44.4: The Wheel

Imagine a wheel of radius r rolling along the ground without slipping. When the wheel makes one complete revolution, the axis will have been directly above each point on the circumference of the wheel exactly once. Therefore the axis of the wheel has traveled a horizontal distance equal to the circumference of the wheel, or $2\pi r$. In other words, each revolution of the wheel causes the axis (or any vehicle attached to the wheel) to move a distance of $2\pi r$.

As a consequence of this observation, we can relate the angular velocity ω of the wheel to the linear velocity v of the axis. Let's say that it takes a time Δt for the wheel to rotate once on its axis. Then the linear velocity of the outer edge of the wheel is $2\pi r/\Delta t$. During that same time, the axis of the wheel has traveled the same distance $2\pi r$, and so the linear velocity of the axis is also $2\pi r/\Delta t$. Both the velocity of the axis and the linear velocity of the outside edge of the wheel are equal to $v = r\omega$. In other words: The linear velocity of the axis with respect to the ground is equal to the linear velocity of the outer edge of the wheel with respect to the axis.

Example 44.4.1

A bicycle with wheels of radius $34\ \text{cm}$ is traveling with a speed of $7\ \text{m/s}$. What is the angular velocity of the wheels?

Solution

From the above discussion, the velocity of the bicycle v is equal to the linear velocity of the outer edge of the wheels, $r\omega$. Therefore $v = r\omega$, so $\omega = v/r = (7\ \text{m/s})/(0.34\ \text{m}) = 20.6\text{rad/s}$.