26.4: Summary

26.4.1: Key Takeaways

The derivative of a function, $f(x)$, with respect to x can be written as:

$$\frac{d}{dx}f(x)=\frac{df}{dx}=f'(x)$$

and measures the rate of change of the function with respect to x. The derivative of a function is generally itself a function. The derivative is defined as:

$$f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Graphically, the derivative of a function represents the slope of the function, and it is positive if the function is increasing, negative if the function is decreasing and zero if the function is flat. Derivatives can always be determined analytically for any continuous function.

A partial derivative measures the rate of change of a multi-variate function, $f(x,y)$, with respect to one of its independent variables. The partial derivative with respect to one of the variables is evaluated by taking the derivative of the function with respect to that variable while treating all other independent variables as if they were constant. The partial derivative of a function (with respect to x) is written as:

$$\frac{\partial f}{\partial x}$$

The gradient of a function, $\nabla f(x,y)$, is a vector in the direction in which that function is increasing most rapidly. It is given by:

$$\nabla f(x,y)=\frac{\partial f}{\partial x}\hat{x}+\frac{\partial f}{\partial y}\hat{y}$$

Given a function, $f(x)$, its anti-derivative with respect to x, $F(x)$, is written:

$$F(x)=\int f(x)dx$$

$F(x)$ is such that its derivative with respect to x is $f(x)$:

$$\frac{dF}{dx}=f(x)$$

The anti-derivative of a function is only ever defined up to a constant, C. We usually write this as:

$$\int f(x)dx=F(x)+C$$

since the derivative of $F(x)+C$ will also be equal to $f(x)$. The anti-derivative is also called the “indefinite integral” of $f(x)$.

The definite integral of a function $f(x)$, between $x=a$ and $x=b$, is written:

$$\int_a^b f(x)dx$$

and is equal to the difference in the anti-derivative evaluated at $x=a$ and $x=b$:

$$\int_a^b f(x)dx=F(b)-F(a)$$

where the constant C no longer matters, since it cancels out. Physical quantities only ever depend on definite integrals, since they must be determined without an arbitrary constant.

Definite integrals are very useful in physics because they are related to a sum. Given a function $f(x)$, one can relate the sum of terms of the form $f(x_i)\Delta x$ over a range of values from $x=a$ to $x=b$ to the integral of $f(x)$ over that range:

$$\lim_{\Delta x \to 0} \sum_{i=1}^{i=N} f(x_{i-1})\Delta x = \int_{x_0}^{x_N} f(x)dx = F(x_N) - F(x_0) =$$