

3.7: Angular Momentum

Notation:

- \mathbf{L}_C = angular momentum of system with respect to centre of mass C.
- \mathbf{L} = angular momentum of system relative to some other origin O.
- $\bar{\mathbf{r}}$ = position vector of C with respect to O.
- \mathbf{P} = linear momentum of system with respect to O.
- (The linear momentum with respect to C is, of course, zero.)

 Theorem:

$$\mathbf{L} = \mathbf{L}_C + \bar{\mathbf{r}} \times \mathbf{P} \quad (3.7.1)$$

Thus:

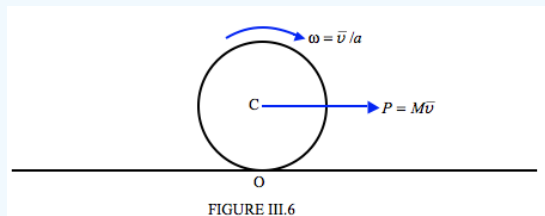
$$\begin{aligned} \mathbf{L} &= \sum \mathbf{r}_i \times \mathbf{p}_i = \sum m_i (\mathbf{r}_i \times \mathbf{v}_i) = \sum m_i (\bar{\mathbf{r}} + \mathbf{r}'_i) \times (\bar{\mathbf{v}} + \mathbf{v}'_i) \\ &= (\bar{\mathbf{r}} \times \bar{\mathbf{v}}) \sum m_i + \bar{\mathbf{r}} \times \sum m_i \mathbf{v}'_i + (\sum m_i \mathbf{r}'_i) \times \bar{\mathbf{v}} + \sum \mathbf{r}'_i \times \mathbf{p}'_i \\ &= M(\bar{\mathbf{r}} \times \bar{\mathbf{v}}) + \bar{\mathbf{r}} \times 0 + 0 \times \bar{\mathbf{v}} + \mathbf{L}_C \end{aligned}$$

therefore

$$\mathbf{L} = \mathbf{L}_C + \bar{\mathbf{r}} \times \mathbf{P}$$

✓ Example 3.7.1

A hoop of radius a rolling along the ground (Figure III.6):



The angular momentum with respect to C is $L_C = I_C \omega$ where I_C is the rotational inertia about C. The angular momentum about O is therefore

$$I = I_C \omega + M \bar{v} a = I_C \omega + M a^2 \omega = (I_C + M a^2) = I \omega$$

where

$$I = I_C + M a^2$$

is the *rotational inertia* about O.

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