41.3: The Conical Pendulum

A conical pendulum is also similar to a simple plane pendulum, except that the pendulum is constrained to move along the surface of a cone, so that the mass m moves in a horizontal circle of radius r, maintaining a constant angle θ from the vertical.

For a conical pendulum, we might ask: what speed v must the pendulum bob have in order to maintain an angle θ from the vertical? To solve this problem, let the pendulum have length L, and let the bob have mass m. A general approach to solving problems involving circular motion like this is to identify the force responsible for keeping the mass moving in a circle, then set that equal to the centripetal force mv^2/r. In this case, the force keeping the mass moving in a circle is the horizontal component of the tension T, which is $T\sin\theta$. Setting that equal to the centripetal force, we have

$$T\sin\theta = \frac{mv^2}{r}. \tag{41.3.1}$$

The vertical component of the tension is

$$T\cos\theta = mg \tag{41.3.2}$$

Dividing Eq. $41.3.1$ by Eq. $41.3.2$,

$$\tan\theta = \frac{v^2}{gr} \tag{41.3.3}$$

From geometry, the radius r of the circle is $L\sin\theta$. Making this substitution, we have

$$\tan\theta = \frac{v^2}{gL\sin\theta} \tag{41.3.4}$$

Solving for the speed v, we finally get

$$v = \sqrt{Lg\sin\theta\tan\theta}. \tag{41.3.5}$$