56.6: Vincenty's Formulæ- Inverse Problem

In the inverse problem, we're given two points on the Earth's surface (ϕ_1, L_1) and (ϕ_2, L_2) and want to calculate the distance s between them, as well as the direction from one to the other. We'll use the constants defining the Earth's ellipsoid as before:

- Earth ellipsoid semi-major axis (i.e. equatorial radius): $a = 6378137.0$ meters.
- Earth flattening factor $f = 1/298.257223563$ This is defined as the difference between semi-major and semi-minor axes, divided by the semi-major axis: $f = (a - b)/a$.

In performing the following calculations, work with all angles in radians. We begin by calculating

$$U_1 = \tan^{-1}[(1-f)\tan\phi_1] \tag{56.6.1}$$

$$U_2 = \tan^{-1}[(1-f)\tan\phi_2] \tag{56.6.2}$$

$$L = L_2 - L_1 \tag{56.6.3}$$

$$b = (1-f)a \tag{56.6.4}$$

Now set an initial value $\lambda = L$. Then iterate on Eqs. (53.28) through (53.35) until λ converges:

$$\sin\sigma = \sqrt{(\cos U_2 \sin\lambda)^2 + (\cos U_1 \sin U_2 - \sin U_1 \cos U_2 \cos\lambda)^{\,2}} \tag{56.6.5}$$

$$\cos\sigma = \sin U_1 \sin U_2 + \cos U_1 \cos U_2 \cos\lambda \tag{56.6.6}$$

$$\sigma = \arctan\frac{\sin\sigma}{\cos\sigma} \tag{56.6.7}$$

$$\sin\alpha = \frac{\cos U_1 \cos U_2 \sin\lambda}{\sin\sigma} \tag{56.6.8}$$

$$\cos^2\alpha = 1 - \sin^2\alpha \tag{56.6.9}$$

$$\cos^2(2\sigma_m) = \cos\sigma - \frac{2\sin U_1 \sin U_2}{\cos^2\alpha} \tag{56.6.10}$$

$$C = \frac{f}{16}\cos^2\alpha\left[4 + f\left(4 - 3\cos^2\alpha\right)\right] \tag{56.6.11}$$

$$\lambda = L + (1-C) f \sin\alpha\left\{\sigma + C\sin\sigma\left[\cos(2\sigma_m) + C\cos\sigma\left(-1 + 2\cos^2(2\sigma_m)\right)\right]\right\} \tag{56.6.12}$$

When λ has converged to the desired degree of accuracy, continue calculating:

$$u^2 = (\cos^2 \alpha)\left(\frac{a^2 - b^2}{b^2}\right) \tag{56.6.13}$$

$$A = 1 + \frac{u^2}{16384}\{4096 + u^2\left[-768 + u^2\left(320 - 175u^2\right)\right]\} \tag{56.6.14}$$

$$B = \frac{u^2}{1024}\{256 + u^2\left[-128 + u^2\left(74 - 47u^2\right)\right]\} \tag{56.6.15}$$

$$\Delta\sigma = B \sin\sigma \left\{\cos(2\sigma_m) + \frac{1}{4}B\left[\cos\sigma\left(-1 + 2\cos^2(2\sigma_m)\right) - \frac{1}{6}B\cos(2\sigma_m)\left(-3 + 4\sin^2\sigma\right)\left(-3 + 4\cos^2(2\sigma_m)\right)\right]\right\} \tag{56.6.16}$$

$$s = bA(\sigma - \Delta\sigma) \tag{56.6.17}$$

$$\alpha_1 = \arctan\left(\frac{\cos U_2 \sin\lambda}{\cos U_1 \sin U_2 - \sin U_1 \cos U_2 \cos\lambda}\right) \tag{56.6.18}$$

$$\alpha_2 = \arctan\left(\frac{\cos U_1 \sin\lambda}{-\sin U_1 \cos U_2 + \cos U_1 \sin U_2 \cos\lambda}\right) \tag{56.6.19}$$

Then s is the distance between the two points.

Example 56.6.1

Find the distance between the sounding rocket in Chesapeake Hall at Prince George's Community College ($38^\circ 53' 16.87'' \mathrm{N}, 76^\circ 49' 23.14'' \mathrm{W}$) and the top (apex) of the Great Pyramid of Giza in Egypt ($29^\circ 58' 45.03'' \mathrm{N}, 31^\circ 08' 03.69'' \mathrm{E}$).

Solution

The given parameters are the coordinates $\phi_1 = 38.888019^\circ, L_1 = -76.823094^\circ, \phi_2 = 29.979175^\circ, L_2 = +31.134358^\circ$. Employing Vincenty’s formulæ (inverse method), we find:

$$\begin{aligned}
U_1 &= 38.794230^\circ \\
U_2 &= 29.895958^\circ \\
L &= 339.15856744^\circ \\
b &= 6356752.3 \text{ meters} \\
\lambda &= 108.139490^\circ \\
u^2 &= 0.00393162979 \\
A &= 1.00098218405082 \\
B &= 9.809796134747123 \times 10^{-4} \\
\Delta\sigma &= 0.054160886^\circ \\
s &= 9351378.858 \text{ meters} \\
\alpha_1 &= 55.910048^\circ \\
\alpha_2 &= 131.801775^\circ
\end{aligned}$$

So the distance $s = 9351.378858$ km (5280 miles, 3576 feet, 10 inches), in the direction 55.910048° (10.91 south of northeast).