54.4: Helmert's Equation

The acceleration due to gravity g is

$$g = G\frac{M_\oplus}{R_\oplus^2} = 9.8\ \mathrm{m/s^2} \tag{54.4.1}$$

to two significant digits, where $M_\oplus$ is the Earth's mass and $R_\oplus$ is its radius. But what if we want to use a more exact value for g ?

You might be tempted to use a value found in some reference books: $g = 9.80665\ \mathrm{m/s^2}$, but that would actually be wrong. This value is just a standard value used for the definitions of some units (for example, in the conversion between pounds-force and newtons). You should never use this value in a physics formula that contains g as the acceleration due to gravity—it's only used when doing certain unit conversions.

The acceleration due to gravity g at the surface of the Earth varies over the surface of the Earth for a number of reasons:

- As you get closer to the equator, the Earth's rotation rate gets larger, resulting in a greater centrifugal force that counteracts gravity. This has the effect of reducing g closer to the equator.
- Also, the Earth has an equatorial bulge due to its rotation, so that you're farther from the center of the Earth near the equator. This also has the effect of reducing g closer to the equator.
- There is also an elevation effect: the higher you are in elevation, the smaller g is.

These effects can be approximately accounted for using an equation called Helmert's equation. According to Helmert's equation, the acceleration due to gravity is given by

$$g = 9.80616 - 0.025928\cos 2\phi + \left(6.9\times 10^{-5}\right)\cos^2 2\phi - \left(3.086\times 10^{-6}\right) H\ \mathrm{m/s^2} \tag{54.4.2}$$

where ϕ is the latitude and H is the elevation (in meters) above sea level. For example, for Largo, Maryland, the latitude ϕ is 38.898 and the elevation H is about 174 feet (53.0 meters). Substituting these values into Helmert's equation, we find g at Largo is about $9.80052\ \mathrm{m/s^2}$. In other cities around the world, the value ranges from $9.779\ \mathrm{m/s^2}$ (Mexico City) to $9.819\ \mathrm{m/s^2}$ (Helsinki). For most problems we just use an average value of $9.8\ \mathrm{m/s^2}$. (You should never round this to $10\ \mathrm{m/s^2}$ unless you're doing a very rough order-ofmagnitude estimation.)