21.7: Summary

21.7.1: Key Takeaways

In order to describe the magnetic force, we introduced the magnetic field, $\vec{B}$. While there are some similarities with the electric field, the key difference in magnetism is that there are no "magnetic charges" (so-called monopoles), and magnets thus always have a North *and* a South pole. As a result, magnetic field lines never end and must always form closed loops. The magnetic field points in the direction of the force that would be exerted on the North pole of a magnet placed at that position.

Electric charges can feel a force from a magnetic field only if they are moving relative to the frame of reference in which the magnetic field is described. If a charge, q, has velocity, $\vec{v}$, in a magnetic field, $\vec{B}$, it will feel a magnetic force given by:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

The magnetic force can do no work, since it always acts in a direction perpendicular to the velocity (and thus to the displacement). The magnetic field acts in opposite directions for charges of opposite signs.

In a uniform magnetic field, a charged particle with charge, q, mass m, and velocity vector, $\vec{v}$, perpendicular to a magnetic field, $\vec{B}$, will undergo uniform circular motion, with a cyclotron radius, R, given by:

$$R = \frac{mv}{qB}$$

A straight wire of length, l, carrying current, I, will experience a magnetic force in a magnetic field, $\vec{B}$:

$$\vec{F}_B = I\vec{l} \times \vec{B}$$

where the vector $\vec{l}$ points in the same direction as the current.

If the wire is curved (or the magnetic field changes direction along the wire), then we can integrate the force, $d\vec{F}$, exerted on each infinitesimal section of wire with length, $d\vec{l}$. Again, the direction of $d\vec{l}$ is in the same direction as the current in the wire. The infinitesimal force on an infinitesimal section of wire, is given by:

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

A closed loop of wire carrying current will experience no net force in a uniform magnetic field. However, it will experience a torque, if the loop is not "aligned" with the magnetic field (the torque is zero if the magnetic field is perpendicular to the plane of the loop). We define the magnetic dipole moment, $\vec{\mu}$ of a loop of wire carrying current, I, to be a vector with magnitude:

$$\mu = IA$$

where A is the area enclosed by the loop. The magnetic dipole moment vector is perpendicular to the plane of the loop, and points in the direction given by the right-hand rule for axial vectors applied to the current (think of the current as rotating in the loop).

The torque from a magnetic field, $\vec{B}$, exerted on a loop with a magnetic dipole moment, $\vec{\mu}$, is given by:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

The torque is zero when the magnetic dipole moment vector is parallel to the magnetic field vector (corresponding to the loop being "aligned" with the magnetic field). One can think of the magnetic dipole moment as a small bar magnet, or the needle of a compass, that always experiences a torque to align it with a magnetic field.

We can define the potential energy of a magnetic dipole moment in a magnetic field as:

$$U = -\vec{\mu} \cdot \vec{B} = \mu B \cos\theta$$

The Hall effect can be observed when current flows through a slab that is immersed in a magnetic field that is perpendicular to the slab. As the electrons move longitudinally through the slab, they will also be pushed to one side by the magnetic force, resulting in an excess of negative charge on that side. An electric potential difference (the "Hall potential") is then established between the two sides of the slab (in the direction perpendicular to the motion of the electrons). The Hall potential is given by:

$$\Delta V_{Hall} = v_d w B$$

where w is the width of the slab in the perpendicular direction, B is the strength of the magnetic field, and v_d is the drift velocity of electrons. The most common use of the Hall effect is to build a Hall probe to measure magnetic fields. However, Hall probes can also measure the drift velocity of electrons and other microscopic properties. The sign of the Hall potential also indicates the sign of the charges moving in the slab.

There are many applications of the magnetic force in our daily lives, including electric motors, loudspeakers, cathode ray tubes, mass spectrometers, and galvanometers.

21.7.2: Important Equations

21.7.2.1: Magnetic force on a moving charge:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

21.7.2.2: Magnetic force on a current-carrying wire:

$$\vec{F}_B = I\vec{l} \times \vec{B}$$

21.7.2.3: Cyclotron radius:

$$R = \frac{mv}{qB}$$

21.7.2.4: Magnetic dipole moment:

$$\mu = IA$$

21.7.2.5: Torque on a magnetic dipole:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

21.7.3: Important Definitions

Definition

Magnetic field: A field used to model the magnetic force. SI units: $[\mathrm{T}]$. Common variable(s): $\vec{B}$.

Definition

Magnetic dipole moment: A property of an object which describes the torque it will experience in a magnetic field. SI units: $[\mathrm{C} \cdot \mathrm{m}^2 \cdot \mathrm{S}^{-1}]$. Common variable(s): $\vec{\mu}$.