11.7: A Useful Exercise

It would probably *not* be a useful exercise to try to memorise the details of the several heat engine cycles described in this chapter. What probably would be a useful exercise is as follows. Note that in each cycle there are four stages, which, in principle at least (if not always in practice) are well defined and separated one from the next. These stages are described by one or another of an isotherm, an adiabat, an isochor or an isobar. It would probably be a good idea to ask oneself, for each stage in each engine, the values of ΔQ, ΔW and ΔU, noting, of course, that in each case, $\Delta U = \Delta Q + \Delta W$. In each case take care to note whether heat is added to or lost from the engine , whether the engine does work or whether work is done on it, and whether the internal energy increases or decreases. By doing this, one could then easily determine how much heat is supplied to the engine, and how much net work it does during the cycle, and hence determine the efficiency of the engine.

The following may serve as useful guidelines. In these guidelines it is assumed that any work done is reversible, and that (except for the steam engine or Rankine cycle) the working substance may be treated as if it were an ideal gas.

Along an *isotherm*, the *internal energy* of an ideal gas is unchanged. That is to say, $\Delta U = 0$. The work done (per mole of working substance) will be an expression of the form $RT \ln(V_2/V_1)$, and the heat lost or gained will then be determined by $\Delta Q + \Delta W = 0$.

Along an *adiabat*, no heat is gained or lost, so that $\Delta Q = 0$. The expression for the work done per mole will be of the form $\frac{R(T_1-T_2)}{\gamma-1} = \frac{P_1V_1-P_2V_2}{\gamma-1}$ where V is the molar volume. Just be sure to understand whether work is done on or by the engine. The change in the internal energy (be sure to understand whether it is an increase or a decrease) is then given by $\Delta U = \Delta W$.

Along an *isochor*, no work is done. That is, $\Delta W = 0$. The heat lost or gained per mole will be of an expression of the form $C_V(T_2 - T_1)$, where C_V is the molar heat capacity at constant volume. The change in the internal energy (be sure to understand whether it is an increase or a decrease) is then given by $\Delta U = \Delta Q$.

Along an *isobar*, none of Q, W or U are unchanged. The work done per mole (*by* or *on* the engine?) will be an expression of the form $\Delta W = P(V_2 - V_1) = R(T_2 - T_1)$.

The heat added to or lost from the engine will be an expression of the form $C_P(T_2 - T_1)$, where C_P is the molar heat capacity at constant pressure. The change in the internal energy (be sure to understand whether it is an increase or a decrease) is then given by $\Delta U = \Delta Q + \Delta W$.

It might also be a good idea to try to draw each cycle in the $T : S$ plane (with the intensive variable T on the vertical axes). Indeed I particularly urge you to do this for the Carnot cycle, which will look particularly simple. Note that, while the *area* inside the cycle in the *$P : V$ plane* is equal to the net work done on the engine during the cycle, the *area* inside the cycle in the $T : S$ plane is equal to the *net heat* supplied to the engine during the cycle.