42.2: Energy

The kinetic energy K of a particle of mass m moving with speed v is defined to be the work required to accelerate the particle from rest to speed v; this is found to be

$$K=\frac{1}{2} m v^2. \tag{42.2.1}$$

From Hooke's law, the potential energy U of a simple harmonic oscillator particle at position x can be shown to be

$$U=\frac{1}{2} k x^2 \tag{42.2.2}$$

The total mechanical energy $E=K+U$ of a simple harmonic oscillator can be found by observing that when $x=\pm A$, we have $v=0$, and therefore the kinetic energy $K=0$ and the total energy is all potential. Since the potential energy at $x=\pm A$ is $U=k A^2 / 2$ (by Eq.$42.2.12$ the total energy must be

$$E=\frac{1}{2} k A^2 \tag{42.2.3}$$

Since total energy is conserved, the energy E is constant and does not change throughout the motion, although the kinetic energy K and potential energy U do change.

In a simple harmonic oscillator, the energy sloshes back and forth between kinetic and potential energy, as shown in Fig. $42.2.1$ At the endpoints of its motion $(x=\pm A)$, the oscillator is momentarily at rest, and the energy is entirely potential; when passing through the equilibrium position $(x=0)$, the energy is entirely kinetic. In between, kinetic energy is being converted to potential energy or vice versa.

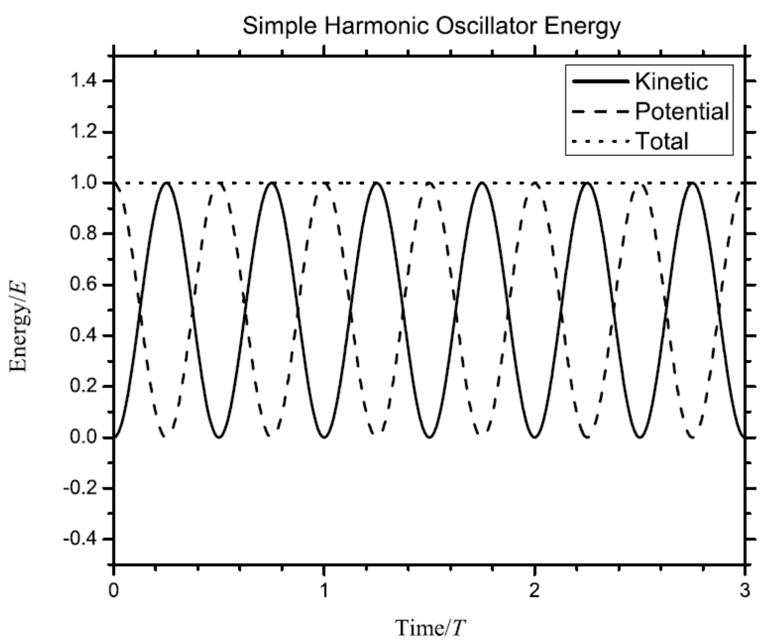

Figure 42.2.1: Kinetic, potential, and total energy of the simple harmonic oscillator as a function of time. The oscillator continuously converts potential energy to kinetic energy and back again, but the total energy E remains constant.

We can find the velocity v of a simple harmonic oscillator as a function of position x (rather than time t) by writing an expression for the conservation of energy:

$$E=K+U \tag{42.2.4}$$

$$\frac{1}{2} k A^2=\frac{1}{2} m v^2+\frac{1}{2} k x^2 \tag{42.2.5}$$

Solving for v, we find

$$v(x)=\pm A\sqrt{\frac{k}{m}}\sqrt{1-\frac{x^2}{A^2}} \tag{42.2.6}$$

This can be simplified somewhat by using Eq. (39.10) to give

$$v(x)=\pm A\omega\sqrt{1-\frac{x^2}{A^2}} \tag{42.2.7}$$

where $A\omega$ is, by inspection of Eq. (39.1.10), the maximum speed of the oscillator (the speed it has while passing through the equilibrium position).