

## 6.1: Electromotive Force

### Learning Objectives

By the end of the section, you will be able to:

- Describe the electromotive force (emf) and the internal resistance of a battery
- Explain the basic operation of a battery

If you forget to turn off your car lights, they slowly dim as the battery runs down. Why don't they suddenly blink off when the battery's energy is gone? Their gradual dimming implies that the battery output voltage decreases as the battery is depleted. The reason for the decrease in output voltage for depleted batteries is that all voltage sources have two fundamental parts—a source of electrical energy and an internal resistance. In this section, we examine the energy source and the internal resistance.

### Introduction to Electromotive Force

Voltage has many sources, a few of which are shown in Figure 6.1.2. All such devices create a **potential difference** and can supply current if connected to a circuit. A special type of potential difference is known as **electromotive force (emf)**. The emf is not a force at all, but the term 'electromotive force' is used for historical reasons. It was coined by Alessandro Volta in the 1800s, when he invented the first battery, also known as the **voltaic pile**. Because the electromotive force is not a force, it is common to refer to these sources simply as sources of emf (pronounced as the letters "ee-em-eff"), instead of sources of electromotive force.

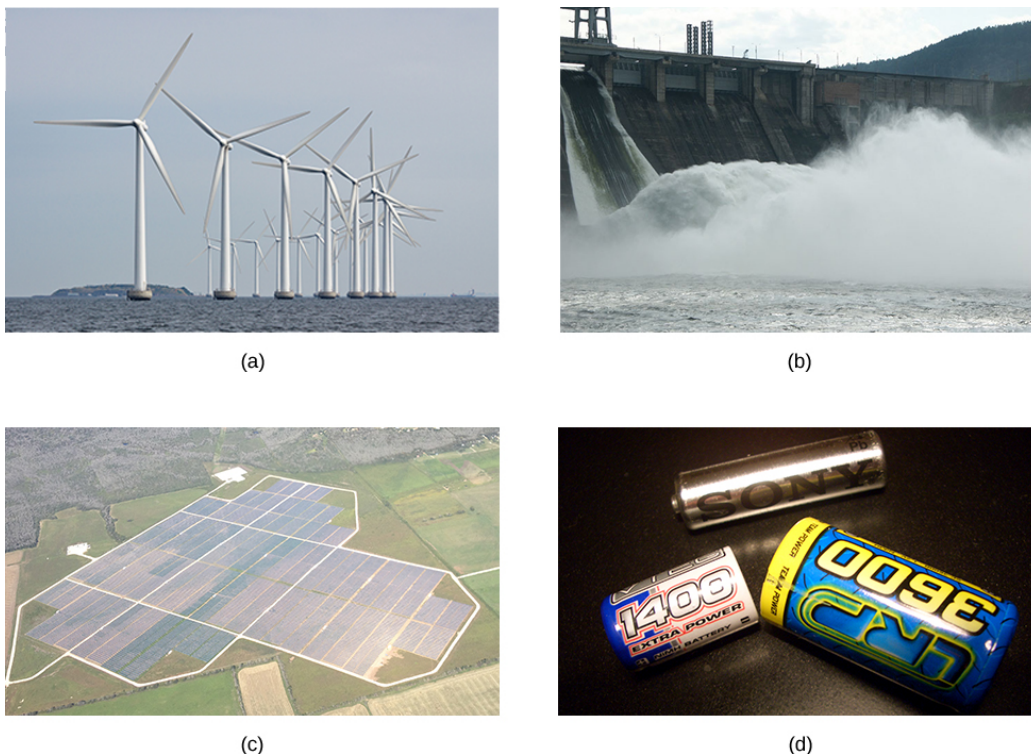


Figure 6.1.1: A variety of voltage sources. (a) The Brazos Wind Farm in Fluvanna, Texas; (b) the Krasnoyarsk Dam in Russia; (c) a solar farm; (d) a group of nickel metal hydride batteries. The voltage output of each device depends on its construction and load. The voltage output equals emf only if there is no load. (credit a: modification of work by "Leaflet"/Wikimedia Commons; credit b: modification of work by Alex Polezhaev; credit c: modification of work by US Department of Energy; credit d: modification of work by Tiaa Monto)

If the electromotive force is not a force at all, then what is the emf and what is a source of emf? To answer these questions, consider a simple circuit of a 12-V lamp attached to a 12-V battery, as shown in Figure 6.1.2. The **battery** can be modeled as a two-terminal device that keeps one terminal at a higher electric potential than the second terminal. The higher electric potential is sometimes called the positive terminal and is labeled with a plus sign. The lower-potential terminal is sometimes called the negative terminal and labeled with a minus sign. This is the source of the emf.

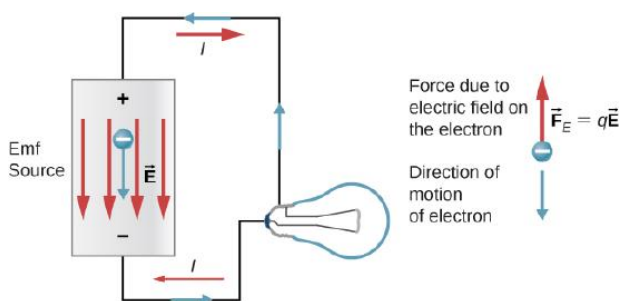


Figure 6.1.2: A source of emf maintains one terminal at a higher electric potential than the other terminal, acting as a source of current in a circuit.

When the emf source is not connected to the lamp, there is no net flow of charge within the emf source. Once the battery is connected to the lamp, charges flow from one terminal of the battery, through the lamp (causing the lamp to light), and back to the other terminal of the battery. If we consider positive (conventional) current flow, positive charges leave the positive terminal, travel through the lamp, and enter the negative terminal.

Positive current flow is useful for most of the circuit analysis in this chapter, but in metallic wires and resistors, electrons contribute the most to current, flowing in the opposite direction of positive current flow. Therefore, it is more realistic to consider the movement of electrons for the analysis of the circuit in Figure 6.1.2. The electrons leave the negative terminal, travel through the lamp, and return to the positive terminal. In order for the emf source to maintain the potential difference between the two terminals, negative charges (electrons) must be moved from the positive terminal to the negative terminal. The emf source acts as a charge pump, moving negative charges from the positive terminal to the negative terminal to maintain the potential difference. This increases the potential energy of the charges and, therefore, the electric potential of the charges.

The force on the negative charge from the electric field is in the opposite direction of the electric field, as shown in Figure 6.1.2. In order for the negative charges to be moved to the negative terminal, work must be done on the negative charges. This requires energy, which comes from chemical reactions in the battery. The potential is kept high on the positive terminal and low on the negative terminal to maintain the potential difference between the two terminals. The emf is equal to the work done on the charge per unit charge ( $\epsilon = \frac{dW}{dq}$ ) when there is no current flowing. Since the unit for work is the joule and the unit for charge is the coulomb, the unit for emf is the volt ( $1 \text{ V} = 1 \text{ J/C}$ ).

The **terminal voltage**  $V_{\text{terminal}}$  of a battery is voltage measured across the terminals of the battery when there is no load connected to the terminal. An ideal battery is an emf source that maintains a constant terminal voltage, independent of the current between the two terminals. An ideal battery has no internal resistance, and the terminal voltage is equal to the emf of the battery. In the next section, we will show that a real battery does have internal resistance and the terminal voltage is always less than the emf of the battery.

## The Origin of Battery Potential

The combination of chemicals and the makeup of the terminals in a battery determine its emf. The **lead acid battery** used in cars and other vehicles is one of the most common combinations of chemicals. Figure 6.1.3 shows a single cell (one of six) of this battery. The cathode (positive) terminal of the cell is connected to a lead oxide plate, whereas the anode (negative) terminal is connected to a lead plate. Both plates are immersed in sulfuric acid, the electrolyte for the system.

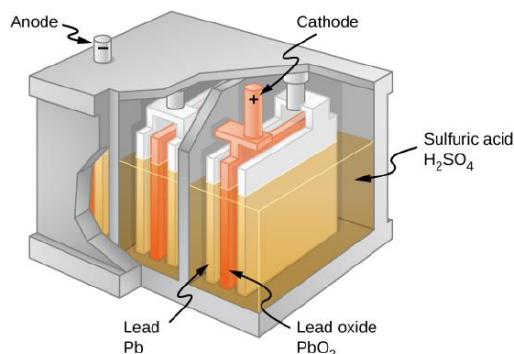


Figure 6.1.3: Chemical reactions in a lead-acid cell separate charge, sending negative charge to the anode, which is connected to the lead plates. The lead oxide plates are connected to the positive or cathode terminal of the cell. Sulfuric acid conducts the charge, as well as participates in the chemical reaction.

Knowing a little about how the chemicals in a lead-acid battery interact helps in understanding the potential created by the battery. Figure 6.1.4 shows the result of a single chemical reaction. Two electrons are placed on the **anode**, making it negative, provided that the **cathode** supplies two electrons. This leaves the cathode positively charged, because it has lost two electrons. In short, a separation of charge has been driven by a chemical reaction.

Note that the reaction does not take place unless there is a complete circuit to allow two electrons to be supplied to the cathode. Under many circumstances, these electrons come from the anode, flow through a resistance, and return to the cathode. Note also that since the chemical reactions involve substances with resistance, it is not possible to create the emf without an internal resistance.

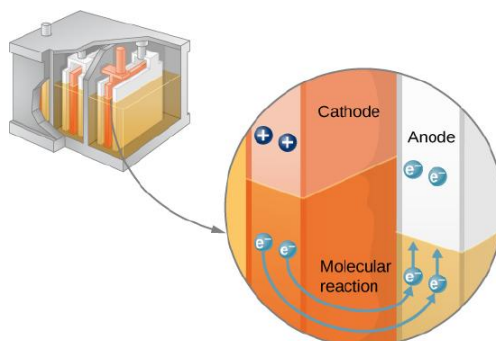


Figure 6.1.4: In a lead-acid battery, two electrons are forced onto the anode of a cell, and two electrons are removed from the cathode of the cell. The chemical reaction in a lead-acid battery places two electrons on the anode and removes two from the cathode. It requires a closed circuit to proceed, since the two electrons must be supplied to the cathode.

## Internal Resistance and Terminal Voltage

The amount of resistance to the flow of current within the voltage source is called the **internal resistance**. The internal resistance  $r$  of a battery can behave in complex ways. It generally increases as a battery is depleted, due to the oxidation of the plates or the reduction of the acidity of the electrolyte. However, internal resistance may also depend on the magnitude and direction of the current through a voltage source, its temperature, and even its history. The internal resistance of rechargeable nickel-cadmium cells, for example, depends on how many times and how deeply they have been depleted. A simple model for a battery consists of an idealized emf source  $\epsilon$  and an internal resistance  $r$  (Figure 6.1.5).

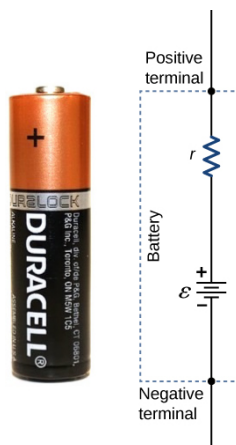


Figure 6.1.5: A battery can be modeled as an idealized emf ( $\epsilon$ ) with an internal resistance ( $r$ ). The terminal voltage of the battery is  $V_{\text{terminal}} = \epsilon - Ir$ .

Suppose an external resistor, known as the load resistance  $R$ , is connected to a voltage source such as a battery, as in Figure 6.1.6. The figure shows a model of a battery with an emf  $\epsilon$ , an internal resistance  $r$ , and a load resistor  $R$  connected across its terminals. Using conventional current flow, positive charges leave the positive terminal of the battery, travel through the resistor, and return to the negative terminal of the battery. The terminal voltage of the battery depends on the emf, the internal resistance, and the current, and is equal to

✓ Note

$$V_{\text{terminal}} = \epsilon - Ir \quad (6.1.1)$$

For a given emf and internal resistance, the terminal voltage decreases as the current increases due to the potential drop  $Ir$  of the internal resistance.

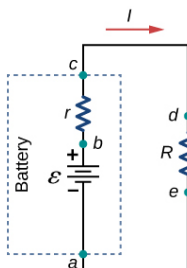


Figure 6.1.6: Schematic of a voltage source and its load resistor  $R$ . Since the internal resistance  $r$  is in series with the load, it can significantly affect the terminal voltage and the current delivered to the load.

A graph of the potential difference across each element the circuit is shown in Figure 6.1.7. A current  $I$  runs through the circuit, and the potential drop across the internal resistor is equal to  $Ir$ . The terminal voltage is equal to  $\epsilon - Ir$ , which is equal to the **potential drop** across the load resistor  $IR = \epsilon - Ir$ . As with potential energy, it is the change in voltage that is important. When the term “voltage” is used, we assume that it is actually the change in the potential, or  $\Delta V$ . However,  $\Delta$  is often omitted for convenience.

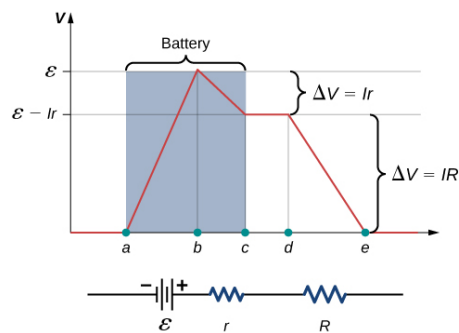


Figure 6.1.7: A graph of the voltage through the circuit of a battery and a load resistance. The electric potential increases the emf of the battery due to the chemical reactions doing work on the charges. There is a decrease in the electric potential in the battery due to the internal resistance. The potential decreases due to the internal resistance  $-Ir$ , making the terminal voltage of the battery equal to  $(\epsilon - Ir)$ . The voltage then decreases by  $(IR)$ . The current is equal to  $I = \frac{\epsilon}{r+R}$ .

The current through the load resistor is  $I = \frac{\epsilon}{r+R}$ . We see from this expression that the smaller the internal resistance  $r$ , the greater the current the voltage source supplies to its load  $R$ . As batteries are depleted,  $r$  increases. If  $r$  becomes a significant fraction of the load resistance, then the current is significantly reduced, as the following example illustrates.

### ✓ Example 6.1.1: Analyzing a Circuit with a Battery and a Load

A given battery has a 12.00-V emf and an internal resistance of 0.100  $\Omega$  (a) Calculate its terminal voltage when connected to a 10.00  $\Omega$  load. (b) What is the terminal voltage when connected to a 0.500  $\Omega$  load? (c) What power does the 0.500  $\Omega$  load dissipate? (d) If the internal resistance grows to 0.500  $\Omega$ , find the current, terminal voltage, and power dissipated by a 0.500  $\Omega$  load.

#### Strategy

The analysis above gave an expression for current when internal resistance is taken into account. Once the current is found, the terminal voltage can be calculated by using the equation  $V_{\text{terminal}} = \epsilon - Ir$ . Once current is found, we can also find the power dissipated by the resistor.

#### Solution

1. Entering the given values for the emf, load resistance, and internal resistance into the expression above yields

$$I = \frac{\epsilon}{R+r} = \frac{12.00 \text{ V}}{10.10 \Omega} = 1.188 \text{ A.} \quad (6.1.2)$$

Enter the known values into the equation  $V_{\text{terminal}} = \epsilon - Ir$  to get the terminal voltage:

$$V_{\text{terminal}} = \epsilon - Ir = 12.00 \text{ V} - (1.188 \text{ A})(0.100 \Omega) = 11.90 \text{ V.} \quad (6.1.3)$$

The terminal voltage here is only slightly lower than the emf, implying that the current drawn by this light load is not significant.

2. Similarly, with  $R_{\text{load}} = 0.500 \Omega$ , the current is

$$I = \frac{\epsilon}{R+r} = \frac{12.00 \text{ V}}{0.600 \Omega} = 20.00 \text{ A.} \quad (6.1.4)$$

The terminal voltage is now

$$V_{\text{terminal}} = \epsilon - Ir = 12.00 \text{ V} - (20.00 \text{ A})(0.100 \Omega) = 10.00 \text{ V.} \quad (6.1.5)$$

The terminal voltage exhibits a more significant reduction compared with emf, implying 0.500  $\Omega$  is a heavy load for this battery. A “heavy load” signifies a larger draw of current from the source but not a larger resistance.

3. The power dissipated by the 0.500  $\Omega$  load can be found using the formula  $P = I^2 R$ . Entering the known values gives

$$P = I^2 R = (20.0 \text{ A})^2 (0.500 \Omega) = 2.00 \times 10^2 \text{ W.} \quad (6.1.6)$$

Note that this power can also be obtained using the expression  $\frac{V^2}{R}$  or  $IV$ , where  $V$  is the terminal voltage (10.0 V in this case).

4. Here, the internal resistance has increased, perhaps due to the depletion of the battery, to the point where it is as great as the load resistance. As before, we first find the current by entering the known values into the expression, yielding

$$I = \frac{\epsilon}{R+r} = \frac{12.00 \text{ V}}{1.00 \Omega} = 12.00 \text{ A}. \quad (6.1.7)$$

Now the terminal voltage is

$$V_{\text{terminal}} = \epsilon - Ir = 12.00 \text{ V} - (12.00 \text{ A})(0.500 \Omega) = 6.00 \text{ V}, \quad (6.1.8)$$

and the power dissipated by the load is

$$P = I^2 R = (12.00 \text{ A})^2 (0.500 \Omega) = 72.00 \text{ W}. \quad (6.1.9)$$

We see that the increased internal resistance has significantly decreased the terminal voltage, current, and power delivered to a load.

### Significance

The internal resistance of a battery can increase for many reasons. For example, the internal resistance of a rechargeable battery increases as the number of times the battery is recharged increases. The increased internal resistance may have two effects on the battery. First, the terminal voltage will decrease. Second, the battery may overheat due to the increased power dissipated by the internal resistance.

### ? Exercise 6.1.1

If you place a wire directly across the two terminal of a battery, effectively shorting out the terminals, the battery will begin to get hot. Why do you suppose this happens?

#### Solution

If a wire is connected across the terminals, the load resistance is close to zero, or at least considerably less than the internal resistance of the battery. Since the internal resistance is small, the current through the circuit will be large,  $I = \frac{\epsilon}{R+r} = \frac{\epsilon}{0+r} = \frac{\epsilon}{r}$ . The large current causes a high power to be dissipated by the internal resistance ( $P = I^2 r$ ). The power is dissipated as heat.

## Battery Testers

Battery testers, such as those in Figure 6.1.8, use small load resistors to intentionally draw current to determine whether the terminal potential drops below an acceptable level. Although it is difficult to measure the internal resistance of a battery, battery testers can provide a measurement of the internal resistance of the battery. If internal resistance is high, the battery is weak, as evidenced by its low terminal voltage.



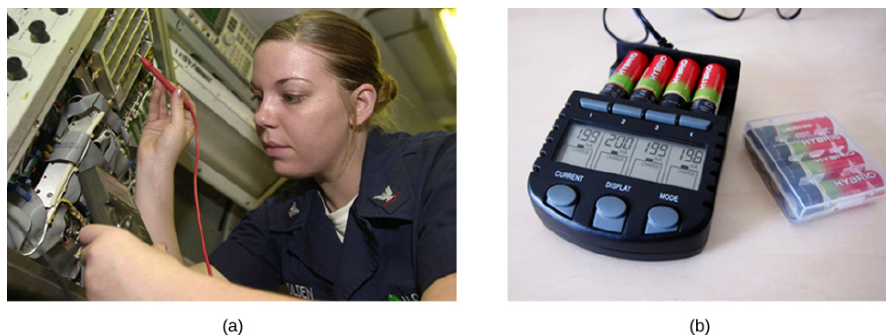


Figure 6.1.8: Battery testers measure terminal voltage under a load to determine the condition of a battery. (a) A US Navy electronics technician uses a battery tester to test large batteries aboard the aircraft carrier USS **Nimitz**. The battery tester she uses has a small resistance that can dissipate large amounts of power. (b) The small device shown is used on small batteries and has a digital display to indicate the acceptability of the terminal voltage. (credit a: modification of work by Jason A. Johnston; credit b: modification of work by Keith Williamson)

Some batteries can be recharged by passing a current through them in the direction opposite to the current they supply to an appliance. This is done routinely in cars and in batteries for small electrical appliances and electronic devices (Figure 6.1.9). The voltage output of the battery charger must be greater than the emf of the battery to reverse the current through it. This causes the terminal voltage of the battery to be greater than the emf, since  $V = \epsilon - Ir$  and  $I$  is now negative.

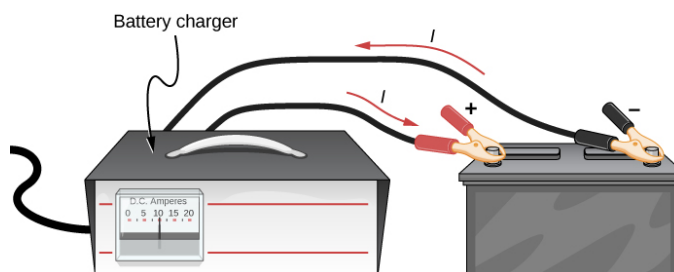


Figure 6.1.9: A car battery charger reverses the normal direction of current through a battery, reversing its chemical reaction and replenishing its chemical potential.

It is important to understand the consequences of the internal resistance of emf sources, such as batteries and solar cells, but often, the analysis of circuits is done with the terminal voltage of the battery, as we have done in the previous sections. The terminal voltage is referred to as simply as  $V$ , dropping the subscript “terminal.” This is because the internal resistance of the battery is difficult to measure directly and can change over time.

## Multiple Voltage Sources

Many devices require more than one battery. Multiple voltage sources, such as batteries, can be connected in series configurations, parallel configurations, or a combination of the two.

In series, the positive terminal of one battery is connected to the negative terminal of another battery. Any number of voltage sources, including batteries, can be connected in series. Two batteries connected in series are shown in Figure 6.1.10. Using Kirchhoff’s loop rule for the circuit in part (b) gives the result

$$\epsilon_1 - Ir_1 + \epsilon_2 - Ir_2 - IR = 0, \quad (6.1.10)$$

$$[(\epsilon_1 + \epsilon_2) - I(r_1 + r_2)] - IR = 0. \quad (6.1.11)$$

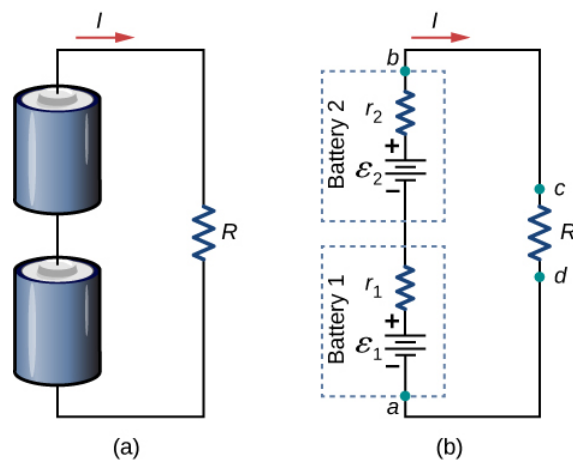


Figure 6.1.10: (a) Two batteries connected in series with a load resistor. (b) The circuit diagram of the two batteries and the load resistor, with each battery modeled as an idealized emf source and an internal resistance.

When voltage sources are in series, their internal resistances can be added together and their emfs can be added together to get the total values. Series connections of voltage sources are common—for example, in flashlights, toys, and other appliances. Usually, the cells are in series in order to produce a larger total emf. In Figure 6.1.10, the terminal voltage is

$$V_{\text{terminal}} = (\epsilon_1 - Ir_1) + (\epsilon_2 - Ir_2) = [(\epsilon_1 + \epsilon_2) - I(r_1 + r_2) - I(r_1 + r_2)] = (\epsilon_1 + \epsilon_2) + Ir_{eq}. \quad (6.1.12)$$

Note that the same current  $I$  is found in each battery because they are connected in series. The disadvantage of series connections of cells is that their internal resistances are additive.

Batteries are connected in series to increase the voltage supplied to the circuit. For instance, an LED flashlight may have two AAA cell batteries, each with a terminal voltage of 1.5 V, to provide 3.0 V to the flashlight.

Any number of batteries can be connected in series. For  $N$  batteries in series, the terminal voltage is equal to

✓ Note

$$V_{\text{terminal}} = (\epsilon_1 + \epsilon_2 + \dots + \epsilon_{N-1} + \epsilon_N) - I(r_1 + r_2 + \dots + r_{N-1} + r_N) = \sum_{i=1}^N \epsilon_i - Ir_{eq} \quad (6.1.13)$$

where the equivalent resistance is

$$r_{eq} = \sum_{i=1}^N r_i \quad (6.1.14)$$

When a load is placed across voltage sources in series, as in Figure 6.1.11, we can find the current:

$$(\epsilon_1 - Ir_1) + (\epsilon_2 - Ir_2) = IR, \quad (6.1.15)$$

$$Ir_1 + Ir_2 + IR = \epsilon_1 + \epsilon_2, \quad (6.1.16)$$

$$I = \frac{\epsilon_1 + \epsilon_2}{r_1 + r_2 + R}. \quad (6.1.17)$$

As expected, the internal resistances increase the equivalent resistance.



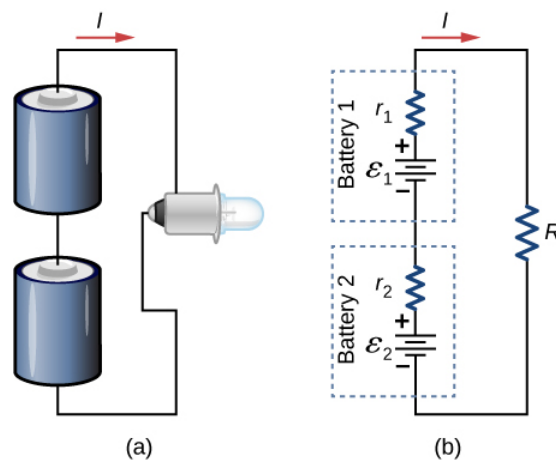


Figure 6.1.11: Two batteries connect in series to an LED bulb, as found in a flashlight.

Voltage sources, such as batteries, can also be connected in parallel. Figure 6.1.12 shows two batteries with identical emfs in parallel and connected to a load resistance. When the batteries are connect in parallel, the positive terminals are connected together and the negative terminals are connected together, and the load resistance is connected to the positive and negative terminals. Normally, voltage sources in parallel have identical emfs. In this simple case, since the voltage sources are in parallel, the total emf is the same as the individual emfs of each battery.

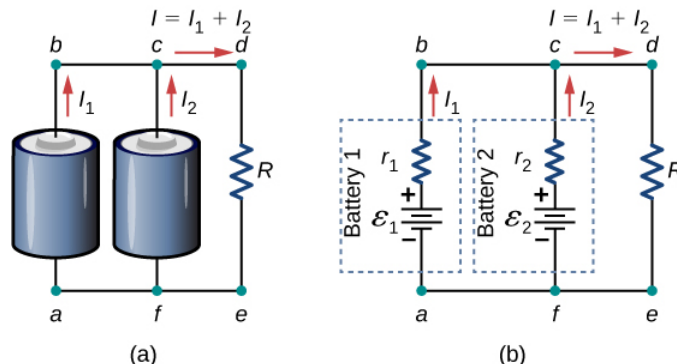


Figure 6.1.12: (a) Two batteries connect in parallel to a load resistor. (b) The circuit diagram shows the shows battery as an emf source and an internal resistor. The two emf sources have identical emfs (each labeled by  $\epsilon$ ) connected in parallel that produce the same emf.

Consider the Kirchhoff analysis of the circuit in Figure 6.1.12b There are two loops and a node at point **b** and  $\epsilon = \epsilon_1 = \epsilon_2$ .

Node b:  $I_1 + I_2 - I = 0$ .

Loop abcfa:  $\epsilon_2 - I_1 r_1 + I_2 r_2 - \epsilon = 0$ ,  $I_1 r_1 = I_2 r_2$ .

Loop fcdef:  $\epsilon_2 - I_2 r_2 - IR = 0$ ,  $\epsilon - I_2 r_2 - IR = 0$ .

Solving for the current through the load resistor results in  $I = \frac{\epsilon}{r_{eq} + R}$ , where  $r_{eq} = \left( \frac{1}{r_1} + \frac{1}{r_2} \right)^{-1}$ . The terminal voltage is equal to the potential drop across the load resistor  $IR = \left( \frac{\epsilon}{r_{eq} + R} \right)$ .

The parallel connection reduces the internal resistance and thus can produce a larger current.

Any number of batteries can be connected in parallel. For **N** batteries in parallel, the terminal voltage is equal to

✓ Note

$$V_{terminal} = \epsilon - I \left( \frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_{N-1}} + \frac{1}{r_N} \right)^{-1} = \epsilon - Ir_{eq} \quad (6.1.18)$$

where the equivalent resistance is

$$r_{eq} = \left( \sum_{i=1}^N \frac{1}{r_i} \right)^{-1} \quad (6.1.19)$$

As an example, some diesel trucks use two 12-V batteries in parallel; they produce a total emf of 12 V but can deliver the larger current needed to start a diesel engine.

In summary, the terminal voltage of batteries in series is equal to the sum of the individual emfs minus the sum of the internal resistances times the current. When batteries are connected in parallel, they usually have equal emfs and the terminal voltage is equal to the emf minus the equivalent internal resistance times the current, where the equivalent internal resistance is smaller than the individual internal resistances. Batteries are connected in series to increase the terminal voltage to the load. Batteries are connected in parallel to increase the current to the load.

## Solar Cell Arrays

Another example dealing with multiple voltage sources is that of combinations of **solar cells** - wired in both series and parallel combinations to yield a desired voltage and current. Photovoltaic generation, which is the conversion of sunlight directly into electricity, is based upon the photoelectric effect. The photoelectric effect is beyond the scope of this chapter and is covered in [Photons and Matter Waves](#), but in general, photons hitting the surface of a solar cell create an electric current in the cell.

Most solar cells are made from pure silicon. Most single cells have a voltage output of about 0.5 V, while the current output is a function of the amount of sunlight falling on the cell (the incident solar radiation known as the insolation). Under bright noon sunlight, a current per unit area of about  $100 \text{ mA}/\text{cm}^2$  of cell surface area is produced by typical single-crystal cells.

Individual solar cells are connected electrically in modules to meet electrical energy needs. They can be wired together in series or in parallel - connected like the batteries discussed earlier. A solar-cell array or module usually consists of between 36 and 72 cells, with a power output of 50 W to 140 W.

Solar cells, like batteries, provide a direct current (dc) voltage. Current from a dc voltage source is unidirectional. Most household appliances need an alternating current (ac) voltage.

## Contributors and Attributions

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