12.4: Shape of the Projectile Path

What is the shape of the projectile's path in Fig. 12.1.1? To find out, we can solve Eq. (12.1.7) for the time t and plug the resulting expression into Eq. (12.1.8) to eliminate t and get an equation for $y(x)$. First solve Eq. (12.1.7) for t :

$$t=\frac{x}{v_0\cos\theta} \tag{12.4.1}$$

Now substitute this into Eq. (12.1.8):

$$y=-\frac{1}{2}g\left(\frac{x}{v_0\cos\theta}\right)^2+(v_0\sin\theta)\left(\frac{x}{v_0\cos\theta}\right) \tag{12.4.2}$$

or

$$y(x)=\left(-\frac{g}{2v_0^2\cos^2\theta}\right)x^2+(\tan\theta)x \tag{12.4.3}$$

This is the equation of a parabola passing through the origin, so the projectile follows a parabolic path.

Actually, this is just an approximation, assuming the acceleration due to gravity is a constant downward in the $-y$ direction. A more careful calculation would have to allow for the curvature of the Earth, which would show the actual path is that of a highly eccentric ellipse. But over relatively short distances where the curvature of the Earth is not important, the elliptical path can be approximated as a parabola.