

66.17: Matrix Properties

This appendix presents a brief summary of the properties of 2×2 and 3×3 matrices.

2×2 Matrices

Determinant

The determinant of a 2×2 matrix is given by the well-known formula:

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc. \quad (66.17.1)$$

Matrix of Cofactors

The matrix of cofactors is the matrix of signed minors; for a 2×2 matrix, this is

$$\text{cof} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \quad (66.17.2)$$

Inverse

Finally, the inverse of a matrix is the transpose of the matrix of cofactors divided by the determinant. For a 2×2 matrix,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \quad (66.17.3)$$

3×3 Matrices

Determinant

The determinant of a 3×3 matrix is given by:

$$\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg) \quad (66.17.4)$$

Matrix of Cofactors

The matrix of cofactors is the matrix of signed minors; for a 3×3 matrix, this is

$$\text{cof} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} ei - fh & fg - di & dh - eg \\ ch - bi & ai - cg & bg - ah \\ bf - ce & cd - af & ae - bd \end{pmatrix} \quad (66.17.5)$$

Inverse

Finally, the inverse of a matrix is the transpose of the matrix of cofactors divided by the determinant. For a 3×3 matrix,

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}^{-1} = \frac{1}{a(ei - fh) - b(di - fg) + c(dh - eg)} \begin{pmatrix} ei - fh & ch - bi & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{pmatrix} \quad (66.17.6)$$

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