16.5: Summary

16.5.1: Key Takeaways

Objects can acquire a net charge if they acquire a net excess or deficit of electrons. Charges are never created, they are only transferred from one object to another. One can charge an object by friction, conduction, or induction. Materials can be classified broadly as conductors, where electrons can move freely in a material, or conductors, in which electrons remain tightly bound to the atoms in the material. If a conducting object acquires a net charge, those charges will migrate to the surface of the conductor.

Coulomb was the first to quantitatively describe the electric force exerted on a point charge, Q_1, by a second point charge, Q_2, located a distance, r, away:

$$\vec{F}_{12} = k\frac{Q_1 Q_2}{r^2}\hat{r}_{21} = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}\hat{r}_{21}$$

where $\hat{r}_{21}$ is the unit vector from Q_2 to Q_1. One can write the force using either Coulomb's constant, k, or the permitivity of free space, ϵ_0. Coulomb's force is attractive if the product $Q_1 Q_2$ is negative, and repulsive if the product is positive. Thus, charges of the same sign exert a repulsive force on each other, whereas opposite charges exert an attractive for on each other.

Mathematically, Coulomb's Law is identical to the gravitational force in Newton's Universal Theory of Gravity, which implies that it is conservative. The electric field vector at some position in space is defined to be the electric force per unit charge at that position in space. That is, at some position in space where the electric field vector is $\vec{E}$, a charge, q, will experience an electric force:

$$\vec{F} = q\vec{E}$$

much like a mass, m, will experience a gravitational force, $m\vec{g}$, in a position in space where the gravitational field is $\vec{g}$. A positive charge will experience a force in the same direction as the electric field, whereas a negative charge will experience a force in the direction opposite of the electric field. The electric field at position, $\vec{r}$, from a point charge, Q, located at the origin, is given by:

$$\vec{E} = k\frac{Q}{r^2}\hat{r}$$

One can visualize an electric field by using "field lines". The field vector at any point in space has a magnitude that is proportional to the number of field lines at that point, and a direction that is tangent to the field lines at that point.

We can model the electric field from a continuous charged object (i.e. not a point charge) by modeling the object as being made up of many point charges. Often, it is easiest to model an N-dimensional object as being the sum of objects of dimension $N-1$ and an infinitesimal length in the remaining dimension. For example, we modeled a line of charge as the sum of point charges that have an infinitesimal length, and we modeled a disk of charge as the the sum of rings that have an infinitesimal thickness. In general, the strategy to model the electric field from a continuous distribution of charge is the same:

1. Make a *good* diagram.
2. Choose a charge element dq.
3. Draw the electric field element, $d\vec{E}$, at the point of interest.
4. Write out the electric field element vector, $d\vec{E}$, in terms of dq and any other relevant variables.
5. Think of symmetry: will any of the component of $d\vec{E}$ sum to zero over all of the dq?
6. Write the total electric field as the sum (integral) of the electric field elements.
7. Identify which variables change as one varies the dq and choose an integration variable to express dq and everything else in terms of that variable and other constants.
8. Do the sum (integral).

Finally, we introduced the electric dipole, which is an object comprised of two equal and opposite charges, $+Q$ and $-Q$, held at fixed distance, l, from each other. One can model an electric dipole using its dipole vector, $\vec{p}$, defined to point in the direction from $-Q$ to $+Q$, with magnitude:

$$p = Ql$$

When a dipole is immersed in a uniform electric field, $\vec{E}$, it will experience a torque given by:

$$\vec{\tau} = \vec{p} \times \vec{E}$$

The torque will act such as to align the vector $\vec{p}$ with the electric field vector. We can define a potential energy, U, to model the energy that is stored in a dipole when it is not aligned with the electric field:

$$U = -\vec{p} \cdot \vec{E}$$

The point of lowest potential energy corresponds to the case when $\vec{p}$ and $\vec{E}$ are parallel, whereas the point of highest potential energy is when the two vectors are anti-parallel.

16.5.2: Important Equations

16.5.2.1: Electric field:

$$\vec{E} = k\frac{Q}{r^2}\vec{r}$$

$$\vec{E} = \int d\vec{E}$$

16.5.2.2: Electric force:

$$\vec{F} = q\vec{E}$$

16.5.2.3: Electric dipole moment:

$$p = Ql$$

16.5.2.4: Torque on a dipole:

$$\vec{\tau} = \vec{p} \times \vec{E}$$

16.5.2.5: Potential energy stored in a dipole:

$$U = -\vec{p} \cdot \vec{E}$$

16.5.3: Important Definitions

Definition

Charge: An object will have a charge if it has an excess or deficit of electrons. SI units: $[\mathrm{C}]$. Common variable(s): Q, q.

Definition

Electric field: The electric field is defined to be the electric force per unit charge. SI units: $[\mathrm{N/C}, \mathrm{V/m}]$. Common variable(s): $\vec{E}$.

Definition

Coulomb's constant: A fundamental physical constant which relates charge and distance to electric field. SI units: $[\mathrm{Nm}^2 C^{-2}]$. Common variable(s): k.

Definition

Electric dipole moment: A vector used to represent an electric dipole. SI units: $[\mathrm{Cm}]$. Common variable(s): $\vec{p}$.

Definition

Linear charge density: The charge per unit length of an object. SI units: $[\mathrm{C/m}]$. Common variable(s): λ.

Definition

Surface charge density: The charge per unit area of an object. SI units: $[\mathrm{Cm}^{-2}]$. Common variable(s): σ.

Definition

Volume charge density: The charge per unit volume of an object. SI units: $[\mathrm{Cm}^{-3}]$. Common variable(s): ρ.

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