

7.12: Solenoids and Toroids

Learning Objectives

By the end of this section, you will be able to:

- Establish a relationship for how the magnetic field of a solenoid varies with distance and current by using both the Biot-Savart law and Ampère's law
- Establish a relationship for how the magnetic field of a toroid varies with distance and current by using Ampère's law

Two of the most common and useful electromagnetic devices are called solenoids and toroids. In one form or another, they are part of numerous instruments, both large and small. In this section, we examine the magnetic field typical of these devices.

Solenoids

A long wire wound in the form of a helical coil is known as a **solenoid**. Solenoids are commonly used in experimental research requiring magnetic fields. A solenoid is generally easy to wind, and near its center, its magnetic field is quite uniform and directly proportional to the current in the wire.

Figure 7.12.1 shows a solenoid consisting of N turns of wire tightly wound over a length L . A current I is flowing along the wire of the solenoid. The number of turns per unit length is N/L ; therefore, the number of turns in an infinitesimal length dy are $(N/L)dy$ turns. This produces a current

$$dI = \frac{NI}{L} dy. \quad (7.12.1)$$

We first calculate the magnetic field at the point P of Figure 7.12.1. This point is on the central axis of the solenoid. We are basically cutting the solenoid into thin slices that are dy thick and treating each as a current loop. Thus, dI is the current through each slice. The magnetic field $d\vec{B}$ due to the current dI in dy can be found with the help of Equation 12.5.3 and Equation 7.12.1:

$$d\vec{B} = \frac{\mu_0 R^2 dI}{2(y^2 + R^2)^{3/2}} \hat{j} = \left(\frac{\mu_0 I R^2 N}{2L} \hat{j} \right) \frac{dy}{(y^2 + R^2)^{3/2}} \quad (7.12.2)$$

where we used Equation 7.12.1 to replace dI . The resultant field at P is found by integrating $d\vec{B}$ along the entire length of the solenoid. It's easiest to evaluate this integral by changing the independent variable from y to θ . From inspection of Figure 7.12.1, we have:

$$\sin \theta = \frac{y}{\sqrt{y^2 + R^2}}. \quad (7.12.3)$$

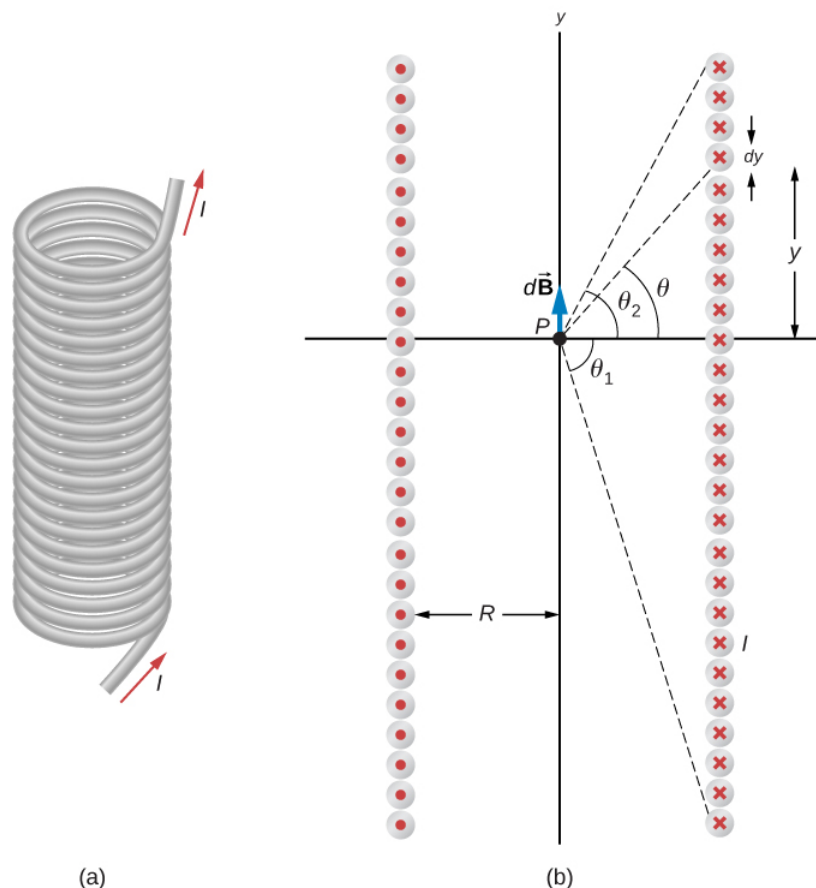


Figure 7.12.1: (a) A solenoid is a long wire wound in the shape of a helix. (b) The magnetic field at the point **P** on the axis of the solenoid is the net field due to all of the current loops.

Taking the differential of both sides of this equation, we obtain

$$\cos \theta d\theta = \left[-\frac{y^2}{(y^2 + R^2)^{3/2}} + \frac{1}{\sqrt{y^2 + R^2}} \right] dy \quad (7.12.4)$$

$$= \frac{R^2 dy}{(y^2 + R^2)^{3/2}}. \quad (7.12.5)$$

When this is substituted into the equation for $d\vec{B}$, we have

$$\vec{B} = \frac{\mu_0 IN}{2L} \hat{j} \int_{\theta_1}^{\theta_2} \cos \theta d\theta = \frac{\mu_0 IN}{2L} (\sin \theta_2 - \sin \theta_1) \hat{j}, \quad (7.12.6)$$

which is the magnetic field along the central axis of a finite solenoid.

Of special interest is the infinitely long solenoid, for which $L \rightarrow \infty$. From a practical point of view, the infinite solenoid is one whose length is much larger than its radius ($L \gg R$). In this case, $\theta_1 = -\frac{\pi}{2}$ and $\theta_2 = \frac{\pi}{2}$. Then from Equation 7.12.6, the magnetic field along the central axis of an infinite solenoid is

$$\vec{B} = \frac{\mu_0 IN}{2L} \hat{j} [\sin(\pi/2) - \sin(-\pi/2)] = \frac{\mu_0 IN}{L} \hat{j} \quad (7.12.7)$$

or

$$\vec{B} = \mu_0 n I \hat{j}, \quad (7.12.8)$$

where **n** is the number of turns per unit length. You can find the direction of \vec{B} with a right-hand rule: Curl your fingers in the direction of the current, and your thumb points along the magnetic field in the interior of the solenoid.

We now use these properties, along with Ampère's law, to calculate the magnitude of the magnetic field at any location inside the infinite solenoid. Consider the closed path of Figure 7.12.2. Along segment 1, \vec{B} is uniform and parallel to the path. Along segments 2 and 4, \vec{B} is perpendicular to part of the path and vanishes over the rest of it. Therefore, segments 2 and 4 do not contribute to the line integral in Ampère's law. Along segment 3, $\vec{B} = 0$ because the magnetic field is zero outside the solenoid. If you consider an Ampère's law loop outside of the solenoid, the current flows in opposite directions on different segments of wire. Therefore, there is no enclosed current and no magnetic field according to Ampère's law. Thus, there is no contribution to the line integral from segment 3. As a result, we find

$$\oint \vec{B} \cdot d\vec{l} = \int_1 \vec{B} \cdot d\vec{l} = Bl. \quad (7.12.9)$$

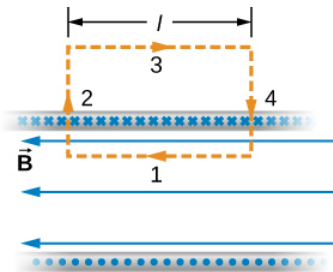


Figure 7.12.2: The path of integration used in Ampère's law to evaluate the magnetic field of an infinite solenoid.

The solenoid has n turns per unit length, so the current that passes through the surface enclosed by the path is nIl . Therefore, from Ampère's law,

$$Bl = \mu_0 nIl \quad (7.12.10)$$

and

✓ Note

$$B = \mu_0 nI \quad (7.12.11)$$

within the solenoid. This agrees with what we found earlier for \mathbf{B} on the central axis of the solenoid. Here, however, the location of segment 1 is arbitrary, so we have found that this equation gives the magnetic field everywhere inside the infinite solenoid.

Outside the solenoid, one can draw an Ampère's law loop around the entire solenoid. This would enclose current flowing in both directions. Therefore, the net current inside the loop is zero. According to Ampère's law, if the net current is zero, the magnetic field must be zero. Therefore, for locations outside of the solenoid's radius, the magnetic field is zero.

When a patient undergoes a **magnetic resonance imaging** (MRI) scan, the person lies down on a table that is moved into the center of a large solenoid that can generate very large magnetic fields. The solenoid is capable of these high fields from high currents flowing through superconducting wires. The large magnetic field is used to change the spin of protons in the patient's body. The time it takes for the spins to align or relax (return to original orientation) is a signature of different tissues that can be analyzed to see if the structures of the tissues is normal (Figure 7.12.3).



Figure 7.12.3: . In an MRI machine, a large magnetic field is generated by the cylindrical solenoid surrounding the patient. (credit: Liz West)

✓ Example 7.12.1: Magnetic Field Inside a SOLenoid

A solenoid has 300 turns wound around a cylinder of diameter 1.20 cm and length 14.0 cm. If the current through the coils is 0.410 A, what is the magnitude of the magnetic field inside and near the middle of the solenoid?

Strategy

We are given the number of turns and the length of the solenoid so we can find the number of turns per unit length. Therefore, the magnetic field inside and near the middle of the solenoid is given by Equation 7.12.11. Outside the solenoid, the magnetic field is zero.

Solution

The number of turns per unit length is

$$n = \frac{300 \text{ turns}}{0.140 \text{ m}} = 2.14 \times 10^3 \text{ turns/m.} \quad (7.12.12)$$

The magnetic field produced inside the solenoid is

$$B = \mu_0 n I = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2.14 \times 10^3 \text{ turns/m})(0.410 \text{ A}) \quad (7.12.13)$$

$$B = 1.10 \times 10^{-3} \text{ T.} \quad (7.12.14)$$

Significance

This solution is valid only if the length of the solenoid is reasonably large compared with its diameter. This example is a case where this is valid.

? Exercise 7.12.1

What is the ratio of the magnetic field produced from using a finite formula over the infinite approximation for an angle θ of (a) 85° ? (b) 89° ? The solenoid has 1000 turns in 50 cm with a current of 1.0 A flowing through the coils

Solution

a. 1.00382; b. 1.00015

Toroids

A toroid is a donut-shaped coil closely wound with one continuous wire, as illustrated in part (a) of Figure 7.12.4 If the toroid has N windings and the current in the wire is I , what is the magnetic field both inside and outside the toroid?

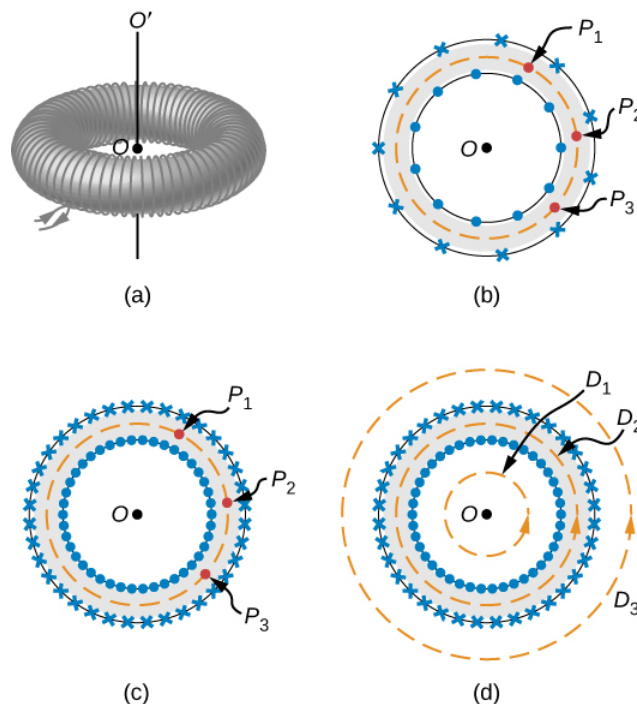


Figure 7.12.4: (a) A toroid is a coil wound into a donut-shaped object. (b) A loosely wound toroid does not have cylindrical symmetry. (c) In a tightly wound toroid, cylindrical symmetry is a very good approximation. (d) Several paths of integration for Ampère's law.

We begin by assuming cylindrical symmetry around the axis OO' . Actually, this assumption is not precisely correct, for as part (b) of Figure 7.12.4 shows, the view of the toroidal coil varies from point to point (for example, P_1 , P_2 and P_3) on a circular path centered around OO' . However, if the toroid is tightly wound, all points on the circle become essentially equivalent [part (c) of Figure 7.12.4], and cylindrical symmetry is an accurate approximation.

With this symmetry, the magnetic field must be tangent to and constant in magnitude along any circular path centered on OO' . This allows us to write for each of the paths D_1 , D_2 and D_3 shown in part (d) of Figure 7.12.4

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r). \quad (7.12.15)$$

Ampère's law relates this integral to the net current passing through any surface bounded by the path of integration. For a path that is external to the toroid, either no current passes through the enclosing surface (path D_1), or the current passing through the surface in one direction is exactly balanced by the current passing through it in the opposite direction (path D_3). In either case, there is no net current passing through the surface, so

$$\oint B(2\pi r) = 0 \quad (7.12.16)$$

and

$$B = 0 \text{ (outside the toroid).} \quad (7.12.17)$$

The turns of a toroid form a helix, rather than circular loops. As a result, there is a small field external to the coil; however, the derivation above holds if the coils were circular.

For a circular path within the toroid (path D_2), the current in the wire cuts the surface N times, resulting in a net current NI through the surface. We now find with Ampère's law,

$$B(2\pi r) = \mu_0 NI \quad (7.12.18)$$

and

$$B = \frac{\mu_0 N I}{2\pi r} \text{ (within the toroid).} \quad (7.12.19)$$

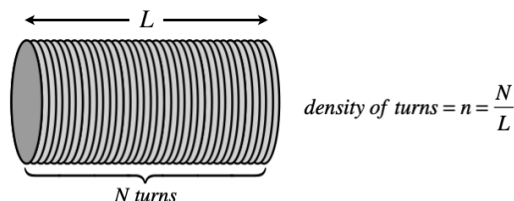
The magnetic field is directed in the counterclockwise direction for the windings shown. When the current in the coils is reversed, the direction of the magnetic field also reverses.

The magnetic field inside a toroid is not uniform, as it varies inversely with the distance r from the axis OO' . However, if the central radius R (the radius midway between the inner and outer radii of the toroid) is much larger than the cross-sectional diameter of the coils r , the variation is fairly small, and the magnitude of the magnetic field may be calculated by Equation 7.12.19 where $r = R$.

Field of a Solenoid

It is possible to stack lots of individual dipoles on top of each other to create a long tube called a **solenoid**. Such a device consists of a number of turns in the coil N , and a length L , resulting in what will be the critical measure, the **turn density**:

Figure 4.4.5 – A Solenoid



How do we compute the field for such an object? Well, first of all, we need to specify what field we want. Like the loop, we will only look on the axis. But we will also simplify it further by assuming we are looking at a point on the axis *inside* the solenoid far from the ends (so essentially it has an infinite length, though the turn density is of course finite).

We treat this as a collection of an infinite number of loops. If we pick an origin (which we can place anywhere along the infinite axis), then we have the field at that point by a loop at a position z on the axis is given by Equation 4.4.10. Then we need to add up the field contributions at the origin due to all of the loops. The problem is, there is not a loop at every point along the z -axis. With a turn density of n , the number of turns in a tiny slice dz would be $n dz$. The total current in that slice would then be this number multiplied by the current through the wound wire (which we will call I):

$$\text{current in a } dz \text{ slice located at } z = I n dz \quad (7.12.20)$$

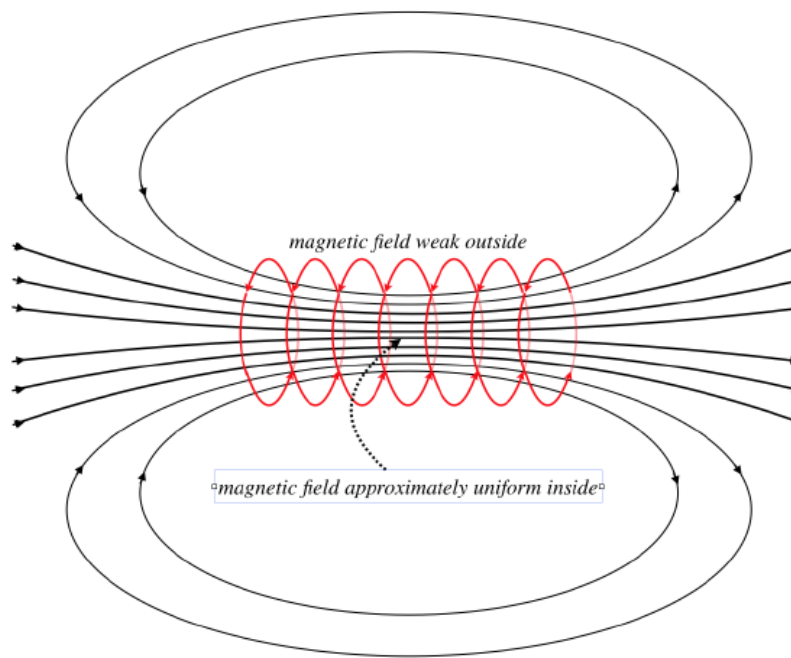
Plugging this into Equation 4.4.10 for the current, gives the tiny contribution to the field by the slice, and adding them all up gives the field. We are not given the radius of the solenoid, but we will call it R , and we'll see that it isn't relevant!

$$B = \int_{-\infty}^{+\infty} \frac{\mu_o (n I dz) R^2}{2(R^2 + z^2)^{\frac{3}{2}}} = \frac{\mu_o n I R^2}{2} \int_{-\infty}^{+\infty} \frac{dz}{(R^2 + z^2)^{\frac{3}{2}}} = \frac{\mu_o n I R^2}{2} \left[\frac{z}{R^2 \sqrt{R^2 + z^2}} \right]_{-\infty}^{+\infty} = \mu_o n I \quad (7.12.21)$$

There are a few particularly interesting aspects of the fields of solenoids, which are not immediately evident from this solution, but which we will state without proof (for now – we have another tool to use later that makes this easier):

- The field within the solenoid doesn't change much (it is pretty much *uniform*). This basically comes from the fact – as we found here – that the field on the axis doesn't depend upon the radius of the solenoid.
- The field just outside the solenoid (on its side, not the end) is very weak (basically it is *zero*).
- The field looks just like that of a bar magnet, but it can be turned on and off by switching the current on or off.

Figure 4.4.6 – Solenoid Magnetic Field



Digression: Electromagnets

Solenoids have lots of practical uses, a common one being something known as an “electromagnet.” For example, junk yards use these to move large chunks of scrap metal. Obviously the ability to cut the current to turn off the magnetic field is key here. If the crane used a permanent magnet, it wouldn’t be able to let go of the crushed car. Another application is for fire doors. Imagine large doors held open in hallways of a building with electromagnets, and if a fire breaks out, the power is cut and the doors close, hopefully slowing the spread of the fire. Gates where you are “buzzed-in” are held shut by a latch that is released with the activation of an electromagnet that draws the latch back with a magnetic field.

Contributors and Attributions

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