

6.3: Kirchhoff's Rules

Learning Objectives

By the end of the section, you will be able to:

- State Kirchhoff's junction rule
- State Kirchhoff's loop rule
- Analyze complex circuits using Kirchhoff's rules

We have just seen that some circuits may be analyzed by reducing a circuit to a single voltage source and an equivalent resistance. Many complex circuits cannot be analyzed with the series-parallel techniques developed in the preceding sections. In this section, we elaborate on the use of Kirchhoff's rules to analyze more complex circuits. For example, the circuit in Figure 6.3.1 is known as a **multi-loop circuit**, which consists of junctions. A junction, also known as a node, is a connection of three or more wires. In this circuit, the previous methods cannot be used, because not all the resistors are in clear series or parallel configurations that can be reduced. Give it a try. The resistors R_1 and R_2 are in series and can be reduced to an equivalent resistance. The same is true of resistors R_4 and R_5 . But what do you do then?

Even though this circuit cannot be analyzed using the methods already learned, two circuit analysis rules can be used to analyze any circuit, simple or complex. The rules are known as **Kirchhoff's rules**, after their inventor Gustav **Kirchhoff** (1824–1887).

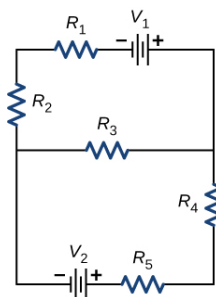


Figure 6.3.1: This circuit cannot be reduced to a combination of series and parallel connections. However, we can use Kirchhoff's rules to analyze it.

✓ Kirchhoff's Rules

- Kirchhoff's first rule—the junction rule. The sum of all currents entering a junction must equal the sum of all currents leaving the junction:

$$\sum I_{in} = \sum I_{out}. \quad (6.3.1)$$

- Kirchhoff's second rule—the loop rule. The algebraic sum of changes in potential around any closed circuit path (loop) must be zero:

$$\sum V = 0. \quad (6.3.2)$$

We now provide explanations of these two rules, followed by problem-solving hints for applying them and a worked example that uses them.

Kirchhoff's First Rule

Kirchhoff's first rule (the **junction rule**) applies to the charge entering and leaving a junction (Figure 6.3.2). As stated earlier, a junction, or node, is a connection of three or more wires. Current is the flow of charge, and charge is conserved; thus, whatever charge flows into the junction must flow out.

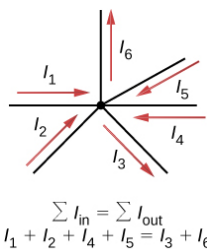


Figure 6.3.2: Charge must be conserved, so the sum of currents into a junction must be equal to the sum of currents out of the junction.

Although it is an over-simplification, an analogy can be made with water pipes connected in a plumbing junction. If the wires in Figure 6.3.2 were replaced by water pipes, and the water was assumed to be incompressible, the volume of water flowing into the junction must equal the volume of water flowing out of the junction.

Kirchhoff's Second Rule

Kirchhoff's second rule (the **loop rule**) applies to potential differences. The loop rule is stated in terms of potential V rather than potential energy, but the two are related since $U = qV$. In a closed loop, whatever energy is supplied by a voltage source, the energy must be transferred into other forms by the devices in the loop, since there are no other ways in which energy can be transferred into or out of the circuit. Kirchhoff's loop rule states that the algebraic sum of potential differences, including voltage supplied by the voltage sources and resistive elements, in any loop must be equal to zero. For example, consider a simple loop with no junctions, as in Figure 6.3.3.

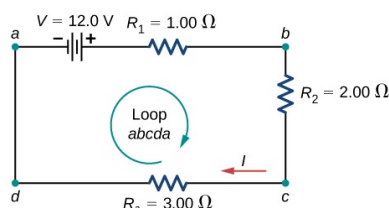


Figure 6.3.3: A simple loop with no junctions. Kirchhoff's loop rule states that the algebraic sum of the voltage differences is equal to zero.

The circuit consists of a voltage source and three external load resistors. The labels **a**, **b**, **c**, and **d** serve as references, and have no other significance. The usefulness of these labels will become apparent soon. The loop is designated as Loop **abcda**, and the labels help keep track of the voltage differences as we travel around the circuit. Start at point **a** and travel to point **b**. The voltage of the voltage source is added to the equation and the potential drop of the resistor R_1 is subtracted. From point **b** to **c**, the potential drop across R_2 is subtracted. From **c** to **d**, the potential drop across R_3 is subtracted. From points **d** to **a**, nothing is done because there are no components.

Figure 6.3.4 shows a graph of the voltage as we travel around the loop. Voltage increases as we cross the battery, whereas voltage decreases as we travel across a resistor. The **potential drop**, or change in the electric potential, is equal to the current through the resistor times the resistance of the resistor. Since the wires have negligible resistance, the voltage remains constant as we cross the wires connecting the components.

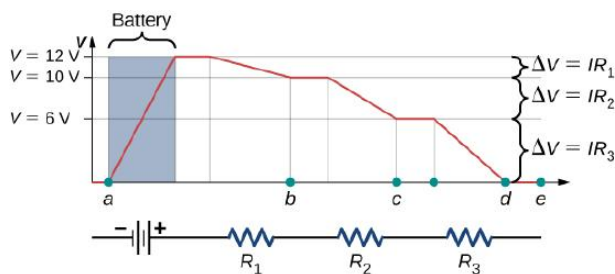


Figure 6.3.4: A voltage graph as we travel around the circuit. The voltage increases as we cross the battery and decreases as we cross each resistor. Since the resistance of the wire is quite small, we assume that the voltage remains constant as we cross the wires connecting the components.

Then Kirchhoff's loop rule states

$$V - IR_1 - IR_2 - IR_3 = 0. \quad (6.3.3)$$

The loop equation can be used to find the current through the loop:

$$I = \frac{V}{R_1 + R_2 + R_3} = \frac{12.00 \text{ V}}{1.00 \Omega + 2.00 \Omega + 3.00 \Omega} = 2.00 \text{ A}. \quad (6.3.4)$$

This loop could have been analyzed using the previous methods, but we will demonstrate the power of Kirchhoff's method in the next section.

Applying Kirchhoff's Rules

By applying Kirchhoff's rules, we generate a set of linear equations that allow us to find the unknown values in circuits. These may be currents, voltages, or resistances. Each time a rule is applied, it produces an equation. If there are as many independent equations as unknowns, then the problem can be solved.

Using Kirchhoff's method of analysis requires several steps, as listed in the following procedure.

✓ Problem-Solving Strategy: Kirchhoff's Rules

1. Label points in the circuit diagram using lowercase letters **a**, **b**, **c**, These labels simply help with orientation.
2. Locate the junctions in the circuit. The junctions are points where three or more wires connect. Label each junction with the currents and directions into and out of it. Make sure at least one current points into the junction and at least one current points out of the junction.
3. Choose the loops in the circuit. Every component must be contained in at least one loop, but a component may be contained in more than one loop.
4. Apply the junction rule. Again, some junctions should not be included in the analysis. You need only use enough nodes to include every current.
5. Apply the loop rule. Use the map in Figure 6.3.5.

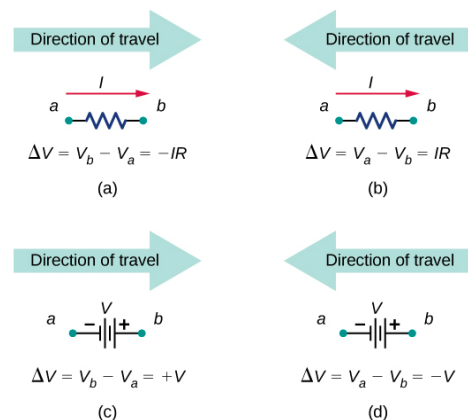


Figure 6.3.5: Each of these resistors and voltage sources is traversed from **a** to **b**. (a) When moving across a resistor in the same direction as the current flow, subtract the potential drop. (b) When moving across a resistor in the opposite direction as the current flow, add the potential drop. (c) When moving across a voltage source from the negative terminal to the positive terminal, add the potential drop. (d) When moving across a voltage source from the positive terminal to the negative terminal, subtract the potential drop.

Let's examine some steps in this procedure more closely. When locating the junctions in the circuit, do not be concerned about the direction of the currents. If the direction of current flow is not obvious, choosing any direction is sufficient as long as at least one current points into the junction and at least one current points out of the junction. If the arrow is in the opposite direction of the conventional current flow, the result for the current in question will be negative but the answer will still be correct.

The number of nodes depends on the circuit. Each current should be included in a node and thus included in at least one junction equation. Do not include nodes that are not linearly independent, meaning nodes that contain the same information.

Consider Figure 6.3.6. There are two junctions in this circuit: Junction **b** and Junction **e**. Points **a**, **c**, **d**, and **f** are not junctions, because a junction must have three or more connections. The equation for Junction **b** is $I_1 = I_2 + I_3$, and the equation for Junction

e is $I_2 + I_3 = I_1$. These are equivalent equations, so it is necessary to keep only one of them.

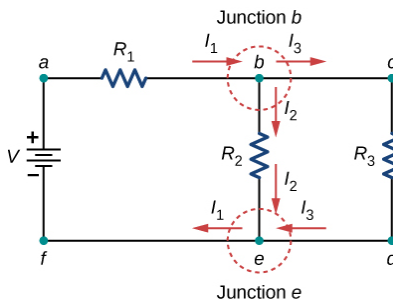


Figure 6.3.6: At first glance, this circuit contains two junctions, Junction **b** and Junction **e**, but only one should be considered because their junction equations are equivalent.

When choosing the loops in the circuit, you need enough loops so that each component is covered once, without repeating loops. Figure 6.3.7 shows four choices for loops to solve a sample circuit; choices (a), (b), and (c) have a sufficient amount of loops to solve the circuit completely. Option (d) reflects more loops than necessary to solve the circuit.

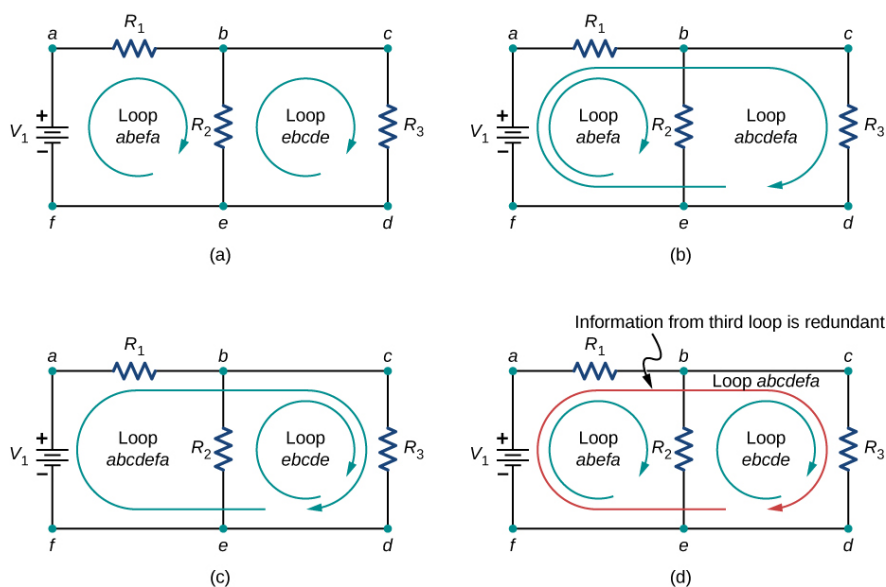


Figure 6.3.7: Panels (a)–(c) are sufficient for the analysis of the circuit. In each case, the two loops shown contain all the circuit elements necessary to solve the circuit completely. Panel (d) shows three loops used, which is more than necessary. Any two loops in the system will contain all information needed to solve the circuit. Adding the third loop provides redundant information.

Consider the circuit in Figure 6.3.8a. Let us analyze this circuit to find the current through each resistor. First, label the circuit as shown in part (b).

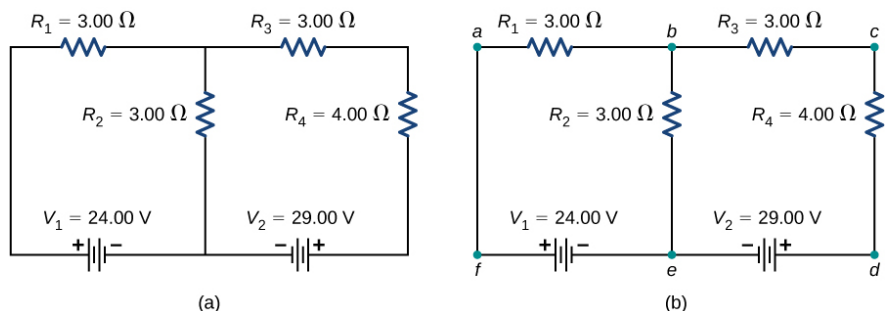


Figure 6.3.8: (a) A multi-loop circuit. (b) Label the circuit to help with orientation.

Next, determine the junctions. In this circuit, points **b** and **e** each have three wires connected, making them junctions. Start to apply Kirchhoff's junction rule ($\sum I_{in} = \sum I_{out}$) by drawing arrows representing the currents and labeling each arrow, as shown in Figure 6.3.9. Junction **b** shows that $I_1 = I_2 + I_3$ and Junction **e** shows that $I_2 + I_3 = I_1$. Since Junction **e** gives the same

information of Junction **b**, it can be disregarded. This circuit has three unknowns, so we need three linearly independent equations to analyze it.

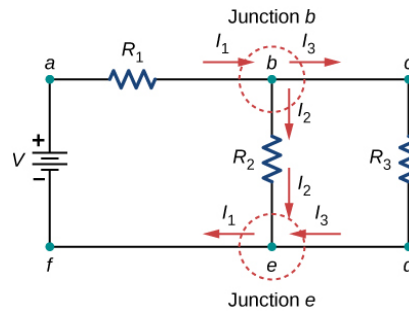


Figure 6.3.9: (a) This circuit has two junctions, labeled b and e, but only node b is used in the analysis. (b) Labeled arrows represent the currents into and out of the junctions.

Next we need to choose the loops. In Figure 6.3.10, Loop **abefa** includes the voltage source V_1 and resistors R_1 and R_2 . The loop starts at point **a**, then travels through points **b**, **e**, and **f**, and then back to point **a**. The second loop, Loop **ebcde**, starts at point **e** and includes resistors R_2 and R_3 , and the voltage source V_2 .

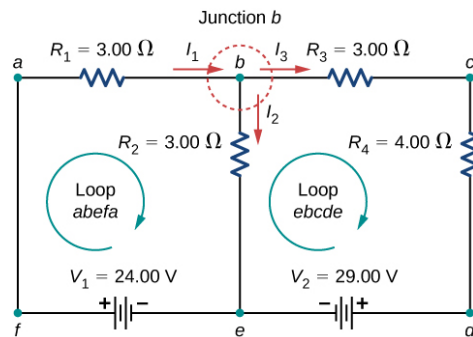


Figure 6.3.10: Choose the loops in the circuit.

Now we can apply Kirchhoff's loop rule, using the map in Figure 6.3.5. Starting at point **a** and moving to point **b**, the resistor R_1 is crossed in the same direction as the current flow I_1 , so the potential drop $I_1 R_1$ is subtracted. Moving from point **b** to point **e**, the resistor R_2 is crossed in the same direction as the current flow I_2 so the potential drop $I_2 R_2$ is subtracted. Moving from point **e** to point **f**, the voltage source V_1 is crossed from the negative terminal to the positive terminal, so V_1 is added. There are no components between points **f** and **a**. The sum of the voltage differences must equal zero:

$$\text{Loop abefa: } -I_1 R_1 - I_2 R_2 + V_1 = 0 \text{ or } V_1 = I_1 R_1 + I_2 R_2. \quad (6.3.5)$$

Finally, we check loop **ebcde**. We start at point **e** and move to point **b**, crossing R_2 in the opposite direction as the current flow I_2 . The potential drop $I_2 R_2$ is added. Next, we cross R_3 and R_4 in the same direction as the current flow I_3 and subtract the potential drops $I_3 R_3$ and $I_3 R_4$. Note that the current is the same through resistors R_3 and R_4 , because they are connected in series. Finally, the voltage source is crossed from the positive terminal to the negative terminal, and the voltage source V_2 is subtracted. The sum of these voltage differences equals zero and yields the loop equation

$$\text{Loop ebcde: } I_2 R_2 - I_3 (R_3 + R_4) - V_2 = 0. \quad (6.3.6)$$

We now have three equations, which we can solve for the three unknowns.

$$\text{Junction b: } I_1 - I_2 - I_3 = 0. \quad (6.3.7)$$

$$\text{Loop abefa: } I_1 R_1 + I_2 R_2 = V_1. \quad (6.3.8)$$

$$\text{Loop ebcde: } I_2 R_2 - I_3 (R_3 + R_4) = V_2. \quad (6.3.9)$$

To solve the three equations for the three unknown currents, start by eliminating current I_2 . First add Equation 6.3.7 times R_2 to Equation 6.3.8. The result is Equation 6.3.11:

$$(R_1 + R_2)I_1 - R_2 I_3 = V_1. \quad (6.3.10)$$

$$6 \Omega I_1 - 3 \Omega I_3 = 24 \text{ V}. \quad (6.3.11)$$

Next, subtract Equation 6.3.9 from Equation 6.3.8. The result is Equation 6.3.13

$$I_1 R_1 + I_3 (R_3 + R_4) = V_1 - V_2. \quad (6.3.12)$$

$$3 \Omega I_1 + 7 \Omega I_3 = -5 \text{ V}. \quad (6.3.13)$$

We can solve Equations 6.3.11 and 6.3.13 for current I_1 . Adding seven times Equation 6.3.11 and three times Equation 6.3.13 results in $51 \Omega I_1 = 153 \text{ V}$, or $I_1 = 3.00 \text{ A}$. Using Equation 6.3.11 results in $I_3 = -2.00 \text{ A}$. Finally, Equation 6.3.7 yields $I_2 = I_1 - I_3 = 5.00 \text{ A}$. One way to check that the solutions are consistent is to check the power supplied by the voltage sources and the power dissipated by the resistors:

$$P_{in} = I_1 V_1 + I_3 V_2 = 130 \text{ W},$$

$$P_{out} = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 + I_3^2 R_4 = 130 \text{ W}.$$

Note that the solution for the current I_3 is negative. This is the correct answer, but suggests that the arrow originally drawn in the junction analysis is the direction opposite of conventional current flow. The power supplied by the second voltage source is 58 W and not -58 W.

✓ Example 6.3.1: Calculating Current by Using Kirchhoff's Rules

Find the currents flowing in the circuit in Figure 6.3.11.

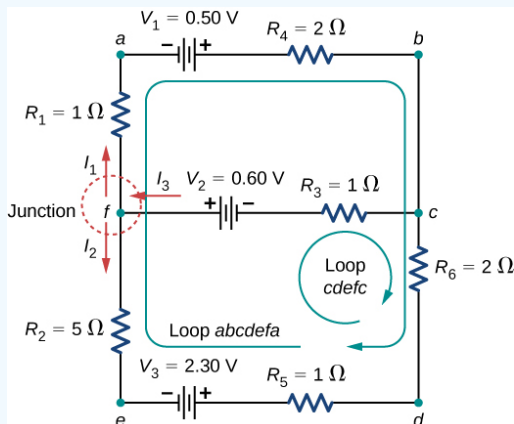


Figure 6.3.11: This circuit is combination of series and parallel configurations of resistors and voltage sources. This circuit cannot be analyzed using the techniques discussed in [Electromotive Force](#) but can be analyzed using Kirchhoff's rules.

Strategy

This circuit is sufficiently complex that the currents cannot be found using Ohm's law and the series-parallel techniques—it is necessary to use Kirchhoff's rules. Currents have been labeled I_1 , I_2 , and I_3 in the figure, and assumptions have been made about their directions. Locations on the diagram have been labeled with letters **a** through **h**. In the solution, we apply the junction and loop rules, seeking three independent equations to allow us to solve for the three unknown currents.

Solution

Applying the junction and loop rules yields the following three equations. We have three unknowns, so three equations are required.

$$\text{Junction c: } I_1 + I_2 = I_3. \quad (6.3.14)$$

$$\text{Loop abcdefa: } I_1(R_1 + R_4) - I_2(R_2 + R_5 + R_6) = V_1 - V_3. \quad (6.3.15)$$

$$\text{Loop cdefc: } I_2(R_2 + R_5 + R_6) + I_3 R_3 = V_2 + V_3. \quad (6.3.16)$$

Simplify the equations by placing the unknowns on one side of the equations.

$$\text{Junction c: } I_1 + I_2 - I_3 = 0. \quad (6.3.17)$$

$$\text{Loop } abcdefa : I_1(3\Omega) - I_2(8\Omega) = 0.5 \text{ V} - 2.30 \text{ V}. \quad (6.3.18)$$

$$\text{Loop } cdefc : I_2(8\Omega) + I_3(1\Omega) = 0.6 \text{ V} + 2.30 \text{ V}. \quad (6.3.19)$$

Simplify the equations. The first loop equation can be simplified by dividing both sides by 3.00. The second loop equation can be simplified by dividing both sides by 6.00.

$$\text{Junction } c : I_1 + I_2 - I_3 = 0. \quad (6.3.20)$$

$$\text{Loop } abcdefa : I_1(3\Omega) - I_2(8\Omega) = -1.8 \text{ V}. \quad (6.3.21)$$

$$\text{Loop } cdefc : I_2(8\Omega) + I_3(1\Omega) = 2.90 \text{ V}. \quad (6.3.22)$$

The results are

$$I_1 = 0.20 \text{ A}, I_2 = 0.30 \text{ A}, I_3 = 0.50 \text{ A}. \quad (6.3.23)$$

Significance

A method to check the calculations is to compute the power dissipated by the resistors and the power supplied by the voltage sources:

$$P_{R_1} = I_1^2 R_1 = 0.04 \text{ W}. \quad (6.3.24)$$

$$P_{R_2} = I_2^2 R_2 = 0.45 \text{ W}. \quad (6.3.25)$$

$$P_{R_3} = I_3^2 R_3 = 0.25 \text{ W}. \quad (6.3.26)$$

$$P_{R_4} = I_1^2 R_4 = 0.08 \text{ W}. \quad (6.3.27)$$

$$P_{R_5} = I_2^2 R_5 = 0.09 \text{ W}. \quad (6.3.28)$$

$$P_{R_6} = I_2^2 R_1 = 0.18 \text{ W}. \quad (6.3.29)$$

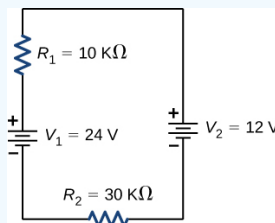
$$P_{\text{dissipated}} = 1.09 \text{ W}. \quad (6.3.30)$$

$$P_{\text{source}} = I_1 V_1 + I_2 V_3 + I_3 V_2 = 0.10 + 0.69 \text{ W} + 0.30 \text{ W} = 1.09 \text{ W}. \quad (6.3.31)$$

The power supplied equals the power dissipated by the resistors.

? Exercise 6.3.1

In considering the following schematic and the power supplied and consumed by a circuit, will a voltage source always provide power to the circuit, or can a voltage source consume power?



Answer

The circuit can be analyzed using Kirchhoff's loop rule. The first voltage source supplies power: $P_{in} = IV_1 = 7.20 \text{ mW}$. The second voltage source consumes power: $P_{out} = IV_2 + I^2 R_1 + I^2 R_2 = 7.2 \text{ mW}$.

✓ Example 6.3.2: Calculating Current by Using Kirchhoff's Rules

Find the current flowing in the circuit in Figure 6.3.12

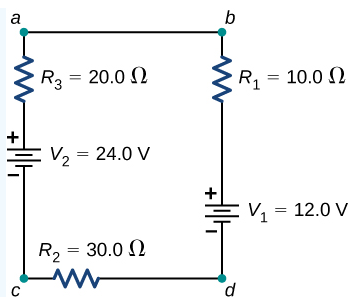


Figure 6.3.12: This circuit consists of three resistors and two batteries connected in series. Note that the batteries are connected with opposite polarities.

Strategy

This circuit can be analyzed using Kirchhoff's rules. There is only one loop and no nodes. Choose the direction of current flow. For this example, we will use the clockwise direction from point **a** to point **b**. Consider Loop **abcda** and use Figure 6.3.5 to write the loop equation. Note that according to Figure 6.3.5 battery V_1 will be added and battery V_2 will be subtracted.

Solution

Applying the junction rule yields the following three equations. We have one unknown, so one equation is required:

$$\text{Loop } abcda : -IR_1 - V_1 - IR_2 + V_2 - IR_3 = 0. \quad (6.3.32)$$

Simplify the equations by placing the unknowns on one side of the equations. Use the values given in the figure.

$$I(R_1 + R_2 + R_3) = V_2 - V_1. \quad (6.3.33)$$

$$I = \frac{V_2 - V_1}{R_1 + R_2 + R_3} = \frac{24 \text{ V} - 12 \text{ V}}{10.0 \Omega + 30.0 \Omega + 10.0 \Omega} = 0.20 \text{ A}. \quad (6.3.34)$$

Significance

The power dissipated or consumed by the circuit equals the power supplied to the circuit, but notice that the current in the battery V_1 is flowing through the battery from the positive terminal to the negative terminal and consumes power.

$$P_{R_1} = I^2 R_1 = 0.40 \text{ W} \quad (6.3.35)$$

$$P_{R_2} = I^2 R_2 = 1.20 \text{ W} \quad (6.3.36)$$

$$P_{R_3} = I^2 R_3 = 0.80 \text{ W} \quad (6.3.37)$$

$$P_{V_1} = IV_1 = 2.40 \text{ W} \quad (6.3.38)$$

$$P_{\text{dissipated}} = 4.80 \text{ W} \quad (6.3.39)$$

$$P_{\text{source}} = IV_2 = 4.80 \text{ W} \quad (6.3.40)$$

The power supplied equals the power dissipated by the resistors and consumed by the battery V_1 .

? Exercise 6.3.2

When using Kirchhoff's laws, you need to decide which loops to use and the direction of current flow through each loop. In analyzing the circuit in Example 6.3.2, the direction of current flow was chosen to be clockwise, from point **a** to point **b**. How would the results change if the direction of the current was chosen to be counterclockwise, from point **b** to point **a**?

Answer

The current calculated would be equal to $I = -0.20 \text{ A}$ instead of $I = 0.20 \text{ A}$. The sum of the power dissipated and the power consumed would still equal the power supplied.

Contributors and Attributions

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