11.3: Torsion Pendulum

A torsion pendulum consists of a mass of rotational inertia I hanging by a thin wire from a fixed point. If we assume that the torque required to twist the wire through an angle θ is proportional to θ and to no higher powers, then the ratio of the torque to the angle is called the torsion constant c. It depends on the shear modulus of the material of which the wire is made, is inversely proportional to its length, and, for a wire of circular cross-section, is proportional to the fourth power of its diameter. A thick wire is much harder to twist than a thin wire. Ribbonlike wires have comparatively small torsion constants. The work required to twist a wire through an angle θ is $\frac{1}{2}c\theta^2$.

When a torsion pendulum is oscillating, its Equation of motion is

$$I\ddot{\theta} = -c\theta. \tag{11.3.1}$$

This is an Equation of the form 11.1.5 and is therefore simple harmonic motion in which $\omega = \sqrt{\frac{c}{I}}$. This example, incidentally, shows that our second definition of simple harmonic motion (i.e. motion that obeys a differential Equation of the form of Equation 11.1.5) is a more general definition than our introductory description as the projection upon a diameter of uniform motion in a circle. In particular, do not imagine that ω here is the same thing as $\dot{\theta}$!

Exercise 11.3.1

Write down the torsional analogues of all the Equations given for linear motion in Sections 11.1 and 11.2.