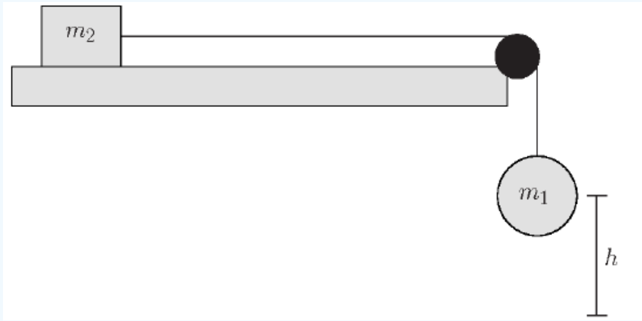
## 8.9: Examples

**Whiteboard Problem 8.9.1: Jupiter Collision**




The figure shows a Half-Atwood machine, with a block of mass $m_2$ = 6.0 kg sliding across a frictionless surface, attached to a hanging mass $m_1$ = 350 g by a string going over a pulley.

1. What is the speed of the sliding block when the hanging mass falls 73 cm? Assume the system starts at rest.
2. Now assume that this system is not actually isolated, but is losing energy due to the contact interaction between the ramp and the block. This energy loss depends on the mass of the block, the distance $d$ that the block slides, and a proportionality constant $\gamma$, like

$$\Delta E = -\gamma m d \tag{8.9.1}$$

If this constant is experimentally found to be $\gamma = 0.35$ m/s$^2$, how fast is the block moving if it falls the same distance?

**Whiteboard Problem 8.9.2: Destroy the Moon!**

A mad scientist wants to destroy the Moon! He constructs a cannon that fires 1500 kg cannonballs, and needs to determine how fast the cannon must be fired to hit the Moon.

1. For his first calculation, he uses the potential energy due to gravity as $U_g = mgh$, and determines the minimum launch speed of the cannonballs to reach the Moon, at a distance of $d_M = 3.84 \times 10^8$ m. What speed does he calculate?
2. What will the initial kinetic energy of the cannonball be?
3. His assistant realizes that since the cannonball is going to be traveling far from the Earth, they must use

$$U_G = -G\frac{m_1 m_2}{r} \tag{8.9.2}$$

for the potential energy due to gravity. When the assistant redoes the calculation, what speed do they get?
4. What will the initial kinetic energy of the cannonball actually have to be?

### Example 8.9.1: Inelastic Collision in the middle of a swing

Tarzan swings on a vine to rescue a helpless explorer (as usual) from some attacking animal or another. He begins his swing from a branch a height of 15 m above the ground, grabs the explorer at the bottom of his swing, and continues the swing, upwards this time, until they both land safely on another branch. Suppose that Tarzan weighs 90 kg and the explorer weighs 70, and that Tarzan doesn't just drop from the branch, but pushes himself off so that he starts the swing with a speed of 5 m/s. How high a branch can he and the explorer reach?

**Solution**

Let us break this down into parts. The first part of the swing involves the conversion of some amount of initial gravitational potential energy into kinetic energy. Then comes the collision with the explorer, which is completely inelastic and we can analyze using conservation of momentum (assuming Tarzan and the explorer form an isolated system for the brief time the collision lasts). After that, the second half of their swing involves the complete conversion of their kinetic energy into gravitational potential energy.

Let $m_1$ be Tarzan's mass, $m_2$ the explorer's mass, $h_i$ the initial height, and $h_f$ the final height. We also have three velocities to worry about (or, more properly in this case, speeds, since their direction is of no concern, as long as they all point the way they are supposed to): Tarzan's initial velocity at the beginning of the swing, which we may call $v_{top}$; his velocity at the bottom of the swing, just before he grabs the explorer, which we may call $v_{bot1}$, and his velocity just after he grabs the explorer, which we may call $v_{bot2}$. (If you find those subscripts confusing, I am sorry, they are the best I could do; please feel free to make up your own.)

- *First part: the downswing*. We apply conservation of energy, in the form Equation (8.2.2), to the first part of the swing. The system we consider consists of Tarzan and the earth, and it has kinetic energy as well as gravitational potential energy. We ignore the source energy and the dissipated energy terms, and consider the system closed despite the fact that Tarzan is holding onto a vine (as we shall see in a couple of chapters, the vine does no "work" on Tarzan—meaning, it does not change his energy, only his direction of motion—because the force it exerts on Tarzan is always perpendicular to his displacement):

$$K_{top} + U^G_{top} = K_{bot1} + U^G_{bot}. \tag{8.9.3}$$

In terms of the quantities I introduced above, this equation becomes:

$$\frac{1}{2} m_1 v^2_{top} + m_1 g h_i = \frac{1}{2} m_1 v^2_{bot1} + 0$$

which can be solved to give

$$v^2_{bot1} = v^2_{top} + 2 g h_i \tag{8.9.4}$$

Substituting, we get

$$v_{bot1} = \sqrt{\left(5 \ \frac{\mathrm{m}}{\mathrm{s}}\right)^2 + 2\left(9.8 \ \frac{\mathrm{m}}{\mathrm{s}^2}\right) \times (15 \ \mathrm{m})} = 17.9 \ \frac{\mathrm{m}}{\mathrm{s}}$$

- *Second part: the completely inelastic collision*. The explorer is initially at rest (we assume he has not seen the wild beast ready to pounce yet, or he has seen it and he is paralyzed by fear!). After Tarzan grabs him they are moving together with a speed $v_{bot2}$. Conservation of momentum gives

$$m_1 v_{bot1} = (m_1 + m_2) v_{bot2} \tag{8.9.5}$$

which we can solve to get

$$v_{bot2} = \frac{m_1 v_{bot1}}{m_1 + m_2} = \frac{(90 \ \mathrm{kg}) \times (17.9 \ \mathrm{m/s})}{160 \ \mathrm{kg}} = 10 \ \frac{\mathrm{m}}{\mathrm{s}}$$

- *Third part: the upswing*. Here we use again conservation of energy in the form

$$K_{bot2} + U^G_{bot} = K_f + U^G_f \tag{8.9.6}$$

where the subscript *f* refers to the very end of the swing, when they both safely reach their new branch, and all their kinetic energy has been converted to gravitational potential energy, so $K_f$ = 0 (which means that is as high as they can go, unless they start climbing the vine!). This equation can be rewritten as

$$\frac{1}{2}(m_1 + m_2) v^2_{bot2} + 0 = 0 + (m_1 + m_2) g h_f$$

and solving for $h_f$ we get

$$h_f = \frac{v_{bot2}^2}{2g} = \frac{(10 \text{ m/s})^2}{2 \times 9.8 \text{ m/s}^2} = 5.15 \text{ m}$$

### Example 8.9.2: Momentum and Energy Review: A Bouncing Superball

A superball of mass 0.25 kg is dropped from rest from a height of h = 1.50 m above the floor. It bounces with no loss of energy and returns to its initial height (Figure 8.9.2).

a. What is the superball's change of momentum during its bounce on the floor?
b. What was Earth's change of momentum due to the ball colliding with the floor?
c. What was Earth's change of velocity as a result of this collision?

(This example shows that you have to be careful about defining your system.)

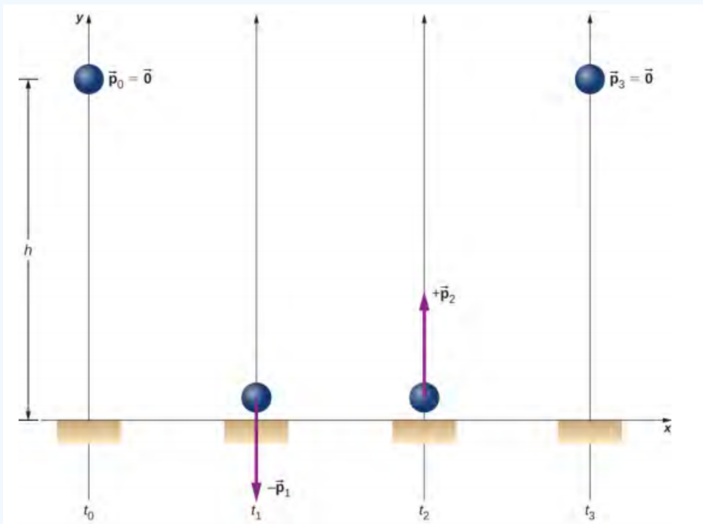



Figure 8.9.2: A superball is dropped to the floor ($t_0$), hits the floor ($t_1$), bounces ($t_2$), and returns to its initial height ($t_3$).

**Strategy**

Since we are asked only about the ball's change of momentum, we define our system to be the ball. But this is clearly not a closed system; gravity applies a downward force on the ball while it is falling, and the normal force from the floor applies a force during the bounce. Thus, we cannot use conservation of momentum as a strategy. Instead, we simply determine the ball's momentum just before it collides with the floor and just after, and calculate the difference. We have the ball's mass, so we need its velocities.

**Solution**

a. Since this is a one-dimensional problem, we use the scalar form of the equations. Let:
   - $p_0$ = the magnitude of the ball's momentum at time $t_0$, the moment it was released; since it was dropped from rest, this is zero.
   - $p_1$ = the magnitude of the ball's momentum at time $t_1$, the instant just before it hits the floor.
   - $p_2$ = the magnitude of the ball's momentum at time $t_2$, just after it loses contact with the floor after the bounce.

The ball's change of momentum is

$$\begin{aligned} \Delta\vec{p} &= \vec{p}_2 - \vec{p}_1 \\ &= p_2\ \hat{j} - (-p_1\ \hat{j}) \\ &= (p_2 + p_1)\hat{j}. \end{aligned}$$

Its velocity just before it hits the floor can be determined from either conservation of energy or kinematics. We use kinematics here; you should re-solve it using conservation of energy and confirm you get the same result.

We want the velocity just before it hits the ground (at time $t_1$). We know its initial velocity $v_0 = 0$ (at time $t_0$), the height it falls, and its acceleration; we don't know the fall time. We could calculate that, but instead we use

$$\vec{v}_1 = -\hat{j}\sqrt{2gy} = -5.4\ m/s\ \hat{j}.$$

Thus the ball has a momentum of

$$\begin{aligned} \vec{p}_1 &= -(0.25\ kg)(-5.4\ m/s\ \hat{j}) \\ &= -(1.4\ kg \cdot m/s)\hat{j}. \end{aligned}$$

We don't have an easy way to calculate the momentum after the bounce. Instead, we reason from the symmetry of the situation.

Before the bounce, the ball starts with zero velocity and falls 1.50 m under the influence of gravity, achieving some amount of momentum just before it hits the ground. On the return trip (after the bounce), it starts with some amount of momentum, rises the same 1.50 m it fell, and ends with zero velocity. Thus, the motion after the bounce was the mirror image of the motion before the bounce. From this symmetry, it must be true that the ball's momentum after the bounce must be equal and opposite to its momentum before the bounce. (This is a subtle but crucial argument; make sure you understand it before you go on.) Therefore,

$$\vec{p}_2 = -\vec{p}_1 = +(1.4\ kg \cdot m/s)\hat{j}.$$

Thus, the ball's change of momentum during the bounce is

$$\begin{aligned} \Delta\vec{p} &= \vec{p}_2 - \vec{p}_1 \\ &= (1.4\ kg \cdot m/s)\hat{j} - (-1.4\ kg \cdot m/s)\hat{j} \\ &= +(2.8\ kg \cdot m/s)\hat{j}. \end{aligned}$$

b. What was Earth's change of momentum due to the ball colliding with the floor? Your instinctive response may well have been either "zero; the Earth is just too massive for that tiny ball to have affected it" or possibly, "more than zero, but utterly negligible." But no—if we re-define our system to be the Superball + Earth, then this system is closed (neglecting the gravitational pulls of the Sun, the Moon, and the other planets in the solar system), and therefore the total change of momentum of this new system must be zero. Therefore, Earth's change of momentum is exactly the same magnitude: $\Delta \vec{p}_{Earth} = -2.8\; kg\; \cdotp m/s\; \hat{j} \ldotp$
c. What was Earth's change of velocity as a result of this collision? This is where your instinctive feeling is probably correct:

$$\begin{aligned} \Delta\vec{v}_{Earth} &= \frac{\Delta\vec{p}_{Earth}}{M_{Earth}} \\ &= -\frac{2.8\ kg \cdot m/s}{5.97 \times 10^{24}\ kg}\ \hat{j} \\ &= -(4.7 \times 10^{-25}\ m/s)\hat{j}. \end{aligned}$$

This change of Earth's velocity is utterly negligible

**Significance**

It is important to realize that the answer to part (c) is not a velocity; it is a change of velocity, which is a very different thing. Nevertheless, to give you a feel for just how small that change of velocity is, suppose you were moving with a velocity of 4.7 x $10^{-25}$ m/s. At this speed, it would take you about 7 million years to travel a distance equal to the diameter of a hydrogen atom.

## Exercise 8.9.4

Would the ball's change of momentum have been larger, smaller, or the same, if it had collided with the floor and stopped (without bouncing)? Would the ball's change of momentum have been larger, smaller, or the same, if it had collided with the floor and stopped (without bouncing)?