

2.2: Momentum

For an object of (inertial) mass m moving, in one dimension, with velocity \vec{v} , we define its *momentum* as

$$\vec{p} = m\vec{v} \quad (2.2.1)$$

(the choice of the letter \vec{p} for momentum is apparently related to the Latin word “impetus”).

We can think of momentum as a sort of extension of the concept of inertia, from an object at rest to an object in motion. When we speak of an object’s inertia, we typically think about what it may take to get it moving; when we speak of its momentum, we typically think of that it may take to stop it (or perhaps deflect it). So, both the inertial mass m and the velocity \vec{v} are involved in the definition.

We may also observe that what looks like inertia in some reference frame may look like momentum in another. For instance, if you are driving in a car towing a trailer behind you, the trailer has only a large amount of inertia, but no momentum, relative to you, because its velocity relative to you is zero; however, the trailer definitely has a large amount of momentum (by virtue of both its inertial mass and its velocity) relative to somebody standing by the side of the road.

2.2.1: Conservation of Momentum; Isolated Systems

For a system of objects, we treat the momentum as an *additive* quantity. So, if two colliding objects, of masses m_1 and m_2 , have initial velocities \vec{v}_{1i} and \vec{v}_{2i} , we say that the total initial momentum of the system is $\vec{p}_i = m_1\vec{v}_{1i} + m_2\vec{v}_{2i}$, and similarly if the final velocities are \vec{v}_{1f} and \vec{v}_{2f} , the total final momentum will be $\vec{p}_f = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$.

We then assert that *the total momentum of the system is not changed by the collision*. Mathematically, this means

$$\vec{p}_i = \vec{p}_f \quad (2.2.2)$$

or

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}. \quad (2.2.3)$$

But this last equation, in fact, follows directly from Equation (2.1.2): to see this, move all the quantities in Equation (2.2.3) having to do with object 1 to one side of the equal sign, and those having to do with object 2 to the other side. You then get

$$\begin{aligned} m_1(\vec{v}_{1i} - \vec{v}_{1f}) &= m_2(\vec{v}_{2f} - \vec{v}_{2i}) \\ -m_1\Delta\vec{v}_1 &= m_2\Delta\vec{v}_2 \end{aligned} \quad (2.2.4)$$

which is just another way to write Equation (2.1.2). Hence, the result (2.1.2) ensures the conservation of the total momentum of a system of any two interacting objects (“particles”), regardless of the form the interaction takes, as long as there are no external forces acting on them.

Momentum conservation is one of the most important principles in all of physics, so let me take a little time to explain how we got here and elaborate on this result. First, as I just mentioned, we have been more or less implicitly assuming that the two interacting objects form an *isolated* system, by which we mean that, throughout, they interact with nothing other than each other. (Equivalently, there are no external forces acting on them.)

It is pretty much impossible to set up a system so that it is *really* isolated in this strict sense; instead, in practice, we settle for making sure that the external forces on the two objects *cancel out*. This is what happens on the air tracks with which you will be doing experiments this semester: gravity is acting on the carts, but that force is balanced out by the upwards push of the air from the track. A system on which there is no *net* external force is as good as isolated for practical purposes, and we will refer to it as such. (It is harder, of course, to completely eliminate friction and drag forces, so we just have to settle for approximately isolated systems in practice.)

Secondly, we have assumed so far that the motion of the two objects is restricted to a straight line—one dimension. In fact, momentum is a *vector* quantity (just like velocity is), which is why we have been writing the previous equations with the small arrows over them.

Conservation of momentum, in general, holds as a vector equation for any isolated system in three dimensions:

$$\vec{p}_i = \vec{p}_f. \quad (2.2.5)$$

What this means, in turn, is that each separate component (x , y and z) of the momentum will be separately conserved (so Equation (2.2.5) is equivalent to three scalar equations, in three dimensions).

Thirdly, although this may not be immediately obvious, for an isolated system of two colliding objects the momentum is truly conserved throughout the whole collision process. It is not just a matter of comparing the initial and final velocities: at any of the times shown in Figures 2.1.1, 2.1.2, or 2.1.3, if we were to measure v_1 and v_2 and compute $m_1v_1 + m_2v_2$, we would obtain the same result. In other words, the total momentum of an isolated system is *constant*: it has the same value at all times.

Finally, all these examples have involved interactions between only two particles. Can we really generalize this to conclude that the total momentum of an isolated system of any number of particles is constant, even when all the particles may be interacting with each other simultaneously? Here, again, the experimental evidence is overwhelmingly in favor of this hypothesis⁴, but much of our confidence on its validity comes in fact from a consideration of the nature of the internal interactions themselves. It is a mathematical fact that all of the interactions so far known to physics have the property of conserving momentum, whether acting individually or simultaneously. No experiments have ever suggested the existence of an interaction that does not have this property.

⁴For an important piece of indirect evidence, just consider that any extended object is in reality a collection of interacting particles, and the experiments establishing conservation of momentum almost always involve such extended objects. See the following section for further thoughts on this matter.

This page titled 2.2: Momentum is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Julio Gea-Banacloche (University of Arkansas Libraries) via source content that was edited to the style and standards of the LibreTexts platform.

- 3.2: Momentum by Julio Gea-Banacloche is licensed CC BY-SA 4.0. Original source: <https://scholarworks.uark.edu/oer/3>.