16.2: Second Law of Motion

Newton's second law of motion states that the net force F on a body is proportional to its resulting acceleration a :

$$F = ma. \tag{16.2.1}$$

Figure 16.2.1: Sir Isaac Newton.

When a force F is applied to a body, it will accelerate with acceleration $a = F/m$-the larger the mass, the smaller the acceleration.

If the force F is a function of position, and using acceleration $a = d^2x/dt^2$, this becomes a differential equation

$$F(x) = m\frac{d^2x}{dt^2} \tag{16.2.2}$$

Solving this differential equation for $x(t)$ gives a complete description of the motion.

As we'll see later when we discuss momentum, the most general form of Newton's second law is not $F = ma$, but $F = dp/dt$, where p is momentum. This reduces to $F = ma$ when mass is constant.

In Newton's second law as given in Eq. $16.2.1$ is only its simple scalar form, and suitable for onedimensional problems. More generally, both force and acceleration are vectors, so that Newton's second law takes the form

$$\mathbf{F} = m\mathbf{a} \tag{16.2.3}$$

Here $\mathbf{F}$ is the net force on the body - that is, the vector sum of all the individual forces acting on it. We might write this more explicitly as

$$\sum_i \mathbf{F}_i = m\mathbf{a} \tag{16.2.4}$$

In other words, the vector sum of all the forces acting on a body equals its mass times the resulting acceleration. This vector formula is really a shorthand for writing three scalar formulas. Taking the x, y, and z components of both sides of Eq. $16.2.4$, we get

$$x: \qquad \sum_i F_{xi} = ma_x \tag{16.2.5}$$

$$y: \qquad \sum_i F_{yi} = ma_y \tag{16.2.6}$$

$$z: \qquad \sum_i F_{zi} = ma_z \tag{16.2.7}$$

(Of course, we omit the z equation when working in only two dimensions.) We'll see some examples of the use of these equations shortly.