# 8.8: Adiabatic Lapse Rate

Earth's atmosphere is not, of course, isothermal. The temperature decreases with height. The temperature lapse rate in an atmosphere is the rate of decrease of temperature with height; that is to say, it is $-dT/dz$.

An *adiabatic* atmosphere is one in which $P/\rho^\gamma$ does not vary with height. In such an atmosphere, if a lump of air is moved adiabatically to a higher level, its pressure and density will change so that $P/\rho^\gamma$ is constant – and will be equal to the ambient pressure and density at the new height. For such an atmosphere, it is possible to calculate the rate at which temperature decreases with height – the *adiabatic lapse rate*. We shall do this calculation, and see how it compares with actual lapse rates.

As in Section 8.7, the condition for hydrostatic equilibrium is

$$dP = -\rho g dz. \tag{8.8.1}$$

Since we are trying to find a relation between T and z for an adiabatic atmosphere (i.e. one in which $P/\rho^\gamma$ doesn't vary with height), we need to find the adiabatic relations between $P$ and $T$ and between ρ and $T$.

These are easily found from the adiabatic relation between $P$ and ρ:

$$P = c\rho^\gamma \tag{8.8.2}$$

and the ideal gas equation of state:

$$P = \frac{\rho R T}{\mu}. \tag{8.8.3}$$

Eliminate $P$:

$$\rho = \left(\frac{RT}{c\mu}\right)^{1/(\gamma-1)}. \tag{8.8.4}$$

Eliminate ρ:

$$P = \frac{R^{\gamma/(\gamma-1)}}{\mu^{\gamma/(\gamma-1)}c^{1/(\gamma-1)}}T^{\gamma/(\gamma-1)}, \tag{8.8.5}$$

from which

$$dP = \frac{\gamma}{\gamma-1}\frac{R^{\gamma/(\gamma-1)}}{\mu^{\gamma/(\gamma-1)}c^{1/(\gamma-1)}}T^{1/(\gamma-1)}dT. \tag{8.8.6}$$

Substitute equations (8.8.4) and (8.8.6) into equation (8.8.1), to obtain, after a little algebra, the following equation for the adiabatic lapse rate:

$$-\frac{dT}{dz} = \left(1-\frac{1}{\gamma}\right)\frac{g\mu}{R}. \tag{8.8.7}$$

This is independent of temperature.

If you take the mean molar mass for air to be 28.8 kg kmole$^{-1}$, and $g$ to be 9.8 m s$^{-2}$ for temperate latitudes, you get for the adiabatic lapse rate for dry air −9.7 K km$^{-1}$. The presence of water vapour in humid air reduces the mean value of μ (and hence the adiabatic lapse rate), and actual lapse rates are usually rather less than the calculated adiabatic lapse rates even for humid air. (The presence of water vapour also increases slightly the value of γ. This would result in a slightly larger lapse rate, but the effect is not as great as the reduction in lapse rate caused by the larger value of μ. Try some numbers to convince yourself of this.) The International Civil Aviation Organization Standard Atmosphere takes the lapse rate in the troposphere (first 11 km) to be −6.3 K km$^{-1}$. What happens if the actual lapse rate is faster than the adiabatic lapse rate? If you imagine a lump of air to be moved adiabatically to a higher level, its pressure and density will change so that $P/\rho^\gamma$ is constant, and it will then find itself in a region where its new density is less that the new ambient density. Consequently, it will continue to rise, and the atmosphere will be convectively unstable, and a storm will ensue. The atmosphere is stable as long as the actual lapse rate is less than the adiabatic lapse rate (which is reduced in humid air) is unstable if the actual lapse rate is greater than the adiabatic lapse rate.