62.7: Energy

Einstein showed that mass is a form of energy, as shown by his most famous equation,

$$E_0 = mc^2. \tag{62.7.1}$$

E_0 is called the rest energy of the particle of mass m. The clearest illustration of this formula is the mutual annihilation of matter and antimatter (a kind of mirror-image of ordinary matter). When a particle of matter collides with a particle of antimatter, the mass of the two particles is converted completely to energy, the amount of energy liberated being given by Eq. $62.7.1$.

As examples, the rest energy of the electron is $511 \mathrm{keV}$, and the rest energy of the proton is $938 \mathrm{MeV}$. ($1 \mathrm{eV}$ is one electron volt, and is equal to $1.602176634 \times 10^{-19}$ J.)

Kinetic Energy

In classical Newtonian mechanics, the kinetic energy is given by $K = mv^2/2$. The relativistic version of this equation is

$$K = (\gamma - 1) mc^2 \tag{62.7.2}$$

It is not obvious that this reduces to the classical expression until we expand γ into a Taylor series:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \frac{5}{16}\frac{v^6}{c^6} + \frac{35}{128}\frac{v^8}{c^8} + \frac{63}{256}\frac{v^{10}}{c^{10}} + \frac{231}{1024}\frac{v^{12}}{c^{12}} + \cdots \tag{62.7.3}$$

Substituting this series expansion for γ into Eq. $62.7.2$, we get

$$K = \frac{1}{2}mv^2 + \frac{3}{8}m\frac{v^4}{c^2} + \frac{5}{16}m\frac{v^6}{c^4} + \frac{35}{128}m\frac{v^8}{c^6} + \frac{63}{256}m\frac{v^{10}}{c^8} + \frac{231}{1024}m\frac{v^{12}}{c^{10}} + \cdots \tag{62.7.4}$$

Unless the speed v is near the speed of light c, all but the first term on the right will be very small and can be neglected, leaving the classical equation.

Total Energy

If the only forms of energy present are the rest energy E_0 and the kinetic energy K, then the total energy E will be the sum of these:

$$E = E_0 + K = \gamma mc^2 \tag{62.7.5}$$

It is often useful to know the total energy of a particle in terms of its momentum p rather than its velocity v. It can be shown that the total energy is given in terms of momentum by

$$E^2 = (pc)^2 + \left(mc^2\right)^2 \tag{62.7.6}$$

In the case where the total energy is much larger than the rest energy ($E \gg E_0$), we may neglect the second term on the right, and use

$$E \approx pc \tag{62.7.7}$$