CHAPTER OVERVIEW

60: Lagrangian Mechanics

In this course we have been studying classical mechanics as formulated by Sir Isaac Newton; this is called Newtonian mechanics. Newtonian mechanics is mathematically fairly straightforward, and can be applied to a wide variety of problems. Newton's formulation of mechanics is not unique, however; other formulations are possible. Here we will look at two common alternative formulations of classical mechanics: Lagrangian mechanics and Hamiltonian mechanics. Langrangian mechanics will be discussed in this chapter; Hamiltonian mechanics will be covered in Chapter 58.

It is important to understand that all of these formulations of mechanics equivalent. In principle, any of them could be used to solve any problem in classical mechanics. The reason theyre important is that in some problems one of the alternative formulations of mechanics may lead to equations that are much easier to solve than the equations that arise from Newtonian mechanics. Unlike Newtonian mechanics, neither Lagrangian nor Hamiltonian mechanics requires the concept of force; instead, these systems are expressed in terms of energy. Although we will be looking at the equations of mechanics in one dimension, all these formulations of mechanics may be generalized to two or three dimensions.

The first alternative to Newtonian mechanics we will look at is Lagrangian mechanics. Using Lagrangian mechanics instead of Newtonian mechanics is sometimes advantageous in certain problems, where the equations of Newtonian mechanics would be quite difficult to solve.

In Lagrangian mechanics, we begin by defining a quantity called the Lagrangian *[Math Processing Error]*, which is defined as the difference between the kinetic energy *[Math Processing Error]* and the potential energy *[Math Processing Error]* :

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Since the kinetic energy is a function of velocity *[Math Processing Error]* and potential energy will typically be a function of position *[Math Processing Error]*, the Lagrangian will (in one dimension) be a function of both *[Math Processing Error]* and *[Math Processing Error]*.

The motion of a particle is then found by solving Lagrange's equation; in one dimension it is

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60.1: Examples