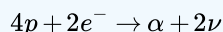


## 24.11: Sample problems and solutions

### ? Exercise 24.11.1

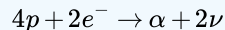
The Sun is powered by nuclear fusion reactions in which, predominantly, hydrogen atoms are fused together into helium atoms. Inside the Sun, the material, mostly hydrogen, is in a form of a plasma, where the electrons are not attached to the nuclei of their atoms. Effectively, one can model the solar fusion reactions<sup>1</sup> as:



where the four protons correspond to the nuclei of four hydrogen atoms,  $\alpha$  is the nucleus of a helium atom, with two neutrons and two protons, and the two  $\nu$  are neutrinos, particles with virtually zero mass. The reaction above is exothermic, and releases energy, because the total mass of particles on the right is less than the total mass on the left. Given that the mass of a proton is  $m_p = 938.3\text{MeV}/c^2$ , the mass of an electron is  $m_e = 0.511\text{MeV}/c^2$ , and the mass of the alpha particle is  $m_\alpha = 3727.4\text{MeV}/c^2$ , how much energy (in and in ) is released in each fusion reaction?

#### Answer

In order to determine the amount of energy released in each reaction, we need to determine the difference in mass between the two sides of the equation:



On the left-hand side, the total mass is:

$$M_{LHS} = 4m_p + 2m_e = 4(938.3\text{MeV}/c^2) + 2(0.511\text{MeV}/c^2) = 3754.22\text{MeV}/c^2$$

whereas on the right-hand side, the total mass is:

$$M_{RHS} = m_\alpha = 3727.4\text{MeV}/c^2$$

Thus, the total energy released in each reaction is given by:

$$\begin{aligned} E &= c^2 \Delta M = c^2 (M_{LHS} - M_{RHS}) = c^2 ((3754.22\text{MeV}/c^2) - (3727.4\text{MeV}/c^2)) \\ &= 26.8\text{MeV} = 4.29 \times 10^{-12}\text{J} \end{aligned}$$

where we showed the answer in both and . Although it may not seem like that much energy per reaction, keep in mind that there are of order reactions per second in the Sun, corresponding to a power output of order  $4 \times 10^{26}\text{W}$ , enough to keep us warm in the summer.

### ? Exercise 24.11.2

A proton is measured by a scientist to have a total energy of  $2.5 \times 10^3\text{MeV}$ .

- What is the speed of the proton?
- How far does the proton travel (in the lab) when 1s goes by in the scientist's frame of reference?
- How far does the proton travel (in the lab) when 1s goes by in the proton's frame of reference?

#### Answer

- From the total energy, we can calculate the gamma factor, which will give us the velocity of the proton (in the reference frame of the scientist):

$$\begin{aligned}
 E &= \gamma m_0 c^2 \\
 \frac{1}{\gamma} &= \frac{m_0 c^2}{E} \\
 \sqrt{1 - \frac{v^2}{c^2}} &= \frac{m_0 c^2}{E} \\
 \frac{v^2}{c^2} &= 1 - \frac{m_0^2 c^4}{E^2} \\
 \therefore v &= \left( \sqrt{1 - \frac{m_0^2 c^4}{E^2}} \right) c \\
 &= 0.92c \\
 &= \left( \sqrt{1 - \frac{(938.3 \text{ MeV}/c^2)^2 c^4}{(2.5 \times 10^3 \text{ MeV})^2}} \right) c \\
 &= \left( \sqrt{1 - \frac{(938.3 \text{ MeV})^2}{(2.5 \times 10^3 \text{ MeV})^2}} \right) c \\
 &= 2.76 \times 10^8 \text{ m/s}
 \end{aligned}$$

b. In the frame of the lab, when one second goes by, the proton will travel a distance:

$$d = vt = (2.76 \times 10^8 \text{ m/s})(1\text{s}) = 2.76 \times 10^8 \text{ m}$$

c. In order to find out how far the proton travels in the lab when one second of proper time goes by in the proton's frame of reference, we need to determine how much time went by in the lab's frame of reference.

The gamma factor for the proton can be obtained from the speed that we determined in part a), or from the total energy directly:

$$\gamma = \frac{E}{m_0 c^2} = \frac{(2.5 \times 10^3 \text{ MeV})}{(938.3 \text{ MeV}/c^2)c^2} = \frac{(2.5 \times 10^3 \text{ MeV})}{(938.3 \text{ MeV})} = 2.66$$

Thus, when  $\Delta t = 1\text{s}$  elapses in the proton's frame of reference, a time dilated time,  $\Delta t'$ , elapses in the lab frame of reference:

$$\Delta t' = \gamma \Delta t = 2.66\text{s}$$

In the lab frame, the proton will travel a distance:

$$d = vt = (2.76 \times 10^8 \text{ m/s})(2.66\text{s}) = 7.34 \times 10^8 \text{ m}$$

### 24.11.1: Footnotes

1. In reality, there are many more reactions involved in getting from hydrogen to helium.

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