1.4: Directions and Notation

The strength of a plane parallel beam of radiation is specified by the radiant flux density **F** watts per square metre such that **F** = dP/dA, where A is the area perpendicular to the direction of propagation. Thus **F** is equal to the net flux πF used by Chandrasekhar, with the important exception that **F** is used only for a plane parallel beam.

Since the equation of transfer deals only in radiances, we will now address the rather intriguing question, "What is the radiance of a plane parallel beam?"

Figure 2 shows a ray of a plane parallel beam of flux density **F** incident on the surface of a scattering medium. We shall let the resulting incident radiance be L_i, which, using Chandrasekhar's notation would be specified in position and direction as

$$L_i = L(0, -\mu_0, \varphi_0), \quad \mu_0 = |\cos\vartheta_0| \tag{1.4.1}$$

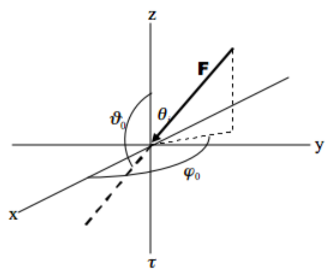

Fig. 2.

Many authors specify the polar direction of this radiation in terms of an *angle of incidence*, say i or θ_i, as the angle between the surface normal and the incident ray, such that $i = \pi - \vartheta_0$ and define μ_0 as $\mu_0 = \cos i$.

The angular distribution of **F** (or, rather, its lack of it) may be specified analytically by making use of the Dirac delta function, which has the property that

$$\int_{-\infty}^{\infty} f(x)\delta(x-a)dx = f(a)$$

and, perhaps more importantly, for any $\varepsilon > 0$

$$\int_{a-\varepsilon}^{a+\varepsilon} f(x)\delta(x-a)dx = f(a).$$

The incident radiance on the surface in the direction $(-\mu_0, \varphi_0)$ is then[1]

$$L_i = \mathbf{F}\delta(\mu-\mu_0)\delta(\varphi-\varphi_0) \tag{1.4.2}$$

Figure 3 shows a reflected beam of radiance L_r where

$$L_r = L(0, +\mu, \varphi), \quad \mu = \cos\vartheta \tag{1.4.3}$$

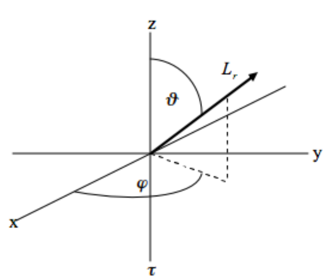

Fig. 3.

Again, many authors define the polar direction of reflection in terms of an *angle of reflection*, the angle between the surface normal and the reflected ray, say θ_r (which is the same as ϑ), and define μ as $\cos\theta_r$.

Figure 4 shows a transmitted ray of radiance L_t where

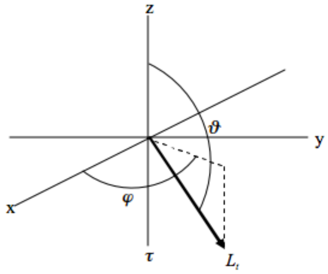

Fig. 4.

The question then arises: are the values of μ_0 and μ, introduced by Chandrasekhar and taken up by others, albeit with different definitions, always positive? The answer to this is, mostly, yes, but care needs to be taken depending on the context in which they occur, as, for example, in the following cases.

Consider the problem of determining the phase angle $0 \le a \le p$ between incident and reflected or transmitted directions μ_0 and μ, where

$$\cos\alpha = \mu_0\mu + \sqrt{\left(1-\mu_0^2\right)\left(1-\mu^2\right)}\cos(\varphi - \varphi_0). \tag{1.4.4}$$

For reflection both μ and μ_0 are positive, but for transmitted rays μ must be negative. The phase function p is often expressed in terms of a or cos a . Or, consider, as in figure 5, the optical path lengths of reflected and transmitted radiation from a depth within a medium of optical thickness t. The total optical path for the reflected ray is $\tau/\mu_0 + \tau/\mu_r$ and for the transmitted ray $\tau/\mu_0 + (t - \tau)/\mu_t$ in which μ_0, μ_r and μ_t are all positive.

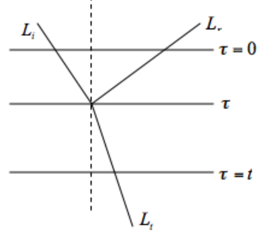

Fig. 5.

[1]If equation 1.4.2 bothers you in that its right hand side does not appear to have units of radiance, then do not worry; in chapter 2 we will demonstrate that indeed it does have such units.