2.8: Dee and Delta

We have discussed the special meanings of the symbols ∂ and đ, but we also need to be clear about the meanings of the more familiar differential symbols Δ, δ, and d. It is often convenient to use the symbol Δ to indicate an increment (not necessarily a particularly small increment) in some quantity. We can then use the symbol δ to mean a *small* increment. We can then say that if, for example, $y = x^2$, and if x were to increase by a small amount δx, the corresponding increment in y would be given approximately by

$$\delta y \cong 2x\delta x \tag{2.8.1}$$

That is,

$$\frac{\partial y}{\partial x} \cong 2x. \tag{2.8.2}$$

This doesn't become exact until we take the limit as \(δ*x*\) and \(δ*y*\) approach zero. We write this limit as $\frac{dy}{dx}$ and then it is *exactly* true that

$$\frac{dy}{dx} = 2x. \tag{2.8.3}$$

There is a valid point of view that would argue that you cannot write dx or dy alone, since both are zero; you can write only the ratio $\frac{dx}{dy}$. It would be wrong, for example, to write

$$dy = 2x\ dx, \tag{2.8.4}$$

or at best it is tantamount to writing 0 = 0. I am not going to contradict that argument, but, at the risk of incurring the wrath of some readers, I am often going to write equations such as Equation 2.8.4, or, more likely, in a thermodynamical context, equations such as

$$dU = TdS - PdV, \tag{2.8.5}$$

even though you may prefer me to say that, for small increments,

$$\delta U \cong T\delta S - P\delta V. \tag{2.8.6}$$

I am going to argue that, in the limit of infinitesimal increments, it is exactly true that $dU = TdS - PdV$. After all, the smaller the increments, the closer it becomes to being true, and, in the limit when the increments are infinitesimally small, it is exactly true, even if it does just mean that zero equals zero. I hope this does not cause too many conceptual problems.

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