14.8: Summary

14.8.1: Key Takeaways

A traveling wave is the propagation of a disturbance with a speed, v, through a medium. Particles in the medium oscillate back and forth, about an equilibrium position, as a wave passes through the medium, but they are not carried with the wave. Only energy is transmitted by a wave.

In a transverse wave, the particles in the medium oscillate in a direction that is perpendicular to the velocity of the wave. In a longitudinal wave, the particles of the medium oscillate in a direction that is co-linear with the velocity of the wave.

A sine wave is described by it frequency, f, its wavelength, λ, its amplitude, A, and its speed, v. We can also use the period of the wave, T, in lieu of the frequency. The frequency and wavelength of a wave are related to each other by the speed of the wave:

$$v = \lambda f$$

Mathematically, a one-dimensional traveling sine wave moving in the positive x direction can be described by:

$$D(x,t) = A\sin(kx - \omega t + \phi)$$

where $D(x,t)$ is the displacement of the particle in the medium at position x at time t. ϕ is the phase of the wave and depends on our choice of when $t = 0$. k is the wave number of the wave, and ω its angular frequency. These are related to the wavelength and frequency, respectively:

$$k = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

If a dynamical model (e.g. Newton's Second Law) of a system/medium leads to an equation with the following form:

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$$

then waves with a speed of v can propagate through the system/medium.

The speed of a wave on a rope of linear mass density, μ, under a tension, F_T, is given by:

$$v = \sqrt{\frac{F_T}{\mu}}$$

Generally, the speed of a wave in a medium depends on the elasticity of the medium when it is deformed and the inertia of the particles in the medium. In order for a wave to propagate through a medium, the particles in the medium must be able to be displaced from their equilibrium position.

A pulse traveling through a rope will get reflected at the end of the rope and travel back in the opposite direction. If the end of the rope is fixed, the reflected pulse will be inverted. If the end of the rope can move, the reflected pulse will be in the same orientation as the incoming pulse.

A one-dimensional wave in a rope of linear mass density, μ, will transfer energy at an average rate:

$$P = \frac{1}{2}\omega^2 \mu A^2 v$$

A three dimensional spherical wave through a medium with density ρ will transfer energy at an average rate:

$$P = 2\pi\rho\omega^2 r^2 v$$

at a distance r from the source of the wave. The amplitude of a spherical wave will decrease as the distance away from the source increases:

$$A = \frac{1}{r}\sqrt{\frac{P}{2\pi\rho\omega^2 v}}$$

The intensity of a spherical wave is defined as the power per unit area transferred by the wave, and is given by:

$$I = \frac{P}{4\pi r^2} = \frac{1}{2}\rho\omega^2 A^2 v$$

The superposition principle states that if $D_1(x,t)$, $D_2(x,t)$, $\ldots$, are functions that satisfy the wave equation, then any linear combination of these functions, $D(x,t)$:

$$D(x,t) = a_1 D_1(x,t) + a_2 D_2(x,t) + a_3 D_3(x,t) + \ldots$$

will also satisfy the wave equation.

Different waves can interfere constructively or destructively in a medium, and the resulting wave is given by the sum of the functions describing the interfering waves.

Standing waves are formed when waves of the same frequency and amplitude traveling in opposite directions interfere. For standing waves on a string, the displacement of a particle on the string is given by:

$$D(x,t) = 2A\sin\left(\frac{n\pi}{L}x\right)\cos(\omega t)$$

where n is the number of the harmonic and L is the length of the string. In particular, a particle at position x will move up and down as a simple harmonic oscillator with amplitude:

$$A(x) = 2A\sin\left(\frac{n\pi}{L}x\right)$$

The condition for a standing wave to exist on a string is that the length of the string must be equal to a multiple of half of the wavelength of the standing wave:

$$\begin{aligned} L &= n\frac{\lambda}{2} \qquad n = 1, 2, 3, \ldots \\ \lambda &= \frac{2L}{n} \\ f &= \frac{nv}{2L} \end{aligned}$$

14.8.2: Important Equations

14.8.2.1: Traveling 1d waves:

$$\begin{aligned} D(x,t) &= A\sin(kx - \omega t + \phi) \\ k &= \frac{2\pi}{\lambda} \\ \omega &= 2\pi f = \frac{2\pi}{T} \\ v &= \lambda f \end{aligned}$$

14.8.2.2: Wave equation:

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$$

14.8.2.3: Wave velocity:

$$v = \sqrt{\frac{F_T}{\mu}} \quad v = \sqrt{\frac{E}{\rho}} \quad v = \sqrt{\frac{B}{\rho}}$$

14.8.2.4: Power (1d wave in a rope):

$$P = \frac{1}{2}\omega^2 \mu A^2 v$$

14.8.2.5: Spherical waves:

$$P = 2\pi\rho\omega^2 r^2 v$$
$$A = \frac{1}{r}\sqrt{\frac{P}{2\pi\rho\omega^2 v}}$$
$$I = \frac{P}{4\pi r^2} = \frac{1}{2}\rho\omega^2 A^2 v$$

14.8.2.6: Standing waves:

$$D(x,t) = 2A\sin\left(\frac{n\pi}{L}x\right)\cos(\omega t)$$
$$A(x) = 2A\sin\left(\frac{n\pi}{L}x\right)$$

14.8.2.7: Standing waves on a string (both ends fixed):

$$L = n\frac{\lambda}{2} \qquad n = 1, 2, 3, \ldots$$
$$\lambda = \frac{2L}{n}$$
$$f = \frac{nv}{2L}$$

14.8.3: Important Definitions

Definition

Wavelength: The distance between two successive maxima ("peaks") or minima (troughs) in a wave. SI units: $[\mathrm{m}]$. Common variable(s): λ.

Definition

Amplitude: The maximal distance that a particle in a medium is displaced from its equilibrium position when a wave passes by. SI units: $[\mathrm{m}]$. Common variable(s): A.

Definition

Frequency: The number of complete oscillations in one second of a particle in a medium as a wave passes by. SI units: $[\mathrm{s}^{-1}]$. Common variable(s): f.

Definition

Bulk modulus: A measurement of an object or substance's resistance to compression. SI units: $[\mathrm{Pa}]$. Common variable(s): B.

Definition

Volume mass density: The mass per unit volume of an object. SI units: $[\mathrm{kg}\cdot\mathrm{m}^{-3}]$. Common variable(s): ρ.

Definition

Intensity: The power per unit area transmitted by a wave. SI units: $[\mathrm{W} \cdot \mathrm{m}^{-2}]$. Common variable(s): I.

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