57.4: Orbital Elements

Now suppose that we want to describe the orbit of one body around another: for example, the Moon around the Earth, or the planet Saturn around the Sun. We first choose an appropriate reference frame, and then we need to describe the orbit. The orbit is specified using a set of seven numbers called the orbital elements of the orbit, which are described here.

Figure $57.4.1$ shows a typical orbit and reference frame. In this figure, the xy-plane is the reference frame (either the equator or the ecliptic), and the x direction is the reference direction (the vernal equinox). The orbit plane intersects the reference plane along a line called the line of nodes. The point where the orbiting body moves from below the reference plane to above the reference plane is called the ascending node, and is marked N in Fig. 1. The opposite point on the line of nodes, where the body moves from above the reference plane to below is called the descending node.

The seven orbital elements are summarized in the table below, and illustrated in Figure $57.4.1$.

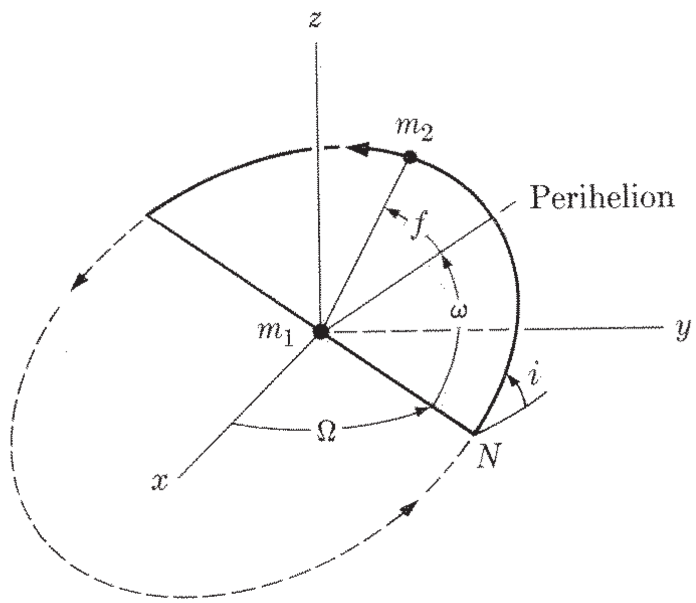

Figure 57.4.1: Orbital elements for a body of mass m_2 orbiting a body of mass m_1. The xy-plane is the reference plane, and x is the direction of the vernal equinox. Shown are the orbital elements i, Ω, and ω, along with the true anomaly f. Point N is the ascending node of the orbit. (From McCuskey, 1963 [12].)

The point of closest approach of the orbiting body to the center body is called the pericenter, and the point of farthest approach is called the apocenter. In the case where the body is orbiting the Earth, these are called the perigee and apogee (respectively); when the body is orbiting the Sun, these points are called the perihelion and aphelion (respectively). The line connecting the pericenter to the apocenter is called the line of apsides.

Now to the orbital elements. First, we need to specify the size of the orbit. Bodies in closed orbits always orbit in ellipses, where the body being orbited is at one of the two foci of the ellipse. The size of the orbit is specified by giving the semi-major axis a of the ellipse.

Second, we need to specify the shape of the orbit. We do this by specifying the eccentricity e of the ellipse. The eccentricity is a number between 0 and 1 , and is measure of how elongated the ellipse is: $e=0$ for a circle, and values of e close to 1 are long, cigar-shaped ellipses. The eccentricity e is related to the semi-major axis a and semi-minor axis b of the ellipse by

$$e=\frac{\sqrt{a^2-b^2}}{a} \tag{57.4.1}$$

Next, we need to specify the orientation of the orbit in space. This requires three angles: (1) the inclination i of the orbit with respect to the reference plane; (2) the longitude of the ascending node Ω, which is the angle between the vernal equinox and the ascending node, measured in the reference plane; and (3) the argument of pericenter ω, which is the angle between the ascending node and the orbit pericenter, measured in the plane of the orbit. These three angles are illustrated in Fig. $57.4.1$.

Now we've completely specified the orbit itself, but we need one more bit of information: where the body is in this orbit. This requires two numbers: an angle, and a time at which the body is at that angle. The angle is called the mean anomaly at epoch M_0, and gives the angle from the pericenter to the body (measured in the plane of the orbit) at a specified epoch time T_0.

Table $57.4.1$ Orbital elements.

Element	Symbol
Semi-major axis	a
Eccentricity	e
Inclination	i
Longitude of ascending node	Ω
Argument of pericenter	ω
Mean anomaly at epoch	M_0