# CHAPTER OVERVIEW

## 61: Hamiltonian Mechanics

Besides Lagrangian mechanics, another alternative formulation of Newtonian mechanics we will look at is Hamiltonian mechanics. In this system, in place of the Lagrangian we define a quantity called the Hamiltonian, to which Hamilton's equations of motion are applied. While Lagrange's equation describes the motion of a particle as a single second-order differential equation, Hamilton's equations describe the motion as a coupled system of two first-order differential equations.

One of the advantages of Hamiltonian mechanics is that it is similar in form to quantum mechanics, the theory that describes the motion of particles at very tiny (subatomic) distance scales. An understanding of Hamiltonian mechanics provides a good introduction to the mathematics of quantum mechanics.

The Hamiltonian $H$ is defined to be the sum of the kinetic and potential energies:

$$H \equiv K + U \tag{61.1}$$

Here the Hamiltonian should be expressed as a function of position $x$ and momentum $p$ (rather than $x$ and $v$, as in the Lagrangian), so that $H = H(x, p)$. This means that the kinetic energy should be written as $K = p^2/2m$, rather than $K = mv^2/2$.

Hamilton's equations in one dimension have the elegant nearly-symmetrical form

$$\frac{dx}{dt} = \frac{\partial H}{\partial p}$$
$$\frac{dp}{dt} = -\frac{\partial H}{\partial x}$$