24.4: Case III- Variable force in the direction of motion

Now let's take another case: suppose the force F is in the direction of motion, but suppose F is not constant, but is a function of position x. Now take the straight-line path over which object moves and divide it into many infinitesimal segments, each of length dx. Then over distance dx, the force F can be considered constant, and the work dW done over distance dx is $F(x)dx$. To get the total work done by the force F, we sum up all these contributions Fdx by doing an integral:

$$W = \int F(x)dx \tag{24.4.1}$$

Example 24.4.1

For a mass on a spring, the work done by the spring force is given by Hooke's law: $F(x) = -kx$, where k is the spring constant. Then the work done by the spring is?

Solution

The work done by the spring is

$$W = \int F(x)dx \tag{24.4.2}$$

$$= \int (-k)x dx \tag{24.4.3}$$

$$= -k \int x dx \tag{24.4.4}$$

$$= -\frac{1}{2}kx^2 \tag{24.4.5}$$

In extending the spring a distance x from equilibrium, a work $-kx^2/2$ is done by the spring; work $+kx^2/2$ is done by you, against the spring.