5.2: SI Units

SI units (which stands for French Système international d'unités) are based on the *meter* as the base unit of length, the *kilogram* as the base unit of mass, and the *second* as the base unit of time. SI units also define four other base units (the *ampere*, *kelvin*, *candela*, and *mole*, to be described later). Any physical quantity that can be measured can be expressed in terms of these seven base units or some combination of them. SI units are summarized in Appendix H.

SI units were originally based mostly on the properties of the Earth and of water. Under the *original* definitions:

- The *meter* was defined to be one ten-millionth the distance from the equator to the North Pole, along a line of longitude passing through Paris.
- The *kilogram* was defined as the mass of 0.001 m^3 of water.
- The *second* was defined as 1/86,400 the length of a day (one rotation of the Earth).
- The definition of the *ampere* is related to electrical properties, ultimately relating to the meter, kilogram, and second.
- The *kelvin* was defined in terms of the thermodynamic properties of water, as well as absolute zero.
- The *candela* was defined by the luminous properties of molten tungsten and the behavior of the human eye.
- The *mole* was defined by the density of the carbon-12 nucleus.

Many of these original definitions have been replaced over time with more precise definitions, as the need for increased precision has arisen. Most recently, on May 20, 2019, there was a major re-definition of SI units, in which the definitions of the kilogram, ampere, kelvin, and mole were all changed. SI units now really have only one unit that is determined experimentally: the unit of time, which is the *second*. The other base units are now defined by defining exact, unchanging values for several of the physical constants.

Length (Meter)

The SI base unit of length, the *meter* (m), has been re-defined more times than any other unit, due to the need for increasing accuracy. Originally (1793) the meter was defined to be *1=10;000;000* the distance from the North Pole to the equator, along a line going through Paris.[1] Then, in 1889, the meter was re-defined to be the distance between two lines engraved on a prototype meter bar kept in Paris. Then in 1960 it was re-defined again: the meter was defined as the distance of *1,650,763.73* wavelengths of the orange-red emission line in the krypton-86 atomic spectrum. Still more stringent accuracy requirements led to the the current definition of the meter, which was implemented in 1983: the meter is now defined to be the distance light in vacuum travels in 1/299,792,458 second. Because of this definition, the speed of light is now exactly 299,792,458 m/s.

U.S. Customary units are legally defined in terms of metric equivalents. For length, the *foot* (ft) is defined to be exactly 0.3048 meter.

Mass (Kilogram)

Originally the *kilogram* (kg) was defined to be the mass of 1 liter $\left(0.001\ \mathrm{m}^3\right)$ of water. The need for more accuracy required the kilogram to be re-defined to be the mass of a standard mass called the *International Prototype Kilogram* which is kept in a vault at the Bureau International des Poids et Mesures (BIPM) in Paris. Each country was given its own copy of the IPK to use as its own national standard.

In 2019, the kilogram was re-defined (somewhat indirectly) by defining *Planck's constant* (used in quantum mechanics) to be *exactly* equal to $h = 6.62607015 \times 10^{-34}\ \mathrm{kg\ m^2\ s^{-1}}$. Since the meter and second are given precise experimental definitions, fixing the value of h has the effect of defining the value for the kilogram.

Another common metric (but non-SI) unit of mass is the metric ton, which is $1000\ \mathrm{kg}$ (a little over 1 short ton).

In U.S. customary units, the pound-mass (lbm) is defined to be exactly $0.45359237\ \mathrm{kg}$

Mass vs. Weight

Mass is not the same thing as *weight*, so it's important not to confuse the two. The *mass* of a body is a measure of the total amount of matter it contains; the *weight* of a body is the gravitational force on it due to the Earth's gravity. At the surface of the Earth, mass m and weight W are proportional to each other:

$$W = mg, \tag{5.2.1}$$

where g is the acceleration due to the Earth's gravity, equal to $9.80 \mathrm{~m/s}^2$. Remember: mass is mass, and is measured in kilograms; weight is a force, and is measured in force units of newtons.

Time (Second)

Originally the base SI unit of time, the *second* (s), was defined to be $1/60$ of 1/60 of 1/24 of the length of a day, so that 60 seconds $= 1$ minute, 60 minutes $= 1$ hour, and 24 hours $= 1$ day. High-precision time measurements have shown that the Earth's rotation rate has short-term irregularities, along with a long-term slowing due to tidal forces. So for a more accurate definition, in 1967 the second was re-defined to be based on a definition using atomic clocks. The second is now defined to be the time required for 9,192,631,770 oscillations of a certain type of radiation emitted from a cesium- 133 atom.

Although officially the symbol for the second is "s", you will also often see people use "sec" to avoid confusing lowercase "s" with the number " 5 ".

The Ampere, Kelvin, and Candela

For this course, most quantities will be defined entirely in terms of meters, kilograms, and seconds. There are four other SI base units, though: the *ampere* (A) (the base unit of electric current); the *kelvin* (K) (the base unit of temperature); the *candela* (cd) (the base unit of luminous intensity, or light brightness); and the *mole* (mol) (the base unit of amount of substance). With the 2019 re-definition of SI units, the *ampere* is now defined by fixing the value of the elementary charge to exactly $e = 1.602176634 \times 10^{-19} \mathrm{~A~s}$. The *kelvin* is now defined by fixing the value of *Boltzmann's constant* to *exactly* $k_B = 1.380649 \times 10^{-23} \mathrm{~J/K}$. The *candela* is a unit that measures the brightness of light, and has a somewhat complex definition that includes a model of the response of the human eye to light of different wavelengths.

Amount of Substance (Mole)

Since we may have a use for the mole in this course, let’s look at its definition in detail. The simplest way to think of it is as the name for a number. Just as “thousand” means *1,000*, “million” means *1,000,000*, and “billion” means 1,000,000,000, in the same way “mole” refers to the number[2] 602,214,076,000,000,000,000,000, or $6.02214076 \times 10^{23}$. You could have a mole of grains of sand or a mole of Volkswagens, but most often the mole is used to count atoms or molecules. There is a reason this number is particularly useful: since each nucleon (proton and neutron) in an atomic nucleus has an average mass of $1.66053906660 \times 10^{-24}$ grams (called an atomic mass unit, or amu), then there are $1/\left(1.66053906660 \times 10^{-24}\right)$ or $6.02214076 \times 10^{23}$ nucleons per gram. In other words, one mole of nucleons has a mass of $1 \mathrm{gram}$. Therefore, if A is the atomic weight of an atom, then A moles of nucleons has a mass of A grams. But A moles of nucleons is the same as 1 mole of atoms, so one mole of atoms has a mass (in grams) equal to the atomic weight. In other words,

$$\text{moles of atoms } = \frac{\text{grams}}{\text{atomic weight}} \tag{5.2.2}$$

Similarly, when counting molecules,

$$\text{moles of molecules } = \frac{\text{grams}}{\text{molecular weight}} \tag{5.2.3}$$

In short, the mole is useful when you need to convert between the mass of a material and the number of atoms or molecules it contains.

It's important to be clear about what exactly you're counting (atoms or molecules) when using moles. It doesn't really make sense to talk about "a mole of oxygen", any more than it would be to talk about "100 of oxygen". It's either a "mole of oxygen atoms" or a "mole of oxygen molecules".[3]

For convenience, sometimes the word entity is used to mean "atom or molecule." Then the formula for determining the number of moles from the mass becomes

$$\text{moles of entities } = \frac{\text{grams}}{\text{entity weight}} \tag{5.2.4}$$

where entity weight means either atomic weight or molecular weight, depending on whether it's atoms or molecules that are being discussed.

Note that although the base SI unit of mass is the kilogram, the mole is defined by having the number of grams equal to the entity weight. Other kinds of "moles" have been defined, such as the pound-mole, ouncemole, and kilogram-mole, in which the indicated unit of mass is numerically equal to the entity weight. For example, 1 kilogram-mole of carbon-12 atoms is 12 kilograms of carbon-12, and contains $6.02214076 \times 10^{26}$ carbon atoms. The SI mole is the same things as a gram-mole.

With the 2019 SI units re-definition, the mole is defined by setting Avogadro's constant equal to exactly $N_A = 6.02214076 \times 10^{23}\ \mathrm{mol}^{-1}$.

Interesting fact: it's estimated that there is roughly one mole of stars in the observable Universe.

SI Derived Units

In addition to the seven base units (m, kg, s, A, K, cd, mol), there are a number of so-called *SI derived units* with special names. We'll introduce these as needed, but a summary of all of them is shown in Appendix 63.8 (Table 2). These are just combinations of base units that occur often enough that it's convenient to give them special names.

Plane Angle (Radian)

One derived SI unit that we will encounter frequently is the SI unit of plane angle. Plane angles are commonly measured in one of two units: *degrees* or *radians*.[4] You're probably familiar with degrees already: one full circle is 360°, a semicircle is 180°, and a right angle is 90°.

The SI unit of plane angle is the radian, which is defined to be that plane angle whose arc length is equal to its radius. This means that a full circle is 2π radians, a semicircle is π radians, and a right angle is $\pi/2$ radians. To convert between degrees and radians, then, we have:

$$\text{degrees} = \text{radians} \times \frac{180}{\pi} \tag{5.2.5}$$

and

$$\text{radians} = \text{degrees} \times \frac{\pi}{180} \tag{5.2.6}$$

The easy way to remember these formulæ is to think in terms of units: 180 has units of degrees and π has units of radians, so in the first equation units of radians cancel on the right-hand side to leave degrees, and in the second equation units of degrees cancel on the right-hand side to leave radians.

Occasionally you will see a formula that involves a "bare" angle that is not the argument of a trigonometric function like the sine, cosine, or tangent. In such cases it is understood that the angle must be in radians. For example, the radius of a circle r, angle θ, and arc length s are related by

$$s = r\theta, \tag{5.2.7}$$

where it is understood that θ is in radians.

See Appendix 63.15 for a further discussion of plane and solid angles.

SI Prefixes

It's often convenient to define both large and small units that measure the same thing. For example, in English units, it's convenient to measure small lengths in inches and large lengths in miles.In SI units, larger and smaller units are defined in a systematic way by the use of *prefixes* to the SI base or derived units. For example, the base SI unit of length is the meter (m), but small lengths may also be measured in centimeters (cm, 0.01 m), and large lengths may be measured in kilometers (km, 1000 m). Table H-3 in Appendix H shows all the SI prefixes and the powers of 10 they represent. You should *memorize* the powers of *10* for all the SI prefixes in this table.

To use the SI prefixes, simply add the prefix to the front of the name of the SI base or derived unit. The symbol for the prefixed unit is the symbol for the prefix written in front of the symbol for the unit. For example, kilometer (km) 10^3 meter, microsecond (μ,s) 10^{-6} s. But put the prefix on the *gram* (g), *not* the kilogram: for example, 1 microgram (μ,g) 10^{-6} g. For historical reasons, the kilogram is the only SI base or derived unit with a prefix.[5]

The 2019 Re-definition of SI Units

On May 20, 2019, a major re-definition of SI units went into effect. With this re-definition, experimental definitions of several of the SI units have been replaced by *defining* the values of several fundamental physical constants, so that these values become fixed and unchanging, no matter how many future experiments are performed. The defined constants are shown in Table $5.2.1$.

Table $5.2.1$: New SI base quantities, defining constants, and definitions.

Base quantity	Defining constant	Definition	Defines SI unit
Frequency	$\Delta v\left({}^{133}\text{Cs}\right)_{\text{hfs}}$	The unperturbed ground-state hyperfine splitting frequency ofthe cesium-133 atom is exactly 9,192,631,770 Hz.	s
Velocity	c	The speed of light in vacuum c is exactly $299,792,458\ \text{m/s}$.	m
Action	h	The Planck constant h is exactly $6.62607015\times 10^{-34}\ \text{J s}$.	kg
Electric charge	e	The elementary charge e is exactly $1.602176634\times 10^{-19}\,\text{C}$.	A
Heat capacity	k_B	The Boltzmann constant k_B is exactly $1.380649\times 10^{-23}\ \text{J/K}$.	K
Amount of substance	N_A	The Avogadro constant N_A is exactly $6.02214076\times 10^{23}\ \text{mol}^{-1}$.	mol
Luminous intensity	K_{cd}	The luminous efficacy K_{cd} of monochromatic radiation of frequency $540\times 10^{12}\ \text{Hz}$ is exactly $683\ \text{lm/W}$.	cd

1. Six hundred two sextillion, two hundred fourteen quintillion, seventy-six quadrillion.

2. Sometimes chemists will refer to a “mole of oxygen” when it’s understood whether the oxygen in question is in the atomic (O) or molecular (O_2) state.

3. A third unit implemented in many calculators is the *grad*: a right angle is 100 grads and a full circle is 400 grads. You may encounter grads in some older literature, such as Laplace’s *Me´canique Ce´leste*. Almost nobody uses grads today, though.

4. Originally, the metric standard of mass was a unit called the *grave* (*GRAH-veh*), equal to 1000 grams. When the metric system was first established by Louis XVI following the French Revolution, the name *grave* was considered politically incorrect, since it resembled the German word *Graf*, or “Count” — a title of nobility, at a time when titles of nobility were shunned. The *grave* was retained as the unit of mass, but under the more acceptable name *kilogram*. The gram itself was too small to be practical as a mass standard.