33.2: Mass Fraction

The fraction of the total initial mass m that is propellant is

$$\frac{\text{propellant mass}}{\text{total initial mass}} = \frac{m - m_e}{m} = 1 - \frac{m_e}{m}. \tag{33.2.1}$$

Solving Eq. (30.1.7) for this fraction, we find

$$1 - \frac{m_e}{m} = 1 - e^{-\Delta v / v_p} \tag{33.2.2}$$

Eq. $33.2.2$ tells what fraction of the rocket's total mass must be fuel in order to achieve a desired change in rocket velocity Δv.

Example 33.2.1

Let's take as an example the launch of a rocket from the Earth's surface to low-Earth orbit.

Solution

In this case, the rocket's velocity will need to change by an amount $\Delta v = 17,000 \text{mph}$, or about $7600\ \text{m/s}$. Let's say we have a rocket that can expel propellant with a speed $v_p = 4000\ \text{m/s}$. Then by Eq. $33.2.2$,

$$1 - \frac{m_e}{m} = 1 - e^{-\Delta v / v_p} = 0.85, \tag{33.2.3}$$

so 85% of the rocket's initial mass must be propellant.