21.1: Horizontal Block and Vertical Block

Consider the following problem: a block of mass m_1 is on a frictionless horizontal surface, and connected by a string, through a pulley, to a mass m_2 hanging vertically (Figure $21.1.1$). (We assume the string is unbreakable, unstretchable, and of negligible mass.) What is the acceleration of the system?

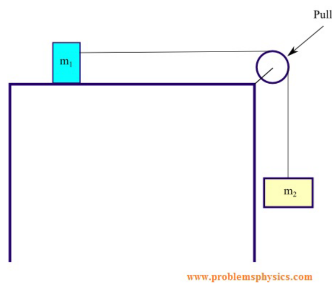

Figure $21.1.1$: Horizontal and vertical pulley connected by a string.

First, we recognize that the block m_1 will accelerate to the right, and block m_2 downward, with the same acceleration a, since the two blocks are tied together. Next, consider the forces on block m_1 : it has a weight $m_1 g$, and is acted upon by a normal force, also of magnitude $m_1 g$, so that the net force in the vertical direction is zero. This is as expected, since the block is not accelerating in the vertical direction. In the horizontal direction, the only force acting on m_1 is the string tension T. Thus for m_1, Newton's second law gives, in the horizontal direction,

$$\Sigma_i F_i = m_1 a \quad \Rightarrow \quad T = m_1 a \tag{21.1.1}$$

There are no horizontal forces acting on mass m_2, but there are two vertical forces: the upward tension T (equal to the tension acting on m_1) and the downward weight force mg. Then Newton's second law for m_2, in the vertical (downward) direction, is

$$\Sigma_i F_i = m_2 a \quad \Rightarrow \quad m_2 g - T = m_2 a \tag{21.1.2}$$

This gives us two simultaneous equations in the two unknowns a and T. Adding the two equations will eliminate the tension T; we can then solve for the acceleration a to find

$$a = \frac{m_2}{m_1 + m_2} g \tag{21.1.3}$$

And then by Eq. $21.1.1$, the tension in the string is

$$T = \frac{m_1 m_2}{m_1 + m_2} g \tag{21.1.4}$$

Now let's consider the same problem, but this time we'll include friction acting on the horizontal block. In this case, Newton's second law for m_1 (Eq. $21.1.1$) will include a frictional force $f = \mu n = \mu m_1 g$ (where μ is the coefficient of (kinetic) friction) acting to the left, and becomes

$$\Sigma_i F_i = m_1 a \quad \Rightarrow \quad T - \mu m_1 g = m_1 a \tag{21.1.5}$$

Newton's second law applied to mass m_2 is the same as before:

$$\Sigma_i F_i = m_2 a \quad \Rightarrow \quad m_2 g - T = m_2 a \tag{21.1.6}$$

Adding these two equations to eliminate the tension T, we find the acceleration a to be

$$a = \frac{m_2 - \mu m_1}{m_1 + m_2} g \tag{21.1.7}$$

and the tension to be (using Eq. $21.1.5$),

$$T = \frac{(1+\mu) m_1 m_2}{m_1 + m_2} g \tag{21.1.8}$$

Notice that these last two equations reduce to their frictionless counterparts when $\mu = 0$.