37.3: Parallel Axis Theorem

There are some theorems that allow us to extend Table 37.2.1 to other rotation axes. The most important of these is called the parallel axis theorem (sometimes called Steiner's theorem).

Theorem 37.3.1

The parallel axis theorem (sometimes called Steiner's theorem) relates the moment of inertia I_{cm} about an axis A passing through the center of mass to the moment of inertia I about another axis parallel to A. If the two rotation axes are separated by a distance h, then

$$I = I_{\text{cm}} + Mh^2 \tag{37.3.1}$$

Example 37.3.1

Consider the fourth example in the previous section, where a hoop was rotated about an axis going through the rim of the hoop. The same result may be found much more simply using the parallel axis theorem.

Solution

From Figure 37.2.1, the moment of inertia of the hoop when rotated about its center is $I_{\text{cm}} = MR^2$. The distance h from the center to the rim is R. Therefore, by the parallel axis theorem,

$$I = MR^2 + MR^2 = 2MR^2 \tag{37.3.2}$$

in agreement with the previous result.

Example 37.3.2

Using the parallel axis theorem, find the moment of inertia of a rod of mass M and length L about an axis perpendicular to the rod and passing through one end.

Solution

From Table 37.2.1, the moment if inertia about an axis perpendicular to the rod and passing through the center of mass is $I_{\text{cm}} = \frac{1}{12}ML^2$. The distance between an axis passing through the center of mass and an axis passing through one end is $h = L/2$. Therefore, by the parallel axis theorem, we have

$$I = I_{\text{cm}} + Mh^2 \tag{37.3.3}$$

$$= \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 \tag{37.3.4}$$

$$= \left(\frac{1}{12} + \frac{1}{4}\right)ML^2 \tag{37.3.5}$$

$$= \frac{1}{3}ML^2, \tag{37.3.6}$$

in agreement with the result in Table 37.2.1.