26.6: Sample problems and solutions

Exercise 26.6.1

You find that the number of customers in your store as a function of time is given by:

$$N(t) = a + bt - ct^2$$

where a, b and c are constants. At what time does your store have the most customers, and what will the number of customers be? (Give the answer in terms of a, b and c).

Answer

We need to find the value of t for which the function $N(t)$ is maximal. This will occur when its derivative with respect to t is zero:

$$\frac{dN}{dt} = b - 2ct = 0$$

$$\therefore t = \frac{b}{2c}$$

At that time, the number of customers will be:

$$N\left(t = \frac{b}{2c}\right) = a + bt - ct^2$$

$$= a + \frac{b^2}{2c} - \frac{b^2}{4c} = a + \frac{3b^2}{4c}$$

Exercise 26.6.2

You measure the speed, $v(t)$, of an accelerating train as function of time, t, to be given by:

$$v(t) = at + bt^2$$

where a and b are constants. How far does the train move between $t = t_0$ and $t = t_1$?

Answer

We are given the speed of the train as a function of time, which is the rate of change of its position:

$$v(t) = \frac{dx}{dt}$$

We need to find how its position, $x(t)$, changes with time, given the speed. In other words, we need to find the anti-derivative of $v(t)$ to get the function for the position as a function of time, $x(t)$:

$$x(t) = \int v(t)dt = \int (at + bt^2)dt$$

$$= \frac{1}{2}at^2 + \frac{1}{3}bt^3 + C$$

where C is an arbitrary constant. The distance covered, Δx, between time t_0 and time t_1 is simply the difference in position at those two times:

$$\Delta x = x(t_1) - x(t_0)$$

$$= \frac{1}{2}at_1^2 + \frac{1}{3}bt_1^3 + C - \frac{1}{2}at_0^2 + \frac{1}{3}bt_0^3 - C$$

$$= \frac{1}{2}a(t_1^2 - t_0^2) + \frac{1}{3}b(t_1^3 - t_0^3)$$