## 52.2: The Continuity Equation

Consider a fluid flowing with velocity $v_1$ through a pipe of cross-sectional area $A_1$. Then in a time $\Delta t$, a volume $v_1 \Delta t A_1$ of fluid passes a fixed on that pipe. Now suppose the pipe flares to a larger or smaller pipe of area $A_2$. If the fluid is incompressible, then the same volume must pass a fixed point in the new pipe in time $\Delta t$. Therefore the fluid velocity in the new pipe must change to a new velocity $v_2$ that satisfies $v_1 \Delta t A_1 = v_2 \Delta t A_2$, or $v_1 A_1 = v_2 A_2$. This implies that for incompressible fluid flow, the flow rate $Av$ must be constant:

$$Av = \text{ constant.} \tag{52.2.1}$$

The flow rate $Av$ has units of volume per unit time ( $\mathrm{m}^3/\mathrm{s}$ ). This relation is called the continuity equation.

You may be familiar with this idea in playing with a garden hose with the nozzle removed. Water flows out of the hose relatively slowly; but if you place your thumb over the opening to block most of the flow, then water squirts out of the small remaining opening at high velocity.