

## 4.3: Electric Potential in a Uniform Electric Field

### Learning Objectives

By the end of this section, you will be able to:

- Describe the relationship between voltage and electric field.
- Derive an expression for the electric potential and electric field.
- Calculate electric field strength given distance and voltage.

In the previous section, we explored the relationship between voltage and energy. In this section, we will explore the relationship between voltage and electric field. For example, a uniform electric field  $\mathbf{E}$  is produced by placing a potential difference (or voltage)  $\Delta V$  across two parallel metal plates, labeled A and B. (Figure 4.3.1) Examining this will tell us what voltage is needed to produce a certain electric field strength; it will also reveal a more fundamental relationship between electric potential and electric field. From a physicist's point of view, either  $\Delta V$  or  $\mathbf{E}$  can be used to describe any charge distribution.  $\Delta V$  is most closely tied to energy, whereas  $\mathbf{E}$  is most closely related to force.  $\Delta V$  is a **scalar** quantity and has no direction, while  $\mathbf{E}$  is a **vector** quantity, having both magnitude and direction. (Note that the magnitude of the electric field strength, a scalar quantity, is represented by  $E$  below.) The relationship between  $\Delta V$  and  $\mathbf{E}$  is revealed by calculating the work done by the force in moving a charge from point A to point B. But, as noted in [Electric Potential Energy: Potential Difference](#), this is complex for arbitrary charge distributions, requiring calculus. We therefore look at a uniform electric field as an interesting special case.

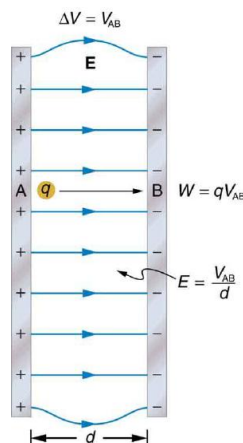


Figure 4.3.1: The relationship between  $V$  and  $E$  for parallel conducting plates is  $E = V/d$ . (Note that  $\Delta V = V_{AB}$  in magnitude. For a charge that is moved from plate A at higher potential to plate B at lower potential, a minus sign needs to be included as follows:  $-\Delta V = V_A - V_B = V_{AB}$ . See the text for details.)

The work done by the electric field in Figure 4.3.1 to move a positive charge  $q$  from A, the positive plate, higher potential, to B, the negative plate, lower potential, is

$$W = -\Delta PE = -q\Delta V \quad (4.3.1)$$

The potential difference between points A and B is

$$-\Delta V = -(V_B - V_A) = V_A - V_B = V_{AB}. \quad (4.3.2)$$

Entering this into the expression for work yields

$$W = qV_{AB}. \quad (4.3.3)$$

Work is  $W = Fd \cos \theta$ ; here  $\cos \theta = 1$ , since the path is parallel to the field, and so  $W = Fd$ . Since  $F = qE$ , we see that  $W = qEd$ . Substituting this expression for work into the previous equation gives

$$qEd = qV_{AB}. \quad (4.3.4)$$

The charge cancels, and so the voltage between points A and B is seen to be

$$\left. \begin{aligned} V_{AB} &= Ed \\ E &= \frac{V_{AB}}{d} \end{aligned} \right\} \text{(uniform } E \text{ -field only),}$$

where  $d$  is the distance from A to B, or the distance between the plates in Figure 4.3.1. Note that the above equation implies the units for electric field are volts per meter. We already know the units for electric field are newtons per coulomb; thus the following relation among units is valid:

$$1\text{N/C} = 1\text{V/m} \quad (4.3.5)$$

#### VOLTAGE BETWEEN POINTS A AND B

$$\left. \begin{aligned} V_{AB} &= Ed \\ E &= \frac{V_{AB}}{d} \end{aligned} \right\} \text{(uniform } E \text{ -field only),}$$

where  $d$  is the distance from A to B, or the distance between the plates.

#### ✓ Example 4.3.1: What Is the Highest Voltage Possible between Two Plates?

Dry air will support a maximum electric field strength of about  $3.0 \times 10^6 \text{ V/m}$ . Above that value, the field creates enough ionization in the air to make the air a conductor. This allows a discharge or spark that reduces the field. What, then, is the maximum voltage between two parallel conducting plates separated by 2.5 cm of dry air?

##### Strategy

We are given the maximum electric field  $E$  between the plates and the distance  $d$  between them. The equation  $V_{AB} = Ed$  can thus be used to calculate the maximum voltage.

##### Solution

The potential difference or voltage between the plates is

$$V_{AB} = Ed.$$

Entering the given values for  $E$  and  $d$  gives

$$V_{AB} = (3.0 \times 10^6 \text{ V/m})(0.025\text{m}) = 7.5 \times 10^4 \text{ V}$$

or

$$V_{AB} = 75\text{kV}.$$

(The answer is quoted to only two digits, since the maximum field strength is approximate.)

##### Discussion

One of the implications of this result is that it takes about 75 kV to make a spark jump across a 2.5 cm (1 in.) gap, or 150 kV for a 5 cm spark. This limits the voltages that can exist between conductors, perhaps on a power transmission line. A smaller voltage will cause a spark if there are points on the surface, since points create greater fields than smooth surfaces. Humid air breaks down at a lower field strength, meaning that a smaller voltage will make a spark jump through humid air. The largest voltages can be built up, say with static electricity, on dry days.



Figure 4.3.2: A spark chamber is used to trace the paths of high-energy particles. Ionization created by the particles as they pass through the gas between the plates allows a spark to jump. The sparks are perpendicular to the plates, following electric field lines between them. The potential difference between adjacent plates is not high enough to cause sparks without the ionization produced by particles from accelerator experiments (or cosmic rays). (credit: Daderot, Wikimedia Commons)

### ✓ Example 4.3.2: Field and Force inside an Electron Gun

- An electron gun has parallel plates separated by 4.00 cm and gives electrons 25.0 keV of energy. What is the electric field strength between the plates?
- What force would this field exert on a piece of plastic with a  $0.500\mu\text{C}$  charge that gets between the plates?

#### Strategy

Since the voltage and plate separation are given, the electric field strength can be calculated directly from the expression  $E = \frac{V_{AB}}{d}$ . Once the electric field strength is known, the force on a charge is found using  $\mathbf{F} = q\mathbf{E}$ . Since the electric field is in only one direction, we can write this equation in terms of the magnitudes,  $F = qE$ .

#### Solution(a)

The expression for the magnitude of the electric field between two uniform metal plates is

$$E = \frac{V_{AB}}{d}.$$

Since the electron is a single charge and is given 25.0 keV of energy, the potential difference must be 25.0 kV. Entering this value for  $V_{AB}$  and the plate separation of 0.0400 m, we obtain

$$E = \frac{25.0\text{kV}}{0.0400\text{m}} = 6.25 \times 10^5 \text{V/m}.$$

#### Solution(b)

The magnitude of the force on a charge in an electric field is obtained from the equation

$$F = qE.$$

Substituting known values gives

$$F = (0.500 \times 10^{-6} \text{C})(6.25 \times 10^5 \text{V/m}) = 0.313 \text{N}.$$

#### Discussion

Note that the units are newtons, since  $1\text{V/m} = 1\text{N/C}$ . The force on the charge is the same no matter where the charge is located between the plates. This is because the electric field is uniform between the plates.

In more general situations, regardless of whether the electric field is uniform, it points in the direction of decreasing potential, because the force on a positive charge is in the direction of  $\mathbf{E}$  and also in the direction of lower potential  $V$ . Furthermore, the magnitude of  $\mathbf{E}$  equals the rate of decrease of  $V$  with distance. The faster  $V$  decreases over distance, the greater the electric field. In equation form, the general relationship between voltage and electric field is

$$E = -\frac{\Delta V}{\Delta s}, \quad (4.3.6)$$

where  $\Delta s$  is the distance over which the change in potential,  $\Delta V$ , takes place. The minus sign tells us that **E** points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

#### RELATIONSHIP BETWEEN VOLTAGE AND ELECTRIC FIELD

In equation form, the general relationship between voltage and electric field is

$$E = -\frac{\Delta V}{\Delta s}, \quad (4.3.7)$$

where  $\Delta s$  is the distance over which the change in potential,  $\Delta V$ , takes place. The minus sign tells us that **E** points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

For continually changing potentials,  $\Delta V$  and  $\Delta s$  become infinitesimals and differential calculus must be employed to determine the electric field.

### Summary

- The voltage between points A and B is

$$\left. \begin{aligned} V_{AB} &= Ed \\ E &= \frac{V_{AB}}{d} \end{aligned} \right\} \text{(uniform } E \text{ -field only),}$$

where  $d$  is the distance from A to B, or the distance between the plates.

- In equation form, the general relationship between voltage and electric field is

$$E = -\frac{\Delta V}{\Delta s}, \quad (4.3.8)$$

where  $\Delta s$  is the distance over which the change in potential,  $\Delta V$ , takes place. The minus sign tells us that **E** points in the direction of decreasing potential.) The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

### Glossary

#### scalar

physical quantity with magnitude but no direction

#### vector

physical quantity with both magnitude and direction

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