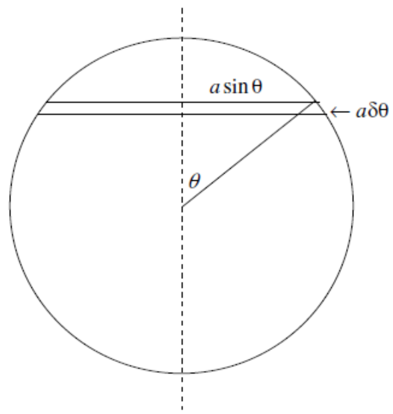
2.7: Three-dimensional Hollow Figures. Spheres, Cylinders, Cones

Hollow*spherical shell, mass m, radius a*



The area of the elemental zone is $2\pi a^2 \sin\theta\delta\theta$. Its mass is

$$m \times \frac{2\pi a^2 \sin\theta\delta\theta}{4\pi a^2} = \frac{1}{2} m \sin\theta\delta\theta$$

Its moment of inertia is $\frac{1}{2} m \sin\theta\delta\theta \times a^2 \sin^2\theta = \frac{1}{2} ma^2 \sin^3\theta\delta\theta$

The moment of inertia of the entire spherical shell is

$$\frac{1}{2} ma^2 \int_0^\pi \sin^3\theta\delta\theta = \frac{2}{3} ma^2$$

This result can be used to calculate, by integration, the moment of inertia $\frac{2}{5} ma^2$ of a solid sphere. Or, if you start with $\frac{2}{5} ma^2$ for a solid sphere, you can differentiate to find the result $\frac{2}{3} ma^2$ for a hollow sphere. Write the moment of inertia for a solid sphere in terms of its density rather than its mass. Then add a layer da and calculate the increase dI in the moment of inertia. We can also use the moment of inertia for a hollow sphere ($\frac{2}{3} ma^2$) to calculate the moment of inertia of a nonuniform solid sphere in which the density varies as $\rho = \rho(r)$. For example, if $\rho = \rho_0 \sqrt{1 - (\frac{r}{a})^2}$, see if you can show that the mass of the sphere is $2.467 \rho_0 a^3$ and that its moment of inertia is $\frac{1}{3} ma^2$. A much easier method will be found in Section 19.

Using methods similar to that given for a solid cylinder, it is left as an exercise to show that the moment of inertia of an open hollow cylinder about an axis perpendicular to its length passing through its centre of mass is $m(\frac{1}{2}a^2 + \frac{1}{3}l^2)$, where a is the radius and $2l$ is the length.

The moment of inertia of a baseless hollow cone of mass m, base radius a, about the axis of the cone could be found by integration. However, those who have an understanding of the way in which the moment of inertia depends on the distribution of mass should readily see, without further ado, that the moment of inertia is $\frac{1}{2} ma^2$. (Look at the cone from above; it looks just like a disc, and indeed it has the same radial mass distribution as a uniform disc.)