15.1: Introduction

One way to cool a gas is as follows. First compress it isothermally. This means compress it in a vessel that isn't insulated, and wait for the gas to lose any heat that is generated so that it returns to room temperature. Then insulate the vessel and allow the gas to expand adiabatically. We could call this cooling by adiabatic decompression.

You can cool a rubber band as follows. First stretch it isothermally. That means, stretch it slowly, so that it has lots of time to lose any heat that is generated. Then, suddenly destretch it, and before it has time to gain any heat from its surrounding, measure its temperature by immediately holding it up to your lips. You will find that it has cooled by adiabatic de-stretching. (If you stretch the band quickly (i.e. adiabatically) and immediately hold it up to your lips, you will find that it is hot. **BUT ...** before you try *that* experiment, **close your eyes tightly**. You don't want the stretched elastic band to break and hit you in the eye. Believe me, you do not want that to happen.)

The method of *adiabatic demagnetization* has been used to obtain extremely low temperatures. A sample of a paramagnetic salt (such as cerium magnesium nitrate), already cooled to low temperatures by other means, is magnetized isothermally. The sample is often suspended in an atmosphere of helium, which can conduct away any heat that is produced, and hence keeps the process isothermal. It is then insulated (by pumping out the helium) and suddenly and adiabatically demagnetized. This process of isothermal magnetization followed by adiabatic demagnetization can be repeated over and over again. Temperatures close to 0 K have been reached in this manner. You could actually reach a temperature of absolute zero if you did this an infinite number of times – but not for any fewer.

In the analysis that follows, I shall have to assume that you are familiar with the concepts of **B**, **H**, magnetic moment and magnetization from electricity and magnetism.

In brief, the *magnetic dipole moment* $\mathbf{p}_m$ of a sample is the maximum torque it experiences in unit field **B**. That is, the torque is given by $\boldsymbol{\tau} = \mathbf{p}_m \times \mathbf{B}$. The magnetization **M** of a specimen is defined by $\mathbf{B} = \mu\mathbf{H} = \mu_0 (\mathbf{H} + \mathbf{M})$. The magnetization is also equal to the *magnetic moment per unit volume*.

Now consider the following.

If the tension in an elastic string is F, the work done **on** the string when its length is increased by dx is $F\,dx$.

If the pressure of a gas is P, the work done **on** the gas when its volume is increased by dV is $-P\,dV$.

And the work done per unit volume **on** an isotropic sample in increasing its magnetization from M to $M + dM$ in a magnetic field B is BdM. (I am assuming here that the sample is isotropic and that the magnetic moment and the magnetic field are in the same direction, and hence I am no longer using boldface to indicate vector quantities.)

Note that, in all of these examples, the work done is the product of an intensive state variable (P, F, B) and the differential of an extensive state variable (dV, dx, dM).

If we add heat to a magnetizable sample, and do work per unit volume on it by putting it in a magnetic field B and thereby increasing its magnetization by dM, then, provided there is no change in volume, the increase in its internal energy per unit volume is given by

$$dU = TdS + BdM, \tag{15.1.1}$$

In this magnetic context, we can define state functions H, A and G per unit volume by

$$H = U - BM \tag{15.1.2}$$

$$A = U - TS \tag{15.1.3}$$

$$G = H - TS = A - BM \tag{15.1.4}$$

In differential form, these become

$$dH = TdS - MdB \tag{15.1.5}$$

$$dA = -SdT + BdM \tag{15.1.6}$$

$$dG = -SdT - MdB \tag{15.1.7}$$

Here M is the dipole moment *per unit volume*, in N m T^{-1} m^{-3}, which is the same as the magnetization, in A m^{-1}. (Other equivalent units for magnetization would be Pa T^{-1} or T m H^{-1}, but I recommend N m T^{-1} m^{-3} as being the most readily understandable in the present context.)

In Section 15.2 I am going to derive an expression for the lowering of the temperature in an adiabatic decompression, $(\partial T/\partial P)_S$. And then, in Section 15.3, I am going to derive an expression, by exactly the same argument, step-by-step, for the lowering of the temperature in an adiabatic demagnetization, $(\partial T/\partial B)_S$.
