52.4: Torricelli’s Theorem

As another example of Bernoulli's equation, consider a cylinder filled with liquid, and with a hole in the side of the cylinder through which the liquid can leak out (Fig. $52.4.1$). With what velocity does the liquid flow out of the hole?

We can analyze this using Bernoulli's equation. At the top surface of the liquid in the cylinder (which we'll call elevation y_1), the pressure will be atmospheric pressure P_0. The liquid level drops here as water flows out of the cylinder, but at a very slow rate, so we'll take the velocity of the liquid here to be approximately zero.

At the hole in the side of the cylinder (where we'll call the elevation y_2), the pressure will also be atmospheric pressure P_0, since the hole is exposed to the atmosphere here also. If the liquid is incompressible with density ρ, then by Bernoulli's equation,

$$\frac{P_0}{\rho g}+\frac{0}{2g}+y_1=\frac{P_0}{\rho g}+\frac{v^2}{2g}+y_2 \tag{52.4.1}$$

$$y_1=\frac{v^2}{2g}+y_2 \tag{52.4.2}$$

$$y_1-y_2=\frac{v^2}{2g} \tag{52.4.3}$$

Calling the difference in elevations $h \equiv y_1-y_2$, we get

$$v=\sqrt{2gh} \tag{52.4.4}$$

This result, called Torricelli's theorem after 17th century Italian physicist Evangelista Torricelli, gives the fluid velocity when the difference between the fluid level in the cylinder and the position in the hole is h. The formula may look familiar: it's the same as the formula for the impact velocity of a point mass dropped from a height h.

In Fig. $52.4.1$, the water leaving the cylinder follows a parabolic path, just as a projectile would. Using the constant-acceleration formulæ, we find that if the hole is a height H above the platform, then the amount

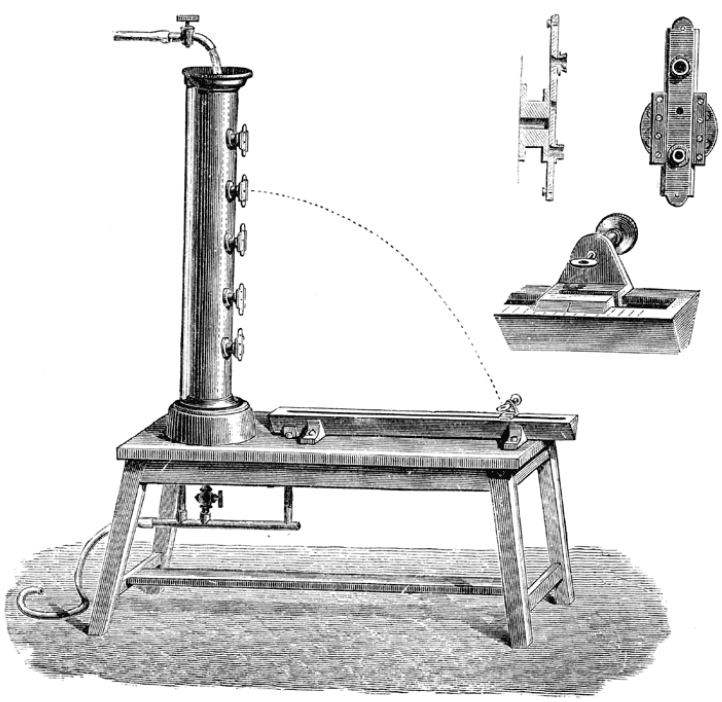

Figure $52.4.1$: Apparatus for demonstrating Torricelli’s theorem. (Ref. [14])

of time required for a parcel of water to fall to the platform will be $t=\sqrt{2H/g}$. Therefore the horizontal distance the water travels will be $x=v\sqrt{2H/g}$. Substituting the horizontal velocity v given by Eq. $52.4.4$, we have the horizontal distance x traveled by the water stream as

$$x=2\sqrt{Hh} \tag{52.4.5}$$

where again H is the height of the hole above the platform, and h is the height of the liquid surface in the cylinder above the hole.

If the cylinder in Fig. $52.4.1$ is filled all the way to the top and all five holes in the cylinder are opened, which stream will travel farthest horizontally? To answer this, let's number the top hole 1 , the bottom hole 5 , and let's choose a coordinate system with $+y$ pointing upward and the origin at the platform. If the distance between the holes is a, then the liquid in the cylinder is at $y=6a$, and so $h=6a-H$; then by Eq. $52.4.5$,

- Hole 1: $H=5a$, $h=a$, so $x=2a\sqrt{5}$.
- Hole 2: $H=4a$, $h=2a$, so $x=2a\sqrt{8}$.
- Hole 3: $H=3a$, $h=3a$, so $x=2a\sqrt{9}$.
- Hole 4: $H=2a$, $h=4a$, so $x=2a\sqrt{8}$.
- Hole 5: $H=a$, $h=5a$, so $x=2a\sqrt{5}$.

The water from the center hole (number 3) will travel farthest, a horizontal distance $x=6a$.

Another way to think about this result is that hole 1 is high above the platform, but the water velocity is low, so it doesn't travel very far horizontally. The water velocity is highest at hole 5 , but the hole is so close to the platform that it also doesn't travel far. Hole 3 is a compromise between height and fluid velocity that gives the maximum horizontal distance.
