40: Newton’s Laws of Motion- Rotational Versions

Newton's three laws of motion have rotational counterparts. The rotational version of Newton's laws of motion are:

Law of Rotational Inertia. A body at rest (non-rotating) will remain at rest, and a body rotating with constant angular velocity will continue rotating with that same angular velocity, unless acted upon by some outside torque.

$\boldsymbol{\tau = I\alpha}$**:** If a torque τ is applied to a body of moment of inertia I, it will accelerate with angular acceleration $\alpha = \tau / I$.

Torques always come in pairs that act in opposite directions. If body 1 acts on body 2 with a torque τ, then body 2 will act back on body 1 with torque τ (equal in magnitude and opposite in direction).

Newton's First Law of Rotational Motion

The rotational form of Newton's first law states that bodies have a property called rotational inertia, which means that once given an initial angular velocity, they will continue spinning with that same angular velocity forever, unless acted upon by some outside torque. Nobody knows why this is; just like with linear inertia, it's just the way the Universe works.

Newton's Second Law of Rotational Motion

The rotational form of Newton's second law of motion states that the torque τ on a body is proportional to its resulting angular acceleration α :

$$\tau = I\alpha \tag{40.1}$$

When a torque τ is applied to a body, its spinning will accelerate with angular acceleration $\alpha = \tau / I$-the larger the moment of inertia, the smaller the angular acceleration.

If the torque τ is a function of angle, and using acceleration $\alpha = d^2\theta / dt^2$, this becomes a differential equation

$$\tau(\theta) = I \frac{d^2\theta}{dt^2} \tag{40.2}$$

Solving this differential equation for $\theta(t)$ gives a complete description of the rotational motion.

The most general form of Newton's second law is not $\tau = I\alpha$, but $\tau = dL/dt$, where L is the angular momentum. This reduces to $\tau = I\alpha$ when the moment of inertia is constant.

The rotational form of Newton's second law may also be expressed in vector form:

$$\tau = I\alpha \tag{40.3}$$

where $\boldsymbol{\alpha}$ is the angular acceleration vector, which lies along the axis of rotation. Most generally, the moment of inertia I is a tensor, i.e. a 3×3 matrix, so that $\boldsymbol{\tau}$ and $\boldsymbol{\alpha}$ do not necessarily lie in the same direction.

Newton's Third of Rotational Motion

The rotational form of Newton's third law of motion states that torques always come in pairs that act in opposite directions. For example, imagine an astronaut floating in space next to a space capsule. He has a wrench in his hand, and wishes to tighten a bolt on the spacecraft. But if he uses the wrench to turn the bolt clockwise, the bolt will, in turn, apply a torque back on him, and the astronaut will rotate himself counterclockwise. To avoid this, the astronaut can anchor his feet to the space capsule. The same thing will still happen, but this time the astronaut and the capsule will rotate counterclockwise. Since the capsule's moment of inertia is so large, the angular acceleration of the capsule $\alpha = \tau / I$ will be very small.

The rotational form of Newton's third law may be used to advantage in controlling spacecraft attitude (orientation). Spacecraft contain a set of spinning wheels called reaction wheels. By applying a torque to one of these wheels, the spacecraft can be rotated in the opposite direction.

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