8.2: Velocity

The velocity of a particle is a measure of how much distance it covers in a given time. [1] SI units of velocity are meters per second ($\mathrm{m/s}$, or $\mathrm{ms^{-1}}$). There are two ways we can talk about velocity: the average velocity over some finite time interval Δt, or the instantaneous velocity at an instant in time t.

Average Velocity

Suppose a particle is at position x_1 at time t_1, and it's at position x_2 at time t_2. Then over the time interval $\Delta t = t_2 - t_1$, the particle undergoes a displacement $\Delta x = x_2 - x_1$. The average velocity v_{ave} of the particle over time interval Δt is defined to be

$$v_{\text{ave}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \tag{8.2.1}$$

Example 8.2.1

If a particle travels 400 meters in 5 seconds what is its average velocity

Solution

Its average velocity is $v_{\text{ave}} = \Delta x/\Delta t = (400\ \mathrm{m})/(5\ \mathrm{s}) = 80\ \mathrm{m/s}$. Remember that $\Delta x = 400\ \mathrm{m}$ means that the particle's position at the end of the time interval is 400 meters beyond its position at the start of the interval. It might have traveled millions of meters in between, but we don't care about that: all that matters is the starting position and ending position.

Instantaneous Velocity

Suppose we want to know the instantaneous velocity at a single instant in time t, rather than an average over a time interval Δt. The calculus gives us a method to do that: we just use Eq. $8.2.1$ to find the average velocity over a time interval Δt, then make the time interval arbitrarily small. Mathematically, this is just the derivative:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{8.2.2}$$

Example 8.2.2

Suppose we have a formula for the position x of a particle at any time t-for example, $x(t) = 5t^2 + 7\ \mathrm{m}$. Then we can get a formula for the velocity v at any time t by taking the derivative: $v(t) = dx/dt = 10t\ \mathrm{m/s}$.

1. The magnitude (absolute value) of velocity is called speed.