66.20: Motion of a Falling Body

Given a body of mass m released from rest at time $t = 0$ from a height h above the floor, we find the following results about the motion. Here the y axis points upward and has its origin at the floor, and so the acceleration due to gravity is $-g$.

- Position y at time t :

$$y(t) = h - \frac{1}{2}gt^2$$

- Velocity v :

$$v(t) = -gt$$
$$v(y) = -\sqrt{2g(h-y)}$$

- Fall time t_f :

$$t_f = \sqrt{\frac{2h}{g}}$$

- Impact velocity v_f :

$$v_f = -\sqrt{2gh}$$

Total energy E :

$$E = mgh$$

- Kinetic energy K :

$$K(t) = \frac{1}{2}mg^2t^2$$
$$K(y) = mg(h-y)$$

- Potential energy U :

$$U(t) = mgh - \frac{1}{2}mg^2t^2$$
$$U(y) = mgy$$

- Time derivatives of kinetic energy:

$$\frac{dK}{dt} = -mgv = mg^2t = mg\sqrt{2g(h-y)}$$
$$\frac{d^2K}{dt^2} = mg^2$$

- Time derivatives of potential energy:

$$\frac{dU}{dt} = mgv = -mg^2t = -mg\sqrt{2g(h-y)}$$
$$\frac{d^2U}{dt^2} = -mg^2$$

- Time averages:

$$\langle v \rangle = -\frac{1}{2}\sqrt{2gh}$$
$$\langle K \rangle = \frac{1}{3}mgh$$
$$\langle U \rangle = \frac{2}{3}mgh$$

- Virial theorem $(n = 0)$:

$$\langle K \rangle = \frac{1}{2} \langle U \rangle$$
