59.1: First Partial Derivatives

You've already learned in a calculus course how to take the derivative of a function of one variable. For example, if

$$f(x) = 3x^2 + 7x^5 \tag{59.1.1}$$

then

$$\frac{df}{dx} = 6x + 35x^4 \tag{59.1.2}$$

But what if f is a function of more that one variable? For example, if

$$f(x, y) = 5x^3y^5 + 4y^2 - 7xy^6 \tag{59.1.3}$$

then how do we take the derivative of f ? In this case, there are two possible first derivatives: one with respect to x, and one with respect to y. These are called partial derivatives, and are indicated using the "backward-6" symbol ∂ in place of the symbol d used for ordinary derivatives.

To compute a partial derivative with respect to x, you simply treat all variables except x as constants. Similarly, for the partial derivative with respect to y, you treat all variables except y as constants. For example, if $g(x, y) = 3x^4y^7$, then the partial derivative of g with respect to x is $\partial g/\partial x = 12x^3y^7$, since both 3 and y^7 are considered constants with respect to x.

As another example, the partial derivatives of Eq. $59.1.3$ are

$$\frac{\partial f}{\partial x} = 15x^2y^5 - 7y^6 \tag{59.1.4}$$

$$\frac{\partial f}{\partial y} = 25x^3y^4 + 8y - 42xy^5 \tag{59.1.5}$$

Notice that in Eq. $59.1.4$, the derivative of the term $4y^2$ with respect to x is 0 , since $4y^2$ is treated as a constant.