44.2: Acceleration

Now let's find the (translational) acceleration of the body down the incline. If the distance down the incline is x, then the velocity v at the bottom of the incline is related to x by

$$v^2 = 2ax \tag{44.2.1}$$

By geometry, $\sin\theta = h/x$, and so $x = h/\sin\theta$; using this to substitute for x, we have

$$v^2 = 2a\frac{h}{\sin\theta}, \tag{44.2.2}$$

or, solving for the acceleration a,

$$a = \frac{v^2 \sin\theta}{2h} \tag{44.2.3}$$

Now let's use Eq. (41.1.12) to substitute for v; the result is an expression for the acceleration of a body rolling down an incline,

$$a = \frac{g\sin\theta}{\beta+1} \tag{44.2.4}$$

Table 44.3.1 shows values of β and a for several common geometries.

Equation $44.2.4$ has some interesting consequences. For example, if you start a solid sphere and a cylindrical shell at the top of a incline and release them at the same time, which one will reach the bottom first? From Table 44.3.1, you can see that the solid sphere will will win: its acceleration (5/7) $g\sin\theta$ is greater than the cylindrical shell's acceleration of $(1/2)g\sin\theta$. What's surprising about this is that all solid spheres will beat all cylindrical shells, regardless of mass or radius. In general, the object with the smaller β will win such a race, since that will give the smallest denominator in Eq. $44.2.4$ and therefore the larger acceleration.