

## 2.2: Geometrical Shapes

### ✓ Learning Objectives

- know what a polygon is
- know what perimeter is and how to find it
- know what the circumference, diameter, and radius of a circle is and how to find each one
- know the meaning of the symbol  $\pi$  and its approximating value
- know what a formula is and four versions of the circumference formula of a circle
- know the meaning and notation for area
- know the area formulas for some common geometric figures
- be able to find the areas of some common geometric figures
- know the meaning and notation for volume
- know the volume formulas for some common geometric objects
- be able to find the volume of some common geometric objects

### Polygons

We can make use of conversion skills with denominate numbers to make measurements of geometric figures such as rectangles, triangles, and circles. To make these measurements we need to be familiar with several definitions.

#### Definition: Polygon

A **polygon** is a closed plane (flat) figure whose sides are line segments (portions of straight lines).

#### Polygons



#### Not polygons



### Perimeter

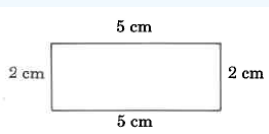
#### Definition: Perimeter

The **perimeter** of a polygon is the distance around the polygon.

To find the perimeter of a polygon, we simply add up the lengths of all the sides.

### ✓ Sample Set A

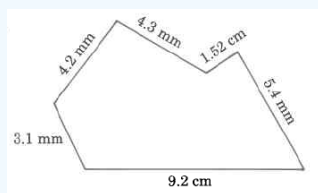
Find the perimeter of each polygon.



#### Solution

$$\begin{aligned}\text{Perimeter} &= 2 \text{ cm} + 5 \text{ cm} + 2 \text{ cm} + 5 \text{ cm} \\ &= 14 \text{ cm}\end{aligned}$$

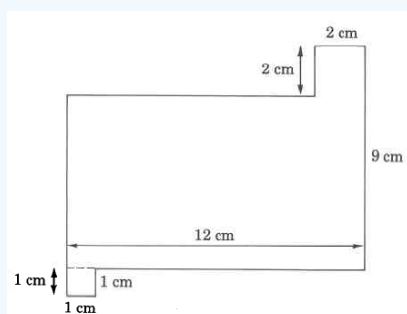
### ✓ Sample Set A



#### Solution

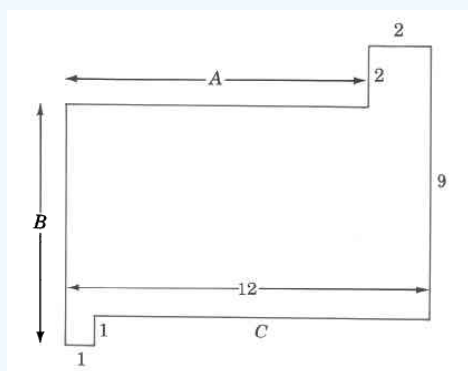
$$\begin{array}{rcl}
 \text{Perimeter} & = & 3.1 \text{ mm} \\
 & & 4.2 \text{ mm} \\
 & & 4.3 \text{ mm} \\
 & & 1.52 \text{ mm} \\
 & & 5.4 \text{ mm} \\
 & + & 9.2 \text{ mm} \\
 \hline
 & = & 27.72 \text{ mm}
 \end{array}$$

### ✓ Sample Set A



#### Solution

Our first observation is that three of the dimensions are missing. However, we can determine the missing measurements using the following process. Let A, B, and C represent the missing measurements. Visualize



$$A = 12\text{m} - 2\text{m} = 10\text{m}$$

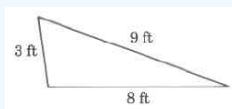
$$B = 9\text{m} + 1\text{m} - 2\text{m} = 8\text{m}$$

$$C = 12\text{m} - 1\text{m} = 11\text{m}$$

$$\begin{array}{rcl}
 \text{Perimeter} & = & 8 \text{ m} \\
 & & 10 \text{ m} \\
 & & 2 \text{ m} \\
 & & 2 \text{ m} \\
 & & 9 \text{ m} \\
 & & 11 \text{ m} \\
 & & 1 \text{ m} \\
 & & \underline{+ 1 \text{ m}} \\
 & & 44 \text{ m}
 \end{array}$$

### Practice Set A

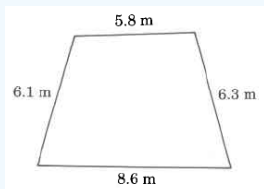
Find the perimeter of each polygon.



**Answer**

20 ft

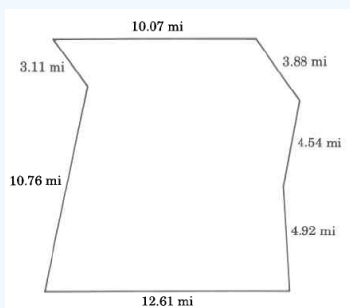
### Practice Set A



**Answer**

26.8 m

### Practice Set A



**Answer**

49.89 mi

## Circumference/Diameter/Radius

### Diameter (d)

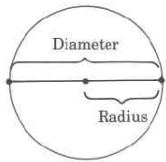
A **diameter** of a circle is any line segment that passes through the center of the circle and has its endpoints on the circle.

### Radius (r)

A **radius** of a circle is any line segment having as its endpoints the center of the circle and a point on the circle. The radius is one half the diameter.

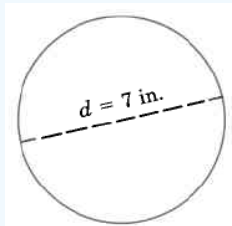
### Circumference (C)

The **circumference** of a circle is the distance around the circle. It is given by  $C = \pi d = 2\pi r$



#### ✓ Sample Set B

Find the circumference of the circle.



#### Solution

Use the formula  $C = \pi d$ .

$$C = \pi \cdot 7 \text{ in.}$$

By commutativity of multiplication,

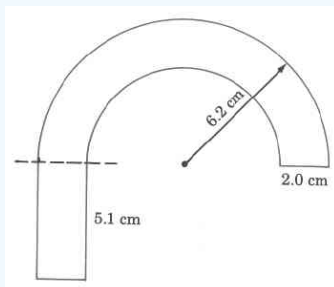
$$C = 7 \text{ in.} \cdot \pi$$

$$C = 7\pi \text{ in., exactly}$$

This result is exact since  $\pi$  has not been approximated.

#### ✓ Sample Set B

Find the perimeter of the figure.



#### Solution

We notice that we have two semicircles (half circles).

The larger radius is 6.2 cm.

The smaller radius is  $6.2 \text{ cm} - 2.0 \text{ cm} = 4.2 \text{ cm}$ .

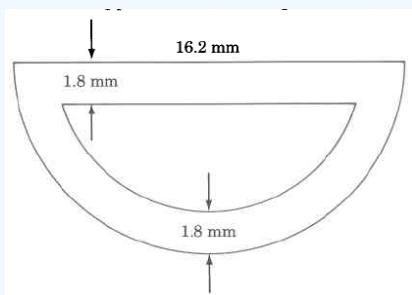
The width of the bottom part of the rectangle is 2.0 cm.

$$\begin{aligned}
 \text{Perimeter} &= 2.0 \text{ cm} \\
 &5.1 \text{ cm} \\
 &2.0 \text{ cm} \\
 &5.1 \text{ cm} \\
 &+ \frac{(0.5) \cdot (2) \cdot (3.14) \cdot (6.2 \text{ cm})}{2} && \text{Circumference of outer semicircle.} \\
 &\quad + \frac{(0.5) \cdot (2) \cdot (3.14) \cdot (4.2 \text{ cm})}{2} && \text{Circumference of inner semicircle.} \\
 &6.2 \text{ cm} - 2.0 \text{ cm} = 4.2 \text{ cm} \\
 &\text{The 0.5 appears because we want the} \\
 &\text{perimeter of only half a circle.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Perimeter} &\approx 2.0 \text{ cm} \\
 &5.1 \text{ cm} \\
 &2.0 \text{ cm} \\
 &5.1 \text{ cm} \\
 &19.468 \text{ cm} \\
 &\underline{+13.188 \text{ cm}} \\
 &48.856 \text{ cm}
 \end{aligned}$$

### Practice Set B

Find the outside perimeter of



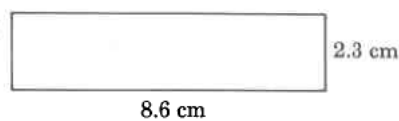
**Answer**

41.634 mm

### Exercises

Find each perimeter or approximate circumference. Use  $\pi = 3.14$ .

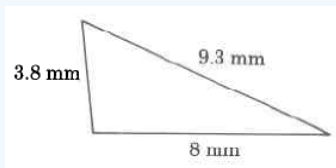
#### Exercise 2.2.1



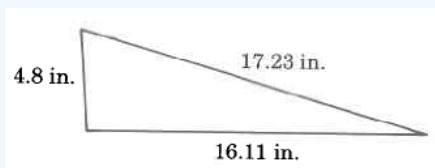
**Answer**

21.8 cm

### Exercise 2.2.2



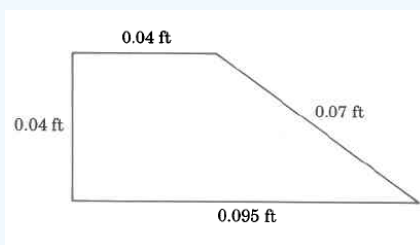
### Exercise 2.2.3



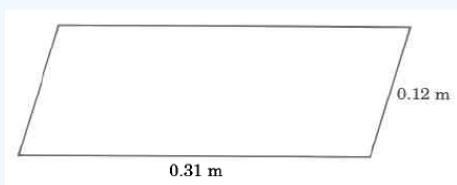
**Answer**

38.14 inches

### Exercise 2.2.4



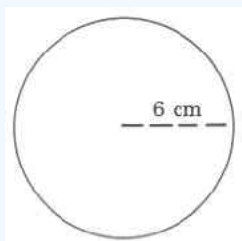
### Exercise 2.2.5



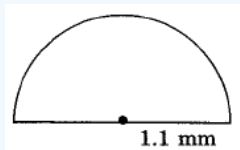
**Answer**

0.86 m

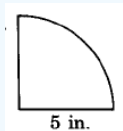
### Exercise 2.2.6



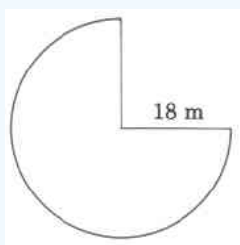
### Exercise 2.2.10



### Exercise 2.2.12



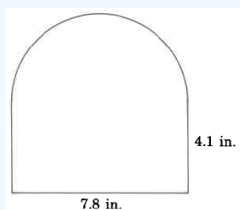
### Exercise 2.2.13



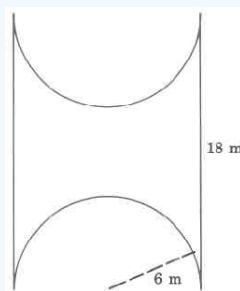
**Answer**

120.78 m

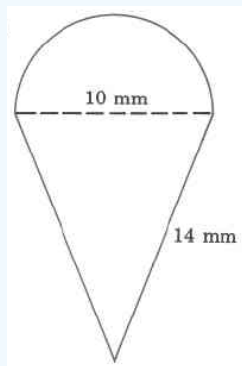
### Exercise 2.2.14



### Exercise 2.2.16



### Exercise 2.2.17



**Answer**

43.7 mm

## The Meaning and Notation for Area

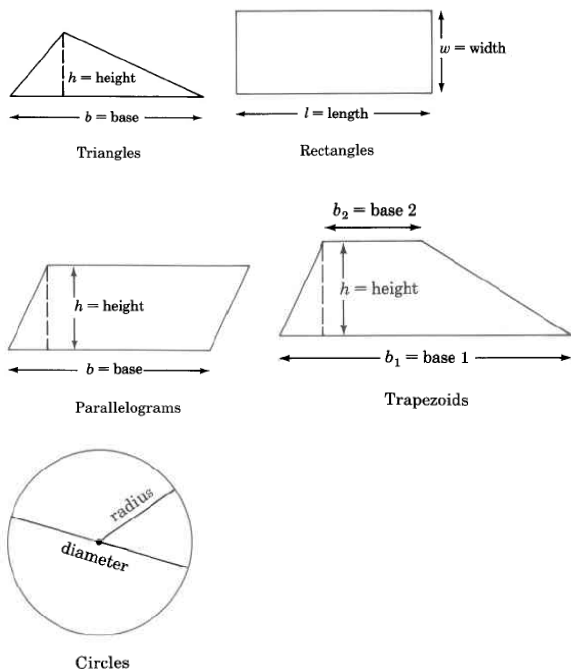
The product  $(\text{length unit}) \cdot (\text{length unit}) = (\text{length unit})^2$ , or, square length unit (sq length unit), can be interpreted physically as the *area* of a surface.

### Area

The **area** of a surface is the amount of square length units contained in the surface.

For example, 3 sq in. means that 3 squares, 1 inch on each side, can be placed precisely on some surface. (The squares may have to be cut and rearranged so they match the shape of the surface.)

We will examine the area of the following geometric figures.


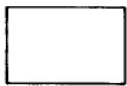


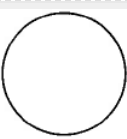


## Area Formulas

We can determine the areas of these geometric figures using the following formulas.

	Figure	Area Formula	Statement

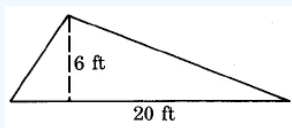


	Triangle	$A_T = \frac{1}{2} \cdot b \cdot h$	Area of a triangle is one half the base times the height.
	Rectangle	$A_R = l \cdot w$	Area of a rectangle is the length times the width.
	Parallelogram	$A_P = b \cdot h$	Area of a parallelogram is base times the height.
	Trapezoid	$A_{Trap} = \frac{1}{2} \cdot (b_1 + b_2) \cdot h$	Area of a trapezoid is one half the sum of the two bases times the height.
	Circle	$A_c = \pi r^2$	Area of a circle is $\pi$ times the square of the radius.

## Finding Areas of Some Common Geometric Figures

### ✓ Sample Set A

Find the area of the triangle.



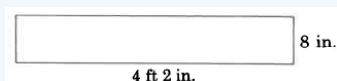
**Solution**

$$\begin{aligned}
 A_T &= \frac{1}{2} \cdot b \cdot h \\
 &= \frac{1}{2} \cdot 20 \cdot 6 \text{ sq ft} \\
 &= 10 \cdot 6 \text{ sq ft} \\
 &= 60 \text{ sq ft} \\
 &= 60 \text{ ft}^2
 \end{aligned}$$

The area of this triangle is 60 sq ft, which is often written as  $60 \text{ ft}^2$ .

### ✓ Sample Set A

Find the area of the rectangle.



**Solution**

Let's first convert 4 ft 2 in. to inches. Since we wish to convert to inches, we'll use the unit fraction  $\frac{12 \text{ in.}}{1 \text{ ft}}$  since it has inches in the numerator. Then,

$$\begin{aligned}
 4 \text{ ft} &= \frac{4 \text{ ft}}{1} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} \\
 &= \frac{4 \cancel{\text{ ft}}}{1} \cdot \frac{12 \text{ in.}}{1 \cancel{\text{ ft}}} \\
 &= 48 \text{ in.}
 \end{aligned}$$

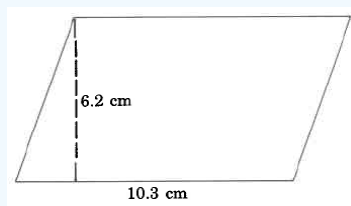
Thus,  $4 \text{ ft } 2 \text{ in.} = 48 \text{ in.} + 2 \text{ in.} = 50 \text{ in.}$

$$\begin{aligned}
 A_R &= l \cdot w \\
 &= 50 \text{ in.} \cdot 8 \text{ in.} \\
 &= 400 \text{ sq in.}
 \end{aligned}$$

The area of this rectangle is 400 sq in.

### ✓ Sample Set A

Find the area of the parallelogram.



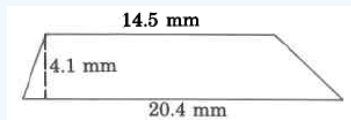
#### Solution

$$\begin{aligned}
 A_P &= b \cdot h \\
 &= 10.3 \text{ cm} \cdot 6.2 \text{ cm} \\
 &= 63.86 \text{ sq cm}
 \end{aligned}$$

The area of this parallelogram is 63.86 sq cm.

### ✓ Sample Set A

Find the area of the trapezoid.



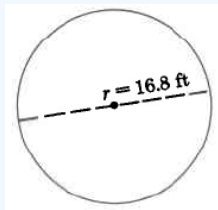
#### Solution

$$\begin{aligned}
 A_{Trap} &= \frac{1}{2} \cdot (b_1 + b_2) \cdot h \\
 &= \frac{1}{2} \cdot (14.5 \text{ mm} + 20.4 \text{ mm}) \cdot (4.1 \text{ mm}) \\
 &= \frac{1}{2} \cdot (34.9 \text{ mm}) \cdot (4.1 \text{ mm}) \\
 &= \frac{1}{2} \cdot (143.09 \text{ sq mm}) \\
 &= 71.545 \text{ sq mm}
 \end{aligned}$$

The area of this trapezoid is 71.545 sq mm.

### ✓ Sample Set A

Find the approximate area of the circle.



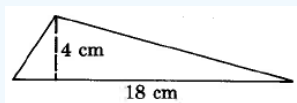
#### Solution

$$\begin{aligned} A_c &= \pi \cdot r^2 \\ &\approx (3.14) \cdot (16.8 \text{ ft})^2 \\ &\approx (3.14) \cdot (282.24 \text{ sq ft}) \\ &\approx 888.23 \text{ sq ft} \end{aligned}$$

The area of this circle is approximately 888.23 sq ft.

### Practice Set A

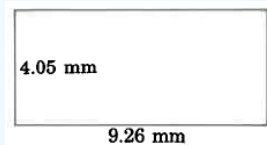
Find the area of each of the following geometric figures.



#### Answer

36 sq cm

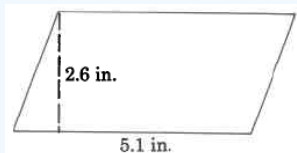
### Practice Set A



#### Answer

37.503 sq mm

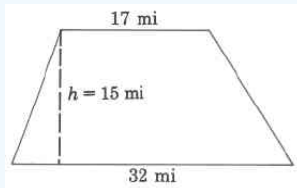
### Practice Set A



#### Answer

13.26 sq in.

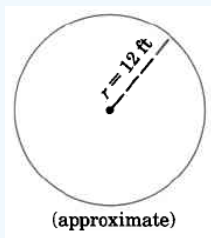
### Practice Set A



**Answer**

367.5 sq mi

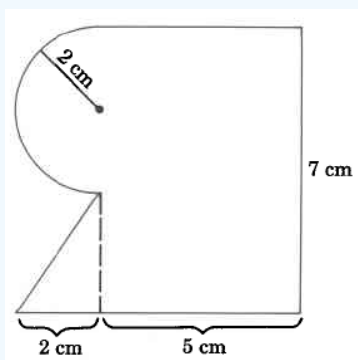
### Practice Set A



**Answer**

452.16 sq ft

### Practice Set A



**Answer**

44.28 sq cm

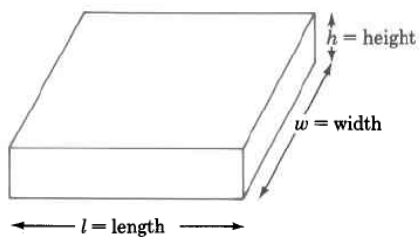
## The Meaning and Notation for Volume

The product (length unit)(length unit)(length unit) = (length unit)<sup>3</sup>, or cubic length unit (cu length unit), can be interpreted physically as the *volume* of a three-dimensional object.

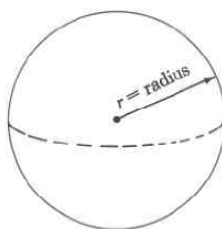
### Volume

The **volume** of an object is the amount of cubic length units contained in the object.

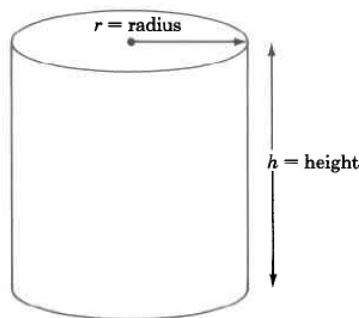
For example, 4 cu mm means that 4 cubes, 1 mm on each side, would precisely fill some three-dimensional object. (The cubes may have to be cut and rearranged so they match the shape of the object.)



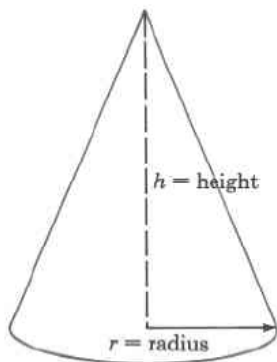
Rectangular solid



Sphere

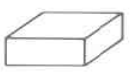

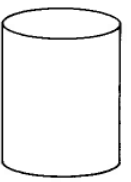



Cylinder



Cone

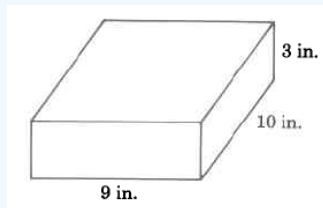
## Volume Formulas

	Figure	Volume Formula	Statement
	Rectangular solid	$V_R = l \cdot w \cdot h$ $= (\text{area of base}) \cdot (\text{height})$	The volume of a rectangular solid is the length times the width times the height.
	Sphere	$V_s = \frac{4}{3} \cdot \pi \cdot r^3$	The volume of a sphere is $\frac{4}{3}$ times $\pi$ times the cube of the radius.
	Cylinder	$V_{Cyl} = \pi \cdot r^2 \cdot h$ $= (\text{area of base}) \cdot (\text{height})$	The volume of a cylinder is $\pi$ times the square of the radius times the height.
	Cone	$V_c = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$ $= (\text{area of base}) \cdot (\text{height})$	The volume of a cone is $\frac{1}{3}$ times $\pi$ times the square of the radius times the height.

## Finding Volumes of Some Common Geometric Objects

### ✓ Sample Set B

Find the volume of the rectangular solid.



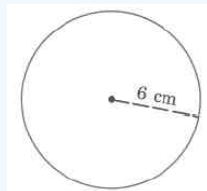
#### Solution

$$\begin{aligned} V_R &= l \cdot w \cdot h \\ &= 9 \text{ in.} \cdot 10 \text{ in.} \cdot 3 \text{ in.} \\ &= 270 \text{ cu in.} \\ &= 270 \text{ in.}^3 \end{aligned}$$

The volume of this rectangular solid is 270 cu in.

### ✓ Sample Set B

Find the approximate volume of the sphere.



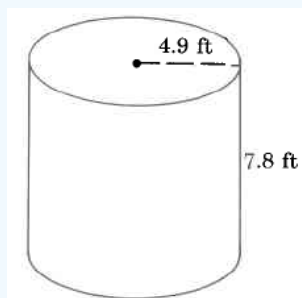
#### Solution

$$\begin{aligned} V_S &= \frac{4}{3} \cdot \pi \cdot r^3 \\ &\approx \left(\frac{4}{3}\right) \cdot (3.14) \cdot (6 \text{ cm})^3 \\ &\approx \left(\frac{4}{3}\right) \cdot (3.14) \cdot (216 \text{ cu cm}) \\ &\approx 904.32 \text{ cu cm} \end{aligned}$$

The approximate volume of this sphere is 904.32 cu cm, which is often written as 904.32 cm<sup>3</sup>.

### ✓ Sample Set B

Find the approximate volume of the cylinder.



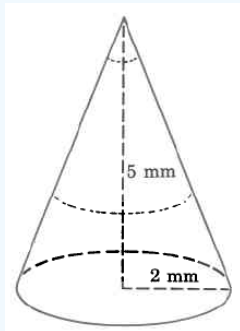
### Solution

$$\begin{aligned}
 V_{Cyl} &= \pi \cdot r^2 \cdot h \\
 &\approx (3.14) \cdot (4.9 \text{ ft})^2 \cdot (7.8 \text{ ft}) \\
 &\approx (3.14) \cdot (24.01 \text{ sq ft}) \cdot (7.8 \text{ ft}) \\
 &\approx (3.14) \cdot (187.278 \text{ cu ft}) \\
 &\approx 588.05292 \text{ cu ft}
 \end{aligned}$$

The volume of this cylinder is approximately 588.05292 cu ft. The volume is approximate because we approximated  $\pi$  with 3.14.

### ✓ Sample Set B

Find the approximate volume of the cone. Round to two decimal places.



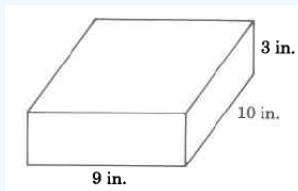
### Solution

$$\begin{aligned}
 V_c &= \frac{1}{3} \cdot \pi \cdot r^2 \cdot h \\
 &\approx \left(\frac{1}{3}\right) \cdot (3.14) \cdot (2 \text{ mm})^2 \cdot (5 \text{ mm}) \\
 &\approx \left(\frac{1}{3}\right) \cdot (3.14) \cdot (4 \text{ sq mm}) \cdot (5 \text{ mm}) \\
 &\approx \left(\frac{1}{3}\right) \cdot (3.14) \cdot (20 \text{ cu mm}) \\
 &\approx 20.9\bar{3} \text{ cu mm} \\
 &\approx 20.93 \text{ cu mm}
 \end{aligned}$$

The volume of this cone is approximately 20.93 cu mm. The volume is approximate because we approximated  $\pi$  with 3.14.

### Practice Set B

Find the volume of each geometric object. If  $\pi$  is required, approximate it with 3.14 and find the approximate volume.

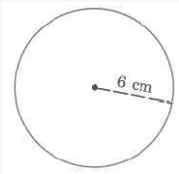


### Answer

21 cu in.

### Practice Set B

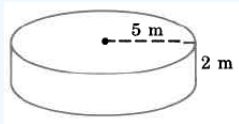
Sphere



**Answer**

904.32 cu ft

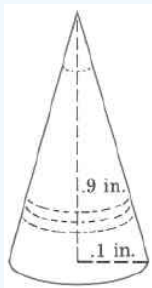
### Practice Set B



**Answer**

157 cu m

### Practice Set B



**Answer**

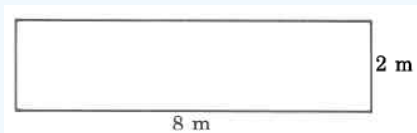
0.00942 cu in.

## Exercises

Find each indicated measurement.

### Exercise 2.2.1

Area



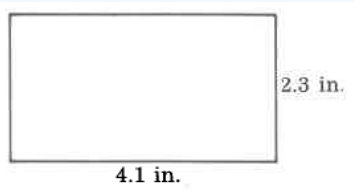
**Answer**

16 sq m



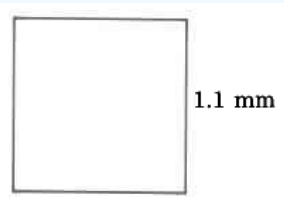
## Exercise 2.2.2

Area



## Exercise 2.2.3

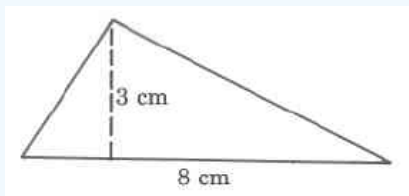
Area

**Answer**

1.21 sq mm

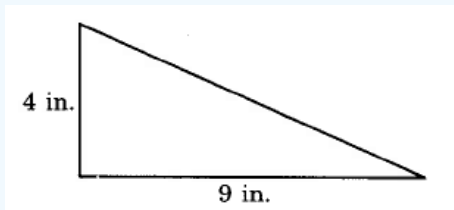
## Exercise 2.2.4

Area



## Exercise 2.2.5

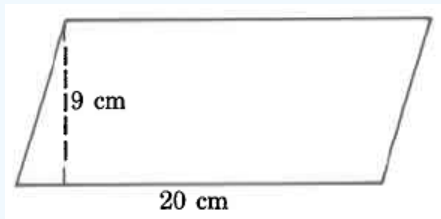
Area

**Answer**

18 sq in.

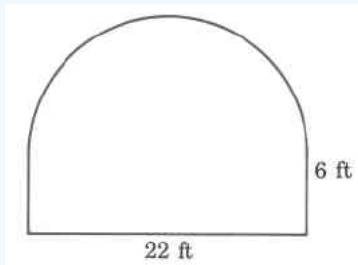
### Exercise 2.2.6

Area



### Exercise 2.2.7

Exact area

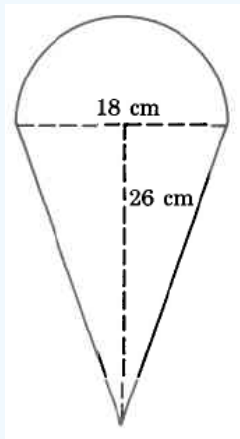


**Answer**

$$(60.5\pi + 132) \text{ sq ft}$$

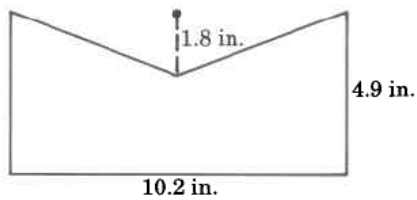
### Exercise 2.2.8

Approximate area



### Exercise 2.2.9

Area

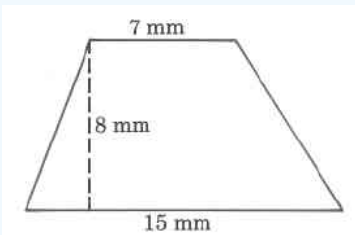


**Answer**

40.8 sq in.

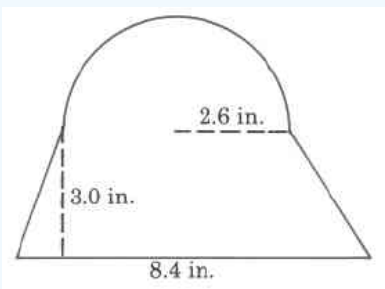
#### Exercise 2.2.10

Area



#### Exercise 2.2.11

Approximate area

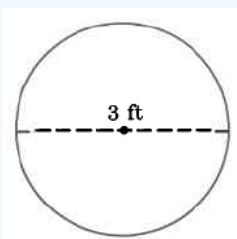


**Answer**

31.0132 sq in.

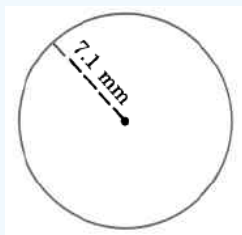
#### Exercise 2.2.12

Exact area



### Exercise 2.2.13

Approximate area

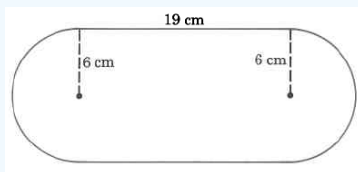


**Answer**

158.2874 sq mm

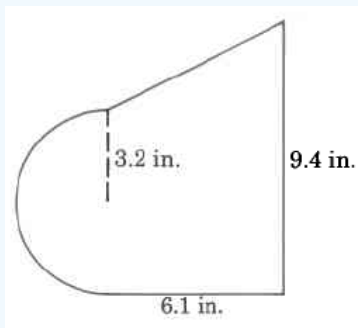
### Exercise 2.2.14

Exact area



### Exercise 2.2.15

Approximate area

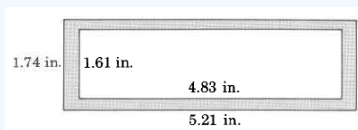


**Answer**

64.2668 sq in.

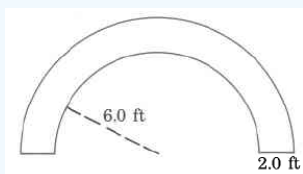
### Exercise 2.2.16

Area



**Exercise 2.2.17**

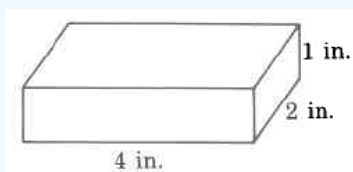
Approximate area

**Answer**

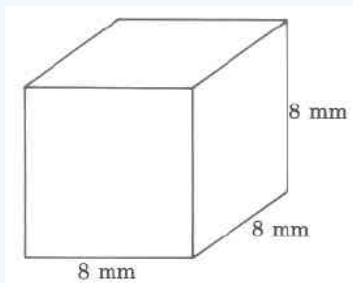
43.96 sq ft

**Exercise 2.2.18**

Volume

**Exercise 2.2.19**

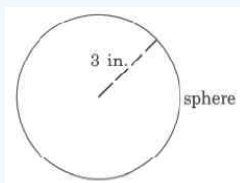
Volume

**Answer**

512 cu cm

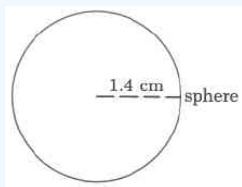
**Exercise 2.2.20**

Exact volume



### Exercise 2.2.21

Approximate volume

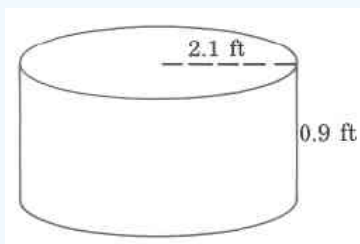


**Answer**

11.49 cu cm

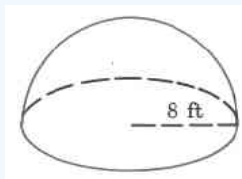
### Exercise 2.2.22

Approximate volume



### Exercise 2.2.23

Exact volume

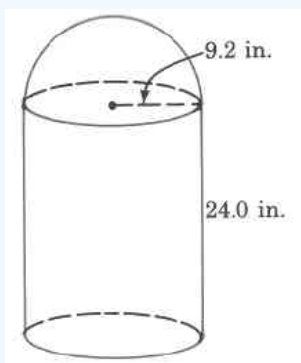


**Answer**

$\frac{1024}{3}\pi$  cu ft

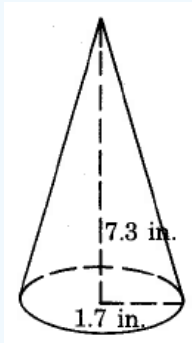
### Exercise 2.2.24

Approximate volume



**Exercise 2.2.25**

Approximate volume

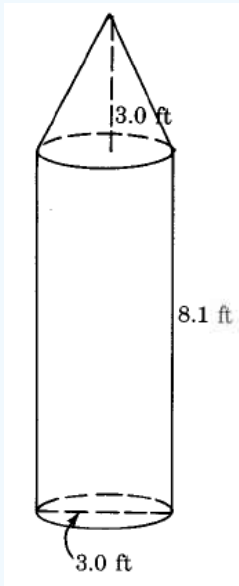


**Answer**

22.08 cu in.

**Exercise 2.2.26**

Approximate volume

**Exercises for Review****Exercise 2.2.27**

In the number 23,426, how many hundreds are there?

**Answer**

4

### Exercise 2.2.28

List all the factors of 32.

### Exercise 2.2.29

Find the value of  $4\frac{3}{4} - 3\frac{5}{6} + 1\frac{2}{3}$ .

**Answer**

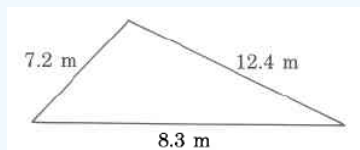
$$\frac{31}{12} = 2\frac{7}{12} = 2.58$$

### Exercise 2.2.30

Find the value of  $\frac{5 + \frac{1}{3}}{2 + \frac{2}{15}}$ .

### Exercise 2.2.31

Find the perimeter.



**Answer**

27.9m

Area (**A**) is measured in square units, perimeter (**P**) is measured in units, and circumference (**C**) is measured in units.

## Square

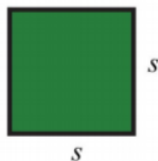


Figure 2.2.1

$$P = 4s \quad (2.2.1)$$

$$A = s^2 \quad (2.2.2)$$

## Rectangle



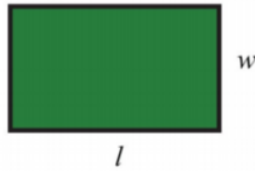


Figure 2.2.2

$$P = 2l + 2w \quad (2.2.3)$$

$$A = lw \quad (2.2.4)$$

## Parallelogram

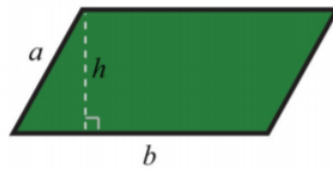


Figure 2.2.3

$$P = 2a + 2b \quad (2.2.5)$$

$$A = bh \quad (2.2.6)$$

## Trapezoid

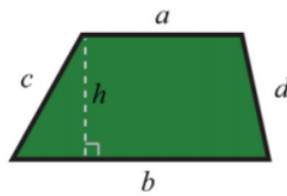


Figure 2.2.4

$$P = a + b + c + d \quad (2.2.7)$$

$$A = \frac{1}{2}h(a + b) \quad (2.2.8)$$

## Triangle

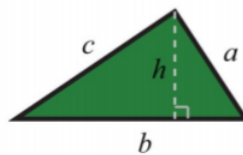


Figure 2.2.5

$$P = a + b + c \quad (2.2.9)$$

$$A = \frac{1}{2}bh \quad (2.2.10)$$

## Circle

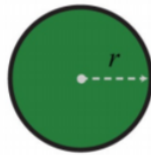


Figure 2.2.6

$$C = 2\pi r \quad (2.2.11)$$

$$A = \pi r^2 \quad (2.2.12)$$

Volume (**V**) is measured in cubic units and surface area (**SA**) is measured in square units.

## Cube

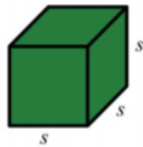


Figure 2.2.1

$$SA = 6s^2 \quad (2.2.13)$$

$$V = s^3 \quad (2.2.14)$$

## Rectangular Solid

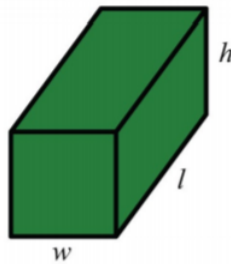


Figure 2.2.2

$$SA = 2lw + 2lh + 2wh \quad (2.2.15)$$

$$V = lwh \quad (2.2.16)$$

## Right Circular Cylinder

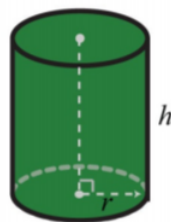


Figure 2.2.3

$$SA = 2\pi r^2 + 2\pi rh \quad (2.2.17)$$

$$V = \pi r^2 h \quad (2.2.18)$$

## Right Circular Cone



Figure 2.2.4

$$SA = \pi r^2 + \pi r s \quad (2.2.19)$$

$$V = \frac{1}{3} \pi r^2 h \quad (2.2.20)$$

## Sphere

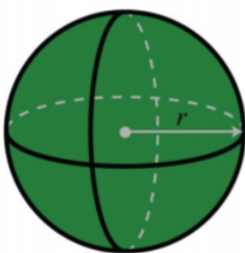


Figure 2.2.5

$$SA = 4\pi r^2 \quad (2.2.21)$$

$$V = \frac{4}{3} \pi r^3 \quad (2.2.22)$$

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