12.5: Hitting a Target on the Ground

Now let's look at the problem of using a projectile to hit a target on the ground at range R. We could do this by fixing the muzzle velocity and varying the launch angle, or by fixing the launch angle and varying the muzzle velocity, or by varying both.

Fixed Launch Angle

The less common situation is to fix the launch angle θ and allow the muzzle velocity v_0 to vary. Beginning with Eq. (12.2.6),

$$R = \frac{v_0^2}{g}\sin 2\theta \tag{12.5.1}$$

we solve for muzzle velocity:

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta}} \tag{12.5.2}$$

There will always be a solution to this equation unless $\theta \geq 90^\circ$, which corresponds to pointing the cannon backwards. In this case v_0 will be imaginary, and there is no muzzle velocity that will allow the projectile to reach the target.

Example 12.5.1

Suppose we have a cannon fixed at an angle of 25° and wish to hit a target at a distance of $R = 250\ \text{m}$. What muzzle velocity v_0 is required?

Solution

By Eq. 12.5.2,

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta}} = 56.55\ \text{m/s} \tag{12.5.3}$$

Fixed Muzzle Velocity

The more common situation is trying to hit a target when the muzzle velocity is fixed and the launch angle is allowed to vary. In this case we solve Eq. (9.1.6) for θ :

$$\theta = \frac{1}{2}\sin^{-1}\left(\frac{gR}{v_0^2}\right) \tag{12.5.4}$$

Example 12.5.2

Suppose the muzzle velocity is $v_0 = 40\ \text{m/s}$ and the target is at a distance of $R = 75\ \text{m}$. What launch angle is needed to hit the target?

Solution

The launch angle is given by

$$\theta = \frac{1}{2}\sin^{-1}\left(\frac{gR}{v_0^2}\right) = 13.67^\circ \text{ and } 76.33^\circ \tag{12.5.5}$$

Recall that the arcsine of a number returns two angles in the range $[0, 2\pi)$, so there will generally be two solutions to Eq. 12.5.4. In this example, the "calculator" solution is 13.67°, and other solution is 76.33°. In general, there will be two complementary launch angles that will both hit the target.

Example 12.5.1

As a second example, suppose the muzzle velocity is $v_0 = 40\ \mathrm{m/s}$ and the target is at a distance of $R = 200\ \mathrm{m}$. What launch angle is needed to hit the target?

Solution

The launch angle is given by

$$\theta = \frac{1}{2}\sin^{-1}\left(\frac{gR}{v_0^2}\right) = \frac{1}{2}\sin^{-1} 1.225 = ??? \tag{12.5.6}$$

The arcsine of a number greater than 1 is not defined [2], so θ cannot be found in this case. Physically, this means that the target is out of range at this muzzle velocity. For a muzzle velocity of $40\ \mathrm{m/s}$, the maximum range is for $\theta = 45^\circ$, which by Eq. (12.2.6) is $163\ \mathrm{m}$ - so $200\ \mathrm{m}$ is out of range.