1.7: Other Reflectance Functions

It is important to distinguish between a reflectance function and the reflectance law it represents. So far we have only considered one such function, the BRDF, so that the Lommel-Seeliger law expressed in terms of the BRDF is given by Equation 1.7.1 and the specific equation for the radiance is given by

$$L_r=\frac{\varpi_0}{4\pi}\frac{1}{\mu_0+\mu}\mu_0\mathbf{F} \tag{1.7.1}$$

Chandrasekhar takes a quite different approach, linking the radiance to the incident flux density through a factor $1/4\mu$, providing a consistent set of **scattering functions** S and **transmission functions** T, so that in the case of reflection from a semi-infinite surface we have

$$L_r=\frac{F}{4\mu}S\left(\mu,\varphi;\mu_0,\varphi_0\right)=\frac{\mathbf{F}}{4\pi\mu}S\left(\mu,\varphi;\mu_0,\varphi_0\right), \tag{1.7.2}$$

where Chandrasekhar *always* uses πF for incident radiant flux density **F**. Although, at least at first sight, this formulation may seem strange, even counterintuitive, there is a reason for it; the μ in the denominator is used to satisfy the *Helmholtz principle of reciprocity* (Chandrasekhar, p171), so that

$$S\left(\mu,\varphi;\mu_0,\varphi_0\right)=S\left(\mu_0,\varphi_0;\mu,\varphi\right). \tag{1.7.3}$$

Comparing Equations 1.7.1 and 1.7.2, it follows that for the Lommel-Seeliger law the Chandrasekhar scattering function is

$$S\left(\mu,\mu_0\right)=\frac{\varpi_0\mu_0\mu}{\mu_0+\mu}, \tag{1.7.4}$$

where it can be seen that the reciprocity principle does indeed hold.

Another function to be found in the literature is the **bidirectional reflectance** r, which links the radiance to the incident flux density, so that the Lommel-Seeliger law is then

$$r\left(\mu_0,\mu\right)=\frac{\varpi_0}{4\pi}\frac{\mu_0}{\mu_0+\mu},\quad L_r=r\mathbf{F} \tag{1.7.5}$$

So, which, if any, of the above functions is the “best” for planetary applications? There does not appear to be any “standard” in use in the literature, indeed the situation would seem to be quite the opposite, many authors making up their own *ad hoc* “reflectance” or “scattering” functions to suit the problem at hand. (This can make for very frustrating reading, especially when words such as “flux”, ”intensity” and “brightness” are used loosely, as, sadly, is often the case).

The author can see no compelling reason to prefer one function over another. What is important is for authors to state clearly and without ambiguity the properties of the reflectance function and rule(s) which they are using.