# 32: The Ballistic Pendulum

How fast does a bullet travel when it leaves the barrel of a rifle? To measure the speed of a bullet, you might imagine an elaborate setup with high-precision timing and stop-action photography, but there's a much simpler method using the ballistic pendulum (Fig. 32.1).

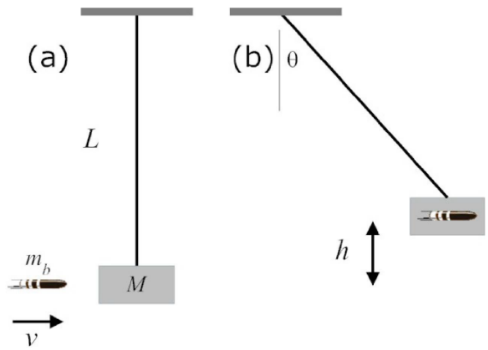


Figure 32.1: The ballistic pendulum. (a) Just before the collision, the pendulum is vertical and at rest; the bullet is moving at speed $v$. (b) After the collision, the bullet has embedded itself into the block. The final position of the pendulum is at an angle $\theta$ from the vertical; the block has moved a vertical height $h$.

The bullet is fired into a wooden block that forms the bob of a pendulum, as shown. The bullet becomes embedded in the block, and the bullet-block combination swings up and is held in its final position with a ratcheting mechanism. The initial speed of the bullet $v$ can then be determined from the angle $\theta$.

Let's determine the relationship between the bullet's initial speed $v$ and the angle $\theta$. First, the bullet embeds itself into the wooden block; this is a perfectly inelastic collision so the speed $v_0$ of the block-bullet combination just after the bullet hits the block is given by Eq. (31.2.4):

$$v_0 = \frac{m_b v + M(0)}{M + m_b} = \frac{m_b v}{M + m_b} \tag{32.1}$$

This relation comes from the conservation of momentum. [1] As the pendulum is hanging vertically, its energy is all kinetic; the pendulum will begin swinging upward, gradually converting its kinetic energy into potential energy until it reaches its maximum height at angle $\theta$, where it is held in place. The block's initial kinetic energy is

$$K_0 = \frac{1}{2}(M + m_b)\, v_0^2 \tag{32.2}$$

$$= \frac{1}{2}(M + m_b)\left(\frac{m_b v}{M + m_b}\right)^2 \tag{32.3}$$

$$= \frac{m_b^2 v^2}{2\,(M + M_b)} \tag{32.4}$$

All of this kinetic energy goes into raising the block-bullet combination by a height $h$, so by conservation of energy,

$$K_0 = U \tag{32.5}$$

$$\frac{m_b^2 v^2}{2\,(M + m_b)} = (M + m_b)\, gh \tag{32.6}$$

Solving for the bullet speed $v$, we find

$$v = \frac{M + m_b}{m_b}\sqrt{2gh} \tag{32.7}$$

Now from geometry, the height $h$ is given in terms of the pendulum length $L$ and the angle $\theta$ by

$$h = L - L\cos\theta = L(1-\cos\theta). \tag{32.8}$$

Substituting this into Eq. $32.7$, we have the initial speed $v$ of the bullet:

$$v = \left(\frac{M}{m_b} + 1\right)\sqrt{2gL(1-\cos\theta)} \tag{32.9}$$

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[1] We cannot use the conservation of energy at this point, because some of the bullet's initial kinetic energy is converted into heat. Using conservation of energy would require knowing things like the increase in the temperature of the block, which we don't know.

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