

66.3: Hyperbolic Trigonometry

Basic Formulæ

$$\begin{aligned}\cosh^2 x - \sinh^2 x &\equiv 1 \\ \operatorname{sech}^2 x &\equiv 1 - \tanh^2 x \\ \operatorname{csch}^2 x &\equiv \coth^2 x - 1\end{aligned}$$

Angle Addition Formulæ

$$\begin{aligned}\sinh(x \pm y) &\equiv \sinh x \cosh y \pm \cosh x \sinh y \\ \cosh(x \pm y) &\equiv \cosh x \cosh y \pm \sinh x \sinh y \\ \tanh(x \pm y) &\equiv \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}\end{aligned}$$

Double-Angle Formulæ

$$\begin{aligned}\sinh 2x &\equiv 2 \sinh x \cosh x \\ \cosh 2x &\equiv \cosh^2 x + \sinh^2 x \\ \tanh 2x &\equiv \frac{2 \tanh x}{1 + \tanh^2 x}\end{aligned}$$

Half-Angle Formulæ

$$\begin{aligned}\sinh \frac{x}{2} &\equiv \pm \sqrt{\frac{\cosh x - 1}{2}} \\ \cosh \frac{x}{2} &\equiv \sqrt{\frac{\cosh x + 1}{2}} \\ \tanh \frac{x}{2} &\equiv \frac{\sinh x}{\cosh x + 1} \equiv \frac{\cosh x - 1}{\sinh x}\end{aligned}$$

Products of Hyperbolic Sines and Cosines

$$\begin{aligned}\sinh x \cosh y &\equiv \frac{1}{2} [\sinh(x + y) + \sinh(x - y)] \\ \cosh x \sinh y &\equiv \frac{1}{2} [\sinh(x + y) - \sinh(x - y)] \\ \cosh x \cosh y &\equiv \frac{1}{2} [\cosh(x + y) + \cosh(x - y)] \\ \sinh x \sinh y &\equiv \frac{1}{2} [\cosh(x + y) - \cosh(x - y)]\end{aligned}$$

Power Reduction Formulæ

$$\begin{aligned}\sinh^2 x &\equiv \frac{1}{2} (\cosh 2x - 1) \\ \cosh^2 x &\equiv \frac{1}{2} (\cosh 2x + 1)\end{aligned}$$

Relations to Plane Trigonometric Functions

$$\begin{aligned}\sinh x &\equiv -i \sin(ix) \\ \cosh x &\equiv \cos(ix) \\ \tanh x &\equiv -i \tan(ix)\end{aligned}$$

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