20.5: The Coefficient of Friction

Some physics textbooks and handbooks include tables of coefficients of friction (μ_s and μ_k) for rubber on wood, metal on metal, etc. These tables are all false, and should be ignored. The coefficient of friction depends on a number of factors, including the smoothness of the surfaces and complex surface interactions (including contaminants from the air sticking on the surfaces), and cannot be simply looked up in a table.

So how do we determine the coefficient of friction? One simple method is to place an object of mass m on an inclined plane (Fig. $20.5.1$).

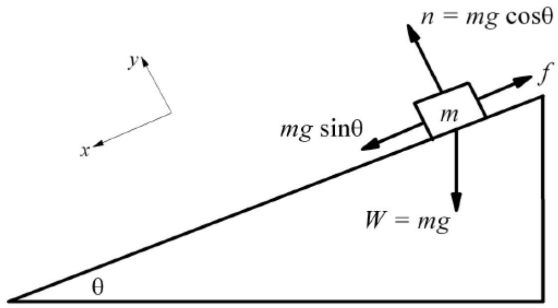

Figure 20.5.1: An object sliding on an inclined plane, with friction included.

Now tilt the plane up to higher and higher angles, gradually increasing θ until just before the object starts to move; let's call this angle θ_s. Now let's apply Newton's second law ($F=ma=0$ since the acceleration is zero) to the x and y components of the motion:

$$x: \quad \sum F_x = ma_x \Rightarrow \qquad mg\sin\theta_s - f_s = 0 \tag{20.5.1}$$

$$y: \quad \sum F_y = ma_y \Rightarrow \qquad n - mg\cos\theta_s = 0 \tag{20.5.2}$$

The x equation (Eq. $20.5.1$) tells us the magnitude of the maximum static frictional force:

$$f_s = \mu_s n = mg\sin\theta_s. \tag{20.5.3}$$

The y equation (Eq. $20.5.2$) tell us the magnitude of the normal force:

$$n = mg\cos\theta_s \tag{20.5.4}$$

Now using Eq. $20.5.4$ to substitute for the normal force n in Eq. $20.5.3$, we have

$$f_s = \mu_s\left(mg\cos\theta_s\right) = mg\sin\theta_s. \tag{20.5.5}$$

To solve for μ_s, we divide through by $mg\cos\theta_s$:

$$\mu_s = \tan\theta_s \tag{20.5.6}$$

So the coefficient of static friction between the object and the inclined plane is just the tangent of angle θ_s.

Now increase the incline angle θ a little more as you give the mass m little taps to get it going. At some angle θ_k, the object will keep moving, at a constant velocity. (Tipping the incline up farther will cause the object to accelerate; you want the angle at which the object moves down the incline at constant velocity, without accelerating.) Once again in this case the acceleration of the object is zero, and the analysis follows just as with the static case. The coefficient of kinetic friction is then

$$\mu_k = \tan\theta_k \tag{20.5.7}$$

So the coefficient of kinetic friction between the object and the inclined plane is just the tangent of angle θ_k.

As a general rule, the coefficient of static friction is greater than the coefficient of kinetic friction; in other words, it generally takes a larger force (acting against friction) to get an object moving than to keep it moving:

$$\mu_s > \mu_k \quad \text{(generally)}. \tag{20.5.8}$$

But as mentioned earlier, under carefully controlled conditions, one finds the relationship tends toward $\mu_s = \mu_k$, so the two frictional forces tend to become indistinguishable.

The coefficient of rolling friction is typically much less than the coefficient of kinetic friction:

$$\mu_r \ll \mu_k \quad \text{(generally)}. \tag{20.5.9}$$
