25.6: Gears

Some writers list the gear as a seventh simple machine, but it isn't. There are only six simple machines. The gear is a compound machine: a combination of the wheel and axle and the pulley.

A system of two connected gears can provide a mechanical advantage, in a rotational sense. The rotational work done by a rotating disk like a gear is [1] $W = \tau\theta$, where τ is the torque applied to the gear, and θ is the angle through which the gear is turned. Since the input work is equal to the output work,

$$W_i = W_o \tag{25.6.1}$$

or

$$\tau_E\theta_E = \tau_R\theta_R \tag{25.6.2}$$

The mechanical advantage of the two gears is then τ_R/τ_E, or

$$M.A. = \frac{\theta_E}{\theta_R} = \frac{\omega_E}{\omega_R} = \frac{r_R}{r_E} = \frac{N_R}{N_E} \tag{25.6.3}$$

Here one of the two gears (E) is the "input" (effort) gear, and the other gear (R) is the "output" (resistance) gear. Therefore the mechanical advantage is the ratio of the input angle rotated (θ_E) to the output angle rotated (θ_R). It is also equal to the ratio of the angular speeds of the input to output gears; to the ratio of the output to input gear radii (r_R/r_E); and to the ratio of the number of teeth in the output gear to the number of teeth in the input gear (N_R/N_E).

The mechanical advantage is also known as the gear ratio.

If the input gear is smaller than the output gear $(r_E < r_R)$, then several turns of the input gear are needed for each turn of the output gear. The mechanical advantage (gear ratio) is greater than 1 , and less input torque is required to do the same work.

If the input gear is larger than the output gear $(r_E > r_R)$, then one turn of the input gear will produce several turns of the output gear. The mechanical advantage (gear ratio) is less than 1 , and more input torque is required to do the same work; this can be used to turn the output gear at high speed while turning the input gear at low speed.

An example of the use of gears is in the bicycle. The input gear (the chainring) is attached to the pedals, and the output gear (the cog) to the rear wheel. In addition, most bicycles provide several gears on both the chainring and the cog, and the rider is able to select a different gear for each. For a bicycle, the gear ratio is usually less than 1 , so that each turn of the pedals will result in more than one turn of the rear wheel. A larger gear ratio (a small front chainring gear used with a large rear cog gear) provides a larger mechanical advantage, and is used for pedaling up hills with less effort. A smaller gear ratio (a large front chainring gear used with a small rear cog gear) provides a smaller mechanical advantage, and is used for pedaling at high speed on level ground or downhill.

[1] Rotational motion is described later in these notes.