5.3: Head-on Collision of a Moving Sphere with an Initially Stationary Sphere

The coefficient of restitution is

$$e = \frac{\text{relative speed of recession after collision}}{\text{relative speed of approach before collision}}. \tag{5.3.1}$$

We suppose that the two masses m_1 and m_2, the initial speed u, and the coefficient of restitution e are known; we wish to find v_1 and v_2.

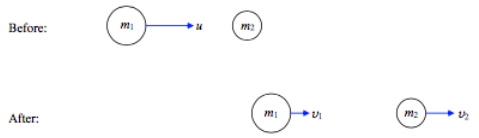

FIGURE V.1

We evidently need two equations. Since there are no *external* forces on the *system*, the linear momentum of the *system* is conserved:

$$m_1 u = m_1 v_1 + m_2 v_2. \tag{5.3.2}$$

The second equation will be the restitution equation (Equation 5.3.1):

$$v_2 - v_1 = eu. \tag{5.3.3}$$

These two equations can be solved to yield

$$v_1 = \left(\frac{m_1 - m_2 e}{m_1 + m_2} \right) u \tag{5.3.4}$$

and

$$v_2 = \left(\frac{m_1 (1 + e)}{m_1 + m_2} \right) u. \tag{5.3.5}$$

The relation between the kinetic energy loss and the coefficient of restitution isn't quite as simple as in Section 5.2.

? Exercise 5.3.1

Show that

$$\frac{\text{kinetic energy (after)}}{\text{kinetic energy (before)}} = \frac{m_1 v_1^2 + m_1 v_2^2}{m_1 u^2} = \frac{m_1 + m_2 e^2}{m_1 + m_2}. \tag{5.3.6}$$

- If $m_2 = \infty$ (as in Section 5.2), this becomes just e^2. If $e = 1$, it becomes unity, so all is well.
- If $m_1 << m_2$ (Ping-pong ball collides with cannon ball), $v_1 = -u, v_2 = 0$.
- If $m_1 = m_2$ (Ping-pong ball collides with ping-pong ball), $v_1 = 0, v_2 = u$.
- If $m_1 >> m_2$ (Cannon ball collides with ping-pong ball), $v_1 = u, v_2 = 2u$.

✓ Example 5.3.1

A moving sphere has a head-on elastic collision with an initially stationary sphere. After collision the kinetic energies of the two spheres are equal. Show that the mass ratio of the two spheres is 0.1716.

Which of the two spheres is the more massive? (I guarantee that your answer to this will be correct.)