17.1: Introduction

A mass m is attached to an elastic spring of force constant k, the other end of which is attached to a fixed point. The spring is supposed to obey Hooke's law, namely that, when it is extended (or compressed) by a distance x from its natural length, the tension (or thrust) in the spring is kx, and the equation of motion is $m\ddot{x} = -kx$. This is simple harmonic motion of period $\frac{2\pi}{\omega}$, where $\omega^2 = \frac{k}{m}$. Most readers will have no difficulty with that problem. But now suppose that, instead of one end of the spring being attached to a fixed point, we have two masses, m_1 and m_2, one at either end of the spring. A diatomic molecule is much the same thing. Can you calculate the period of simple harmonic oscillations? It looks like an easy problem, but it somehow seems difficult to get a hand on it by conventional newtonian methods. In fact it can be done quite readily by newtonian methods, but this problem, as well as more complicated problems where you have several masses connected by several springs and several possible modes of vibration, is particularly suitable by lagrangian methods, and this chapter will give several examples of vibrating systems tackled by lagrangian methods.