5.1: Conservation of Linear Momentum

Learning Objectives

- Explain the meaning of "conservation of momentum"
- Correctly identify if a system is, or is not, closed
- Define a system whose momentum is conserved
- Mathematically express conservation of momentum for a given system
- Calculate an unknown quantity using conservation of momentum

Recall what we learned about momentum transfer in Chapter 2: when objects interact with each other (like in a collision), they do so by *transferring momentum between each other*. What's more, this momentum is conserved - in that chapter, we expressed this conservation by noting the amount that one object lost was the same that the other object gained. Mathematically, we could write this as

$$\Delta \vec{p}_1 = -\Delta \vec{p}_2, \tag{5.1.1}$$

for two objects. If this collison happens over some time period Δt, we can write this change as rate of change,

$$\frac{\Delta \vec{p}_1}{\Delta t} = -\frac{\Delta \vec{p}_2}{\Delta t}. \tag{5.1.2}$$

So now one object gains or looses momentum as the same rate that the other looses or gains it. Now performing some simple manipulations on this expression:

$$\frac{\Delta \vec{p}_1}{\Delta t} = -\frac{\Delta \vec{p}_2}{\Delta t} \rightarrow \frac{\Delta \vec{p}_1}{\Delta t} + \frac{\Delta \vec{p}_2}{\Delta t} = 0 \rightarrow \frac{\Delta \vec{p}_1 + \Delta \vec{p}_2}{\Delta t} = 0. \tag{5.1.3}$$

Using the definition of Δ, the top of this expression can be written as

$$\Delta \vec{p}_1 + \Delta \vec{p}_2 = (\vec{p}_{1f} - \vec{p}_{1i}) + (\vec{p}_{2f} - \vec{p}_{2i}) = (\vec{p}_{1f} + \vec{p}_{2f}) - (\vec{p}_{1i} + \vec{p}_{2i}). \tag{5.1.4}$$

Now looking at this expression, if we now define the total momentum of the system to be $\vec{P}_{sys} = \vec{p}_1 + \vec{p}_2$, we can see that what we just wrote was

$$\vec{P}_{sys,f} - \vec{P}_{sys,i}, \tag{5.1.5}$$

and combined with equation 5.1.3, we see this gives us

$$\vec{P}_{sys,f} - \vec{P}_{sys,i} = 0 \rightarrow \Delta \vec{P}_{sys} = 0. \tag{5.1.6}$$

Or, in other words, the **total momentum of the system does not change in time**. This is the best statement of conservation of momentum we have gotten so far, and is the best one to remember going forward. It does not matter "when" the initial and final states happen, all that matters is the amount of momentum does not change between those initial and final states.

Since momentum is a vector, both the magnitude and direction of this momentum must be conserved, as shown in Figure 5.1.1, the total momentum of the system before and after the collision remains the same, in both magnitude and direction.

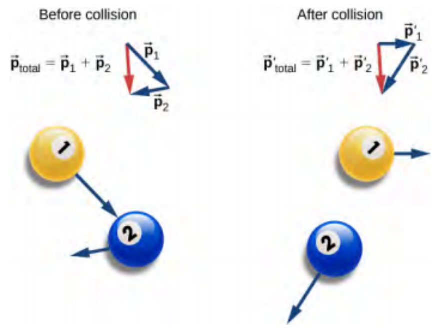

Figure 5.1.1: Before the collision, the two billiard balls travel with momenta $\vec{p}_1$ and $\vec{p}_2$. The total momentum of the system is the sum of these, as shown by the red vector labeled $\vec{p}_{total}$ on the left. After the collision, the two billiard balls travel with different momenta $\vec{p}\,'_1$ and $\vec{p}\,'_2$. The total momentum, however, has not changed, as shown by the red vector arrow $\vec{p}\,'_{total}$ on the right.

Generalizing this result to N objects, we obtain

$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots + \vec{p}_N = constant \tag{5.1.7}$$

$$\sum_{j=1}^{N} \vec{p}_j = constant. \tag{5.1.8}$$

Equation 5.1.8 is the definition of the total (or net) momentum of a system of N interacting objects, along with the statement that the total momentum of a system of objects is constant in time—that is, *momentum is conserved*.

Conservation Laws

If the value of a physical quantity is constant in time, we say that the quantity is conserved.

Requirements for Momentum Conservation

There is a complication, however. A system must meet two requirements for its momentum to be conserved:

1. **The mass of the system must remain constant during the interaction.** As the objects interact (apply forces on each other), they may transfer mass from one to another; but any mass one object gains is balanced by the loss of that mass from another. The total mass of the system of objects, therefore, remains unchanged as time passes:

$$\left[\frac{dm}{dt}\right]_{system} = 0. \tag{5.1.9}$$

2. **The net external force on the system must be zero**. As the objects collide, or explode, and move around, they exert forces on each other. However, all of these forces are internal to the system, and thus each of these internal forces is balanced by another internal force that is equal in magnitude and opposite in sign. As a result, the change in momentum caused by each internal force is cancelled by another momentum change that is equal in magnitude and opposite in direction. Therefore, internal forces cannot change the total momentum of a system because the changes sum to zero. However, if there is some external force that acts on all of the objects (gravity, for example, or friction), then this force changes the momentum of the system as a whole; that is to say, the momentum of the system is changed by the external force. Thus, for the momentum of the system to be conserved, we must have

$$\vec{F}_{ext} = \vec{0}. \tag{5.1.10}$$

A system of objects that meets these two requirements is said to be a **closed system** (also called an isolated system). Another way to state equation 5.1.6 is

Law of Conservation of Momentum

The total momentum of a closed system is conserved:

$$\sum_{j=1}^{N} \vec{p}_j = constant. \tag{5.1.11}$$

This statement is called the **Law of Conservation of Momentum**. Along with the conservation of energy, it is one of the foundations upon which all of physics stands. All our experimental evidence supports this statement: from the motions of galactic clusters to the quarks that make up the proton and the neutron, and at every scale in between. In **a closed system, the total momentum never changes**.

Note that there absolutely **can** be external forces acting on the system; but for the system's momentum to remain constant, these external forces have to cancel, so that the **net** external force is zero. Billiard balls on a table all have a weight force acting on them, but the weights are balanced (canceled) by the normal forces, so there is no net force.

The Meaning of 'System'

A **system** (mechanical) is the collection of objects in whose motion (kinematics and dynamics) you are interested. If you are analyzing the bounce of a ball on the ground, you are probably only interested in the motion of the ball, and not of Earth; thus, the ball is your system. If you are analyzing a car crash, the two cars together compose your system (Figure 5.1.2).

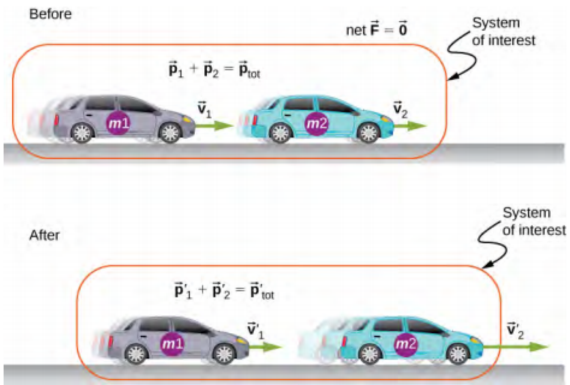

Figure 5.1.2: The two cars together form the system that is to be analyzed. It is important to remember that the contents (the mass) of the system do not change before, during, or after the objects in the system interact.