57.16: Parabolic Orbits

Suppose we wish to calculate the position of a body that is in a parabolic or near-parabolic orbit ($e \approx 1$), as is the case with some comets in orbit around the Sun. The procedure is the same as outlined in Section 57.6, except for Eq. 57.6.1 through 57.6.5.

For parabolic orbits, in place of the semi-major axis of the ellipse a, we use the perihelion distance q, and in place of the epoch time we use the time of perihelion passage T_p. Then the true anomaly f at time t is given by Barker's equation,

$$\tan\left(\frac{f}{2}\right)+\frac{1}{3}\tan^3\left(\frac{f}{2}\right)=\sqrt{\frac{GM}{2q^3}}\,(t-T_p) \tag{57.16.1}$$

In the case of a body orbiting the Sun, GM is the graviational constant of the Sun, equal to $1.32712440041\times10^{20}$ $\mathrm{m}^3\ \mathrm{s}^{-2}$. It is possible to solve Barker's equation $57.16.1$ for the true anomaly f directly (see e.g. McCuskey [12]) in just a few steps. Let K be the right-hand side of Eq. $57.16.1$:

$$K\equiv\sqrt{\frac{GM}{2q^3}}\,(t-T_p) \tag{57.16.2}$$

Then the true anomaly f is found through a series of steps:

$$\cot s=\frac{3}{2}|K|=\frac{3\sqrt{GM}}{(2q)^{3/2}}|t-T_p| \tag{57.16.3}$$

$$\cot\left(\frac{s}{2}\right)=\sqrt{1+\cot^2 s}+\cot s \tag{57.16.4}$$

$$\cot w=\sqrt[3]{\cot\left(\frac{s}{2}\right)} \tag{57.16.5}$$

$$\cot 2w=\frac{\cot^2 w-1}{2\cot w} \tag{57.16.6}$$

$$\tan\left(\frac{f}{2}\right)=(2\cot 2w)\times\operatorname{sgn}(t-T_0) \tag{57.16.7}$$

Here $\operatorname{sgn}(x)$ is the signum function, and is defined as

$$\operatorname{sgn}(x)=\begin{cases}-1\,(x<0)\\ 0\,(x=0)\\ +1\,(x>0)\end{cases}$$

Once the true anomaly f has been found, the radial distance from the Sun to the body is found by this replacement for Eq. 57.6.5:

$$r=q\sec^2\left(\frac{f}{2}\right) \tag{57.16.8}$$

The rest of position calculation is the same as described in Section 57.6 for elliptical orbits.