11.5iii: Critical damping- [Math Processing Error] γ=2ω0

Before embarking on this section, you might just want to refresh your memory of differential equations as described in Section 11.4.

In this case, *[Math Processing Error]* and *[Math Processing Error]* are each equal to *[Math Processing Error]*. As discussed in Section 4, the general solution is of the form

[Math Processing Error]

which can also be written in the form

[Math Processing Error]

Either way, there are two arbitrary constants, which can be determined by the initial values of the displacement and speed. It is easy to show that

[Math Processing Error]

The particle will not go through zero unless

[Math Processing Error]

I'll leave it to the reader to draw a graph of Equation 11.5.19.

Ideally the hydraulic door closer that you see near the tops of doors in public buildings should be critically damped. This will cause the door to close fastest without slamming. And we have already used the physics of impulsive forces in Problem 2.1 of Chapter 8 to work out where to place a door stop for minimum reaction on the hinges. Truly an understanding of physics is of enormous importance in achieving the task of closing a door!

A more subtle example is in the design of a moving-coil ammeter. In this instrument, the electric current is passed through a coil between the poles of a magnet, and the coil then swings around against the restoring force of a little spiral spring. The coil is wound on a light aluminium frame called a former, and, as the coil (and hence the former) moves in the magnetic field, a little current is induced in the former, and this damps the motion of the coil. In order that the coil and the pointer should move to the equilibrium position in the fastest possible time without oscillating, the system should be critically damped - which means that the rotational inertia and the electrical resistance of the little aluminium former has to be carefully designed to achieve this.