

## 11.8: Heat Engines and Refrigerators

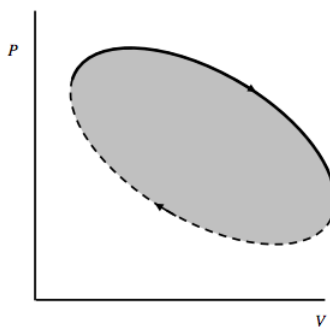


FIGURE XI.8

Figure XI.8 illustrates schematically the path taken by the state of a working substance in a generalized heat engine. In the upper part of the cycle (continuous curve) the working substance is expanding, and the machine is doing work. The work done **by** the engine is  $\int P dV$ , or the area under that part of the curve. In the lower part of the cycle (dashed curve) the working substance is being compressed; work is being done **on** it. This work is the area under the dashed portion of the cycle. The **net** work done **by** the engine during the cycle is the work done **by** the engine while it is expanding *minus* the work done **on** it during the compression part of the cycle, and this is the *area enclosed* by the cycle.

During one part of any heat engine cycle, heat is supplied **to** the engine, and during other parts, heat is lost **from** it. As described in Section 11.1, the *efficiency*  $\eta$  of a heat engine is defined by

$$\eta = \frac{\text{net external work done by the engine during a cycle}}{\text{heat supplied to the engine during a cycle.}} \quad (11.8.1)$$

Note that the word “net” does not appear in the denominator. The efficiency can also be *calculated* from

$$\eta = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}}, \quad (11.8.2)$$

though I stress that this is not a *definition*.

In the *Carnot engine*, which is the most efficient conceivable engine for given source and sink temperature, the efficiency is

$$\eta = \frac{T_2 - T_1}{T_2}, \quad (11.8.3)$$

where  $T_2$  and  $T_1$  are respectively the temperatures of the hot source and cold sink.

If the working substance is taken round a cycle in the  $PV$ -plane in the *counterclockwise* direction, the device is a *refrigerator*.

In that case the area enclosed by the cycle is equal to the net work that is done **on** the working substance. If the refrigerator operates on a reverse *Carnot* cycle, the working substance **takes in** (from whatever it is that it is trying to cool) a quantity of heat  $Q_1$  as it expands isothermally from  $d$  to  $c$  (see figure XI.1, but with the arrows reversed) and **expels a** (greater) quantity of heat  $Q_2$  as it is compressed isothermally from  $b$  to  $a$ . This quantity  $Q_2$  is expelled into the room – which is why the room gets warmer when you switch on the fridge. (What – you never noticed?) The *refrigerating effect* is  $Q_1$ , since this is the quantity of heat taken in by the refrigerator from the body that is to be cooled.

The *coefficient of performance* of a refrigerator is defined by

$$\frac{\text{refrigerating effect}}{\text{net work done on the engine during the cycle.}} \quad (11.8.4)$$

By the first law of thermodynamics, the denominator of the expression is  $Q_2 - Q_1$ , and for a reversible Carnot cycle, the entropy in equals the entropy out, so that  $Q_2/Q_1 = T_2/T_1$ . Therefore the coefficient of performance for a Carnot refrigeration cycle can be calculated from

$$\frac{T_1}{T_2 - T_1}. \quad (11.8.5)$$

This, of course, can be much greater than 1 – but no refrigerator working between the same source and sink temperatures can have a coefficient of performance greater than that of a reversible Carnot refrigerator.

Of course the working substance in a real refrigerator (“fridge”) is not an ideal gas, nor does one follow a Carnot cycle – there are too many practical difficulties in the way of achieving this ideal dream. As mentioned elsewhere in this course, I am not a practical man and I am not suited to describing real, practical machines. The fundamental principles described in this section do, of course, still apply in the real world! In a real refrigerator, the working substance (the *refrigerant*) is a volatile fluid which is vaporized in one part of the operation and condensed to a liquid in another part. In industrial refrigerators, the refrigerant may be ammonia, but this is considered to be too dangerous for domestic use. “Freon”, which was a mixture of chlorofluorocarbons, such as  $\text{CCl}_2\text{F}_2$ , was in fashion for a while, but escaping chlorofluorocarbons have been known for some time to cause breakdown of ozone ( $\text{O}_3$ ) in the atmosphere, thus destroying our protection against ultraviolet radiation from the Sun. The chlorofluorocarbons have been largely replaced by hydrofluorocarbons, such as  $\text{C}_2\text{H}_2\text{F}_4$ , which are believed to be less damaging to the ozone layer. The exact formula or mixture is doubtless a trade secret.

The fluid is forced around a system of tubes by a pump called the *compressor*. Shortly before the fluid reaches the freezer it is in liquid form, moving along some rather narrow pipes. It is then forced through a nozzle into a system of wider pipes (the *evaporator*) surrounding the freezer, and there it vaporizes, taking heat from the food and from the air in the freezer. A fan may also distribute the cooled air throughout the rest of the refrigerator. After leaving the freezer, the vapour returns to the compressor, where it is, of course, compressed (which is why the pump is called the compressor). This produces heat, which is dissipated into the room as the fluid is forced through a series of pipes and vanes, known as the condenser, at the rear of the fridge, where the fluid condenses into liquid form again. The cycle then starts anew.

The following summary of Carnot heat engines and refrigerators may be helpful. (But just remember that, while Carnot cycles are the most efficient engines and refrigerators for given source and sink temperatures, the practical realization of a real engine or refrigerator may not be identical to this theoretical ideal.)

Notation:

$T_2$  = hotter temperature

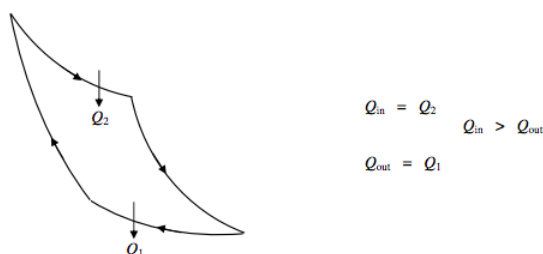
$T_1$  = cooler temperature

$Q_2$  = heat gained or lost at  $T_2$

$Q_1$  = heat gained or lost at  $T_1$

$$\Delta S = 0 \quad \frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

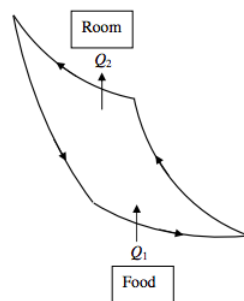
Heat Engine:



$$\Delta U = 0 \quad \text{Net work done by engine} = Q_2 - Q_1.$$

$$\text{Efficiency } \eta = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}} = \frac{Q_2 - Q_1}{Q_2} = \frac{T_2 - T_1}{T_2}$$

Refrigerator:



$$Q_{in} = Q_1$$

$$Q_{out} = Q_2$$

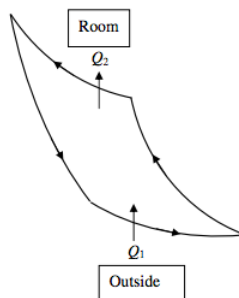
$$Q_{out} > Q_{in}$$

$$\Delta U = 0 \quad \text{Net work done on refrigerator} = Q_2 - Q_1$$

$$\text{Coefficient of Performance } P = \frac{Q_{in}}{Q_{out} - Q_{in}} = \frac{Q_1}{Q_2 - Q_1} = \frac{T_1}{T_2 - T_1}$$

### Heat Pump:

The principle of a heat pump is the same as that of a refrigerator, except that its purpose is different. The purpose of a refrigerator is to extract heat from something (e.g. food) and so to make it colder. That the heat so extracted goes into the room to make the room warmer (at least in principle) is incidental. The important thing is how much heat is extracted from the food, and that is why it is appropriate to define the coefficient of performance of a refrigerator as the *refrigerating effect* (i.e.  $Q_1$ ) divided by the net work done on the refrigerator, per cycle. But with a heat pump, the object is to *heat the room* by extracting heat from outside. That the outside may become cooler (at least in principle) is incidental. Thus, for a heat pump, the appropriate definition of the coefficient of performance is the *heating effect* (i.e.  $Q_2$ ) divided by the net work done on the refrigerator, per cycle.



$$Q_{in} = Q_1$$

$$Q_{out} = Q_2$$

$$Q_{out} > Q_{in}$$

$$\Delta U = 0 \quad \text{Net work done on heat pump} = Q_2 - Q_1$$

$$\text{Coefficient of Performance } P = \frac{Q_{out}}{Q_{out} - Q_{in}} = \frac{Q_2}{Q_2 - Q_1} = \frac{T_2}{T_2 - T_1}$$

You can see from this equation that, the warmer it is outside ( $T_1$ ), the greater the coefficient of performance. You may therefore wonder if it is practical to use a heat pump to heat a building in a cold climate, such as the Quebec winter. And, if it isn't, can one devise an engine that is simultaneously a refrigerator and a heat pump; that is to say, it extracts heats from (i.e. cools) the food, and transfers this heat (plus a little bit more because of the work that is done on the refrigerator/heat pump) into the room in order to heat the room effectively. There's an answer to that in an article in the Victoria Times-Colonist of June 11, 2006, which I reproduce, with permission, below.

# Fridge used to heat

Energy-efficient heating allows supermarket to stay warm during bone-chilling winters

By MIKE DE SOUZA  
CanWest News Service

OTTAWA— The manager of a large grocery supermarket in Quebec was skeptical when Natural Resources Canada told him he could run his 9,450-square-metre store during bone-chilling winters without a furnace.

Heading into the Christmas holiday season in 2004, government researchers helped design the brand new grocery store for Loblaws, in Repentigny, northeast of Montreal, with a unique, revolutionary system that allows it to use its refrigerators to heat the building.

"In December, and the months that followed the opening in 2004, it was an especially cold period, -26 C and -30 C," said Dr. Sophie Hosatte, a senior NRCan researcher.

"It's a critical period for the supermarket in terms of sales, [but] there was absolutely no problem. The comfort-level was excellent inside the supermarket."

More than two years later, the system continues to keep the store running through winters, capturing heat and energy released by the refrigerators and pumping it back into the building. As a result it drastically slashes costs for heating and maintenance, as well as reducing greenhouse gas emissions, Hosatte said.

In a single store, NRCan estimates the system can cut greenhouse gas emissions by 1.5 megatonnes, the equivalent of 300 cars driving 20,000 kilometres in a year. But since the new government is in the midst of reviewing all climate-change programs and policies developed when the federal Liberals were in power, the future of the technology is in limbo.

"We are in a transition year," Hosatte said. "For the future, we have to wait for the new climate change plan."

NRCan continuously monitors and measures the performance of the system throughout the Repentigny store, and so far, Hosatte said the results are meeting their targets perfectly. The technology has also been adapted in two Loblaws grocery stores that retrofitted their heating systems in the Ottawa area, along with about a dozen ice rinks and arenas across the country.

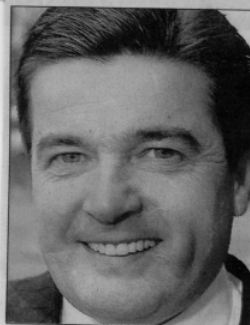
While her department has demonstrated the system works well, she said it still needs to provide more technical support to help private sector partners apply and adapt the technology.

"That's our role, presently, to develop tools, guidelines and to help supermarkets adapt these technologies," she said. "Otherwise there would be reluctance because of the risk."

The researchers may take comfort the

TIMES COLONIST SUNDAY JUNE 11 2006 A5

## Quebec grocery store



Gary Lunn wants to promote energy efficiency.  
John McKay/TC

Conservative government has pledged to encourage more energy savings in the construction and transportation industry. But there are no guarantees which programs are going to stick around, once the Tories finalize their new climate change program in the fall.

"There's lots of ways that we could promote energy efficiency, and we're going to be there to support that," Natural Resources Minister Gary Lunn told a Commons committee last week.

"Sometimes, we'll have to spend more at the front end, to get the big benefits at

the back end, and we'll evaluate all those programs. But at the end of the day, we want to ensure the Canadian taxpayer is getting the best value for their tax dollars."

Lunn also suggested ground source heat pumps could be encouraged as an alternative system to help home owners reduce energy consumption for both heating and cooling, by capturing energy from the ground and pumping it into a building.

"The technology is there, it's been there for years, and I think that we need to move the entire construction industry, and the renovation industry to getting into these forms of energy efficiency," said Lunn, MP for Saanich-Gulf Islands.

Some researchers say there's enough technology available for Canada to effectively tackle its target under the international Kyoto protocol on climate change to reduce greenhouse gas emissions by six per cent below 1990 levels. But it takes a political will and economic incentives such as reducing energy costs to apply those technologies, said Frederic Genest, an engineer with Pageau Morel and Associates.

Genest's firm recently designed a hybrid ground source heating and cooling system for a new Mountain Equipment Co-op store in Montreal, that is boasting 65 per cent savings in energy consumption.

"There are a lot of possibilities, there are many things that we could do," said Genest.

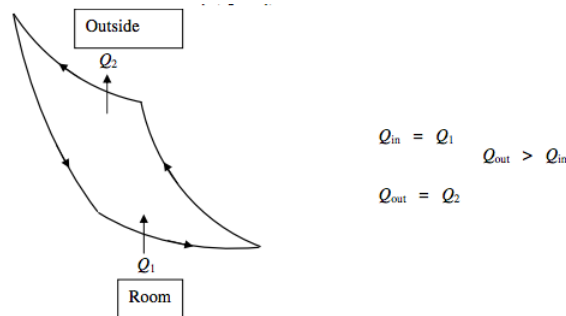
### Air Conditioner

The purpose of a *refrigerator* ("fridge") is to pump some heat  $Q_1$  from the food (or whatever is to be kept cool). The quantity  $Q_1$  is the "refrigerating effect". During the operation of the fridge, a somewhat greater quantity  $Q_2$  of heat is expelled into the room,

though this should not result in a very noticeable rise in temperature of the room, partly because the room has a large thermal capacity, and partly because much of this heat will be lost through the windows. The *coefficient of performance* of the fridge is the refrigerating effect per cycle,  $Q_1$ , divided by the net work done on the fridge per cycle, and, for a Carnot cycle it can be calculated from  $T_1/(T_2 - T_1)$ .

The purpose of a *heat pump* is to pump some heat  $Q_1$  from outside, and (from the work done on the pump) to pump a larger quantity  $Q_2$  of heat into the room – large enough, indeed to warm the room appreciably, supposing that you don't keep all the windows wide open. The coefficient of performance must therefore be defined as  $Q_2$  divided by the net work done on the fridge per cycle. For a Carnot cycle it can be calculated from  $T_2/(T_2 - T_1)$ .

There is a third possibility, namely an air conditioner. This will incorporate a dehumidifier, but, in our present context we regard it as a device whose purpose is to pump heat from the room to the outside, rather than from outside to the room. If it is successful, the room will become cooler than the outside. Thus an air conditioner is more like a refrigerator, in that the coefficient of performance is the heat  $Q_1$  extracted per cycle from the room divided by the net work done on the machine per cycle. For a Carnot cycle it can be calculated from  $T_1/(T_2 - T_1)$ .



$$\Delta U = 0 \quad \text{Net work done on air conditioner} = Q_2 - Q_1$$

$$\text{Coefficient of Performance } P = \frac{Q_{\text{in}}}{Q_{\text{out}} - Q_{\text{in}}} = \frac{Q_1}{Q_2 - Q_1} = \frac{T_1}{T_2 - T_1}$$

Those who have read thus far will have an idea that there are things called *heat engines*, *refrigerators*, *heat pumps* and *air conditioners*, which are represented by Carnot cycles or similar cycles, with arrows going in different directions, a few equations with different subscripts, and subtly different definitions of efficiency or coefficient of performance. Since I prepared these notes I have discovered that there actually exist in the real world, real, solid machines called *heat engines*, *refrigerators*, *heat pumps* and *air conditioners*. I have discovered two very nice little pamphlets describing real heat pumps and real air conditioners, and how you might install them to heat or to cool your home. They are called *Heating and Cooling with a Heat Pump*, and *Air Conditioning your Home*, each about 50 pages. My copies are dated 1996, revised 2004, though I dare say you might be able to get more recent ones. They are available free from Energy Publications, Office of Energy Efficiency, Natural Resources Canada, c/o S.J.D.S., 1779 Pink Road, Gatineau, Province of Québec, Canada J9J 3N7. I found them fascinating.

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