12.8: The Monkey and the Hunter Problem

A famous problem involving projectile motion is the "monkey and hunter problem" (Fig. 12.8.1). A hunter spots a monkey hanging from a tree branch, aims his rifle directly at the monkey, and fires. The monkey, hearing the shot, lets go of the branch at the same instant the hunter fires the rifle, hoping to escape by falling to the ground. Will the monkey escape? The unexpected answer is "no": the bullet will always hit the monkey anyway, regardless of the angle of the rifle, the speed of the bullet, or the distance to the monkey, as long as the monkey is in range.

To show that this is so, let's first define a coordinate system. Let the origin be at the end of the rifle, with the x axis pointing horizontally to the right, and y pointing vertically upward. Let D be the horizontal distance of the monkey from the origin, and H be the initial height of the monkey (Fig. 12.8.1).

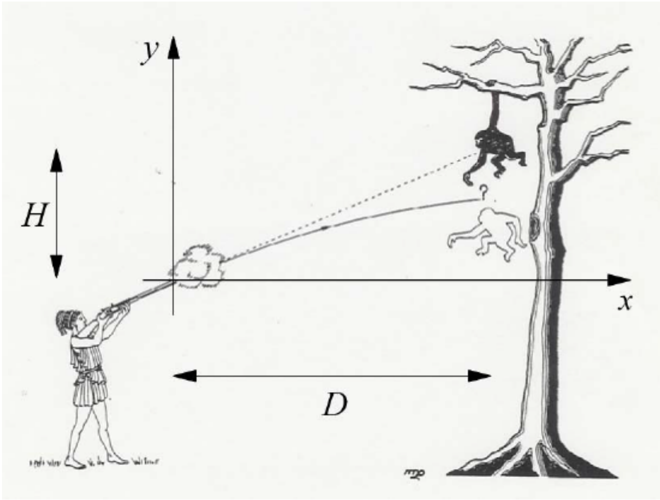

Figure 12.8.1: The “monkey and hunter” problem. (Credit: The English School, Fahaheel, Kuwait.)

Now we'll show that at $x = D$, the monkey and the bullet will both be at the same height y. Let the muzzle velocity of the rifle be v_0, and let the angle of fire of the rifle from the horizontal be θ. Let's begin by finding the time t needed for the bullet to travel the horizontal distance D from the rifle. Since the horizontal component of the velocity is a constant $v_0 \cos\theta$ (Eq. 12.1.7), the x coordinate of the bullet at time t is

$$x_b(t) = (v_0 \cos\theta)\, t \tag{12.8.1}$$

Setting $x_b(t) = D$ and solving for the time t, we find (calling this time t_f)

$$t_f = \frac{D}{v_0 \cos\theta} \tag{12.8.2}$$

Next, let's find the y coordinate of the bullet at this time t_f. By Eq. (12.1.8)

$$y_b(t) = -\frac{1}{2} g t^2 + (v_0 \sin\theta)\, t. \tag{12.8.3}$$

Substituting $t = t_f = D/\left(v_0 \cos\theta\right)$, we have

$$\begin{aligned} y_b &= -\frac{1}{2} g \left(\frac{D}{v_0 \cos\theta}\right)^2 + (v_0 \sin\theta)\left(\frac{D}{v_0 \cos\theta}\right) \\ &= -\frac{g D^2}{2 v_0^2 \cos^2\theta} + D \tan\theta. \end{aligned}$$

But from trigonometry, $\tan\theta = H/D$; making this substitution in the second term on the right, we have

$$y_b = H - \frac{g D^2}{2 v_0^2 \cos^2\theta} \quad \text{(bullet)}. \tag{12.8.4}$$

Finally, let's find the y coordinate of the monkey at time t_f. The monkey falls in one dimension; its y coordinate at time t is (using Eq. (8.7.11) with y instead of x, and with $a=-g, v_0=0$ and $y_0=H$):

$$y_m(t)=H-\frac{1}{2} g t^2. \tag{12.8.5}$$

Now substituting $t=t_f=D/\left(v_0 \cos \theta\right)$,

$$y_b=-\frac{1}{2} g\left(\frac{D}{v_0 \cos \theta}\right)^2+\left(v_0 \sin \theta\right)\left(\frac{D}{v_0 \cos \theta}\right) \tag{12.8.6}$$

$$=-\frac{g D^2}{2 v_0^2 \cos ^2 \theta}+D \tan \theta. \tag{12.8.7}$$

Comparing Eqs. $12.8.6$ and $12.8.9$, you can see that the monkey and bullet will have the same y coordinate when $x=D$, so the monkey will always get hit, regardless of the values of D, H, v_0, or θ. Q.E.D. [1]

Essentially what's happening here is that the monkey and bullet are both accelerated by the same amount, $g=9.8 \mathrm{~m} / \mathrm{s}^2$, so for a given amount of time, the monkey will fall the same distance as the bullet falls from the straight-line path it would take if there were no gravity. Therefore, the bullet always hits the monkey.

[1] Q.E.D. is an abbreviation for the Latin phrase quod erat demonstrandum, meaning "which was to be demonstrated."