mathbf{r}}_i \times \dot{\mathbf{p}}_i \quad (3.11.2)$$

But the first term is zero, because  $\dot{\mathbf{r}}$  and  $\mathbf{p}_i$  are parallel.

Also

$$\dot{\mathbf{r}}_i = \mathbf{F}_i + \sum_j \mathbf{F}_{ij} \quad (3.11.3)$$

$$\dot{\mathbf{L}}_i = \sum_i \mathbf{r}_i \times (\mathbf{r}_i + \sum_j \mathbf{F}_{ij}) = \sum_i \mathbf{r}_i \times \mathbf{F}_i + \sum_i \mathbf{r}_i \times \sum_j \mathbf{F}_{ij}$$

$$\therefore \quad \sum_i \mathbf{r}_i \times \mathbf{F}_i + \sum_i \mathbf{r}_i \times \sum_j \mathbf{F}_{ij}$$

But  $\sum_i \sum_j \mathbf{F}_{ij} = 0$  by Newton's third law of motion, and so  $\sum_i \sum_j \mathbf{r}_i \times \mathbf{F}_{ij} = 0$ .

Also  $\sum_i \mathbf{r}_i \times \mathbf{F}_i = \boldsymbol{\tau}$ , and so we arrive at

$$\dot{\mathbf{L}} = \boldsymbol{\tau} \quad (3.11.4)$$

which was to be demonstrated.

#### Corollary: Law of Conservation of Angular Momentum

If the sum of the external torques on a system is zero, the angular momentum is constant.

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