21.2: Inclined Block and Vertical Block

Now let's generalize the previous problem by placing block m_1 on an upward inclined plane that makes an angle θ to the horizontal (Figure $21.2.1$). We'll begin with the case where the inclined plane is frictionless. The forces on mass m_1 are the upward tension T as before, plus a downward acceleration $m_1 g \sin\theta$ down the inclined plane.

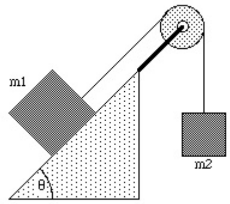

Figure 21.2.1: Inclined block and vertical block.

Taking upslope as positive and downslope as negative, Newton's second law for m_1 is then

$$\Sigma_i F_i = m_1 a \quad \Rightarrow \quad T - m_1 g \sin\theta = m_1 a \tag{21.2.1}$$

Newton's second law for m_2, in the vertical (downward) direction, is the same as before:

$$\Sigma_i F_i = m_2 a \quad \Rightarrow \quad m_2 g - T = m_2 a \tag{21.2.2}$$

As before, we add these two equations to eliminate the tension T and solve for the acceleration a. We find

$$a = \frac{m_2 - m_1 \sin\theta}{m_1 + m_2} g \tag{21.2.3}$$

and then solving for the tension T, we find

$$T = \frac{m_1 m_2 (1 + \sin\theta)}{m_1 + m_2} g \tag{21.2.4}$$

Notice that these equations reduce to the equations for m_1 on a horizontal surface (Section 18.1) when we set $\theta = 0$, as expected.

Note particularly how we chose the signs in this problem. When the system is released, the vertical block will fall downward; we'll choose to call this the positive $(+a)$ direction. Since this will result in the block on the plane accelerating upslope, this means we must choose upslope to be the positive direction to keep the signs consistent.

Now let's generalize this even further by adding friction to the inclined plane. In this case, mass m_1 will experience an upslope force equal to the tension T and a downslope force $m_1 g \sin\theta$. In addition, there will be a frictional force $f = \mu n = \mu m_1 g \cos\theta$ acting opposite the direction of motion (downslope). Thus

$$\Sigma_i F_i = m_1 a \quad \Rightarrow \quad T - m_1 g \sin\theta - \mu m_1 g \cos\theta = m_1 a \tag{21.2.5}$$

Newton's second law for m_2, in the vertical (downward) direction, is the same as before:

$$\Sigma_i F_i = m_2 a \quad \Rightarrow \quad m_2 g - T = m_2 a \tag{21.2.6}$$

As before, we add these two equations to eliminate the tension T and solve for the acceleration a :

$$a = \frac{m_2 - m_1(\mu \cos\theta + \sin\theta)}{m_1 + m_2} g \tag{21.2.7}$$

and we find the tension to be

$$T = \frac{m_1 m_2 (1 + \mu \cos\theta + \sin\theta)}{m_1 + m_2} g \tag{21.2.8}$$

The last two equations are generalizations of all the previous problems. Setting $\theta = 0$ recovers the equations for m_1 on a horizontal surface, and setting $\mu = 0$ recovers the frictionless formulas. Furthermore, setting $\mu = 0$ and $\theta = 90^\circ$ produces the equations for the acceleration and tension for the Atwood's machine discussed in Chapter 18.