15.5: Summary

15.5.1: Key Takeaways

The pressure from a force, $\vec{F}$, exerted over a surface with area, A, is a scalar quantity defined as:

$$P=\frac{F_{\perp}}{A}$$

where $F_{\perp}$ is the component of the force perpendicular to the surface.

If a force is exerted on the particles in a fluid (e.g. gravity), a pressure will exist everywhere in the fluid. If the fluid is placed in a container, that pressure leads to an external force on all surfaces of the container.

If two fluids at different pressures exist on either side of an interface/object, the net force on that interface/object from the pressures of the fluids will be proportional to the difference in pressure of the fluids on either side.

A fluid is in hydrostatic equilibrium if the sum of the forces on any fluid element is zero. In the presence of gravity, this always leads to a vertical pressure gradient

$$\frac{dP}{dy}=-\rho g$$

where ρ is the density of the fluid, g is the magnitude of the Earth's gravitational field, and the y axis is positive upwards.

If the fluid is incompressible, then the difference in pressure between two points at heights y_1 and y_2 is given by:

$$P(y_2)-P(y_1)=-\rho g(y_2-y_1)$$

Pascal's Principle states that if an external pressure, P, is applied to one location in a fluid, then the pressure everywhere in the fluid increases by P.

If an object is immersed in a fluid, it will experience a force of buoyancy that is in the opposite direction to the gravitational field in that fluid. The magnitude of the buoyancy force is given by Archimedes' Principle:

$$F_B=\rho V g$$

where, ρ, is the density of the fluid and, V, is the volume of the fluid displaced by the object (i.e. the volume of the part of the object that is immersed in the fluid).

We can distinguish between laminar and turbulent flow of fluids. In laminar flow, individual particles in the fluid follow well-defined streamlines. In turbulent flow, individual particle follow complicated paths that usually involve Eddy currents. In general, it is much easier to model the laminar flow of fluids.

The equation of continuity states that the mass flow rate of a fluid through a closed system must be the same everywhere in the system (no fluid can appear or disappear). For laminar flow of a fluid with density, ρ, flowing at speed, v, through a pipe with cross section, A, the mass flow rate is a constant:

$$\rho A v=\text{constant}$$

A fluid is said to be incompressible if it has constant density. For a fluid of constant density, the volume flow rate, Q, must be constant everywhere in a closed system:

$$Q=Av=\text{constant}$$

Bernoulli's Principle, which is based on the conservation of mechanical energy, states that the following quantity is a constant:

$$P+\frac{1}{2}\rho v^2+\rho g y=\text{constant}$$

for the laminar flow of a fluid with no viscosity. P is the internal pressure of the fluid, v its speed, and y the height of the fluid relative to a fixed coordinate system. In particular, Bernoulli's Principle implies that, for a constant height, the internal pressure of a fluid must decrease if its speed increases.

Viscosity, η, for the laminar flow of a fluid can be modeled as the result of the internal friction force between layers of the fluid. Because of viscosity, a fluid cannot flow in a horizontal pipe unless there is a difference in pressure across the pipe. Similarly, there will be no horizontal pressure gradient through a fluid unless the fluid is flowing. In general, the volume flow rate, Q, of an incompressible fluid through a pipe with resistance, R, is given by:

$$Q=\frac{\Delta P}{R}$$

For the laminar flow of a fluid with viscosity, η, through a horizontal cylindrical pipe of length, L, and radius, r, the flow rate is given by Poiseuille's equation:

$$Q=\frac{\pi r^4}{8\eta L}\Delta P$$

15.5.2: Important Equations

15.5.2.1: In the presence of gravity:

$$\begin{aligned} \frac{dP}{dy} &= -\rho g \\ P(y_2)-P(y_1) &= -\rho g(y_2-y_1) \\ F_B &= \rho V g \end{aligned}$$

15.5.2.2: Equation of continuity:

$$\begin{aligned} \rho A v &= \text{constant} \\ Q &= Av = \text{constant} \quad (\text{if incompressible}) \end{aligned}$$

15.5.2.3: Bernoulli:

$$P+\frac{1}{2}\rho v^2+\rho g y=\text{constant}$$

15.5.2.4: Viscosity:

$$\begin{aligned} Q &= \frac{\Delta P}{R} \\ Q &= \frac{\pi r^4}{8\eta L}\Delta P \quad (\text{Poiseuille}) \end{aligned}$$

15.5.3: Important Definitions

Definition

Pressure: A measurement of force per unit area. SI units: $[\mathrm{Pa}]$. Common variable(s): P.

Definition

Viscosity: A measurement of a fluid's resistance to flow. SI units: $[\mathrm{Pa\,s}]$. Common variable(s): η.

Definition

Flow rate: Measurement of a fluid's motion, in volume per unit time. SI units: $[\mathrm{m^3 s^{-1}}]$. Common variable(s): Q.