22.5: Summary

22.5.1: Key Takeaways

Magnetic fields are created by moving charges. The Biot-Savart Law allows us to determine the infinitesimal magnetic field, $d\vec B$, that is produced by the current, I, flowing in an infinitesimal section of wire, $d\vec l$:

$$d\vec B = \frac{\mu_0 I}{4\pi}\frac{d\vec l\times\hat r}{r^2}$$

where μ_0 is a constant called the permeability of free space. The vector $\vec r$ points from the wire element, $d\vec l$, to the point at which we want to determine the magnetic field. In order to determine the magnetic field from a finite wire, one must sum (integrate) the contributions that come from each section of wire. It is often easier to work with the Biot-Savart law written without the unit vector, $\hat r$:

$$d\vec B = \frac{\mu_0 I}{4\pi}\frac{d\vec l\times\vec r}{r^3}$$

The magnetic field at a distance, h, from an infinitely long wire carrying current, I, is given by:

$$B = \frac{\mu_0 I}{2\pi h}$$

The magnetic field from a straight current-carrying wire forms concentric circles centered around the wire. The direction of the magnetic field is given by the right-hand rule for axial vectors; with the thumb pointing in the direction of current, the fingers curl in the direction of the magnetic field.

The magnitude of the magnetic field, a distance, h, from the center of a circular loop of wire with radius, R, carrying current, I, along the axis of symmetry of the loop is given by:

$$B = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+h^2)^{\frac{3}{2}}}$$

The direction of the magnetic field can also be found using the right-hand rule for axial currents. In this case, if your fingers curl in the direction of the current loop, your thumb points in the same direction as the magnetic field at the center of the loop.

Two parallel wires carrying currents, I_1 and I_2, separated by a distance, h, will exert equal and opposite forces on each other with a magnitude:

$$F = \frac{\mu_0 I_1 I_2}{2\pi h}$$

The force is attractive if the two currents flow in the same direction and repulsive otherwise.

Ampere’s Law is the magnetism analogue to Gauss’ Law. Just like Gauss’ Law, it requires a high degree of symmetry to be applied analytically, although it is always valid. Ampere’s Law relates the circulation of the magnetic field around a closed path to the current enclosed by that path:

$$\oint \vec B\cdot d\vec l = \mu_0 I^{enc}$$

In order to apply Ampere’s Law, we must first choose an Amperian loop over which to compute the closed path integral (instead of choosing a Gaussian surface to calculate the flux of the electric field on a closed surface). The circulation integral will be straightforward to evaluate if:

1. **The angle between $\vec B$ and $d\vec l$ is constant along the path**, so that:

$$\oint \vec B\cdot d\vec l = \oint Bdl\cos\theta = \cos\theta\oint Bdl$$

where θ is the angle between $\vec B$ and $d\vec l$.

2. **The magnitude of $\vec{B}$ is constant along the path**, so that:

$$\cos\theta \oint Bdl = B\cos\theta \oint dl$$

The current enclosed, I^{enc}, corresponds to the net current that crosses the surface that is defined by the Amperian loop (a closed path always defines a surface).

Ampere's Law is straightforward to use in situations with a high degree of symmetry, such as infinitely long wires carrying current.

Solenoids are formed by combining many loops of current together, in order to form a strong and uniform magnetic field. The magnetic field inside of a solenoid has a magnitude of:

$$B = \mu_0 nI$$

where, I, is the current in the solenoid, and n, is the number of loops per unit length in the solenoid. The magnetic field just outside of a solenoid is zero, and generally, the magnetic field is negligible outside of a solenoid.

A toroid is formed by bending a solenoid into a circle to form a torus. The magnetic field lines inside of a toroid form concentric circles. The magnetic field decreases with radius inside of a toroid and is identically zero everywhere outside a toroid.

22.5.2: Important Equations

22.5.2.1: Biot-Savart law:

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{r}}{r^3}$$

22.5.2.2: Magnetic field from a finite wire:

$$B = \frac{\mu_0 I}{2\pi h}$$

22.5.2.3: Magnetic field from an infinitely long wire:

$$B = \frac{\mu_0 I}{2\pi h}$$

22.5.2.4: Magnetic field from a circular loop of current:

$$B = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+h^2)^{\frac{3}{2}}}$$

22.5.2.5: Force between two wires:

$$F = \frac{\mu_0 I_1 I_2}{2\pi h}$$

22.5.2.6: Ampere's Law:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I^{enc}$$