19.3: The Catenary

Consider a chain elevated above ground, attached only at its two ends, both ends at the same height, and hanging under its own weight. The chain will sag, forming a hyperbolic cosine curve called a catenary. With a coordinate system defined as shown in Figure $19.3.1$, the equation of the catenary is found to be

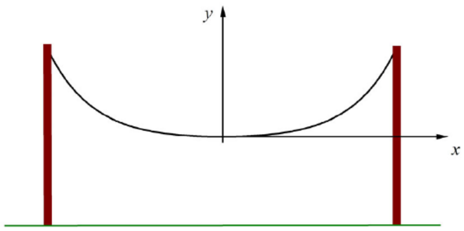

Figure 19.3.1: A chain hanging under its own weight, forming a catenary curve.

$$y=a\cosh\left(\frac{x}{a}\right)-a \tag{19.3.1}$$

where $a=H/w, H$ is the horizontal tension in the chain at the pole (in newtons), and w is the linear weight density of the chain (in newtons per meter).

The arc length s of the catenary from $x=0$ to x is given by

$$s(x)=a\sinh\left(\frac{x}{a}\right) \tag{19.3.2}$$

so that if the poles are separated by a distance d, the total arc length s_t is

$$s_t=2a\sinh\left(\frac{d}{2a}\right) \tag{19.3.3}$$

Note that if the horizontal tension H is very large (the chain is pulled very taut), then $a=H/w$ is very large, $d/2a$ is very small, and so $\sinh(d/2a)\approx d/2a$, so that $s_t\approx d$, as expected.