53.1: Hydraulics- The Hydraulic Press

The properties of liquids may be exploited to make it possible to lift large, heavy objects using a machine called a hydraulic press (Fig. $53.1.1$). Referring to the figure, we know by Pascal's law that pressure P_1 must be equal to pressure P_2 :

$$P_1 = P_2. \tag{53.1.1}$$

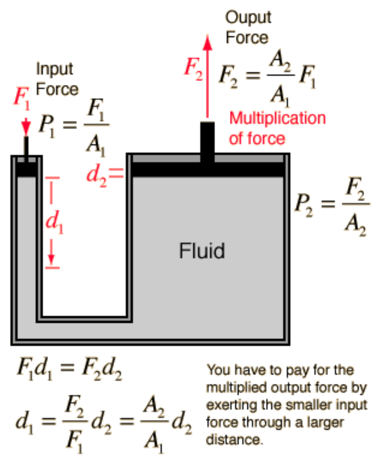

Figure $53.1.1$: The hydraulic press. (Credit: HyperPhysics project, Georgia State University, Ref. [7]).

Therefore

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \tag{53.1.2}$$

where A_1 and A_2 are the cross-sectional areas of the pistons on the left and right. Solving for F_1, we find

$$F_1 = F_2 \frac{A_1}{A_2} \tag{53.1.3}$$

Since $A_2 > A_1$, the force F_1 is multiplied by the factor A_2/A_1. One may place a heavy object like an automobile on the right, and lift it by applying a relatively small force on the left. The price for gaining this multiplication of force is that the piston on the left must be moved through a greater distance than the object on the right will be raised. To find the distance d_1 through which the piston on the left must be moved in order to lift the object on the right a distance d_2, we note that the liquid is essentially incompressible; therefore the volume change on the left must equal the volume change on the right:

$$A_1 d_1 = A_2 d_2 \tag{53.1.4}$$

Therefore the distance d_1 is

$$d_1 = d_2 \frac{A_2}{A_1} \tag{53.1.5}$$

We can find d_1 in terms of the ratio of forces using Eq. $53.1.3$ to substitute for A_2/A_1; we get

$$d_1 = d_2 \frac{F_2}{F_1} \tag{53.1.6}$$

The force in the small cylinder must be exerted over a much larger distance. A small force exerted over a large distance is traded for a large force over a small distance.

Example $53.1.1$

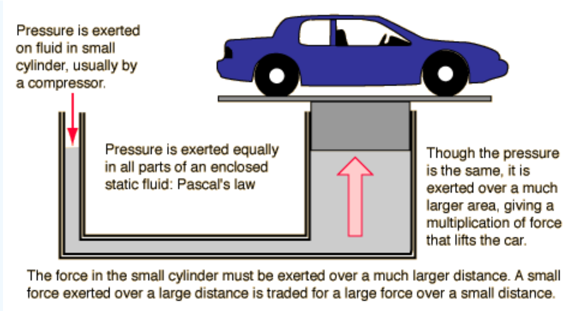

Figure 53.1.2: An automobile on a hydraulic press. (Credit: HyperPhysics project, Georgia State University, Ref. [7]).

Suppose the piston on the left has a diameter of 10 cm, and the piston on the right has a diameter of $1 \mathrm{~m}$. What force must be applied on the left to lift a 1000-kg automobile on the right? (See Fig. \(\PageIndex{2}\).)

Solution

The automobile has a weight $F_2=mg=(1000 \mathrm{~kg})\left(9.8 \mathrm{~m/s}^2\right)=9.8 \times 10^3 \mathrm{~N}$. The area $A_1=\pi r^2/4=(\pi/4)(0.1 \mathrm{~m})^2=(\pi/4) \times 10^{-2} \mathrm{~m}^2$. The area $A_2=\pi r^2/4=(\pi/4)(1 \mathrm{~m})^2=\pi/4 \mathrm{~m}^2$. The force F_1 is then

$$F_1=F_2 \frac{A_1}{A_2} \tag{53.1.7}$$

$$=\left(9.8 \times 10^3 \mathrm{~N}\right) \frac{\pi/4 \times 10^{-2} \mathrm{~m}^2}{\pi/4 \mathrm{~m}^2} \tag{53.1.8}$$

$$=98 \mathrm{~N} \tag{53.1.9}$$

In this case, the piston on the left must be pushed in $1 \mathrm{~m}$ to lift the car by 1 cm.