24.5: Case IV (General Case)- Variable force along any direction

In the most general case, an object moves through some arbitrary path in space, and the force $\mathbf{F}$ is variable, which we'll write as $\mathbf{F}(\mathbf{r})$. Then we'll divide the path into infinitesimal segments $d\mathbf{r}$, and the force along $d\mathbf{r}$ can be considered constant over that short distance. The work done by the force along $d\mathbf{r}$ is $dW = F(\mathbf{r}) \cdot d\mathbf{r}$. Then the total work done by the force is computed with an integral:

$$W = \int \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r} \tag{24.5.1}$$

This means that you imagine dividing the entire path into infinitesimal segments $d\mathbf{r}$; at each segment, you compute the force $\mathbf{F}(\mathbf{r})$ at that segment, and take the dot product $\mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$. You then add together all those dot products with an integral to get the total work done by the force.

This general expression for work reduces to the other formulæ under the special conditions mentioned earlier. For example, if $\mathbf{F}$ acts in the direction of motion, then $\mathbf{F}(\mathbf{r}) \cdot d\mathbf{r} = F(x)dx$, and we get Eq. (24.4.1). If the force $\mathbf{F}$ in Eq. $24.5.1$ is constant, then $\mathbf{F}$ can be taken outside the integral, and we recover Eq. (24.3.1). And if $\mathbf{F}$ and $\mathbf{r}$ are parallel, then Eq. (24.3.1) reduces to Eq. (24.2.1).