3.6: Summary

3.6.1: Key Takeaways

To describe motion in one dimension, we must define an axis with:

1. An origin (where $x=0$).
2. A direction (the direction in which x increases).
3. A unit for the length.

We describe the position of an object with a function $x(t)$ that *depends* on time. The rate of change of position is called "velocity", $v_x(t)$ and is a function given by the time-derivative of position. The rate of change of velocity is called "acceleration", $a_x(t)$ and is a function given by the time-derivative of velocity.

Given a function for acceleration, $a_x(t)$, one can use its anti-derivative to determine velocity.

$$v_x(t)=\lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t}=\frac{dx}{dt}$$

Given a function for velocity, $v_x(t)$, one can use its anti-derivative to determine position.

$$a_x(t)=\lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t}=\frac{dv_x}{dt}$$

With a constant acceleration, $a_x(t)=a_x$, if the object had velocity v_{0x} and position x_0 at $t=0$:

$$v_x(t)=v_{0x}t+a_xt$$

$$x(t)=x_0+v_{0x}t+\frac{1}{2}a_xt^2$$

$$v^2-v_0^2=2a(x-x_0)$$

An inertial frame of reference is one that is moving with a constant velocity. It is impossible to define a frame of reference that is truly "at rest", so we consider inertial frames of reference only relative to other frames of reference that we also consider to be inertial. If an object has a position $x^A(t)$ in a given inertial frame of reference, x, that is moving with a velocity $v^{'B}$ compared to a different inertial frame of reference, $x^{'}$, then the position of the object in the $x^{'}$ frame of reference is given by $x'(t)=v^{'B}+x^A(t)$.

$$x^{'A}(t)=v^{'B}t+x^A(t)$$

$$v^{'A}(t)=v^{'B}+v^A(t)$$

$$a^{'A}(t)=a(t)$$

3.6.2: Important equations

If the position of an object is described by a function $x(t)$, then, its velocity, $v_x(t)$, and acceleration, $a_x(t)$, are given by:

$$v_x(t)=\lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t}=\frac{dx}{dt}$$

$$a_x(t)=\lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t}=\frac{dv_x}{dt}$$

Conversely, given the acceleration, $a_x(t)$, on can find the velocity and position:

$$v_x(t)=\lim_{\Delta t\to 0}\sum_i a_x(t_i)\Delta t+C=\int a_x(t)dt+C$$

$$x(t)=\lim_{\Delta t\to 0}\sum_i v_x(t_i)\Delta t+C=\int v_x(t)dt+C$$

With a constant acceleration, $a_x(t)=a_x$, if the object had velocity v_{0x} and position x_0 at $t=0$:

$$\begin{aligned} v_x(t) &= v_{0x}t + a_x t \\ x(t) &= x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \\ v^2 - v_0^2 &= 2a(x - x_0) \end{aligned}$$

If an object has position x^A as measured in a frame of reference x that is moving at constant speed v'^B as measured in a second frame of reference x', then in the x' reference frame, the kinematic quantities for the object are obtained by the Galilean transformation:

$$\begin{aligned} x'^A(t) &= v'^B t + x^A(t) \\ v'^A(t) &= v'^B + v^A(t) \\ a'^A(t) &= a(t) \end{aligned}$$

3.6.3: Important Definitions

Definition

Position: The distance between the defined coordinate system's origin and an object. SI units: $[\mathrm{m}]$. Common variable(s): $\vec{x}, \vec{r}$.

Definition

Velocity: The rate at which position changes with respect to time. SI units: $[\mathrm{ms}^{-1}]$. Common variable(s): $\vec{v}$.

Definition

Acceleration: the rate at which velocity changes with respect to time. SI units: $[\mathrm{ms}^{-2}]$. Common variable(s): $\vec{a}$.

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