10.2: Component Form

Suppose we have two vectors in rectangular form. What is the dot product of the two in terms of their components? To answer this, we begin with the definition of the dot product, Eq. (7.1.1):

$$\mathbf{A} \cdot \mathbf{B} = AB\cos(\beta - \alpha), \tag{10.2.1}$$

where α is the angle vector A makes with respect to the x axis, and β is the angle vector $\mathbf{B}$ makes with respect to the x axis, so that $\beta - \alpha$ is the angle between the two vectors (Fig. $10.2.1$). We now use a trigonometric identity to expand the argument of the cosine:

$$\mathbf{A} \cdot \mathbf{B} = AB(\cos\beta\cos\alpha + \sin\beta\sin\alpha) \tag{10.2.2}$$

Now making use of the relations $\cos\theta = \text{adj} / \text{ hyp}$ and $\sin\theta = \text{opp/hyp}$, we have

$$\cos\alpha = \frac{A_x}{A}; \quad \cos\beta = \frac{B_x}{B}; \quad \sin\alpha = \frac{A_y}{A}; \quad \sin\beta = \frac{B_y}{B} \tag{10.2.3}$$

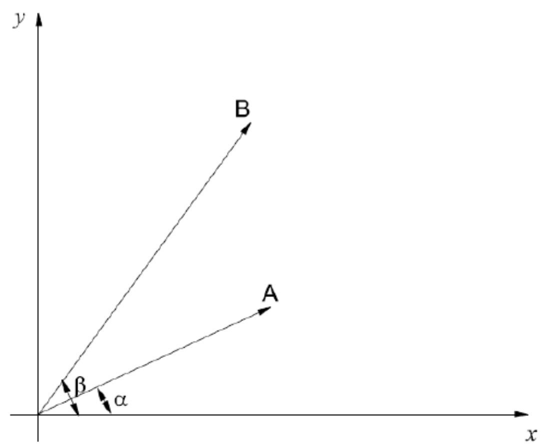

Figure 10.2.1: The two vectors A and B are to be multiplied using the dot product to get A B.

Making these substitutions into Eq. $10.2.2$, we have

$$\mathbf{A} \cdot \mathbf{B} = AB\left(\frac{B_x}{B}\frac{A_x}{A} + \frac{B_y}{B}\frac{A_y}{A}\right) \tag{10.2.4}$$

$$= A_xB_x + A_yB_y \tag{10.2.5}$$

This result can be generalized from two to three dimensions to get

$$\mathbf{A} \cdot \mathbf{B} = A_xB_x + A_yB_y + A_zB_z \tag{10.2.6}$$

Example 10.2.1

Suppose vectors $\mathbf{A} = 3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ and $\mathbf{B} = \mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$. What is the dot product of the two vectors

Solution

Then the dot product of the two vectors is

$$\mathbf{A} \cdot \mathbf{B} = (3)(1) + (4)(-5) + (-2)(2) = -21$$

Notice that the final result is a *scalar*, not a vector.