# 58.1: Circular Orbits

As a simple example, suppose we wish to place a spacecraft of mass $m$ into a circular orbit around the Earth. If the orbit radius is $r$, then the potential energy $U$ of the spacecraft (with $U=0$ at $r=\infty$ ) is

$$U_c=-G\frac{M_\oplus m}{r} \tag{58.1.1}$$

where $G$ is Newton's gravitational constant and $M_\oplus$ is the mass of the Earth. The kinetic energy of the spacecraft is

$$K_c=\frac{1}{2}mv^2 \tag{58.1.2}$$

Here the orbit velocity $v$ at orbital radius $r$ is found by setting the centripetal force $mv^2/r$ equal to the gravitational force $GM_\oplus m/r^2$ :

$$\frac{mv^2}{r}=\frac{GM_\oplus m}{r^2} \tag{58.1.3}$$

so, solving for $v$,

$$v_c=\sqrt{\frac{GM_\oplus}{r}} \tag{58.1.4}$$

Substituting this result into Eq. $58.1.2$, we have an expression for the kinetic energy $K$ in terms of the orbit radius $r$ :

$$K=G\frac{M_\oplus m}{2r} \tag{58.1.5}$$

From Eqs. $58.1.1$ and $58.1.5$, we find the total orbit energy $E$ is

$$E_c=U_c+K_c=-G\frac{M_\oplus m}{2r} \tag{58.1.6}$$

This is an important result, since total energy is conserved. Another important result is the angular momentum of the spacecraft, since that's also conserved. The angular momentum of the spacecraft in a circular orbit is $L=mvr$; using Eq. $58.1.4$, we have

$$L=m\sqrt{GM_\oplus r} \tag{58.1.7}$$

## Launch Velocity

Suppose we wish to launch a spacecraft from the surface of the Earth into a circular orbit of radius $r$, using only a single blast of the engines on the ground and coasting the rest of the way. The initial velocity with which the spacecraft is launched is called the launch velocity, and can be found using the conservation of energy:

$$\begin{aligned} E &= U+K \\ &= -G\frac{M_\oplus m}{r}+\frac{1}{2}mv^2 \end{aligned}$$

and so solving for $v$ gives the launch velocity $v_L$ :

$$v_L=\sqrt{2\left(\frac{E}{m}+G\frac{M_\oplus}{r}\right)} \tag{58.1.8}$$

In real life, however, there are a number of complications that require an analysis more complex that this:

- Spacecraft are not launched with a single initial blast and allowed to coast. Instead, the engines are continuously burned over some extended period.
- The mass of the spacecraft decreases during launch, as fuel is burned, so that the rocket equation must be employed. (See Chapter 30.)
- Most spacecraft are staged in some way (as described in Chapter 30), which also causes the spacecraft mass to decrease with time during launch.

- The drag due to the Earth's atmosphere must be accounted for, which we have not done here.

There's another issue here. The above analysis assumes the spacecraft is launched from a non-rotating Earth. In real life, we launch from a rotating Earth, which we can use to our advantage. Since the Earth is rotating, we can use its rotational velocity to contribute to the needed launch velocity, as long as the spacecraft is launched to the east so that it orbits the Earth in the same sense as the Earth's rotation. The linear velocity of the Earth due to its rotation is $R_{\oplus}\omega$, where $R_{\oplus}$ is the radius of the Earth (about $6378 \mathrm{~km}$ ) and $\omega$ is the angular velocity of the Earth:

$$\omega=\frac{1 \mathrm{rev}}{24 \mathrm{hr}}=\frac{2 \pi \mathrm{rad}}{86400 \mathrm{sec}}=7.2722 \times 10^{-5} \mathrm{rad} / \mathrm{s} \tag{58.1.9}$$

At latitude $\phi$, the linear velocity of the Earth is

$$v=R_{\oplus} \omega \cos \phi \tag{58.1.10}$$

The closer the launch site is the the equator $(\phi=0)$, the larger $v$ is, and the more we can take advantage of the Earth's rotation in helping to achieve the desired launch velocity. This is why the Kennedy Space Center is located in Florida: it's in the southern United States, about as close to the equator as we can get within the United States. [1] The latitude of the Kennedy Space Center is $\phi=28.5^{\circ}$ , which gives $v=408 \mathrm{~km} / \mathrm{s}$ that we get "for free" from the Earth toward the launch velocity.

To take maximum advantage of the Earth's rotation, a spacecraft would be launched due east from the Kennedy Space Center. Once the spacecraft is in orbit, it cannot just orbit the Earth at the latitude of the launch site; the laws of physics require the plane of the orbit to pass through the center of the Earth. The result is a circular orbit, inclined with respect to the equator by an angle equal to the latitude of the launch site (28.5 ° for a launch from Kennedy). Many launches from the Kennedy space center are therefore circular (or near-circular) orbits with an inclination of $28.5^{\circ}$ with respect to the equator.

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[1] Also, by using the east coast of Florida and launching to the east, we launch out over the Atlantic Ocean instead of over populated areas. This is another important factor that makes the east coast of Florida a desirable launch site.

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