30: Impulse

When two bodies move toward each other until they come into direct contact, the event it called a collision. The time during which the two bodies are in direct contact with one another [1] is actually quite short, and during that short time the force between the bodies is very large (Fig. 30.1). We can characterize such a collision by the impulse I, which is defined as the area under the force vs. time curve:

$$I=\int F(t) d t \tag{30.1}$$

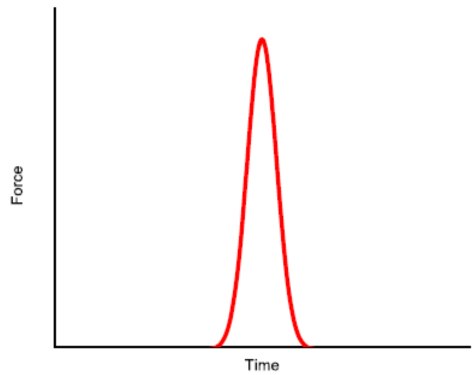

Figure 30.1: Force between two colliding bodies vs. time. There is a large force between the bodies, but it lasts only for a short time. The area under the curve is the impluse I.

The SI units of impulse are newton-seconds (Ns).

The impulse is closely related to the average force between the bodies during the collision. Recall from the calculus that the average of a function $f(x)$ over the interval $x=a$ to $x=b$ is

$$\overline{f(x)}=\frac{1}{b-a} \int_a^b f(x) d x \tag{30.2}$$

Therefore the average of a force $F(t)$ over the time interval $t=t_1$ to $t=t_2$ is given by

$$F_{\text {ave }}=\frac{1}{t_2-t_1} \int_{t_1}^{t_2} F(t) d t . \tag{30.3}$$

The integral is just the impulse I. Writing $\Delta t=t_2-t_1$, we then have the average force $F_{\text {ave }}$ as

$$F_{\text {ave }}=\frac{I}{\Delta t} \tag{30.4}$$

There is a close relationship between impulse and momentum. Recall by Newton's second law (Eq. 29.2.1) that $F=d p / d t$; substituting this into Eq. 30.1 gives

$$I=\int F d t=\int \frac{d p}{d t} d t=\int d p=\Delta p \tag{30.5}$$

and so the impulse is just the change in momentum of the body during the collision:

$$I=\Delta p \tag{30.6}$$

For many collision problems, the large force at work during the collision is so much larger than other forces present (friction, etc.) that the other forces can be neglected. Also, the duration Δt of the collision is so short that the motion of the bodies during the collision can be neglected; in other words, we can consider the collision to be essentially instantaneous. These assumptions together are referred to as the impulse approximation.

Example 30.1

Suppose you hit a golf ball with a driver, giving it an initial velocity of 134 miles per hour. The club is in contact with the ball for 0.5 ms. What is the average force of the club on the ball?

Solution

From the change in the ball's momentum we can find the impulse, and from the impulse we can find the average force. The ball is initially at rest, so its initial momentum is zero. After being hit by the driver, the ball has a velocity of 134 miles per hour $= 59.9 \text{ m/s}$. The mass of a golf ball is 45.0 g, so its final momentum is $(59.9 \text{ m/s})(0.045 \text{ kg}) = 2.6955 \text{ kg m/s}$ Therefore the impulse for the collision is

$$I = \Delta p = p_f - p_i = 2.6955 \text{ kg m/s} - 0 \tag{30.7}$$

$$= 2.6955 \text{ kg m/s} \tag{30.8}$$

Then by Eq. 30.4, the average force during the collision is

F_{\text {ave }} & =\frac{I}{\Delta t}

$$= \frac{2.6955 \text{ kg m/s}}{0.5 \times 10^{-3} \text{ s}} \tag{30.9}$$

$$= 5391 \text{ N} \tag{30.10}$$

$$= 1212 \text{lbf}. \tag{30.11}$$

[1] Two colliding bodies normally never come into direct contact with each other; rather the outermost electrons in their outermost atomic layers repel each other under the electrostatic force.