39.1: Torque Method

The first method involves building a device specifically for the purpose (Figure 39.1.1).

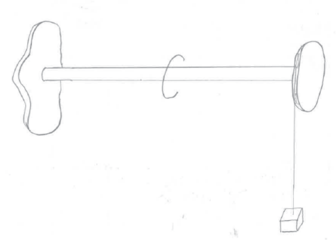

Figure 39.1.1: A simple device for measuring the moment of inertia.

A rotating rod has a pulley at one end and the body to be measured attached to the other end. A string with a weight of mass m at one end is wrapped around the pulley, so that the falling weight will unwrap the string. If the pulley has radius r_p, then the falling weight will apply a force mg to the pulley, which will result in a torque mgr_p on the pulley. This torque is then applied to the pulley, to the rod, and to the test body at the other end of the rod. The rotation angle of the pulley at any time t is thus given by

$$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\frac{\tau}{I}t^2 \tag{39.1.1}$$

where α is the angular acceleration, which, by the rotational version of Newton's second law, is equal to τ/I, where $\tau = mgr_p$ is the torque and I is the total moment of inertia, including the pulley, the test body, and the rod. Let's write this total moment of inertia as

$$I = I_p + I_r + I_b \tag{39.1.2}$$

where I_p is the moment of inertia of the pulley, I_r is the moment of inertia of the rod, and I_b is the moment of inertia of the test body, which is what we're trying to measure. The rotation angle θ is given by $\theta = 2\pi N$, where N is the number of revolutions of the pulley. But N is also equal to the total length L of the string divided by the circumference of the pulley: $N = L/(2\pi r_p)$. Thus

$$\theta = 2\pi\frac{L}{2\pi r_p} = \frac{L}{r_p} \tag{39.1.3}$$

Combining all these results, Eq. 39.1.1 becomes

$$\frac{L}{r_p} = \frac{1}{2}\frac{mgr_p}{I_p + I_r + I_b}t^2 \tag{39.1.4}$$

Solving for the moment of inertia of the body,

$$I_b = \frac{mgr_p^2t^2}{2L} - I_p - I_r \tag{39.1.5}$$

The pulley and rod are both disks, so their respective moments of inertia are $I_p = \frac{1}{2}m_pr_p^2$ and $I_r = \frac{1}{2}m_rr_r^2$, where m_p and r_p are the mass and radius of the pulley, and m_r and r_r the mass and radius of the rod. Equation 39.1.6 then becomes

$$I_b = \frac{r_p^2}{2}\left(\frac{mgt^2}{L} - m_p\right) - \frac{1}{2}m_rr_r^2 \tag{39.1.6}$$

To use the machine, we attach the test body to the end of the rod opposite the weight, wrap the string around the pulley, release the weight, and measure how much time t it takes the string to completely unwind. The moment of inertia of the test body is then given by Eq. 39.1.1). The weight m can be adjusted so that the unwinding time t is long enough to be measured easily (say, several seconds).