44.3: Kinetic Energy

As a body rolls down an incline, its potential energy is converted partly into translational kinetic energy, and partly into rotational kinetic energy. How much goes into translational kinetic energy, and how much into rotational form?

First, let's compute the translational kinetic energy, $K_t = Mv^2/2$. Using Eq. 44.1.12 to substitute for v gives

$$K_t = \frac{1}{2}Mv^2 = \frac{1}{2}M\left(\frac{2gh}{\beta+1}\right) \tag{44.3.1}$$

or

$$K_t = \frac{Mgh}{\beta+1} \tag{44.3.2}$$

Now let's find the rotational kinetic energy, $K_r = I_{\text{cm}}\omega^2/2$. Using $\omega = v/R$,

$$K_r = \frac{1}{2}I_{\text{cm}}\left(\frac{v}{R}\right)^2 \tag{44.3.3}$$

Again using Eq. 44.1.12 to substitute for v,

$$K_r = \frac{1}{2}\frac{I_{\text{cm}}}{R^2}\frac{2gh}{\beta+1} \tag{44.3.4}$$

Multiplying the numerator and denominator by M,

$$K_r = \frac{I_{\text{cm}}}{MR^2}\frac{Mgh}{\beta+1} \tag{44.3.5}$$

The first factor on the right is just β, so we finally have for the rotational kinetic energy

$$K_r = \text{Mgh}\left(\frac{\beta}{\beta+1}\right) = \beta K_t \tag{44.3.6}$$

Knowing that the total kinetic energy is $K = Mgh$, we can now use Eqs. $44.3.2$ and $44.3.6$ to find the ratio of the translational kinetic energy to the total kinetic energy:

$$\frac{K_t}{K} = \frac{1}{\beta+1} \tag{44.3.7}$$

Similarly, the ratio of the rotational to total kinetic energy is given by

$$\frac{K_r}{K} = \frac{\beta}{\beta+1} \tag{44.3.8}$$

Values of these ratios for common body geometries are shown in Table $44.3.1$. It is interesting to note that substituting $\beta = 0$ into the formulæ we've derived here recovers the formulæ for an object sliding down an incline without rolling, as shown in the last line of the table.

Table $44.3.1$: Accelerations and energy ratios for rolling bodies.

Body	β	a	K_t/K	K_r/K
Cylindrical shell	1	$(1/2)g\sin\theta$	1/2	1/2
Solid cylinder	1/2	$(2/3)g\sin\theta$	2/3	1/3
Spherical shell	2/3	$(3/5)g\sin\theta$	3/5	2/5
Solid sphere	2/5	$(5/7)g\sin\theta$	5/7	2/7
Sliding object	0	$g\sin\theta$	1	0