9.1: Potential Energy of a System

Types of Potential Energy

For each type of interaction present in a system, you can label a corresponding type of potential energy. The total potential energy of the system is the sum of the potential energies of all the types. Let's look at some particular examples of two important types of potential energy.

Gravitational Potential Energy Near Earth's Surface

The system of interest consists of our planet, Earth, and one or more particles near its surface (or bodies small enough to be considered as particles, compared to Earth). The gravitational force on each particle (or body) is just its weight *mg* near the surface of Earth, acting vertically down. As the Earth pulls downward on the object, the object is also pulling upwards on the Earth (that's Newton's third law, which we will cover in the second half of this text). However, this force of the object on the Earth is very small, so we will generally ignore it. Therefore, we consider this system to be a group of single-particle systems, subject to the uniform gravitational force of Earth.

As we've seen in the previous chapter, the gravitational potential energy function, near Earth's surface, is

$$U_g(y) = mgy \tag{9.1.1}$$

A particularly important aspect of this formula is the choice of $y=0$. This is a coordinate system choice, but it's more significant because it means you can *choose were the zero of gravitational potential energy is*. We can see this is true by considering the following rewriting of the conservation of energy formula:

$$E_f - E_i = 0 \rightarrow (K_f + U_f) - (K_i + U_i) = 0 \rightarrow (K_f - K_i) + (U_f - U_i) = 0 \rightarrow \Delta K + \Delta U = 0 \rightarrow \Delta K = -\Delta U. \tag{9.1.2}$$

That last expression tells us what we care about - the change of the kinetic energy only depends on the change in the potential, not the absolute value. An object loosing 10 J of potential with always gain 10 J of kinetic, no matter if it started with 10 J and went to 0 J or started with 1600 J and went to 1590 J. Since this is a choice you can make, usually it's possible to make an "easy" choice; this is often when "the height of the object is zero", leading to the specific form of the expression 9.1.1.

Example 9.1.2: Gravitational PotentIAL Energy of a hiker

The summit of Great Blue Hill in Milton, MA, is 147 m above its base and has an elevation above sea level of 195 m (Figure 9.1.2). (Its Native American name, *Massachusett*, was adopted by settlers for naming the Bay Colony and state near its location.) A 75-kg hiker ascends from the base to the summit. What is the gravitational potential energy of the hiker-Earth system with respect to zero gravitational potential energy at base height, when the hiker is (a) at the base of the hill, (b) at the summit, and (c) at sea level, afterward?

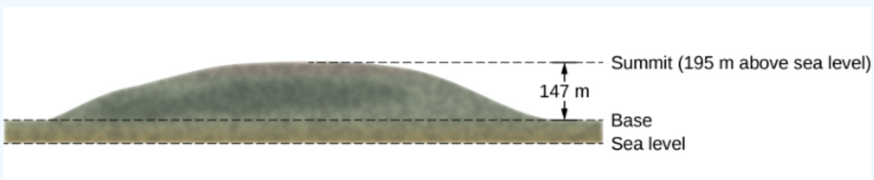

Figure 9.1.2: Sketch of the profile of Great Blue Hill, Milton, MA. The altitudes of the three levels are indicated.

Strategy

First, we need to pick an origin for the *y*-axis and then determine the value of the constant that makes the potential energy zero at the height of the base. Then, we can determine the potential energies from Equation 9.1.1, based on the relationship between the zero potential energy height and the height at which the hiker is located.

Solution

a. Let's choose the origin for the *y*-axis at base height, where we also want the zero of potential energy to be. This choice makes the constant equal to zero and

$$U(\text{ base }) = U(0) = 0$$

b. At the summit, y = 147 m, so

$$U(\text{ summit }) = U(147\text{ m}) = mgh = (75 \times 9.8\text{ N})(147\text{ m}) = 108\text{ kJ}.$$

c. At sea level, y = (147 - 195) m = -48 m, so

$$U \text{ (sea-level) } = (75 \times 9.8\text{N})(-48\text{m}) = -35.3\text{kJ}.$$

Significance

Besides illustrating the use of Equation 9.1.1, the values of gravitational potential energy we found are reasonable. The gravitational potential energy is higher at the summit than at the base, and lower at sea level than at the base. The numerical values of the potential energies depend on the choice of zero of potential energy, but the physically meaningful differences of potential energy do not.

Exercise 9.1.2

What are the values of the gravitational potential energy of the hiker at the base, summit, and sea level, with respect to a sea-level zero of potential energy?

Elastic Potential Energy

The next form of potential energy we are going to look at is the energy stored in a spring, or the elastic potential energy. A spring is an object which be compressed a distance x (from equilibrium) or stretched a distance x (again, from equilibrium). In either case, the potential energy stored in such an object is

$$U_s(x) = \frac{1}{2}kx^2 \tag{9.1.3}$$

If the spring force is the only force acting, it is simplest to take the zero of potential energy at x = 0, when the spring is at its unstretched length - this has actually been done in the previous expression.

Example 9.1.3: Spring Potential Energy

A system contains a perfectly elastic spring, with an unstretched length of 20 cm and a spring constant of 4 N/cm. (a) How much elastic potential energy does the spring contribute when its length is 23 cm? (b) How much more potential energy does it contribute if its length increases to 26 cm?

Strategy

When the spring is at its unstretched length, it contributes nothing to the potential energy of the system, so we can use Equation 9.1.3 with the constant equal to zero. The value of *x* is the length minus the unstretched length. When the spring is expanded, the spring's displacement or difference between its relaxed length and stretched length should be used for the *x*-value in calculating the potential energy of the spring.

Solution

a. The displacement of the spring is x = 23 cm − 20 cm = 3 cm, so the contributed potential energy is $U = \frac{1}{2}kx^2 = \frac{1}{2}(4\text{ N/cm})(3\text{ cm})^2 = 0.18\text{ J}$.
b. When the spring's displacement is x = 26 cm − 20 cm = 6 cm, the potential energy is $U = \frac{1}{2}kx^2 = \frac{1}{2}(4\text{ N/cm})(6\text{ cm})^2 = 0.72\text{ J}$, which is a 0.54-J increase over the amount in part (a).

Significance

Calculating the elastic potential energy and potential energy differences from Equation 9.1.3 involves solving for the potential energies based on the given lengths of the spring. Since *U* depends on x^2, the potential energy for a compression (negative *x*) is the same as for an extension of equal magnitude.

Exercise 9.1.3

When the length of the spring in Example 8.2.3 changes from an initial value of 22.0 cm to a final value, the elastic potential energy it contributes changes by −0.0800J. Find the final length.

Gravitational and Elastic Potential Energy

A simple system embodying both gravitational and elastic types of potential energy is a one-dimensional, vertical mass-spring system. This consists of a massive particle (or block), hung from one end of a perfectly elastic, massless spring, the other end of which is fixed, as illustrated in Figure 9.1.1.

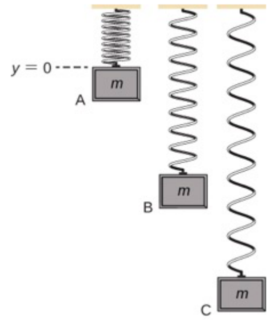

Figure 9.1.1: A vertical mass-spring system, with the positive *y*-axis pointing upward. The mass is initially at an unstretched spring length, point A. Then it is released, expanding past point B to point C, where it comes to a stop.

First, let's consider the potential energy of the system. We need to define the constant in the potential energy function of Equation 9.1.1. Often, the ground is a suitable choice for when the gravitational potential energy is zero; however, in this case, the highest point or when y = 0 is a convenient location for zero gravitational potential energy. Note that this choice is arbitrary, and the problem can be solved correctly even if another choice is picked.

We must also define the elastic potential energy of the system and the corresponding constant, as detailed in Equation 9.1.3. This is where the spring is unstretched, or at the y = 0 position.

If we consider that the total energy of the system is conserved, then the energy at point A equals point C. The block is placed just on the spring so its initial kinetic energy is zero. By the setup of the problem discussed previously, both the gravitational potential energy and elastic potential energy are equal to zero. Therefore, the initial energy of the system is zero. When the block arrives at point C, its kinetic energy is zero. However, it now has both gravitational potential energy and elastic potential energy. Therefore, we can solve for the distance y that the block travels before coming to a stop:

$$\begin{aligned} K_{\mathrm{A}} + U_{A} &= K_{C} + U_{\mathrm{C}} \\ 0 &= 0 + mgy_{C} + \frac{1}{2}k(y_{C})^2 \\ y_{\mathrm{C}} &= \frac{-2mg}{k} \end{aligned}$$

Figure 9.1.4: A bungee jumper transforms gravitational potential energy at the start of the jump into elastic potential energy at the bottom of the jump.

Example 9.1.4: Potential energy of a vertical mass-spring system

A block weighing 1.2 N is hung from a spring with a spring constant of 6.0 N/m, as shown in Figure 9.1.3. (a) What is the maximum expansion of the spring, as seen at point C? (b) What is the total potential energy at point B, halfway between A and C? (c) What is the speed of the block at point B?

Strategy

In part (a) we calculate the distance y_C as discussed in the previous text. Then in part (b), we use half of the y value to calculate the potential energy at point B using equations Equation 9.1.1 and Equation 9.1.3. This energy must be equal to the kinetic energy, Equation 8.1.1, at point B since the initial energy of the system is zero. By calculating the kinetic energy at point B, we can now calculate the speed of the block at point B.

Solution

a. Since the total energy of the system is zero at point A as discussed previously, the maximum expansion of the spring is calculated to be:

$$\begin{aligned} y_{\mathrm{C}} &= \frac{-2mg}{k} \\ y_{\mathrm{C}} &= \frac{-2(1.2\ \mathrm{N})}{(6.0\ \mathrm{N/m})} = -0.40\ \mathrm{m} \end{aligned} \tag{9.1.4}$$

b. The position of y_B is half of the position at y_C or -0.20 m. The total potential energy at point B would therefore be:

$$\begin{aligned} U_B &= mgy_B + \left(\frac{1}{2}ky_B\right)^2 \\ U_B &= (1.2\ \mathrm{N})(-0.20\ \mathrm{m}) + \frac{1}{2}(6\ \mathrm{N/m})(-0.20\ \mathrm{m})^2 \\ U_B &= -0.12\ \mathrm{J} \end{aligned}$$

c. The mass of the block is the weight divided by gravity.

$$m = \frac{F_w}{g} = \frac{1.2\ \mathrm{N}}{9.8\ \mathrm{m/s^2}} = 0.12\ \mathrm{kg}$$

The kinetic energy at point B therefore is 0.12 J because the total energy is zero. Therefore, the speed of the block at point B is equal to

$$\begin{aligned} K &= \tfrac{1}{2}mv^2 \\ v &= \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(0.12\ \mathrm{J})}{(0.12\ \mathrm{kg})}} = 1.4\ \mathrm{m/s} \end{aligned} \tag{9.1.5}$$

Significance

Even though the potential energy due to gravity is relative to a chosen zero location, the solutions to this problem would be the same if the zero energy points were chosen at different locations.

? Exercise 9.1.4

Suppose the mass in Equation 9.1.5 is doubled while keeping the all other conditions the same. Would the maximum expansion of the spring increase, decrease, or remain the same? Would the speed at point B be larger, smaller, or the same compared to the original mass?

Simulation

View this simulation to learn about conservation of energy with a skater! Build tracks, ramps and jumps for the skater and view the kinetic energy, potential energy and friction as he moves. You can also take the skater to different planets or even space!

A sample chart of a variety of energies is shown in Table 9.1.1 to give you an idea about typical energy values associated with certain events. Some of these are calculated using kinetic energy, whereas others are calculated by using quantities found in a form of potential energy that may not have been discussed at this point.

Table 9.1.1: Energy of Various Objects and Phenomena

Object/phenomenon	Energy in joules
Big Bang	10^{68}
Annual world energy use	4.0×10^{20}

Object/phenomenon	Energy in joules
Large fusion bomb (9 megaton)	3.8×10^{16}
Hiroshima-size fission bomb (10 kiloton)	4.2×10^{13}
1 barrel crude oil	5.9×10^{9}
1 ton TNT	4.2×10^{9}
1 gallon of gasoline	1.2×10^{8}
Daily adult food intake (recommended)	1.2×10^{7}
1000-kg car at 90 km/h	3.1×10^{5}
Tennis ball at 100 km/h	22
Mosquito (10^{-2} g at 0.5 m/s)	1.3×10^{-6}
Single electron in a TV tube beam	4.0×10^{-15}
Energy to break one DNA strand	10^{-19}