3.12: Electric Charges and Fields (Answer)

Check Your Understanding

5.1. The force would point outward.

5.2. The net force would point **58°** below the **−x**-axis.

5.3. $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

5.4. We will no longer be able to take advantage of symmetry. Instead, we will need to calculate each of the two components of the electric field with their own integral.

5.5. The point charge would be $Q = \sigma ab$ where a and b are the sides of the rectangle but otherwise identical.

5.6. The electric field would be zero in between, and have magnitude $\frac{\sigma}{\varepsilon_0}$ everywhere else.

Conceptual Questions

1. There are mostly equal numbers of positive and negative charges present, making the object electrically neutral.

3. a. yes;

b. yes

5. Take an object with a known charge, either positive or negative, and bring it close to the rod. If the known charged object is positive and it is repelled from the rod, the rod is charged positive. If the positively charged object is attracted to the rod, the rod is negatively charged.

7. No, the dust is attracted to both because the dust particle molecules become polarized in the direction of the silk.

9. Yes, polarization charge is induced on the conductor so that the positive charge is nearest the charged rod, causing an attractive force.

11. Charging by conduction is charging by contact where charge is transferred to the object. Charging by induction first involves producing a polarization charge in the object and then connecting a wire to ground to allow some of the charge to leave the object, leaving the object charged.

13. This is so that any excess charge is transferred to the ground, keeping the gasoline receptacles neutral. If there is excess charge on the gasoline receptacle, a spark could ignite it.

15. The dryer charges the clothes. If they are damp, the presence of water molecules suppresses the charge.

17. There are only two types of charge, attractive and repulsive. If you bring a charged object near the quartz, only one of these two effects will happen, proving there is not a third kind of charge.

19. a. No, since a polarization charge is induced. b. Yes, since the polarization charge would produce only an attractive force.

21. The force holding the nucleus together must be greater than the electrostatic repulsive force on the protons.

23. Either sign of the test charge could be used, but the convention is to use a positive test charge.

25. The charges are of the same sign.

27. At infinity, we would expect the field to go to zero, but because the sheet is infinite in extent, this is not the case. Everywhere you are, you see an infinite plane in all directions.

29. The infinite charged plate would have $E = \frac{\sigma}{2\varepsilon_0}$ everywhere. The field would point toward the plate if it were negatively charged and point away from the plate if it were positively charged. The electric field of the parallel plates would be zero between them if they had the same charge, and **E** would be $E = \frac{\sigma}{\varepsilon_0}$ everywhere else. If the charges were opposite, the situation is reversed, zero outside the plates and $E = \frac{\sigma}{\varepsilon_0}$ between them.

31. yes; no

33. At the surface of Earth, the gravitational field is always directed in toward Earth's center. An electric field could move a charged particle in a different direction than toward the center of Earth. This would indicate an electric field is present.

35. 10

Problems

37. a. $2.00 \times 10^{-9} C(\frac{1}{1.602 \times 10^{-19}} e/C) = 1.248 \times 10^{10} electrons2$;

b. $0.500 \times 10^{-6} C(\frac{1}{1.602 \times 10^{-19}} e/C) = 3.121 \times 10^{12} electrons$

39. $\frac{3.750 \times 10^{21} e}{6.242 \times 10^{18} e/C} = -600.8C$

41. a. $2.0 \times 10^{-9} C(6.242 \times 10^{18} e/C) = 1.248 \times 10^{10} e$;

b. $9.109 \times 10^{-31} kg(1.248 \times 10^{10} e) = 1.137 \times 10^{-20} kg, \frac{1.137 \times 10^{-20} kg}{2.5 \times 10^{-3} kg} = 4.548 \times 10^{-18}$ or 4.545×10^{-16}

43. $5.00 \times 10^{-9} C(6.242 \times 10^{18} e/C) = 3.121 \times 10^{10} e; 3.121 \times 10^{10} e + 1.0000 \times 10^{12} e = 1.0312 \times 10^{12} e$.

45. atomic mass of copper atom times $1u = 1.055 \times 10^{-25} kg$; number of copper atoms = $4.739 \times 10^{23} atoms$; number of electrons equals 29 times number of atoms or $1.374 \times 10^{25} electrons$; $\frac{2.00 \times 10^{-6} C(6.242 \times 10^{18} e/C)}{1.374 \times 10^{25} e} = 9.083 \times 10^{-13}$ or 9.083×10^{-11}.

47. $244.00u(1.66 \times 10^{-27} kg/u) = 4.050 \times 10^{-25} kg$; $\frac{4.00kg}{4.050 \times 10^{-25} kg} = 9.877 \times 10^{24} atoms$ $9.877 \times 10^{24}(94) = 9.284 \times 10^{26} protons$ $9.284 \times 10^{26} protons; 9.284 \times 10^{26}(1.602 \times 10^{-19} C/p) = 1.487 \times 10^{8} C$

49. a. charge 1 is $3\mu C$; charge 2 is $12\mu C$, $F_{31} = 2.16 \times 10^{-4} N$ to the left,

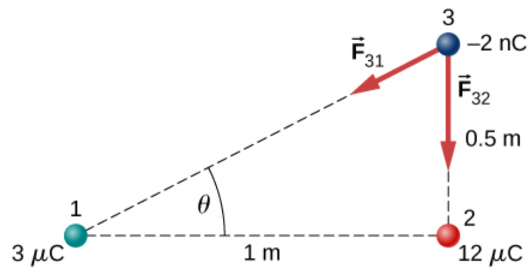
$F_{32} = 8.63 \times 10^{-4} N$ to the right,

$F_{net} = 6.47 \times 10^{-4} N$ to the right;

b. $F_{31} = 2.16 \times 10^{-4} N$ to the right,

$F_{32} = 9.59 \times 10^{-5} N$ to the right,

$F_{net} = 3.12 \times 10^{-4} N$ to the right,



c. $\vec{F}_{31x} = -2.76 \times 10^{-5} N\hat{i}$,

$\vec{F}_{31y} = -1.38 \times 10^{-5} N\hat{j}$,

$\vec{F}_{32y} = -8.63 \times 10^{-4} N\hat{j}$,

$\vec{F}_{net} = -2.76 \times 10^{-5} N\hat{i} - 8.77 \times 10^{-4} N\hat{j}$

51. $F = 230.7N$

53. $F = 53.94N$

55. The tension is $T = 0.049N$. The horizontal component of the tension is $0.0043N$

$d = 0.088m, q = 6.1 \times 10^{-8}C$.

The charges can be positive or negative, but both have to be the same sign.

57. Let the charge on one of the spheres be **rQ**, where r is a fraction between 0 and 1. In the numerator of Coulomb's law, the term involving the charges is $rQ(1-r)Q$. This is equal to $(r-r^2)Q^2$. Finding the maximum of this term gives $1-2r = 0 \Rightarrow r = \frac{1}{2}$

59. Define right to be the positive direction and hence left is the negative direction, then $F = -0.05N$

61. The particles form triangle of sides 13, 13, and 24 cm. The **x**-components cancel, whereas there is a contribution to the y-component from both charges 24 cm apart. The y-axis passing through the third charge bisects the 24-cm line, creating two right triangles of sides 5, 12, and 13 cm. $F_y = 2.56N$ in the negative **y**-direction since the force is attractive. The net force from both charges is $\vec{F}_{net} = -5.12N\hat{j}$

63. The diagonal is $\sqrt{2}a$ and the components of the force due to the diagonal charge has a factor $cos\theta = \frac{1}{\sqrt{2}}$;

$\vec{F}_{net} = [k\frac{q^2}{a^2} + k\frac{q^2}{2a^2}\frac{1}{\sqrt{2}}]\hat{i} - [k\frac{q^2}{a^2} + k\frac{q^2}{2a^2}\frac{1}{\sqrt{2}}]\hat{j}$

65. $a.\ E = 2.0 \times 10^{-2}\frac{N}{C}$;

$b.\ F = 2.0 \times 10^{-19}N$

67. a. $E = 2.88 \times 10^{11} N/C$;

b. $E = 1.44 \times 10^{11} N/C$;

c. $F = 4.61 \times 10^{-8}N$ on alpha particl

$F = 4.61 \times 10^{-8}N$ on electron

69. $E = (-2.0\hat{i} + 3.0\hat{j})N$

71. $F = 3.204 \times 10^{-14}N$,

$a = 3.517 \times 10^{16} m/s^2$

73. $q = 2.78 \times 10^{-9}C$

75. a. $E = 1.15 \times 10^{12} N/C$;

b. $F = 1.47 \times 10^{-6}N$

77. If the q_2 is to the right of q_1, the electric field vector from both charges point to the right.

a. $E = 2.70 \times 10^6 N/C$;

b. $F = 54.0N$

79. There is **45°** right triangle geometry. The **x**-components of the electric field at $y = 3m$ cancel. The **y**-components give $E(y = 3m) = 2.83 \times 10^3 N/C$.

At the origin we have a a negative charge of magnitide $q = -2.83 \times 10^{-6}C$

81. $\vec{E}(z) = 3.6 \times 10^4 N\hat{k}$

83. $dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x+a)^2}, E = \frac{\lambda}{4\pi\varepsilon_0}[\frac{1}{l+a} - \frac{1}{a}]$

85. $\sigma = 0.02C/m^2\ E = 2.26 \times 10^9 N/C$

87. At P_1 : $\vec{E}(y) = \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{y\sqrt{y^2+\frac{L^2}{4}}}\hat{j} \Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q}{\frac{a}{2}\sqrt{(\frac{a}{2})^2+\frac{L^2}{4}}}\hat{j} = \frac{1}{\pi\varepsilon_0}\frac{q}{a\sqrt{a^2+L^2}}\hat{j}$

At P_2: Put the origin at the end of **L**.

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x+a)^2}, \vec{E} = -\frac{q}{4\pi\varepsilon_0 l}[\frac{1}{l+a} - \frac{1}{a}]\hat{i}$$

89. a. $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda_x}{a}\hat{i} + \frac{1}{4\pi\varepsilon_0}\frac{2\lambda_y}{b}\hat{j}$;

b. $\frac{1}{4\pi\varepsilon_0}\frac{2(\lambda_x+\lambda_y)}{c}\hat{k}$

91. a. $\vec{F} = 3.2 \times 10^{-17} N\hat{i}$,

$\vec{a} = 1.92 \times 10^{10} m/s^2\hat{i}$;

b. $\vec{F} = -3.2 \times 10^{-17} N\hat{i}$,

$\vec{a} = -3.51 \times 10^{13} m/s^2\hat{i}$

93. $m = 6.5 \times 10^{-11} kg$,

$E = 1.6 \times 10^7 N/C$

95. $E = 1.70 \times 10^6 N/C$,

$F = 1.53 \times 10^{-3} NTcos\theta = mgTsin\theta = qE$,

$tan\theta = 0.62 \Rightarrow \theta = 32.0^\circ$,

This is independent of the length of the string.

97. circular arc $dE_x(-\hat{i}) = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{r^2}cos\theta(-\hat{i}$,

$\vec{E}_x = \frac{\lambda}{4\pi\varepsilon_0 r}(-\hat{i})$,

$dEy(-\hat{i}\,\hat{}) = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{r^2}sin\theta(-\hat{j})$,

$\vec{E}_y = \frac{\lambda}{4\pi\varepsilon_0 r}(-\hat{j})$;

y-axis: $\vec{E}_x = \frac{\lambda}{4\pi\varepsilon_0 r}(-\hat{i})$;

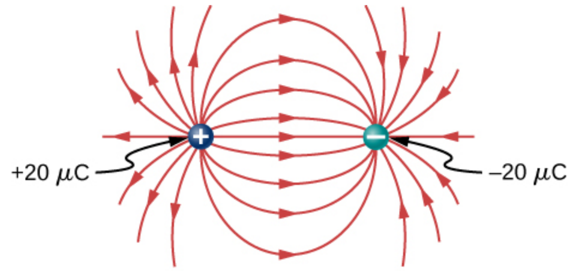
x-axis: $\vec{E}_y = \frac{\lambda}{4\pi\varepsilon_0 r}(-\hat{j})$,

$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}(-\hat{i}) + \frac{\lambda}{2\pi\varepsilon_0 r}(-\hat{j})$

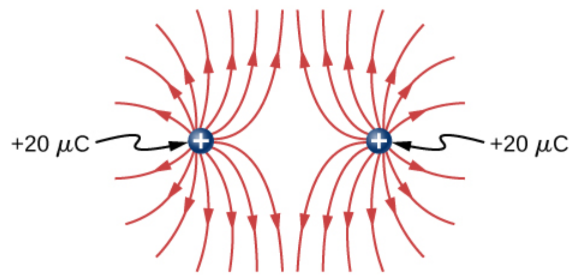
99. a. $W = \frac{1}{2}m(v^2 - v_0^2), \frac{Qq}{4\pi\varepsilon_0}(\frac{1}{r} - \frac{1}{r_0}) = \frac{1}{2}m(v^2 - v_0^2) \Rightarrow r_0 - r = \frac{4\pi\varepsilon_0}{Qq}\frac{1}{2}rr_0 m(v^2 - v_0^2)$;

b. $r_0 - r$ is negative; therefore, $v_0 > v, r \to \infty$, and $v \to 0$: $\frac{Qq}{4\pi\varepsilon_0}(-\frac{1}{r_0}) = -\frac{1}{2}mv_0^2 \Rightarrow v_0 = \sqrt{\frac{Qq}{2\pi\varepsilon_0 m r_0}}$

101.



(a)



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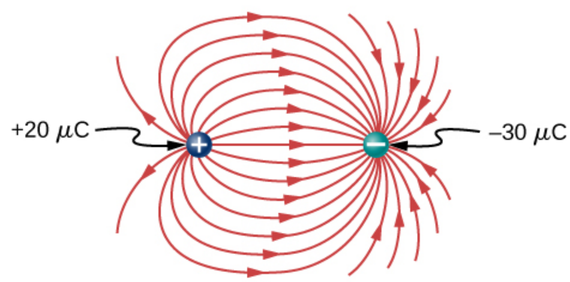
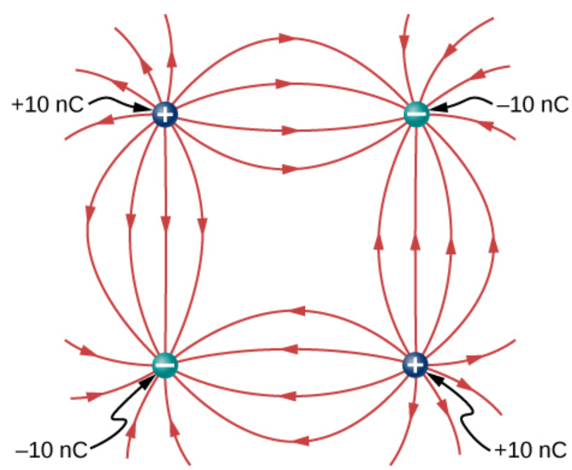


(c)

103.



105. $E_x = 0, E_y = \frac{1}{4\pi\varepsilon_0}[\frac{2q}{(x^2+a^2})\frac{a}{\sqrt{(x^2+a^2)}} \Rightarrow x \gg a \Rightarrow \frac{1}{2\pi\varepsilon_0}\frac{qa}{x^3}$

$$E_y = \frac{q}{4\pi\varepsilon_0}[\frac{2ya+2ya}{(y-a)^2(y+a)^2}] \Rightarrow y \gg a \Rightarrow \frac{1}{\pi\varepsilon_0}\frac{qa}{y^3}$$

107. The net dipole moment of the molecule is the vector sum of the individual dipole moments between the two O-H. The separation O-H is 0.9578 angstroms:

$$\vec{p} = 1.889 \times 10^{-29} Cm\hat{i}$$

Additional Problems

109. $\vec{F}_{net} = [-8.99 \times 10^9 \frac{3.0 \times 10^{-6}(5.0 \times 10^{-6})}{(3.0m)^2} - 8.99 \times 10^9 \frac{9.0 \times 10^{-6}(5.0 \times 10^{-6})}{(3.0m)^2}]\hat{i}, -8.99$

$\times 10^9 \frac{6.0 \times 10^{-6}(5.0 \times 10^{-6})}{(3.0m)^2}\hat{j} = -0.06N\hat{i} - 0.03N\hat{j}$

111. Charges **Q** and **q** form a right triangle of sides 1 m and $3+\sqrt{3}m.$ Charges **2Q** and **q** form a right triangle of sides 1 m and $\sqrt{3}m.$

$F_x = 0.049N,$

$F_y = 0.09N,$

$\vec{F}_{net} = 0.049N\hat{i} + 0.09N\hat{j}$

113. W=0.054J

115. a. $\vec{E} = \frac{1}{4\pi\varepsilon_0}(\frac{q}{(2a)^2} - \frac{q}{a^2})\hat{i}$;

b. $\vec{E} = \frac{\sqrt{3}}{4\pi\varepsilon_0}\frac{q}{a^2}(-\hat{j})$;

c. $\vec{E} = \frac{2}{\pi\varepsilon_0}\frac{q}{a^2}\frac{1}{\sqrt{2}}(-\hat{j})$

117. $\vec{E} = 6.4 \times 10^6(\hat{i}) + 1.5 \times 10^7(\hat{j})N/C$

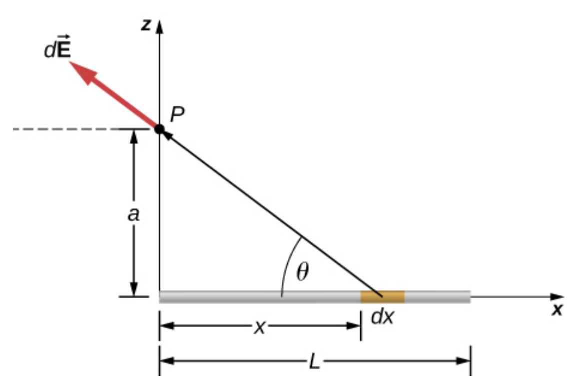
119. $F = qE_0(1 + x/a)$ $W = \frac{1}{2}m(v^2 - v_0^2)$,

$\frac{1}{2}mv^2 = qE_0(\frac{15a}{2})J$

121. Electric field of wire at **x**: $\vec{E}(x) = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda_y}{x}\hat{i}$,

$dF = \frac{\lambda_y\lambda_x}{2\pi\varepsilon_0}(lnb - lna)$

123.



$$dEx = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x^2 + a^2)}\frac{x}{\sqrt{x^2 + a^2}},$$

$$\vec{E}_x = \frac{\lambda}{4\pi\varepsilon_0}[\frac{1}{\sqrt{L^2 + a^2}} - \frac{1}{a}]\hat{i},$$

$$dE_z = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x^2+a^2)}\frac{a}{\sqrt{x^2+a^2}},$$
$$\vec{E}_z = \frac{\lambda}{4\pi\varepsilon_0 a}\frac{L}{\sqrt{L^2+a^2}}\hat{k},$$

Substituting z for a, we have:

$$\vec{E}(z) = \frac{\lambda}{4\pi\varepsilon_0}[\frac{1}{\sqrt{L^2+z^2}} - \frac{1}{z}]\hat{i} + \frac{\lambda}{4\pi\varepsilon_0 z}\frac{L}{\sqrt{L^2+z^2}}\hat{k}$$

125. There is a net force only in the **y**-direction. Let θ be the angle the vector from **dx** to **q** makes with the **x**-axis. The components along the **x**-axis cancel due to symmetry, leaving the **y**-component of the force.

$$dF_y = \frac{1}{4\pi\varepsilon_0}\frac{aq\lambda dx}{(x^2+a^2)^{3/2}},$$
$$Fy = \frac{1}{2\pi\varepsilon_0}\frac{q\lambda}{a}[\frac{l/2}{((l/2)^2+a^2)^{1/2}}]$$

Check Your Understanding

6.1. Place it so that its unit normal is perpendicular to $\vec{E}$.

6.2. $mab^2/2$

6.3 a. $3.4\times10^5 N\cdot m^2/C$;

b. $-3.4\times10^5 N\cdot m^2/C$;

c. $3.4\times10^5 N\cdot m^2/C$;

d. 0

6.4. In this case, there is only $\vec{E}_{out}$. So,yes.

6.5. $\vec{E} = \frac{\lambda_0}{2\pi\varepsilon_0}\frac{1}{d}\hat{r}$; This agrees with the calculation of Example 5.5 where we found the electric field by integrating over the charged wire. Notice how much simpler the calculation of this electric field is with Gauss's law.

6.6. If there are other charged objects around, then the charges on the surface of the sphere will not necessarily be spherically symmetrical; there will be more in certain direction than in other directions.

Conceptual Questions

1. a. If the planar surface is perpendicular to the electric field vector, the maximum flux would be obtained. b. If the planar surface were parallel to the electric field vector, the minimum flux would be obtained.

3. true

5. Since the electric field vector has a $\frac{1}{r^2}$ dependence, the fluxes are the same since $A = 4\pi r^2$.

7. a. no;

b. zero

9. Both fields vary as $\frac{1}{r^2}$. Because the gravitational constant is so much smaller than $\frac{1}{4\pi\varepsilon_0}$, the gravitational field is orders of magnitude weaker than the electric field.

11. No, it is produced by all charges both inside and outside the Gaussian surface.

13. No, since the situation does not have symmetry, making Gauss's law challenging to simplify.

15. Any shape of the Gaussian surface can be used. The only restriction is that the Gaussian integral must be calculable; therefore, a box or a cylinder are the most convenient geometrical shapes for the Gaussian surface.

17. yes

19. Since the electric field is zero inside a conductor, a charge of $-2.0\mu C$ is induced on the inside surface of the cavity. This will put a charge of $+2.0\mu C$ on the outside surface leaving a net charge of $-3.0\mu C$ on the surface.

Problems

21. $\Phi = \vec{E}\cdot\vec{A} \rightarrow EAcos\theta = 2.2\times10^4 N\cdot m^2/C$ electric field in direction of unit normal; $\Phi = \vec{E}\cdot\vec{A} \rightarrow EAcos\theta = -2.2\times10^4 N\cdot m^2/C$ electric field opposite to unit normal

23. $\frac{3\times10^{-5} N\cdot m^2/C}{(0.05m)^2} = E \Rightarrow \sigma = 2.12\times10^{-13} C/m^2$

25. a. $\Phi = 0.17 N\cdot m^2/C$;

b. $\Phi = 0$;

c. $\Phi = EAcos0^\circ = 1.0\times10^3 N/C(2.0\times10^{-4}m)^2 cos0^\circ = 0.20 N\cdot m^2/C$

27. $\Phi = 3.8\times10^4 N\cdot m^2/C$

29. $\vec{E}(z) = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}\hat{k}, \int \vec{E}\cdot\hat{n}dA = \frac{\lambda}{\varepsilon_0}l$

31. a. $\Phi = 3.39\times10^3 N\cdot m^2/C$;

b. $\Phi = 0$;

c. $\Phi = -2.25\times10^5 N\cdot m^2/C$;

d. $\Phi = 90.4 N\cdot m^2/C$

33. $\Phi = 1.13\times10^6 N\cdot m^2/C$

35. Make a cube with **q** at the center, using the cube of side a. This would take four cubes of side a to make one side of the large cube. The shaded side of the small cube would be 1/24th of the total area of the large cube; therefore, the flux through the shaded area would be $\Phi = \frac{1}{24}\frac{q}{\varepsilon_0}$.

37. $q = 3.54\times10^{-7} C$

39. zero, also because flux in equals flux out

41. $r > R, E = \frac{Q}{4\pi\varepsilon_0 r^2}; r < R, E = \frac{qr}{4\pi\varepsilon_0 R^3}$

43. $EA = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = 4.50\times10^7 N/C$

45. a. 0;

b. 0;

c. $\vec{E} = 6.74\times10^6 N/C(-\hat{r})$

47. a. 0;

b. $E = 2.70\times10^6 N/C$

49. a. Yes, the length of the rod is much greater than the distance to the point in question.

b. No, The length of the rod is of the same order of magnitude as the distance to the point in question.

c. Yes, the length of the rod is much greater than the distance to the point in question.

d. No. The length of the rod is of the same order of magnitude as the distance to the point in question.

51. a. $\vec{E} = \frac{R\sigma_0}{\varepsilon_0}\frac{1}{r}\hat{r} \Rightarrow \sigma_0 = 5.31\times10^{-11} C/m^2, \lambda = 3.33\times10^{-12} C/m$;

b. $\Phi = \frac{q_{enc}}{\varepsilon_0} = \frac{3.33\times10^{-12} C/m(0.05m)}{\varepsilon+0} = 0.019 N\cdot m^2/C$

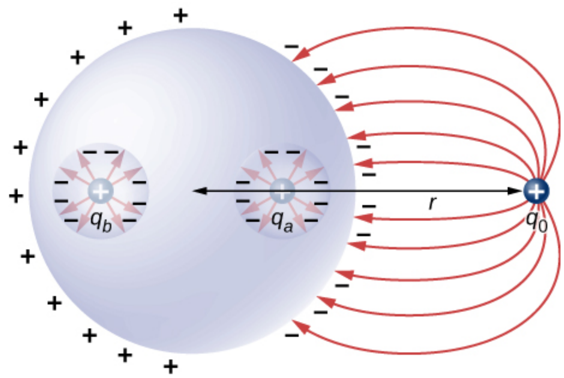
53. $E2\pi rl = \frac{\rho\pi r^2 l}{\varepsilon_0} \Rightarrow E = \frac{\rho r}{2\varepsilon_0}(r \le R); E2\pi rl = \frac{\rho\pi R^2 l}{\varepsilon_0} \Rightarrow E = \frac{\rho R^2}{2\varepsilon_0 r}(r \ge R)$

55. $\Phi = \frac{q_{enc}}{\varepsilon_0} \Rightarrow q_{enc} = -1.0 \times 10^{-9} C$

57. $q_{enc} = \frac{4}{5}\pi\alpha r^5, E4\pi r^2 = \frac{4\pi\alpha r^5}{5\varepsilon_0} \Rightarrow E = \frac{\alpha r^3}{5\varepsilon_0}(r \le R), q_{enc} = \frac{4}{5}\pi\alpha R^5, E4\pi r^2 = \frac{4\pi\alpha R^5}{5\varepsilon_0} \Rightarrow E = \frac{\alpha R^5}{5\varepsilon_0 r^2}(r \ge R)$

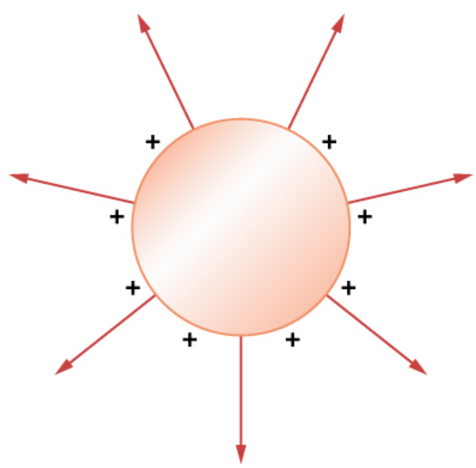
59. integrate by parts: $q_{enc} = 4\pi\rho_0[-e^{-\alpha r}(\frac{(r)^2}{\alpha} + \frac{2r}{\alpha^2} + \frac{2}{\alpha^3}) + \frac{2}{\alpha^3}] \Rightarrow E = \frac{\rho_0}{r^2\varepsilon_0}[-e^{-\alpha r}(\frac{(r)^2}{\alpha} + \frac{2r}{\alpha^2} + \frac{2}{\alpha^3}) + \frac{2}{\alpha^3}]$

61.



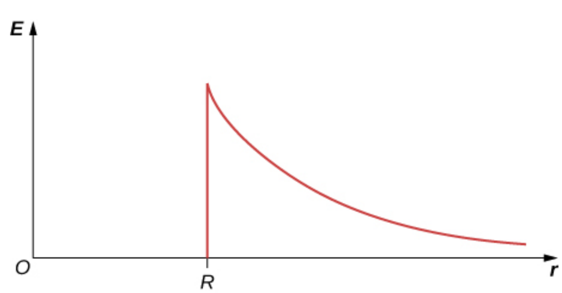
63. a. Outside: $E2\pi rl = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{3.0C/m}{2\pi\varepsilon_0 r}$; Inside $E_{in} = 0$;

b.



65. a. $E2\pi rl = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r} r \ge R$ **E** inside equals 0;

b.



67. $E=5.65\times10^4 N/C$

69. $\lambda=\frac{\lambda l}{\varepsilon_0}\Rightarrow E=\frac{a\sigma}{\varepsilon_0 r}r\geq a, E=0$ inside since q **enclosed**=0

71. a. $E=0$;

b. $E2\pi rL=\frac{Q}{\varepsilon_0}\Rightarrow E=\frac{Q}{2\pi\varepsilon_0 rL}$; c. $E=0$ since **r** would be either inside the second shell or if outside then **q** enclosed equals 0.

Additional Problems

73. $\int\vec{E}\cdot\hat{n}dA=a^4$

75. a. $\int\vec{E}\cdot\hat{n}dA=E_0r^2\pi$; b. zero, since the flux through the upper half cancels the flux through the lower half of the sphere

77. $\Phi=\frac{q_{enc}}{\varepsilon_0}$; There are two contributions to the surface integral: one at the side of the rectangle at $x=0$ and the other at the side at $x=2.0m$; $-E(0)[1.5m^2]+E(2.0m)[1.5m^2]=\frac{q_{enc}}{\varepsilon_0}=-100Nm2^/C$

where the minus sign indicates that at $x=0$, the electric field is along positive **x** and the unit normal is along negative **x**. At $x=2$, the unit normal and the electric field vector are in the same direction: $q_{enc}=\varepsilon_0\Phi=-8.85\times10^{-10}C$

79. didn't keep consistent directions for the area vectors, or the electric fields

81. a. $\sigma=3.0\times10^{-3}C/m^2$, $+3\times10^{-3}C/m^2$ on one and $-3\times10^{-3}C/m^2$ on the other;

b. $E=3.39\times10^8 N/CE=3.39\times10^8N/C$

83. Construct a Gaussian cylinder along the **z**-axis with cross-sectional area **A.**

$|z|\geq\frac{a}{2}q_{enc}=\rho Aa,\Phi=\frac{\rho Aa}{\varepsilon_0}\Rightarrow E=\frac{\rho a}{2\varepsilon_0}$,

$|z|\leq\frac{a}{2}q_{enc}=\rho A2z, E(2A)=\frac{\rho A2z}{\varepsilon_0}\Rightarrow E=\frac{\rho z}{\varepsilon_0}$

85. a. $r>b_2$ $E4\pi r^2=\frac{\frac{4}{3}\pi[\rho_1(b_1^3-a_1^3)+\rho_2(b_2^3-a_2^3)}{\varepsilon_0}\Rightarrow E=\frac{\rho_1(b_1^3-a_1^3)+\rho_2(b_2^3-a_2^3)}{3\varepsilon_0r^2}$;

b. $a_2<r<b_2$ $E4\pi r^2=\frac{\frac{4}{3}\pi[\rho_1(b_1^3-a_1^3)+\rho_2(r^3-a_2^3)]}{\varepsilon_0}\Rightarrow E=\frac{\rho_1(b_1^3-a_1^3)+\rho_2(r^3-a_2^3)}{3\varepsilon_0r^2}$;

c. $b_1<r<a_2$ $E4\pi r^2=\frac{\frac{4}{3}\pi\rho_1(b_1^3-a_1^3)}{\varepsilon_0}\Rightarrow E=\frac{\rho_1(b_1^3-a_1^3)}{3\varepsilon_0r^2}$;

d. $a_1<r<b_1$ $E4\pi r^2=\frac{\frac{4}{3}\pi\rho_1(r^3_-a_1^3)}{\varepsilon_0}\Rightarrow E=\frac{\rho_1(r^3-a_1^3)}{3\varepsilon_0r^2}$;

e. 0

87. Electric field due to plate without hole: $E=\frac{\sigma}{2\varepsilon_0}$.

Electric field of just hole filled with \(\displaystyle −σE=\frac{−σ}{2ε_0}(1−\frac{z}{\sqrt{R^2+z^2}}).

Thus, $E_{net}=\frac{\sigma}{2\varepsilon_0}\frac{h}{\sqrt{R^2+h^2}}$

89. a. $E=0$; b. $E=\frac{q_1}{4\pi\varepsilon_0r^2}$; c. $E=\frac{q_1+q_2}{4\pi\varepsilon_0r^2}$; d. 0 q_1-q_1, q_1+q_2

Challenge Problems

91. Given the referenced link, using a distance to Vega of $237 \times 10^{15} m$[4] and a diameter of 2.4 m for the primary mirror,[5] we find that at a wavelength of 555.6 nm, Vega is emitting $2.44 \times 10^{24} J/s$ at that wavelength. Note that the flux through the mirror is essentially constant.

93. The symmetry of the system forces $\vec{E}$ to be perpendicular to the sheet and constant over any plane parallel to the sheet. To calculate the electric field, we choose the cylindrical Gaussian surface shown. The cross-section area and the height of the cylinder are **A** and **2x**, respectively, and the cylinder is positioned so that it is bisected by the plane sheet. Since **E** is perpendicular to each end and parallel to the side of the cylinder, we have **EA** as the flux through each end and there is no flux through the side. The charge enclosed by the cylinder is σA, so from Gauss's law, $2EA = \frac{\sigma A}{\varepsilon_0}$, and the electric field of an infinite sheet of charge is

$E = \frac{\sigma}{2\varepsilon_0}$, in agreement with the calculation of in the text.

95. There is **Q/2** on each side of the plate since the net charge is $Q : \sigma = \frac{Q}{2A}$,

$$\oint_S \vec{E} \cdot \hat{n} dA = \frac{2\sigma \Delta A}{\varepsilon_0} \Rightarrow E_P = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 2A}$$

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