# 1.3: Diffuse Reflection and Transmission

The fundamental problem of planetary photometry is the diffuse reflection and transmission of a *plane parallel* beam of radiation by a *scattering medium*, which we would understand as a planetary atmosphere and/or surface or a planetary layer such as the rings of Saturn. Such media may be idealised as locally plane parallel strata in which physical properties are uniform throughout a given layer. In such cases we may use a hybrid Cartesian and spherical frame of reference in which the Oxy plane is the surface and z-axis points in the direction of the surface normal. Directions are then specified by the polar and azimuthal angles $\vartheta$ and $\varphi$ ("curly theta" and "curly phi") respectively. Further, with problems of this kind, rather than working in actual physical distances it is preferable to work in terms of normal optical thickness t , measured downwards from $z = 0$, such that $d\tau = -\kappa \rho dz$. Radiation that has traversed a path of optical thickness t is attenuated by a factor of exp(- t )

Using the direction cosine $m$ = cos ϑ the *standard form* of the equation of transfer for *plane parallel media* is

$$\mu \frac{dL(\tau, \mu, \varphi)}{d\tau} = L(\tau, \mu, \varphi) - \Im(\tau, \mu, \varphi) \tag{1.3.1}$$

For a scattering medium, the only contribution to the source function is the scattering of that radiation which has been incident on the medium from *external* sources, so that, by totalling the contributions impinging on level τ from all directions, the source function is

$$\Im(\tau, \mu, \varphi) = \frac{1}{4\pi} \int_{-1}^{1} \int_{0}^{2\pi} p\left(\mu, \varphi; \mu', \varphi'\right) L\left(\tau, \mu', \varphi'\right) d\varphi' d\mu' \tag{1.3.2}$$

where $p$ is the *normalised phase function* which determines the angular distribution of the scattering. A convenient way to think of $p$ is that $\frac{p}{4\pi} d\omega$ is the probability that a photon travelling in the direction (μ', φ') would be scattered into an elemental solid angle $dw$ in the direction (μ, φ).

Radiation traversing a normal optical thickness $dt$ in the direction (μ, φ) will be attenuated by the amount $\delta L = L\delta\tau/\mu$. Of this amount, a fraction can be attributed to that caused by scattering alone – this fraction is called the *single scattering albedo* $\varpi_0$. It then follows that the phase function $p$ must be normalised according to

$$\int_{4\pi} \frac{p}{4\pi} d\omega = 0 \leq \varpi_0 \leq 1 \tag{1.3.3}$$

and if $p$ is a constant, then $p = \varpi_0$ and the scattering is isotropic.