23.2: Centripetal Force

By Newton's first law of motion, a body in motion will tend to continue that motion in a straight line, unless acted upon by some outside force. Therefore, if a body is moving in a circle, there must be some force present that is causing it to move in a circle. Whatever force is responsible for making the body move in a circle we identify as the centripetal force. The magnitude of the centripetal force is equal to the mass of the body times the centripetal acceleration v^2/r :

$$F_c = \frac{mv^2}{r} \tag{23.2.1}$$

For example, for a satellite orbiting the Earth, the centripetal force is the gravitational force. If you tie a small weight to the end of a string and swing it over your head in a circle, then the centripetal force is the tension in the string. Typically the way we approach problems involving uniform circular motion is to write down an expression for the centripetal force, and set it equal to mv^2/r.

Exercise 23.2.1

The International Space Station orbits the Earth at an altitude of about 350 km. How fast is it moving?

Answer

We'll write down an expression for the gravitational force and set it equal to mv^2/r. The radius of the Earth is 6378.140 km, if you add that to the altitude of the Space Station above the Earth's surface, you find the radius of its orbit $r = 6378.140 \text{ km} + 350 \text{ km} = 6728.140 \text{ km} = 6728140 \text{ m}$The centripetal force in this case is the gravitational force, which (as will be seen later) is given by $F = GM_\oplus m/r^2$, where $M_\oplus$ is the mass of the Earth, m is the mass of the space station, r is the radius of the orbit, and G is the universal gravitational constant. Setting this expression for the gravitational force equal to the centripetal force mv^2/r, we have

$$G\frac{M_\oplus m}{r^2} = \frac{mv^2}{r} \tag{23.2.2}$$

Multiplying both sides by r/m,

$$v^2 = G\frac{M_\oplus}{r} \tag{23.2.3}$$

and so

$$v = \sqrt{\frac{GM_\oplus}{r}} \tag{23.2.4}$$

Using the orbital radius $r = 6728140$ mand $GM_\oplus = 3.986005 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$, we have the velocity of the Space Station as

$$v = \sqrt{\frac{GM_\oplus}{r}} \tag{23.2.5}$$

$$= \sqrt{\frac{3.986005 \times 10^{14} \text{ m}^3 \text{ s}^{-2}}{6728140 \text{ m}}} \tag{23.2.6}$$

$$= 7697 \text{ m/s} \tag{23.2.7}$$

$$= 17,200 \text{mph} \tag{23.2.8}$$

A note about Eq. $23.2.6$ the product $GM_\oplus$ is known to higher accuracy than either G or $M_\oplus$ individually; therefore we use the product here. See Appendix L for a listing of common physical constants.