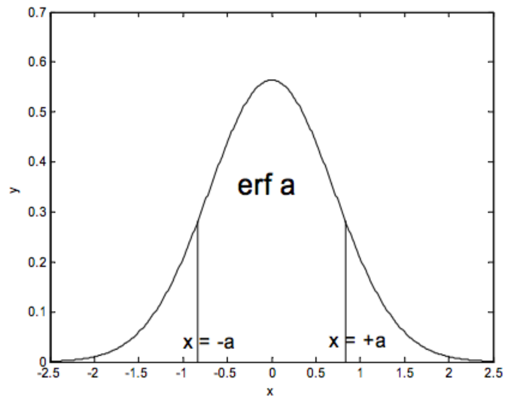
4.1: Error Function

Before we start this chapter, let's just make sure that we are familiar with the error function erf a. We may need it during this chapter.

Here is a graph of the gaussian function

$$y=\frac{1}{\sqrt{\pi}}e^{-x^2}. \tag{4.1.1}$$



I have chosen the coefficient $1/\sqrt{\pi}$ so that the area under the curve, from $-\infty$ to $+\infty$ is 1. The maximum value, which occurs at x = 0, is $1/\sqrt{\pi}=0.5642$, and it is easy to show that the half width at half the maximum is $\sqrt{\ln 2}=0.8326$. Also of some interest (though not particularly in this chapter) is the square root of the second moment of area around the y-axis. In a mechanical context this would be called the **radius of gyration**. In a statistical context it would be called the standard deviation. Either way, its value is $1/\sqrt{2}=0.7071$. We shall meet the gaussian function again in Chapter 6.

In the present chapter we shall need to make use of the **error function** erf *a*. This is the area under the gaussian curve from *x* = -*a* to *x* = +*a*:

$$\operatorname{erf} a=\frac{1}{\sqrt{\pi}}\int_{-a}^{+a}e^{-x^2}\,dx. \tag{4.1.2}$$

The area outside the limits x = ±a, which is the area under the two "tails" of the gaussian function, is sometimes called the *complementary error function*:

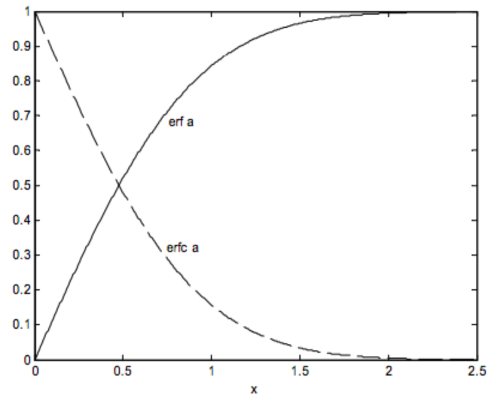
$$\operatorname{erfc} a=1-\operatorname{erf} a \tag{4.1.3}$$

It will be clear that erf a goes from 0 to 1 as a goes from 0 to infinity. Note also that

erfc (one standard deviation) = 0.3173

erfc (two standard deviations) = 0.0455.

Here are graphs of erf *a* (continuous line) and erfc *a* (dashed line) versus *a*.


erf a
erfc a
x