54.6: Escape Velocity

The escape velocity is the initial velocity a particle must have to escape the gravity of its parent body. Typically it refers to the initial velocity a particle must have at the surface of a planet in order to leave the planet forever, and never be pulled back by the planet's gravity. If a particle leaves the surface of a planet with an initial velocity equal to the escape velocity, then the body will move more and more slowly as the particle moves farther from the planet, finally reaching a velocity of zero at $r = \infty$. (We assume only the particle and the planet are present, and ignore all other bodies.)

To compute the escape velocity, consider running the problem with time running backwards: the body starts at $r = \infty$ with zero velocity and falls toward the planet. The impact velocity from infinity will be the same as the escape velocity. Now at $r = \infty$, the potential energy $U = -GM_p m/r = 0$, where M_p is the mass of the planet and m is the mass of the particle. Since the particle is at rest at $r = \infty$, the kinetic energy there is also zero, so the total mechanical energy $K + U = 0$. Now let the particle begin falling from $r = \infty$ under the influence of the planet's gravity, until it impacts the planet at $r = R_p$, where R_p is the radius of the planet. At the point of impact the potential energy is $U = -GM_p m/R_p$, and its kinetic energy will be $K = mv_e^2/2$, where v_e is the impact (escape) velocity. By the law of conservation of energy, the total mechanical energy at $r = \infty$ must be the same as it is at $r = R_p$:

$$K + U = \frac{1}{2}mv_e^2 - \frac{GM_p m}{R_p} = 0 \tag{54.6.1}$$

Solving for the escape velocity v_e, we find

$$v_e = \sqrt{\frac{2GM_p}{R_p}} \tag{54.6.2}$$

For the Earth, for example, we have (from Appendix L) $GM_p = 3.986005 \times 10^{14}\ \mathrm{m}^3\ \mathrm{s}^{-2}$ and $R_p =\ 6378.140 \times 10^3\ \mathrm{m}$; substituting these values into Eq. $54.6.1$, we find the escape velocity for Earth is $v_e = 11.2\ \mathrm{km/s}$. In other words, if you were to fire a projectile from the surface of the Earth with an initial velocity of $11.2\ \mathrm{km/s}$, it would be able to escape the Earth's gravity, going more and more slowly the higher it goes, finally coming to rest at $r = \infty$.

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