5.4: More Examples

Problem-Solving Strategy: Conservation of Momentum

Using conservation of momentum requires four basic steps. The first step is crucial:

1. Identify a closed system (total mass is constant, no net external force acts on the system).
2. Write down an expression representing the total momentum of the system before the "event" (explosion or collision).
3. Write down an expression representing the total momentum of the system after the "event."
4. Set these two expressions equal to each other, and solve this equation for the desired quantity

Example 5.4.1: Colliding Carts

Two carts in a physics lab roll on a level track, with negligible friction. These carts have small magnets at their ends, so that when they collide, they stick together (Figure 5.4.1). The first cart has a mass of 675 grams and is rolling at 0.75 m/s to the right; the second has a mass of 500 grams and is rolling at 1.33 m/s, also to the right. After the collision, what is the velocity of the two joined carts?

Figure 5.4.1: Two lab carts collide and stick together after the collision.

Strategy

We have a collision. We're given masses and initial velocities; we're asked for the final velocity. This all suggests using conservation of momentum as a method of solution. However, we can only use it if we have a closed system. So we need to be sure that the system we choose has no net external force on it, and that its mass is not changed by the collision.

Defining the system to be the two carts meets the requirements for a closed system: The combined mass of the two carts certainly doesn't change, and while the carts definitely exert forces on each other, those forces are internal to the system, so they do not change the momentum of the system as a whole. In the vertical direction, the weights of the carts are canceled by the normal forces on the carts from the track.

Solution

Conservation of momentum is

$$\vec{p}_f = \vec{p}_i .$$

Define the direction of their initial velocity vectors to be the +x-direction. The initial momentum is then

$$\vec{p}_i = m_1 \begin{bmatrix} v_{1,x} \\ 0 \\ 0 \end{bmatrix} + m_2 \begin{bmatrix} v_{2,x} \\ 0 \\ 0 \end{bmatrix} .$$

The final momentum of the now-linked carts is

$$\vec{p}_f = (m_1 + m_2) \begin{bmatrix} v_{f,x} \\ v_{f,y} \\ v_{f,z} \end{bmatrix} .$$

Once again, our y and z equations are not interesting. We can concentrate on the x direction. Using the x equation:

$$(m_1 + m_2)\vec{v}_{f,x} = m_1 v_{1,x} + m_2 v_{2,x}$$

$$\vec{v}_f = \left(\frac{m_1 v_{1,x} + m_2 v_{2,x}}{m_1 + m_2} \right) .$$

Substituting the given numbers:

$$\vec{v}_{f,x} = \left[\frac{(0.675\ kg)(0.75\ m/s) + (0.5\ kg)(1.33\ m/s)}{1.175\ kg}\right]\hat{i}$$

$$= (0.997\ m/s).$$

Significance

The principles that apply here to two laboratory carts apply identically to all objects of whatever type or size. Even for photons, the concepts of momentum and conservation of momentum are still crucially important even at that scale. (Since they are massless, the momentum of a photon is defined very differently from the momentum of ordinary objects. You will learn about this when you study quantum physics.)

? Exercise 5.4.2

Suppose the second, smaller cart had been initially moving to the left. What would the sign of the final velocity have been in this case?

✓ Example 5.4.3: Ice Hockey 1

Two hockey pucks of identical mass are on a flat, horizontal ice hockey rink. The red puck is motionless; the blue puck is moving at 2.5 m/s to the left (Figure 5.4.3). It collides with the motionless red puck. The pucks have a mass of 15 g. After the collision, the red puck is moving at 2.5 m/s, to the left. What is the final velocity of the blue puck?

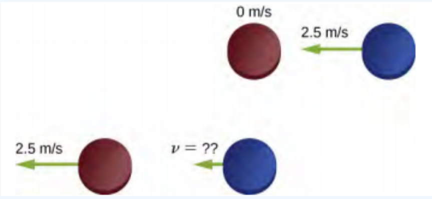

Figure 5.4.3: Two identical hockey pucks colliding. The top diagram shows the pucks the instant before the collision, and the bottom diagram show the pucks the instant after the collision. The net external force is zero.

Strategy

We're told that we have two colliding objects, we're told the masses and initial velocities, and one final velocity; we're asked for both final velocities. Conservation of momentum seems like a good strategy. Define the system to be the two pucks; there's no friction, so we have a closed system.

Before you look at the solution, what do you think the answer will be?

The blue puck final velocity will be:

a. zero
b. 2.5 m/s to the left
c. 2.5 m/s to the right
d. 1.25 m/s to the left
e. 1.25 m/s to the right
f. something else

Solution

Define the +x-direction to point to the right. Conservation of momentum then reads

$$\vec{p_f} = \vec{p_i}$$

$$m\begin{bmatrix} v_{r_{f,x}} \\ v_{r_{f,y}} \\ v_{r_{f,z}} \end{bmatrix} + m\begin{bmatrix} v_{b_{f,x}} \\ v_{b_{f,y}} \\ v_{b_{f,z}} \end{bmatrix} = m\begin{bmatrix} v_{r_{i,x}} \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} v_{b_{i,x}} \\ 0 \\ 0 \end{bmatrix}$$

As in the other examples, only the x direction is of interest. Before the collision, the momentum of the system is entirely and only in the blue puck. Thus,

$$mv_{r_{f,x}} + mv_{b_{f,x}} = -mv_{b_{i,x}}$$

$$v_{r_{f,x}} + v_{b_{f,x}} = -v_{b_{i,x}}.$$

(Remember that the masses of the pucks are equal.) Substituting numbers:

$$-(2.5\ m/s) + v_{b_{f,x}} = -(2.5\ m/s)$$
$$v_{b_{f,x}} = 0.$$

Significance

Evidently, the two pucks simply exchanged momentum. The blue puck transferred all of its momentum to the red puck. In fact, this is what happens in similar collision where $m_1 = m_2$.

? Exercise 5.4.4

Even if there were some friction on the ice, it is still possible to use conservation of momentum to solve this problem, but you would need to impose an additional condition on the problem. What is that additional condition?

✓ Example 5.4.5: Philae

On November 12, 2014, the European Space Agency successfully landed a probe named **Philae** on Comet 67P/Churyumov/Gerasimenko (Figure 5.4.4). During the landing, however, the probe actually landed three times, because it bounced twice. Let's calculate how much the comet's speed changed as a result of the first bounce.

Figure 5.4.4: An artist's rendering of Philae landing on a comet. (credit: modification of work by "DLR German Aerospace Center"/Flickr)

Let's define upward to be the +y-direction, perpendicular to the surface of the comet, and y = 0 to be at the surface of the comet. Here's what we know:

- The mass of Comet 67P: $M_c = 1.0 \times 10^{13}$ kg
- The acceleration due to the comet's gravity: $\vec{a} = -(5.0 \times 10^{-3}\ m/s^2)$ in y direction
- **Philae**'s mass: $M_p = 96$ kg
- Initial touchdown speed: $\vec{v}_1 = -(1.0\ m/s)$ in y direction
- Initial upward speed due to first bounce: $\vec{v}_2 = (0.38\ m/s)$ in y direction.
- Landing impact time: $\Delta t = 1.3$ s

Strategy

We're asked for how much the comet's speed changed, but we don't know much about the comet, beyond its mass and the acceleration its gravity causes. However, we are told that the **Philae** lander collides with (lands on) the comet, and bounces off of it. A collision suggests momentum as a strategy for solving this problem.

If we define a system that consists of both **Philae** and Comet 67/P, then there is no net external force on this system, and thus the momentum of this system is conserved. (We'll neglect the gravitational force of the sun.) Thus, if we calculate the change of momentum of the lander, we automatically have the change of momentum of the comet. Also, the comet's change of velocity is directly related to its change of momentum as a result of the lander "colliding" with it.

Solution

Let $\vec{p}_1$ be **Philae**'s momentum at the moment just before touchdown, and $\vec{p}_2$ be its momentum just after the first bounce. Then its momentum just before landing was

$$\begin{aligned}\vec{p}_1 = M_p\vec{v}_1 &= \begin{bmatrix} 0 \\ (96\ kg)(-1.0\ m/s) \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ -(96\ kg \cdot m/s) \\ 0 \end{bmatrix}\end{aligned}$$

and just after was

$$\vec{p}_2 = M_p\vec{v}_2 = \begin{bmatrix} 0 \\ (96\ kg)(+0.38\ m/s) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ (36.5\ kg \cdot m/s) \\ 0 \end{bmatrix}.$$

Therefore, the lander's change of momentum during the first bounce is

$$\begin{aligned}\Delta\vec{p} &= \vec{p}_2 - \vec{p}_1 \\ &= \begin{bmatrix} 0 \\ (36.5\ kg \cdot m/s) \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -(96\ kg \cdot m/s) \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ (133\ kg \cdot m/s) \\ 0 \end{bmatrix}\end{aligned}$$

Notice how important it is to include the negative sign of the initial momentum.

Now for the comet. Since momentum of the system must be conserved, the comet's momentum changed by exactly the negative of this:

$$\Delta\vec{p}_c = -\Delta\vec{p} = \begin{bmatrix} 0 \\ (-133\ kg \cdot m/s) \\ 0 \end{bmatrix}.$$

Therefore, its change of velocity (entirely in the y) direction is

$$\Delta v_{c,y} = \frac{\Delta p_{c,y}}{M_c} = \frac{-(133\ kg \cdot m/s)}{1.0 \times 10^{13}\ kg} = -(1.33 \times 10^{-11}\ m/s).$$

Significance

This is a very small change in velocity, about a thousandth of a billionth of a meter per second. Crucially, however, it is **not** zero.

Exercise 5.4.6

The changes of momentum for **Philae** and for Comet 67/P were equal (in magnitude). Were the impulses experienced by **Philae** and the comet equal? How about the forces? How about the changes of kinetic energies?