

CHAPTER OVERVIEW

10: C10) Work

- 10.1: Introduction- Work and Impulse
- 10.2: Work on a Single Particle
- 10.3: The "Center of Mass Work"
- 10.4: Examples
- 10.E: Work and Kinetic Energy (Exercises)

In the previous chapters, we have been considering the phenomena of *conservation of energy*, which is the idea that the total energy of a closed system does not change in time. We utilized this idea for solving problems by identifying all the interactions in the system, associating them with sources of energy, and setting the initial energy equal to the final energy. This idea is very powerful, but it certainly requires you to know how to associate a potential energy to a particular interaction. So far, we've only done this with two interactions - the force of gravity near the Earth ($U_g = mgh$) and springs ($U_s = \frac{1}{2}m\Delta x^2$). So what happens when you have an interaction whose potential you don't know - or even worse, that doesn't even exist? (That's called a *nonconservative force*, we will cover those in [Chapter N8](#)).

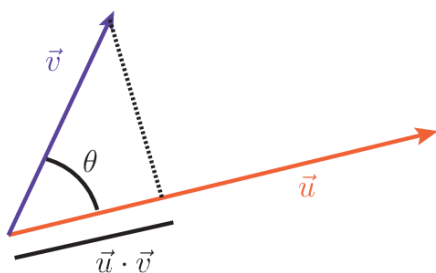
We certainly know that forces can be used to store energy, since that's what both the gravitational force and the spring force do. But what about some other force - say, a force pushing a box along the floor? Does this store energy? It's pretty easy to see that it does, because that box might go from moving to not moving, demonstrating that it now has kinetic energy, which it clearly got from your pushing. It turns out to be pretty easy to describe how much energy is transferred into the system from this force - **that quantity is called work**.

For a constant force, the work is a simple formula,

$$W = \vec{F} \cdot \Delta \vec{r}, \quad (10.1)$$

where \vec{F} is the force and $\Delta \vec{r}$ is the change in position of the center of mass of the object. But notice the mathematical operation being performed here - it's a dot product between two vectors. For any two vectors $\vec{u} = u_x\hat{x} + u_y\hat{y}$ and $\vec{v} = v_x\hat{x} + v_y\hat{y}$, the dot product can be written in two different ways:

$$\vec{u} \cdot \vec{v} = \|\vec{v}\| \|\vec{u}\| \cos \theta, \text{ or } \vec{u} \cdot \vec{v} = u_x v_x + u_y v_y. \quad (10.2)$$



We have to be very careful with that first formula - the angle θ must be *the angle between the vectors \vec{u} and \vec{v} when they are arranged tail to tail (see the figure)*. Generally, if we have a picture, this first formula is the one we want to use. However, sometimes we might just have the components, and then it's much easier to use the second formula. In either case, notice carefully that the result is a scalar - in fact, the result is *the projection of the second vector onto the first*. We won't need that fact much in this class, but it's true geometrically.

So let's consider a few simple examples, focusing on the first of the two formula above. Let's act on our block with a force of 10 N.

1. If we push the block horizontally, a distance 10 m, we get $W = (10 \text{ N})(10 \text{ m}) \cos(0) = 100 \text{ J}$. (Notice the energy units!) The angle was zero here because we are *pushing in the same direction as the motion of the block*.
2. Let's push against the motion of the block - so it's moving forwards 10 m, but we are pushing the other way. Then we get $W = (10 \text{ N})(10 \text{ m}) \cos(180^\circ) = -100 \text{ J}$. Notice how the angle changed - the force vector \vec{F} was 180° from the displacement vector $\Delta \vec{r}$, and the work done was negative.
3. Finally, let's push straight downwards on the block as it moves the same 10 m. The work is now $W = (10 \text{ N})(10 \text{ m}) \cos(90^\circ) = 0$. The work done is zero here, because if you push straight downwards (and there is no friction), you can't change the energy of the block!