# 54.5: Earth Density Model

Suppose we have a uniform, spherical body (such as a planet) of radius $R$ and mass $M$. What is the acceleration $g$ due to gravity as a function of $r$ for $r$ both inside and outside the body $(0 \leq r < \infty)$ ?

First, consider the case where we're inside the body $(r \leq R)$. In this case, the acceleration due to gravity at $r$ is $g(r) = Gm/r^2$, where $m$ is the total mass inside a sphere of radius $r$ :

$$m = \frac{4}{3}\pi r^3 \rho \tag{54.5.1}$$

where the (uniform) density $\rho = M/\left(\frac{4}{3}\pi R^3\right)$. Thus

$$g(r) = \frac{GM}{R^3} r \quad (0 \leq r < R) \tag{54.5.2}$$

so inside the body, $g \propto r$.

Second, consider the case where we're outside the body $(r > R)$. In this case, the total mass inside a sphere of radius $r$ is $M$, and so

$$g(r) = \frac{GM}{r^2} \quad (r \geq R) \tag{54.5.3}$$

so that outside the body, $g \propto 1/r^2$. The maximum value of $g$ is at the surface, $g = GM/R^2$ at $r = R$ (Figure 54.5.1).

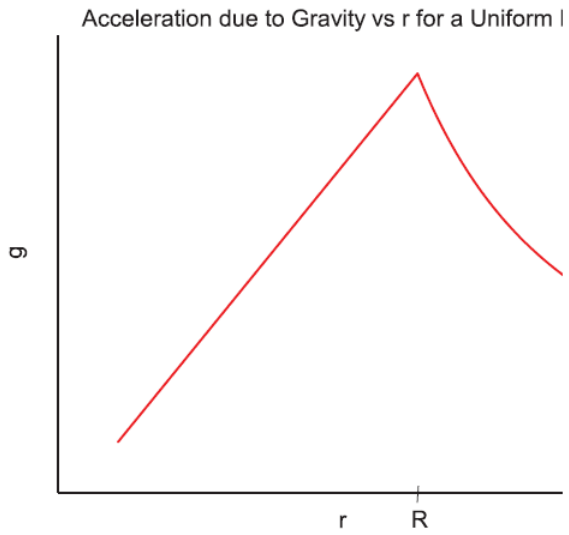


Figure 54.5.1: Acceleration due to gravity for a uniform sphere.

However, planetary bodies are generally not uniform. For example, the Earth has a higher density closer to its core, and its density decreases closer to the surface. One density model of the Earth given by Dziewonski and Anderson [1] is shown in Figure 54.5.2 We can use this density model to compute a more realistic model of $g(r)$ inside the Earth:

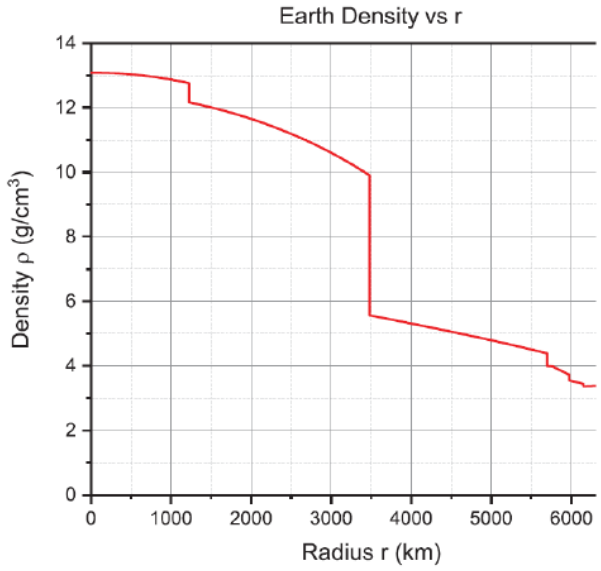


Figure 54.5.1: Earth density model (Dziewonski and Anderson, 1981.)

$$g(r) = \int_0^r \frac{G\rho(r)}{r^2} dV = \int_0^r \frac{G\rho(r)}{r^2} \left(4\pi r^2\right) dr = 4\pi G \int_0^r \rho(r) dr \tag{54.5.4}$$

The result is plotted in Figure 54.5.3. We see that in a more realistic model of the Earth's interior, the maximum value of the acceleration to to gravity $g$ occurs just outside the outer core, where $g = 10.7\ \mathrm{m/s^2}$.

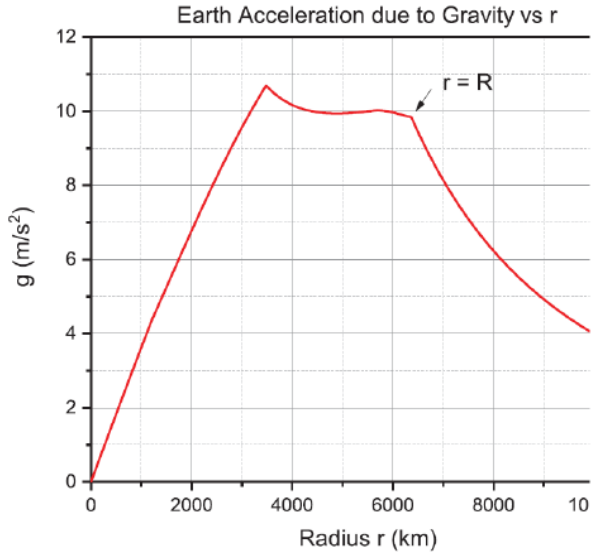


Figure 54.5.1: Modeled acceleration due to gravity for Earth.

[1] Dziewonski, A.M., and Anderson, D.L., Preliminary Earth reference model. Physics of the Earth and Planetary Interiors, 25 (1981) 297 – 356.