39.2: Pendulum Method

A second method for measuring a body's moment of inertia has been described by Rhett Allain of Southeastern Louisiana University. [1]. The idea of this method is to attach the body to be measured to a long string, forming a physical pendulum. One measures the period T of the pendulum at a variety of different lengths L. Now recall that the period T of a physical pendulum is given by

$$T = 2\pi\sqrt{\frac{d}{mgL}} \tag{39.2.1}$$

Solving for the moment of inertia, we get

$$\ell = \frac{T^2 mgL}{4\pi^2} \tag{39.2.2}$$

By the parallel-axis theorem,

$$d = I + mL^2 \tag{39.2.3}$$

Combining these equations, we get

$$\frac{T^2 mgL}{4\pi^2} = I + mL^2 \tag{39.2.4}$$

If we plot the left-hand side vs. L^2, we will get a straight line of slope m and ordinate intercept equal to the moment of inertia I.

In summary, the steps for measuring the moment of inertia are:

1. Attach the test object to the end of a string, forming a physical pendulum.
2. Measure the period T of the pendulum at various lengths L.
3. Perform a linear regression analysis on the data (treating L^2 as the independent variable, and $T^2 mgL/4\pi^2$ as the dependent variable).
4. The ordinate intercept is then the desired moment of inertia.

[1] https://www.wired.com/2017/05/physic...dget-spinner/