13.1: Appendix F- Mathematical Formulas

Quadratic formula

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Table E1 - Geometry

Triangle of base b and height h	Area = $\frac{1}{2}bh$	
Circle of radius r	Circumference = $2\pi r$	Area = πr^2
Sphere of radius r	Surface Area = $2\pi r^2$	Volume = $\frac{4}{3}\pi r^3$
Cylinder of radius r and height h	Area of curved surface = $2\pi rh$	Volume = $\pi r^2 h$

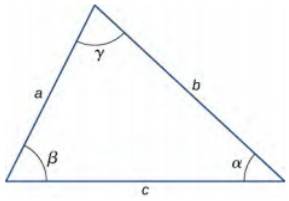
Trigonometry

Trigonometric Identities

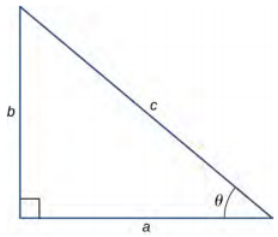
1. $\sin\theta = \frac{1}{\csc\theta}$
2. $\cos\theta = \frac{1}{\sec\theta}$
3. $\tan\theta = \frac{1}{\cot\theta}$
4. $\sin(90° - \theta) = \cos\theta$
5. $\cos(90° - \theta) = \sin\theta$
6. $\tan(90° - \theta) = \cot\theta$
7. $\sin^2\theta + \cos^2\theta = 1$
8. $\sec^2\theta - \tan^2\theta = 1$
9. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
10. $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
11. $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
12. $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$
13. $\sin 2\theta = 2\sin\theta\cos\theta$
14. $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
15. $\sin\alpha + \sin\beta = 2\sin\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$
16. $\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$

Triangles

1. Law of sines: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$
2. Law of cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$



3. Pythagorean theorem: $a^2 + b^2 = c^2$



Series expansions

1. Binomial theorem: $(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)a^{n-2}b^2}{2!} + \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} + \cdots$
2. $(1 \pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} \pm \cdots \quad (x^2 < 1)$
3. $(1 \pm x)^{-n} = 1 \mp \frac{nx}{1!} + \frac{n(n+1)x^2}{2!} \mp \cdots \quad (x^2 < 1)$
4. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$
5. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$
6. $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$
7. $e^x = 1 + x + \frac{x^2}{2!} + \cdots$
8. $\ln(1 + x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots \quad (|x| < 1)$

Derivatives

1. $\frac{d}{dx}[a\, f(x)] = a \frac{d}{dx} f(x)$
2. $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$
3. $\frac{d}{dx}[f(x)g(x)] = f(x) \frac{d}{dx} g(x) + g(x) \frac{d}{dx} f(x)$
4. $\frac{d}{dx} f(u) = [\frac{d}{du} f(u)] \frac{du}{dx}$
5. $\frac{d}{dx} x^m = mx^{m-1}$
6. $\frac{d}{dx} \sin x = \cos x$
7. $\frac{d}{dx} \cos x = -\sin x$
8. $\frac{d}{dx} \tan x = \sec^2 x$
9. $\frac{d}{dx} \cot x = -\csc^2 x$
10. $\frac{d}{dx} \sec x = \tan x \sec x$
11. $\frac{d}{dx} \csc x = -\cot x \csc x$
12. $\frac{d}{dx} e^x = e^x$
13. $\frac{d}{dx} \ln x = \frac{1}{x}$
14. $\frac{d}{dx} \sin^{-1} x = \frac{1}{1-x^2}$
15. $\frac{d}{dx} \cos^{-1} x = -\frac{1}{1-x^2}$
16. $\frac{d}{dx} \tan^{-1} x = -\frac{1}{1+x^2}$

Integrals

1. $\int a\, f(x)dx = a \int f(x)dx$
2. $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$
3. $\int x^m\, dx = \frac{x^{m+1}}{m+1}$ for $(m \neq -1) = \ln x$ for $(m = -1)$
4. $\int \sin x\, dx = -\cos x$
5. $\int \cos x\, dx = \sin x$
6. $\int \tan x\, dx = \ln|\sec x|$
7. $\int \sin^2 (ax)\, dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$
8. $\int \cos^2 (ax)\, dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$
9. $\int \sin (ax) \cos (ax)\, dx = -\frac{\cos 2ax}{4a}$
10. $\int e^{ax}\, dx = \frac{1}{a} e^{ax}$
11. $\int xe^{ax}\, dx = \frac{e^{ax}}{a^2}(ax - 1)$

12. $\int \ln ax \, dx = x \ln ax - x$
13. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$
14. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right|$
15. $\int \frac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1}\frac{x}{a}$
16. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$
17. $\int \sqrt{a^2+x^2}\ dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\sinh^{-1}\frac{x}{a}$
18. $\int \sqrt{a^2-x^2}\ dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$

Contributors and Attributions