

2.12: Principal Planes

Consider a thick lens, or a system of two separated lenses. In Figure II.21, F_1 is the first focal point and H_1 is the first principal plane. In Figure II.22, F_2 is the second focal point and H_2 is the second principal plane.

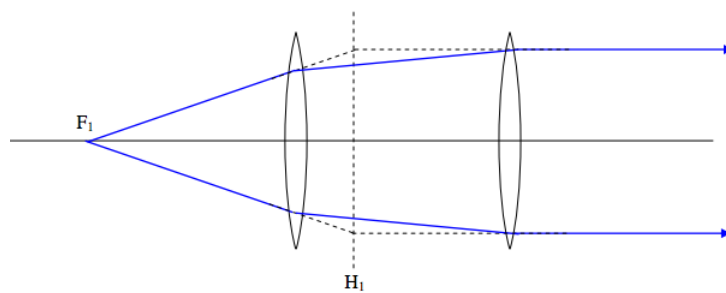
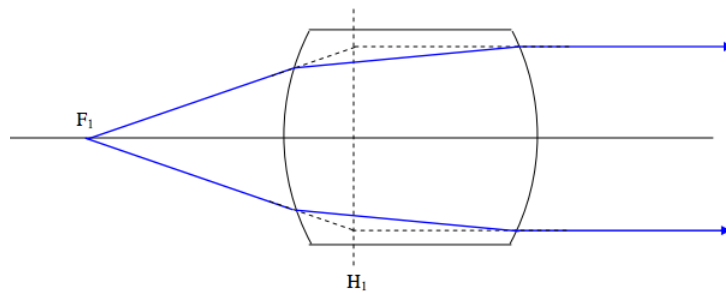


FIGURE II.21

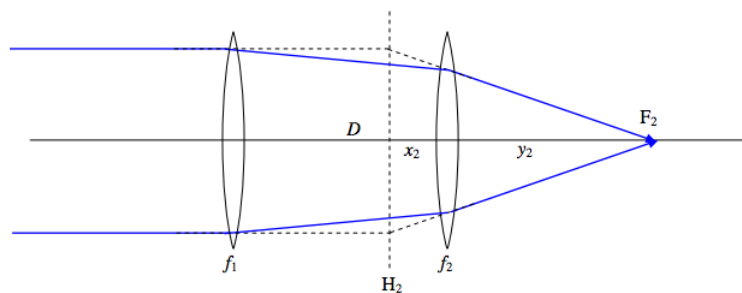
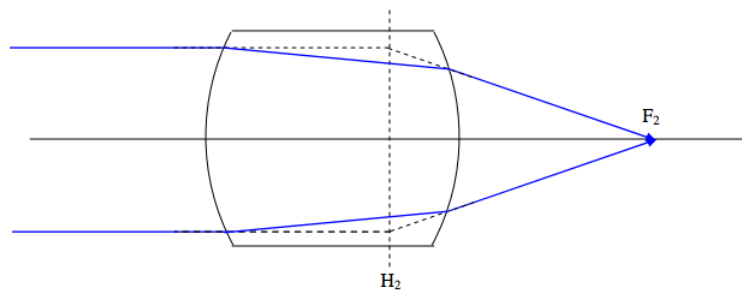


FIGURE II.22

I refer now to the second part of Figure II.22, and I suppose that the focal lengths of the two lenses are f_1 and f_2 , and the distance between them is D . I now invite the reader to calculate the distances x_2 and y_2 . The distance x_2 can be calculated by consideration of some similar triangles (which the reader will have to add to the drawing), and the distance y_2 can be calculated by calculating the convergences C_1, C_2, C_3, C_4 in the manner which is by now familiar. You should get

$$x_2 = \frac{Df_2}{f_1 + f_2 - D}. \quad (2.12.1)$$

and

$$y_2 = \frac{f_2(f_1 - D)}{f_1 + f_2 - D}. \quad (2.12.2)$$

I further invite the reader to imagine that the two lenses are to be replaced by a single lens situated in the plane H_2 so as to bring the light to the same focus F_2 as was obtained by the two original lenses. The question is: what must be the focal length f of this single lens? The answer is obviously $x_2 + y_2$, which comes to

$$f = \frac{f_1 f_2}{f_1 + f_2 - D}. \quad (2.12.3)$$

The eyepiece of an optical instrument such as a telescope or a microscope is generally a combination of two (or more) lenses, called the *field lens* and the *eye lens*. They are generally arranged so that the distance between the two is equal to half the sum of the focal lengths of the two lenses. We shall now see that this arrangement, with two lenses made of the same glass, is relatively free from chromatic aberration.

Let us remind ourselves that the power of a lens in air is given by

$$P = \frac{1}{f} = (n - 1) \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \quad (2.12.4)$$

Here r_1 and r_2 are the radii of curvature of the two surfaces, and n is the refractive index of the glass. For short, I am going to write Equation 2.12.4 as

$$P = \frac{1}{f} = \mu S, \quad (2.12.5)$$

where $\mu = n - 1$ and $S = \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$. That being so, Equation 2.12.3 can be written

$$P = \mu(S_1 + S_2) - \mu^2 S D_1 S_2 \quad (2.12.6)$$

This equation shows how the position of the focus F_2 varies with colour. In particular,

$$\frac{dP}{d\mu} = S_1 + S_2 - 2\mu D S_1 S_2, \quad (2.12.7)$$

which shows that the position of F_2 doesn't vary with colour provided that the distance between the lenses is

$$D = \frac{S_1 + S_2}{2\mu S_1 S_2}. \quad (2.12.8)$$

On going back to Equation 2.12.5, we see that this translates to

$$D = \frac{1}{2}(f_1 + f_2). \quad (2.12.9)$$

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