37.4: Plane Figure Theorem

Another, lesser known, theorem involving moments of inertia is the plane figure theorem, and relates to the moment of inertia of a two-dimensional (plane) figure.

Theorem 37.4.1

The plane figure theorem states that given the moments of inertia I_x and I_y of the figure about two perpendicular axes in the plane of the figure, the moment of inertia I_z about an axis perpendicular to the first two is given by

$$I_z = I_x + I_y. \tag{37.4.1}$$

Example 37.4.1

What is the moment of inertia of a uniform disk of mass M and radius R when rotated about an axis passing through the center of the disk and lying in the plane of the disk?

Solution

Define a coordinate system such that the x axis lies along the rotation axis, and the z axis is perpendicular to the disk and passing through the center of the disk. Then by symmetry, the desired moment of inertia $I = I_x = I_y$. Furthermore, we know from Table 37.2.1 that $I_z = \frac{1}{2}MR^2$. Therefore, by the plane figure theorem,

$$I_z = I_x + I_y. \tag{37.4.2}$$

which becomes

$$\frac{1}{2}MR^2 = I + I \tag{37.4.3}$$

So

$$I = \frac{1}{4}MR^2 \tag{37.4.4}$$