

56.3: The Cosine Formula

The cosine formula from spherical trigonometry gives the angular separation between two points on the surface of a sphere, where the apex of the angle is at the center of the sphere. If the two points have latitudes ϕ_1 and ϕ_2 and longitudes L_1 and L_2 , then the cosine formula gives the angular separation of the two points θ :

$$\cos \theta = \sin \phi_1 \sin \phi_2 + \cos \phi_1 \cos \phi_2 \cos(L_1 - L_2) \quad (56.3.1)$$

This formula may be used to compute, for example, the angular separation between two stars in the sky, where $\phi = \delta$ and $L = \alpha$ are the celestial counterparts of latitude and longitude, called declination and right ascension, respectively (see Chapter 54). To find the distance s between two points on the Earth's surface, convert θ to radians and use $s = R_{\oplus} \theta$, where $R_{\oplus} = 6371.0$ km is the average radius of the Earth.

If the angular separation θ between the two points is small, better accuracy may be obtained by using the haversine function, $\text{hav}(x)$. The haversine is defined by

$$\text{hav } \theta \equiv \sin^2 \left(\frac{\theta}{2} \right) \quad (56.3.2)$$

and so the inverse haversine function is given by

$$\text{hav}^{-1} y \equiv 2 \sin^{-1} \sqrt{y}. \quad (56.3.3)$$

Using the haversine function, the cosine formula can be replaced by

$$\text{hav } \theta = \text{hav}(\phi_1 - \phi_2) + \cos \phi_1 \cos \phi_2 \text{hav}(L_1 - L_2) \quad (56.3.4)$$

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