2.3: Implicit Differentiation

Equation 2.2.5 can be used to solve the problem of differentiation of an implicit function. Consider, for example, the unlikely equation

$$\ln(xy) = x^2y^3 \tag{2.3.1}$$

Calculate the derivative *dy/dx*. It would be easy if only one could write this in the form *y* = something; but it is difficult (impossible as far as I know) to write *y explicitly* as a function of x. Equation 2.3.1 implicitly relates y to x. How are we going to calculate dy/dx?

The curve $f(x,y)=0$ might be considered as being the intersection of the surface $z=f(x,y)$ with the plane $z=0$. Seen thus, the derivative dy/dx can be thought of as the limit as δx and δy approach zero of the ratio $\delta y/\delta x$ within the plane $z=0$; that is, keeping z constant and hence δz equal to zero. Thus equation 2.2.5 gives us that

$$\frac{dy}{dx} = -\left(\frac{\partial f}{\partial x}\right)/\left(\frac{\partial f}{\partial y}\right). \tag{2.3.2}$$

For example, show that, for Rquation 2.3.1,

$$\frac{dy}{dx} = \frac{y(2x^2y^3-1)}{x(1-3x^2y^3)}. \tag{2.3.3}$$