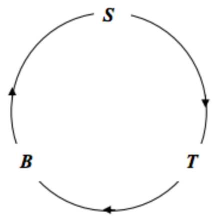
15.3: Adiabatic Demagnetization

We are now going to do the same argument for adiabatic demagnetization.

We are going to calculate an expression for $(\partial T/\partial B)_S$. The expression will be positive, since T and B increase together. We shall consider the entropy as a function of temperature and magnetic field, and, with the variables



we shall start with the cyclic relation

$$\left(\frac{\partial S}{\partial T}\right)_B\left(\frac{\partial T}{\partial B}\right)_S\left(\frac{\partial B}{\partial S}\right)_T=-1. \tag{15.3.1}$$

The middle term is the one we want. Let's find expressions for the first and third partial derivatives in terms of things that we can measure.

In a reversible process $dS = dQ/T$, and, in a constant magnetic field, $dQ = C_B dT$. Here I am taking S to mean the entropy *per unit volume*, and C_B is the heat capacity per *unit volume* (i.e. the heat required to raise the temperature of unit volume by one degree) in a constant magnetic field.

Thus we have $\left(\frac{\partial S}{\partial T}\right)_B=\frac{C_B}{T}$.

The Maxwell relation corresponding to $\left(\frac{\partial S}{\partial P}\right)_T=-\left(\frac{\partial V}{\partial T}\right)_P$ is $\left(\frac{\partial S}{\partial B}\right)_T=\left(\frac{\partial M}{\partial T}\right)_B$. Thus Equation 15.3.1 becomes

$$\left(\frac{\partial T}{\partial B}\right)_S=-\frac{T}{C_B}\left(\frac{\partial M}{\partial T}\right)_B \tag{15.3.2}$$

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Now for a paramagnetic material, the magnetization, for a given field, is proportional to B and it falls off inversely as the temperature (that's the *equation of state*). That is, M = *aB*/*T*. and therefore $\left(\frac{\partial M}{\partial T}\right)_B=-\frac{aB}{T^2}=-\frac{M}{T}$. Equation 15.3.2 therefore becomes

$$\left(\frac{\partial T}{\partial B}\right)_s=\frac{M}{C_B}. \tag{15.3.3}$$

You should check the dimensions of this equation.

The cooling effect is particularly effective at low temperatures, when C_B is small.