57.15: Hyperbolic Orbits

Suppose we wish to calculate the position of a body that is in a hyperbolic orbit $(e > 1)$, as is the case with some comets in orbit around the Sun. The procedure is the same as outlined in Section 57.6, except for Equations 57.6.3 through 57.6.5.

For hyperbolic orbits, in place of Kepler's equation (Eq. 57.6.3), we use the hyperbolic Kepler's equation:

$$M = e \sinh F - F \tag{57.15.1}$$

where M is the mean anomaly (in radians), and F is a variable that takes the place of the eccentric anomaly. As with the elliptical Kepler's equation, the hyperbolic version cannot be solved for F in closed form; instead we must rely on some numerical method like Newton's method to solve for F. Once we have found F, we solve for the true anomaly f using this replacement for Eq. 57.6.4:

$$\tan\left(\frac{f}{2}\right) = \sqrt{\frac{e+1}{e-1}} \tanh\left(\frac{F}{2}\right) \tag{57.15.2}$$

Finally, the radial distance from the Sun to the body is found by this replacement for Eq. 57.6.5:

$$r = a(e \cosh F - 1) \tag{57.15.3}$$

where for a hyperbola a is the distance from the center of the hyperbola to either vertex. The rest of position calculation is the same as described in Section 57.6 for elliptical orbits.