41.5: The Physical Pendulum

A physical pendulum consists of an extended body that allowed to swing back and forth around some pivot point. If the pivot point is at the center of mass, the body will not swing, so the pivot point should be displaced from the center of mass. As an example, you can form a physical pendulum by suspending a meter stick from one end and allowing to swing back and forth.

In a physical pendulum of mass M, there is a force Mg acting on the center of mass. Suppose the body is suspended from a point that is a distance h

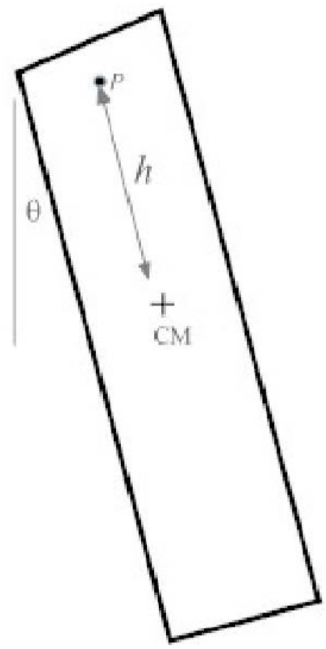

Figure 41.5.1: A physical pendulum. The object has mass M and is suspended from point P; h is the distance between P and the center of mass.

from the center of mass (Fig. $41.5.1$). Then there is a weight force Mg acting on the center of mass of the body, which creates a torque $-Mgh\sin\theta$ about the pivot point. Then by the rotational version of Newton's second law,

$$\tau = I\alpha \tag{41.5.1}$$

$$-Mgh\sin\theta = I\frac{d^2\theta}{dt^2} \tag{41.5.2}$$

where I is the moment of inertia of the body when rotated about its pivot point. As with the simple plane pendulum, this is a second-order differential equation that is difficult to solve. But if we constrain the oscillations to small amplitudes, we can make the approximation $\sin\theta \approx \theta$ as before, and the equation becomes

$$\frac{d^2\theta}{dt^2} = -\frac{Mgh}{I}\theta \tag{41.5.3}$$

We can solve this second-order differential equation as before, and get

$$\theta(t) = \theta_0\cos(\omega t + \delta) \tag{41.5.4}$$

where θ_0 is the (angular) amplitude of the motion (in radians), $\omega = \sqrt{Mgh/I}$ is the angular frequency of the motion ($\mathrm{rad/s}$), and δ is an arbitrary integration constant (seconds). The solution can be verified by direct substitution into Eq. (38.16).

The period T of the motion (the time required for one complete back-and-forth cycle) is given by

$$T = \frac{2\pi}{\omega} \tag{41.5.5}$$

or

$$T=2\pi\sqrt{\frac{I}{Mgh}}. \tag{41.5.6}$$