2.15: Solid Body

The moments of inertia of a collection of point masses distributed in three-dimensional space (or of a solid three-dimensional body, which, after all, is a collection of point masses (atoms)) with respect to axes Oxyz are:

$$\begin{array}{ll} A=\sum m(y^2+z^2) & F=\sum myz \\ B=\sum m(z^2+x^2) & G=\sum mzx \\ C=\sum m(x^2+y^2) & H=\sum mxy \end{array}$$

Suppose that A, B, C, F, G, H, are the moments and products of inertia with respect to axes whose origin is at the centre of mass. The *parallel axes theorems* (which the reader should prove) are as follows: Let P be some point not at the centre of mass, such that the coordinates of the centre of mass with respect to axes parallel to the axes Oxyz but with origin at P are $(\overline{x}, \overline{y}, \overline{z})$.

Then the moments and products of inertia with respect to the axes through P are

$$\begin{array}{ll} A+M(\overline{y}^2+\overline{z}^2) & F+M\overline{yz} \\ B+M(\overline{z}^2+\overline{x}^2) & G+M\overline{zx} \\ C+M(\overline{x}^2+\overline{y}^2) & H+M\overline{yx} \end{array}$$

where M is the total mass.

Unless stated otherwise, in what follows we shall suppose that the moments and products of inertia under discussion are referred to a set of axes with the centre of mass as origin.