54.1: Newton’s Law of Gravity

The English physicist Sir Isaac Newton developed his theory of the gravitational force in his famous work Philosophice Naturalis Principia Mathematica. In modern language and notation, it states that the force F between two point masses m_1 and m_2 separated by a distance r is given by

$$F=-G\frac{m_1 m_2}{r^2} \tag{54.1.1}$$

where G is the universal gravitational constant, $6.67430\times 10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}$. Here we take the usual convention in one dimension, where a negative force is attractive, and a positive force is repulsive. Since mass is always positive, the gravitational force is always attractive.

In vector form, Newton's law of gravity becomes

$$\mathbf{F}_{12}=G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}_{12} \tag{54.1.2}$$

where $\mathbf{F}_{12}$ is the force on mass 1 due to mass 2, and $\hat{\mathbf{r}}_{12}$ is a unit vector pointing from mass 1 to mass 2 .

From Newton's law of gravity, we can deduce the acceleration due to gravity at the Earth's surface. The gravitational force between the Earth of mass $M_\oplus$ and an object on the surface of mass m is (in magnitude)

$$F=G\frac{M_\oplus m}{R_\oplus^2} \tag{54.1.3}$$

where $R_\oplus$ is the radius of the Earth. By Newton's second law, the gravitational force on m at the Earth's surface is $F=ma=mg$, so $g=F/m$, and we have

$$g=\frac{GM_\oplus}{R_\oplus^2}=9.8\ \mathrm{m/s}^2 \tag{54.1.4}$$