31.6: Collisions in Two Dimensions

Now consider a collision in two dimensions between two masses m_1 and m_2 (Fig. 31.6.1). Without loss of generality, we can work in a coordinate system that is at rest with respect to mass m_2, and in which mass m_1 is moving in the $+x$ direction, as shown in the figure. Then before the collision, mass m_1 is moving with velocity $\mathbf{v}_{1i} = v_{1i}\mathbf{i}$. After the collision, mass m_1 moves with velocity $\mathbf{v}_{1f} = (v_{1f}\cos\theta_1)\,\mathbf{i} - (v_{1f}\sin\theta_1)\,\mathbf{j}$; mass m_2 moves with velocity $\mathbf{v}_{2f} = (v_{2f}\cos\theta_2)\,\mathbf{i} + (v_{2f}\sin\theta_2)\,\mathbf{j}$.

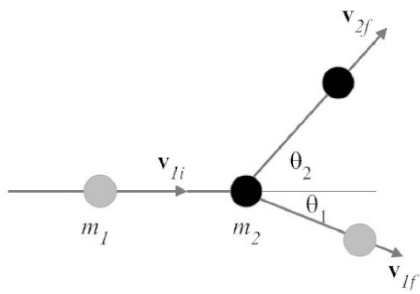

Figure 31.6.1: A collision in two dimensions.

By conservation of momentum, we know that both the x and y components of the total system momentum are independently conserved. This gives two equations: in the x direction,

$$p_{ix} = p_{fx} \tag{31.6.1}$$

$$m_1v_{1i} = m_1v_{1f}\cos\theta_1 + m_2v_{2f}\cos\theta_2 \tag{31.6.2}$$

and in the y direciton,

$$p_{iy} = p_{fy} \tag{31.6.3}$$

$$0 = -m_1v_{1f}\sin\theta_1 + m_2v_{2f}\sin\theta_2. \tag{31.6.4}$$

So Eqs. 31.6.2 and 31.6.4 give us two equations - but in this case there are four unknowns (v_{1f}, v_{2f}, θ_1, and θ_2). To determine the four unknowns, we need as many equations as we have unknowns, so we're two equations short and we need to provide some more information. For example, if we assume that the collision is perfectly elastic, then we can add another equation, since kinetic energy will be conserved in this case:

$$K_i = K_f \tag{31.6.5}$$

$$\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \tag{31.6.6}$$

Now we have three equations (Eqs. 31.6.2, 31.6.4, and 31.6.6), but we still have four unknowns—we still need more information to find the final velocities. To solve the problem, we could be given one of the four unknowns, for example. But the piece of information that's really missing here is the impact parameter of the collision, which is the perpendicular distance between the center of mass m_2 and the line along the the initial velocity vector $\mathbf{v}_{1i}$. If the impact parameter is zero, then mass m_1 hits mass m_2 head-on. If the impact parameter is equal to the sum of the radii of m_1 and m_2, then the two masses will barely touch in a glancing blow. Knowing the impact parameter is necessary for finding the angles θ_1 and θ_2.

Collisions in two dimensions are more general that you might think: under a central-force law, motion will be in a plane, so the particles will move in two dimensions. Analyzing two-dimensional collisions of this type is common in particle physics. There the particles typically do not actually touch, but are repelled or attracted by the electrostatic force. The same laws apply in particle physics as what we've described here.