34.1: Discrete Masses

For a collection of discrete point masses in one dimension, the center of mass x_{cm} is defined to be

$$x_{\text{cm}} = \frac{\sum_i m_i x_i}{\sum_i m_i} \tag{34.1.1}$$

where the summations are over all of the point masses. This is just the weighted average of the positions of the masses, where the "weights" are the masses. Note that the denominator is the total mass of all the point masses.

Example 34.1.1

Suppose there is a mass of $3\ \text{kg}$ at $x = 1\ \text{m}$, a mass of $2\ \text{kg}$ at the origin, and a mass of $4\ \text{kg}$ at $x = 2\ \text{m}$. Where is the center of mass?

Solution

Let's put the data in a table:

i	$m_i(\ \text{kg})$	$x_i(\ \text{m})$
1	3.0	1.0
2	2.0	0.0
3	4.0	2.0

Then by Eq. 34.1.1,

$$x_{\text{cm}} = \frac{(3\ \text{kg})(1\ \text{m}) + (2\ \text{kg})(0\ \text{m}) + (4\ \text{kg})(2\ \text{m})}{3\ \text{kg} + 2\ \text{kg} + 4\ \text{kg}} \tag{34.1.2}$$

$$= 1.222\ \text{m}. \tag{34.1.3}$$

In two or three dimensions, the x, y, and z coordinates of the center of mass are calculated independently:

$$x_{\text{cm}} = \frac{\sum_i m_i x_i}{\sum_i m_i} \tag{34.1.4}$$

$$y_{\text{cm}} = \frac{\sum_i m_i y_i}{\sum_i m_i} \tag{34.1.5}$$

$$z_{\text{cm}} = \frac{\sum_i m_i z_i}{\sum_i m_i} \tag{34.1.6}$$

Example 34.1.2

In two dimensions: suppose there is a mass of $3\ \text{kg}$ at $(x, y) = (1, 3)\text{m}$, a mass of $2\ \text{kg}$ at the origin, and a mass of $4\ \text{kg}$ at $(x, y) = (5, -1)\text{m}$. Where is the center of mass?

Solution

Let's put the data in a table:

i	$m_i(\ \text{kg})$	$x_i(\ \text{m})$	$y_i(\ \text{m})$
1	3.0	3.0	3.0

i	m_i (kg)	x_i (m)	y_i (m)
1	3.0	1.0	3.0
2	2.0	0.0	0.0
3	4.0	5.0	-1.0

Then by Eqs. 34.1.4and 34.1.5,

$$x_{\text{cm}} = \frac{(3\text{ kg})(1\text{ m}) + (2\text{ kg})(0\text{ m}) + (4\text{ kg})(5\text{ m})}{3\text{ kg} + 2\text{ kg} + 4\text{ kg}} \tag{34.1.7}$$

$$= 2.556\text{ m}. \tag{34.1.8}$$

and

$$y_{\text{cm}} = \frac{(3\text{ kg})(3\text{ m}) + (2\text{ kg})(0\text{ m}) + (4\text{ kg})(-1\text{ m})}{3\text{ kg} + 2\text{ kg} + 4\text{ kg}} \tag{34.1.9}$$

$$= 0.556\text{ m}. \tag{34.1.10}$$

The center of mass is at $(x_{\text{cm}}, y_{\text{cm}}) = (2.556, 0.556)\text{m}$