17: The Inclined Plane

An inclined plane (Fig. 17.1) is one of the classical simple machines. [1] Let's consider the motion of a block sliding down a frictionless inclined plane.

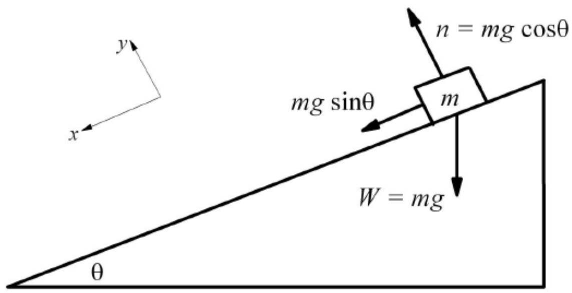

Figure 17.1: An object sliding on an inclined plane.

First, let's define a coordinate system: let's take the origin at the block's initial position, $+x$ pointing down the plane (in the direction of motion), and $+y$ pointing upward perpendicular to the plane. Let's apply Newton's second law to the x and y directions:

$$x: \quad \sum F_x = mg\sin\theta = ma \tag{17.1}$$

$$y: \quad \sum F_y = n - mg\cos\theta = 0 \tag{17.2}$$

In Eq. 17.1, the sum of the forces in the x direction is $mg\sin\theta$; solving this equation gives the acceleration of a block down an incline:

$$a = g\sin\theta \tag{17.3}$$

In Eq. 17.2, the forces in the y direction are n (in the $+y$ direction) and the y component of the weight ($mg\cos\theta$) in the $-y$ direction. Solving this equation gives the magnitude of the normal force: $n = mg\cos\theta$.

From this example, we can see the general procedure for solving problems like this:

1. Define a coordinate system. You're free to define the direction and origin however you wish, so choose something convenient that will make the equations simple.
2. Identify all the forces acting on the body. You may wish to draw a free-body diagram if it helps you to identify the forces.
3. Find the projection of each force onto the coordinate axes you defined.
4. Apply Newton's second law ($\sum_i F_i = ma$) in both the x and y directions.
5. Solve the equations to find whatever you're asked to find.

[1] The others are the lever, the wheel and axle, the pulley, the wedge, and the screw. See Chapter 22.