12.1: Introduction to Projectile Motion

An important example of two-dimensional motion under constant acceleration is the motion of a projectile (e.g. a cannonball fired from a cannon) at the surface of the Earth (Fig. $12.1.1$).

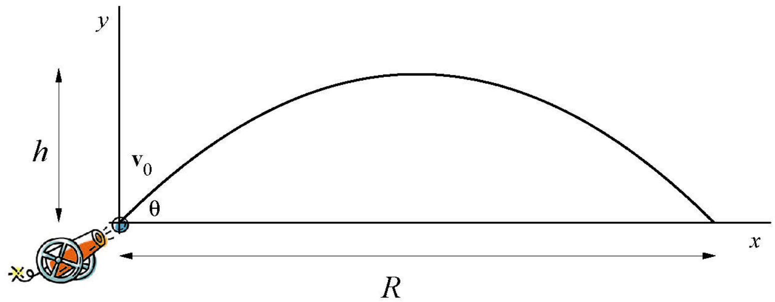

Figure 12.1.1: Parabolic path of a projectile launched with muzzle velocity v_0 at angle θ. Here the x axis is along the ground, R is the range, and h is the maximum altitude.

The acceleration in this case is the acceleration due to gravity, so the constant-acceleration equations apply. The position vector as a function of time is given by Eq. (11.2.7):

$$\mathbf{r}(t)=\frac{1}{2}\mathbf{a}t^2+\mathbf{v}_0t+\mathbf{r}_0 \tag{12.1.1}$$

where $\mathbf{v}_0$ is the initial velocity of the cannonball, called the muzzle velocity. Let's take time $t=0$ to be the instant the cannonball leaves the cannon. Then if we choose the origin to be at the cannon (Fig.$12.1.1$), then $\mathbf{r}_0=\mathbf{0}$. The acceleration in this case is in the $-y$ direction, so $\mathbf{a}=-g\mathbf{j}$, and Eq. $12.1.1$ becomes

$$\mathbf{r}(t)=-\frac{1}{2}gt^2\mathbf{j}+\mathbf{v}_0t \tag{12.1.2}$$

where the initial velocity $\mathbf{v}_0=v_{x0}\mathbf{i}+v_{y0}\mathbf{j}$. This vector equation actually represents two scalar equations: one for $x(t)$ and one for $y(t)$:

$$x(t)=v_{x0}t \tag{12.1.3}$$

$$y(t)=-\frac{1}{2}gt^2+v_{y0}t \tag{12.1.4}$$

Typically in real life you will not know the cartesian components of the velocity vector (v_{x0} and v_{y0}); instead you are more likely to know the magnitude of the muzzle velocity v_0 and the launch angle θ. Converting the muzzle velocity vector from rectangular to polar form,

$$v_{0x}=v_0\cos\theta \tag{12.1.5}$$

$$v_{0y}=v_0\sin\theta \tag{12.1.6}$$

Equations $12.1.3$ and $12.1.4$ then become

$$x(t)=(v_0\cos\theta)\,t \tag{12.1.7}$$

$$y(t)=-\frac{1}{2}gt^2+(v_0\sin\theta)\,t \tag{12.1.8}$$

These equations give the x and y coordinates of the projectile at any time t.

Now let's consider a few questions we can ask about the motion of a projectile.