2.9: Spheres - Bond Albedo, Phase Integral and Geometrical Albedo

Originally defined for a sphere, the Bond albedo is defined as the ratio of the total power P_r scattered by the sphere to the total power P_i intercepted by it.

If we let the intensity of the sphere as a function of solar phase angle α be I(α) watts per steradian, then the total scattered flux may be obtained by multiplying by 2π sin α dα and integrating over α from 0 to π

$$P_r = 2\pi \int_0^\pi I(\alpha) \sin\alpha d\alpha, \tag{2.9.1}$$

which can be expressed in terms of the *normalised phase law* ψ(α) = I(α)/I(0)

$$P_r = 2\pi I(0) \int_0^\pi \phi(\alpha) \sin\alpha d\alpha. \tag{2.9.2}$$

For a sphere of radius α, the intercepted flux is simply $P_i = \pi\, \alpha^2\, \mathbf{F}$, so that the Bond albedo may be expressed as

$$A = \frac{I(0)}{\alpha^2 \mathbf{F}} \times 2 \int_0^\pi \phi(\alpha) \sin\alpha d\alpha = p \times q \tag{2.9.3}$$

in which it may be seen as the product of two factors, the second of which,

$$q = 2 \int_0^\pi \phi(\alpha) \sin\alpha d\alpha, \tag{2.9.4}$$

is called the *phase integral*, which depends only on the directional reflecting properties of the planet. The first factor

$$p = \frac{I(0)}{\alpha^2 \mathbf{F}} \tag{2.9.5}$$

depends only on the geometrical and photometric properties of the planet when observed at full phase. The quantity p is itself a (kind of) albedo since $\alpha^2\mathbf{F}$ can be seen as the intensity, scattered back towards the source, of a normally irradiated *lossless* (ϖ_0=1) Lambertian disc of the same radius as the planet. The factor p is called the *geometrical albedo*. [When albedo is used without qualification in the context of the photometry of asteroids it (usually) means geometrical albedo, in particular that observed in the Johnson V-band, p_V, the *visual geometrical albedo*].

For the reflectance rules we have considered so far, *i.e.* Lambert's law and the Lommel-Seeliger law, analytical expressions for A, p and q are readily found, as summarised in Table II.

Table II. Properties of Spheres

	Lambertian	Lommel-Seeliger
q	$\frac{3}{2}$	$\frac{16}{3}(1-\ln 2)$
p	$2\varpi_0/3$	$\varpi_{0/8}$
A	ϖ_0	$\frac{3}{2}\varpi_0(1-\ln 2)$

More complicated reflectance laws, in particular those which address the problem of the *opposition effect* for atmosphereless bodies do not readily lend themselves to analytical solutions. In general, such laws exhibit a BRDF which depends on phase angle α and a possible set of *reflectance parameters*, symbolised by the ellipsis, so that the BRDF would be generally expressed in the form

$$f_r = f_r\left(\mu_0, \mu, \alpha; \ldots\right), \tag{2.9.6}$$

where the dependence on φ and φ_0 has been replaced by α, the angle between the incident and scattered radiation, *i.e.* α does not always mean *solar* phase angle.