2.7.1: Solving Linera Equations

Learning Objectives

By the end of this section, you will be able to:

- Solve equations using a general strategy

It is time now to lay out one overall strategy that can be used to solve any linear equation. Some equations we solve will not require all these steps to solve, but many will.

Beginning by simplifying each side of the equation makes the remaining steps easier.

Exercise 2.7.1.1: How to Solve Linear Equations Using the General Strategy

Solve: $-6(x+3)=24$.

Answer

Step 1. Simplify each side of the equation as much as possible.	Use the Distributive Property. Notice that each side of the equation is simplified as much as possible.	$-6(x+3)=24$ $-6x-18=24$
Step 2. Collect all variable terms on one side of the equation.	Nothing to do – all *x*'s are on the left side.	
Step 3. Collect constant terms on the other side of the equation.	To get constants only on the right, add 18 to each side. Simplify.	$-6x-18+18=24+18$ $-6x=42$
Step 4. Make the coefficient of the variable term to equal to 1.	Divide each side by -6. Simplify.	$\frac{-6x}{-6}=\frac{42}{-6}$ $x=-7$
Step 5. Check the solution.	Let $x=-7$ Simplify. Multiply.	**Check:** $-6(x+3)=24$ $-6(-7+3)\stackrel{?}{=}24$ $-6(-4)\stackrel{?}{=}24$ $24=24\checkmark$

Exercise 2.7.1.2

Solve: $5(x+3)=35$

Answer

$x=4$

Exercise 2.7.1.3

Solve: $6(y-4)=-18$

Answer

$y=1$

GENERAL STRATEGY FOR SOLVING LINEAR EQUATIONS.

1. **Simplify each side of the equation as much as possible.**
   Use the Distributive Property to remove any parentheses.
   Combine like terms.
2. **Collect all the variable terms on one side of the equation.**
   Use the Addition or Subtraction Property of Equality.
3. **Collect all the constant terms on the other side of the equation.**
   Use the Addition or Subtraction Property of Equality.
4. **Make the coefficient of the variable term to equal to 1.**
   Use the Multiplication or Division Property of Equality.
   State the solution to the equation.
5. **Check the solution.** Substitute the solution into the original equation to make sure the result is a true statement.

Exercise 2.7.1.4

Solve: $-(y+9)=8$

Answer

		$-(y+9)=8$
Simplify each side of the equation as much as possible by distributing.		$-y-9=8$
The only y term is on the left side, so all variable terms are on the left side of the equation.		
Add 9 to both sides to get all constant terms on the right side of the equation.		$-y-9+9=8+9$
Simplify.		$-y=17$
Rewrite −y as −1y.		$-1y=17$
Make the coefficient of the variable term to equal to 1 by dividing both sides by −1.		$\frac{-1y}{-1}=\frac{17}{-1}$
Simplify.		$y=-17$
Check:	$-(y+9)=8$	
Let y=−17.	$-(-17+9)\stackrel{?}{=}8$	
	$-(-8)\stackrel{?}{=}8$	
	$8=8\checkmark$	

Exercise 2.7.1.5

Solve: $-(y+8)=-2$

Answer

$y=-6$

Exercise 2.7.1.6

Solve: $-(z+4)=-12$

Answer

$z=8$

Exercise 2.7.1.7

Solve: $5(a-3)+5=-10$

Answer

		$5(a-3)+5=-10$
Simplify each side of the equation as much as possible.		
Distribute.		$5a-15+5=-10$
Combine like terms.		$5a-10=-10$
The only a term is on the left side, so all variable terms are on one side of the equation.		
Add 10 to both sides to get all constant terms on the other side of the equation.		$5a-10+10=-10+10$
Simplify.		$5a=0$
Make the coefficient of the variable term to equal to 11 by dividing both sides by 55.		$\frac{5a}{5}=\frac{0}{5}$
Simplify.		$a=0$
Check:	$5(a-3)+5=-10$	
Let a=0.	$5(0-3)+5\stackrel{?}{=}-10$	
	$5(-3)+5\stackrel{?}{=}-10$	
	$-15+5\stackrel{?}{=}-10$	
	$-10=-10\checkmark$	

Exercise 2.7.1.8

Solve: $2(m-4)+3=-1$

Answer

$m=2$

Exercise 2.7.1.9

Solve:$7(n-3)-8=-15$

Answer

$n=2$

Exercise 2.7.1.10

Solve: $\frac{2}{3}(6m-3)=8-m$

Answer

		$\frac{2}{3}(6m-3)=8-m$
Distribute.		$4m-2=8-m$
Add m to get the variables only to the left.		$4m+m-2=8-m+m$
Simplify.		$5m-2=8$
Add 2 to get constants only on the right.		$5m-2+2=8+2$
Simplify.		$5m=10$
Divide by 5.		$\frac{5m}{5}=\frac{10}{5}$
Simplify.		$m=2$
Check:	$\frac{2}{3}(6m-3)=8-m$	
Let m=2.	$\frac{2}{3}(6\cdot 2-3)\stackrel{?}{=}8-2$	
	$\frac{2}{3}(12-3)\stackrel{?}{=}6$	
	$\frac{2}{3}(9)\stackrel{?}{=}6$	
	$6=6\checkmark$	

? Exercise 2.7.1.11

Solve: $\frac{1}{3}(6u+3)=7-u$

Answer

$u=2$

? Exercise 2.7.1.12

Solve: $\frac{2}{3}(9x-12)=8+2x$

Answer

$x=4$

? Exercise 2.7.1.13

Solve: $8-2(3y+5)=0$

Answer

	$8-2(3y+5)=0$
Simplify—use the Distributive Property.	$8-6y-10=0$
Combine like terms.	$-6y-2=0$
Add 2 to both sides to collect constants on the right.	$-6y-2+2=0+2$
Simplify.	$-6y=2$
Divide both sides by −6−6.	$\frac{-6y}{-6}=\frac{2}{-6}$

Simplify.	$y=-\frac{1}{3}$
Check: Let y=−13. $8-2(3y+5)=0$ $8-2\left[3\left(-\frac{1}{3}\right)+5\right]=0$ $8-2(-1+5)\stackrel{?}{=}0$ $8-2(4)\stackrel{?}{=}0$ $8-8\stackrel{?}{=}0$ $0=0✓$	

? Exercise 2.7.1.14

Solve: $12-3(4j+3)=-17$

Answer

$j=\frac{5}{3}$

? Exercise 2.7.1.15

Solve: $-6-8(k-2)=-10$

Answer

$k=\frac{5}{2}$

? Exercise 2.7.1.16

Solve: $4(x-1)-2=5(2x+3)+6$

Answer

		$4(x-1)-2=5(2x+3)+6$
Distribute.		$4x-4-2=10x+15+6$
Combine like terms.		$4x-6=10x+21$
Subtract 4x to get the variables only on the right side since $10>4$.		$4x-4x-6=10x-4x+21$
Simplify.		$-6=6x+21$
Subtract 21 to get the constants on left.		$-6-21=6x+21-21$
Simplify.		$-27=6x$
Divide by 6.		$\frac{-27}{6}=\frac{6x}{6}$
Simplify.		$-\frac{9}{2}=x$
Check:	$4(x-1)-2=5(2x+3)+6$	
Let $x=-\frac{9}{2}$.	$4\left(-\frac{9}{2}-1\right)-2\stackrel{?}{=}5\left[2\left(-\frac{9}{2}\right)+3\right]+6$	

	$4\left(-\frac{11}{2}\right)-2\stackrel{?}{=}5(-9+3)+6$	
	$-22-2\stackrel{?}{=}5(-6)+6$	
	$-24\stackrel{?}{=}-30+6$	
	$-24=-24✓$	

? Exercise 2.7.1.17

Solve: $6(p-3)-7=5(4p+3)-12$

Answer

$p=-2$

? Exercise 2.7.1.18

Solve: $8(q+1)-5=3(2q-4)-1$

Answer

$q=-8$

? Exercise 2.7.1.19

Solve: $10[3-8(2s-5)]=15(40-5s)$

Answer

		$10[3-8(2s-5)]=15(40-5s)$
Simplify from the innermost parentheses first.		$10[3-16s+40]=15(40-5s)$
Combine like terms in the brackets.		$10[43-16s]=15(40-5s)$
Distribute.		$430-160s=600-75s$
Add 160s to get the s's to the right.		$430-160s+160s=600-75s+160s$
Simplify.		$430=600+85s$
Subtract 600 to get the constants to the left.		$430-600=600+85s-600$
Simplify.		$-170=85s$
Divide.		$\frac{-170}{85}=\frac{85s}{85}$
Simplify.		$-2=s$
Check:	$10[3-8(2s-5)]=15(40-5s)$	
Substitute s=−2.	$10[3-8(2(-2)-5)]\stackrel{?}{=}15(40-5(-2))$	
	$10[3-8(-4-5)]\stackrel{?}{=}15(40+10)$	
	$10[3-8(-9)]\stackrel{?}{=}15(50)$	
	$10[3+72]\stackrel{?}{=}750$	
	$10[75]\stackrel{?}{=}750$	

	$750=750$ ✓	

? Exercise 2.7.1.20

Solve: $6[4-2(7y-1)]=8(13-8y)$.

Answer

$y=-\frac{17}{5}$

? Exercise 2.7.1.21

Solve: $12[1-5(4z-1)]=3(24+11z)$.

Answer

$z=0$

? Exercise 2.7.1.22

Solve: $0.36(100n+5)=0.6(30n+15)$.

Answer

		$0.36(100n+5)=0.6(30n+15)$
Distribute.		$36n+1.8=18n+9$
Subtract 18n to get the variables to the left.		$36n-18n+1.8=18n-18n+9$
Simplify.		$18n+1.8=9$
Subtract 1.8 to get the constants to the right.		$18n+1.8-1.8=9-1.8$
Simplify.		$18n=7.2$
Divide.		$\frac{18n}{18}=\frac{7.2}{18}$
Simplify.		$n=0.4$
Check:	$0.36(100n+5)=0.6(30n+15)$	
Let n=0.4.	$0.36(100(0.4)+5)\stackrel{?}{=}0.6(30(0.4)+15)$	
	$0.36(40+5)\stackrel{?}{=}0.6(12+15)$	
	$0.36(45)\stackrel{?}{=}0.6(27)$	
	$16.2=16.2$ ✓	

? Exercise 2.7.1.23

Solve: $0.55(100n+8)=0.6(85n+14)$.

Answer

$n=1$

Exercise 2.7.1.24

Solve: $0.15(40m-120)=0.5(60m+12)$.

Answer

$m=-1$

Key Concepts

- **General Strategy for Solving Linear Equations**
  1. Simplify each side of the equation as much as possible.
     Use the Distributive Property to remove any parentheses.
     Combine like terms.
  2. Collect all the variable terms on one side of the equation.
     Use the Addition or Subtraction Property of Equality.
  3. Collect all the constant terms on the other side of the equation.
     Use the Addition or Subtraction Property of Equality.
  4. Make the coefficient of the variable term to equal to 1.
     Use the Multiplication or Division Property of Equality.
     State the solution to the equation.
  5. Check the solution.
     Substitute the solution into the original equation.

Practice Makes Perfect

In the following exercises, solve each linear equation.

Exercise 2.7.1.1

$15(y-9)=-60$

Exercise 2.7.1.2

$21(y-5)=-42$

Answer

$y=3$

Exercise 2.7.1.3

$-9(2n+1)=36$

Exercise 2.7.1.4

$-16(3n+4)=32$

Answer

$n=-2$

Exercise 2.7.1.5

$8(22+11r)=0$

Exercise 2.7.1.6

$5(8+6p)=0$

Answer

$p=-\frac{4}{3}$

Exercise 2.7.1.7

$-(w-12)=30$

Exercise 2.7.1.8

$-(t-19)=28$

Answer

$t=-9$

Exercise 2.7.1.9

$9(6a+8)+9=81$

Exercise 2.7.1.10

$8(9b-4)-12=100$

Answer

$b=2$

Exercise 2.7.1.11

$32+3(z+4)=41$

Exercise 2.7.1.12

$21+2(m-4)=25$

Answer

$m=6$

Exercise 2.7.1.13

$51+5(4-q)=56$

Exercise 2.7.1.14

$-6+6(5-k)=15$

Answer

$k=\frac{3}{2}$

Exercise 2.7.1.15

$2(9s-6)-62=16$

Exercise 2.7.1.16

$8(6t-5)-35=-27$

Answer

$t=1$

Exercise 2.7.1.17

$3(10-2x)+54=0$

Exercise 2.7.1.18

$-2(11-7x)+54=4$

Answer

$x=-2$

Exercise 2.7.1.19

$\frac{2}{3}(9c-3)=22$

Exercise 2.7.1.20

$\frac{3}{5}(10x-5)=27$

Answer

$x=5$

Exercise 2.7.1.21

$\frac{1}{5}(15c+10)=c+7$

Exercise 2.7.1.22

$\frac{1}{4}(20d+12)=d+7$

Answer

$d=1$

Exercise 2.7.1.23

$18-(9r+7)=-16$

Exercise 2.7.1.24

$15-(3r+8)=28$

Answer

$r=-7$

Exercise 2.7.1.25

$5-(n-1)=19$

Exercise 2.7.1.26

$-3-(m-1)=13$

Answer

$m=-15$

Exercise 2.7.1.27

$11-4(y-8)=43$

Exercise 2.7.1.28

$18-2(y-3)=32$

Answer

$y=-4$

Exercise 2.7.1.29

$24-8(3v+6)=0$

Exercise 2.7.1.30

$35-5(2w+8)=-10$

Answer

$w=\frac{1}{2}$

Exercise 2.7.1.31

$4(a-12)=3(a+5)$

Exercise 2.7.1.32

$-2(a-6)=4(a-3)$

Answer

$a=4$

Exercise 2.7.1.33

$2(5-u)=-3(2u+6)$

Exercise 2.7.1.34

$5(8-r)=-2(2r-16)$

Answer

$r=8$

Exercise 2.7.1.35

$3(4n-1)-2=8n+3$

Exercise 2.7.1.36

$9(2m-3)-8=4m+7$

Answer

$m=3$

Exercise 2.7.1.37

$12+2(5-3y)=-9(y-1)-2$

Exercise 2.7.1.38

$-15+4(2-5y)=-7(y-4)+4$

Answer

$y=-3$

Exercise 2.7.1.39

$8(x-4)-7x=14$

Exercise 2.7.1.40

$5(x-4)-4x=14$

Answer

$x=34$

Exercise 2.7.1.41

$5+6(3s-5)=-3+2(8s-1)$

Exercise 2.7.1.42

$-12+8(x-5)=-4+3(5x-2)$

Answer

$x=-6$

Exercise 2.7.1.43

$4(u-1)-8=6(3u-2)-7$

Exercise 2.7.1.44

$7(2n-5)=8(4n-1)-9$

Answer

$n=-1$

Exercise 2.7.1.45

$4(p-4)-(p+7)=5(p-3)$

Exercise 2.7.1.46

$3(a-2)-(a+6)=4(a-1)$

Answer

$a=-4$

Exercise 2.7.1.47

$$\begin{aligned}&-(9y+5)-(3y-7)\\&=16-(4y-2)\end{aligned}$$

Exercise 2.7.1.48

$$\begin{aligned}&-(7m+4)-(2m-5)\\&=14-(5m-3)\end{aligned}$$

Answer

$m=-4$

Exercise 2.7.1.49

$$\begin{aligned}&4[5-8(4c-3)]\\&=12(1-13c)-8\end{aligned}$$

Exercise 2.7.1.50

$$\begin{aligned}&5[9-2(6d-1)]\\&=11(4-10d)-139\end{aligned}$$

Answer

$d = -3$

? Exercise 2.7.1.51

$3[-9+8(4h-3)]$
$=2(5-12h)-19$

? Exercise 2.7.1.52

$3[-14+2(15k-6)]$
$=8(3-5k)-24$

Answer

$k = \frac{3}{5}$

? Exercise 2.7.1.53

$5[2(m+4)+8(m-7)]$
$=2[3(5+m)-(21-3m)]$

? Exercise 2.7.1.54

$10[5(n+1)+4(n-1)]$
$=11[7(5+n)-(25-3n)]$

Answer

$n = -5$

? Exercise 2.7.1.55

$5(1.2u-4.8) = -12$

? Exercise 2.7.1.56

$4(2.5v-0.6) = 7.6$

Answer

$v = 1$

? Exercise 2.7.1.57

$0.25(q-6) = 0.1(q+18)$

? Exercise 2.7.1.58

$0.2(p-6) = 0.4(p+14)$

Answer

$p = -34$

Exercise 2.7.1.59

$0.2(30n+50)=28$

Exercise 2.7.1.60

$0.5(16m+34)=-15$

Answer

$m=-4$

Classify Equations

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction and then state the solution.

Exercise 2.7.1.61

$23z+19=3(5z-9)+8z+46$

Exercise 2.7.1.62

$15y+32=2(10y-7)-5y+46$

Answer

identity; all real numbers

Exercise 2.7.1.63

$5(b-9)+4(3b+9)=6(4b-5)-7b+21$

Exercise 2.7.1.64

$9(a-4)+3(2a+5)=7(3a-4)-6a+7$

Answer

identity; all real numbers

Exercise 2.7.1.65

$18(5j-1)+29=47$

Exercise 2.7.1.66

$24(3d-4)+100=52$

Answer

conditional equation; $d=\frac{2}{3}$

Exercise 2.7.1.67

$22(3m-4)=8(2m+9)$

Exercise 2.7.1.68

$30(2n-1)=5(10n+8)$

Answer

conditional equation; $n=7$

Exercise 2.7.1.69

$7v+42=11(3v+8)-2(13v-1)$

Exercise 2.7.1.70

$18u-51=9(4u+5)-6(3u-10)$

Answer

contradiction; no solution

Exercise 2.7.1.71

$3(6q-9)+7(q+4)=5(6q+8)-5(q+1)$

Exercise 2.7.1.72

$5(p+4)+8(2p-1)=9(3p-5)-6(p-2)$

Answer

contradiction; no solution

Exercise 2.7.1.73

$12(6h-1)=8(8h+5)-4$

Exercise 2.7.1.74

$9(4k-7)=11(3k+1)+4$

Answer

conditional equation; $k=26$

Exercise 2.7.1.75

$45(3y-2)=9(15y-6)$

Exercise 2.7.1.76

$60(2x-1)=15(8x+5)$

Answer

contradiction; no solution

Exercise 2.7.1.77

$16(6n+15)=48(2n+5)$

Exercise 2.7.1.78

$36(4m+5)=12(12m+15)$

Answer

identity; all real numbers

Exercise 2.7.1.79

$9(14d+9)+4d=13(10d+6)+3$

Exercise 2.7.1.80

$11(8c+5)-8c=2(40c+25)+5$

Answer

identity; all real numbers

Everyday Math

Exercise 2.7.1.81

Fencing Micah has 44 feet of fencing to make a dog run in his yard. He wants the length to be 2.5 feet more than the width. Find the length, *L*, by solving the equation 2L+2(L−2.5)=44.

Exercise 2.7.1.82

Coins Rhonda has $\$1.90$ in nickels and dimes. The number of dimes is one less than twice the number of nickels. Find the number of nickels, n, by solving the equation $0.05n+0.10(2n-1)=1.90$.

Answer

8 nickels

Writing Exercises

Exercise 2.7.1.83

Using your own words, list the steps in the general strategy for solving linear equations.

Exercise 2.7.1.84

Explain why you should simplify both sides of an equation as much as possible before collecting the variable terms to one side and the constant terms to the other side.

Answer

Answers will vary.

Exercise 2.7.1.85

What is the first step you take when solving the equation $3-7(y-4)=38$? Why is this your first step?

Exercise 2.7.1.86

Solve the equation $\frac{1}{4}(8x+20)=3x-4$ explaining all the steps of your solution as in the examples in this section.

Answer

Answers will vary.

Use the Distance, Rate, and Time Formula

One formula you will use often in algebra and in everyday life is the formula for distance traveled by an object moving at a constant rate. Rate is an equivalent word for “speed.” The basic idea of rate may already familiar to you. Do you know what distance you travel if you drive at a steady rate of 60 miles per hour for 2 hours? (This might happen if you use your car’s cruise control while driving on the highway.) If you said 120 miles, you already know how to use this formula!

DISTANCE, RATE, AND TIME

For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula:

$$\begin{array}{lrcl} d=rt \quad \text{where} & d & = & \text{distance} \\ & r & = & \text{rate} \\ & t & = & \text{time} \end{array} \tag{2.7.1.1}$$

We will use the Strategy for Solving Applications that we used earlier in this chapter. When our problem requires a formula, we change Step 4. In place of writing a sentence, we write the appropriate formula. We write the revised steps here for reference.

SOLVE AN APPLICATION (WITH A FORMULA).

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. Write the appropriate formula for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

You may want to create a mini-chart to summarize the information in the problem. See the chart in this first example.

Exercise 2.7.1.1

Jamal rides his bike at a uniform rate of 12 miles per hour for $3\frac{1}{2}$ hours. What distance has he traveled?

Answer

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	distance traveled
Step 3. Name. Choose a variable to represent it.	Let d = distance.
Step 4. Translate: Write the appropriate formula.	$d = rt$
	$d = ?$ $r = 12$ mph $t = 3\frac{1}{2}$ hours
Substitute in the given information.	$d = 12 \cdot 3\frac{1}{2}$
Step 5. Solve the equation.	$d = 42$ miles
Step 6. Check	
Does 42 miles make sense?	
Jamal rides:	
12 miles in 1 hour, 24 miles in 2 hours, 36 miles in 3 hours, 42 miles in $3\frac{1}{2}$ hours is reasonable 48 miles in 4 hours.	
Step 7. Answer the question with a complete sentence.	Jamal rode 42 miles.

Exercise 2.7.1.2

Lindsay drove for $5\frac{1}{2}$ hours at 60 miles per hour. How much distance did she travel?

Answer

330 miles

Exercise 2.7.1.3

Trinh walked for $2\frac{1}{3}$ hours at 3 miles per hour. How far did she walk?

Answer

7 miles

Exercise 2.7.1.4

Rey is planning to drive from his house in San Diego to visit his grandmother in Sacramento, a distance of 520 miles. If he can drive at a steady rate of 65 miles per hour, how many hours will the trip take?

Answer

Step 1. Read the problem.	

Step 2. Identify what you are looking for.	How many hours (time)
Step 3. Name. Choose a variable to represent it.	Let t = time.
	$d = 520$ miles $r = 65$ mph $t = ?$ hours
Step 4. Translate. Write the appropriate formula.	$d = rt$
Substitute in the given information.	$520 = 65t$
Step 5. Solve the equation.	$t = 8$
Step 6. Check. Substitute the numbers into the formula and make sure the result is a true statement.	
$\begin{aligned} d &= rt \\ 520 &\stackrel{?}{=} 65 \cdot 8 \\ 520 &= 520\checkmark \end{aligned}$	
Step 7. Answer the question with a complete sentence. Rey's trip will take 8 hours.	

? Exercise 2.7.1.5

Lee wants to drive from Phoenix to his brother's apartment in San Francisco, a distance of 770 miles. If he drives at a steady rate of 70 miles per hour, how many hours will the trip take?

Answer

11 hours

? Exercise 2.7.1.6

Yesenia is 168 miles from Chicago. If she needs to be in Chicago in 3 hours, at what rate does she need to drive?

Answer

56 mph

Solve a Formula for a Specific Variable

You are probably familiar with some geometry formulas. A formula is a mathematical description of the relationship between variables. Formulas are also used in the sciences, such as chemistry, physics, and biology. In medicine they are used for calculations for dispensing medicine or determining body mass index. Spreadsheet programs rely on formulas to make calculations. It is important to be familiar with formulas and be able to manipulate them easily.

In Exercise $2.7.1.1$ and Exercise $2.7.1.4$, we used the formula $d = rt$. This formula gives the value of d, distance, when you substitute in the values of r and t, the rate and time. But in Exercise $2.7.1.4$, we had to find the value of t. We substituted in values of d and r and then used algebra to solve for tt. If you had to do this often, you might wonder why there is not a formula that gives the value of t when you substitute in the values of d and r. We can make a formula like this by solving the formula $d = rt$ for t.

To solve a formula for a specific variable means to isolate that variable on one side of the equals sign with a coefficient of 1. All other variables and constants are on the other side of the equals sign. To see how to solve a formula for a specific variable, we will start with the distance, rate and time formula.

Exercise 2.7.1.7

Solve the formula d=rt for t:

1. when d=520 and r=65
2. in general

Answer

We will write the solutions side-by-side to demonstrate that solving a formula in general uses the same steps as when we have numbers to substitute.

1. when d=520 and r=65		2. in general	
Write the formula.	$d=rt$	Write the formula.	$d=rt$
Substitute.	$520=65t$		
Divide, to isolate t.	$\frac{520}{65}=\frac{65t}{65}$	Divide, to isolate tt.	$\frac{d}{r}=\frac{rt}{t}$
Simplify.	$8=t$	Simplify.	$\frac{d}{r}=t$

We say the formula $t=\frac{d}{r}$ is solved for t.

Exercise 2.7.1.8

Solve the formula $d=rt$ for r:

1. when d=180 and t=4
2. in general

Answer

1. $r=45$
2. $r=\frac{d}{t}$

Exercise 2.7.1.9

Solve the formula $d=rt$ for r:

1. when d=780 and t=12
2. in general

Answer

1. $r=65$
2. \(r = \frac{d}{rt\)

Exercise 2.7.1.10

Solve the formula $A=\frac{1}{2}bh$ for h:

1. when $A=90$ and $b=15$
2. in general

Answer

1. when $A=90$ and $b=15$		2. in general	
Write the formula.	$A=\frac{1}{2}bh$	Write the formula.	$A=\frac{1}{2}bh$

Substitute.	$90=\frac{1}{2}\cdot 15\cdot h$		
Clear the fractions.	$2\cdot 90=2\cdot\frac{1}{2}15h$	Clear the fractions.	$2\cdot A=2\cdot\frac{1}{2}bh$
Simplify.	$180=15h$	Simplify.	$2A=bh$
Solve for h.	$12=h$	Solve for hh.	$\frac{2A}{b}=h$

We can now find the height of a triangle, if we know the area and the base, by using the formula $h=\frac{2A}{b}$

? Exercise 2.7.1.11

Solve the formula $A=\frac{1}{2}bh$ for h:

1. when $A=170$ and $b=17$
2. in general

Answer

1. $h=20$
2. $h=\frac{2A}{b}$

? Exercise 2.7.1.12

Solve the formula $A=\frac{1}{2}bh$ for h:

1. when $A=62$ and $h=31$
2. in general

Answer

1. $b=4$
2. $b=\frac{2A}{h}$

The formula $I=Prt$ is used to calculate simple interest, *I*, for a principal, *P*, invested at rate, *r*, for *t* years.

? Exercise 2.7.1.13

Solve the formula I=Prt to find the principal, P:

1. when I=$5,600, r=4% ,t=7years
2. in general

Answer

1. I=$5,600, r=4% ,t=7years		2. in general	
Write the formula.	$I=Prt$	Write the formula.	$I=Prt$
Substitute.	$5600=P(0.04)(7)$		
Simplify.	$5600=P(0.28)$	Simplify.	$I=P(rt)$
Divide, to isolate *P*.	$\frac{5600}{0.28}=\frac{P(0.28)}{0.28}$	Divide, to isolate *P*.	$\frac{I}{rt}=\frac{P(rt)}{rt}$
Simplify.	$20{,}000=P$	Simplify.	$\frac{I}{rt}=P$
The principal is	$20,000		$P=\frac{I}{rt}$

? Exercise 2.7.1.14

Solve the formula I=Prt to find the principal, P:

1. when I=\$2160, r=6% ,t=3 years
2. in general

Answer

1. \$12000
2. $P = \frac{1}{rt}$

? Exercise 2.7.1.15

Solve the formula I=Prt to find the principal, P:

1. when I=\$5400, r= 12% ,t=5 years
2. in general

Answer

1. \$9000
2. $P = \frac{1}{rt}$

Later in this class, and in future algebra classes, you'll encounter equations that relate two variables, usually *x* and *y*. You might be given an equation that is solved for *y* and need to solve it for *x*, or vice versa. In the following example, we're given an equation with both *x* and *y* on the same side and we'll solve it for *y*.

? Exercise 2.7.1.16

Solve the formula 3x+2y=18 for *y*:

1. when x=4
2. in general

Answer

1. when x=4		2. in general	
	$3x+2y=18$		$3x+2y=18$
Substitute.	$3(4)+2y=18$		
Subtract to isolate the y-term.	$12-12+2y=18-12$	Subtract to isolate the y-term.	$3x-3x+2y=18-3x$
Divide.	$\frac{2y}{2}=\frac{6}{2}$	Divide.	$\frac{2y}{2}=\frac{18}{2}-\frac{3x}{2}$
Simplify.	$y=3$	Simplify.	$y=-\frac{3x}{2}+9$

? Exercise 2.7.1.17

Solve the formula 3x+4y=10 for *y*:

1. when $x = \frac{14}{3}$
2. in general

Answer

1. $y=-1$
2. $y=\frac{10-3x}{4}$

Exercise 2.7.1.18

Solve the formula 5x+2y=18 for *y*:

1. when $x=4$
2. in general

Answer

1. $y=-1$
2. $y=\frac{18-5x}{2}$

In Exercise 2.7.1.7through Exercise 2.7.1.18we used the numbers in part 1 as a guide to solving in general in part 2. Now we will solve a formula in general without using numbers as a guide.

Exercise 2.7.1.19

Solve the formula P=a+b+c for a.

Answer

We will isolate aa on one side of the equation.	$P=a+b+c$
Both b and c are added to a, so we subtract them from both sides of the equation.	$P-b-c=a+b+c-b-c$
Simplify.	$P-b-c=a$ $a=P-b-c$

Exercise 2.7.1.20

Solve the formula P=a+b+c for *b*.

Answer

b=P−a−c

Exercise 2.7.1.21

Solve the formula P=a+b+c for *c*.

Answer

c=P−a−b

Exercise 2.7.1.22

Solve the formula 6x+5y=13 for *y*.

Answer

	$6x+5y=13$
Subtract 6x from both sides to isolate the term with y.	$6x-6x+5y=13-6x$

Simplify.	$5y = 13 - 6x$
Divide by 5 to make the coefficient 1.	$\frac{5y}{5} = \frac{13-6x}{5}$
Simplify.	$y = \frac{13-6x}{5}$

The fraction is simplified. We cannot divide $13-6x$ by 5.

Exercise 2.7.1.23

Solve the formula $4x+7y=9$ for *y*.

Answer

$y = \frac{9-4x}{7}$

Exercise 2.7.1.24

Solve the formula $5x+8y=1$ for *y*.

Answer

$y = \frac{1-5x}{8}$

Key Concepts

- **To Solve an Application (with a formula)**
  1. **Read** the problem. Make sure all the words and ideas are understood.
  2. **Identify** what we are looking for.
  3. **Name** what we are looking for. Choose a variable to represent that quantity.
  4. **Translate** into an equation. Write the appropriate formula for the situation. Substitute in the given information.
  5. **Solve** the equation using good algebra techniques.
  6. **Check** the answer in the problem and make sure it makes sense.
  7. **Answer** the question with a complete sentence.
- **Distance, Rate and Time**
  For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula: $d=rt$ where *d* = distance, *r* = rate, *t* = time.
- **To solve a formula for a specific variable** means to get that variable by itself with a coefficient of 1 on one side of the equation and all other variables and constants on the other side.