9.2: Vector Arithmetic- Graphical Methods

Vector arithmetic can be done graphically, by drawing the vectors as arrows on graph paper, and measuring the results with a ruler and protractor. The advantage of the graphical methods are that they give a good intuitive picture of what's going on to help you visualize what you're trying to do. The disadvantages are that the graphical methods can be time-consuming, and not very accurate.

In practice, the graphical methods are usually used to make a quick sketch, to help organize and clarify your thinking, so you can be clear that you're doing things correctly. The algebraic methods are then used for the actual calculations.

When drawing vectors, you are free to move the vector around the page however you want, as long as you don't change the direction or magnitude.

Addition

We'll begin with addition. There are two methods available to add two vectors together: the first is called the parallelogram method. In this method, you draw the two vectors to be added with their tail end points at the same point. This figure forms half a parallelogram; draw two additional lines to complete the parallelogram. Now draw a vector from the tail endpoint across the diagonal of the parallelogram. This diagonal vector is the sum of the two original vectors (Fig. 9.2.1(a)).

The second graphical method of vector addition is called the triangle method. In this method, you first draw one vector, then draw the second so that its tail is at the head of the first vector. To find the sum of the two vectors, draw a vector from the tail of the first vector to the head of the second (Fig. 9.2.1(b)).

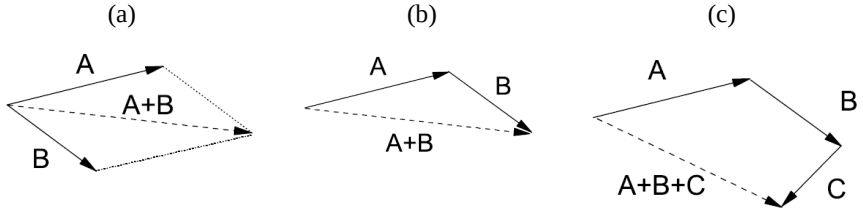
The triangle method can be extended to add any number of vectors together. Just draw the vectors one by one, with the tail of each vector at the head of the previous one. The sum of all the vectors is then found by drawing a vector from the tail of the first vector in the chain to the head of the last one (Fig. 9.2.1(c)). This is called the polygon method.

Subtraction

To subtract two vectors graphically, draw the two vectors so that their tail endpoints are at the same point. To draw the difference vector, draw a vector from the head of the subtrahend vector to the head of the minuend vector (Fig. 9.2.1(d)).

Scalar Multiplication

Multiplying a vector by a scalar will change the length of the vector. Multiplying by a scalar greater than 1 (in absolute value) will lengthen the vector; multiplying by a scalar less than 1 in absolute value will shrink the vector. If the scalar is positive, the product vector will have the same direction as the original; if the scalar is negative, the product vector will be opposite the direction of the original (Fig. 9.2.1(e)).



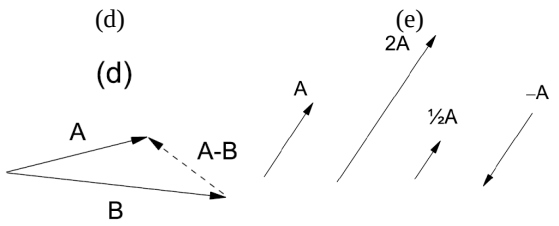


Figure 9.2.1: Graphical methods for vector arithmetic. (a) Addition of vectors $\mathbf{A}$ and $\mathbf{B}$ using the parallelogram method. (b) Addition of the same vectors $\mathbf{A}$ and $\mathbf{B}$ using the triangle method. (c) Addition of vectors $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}$ using a generalization of the triangle method called the polygon method. The sum vector points from the tail of the first vector to the head of the last. (d) Vector subtraction: $\mathbf{A} - \mathbf{B}$ points from the head of $\mathbf{B}$ to the head of A. (e) Multiplication of a vector A by various scalars. Multiplying by a scalar greater than 1 makes the vector longer; multiplying by a scalar less than 1 makes it shorter. The resulting vector will be in the same direction as $\mathbf{A}$ unless the scalar is negative, in which case the result will point opposite the direction of $\mathbf{A}$.