15.1: Forces and Newton's Three Laws

Newton's Laws

Up to this point in the semester, we've been studying the interaction between objects by modeling the interactions with energy and momentum. The transfer and conservation of these quantities allowed us to determine their motion. Now, we would like to describe the interactions between objects using **forces**. In some ways, this description of the physical world is more intuitive; forces push and pull on objects, much like how we interaction with objects in our everyday lives. Of course, momentum and energy is still being transferred around, but the force description gives us a different perspective, and intuition about the motion is often more direct. On the other hand, since forces are vectors, it requires more mathematical sophistication and care then when dealing with energy.

Forces Are Vectors

When you push or pull on an object, it matters what direction you are pushing or pulling it. This is very natural; if you push in one direction and your friend pushes just as hard in the opposite direction, the object will not move. But what happens if you push in one direction and your friend pushes just as hard, but not *quite* in the opposite direction? The object might move in some other direction, and that's what we want to know about. All these various direction-and-magnitude complexities can be easily dealt with by **modeling all forces as vectors**. They must be written as $\vec{F}=\begin{bmatrix}F_x\\F_y\\F_z\end{bmatrix}$ in the coordinate system you specify in order to get any answers right!

Newton's First Law

Newton's first law can be summarized as "an object in motion tends to stay in motion unless acted on by a net external force". The converse is also true; "an object at rest tends to stay at rest unless acted on by a net external force". An important word here is *net*, which means *sum of all*. A hockey puck sliding across the ice will continue to slide forever if there is no friction, but it *does have external forces acting on it* (gravity and the normal force, in this case). But these forces balance out, so there is no net force on the hockey puck. Newton's first law does not really help us solve problems, but rather it helps with our modeling process. It tells us when we should expect objects to exhibit motion.

Newton's Second Law

Newton's second law is the primary tool we will use to determine the motion of an object given some forces acting on it. We usually remember it as

$$\sum \vec{F}=m\vec{a}, \tag{15.1.1}$$

where the $\sum$ symbol means "add up all the forces". This is an important thing to remember - an object can have several forces acting on it, but a single object only ever has one acceleration $\vec{a}$. A very common mistake is to think "each force makes an acceleration F/m, and I will add them all up to get the acceleration of the object", but that is incorrect. A single object has only a single path in space, and therefore only has a single acceleration.

We can actually derive Newton's second law from the definition of force and momentum we have already encountered, namely

$$\vec{F}_{net}=\frac{d\vec{p}}{dt}. \tag{15.1.2}$$

(Recall that "net" means "sum of all", which is mathematically the same thing as the symbol Σ.)To do this, we just use the definition of momentum, $\vec{p}=m\vec{v}$, and assume the mass is constant in time (as it often is). Then we get:

$$\vec{F}_{net}=\frac{d}{dt}(m\vec{v})=m\frac{d\vec{v}}{dt}=m\vec{a}. \tag{15.1.3}$$

I typically write Newton's second law in a slightly different way,

$$\boxed{\vec{a} = \frac{\sum \vec{F}}{m}}, \tag{15.1.4}$$

which mathematically identical, but reads more like "the acceleration is the sum of the forces divided by the mass", which is more like how we use Newton's second law.

Finally, since this is a vector equation, it actually contains several independent equations inside it, one for each direction. You will actually have components in each direction, which in column vector form looks like

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \frac{\sum F_x}{m} \\ \frac{\sum F_y}{m} \\ \frac{\sum F_z}{m} \end{bmatrix} \tag{15.1.5}$$

or if we separate it into components, we get

$$a_x = \frac{\sum F_x}{m}, \qquad a_y = \frac{\sum F_y}{m}, \qquad a_z = \frac{\sum F_z}{m}, \tag{15.1.6}$$

. Either way you will have to solve all three equations for the components independently.

Newton's Third Law

Newton's third law is "for every action there is an equal and opposite reaction". In this case, our "actions" are forces. The typical example of this is "I push on the wall with a force $\vec{F}$, so the wall pushes on me with a force $-\vec{F}$". Mathematically, if we have a force $\vec{F}_{AB}$ acting from object A to object B, Newton's third law tells us that we know there must be a force $\vec{F}_{BA}$ acting from object B to object A. The magnitudes of these forces are equal, and their directions are opposite:

$$|\vec{F}_{AB}| = |\vec{F}_{BA}|, \qquad \vec{F}_{AB} = -\vec{F}_{BA}. \tag{15.1.7}$$

Notice the way we've notated this - each force corresponds with *a pair of objects*, one that creates the force and one that experiences it. All forces have both - you push on a wall (*you* and *wall* are the objects), the force of the floor pushing up on you, etc. In the case of the forces above, we're writing $\vec{F}_{AB}$ to mean "the force created by A, acting on B", or "the force from A to B". The order of these subscripts is not always that important (since Newton's third law tells us that $|F_{AB}| = |F_{BA}|$), but we will try to be careful when we are writing them.