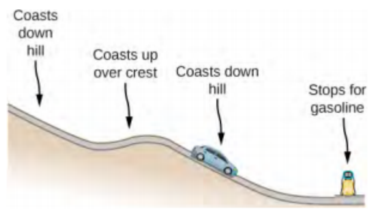
9.E: Potential Energy and Conservation of Energy (Exercises)

Conceptual Questions

1. The force exerted by a diving board is conservative, provided the internal friction is negligible. Assuming friction is negligible, describe changes in the potential energy of a diving board as a swimmer drives from it, starting just before the swimmer steps on the board until just after his feet leave it.
2. Describe the gravitational potential energy transfers and transformations for a javelin, starting from the point at which an athlete picks up the javelin and ending when the javelin is stuck into the ground after being thrown.
3. Consider the following scenario. A car for which friction is not negligible accelerates from rest down a hill, running out of gasoline after a short distance (see below). The driver lets the car coast farther down the hill, then up and over a small crest. He then coasts down that hill into a gas station, where he brakes to a stop and fills the tank with gasoline. Identify the forms of energy the car has, and how they are changed and transferred in this series of events.

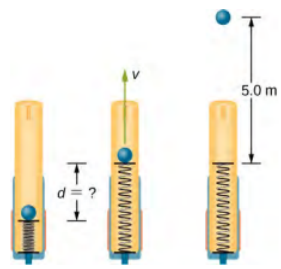


4. A box is dropped onto a spring at its equilibrium position. The spring compresses with the box attached and comes to rest. Since the spring is in the vertical position, does the change in the gravitational potential energy of the box while the spring is compressing need to be considered in this problem?

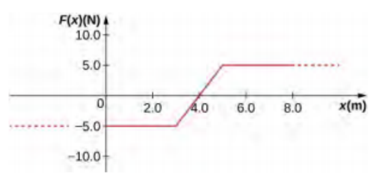
Problems

5. A force F(x) = (3.0/x) N acts on a particle as it moves along the positive x-axis. (a) How much work does the force do on the particle as it moves from x = 2.0 m to x = 5.0 m? (b) Picking a convenient reference point of the potential energy to be zero at x = ∞, find the potential energy for this force.
6. A force F(x) = ($-5.0x^2$ + 7.0x) N acts on a particle. (a) How much work does the force do on the particle as it moves from x = 2.0 m to x = 5.0 m? (b) Picking a convenient reference point of the potential energy to be zero at x = ∞, find the potential energy for this force.
7. Find the force corresponding to the potential energy U(x) = $-\frac{a}{x}+\frac{b}{x^2}$.
8. The potential energy function for either one of the two atoms in a diatomic molecule is often approximated by U(x) = $-\frac{a}{x^{12}}-\frac{b}{x^6}$ where x is the distance between the atoms. (a) At what distance of separation does the potential energy have a local minimum (not at x = ∞)? (b) What is the force on an atom at this separation? (c) How does the force vary with the separation distance?
9. A particle of mass 2.0 kg moves under the influence of the force F(x) = $\left(\frac{3}{\sqrt{x}}\right)$ N. If its speed at x = 2.0 m is v = 6.0 m/s, what is its speed at x = 7.0 m?
10. A particle of mass 2.0 kg moves under the influence of the force F(x) = ($-5x^2$ + 7x) N. If its speed at x = −4.0 m is v = 20.0 m/s, what is its speed at x = 4.0 m?
11. A crate on rollers is being pushed without frictional loss of energy across the floor of a freight car (see the following figure). The car is moving to the right with a constant speed v0 . If the crate starts at rest relative to the freight car, then from the work-energy theorem, Fd = $\frac{mv^2}{2}$, where d, the distance the crate moves, and v, the speed of the crate, are both measured relative to the freight car. (a) To an observer at rest beside the tracks, what distance d′ is the crate pushed when it moves the distance d in the car? (b) What are the crate's initial and final speeds v_0' and v′ as measured by the observer beside the tracks? (c) Show that Fd′ = $\frac{m(v')^2}{2}-\frac{m(v_0')^2}{2}$ and, consequently, that work is equal to the change in kinetic energy in both reference systems.

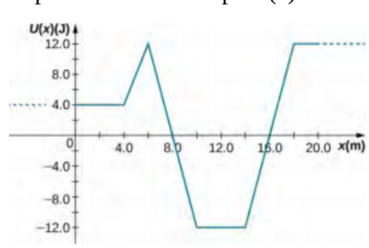
12. Assume that the force of a bow on an arrow behaves like the spring force. In aiming the arrow, an archer pulls the bow back 50 cm and holds it in position with a force of 150 N. What is the spring constant of the bow? If the mass of the arrow is 50 g and the "spring" is massless, what is the speed of the arrow immediately after it leaves the bow?
13. The massless spring of a spring gun has a force constant k = 12 N/cm. When the gun is aimed vertically, a 15-g projectile is shot to a height of 5.0 m above the end of the expanded spring. (See below.) How much was the spring compressed initially?



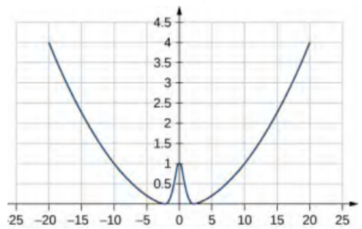
14. A mysterious constant force of 10 N acts horizontally on everything. The direction of the force is found to be always pointed toward a wall in a big hall. Find the potential energy of a particle due to this force when it is at a distance x from the wall, assuming the potential energy at the wall to be zero.
15. A single force F(x) = −4.0x (in newtons) acts on a 1.0-kg body. When x = 3.5 m, the speed of the body is 4.0 m/s. What is its speed at x = 2.0 m?
16. A particle of mass 4.0 kg is constrained to move along the x-axis under a single force $F(x) = -cx^3$, where $c = 8.0\ \text{N/m}^3$. The particle's speed at A, where $x_A = 1.0$ m, is 6.0 m/s. What is its speed at B, where $x_B = -2.0$ m?
17. The force on a particle of mass 2.0 kg varies with position according to $F(x) = -3.0x^2$ (x in meters, F(x) in newtons). The particle's velocity at x = 2.0 m is 5.0 m/s. Calculate the mechanical energy of the particle using (a) the origin as the reference point and (b) x = 4.0 m as the reference point. (c) Find the particle's velocity at x = 1.0 m. Do this part of the problem for each reference point.
18. A 4.0-kg particle moving along the x-axis is acted upon by the force whose functional form appears below. The velocity of the particle at x = 0 is v = 6.0 m/s. Find the particle's speed at x = (a) 2.0 m, (b) 4.0 m, (c) 10.0 m, (d) Does the particle turn around at some point and head back toward the origin? (e) Repeat part (d) if v = 2.0 m/s at x = 0.



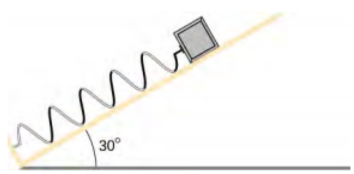
19. A particle of mass 0.50 kg moves along the x-axis with a potential energy whose dependence on x is shown below. (a) What is the force on the particle at x = 2.0, 5.0, 8.0, and 12 m? (b) If the total mechanical energy E of the particle is −6.0 J, what are the minimum and maximum positions of the particle? (c) What are these positions if E = 2.0 J? (d) If E = 16 J, what are the speeds of the particle at the positions listed in part (a)?



20. (a) Sketch a graph of the potential energy function U(x) = $\frac{kx^2}{2} + Ae^{-\alpha x^2}$, where k, A, and α are constants. (b) What is the force corresponding to this potential energy? (c) Suppose a particle of mass m moving with this potential energy has a velocity v_a when its position is x = a. Show that the particle does not pass through the origin unless $A \leq \frac{mv_a^2 + ka^2}{2\left(1 - e^{-\alpha a^2}\right)}$.

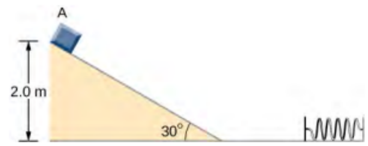


21. In the Back to the Future movies (https://openstaxcollege.org/l/21bactofutclip), a DeLorean car of mass 1230 kg travels at 88 miles per hour to venture back to the future. (a) What is the kinetic energy of the DeLorian? (b) What spring constant would be needed to stop this DeLorean in a distance of 0.1m?
22. In a Coyote/Road Runner cartoon clip (https://openstaxcollege.org/l/21coyroadcarcl), a spring expands quickly and sends the coyote into a rock. If the spring extended 5 m and sent the coyote of mass 20 kg to a speed of 15 m/s, (a) what is the spring constant of this spring? (b) If the coyote were sent vertically into the air with the energy given to him by the spring, how high could he go if there were no non-conservative forces?
23. In the movie Monty Python and the Holy Grail (https://openstaxcollege.org/l/21monpytmovcl) a cow is catapulted from the top of a castle wall over to the people down below. The gravitational potential energy is set to zero at ground level. The cow is launched from a spring of spring constant 1.1×10^4 N/m that is expanded 0.5 m from equilibrium. If the castle is 9.1 m tall and the mass of the cow is 110 kg, (a) what is the gravitational potential energy of the cow at the top of the castle? (b) What is the elastic spring energy of the cow before the catapult is released? (c) What is the speed of the cow right before it lands on the ground?
24. A 5.00×10^5-kg subway train is brought to a stop from a speed of 0.500 m/s in 0.400 m by a large spring bumper at the end of its track. What is the spring constant k of the spring?
25. A pogo stick has a spring with a spring constant of 2.5×10^4 N/m, which can be compressed 12.0 cm. To what maximum height from the uncompressed spring can a child jump on the stick using only the energy in the spring, if the child and stick have a total mass of 40 kg?
26. A block of mass 500 g is attached to a spring of spring constant 80 N/m (see the following figure). The other end of the spring is attached to a support while the mass rests on a rough surface with a coefficient of friction of 0.20 that is inclined at angle of 30°. The block is pushed along the surface till the spring compresses by 10 cm and is then released from rest. (a) How much potential energy was stored in the block-spring-support system when the block was just released? (b) Determine the speed of the block when it crosses the point when the spring is neither compressed nor stretched. (c) Determine the position of the block where it just comes to rest on its way up the incline.

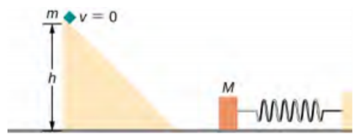


27. A block of mass 200 g is attached at the end of a massless spring at equilibrium length of spring constant 50 N/m. The other end of the spring is attached to the ceiling and the mass is released at a height considered to be where the gravitational potential energy is zero. (a) What is the net potential energy of the block at the instant the block is at the lowest point? (b) What is the net potential energy of the block at the midpoint of its descent? (c) What is the speed of the block at the midpoint of its descent?
28. A child (32 kg) jumps up and down on a trampoline. The trampoline exerts a spring restoring force on the child with a constant of 5000 N/m. At the highest point of the bounce, the child is 1.0 m above the level surface of the trampoline. What is the compression distance of the trampoline? Neglect the bending of the legs or any transfer of energy of the child into the trampoline while jumping.

29. A massless spring with force constant k = 200 N/m hangs from the ceiling. A 2.0-kg block is attached to the free end of the spring and released. If the block falls 17 cm before starting back upwards, how much work is done by friction during its descent?
30. An object of mass 10 kg is released at point A, slides to the bottom of the 30° incline, then collides with a horizontal massless spring, compressing it a maximum distance of 0.75 m. (See below.) The spring constant is 500 M/m, the height of the incline is 2.0 m, and the horizontal surface is frictionless. (a) What is the speed of the object at the bottom of the incline? (b) What is the work of friction on the object while it is on the incline? (c) The spring recoils and sends the object back toward the incline. What is the speed of the object when it reaches the base of the incline? (d) What vertical distance does it move back up the incline?



31. A block of mass m, after sliding down a frictionless incline, strikes another block of mass M that is attached to a spring of spring constant k (see below). The blocks stick together upon impact and travel together. (a) Find the compression of the spring in terms of m, M, h, g, and k when the combination comes to rest. (b) The loss of kinetic energy as a result of the bonding of the two masses upon impact is stored in the so-called binding energy of the two masses. Calculate the binding energy.



32. A block of mass 300 g is attached to a spring of spring constant 100 N/m. The other end of the spring is attached to a support while the block rests on a smooth horizontal table and can slide freely without any friction. The block is pushed horizontally till the spring compresses by 12 cm, and then the block is released from rest. (a) How much potential energy was stored in the block-spring support system when the block was just released? (b) Determine the speed of the block when it crosses the point when the spring is neither compressed nor stretched. (c) Determine the speed of the block when it has traveled a distance of 20 cm from where it was released.
33. Consider a block of mass 0.200 kg attached to a spring of spring constant 100 N/m. The block is placed on a frictionless table, and the other end of the spring is attached to the wall so that the spring is level with the table. The block is then pushed in so that the spring is compressed by 10.0 cm. Find the speed of the block as it crosses (a) the point when the spring is not stretched, (b) 5.00 cm to the left of point in (a), and (c) 5.00 cm to the right of point in (a).

Contributors and Attributions

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