62.6: Addition of Velocities

Let's suppose that we have two bodies moving in one dimension. The first is moving at speed u, and the second is moving at speed v. What is the speed of the second relative to the first? In other words, what will you measure as the speed of the second body if you're sitting on the first body?

In classical Newtonian mechanics, the speed w of the second body relative to the first is simply

$$w = v - u. \tag{62.6.1}$$

For example, if the first body is moving to the right with speed $u = 10\ \mathrm{m/s}$, and the second body is moving toward it to the left with speed $v = -20\ \mathrm{m/s}$, then an observer on the first body will see the second body moving toward it with a speed of $w = 30\ \mathrm{m/s}$.

In the special theory of relativity, this seemingly self-evident equation for adding velocities must be modified as follows:

$$w = \frac{v-u}{1-uv/c^2} \tag{62.6.2}$$

This reduces to Eq. $62.6.1$ unless the speeds involved are near the speed of light. For the above example, where $u = 10\ \mathrm{m/s}$ and $v = -20\ \mathrm{m/s}$, Eq. $62.6.2$ gives $w = 29.99999999999993324\ \mathrm{m/s}$, rather than $w = 30\ \mathrm{m/s}$ given by Eq. $62.6.1$. As you can see, for many applications, the difference between the classical formula (Eq. $62.6.1$) and the exact relativistic formula (Eq. $62.6.2$) is not enough to justify the extra complexity of using the relativistic formula.

But for speeds near the speed of light, using the relativistic formula is important. For example, if $u = 0.99c$ and $v = -0.99c$, then the classical formula of Eq. $62.6.1$ would give $w = 1.98c > c$, in violation of special relativity; but using the exact expression in Eq. $62.6.2$ gives the correct answer, $w = 0.9999494975c$

Eq. $62.6.2$ makes it impossible for the the relative speeds to be greater than the speed of light c. In the extreme case $u = c$ and $v = -c$, Eq. $62.6.2$ gives $w = c$, in agreement with the Einstein's second postulate.