## 59.2: Higher-Order Partial Derivatives

It is similarly possible to take higher-order partial derivatives. For a function of two variables $f(x, y)$, there are three possible second derivatives:

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right); \quad \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right); \quad \text{and} \quad \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right). \tag{59.2.1}$$

In the second case, the order of differentiation doesn't matter: $\partial^2 f/(\partial x \partial y) \equiv \partial^2 f/(\partial y \partial x)$. This property is known as Clairaut's theorem.

For example, suppose $f(x, y)$ is as given by Eq. 59.1.3. Then the second partial derivatives of $f$ are found by taking partial derivatives of Eqs. 59.1.4 and 59.1.5:

$$\frac{\partial^2 f}{\partial x^2} = 30xy^5 \tag{59.2.2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = 75x^2y^4 - 42y^5 \tag{59.2.3}$$

$$\frac{\partial^2 f}{\partial y^2} = 100x^3y^3 + 8 - 210xy^4 \tag{59.2.4}$$