# 3.2: Vector Algebra in 1 Dimension

## Learning Objectives

- Explain the effect of multiplying a vector quantity by a scalar.
- Describe how one-dimensional vector quantities are added or subtracted.
- Explain the geometric construction for the addition or subtraction of vectors in a plane.
- Distinguish between a vector equation and a scalar equation.

Suppose your friend walks from the campsite at A to the fishing pond at B and then walks back: from the fishing pond at B to the campsite at A. The magnitude of the displacement vector $\vec{D}_{AB}$ from A to B is the same as the magnitude of the displacement vector $\vec{D}_{BA}$ from B to A (it equals 6 km in both cases), so we can write $\vec{D}_{AB} = \vec{D}_{BA}$. However, vector $\vec{D}_{AB}$ is not equal to vector $\vec{D}_{BA}$ because these two vectors have different directions: $\vec{D}_{AB} \neq \vec{D}_{BA}$. In Figure 2.3, vector $\vec{D}_{BA}$ would be represented by a vector with an origin at point B and an end at point A, indicating vector $\vec{D}_{BA}$ points to the southwest, which is exactly 180° opposite to the direction of vector $\vec{D}_{AB}$. We say that vector $\vec{D}_{BA}$ is **antiparallel** to vector $\vec{D}_{AB}$ and write $\vec{D}_{AB} = -\vec{D}_{BA}$, where the minus sign indicates the antiparallel direction.

Two vectors that have identical directions are said to be **parallel vectors**—meaning, they are **parallel** to each other. Two parallel vectors $\vec{A}$ and $\vec{B}$ are equal, denoted by $\vec{A} = \vec{B}$, if and only if they have equal magnitudes $|\vec{A}| = |\vec{B}|$. Two vectors with directions perpendicular to each other are said to be **orthogonal vectors**. These relations between vectors are illustrated in Figure $3.2.4$.

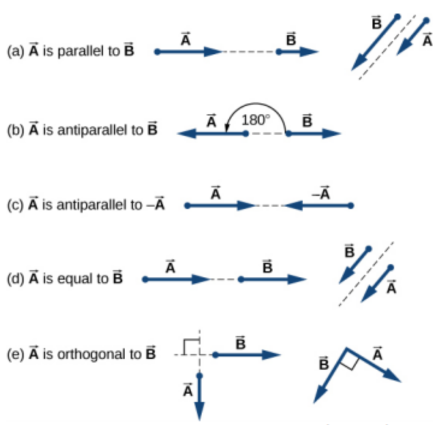


Figure $3.2.4$: Various relations between two vectors $\vec{A}$ and $\vec{B}$. (a) $\vec{A} \neq \vec{B}$ because $A \neq B$ . (b) $\vec{A} \neq \vec{B}$ because they are not parallel and $A \neq B$ . (c) $\vec{A} \neq -\vec{A}$ because they have different directions (even though $|\vec{A}| = |-\vec{A}| = A)$ . (d) $\vec{A} = \vec{B}$ because they are parallel and have identical magnitudes A = B. (e) $\vec{A} \neq \vec{B}$ because they have different directions (are not parallel); here, their directions differ by 90° —meaning, they are orthogonal.

## Exercise 2.1

Two motorboats named **Alice** and **Bob** are moving on a lake. Given the information about their velocity vectors in each of the following situations, indicate whether their velocity vectors are equal or otherwise.

a. **Alice** moves north at 6 knots and **Bob** moves west at 6 knots.
b. **Alice** moves west at 6 knots and **Bob** moves west at 3 knots.
c. **Alice** moves northeast at 6 knots and **Bob** moves south at 3 knots.
d. **Alice** moves northeast at 6 knots and **Bob** moves southwest at 6 knots.
e. **Alice** moves northeast at 2 knots and **Bob** moves closer to the shore northeast at 2 knots.

## Algebra of Vectors in One Dimension

Vectors can be multiplied by scalars, added to other vectors, or subtracted from other vectors. We can illustrate these vector concepts using an example of the fishing trip seen in Figure $3.2.5$.

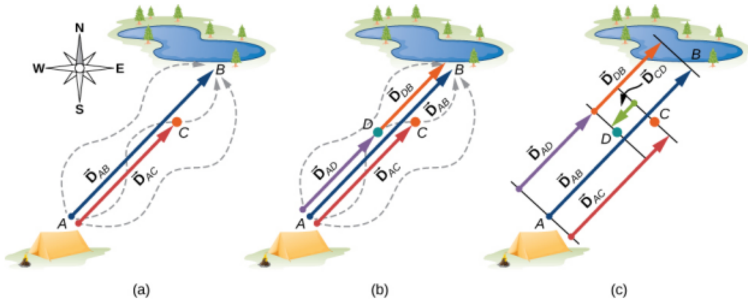


Figure 3.2.5: Displacement vectors for a fishing trip. (a) Stopping to rest at point C while walking from camp (point A) to the pond (point B). (b) Going back for the dropped tackle box (point D). (c) Finishing up at the fishing pond.

Suppose your friend departs from point A (the campsite) and walks in the direction to point B (the fishing pond), but, along the way, stops to rest at some point C located three-quarters of the distance between A and B, beginning from point A (Figure 3.2.5$a$). What is his displacement vector $\vec{D}_{AC}$ when he reaches point C? We know that if he walks all the way to B, his displacement vector relative to A is $\vec{D}_{AB}$, which has magnitude $D_{AB}$ = 6 km and a direction of northeast. If he walks only a 0.75 fraction of the total distance, maintaining the northeasterly direction, at point C he must be 0.75 $D_{AB}$ = 4.5 km away from the campsite at A. So, his displacement vector at the rest point C has magnitude $D_{AC}$ = 4.5 km = 0.75 $D_{AB}$ and is parallel to the displacement vector $\vec{D}_{AB}$. All of this can be stated succinctly in the form of the following **vector equation**:

$$\vec{D}_{AC} = 0.75\ \vec{D}_{AB}.$$

In a vector equation, both sides of the equation are vectors. The previous equation is an example of a vector multiplied by a positive scalar (number) $\alpha$ = 0.75. The result, $\vec{D}_{AC}$, of such a multiplication is a new vector with a direction parallel to the direction of the original vector $\vec{D}_{AB}$. In general, when a vector $\vec{D}_A$ is multiplied by a positive scalar $\alpha$, the result is a new vector $\vec{D}_B$ that is parallel to $\vec{D}_A$:

$$\vec{B} = \alpha \vec{A} \tag{3.2.1}$$

The magnitude $|\vec{B}|$ of this new vector is obtained by multiplying the magnitude $|\vec{A}|$ of the original vector, as expressed by the **scalar equation**:

$$B = |\alpha| A. \tag{3.2.2}$$

In a scalar equation, both sides of the equation are numbers. Equation 3.2.2 is a scalar equation because the magnitudes of vectors are scalar quantities (and positive numbers). If the scalar $\alpha$ is **negative** in the vector equation Equation 3.2.1, then the magnitude $|\vec{B}|$ of the new vector is still given by Equation 3.2.2, but the direction of the new vector $\vec{B}$ is **antiparallel** to the direction of $\vec{A}$. These principles are illustrated in Figure 3.2.6$a$ by two examples where the length of vector $\vec{A}$ is 1.5 units. When $\alpha$ = 2, the new vector $\vec{B} = 2\vec{A}$ has length B = 2A = 3.0 units (twice as long as the original vector) and is parallel to the original vector. When $\alpha$ = −2, the new vector $\vec{C} = -2\vec{A}$ has length C = |−2| A = 3.0 units (twice as long as the original vector) and is antiparallel to the original vector.

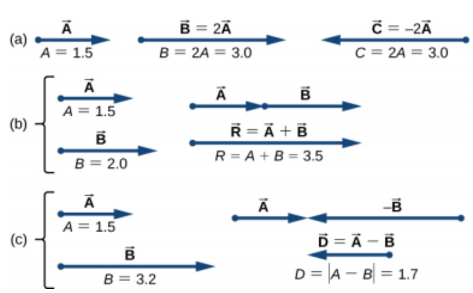


Figure 3.2.6: Algebra of vectors in one dimension. (a) Multiplication by a scalar. (b) Addition of two vectors ($\vec{R}$ is called the resultant of vectors ($\vec{A}$ and ($\vec{B}$). (c) Subtraction of two vectors ($\vec{D}$ is the difference of vectors ($\vec{A}$ and $\vec{B}$).

Now suppose your fishing buddy departs from point A (the campsite), walking in the direction to point B (the fishing hole), but he realizes he lost his tackle box when he stopped to rest at point C (located three-quarters of the distance between A and B, beginning from point A). So, he turns back and retraces his steps in the direction toward the campsite and finds the box lying on the path at some point D only 1.2 km away from point C (see Figure $3.2.5b$). What is his displacement vector $\vec{D}_{AD}$ when he finds the box at point D? What is his displacement vector $\vec{D}_{DB}$ from point D to the hole? We have already established that at rest point C his displacement vector is $\vec{D}_{AC}$ = 0.75 $\vec{D}_{AB}$. Starting at point C, he walks southwest (toward the campsite), which means his new displacement vector $\vec{D}_{CD}$ from point C to point D is antiparallel to $\vec{D}_{AB}$. Its magnitude $|\vec{D}_{CD}|$ is $D_{CD}$ = 1.2 km = 0.2 $D_{AB}$, so his second displacement vector is $\vec{D}_{CD}$ = −0.2 $\vec{D}_{AB}$. His total displacement $\vec{D}_{AD}$ relative to the campsite is the vector sum of the two displacement vectors: vector $\vec{D}_{AC}$ (from the campsite to the rest point) and vector $\vec{D}_{CD}$ (from the rest point to the point where he finds his box):

$$\vec{D}_{AD} = \vec{D}_{AC} + \vec{D}_{CD}. \tag{3.2.3}$$

The vector sum of two (or more vectors is called the **resultant vector** or, for short, the **resultant**. When the vectors on the right-hand-side of Equation 3.2.3 are known, we can find the resultant $\vec{D}_{AD}$ as follows:

$$\vec{D}_{AD} = \vec{D}_{AC} + \vec{D}_{CD} = 0.75\ \vec{D}_{AB} - 0.2\ \vec{D}_{AB} = (0.75 - 0.2)\vec{D}_{AB} = 0.55\vec{D}_{AB}. \tag{3.2.4}$$

When your friend finally reaches the pond at B, his displacement vector $\vec{D}_{AB}$ from point A is the vector sum of his displacement vector $\vec{D}_{AD}$ from point A to point D and his displacement vector $\vec{D}_{DB}$ from point D to the fishing hole: $\vec{D}_{AB}$ = $\vec{D}_{AD}$ + $\vec{D}_{DB}$ (see Figure $3.2.5c$). This means his displacement vector $\vec{D}_{DB}$ is the difference of two vectors:

$$\vec{D}_{DB} = \vec{D}_{AB} - \vec{D}_{AD} = \vec{D}_{AB} + (-\vec{D}_{AD}). \tag{3.2.5}$$

Notice that a difference of two vectors is nothing more than a vector sum of two vectors because the second term in Equation 3.2.5 is vector $-\vec{D}_{AD}$ (which is antiparallel to $\vec{D}_{AD}$). When we substitute Equation 3.2.4 into Equation 3.2.5, we obtain the second displacement vector:

$$\vec{D}_{DB} = \vec{D}_{AB} - \vec{D}_{AD} = \vec{D}_{AB} - 0.55\ \vec{D}_{AB} = (1.0 - 0.55)\ \vec{D}_{AB} = 0.45\ \vec{D}_{AB}. \tag{3.2.6}$$

This result means your friend walked $D_{DB}$ = 0.45 $D_{AB}$ = 0.45(6.0 km) = 2.7 km from the point where he finds his tackle box to the fishing hole.

When vectors $\vec{A}$ and $\vec{B}$ lie along a line (that is, in one dimension), such as in the camping example, their resultant $\vec{R}$ = $\vec{A}$ + $\vec{B}$ and their difference $\vec{D}$ = $\vec{A}$ − $\vec{B}$ both lie along the same direction. We can illustrate the addition or subtraction of vectors by drawing the corresponding vectors to scale in one dimension, as shown in Figure $3.2.6$.

To illustrate the resultant when $\vec{A}$ and $\vec{B}$ are two parallel vectors, we draw them along one line by placing the origin of one vector at the end of the other vector in head-to-tail fashion (see Figure (\PageIndex{6b}\)). The magnitude of this resultant is the sum of their magnitudes: R = A + B. The direction of the resultant is parallel to both vectors. When vector $\vec{A}$ is antiparallel to vector $\vec{B}$, we draw them along one line in either head-to-head fashion (Figure (\PageIndex{6c}\)) or tail-to-tail fashion. The magnitude of the vector difference, then, is the **absolute value** D = |A − B| of the difference of their magnitudes. The direction of the difference vector $\vec{D}$ is parallel to the direction of the longer vector.

In general, in one dimension—as well as in higher dimensions, such as in a plane or in space—we can add any number of vectors and we can do so in any order because the addition of vectors is **commutative**,

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}. \tag{3.2.7}$$

and **associative,**

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C}). \tag{3.2.8}$$

Moreover, multiplication by a scalar is **distributive**:

$$\alpha_1\vec{A} + \alpha_2\vec{A} = (\alpha_1 + \alpha_2)\vec{A}. \tag{3.2.9}$$

We used the distributive property in Equation 3.2.4 and Equation 3.2.6.