13.A: Atomic Structure (Answers)

Check Your Understanding

8.1. No. The quantum number $m = -l, -l+1, \ldots, 0, \ldots, l-1, l$. Thus, the magnitude of L_z is always less than **L** because $< \sqrt{l(l+1)}$

8.2. $s = 3/2 <$

8.3. frequency quadruples

Conceptual Questions

1. n (principal quantum number) → total energy

l (orbital angular quantum number) → total absolute magnitude of the orbital angular momentum

m (orbital angular projection quantum number) → z-component of the orbital angular momentum

3. The Bohr model describes the electron as a particle that moves around the proton in well-defined orbits. Schrödinger's model describes the electron as a wave, and knowledge about the position of the electron is restricted to probability statements. The total energy of the electron in the ground state (and all excited states) is the same for both models. However, the orbital angular momentum of the ground state is different for these models. In Bohr's model, $L(groundstate) = 1$, and in Schrödinger's model, $L(groundstate) = 0$.

5. a, c, d; The total energy is changed (Zeeman splitting). The work done on the hydrogen atom rotates the atom, so the **z**-component of angular momentum and polar angle are affected. However, the angular momentum is not affected.

7. Even in the ground state $(l = 0)$, a hydrogen atom has magnetic properties due the intrinsic (internal) electron spin. The magnetic moment of an electron is proportional to its spin.

9. For all electrons, $s = ½$ and $m_s = \pm ½$. As we will see, not all particles have the same spin quantum number. For example, a photon as a spin 1 ($s = 1$), and a Higgs boson has spin 0 ($s = 0$).

11. An electron has a magnetic moment associated with its intrinsic (internal) spin. Spin-orbit coupling occurs when this interacts with the magnetic field produced by the orbital angular momentum of the electron.

13. Elements that belong in the same column in the periodic table of elements have the same fillings of their outer shells, and therefore the same number of valence electrons. For example:

Li: $1s^2 2s^1$ (one valence electron in the $n = 2$ shell)

Na: $1s^2 2s 2p^6 3s^1$ (one valence electron in the $n = 2$ shell)

Both, Li and Na belong to first column.

15. Atomic and molecular spectra are said to be "discrete," because only certain spectral lines are observed. In contrast, spectra from a white light source (consisting of many photon frequencies) are continuous because a continuous "rainbow" of colors is observed.

17. UV light consists of relatively high frequency (short wavelength) photons. So the energy of the absorbed photon and the energy transition (ΔE) in the atom is relatively large. In comparison, visible light consists of relatively lower-frequency photons. Therefore, the energy transition in the atom and the energy of the emitted photon is relatively small.

19. For macroscopic systems, the quantum numbers are very large, so the energy difference (ΔE) between adjacent energy levels (orbits) is very small. The energy released in transitions between these closely space energy levels is much too small to be detected.

21. Laser light relies on the process of stimulated emission. In this process, electrons must be prepared in an excited (upper) metastable state such that the passage of light through the system produces de-excitations and, therefore, additional light.

23. A Blu-Ray player uses blue laser light to probe the bumps and pits of the disc and a CD player uses red laser light. The relatively short-wavelength blue light is necessary to probe the smaller pits and bumps on a Blu-ray disc; smaller pits and bumps correspond to higher storage densities.

25. $(r, \theta, \phi) = (\sqrt{6}, 66^\circ, 27^\circ)$.

27. $\pm 3, \pm 2, \pm 1, 0$ are possible

29. 18

31. $F = -k\frac{Qq}{r^2}$

33. (1, 1, 1)

35. For the orbital angular momentum quantum number, **l**, the allowed values of:

$m = -l, -l+1, \ldots 0, \ldots l-1, l$.

With the exception of $m = 0$, the total number is just **2l** because the number of states on either side of $m = 0$ is just **l**. Including $m = 0$, the total number of orbital angular momentum states for the orbital angular momentum quantum number, **l**, is: $2l+1$. Later, when we consider electron spin, the total number of angular momentum states will be found to twice this value because each orbital angular momentum states is associated with two states of electron spin: spin up and spin down).

37. The probability that the **1s** electron of a hydrogen atom is found outside of the Bohr radius is $\int_{a_0}^{\infty} P(r)dr \approx 0.68$

39. For $n = 2, l = 0$ (1 state), and $l = 1$ (3 states). The total is 4.

41. The **3p** state corresponds to $n = 3, l = 2$. Therefore, $\mu = \mu_B\sqrt{6}$

43. The ratio of their masses is 1/207, so the ratio of their magnetic moments is 207. The electron's magnetic moment is more than 200 times larger than the muon.

45. a. The 3d state corresponds to $n = 3, l = 2$. So,

$I = 4.43 \times 10^{-7} A$.

b. The maximum torque occurs when the magnetic moment and external magnetic field vectors are at right angles $(sin\theta = 1)$. In this case:

$|\vec{\tau}| = \mu B.$

$\tau = 5.70 \times 10^{-26} N \cdot m$ ..

47. A **3p** electron is in the state $n = 3$ and $l = 1$. The minimum torque magnitude occurs when the magnetic moment and external magnetic field vectors are most parallel (antiparallel). This occurs when $m = \pm 1$. The torque magnitude is given by

$|\vec{\tau}| = \mu B sin\theta$,

Where

$\mu = (1.31 \times 10^{-24} J/T)$.

For $m = \pm 1$, we have:

$|\overrightarrow{\tau}| = 2.32 \times 10^{21} N \cdot m$.

49. An infinitesimal work **dW** done by a magnetic torque τ to rotate the magnetic moment through an angle $-d\theta$:

$dW = \tau(-d\theta)$,

where $\tau = |\vec{\mu} \times \vec{B}|$. Work done is interpreted as a drop in potential energy U, so

$dW = -dU.$

The total energy change is determined by summing over infinitesimal changes in the potential energy:

$U = -\mu B cos\theta$

$U = -\vec{\mu} \cdot \vec{B}$.

51. Spin up (relative to positive **z**-axis):

$\theta = 55^{\circ}$.

Spin down (relative to positive **z**-axis):

$$\theta = cos^{-1}\left(\frac{S_z}{S}\right) = cos^{-1}\left(\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = cos^{-1}\left(\frac{-1}{\sqrt{3}}\right) = 125^{\circ}.$$

53. The spin projection quantum number is $m_s = \pm½$, so the z-component of the magnetic moment is

$\mu_z = \pm\mu_B$.

The potential energy associated with the interaction between the electron and the external magnetic field is

$U = \mp\mu_B B$.

The energy difference between these states is $\Delta E = 2\mu_B B$, so the wavelength of light produced is

$\lambda = 5.36 \times 10^{-5} m \approx 53.6 \mu m$

55. It is increased by a factor of 2.

57. a. 32;

b.

ℓ (2ℓ+1)

0 s 2(0+1) =2

1 p 2(2+1) =6

2 d 2(4+1) =10

3 f 2(6+1) =14
32

59. a. and e. are allowed; the others are not allowed.

b. $l = 3$ not allowed for $n = 1, l \leq (n-1)$.

c. Cannot have three electrons in **s** subshell because $3 > 2(2l+1) = 2$.

d. Cannot have seven electrons in **p** subshell (max of 6) $2(2l+1) = 2(2+1) = 6$.

61. $[Ar]4s^2 3d^6$

63. a. The minimum value of ℓ is $l = 2$ to have nine electrons in it.

b. $3d^9$.

65. $[He]2s^2 2p^2$

67. For He^+, one electron "orbits" a nucleus with two protons and two neutrons ($Z = 2$). Ionization energy refers to the energy required to remove the electron from the atom. The energy needed to remove the electron in the ground state of He+He+ ion to infinity is negative the value of the ground state energy, written:

$E = -54.4 eV$.

Thus, the energy to ionize the electron is $+54.4 eV$.

Similarly, the energy needed to remove an electron in the first excited state of Li^{2+} ion to infinity is negative the value of the first excited state energy, written:

$E = -30.6 eV$.

The energy to ionize the electron is 30.6 eV.

69. The wavelength of the laser is given by:

$$\lambda = \frac{hc}{-\Delta E},$$

where E_γ is the energy of the photon and ΔE is the magnitude of the energy difference. Solving for the latter, we get:

$$\Delta E = -2.795 eV.$$

The negative sign indicates that the electron lost energy in the transition.

71. $\Delta E_{L \to K} \approx (Z-1)^2(10.2 eV) = 3.68 \times 10^3 eV$.

73. According to the conservation of the energy, the potential energy of the electron is converted completely into kinetic energy. The initial kinetic energy of the electron is zero (the electron begins at rest). So, the kinetic energy of the electron just before it strikes the target is:

$$K = e\Delta V.$$

If all of this energy is converted into braking radiation, the frequency of the emitted radiation is a maximum, therefore:

$$f_{max} = \frac{e\Delta V}{h}.$$

When the emitted frequency is a maximum, then the emitted wavelength is a minimum, so:

$$\lambda_{min} = 0.1293 nm.$$

75. A muon is 200 times heavier than an electron, but the minimum wavelength does not depend on mass, so the result is unchanged.

77. $4.13 \times 10^{-11} m$

79. 72.5 keV

81. The atomic numbers for Cu and Au are $Z = 29$ and 79, respectively. The X-ray photon frequency for gold is greater than copper by a factor:

$$(\frac{f_{Au}}{f_{Cu}})^2 = (\frac{79-1}{29-1})^2 \approx 8 .$$

Therefore, the X-ray wavelength of Au is about eight times shorter than for copper.

83. a. If flesh has the same density as water, then we used 1.34×10^{23} photons.

b. 2.52 MW

Additional Problems

85. The smallest angle corresponds to $l = n-1$ and $m = l = n-1$. Therefore $\theta = cos^{-1}(\sqrt{n-1} n)$.

87. a. According to Equation 8.1, when $r = 0, U(r) = -\infty$, and when $r = +\infty, U(r) = 0$. The former result suggests that the electron can have an infinite negative potential energy. The quantum model of the hydrogen atom avoids this possibility because the probability density at $r = 0$ is zero.

89. A formal solution using sums is somewhat complicated. However, the answer easily found by studying the mathematical pattern between the principal quantum number and the total number of orbital angular momentum states.

For $n = 1$, the total number of orbital angular momentum states is 1; for $n = 2$, the total number is 4; and, when $n = 3$, the total number is 9, and so on. The pattern suggests the total number of orbital angular momentum states for the nth shell is n^2.

(Later, when we consider electron spin, the total number of angular momentum states will be found to be $2n^2$, because each orbital angular momentum states is associated with two states of electron spin; spin up and spin down).

91. 50

93. The maximum number of orbital angular momentum electron states in the nth shell of an atom is n^2. Each of these states can be filled by a spin up and spin down electron. So, the maximum number of electron states in the nth shell is $2n^2$.

95. a., c., and e. are allowed; the others are not allowed.

b. $l > n$ is not allowed.

d. $7 > 2(2l+1)$

97. $f = 1.8 \times 10^9 Hz$

99. The atomic numbers for Cu and Ag are $Z = 29$ and 47, respectively. The X-ray photon frequency for silver is greater than copper by the following factor:

$$(\frac{f_{Ag}}{f_{Cu}})^2 = 2.7.$$

Therefore, the X-ray wavelength of Ag is about three times shorter than for copper.

101. a. 3.24;

b. n_i is not an integer. c. The wavelength must not be correct. Because $n_i > 2$, the assumption that the line was from the Balmer series is possible, but the wavelength of the light did not produce an integer value for n_i. If the wavelength is correct, then the assumption that the gas is hydrogen is not correct; it might be sodium instead.