51.2: Floating Bodies

If a solid body is placed in a fluid, it will float if its density is less than the fluid density. Suppose we have a body of mass m_b and volume V_b floating in a liquid of density ρ_0. Then part of the body will be submerged, and part will be above the surface of the liquid. How much of the body will be submerged?

To answer this, let V_s be the submerged volume of the body. Then if the body is in equilibrium, the weight of the body $m_b g$ (acting downward) must equal the upward buoyant force. But by Archimedes' principle, the buoyant force is equal to the weight of the displaced liquid. Now the mass of the displaced liquid is the displaced volume V_s times the density of the liquid, ρ_0, and so the weight of displaced fluid is $V_s \rho_0 g$. Then since the weight of the body equals the upward buoyant force, we have

$$m_b g = V_s \rho_0 g \tag{51.2.1}$$

$$m_b = V_s \rho_0 \tag{51.2.2}$$

To write this another way, note that the mean density of the body is $\rho_b = m_b / V_b$. Using this to substitute for m_b in the above equation, we get

$$V_b \rho_b = V_s \rho_0 \tag{51.2.3}$$

and so

$$\frac{V_s}{V_b} = \frac{\rho_b}{\rho_0} \tag{51.2.4}$$

In other words, the fraction of the body's volume that is submerged is equal to the mean density of the body divided by the density of the liquid. The less dense the body, the higher it will "ride" in the liquid; the denser the body, the lower it will be submerged.

Example 51.2.1

We use the phrase "the tip of the iceberg" to indicate a small part of something much larger. The phrase has its origin in observation that an iceberg floating in water has only a small part of its volume visible above the water surface. In a real iceberg, how much of the iceberg is above water, and how much is below water?

Solution

First, note that the iceberg itself is made of fresh water, and is typically floating in sea water. The density of ice is about $\rho_b = 0.9169 \text{ g/cm}^3$, and the density of sea water is about 1.025 g/cm^3. Therefore the fraction of the iceberg that is submerged is $\rho_b / \rho_0 = 0.9169/1.025 = 0.895$ So an iceberg has about 90% of its volume submerged below water, and about only about 10% above water.