23.5: Examples

Example 23.5.1

Motion in a horizontal circle.

Solution

Suppose you spin a mass m in a horizontal circle of radius r over your head; then the centripetal force (the tension in the string) is mv^2/r, where v is the speed of the mass.

Assume there is no gravity present; then what happens if the string suddenly breaks? Then the mass will immediately move in a straight line tangent to the circle.

Example 23.5.2

Motion in a vertical circle.

Solution

If you spin a bucket of water in a circle in a vertical plane (Fig. 23.5.1, then (if you're spinning it fast enough) the centrifugal force (i.e. inertia) will keep the water in the bucket. How fast must you spin the bucket?

Figure 23.5.1: A bucket of water being spun in a vertical circle. Inertia (sometimes thought of as a fictitious "centrifugal force") keeps the water in the bucket, even when upside-down. (Ref. [18]).

At top of the swing (when the string is vertical and the bucket is upside-down), the outward centrifugal force mv^2/r must be greater than or equal to the weight of the water mg; so the minimum speed v of the bucket is given by

$$\frac{mv^2}{r} = mg \tag{23.5.1}$$

or

$$v = \sqrt{gr} \tag{23.5.2}$$

The time T required for the bucket to make one complete circle (called the period of the motion) is then

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{gr}} \tag{23.5.3}$$

or

$$T = 2\pi\sqrt{\frac{r}{g}} \tag{23.5.4}$$

For example, if the bucket is swung in a circle of radius 0.8 meters, this formula gives a period of 1.80 seconds; in other words, if you swing the bucket in a vertical circle at a constant speed so that it completes each circle in not more that 1.80 seconds, the water will stay in the bucket, even at the top of the swing.