3.5: Electric Field- Concept of a Field Revisited

Learning Objectives

By the end of this section, you will be able to:

- Describe a force field and calculate the strength of an electric field due to a point charge.
- Calculate the force exerted on a test charge by an electric field.
- Explain the relationship between electrical force ($\vec{F}$) on a test charge and electrical field strength ($\vec{E}$).

Contact forces, such as between a baseball and a bat, are explained on the small scale by the interaction of the charges in atoms and molecules in close proximity. They interact through forces that include the **Coulomb force**. Action at a distance is a force between objects that are not close enough for their atoms to "touch." That is, they are separated by more than a few atomic diameters.

For example, a charged rubber comb attracts neutral bits of paper from a distance via the Coulomb force. It is very useful to think of an object being surrounded in space by a **force field**. The force field carries the force to another object (called a test object) some distance away.

Concept of a Field

A field is a way of conceptualizing and mapping the force that surrounds any object and acts on another object at a distance without apparent physical connection. For example, the gravitational field surrounding the earth (and all other masses) represents the gravitational force that would be experienced if another mass were placed at a given point within the field.

In the same way, the Coulomb force field surrounding any charge extends throughout space. Using Coulomb's law, $\vec{F} = k\frac{q_1 q_2}{r_{12}^2}\hat{r_{12}}$ its magnitude is given by the equation $F = k|qQ|/r_{12}^2$, for a **point charge** (a particle having a charge q) acting on a **test charge** Q at a distance r (Figure $3.5.1$). Both the magnitude and direction of the Coulomb force field depend on q and the test charge Q.

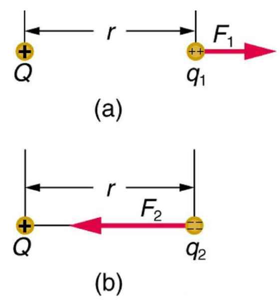

Figure $3.5.1$: The Coulomb force field due to a positive charge Q is shown acting on two different charges. Both charges are the same distance from Q. (a) Since q_1 is positive, the force F_1 acting on it is repulsive. (b) The charge q_2 is negative and greater in magnitude than q_1, and so the force F_2 acting on it is attractive and stronger than F_1. The Coulomb force field is thus not unique at any point in space, because it depends on the test charges q_1 and q_2 as well as the charge Q.

To simplify things, we would prefer to have a field that depends only on q and not on the test charge Q. The electric field is defined in such a manner that it represents only the charge creating it and is unique at every point in space. Specifically, the electric field $\vec{E}$ is defined to be the ratio of the Coulomb force to the test charge:

$$\vec{E} = \frac{\vec{F}}{Q} \tag{3.5.1}$$

where

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \tag{3.5.2}$$

This expression is called the electric field at position $P = P(x, y, z)$ due to charge q. Here, P is the location of the point in space where you are calculating the field and is relative to the positions $\vec{r}_i$ of charge q. Note that we have to impose a coordinate system to solve actual problems.

Notice that the calculation of the electric field makes no reference to the test charge. Thus, the physically useful approach is to calculate the electric field using 3.5.2 and then use it to calculate the force on some test charge later, if needed. Different test charges experience different forces Equation 3.5.1. That being said, recall that there is no fundamental difference between a test charge and a source charge; these are merely convenient labels for the system of interest. Any charge produces an electric field; however, just as Earth's orbit is not affected by Earth's own gravity, a charge is not subject to a force due to the electric field it generates. Charges are only subject to forces from the electric fields of other charges.

In this respect, the electric field $\vec{E}$ of a point charge is similar to the gravitational field $\vec{g}$ of Earth; once we have calculated the gravitational field at some point in space, we can use it any time we want to calculate the resulting force on any mass we choose to place at that point. In fact, this is exactly what we do when we say the gravitational field of Earth (near Earth's surface) has a value of $9.81\,m/s^2$ and then we calculate the resulting force (i.e., weight) on different masses. Also, the general expression for calculating $\vec{g}$ at arbitrary distances from the center of Earth (i.e., not just near Earth's surface) is very similar to the expression for E:

$$\vec{g} = G\frac{M}{r^2}\hat{r}$$

where G is a proportionality constant, playing the same role for $\vec{g}$ as $k = \dfrac{1}{4\pi\epsilon_0}$ does for $\vec{E}$. The value of $\vec{g}$ is calculated once and is then used in an endless number of problems.

To push the analogy further, notice the units of the electric field: From $\vec{F} = Q\vec{E}$, the units of $\vec{E}$ are newtons per coulomb, N/C, that is, the electric field applies a force on each unit charge. Now notice the units of $\vec{g}$: From $\vec{\omega} = m\vec{g}$ the units of $\vec{g}$ are newtons per kilogram, N/kg, that is, the gravitational field applies a force on each unit mass. We could say that the gravitational field of Earth, near Earth's surface, has a value of 9.81 N/kg.

The Meaning of "Field

Recall from your studies of gravity that the word "field" in this context has a precise meaning. A field, in physics, is a physical quantity whose value depends on (is a function of) position, relative to the source of the field. In the case of the electric field, Equation 3.5.2 shows that the value of $\vec{E}$ (both the magnitude and the direction) depends on where in space the point P is located, measured from the locations $\vec{r}$ of the source charges q.

In addition, since the electric field is a vector quantity, the electric field is referred to as a vector field. (The gravitational field is also a vector field.) In contrast, a field that has only a magnitude at every point is a **scalar field.** The temperature in a room is an example of a scalar field. It is a field because the temperature, in general, is different at different locations in the room, and it is a scalar field because temperature is a scalar quantity.

Also, as you did with the gravitational field of an object with mass, you should picture the electric field of a charge-bearing object (the source charge) as a continuous, immaterial substance that surrounds the source charge, filling all of space—in principle, to $\pm\infty$ in all directions. The field exists at every physical point in space. To put it another way, the electric charge on an object alters the space around the charged object in such a way that all other electrically charged objects in space experience an electric force as a result of being in that field. The electric field, then, is the mechanism by which the electric properties of the source charge are transmitted to and through the rest of the universe. (Again, the range of the electric force is infinite.)

We will see in subsequent chapters that the speed at which electrical phenomena travel is the same as the speed of light. There is a deep connection between the electric field and light.

Superposition

Yet another experimental fact about the field is that it obeys the superposition principle. In this context, that means that we can (in principle) calculate the total electric field of many source charges by calculating the electric field of only q_1 at position **P**, then calculate the field of q_2 at **P**, while—and this is the crucial idea—ignoring the field of, and indeed even the existence of, q_1. We can repeat this process, calculating the field of each individual source charge, independently of the existence of any of the other charges. The total electric field, then, is the vector sum of all these fields.

In the next section, we describe how to determine the shape of an electric field of a source charge distribution and how to sketch it.

The Direction of the Field

Equation 3.5.2 enables us to determine the magnitude of the electric field, but we need the direction also. We use the convention that the direction of any electric field vector is the same as the direction of the electric force vector that the field would apply to a positive test charge placed in that field. Such a charge would be repelled by positive source charges (the force on it would point away from the positive source charge) but attracted to negative charges (the force points toward the negative source).

Direction of the Electric Field

By convention, all electric fields $\vec{E}$ point away from positive source charges and point toward negative source charges.

Example 3.5.1: The E-field of an Atom

In an ionized helium atom, the most probable distance between the nucleus and the electron is $r = 26.5 \times 10^{-12} m$. What is the electric field due to the nucleus at the location of the electron?

Strategy

Note that although the electron is mentioned, it is not used in any calculation. The problem asks for an electric field, not a force; hence, there is only one charge involved, and the problem specifically asks for the field due to the nucleus. Thus, the electron is a red herring; only its distance matters. Also, since the distance between the two protons in the nucleus is much, much smaller than the distance of the electron from the nucleus, we can treat the two protons as a single charge +2**e** (Figure 3.5.2).

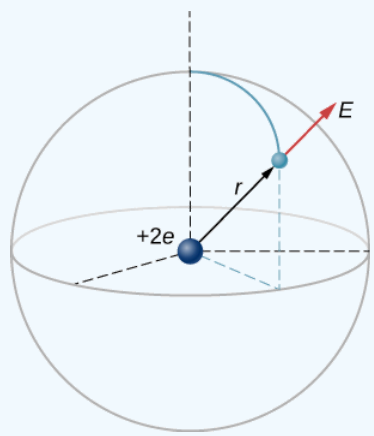

Figure 3.5.2: A schematic representation of a helium atom. Again, helium physically looks nothing like this, but this sort of diagram is helpful for calculating the electric field of the nucleus.

Solution

The electric field is calculated by

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}.$$

Here, $q = 2e = 2(1.6 \times 10^{-19}\,C)$ (since there are two protons) and **r** is given; substituting gives

$$\begin{aligned}\vec{E} &= \frac{1}{4\pi\left(8.85 \times 10^{-12}\dfrac{C^2}{N\cdot m^2}\right)}\frac{2(1.6\times 10^{-19} C)}{26.5\times 10^{-12} m)^2}\hat{r} \\ &= 4.1\times 10^{12}\frac{N}{C}\hat{r}.\end{aligned}$$

The direction of $\vec{E}$ is radially away from the nucleus in all directions. Why? Because a positive test charge placed in this field would accelerate radially away from the nucleus (since it is also positively charged), and again, the convention is that the direction of the electric field vector is defined in terms of the direction of the force it would apply to positive test charges.

The electric field is thus seen to depend only on the charge Q and the distance r; it is completely independent of the test charge$q$.

✓ Example 3.5.2: Calculating the Electric Field of a Point Charge

Calculate the strength and direction of the electric field E due to a point charge of 2.00 nC (nano-Coulombs) at a distance of 5.00 mm from the charge.

Strategy

We can find the electric field created by a point charge by using the equation $E = kQ/r^2$.

Solution

Here $Q = 2.00 \times 10^{-9} C$ and $r = 5.00 \times 10^{-3} m$. Entering those values into the above equation gives

$$\begin{aligned} E &= k\frac{Q}{r^2} \\ &= (8.99 \times 10^9 N \cdot m^2/C^2) \times \frac{(2.00 \times 10^{-9} C)}{(5.00 \times 10^{-3} m)^2} \\ &= 7.19 \times 10^5 N/C. \end{aligned}$$

Discussion

This **electric field strength** is the same at any point 5.00 mm away from the charge Q that creates the field. It is positive, meaning that it has a direction pointing away from the charge Q.

✓ Example 3.5.3: Calculating the Force Exerted on a Point Charge by an Electric Field

What force does the electric field found in the previous example exert on a point charge of$-0.250\mu C$?

Strategy

Since we know the electric field strength and the charge in the field, the force on that charge can be calculated using the definition of electric field $\mathbf{E} = \mathbf{F}/q$ rearranged to $\mathbf{F} = q\mathbf{E}$.

Solution

The magnitude of the force on a charge $q = -.250\mu C$ exerted by a field of strength $E = 7.20 \times 10^5 N/C$ is thus,

$$\begin{aligned} F &= -qE \\ &= (0.250 \times 10^{-6} C)(7.20 \times 10^5 N/C) \\ &= 0.180 N. \end{aligned}$$

Because q is negative, the force is directed opposite to the direction of the field.

Discussion

The force is attractive, as expected for unlike charges. (The field was created by a positive charge and here acts on a negative charge.) The charges in this example are typical of common static electricity, and the modest attractive force obtained is similar to forces experienced in static cling and similar situations.

Field due to Many Charges

Suppose we have N source charges $q_1, q_2, q_3, \dots q_N$ located at positions $vecr_1, \vec{r}_2, \vec{r}_3, \dots \vec{r}_N$, the electric field due to all charges is the vector sum of the electric fields due to each of the charges:

$$\vec{E} \equiv \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \frac{q_3}{r_3^2}\hat{r}_3 + \dots + \frac{q_N}{r_1^2}\hat{r}_N\right) \tag{3.5.3}$$

or, more compactly,

$$\vec{E}(P) \equiv \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{r}_i. \tag{3.5.4}$$

This expression is called the electric field at position $P = P(x, y, z)$ of the N source charges. Here, P is the location of the point in space where you are calculating the field and is relative to the positions $\vec{r}_i$ of the source charges (Figure $3.5.1$). Note that we have to impose a coordinate system to solve actual problems.

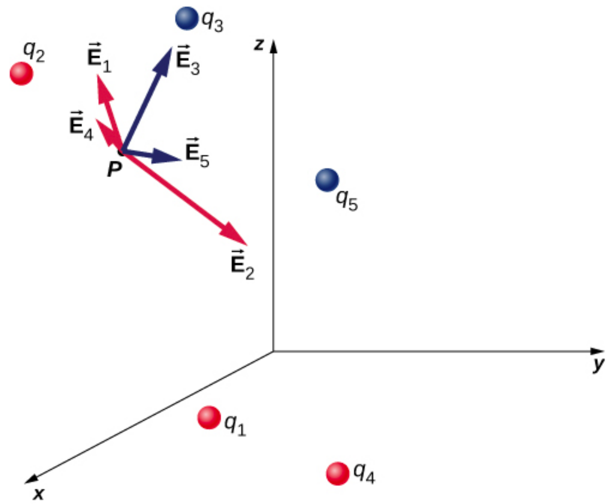

Figure $3.5.3$: Each of these five source charges creates its own electric field at every point in space; shown here are the field vectors at an arbitrary point P. Like the electric force, the net electric field obeys the superposition principle.

Notice that the calculation of the electric field makes no reference to the test charge. Thus, the physically useful approach is to calculate the electric field and then use it to calculate the force on some test charge later, if needed. Different test charges experience different forces Equation $3.5.1$, but it is the same electric field Equation $3.5.3$. That being said, recall that there is no fundamental difference between a test charge and a source charge; these are merely convenient labels for the system of interest. Any charge produces an electric field; however, just as Earth's orbit is not affected by Earth's own gravity, a charge is not subject to a force due to the electric field it generates. Charges are only subject to forces from the electric fields of other charges.

Example $3.5.4$: Adding Electric Fields

Find the magnitude and direction of the total electric field due to the two point charges, q_1 and q_2, at the origin of the coordinate system .

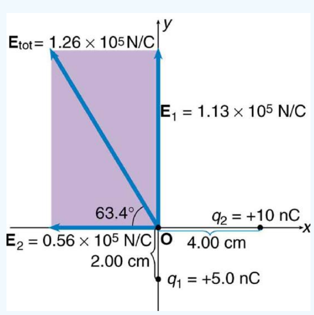

Figure $3.5.4$: The electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ at the origin O add to $\mathbf{E}_{tot}$.

Strategy

Since the electric field is a vector (having magnitude and direction), we add electric fields with the same vector techniques used for other types of vectors. We first must find the electric field due to each charge at the point of interest, which is the origin of the coordinate system (O) in this instance. We pretend that there is a positive test charge, q, at point O, which allows us to determine the direction of the fields $\mathbf{E}_1$ and $\mathbf{E}_2$. Once those fields are found, the total field can be determined using **vector addition**.

Solution

The electric field strength at the origin due to q_1 is labeled E_1 and is calculated:

$$E_1 = k\frac{q_1}{r_1^2} = (8.99 \times 10^9 N \cdot m^2/C^2)\frac{(5.00 \times 10^{-9} C)}{(2.00 \times 10^{-2} m)^2} \tag{3.5.5}$$

$$E_1 = 1.124 \times 10^5 N/C. \tag{3.5.6}$$

Similarly, E_2 is

$$E_2 = k\frac{q_2}{r_2^2} = (8.99 \times 10^9 N \cdot m^2/C^2)\frac{(10.0 \times 10^{-9} C)}{(4.00 \times 10^{-2} m)^2} \tag{3.5.7}$$

$$E_2 = 0.5619 \times 10^5 N/C. \tag{3.5.8}$$

Four digits have been retained in this solution to illustrate that E_1 is exactly twice the magnitude of E_2. Now arrows are drawn to represent the magnitudes and directions of $\mathbf{E}_1$ and $\mathbf{E}_2$. (Figure $3.5.3$) The direction of the electric field is that of the force on a positive charge so both arrows point directly away from the positive charges that create them. The arrow for $\mathbf{E}_1$ is exactly twice the length of that for $\mathbf{E}_2$. The arrows form a right triangle in this case and can be added using the Pythagorean theorem. The magnitude of the total field E_{tot} is

$$E_{tot} = (E_1^2 + E_2^2)^{1/2} \tag{3.5.9}$$

$$= [(1.124 \times 10^5 N/C)^2 + (0.5619 \times 10^5 N/C)^2]^{1/2} \tag{3.5.10}$$

$$= 1.26 \times 10^5 N/C. \tag{3.5.11}$$

The direction is

$$\theta = \tan^{-1}(\frac{E_1}{E_2}) \tag{3.5.12}$$

$$= \tan^{-1}(\frac{1.124 \times 10^5 N/C}{0.5619 \times 10^5 N/C}) \tag{3.5.13}$$

$$= 63.4^\circ, \tag{3.5.14}$$

or 63.4° above the *x*-axis.

Discussion

In cases where the electric field vectors to be added are not perpendicular, vector components or graphical techniques can be used. The total electric field found in this example is the total electric field at only one point in space. To find the total electric field due to these two charges over an entire region, the same technique must be repeated for each point in the region. This impossibly lengthy task (there are an infinite number of points in space) can be avoided by calculating the total field at representative points and using some of the unifying features noted next.

✓ Example $3.5.5$: The E-Field above Two Equal Charges

a. Find the electric field (magnitude and direction) a distance **z** above the midpoint between two equal charges $+q$ that are a distance **d** apart (Figure $3.5.3$). Check that your result is consistent with what you'd expect when $z \gg d$.
b. The same as part (a), only this time make the right-hand charge $-q$ instead of $+q$.

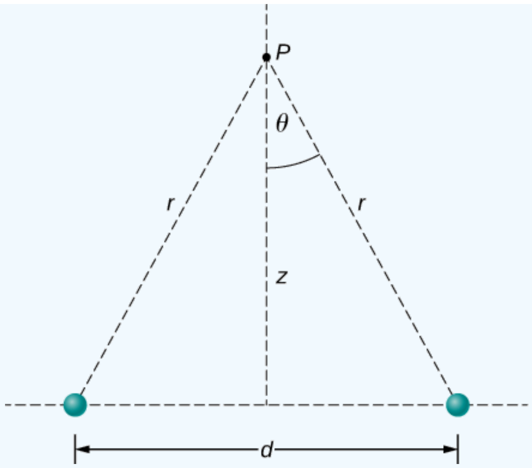

Figure 3.5.5: Finding the field of two identical source charges at the point P. Due to the symmetry, the net field at \(**P**\) is entirely vertical. (Notice that this is **not** true away from the midline between the charges.)

Strategy

We add the two fields as vectors, per Equation 3.5.3. Notice that the system (and therefore the field) is symmetrical about the vertical axis; as a result, the horizontal components of the field vectors cancel. This simplifies the math. Also, we take care to express our final answer in terms of only quantities that are given in the original statement of the problem: q, z, d, and constants (π, ϵ_0).

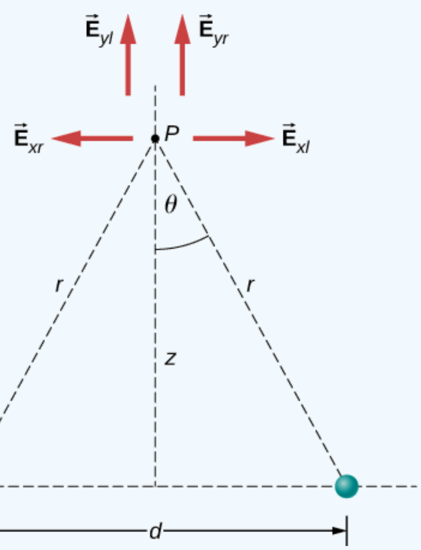

Figure 3.5.6. **Note that the horizontal components of the electric fields from the two charges cancel each other out, while the vertical components add together.**

Solution

a. By symmetry, the horizontal (**x**)-components of $\vec{E}$ cancel (Figure 3.5.4);

$$\begin{aligned} E_x &= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\sin\theta - \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\sin\theta \\ &= 0. \end{aligned}$$

The vertical (**z**)-component is given by

$$\begin{aligned} E_z &= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\cos\theta + \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\cos\theta \\ &= \frac{1}{4\pi\epsilon_0}\frac{2q}{r^2}\cos\theta. \end{aligned}$$

Since none of the other components survive, this is the entire electric field, and it points in the $\hat{k}$ direction. Notice that this calculation uses the principle of superposition; we calculate the fields of the two charges independently and then add them together.

What we want to do now is replace the quantities in this expression that we don't know (such as r), or can't easily measure (such as $cos\,\theta$ with quantities that we do know, or can measure. In this case, by geometry,

$$r^2 = z^2 + \left(\frac{d}{2}\right)^2$$

and

$$\cos\,\theta = \frac{z}{R} = \frac{z}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{1/2}}.$$

Thus, substituting,

$$E_z = \frac{1}{4\pi\epsilon_0}\frac{2q}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^2}\frac{z}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{1/2}}.$$

Simplifying, the desired answer is

$$E_z = \frac{1}{4\pi\epsilon_0}\frac{2qz}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}}. \tag{3.5.15}$$

b. If the source charges are equal and opposite, the vertical components cancel because

$$E_z = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,cos\,\theta - \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\cos\,\theta = 0$$

and we get, for the horizontal component of $\vec{E}$.

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{qd}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}}. \tag{3.5.16}$$

Summary

- The electrostatic force field surrounding a charged object extends out into space in all directions.
- The electrostatic force exerted by a point charge on a test charge at a distance$r$ depends on the charge of both charges, as well as the distance between the two.
- The electric field $\mathbf{E}$ is defined to be $\mathbf{E} = \frac{\mathbf{F}}{q}$, where $\mathbf{F}$ is the Coulomb or electrostatic force exerted on a small positive test charge q. $\mathbf{E}$ has units of N/C.
- The magnitude of the electric field $\mathbf{E}$ created by a point charge Q is $\mathbf{E} = k\frac{|Q|}{r^2}$. where r is the distance from Q. The electric field $\mathbf{E}$ is a vector and fields due to multiple charges add like vectors.

Glossary

field

a map of the amount and direction of a force acting on other objects, extending out into space

point charge

A charged particle, designated Q, generating an electric field

test charge

A particle (designated q) with either a positive or negative charge set down within an electric field generated by a point charge

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