49.1: Precession

The gyroscope can be used to illustrate some properties of rotating bodies. For example, suppose the axis of the gyroscope is held vertical, and the axle is supported from the bottom end only. If the gyroscope is not spinning, then the instrument is unstable: the slightest movement from a perfectly balanced vertical position will cause it to topple over. But suppose we set the gyroscope spinning first, then set it down so the axle is vertical and supported from the bottom end. The instrument will still tend to topple over, but in doing so it will pivot about the bottom end of the axle, creating a torque about that point. The spinning gyroscope already has an angular momentum $\mathbf{L}$; the torque $\boldsymbol{\tau} = d\mathbf{L}/dt$ due to gyroscope wanting to tip over causes the instrument's angular momentum to change with time, causing it to move in a circle.

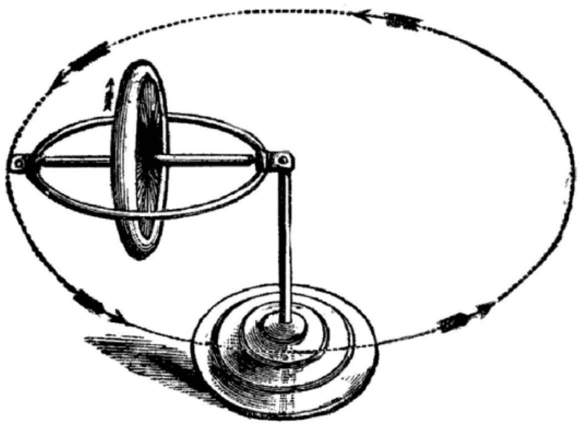

Figure 49.1.1: Motion of a gyroscope. By the right-hand rule, the angular momentum of the wheel is to the left. The torque vector due to the gyroscope tipping over is horizontal, toward the observer. This torque vector "pushes" the angular momentum vector around counterclockwise, as shown; the resulting motion is called precession. (From Ref. [13])

For example, suppose the gyroscope is vertical and spinning counterclockwise as seen from above. Then by the right-hand rule, its angular momentum vector $\mathbf{L}$ points upward. If you're watching the gyroscope from the side and it begins to topple over to the right, then there is a torque vector $\boldsymbol{\tau}$ pointing away from you. Since $\boldsymbol{\tau} = d\mathbf{L}/dt$, this means the torque and the change in angular momentum will be in the same direction, so the gyroscope will start to rotate away from you. Essentially the falling over of the gyroscope is turned sideways, causing the gyroscope to describe a circular motion called precession.

The angular velocity vector $\boldsymbol{\omega}_P$ of this precession is found to satisfy

$$\boldsymbol{\tau} = \omega_P \times \mathbf{L} \tag{49.1.1}$$

Solving for the magnitude of the angular velocity of the precession ω_P, we find

$$\omega_P = \frac{MgD}{L\sin\theta} \tag{49.1.2}$$

where M is the mass of the gyroscope wheel, D is the distance between the bottom end of the axle and the wheel, L is the angular momentum of the gyroscope about its axis, and θ is the angle of the gyroscope axis from the vertical.