52.3: Bernoulli's Equation

Bernoulli's equation was developed by 18th-century Swiss physicist Daniel Bernoulli. Given fluid flow in a pipe that varies in elevation, the equation relates the velocity, pressure, and elevation as the fluid flows through the pipe. It states

$$\frac{P}{\rho g}+\frac{v^2}{2g}+y=\text{ constant}, \tag{52.3.1}$$

where P is the pressure, v is the fluid velocity, y is elevation, ρ is the fluid density, and g is the acceleration due to gravity. Each term in Bernoulli's equation has units of length and is called a head: the $P/(\rho g)$ term is called the pressure head, the $v^2/(2g)$ term is called the velocity head, and the y term is called the elevation head.

Example 52.3.1

Suppose we have a vertical pipe containing a stationary incompressible fluid of density ρ. How does the pressure P vary with depth h ?

Solution

Let the pressure at depth $h=0$ be P_0. Since the fluid is stationary, the fluid velocity v is zero everywhere. Then Bernoulli's equation becomes (with $y=-h$)

$$\frac{P_0}{\rho g}+\frac{0}{2g}+0=\frac{P}{\rho g}+\frac{0}{2g}-h \tag{52.3.2}$$

$$\frac{P_0}{\rho g}=\frac{P}{\rho g}-h \tag{52.3.3}$$

$$P_0=P-\rho g h \tag{52.3.4}$$

P & =P_{0}+\rho g h,\]

in agreement with Eq. 51.4.5.