54.8: General Relativity

Our best theory of gravity to date is Albert Einstein's general theory of relativity. A full description of general relativity is beyond the scope of this course, as it makes use of advanced mathematical ideas such as differential geometry. But briefly, the idea is that mass causes space and time to become distorted, and it is this distortion that is the nature of the gravitational force.

The central equation governing general relativity is called the Einstein field equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{54.8.1}$$

The indices μ and v range from 0 to 3 , and stand for the coordinates t, x, y, and z, so that each side of the equation is a 4×4 matrix. Broadly speaking, the left-hand side of this equation represents the curvature of space time, and the right-hand side represents the distribution of mass. Here:

- $R_{\mu\nu}$ is the Ricci curvature tensor, and describes the curvature of space-time.
- R is the scalar curvature, and is an overall average curvature of space-time.
- $g_{\mu\nu}$ is the metric tensor, and defines the "distance" between neighboring points in space-time.
- $T_{\mu\nu}$ is the stress-energy tensor, and measures the mass density of matter.
- G is the gravitational constant, and c is the speed of light in vacuum.

In the special case where the gravitational field is weak, it can be shown that Einstein's field equation reduces to Gauss's law for gravity (Eq. 54.7.1), i.e. Newtonian gravity.

A few consequences of general relativity are:

- Time moves more slowly in a strong gravitational field than in a weak field. For example, clocks run more slowly at sea level than at the top of a mountain.
- Light can be bent by gravity. This was an important early test of general relativity: the amount of light bending predicted by general relativity was confirmed by measuring the positions of stars near the Sun during a solar eclipse in 1919. This effect has been observed recently by the Hubble Space Telescope in the form of gravitational lensing: the gravity of a relatively nearby galaxy will bend the light from more distant objects, producing multiple images of the distant object.
- Gravitational redshift: light emitted by a massive object will tend to be redder than it would be if the gravity were not present.
- Orbit precession: the orbits of planets "precess" due to gravitational effects, causing the perihelion position to slowly move around the Sun. The amount of this precession predicted by general relativity is slightly different than what would be predicted by Newtonian gravity. The effect is very slight, and most noticeable in the orbit of Mercury.
