12.3: Maximum Altitude

Let's look at another question: what is the maximum altitude reached by the projectile? Let's think about what is unique about the point where the projectile is at its maximum altitude: the y component of the velocity is momentarily zero at that point. Eq. (11.2.3) gives the velocity vector of the projectile at any time t :

$$\mathbf{v}(t) = \mathbf{a}t + \mathbf{v}_0 \tag{12.3.1}$$

$$= -gt\mathbf{j} + \mathbf{v}_0 \tag{12.3.2}$$

which is equivalent to the two scalar equations

$$v_x(t) = v_{x0} = v_0 \cos\theta \tag{12.3.3}$$

$$v_y(t) = -gt + v_{y0} = -gt + v_0 \sin\theta \tag{12.3.4}$$

To find the maximum altitude, we want to set $v_y = 0$:

$$0 = -gt + v_0 \sin\theta. \tag{12.3.5}$$

Solving for time t,

$$t = \frac{v_0}{g}\sin\theta. \tag{12.3.6}$$

This is the amount of time it takes the projectile to reach the point where $v_y = 0$, which is the point of maximum altitude. Note that this is half of the time in flight (Eq. (12.1.4)), so the projectile reaches its maximum height half-way through its flight. (You could also arrive at this same result by using Eq. (12.1.8) for $y(t)$, then setting $dy/dt = 0$ by the first derivative test.)

Plugging this time into Eq. (12.1.8) gives the maximum altitude h :

$$h = y\left(\frac{t_f}{2}\right) = -\frac{1}{2}g\left(\frac{v_0}{g}\sin\theta\right)^2 + (v_0\sin\theta)\left(\frac{v_0}{g}\sin\theta\right) \tag{12.3.7}$$

$$= -\frac{1}{2}\frac{v_0^2\sin^2\theta}{g} + \frac{v_0^2\sin^2\theta}{g} \tag{12.3.8}$$

so the maximum altitude is

$$h = \frac{v_0^2\sin^2\theta}{2g} \tag{12.3.9}$$