3.4: Vector Algebra in Multiple Dimensions- Calculations

Adding Vectors

Luckily, adding vectors is very easy when we use our column vector notation. We simply add the components together, which is the same as just adding across each row of the matrix. Let's say we have two vectors, $\vec{A}$ and $\vec{B}$, and we want to add them together to create a new vector, $\vec{C}=\vec{A}+\vec{B}$. We can easily find the components of the new vector.

$$\vec{C}=\vec{A}+\vec{B}=\begin{bmatrix}A_x\\A_y\\A_z\end{bmatrix}+\begin{bmatrix}B_x\\B_y\\B_z\end{bmatrix}=\begin{bmatrix}A_x+B_x\\A_y+B_y\\A_z+B_z\end{bmatrix}\tag{3.4.1}$$

Example 3.4.1

Vector $\vec{A}$ has components $A_x=3$ m, $A_y=7$ m, $A_z=-4$ m. Vector $\vec{B}$ has components $B_x=5$ m, $B_y=-2$ m, $B_z=8$ m. What is $\vec{C}=\vec{A}+\vec{B}$?

Solution

$$\vec{C}=\vec{A}+\vec{B}=\begin{bmatrix}A_x\\A_y\\A_z\end{bmatrix}+\begin{bmatrix}B_x\\B_y\\B_z\end{bmatrix}=\begin{bmatrix}3\text{ m}+5\text{ m}\\7\text{ m}+(-2)\text{ m}\\-4\text{ m}+8\text{ m}\end{bmatrix}\tag{3.4.2}$$

$$\vec{C}=\begin{bmatrix}8\text{ m}\\5\text{ m}\\4\text{ m}\end{bmatrix}\tag{3.4.3}$$

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For multiplication, we simply multiply each component. So:

$$3\vec{A}=\begin{bmatrix}3A_x\\3A_y\\3A_z\end{bmatrix}\tag{3.4.4}$$

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Subtraction is just the same as adding the negative of the matrix that is being subtracted.

You cannon multiply two vectors together in a normal way. We will cover different types of vector multiplication later in this book.

Finally, you cannot divide a vector by another vector.

Let's look at a little more realistic example.

Example 3.4.2 (Adapted from OpenStax)

During a takeoff of IAI Heron, its position with respect to a control tower is 100 m above the ground, 300 m to the east, and 200 m to the north. One minute later, its position is 250 m above the ground, 1200 m to the east, and 2100 m to the north. What is the drone’s displacement vector with respect to the control tower? What is the magnitude of its displacement vector?

Figure The drone IAI Heron in flight. (credit: SSgt Reynaldo Ramon, USAF)

Solution

We are given two position vectors. Let's call them $\vec{r}_i$ and $\vec{r}_f$. Let us call east +x, north +y, and "above the ground" +z. Then these will be our two position vectors:

$$\vec{r}_i = \begin{bmatrix} 300\ \mathrm{m} \\ 200\ \mathrm{m} \\ 100\ \mathrm{m} \end{bmatrix} \text{ and } \vec{r}_f = \begin{bmatrix} 1200\ \mathrm{m} \\ 2100\ \mathrm{m} \\ 250\ \mathrm{m} \end{bmatrix} \tag{3.4.5}$$

To find the displacment vector of the drone with respect to the control tower, we need to subtract the initial position $\vec{r}_i$ of the drone from the final position, $\vec{r}_f$.

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i = \begin{bmatrix} 1200\ \mathrm{m} \\ 2100\ \mathrm{m} \\ 250\ \mathrm{m} \end{bmatrix} - \begin{bmatrix} 300\ \mathrm{m} \\ 200\ \mathrm{m} \\ 100\ \mathrm{m} \end{bmatrix} \tag{3.4.6}$$

$$\boxed{\Delta\vec{r} = \begin{bmatrix} 900\ \mathrm{m} \\ 1900\ \mathrm{m} \\ 150\ \mathrm{m} \end{bmatrix}} \tag{3.4.7}$$

Finally, if we want the magnitude of the displacement, we should use our 3-dimensional Pythagorean theorem:

$$|\Delta\vec{r}| = \sqrt{(900\ \mathrm{m})^2 + (1900\ \mathrm{m})^2 + (150\ \mathrm{m})^2} \tag{3.4.8}$$

$$|\Delta\vec{r}| = 2108\ \mathrm{m} \tag{3.4.9}$$

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which sounds reasonable!