23.7: Summary

23.7.1: Key Takeaways

Faraday's Law connects a **changing** magnetic flux to an induced voltage:

$$\Delta V = -\frac{d\Phi_B}{dt}$$

The magnetic flux, Φ_B, is calculated as the flux of the magnetic field through an open surface, S:

$$\Phi_B = \int_S \vec{B}\cdot d\vec{A}$$

The induced voltage, ΔV, is the potential difference that is induced along the closed path (a "loop") that bounds the surface, S. If a charge, q, were to move around that closed path, it would gain (or lose) energy, $q\Delta V$. Note that the potential difference that is induced corresponds to a non-conservative electric force, as a charge can gain/lose energy by moving along a closed path. The induced voltage is often called an induced electromotive force (emf), even if it is a voltage.

The minus sign in Faraday's Law is sometime referred to as "Lenz'a Law", since it indicates in which direction the induced voltage will be. It is easiest to think of the closed path as a physical wire (e.g. a loop of wire) through which a current will be induced as a result of the induced voltage. The minus sign is easiest to interpret in terms of the relative direction between the area vector used to define the flux, and the magnetic dipole moment vector, $\vec{\mu}$, associated with the induced current (which points in the same direction as the magnetic field that is produced by the induced current).

When calculating the flux of the magnetic field, the surface element vector $d\vec{A}$, must be perpendicular to the surface through which the flux is calculated, which leads to two possible choices. Once a choice is made, and Faraday's Law has been applied, the sign of ΔV will indicate if the magnetic dipole moment of the induced current points in the same direction as $d\vec{A}$ (positive ΔV) or in the opposite direction (negative ΔV).

If N loops of wire are combined together into a coil, the voltages across each loop sum together, so that the voltage induced across the coil is given by:

$$\Delta V = -N\frac{d\Phi_B}{dt}$$

Lenz's Law is a statement about conservation of energy. Indeed, the induced current must create a magnetic field that **opposes** the change in flux, otherwise, the induced current would grow indefinitely. Lenz's Law can be summarized as follows:

- If the magnitude of the magnetic **flux is increasing** in the loop, then the induced current produces a magnetic field that is in the **opposite direction** from the original magnetic field.
- If the magnitude of the magnetic **flux is decreasing** in the loop, then the induced current produces a magnetic field that is in the **same direction** as the original magnetic field.

A voltage is induced along a closed path any time that the flux of the magnetic field through the corresponding surface changes. The flux can change either because the magnetic field is changing, or because the loop is changing (in size or orientation relative to the magnetic field). In the latter case (changing loop), one speaks of a "motional emf". A generator creates a motional emf by rotating a coil (with N loops, each with area, A), inside a fixed uniform magnetic field, $\vec{B}$. The voltage produced by a generator is given by:

$$\Delta V = NAB\omega\sin(\omega t)$$

where ω is the angular speed of the coil. A generator thus produces alternating voltage/current. The current that is induced in the coil of the generator will dissipate energy as it flows through a resistance, R. Thus, one must do work in order to keep the generator spinning. The current induced in the coil of the generator will also result in a magnetic moment, and a "counter torque" will be exerted on the coil. One must thus exert a torque in order to keep the generator spinning (and the work done by exerting that torque is converted into the electrical energy dissipated in the resistor). The counter torque on the generator is always in the same direction, and has a magnitude:

$$\tau=\frac{NA^2B^2\omega\sin^2(\omega t)}{R}$$

When an electric motor is used, a "back emf" is induced in the coil of the motor. The back emf is such that it resists the direction of current (Lenz's Law), or else the motor would spin infinitely fast. As the motor spins faster, the back emf grows, until it reaches an equilibrium. Motors thus draw a large current when they first start up, since at low speed, they have no back emf.

Since a changing magnetic flux induces a voltage, an electric field is also induced. We can replace the voltage in Faraday's Law with the circulation of the electric field to write a more general version of Faraday's Law:

$$\oint\vec E\cdot d\vec l=-\frac{d\Phi_B}{dt}$$

The induced electric field forms closed field lines, and is different than the electric field that is produced by static charges, since the latter will have field lines that start and end on charges. The force associated with the induced electric field is not conservative.

When a metallic object passes through a region of magnetic field, the induced electric field will induce current loops in the material called eddy currents. The magnetic field will also exert a force on these eddy currents to oppose the motion that is creating the currents (Lenz's Law); as the eddy currents dissipate electrical energy in the material, the metallic object must lose kinetic energy unless a force is acting on it. Magnetic brakes make use of this principle.

Transformers are used to convert an alternating voltage, ΔV_p, into a different alternating voltage, ΔV_s. A "primary" coil, with N_p windings, creates a changing magnetic flux that is guided (e.g. by an iron core) to a "secondary" coil, with N_s windings. The voltage induced in the secondary coil is given by:

$$\Delta V_s=\frac{N_p}{N_s}\Delta V_p$$

Maxwell's four equations form our best classical theory of electromagnetism. Those equations imply that a changing magnetic field produces an electric field (Faraday's Law), while a changing electric field can produce a magnetic field ('s Law). By combining Maxwell's equation (with some heavy vector calculus), one can show that this leads to the formation of electromagnetic waves, that propagate with a speed, c, given by:

$$c=\frac{1}{\sqrt{\epsilon_0\mu_0}}$$

23.7.2: Important Equations

23.7.2.1: Magnetic flux:

$$\Phi_B=\int_S\vec B\cdot d\vec A$$

23.7.2.2: Faraday's Law:

$$\Delta V=-N\frac{d\Phi_B}{dt}$$

23.7.2.3: Faraday's Law:

$$\oint\vec E\cdot d\vec l=-\frac{d\Phi_B}{dt}$$

23.7.2.4: Voltage produced by a generator:

$$\Delta V=NAB\omega\sin(\omega t)$$

23.7.2.5: Counter torque on a generator:

$$\tau=\frac{NA^2B^2\omega\sin^2(\omega t)}{R}$$

23.7.2.6: Secondary voltage in a transformer:

$$\Delta V_s = \frac{N_p}{N_s}\Delta V_p$$
