2.3: Force and Impulse

We have already seen how we can think of interacting objects as objects that transfer momentum between each other. We've written this transfer of momentum as $\Delta\vec{p}$, and we've discussed how this momentum cannot be lost (a phenomena known as **conservation**); if one object gains $\Delta\vec{p}$, the object that it's interacting with must have lost that same amount of momentum. For a two-object system, that means precisely that $\Delta\vec{p}_1 = -\Delta\vec{p}_2$.

Fundamentally, you can imagine momentum as a "currency" that is constantly being exchanged between objects. When two objects interact, one object can gain momentum only by the other losing momentum. There is no way to create "new" momentum. (Unfortunately, in my experience, this is also true with money. I have not yet found a way to increase the amount I have without getting it from some other person or entity.)

We often give a name to the momentum that is exchanged between two objects. It is called "impulse". If object A interacts with object B, A might have a change in momentum, $\Delta\vec{p}_A$ that it has received from object B. This value is the "impulse" that object A has received. We know from conservation of momentum (and our currency analogy) that B will also receive an impulse, $\Delta\vec{p}_B$, that is equal to $-\Delta\vec{p}_A$. ($\Delta\vec{p}_B = -\Delta\vec{p}_A$).

But of course we know that this is not the only way to describe interacting objects - in fact, perhaps the most intuitive way to describe two objects interacting is actually using a **force between them**. We are going to talk a lot more about forces in the second half of the this book, but right now we just want to acknowledge that if interactions can be described with either forces or momentum transfer, there must be some relationship between these two quantities. In fact, the relationship can be made concrete once you know the **time period $\boldsymbol{\Delta t}$** over which the interaction occurs.

Specifically, if you have a change in momentum $\Delta\vec{p}$ (also called an "impulse") resulting from an interaction that happens over a time period Δt, we can associate a force with this interaction via

$$\boxed{\vec{F}_{ave} = \frac{\Delta\vec{p}}{\Delta t}} \tag{2.3.1}$$

Notice that this definition only really works for an *average* force $\vec{F}_{ave}$, since we are talking about the changes in time being possibly relatively large. If the change in time is very small (in the sense of an infinitesimal dt from calculus), we can actually write

$$\vec{F} = \frac{d\vec{p}}{dt} \tag{2.3.2}$$

for the *instantaneous* force, which is the usual notion of force we are used to (this relationship can also be derived from Newton's second law).

All of the above is a perfectly fine description of a force that does not depend on time, which is often what we are dealing with. However, if a force does depend on time (think of throwing a baseball, or a car crash - the force over the time period of those interactions may not be constant at all), we can still find the impulse delivered by performing an integration,

$$\vec{J} = \int \vec{F} dt. \tag{2.3.3}$$

2.3.1: Gravity

We will be looking at many different forces in future chapters, but for now we will just learn one of the most important forces, the force of gravity. Actually, we should be a little more specific. We will learn about a special force which is the force of gravity on (and near) the surface of the Earth. You are probably aware of this force. It is the reason you fall when you trip, why you shouldn't hold your ice cream cone upside down, and (at least partially) why baseball is an interesting sport. It is the force that pulls things down. In fact, I often tell my students, gravity is actually how we know which direction *is* down. "Down" is generally agreed upon as the direction that gravity pulls things.

Close to earth, the force of gravity has a very simple form:

$$\vec{F}_g = m\vec{g} \tag{2.3.4}$$

Where $\vec{g} = 9.8\mathrm{m/s}^2$ is the acceleration due to gravity and points "down" (generally towards the center of the Earth). The S.I. unit for force is a Newton (N).

So what does this mean in terms of momentum and impulse? Let's take a 1 kg weight that is being acted on only by the force of gravity. Using our Equation 2.3.4, we know that the magnitude of force it experiences will be $(1 \text{ kg})(9.8 m/s) = 9.8 \text{ N}$. We know from our Equation 2.3.1 that in every second (1 s), we will have a change of momentum (impulse) of:

$$|\Delta\vec{p}| = |\vec{F}|\Delta t = (9.8 \text{ N})(1 \text{ s}) = 9.8 \mathrm{kg} \cdot \mathrm{m/s} \tag{2.3.5}$$

where I have used absolute value signs around our momentum and force to show that I am just interested in the magnitude (not direction). Besides, we already know that the impulse from the Earth will pull us *down*.

Since we knew from the beginning that we have a 1 kg weight, we can also calculate our change in speed in one second:

$$|\Delta\vec{v}| = |\Delta\vec{p}|/m = 9.8 \text{ m/s} \tag{2.3.6}$$

which, we will find later, we could have discovered directly just by knowing the acceleration due to gravity.

Hopefully, this gives some idea of how momentum, change in momentum, impulse, and force are all related.