

19.9: Sample problems and solutions

? Exercise 19.9.1

A cylindrical wire has a current density that increases with radius as $j(r) = ar$, where r , is the radial distance from the center of the wire, and a , is a constant. If the wire has a radius of $R = 1.5\text{cm}$, what is the total current in the wire?

Answer

To determine the current through the entire cross section of the wire, we first divide the cross-section of the wire into infinitesimally small concentric rings of radius, r , and width, dr . The cross-sectional area of one ring is given by:

$$dA = 2\pi r dr$$

so that the current through one ring is given by:

$$dI = j(r)dA = 2\pi ar^2 dr$$

The current through the whole wire is then found by summing the currents through each ring:

$$I = \int dI = \int_0^R 2\pi ar^2 dr = \frac{2}{3}\pi aR^3$$

? Exercise 19.9.2

A resistor is measured to have a resistance of $R_1 = 103.4\Omega$ at a temperature of $T_1 = 30^\circ\text{C}$, and a resistance of $R_2 = 106.8\Omega$ at a temperature of $T_2 = 40^\circ\text{C}$. Using the values in *Table 19.3.1*, determine the material from which the resistor is made.

Answer

To determine the material of the resistor, we can find the temperature coefficient, α , since we are given measurements of resistance, R_1 and R_2 , at two different temperatures, T_1 , and T_2 , respectively. The reference temperature is set to be $T_0 = 20^\circ\text{Celsius}$, so that we can compare with *Table 19.3.1*.

We know that the resistance will vary with temperature, since the resistivity is temperature-dependent. The temperature dependence of resistivity is given by:

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

If the resistor has length, L , and cross-sectional area, A , it will have resistance, R , given by:

$$R(T) = \rho(T)\frac{L}{A} = \frac{\rho_0 L}{A}[1 + \alpha(T - T_0)] = R_0[1 + \alpha(T - T_0)]$$

where R_0 is the resistance at the reference temperature, T_0 . Since we are given the resistance at two different temperatures, we can determine both α and R_0 , for a choice of $T_0 = 20^\circ\text{C}$:

$$\begin{aligned}
 R_1 &= R_0[1 + \alpha(T_1 - T_0)] \\
 R_2 &= R_0[1 + \alpha(T_2 - T_0)] \\
 \therefore \frac{R_1}{R_2} &= \frac{1 + \alpha(T_1 - T_0)}{1 + \alpha(T_2 - T_0)} \\
 R_1[1 + \alpha(T_2 - T_0)] &= R_2[1 + \alpha(T_1 - T_0)] \\
 \alpha(R_1(T_2 - T_0) - R_2(T_2 - T_0)) &= R_2 - R_1 \\
 \therefore \alpha &= \frac{R_2 - R_1}{R_1(T_2 - T_0) - R_2(T_1 - T_0)} \\
 &= \frac{(106.8\Omega) - (103.4\Omega)}{(103.4\Omega)((40^\circ\text{Celsius}) - (20^\circ\text{Celsius})) - (106.8\Omega)((30^\circ\text{Celsius}) - (20^\circ\text{Celsius}))} \\
 &= 0.0034
 \end{aligned}$$

Referring to *Table 19.3.1*, the material could likely be gold.

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