1.5: Unit Conversion

Learning Objectives

- Use conversion factors to express the value of a given quantity in different units.

It is often necessary to convert from one unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters and you need to convert them to cups. Or perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you may need to convert units of feet or meters to miles.

1.5.1 Dimensional Homogeneity

All theoretically derived equations that describe physical phenomena must be dimensionally homogeneous. An equation is **dimensionally homogeneous** if the dimensions of both sides of the equation are the same and all additive terms have the same dimensions.

1.5.2 Converting Units

The most common errors in using units occur when converting a physical quantity from one set of units to another set. When you convert units you are not changing the size of the physical quantity, only the numerical value associated with the units in which it is measured.

The relationship between two units for the same dimension are typically found in a handbook as an **equivalence relation**, such as $1 \text{ ft} = 12 \text{ in}$. Note again that the unit symbols are mathematical entities and cannot be neglected.

The key to converting units is to recall that multiplying a mathematical expression by unity (1) does not change the magnitude of the mathematical expression. A **unit conversion factor** equals unity and can be constructed from an equivalence relation. Example B.1 shows how to convert equivalence statements into unit conversion factors.

Example 1.5.1

Convert the given equivalence relations into unit conversion factors.

$$\begin{aligned} 1 \text{ ft} = 12 \text{ in} \quad &\Rightarrow \quad 1 = 12 \, \frac{\text{in}}{\text{ft}} \\ 1 \text{ slug} = 32.174 \text{ lbm} \quad &\Rightarrow \quad 1 = 32.174 \, \frac{\text{lbm}}{\text{slug}} \\ 1 \text{ mol} = 0.001 \text{ kmol} \quad &\Rightarrow \quad 1 = 0.001 \, \frac{\text{kmol}}{\text{mol}} \\ 1\text{N} = 1 \, \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \quad &\Rightarrow \quad 1 = 1 \, \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \end{aligned}$$

The left-hand column shows the equivalence relations from a handbook and the right-hand column shows the resulting unit conversion factors. Notice how it would be mathematically incorrect to just drop the unit symbols.

The following Example illustrates how to perform a simple unit conversion for pressure, now that we have the unit conversion factors.

Example 1.5.2

Given a pressure of $13.0 \text{ lbf}/\text{in}^2$, convert the pressure to lbf/ft^2.

Answer

$$p = 13.0 \, \frac{\text{lbf}}{\text{in}^2} = 13.0 \, \frac{\text{lbf}}{\text{in}^2} \times \left(12 \, \frac{\text{in}}{\text{ft}}\right)^2 = 13.0 \times 12 \left(\frac{\text{lbf}}{\cancel{\text{in}^2}} \times \frac{\cancel{\text{in}^2}}{\text{ft}^2} \right) = 1872.0 \, \frac{\text{lbf}}{\text{ft}^2}$$

Now convert the pressure value to $\mathrm{N/m^2}$.

Answer

$$p = 13.0\ \frac{\mathrm{lbf}}{\mathrm{in}^2} \times \underbrace{\left(4.448\ \frac{\mathrm{N}}{\mathrm{lbf}}\right)}_{1} \times \underbrace{\left(\frac{1\ \mathrm{in}}{0.0254\ \mathrm{m}}\right)^2}_{1} = 89,627\ \frac{\mathrm{N}}{\mathrm{m}^2}$$

If done, correctly the intermediate units should cancel. Check this out by drawing lines through the units that cancel.

Let's consider a simple example of how to convert units. Suppose we want to convert 80 m to kilometers. The first thing to do is to list the units you have and the units to which you want to convert. In this case, we have units in meters and we want to convert to kilometers. Next, we need to determine a conversion factor relating meters to kilometers. A **conversion factor** is a ratio that expresses how many of one unit are equal to another unit. For example, there are 12 in. in 1 ft, 1609 m in 1 mi, 100 cm in 1 m, 60 s in 1 min, and so on. In this case, we know that there are 1000 m in 1 km. Now we can set up our unit conversion. We write the units we have and then multiply them by the conversion factor so the units cancel out, as shown:

$$80\ \cancel{m} \times \frac{1\ km}{1000\ \cancel{m}} = 0.080\ km. \tag{1.5.1}$$

Note that the unwanted meter unit cancels, leaving only the desired kilometer unit. You can use this method to convert between any type of unit. Now, the conversion of 80 m to kilometers is simply the use of a metric prefix, as we saw in the preceding section, so we can get the same answer just as easily by noting that

$$80\ m = 8.0 \times 10^1\ m = 8.0 \times 10^{-2}\ km = 0.080\ km, \tag{1.5.2}$$

since "kilo-" means 10^3 and 1 = −2 + 3. However, using conversion factors is handy when converting between units that are not metric or when converting between derived units, as the following examples illustrate.

Example 1.5.3: Converting Nonmetric Units to Metric

The distance from the university to home is 10 mi and it usually takes 20 min to drive this distance. Calculate the average speed in meters per second (m/s). (**Note**: Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units, then we can get the average speed into the desired units by picking the correct conversion factors and multiplying by them. The correct conversion factors are those that cancel the unwanted units and leave the desired units in their place. In this case, we want to convert miles to meters, so we need to know the fact that there are 1609 m in 1 mi. We also want to convert minutes to seconds, so we use the conversion of 60 s in 1 min.

Strategy

First we calculate the average speed using the given units, then we can get the average speed into the desired units by picking the correct conversion factors and multiplying by them. The correct conversion factors are those that cancel the unwanted units and leave the desired units in their place. In this case, we want to convert miles to meters, so we need to know the fact that there are 1609 m in 1 mi. We also want to convert minutes to seconds, so we use the conversion of 60 s in 1 min.

Solution

1. Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now. Average speed and other motion concepts are covered in later chapters.) In equation form, $Average\; speed = \frac{Distance}{Time} \ldotp \nonumber $
2. Substitute the given values for distance and time: $Average\; speed = \frac{10\; mi}{20\; min} = 0.50\; \frac{mi}{min} \ldotp \nonumber $
3. Convert miles per minute to meters per second by multiplying by the conversion factor that cancels miles and leave meters, and also by the conversion factor that cancels minutes and leave seconds: $0.50\; \frac{\cancel{mile}}{\cancel{min}} \times \frac{1609\; m}{1\; \cancel{mile}} \times \frac{\cancel{1\; min}}{60\; s} = \frac{(0.50)(1609)}{60}\; m/s = 13\;

m/s \ldotp \nonumber$

Significance

Check the answer in the following ways:

1. Be sure the units in the unit conversion cancel correctly. If the unit conversion factor was written upside down, the units do not cancel correctly in the equation. We see the “miles” in the numerator in 0.50 mi/min cancels the “mile” in the denominator in the first conversion factor. Also, the “min” in the denominator in 0.50 mi/min cancels the “min” in the numerator in the second conversion factor.
2. Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of meters per second and, after the cancelations, the only units left are a meter (m) in the numerator and a second (s) in the denominator, so we have indeed obtained these units.

? Exercise 1.5.1

Light travels about 9 Pm in a year. Given that a year is about 3 x 10^7 s, what is the speed of light in meters per second?

Answer

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✓ Example 1.5.4: Converting between Metric Units

The density of iron is 7.86 g/cm^3 under standard conditions. Convert this to kg/m^3.

Strategy

We need to convert grams to kilograms and cubic centimeters to cubic meters. The conversion factors we need are 1 kg = 10^3 g and 1 cm = 10^{-2}m. However, we are dealing with cubic centimeters (cm^3 = cm x cm x cm), so we have to use the second conversion factor three times (that is, we need to cube it). The idea is still to multiply by the conversion factors in such a way that they cancel the units we want to get rid of and introduce the units we want to keep.

Solution

$$7.86 \frac{\cancel{g}}{\cancel{cm^3}} \times \frac{kg}{10^3 \ \cancel{g}} \times \left(\frac{\cancel{cm}}{10^{-2} \ m} \right)^3 = \frac{7.86}{(10^3)(10^{-6})} \ kg/m^3 = 7.86 \times 10^3 \ kg/m^3$$

Significance

Remember, it’s always important to check the answer.

1. Be sure to cancel the units in the unit conversion correctly. We see that the gram (“g”) in the numerator in 7.86 g/cm^3 cancels the “g” in the denominator in the first conversion factor. Also, the three factors of “cm” in the denominator in 7.86 g/cm^3 cancel with the three factors of “cm” in the numerator that we get by cubing the second conversion factor.
2. Check that the units of the final answer are the desired units. The problem asked for us to convert to kilograms per cubic meter. After the cancelations just described, we see the only units we have left are “kg” in the numerator and three factors of “m” in the denominator (that is, one factor of “m” cubed, or “m^3”). Therefore, the units on the final answer are correct.

? Exercise 1.5.2

We know from Figure 1.4 that the diameter of Earth is on the order of 10^7 m, so the order of magnitude of its surface area is 10^{14} m^2. What is that in square kilometers (that is, km^2)? (Try doing this both by converting 10^7 m to km and then squaring it and then by converting 10^{14} m^2 directly to square kilometers. You should get the same answer both ways.)

Answer

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Unit conversions may not seem very interesting, but not doing them can be costly. One famous example of this situation was seen with the **Mars Climate Orbiter**. This probe was launched by NASA on December 11, 1998. On September 23, 1999, while attempting to guide the probe into its planned orbit around Mars, NASA lost contact with it. Subsequent investigations showed a piece of software called SM_FORCES (or “small forces”) was recording thruster performance data in the English units of pound-seconds (lb • s). However, other pieces of software that used these values for course corrections expected them to be recorded in the SI units of newton-seconds (N • s), as dictated in the software interface protocols. This error caused the probe to follow a very different trajectory from what NASA thought it was following, which most likely caused the probe either to burn up in the Martian atmosphere or to shoot out into space. This failure to pay attention to unit conversions cost hundreds of millions of dollars, not to mention all the time invested by the scientists and engineers who worked on the project.

Exercise 1.5.3

Given that 1 lb (pound) is 4.45 N, were the numbers being output by SM_FORCES too big or too small?

Example 1.5.5

A tank contains 15 mol of an ideal gas. The pressure in the tank is $1500\ \mathrm{kPa}$ and the volume of the tank is $10\ \mathrm{m}^3$. The ideal gas constant is $8.314\ \mathrm{kJ}/(\mathrm{kmol}\cdot\mathrm{K})$. Determine the temperature of the gas in the tank.

$$\text{The ideal gas equation is } pV = n\bar{R}T.$$

$$\text{We solve for } T = \frac{pV}{n\bar{R}}$$

$$\therefore T = \frac{pV}{n\bar{R}} = \frac{(1500\ \mathrm{kPa})\times(10\ \mathrm{m}^3)}{(15\ \mathrm{kmol})\times\left(8.314\ \dfrac{\mathrm{kJ}}{\mathrm{kmol}\cdot\mathrm{K}}\right)} = \frac{15000\ \mathrm{kPa}\cdot\mathrm{m}^3}{124.71\ \dfrac{\mathrm{kJ}}{\mathrm{K}}}\times\underbrace{\left[\frac{\mathrm{kJ}}{\mathrm{kPa}\cdot\mathrm{m}^3}\right]}_{1} = 120.3\ \mathrm{K}$$

Again, the check is to see if the appropriate units cancel out.

1.5.3 Weight and Mass

People frequently confuse the terms weight and mass. The weight of an object is the force exerted by the earth's gravitational field on the object. Mathematically, $W = mg$, where m is the mass of the object and g is the local gravitational field strength. The local gravitational field strength is also referred to as the local acceleration of gravity.

Standard values for the local gravitational field strength are

$$g = 9.80665\ \mathrm{N}/\mathrm{kg} = 32.174\ \mathrm{lbf}/\mathrm{slug} = 1.000\ \mathrm{lbf}/\mathrm{lbm}.$$

Standard values for the local acceleration of gravity are

$$g = 9.80665\ \mathrm{m}/\mathrm{s}^2 = 32.174\mathrm{ft}/\mathrm{s}^2.$$

TEST YOURSELF: Why do these two interpretations for g come up with similar numbers but different units?

Much of the confusion about mass and weight can be directly attributed to the fact that the mass and force units in the American Engineering System are both called "pounds." To eliminate this problem, it is highly recommended that you only talk about pound-force (lbf) or a pound-mass (lbm). You would never confuse a newton with a kilogram, but then they have different names. Unfortunately, you will still find "pound" and "lb" used frequently to mean both mass and weight. Always approach "pounds" with caution when doing calculations. Remember that the weight of an object is always a function of the local gravitational field strength, but its mass is independent of the gravitational field.