35.1: Definition and Forms

The cross product (sometimes called the vector product) is indicated with a cross sign $(\mathbf{A}\times\mathbf{B})$ and is pronounced "A cross B." When you take the cross product of two vectors, you get back another vector, whose magnitude is

$$|\mathbf{A}\times\mathbf{B}| = AB\sin\theta \tag{35.1.1}$$

where θ is the angle separating vectors $\mathbf{A}$ and $\mathbf{B}$[1]

The direction of the vector $\mathbf{A}\times\mathbf{B}$ is perpendicular to the plane of vectors $\mathbf{A}$ and $\mathbf{B}$. But there are two possible choices for direction of a vector perpendicular to a plane; which one do we choose? By convention, we choose the one given by a right-hand rule: if you curl the fingers of your right and from vector $\mathbf{A}$ toward vector $\mathbf{B}$, then the thumb of your right hand points in the direction of $\mathbf{A}\times\mathbf{B}$ (Fig. 35.1.1).

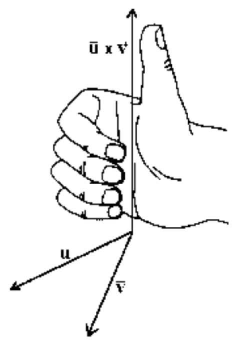

Figure 35.1.1: The vector cross product $\mathbf{A}\times\mathbf{B}$ is perpendicular to the plane of $\mathbf{A}$ and $\mathbf{B}$, and in the right-hand sense. (Credit: “Connected Curriculum Project”, Duke University.)

Since $\mathbf{A}\times\mathbf{B}$ is perpendicular to the plane formed by vectors $\mathbf{A}$ and $\mathbf{B}$, it is also perpendicular to both vectors $\mathbf{A}$ and $\mathbf{B}$:

$$(\mathbf{A}\times\mathbf{B}) \perp \mathbf{A} \tag{35.1.2}$$

$$(\mathbf{A}\times\mathbf{B}) \perp \mathbf{B} \tag{35.1.3}$$

Component Form

A convenient mnemonic for finding the rectangular components of the cross product is through a matrix determinant:

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \tag{35.1.4}$$

$$= (A_yB_z - A_zB_y)\,\mathbf{i} - (A_xB_z - A_zB_x)\,\mathbf{j} + (A_xB_y - A_yB_x)\,\mathbf{k}. \tag{35.1.5}$$

For example, if $\mathbf{A} = 3\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ and $\mathbf{B} = 2\mathbf{i}-\mathbf{j}+4\mathbf{k}$, then $\mathbf{A}\times\mathbf{B} = (20-(-2))\mathbf{i} - (12-4)\mathbf{j} + (-3-10)\mathbf{k} = 22\mathbf{i} - 8\mathbf{j} - 13\mathbf{k}$.

Matrix Formulation

Another way to represent the components of the cross product is to write the components of vector $\mathbf{A}$ into an antisymmetric 3×3 matrix, then multiply that matrix by the column vector $\mathbf{B}$:

$$\mathbf{A}\times\mathbf{B} = \begin{pmatrix} 0 & -A_z & A_y \\ A_z & 0 & -A_x \\ -A_y & A_x & 0 \end{pmatrix} \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} \tag{35.1.6}$$

$$= \begin{pmatrix} A_y B_z - A_z B_y \\ A_z B_x - A_x B_z \\ A_x B_y - A_y B_x \end{pmatrix}. \tag{35.1.7}$$

[1] An old physics joke: What do you get when you cross an elephant with a banana? Ans. "Elephant banana sine θ."