36.1: Translational vs. Rotational Motion

There are some important relations between translational and rotational motion. Recall the relation between an angle θ (in radians) and arc length s :

$$s=r\theta \tag{36.1.1}$$

where r is the radius of rotation. Taking derivatives of both sides with respect to time and using $ds/dt=v$, $d\theta/dt=\omega$, and r is constant, we get a relation between linear and angular velocities:

$$v=r\omega \tag{36.1.2}$$

since the radius of rotation r is constant. Taking derivatives with respect to time again, we get a relation between the learn and angular accelerations:

$$a=r\alpha \tag{36.1.3}$$

Many of the formulæ involving rotational motion are similar to the formulæ we saw in translational motion, and we can use the same methods for working with them. Each of the quantities we encountered in translational motion has a rotational counterpart, as shown in Table $36.1.1$. (Time t is the same in both translational and rotational motion.)

Table $36.1.1$: Translational and rotational quantities. This table shows several quantities related to translational motion, along with their counterparts in rotational motion and how the two are related.

Translational Motion		Rotational Motion		Relationship
Name	Symbol	Name	Symbol	Relationship
Position	x	Angle	θ	$\theta=s/r$
Velocity	v	Angular velocity	ω	$\omega=v_t/r$
Acceleration	a	Angular acceleration	(\alpha\)	$\alpha=a_t/r$
Mass	m	Moment of inertia	I	$I=\int r^2dm$
Force	F	Torque	τ	$\tau=\mathbf{r}\times\mathbf{F}$
Momentum	p	Angular momentum	L	$\mathbf{L}=\mathbf{r}\times\mathbf{p}$

(In the first three lines, s is arc length, and v_t and a_t are the tangential components of the velocity and acceleration, respectively.)

Many of the translational formulæ we've encountered so far have a similar formula in rotational motion. We can generally find these rotational formulæ by replacing the translational variables with the corresponding rotational variables from Table 33-1. Examples of such formulæ are shown in Table $36.1.2$.

Table $36.1.2$: Translational and rotational formulæ. This table shows a number of formulæ from translational mechanics, along with their rotational counterparts.

Description	Translational Motion	Rotational Motion
Velocity	$v=dx/dt$	$\omega=d\theta/dt$
Acceleration	$a=dv/dt$	$\alpha=d\omega/dt$
Constant acceleration	$x=\frac{1}{2}at^2+v_0t+x_0$	$\theta=\frac{1}{2}\alpha t^2+\omega_0t+\theta_0$
»	$v=at+v_0$	$\omega=\alpha t+\omega_0$
»	$v=at+v_0$	$\omega=\alpha t+\omega_0$
»	$v^2=v_0^2+2a\left(x-x_0\right)$	$\omega^2=\omega_0^2+2\alpha\left(\theta-\theta_0\right)$
Newton's 2nd law (const. mass)	$F=ma$	$\tau=I\alpha$

Description	Translational Motion	Rotational Motion
Newton's 2nd law (general)	$F = dp/dt$	$\tau = dL/dt$
Momentum	$p = mv$	$L = I\omega$
Work	$W = Fx$	$W = \tau\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\omega^2$
»	$K = p^2/2m$	$K = L^2/2I$
Hooke's Law	$F = -kx$	$\tau = -\kappa\theta$
Potential energy (spring)	$U_s = \frac{1}{2}kx^2$	$U_s = \frac{1}{2}\kappa\theta^2$
Power	$P = Fv$	$P = \tau\omega$