

13.1: The Method

Newton's method is a numerical method for finding the root(s) x of the equation

$$f(x) = 0. \quad (13.1.1)$$

The method requires that you first make an initial estimate x_0 of the root. From that initial estimate, you calculate a better estimate using the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (13.1.2)$$

Applying this formula (with $n = 0$) to the initial estimate x_0 gives a better estimate x_1 . This better estimate x_1 is then run through the formula again ($n = 1$) to get an even better estimate x_2 , etc. The process may be repeated indefinitely to yield a solution to whatever accuracy is desired.

If the equation $f(x) = 0$ has more than one root, then the method will generally find the one closest to the initial estimate. Choosing different initial estimates closer to the other roots will find those other roots.

If there is no root (for example, $f(x) = x^2 + 1 = 0$), the method will tend to "blow up": instead of converging to a solution, you may just get bigger and bigger numbers, or you may get a series of different numbers that show no sign of converging to a single value.

13.1: The Method is shared under a [CC BY-NC-SA 4.0](https://creativecommons.org/licenses/by-nc-sa/4.0/) license and was authored, remixed, and/or curated by LibreTexts.