8.1: Faraday and Lenz's Laws

Learning Objectives

By the end of this section, you will be able to:

- Determine the magnetic flux through a surface, knowing the strength of the magnetic field, the surface area, and the angle between the normal to the surface and the magnetic field
- Use Faraday's law to determine the magnitude of induced emf in a closed loop due to changing magnetic flux through the loop
- Use Lenz's law to determine the direction of induced emf whenever a magnetic flux changes
- Use Faraday's law with Lenz's law to determine the induced emf in a coil and in a solenoid

We have been considering electric fields created by fixed charge distributions and magnetic fields produced by constant currents, but electromagnetic phenomena are not restricted to these stationary situations. Most of the interesting applications of electromagnetism are, in fact, time-dependent. To investigate some of these applications, we now remove the time-independent assumption that we have been making and allow the fields to vary with time. In this and the next several chapters, you will see a wonderful symmetry in the behavior exhibited by time-varying electric and magnetic fields. Mathematically, this symmetry is expressed by an additional term in Ampère's law and by another key equation of electromagnetism called Faraday's law. We also discuss how moving a wire through a magnetic field produces an emf or voltage. Lastly, we describe applications of these principles.

Faraday's Law

The first productive experiments concerning the effects of time-varying magnetic fields were performed by Michael **Faraday** in 1831. One of his early experiments is represented in Figure 8.1.1. An **emf** is induced when the magnetic field in the coil is changed by pushing a bar magnet into or out of the coil. Emfs of opposite signs are produced by motion in opposite directions, and the directions of emfs are also reversed by reversing poles. The same results are produced if the coil is moved rather than the magnet—it is the relative motion that is important. The faster the motion, the greater the emf, and there is no emf when the magnet is stationary relative to the coil.

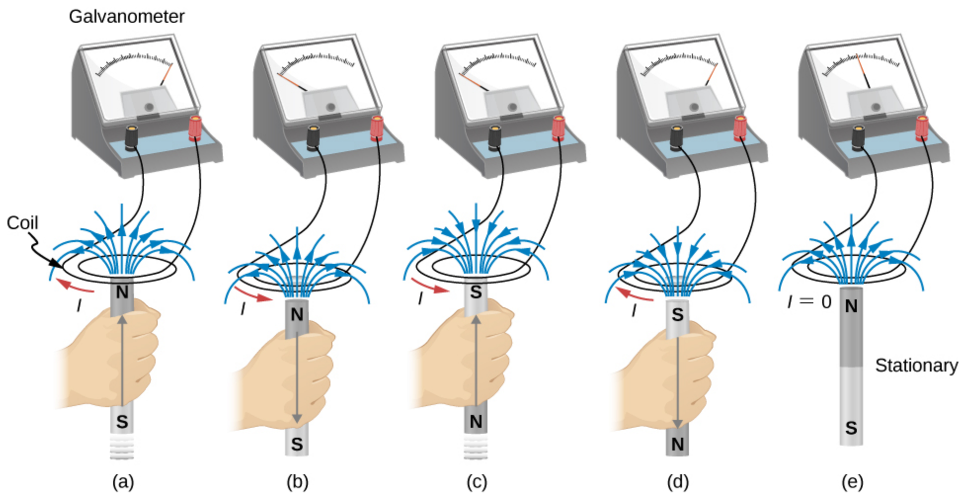


Figure 8.1.1: Movement of a magnet relative to a coil produces emfs as shown (a–d). The same emfs are produced if the coil is moved relative to the magnet. This short-lived emf is only present during the motion. The greater the speed, the greater the magnitude of the emf, and the emf is zero when there is no motion, as shown in (e).

Faraday also discovered that a similar effect can be produced using two circuits—a changing current in one circuit induces a current in a second, nearby circuit. For example, when the switch is closed in circuit 1 of Figure 8.1.1a, the ammeter needle of circuit 2 momentarily deflects, indicating that a short-lived current surge has been induced in that circuit. The ammeter needle quickly returns to its original position, where it remains. However, if the switch of circuit 1 is now suddenly opened, another short-lived current surge in the direction opposite from before is observed in circuit 2.

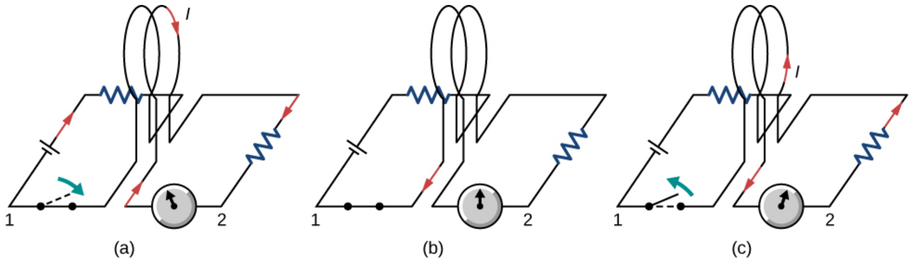


Figure 8.1.2: (a) Closing the switch of circuit 1 produces a short-lived current surge in circuit 2. (b) If the switch remains closed, no current is observed in circuit 2. (c) Opening the switch again produces a short-lived current in circuit 2 but in the opposite direction from before.

Faraday realized that in both experiments, a current flowed in the circuit containing the ammeter only when the magnetic field in the region occupied by that circuit was **changing**. As the magnet of the figure was moved, the strength of its magnetic field at the loop changed; and when the current in circuit 1 was turned on or off, the strength of its magnetic field at circuit 2 changed. Faraday was eventually able to interpret these and all other experiments involving magnetic fields that vary with time in terms of the following law.

Faraday's Law

The emf ϵ induced is the negative change in the magnetic flux Φ_m per unit time. Any change in the magnetic field or change in orientation of the area of the coil with respect to the magnetic field induces a voltage (emf).

The magnetic flux is a measurement of the amount of magnetic field lines through a given surface area, as seen in Figure 8.1.3. This definition is similar to the electric flux studied earlier. This means that if we have

$$\Phi_m = \int_S \vec{B} \cdot \hat{n} dA, \tag{8.1.1}$$

then the **induced emf** or the voltage generated by a conductor or coil moving in a magnetic field is

$$\epsilon = -\frac{d}{dt} \int_S \vec{B} \cdot \hat{n} dA = -\frac{d\Phi_m}{dt}. \tag{8.1.2}$$

The negative sign describes the direction in which the induced emf drives current around a circuit. However, that direction is most easily determined with a rule known as Lenz's law, which we will discuss shortly.

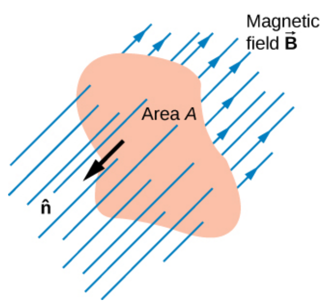


Figure 8.1.3: The magnetic flux is the amount of magnetic field lines cutting through a surface area **A** defined by the unit area vector $\hat{n}$. If the angle between the unit area $\hat{n}$ and magnetic field vector $\vec{B}$ are parallel or antiparallel, as shown in the diagram, the magnetic flux is the highest possible value given the values of area and magnetic field.

$8.1.1a$ depicts a circuit and an arbitrary surface **S** that it bounds. Notice that **S** is an **open surface**. It can be shown that **any** open surface bounded by the circuit in question can be used to evaluate Φ_m. For example, Φ_m is the same for the various surfaces $S_1, S_2, \ldots$ of part (b) of the figure.

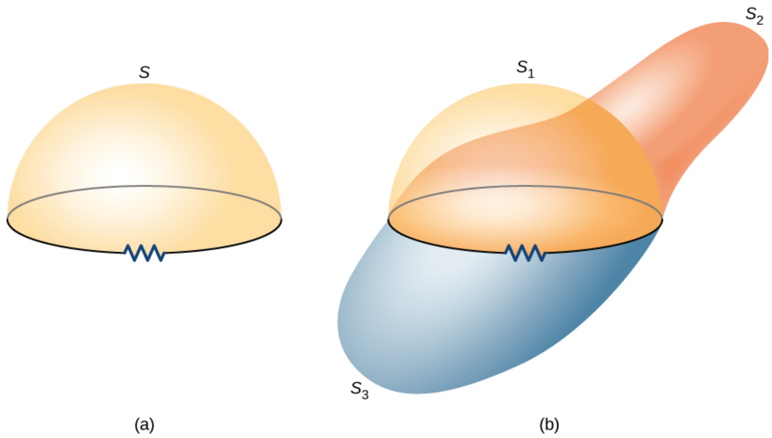


Figure 8.1.4: (a) A circuit bounding an arbitrary open surface **S**. The planar area bounded by the circuit is not part of **S**. (b) Three arbitrary open surfaces bounded by the same circuit. The value of Φ_m is the same for all these surfaces.

The SI unit for magnetic flux is the weber (Wb),

$$1\,Wb = 1\,T \cdot m^2. \tag{8.1.3}$$

Occasionally, the magnetic field unit is expressed as webers per square meter (Wb/m^2) instead of teslas, based on this definition. In many practical applications, the circuit of interest consists of a number **N** of tightly wound turns (Figure 8.1.5). Each turn experiences the same magnetic flux. Therefore, the net magnetic flux through the circuits is **N** times the flux through one turn, and Faraday’s law is written as

$$\epsilon = -\frac{d}{dt}(N\Phi_m) = -N\frac{d\Phi_m}{dt}. \tag{8.1.4}$$

Lenz's Law

The direction in which the induced emf drives current around a wire loop can be found through the negative sign. However, it is usually easier to determine this direction with Lenz’s law, named in honor of its discoverer, Heinrich Lenz (1804–1865). (Faraday also discovered this law, independently of Lenz.) We state Lenz’s law as follows:

Lenz's Law

The direction of the induced emf drives current around a wire loop to always **oppose** the change in magnetic flux that causes the emf.

Lenz’s law can also be considered in terms of conservation of energy. If pushing a magnet into a coil causes current, the energy in that current must have come from somewhere. If the induced current causes a magnetic field opposing the increase in field of the magnet we pushed in, then the situation is clear. We pushed a magnet against a field and did work on the system, and that showed up as current. If it were not the case that the induced field opposes the change in the flux, the magnet would be pulled in produce a current without anything having done work. Electric potential energy would have been created, violating the conservation of energy.

To determine an induced emf ϵ, you first calculate the magnetic flux Φ_m and then obtain $d\Phi_m/dt$. The magnitude of ϵ is given by

$$\epsilon = \left|\frac{d\Phi_m}{dt}\right|. \tag{8.1.5}$$

Finally, you can apply Lenz’s law to determine the sense of ϵ. This will be developed through examples that illustrate the following problem-solving strategy.

Problem-Solving Strategy: Lenz’s Law

To use Lenz’s law to determine the directions of induced magnetic fields, currents, and emfs:

- Make a sketch of the situation for use in visualizing and recording directions.

- Determine the direction of the applied magnetic field $\vec{B}$.
- Determine whether its magnetic flux is increasing or decreasing.
- Now determine the direction of the induced magnetic field $\vec{B}$. The induced magnetic field tries to reinforce a magnetic flux that is decreasing or opposes a magnetic flux that is increasing. Therefore, the induced magnetic field adds or subtracts to the applied magnetic field, depending on the change in magnetic flux.
- Use **right-hand rule 2** (RHR-2; see Magnetic Forces and Fields) to determine the direction of the induced current **I** that is responsible for the induced magnetic field $\vec{B}$.
- The direction (or polarity) of the induced emf can now drive a conventional current in this direction.

Let's apply Lenz's law to the system of Figure $8.1.6a$. We designate the "front" of the closed conducting loop as the region containing the approaching bar magnet, and the "back" of the loop as the other region. As the north pole of the magnet moves toward the loop, the flux through the loop due to the field of the magnet increases because the strength of field lines directed from the front to the back of the loop is increasing. A current is therefore induced in the loop. By Lenz's law, the direction of the induced current must be such that its own magnetic field is directed in a way to **oppose** the changing flux caused by the field of the approaching magnet. Hence, the induced current circulates so that its magnetic field lines through the loop are directed from the back to the front of the loop. By RHR-2, place your thumb pointing against the magnetic field lines, which is toward the bar magnet. Your fingers wrap in a counterclockwise direction as viewed from the bar magnet. Alternatively, we can determine the direction of the induced current by treating the current loop as an electromagnet that **opposes** the approach of the north pole of the bar magnet. This occurs when the induced current flows as shown, for then the face of the loop nearer the approaching magnet is also a north pole.

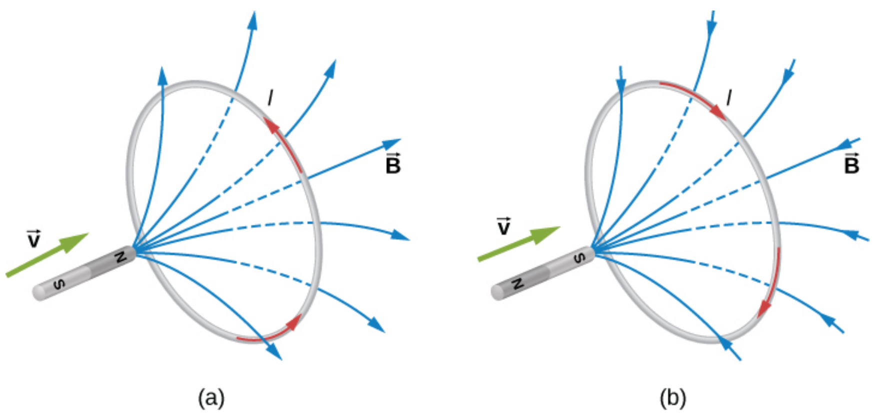
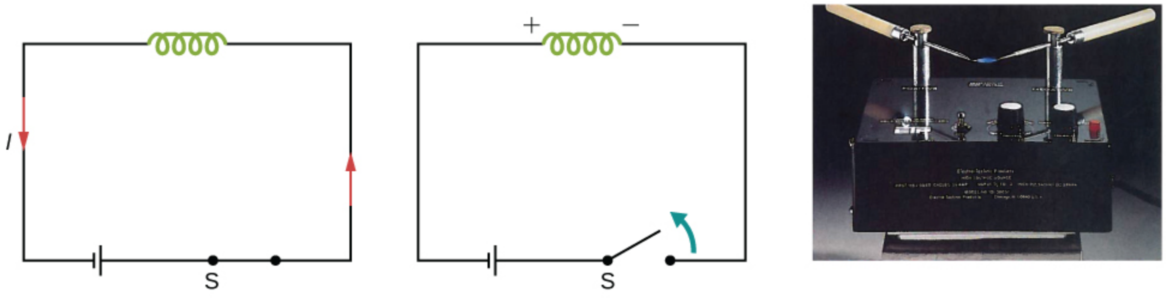


Figure $8.1.6$: The change in magnetic flux caused by the approaching magnet induces a current in the loop. (a) An approaching north pole induces a counterclockwise current with respect to the bar magnet. (b) An approaching south pole induces a clockwise current with respect to the bar magnet.

Part (b) of the figure shows the south pole of a magnet moving toward a conducting loop. In this case, the flux through the loop due to the field of the magnet increases because the number of field lines directed from the back to the front of the loop is increasing. To oppose this change, a current is induced in the loop whose field lines through the loop are directed from the front to the back. Equivalently, we can say that the current flows in a direction so that the face of the loop nearer the approaching magnet is a south pole, which then repels the approaching south pole of the magnet. By RHR-2, your thumb points away from the bar magnet. Your fingers wrap in a clockwise fashion, which is the direction of the induced current.

Another example illustrating the use of Lenz's law is shown in Figure $8.1.7$. When the switch is opened, the decrease in current through the solenoid causes a decrease in magnetic flux through its coils, which induces an emf in the solenoid. This emf must oppose the change (the termination of the current) causing it. Consequently, the induced emf has the polarity shown and drives in the direction of the original current. This may generate an arc across the terminals of the switch as it is opened.



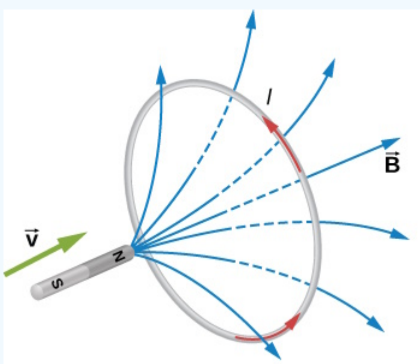
(a) (b) (c)

Figure 8.1.7: (a) A solenoid connected to a source of emf. (b) Opening switch S terminates the current, which in turn induces an emf in the solenoid. (c) A potential difference between the ends of the sharply pointed rods is produced by inducing an emf in a coil. This potential difference is large enough to produce an arc between the sharp points.

Examples and Exercises

Example 8.1.1: Calculating Emf: How Great is the Induced Emf?

Calculate the magnitude of the induced emf when the magnet in the Figure is thrust into the coil, given the following information: the single loop coil has a radius of 6.00 cm and the average value of $B\cos\theta$ (this is given, since the bar magnet's field is complex) increases from 0.0500 T to 0.250 T in 0.100 s.



Strategy

To find the *magnitute* of emf, we use Faraday's law of induction as stated by $emf = -N\frac{\Delta\Phi}{\Delta t}$, but without the minus sign that indicates direction:

$$emf = N\frac{\Delta\Phi}{\Delta t}. \tag{8.1.6}$$

Solution

We are given that $N = 1$ and $\Delta t = 0.100s$ but we must determine the change in flux $\Delta\Phi$ before we can find emf. Since the area of the loop is fixed, we see that

$$\Delta\Phi\left(BA\cos\theta\right) = A\Delta\left(B\cos\theta\right). \tag{8.1.7}$$

Now $\Delta\left(B\cos\theta\right) = 0.200T$, since it was given that $B\cos\theta$ changes from 0.0500 to 0.250 T. The area of the loop is $A = \pi r^2 = (3.14...)\left(0.060m\right)^2 = 1.13\times 10^{-2} m^2$. Thus,

$$\Delta\Phi = \left(1.13\times 10^{-2} m^2\right)\left(0.200T\right). \tag{8.1.8}$$

Entering the determined values into the expression for emf gives

$$Emf = N\frac{\Delta\Phi}{\Delta t} = \frac{\left(1.13\times 10^{-2} m^2\right)\left(0.200T\right)}{0.100s} = 22.6mV. \tag{8.1.9}$$

Discussion:

While this is an easily measured voltage, it is certainly not large enough for most practical applications. More loops in the coil, a stronger magnet, and faster movement make induction the practical source of voltages that it is.

Example $8.1.2$: A Square Coil in a Changing Magnetic Field

The square coil of Figure $8.1.1$ has sides $l = 0.25\,m$ long and is tightly wound with $N = 200$ turns of wire. The resistance of the coil is $R = 5.0\,\Omega$ The coil is placed in a spatially uniform magnetic field that is directed perpendicular to the face of the coil and whose magnitude is decreasing at a rate $dB/dt = -0.040\,T/s$. (a) What is the magnitude of the emf induced in the coil? (b) What is the magnitude of the current circulating through the coil?

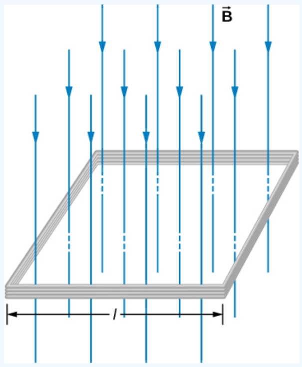


Figure $8.1.5$: A square coil with **N** turns of wire with uniform magnetic field $\vec{B}$ directed in the downward direction, perpendicular to the coil.

Strategy

The area vector, or $\hat{n}$ direction, is perpendicular to area covering the loop. We will choose this to be pointing downward so that $\vec{B}$ is parallel to $\hat{n}$ and that the flux turns into multiplication of magnetic field times area. The area of the loop is not changing in time, so it can be factored out of the time derivative, leaving the magnetic field as the only quantity varying in time. Lastly, we can apply Ohm’s law once we know the induced emf to find the current in the loop.

Solution

1. The flux through one turn is

$$\Phi_m = BA = Bt^2, \tag{8.1.10}$$

so we can calculate the magnitude of the emf from Faraday’s law. The sign of the emf will be discussed in the next section, on Lenz’s law:

$$|\epsilon| = \left| -N \frac{d\Phi_m}{dt} \right| = Nl^2 \frac{dB}{dt} \tag{8.1.11}$$

$$= (200)(0.25\,m)^2(0.040\,T/s) = 0.50\,V. \tag{8.1.12}$$

The magnitude of the current induced in the coil is

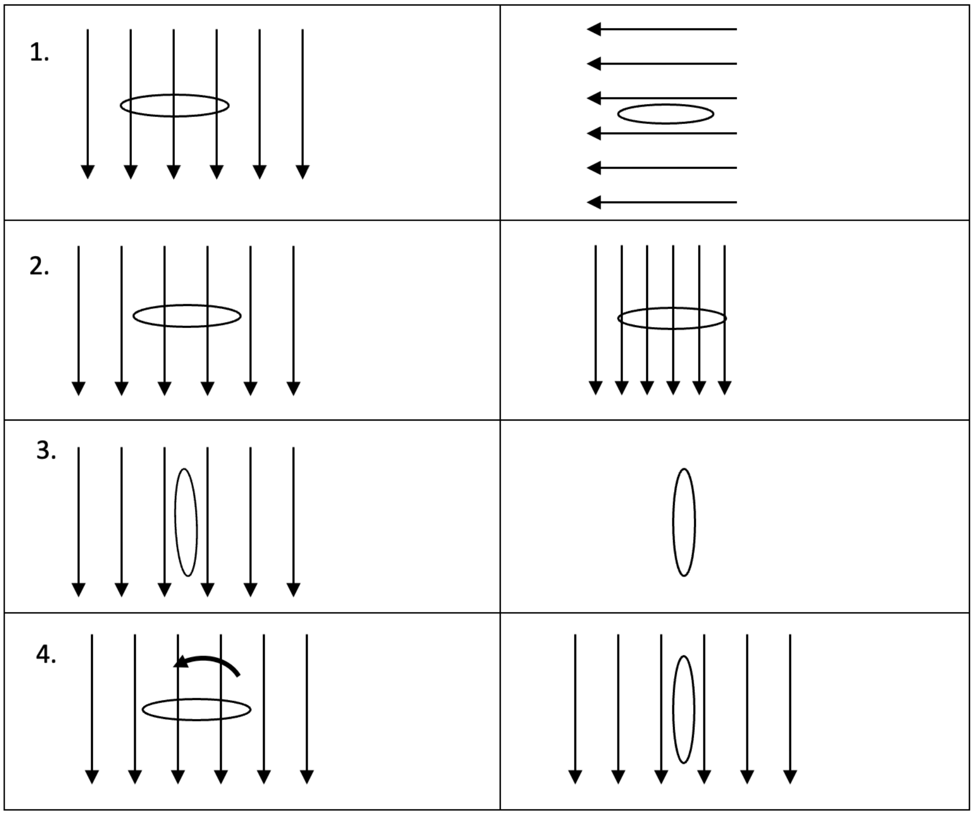
$$I = \frac{\epsilon}{R} = \frac{0.50\,V}{5.0\,\Omega} = 0.10\,A. \tag{8.1.13}$$

Significance

If the area of the loop were changing in time, we would not be able to pull it out of the time derivative. Since the loop is a closed path, the result of this current would be a small amount of heating of the wires until the magnetic field stops changing. This may increase the area of the loop slightly as the wires are heated.

Example 8.1.3:

Below are 4 cases: the left panels show the initial and the right panels show the final configurations. The arrows indicate the direction and magnitude of the external magnetic field.



Fill in the table below for each case

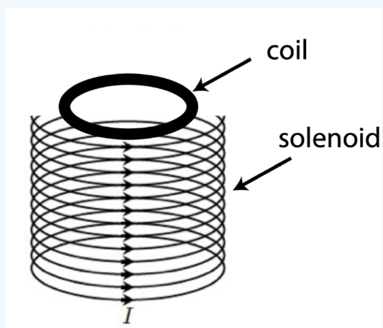
Case	Change in Flux (Yes or No). If yes, what changed (A, B, or θ)	Direction of induced Field	Direction of induced Current	Brief Explanation
1				
2				
3				
4				

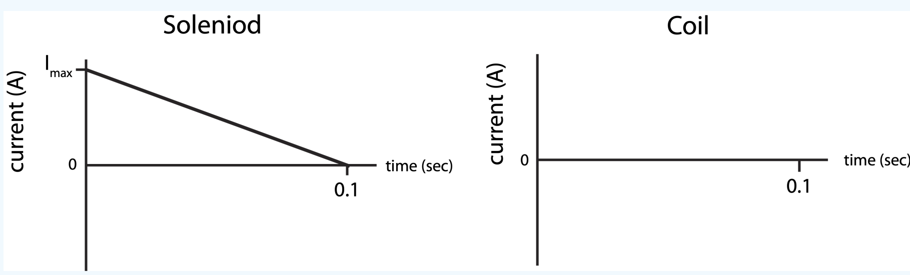
Solution

Case	Change in Flux (Yes or No). If yes, what changed (A, B or θ)	Direction of induced Field	Direction of induced Current	Brief Explanation
1	*Yes: angle*	*Down*	*Clockwise from above*	*B field down goes to zero, so the induced field is down. Using RHR results in CW current from above.*
2	*Yes: B field*	*Up*	*Counterclockwise from above*	*Density of field down increases, to oppose that induced field is up. Using RH results in CCW current from above.*
3	*No*	*N/A*	*N/A*	*Flux is not changing, so there is no induced current*
4	*Yes: angle*	*To the right*	*Clockwise from the left*	*B field down goes to zero, so induced field is to the right as the loop rotates. Using RHR results in CW current viewed from the left.*

Example 8.1.4:

Consider a magnetic field is created by a large solenoid magnet. The solenoid is 1.5 meters long, has 5000 turns, a resistance of 4Ω, and a one-meter radius. A coil located inside the solenoid has a single loop (as depicted below), a resistance of 0.6Ω, and a 0.4 m radius. The solenoid initially has a current which produces a 0.2 T magnetic field. The solenoid's current is then reduced linearly to zero in 0.1 seconds as shown in the left plot below.





a) Calculate I_{max} which is marked on the plot below.

b) Make a graph of the current in the coil on the right plot below. Make sure to indicate numerical values and explain your choice of sign for the current.

Solution

a) The magnetic field for a solenoid is $B = \frac{\mu_o I N}{L}$. *The maximum current with be when the magnetic field is at 0.2 T, since the current is reduced with time:*

$$I_{max} = \frac{BL}{\mu_o N} = \frac{0.2T \times 1.5m}{4 \times 10^{-7} \frac{N}{A^2} \times 5000} = 47.7\ A$$

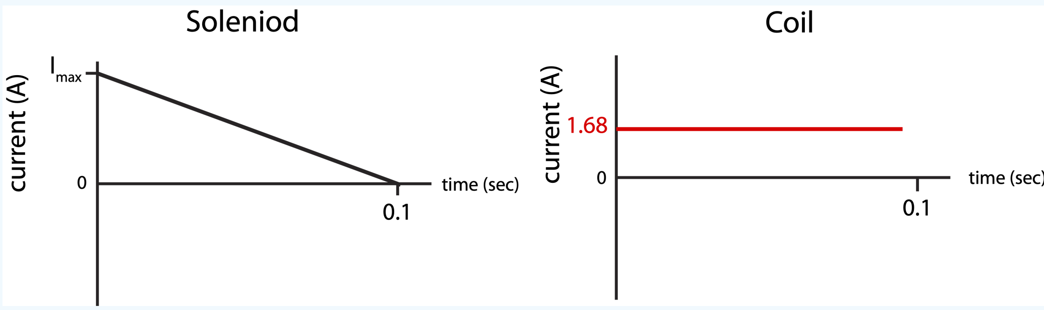
b) The magnetic field is upward using the current RHR, which is reduced to zero. Thus, the induced magnetic field will also be upward to oppose the change of flux decreasing in the upward direction. Using the same RHR again, this results in an induced current which is counterclockwise as viewed from top. In other words, the direction of the current in the coil is the same as the direction of the current in the solenoid.

Now that we have established the direction of current, we just need to worry about the magnitude of the induced emf and current. The magnitude of induced emf for the wire which only contains one loop is:

$$|\mathcal{E}| = \frac{d\Phi}{dt} = A\frac{\Delta B}{\Delta t} = \pi r^2 \frac{dB}{dt} = (0.4^2\pi)m^2 \frac{0.2T}{0.1s} = 1.68A$$

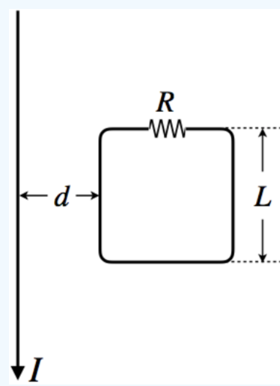
Solving for induced current:

$$I_{coil} = \frac{|\mathcal{E}|}{R} = \frac{1.0V}{0.6\Omega} = 1.68A$$



Example 8.1.5:

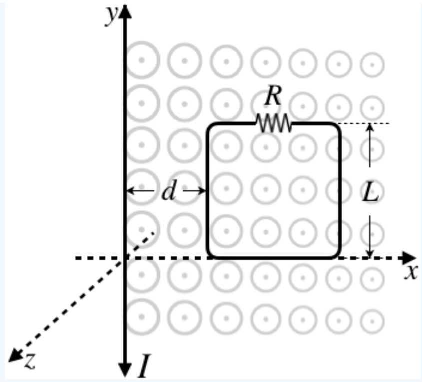
A square loop of wire with a total resistance R has a side-length of L. It resides near a long, straight wire such that the long wire is in the same plane as the loop, with one of its sides parallel to the wire a distance d from it, as shown in the diagram.



a. *Compute the magnetic flux through the loop due to a current I flowing in the direction shown in the diagram.*
b. *If the current in the long wire is changing, then the magnetic field flux through the loop will also be changing. Suppose that the current in the long wire is increasing at a constant rate of $\frac{dI}{dt} = \alpha$. Find the current induced in the loop, including its direction.*

Solution

a. We know the magnetic field for a long-straight wire gets weaker at greater distances, which means we have to perform the flux integral. The diagram below shows the magnetic field and introduces some axes for performing the math required.



The magnetic field strength in the x-y plane in terms of the distance x from the long wire is given by:

$$B = \frac{\mu_o I}{2\pi x}$$

We take as a differential area element a thin vertical slice down the length of the circuit. This slice has an area of $dA = Ldx$, and the field is constant throughout the slice. Also, the area vector is parallel to the magnetic field, so choosing the loop direction as counterclockwise, the angle between the field and the area vector is 0^o. The flux integral is therefore:

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int BdA\cos\theta = \int_{x=d}^{x=d+L} \left(\frac{\mu_o I}{2\pi x}\right) Ldx\cos 0^o = \frac{\mu_o IL}{2\pi} \ln\left[\frac{d+L}{d}\right]$$

b. The emf induced in the loop (which has only one turn in it) is found using Faraday's law:

$$\mathcal{E}_{induced} = -\frac{d\Phi_B}{dt} = -\frac{\mu_o L}{2\pi} \ln\left[\frac{d+L}{d}\right] \frac{dI}{dt} = -\frac{\mu_o L\alpha}{2\pi} \ln\left[\frac{d+L}{d}\right]$$

The magnitude of the current is this emf divided by the resistance:

$$\frac{\mu_o L\alpha}{2\pi R} \ln\left[\frac{d+L}{d}\right]$$

The direction of the current will be such that it provides a field that will seek to counter the change. The current that is causing the field is increasing, so the flux out of the page in increasing. The induced current will therefore produce a magnetic field inside the loop that points into the page, which means it must flow clockwise.

✔ Example $8.1.2A$: A Circular Coil in a Changing Magnetic Field

A magnetic field $\vec{B}$ is directed outward perpendicular to the plane of a circular coil of radius $r = 0.50\,m$ (Figure $8.1.3$). The field is cylindrically symmetrical with respect to the center of the coil, and its magnitude decays exponentially according to $B = (1.5T)e^{(5.0s^{-1})t}$, where **B** is in teslas and **t** is in seconds. (a) Calculate the emf induced in the coil at the times $t_1 = 0$, $t_2 = 5.0 \times 10^{-2}\,s$, and $t_3 = 1.0\,s$. (b) Determine the current in the coil at these three times if its resistance is $10\,\Omega$.

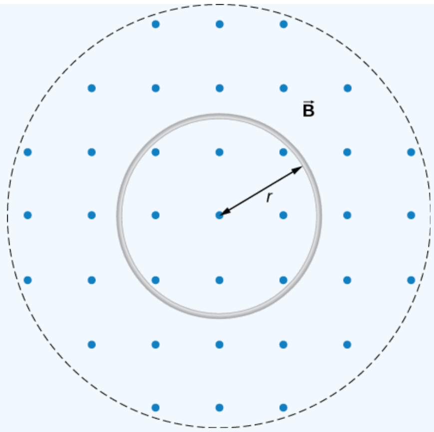


Figure 8.1.8: A circular coil in a decreasing magnetic field.

Strategy

Since the magnetic field is perpendicular to the plane of the coil and constant over each spot in the coil, the dot product of the magnetic field $\vec{B}$ and normal to the area unit vector $\hat{n}$ turns into a multiplication. The magnetic field can be pulled out of the integration, leaving the flux as the product of the magnetic field times area. We need to take the time derivative of the exponential function to calculate the emf using Faraday's law. Then we use Ohm's law to calculate the current.

Solution

1. Since $\vec{B}$ is perpendicular to the plane of the coil, the magnetic flux is given by

$$\Phi_m = B\pi r^2 = (1.5e^{-5.0t}T)\pi(0.50\,m)^2 \tag{8.1.14}$$

$$= 1.2e^{-(5.0s^{-1})t}Wb. \tag{8.1.15}$$

From Faraday's law, the magnitude of the induced emf is

$$\epsilon = \left|\frac{d\Phi_m}{dt}\right| = \left|\frac{d}{dt}(1.2e^{-(5.0s^{-1})t}Wb)\right| = 6.0e^{-(5.0s^{-1})t}V. \tag{8.1.16}$$

Since $\vec{B}$ is directed out of the page and is decreasing, the induced current must flow counterclockwise when viewed from above so that the magnetic field it produces through the coil also points out of the page. For all three times, the sense of e is counterclockwise; its magnitudes are

$$\epsilon(t_1) = 6.0V;\ \epsilon(t_2) = 4.7\,V;\ \epsilon(t_3) = 0040\,V. \tag{8.1.17}$$

2. From Ohm's law, the respective currents are

$$I(t_1) = \frac{\epsilon(t_1)}{R} = \frac{6.0\,V}{10\,\Omega} = 0.60\,A; \tag{8.1.18}$$

$$I(t_2) = \frac{4.7\,V}{10\,\Omega} = 0.47\,A; \tag{8.1.19}$$

and

$$I(t_3) = \frac{0.040\,V}{10\,\Omega} = 4.0 \times 10^{-3}\,A. \tag{8.1.20}$$

Significance

An emf voltage is created by a changing magnetic flux over time. If we know how the magnetic field varies with time over a constant area, we can take its time derivative to calculate the induced emf.

Example $8.1.2B$: Changing Magnetic Field Inside a Solenoid

The current through the windings of a solenoid with $n = 2000$ turns per meter is changing at a rate $dI/dt = 3.0\,A/s$. (See Sources of Magnetic Fields for a discussion of solenoids.) The solenoid is 50-cm long and has a cross-sectional diameter of 3.0 cm. A small coil consisting of $N = 20$ closely wound turns wrapped in a circle of diameter 1.0 cm is placed in the middle of the solenoid such that the plane of the coil is perpendicular to the central axis of the solenoid. Assuming that the infinite-solenoid approximation is valid at the location of the small coil, determine the magnitude of the emf induced in the coil.

Strategy

The magnetic field in the middle of the solenoid is a uniform value of $\mu_0 nI$. This field is producing a maximum magnetic flux through the coil as it is directed along the length of the solenoid. Therefore, the magnetic flux through the coil is the product of the solenoid's magnetic field times the area of the coil. Faraday's law involves a time derivative of the magnetic flux. The only quantity varying in time is the current, the rest can be pulled out of the time derivative. Lastly, we include the number of turns in the coil to determine the induced emf in the coil.

Solution

Since the field of the solenoid is given by $B = \mu_0 nI$, the flux through each turn of the small coil is

$$\Phi_m = \mu_0 nI\left(\frac{\pi d^2}{4}\right), \tag{8.1.21}$$

where **d** is the diameter of the coil. Now from Faraday's law, the magnitude of the emf induced in the coil is

$$\epsilon = \left|N\frac{d\Phi_m}{dt}\right| = \left|N\mu_0 n\frac{\pi d^2}{4}\frac{dI}{dt}\right| \tag{8.1.22}$$

$$= 20(4\pi \times 10^{-7} T\cdot m/s)(2000\,m^{-1})\frac{\pi(0.010\,m)^2}{4}(3.0\,A/s) \tag{8.1.23}$$

$$= 1.2 \times 10^{-5}\,V. \tag{8.1.24}$$

Significance

When the current is turned on in a vertical solenoid, as shown in Figure $8.1.9$, the ring has an induced emf from the solenoid's changing magnetic flux that opposes the change. The result is that the ring is fired vertically into the air.

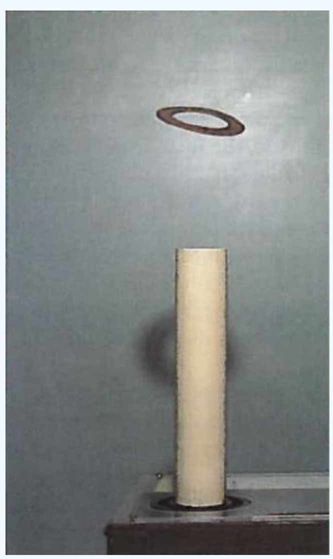


Figure $8.1.9$: The jumping ring. When a current is turned on in the vertical solenoid, a current is induced in the metal ring. The stray field produced by the solenoid causes the ring to jump off the solenoid.

Exercise 8.1.1

A closely wound coil has a radius of 4.0 cm, 50 turns, and a total resistance of $40\,\Omega$. At what rate must a magnetic field perpendicular to the face of the coil change in order to produce Joule heating in the coil at a rate of 2.0 mW?

Solution

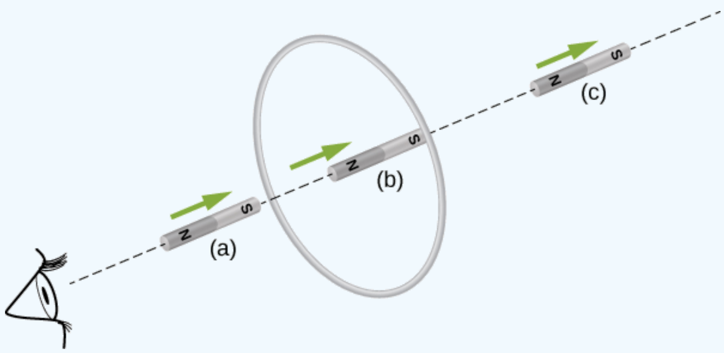
1.1 T/s

Exercise 8.1.2

Find the direction of the induced current in the wire loop shown below as the magnet enters, passes through, and leaves the loop.

Solution

To the observer shown, the current flows clockwise as the magnet approaches, decreases to zero when the magnet is centered in the plane of the coil, and then flows counterclockwise as the magnet leaves the coil.



Applications of Electromagnetic Induction

There are many applications of Faraday’s Law of induction, as we will explore in this chapter and others. At this juncture, let us mention several that have to do with data storage and magnetic fields. A very important application has to do with audio and video *recording tapes*. A plastic tape, coated with iron oxide, moves past a recording head. This recording head is basically a round iron ring about which is wrapped a coil of wire—an electromagnet (Figure 2). A signal in the form of a varying input current from a microphone or camera goes to the recording head. These signals (which are a function of the signal amplitude and frequency) produce varying magnetic fields at the recording head. As the tape moves past the recording head, the magnetic field orientations of the iron oxide molecules on the tape are changed thus recording the signal. In the playback mode, the magnetized tape is run past another head, similar in structure to the recording head. The different magnetic field orientations of the iron oxide molecules on the tape induces an emf in the coil of wire in the playback head. This signal then is sent to a loudspeaker or video player.

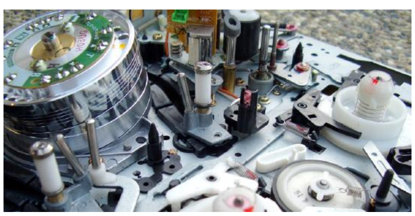


Figure 8.1.2: Recording and playback heads used with audio and video magnetic tapes. (credit: Steve Jurvetson)

Similar principles apply to computer hard drives, except at a much faster rate. Here recordings are on a coated, spinning disk. Read heads historically were made to work on the principle of induction. However, the input information is carried in digital rather than analog form – a series of 0’s or 1’s are written upon the spinning hard drive. Today, most hard drive readout devices do not work on the principle of induction, but use a technique known as *giant magnetoresistance*. (The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of nanotechnology.) Another application of induction is found on the magnetic stripe on the back of your personal credit card as used at the grocery store or the ATM machine. This works on the same principle as the audio or video tape mentioned in the last paragraph in which a head reads personal information from your card.

Another application of electromagnetic induction is when electrical signals need to be transmitted across a barrier. Consider the *cochlear implant* shown below. Sound is picked up by a microphone on the outside of the skull and is used to set up a varying magnetic field. A current is induced in a receiver secured in the bone beneath the skin and transmitted to electrodes in the inner ear. Electromagnetic induction can be used in other instances where electric signals need to be conveyed across various media.

Figure 8.1.3: Electromagnetic induction used in transmitting electric currents across mediums. The device on the baby's head induces an electrical current in a receiver secured in the bone beneath the skin. (credit: Bjorn Knetsch)

Another contemporary area of research in which electromagnetic induction is being successfully implemented (and with substantial potential) is transcranial magnetic simulation. A host of disorders, including depression and hallucinations can be traced to irregular localized electrical activity in the brain. In *transcranial magnetic stimulation*, a rapidly varying and very localized magnetic field is placed close to certain sites identified in the brain. Weak electric currents are induced in the identified sites and can result in recovery of electrical functioning in the brain tissue.

Sleep apnea ("the cessation of breath") affects both adults and infants (especially premature babies and it may be a cause of sudden infant deaths [SID]). In such individuals, breath can stop repeatedly during their sleep. A cessation of more than 20 seconds can be very dangerous. Stroke, heart failure, and tiredness are just some of the possible consequences for a person having sleep apnea. The concern in infants is the stopping of breath for these longer times. One type of monitor to alert parents when a child is not breathing uses electromagnetic induction. A wire wrapped around the infant's chest has an alternating current running through it. The expansion and contraction of the infant's chest as the infant breathes changes the area through the coil. A pickup coil located nearby has an alternating current induced in it due to the changing magnetic field of the initial wire. If the child stops breathing, there will be a change in the induced current, and so a parent can be alerted.

Other applications include:

- **Seismograph:** One way to exploit Faraday's Law is to attach a magnet to anything that moves and place it near a loop of wire; any movement or oscillation in the object can be detected as an induced current in the wire loop. In this way we can translate physical movements and oscillations into electrical impulses. In all devices of this kind, the movement or oscillation is measured between the position of a coil relative to a magnet, whose movement causes the current in the coil to vary, generating an electrical signal. For example, as the vibrations produced by an earthquake pass through a seismograph, a magnet's vibrations produce a current that can be amplified to drive a plotting pen. This is how the seismograph operates.
- **Guitar Pickup:** Les Paul, a pioneer musician of pop-jazz guitar, applied Faraday's Law to the making of musical instruments and invented the first *electric guitar*. The "pickup" of an electric guitar consists of a permanent magnet with a coil of wire wrapped around it several times. The permanent magnet is placed very close to the metal guitar strings. The magnetic field of the permanent magnet causes a part of the metal string of the guitar to become magnetized. When one plucks the string, it vibrates, creating a changing magnetic flux through the coil of wire surrounding the permanent magnet. The coil "picks up" the vibrations that generate an induced current and sends the signal to an amplifier, to the pleasure of rock fans everywhere.
- **Electric Generator:** An electric generator is used to efficiently convert mechanical energy to electrical energy. The mechanical energy can be provided by any number of means, such as falling water (like in a hydroelectric generator), expanding steam (as in coal, oil, and nuclear power plants), or wind (as in wind turbine generators). In all cases, the principle is the same, the mechanical energy is used to move a conducting wire coil inside a magnetic field (usually by rotating the wire). In this case, the area of the coil is the constant, the magnitude of the field is constant, so the angle term in the equation for Faraday's law is responsible for the changing flux. This is caused by the change in the relative orientation between the magnetic field and the normal to the area of the coil. Consider the simple scenario where we rotate the coil with constant angular speed ω. The rotation angle is given by $\theta = \omega t$, and the flux will be proportional to $\cos \omega t$. Using calculus, the time rate of change of the flux will then be proportional to $\omega \sin \omega t$. This means the induced current will oscillate *sinusoidally*. In other words, the current in the coil alternates in direction, flowing in one direction for half the cycle and flowing the other direction for the other half. This

kind of generator is referred to as an *alternating current generator*, or simply an AC generator. The standard plugs you use to power all of your electrical appliances are all powered by an electric generator of this form.
- **Electric Motor:** Electric motors work in basically the reverse principle that operates electric generators: an alternating electric current causes an electromagnetic cylinder to periodically switch poles, which interacts with the field of an inlaid magnet to turn it. Some motors use electromagnets for both components, but the principle is the same. The stationary magnetic piece is called the *stator* and the magnetic piece that rotates is called that *rotor*
- **Hybrid Cars**: Regenerative breaking discussed above.

MAKING CONNECTIONS: CONSERVATION OF ENERGY:

Lenz's law is a manifestation of the conservation of energy. The induced emf produces a current that opposes the change in flux, because a change in flux means a change in energy. Energy can enter or leave, but not instantaneously. Lenz's law is a consequence. As the change begins, the law says induction opposes and, thus, slows the change. In fact, if the induced emf were in the same direction as the change in flux, there would be a positive feedback that would give us free energy from no apparent source—conservation of energy would be violated.

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