47.1: Introduction to Angular Momentum

The rotational counterpart of momentum is called angular momentum. Just as linear momentum is defined as the product of mass and velocity $(p = mv)$, angular momentum L is defined as the product of moment of inertia and angular velocity:

$$L = I\omega. \tag{47.1.1}$$

More generally, angular momentum, like linear momentum is a vector quantity:

$$\mathbf{L} = I\omega. \tag{47.1.2}$$

SI units for angular momentum are $\mathrm{kgm}^2\ \mathrm{s}^{-1}$, or Nms.

Angular momentum $\mathbf{L}$ is related to linear momentum $\mathbf{p}$ according to

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \tag{47.1.3}$$

If you recall, Newton's second law of motion states that $F = dp/dt$, where F is force and p is momentum; in the special case where mass is constant, this reduces to $F = ma$, where a is the acceleration. There are analogous formulæ in rotational motion, which can be derived by taking the time derivative of Eq. $47.1.3$:

$$\frac{d\mathbf{L}}{dt} = \mathbf{r} \times \frac{d\mathbf{p}}{dt} \tag{47.1.4}$$

The right-hand side is the torque; the result is the rotational form of Newton's second law:

$$\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt} \tag{47.1.5}$$

where $\boldsymbol{\tau}$ is torque and $\mathbf{L}$ is angular momentum. In the case where the moment of inertia is constant, this reduces to $\boldsymbol{\tau} = I\boldsymbol{\alpha}$, where $\boldsymbol{\alpha}$ is the angular acceleration.