52.9: Fluid Flow through a Pipe

If a viscous fluid is flowing through a pipe, then there is an additional term called the friction head introduced into Bernoulli's equation:

$$\frac{P}{\rho g}+f\frac{L}{D}\frac{v^2}{2g}+\frac{v^2}{2g}+y=\text{ constant} \tag{52.9.1}$$

where the second term on the left is the friction head; f is a dimensionless constant called the friction factor, [1] L is the pipe length, D is the pipe diameter, and v is the average fluid velocity (the fluid will flow faster at the center of the pipe than near the edges).

For laminar flow, the friction factor f is given simply by

$$f=\frac{64}{\mathrm{Re}}\qquad(\text{laminar flow}), \tag{52.9.2}$$

where Re is the Reynolds number. For a nonviscous fluid, the viscosity $\mu=0$, the Reynolds number $\mathrm{Re}=\infty$, and so $f=0$, so that Eq. $52.9.1$ reduces to the previous form of Bernoulli's equation, Eq. 52.3.1.

For turbulent flow, the analysis to find the friction factor is more complicated and depends on the Reynolds number and the ratio of the pipe surface roughness to pipe diameter. There is a general formula due to S.W. Churchill that gives the friction factor f for all values of Reynolds numbers and all types of flow (laminar, transitional, and turbulent) through both rough and smooth pipes. Churchill's equation (as modified by L.L. Simpson to produce accurate results for turbulent flow) is

$$f=\left|\left(\frac{64}{\mathrm{Re}}\right)^{12}+\left\{\left[2\log_{10}\left(\frac{\varepsilon}{3.7D}-\frac{5.02}{\mathrm{Re}}\log_{10}\left(\frac{\varepsilon}{3.7D}+\left(\frac{7}{\mathrm{Re}}\right)^{0.9}\right)\right)\right]^{16}+\left(\frac{13269}{\mathrm{Re}}\right)^{16}\right\}^{-3/2}\right|^{1/12} \tag{52.9.3}$$

where ε is the pipe roughness and D is the pipe diameter. The friction factor vs. Reynolds number is shown in Figure $52.9.1$

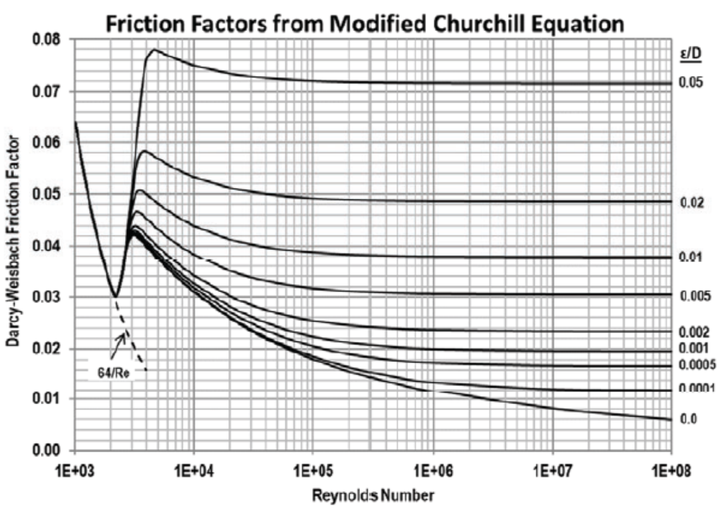

Figure 52.9.1: Friction factor as a function of Reynolds number, for both laminar and turbulent flow. (Ref. [11])

[1] Sometimes f is called the Moody friction factor, Weisbach friction factor, or Darcy friction factor. One sometimes also encounters the Fanning friction factor equal to $f/4$, and the Stanton friction factor equal to $f/8$. The Moody friction factor used here is the most common.