8.7: Scale Height in an Isothermal Atmosphere

The material in this chapter doubtless has countless applications, most of which I am unaware of, in meteorology. Two simple topics are easy to mention, namely the *scale height in an isothermal atmosphere*, dealt with in this section, and the adiabatic lapse rate dealt with in the next section.

Let us imagine a column of air of cross-sectional area A in an isothermal atmosphere – that is to say the temperature T is uniform throughout. Consider the equilibrium of the portion of the air between heights z and $z + dz$. The weight of this portion is $\rho gAdz$. Let P be the pressure at height z and $P + dP$ be the pressure at height $z + dz$. (Note that dP is negative.) The net upward force on the portion dz of the air is $-AdP$. Therefore $dP = -\rho gdz$. But if we regard air as an ideal gas, it obeys the equation of state for an ideal gas, equation 6.1.7: $P = \rho RT/\mu$ where ρ and μ are respectively the density and the "molecular weight" (molar mass) of the gas. Therefore $\frac{RT}{\mu}d\rho = -\rho gdz$, or $\frac{d\rho}{\rho} = -\frac{\mu g}{RT}dz$. Integrate to obtain

$$\rho = \rho e^{-z/H} \tag{8.7.1}$$

where $H = \frac{RT}{\mu g}$ is the *scale height*. It is large if the temperature is high, the gas light and the planet's gravity feeble. It is the height at which the density is reduced to a fraction $1/e$, or 36.8%. of its ground value. What would it be, in kilometres, for an atmosphere consisting of 80% N_2 and 20% O_2, at a temperature of 20 ºC, where the gravitational acceleration is 9.8 m s^{-2}? What fraction is this of the radius of Earth? If you made a model of Earth one metre in diameter (radius = 50 cm), how thick would be the atmosphere? You'd better look after it - our atmosphere is a very thin skin clinging to the surface!

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