12.2: Range

The first question we'll look at is: how far will the projectile go? This is called the range, and is shown as R in Fig. 9.1. How do we find this? We need to look at what conditions are unique to the problem we're trying to solve; in this case, what's unique about the range R is that it's the x coordinate of the projectile when $y=0$. So let's set $y=0$ in Eq. (12.1.8) and see what happens:

$$0=-\frac{1}{2} g t^{2}+\left(v_{0} \sin \theta\right) t \tag{12.2.1}$$

What we're after is the value of x when $y=0$, so let's try solving this for time t, then plugging that into Eq. (12.1.7). Solving Eq. $12.2.1$ for t by factoring out a t, we have [1]

$$0=\left[-\frac{1}{2} g t+v_{0} \sin \theta\right] t \tag{12.2.2}$$

This means that for $y=0$, either $t=0$ (which it is at launch), or else $-g t / 2+v_{0} \sin \theta=0$. The second case is the one we're interested in:

$$0=-\frac{1}{2} g t+v_{0} \sin \theta \tag{12.2.3}$$

or

$$t=\frac{2}{g} v_{0} \sin \theta \tag{12.2.4}$$

This is the total time the projectile is in the air, and is called the time in flight $\left(t_{f}\right)$. Substituting this time into Eq. (12.1.7) gives the range:

$$R=x\left(t_{f}\right)=\left(v_{0} \cos \theta\right)\left(\frac{2}{g} v_{0} \sin \theta\right) \tag{12.2.5}$$

Using the identity $\sin 2 \theta \equiv 2 \sin \theta \cos \theta$, this becomes

$$R=\frac{v_{0}^{2}}{g} \sin 2 \theta \tag{12.2.6}$$

A related question is: at what launch angle θ do you get the maximum range for a fixed muzzle velocity v_{0} ? Examining Eq. $12.2.6$, the largest value the sine can have is 1 , so

$$\sin 2 \theta=1 \tag{12.2.7}$$

$$2 \theta=90^{\circ} \tag{12.2.8}$$

$$\theta=45^{\circ} \tag{12.2.9}$$

So a projectile will go the farthest if launched at an angle of 45° from the horizontal. Another way to arrive at the same result is to use the first derivative test: Eq. $12.2.6$ gives $R(\theta)$, so to find the value of θ that gives the maximum range R, we set $d R / d \theta=0$:

$$\frac{d R}{d \theta}=\frac{d}{d \theta}\left(\frac{v_{0}^{2}}{g} \sin 2 \theta\right)=0 \tag{12.2.10}$$

or, using the chain rule,

$$\frac{2 v_{0}^{2}}{g} \cos 2 \theta=0 \tag{12.2.11}$$

$$\cos 2 \theta=0 \tag{12.2.12}$$

Now $\cos 2 \theta=0$ implies $2 \theta=90^{\circ}$ or $2 \theta=270^{\circ}$, and therefore $\theta=45^{\circ}$ or $\theta=135^{\circ}$. We discard the solution $\theta=135^{\circ}$ on physical grounds: it corresponds to pointing the cannon backwards at 45° from the horizontal, which is a solution we're not interested in.

1. Note that we cannot divide Eq. (9.9) by the variable t without losing roots. The proper procedure is to factor out a factor of t, then use the fact that if the product of two factors is zero, then one or both factors must be zero. It is OK to divide through by a nonzero constant, though.