57.10: The Vis Viva Equation

When an object of small mass m orbits a body of much larger mass M, we can use conservation of energy considerations to find the smaller body's velocity v at radial distance r. We have for the small body m :

$$K=\frac{1}{2}mv^2 \text{ (kinetic energy)} \tag{57.10.1}$$

$$U=-\frac{GMm}{r} \text{ (potential energy)} \tag{57.10.2}$$

$$E=-\frac{GMm}{2a} \text{ (total energy)} \tag{57.10.3}$$

where the quantity a is the radius for a circular orbit, the semi-major axis for an elliptical orbit, the negative of the semi-major axis for a hyperbolic orbit, or infinity for a parabolic orbit.

By conservation of energy,

$$E=K+U \tag{57.10.4}$$

$$-\frac{GMm}{2a}=\frac{1}{2}mv^2-\frac{GMm}{r}. \tag{57.10.5}$$

Solving for the orbit speed v, we find

$$v=\sqrt{GM\left(\frac{2}{r}-\frac{1}{a}\right)} \tag{57.10.6}$$

This result is known as the vis viva equation (Latin for "live force").

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