58.2: Geosynchronous Orbits

Consider the motion of an artificial satellite in a circular orbit of radius r around the Earth. In order to be orbiting at radius r, it will have orbital speed v given by setting the centripetal force equal to the gravitational force:

$$\frac{mv^2}{r} = \frac{GM_\oplus m}{r^2} \tag{58.2.1}$$

where G is Newton's gravitational constant and $M_\oplus$ is the mass of the Earth. Solving for the orbital velocity v,

$$v = \sqrt{\frac{GM_\oplus}{r}} \tag{58.2.2}$$

Notice the one-to-one correspondence between r and v : for each orbital radius r there is a specific orbital velocity v for any object in that orbit.

The period T is the time required to complete one orbit, and is equal to the length of one orbit $2\pi r$ divided by the orbit velocity v :

$$T = \frac{2\pi r}{v}. \tag{58.2.3}$$

Using Eq. $58.2.2$ to substitute for v, we find the period of an orbit at radius r to be

$$\begin{aligned} T &= \frac{2\pi r}{v} \\ &= 2\pi r\sqrt{\frac{r}{GM_\oplus}} \\ &= 2\pi\sqrt{\frac{r^3}{GM_\oplus}} \end{aligned}$$

This shows that at any given orbital radius r, there is a specific orbital period for a body in a circular orbit of that radius.

Now suppose an artificial satellite is orbiting directly above the Earth's equator, and in the same sense as the Earth's rotation (counterclockwise as seen from above the north pole). If the period T is 24 hours, the satellite will stay directly above the same point on the equator as it orbits the Earth, and will appear to "hover" above the Earth. Such an orbit is called a geosynchronous orbit.

The radius of a geosynchronous orbit can be found by solving Eq. $58.2.6$ for r :

$$\frac{T^2}{4\pi^2} = \frac{r^3}{GM_\oplus} \tag{58.2.4}$$

or

$$r = \sqrt[3]{\frac{GM_\oplus T^2}{4\pi^2}} \tag{58.2.5}$$

Now setting $T = 24 \text{ hours} = 86400 \text{sec}$ and $GM_\oplus = 3.986005 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$, we find the radius of a geosynchronous orbit to be

$$r = 42,241 \text{ km} = 6.62 R_\oplus, \tag{58.2.6}$$

where $R_\oplus = 6378.140 \text{ km}$ is the equatorial radius of the Earth. The altitude of a geosynchronous orbit is

$$r - R_\oplus = 35,863 \text{ km} = 22,284 \text{ miles}. \tag{58.2.7}$$

(This number is the origin of the address of the former COMSAT Laboratories: 22300 Comsat Drive, Clarksburg, Maryland.)

Geosynchronous orbits are often used for communications satellites and satellite television. Since the satellites appear to hover over the equator, the satellite antenna dish need only be pointed at the satellite once; the satellite will not move appreciably from the point of view of the observer. Three geosynchronous satellites placed over the equator 120° in longitude apart are sufficient to cover the whole Earth (except for regions near the poles).

Some people have proposed the construction of space elevators to move people and cargo into space. A strong light cable would connect a geosynchronous satellite to the surface of the Earth, and elevator cars would move up and down the cable. The technology necessary to construct a space elevator is still some distance in the future, though.