45.2: Modern Treatment

Developments in the theory of classical mechanics since Galileo's time allow us to investigate his experiment in more detail. For one thing, we now know that the proportionality constant in Eq. 45.1.1 is $a/2$, where a is the acceleration of the ball down the incline; Galileo's law then becomes

$$x = \frac{1}{2}at^2. \tag{45.2.1}$$

Furthermore, we now know that the acceleration a of a solid ball rolling down an inclined plane is given by

$$a = \frac{g\sin\theta}{1+\frac{I_{\text{cm}}}{MR^2}} = \frac{g\sin\theta}{1+\beta} \tag{45.2.2}$$

where g is the acceleration due to gravity $\left(9.8 \text{ m/s}^2\right)$, θ is the inclination of the inclined plane, I_{cm} is the moment of inertia of the ball about its center of mass, M is the mass of the ball, R is the radius of the ball, and $\beta \equiv I_{\text{cm}}/\left(MR^2\right)$. For a solid spherical ball, we know

$$I_{\text{cm}} = \frac{2}{5}MR^2 \tag{45.2.3}$$

so $\beta \equiv I_{\text{cm}}/\left(MR^2\right) = 2/5$ the acceleration of a solid ball down an inclined plane is therefore

$$a = \frac{5}{7}g\sin\theta. \tag{45.2.4}$$

Galileo's law for a solid ball rolling down an incline then becomes

$$x = \frac{1}{2}at^2 \tag{45.2.5}$$

$$= \frac{1}{2}\left(\frac{5}{7}g\sin\theta\right)t^2 \tag{45.2.6}$$

$$= \frac{5}{14}(g\sin\theta)t^2. \tag{45.2.7}$$

Using $g = 9.8 \text{ m/s}^2$ and $\theta = 4^\circ$ for Galileo's incline, we get

$$x = 0.244t^2 \tag{45.2.8}$$

where x is in meters and t is in seconds.

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