# 7.5: Power

We finish the chapter by introducing the concept of "power", which is the rate at which work is done on an object, or more generally, the rate at which energy is being converted from one form to another. If an amount of work, $\Delta W$, was done in a period of time $\Delta t$, then the work was done at a rate of:

$$P=\frac{\Delta W}{\Delta t} \tag{7.5.1}$$

where $P$ is called the power. The SI unit for power is the "Watt", abbreviated $\mathrm{W}$, which corresponds to $\mathrm{J/s = kg\ m^2/s^3}$ in base SI units. If the rate at which work is being done changes with time, then the instantaneous power is defined as:

$$P=\frac{dW}{dt} \tag{7.5.2}$$

You have probably already encountered power in your everyday life. For example, your $1000\mathrm{W}$ hair dryer consumes "electrical energy" at a rate of $1000\mathrm{J}$ per second and converts it into the kinetic energy of the fan as well as the thermal energy to heat up the air. Horsepower (hp) is an imperial unit of power that is often used for vehicles, the conversion being $1\mathrm{hp}=746\mathrm{W}$. A $100\mathrm{hp}$ car thus has an engine that consumes the chemical energy released by burning gasoline at a rate of $7.46\times 10^4\mathrm{J}$ per second and converts it into work done on the car as well as into heat.

## Exercise 7.5.1

Two cranes lift two identical boxes off of the ground. One crane is twice as powerful as the other. Both cranes do the same amount of work on the boxes and operate at full power. Which of the following statements is true of the boxes, once the cranes have done work on them?

A. One box has been lifted twice as high as the other.
B. The boxes are lifted to the same height in the same amount of time.
C. The boxes are lifted to the same height, but it takes one of the boxes twice as long to get there.
D. One box is lifted twice as high as the other, but it takes the same amount of time to get there.

**Answer**

C.

## Example 7.5.1

If a car engine can do work on the car with a power of $P$, what will be the speed of the car at some time $t$ if the car was at rest at time $t=0$?

### Solution

First, we need to calculate how much total work was done on the car:

$$W=Pt$$

Then, using the Work-Energy Theorem, we can find the speed of the car at some time $t$:

$$\begin{aligned} W&=\frac{1}{2}mv_f^2-\frac{1}{2}mv_i^2 \\ Pt&=\frac{1}{2}mv_f^2 \\ \therefore v_f&=\sqrt{\frac{2Pt}{m}} \end{aligned}$$

**Discussion**:

The model for the final speed of the car makes sense because:

- The dimension of the expression for $v_f$ is speed (you should check this!).

- The speed is greater if either the time or power are greater (so the speed is larger if more work is done on the car).
- The speed is smaller if the mass of the car is greater (the acceleration of the car will be less if the mass of the car is larger).

**Example 7.5.2**

You are pushing a crate along a horizontal surface at constant speed, $v$. You find that you need to exert a force of $\vec F$ on the crate in order to overcome the friction between the crate and the ground. How much power are you expending by pushing on the crate?

**Solution**

We need to calculate the rate at which the force, $\vec F$, that you exert on the crate does work. If the crate is moving at constant speed, $v$, then in a time $\Delta t$, it will cover a distance, $d = v\Delta t$ . Since you exert a force in the same direction as the motion of the crate, the work done over that distance $d$ is:

$$\Delta W = \vec F \cdot \vec d = Fd\cos(0) = Fv\Delta t$$

The power corresponding to the work done in that period of time is thus:

$$P = \frac{\Delta W}{\Delta t} = Fv$$

This is quite a general result for the rate at which a force does work when it is exerted on an object moving at constant speed.

**Olivia's Thoughts**

*Example 7.3.2* ties into what I brought up earlier. If you think to yourself: "The velocity is constant, so the work must be zero", the formula,

$$P = \frac{\Delta W}{\Delta t} = Fv$$

wouldn't make any sense. Since $v$ is a constant velocity, the power would always be equal to zero, which of course isn't right. Again, remember that when the velocity is constant, it is only the **net work** that is equal to zero. In *Example 7.3.2*, it's asking for the power that **you** are expending by pushing on the crate (which is the same as asking for the rate of the work done **by** you **on** the crate). So, the formula does indeed make sense.