56.1: Introduction to Geodesy

Geodesy is the study of the measurement of the Earth, its precise shape, and the details of its gravitational field. With precise determinations of latitude and longitude around the globe, geodesists provide a means to tie together results of local land surveys into a consistent, coherent, global system.

Although the Earth is often considered to be a sphere, it is much closer to being an oblate spheroid, bulging slightly at the equator due rotation over time. The equatorial and polar radii, set by the WGS 84 standard [1], are $a = 6378.1370\text{ km}$ and $b = 6356.7523142\text{ km}$, respectively. The polar radius b is calculated from the equatorial radius a and a flattening parameter. [2] These two radii approximate the Earth's size at sea level. From them we can calculate the constants given in Table Table $|PageIndex1$.

Table $|PageIndex1$. WGS 84 Derived Constants.

Surface area of oblate spheroid	$5.10065622 \times 10^8\text{ km}^2$
Volume of oblate spheroid	$3.44795987 \times 10^{11}\text{ km}^3$
Mean radius of semi-axes	6371.00877 km
Radius of sphere of equal area	6371.00718 km
Radius of sphere of equal volume	6371.00079 km
Equatorial circumference	40075.0167 km
Polar circumference	40007.8629 km

This oblate spheroid is part of the satellite-based Global Positioning System (GPS) and serves as the reference for coordinate calculations. GPS can also be used to estimate elevation; however the accuracy is reduced, in large part because sea level is not the shape of an oblate spheroid, but varies with the gravity, which in turn varies with latitude and local terrain and ground composition. Sea level is modeled as an irregularly shaped surface known as the geoid. Its local value is referenced to the spheroid.

The Earth's volume exceeds that of the oblate spheroid due to the land volume above sea level, which is estimated to be $3.755 \times 10^7\text{ km}^3$, based on the global mean elevation of 231.3 meters.Note that one fourth of the polar circumference is less the 2 km more than the pole-to-equator dimension used to define the original meter. A large part of this discrepancy is due to a calculation error made during

the survey that was made in the late 18th century to determine the distance from the equator to the North Pole for the purpose of defining the meter. The resulting error led to our current meter being about 0.2 mm shorter than intended. [3]

In this chapter, we'll examine some formulæ used in geodesy to find the distance between two points on the globe. The so-called cosine formula is a fairly simple method for determining distance along the Earth's surface, under the assumption that the Earth is a perfect sphere. More precise results may be obtained by using Vincenty's formule, which model the Earth as an ellipsoid. Vincenty's formulæ are much more complicated, but they have two advantages over the cosine formula: they give more accurate results,

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[1] 1984 World Geodetic System.

[2] The flattening factor $f = (a - b)/a = 1/298.257223563$. so $b = a(1 - f)$.

[3] The story of this survey is described in the book The Measure of All Things by Ken Alder.