2.11: Gaussian Triple Integral Algorithm

To approximate the integral

$$I=\int_a^b\int_c^d\int_e^f F(x,y,z)dzdydx \tag{2.11.1}$$

where it is assumed that the roots R and coefficients C are stored in two-dimensional arrays.

```
BEGIN
    h1 = (b – a)/2
    h2 = (b + a)/2
    I = 0
    FOR i = 1, 2,..., m DO
        Ix = 0
        x = h1*R[m][i] + h2
        k1 = (d – c)/2
        k2 = (d + c)/2
        FOR j = 1, 2,..., n DO
            Iy = 0
            y = k1*R[n][i] + k2
            l1 = (f – e)/2
            l2 = (f + e)/2
            FOR k = 1, 2,..., p DO
                z = l1*R[p][k] + l2
                Iy = Iy + C[p][k]*F(x, y, z)
            END FOR { k-loop }
            Ix = Ix + C[n][j]*l1*Iy
        END FOR { j-loop }
        I = I + C[m][i]*k1*Ix
    END FOR { i-loop }
    I = h1*I
    PRINT I
END
```

This algorithm may be generalised further by allowing limits e and f to be functions e(x,y) and f(x,y) and c and d to be functions c(x) and d(x). For our purposes the limits of integration are fixed values.

Applying this algorithm to equation (28) for the Bond albedo and identifying μ with x, we see that

$$\frac{A}{2}=\int_0^1\int_0^{2\pi}\int_0^1 \times f_r(x,\mu,\alpha;\ldots)\mu d\mu d\phi dx \tag{2.11.2}$$

and by further identifying *z* with μ and *y* with φ

$$\mathrm{F(x,y,z)=2xz}f_r\mathrm{(x,z},\alpha;\ldots) \tag{2.11.3}$$

where a is itself a function of x,y and z [cf. equation (26)]

$$\alpha=\cos^{-1}\left[\mathrm{xz}+\sqrt{\left(1-\mathrm{x}^{2}\right)\left(1-\mathrm{z}^{2}\right)}\cos\mathrm{y}\right]. \tag{2.11.4}$$

For the phase integral, there is no need to invoke the likes of equation (32) since the intensity I(α) is explicitly expressed in terms of α and one stage of the integration is with respect to α. The parameters, … , are, of course, not variables since they retain their values for the duration of the integration.

When applying these integrals it is strongly suggested that *A*, *p* and *q* each be calculated independently in order to verify that the relationship *A* = *p q* holds. Taking shortcuts may bury insidious bugs, some possibly as simple as a typo., inside a program and result in at least two undetected erroneous results.