11.1: Position, Velocity, Acceleration

Position

Let's begin with the position vector. First define a two-dimensional coordinate system (or a three-dimensional system for a three-dimensional problem), placing the origin and axis directions in any way that's convenient. Then the position vector $\mathbf{r}$ of a particle is a vector pointing from the origin to the particle.

Velocity

We define the velocity vector $\mathbf{v}$ in a way that's analogous to the definition of the scalar velocity, using the vector version of the definition of a derivative:

$$\mathbf{v}=\lim_{\Delta t\to 0}\frac{\Delta\mathbf{r}}{\Delta t}=\frac{d\mathbf{r}}{dt} \tag{11.1.1}$$

where $\Delta\mathbf{r}=\mathbf{r}_2-\mathbf{r}_1$ is the difference in the position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ at two closely spaced times t_1 and t_2, respectively.

Acceleration

Similarly, the acceleration vector a is defined as

$$\mathbf{a}=\lim_{\Delta t\to 0}\frac{\Delta\mathbf{v}}{\Delta t}=\frac{d\mathbf{v}}{dt}=\frac{d^2\mathbf{r}}{dt^2} \tag{11.1.2}$$

where $\Delta\mathbf{v}=\mathbf{v}_2-\mathbf{v}_1$ is the difference in the velocity vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ at two closely spaced times t_1 and t_2, respectively.

Inverse Relationships

Equations (11.1.1) and (11.1.2) may be inverted, as was done in one dimension:

$$\mathbf{r}(t)=\int\mathbf{v}(t)dt \tag{11.1.3}$$

$$\mathbf{v}(t)=\int\mathbf{a}(t)dt \tag{11.1.4}$$