13.8: Volume, Temperature and the Grüneisen Parameter

If you compress a material adiabatically and reversibly (i.e. isentropically) its temperature goes up. The amount by which it goes up can be represented by the partial derivative $\left(\frac{\partial T}{\partial V}\right)_S$. Here, V could mean the total volume , the specific volume or the molar volume, according to context, and you would have to specify your units accordingly. The derivative is negative, because the temperature goes up as the volume is decreased.

[Compare this with the definition of the volume coefficient of expansion $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$, which is positive. Think about the difference.]

A dimensionless version which also expresses the variation of temperature with volume would be $\frac{V}{T}\left(\frac{\partial T}{\partial V}\right)_S = \left(\frac{\partial \ln T}{\partial \ln V}\right)_S$, and here there is no need to specify whether V means total, specific or molar. The derivative could also be written as $-\left(\frac{\partial \ln T}{\partial \ln \rho}\right)_S$, where ρ is the density. The positive value, $-\left(\frac{\partial \ln T}{\partial \ln V}\right)_S = +\left(\frac{\partial \ln T}{\partial \ln \rho}\right)_S$ is called the *Grüneisen parameter*. We have already used the symbols G, g, Γ and γ for various things in these notes, so I am stuck for a suitable symbol. Sometimes non-italic symbols are used for dimensionless parameters, such as R for Reynolds number in aerodynamics. Let's try Gr for the Grüneisen parameter.

For an *ideal gas*, the relation between volume and temperature in a reversible adiabatic expansion is $TV^{\gamma-1}$ = constant, and therefore the Grüneisen parameter for an ideal gas is $\gamma - 1$.

In thinking about volume and temperature changes, we often have some sort of a gas (ideal or otherwise) in mind. However, geophysicists have to deal with very large pressures in the interior of the Earth, where volume and temperature changes of solids under pressure are not negligible, and geophysicists often make use of the Grüneisen parameter for solid materials.

For a bit of practice in deriving relationships between some of the quantities described in this chapter, see if you can show that

$$\mathrm{Gr} = \frac{\beta}{\rho C_V \kappa_{\mathrm{iso}}} = \frac{\beta}{\rho C_P \kappa_{\mathrm{ad}}} \tag{13.8.1}$$

and

$$\gamma = 1 + \mathrm{Gr}\,\beta T. \tag{13.8.2}$$

If ρ in these questions stands for density (mass per unit volume), what, precisely, are C_V and C_P? Total, specific or molar? Or does it not matter? What do these equations become in the case of an ideal gas?