1.7: How to Solve Problems in this Course

Learning Objectives

- Describe the process for developing a problem-solving strategy.
- Explain how to find the numerical solution to a problem.
- Summarize the process for assessing the significance of the numerical solution to a problem.

Problem-solving skills are clearly essential to success in a quantitative course in physics. More important, the ability to apply broad physical principles—usually represented by equations—to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in this text and for applying physics in everyday life.

As you are probably well aware, a certain amount of creativity and insight is required to solve problems. No rigid procedure works every time. Creativity and insight grow with experience. With practice, the basics of problem solving become almost automatic. One way to get practice is to work out the text's examples for yourself as you read. Another is to work as many end-of-section problems as possible, starting with the easiest to build confidence and then progressing to the more difficult. After you become involved in physics, you will see it all around you, and you can begin to apply it to situations you encounter outside the classroom, just as is done in many of the applications in this text.

Although there is no simple step-by-step method that works for every problem, the following processes facilitates problem solving and make it more meaningful. Two approaches are provided:

Physics Approach

For this approach, a three-stage process is usedl. The three stages are strategy, solution, and significance. This process is used in examples throughout the book. Here, we look at each stage of the process in turn.

1.7.1 Strategy

Strategy is the beginning stage of solving a problem. The idea is to figure out exactly what the problem is and then develop a strategy for solving it. Some general advice for this stage is as follows:

- **Examine the situation to determine which physical principles are involved**. It often helps to **draw a simple sketch** at the outset. You often need to decide which direction is positive and note that on your sketch. When you have identified the physical principles, it is much easier to find and apply the equations representing those principles. Although finding the correct equation is essential, keep in mind that equations represent physical principles, laws of nature, and relationships among physical quantities. Without a conceptual understanding of a problem, a numerical solution is meaningless.
- **Make a list of what is given or can be inferred from the problem as stated (identify the "knowns")**. Many problems are stated very succinctly and require some inspection to determine what is known. Drawing a sketch be very useful at this point as well. Formally identifying the knowns is of particular importance in applying physics to real-world situations. For example, the word stopped means the velocity is zero at that instant. Also, we can often take initial time and position as zero by the appropriate choice of coordinate system.
- **Identify exactly what needs to be determined in the problem (identify the unknowns).** In complex problems, especially, it is not always obvious what needs to be found or in what sequence. Making a list can help identify the unknowns.
- **Determine which physical principles can help you solve the problem**. Since physical principles tend to be expressed in the form of mathematical equations, a list of knowns and unknowns can help here. It is easiest if you can find equations that contain only one unknown—that is, all the other variables are known—so you can solve for the unknown easily. If the equation contains more than one unknown, then additional equations are needed to solve the problem. In some problems, several unknowns must be determined to get at the one needed most. In such problems it is especially important to keep physical principles in mind to avoid going astray in a sea of equations. You may have to use two (or more) different equations to get the final answer.

1.7.2 Solution

The solution stage is when you do the math. **Substitute the knowns (along with their units) into the appropriate equation and obtain numerical solutions complete with units**. That is, do the algebra, calculus, geometry, or arithmetic necessary to find the unknown from the knowns, being sure to carry the units through the calculations. This step is clearly important because it produces the numerical answer, along with its units. Notice, however, that this stage is only one-third of the overall problem-solving process.

1.7.3 Significance

After having done the math in the solution stage of problem solving, it is tempting to think you are done. But, always remember that physics is not math. Rather, in doing physics, we use mathematics as a tool to help us understand nature. So, after you obtain a numerical answer, you should always assess its significance:

- **Check your units**. If the units of the answer are incorrect, then an error has been made and you should go back over your previous steps to find it. One way to find the mistake is to check all the equations you derived for dimensional consistency. However, be warned that correct units do not guarantee the numerical part of the answer is also correct.
- **Check the answer to see whether it is reasonable. Does it make sense?** This step is extremely important: –the goal of physics is to describe nature accurately. To determine whether the answer is reasonable, check both its magnitude and its sign, in addition to its units. The magnitude should be consistent with a rough estimate of what it should be. It should also compare reasonably with magnitudes of other quantities of the same type. The sign usually tells you about direction and should be consistent with your prior expectations. Your judgment will improve as you solve more physics problems, and it will become possible for you to make finer judgments regarding whether nature is described adequately by the answer to a problem. This step brings the problem back to its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than just being able to solve a problem mechanically.
- **Check to see whether the answer tells you something interesting. What does it mean?** This is the flip side of the question: Does it make sense? Ultimately, physics is about understanding nature, and we solve physics problems to learn a little something about how nature operates. Therefore, assuming the answer does make sense, you should always take a moment to see if it tells you something about the world that you find interesting. Even if the answer to this particular problem is not very interesting to you, what about the method you used to solve it? Could the method be adapted to answer a question that you do find interesting? In many ways, it is in answering questions such as these science that progresses.

Engineering Approach

The engineering approach is not fundamentally different from the approach described above. In other words, either approach works.

Engineering problem solving is based on the study of models that describe real systems. In every case, the real system must be modeled by making simplifying assumptions before any mathematical or empirical analysis can be performed. Realistic and useful answers can only be obtained if the modeling assumptions "catch" the important features of the problem. The behavior of any model is constrained by the physical laws it incorporates and the modeling assumptions used in its development. Two different models for the same system may behave in entirely different ways. The engineers' job is to develop the "best" model for the problem at hand.

Because most mistakes are made in the process of developing the model it is essential that you learn to solve problems in a methodical fashion that documents your solution process including your modeling assumptions. Engineering calculations are part of the archival record of any engineering project and are frequently referred to years after the original work is completed. Many a junior engineer begins a new job by reviewing engineering calculations performed by others.

To help you develop your engineering problem solving skills, a multi-step process is proposed to help you (1) organize your thoughts, (2) document your solution, and (3) improve your ability to solve *new* problems. A summary of the steps is presented in Figure A-1. A sample problem showing the format can be found at the end of this appendix. As with any heuristic, this one does not guarantee a solution; however, its usefulness has been proven so frequently that we want you to use it in this course.

Figure A-1

SUMMARY OF PROBLEM SOLVING STEPS

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KNOWN: In your own words, state briefly what is known. (Step #1)
FIND: State concisely what you are trying to find. (Step #2)
GIVEN: Translate the problem word statement into sketches and symbolic notation. All pertinent information given explicitly in the problem statement should be listed here. (Step #3)
ANALYSIS: Develop a model and solve for desired information.

- Develop a strategy. (STRATEGY) (Step #4)
- Make modeling assumptions. (Clearly identified.) (Step #5)
- Develop and solve the model. (Step #6)
  - Develop symbolic solutions.
  - Calculate numerical values.
  - Check the reasonableness of your answers.

COMMENT: Discuss your results. (Step #7)

A more detailed discussion of each step is presented in the following sections. (Based on material in *Fundamentals of Engineering Thermodynamics* by M. J. Moran and H. N. Shapiro, J. Wiley & Sons, Inc., New York, 1988.)

KNOWN: In your own words, state briefly what is known. Read the problem statement and think about what it says. Do not just blindly copy the problem statement over again or list every detail of the problem. Construct a short sentence that summarizes the situation.

FIND: State concisely what you are trying to find. (If you don't know what you are looking for, how do you know when you've found it?) Do not just copy (a)...., (b)...., etc. from the problem and do not assume that you must find things in the order implied in the problem statement.

GIVEN: Translate the word statement of the problem into engineering sketches and symbolic notation. When completed, you should be able to throw away the original problem statement because you have recorded all of the pertinent information.

Draw and label a sketch of the physical system or device. (If you cannot visualize the problem, you probably can't solve it!) If you anticipate using a conservation or accounting principle, identify the boundaries (control surfaces) of the system you select for your analysis and identify the interactions between this system and the surroundings, e.g. forces, work, mass flow, etc.

Define symbols for the important variables and parameters of the problem. Record the numerical values given for the important variables and parameters.

Label the diagram with all relevant information from the problem statement. This is where you record all of the information explicitly given in the problem statement.

Be especially wary of making implicit assumptions as you prepare this section. Recognize the difference between information that is given explicitly in the problem and your interpretation of the information.

ANALYSIS: It is in this section that an appropriate mathematical model is developed and used to find the desired information. As you prepare this section, carefully annotate your solution with words that describe what you are doing. This commentary is invaluable in exposing your thought processes and if need be in recreating it at a later time.

- **Develop a strategy.** Every solution should include some initial statements that reveal your plan for solving the problem. As a starting point, clearly state what you believe to be the physical laws or concepts that will be important in solving this problem. What's the property to be counted? What’s the appropriate system? What's the appropriate time period? What constitutive relationships may be required?
  Your initial strategy may not be the best approach or the only approach. It may not even be correct approach, but as you proceed through the analysis process your plans may change. As they do just document them.
  To stress the importance of consciously thinking about the problem, every analysis section should start with a brief subsection labeled STRATEGY.
- **Make modeling assumptions.** Every problem solution requires that you make modeling assumptions. These assumptions are based on the information given in the problem statement, your interpretation of the given information, and your understanding of the underlying phenomena. Every model begins with universally accepted natural laws, and the assumptions provide the traceable link between the fundamental laws and problem-specific model you have developed. *All assumptions should be*

clearly identified as they are applied. You should be able to give a logical reason for every modeling assumption you make. If you cannot, it probably is an incorrect assumption.
Some problem solving formats call for a separate section listing all assumptions before you begin your analysis. There are two problems with this approach. First, experience shows that it is often difficult to know exactly what assumptions to make until you are building the model. Secondly, separating the assumptions from their application in the model tends to hide how they influence the modeling process. If a summary list is desired, it should be prepared after the analysis is completed.

- **Develop symbolic solutions.** Symbolic solutions are critical in engineering analysis and should always be developed and examined before you insert numerical values. The physics is in the symbolic solution, not the numerical answer. If the symbolic solution is incorrect, there's no hope for the numbers. If possible, solve an equation for the unknown quantity and isolate it on one side of the equal sign. *It is desirable to work with symbolic equations as long as possible before substituting in numbers for many reasons.* Symbolic solutions are especially useful when you are looking for errors, for solving parametric problems where certain parameters change, and are much easier to modify as your model develops. Look for groups of terms or ways to rearrange you symbolic answer that simplify the equation and allow you to check for dimensional consistency. Groups of terms with physical meaning or logical intermediate values should be assigned a unique symbol. Numerical values for these intermediate answers can then be calculated and checked separately.
- **Calculate numerical values.** Examine your symbolic solution and see if it makes sense. Once you are satisfied with the symbolic solution, substitute in the numbers and calculate the numerical answer. It is good practice to identify the source, e.g. table, chart, or book, of all numerical data used in the solution, especially if it is not common knowledge. It is also good practice to calculate intermediate or partial numerical answers when you are faced with a very long computation or complicated equation. This prevents calculator errors from creeping into a problem and gives you an opportunity to check the answers against your physical intuition.
- **Check the reasonableness of your answers.** Once you have a numerical answer, consider the magnitude and sign of all values and decide whether they are reasonable. One way to do this is to compare your answer against the results of a simpler model or models that would be expected to bracket your answer. Try different units for the answer, say gallons per minute instead of liters per second, to match your experience.

As you prepare the analysis, *do not waste time recopying the solution over again if you reach a dead end or make a mistake*. Just cross out the error, clearly identify the mistake, and keep going. Textbook examples and professors' notes give the mistaken impression that problem solving is a linear process that follows a single path with no mistakes and no side trips. Everyone makes mistakes, takes unexpected side trips, and forgets to make an important assumption.

Successful problem solvers acknowledge these diversions and learn from them. You should never start a problem more than once; however, your solution may take several turns before you are satisfied with the answer. The record of your journey is important. Don't "clean up" the solution. Clean up your standard problem solving method because a sloppy solution is usually the result of sloppy thinking. Get in the habit of attacking every problem in the same way. Scrap paper is meant for doodles, not engineering calculations.

COMMENTS: Discuss your results briefly. Comment on what you learned, identify key aspects of the solution, and indicate how your model might be improved by changing assumptions. Consciously check the validity of your answer by considering simpler models. Don't wait for someone else (like your boss or instructor) to find an error in your work by performing a five-minute "back-of-the-envelope" calculation you could have performed before submitting your answer.