9.4: Vector Operations

The Zero Vector

The zero vector is the vector $\mathbf{0} = 0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$. It has zero magnitude, and its direction is undefined. The zero vector is not the same thing as the scalar 0 : $\mathbf{0} \neq 0$. One is a vector, and the other is a scalar.

Derivatives of Vectors

You can take the derivative of a vector component-by-component. For example, if a vector $\mathbf{A}(t)$ is a function of time t, then $\mathbf{A}(t) = A_x(t)\mathbf{i} + A_y(t)\mathbf{j} + A_z(t)\mathbf{k}$, and

$$\frac{d\mathbf{A}(t)}{dt} = \frac{dA_x(t)}{dt}\mathbf{i} + \frac{dA_y(t)}{dt}\mathbf{j} + \frac{dA_z(t)}{dt}\mathbf{k} \tag{9.4.1}$$

It's possible to take other kinds of derivatives of vectors, known as the divergence $(\nabla\cdot)$ and $\operatorname{curl}(\nabla\times)$. You'll learn about these in a course on vector calculus.

Integrals of Vectors

Integrating a vector is similarly done term-by-term. If a vector $\mathbf{A}(t)$ is a function of time t, then $\mathbf{A}(t) = A_x(t)\mathbf{i} + A_y(t)\mathbf{j} + A_z(t)\mathbf{k}$, and

$$\int \mathbf{A}(t)dt = \int A_x(t)dt\mathbf{i} + \int A_y(t)dt\mathbf{j} + \int A_z(t)dt\mathbf{k} \tag{9.4.2}$$

Other Vector Operations

Other mathematical operations with vectors are possible. For example, is it possible to add a vector and a scalar together? The answer is: sort of. You get something similar to a quaternion, which is a hypercomplex number of the form $a + bi + cj + dk$ (where $i^2 = j^2 = k^2 = -1$). Quaternions are sometimes used in aeronautical and astronautical engineering to describe the rotation of one coordinate system with respect to another.

What about multiplying a vector by another vector? Yes, this is possible. In fact, there are three different kinds of multiplication that can be used to multiply two vectors together, as described in the next chapter.

How about division - can you divide by a vector? No; division by a vector is not defined. A vector may be a dividend, but not a divisor. But you can divide a vector by a scalar by simply multiplying by the reciprocal of the scalar:

$$\frac{\mathbf{A}}{c} = \frac{1}{c}(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \tag{9.4.3}$$

$$= \frac{A_x}{c}\mathbf{i} + \frac{A_y}{c}\mathbf{j} + \frac{A_z}{c}\mathbf{k}. \tag{9.4.4}$$