26.1: Kinetic Energy

Kinetic energy is the energy a body has as a consequence of it being in motion. If a body is at rest, it has zero kinetic energy; if it is in motion, it has more kinetic energy the faster it's going.

Kinetic energy is defined to be the amount of work required to accelerate a body of mass m from rest to velocity v. We can compute an explicit formula for it as follows: by definition, the kinetic energy K is, by Eq. (24.4.1),

$$K = W = \int F dx \tag{26.1.1}$$

Applying Newton's second law $F = ma$,

$$K = \int ma dx \tag{26.1.2}$$

$$= \int m \frac{dv}{dt} dx \tag{26.1.3}$$

Now applying the chain rule,

$$K = \int m \frac{dv}{dx} \frac{dx}{dt} dx \tag{26.1.4}$$

Cancelling dx in denominator with the final dx,

$$K = \int_0^v m \frac{dx}{dt} dv \tag{26.1.5}$$

Now dx/dt is just the velocity v, so

$$K = \int_0^v mv dv \tag{26.1.6}$$

$$= m \int_0^v v dv \tag{26.1.7}$$

or

$$K = \frac{1}{2} m v^2. \tag{26.1.8}$$

Sometimes it's useful to write this in vector form:

$$K = \frac{1}{2} m \mathbf{v} \cdot \mathbf{v} \tag{26.1.9}$$