11.3: Vertical vs. Horizontal Motion

Consider the experiment shown in Figure 11.3.1: two balls are initially at the same height above the floor; both are released at the same time, but one is allowed to fall vertically, while the other is given an initial velocity v_0 in the horizontal direction. Which ball hits the floor first?

You might be inclined to think that the ball that falls vertically would hit the floor first, because it doesn't have as far to go. But the correct answer is that both balls land at the same time. The reason is that the horizontal and vertical components of the motion are independent-the horizontal motion of the second ball has no effect on its vertical motion. Let's set up a coordinate whose origin is at the release point, with $+x$ to the right and $+y$ upward. Then for the first ball (the one falling vertically), we have $\mathbf{a} = -g\mathbf{j}$, $\mathbf{v}_0 = \mathbf{0}$, and $\mathbf{r}_0 = \mathbf{0}$; therefore

$$\mathbf{r}(t) = \frac{1}{2}\mathbf{a}t^2 + \mathbf{v}_0 t + \mathbf{r}_0 \tag{11.3.1}$$

$$= -\frac{1}{2}gt^2\mathbf{j} \tag{11.3.2}$$

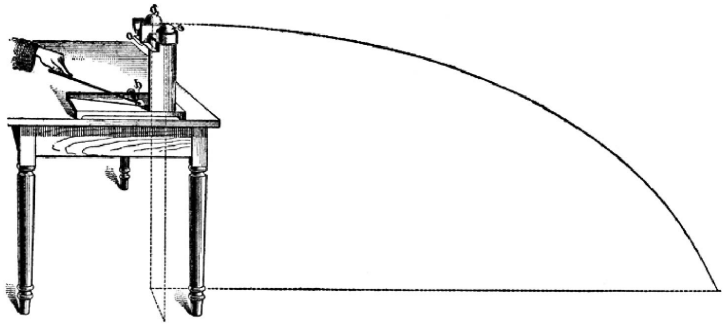

Figure 11.3.1: Vertical vs. horizontal motion. If two objects are released simultaneously (one falling vertically and one given an initial horizontal velocity), then they both land on the floor at the same time. (Ref. [9])

or

$$x(t) = 0 \tag{11.3.3}$$

$$y(t) = -\frac{1}{2}gt^2. \tag{11.3.4}$$

For the second ball (the one given an initial horizontal velocity v_0), we have $\mathbf{a} = -g\mathbf{j}$, $\mathbf{v}_0 = v_0\mathbf{i}$, and $\mathbf{r}_0 = \mathbf{0}$; therefore

$$\mathbf{r}(t) = \frac{1}{2}\mathbf{a}t^2 + \mathbf{v}_0 t + \mathbf{r}_0 \tag{11.3.5}$$

$$= v_0 t\mathbf{i} - \frac{1}{2}gt^2\mathbf{j} \tag{11.3.6}$$

or

$$x(t) = v_0 t \tag{11.3.7}$$

$$y(t) = -\frac{1}{2}gt^2 \tag{11.3.8}$$

So both balls have the same vertical (y) component of motion. Both balls fall together vertically, but the second ball has a uniform horizontal motion superimposed on its vertical motion; the combination of horizontal and vertical motions gives the second ball a parabolic path, as we'll see in Chapter 12.