66.2: Trigonometry

Basic Formulæ

$$\begin{aligned} \sin^2\theta + \cos^2\theta &\equiv 1 \\ \sec^2\theta &\equiv 1+\tan^2\theta \\ \csc^2\theta &\equiv 1+\cot^2\theta \end{aligned}$$

Angle Addition Formulæ

$$\begin{aligned} \sin(\alpha\pm\beta) &\equiv \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \\ \cos(\alpha\pm\beta) &\equiv \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \\ \tan(\alpha\pm\beta) &\equiv \frac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta} \end{aligned}$$

Double-Angle Formulæ

$$\begin{aligned} &\sin 2\theta \equiv 2\sin\theta\cos\theta \equiv \frac{2\tan\theta}{1+\tan^2\theta} \\ &\cos 2\theta \equiv \cos^2\theta - \sin^2\theta \equiv 1-2\sin^2\theta \equiv 2\cos^2\theta - 1 \equiv \frac{1-\tan^2\theta}{1+\tan^2\theta} \\ &\tan 2\theta \equiv \frac{2\tan\theta}{1-\tan^2\theta} \end{aligned}$$

Triple-Angle Formulæ

$$\begin{aligned} \sin 3\theta &\equiv 3\sin\theta - 4\sin^3\theta \\ \cos 3\theta &\equiv 4\cos^3\theta - 3\cos\theta \\ \tan 3\theta &\equiv \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta} \\ \cot 3\theta &\equiv \frac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1} \end{aligned}$$

Quadruple-Angle Formulæ

$$\begin{aligned} \sin 4\theta &\equiv 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta \\ \cos 4\theta &\equiv \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta \\ \tan 4\theta &\equiv \frac{4\tan\theta - 4\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta} \\ \cot 4\theta &\equiv \frac{\cot^4\theta - 6\cot^2\theta + 1}{4\cot^3\theta - 4\cot\theta} \end{aligned}$$

Half-Angle Formulæ

$$\begin{aligned} \sin\frac{\theta}{2} &\equiv \pm\sqrt{\frac{1-\cos\theta}{2}} \\ \cos\frac{\theta}{2} &\equiv \pm\sqrt{\frac{1+\cos\theta}{2}} \\ \tan\frac{\theta}{2} &\equiv \frac{\sin\theta}{1+\cos\theta} \equiv \frac{1-\cos\theta}{\sin\theta} \end{aligned}$$

Products of Sines and Cosines

$$\begin{aligned} \sin\alpha\cos\beta &\equiv \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)] \\ \cos\alpha\sin\beta &\equiv \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)] \\ \cos\alpha\cos\beta &\equiv \frac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)] \\ \sin\alpha\sin\beta &\equiv -\frac{1}{2}[\cos(\alpha+\beta)-\cos(\alpha-\beta)] \end{aligned}$$

Sums and Differences of Sines and Cosines

$$\begin{aligned} \sin\alpha+\sin\beta &\equiv 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\ \sin\alpha-\sin\beta &\equiv 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\ \cos\alpha+\cos\beta &\equiv 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\ \cos\alpha-\cos\beta &\equiv -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \end{aligned}$$

Power Reduction Formulæ

$$\begin{aligned} \sin^2\theta &\equiv \frac{1}{2}(1-\cos 2\theta) \\ \cos^2\theta &\equiv \frac{1}{2}(1+\cos 2\theta) \\ \tan^2\theta &\equiv \frac{1-\cos 2\theta}{1+\cos 2\theta} \end{aligned}$$

Other Formulæ

$$\tan\theta \equiv \cot\theta - 2\cot 2\theta \tag{66.2.1}$$

Trig Cheat Sheet Paul Dawkins

https://tutorial.math.lamar.edu/pdf/trig_cheat_sheet.pdf

Exact values of trigonometric functions at 3° intervals. (Ref. [6])

\theta	\sin \theta	\cos \theta
\(0^{\circ}=0 \pi	0	1
$3^{\circ}=\frac{\pi}{60}$	$\frac{1}{16}[(\sqrt{6}+\sqrt{2})(\sqrt{5}-1)-2(\sqrt{3}-1)\sqrt{5+\sqrt{5}}]$	$\frac{1}{16}[2(\sqrt{3}+1)\sqrt{5+\sqrt{5}}+(\sqrt{6}-\sqrt{2})(\sqrt{5}-1)]$
$6^{\circ}=\frac{\pi}{30}$	$\frac{1}{8}(\sqrt{30-6\sqrt{5}}-\sqrt{5}-1)$	$\frac{1}{8}(\sqrt{15}+\sqrt{3}+\sqrt{10-2\sqrt{5}})$
$9^{\circ}=\frac{\pi}{20}$	$\frac{1}{8}(\sqrt{10}+\sqrt{2}-2\sqrt{5-\sqrt{5}})$	$\frac{1}{8}(\sqrt{10}+\sqrt{2}+2\sqrt{5-\sqrt{5}})$
$12^{\circ}=\frac{\pi}{15}$	$\frac{1}{8}(\sqrt{10+2\sqrt{5}}-\sqrt{15}+\sqrt{3})$	$\frac{1}{8}(\sqrt{30+6\sqrt{5}}+\sqrt{5}-1)$
$15^{\circ}=\frac{\pi}{12}$	$\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$\frac{1}{4}(\sqrt{6}+\sqrt{2})$
$18^{\circ}=\frac{\pi}{10}$	$\frac{1}{4}(\sqrt{5}-1)$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$
$21^{\circ}=\frac{7\pi}{60}$	$\frac{1}{16}[2(\sqrt{3}+1)\sqrt{5-\sqrt{5}}-(\sqrt{6}-\sqrt{2})(\sqrt{5}+1)]$	$\frac{1}{16}[(\sqrt{6}+\sqrt{2})(\sqrt{5}+1)+2(\sqrt{3}-1)\sqrt{5-\sqrt{5}}]$
$24^{\circ}=\frac{2\pi}{15}$ \)	$\frac{1}{8}(\sqrt{15}+\sqrt{3}-\sqrt{10-2\sqrt{5}})$	$\frac{1}{8}(\sqrt{30-6\sqrt{5}}+\sqrt{5}+1)$
$27^{\circ}=\frac{3\pi}{20}$	$\frac{1}{8}(2\sqrt{5+\sqrt{5}}-\sqrt{10}+\sqrt{2})$	$\frac{1}{8}(2\sqrt{5+\sqrt{5}}+\sqrt{10}-\sqrt{2})$
$30^{\circ}=\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$
$33^{\circ}=\frac{11\pi}{60}$	$\frac{1}{16}[(\sqrt{6}+\sqrt{2})(\sqrt{5}-1)+2(\sqrt{3}-1)\sqrt{5+\sqrt{5}}]$	$\frac{1}{16}[2(\sqrt{3}+1)\sqrt{5+\sqrt{5}}-(\sqrt{6}-\sqrt{2})(\sqrt{5}-1)]$
$36^{\circ}=\frac{\pi}{5}$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$	$\frac{1}{4}(\sqrt{5}+1)$
$39^{\circ}=\frac{13\pi}{60}$	$\frac{1}{16}[(\sqrt{6}+\sqrt{2})(\sqrt{5}+1)-2(\sqrt{3}-1)\sqrt{5-\sqrt{5}}]$	$\frac{1}{16}[2(\sqrt{3}+1)\sqrt{5-\sqrt{5}}+(\sqrt{6}-\sqrt{2})(\sqrt{5}+1)]$
$42^{\circ}=\frac{7\pi}{30}$	$\frac{1}{8}(\sqrt{30+6\sqrt{5}}-\sqrt{5}+1)$	$\frac{1}{8}(\sqrt{10+2\sqrt{5}}+\sqrt{15}-\sqrt{3})$
$45^{\circ}=\frac{\pi}{4}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{2}$
$48^{\circ}=\frac{4\pi}{15}$	$\frac{1}{8}(\sqrt{10+2\sqrt{5}}+\sqrt{15}-\sqrt{3})$	$\frac{1}{8}(\sqrt{30+6\sqrt{5}}-\sqrt{5}+1)$
$51^{\circ}=\frac{17\pi}{60}$	$\frac{1}{16}[2(\sqrt{3}+1)\sqrt{5-\sqrt{5}}+(\sqrt{6}-\sqrt{2})(\sqrt{5}+1)]$	$\frac{1}{16}[(\sqrt{6}+\sqrt{2})(\sqrt{5}+1)-2(\sqrt{3}-1)\sqrt{5-\sqrt{5}}]$
$54^{\circ}=\frac{3\pi}{10}$	$\frac{1}{4}(\sqrt{5}+1)$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$
$57^{\circ}=\frac{19\pi}{60}$	$\frac{1}{16}[2(\sqrt{3}+1)\sqrt{5+\sqrt{5}}-(\sqrt{6}-\sqrt{2})(\sqrt{5}-1)]$	$\frac{1}{16}[(\sqrt{6}+\sqrt{2})(\sqrt{5}-1)+2(\sqrt{3}-1)\sqrt{5+\sqrt{5}}]$
$60^{\circ}=\frac{\pi}{3}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}$
$63^{\circ}=\frac{7\pi}{20}$	$\frac{1}{8}(2\sqrt{5+\sqrt{5}}+\sqrt{10}-\sqrt{2})$	$\frac{1}{8}(2\sqrt{5+\sqrt{5}}-\sqrt{10}+\sqrt{2})$
$66^{\circ}=\frac{11\pi}{30}$	$\frac{1}{8}(\sqrt{30-6\sqrt{5}}+\sqrt{5}+1)$	$\frac{1}{8}(\sqrt{15}+\sqrt{3}-\sqrt{10-2\sqrt{5}})$
$69^{\circ}=\frac{23}{60}\pi$	$\frac{1}{16}[(\sqrt{6}+\sqrt{2})(\sqrt{5}+1)+2(\sqrt{3}-1)\sqrt{5-\sqrt{5}}]$	$\frac{1}{16}[2(\sqrt{3}+1)\sqrt{5-\sqrt{5}}-(\sqrt{6}-\sqrt{2})(\sqrt{5}+1)]$
$75^{\circ}=\frac{5\pi}{12}$	$\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$\frac{1}{4}(\sqrt{6}-\sqrt{2})$
$78^{\circ}=\frac{13\pi}{30}$	$\frac{1}{8}(\sqrt{30+6\sqrt{5}}+\sqrt{5}-1)$	$\frac{1}{8}(\sqrt{10+2\sqrt{5}}-\sqrt{15}+\sqrt{3})$
$81^{\circ}=\frac{19\pi}{20}$	$\frac{1}{8}(\sqrt{10}+\sqrt{2}+2\sqrt{5-\sqrt{5}})$	$\frac{1}{8}(\sqrt{10}+\sqrt{2}-2\sqrt{5-\sqrt{5}})$
$84^{\circ}=\frac{7\pi}{15}$	$\frac{1}{8}(\sqrt{15}+\sqrt{3}+\sqrt{10-2\sqrt{5}})$	$\frac{1}{8}(\sqrt{30-6\sqrt{5}}-\sqrt{5}-1)$
$87^{\circ}=\frac{29\pi}{60}$	$\frac{1}{16}[2(\sqrt{3}+1)\sqrt{5+\sqrt{5}}+(\sqrt{6}-\sqrt{2})(\sqrt{5}-1)]$	$\frac{1}{16}[(\sqrt{6}+\sqrt{2})(\sqrt{5}-1)-2(\sqrt{3}-1)$ \sqrt{5+\sqrt{5}
$90^{\circ}=\frac{\pi}{2}$	1	0