6.E: Thermodynamics (Exercises)

1. (a) Show that under conditions of standard pressure and temperature, the volume of a sample of an ideal gas depends only on the number of molecules in it.

(b) One mole is defined as 6.0×10^{23} atoms. Find the volume of one mole of an ideal gas, in units of liters, at standard temperature and pressure ($0^\circ \mathrm{C}$ and 101 kPa). (answer check available at lightandmatter.com)

2. A gas in a cylinder expands its volume by an amount dV, pushing out a piston. Show that the work done by the gas on the piston is given by $dW = PdV$.

3. (a) A helium atom contains 2 protons, 2 electrons, and 2 neutrons. Find the mass of a helium atom. (answer check available at lightandmatter.com)
(b) Find the number of atoms in 1.0 kg of helium. (answer check available at lightandmatter.com)
(c) Helium gas is monoatomic. Find the amount of heat needed to raise the temperature of 1.0 kg of helium by 1.0 degree C. (This is known as helium's heat capacity at constant volume.) (answer check available at lightandmatter.com)

4. A sample of gas is enclosed in a sealed chamber. The gas consists of molecules, which are then split in half through some process such as exposure to ultraviolet light, or passing an electric spark through the gas. The gas returns to thermal equilibrium with the surrounding room. How does its pressure now compare with its pressure before the molecules were split?

5. Most of the atoms in the universe are in the form of gas that is not part of any star or galaxy: the intergalactic medium (IGM). The IGM consists of about 10^{-5} atoms per cubic centimeter, with a typical temperature of about 10^3 K. These are, in some sense, the density and temperature of the universe (not counting light, or the exotic particles known as "dark matter"). Calculate the pressure of the universe (or, speaking more carefully, the typical pressure due to the IGM).(answer check available at lightandmatter.com)

6. Estimate the pressure at the center of the Earth, assuming it is of constant density throughout. Note that g is not constant with respect to depth --- as shown in example 19 on page 105, g equals Gmr/b^3 for r, the distance from the center, less than b, the earth's radius.
(a) State your result in terms of G, m, and b.(answer check available at lightandmatter.com)
(b) Show that your answer from part a has the right units for pressure.
(c) Evaluate the result numerically. (answer check available at lightandmatter.com)
(d) Given that the earth's atmosphere is on the order of one thousandth the earth's radius, and that the density of the earth is several thousand times greater than the density of the lower atmosphere, check that your result is of a reasonable order of magnitude.

7. (a) Determine the ratio between the escape velocities from the surfaces of the earth and the moon. (answer check available at lightandmatter.com)
(b) The temperature during the lunar daytime gets up to about 130°\textup{C}. In the extremely thin (almost nonexistent) lunar atmosphere, estimate how the typical velocity of a molecule would compare with that of the same type of molecule in the earth's atmosphere. Assume that the earth's atmosphere has a temperature of 0°\textup{C}. (answer check available at lightandmatter.com)
(c) Suppose you were to go to the moon and release some fluorocarbon gas, with molecular formula $\mathrm{C}_n\mathrm{F}_{2n+2}$. Estimate what is the smallest fluorocarbon molecule (lowest n) whose typical velocity would be lower than that of an N_2 molecule on earth in proportion to the moon's lower escape velocity. The moon would be able to retain an atmosphere made of these molecules. (answer check available at lightandmatter.com)

8. Refrigerators, air conditioners, and heat pumps are heat engines that work in reverse. You put in mechanical work, and the effect is to take heat out of a cooler reservoir and deposit heat in a warmer one: $Q_L + W = Q_H$. As with the heat engines discussed previously, the efficiency is defined as the energy transfer you want (Q_L for a refrigerator or air conditioner, Q_H for a heat pump) divided by the energy transfer you pay for (W).

Efficiencies are supposed to be unitless, but the efficiency of an air conditioner is normally given in terms of an EER rating (or a more complex version called an SEER). The EER is defined as Q_L/W, but expressed in the barbaric units of of Btu/watt-hour. A typical EER rating for a residential air conditioner is about 10 Btu/watt-hour, corresponding to an efficiency of about 3. The standard temperatures used for testing an air conditioner's efficiency are $80^\circ F$ (27°C) inside and $95^\circ F$ (35°C) outside.

(a) What would be the EER rating of a reversed Carnot engine used as an air conditioner? (answer check available at lightandmatter.com)

(b) If you ran a 3-kW residential air conditioner, with an efficiency of 3, for one hour, what would be the effect on the total entropy of the universe? Is your answer consistent with the second law of thermodynamics? (answer check available at lightandmatter.com)

9. Even when resting, the human body needs to do a certain amount of mechanical work to keep the heart beating. This quantity is difficult to define and measure with high precision, and also depends on the individual and her level of activity, but it's estimated to be about 1 to 5 watts. Suppose we consider the human body as nothing more than a pump. A person who is just lying in bed all day needs about 1000 kcal/day worth of food to stay alive. (a) Estimate the person's thermodynamic efficiency as a pump, and (b) compare with the maximum possible efficiency imposed by the laws of thermodynamics for a heat engine operating across the difference between a body temperature of $37^{\circ}\mathrm{C}$ and an ambient temperature of $22^{\circ}\mathrm{C}$. (c) Interpret your answer.\hwans{hwans:heart-efficiency}

10. Example 25 on page 332 suggests analyzing the resonance of a violin at 300 Hz as a Helmholtz resonance. However, we might expect the equation for the frequency of a Helmholtz resonator to be a rather crude approximation here, since the f-holes are not long tubes, but slits cut through the face of the instrument, which is only about 2.5 mm thick. (a) Estimate the frequency that way anyway, for a violin with a volume of about 1.6 liters, and f-holes with a total area of 10 cm^2. (b) A common rule of thumb is that at an open end of an air column, such as the neck of a real Helmholtz resonator, some air beyond the mouth also vibrates as if it was inside the tube, and that this effect can be taken into account by adding 0.4 times the diameter of the tube for each open end (i.e., 0.8 times the diameter when both ends are open). Applying this to the violin's f-holes results in a huge change in L, since the ~ 7 mm width of the f-hole is considerably greater than the thickness of the wood. Try it, and see if the result is a better approximation to the observed frequency of the resonance.\hwans{hwans:violin-helmholtz}

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