PHYSICAL SCIENCE FOR EDUCATORS (CID: PHYS 14)

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Physical Science for Educators (CID: PHYS 14)

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CHAPTER OVERVIEW

1: Elemental Beginnings- Foundations of Physics and Chemistry

This lesson is based on Chapter 1 of the OpenStax College Physics textbook. Each section of the chapter will include my lecture slides as well as an embedded link to my YouTube video for that lecture.

1.1: Introduction and Learning Objectives

Chapter Objectives

Learning Objectives

- Explore Chemistry in Context: Analyze the historical development of chemistry, from its early practices in alchemy to its current role as a central science in various fields, highlighting the interconnectedness of chemistry with other STEM disciplines.
- Understanding the Scientific Method: Recognize the key components of the scientific method, including observation, hypothesis formulation, experimentation, and analysis, and how they contribute to scientific inquiry.
- Differentiating Observations: Distinguish between qualitative and quantitative observations, understanding their roles in forming hypotheses and conducting experiments.
- Formulating Hypotheses: Develop the skill to create testable hypotheses based on observations, understanding their importance as a fundamental step in the scientific method.
- Designing and Conducting Experiments: Learn to design controlled experiments to test hypotheses, focusing on the manipulation of independent variables and measurement of dependent variables.
- Interpreting Experimental Results: Develop the ability to analyze experimental data, determining whether to accept, reject, or modify hypotheses based on empirical evidence.
- Advancing Hypotheses to Theories and Laws: Understand the process by which repeated experimental validation of hypotheses leads to the development of scientific theories and laws.
- Evaluating Science in Media: Critically assess scientific information presented in media, distinguishing between sound science and "junk science," and recognizing the influence of political or commercial interests.
- Identifying and Avoiding Pseudoscience: Learn to recognize pseudoscience and understand its distinction from legitimate scientific inquiry, emphasizing the importance of controlled and interpretable experiments.
- Applying the Scientific Method in Real-World Contexts: Explore the application of the scientific method in real-world situations, including observational and manipulative experiments across various scientific fields.
- Scientific Literacy and Critical Thinking: Promote scientific literacy and critical thinking, empowering educators to teach the scientific method as a tool for understanding and investigating the natural world.

1.2: Chemistry and Physics in Context

Throughout human history, our relentless curiosity has driven us to understand the world around us, leading to the exploration and cataloging of a vast array of phenomena. From the basic actions of chipping flint in the Stone Age to the complex processes of isolating chemical substances and creating metallic alloys, humans have sought to manipulate matter to better understand and utilize it. This journey of discovery extends from the mundane to the cosmic—from the flight of birds and the colors of flowers to the clusters of galaxies and the fundamental questions of the universe's creation.

1.1.1: This portrayal shows an alchemist’s workshop circa 1580. Although alchemy made some useful contributions to how to manipulate matter, it was not scientific by modern standards. (credit: Chemical Heritage Foundation).

A sketch depicts 4 people stirring and handling chemicals. The chemicals are held in a variety of barrels and large cylinders. Several of the containers are being heated over burning embers. A large stove in the laboratory is filled with burning embers. There is also a large chest in the corner that is producing steam.

Physics and chemistry, as pivotal scientific disciplines, delve into these phenomena by explaining the interactions of energy, matter, space, and time. They uncover the surprisingly simple and unified physical laws that underpin the apparent complexity of the universe. For example, the law of conservation of energy connects seemingly unrelated concepts such as food calories, batteries, car engines, and electric vehicles, illustrating how energy transforms from one form to another without loss. This principle is a cornerstone of both physics and chemistry, illuminating everything from the metabolic processes in biology to the energy transformations in chemical reactions and technological devices.

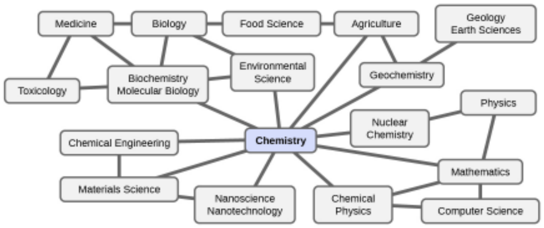

Figure 1.1.2 Knowledge of chemistry is central to understanding a wide range of scientific disciplines. This diagram shows just some of the interrelationships between chemistry and other fields.

A flowchart shows a box containing chemistry at its center. Chemistry is connected to geochemistry, nuclear chemistry, chemical physics, nanoscience and nanotechnology, materials science, chemical engineering, biochemistry and molecular biology, environmental science, agriculture, and mathematics. Each of these disciplines is further connected to other related fields including medicine, biology, food science, geology, earth sciences, toxicology, physics, and computer science.

The role of chemistry in understanding the material composition of the universe complements the principles of physics. Chemistry, often termed the "central science," bridges the physical sciences with life sciences and applied sciences such as medicine and engineering. The interplay between chemistry and physics is evident in fields such as biochemistry and chemical engineering, where the behavior of atoms and molecules is crucial for designing new materials and drugs.

In the laboratory, early alchemists mixed substances, hoping to transform lead into gold, unaware that they were laying the groundwork for modern chemistry. Similarly, physicists from ancient times to the present have sought to understand the forces that govern the motion of the planets and the properties of light and gravity that influence our everyday lives. This foundational knowledge helps engineers design smartphones and GPS systems that rely on precise interactions of electrical circuits and the laws of motion and relativity.

Science thrives on this interconnectedness, where a discovery in one field can illuminate concepts in another. The shared methodologies and conceptual frameworks of physics and chemistry enable us to solve complex problems and innovate solutions that traverse multiple disciplines. Whether analyzing the chemical properties of new material or studying the forces that bind the cosmos, scientists rely on an integrated approach that respects the underlying simplicity and order of nature.

Understanding these connections enhances our appreciation of the natural world and equips us with the analytical skills necessary for a wide range of careers. By applying the laws of physics and chemistry, we not only decipher how the universe operates but also learn to shape it to improve our quality of life. This educational journey empowers us with a comprehensive toolkit to explore, describe, and manipulate the world around us, ensuring that we continue to innovate and advance our understanding of the universe.

In conclusion, the quest for knowledge in chemistry and physics exemplifies the human desire to seek order and understand the underlying principles of the universe. As we continue to explore these fundamental sciences, we unlock the mysteries of the material world and harness its powers to foster technological advances and improve our everyday lives. This integrated approach in science education highlights the importance of understanding the interrelationships among various scientific disciplines and the real-world applications that these relationships enable.

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1.3: Using the Scientific Method

Learning Objectives

- To identify the components of the scientific method
- Classify measurements as being quantitative or qualitative.
- Evaluate science in the media.

The Scientific Method

Scientists search for answers to questions and solutions to problems by using a procedure called the **scientific method**. This procedure consists of making observations, formulating hypotheses, and designing experiments, which in turn lead to additional observations, hypotheses, and experiments in repeated cycles (Figure $1.3.1$).

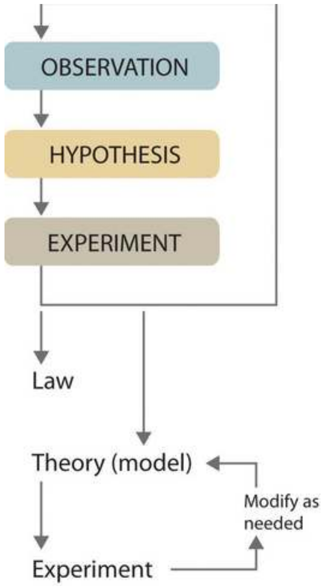


Figure $1.3.1$: The Steps in the Scientific Method. (CC BY-SA-NC; anonymous).

Step 1: Make observations

Observations can be qualitative or quantitative. **Qualitative observations** describe properties or occurrences in ways that do not rely on numbers. Examples of qualitative observations include the following: the outside air temperature is cooler during the winter season, table salt is a crystalline solid, sulfur crystals are yellow, and dissolving a penny in dilute nitric acid forms a blue solution and a brown gas. **Quantitative observations** are measurements, which by definition consist of both a number and a unit. Examples of quantitative observations include the following: the melting point of crystalline sulfur is 115.21° Celsius, and 35.9 grams of table salt—whose chemical name is sodium chloride—dissolves in 100 grams of water at 20° Celsius. For the question of the dinosaurs' extinction, the initial observation was quantitative: iridium concentrations in sediments dating to 66 million years ago were 20–160 times higher than normal.

Step 2: Formulate a hypothesis

After deciding to learn more about an observation or a set of observations, scientists generally begin an investigation by forming a **hypothesis**, a tentative explanation for the observation(s). The hypothesis may not be correct, but it puts the scientist's understanding of the system being studied into a form that can be tested. For example, the observation that we experience alternating periods of light and darkness corresponding to observed movements of the sun, moon, clouds, and shadows is consistent with either of two hypotheses:

a. Earth rotates on its axis every 24 hours, alternately exposing one side to the sun, or
b. the sun revolves around Earth every 24 hours.

Suitable experiments can be designed to choose between these two alternatives. For the disappearance of the dinosaurs, the hypothesis was that the impact of a large extraterrestrial object caused their extinction. Unfortunately (or perhaps, fortunately), this hypothesis does not lend itself to direct testing by any obvious experiment, but scientists can collect additional data that either support or refute it.

Step 3: Design and perform experiments

After developing a hypothesis, scientists design and conduct experiments to test its validity. **Experiments** are systematic methods of making observations or measurements under controlled conditions, ideally with only one variable altered at a time. This approach allows researchers to isolate the effects of one variable on another in a clear, structured environment.

In any experiment, two primary types of variables are involved:

> **Independent Variable:** This is the variable that the scientist changes deliberately to observe its effect. It's the cause part of cause and effect.
>
> **Dependent Variable:** This variable is what the scientist measures or observes. It's the effect that is potentially influenced by changes in the independent variable.

Experiments typically have at least two groups:

> **Control Group:** This group does not receive the experimental treatment or change in the independent variable. It serves as a baseline to compare the effects of the independent variable. Some experiments include things called **positive controls** and **negative controls**. These are slightly different from control groups. Positive and negative controls serve to show that the experiment is working correctly. A positive control is a part of the experiment that is deliberately designed to give a positive result. It shows that the experiment is capable of producing a positive result when it is supposed to. A negative control is a part of the experiment that is designed to give a negative result. It shows that the experiment is capable of producing a negative result when it is supposed to.
>
> **Treatment Group(s):** In these groups, the independent variable is manipulated to various degrees, allowing the scientist to observe the changes that occur in the dependent variable.

After the experiment, scientists collect data, often quantifying it in graphs or tables for analysis. This data analysis helps determine if there is a statistical relationship between the dependent and independent variables, thereby supporting or refuting the hypothesis.

Step 4: Accept or modify the hypothesis

A properly designed and executed experiment enables a scientist to determine whether the original hypothesis is valid. In which case he can proceed to step 5. In other cases, experiments often demonstrate that the hypothesis is incorrect or that it must be modified thus requiring further experimentation.

Step 5: Development into law and/or theory

More experimental data are then collected and analyzed, at which point a scientist may begin to think that the results are sufficiently reproducible (i.e., dependable) to merit being summarized in **law**, a verbal or mathematical description of a phenomenon that allows for general predictions. A law simply says what happens; it does not address the question of why.

One example of a law, the law of definite proportions, which was discovered by the French scientist Joseph Proust (1754–1826), states that a chemical substance always contains the same proportions of elements by mass. Thus, sodium chloride (table salt) always contains the same proportion by mass of sodium to chlorine, in this case, 39.34% sodium and 60.66% chlorine by mass, and sucrose (table sugar) is always 42.11% carbon, 6.48% hydrogen, and 51.41% oxygen by mass.

Whereas a law states only what happens, a **theory** attempts to explain why nature behaves as it does. Laws are unlikely to change greatly over time unless a major experimental error is discovered. In contrast, a theory, by definition, is incomplete and imperfect, evolving with time to explain new facts as they are discovered.

Because scientists can enter the cycle shown in Figure $1.3.1$ at any point, the actual application of the scientific method to different topics can take many different forms. For example, a scientist may start with a hypothesis formed by reading about work done by others in the field, rather than by making direct observations.

A Real-World Application of the Scientific Method

In 2007, my husband and I journeyed to China to adopt our daughter. Upon arrival in Beijing, I became violently ill. Due to her visa paperwork, my husband, daughter, and I were required to stay in China for two weeks. Unfortunately, I was ill the entire time. Once the two-week period was up, the three of us flew back to the United States where I continued to be sick. For the next year, I remained ill and lost a total of 30 pounds. The picture below shows me holding my daughter eight months after we returned home from China.

I would like you to attempt to perform the scientific method on my situation described above. List the steps of the scientific method along with some plausible explanations. * Please have this ready to discuss in class.*

✓ Exercise (PageIndex{1})

Classify each statement as a law, a theory, an experiment, a hypothesis, a qualitative observation, or a quantitative observation.

a. Ice always floats on liquid water.
b. Birds evolved from dinosaurs.
c. According to Albert Einstein, mass X speed of light = energy
d. When 10 g of ice was added to 100 mL of water at 25°C, the temperature of the water decreased to 15.5°C after the ice melted.
e. The ingredients of Ivory soap were analyzed to see whether it really is 99.44% pure, as advertised.

Solution

a. This is a general statement of a relationship between the properties of liquid and solid water, so it is a law.
b. This is an educated guess regarding the origin of birds, so it is a hypothesis.
c. This is a theory that explains an explanation of events and can be disproven at any time.
d. The temperature is measured before and after a change is made in a system, so these are quantitative observations.
e. This is an analysis designed to test a hypothesis (in this case, the manufacturer’s claim of purity), so it is an experiment.

? Exercise (PageIndex{2})

Classify each statement as a law, a theory, an experiment, a hypothesis, a qualitative observation, or a quantitative observation.

a. Measured amounts of acid were added to a Rolaids tablet to see whether it really “consumes 47 times its weight in excess stomach acid.”
b. Heat always flows from hot objects to cooler ones, not in the opposite direction.
c. The universe was formed by a massive explosion that propelled matter into a vacuum.
d. Michael Jordan is the greatest pure shooter ever to play professional basketball.
e. Limestone is relatively insoluble in water but dissolves readily in dilute acid with the evolution of a gas.
f. Gas mixtures that contain more than 4% hydrogen in air are potentially explosive.

Answer a: experiment

Answer b: law

Answer c: theory

Answer d: hypothesis

Answer e: qualitative observation

Answer f: quantitative observation

? Exercise 1.3.1

Classify each statement as a law, a theory, an experiment, a hypothesis, a qualitative observation, or a quantitative observation.

a. When 10 g of ice were added to 100 mL of water at 25 °C, the temperature of the water decreased to 15.5 °C after the ice melted.
b. Litmus paper dipped in lemon juice turns red.
c. A prism separates white light into a spectrum of colors.
d. Limestone is relatively insoluble in water but dissolves readily in dilute acid with the evolution of a gas.
e. Gas mixtures that contain more than 4% hydrogen in air are potentially explosive.

Answer a: quantitative observation

Answer b: qualitative observation

Answer c: qualitative observation

Answer d: qualitative observation

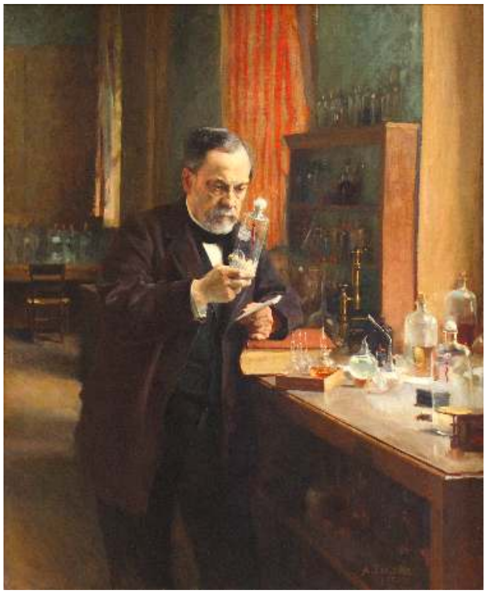
Answer e: quantitative observation

The Scientific Methods: Crash Course History of Science #14

One famous example is the experiment conducted by Louis Pasteur to test the hypothesis of **spontaneous generation**. This was the once popular idea that non-living material can spontaneously transform into living organisms. For example, if meat were left out too long, people would find that it was infested with maggots. Many people assumed that the meat was transforming into live maggots. After the discovery of microorganisms like bacteria, people questioned where they came from. Were they being spontaneously generated or were they reproducing? In other words, did microorganisms produce new microorganisms?

To test the hypothesis of spontaneous generation, Pasteur devised a simple but ingenious experiment. He filled two flasks with chicken broth. One was an open-necked flask, and the other was a swan-necked flask. The open-necked flask would allow bacteria from the air to enter the broth. On the other hand, the swan-necked flask would trap any bacteria in the elbow of the neck, thus preventing the bacteria from reaching the broth. He then boiled both broths to sterilize them. If spontaneous generation was valid, it should not have mattered whether bacteria from the outside were able to enter the broth. There should be bacteria growing in both flasks.

On the other hand, if bacteria only came from the outside, then only the open-necked flask should have bacteria growing it. After a few days, Pasteur examined the broth in both flasks and found only new bacteria growing in the open-necked flask. Since the swan-neck flask did not show any bacteria growing in it, the hypothesis of spontaneous generation was rejected, and we now know that for maggots to grow on meat, flies must first lay their eggs on the meat. The maggots then hatch out of the eggs and feed on the rotting meat. The meat itself did not spontaneously generate the maggots.



Albert_Edelfelt_-_Louis_Pasteur_-_1885.jpg Photo from Wikimedia Commons

To put Pasteur’s experiment into the above terms, the shape of the flask’s neck was the independent variable and the amount of bacterial growing in each flask was the dependent variable that Pasteur tested for. The open-neck flask was the control group used to compare the swan-neck flask that was used for the treatment group.

Wikimedia commons L0057281 Copy of Pasteur's flask used in his experiments on spontaneous generation Credit: Science Museum, London. Wellcome Images images@wellcome.ac.uk http://wellcomeimages.orgCopy_of_Pas...e_L0057281.jpg

Erlenmyer flask By Hannes Grobe 19:04, 3 September 2006 (UTC) - Own work, CC BY-SA 2.5,https://commons.wikimedia.org/w/index.php

The above is an example of a **manipulative experiment** in which a single variable is to be changed under controlled conditions. Many people think that manipulative experiments are the only way in which science is done, but there are many cases in which the phenomena being studied are too big or too distant in space or time to be studied by a controlled experiment in the lab. In these cases, scientists learn by doing **observational science**. For example, we cannot build a star in a lab and study it up close. Fortunately, our galaxy is filled with billions of stars of different sizes, ages, and temperatures. By collecting data from many stars in various stages of stellar evolution, astronomers can build a model for how stars form and change over time. Astronomy is, therefore, largely an observational science.

A scientific experiment must be repeatable, that is, you or someone else conducting the same study should get similar results. One case might be a fluke, but if several repeats of the experiment yield similar results, then the hypothesis is better supported.

Once the scientist has completed their study, the usual practice is for them to write a paper and submit it to a peer-reviewed journal. Peer review is the process by which scientists in the same field evaluate each other’s work. When a journal receives a proposed paper, it sends it out to several other scientists (the “peers”) who review it and recommend whether to publish it. If the peers conclude that the research followed good scientific methodology and the data support the conclusions, they recommend publication. Otherwise, the paper is rejected, and the scientist has to do more work before resubmitting.

Unfortunately, there are many places where “scientific” papers can be published without peer review. Many of these are pay-for-publishing journals that will publish almost any paper if the researcher pays a fee. Also, some organizations with a political agenda may self-publish what looks like legitimate research that is slanted to reach a predetermined conclusion.

Today, few people dispute the fact that there is a strong link between smoking tobacco and certain forms of cancer. However, for decades, doubt was sown by an organization called the Tobacco Institute, which published many convincing-looking papers. All these papers came to the same conclusion: “Gosh, we just can’t find any link between tobacco and cancer.” This was the opposite of what nearly every other researcher in the field concluded. However, to be able to come to different results? Well, the Tobacco Institute was funded by the tobacco industry, and we now know that they were under orders by their sponsors to come to predetermined conclusions no matter what the data said.

More recently, a research paper in a pay-to-publish journal created a sensation in the news media by announcing that you can lose weight by eating chocolate. Wouldn’t that be great? Sounds too good to be true, right? Well, it was. The researcher behind it came forward to admit that the entire paper was a hoax. He did it to highlight how easy it is to get bogus studies published and how readily the media can hype what seem like sensational results. Sadly, most journalists are not trained to discern the difference between sound science and what we can label **“junk science”** or **pseudoscience**.

Science plays a big role in our understanding of the world around us. It is, therefore, imperative that we have a scientifically literate society that is capable of discerning good science from junk science, especially when making decisions relating to their health and well-being.

One final word on science and relates to how people use the word **theory**. Many non-scientists use the word to mean a guess or a hunch based on incomplete information. However, this is now how scientists use the term. In science, a theory is a broad explanation for a phenomenon that has been well-tested, shown to be supported repeatedly by experiments, and has gained wide acceptance. It is not a single hunch or guess. The way non-scientists use the term theory is more akin to how scientists use the word hypothesis. Another misconception is that if a hypothesis is validated by experiment, it may be “promoted” to become a theory. That is not accurate either. A **theory** is a broader explanation, while a hypothesis is generally narrower in scope. Indeed, a single theory may encompass several hypotheses. A theory offers a deeper, more detailed explanation for why things happen as they do in nature, like Einstein's Theory of Relativity explaining gravity through the curvature of space-time. It’s not just an isolated observation; it's a comprehensive model that explains a wide range of phenomena.

On the other hand, a law describes what happens based on consistent observational data, such as Newton's Laws of Motion that describe how objects move but do not explain why they move that way. Laws provide a description of an observed phenomenon without looking into the mechanics behind it. They are universally accepted as true under consistent conditions and help predict outcomes.

It’s important to note that in science, a hypothesis does not simply "evolve" into a theory through validation. A theory is a broader and more complex explanation that often encompasses several hypotheses. The transition from hypothesis to theory involves a significant accumulation of evidence and broader acceptance in the scientific community.

Keep that in mind when someone dismisses a scientific principle as “just a theory.” Nearly everything you are taught in a science class is based on theory. Of course, just because a theory has gained broad acceptance does not always mean it is true, but it takes a lot of evidence to overturn a theory that has been repeatedly validated by observation. Thus, much of what you learn in science classes is based on theories—well-established, widely tested, and crucial for advancing our understanding of the natural world.

Evaluating Science in the Media

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1.4: Limitations of the Scientific Method

Learning Objectives

- To understand the limitations in the scientific method, one must become familiar with the scientific method and its components.

Pseudo-science, basically "fake"-science," consists of scientific claims which are made to appear factual when they are actually false. Many people question whether Pseudo-science should even contain the word "science" as Pseudo-science isn't really even an imitation of science; it pretty much disregards the scientific method all together. Also known as alternative or fringe-science, Pseudo-science relies on invalid arguments called sophisms, a word Webster dictionary defines as "an argument apparently correct in form but actually invalid; especially : such an argument used to deceive". Pseudo-science usually lacks supporting evidence and does not abide by the scientific method. That is, pseudo-theories fail to use carefully cultivated and controlled experiments to test a hypothesis. A scientific hypothesis must include observable, empirical and testable data, and must allow other experts to test the hypothesis. Pseudo-science does not accomplish these goals. Several examples of Pseudo-Science include phrenology, astrology, homeopathy, reflexology and iridology.

Distinguishing Pseudo-Science

In order to distinguish a pseudoscience, one must look at the definition of science, and the aspects that make science what it is. Science is a process based on observations, conjectures, and assessments to provide better understanding of the natural phenomena of the world. Science generally always follows a formal system of inquiry which consists of observations, explanations, experiments, and lastly, hypothesis and predictions. Scientific theories are always challenged by experts and revised to fit new theories. Pseudo-science, however, is mostly based on beliefs and it greatly opposes contradictions. Their hypothesis are never revised to fit new data or information. Scientist continually disprove ideas to achieve a better understanding of the physical world, whereas pseudo-scienctists focus on proving theories to make their claims seem plausible. For example, science text books come out with new editions every couple of years to correct typos, update information, add new illustrations, etc. However, it has been observed that pseudo-science textbooks only come out with one edition, and is never updated or revised even if their theory has been proven to be false.

Pseudo-science beliefs often tend to be greatly exaggerated and very vague. Complicated technical language is often used to sound impressive but it is usually meaningless. For example, a phrase like "energy vibrations" is used to sound remarkable but a phrase like this is insignificant and doesn't really explain anything. Furthermore, Pseudo-science often consists of outrageous, yet unprovable claims. Thus, pseudo-scientists tend to focus on confirming their ideas, rather than finding evidence that refutes them. The following dialogue contains the thought-processes behind Pseudo-Science.

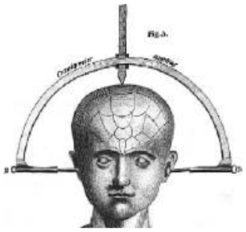
1. My friend and I think unicorns exist
2. Science has no evidence about unicorns.
3. Science therefore cannot prove if unicorns do or do not exist.
4. One day my friend, a very trustworthy person, said she saw a unicorn in the field by her house. There is no other evidence, other than the fact that my friend saw it.
5. Unicorns exist and any scientist who tries to deny the existence of unicorns is a fun-sucking, hostile human being.

The dialogue above features many key characteristics of Pseudo-Science. The speaker makes his or her point valid though the two facts alone that her friend had a personal experience and that science has no proof to prove the theory wrong. Finally, the speaker insults anyone who would challenge the theory. In science, challenges to a theory are accepted as everyone has the same common goal of improving the understanding of the natural word. Below is a table that lays out the key characteristics of Science and Pseudo-Science

SCIENCE	PSEUDO-SCIENCE
Science never proves anything.	Pseudoscience aimes to prove an idea.
Self-correcting methodology which involves critical thinking.	Starts with a conculsion and gives easy answers to complex problems.

SCIENCE	PSEUDO-SCIENCE
An on-going process to develop a better understanding of the physical world by testing all possible hypotheses.	Often driven by social, political or commercial goals.
Involves a continual expansion of knowledge due to intense research.	A field has not evolved a lot since the beginning. If any research is done, it is done to justify the claims, rather that expand them.
Scientists constantly attempt to refute other scientists' works.	An attempt to disprove the beliefs is considered hostile and unacceptable.
When results or observations are not consistent with a scientific understanding, intense research follows.	Results or observations that are not consistent with current beliefs are ignored.
Remains questionable at any time. There are two types of theories: those that have been proven wrong by experimentation and data, and every other theory. Thus, no theory can be proven correct; every theory is also subject to being refuted.	Beliefs of the field can not usually be tested empirically so will likely not ever be proven wrong; Thus, Pseudo-scientists believe that they are right just because no one can prove them wrong.
Concepts are based on previous understandings or knowledge.	Pseudo-Scientists are often not in touch with main-stream science and are often driven by the egos of the "scientists". Furthermore, famous names and testimonials are often used for support rather than scientific evidence.
Findings must be stated in unambiguous, clear language.	Pseudoscience often uses very vague, yet seemingly technical terms.

Phrenology



Phrenology, also known as craniology, was a "science" popular during the early 1800s that was centered around the idea that the brain was an organ of the mind. During this time, most people believed that the brain was divided into distinct sections that all controlled different parts of a person's personality or intelligence. The basis of phrenology revolves around the concept that the brain mirrors a muscle and those parts of the brain which are "exercised" the most, will be proportionally larger than those parts of the brain that aren't often used. Thus, the scientists pictured the brain as a bumpy surface, with the make-up of the surface differing for every person depending on their personality and intelligence. By the mid 19th century, automated phrenology machines existed, which was basically a set of spring loaded probes that were placed on the head to measure the topography of one's skull. The machine then gave an automated reading about a person's characteristics based on this.

Let's consider some of the key characteristics of pseudo-science from our chart, and see how they apply to phrenology.

- Pseudo-Scientists are often not in touch with main-stream science: Scientific research has since the 1800s shown how though the brain is indeed divided into sections, each section does not determine a characteristic or personality trait, but instead controls a specific function such as memory or motor skills. Likewise, it has been concluded that the brain conforms to the shape of the skull , rather than the skull conforming to the shape of the brain (meaning the bumps of a persons skull have nothing to do with the shape of the brain). Back in the 1800s, little knowledge existed about the realities of brain structure and function, so the concept wasn't as reflexive of pseudo-science as it is today. However, some doctors and scientist still believe in the basic tenets of phrenology. Phrenology today exists as a classic form of pseudo-science as it goes against the common understanding about how the brain functions.
- Often driven by social, political or commercial goals- Indeed, the main goal of phrenology was a political and social one: to prove the dominance of the white race over other races. "Scientists" measured the brains of both races and concluded that the brains of white people were larger then that of people of African descent. Therefore, they concluded, they were smarter and superior. It was later revealed that the scientists were biased while conducting the experiment and that they were previously aware of what race each brain belonged to. The experiment was repeated and this time the scientists were not aware of the race and they concluded that the brains were of equal size. The second experiment better conforms to the scientific method, as in this case the scientists objectively measured the brains, while in the first case the bias of the scientists lead to their conclusions. Thus, this situation demonstrates a two-fold level of defective science because not

only was the idea of measuring the brains to determine personality and intelligence not correct all together, but the methods in which the scientists were doing this was also flawed. Phrenology was also commercially driven, since phrenology parlors where very wide spread and many devices were on the market to be used to measure.

- Pseudo-Scientists are often driven by the egos of the "scientists"- In the book *Phrenology and the origins of Victorian Scientific Naturalism* by John Van Whye, Van Whye quotes about the main discoverer of Phrenology Franz Joseph Gall, that " the peculiar incentive behind Gall's fascination with explaining individuals' differences may have lain in his hubris" (Van Whye 18). Of the 12 children in his family, Gall was the sharpest and brightest and naturally interested in distinguishing factors between children. Even as a young school boy, Gall noticed that the other children who were just as good at memorization as he was all had protruding eyes, which lead him to the idea of the basis of phrenology, that the characteristics of one's head indicates his or her intelligence.

Reflexology

Reflexology is a way of treatment that involves physically applying pressure to the feet or hands with the belief that each are divided up into different zones that are "connected" to other parts of the body. Thus, reflexologists assert that they can make physical changes throughout the body simply by rubbing ones hands or feet. Like we did with phrenology, lets go through some of the main characteristics of Pseudo-Science and see how they apply to reflexology.

- Pseudo-Scientists are often not in touch with main-stream science: No Scientific research has proven the validity of reflexology and how in fact it would actually work. In 2009, the Australian Medical Journal conducted an extensive study on reflexology and concluded "The best evidence available to date does not demonstrate convincingly that reflexology is an effective treatment for any medical condition". However, despite this lack of evidence, Reflexology continues.
- Pseudoscience often uses very vague, yet seemingly technical terms terms A main focus of reflexology is that the pressure on the foot removes any blockage of Qi, the "life energy force" and restores balance to lead to better health. Terms like "vital energy" or "energy blockage" which are used to talk about reflexology are classic pseudo-science terms; they sound impressive yet have no meaning to us
- Furthermore, famous names and testimonials are often used for support rather than scientific evidence. Because pseudo-science beliefs do not use scientific data for support, they must rely on individual circumstances when their product, idea, etc. appeared to have worked. For example, on the home page of well-known reflexologist Laura Norman's home page, she has a quote of Regis Philben (past host of W*ho Wants to be a Millionaire?*) saying "Laura Norman's Reflexology spared me from a kidney stone operation and saved my life.", opposed to a quote from say, a medical journal, that would cite how many studies say reflexology is an extremely effective form of treatment.

Distinguishing Pseudo-Science from other types of invalid science

An important distinction should be made between Pseudo-science and other types of defective science. Take for example, the "discovery" of N-rays. While attempting to polarize X-rays, physicist René Prosper Blondlot claimed to have discovered a new type of radiation he called N-rays. After Blondlot shared with others his exciting discovery, many other scientists confirmed his beliefs by saying they too had saw the N-rays. Though he claimed N-rays contained impossible properties, Blondlot asserted when he put a hot wire in an iron tube, he was able to detect the N-rays when he used a thread of calcium sulfite that glowed slightly when the rays were sent through a prism of aluminum. Blondlot claimed that all substances except some treated metals and green wood emit N-rays. However, Nature magazine was skeptical of Blondlot and sent physicist Robert Wood to investigate. Before Blondlot was about to show Wood the rays, Wood removed the aluminum prism from the machine without telling Blondlot. Without the prism, the rays would be impossible to detect. However, Blondlot claimed to still see the N-rays, demonstrating how the N-rays did not exist; Blondlot just wanted them to exist. This is an example of Pathological science, a phenomenon which occurs when scientists practice wishful data interpretation and come up with results they want to see. This case of Pathological science and Pseudo-science differ. For one, Blondlot asked for a confirmation by other experts, something Pseudo-science usually lacks. More importantly, in pathological science, a scientist starts by following the scientific method; Blondlot was indeed doing an experiment when he made his discovery and proceeded to experiment when he found the substances that did not emit the rays. However, Pseudo-science usually includes a complete disregard of the scientific method, while Pathological scientists includes following the scientific method but seeing the results you wish to see.

Another type of invalid science, called hoax science occurred in 1999 when a team at the Lawrence Berkeley National Laboratory claimed to have discovered elements 116 and 118 when they bombarded Lead with Krypton particles. However, by 2002 it had been discovered that physicist Victor Ninov had intentionally fudged the data to get the ideal results. Thus, the concept of hoax science, which occurs when the data is intentionally falsified, differs both from pathological and pseudo science. In pathological science, scientists wishfully interpret the data and legitimately think they see what they want to see. However, in Hoax science, scientists know they don't see what they want to see, but just say they did. Finally, in Pseudo-Science, scientists don't consider the scientific method at all as they don't use valid experiments to back up their data in the first place.

From Pseudo-Science to Science

There have been incidents where what was once considered pseudo-science became a respectable theory. In 1911, German astronomer and meteorologist Alfred Wegener first began developing the idea of Continental Drift. The observation that the coastlines of African and South American seemed to fit together was not a new observation: scientists just couldn't believe that the continents could have drifted so far to cross the 5,000 mile Atlantic Ocean. At the time, it was a common theory that a land bridge had existed between Africa and Brazil. However, one day in the library Wegener read a study about a certain species that could not have crossed the ocean, yet had fossils appeared on both sides of the supposed land bridge. This piece of evidence lead Wegener to believe that our world had once been one piece, and had since drifted apart. However, Wegener's theory encountered much hostility and disbelief. In this time, it was the norm for scientists to stay within the scopes of their fields, meaning biologists did not study physics, chemists did not study oceanology and of course, meteorologists/astronomers like Wegener did not study geology. Thus, Wegener's theory faced much criticism just due to the fact that he was not a geologist. Also, Wegener could not explain why the continents moved, just that they did. This lack of reasoning lead to more skepticism about the theory and all these factors combined lead to the viewing of continental drift as Pseudo-Science. However, today much evidence exists that shows that Continental Drift is a perfectly acceptable scientific theory. Today, the modern ideas of plate tectonics can help explain Continental Drift, as the Plate Tectonic Theory presents the idea that the earth's surface is made up of several large plates that often move up to a few inches every year. Also, the development of paleomagnetism, which allows us to determine the earth's magnetic poles at the time a rock formed, suggests that the earth's magnetic poles have changed many times in the last 175 million years and that at one time South America and Africa were connected.

Limitations of the Scientific Method

Due to the need to have completely controlled experiments to test a hypothesis, science can not prove everything. For example, ideas about God and other supernatural beings can never be confirmed or denied, as no experiment exists that could test their presence. Supporters of Intelligent Design attempt to convey their beliefs as scientific, but nonetheless the scientific method can never prove this. Science is meant to give us a better understanding of the mysteries of the the natural world, by refuting previous hypotheses, and the existence of supernatural beings lies outside of science all together. Another limitation of the scientific method is when it comes making judgements about whether certain scientific phenomenons are "good" or "bad". For example, the scientific method cannot alone say that global warming is bad or harmful to the world, as it can only study the objective causes and consequences. Furthermore, science cannot answer questions about morality, as scientific results lay out of the scope of cultural, religious and social influences.

Concept Assessment

Determine if each statement is true or false (see answers at bottom of the page)

1. What is considered Pseudo-Science today will always be considered Pseudo-Science
2. A person has a cold and decides to seek reflexology treatment. The next day, the person gets better. This means reflexology is a valid scientific theory
3. Just because "science" is immoral or defective does not necessarily mean it is Pseudo-Science
4. Famous people are used in advertisements for products such as gatorade. This means these products are Pseudo-Science
5. Medically based Pseudo-Science such as homeopathy, reflexology or acupuncture have absolutely no benefits to people

References

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2. Van, Wyhe John. *Phrenology and the Origins of Victorian Scientific Naturalism*. Aldershot, Hanst, England: Ashgate, 2004. Print.
3. Yount, Lisa. *Alfred Wegener: Creator of the Continental Drift Theory*. New York: Chelsea House, 2009. Print.

Answers to concept assessment

1. False- just because something is considered pseudo-science today, does not mean it will always be. Take for example, our discussion about Continental Drift. Continental Drift used to be considered Pseudo-Science, but now since there is scientific evidence to prove it, the theory is considered a product of science.
2. False- Just because a person got better after having reflexology treatment does not mean the treatment, which has no scientific evidence behind it, is the sole reason for a person's recovery. Many other factors could have lead to a person's healing, such as medication or time to let the body fight by itself so it would be impossible to determine that reflexology caused a person to get over a cold
3. True- Pseudo-Science is a specific type of defective. See the discussion about pathological and hoax science to learn how to distinguish Pseudo-Science from other types of invalid Science.
4. False- The common characteristic of relying on testimonials or celebrity support of Pseudo-Science is just one of the many characteristics on Pseudo-Science. Before declaring something as Pseudo-Science or science, it is important to consider various characteristics of both and focus on whether or not the ideas have experimentally determined data to support them. There has indeed been Scientific Data to support the use of Gatorade.
5. False- though there is little scientific evidence to support these types of medical treatment, it does not mean that they have no value. The Placebo effect may be relevant here, as people may believe that the methods are working, which may trigger the body to actually feel better.

1.5: Teaching the Scientific Method

What is a Lesson Plan?

A lesson plan is a detailed outline of the teaching objectives, materials, activities, and assessment methods for a specific lesson. It serves as a guide for teachers to ensure that instructional goals are met and provides a structured approach to teaching a particular topic. Here’s what a typical lesson plan includes:

- **Objectives:** Clear statements about what students will learn and be able to do by the end of the lesson.
- **Materials:** A list of all the supplies and resources needed for the lesson.
- **Procedure:** Step-by-step instructions on how to conduct the lesson, including introduction, main activities, and conclusion.
- **Assessment:** Methods for evaluating whether students have understood the lesson objectives, such as worksheets, quizzes, or class discussions.
- **Extensions:** Additional activities or projects to reinforce the lesson or explore the topic further.

By using a lesson plan, teachers can ensure that they cover all necessary content, engage students effectively, and assess their understanding in a structured and organized manner.

Teaching the Scientific Method to K-5th Graders

The scientific method is a fundamental process in science that helps students understand how scientists investigate and learn about the world. Teaching this method to K-5th graders involves breaking down complex concepts into simpler, engaging activities that foster curiosity and critical thinking. Here’s how to effectively introduce the scientific method to young students using a structured lesson plan.

Lesson Plan: Introduction to the Scientific Method

Objective: Students will learn the steps of the scientific method and apply them in a simple, hands-on experiment.

Materials:

- Clear plastic cups
- Water
- Food coloring
- Paper towels
- Small objects (e.g., coins, pebbles)
- Worksheet for recording observations

Steps of the Scientific Method:

1. **Make Observations:** Begin with a discussion about observations. Explain that scientists use their senses to learn about the world around them. Show students a cup of water and ask them to describe what they see (e.g., “The water is clear,” “It’s in a plastic cup”).
2. **Formulate a Hypothesis:** Introduce the concept of a hypothesis as an educated guess about what might happen in an experiment. For example, ask, “What do you think will happen if we add food coloring to the water?” Guide students to form simple hypotheses like, “The water will turn blue.”
3. **Design and Perform Experiments:** Explain that experiments test hypotheses. Divide students into small groups and give each group a cup of water, a drop of food coloring, and a paper towel. Instruct them to add a drop of food coloring to the water and observe what happens. Encourage them to record their observations on the worksheet.
4. **Collect Data:** Teach students how to collect and record data. For this experiment, they can draw what the water looks like before and after adding the food coloring.
5. **Analyze Results:** Discuss the results as a class. Did the water change color as expected? Why or why not? Help students understand that their observations either support or refute their hypotheses.
6. **Accept or Modify Hypothesis:** If the results support the hypothesis, explain that this means their guess was correct. If not, discuss what might have gone wrong and how they could change their hypothesis and try again.
7. **Develop into Law or Theory:** For K-5 students, simplify this step by explaining that when scientists see the same results over and over, they can make rules about how things work. For example, “Every time we add blue food coloring to water,

it turns blue.”

Example Activity: Floating and Sinking

Objective: Students will understand the concept of density and apply the scientific method to predict and test which objects will float or sink in water.

Materials:

- Clear plastic bin filled with water
- Various small objects (e.g., coin, cork, plastic toy, pebble)
- Prediction worksheet

Procedure:

1. **Observation:** Show students the objects and the bin of water. Ask them to observe the objects and think about which ones might float or sink.
2. **Hypothesis:** Have each student pick an object and predict whether it will float or sink. Record their predictions on the worksheet.
3. **Experiment:** One by one, place each object in the water. Ask students to observe what happens and record the results.
4. **Data Collection:** Create a chart on the board with two columns: “Float” and “Sink.” Fill in the chart as students share their observations.
5. **Analysis:** Discuss the results. Were their predictions correct? Why did some objects float while others sank? Introduce the concept of density in simple terms.
6. **Conclusion:** Help students understand that their hypotheses can be tested and revised. If an object didn’t behave as predicted, discuss what might have been different about that object (e.g., weight, material).

1.6: End of Chapter Activity- Develop a Lesson Plan on the Scientific Method

Assignment: Develop a Lesson Plan on the Scientific Method

Objective: Create your lesson plan on the scientific method to teach a different concept or experiment suitable for K-5th grade students.

Assignment:

1. **Choose a Topic:**
    - Select a simple scientific concept or age-appropriate experiment for K-5th grade students.
    - Examples: Effects of sunlight on plant growth, how different liquids affect objects or simple weather experiments.
2. **Create a Lesson Plan:**
    - **Introduction:**
        - Start with a discussion about the chosen topic and why it is interesting or important.
    - **Explain the Steps of the Scientific Method:**
        - Define and explain each step with examples relevant to the chosen topic.
    - **Plan the Experiment:**
        - Detail how the experiment will be conducted, what materials are needed, and how data will be collected.
    - **Conduct the Experiment:**
        - Outline how students will perform the experiment, ensuring it is safe and manageable.
    - **Data Analysis:**
        - Describe how students will analyze the data and what they should look for.
    - **Conclusion:**
        - Plan for students to present their findings and discuss what they learned.
3. **Reflection:**
    - After creating the lesson plan, write a brief reflection on how the scientific method can help students understand and explore scientific concepts.

Developing your own lesson plans, will reinforce your understanding of the scientific method and teach you how to effectively teach this important concept to young students. This hands-on approach not only engages students but also fosters a curiosity and love for science from an early age.

1.7: End of Chapter Key Terms

Elemental Beginnings: Foundations of Physics and Chemistry Key Terms

1. **Control Group:** A group in an experiment that does not receive the experimental treatment or change in the independent variable, serving as a baseline to compare the effects of the independent variable.
2. **Dependent Variable:** The variable that is measured or observed in an experiment, potentially influenced by changes in the independent variable.
3. **Experiments**: Systematic methods of making observations or measurements under controlled conditions, ideally with only one variable altered at a time.
4. **Hypothesis:** A tentative explanation for an observation or set of observations that can be tested through further investigation.
5. **Independent Variable:** The variable that the scientist deliberately changes to observe its effect, representing the cause in a cause-and-effect relationship.
6. **Junk Science or Pseudoscience:** Research or claims presented as scientific but lacking rigorous standards and peer review, often with predetermined conclusions or biased interpretations.
7. **Law:** A verbal or mathematical description of a phenomenon that allows for general predictions, describing what happens without explaining why.
8. **Manipulative Experiment:** An experiment in which a single variable is deliberately changed under controlled conditions to isolate its effect on another variable.
9. **Negative Controls:** Parts of an experiment designed to give a negative result, showing that the experiment is capable of producing a negative result when it is supposed.
10. **Observational Science:** A scientific approach where phenomena are studied without manipulation, often due to the impracticality of controlled experiments, such as in astronomy.
11. **Observations:** Initial steps in the scientific method, which can be qualitative (describing properties or occurrences without numbers) or quantitative (measurements consisting of numbers and units).
12. **Positive Controls:** Parts of an experiment designed to give a positive result, demonstrating that the experiment is capable of producing a positive result when it is supposed to.
13. **Qualitative Observations:** Descriptions of properties or occurrences that do not rely on numerical measurements, such as the color of sulfur crystals.
14. **Quantitative Observations:** Measurements that consist of both a number and a unit, such as the melting point of a substance.
15. **Scientific Method:** A procedure for searching for answers to questions and solutions to problems through making observations, formulating hypotheses, designing experiments, and collecting data in repeated cycles.
16. **Spontaneous Generation:** An outdated hypothesis suggesting that non-living material can spontaneously transform into living organisms, disproven by experiments such as those conducted by Louis Pasteur.
17. **Theory:** A broad explanation for a phenomenon that has been well-tested, supported by repeated experiments, and widely accepted, explaining why nature behaves as it does.
18. **Treatment Group(s)**: Groups in an experiment where the independent variable is manipulated to various degrees to observe the changes that occur in the dependent variable.

CHAPTER OVERVIEW

2: Units, Measurement, Graphing, and Calculation

2.1: Introduction and Learning Objectives

Introduction to Units, Measurements, Graphing, and Calculation

Welcome to the fascinating world of physical science, where precision, accuracy, and methodical problem-solving lay the foundation for discovery and innovation. As future educators, you will be guiding young minds through the complexities of scientific principles, fostering their curiosity, and nurturing their understanding of the natural world. This chapter, "Units, Measurements, Graphing, and Calculation," is designed to equip you with the essential mathematical and analytical skills necessary to teach physical science effectively.

We begin with a thorough review of fundamental math concepts, from the order of operations and handling negative numbers to mastering decimals and fractions. These basics form the bedrock of scientific calculations and are crucial for ensuring that students grasp more complex topics down the line. You'll learn how to manipulate formulas, calculate perimeters and circumferences, and work with percentages in various contexts, including error analysis.

Moving forward, we'll delve into geometry, exploring angles, triangles, polygons, and composite figures. You'll gain the ability to calculate areas and volumes of common solids, skills that are indispensable in both classroom demonstrations and real-world applications. Understanding and converting units of measurement is another critical component of this chapter, as it ensures consistency and accuracy in scientific inquiry.

Graphing is another vital skill you'll develop here. Accurate data representation through various types of graphs is not only a fundamental scientific practice but also an engaging way to help students visualize and interpret data. You'll learn how to choose the appropriate graph type for different data sets and ensure clarity and precision in your presentations.

Dimensional analysis, precision, accuracy, and significant figures will round out your mathematical toolkit, enabling you to perform and teach scientific calculations with confidence. By mastering these concepts, you'll be prepared to guide your students through the intricacies of physical science, helping them to develop critical thinking and analytical skills that will serve them well in their academic and professional futures.

This chapter is not just about learning techniques and formulas; it's about building a strong foundation for scientific literacy. As you engage with the material, remember that your role as an educator is to inspire and empower the next generation of scientists, engineers, and informed citizens. Let's embark on this journey together, fostering a deep understanding of the principles that govern our physical world.

Learning Objectives

Chapter 2: Units, Measurements, Graphing, and Calculation

1. Review mathematical concepts.
2. Analyze errors using percentages.
3. Calculate and interpret mean, median, and mode in data sets.
4. Calculate and apply basic probability concepts.
5. Calculate and interpret standard deviation in data sets.
6. Apply the rules of exponents in mathematical calculations.
7. Use scientific notation to express large and small numbers.
8. Differentiate between precision and accuracy in measurements.
9. Apply significant figures in scientific calculations.
10. Perform accurate measurements using various tools.
11. Differentiate between measurement systems and apply them.
12. Use and convert US measurement units in scientific calculations.
13. Apply metric units in scientific calculations.
14. Perform conversions between different measurement systems.
15. Create and interpret graphs to represent data.
16. Choose appropriate graph types for different data sets.
17. Use dimensional analysis for unit conversions.
18. Apply chapter concepts in practical, real-world activities.
19. Review and apply key terms from the chapter in scientific contexts.

SECTION OVERVIEW

2.2: Math Review

2.2.1: Order of Operations

To evaluate an expression means to simplify it and find its value.

Exercises (PageIndex{1})

1. Evaluate by performing the addition first: $12-2+3$

2. Evaluate by performing the subtraction first: $12-2+3$

Answer

1. 7
2. 13

When we evaluate an expression, we want to have a single correct answer. It isn't very helpful for the answer to be "maybe 7, or maybe 13". Mathematicians have decided on an order of operations, which tells us which steps should be done before other steps. Think of them as the rules of the road.

Order of Operations: PEMDAS

P: Work inside of **parentheses** or grouping symbols, following the order PEMDAS as necessary inside the grouping symbols.

E: Evaluate **exponents**.

MD: Perform **multiplications** and **divisions** from left to right.

AS: Perform **additions** and **subtractions** from left to right.

Exercises (PageIndex{1})

3. $12-(2+3)$

4. $12-2+3$

Answer

3. 7
4. 13

Based on Exercises 3 & 4, we can see that Exercise 1 told us to use the wrong order of operations. If there are no parentheses, we must evaluate (12-2+3) by *first* performing the subtraction and *then* performing the addition.

Before we move on, you should be aware that there are a handful of ways to show multiplication. All of the following represent (3times4):

(3cdot4) (3*4) (3(4)) ((3)4) ((3)(4))

In this textbook, you will most often see the dot, like (3cdot4), or parentheses directly next to a number, like (3(4)). We tend to avoid using the (3times4) symbol because it can be mistaken for the letter x.

Exercises (PageIndex{1})

Simplify each expression.

5. (12div(3cdot2))

6. (12div3cdot2)

7. (5(1+3)-2)

8. (5(1)+(3-2))

Answer

5. 2

6. 8

7. 18

8. 6

An ***exponent*** indicates repeated multiplication. For example, (6^2=6cdot6=36) and (4^3=4cdot4cdot4=64). The exponent tells us how many factors of the base are being multiplied together.

Exercises (PageIndex{1})

Simplify each expression.

9. (3^2+4^2)

10. ((3 + 4)^2)

11. ((7+3)(7-5)^3)

12. (7+3(7-5)^3)

Answer

9. 25

10. 49

11. 80

12. 31

In the next set of exercises, the only differences are the parentheses, but every exercise has a different answer.

Exercises (PageIndex{1})

Simplify each expression.

13. (39-7cdot2+3)

14. ((39-7)cdot2+3)

15. (39-(7cdot2+3))

16. (39-7cdot(2+3))

17. ((39-7)cdot(2+3))

Answer

13. 28

14. 67

15. 22

16. 4

17. 160

It is possible to have grouping symbols nested within grouping symbols; for example, $7+(5^2-(3(17-12\div 4)+2\cdot 5)\div 4)$.

To make it somewhat easier to match up the pairs of left and right parentheses, we can use square brackets instead: $7+(5^2-[3(17-12\div 4)+2\cdot 5]\div 4)$.

Exercises (PageIndex{1})

Simplify the expression.

18. $7+(5^2-[3(17-12\div 4)+2\cdot 5]\div 4)$

Answer

18. 19

A fraction bar is another grouping symbol; it tells us to perform all of the steps on the top and separately perform all of the steps down below. The final step is to divide the top number by the bottom number.

Exercises (PageIndex{1})

Simplify each expression.

19. $\frac{15-1}{6+1}$

20. $\frac{(7+2)\cdot 4}{18\div(3+3)}$

21. $\frac{5\cdot 4^2}{2}$

22. $\frac{(5\cdot 4)^2}{2}$

23. $\frac{(5-1)^2}{2+6}$

24. $(5-1)^2\div 2+6$

Answer

19. 2

20. 12

21. 40

22. 200

23. 2

24. 14

We will look at formulas in a later module, but let’s finish by translating from words to a mathematical expression.

Exercises (PageIndex{1})

25. You can find the approximate Fahrenheit temperature by doubling the Celsius temperature and adding 30. If the temperature is 9°C, what is the approximate Fahrenheit temperature? Write an expression and simplify it.

26. You can find the approximate Celsius temperature by subtracting 30 from the Fahrenheit temperature and then dividing by 2. If the temperature is $72°$F, what is the approximate Celsius temperature? Write an expression and simplify it.

Answer

25. $9 \cdot 2 + 30 = 48$ °F

26. $(72 - 30) \div 2 = 21$ °C

2.2.2: Negative Numbers

Negative numbers are a fact of life, from winter temperatures to our bank accounts. Let's practice evaluating expressions involving negative numbers.

Absolute Value

The ***absolute value*** of a number is its distance from 0. You can think of it as the size of a number without identifying it as positive or negative. Numbers with the same absolute value but different signs, such as 3 and -3, are called ***opposites***. The absolute value of -3 is 3, and the absolute value of 3 is also 3.

We use a pair of straight vertical bars to indicate absolute value; for example,$|-3|=3$ and $|3|=3$.

? Exercises 2.2.2.1

Evaluate each expression

1. $|-5|$

2. $|5|$

Answer

1. 5
2. 5

Adding Negative Numbers

To add two negative numbers, add their absolute values (i.e., ignore the negative signs) and make the final answer negative.

? Exercises 2.2.2.1

Perform each addition.

3. $-8+(-7)$

4. $-13+(-9)$

Answer

3. -15
4. -22

To add a positive number and a negative number, we *subtract* the smaller absolute value from the larger. If the positive number has the larger absolute value, the final answer is positive. If the negative number has the larger absolute value, the final answer is negative.

Exercises 2.2.2.1

Perform each addition.

5. $7+(-3)$

6. $-7+3$

7. $14+(-23)$

8. $-14+23$

9. The temperature at noon on a chilly Monday was -7°F. By the next day at noon, the temperature had risen 25°F. What was the temperature at noon on Tuesday?

Answer

5. 4
6. -4
7. -9
8. 9
9. 18°F

If an expression consists of only additions, we can break the rules for order of operations and add the numbers in whatever order we choose.

Exercises 2.2.2.1

Evaluate each expression using any shortcuts that you notice.

10. $-10+4+(-4)+3+10$

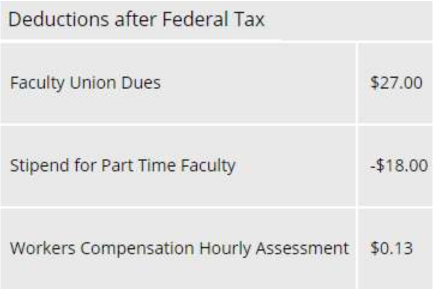
11. $-291+73+(-9)+27$

Answer

10. 3
11. -200

Subtracting Negative Numbers

The image below shows part of a paystub in which an $\$\ 18$ payment needed to be made, but the payroll folks wanted to track the payment in the deductions category. Of course, a positive number in the deductions will subtract money away from the paycheck. Here, though, a deduction of negative 18 dollars has the effect of *adding*18 dollars to the paycheck. Subtracting a negative amount is equivalent to adding a positive amount.



Deductions after Federal Tax	
Faculty Union Dues	$27.00
Stipend for Part Time Faculty	-$18.00
Workers Compensation Hourly Assessment	$0.13

To subtract two signed numbers, we **add** the first number to the **opposite** of the second number.

? Exercises 2.2.2.1

Perform each subtraction.

12. $5-2$

13. $2-5$

14. $-2-5$

15. $-5-2$

16. $2-(-5)$

17. $5-(-2)$

18. $-2-(-5)$

19. $-5-(-2)$

20. One day in February, the temperature in Portland, Oregon is 43°F, and the temperature in Portland, Maine is -12°F. What is the difference in temperature?

Answer

12. 3

13. -3

14. -7

15. -7

16. 7

17. 7

18. 3

19. -3

20. 55°F difference

Multiplying Negative Numbers

Suppose you spend 3 dollars on a coffee every day. We could represent spending 3 dollars as a negative number, -3 dollars. Over the course of a 5-day work week, you would spend 15 dollars, which we could represent as -15 dollars. This shows that $-3\cdot 5=-15$, or $5\cdot -3=-15$.

If two numbers with **opposite** signs are multiplied, the product is negative.

? Exercise 2.2.2.1

Find each product.

21. $-4\cdot 3$

22. $5(-8)$

Answer

21. -12

22. -40

Going back to our coffee example, we saw that $5(-3)=-15$. Therefore, the *opposite* of $5(-3)$ must be positive 15. Because -5 is the opposite of 5, this implies that $-5(-3)=15$.

If two numbers with the **same** sign are multiplied, the product is positive.

WARNING! These rules are different from the rules for addition; be careful not to mix them up.

? Exercise 2.2.2.1

Find each product.

23. $-2(-9)$

24. $-3(-7)$

Answer

23. 18

24. 21

Recall that an exponent represents a repeated multiplication. Let's see what happens when we raise a negative number to an exponent.

? Exercises 2.2.2.1

Evaluate each expression.

25. $(-2)^2$

26. $(-2)^3$

27. $(-2)^4$

28. $(-2)^5$

Answer

25. 4

26. -8

27. 16

28. -32

If a negative number is raised to an **odd** power, the result is negative.
If a negative number is raised to an **even** power, the result is positive.

Dividing Negative Numbers

Let's go back to the coffee example we saw earlier: $-3\cdot 5=-15$. We can rewrite this fact using division and see that $-15\div 5=-3$; a negative divided by a positive gives a negative result. Also, $-15\div -3=5$; a negative divided by a negative gives a positive result. This means that the rules for division work exactly like the rules for multiplication.

> If two numbers with **opposite** signs are divided, the quotient is negative.
> If two numbers with the **same** sign are divided, the quotient is positive.

? Exercises 2.2.2.1

Find each quotient.

29. $-42 \div 6$

30. $32 \div (-8)$

31. $-27 \div (-3)$

32. $0 \div 4$

33. $0 \div (-4)$

34. $4 \div 0$

Answer

29. -7

30. -4

31. 9

32. 0

33. 0

34. undefined

Go ahead and check those last three exercises with a calculator. Any surprises?

- 0 divided by another number is 0.
- A number divided by 0 is undefined, or not a real number.

Here's a quick explanation of why $4 \div 0$ can't be a real number. Suppose that there is a mystery number, which we'll call n, such that $4 \div 0 = n$. Then we can rewrite this division as a related multiplication, $n \cdot 0 = 4$. But because 0 times any number is 0, the left side of this equation is 0, and we get the result that $0 = 4$, which doesn't make sense. Therefore, there is no such number n, and $4 \div 0$ cannot be a real number.

Order of Operations with Negative Numbers

P: Work inside of **parentheses** or grouping symbols, following the order PEMDAS as necessary.

E: Evaluate **exponents**.

MD: Perform **multiplications** and **divisions** from left to right.

AS: Perform **additions** and **subtractions** from left to right.

Let's finish up this module with some order of operations practice.

? Exercises 2.2.2.1

Evaluate each expression using the order of operations

35. \((2-5)^2\cdot2+1\

36. $2 - 5^2 \cdot (2 + 1)$

37. $[7(-2) + 16] \div 2$

38. $7(-2) + 16 \div 2$

39. $\dfrac{1 - 3^4}{2(5)}$

40. $\dfrac{(1 - 3)^4}{2} \cdot 5$

Answer

35. 19

36. -73

37. 1

38. -6

39. -8

40. 40

2.2.3: Decimals

Decimal notation is based on powers of 10: 0.1 is one tenth, 0.01 is one hundredth, 0.001 is one thousandth, and so on.

thousands	hundreds	tens	ones/units	.	tenths	hundredths	thousandths

Exercises 2.2.3.1

Write each number.

1. ninety and twenty-three hundredths

2. seven and fifty-six thousandths

Answer

1. 90.23
2. 7.056

Adding & Subtracting Decimals

Before you add or subtract decimals, you must line up the decimal points.

Exercises 2.2.3.1

Add each pair of numbers.

3. $3.75+12.8$

4. $71.085+112.93$

Answer

3. 16.55
4. 184.015

When subtracting, you may need to add zeros to the first number so you can borrow correctly.

Exercises 2.2.3.1

Subtract each pair of numbers.

5. $46.57-38.29$

6. $82.78-67.024$

Answer

5. 8.28

6. 15.756

Multiplying Decimals

To multiply decimal numbers:

1. Temporarily ignore the decimal points.
2. Multiply the numbers as though they are whole numbers.
3. Add the total number of decimal digits in the two numbers you multiplied. The result will have that number of digits to the right of the decimal point.

Note: You do NOT need to line up the decimal points when you are multiplying.

Exercises $2.2.3.1$

Multiply each pair of numbers.

7. $13.5 \cdot 2.9$

8. $4.18 \cdot 3.7$

9. Evie worked 37.5 hours at a pay rate of $\$\,17.50$ per hour. How much did she earn in total?

Answer

7. 39.15

8. 15.466

9. $\$\,656.25$

Dividing Decimals

Let's review everyone's favorite topic, long division. The three parts of a division are named as follows: dividend ÷ divisor = quotient. When this is written with a long division symbol, the dividend is inside the symbol, the divisor is on the left, and the quotient is the answer we create on top.

$$\begin{array}{r} \text{quotient} \\ \text{divisor}\,\overline{)\,\text{dividend}} \end{array}$$

To divide by a decimal:

1. Write in long division form.
2. Move the decimal point of the divisor until it is a whole number.
3. Move the decimal point of the dividend the same number of places to the right.
4. Place the decimal point in the quotient directly above the decimal point in the dividend. Divide the numbers as though they are whole numbers.
5. If necessary, add zeros to the right of the last digit of the dividend to continue.

Exercise $2.2.3.1$

Divide each pair of numbers.

10. $97.4 \div 0.4$

11. $9.74 \div 0.04$

Answer

10. 243.5

11. $2,435$

Rounding Numbers

It is often necessary to round a number to a specified place value. We will discuss this in much more depth in a future module, but let's practice rounding now.

Rounding a number:

1. Locate the **rounding digit** in the place to which you are rounding.
2. Look at the **test digit** directly to the right of the rounding digit.
3. If the test digit is 5 or greater, increase the rounding digit by 1 and drop all digits to its right. If the test digit is less than 5, keep the rounding digit the same and drop all digits to its right.

? Exercises 2.2.3.1

Round each number to the indicated place value.

12. $6,473$ (thousands)

13. $6,473$ (hundreds)

14. $6,473$ (tens)

15. 0.7049 (tenths)

16. 0.7049 (hundredths)

17. 0.7049 (thousandths)

Answer

12. $6,000$
13. $6,500$
14. $6,470$
15. 0.7
16. 0.70
17. 0.705

If a decimal answer goes on and on, it may be practical to round it off.

? Exercises 2.2.3.1

18. Jerry drove 257 miles using 11 gallons of gas. How many miles per gallon did his car get? Round your result to the nearest tenth.

Answer

18. 23.4 miles per gallon

2.2.4: Fractions

5 out of 4 people have trouble with fractions.

A fraction describes equal parts of a whole: $\frac{\text{part}}{\text{whole}}$

Using official math vocabulary: $\frac{\text{numerator}}{\text{denominator}}$

Exercises 2.2.4.1

The month of April had 11 rainy days and 19 days that were not rainy.
1. What fraction of the days were rainy?
2. What fraction of the days were not rainy?

Answer

1. $\frac{11}{30}$
2. $\frac{19}{30}$

Simplifying Fractions

Two fractions are equivalent if they represent the same number. (The same portion of a whole.) To build an equivalent fraction, multiply the numerator and denominator by the same number.

Exercises 2.2.4.1

3. Write $\frac{4}{5}$ as an equivalent fraction with a denominator of 15.

4. Write $\frac{2}{3}$ as an equivalent fraction with a denominator of 12.

Answer

3. $\frac{12}{15}$
4. $\frac{8}{12}$

Many fractions can be ***simplified***, or reduced. Here are four special cases.

Exercises 2.2.4.1

Simplify each fraction, if possible.

5. $\frac{7}{1}$

6. $\frac{7}{7}$

7. $\frac{0}{7}$

8. $\frac{7}{0}$

Answer

5. 7

6. 1

7. 0

8. undefined

A fraction is completely reduced, or in simplest form, or in lowest terms, when the numerator and denominator have no common factors other than 1. To reduce a fraction, divide the numerator and denominator by the same number.

? Exercises 2.2.4.1

Reduce each fraction to simplest form.

9. $\frac{9}{12}$

10. $\frac{10}{6}$

Answer

9. $\frac{3}{4}$

10. $\frac{5}{3}$

Multiplying Fractions

To multiply fractions, multiply the numerators and multiply the denominators straight across. If possible, simplify your answer.

? Exercises 2.2.4.1

Multiply each pair of numbers. Be sure that each answer is in simplest form.

11. $8 \cdot \frac{1}{4}$

12. $\frac{6}{7} \cdot \frac{7}{12}$

13. $\frac{5}{8} \cdot \frac{2}{3}$

14. $\frac{6}{5} \cdot \frac{10}{12}$

Answer

11. 2

12. $\frac{1}{2}$

13. $\frac{5}{12}$

14. 1

To find a fraction **of** a number, multiply.

? Exercise 2.2.4.1

15. To pass his workplace training, Nathan must correctly answer at least $\frac{9}{10}$ of 50 questions. How many questions must he answer correctly to pass the training?

Answer

15. at least 45 questions

Dividing Fractions

To divide by a fraction, multiply by the reciprocal of the second number. (Flip the second fraction upside-down.)

? Exercises 2.2.4.1

Divide. Be sure that each answer is in simplest form.

16. $12 \div \frac{3}{4}$

17. $\frac{3}{10} \div \frac{1}{2}$

18. Suppose you need to measure 2 cups of flour, but the only scoop you can find is $\frac{1}{3}$ cup. How many scoops of flour will you need?

Answer

16. 16

17. $\frac{3}{5}$

18. 6 scoops

Comparing Fractions

If two fractions have the same denominator, we can simply compare their numerators.

If two fractions have different denominators, we can rewrite them with a common denominator and then compare their numerators.

? Exercise 2.2.4.1

19. Cookie recipe A requires $\frac{3}{4}$ cup of sugar, whereas cookie recipe B requires $\frac{2}{3}$ cup of sugar. Which recipe requires more sugar?

Answer

19. A requires $\frac{1}{12}$ cup more than B

Adding & Subtracting Fractions

To add or subtract two fractions with the same denominator, add or subtract the numerators and keep the common denominator.

❓ Exercises 2.2.4.1

20. Jack ate $\frac{3}{8}$ of a pizza. Mack ate $\frac{1}{8}$ of the pizza. What fraction of the pizza did they eat together?

21. Tracy ate $\frac{5}{6}$ of a pizza. Stacy ate $\frac{1}{6}$ of the pizza. How much more of the pizza did Tracy eat?

Answer

20. $\frac{1}{2}$ of the pizza

21. $\frac{2}{3}$ more

To add or subtract two fractions with different denominators, first write them with a common denominator. Then add or subtract them.

❓ Exercises 2.2.4.1

A $\frac{3}{8}$-inch thick sheet of plywood is going to be laid onto a $\frac{1}{4}$-inch thick sheet of plywood.

22. What is the combined thickness of the two sheets?

23. What is the difference in thickness of the two sheets of plywood?

Jacqueline budgets $\frac{1}{4}$ of her monthly income for food and $\frac{1}{3}$ of her monthly income for rent.

24. What fraction of her monthly income does she budget for these two expenses combined?

25. What fraction more of her monthly income does she budget for her rent than for her food?

Answer

22. $\frac{5}{8}$ inches combined

23. $\frac{1}{8}$ inches difference

24. $\frac{7}{12}$ combined

25. $\frac{1}{12}$ more

Fractions and Decimals

To write a fraction as a decimal, divide the numerator by the denominator.

A decimal that ends (eventually has a remainder of 0) is called a terminating decimal. Fun fact: If the denominator of a fraction has no prime factors other than 2‘s and 5‘s, the decimal will terminate. Also, the fraction can be built up to have a denominator of 10, or 100, or $1,000\ldots$

❓ Exercises 2.2.4.1

Write each fraction as a decimal.

26. $\frac{11}{4}$

27. $\frac{7}{20}$

Answer

26. 2.75

27. 0.35

A decimal that continues a pattern of digits is called a repeating decimal. We can represent the repeating digits by using either an overbar or ellipsis (three dots)…

? Exercises 2.2.4.1

Write each fraction as a decimal.

28. $\frac{5}{9}$

29. $\frac{18}{11}$

Answer

28. $0.\overline{5}$ or $0.555\ldots$

29. $1.\overline{63}$ or $1.636363\ldots$

Mixed Numbers

A mixed number represents a **sum**. For example, $6\frac{2}{3}$ means $6+\frac{2}{3}$.

To write a mixed number as an improper fraction:

1. Multiply the whole number part by the denominator.
2. Add this result to the original numerator to get the new numerator.
3. Keep the same denominator.

? Exercises 2.2.4.1

Rewrite each mixed number as an improper fraction.

30. $2\frac{1}{5}$

31. $6\frac{2}{3}$

Answer

30. $\frac{11}{5}$

31. $\frac{20}{3}$

To write an improper fraction as a mixed number:

1. Divide the numerator by the denominator to get the whole number part.
2. The remainder after dividing is the new numerator.
3. Keep the same denominator.

Exercises 2.2.4.1

Rewrite each improper fraction as a mixed number.

32. $\frac{23}{2}$

33. $\frac{14}{3}$

Answer

32. $11\frac{1}{2}$

33. $4\frac{2}{3}$

Adding or subtracting mixed numbers can be fairly simple or more complicated, depending on the numbers. If adding two mixed numbers would give you an improper fraction as part of your result, you'll need to carry; if subtracting two mixed numbers would give you a negative fraction as part of your result, you'll need to borrow.

Exercises 2.2.4.1

34. Add: $7\frac{1}{3}+2\frac{3}{4}$

35. Subtract: $7\frac{1}{3}-2\frac{3}{4}$

Answer

34. $10\frac{1}{12}$

35. $4\frac{7}{12}$

Multiplying or dividing mixed numbers is tricky. Change any mixed numbers into improper fractions before doing the calculation, then change the answer back to a mixed number if possible.

Exercises 2.2.4.1

36. Multiply: $3\frac{1}{2}\cdot 2\frac{1}{3}$

37. $5\frac{1}{2}$ cups of water will be divided equally into 3 jars. How much water will go into each jar?

Answer

36. $8\frac{1}{6}$

37. $1\frac{5}{6}$ cup

- **1.4: Fractions** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

2.2.5: Formulas

You may use a calculator throughout this module if needed.

Formulas

A formula is an equation or set of calculations that takes a number (or numbers) as input, and produces an output. The output is often a number, but it could also be a decision such as yes or no. The numbers in a formula are usually represented with letters of the alphabet, which are called ***variables*** because their values can vary. To evaluate a formula, we substitute a number (or numbers) into the formula and then perform the steps using the order of operations.

> Note: When a number is written directly next to a variable, it indicates multiplication. For example, $2H$ means $2 \cdot H$.

Exercises 2.2.5.1

The cost, in dollars, of mailing a large envelope weighing w ounces is calculated by the formula $C = 0.20w + 0.80$.[1]

1. Find the cost of mailing a 3-ounce envelope.

2. Find the cost of mailing a 9-ounce envelope.

Radio Cab charges the following rates for a taxi ride: a fixed fee of $\$\ 3.50$ plus a rate of $\$\ 2.60$ per mile. The total cost, in dollars, of a ride m miles long can be represented by the formula $C = 3.50 + 2.60m$.[2]

3. Find the cost of a 5-mile ride.

4. Find the cost of a 7.5-mile ride.

5. Find the cost of getting in the taxi, then changing your mind and getting out without riding anywhere.

The number of members a state has in the U.S. House of Representatives can be approximated by the formula $R = P \div .7$,[3] where P is the population in millions. The 2010 populations of three states are as follows:[4]

Oregon	3.8 million
Washington	6.7 million
California	37.2 million

Round all answers to the nearest whole number.

6. How many U.S. Representatives does Oregon have?

7. How many U.S. Representatives does Washington have?

8. How many U.S. Representatives does California have?

The number of electoral votes a state has can be approximated by the formula $E = P \div 0.7 + 2$, where P is the population in millions.

9. How many electoral votes does Oregon have?

10. How many electoral votes does Washington have?

11. How many electoral votes does California have?

Answer

1. $\$ 1.40$
2. $\$ 2.60$
3. $\$ 16.50$
4. $\$ 23.00$
5. $\$ 3.50$
6. 5 representatives (rounded down from 5.43)
7. 10 representatives (rounded up from 9.57)
8. 53 representatives (rounded down from 53.14)
9. 7 electoral votes
10. 12 electoral votes
11. 55 electoral votes

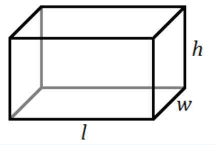
Some formulas require more than one number for the input.

Exercises 2.2.5.1

When a patient's blood pressure is checked, they are usually told two numbers: the systolic blood pressure (SBP) and the diastolic blood pressure (DBP). The mean arterial pressure (MAP) can be estimated by the following formula: $MAP = \dfrac{SBP + 2 \cdot DBP}{3}$. (The units are mm Hg, or millimeters of mercury.) Calculate the mean arterial pressure for each patient.

12. SBP = 120, DBP = 75

13. SBP = 140, DBP = 90



UPS uses this formula to determine the "measurement" of a package with length l, width w, and height h: $m = l + 2w + 2h$.[5] Determine the measurement of a package with the following dimensions.

14. length 18 inches, width 12 inches, height 14 inches

15. length 16 inches, width 14 inches, height 15 inches

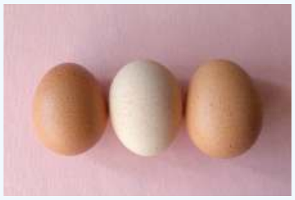
Answer

12. 90 mm Hg
13. around 107 mm Hg
14. 70 in
15. 74 in

The next set of exercises involves a formula that gives a yes or no answer.

? Exercises 2.2.5.1

In Australia, a chicken egg is designated “large” if its mass, in grams, satisfies the following formula: $|m-54.1| \leq 4.1$. Determine whether each egg qualifies as large.[6]



16. Egg 1’s mass is 57.8 grams.

17. Egg 2’s mass is 58.3 grams.

18. Egg 3’s mass is 49.8 grams.

19. Egg 4’s mass is 50.0 grams.

Answer

16. yes
17. no; too large
18. no; too small
19. yes

Temperature

The Celsius temperature scale is based on the freezing point of water (0°C = 32°F), and the boiling point of water (100°C = 212°F). By subtracting these numbers, we can see that a difference of 180°F is equivalent to 100°C. The ratio $\frac{180}{100}$ reduces to $\frac{9}{5}$, which means that 9 degrees on the Fahrenheit scale is equivalent to 5 degrees on the Celsius scale. (Of course, $\frac{180}{100}$ is also equal to 1.8, which means that 1 degree Celsius is equivalent to 1.8 degrees Fahrenheit.) Because Fahrenheit and Celsius do not have the same zero point, however, we must add or subtract 32 as well. See the formulas below.

Temperature Formulas

$$F=\frac{9}{5}C+32 \text{ or } F=1.8C+32$$

$$C=\frac{5}{9}(F-32) \text{ or } C=(F-32)\div 1.8$$

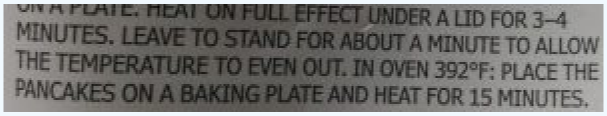
? Exercises 2.2.5.1

20. The temperature on a cool day is 10°C. Convert this temperature to Fahrenheit.

21. Normal body temperature is 98.6°F. What is this temperature in Celsius?

22. The FDA recommends that a freezer be set below -18°C. What is the Fahrenheit equivalent?

23. A package of frozen pancakes from IKEA calls for the oven to be set to 392°F. Clearly, this was originally calculated in Celsius. What is the corresponding Celsius temperature?



ON A PLATE. HEAT ON FULL EFFECT UNDER A LID FOR 3–4
MINUTES. LEAVE TO STAND FOR ABOUT A MINUTE TO ALLOW
THE TEMPERATURE TO EVEN OUT. IN OVEN 392°F: PLACE THE
PANCAKES ON A BAKING PLATE AND HEAT FOR 15 MINUTES.

Answer

20. $50°C$

21. $37°C$

22. $-0.4°F$

23. $200°C$

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1. pe.usps.com/text/dmm300/Notice123.htm#_c037 ↩
2. https://www.radiocab.net/services-radio-cab/ ↩
3. https://www.pewresearch.org/fact-tank/2018/05/31/u-s-population-keeps-growing-but-house-of-representatives-is-same-size-as-in-taft-era/ ↩
4. https://en.Wikipedia.org/wiki/List_of_states_and_territories_of_the_United_States_by_population ↩
5. www.ups.com/us/en/help-center/packaging-and-supplies/prepare-overize.page ↩
6. https://en.Wikipedia.org/wiki/Chicken_egg_sizes ↩

2.2.6: Perimeter and Circumference

You may use a calculator throughout this module if needed.

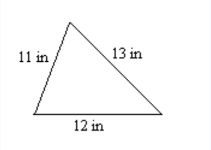
Perimeter

A polygon is a closed geometric figure with straight sides. Common polygons include triangles, squares, rectangles, parallelograms, trapezoids, pentagons, hexagons, octagons… The perimeter of a polygon is the distance around the outside. In general, to find the perimeter of a polygon, you can add up the lengths of all of its sides.

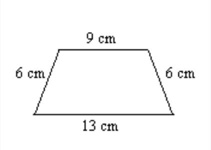
Also, if you haven't already, now is the time to get in the habit of including units in your answers.

? Exercises 2.2.6.1

1. Find the perimeter of the triangle.



2. Find the perimeter of the trapezoid.



Answer

1. 36 in
2. 34 cm

If we know that some of the sides of a polygon are equal, we can use a formula as an alternative to adding up all of the lengths individually. The first formula shown below uses the variable s for the side of a square. The rectangle formulas use l for length and w for width, or b for base and h for height; these terms are interchangeable.

Perimeter Formulas

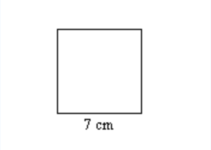
Square: $P=4s$

Rectangle: $P=2l+2w$ or $P=2b+2h$

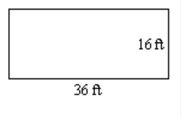
Rectangle: $P=2(l+w)$ or $P=2(b+h)$

? Exercises 2.2.6.1

3. Find the perimeter of the square.



4. Find the perimeter of the rectangle.



5. A storage area, which is a rectangle that is 45 feet long and 20 feet wide, needs to be fenced around all four sides. How many feet of fencing is required? (To keep it simple, ignore any gates or other complications.)

6. Giancarlo is putting crown molding around the edge of the ceiling of his living room. If the room is a 12-foot by 16-foot rectangle, how much crown molding does he need?

Answer

3. $28\,\text{cm}$

4. $104\,\text{ft}$

5. $130\,\text{ft}$

6. 56 ft

The sides of a regular polygon are all equal in length. Therefore, multiplying the length of a side by the number of sides will give us the perimeter.

Perimeter Formula

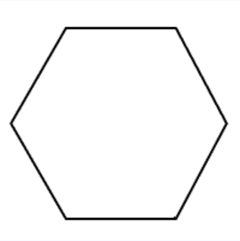
Regular Polygon with n sides of length s:

$$P = n \cdot s$$

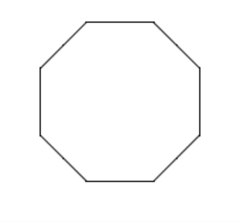
Exercises 2.2.6.1

Find the perimeter of each regular polygon.

7. Each side of the hexagon is 4 inches long.



8. Each side of the octagon is 2.5 centimeters long.



Answer

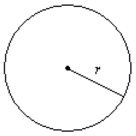
7. $24\,\text{in}$

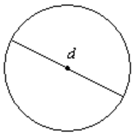
8. $20\,\text{cm}$

Circumference

Instead of calling it the perimeter, the distance around the outside of circle is called the ***circumference***. Let's review some circle vocabulary before moving on.

Every point on a circle is the same distance from its center. This distance from the center to the edge of the circle is called the ***radius***. The distance from one edge to another, through the center of the circle, is called the ***diameter***. As you can see, the diameter is twice the length of the radius.





Throughout history, different civilizations have discovered that the circumference of a circle is slightly more than 3 times the length of its diameter. By the year 2000 BCE, the Babylonians were using the value $3\frac{1}{8}=3.125$ and the Egyptians were using the value $3\frac{13}{81}\approx 3.1605$.[1] The value $3\frac{1}{7}\approx 3.1429$ is an even better approximation for the ratio of the circumference to the diameter. However, the actual value cannot be written as an exact fraction. It is the irrational number π, pronounced "pie", which is approximately 3.14159.

Circumference Formulas

$$C=\pi d$$

$$C=2\pi r$$

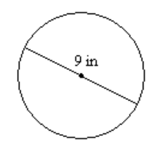
Any scientific calculator will have a π key; using this will give you the most accurate result, although you should be sure to round your answer appropriately. (See this module if you need a refresher on rounding with multiplying or dividing.) Many people use 3.14 as an approximation for π, but this can lead to round-off error; if you must use an approximation, 3.1416 is better than 3.14.

Sometimes we bend the rules in this textbook and ask you to round to a certain place value instead of rounding to a certain number of significant digits.

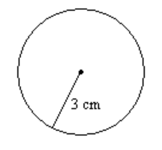
Exercises 2.2.6.1

Calculate the circumference of each circle. Round to the nearest tenth.

9.



10.



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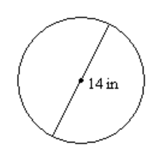

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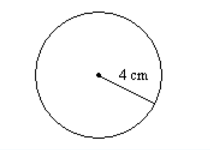


Answer

9. 28.3in

10. 18.8cm

11. 44.0in

12. 25.1cm

---

1. This information comes from Chapter 1 of the book *A History of Pi* by Petr Beckmann. It is a surprisingly interesting read. ↩

2.2.7: Percents Part 1

Percent Basics

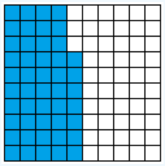
Percent means “per one hundred”. A percent is a ratio or fraction with a denominator of 100.

? Exercises 2.2.7.1

During Super Bowl XLIX between the Seahawks and Patriots, 89 out of 100 television sets in Seattle were tuned to the game.[1]

1. What percent of the television sets were tuned to the game?

2. What percent of the television sets were not tuned to the game?



3. What percent of the squares are shaded?

4. What percent of the squares are not shaded?

Answer

1. 89%
2. 11%
3. 47%
4. 53%

To write a percent as a fraction: drop the percent sign, write the number over 100, and simplify if possible.

Tip: If a percent is greater than 100%, the fraction will be greater than 1. If a percent is less than 1%, the fraction will be less than $\frac{1}{100}$.

? Exercises 2.2.7.1

Write each percent as a fraction, and simplify if possible.

5. About 71% of Earth’s surface is covered by water.[2]

6. About 1.3% of Earth’s land surface is permanent cropland.[3]

7. About 0.04% of Earth’s atmosphere is carbon dioxide.[4]

8. The worldwide number of active Facebook users in the fourth quarter of 2018 was approximately 102% of the number of users in the third quarter of 2018.[5]

Answer

5. $\frac{71}{100}$
6. $\frac{1.3}{100}=\frac{13}{1000}$
7. $\frac{0.04}{100}=\frac{1}{2500}$
8. $\frac{102}{100}=\frac{51}{50}$

To write a percent as a decimal: drop the percent sign and move the decimal point two places to the left.

? Exercises 2.2.7.1

Write each percent from Exercises 5 through 8 as a decimal.

9. 71%

10. 1.3%

11. 0.04%

12. 102%

Answer

9. 0.71

10. 0.013

11. 0.0004

12. 1.02

To write a decimal as a percent: move the decimal point two places to the right and insert a percent sign.

? Exercises 2.2.7.1

Write each decimal number as a percent.

13. 0.23

14. 0.07

15. 0.085

16. 2.5

Answer

13. 23%

14. 7%

15. 8.5%

16. 250%

To write a fraction as a percent, write the fraction as a decimal by dividing the numerator by the denominator, then move the decimal point two places to the right and insert a percent sign.

Alternate method: Recall from the fractions module that if the denominator of a fraction has no prime factors other than 2‘s and 5‘s, then the fraction can be built up to have a denominator of 10, or 100, or $1,000\ldots$

? Exercises 2.2.7.1

17. 7 out of 25 students were tardy on Wednesday. Write $\frac{7}{25}$ as a percent.

18. A package of 24 m&m's contained 3 orange m&m's. Write $\frac{3}{24}$ as a percent.

Answer

17. 28%

18. 12.5%

Solving Percent Problems: Finding the Amount

You may use a calculator throughout the remainder of this module.

We often use the words ***amount*** and ***base*** in a percent problem. The ***amount*** is the answer we get after finding the percent of the original number. The ***base*** is the original number, the number we find the percent of. (You may also think of the amount as the part, and the base as the whole.) We can call the percent the ***rate***.

$$\text{Amount} = \text{Rate} \cdot \text{Base}$$

$$A = R \cdot B$$

Be sure to change the percent to a decimal before multiplying.

? Exercises 2.2.7.1

19. What is 9% of 350?

20. 30% of 75 is what number?

21. Find 13.5% of 500.

22. 125% of 80 is equal to what amount?

23. What number is 40% of 96.5?

24. Calculate 0.5% of 450.

Suppose you buy an electric drill with a retail price of $\$\ 109.97$ in a city with 8.5% sales tax.

25. Find the amount of the tax. Round to the nearest cent, if necessary.

26. How much do you pay in total?

Answer

19. 31.5

20. 22.5

21. 67.5

22. 100

23. 38.6

24. 2.25

25. $\$\ 9.35$

26. $\$\ 119.32$

---

1. https://twitter.com/darrenrovell/status/562258101337067521 ↩
2. https://en.Wikipedia.org/wiki/Earth#Surface ↩

3. https://en.Wikipedia.org/wiki/Earth#Surface ↵
4. https://en.Wikipedia.org/wiki/Atmosphere_of_Earth#Composition ↵
5. https://www.statista.com/statistics/264810/number-of-monthly-active-facebook-users-worldwide/ ↵

2.2.8: Ratios, Rates, Proportions

Ratios & Rates

A ratio is the quotient of two numbers or the quotient of two quantities with the same units.

When writing a ratio as a fraction, the first quantity is the numerator and the second quantity is the denominator.

? Exercise 2.2.8.1

1. Find the ratio of 45 minutes to 2 hours. Simplify the fraction, if possible.

Answer

1. $\frac{3}{8}$

A rate is the quotient of two quantities with different units. You must include the units.

When writing a rate as a fraction, the first quantity is the numerator and the second quantity is the denominator. Simplify the fraction, if possible. Include the units in the fraction.

? Exercise 2.2.8.1

2. A car travels 105 miles in 2 hours. Write the rate as a fraction.

Answer

2. $\frac{105 \text{ mi}}{2 \text{ hr}}$

A unit rate has a denominator of 1. If necessary, divide the numerator by the denominator and express the rate as a mixed number or decimal.

? Exercise 2.2.8.1

3. A car travels 105 miles in 2 hours. Write as a unit rate.

Answer

3. $\frac{52.5 \text{ mi}}{1 \text{ hr}}$ or 52.5 miles per hour

A unit price is a rate with the price in the numerator and a denominator equal to 1. The unit price tells the cost of one unit or one item. You can also simply divide the cost by the size or number of items.

❓ Exercises 2.2.8.1

4. An 18-ounce box of cereal costs $\$ 3.59$. Find the unit price.

5. A 12-ounce box of cereal costs $\$ 2.99$. Find the unit price.

6. Which box has a lower unit price?

Answer

4. $\$ 0.199$/oz, or around 20 cents per ounce
5. $\$ 0.249$/oz, or around 25 cents per ounce
6. the 18-ounce box has the lower unit price

Proportions

A proportion says that two ratios (or rates) are equal.

❓ Exercises 2.2.8.1

Determine whether each proportion is true or false by simplifying each fraction.

7. $\frac{6}{8}=\frac{21}{28}$

8. $\frac{10}{15}=\frac{16}{20}$

Answer

7. $\frac{3}{4}=\frac{3}{4}$; true
8. $\frac{2}{3}\neq\frac{4}{5}$; false

A common method of determining whether a proportion is true or false is called cross-multiplying or finding the ***cross products***. We multiply diagonally across the equal sign. In a true proportion, the cross products are equal.

$$\frac{a}{b}=\frac{c}{d}\ \rightarrow a\cdot d=b\cdot c$$

Exercises 2.2.8.1

Determine whether each proportion is true or false by cross-multiplying.

9. $\frac{6}{8} = \frac{21}{28}$

10. $\frac{10}{15} = \frac{16}{20}$

11. $\frac{14}{4} = \frac{15}{5}$

12. $\frac{0.8}{4} = \frac{5}{25}$

Answer

9. $168 = 168$; true

10. $200 \neq 240$; false

11. $70 \neq 60$; false

12. $20 = 20$; true

As we saw in a previous module, we can use a variable to stand for a missing number. If a proportion has a missing number, we can use cross multiplication to solve for the missing number. This is as close to algebra as we get in this textbook.

To solve a proportion for a variable:

1. Set the cross products equal to form an equation of the form $a \cdot d = b \cdot c$.
2. Isolate the variable by rewriting the multiplication equation as a division equation.
3. Check the solution by substituting the answer into the original proportion and finding the cross products.

You may discover slightly different methods that you prefer.[1] If you think "Hey, can't I do this a different way?", you may be correct.

Exercises 2.2.8.1

Solve for the variable.

13. $\frac{8}{10} = \frac{x}{15}$

14. $\frac{3}{2} = \frac{7.5}{n}$

15. $\frac{3}{k} = \frac{18}{24}$

16. $\frac{w}{6} = \frac{15}{9}$

17. $\frac{5}{4} = \frac{13}{x}$

18. $\frac{3.2}{7.2} = \frac{m}{4.5}$ (calculator recommended)

Answer

13. $x = 12$

14. $n = 5$

15. $k = 4$

16. $w = 10$

17. $x = 10.4$

18. $m = 2.0$

Problems that involve rates, ratios, scale models, etc. can be solved with proportions. When solving a real-world problem using a proportion, be consistent with the units.

Exercises 2.2.8.1

19. Tonisha drove her car 320 miles and used 12.5 gallons of gas. At this rate, how far could she drive using 10 gallons of gas?

20. Marcus worked 14 hours and earned $\$\ 210$. At the same rate of pay, how long would he have to work to earn $\$\ 300$?

21. A picture of your author appearing on *Jeopardy!* that is 375 pixels high and 475 pixels wide needs to be reduced in size so that it is 150 pixels high. If the height and width are kept proportional, what is the width of the picture after it has been reduced?

Answer

19. 256 miles

20. 20 hours

21. 190 pixels wide

---

1. The steps in the box are designed to avoid mentioning the algebraic step of dividing both sides of the equation by a number. If you are comfortable with basic algebra, then you would phrase step 2 differently. ↩

- **1.10: Ratios, Rates, Proportions** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

2.2.9: Percents Part 2 and Error Analysis

You may use a calculator throughout this module.

Recall: The amount is the answer we get after finding the percent of the original number. The base is the original number, the number we find the percent of. We can call the percent the rate.

When we looked at percents in a previous module, we focused on finding the amount. In this module, we will learn how to find the percentage rate and the base.

$$\text{Amount} = \text{Rate} \cdot \text{Base}$$

$$A = R \cdot B$$

We can translate from words into algebra.

- “is” means equals
- “of” means multiply
- “what” means a variable

Solving Percent Problems: Finding the Rate

Suppose you earned 56 points on a 60-point quiz. To figure out your grade as a percent, you need to answer the question “56 is what percent of 60?” We can translate this sentence into the equation $56 = R \cdot 60$.

Exercises 2.2.9.1

1. 56 is what percent of 60?

2. What percent of 120 is 45?

Answer

1. 93% or 93.3%
2. 37.5%

Be aware that this method gives us the answer in decimal form and we must move the decimal point to convert the answer to a percent.

Also, if the instructions don’t explicitly tell you how to round your answer, use your best judgment: to the nearest whole percent or nearest tenth of a percent, to two or three significant figures, etc.

Solving Percent Problems: Finding the Base

Suppose you earn 2% cash rewards for the amount you charge on your credit card. If you want to earn $\$\,50$ in cash rewards, how much do you need to charge on your card? To figure this out, you need to answer the question “50 is 2% of what number?” We can translate this into the equation $50 = 0.02 \cdot B$.

? Exercises 2.2.9.1

3. $\$ 50$ is 2% of what number?

4. 5% of what number is 36?

Answer

3. $\$ 2,500$
4. 720

Solving Percent Problems: Using Proportions

Recall that a percent is a ratio, a fraction out of 100. Instead of translating word for word as we have just been doing, we can set up a proportion with the percentage rate over 100. Because the base is the original amount, it corresponds to 100%.

$$\frac{\text{amount}}{\text{base}}=\frac{\text{percent}}{100}$$

Let's try Exercises 1 through 4 again, using proportions.

? Exercises 2.2.9.1

5. 56 is what percent of 60?

6. What percent of 120 is 45?

7. $\$ 50$ is 2% of what number?

8. 5% of what number is 36?

Answer

5. 93% or 93.3%
6. 37.5%
7. $\$ 2,500$
8. 720

Now that we have looked at both methods, you are free to use whichever method you prefer: percent equations or proportions.

? Exercises 2.2.9.1

9. An 18% tip will be added to a dinner that cost $\$ 107.50$. What is the amount of the tip?

10. The University of Oregon women's basketball team made 13 of the 29 three-points shots they attempted during a game against UNC. What percent of their three-point shots did the team make?

11. 45% of the people surveyed answered "yes" to a poll question. If 180 people answered "yes", how many people were surveyed altogether?

Answer

9. $\$ 19.35$
10. 44.8% or 45%
11. 400 people were surveyed

Solving Percent Problems: Percent Increase

When a quantity changes, it is often useful to know by what percent it changed. If the price of a candy bar is increased by 50 cents, you might be annoyed because it's it's a relatively large percentage of the original price. If the price of a car is increased by 50 cents, though, you wouldn't care because it's such a small percentage of the original price.

To find the percent of increase:

1. Subtract the two numbers to find the amount of increase.
2. Using this result as the amount and the **original** number as the base, find the unknown percent.

Notice that we always use the ***original*** number for the base, the number that occurred earlier in time. In the case of a percent increase, this is the smaller of the two numbers.

Exercises 2.2.9.1

12. The price of a candy bar increased from $\$ 0.89$ to $\$ 1.39$. By what percent did the price increase?

13. The population of Portland in 2010 was $583,793$. The estimated population in 2019 was $654,741$. Find the percent of increase in the population.[1]

Answer

12. 56.2%increase

13. 12.2%increase

Solving Percent Problems: Percent Decrease

Finding the percent decrease in a number is very similar.

To find the percent of decrease:

1. Subtract the two numbers to find the amount of decrease.
2. Using this result as the amount and the **original** number as the base, find the unknown percent.

Again, we always use the ***original*** number for the base, the number that occurred earlier in time. For a percent decrease, this is the larger of the two numbers.

Exercises 2.2.9.1

14. During a sale, the price of a candy bar was reduced from $\$ 1.39$ to $\$ 0.89$. By what percent did the price decrease?

15. The number of students enrolled at Clackamas Community College decreased from $7,439$ in Summer 2019 to $4,781$ in Summer 2020. Find the percent of decrease in enrollment.

Answer

14. 36.0%decrease

15. 35.7%decrease

Relative Error

In an earlier module, we said that a measurement will always include some error, no matter how carefully we measure. It can be helpful to consider the size of the error relative to the size of what is being measured. As we saw in the examples above, a difference of 50 cents is important when we're pricing candy bars but insignificant when we're pricing cars. In the same way, an error of an eighth of an inch could be a deal-breaker when you're trying to fit a screen into a window frame, but an eighth of an inch is insignificant when you're measuring the length of your garage.

The ***expected outcome*** is what the number would be in a perfect world. If a window screen is supposed to be exactly 25 inches wide, we call this the expected outcome, and we treat it as though it has infinitely many significant digits. In theory, the expected outcome is $25.000000\ldots$

To find the ***absolute error***, we subtract the measurement and the expected outcome. Because we always treat the expected outcome as though it has unlimited significant figures, the absolute error should have the same precision (place value) as the *measurement*, not the expected outcome.

To find the ***relative error***, we divide the absolute error by the expected outcome. We usually express the relative error as a percent. In fact, the procedure for finding the relative error is identical to the procedures for finding a percent increase or percent decrease!

To find the relative error:

1. Subtract the two numbers to find the absolute error.
2. Using the **absolute error** as the amount and the **expected outcome** as the base, find the unknown percent.

Exercisew 2.2.9.1

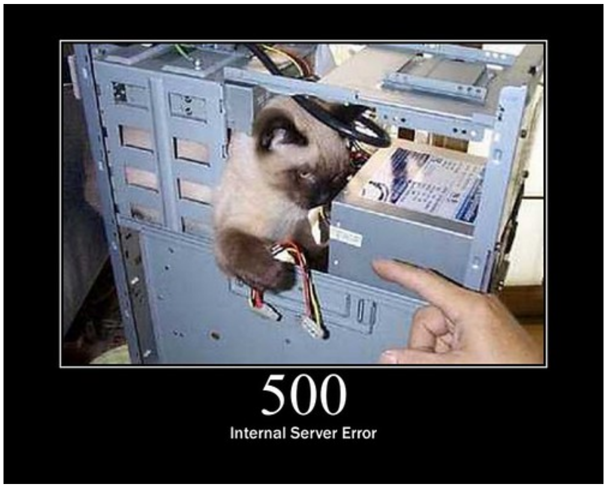
16. A window screen is measured to be $25\frac{3}{16}$ inches wide instead of the advertised 25 inches. Determine the relative error, rounded to the nearest tenth of a percent.

17. The contents of a box of cereal are supposed to weigh 10.8 ounces, but they are measured at 10.67 ounces. Determine the relative error, rounded to the nearest tenth of a percent.

Answer

16. $0.1875 \div 25 \approx 0.8\%$

17. $0.13 \div 10.8 \approx 1.2\%$



Tolerance

The ***tolerance*** is the maximum amount that a measurement is allowed to differ from the expected outcome. For example, the U.S. Mint needs its coins to have a consistent size and weight so that they will work in vending machines. A dime (10 cents) weighs 2.268 grams, with a tolerance of ± 0.091 grams.[2] This tells us that the minimum acceptable weight is $2.268 - 0.091 = 2.177$ grams, and the maximum acceptable weight is $2.268 + 0.091 = 2.359$ grams. A dime with a weight outside of the range $2.177 \leq \text{weight} \leq 2.359$ would be unacceptable.

Exercises 2.2.9.1

A U.S. nickel (5 cents) weighs 5.000 grams with a tolerance of ± 0.194 grams.

18. Determine the lowest acceptable weight and highest acceptable weight of a nickel.

19. Determine the relative error of a nickel that weighs 5.21 grams.

A U.S. quarter (25 cents) weighs 5.670 grams with a tolerance of ± 0.227 grams.

20. Determine the lowest acceptable weight and highest acceptable weight of a quarter.

21. Determine the relative error of a quarter that weighs 5.43 grams.

Answer

18. 4.806g; 5.194g

19. $0.21 \div 5.000 = 4.2\%$

20. 5.443g; 5.897g

21. $0.24 \div 5.670 \approx 4.2\%$

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1. www.census.gov/quickfacts/fact/table/portlandcityoregon,OR,US/PST045219 ↵
2. https://www.usmint.gov/learn/coin-and-medal-programs/coin-specifications and https://www.thesprucecrafts.com/how-much-do-coins-weigh-4171330↵

2.2.10: Percents Part 3

You may use a calculator throughout this module.

There is one more situation involving percents that often trips people up: working backwards from the result of a percent change to find the original value.

$$\text{Amount} = \text{Rate} \cdot \text{Base}$$

$$A = R \cdot B$$

Finding the Base After Percent Increase

Suppose a 12% tax is added to a price; what percent of the original is the new amount?

Well, the original number is 100% of itself, so the new amount must be $100\% + 12\% = 112\%$ of the original.

As a proportion, $\frac{A}{B} = \frac{112}{100}$. As an equation, $A = 1.12 \cdot B$.

If a number is **increased** by a percent, add that percent to 100% and use that result for R.

The most common error in solving this type of problem is applying the percent to the new number instead of the original. For example, consider this question: “After a 12% increase, the new price of a computer is $\$1,120$. What was the original price?”

People often work this problem by finding 12% of $\$1,120$ and subtracting that away: 12% of $1,120$ is 134.40, and $1,120 - 134.40 = 985.60$ It appears that the original price was $\$985.60$, but if we check this result, we find that the numbers don’t add up. 12% of 985.60 is 118.272, and $985.60 + 118.272 = 1,103.872$, not $1,120$.

The correct way to think about this is $1,120 = 1.12 \cdot B$. Dividing $1,120$ by 1.12 gives us the answer $1,000$, which is clearly correct because we can find that 12% of $1,000$ is 120, making the new amount $1,120$. The original price was $\$1,000$.

To summarize, we cannot subtract 12% from the new amount; we must instead ***divide*** the new amount by 112%.

? Exercises 2.2.10.1

1. A sales tax of 8% is added to the selling price of a lawn tractor, making the total price $\$1,402.92$ What is the selling price of the lawn tractor without tax?

2. The U.S. population in 2018 was estimated to be 327.2 million, which represents a 7.6% increase from 2008. What was the U.S. population in 2008?

Answer

1. $\$1,299.00$
2. 304.1 million

Finding the Base After Percent Decrease

Suppose a 12% discount is applied to a price; what percent of the original is the new amount?

As above, the original number is 100% of itself, so the new amount must be $100\% - 12\% = 88\%$ of the original.

As a proportion, $\frac{A}{B}=\frac{88}{100}$. As an equation, $A=0.88\cdot B$.

If a number is **decreased** by a percent, subtract that percent from 100% and use that result for R.

As above, the most common error in solving this type of problem is applying the percent to the new number instead of the original. For example, consider this question: “After a 12% decrease, the new price of a computer is $\$880$. What was the original price?”

People often work this problem by finding 12% of 880 and adding it on: 12% of 880 is 105.60, and $880+105.60=985.60$. It appears that the original price was $\$985.60$, but if we check this result, we find that the numbers don’t add up. 12% of 985.60 is 118.272, and $985.60-118.272=867.328$, not 880.

The correct way to think about this is $880=0.88\cdot B$. Dividing 880 by 0.88 gives us the answer $1,000$, which is clearly correct because we can find that 12% of $1,000$ is 120, making the new amount 880. The original price was $\$1,000$.

To summarize, we cannot add 12% to the new amount; we must instead ***divide*** the new amount by 88%.

Exercises 2.2.10.1

3. A city department’s budget was cut by 5% this year. If this year’s budget is $\$3.04$ million, what was last year’s budget?

4. CCC’s enrollment in Summer 2019 was $9,116$ students, which was a decrease of 2.17% from Summer 2018. What was the enrollment in Summer 2018? (Round to the nearest whole number.)[1]

5. An educational website claims that by purchasing access for $\$5$, you’ll save 69% off the standard price. What was the standard price? (Use you best judgment when rounding your answer.)

Answer

3. $\$3.20$ million

4. $9,318$ students

5. $\$16.13$, or more likely, just $\$16$.

---

[1] These enrollment numbers don't match those in Percents Part 2, which makes me wonder how accurate the yearly reports are. Or maybe I inadvertently grabbed data from two different ways that enrollment was being counted. ↵

2.2.11: Angles

You will need a calculator near the end of this module.

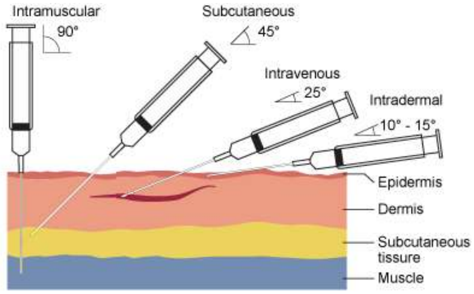
an angle grinder

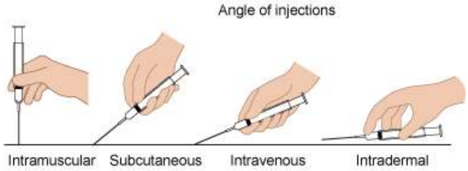
Angle measurement is important in construction, surveying, physical therapy, and many other fields. We can visualize an angle as the figure formed when two line segments share a common endpoint. We can also think about an angle as a measure of rotation. A full rotation or a full circle is 360°, so a half rotation or U-turn is 180°, and a quarter turn is 90°.

We often classify angles by their size.

- **Acute Angle:** between 0° and 90°
- **Right Angle:** exactly 90°
- **Obtuse Angle:** between 90° and 180°
- **Straight Angle:** exactly 180°
- **Reflexive Angle:** between 180° and 360°

Lines that form a 90° angle are called ***perpendicular***. As shown below, the needle should be perpendicular to the body surface for an intramuscular injection.

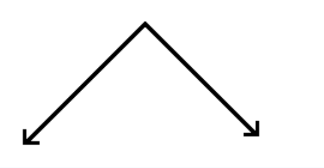


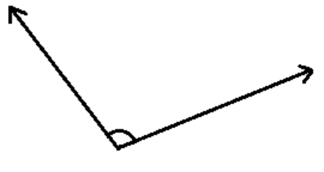


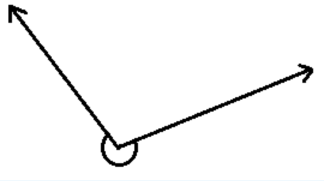
Exercises 2.2.11.1

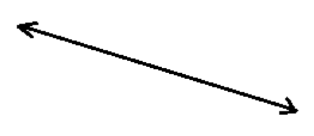
Identify each angle shown below as acute, right, obtuse, straight, or reflexive.

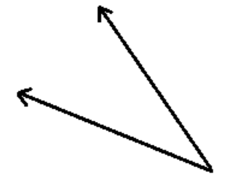
1.



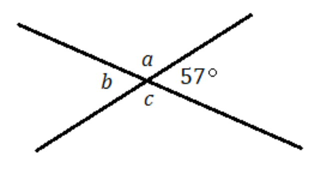
2. 

3. 

4. 

5. 

Find the measure of each unknown angle.

6. 

Answer

1. right angle
2. obtuse angle
3. reflexive angle
4. straight angle
5. acute angle
6. $a = 123^\circ$; $b = 57^\circ$; $c = 123^\circ$

Angles in Triangles

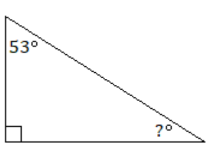
If you need to find the measures of the angles in a triangle, there are a few rules that can help.

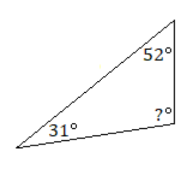
The sum of the angles of every triangle is 180°.

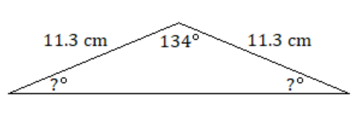
If any sides of a triangle have equal lengths, then the angles opposite those sides will have equal measures.

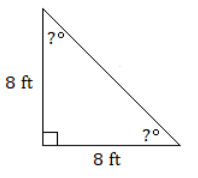
? Exercises 2.2.11.1

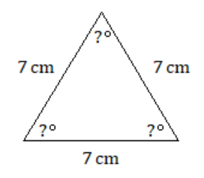
Find the measures of the unknown angles in each triangle.

7. 

8. 

9. 

10. 

11. 

Answer

7. 37°

8. 97°

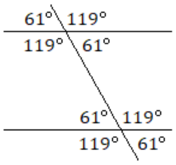
9. 23° each

10. 45° each

11. 60° each

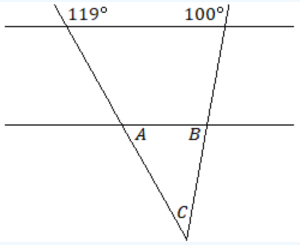
Angles and Parallel Lines

Two lines that point in the exact same direction and will never cross are called parallel lines. If two parallel lines are crossed by a third line, sets of equally-sized angles will be formed, as shown in the following diagram. All four acute angles will be equal in measure, all four obtuse angles will be equal in measure, and any acute angle and obtuse angle will have a combined measure of 180°.



Example 2.2.11.1

Find the measures of angles A, B, and C.



Solution

A. 61°

B. 80°

C. 39°

Degrees, Minutes, Seconds

It is possible to have angle measures that are not a whole number of degrees. It is common to use decimals in these situations, but the older method—called the ***degrees-minutes-seconds*** or DMS system—divides a degree using fractions out of 60: a minute is $\frac{1}{60}$ of a degree, and a second is $\frac{1}{60}$ of a minute, which means a second is $\frac{1}{3,600}$ of a degree. (Fortunately, these conversions work exactly like time; think of 1 degree as 1 hour.) For example, $2.5^\circ = 2^\circ 30'$.

We will look at the procedure for converting between systems, but there are online calculators such as the one at https://www.fcc.gov/media/radio/dms-decimal which will do the conversions for you.

If you have latitude and longitude in DMS, like N $18^\circ 54' 40''$ W $155^\circ 40' 51''$, and need to convert it to decimal degrees, the process is fairly simple with a calculator.

Converting from DMS to Decimal Degrees

Enter $\text{degrees} + \text{minutes} \div 60 + \text{seconds} \div 3600$ in your calculator. Round the result to the fourth decimal place, if necessary.[1]

Exercises 2.2.11.1

Convert each angle measurement from degrees-minutes-seconds into decimal form. Round to the nearest ten-thousandth, if necessary.

13. $67^\circ 48' 54''$

14. $19^\circ 37' 25''$

15. $34^\circ 14' 12''$

Answer

13. 67.815°

14. 19.6236°

15. 34.2367°

Going from decimal degrees to DMS is a more complicated process.

Converting from Decimal Degrees to DMS

1. The whole-number part of the angle measurement gives the number of degrees.
2. Multiply the decimal part by 60. The whole number part of this result is the number of minutes.
3. Multiply the decimal part *of the minutes* by 60. This gives the number of seconds (including any decimal part of seconds).

For example, let's convert 15.374°.

1. The *degrees* part of our answer will be 15.
2. The decimal part times 60 is $0.374 \cdot 60 = 22.44$ minutes. The *minutes* part of our answer will be 22.
3. The decimal part times 60 is $0.44 \cdot 60 = 26.4$ seconds. The *seconds* part of our answer will be 26.4.

So $15.374^\circ = 15^\circ 22' 26.4''$.

? Exercises 2.2.11.1

Convert each angle measurement from decimal into degrees-minutes-seconds form.

16. 26.785°

17. 58.216°

18. 41.13°

Answer

16. $26^\circ 47' 6''$

17. $58^\circ 12' 57.6''$

18. $41^\circ 7' 48''$

---

1. We round to four decimal places because 1 second of angle is $\frac{1}{3,600}$ of a degree. This is a smaller fraction than $\frac{1}{1,000}$ so our precision is slightly better than the thousandths place. ↩

2.2.12: Triangles

You may use a calculator throughout this module as needed.

Classifying Triangles

We can classify triangles into three categories based on the lengths of their sides.

- Equilateral triangle: all three sides have the same length
- Isosceles triangle: exactly two sides have the same length
- Scalene triangle: all three sides have different lengths

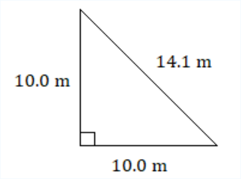
We can also classify triangles into three categories based on the measures of their angles.

- Obtuse triangle: one of the angles is an obtuse angle
- Right triangle: one of the angles is a right angle
- Acute triangle: all three of the angles are acute

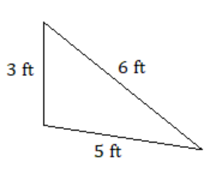
? Exercises 2.2.12.1

Classify each triangle by angle and side. For example, “acute scalene”.)

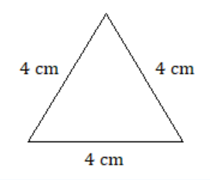
1.



2.



3.



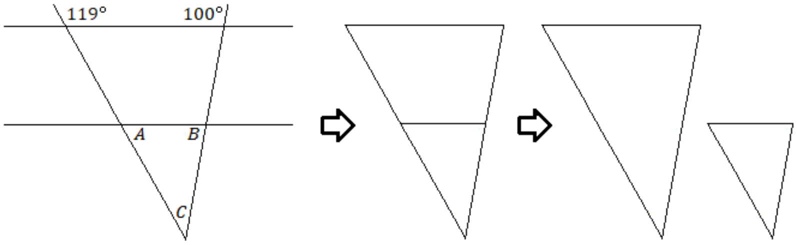
Answer

1. right isosceles triangle
2. obtuse scalene triangle

3. acute equilateral triangle (yes, an equilateral triangle will always be acute)

Similar Triangles

In one of the diagrams in the previous module, the parallel lines included two similar triangles, although they may be hard to see.

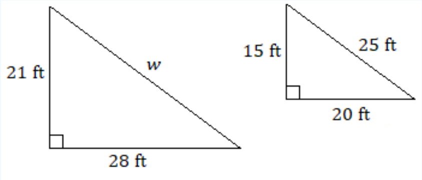


Two triangles are similar if the three angles of one triangle have the same measure as the three angles of the second triangle. The lengths of the sides of similar triangles will be in the same proportion. The triangles will have the same shape but the lengths will be scaled up or down.

? Exercises 2.2.12.1

Assume that each pair of triangles are similar. Use a proportion to find each unknown length.

4.



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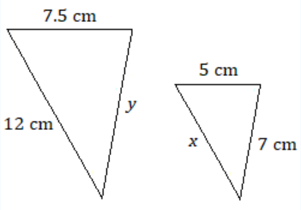


Answer

4. $w = 35$ ft

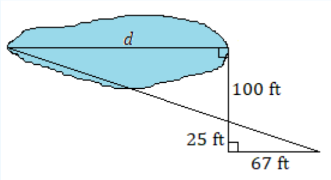
5. $x = 8$ cm; $y = 10.5$ cm

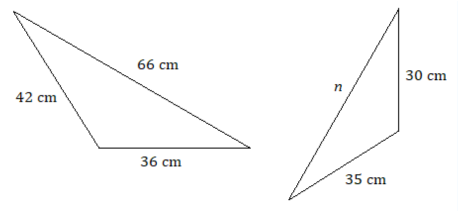
Recognizing corresponding sides can be more difficult when the figures are oriented differently.

? Exercises 2.2.12.1

Assume that each pair of triangles are similar. Use a proportion to find each unknown length.

6.



7. 

Answer

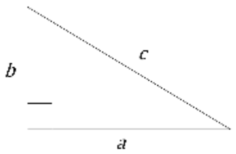
6. $d = 268$ ft

7. $n = 55$ cm

Right Triangles

In a right triangle, the two sides that form the right angle are called the ***legs***. The side opposite the right angle, which will always be the longest side, is called the ***hypotenuse***.

The Pythagorean theorem says that the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.



Theorem 2.2.12.1: The Pythagorean Theorem

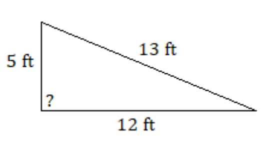
In a right triangle with legs a and b and hypotenuse c,

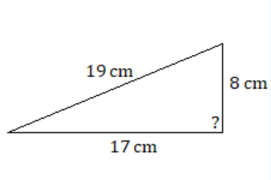
$$a^2 + b^2 = c^2$$

If you know the lengths of all three sides of a triangle, you can use the Pythagorean theorem to verify whether the triangle is a right triangle or not. The ancient Egyptians used this method for surveying when they needed to redraw boundaries after the yearly flooding of the Nile washed away their previous markings.[1]

Exercises 2.2.12.1

Use the Pythagorean theorem to determine whether either of the following triangles is a right triangle.

8. 

9. 

Answer

8. right triangle, because $5^2 + 12^2 = 13^2$

9. not a right triangle, because $8^2+17^2 \neq 19^2$

Before we continue, we need to briefly discuss square roots. Calculating a square root is the opposite of squaring a number. For example, $\sqrt{49}=7$ because $7^2=49$. If the number under the square root symbol is not a perfect square like 49, then the square root will be an irrational decimal that we will round off as necessary.

Exercises 2.2.12.1

Use a calculator to find the value of each square root. Round to the hundredths place.

10. $\sqrt{50}$

11. $\sqrt{296}$

12. $\sqrt{943}$

Answer

10. 7.07

11. 17.20

12. 30.71

We most often use the Pythagorean theorem to calculate the length of a missing side of a right triangle. Here are three different versions of the Pythagorean theorem arranged to find a missing side, so you don't have to use algebra with $a^2+b^2=c^2$.

The Pythagorean Theorem, three other versions

$c=\sqrt{a^2+b^2}$

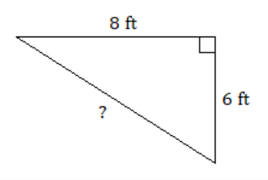
$b=\sqrt{c^2-a^2}$

$a=\sqrt{c^2-b^2}$

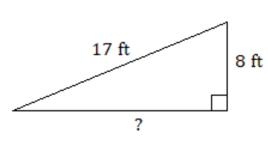
Exercises 2.2.12.1

Find the length of the missing side for each of these right triangles. Round to the nearest tenth, if necessary.

13.



14.



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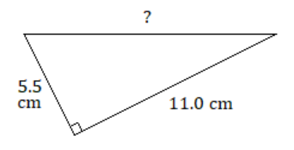
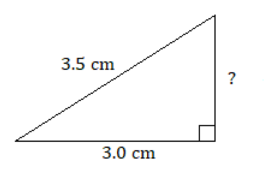


16. 

Answer

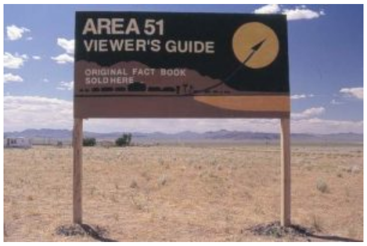
13. 10 ft
14. 15 ft
15. 12.3 cm
16. 1.8 cm

---

1. The surveyors were called "rope-stretchers" because they used a loop of rope 12 units long with 12 equally-spaced knots. Three rope-stretchers each held a knot, forming a triangle with lengths 3, 4, and 5 units. When the rope was stretched tight, they knew that the angle between the 3-unit and 4-unit sides was a right angle because $3^2+4^2=5^2$. From *Discovering Geometry: an Inductive Approach* by Michael Serra, Key Curriculum Press, 1997. ↵

2.2.13: Area of Polygons and Circles

You may use a calculator for most of this module as needed.

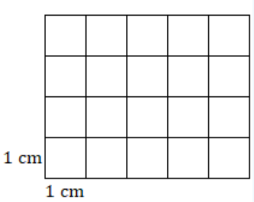


We have seen that the perimeter of a polygon is the distance around the outside. Perimeter is a length, which is one-dimensional, and so it is measured in linear units (feet, centimeters, miles, etc.). The area of a polygon is the amount of two-dimensional space inside the polygon, and it is measured in square units: square feet, square centimeters, square miles, etc.

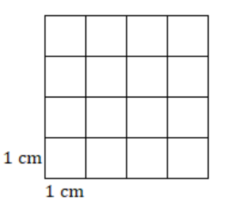
You can always think of area as the number of squares required to completely fill in the shape.

Exercises 2.2.13.1

1. Find the area of this rectangle.



2. Find the area of this square.



Answer

1. 20 cm^2
2. 16 cm^2

Rectangles and Squares

There are of course formulas for finding the areas of rectangles and squares; we don’t have to count little squares.

Area of a Rectangle

$A = lw$[1] or $A = bh$

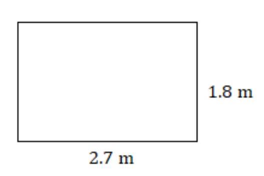
Area of a Square

$$A = s^2$$

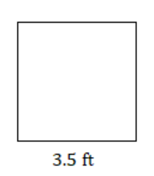
? Exercises 2.2.13.1

Find the area of each figure.

3.



4.



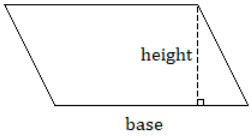
Answer

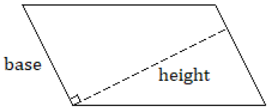
3. $4.86\ \mathrm{m}^2$

4. $12.25\ \mathrm{ft}^2$

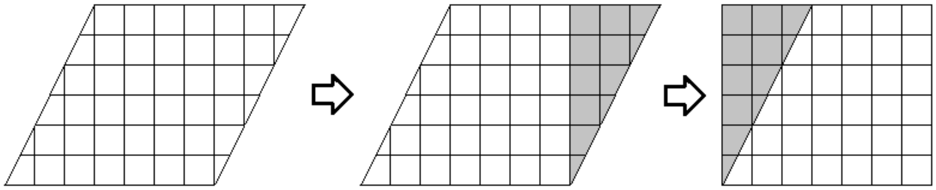
Parallelograms

Another common polygon is the ***parallelogram***, which looks like a tilted rectangle. As the name implies, the pairs of opposite sides are parallel and have the same length. Notice that, if we label one side as a base of the parallelogram, we have a perpendicular height which is *not* the length of the other sides.





The following set of diagrams shows that we can cut off part of a parallelogram and rearrange the pieces into a rectangle with the same base and height as the original parallelogram. A parallelogram with a base of 7 units and a vertical height of 6 units is transformed into a 7 by 6 rectangle, with an area of 42 square units.



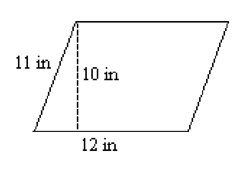
Therefore, the formula for the area of a parallelogram is identical to the formula for the area of a rectangle, provided that we are careful to use the base and the height, which must be perpendicular.

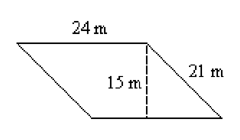
Area of a Parallelogram

$A = bh$

? Exercises 2.2.13.1

Find the area of each parallelogram.

5. 

6. 

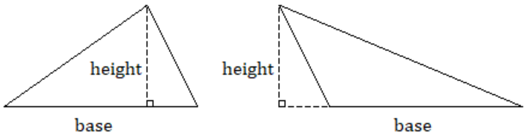
Answer

5. 120 in^2

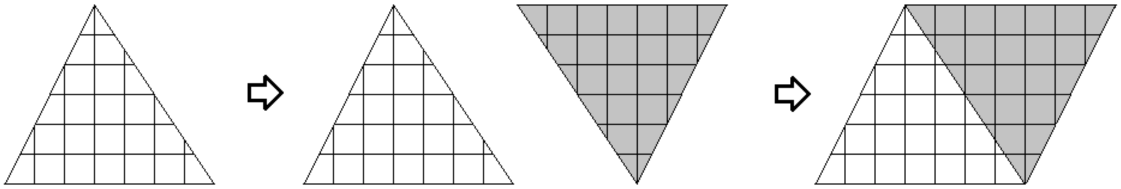
6. 360 m^2

Triangles

When we need to find the area of a triangle, we need to identify a base and a height that is perpendicular to that base. If the triangle is obtuse, you may have to imagine the height outside of the triangle and extend the base line to meet it.



As shown below, any triangle can be doubled to form a parallelogram. Therefore, the area of a triangle is one half the area of a parallelogram with the same base and height.



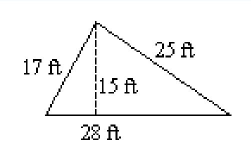
Area of a Triangle

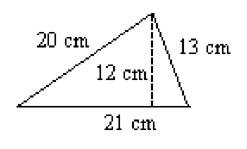
$A = \frac{1}{2}bh$ or $A = bh \div 2$

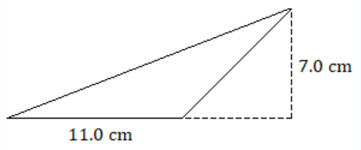
As with a parallelogram, remember that the height must be perpendicular to the base.

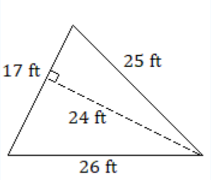
Exercises 2.2.13.1

Find the area of each triangle.

7. 

8. 

9. 

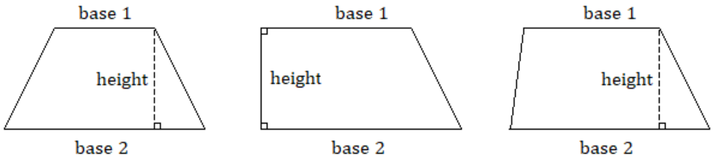
10. 

Answer

7. 210 ft^2
8. 126 cm^2
9. 38.5 cm^2
10. 204 ft^2

Trapezoids

A somewhat less common quadrilateral is the ***trapezoid***, which has exactly one pair of parallel sides, which we call the bases. The first example shown below is called an isosceles trapezoid because, like an isosceles triangle, its two nonparallel sides have equal lengths.



There are a number of ways to show where the area formula comes from, but the explanations are better in video because they can be animated.[2][3][4]

Area of a Trapezoid

$$A = \frac{1}{2}h(b_1 + b_2)$$

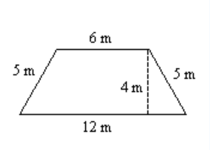
or

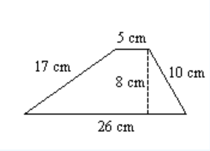
$$A = (b_1 + b_2)h \div 2$$

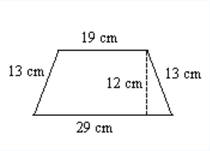
Don’t be intimidated by the subscripts on b_1 and b_2; it’s just a way to name two different measurements using the same letter for the variable. (Many people call the bases a and b instead; feel free to write it whichever way you prefer.) Whatever you call them, you just add the two bases, multiply by the height, and take half of that.

Exercises 2.2.13.1

Find the area of each trapezoid.

11. 

12. 

13. 

Answer

11. 36 m^2

12. 124 cm^2

13. 288 cm^2

Circles

The area of a circle is π times the square of the radius: $A=\pi r^2$. The units are still square units, even though a circle is round. (Think of the squares on a round waffle.) Because we can't fit a whole number of squares—or an exact fraction of squares—inside the circle, the area of a circle will be an approximation.

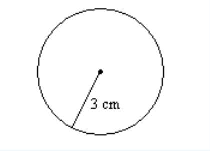
Area of a Circle

\[A=\pi{r^2}\)]

Remember that $\pi \approx 3.1416$.

Exercises 2.2.13.1

Find the area of each circle. Round to the nearest tenth or to three significant figures, whichever seems appropriate.

14. 

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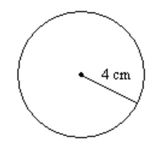

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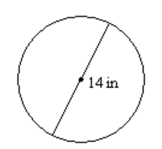
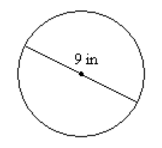
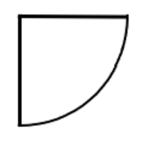


17.

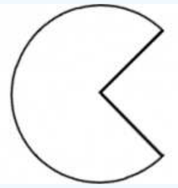


Each figure is a fraction of a circle. Calculate each area.

18. The radius of the quarter circle is 5 meters.



19. A quarter circle has been removed from a circle with a diameter of 7 feet.



Answer

14. 28.3 cm^2
15. 50.3 cm^2
16. 153.9 in^2
17. 63.6 in^2
18. 19.6 m^2
19. 28.9 ft^2

---

1. You might choose to use capital letters for the variables here because a lowercase letter "l" can easily be mistaken for a number "1". ↩
2. https://youtu.be/yTnYRpcZA9c ↩
3. https://youtu.be/WZtO3oERges ↩
4. https://youtu.be/uLHc6Br2veg ↩

- **1.19: Area of Polygons and Circles** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

2.2.14: Composite Figures

This sunburst pattern is composed of (276) geometric tiles

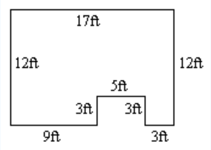
You may use a calculator throughout this module as needed.

Many objects have odd shapes made up of simpler shapes. A composite figure is a geometric figure which is formed by—or *composed* of—two or more basic geometric figures. We will look at a handful of fairly simple examples, but this concept can of course be extended to much more complicated figures.

To find the area of a composite figure, it is generally a good idea to divide it into simpler shapes and either add or subtract their areas as necessary.

? Exercises 2.2.14.1

A floor plan of a room is shown. The room is a 12-foot by 17-foot rectangle, with a 3-foot by 5-foot rectangle cut out of the south side.



1. Determine the amount of molding required to go around the perimeter of the room.

2. Determine the amount of flooring required to cover the entire area.

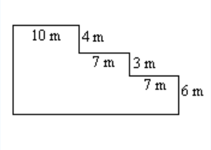
Answer

1. 64 ft
2. 189 ft^2

You may need to figure out some unmarked dimensions.

? Exercises 2.2.14.1

A plan for an irregular parking lot is shown.



3. Calculate the perimeter.

4. Calculate the area.

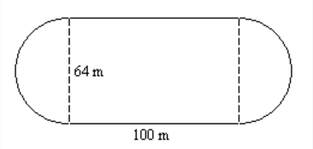
Answer

3. 74 m
4. 235 m^2

Sometimes, composite figures involve parts of circles coupled with polygons.

? Exercises 2.2.14.1

A high school is building a track at its athletic fields. The track, which is formed by two straight sides and two semicircles as shown in the plans below, is supposed to have a total length of 400 meters.



5. Determine the distance around the track. Will the track be the right length?

6. After the new track is built, landscapers need to lay sod on the field inside the track. What is the area of the field inside the track?

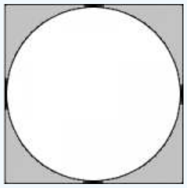
Answer

5. Based on the stated measurements, the distance around the track will be 401 meters, which appears to be 1 meter too long. In real life, precision would be very important here, and you might ask for the measurements to be given to the nearest tenth of a meter.

6. around $9,620\ \mathrm{m}^2$

If we need to determine a fraction or percent out of the whole, we may be able to solve the problem without knowing any actual measurements.

? Exercise 2.2.14.1



7. Circular disks are being cut from squares of sheet metal, with the remainder around the corners being discarded. Assuming that the circles are made as large as possible, what percent of the sheet metal will be discarded?

Answer

around 21.5%(Hint: Make up an easy number for the side of the square, like 2 or 10.)

2.2.15: Surface Area of Common Solids

You may use a calculator throughout this module.

We will now turn our attention from two-dimensional figures to three-dimensional figures, which we often call ***solids***, even if they are hollow inside. In this module, we will look the surface areas of some common solids. (We will look at volume in a later module.) Surface area is what it sounds like: it's the sum of the areas of all of the outer surfaces of the solid. When you are struggling to wrap a present because your sheet of wrapping paper isn't quite big enough, you are dealing with surface area.

There are two different kinds of surface area that are important: the ***lateral surface area*** (LSA) and ***total surface area*** (TSA). To visualize the difference between LSA and TSA, consider a can of soup. The lateral surface area would be used to measure the size of the paper label around the can. The total surface area would be used to measure the amount of sheet metal needed to make the can. In other words, the total surface area includes the top and bottom, whereas the lateral surface area does not.

Rectangular Solids

A ***rectangular solid*** looks like a rectangular box. It has three pairs of equally sized rectangles on the front and back, on the left and right, and on the top and bottom. A ***cube*** is a special rectangular solid with equally-sized squares for all six faces.

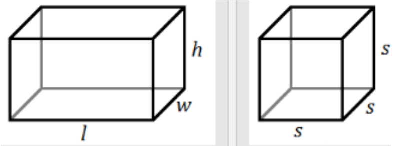


Figure $2.2.15.1$: (left) A rectangular solid and (right) a cube.

The lateral surface area is the combined total area of the four vertical faces of the solid, but not the top and bottom. If you were painting the four walls of a room, you would be thinking about the lateral surface area.

The total surface area is the combined total area of all six faces of the solid. If you were painting the four walls, the floor, and the ceiling of a room, you would be thinking about the total surface area.

For a rectangular solid with length l, width w, and height $h\ldots$[1]

$$LSA = 2lh + 2wh$$

$$TSA = 2lh + 2wh + 2lw$$

$$TSA = LSA + 2lw$$

For a cube with side length $s\ldots$

$$LSA = 4s^2$$

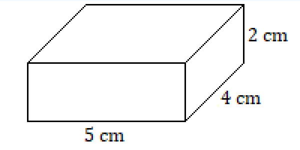
$$TSA = 6s^2$$

Note: These dimensions are sometimes called base, depth, and height.

Exercises 2.2.15.1

1. Find the lateral surface area of this rectangular solid.

2. Find the total surface area of this rectangular solid.



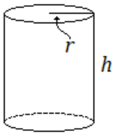
Answer

1. 36 cm^2

2. 76 cm^2

Cylinders

As mentioned earlier in this module, the lateral surface area of a soup can is the paper label, which is a rectangle. Therefore, the lateral surface area of a ***cylinder*** is a rectangle; its width is equal to the circumference of the circle, $2\pi r$, and its height is the height of the cylinder.



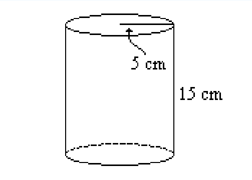
Since a cylinder has equal-sized circles at the top and bottom, its total surface area is equal to the lateral surface area plus twice the area of one of the circles.

For a cylinder with radius r and height $h\ldots$

- $LSA = 2\pi rh$
- $TSA = 2\pi rh + 2\pi r^2$
- $TSA = LSA + 2\pi r^2$

Be aware that if you are given the diameter of the cylinder, you will need to cut it in half before using these formulas.

Exercises 2.2.15.1



3. Find the lateral surface area of this cylinder.

4. Find the total surface area of this cylinder.

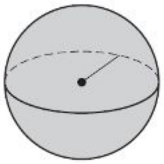
Answer

3. 471 cm^2

4. 628 cm^2

Spheres

The final solid of this module is the ***sphere***, which can be thought of as a circle in three dimensions: every point on the surface of a sphere is the same distance from the center. Because of this, a sphere has only one important measurement: its radius. Of course, its diameter could be important also, but the idea is that a sphere doesn't have different dimensions such as length, width, and height. A sphere has the same radius (or diameter) in every direction.



We would need to use calculus to derive the formula for the surface area of a sphere, so we'll just assume it's true and get on with the business at hand. Notice that, because a sphere doesn't have top or bottom faces, we don't need to worry about finding the lateral surface area. The only surface area is the total surface area.

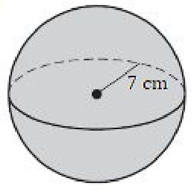
For a sphere with radius r or diameter d…

$$SA = 4\pi r^2 \text{ or } SA = \pi d^2$$

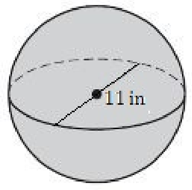
Coincidentally, the surface area of a sphere is 4 times the area of the cross-sectional circle at the sphere's widest part. You may find it interesting to try to visualize this, or head to the kitchen for a demonstration: if you cut an orange into four quarters, the peel on one of those quarter oranges has the same area as the circle formed by the first cut.

? Exercises 2.2.15.1

5. Find the surface area of this sphere.



6. Find the surface area of this sphere.



Answer

5. 616 cm^2
6. 380 cm^2

---

1. You might choose to use capital letters for the variables here because a lowercase letter "l" can easily be mistaken for a number "1". ↩

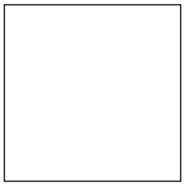
2.2.16: Converting Units of Area

You may use a calculator throughout this module.

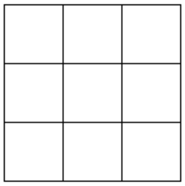
Converting between units of area requires us to be careful because square units behave differently than linear units.

U.S. System: Converting Measurements of Area

Consider a square yard; the area of a square with sides 1 yard long.



1 yard = 3 feet, so we can divide the square into three sections vertically and three sections horizontally to convert both dimensions of the square from yards to feet. This forms a 3 by 3 grid, which shows us visually that 1 square yard equals 9 square feet, not 3 square feet!



The linear conversion ratio of 1 to 3 means that that the conversion ratio for the areas is 1 to 3^2, or 1 to 9.

Here's another way to think about it without a diagram: $1\text{ yd} = 3\text{ ft}$, so $(1\text{ yd})^2 = (3\text{ ft})^2$. To remove the parentheses, we must square the number *and* square the units: $(3\text{ ft})^2 = 3^2\text{ ft}^2 = 9\text{ ft}^2$.

More generally, we need to **square** the linear conversion factors when converting units of area. If the linear units have a ratio of 1 to n, the square units will have a ratio of 1 to n^2.

? Exercises 2.2.16.1

1. An acre is defined as the area of a 660 foot by 66 foot rectangle. (That's a furlong by a chain, if you were curious.) How many square feet are in 1 acre?

2. How many square yards are in 1 acre?

3. How many square inches equals 1 square foot?

Answer

1. $43,560\text{ ft}^2$
2. $4,840\text{yd}^2$
3. 144 in^2

It should be no surprise that this module will be full of conversion ratios. As always, if you discover other conversion ratios that aren't provided here, it would be a good idea to write them down so you can use them as needed.

- $1\text{ ft}^2 = 144\text{ in}^2$
- $1\text{ yd}^2 = 9\text{ ft}^2$
- $1\text{ acre (ac)} = 43,560\text{ ft}^2$
- $1\text{ ac} = 4,840\text{ yd}^2$
- $1\text{ mi}^2 = 27,878,400\text{ ft}^2$
- $1\text{ mi}^2 = 3,097,600\text{ yd}^2$

- $1\ \text{mi}^2 = 640\ \text{ac}$

An acre is defined as a unit of area; it would be wrong to say “acres squared” or put an exponent of 2 on the units.

? Exercises 2.2.16.1

4. A hallway is 9 yards long and 2 yards wide. How many square feet of linoleum are needed to cover the hallway?

5. A proposed site for an elementary school is 600 feet by 600 feet. Find its area, in acres.

Answer

4. $162\ \text{ft}^2$

5. $8.3\ \text{ac}$

Metric System: Converting Measurements of Area

- $1\ \text{cm}^2 = 100\ \text{mm}^2$
- $1\ \text{m}^2 = 1,000,000\ \text{mm}^2$
- $1\ \text{m}^2 = 10,000\ \text{cm}^2$
- $1\ \text{hectare (ha)} = 10,000\ \text{m}^2$
- $1\ \text{km}^2 = 1,000,000\ \text{m}^2$
- $1\ \text{km}^2 = 100\ \text{ha}$

A hectare is defined as a square with sides 100 meters long. Dividing a square kilometer into ten rows and ten columns will make a 10 by 10 grid of 100 hectares. As with acres, it would be wrong to say “hectares squared” or put an exponent of 2 on the units.

? Exercises 2.2.16.1

6. A hallway is 9 meters long and 2 meters wide. How many square centimeters of linoleum are needed to cover the hallway?

7. A proposed site for an elementary school is 200 meters by 200 meters. Find its area, in hectares.

Answer

6. $180,000\ \text{cm}^2$

7. $4\ \text{ha}$

Both Systems: Converting Measurements of Area

Converting between the U.S. and metric systems will involve messy decimal values. For example, because $1\ \text{in} = 2.54\ \text{cm}$, we can square both numbers and find that $(1\ \text{in})^2 = (2.54\ \text{cm})^2 = 6.4516\ \text{cm}^2$. The conversions are rounded to three or four significant digits in the table below.

- $1\ \text{in}^2 \approx 6.45\ \text{cm}^2 \leftrightarrow 1\ \text{cm}^2 \approx 0.155\ \text{in}^2$
- $1\ \text{in}^2 \approx 6.45\ \text{cm}^2 \leftrightarrow 1\ \text{cm}^2 \approx 0.155\ \text{in}^2$
- $1\ \text{yd}^2 \approx 0.836\ \text{m}^2 \leftrightarrow 1\ \text{m}^2 \approx 1.196\ \text{yd}^2$
- $1\ \text{mi}^2 \approx 2.59\ \text{km}^2 \leftrightarrow 1\ \text{km}^2 \approx 0.386\ \text{mi}^2$
- $1\ \text{ac} \approx 0.405\ \text{ha} \leftrightarrow 1\ \text{ha} \approx 2.47\ \text{ac}$

? Exercises 2.2.16.1

8. The area of Portland is $145\ \text{mi}^2$. Convert this area to square kilometers.

9. How many hectares is a $5,000$ acre ranch?

10. A sheet of paper measures 8.5 inches by 11 inches. What is the area in square centimeters?

11. A soccer field is 100 meters long and 70 meters wide. What is its area in square feet?

Answer

8. $376\ \text{km}^2$

9. $2,000\ \text{ha}$(to one sig fig) or $2,\bar{0}00\ \text{ha}$(to two sig figs)

10. $6\bar{0}0\ \text{cm}^2$ (to two sig figs)

11. $75,300\ \text{ft}^2$

Areas of Similar Figures

Earlier in this module, it was stated that if the linear units have a ratio of 1 to n, the square units will have a ratio of 1 to n^2. This applies to similar figures as well.

If the linear dimensions of two similar figures have a ratio of 1 to n, then the areas will have a ratio of 1 to n^2. This is true for circles, similar triangles, similar rectangles, similar hexagons, you name it. We'll verify this in the following exercises.

? Exercises 2.2.16.1

A personal pizza has a 7-inch diameter. A medium pizza has a diameter twice that of a personal pizza.

12. Determine the area of the medium pizza.

13. Determine the area of the personal pizza.

14. What is the ratio of the areas of the two pizzas?

Right triangle ABC has legs 3 cm and 4 cm long. Right triangle DEF has legs triple the length of ABC's.

15. Determine the area of the larger triangle, DEF.

16. Determine the area of the smaller triangle, ABC.

17. What is the ratio of the areas of the two triangles?

Answer

12. $154\ \text{in}^2$

13. $38.5\ \text{in}^2$

14. 4 to 1

15. $54\ \text{cm}^2$

16. $6\ \text{cm}^2$

17. 9 to 1

2.2.17: Volume of Common Solids

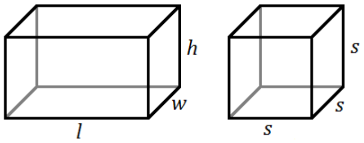
Wanda Ortiz of The Iron Maidens turns up the volume.

You may use a calculator throughout this module.

Note: We will not necessarily follow the rules for rounding (precision and accuracy) in this module. Many of these figures have dimensions with only one significant figure, but we would lose a lot of information if we rounded the results to only one sig fig.

In the answer key, we will often round to the nearest whole number, or to the nearest tenth, or to two or three significant figures as we deem appropriate.

The surface area of a solid is the sum of the areas of all its faces; therefore, surface area is two-dimensional and measured in square units. The volume is the amount of space inside the solid. Volume is three-dimensional, measured in cubic units. You can imagine the volume as the number of cubes required to completely fill up the solid.



Volume of a Rectangular Solid

For a rectangular solid with length l, width w, and height h:

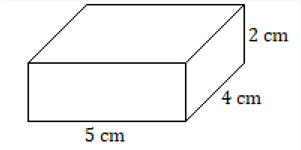
$$V = lwh$$

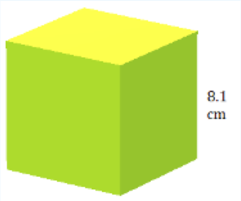
For a cube with side length s:

$$V = s^3$$

Exercises 2.2.17.1

Find the volume of each solid.

1. 

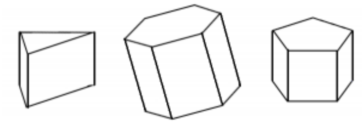
2. 

Answer

1. 40 cm^3

2. 531 cm^3

A solid with two equal sized polygons as its bases and rectangular lateral faces is called a right-angle prism. Some examples are shown below. We will refer to them simply as ***prisms*** in this textbook. (We will not be working with oblique prisms, which have parallelograms for the lateral faces.)



If you know the area of one of the bases, multiplying it by the height gives you the volume of the prism. In the formula below, we are using a capital B to represent the *area* of the base.

Volume of a Prism

For a prism with base area B and height h:

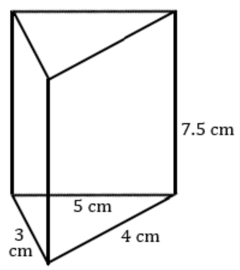
$$V = Bh$$

If the prism is lying on its side, the "height" will look like a length. No matter how the prism is oriented, the height is the dimension that is perpendicular to the planes of the two parallel bases.

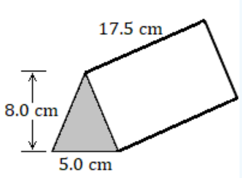
Exercises 2.2.17.1

Find the volume of each prism.

3.



4.



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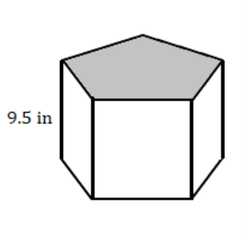
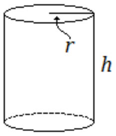


Answer

3. 45 cm^3
4. 350 cm^3
5. 520 cm^3

The area of the pentagon is 55 square inches.

A cylinder can be thought of as a prism with bases that are circles, rather than polygons. Just as with a prism, the volume is the area of the base multiplied by the height.



Volume of a Cylinder

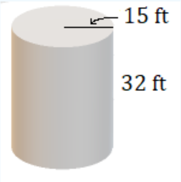
For a cylinder with radius r and height h:

$$V = \pi r^2 h$$

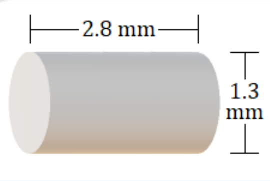
Exercises 2.2.17.1

Find the volume of each cylinder.

6.



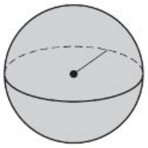
7.



Answer

6. $22,600\ \text{ft}^3$
7. $3.7\ \text{mm}^3$

As with surface area, we would need to use calculus to derive the formula for the volume of a **sphere**. Just believe it. ¯_(ツ)_/¯



Volume of a Sphere

For a sphere with radius r:

$$V = \frac{4}{3}\pi r^3$$

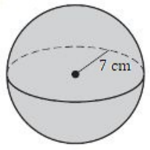
or

$$V = 4\pi r^3 \div 3$$

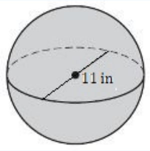
? Exercises 2.2.17.1

Find the volume of each sphere.

8.



9.



Answer

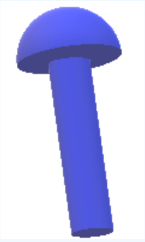
8. $1,440\ \text{cm}^3$

9. $7\bar{0}0\ \text{in}^3$

Composite Solids

Of course, not every three-dimensional object is a prism, cylinder, or sphere. A composite solid is made up of two or more simpler solids. As with two-dimensional composite figures, breaking the figure into recognizable solids is a good first step.

? Exercise 2.2.17.1



10. A rivet is formed by topping a cylinder with a hemisphere. The width of the cylindrical part (the rivet pin) is 1.6 cm and the length is 7 cm. The width of the hemisphere-shaped top (the rivet head) is 3.2 cm. Find the rivet's volume.

Answer

$31.2\ \text{cm}^3$(the cylinder's volume $\approx 14.07\ \text{cm}^3$and the hemisphere's volume $\approx 17.16\ \text{cm}^3$.)

2.2.18: Area of Regular Polygons

You may use a calculator throughout this module.

The Pentagon building spans 28.7 acres $(116,000\ \mathrm{m}^2)$, and includes an additional 5.1 acres $(21,000\ \mathrm{m}^2)$ as a central courtyard.[1] A pentagon is an example of a regular polygon.

A regular polygon has all sides of equal length and all angles of equal measure. Because of this symmetry, a circle can be inscribed—drawn inside the polygon touching each side at one point—or circumscribed—drawn outside the polygon intersecting each vertex. We'll focus on the inscribed circle first.

Let's call the radius of the inscribed circle lowercase r; this is the distance from the center of the polygon perpendicular to one of the sides.[2]

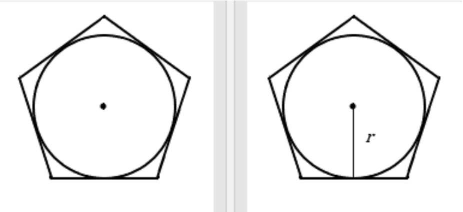


Figure $2.2.18.1$: (left) inscribed circle (right) inscribed circle with radius.

Area of a Regular Polygon (with a radius drawn to the center of one side) [3]

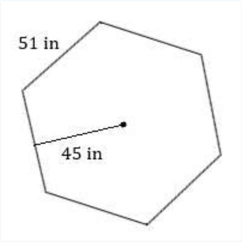
For a regular polygon with n sides of length s, and inscribed (inner) radius r,

$$A = nsr \div 2$$

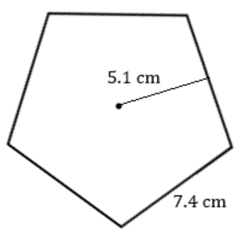
Note: This formula is derived from dividing the polygon into n equally-sized triangles and combining the areas of those triangles.

Exercises $2.2.18.1$

1. Calculate the area of this regular hexagon.



2. Calculate the pentagon's area of this regular pentagon.



3. A stop sign has a a height of 30 inches, and each edge measures 12.5 inches. Find the area of the sign.

Answer

1. $6,900 \text{ in}^2$
2. 94 cm^2
3. 750 in^2

Okay, but what if we know the distance from the center to one of the corners instead of the distance from the center to an edge? We'll need to imagine a circumscribed circle.

Let's call the radius of the circumscribed circle capital R; this is the distance from the center of the polygon to one of the vertices (corners).

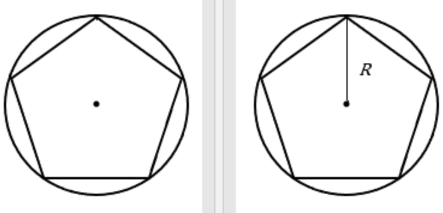


Figure $2.2.18.2$ (left) circumscribed circle and (right) circumscried circles with circle capital.

Area of a Regular Polygon (with a radius drawn to a vertex) [4]

For a regular polygon with n sides of length s, and circumscribed (outer) radius R,

$$A = 0.25ns\sqrt{4R^2 - s^2}$$

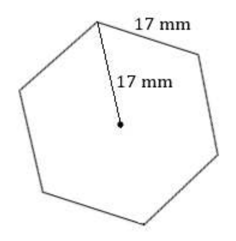
or

$$A = ns\sqrt{4R^2 - s^2} \div 4$$

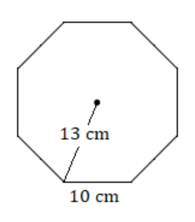
Note: This formula is also derived from dividing the polygon into n equally-sized triangles and combining the areas of those triangles. This formula includes a square root because it involves the Pythagorean theorem.

? Exercises $2.2.18.1$

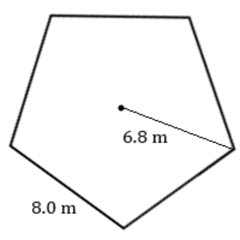
4. Calculate the area of this regular hexagon.



5. Calculate the area of this regular octagon.



6. Calculate the area of this regular pentagon.



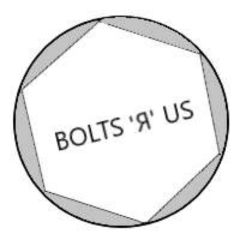
Answer

4. 750 mm^2
5. 480 cm^2
6. 110 m^2

As you know, a composite figure is a geometric figure which is formed by joining two or more basic geometric figures. Let's look at a composite figure formed by a circle and a regular polygon.

Exercise 2.2.18.1

7. The hexagonal head of a bolt fits snugly into a circular cap with a circular hole with inside diameter 46 mm as shown in this diagram. Opposite sides of the bolt head are 40 mm apart. Find the total empty area in the hole around the edges of the bolt head.



Answer

280 mm^2 (the area of the circle $\approx 1,660\text{ mm}^2$ and the area of the hexagon is $1,380\text{ mm}^2$)

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1. https://en.Wikipedia.org/wiki/The_Pentagon ↵

2. The inner radius is more commonly called the *apothem* and labeled a, but we are trying to keep the jargon to a minimum in this textbook. ↵
3. This formula is more commonly written as one-half the apothem times the perimeter: $A = \frac{1}{2}ap$ ↵
4. Your author created this formula because every other version of it uses trigonometry, which we aren't covering in this textbook. ↵

2.2.19: Pyramids and Cones

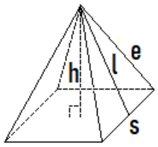
You may use a calculator throughout this module.

> *Note:* We will not necessarily follow the rules for rounding (precision and accuracy) in this module. Many of these figures have dimensions with only one significant figure, but we would lose a lot of information if we rounded the results to only one sig fig.

In the answer key, we will often round to the nearest whole number, or to the nearest tenth, or to two or three significant figures as we deem appropriate.

Pyramids

A **pyramid** is a geometric solid with a polygon base and triangular faces with a common vertex (called the *apex* of the pyramid). Pyramids are named according to the shape of their bases. The most common pyramids have a square or another regular polygon for a base, making all of the faces identical isosceles triangles. The height, h, is the distance from the apex straight down to the center of the base. Two other measures used with pyramids are the edge length e, the sides of the triangular faces, and the slant height *l*, the height of the triangular faces.



Volume of a Pyramid

In general, the volume of a pyramid with base of area B and height h is

$$V=\frac{1}{3}Bh$$

or

$$V=Bh\div 3$$

If the base is a square with side length s, the volume is

$$V=\frac{1}{3}s^2h$$

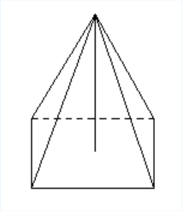
or

$$V=s^2h\div 3$$

Interestingly, the volume of a pyramid is $\frac{1}{3}$ the volume of a prism with the same base and height.

Exercises 2.2.19.1

1. A pyramid has a square base with sides 16 centimeters long, and a height of 15 centimeters. Find the volume of the pyramid.



2. The Great Pyramid at Giza in Egypt has a height of 137 meters and a square base with sides 230 meters long.[1] Find the volume of the pyramid.

Answer

1. $1,280\ \mathrm{cm}^3$
2. 2.4 million m^3

The lateral surface area (LSA) of a pyramid is found by adding the area of each triangular face.

Lateral Surface Area of a Pyramid

If the base of a pyramid is a regular polygon with n sides each of length s, and the slant height is l, then

$$LSA = \frac{1}{2}nsl$$

or

$$LSA = nsl \div 2$$

If the base is a square, then

$$LSA = 2sl$$

The total surface area (TSA) is of course found by adding the area of the base B to the lateral surface area. If the base is a regular polygon, you will need to use the techniques we studied in a previous module.

Total Surface Area of a Pyramid

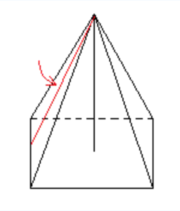
$$TSA = LSA + B$$

If the base is a square, then

$$TSA = 2sl + s^2$$

❓ Exercises 2.2.19.1

3. A pyramid has a square base with sides \(16\) centimeters long, and a slant height of 17 centimeters. Find the lateral surface area and total surface area of the pyramid.



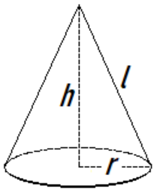
4. The Great Pyramid at Giza has a slant height of 179 meters and a square base with sides 230 meters long. Find the lateral surface area of the pyramid.

Answer

3. $544\ \mathrm{cm}^2$; $80\overline{0}\ \mathrm{cm}^2$
4. $82,300\ \mathrm{m}^2$; $135,000\ \mathrm{m}^2$

Cones

A cone is like a pyramid with a circular base.



You may be able to determine the height h of a cone (the altitude from the apex, perpendicular to the base), or the slant height l (which is the length from the apex to the edge of the circular base). Note that the height, radius, and slant height form a right triangle with the slant height as the hypotenuse. We can use the Pythagorean theorem to determine the following equivalences.

The slant height$l$, height h, and radius r of a cone are related as follows:

$$l=\sqrt{r^2+h^2}$$

$$h=\sqrt{l^2-r^2}$$

$$r=\sqrt{l^2-h^2}$$

Just as the volume of a pyramid is $\frac{1}{3}$ the volume of a prism with the same base and height, the volume of a cone is $\frac{1}{3}$ the volume of a cylinder with the same base and height.

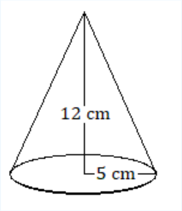
Volume of a Cone

The volume of a cone with a base radius r and height h is

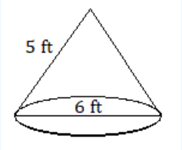
$$V=\frac{1}{3}\pi r^2 h \backslash) or \backslash(V=\pi r^2 h \div 3$$

Exercises 2.2.19.1

5. The base of a cone has a radius of 5 centimeters, and the vertical height of the cone is 12 centimeters. Find the volume of the cone.



6. The base of a cone has a diameter of 6 feet, and the slant height of the cone is 5 feet. Find the volume of the cone.



Answer

5. 314 cm^3
6. 37.7 ft^3

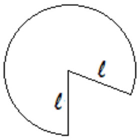
For the surface area of a cone, we have the following formulas.

Surface Area of a Cone

$$LSA = \pi r l$$

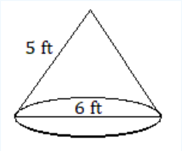
$$TSA = LSA + \pi r^2 = \pi r l + \pi r^2$$

It's hard to explain the justification for the LSA formula in words, but here goes. The lateral surface of a cone, when flattened out, is a circle with radius l that is missing a wedge. The circumference of this partial circle, because it matched the circumference of the circular base, is $2\pi r$. The circumference of the entire circle with radius l would be $2\pi l$, so the part we have is just a fraction of the entire circle. To be precise, the fraction is $\frac{2\pi r}{2\pi l}$, which reduces to $\frac{r}{l}$. The area of the entire circle with radius l would be πl^2. Because the partial circle is the fraction $\frac{r}{l}$ of the entire circle, the area of the partial circle is $\pi l^2 \cdot \frac{r}{l} = \pi r l$.

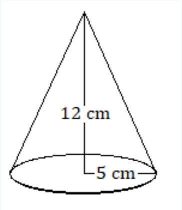


Exercises 2.2.19.1

7. The base of a cone has a diameter of 6 feet, and the slant height of the cone is 5 feet. Find the lateral surface area and total surface area of the cone.



8. The base of a cone has a radius of 5 centimeters, and the vertical height of the cone is 12 centimeters. Find the lateral surface area and total surface area of the cone.



Answer

7. 47.1 ft^2; 75.4 ft^2

8. 204 cm^2; 283 cm^2

Now that we have looked at the five major solids—prism, cylinder, sphere, pyramid, cone—you should be able to handle composite solids made from these shapes. Just remember to take them in pieces.

Exercise 2.2.19.1

A 250-gallon propane tank is roughly in the shape of a cylinder with a hemisphere on each end. The length of the cylindrical part is 6 feet long, and the cross-sectional diameter of the tank is 2.5 feet.

9. Calculate the volume of the tank in cubic feet.

10. Verify that the tank can hold 250 gallons of liquid propane.

Answer

9. 37.6 ft^3 (the cylinder's volume $\approx 29.45 \text{ ft}^3$ and the two hemispheres' combined volume $\approx 8.18 \text{ ft}^3$)

10. $37.6 \text{ ft}^3 \approx 281.5$ gal, which is more than 250 gal

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1. https://en.Wikipedia.org/wiki/Great_Pyramid_of_Giza ↵

2.2.20: Mean, Median, Mode

You may use a calculator throughout this module.

We often describe data using a ***measure of central tendency***. This is a number that we use to describe the typical data value. We will now look at the ***mean***, the ***median***, and the ***mode***.

Mean

The mean of a set of data is what we commonly call the average: add up all of the numbers and then divide by how many numbers there were.

Exercises 2.2.20.1

1. The table below shows the average price of a gallon of regular unleaded gasoline in the Seattle metro area for ten weeks in Fall 2019. Compute the mean price over this time period.[1]

Sep 16, 2019	$ 3.24
Sep 23, 2019	$ 3.25
Sep 30, 2019	$ 3.27
Oct 07, 2019	$ 3.30
Oct 14, 2019	$ 3.35
Oct 21, 2019	$ 3.41
Oct 28, 2019	$ 3.44
Nov 04, 2019	$ 3.43
Nov 11, 2019	$ 3.42
Nov 18, 2019	$ 3.40

2. The table below shows the average price of a gallon of regular unleaded gasoline in the Seattle metro area for ten weeks in Fall 2008. Compute the mean price over this time period.[2]

Sep 15, 2008	$ 3.77
Sep 22, 2008	$ 3.70
Sep 29, 2008	$ 3.65
Oct 06, 2008	$ 3.54
Oct 13, 2008	$ 3.36
Oct 20, 2008	$ 3.09
Oct 27, 2008	$ 2.78
Nov 03, 2008	$ 2.54
Nov 10, 2008	$ 2.38
Nov 17, 2008	$ 2.24

Answer

1. $ 3.35
2. $ 3.11

Median

The median is the middle number in a set of data; it has an equal number of data values below it as above it. The numbers must be arranged in order, usually smallest to largest but largest to smallest would also work. Then we can count in from both ends of the list and find the median in the middle.

If there are an odd number of data values, there will be one number in the middle, which is the median.

If there are an even number of data values, there will be two numbers in the middle. The mean of these two numbers is the median.

Exercises 2.2.20.1

The houses on a block have these property values: $\$\,250,000$; $\$\,300,000$; $\$\,320,000$; $\$\,190,000$; $\$\,220,000$

3. Find the mean property value.

4. Find the median property value.

A new house is built on the block, making the property values $\$\,250,000$; $\$\,300,000$; $\$\,320,000$; $\$\,190,000$; $\$\,220,000$ and $\$\,750,000$

5. Find the mean property value.

6. Find the median property value.

7. Which of these measures appears to give a more accurate representation of the typical house on the block?

Answer

3. $\$\,256,000$
4. $\$\,250,000$
5. $\$\,338,000$
6. $\$\,275,000$
7. the median is more representative because the mean is higher than five of the six home values.

The mean is better to work with when we do more complicated statistical analysis, but it is sensitive to extreme values; in other words, one very large or very small number can have a significant effect on the mean. The median is not sensitive to extreme values, which can make it a better measure to use when describing data that has one or two numbers very different from the remainder of the data.

For example, suppose you had ten quizzes, and you scored 100 on nine of them but missed one quiz and received a score of 0. You earned a total of 900 points over 10 quizzes, making your mean score 90. However, your median score would be 100 because the median is calculated based on your fifth and sixth highest scores.

Mode

In the example above, 100 is also the mode of your scores because it is the most common quiz score in your gradebook. The mode is the value that appears **most** frequently in the data set. On the game show *Family Feud*, the goal is to guess the mode: the most popular answer.

If no numbers are repeated, then the data set has no mode. If there are two values that are tied for most frequently occurring, then they are both considered a mode and the data set is called bimodal. If there are more than two values tied for the lead, we usually say that there is no mode.[3] (It's like in sports: there is usually one MVP, but occasionally there are two co-MVPs. Having three or more MVPs would start to get ridiculous.)

Exercises 2.2.20.1

8. One hundred cell phone owners are asked which carrier they use. What is the mode of the data?

AT&T Mobility	Verizon Wireless	T-Mobile US	Dish Wireless	U.S. Cellular
43	29	24	2	2

9. Fifty people are asked what their favorite type of Girl Scout cookie is. What is the mode?

S'Mores	Samoas	Tagalongs	Trefoils	Thin Mints
4	16	5	9	16

Answer

8. AT&T Mobility

9. Samoas and Thin Mints

Let's put it all together and find the mean, median, and mode of some data sets. Sportsball!

From 2001-2019, these are the numbers of games won by the New England Patriots each NFL season.[4]

year	wins
2001	11
2002	9
2003	14
2004	14
2005	10
2006	12
2007	16
2008	11
2009	10
2010	14
2011	13
2012	12
2013	12
2014	12
2015	12
2016	14
2017	13
2018	11
2019	12

Exercises 2.2.20.1

10. Find the mean number of games won from 2001 to 2019.

11. Find the median number of games won from 2001 to 2019.

12. Find the mode of the number of games won from 2001 to 2019.

13. Do any of these measures appear to be misleading, or do they all represent the data fairly well?

Answer

10. 12.2 games

11. 12 games

12. 12 games

13. they all represent the data fairly well; 12 wins represents a typical Patriots season.

From 2001-2019, these are the numbers of games won by the Buffalo Bills each NFL season.[5]

year	wins
2001	3
2002	8
2003	6
2004	9
2005	5
2006	7
2007	7
2008	7
2009	6
2010	4
2011	6
2012	6
2013	6
2014	9
2015	8
2016	7
2017	9
2018	6
2019	10

Exercises 2.2.20.1

14. Find the mean number of games won from 2001 to 2019.

15. Find the median number of games won from 2001 to 2019.

16. Find the mode of the number of games won from 2001 to 2019.

17. Do any of these measures appear to be misleading, or do they all represent the data fairly well?

Answer

14. 6.8 games

15. 7 games

16. 6 games

17. they all represent the data fairly well; 6 or 7 wins represents a typical Bills season.

Some sets of data may not be easy to describe with one measure of central tendency.

? Exercises 2.2.20.1

Thirteen clementines are weighed. Their masses, in grams, are
$82, 90, 90, 92, 93, 94, 94, 102, 107, 107, 108, 109, 109.$

18. Determine the mean. Does the mean appear to represent the mass of a typical clementine?

19. Determine the median. Does the median appear to represent the mass of a typical clementine?

20. Determine the mode. Does the mode appear to represent the mass of a typical clementine?

Suppose that the 108-gram clementine is a tiny bit heavier and the masses are actually
$82, 90, 90, 92, 93, 94, 94, 102, 107, 107, 109, 109, 109.$

21. Determine the new mean. Is the new mean different from the original mean?

22. Determine the new median. Is the new median different from the original median?

23. Determine the new mode. Is the new mode different from the original mode? Does it represent the mass of a typical clementine?

Answer

18. 98.2 grams; the mean doesn't seem to represent a typical clementine because there is a group of smaller ones (from 82 to 94 grams) and a group of larger ones (from 102 to 109 grams) with none in the middle.

19. 94 grams; for the same reason, the median doesn't represent a typical clementine, but you could say it helps split the clementines into a lighter group and a heavier group.

20. there is no mode because too many values appear twice.

21. 98.3 grams; this is a small increase over the previous mean.

22. 94 grams; the median does not change when one of the highest numbers increases.

23. 109 grams; you might say it represents the mass of a typical large clementine, but it doesn't represent the entire group.

One of your author’s cats is named Clementine. She weighs more than 109 grams.

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1. Source: www.eia.gov/petroleum/gasdiesel/↵
2. Source: www.eia.gov/petroleum/gasdiesel/↵
3. The concepts of trimodal and multimodal data exist, but we aren't going to consider anything beyond bimodal in this textbook. ↵
4. Source: https://www.pro-football-reference.com/teams/nwe/index.htm ↵
5. Source: https://www.pro-football-reference.com/teams/buf/index.htm ↵

2.2.21: Probability

Probability is the likelihood that some event occurs. If the event occurs, we call that a favorable outcome. The set of all possible events (or outcomes) is called the sample space of the event. We will limit our focus to independent events, which do not influence each other. For example, if we roll a 5 on one die, that does not affect the probability of rolling a 5 on the other die. (We will not be studying *dependent* events, which do influence each other.)

If we are working with something simple like dice, cards, or coin flips where we know all of the possible outcomes, we can calculate the theoretical probability of an event occurring. To do this, we divide the number of ways the event can occur by the total number of possible outcomes. We may choose to write the probability as a fraction, a decimal, or a percent depending on what form seems most useful.

Theoretical probability of an event:

$$P(\text{event})=\frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

Suppose two six-sided dice, numbered 1 through 6, are rolled. There are $6\cdot 6=36$ possible outcomes in the sample space. If we are playing a game where we take the sum of the dice, the only possible outcomes are 2 through 12. However, as the following table shows, these outcomes are not all equally likely. For example, there are two different ways to roll a 3, but only one way to roll a 2.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

? Exercises 2.2.21.1

Two six-sided dice numbered 1 through 6 are rolled. Find the probability of each event occurring.

1. The sum of the dice is 7.

2. The sum of the dice is 11.

3. The sum of the dice is 7 or 11.

4. The sum of the dice is greater than 1.

5. The sum of the dice is 13.

Answer

1. $\frac{6}{36}=\frac{1}{6}$

2. $\frac{2}{36} = \frac{1}{18}$
3. $\frac{8}{36} = \frac{2}{9}$
4. $\frac{36}{36} = 1$
5. $\frac{0}{36} = 0$

Some things to notice…

If an event is impossible, its probability is 0% or 0.

If an event is certain to happen, its probability is 100% or 1.

If it will be tedious to count up all of the favorable outcomes, it may be easier to count up the unfavorable outcomes and subtract from the total.

Exercises 2.2.21.1

Two six-sided dice numbered 1 through 6 are rolled. Find the probability of each event occuring.

6. The sum of the dice is 5.

7. The sum of the dice is not 5.

8. The sum of the dice is greater than 9.

9. The sum of the dice is 9 or lower.

Answer

6. $\frac{4}{36} = \frac{1}{9}$
7. $\frac{32}{36} = \frac{8}{9}$
8. $\frac{6}{36} = \frac{1}{6}$
9. $\frac{30}{36} = \frac{5}{6}$

The set of outcomes in which an event does not occur is called the ***complement*** of the event. The event “the sum is not 5” is the complement of “the sum is 5”. Two complements *complete* the sample space.

If the probability of an event happening is p, the probability of the complement is $1 - p$.

Exercises 2.2.21.1

A bowl of 60 Tootsie Rolls Fruit Chews contains the following: 15 cherry, 14 lemon, 13 lime, 11 orange, 7 vanilla.

10. If one Tootsie Roll is randomly selected from the bowl, what is the probability that it is cherry?

11. What is the probability that a randomly selected Tootsie Roll is either lemon or lime?

12. What is the probability that a randomly selected Tootsie Roll is not orange or vanilla?

Answer

10. $\frac{15}{60} = \frac{1}{4}$
11. $\frac{27}{60} = \frac{9}{20}$
12. $\frac{42}{60} = \frac{7}{10}$

Here's where we try to condense the basics of genetic crosses into one paragraph.

Each parent gives one allele to their child. The allele for brown eyes is *B*, and the allele for blue eyes is *b*. If two parents both have genotype *Bb*, the table below (which biologists call a Punnett square) shows that there are four equally-likely outcomes: *BB*, *Bb*, *Bb*, *bb*. The allele for brown eyes, *B*, is dominant over the gene for blue eyes, *b*, which means that if a child has any *B* alleles, they will have brown eyes. The only genotype for which the child will have blue eyes is *bb*.

	B	*b*
B	*BB*	*Bb*
b	*Bb*	*bb*

? Exercises 2.2.21.1

Two parents have genotypes *Bb* and *Bb*. (*B* = brown, *b* = blue)

13. What is the probability that their child will have blue eyes?

14. What is the probability that their child will have brown eyes?

Answer

13. $\frac{1}{4} = 25\%$
14. $\frac{3}{4} = 75\%$

Now suppose that one parent has genotype *Bb* but the other parent has genotype *bb*. The Punnett square will look like this.

	B	*b*
b	*Bb*	*bb*
b	*Bb*	*bb*

? Exercises 2.2.21.1

Two parents have genotypes *Bb* and *bb*. (*B* = brown, *b* = blue)

15. What is the probability that their child will have blue eyes?

16. What is the probability that their child will have brown eyes?

Answer

15. $\frac{2}{4} = 50\%$
16. $\frac{2}{4} = 50\%$

The previous methods work when we know the total number of outcomes and we can assume that they are all equally likely. (The dice aren't loaded, for example.) However, life is usually more complicated than a game of dice or a bowl of Tootsie Rolls. In many situations, we have to observe what has happened in the past and use that data to predict what might happen in the future. If someone predicts that an Alaska Airlines flight has a 95.5% of arriving on time, that is of course based on Alaska's past rate of success.[1] When we calculate the probability this way, by observation, we call it an empirical probability.

Empirical probability of an event:

$$P(\text{event}) = \frac{\text{number of favorable observations}}{\text{total number of observations}}$$

Although the wording may seem complicated, we are still just thinking about $\frac{\text{part}}{\text{whole}}$.

Exercises 2.2.21.1

A photocopier makes 250 copies, but 8 of them are unacceptable because they have toner smeared on them.

17. What is the empirical probability that a copy will be unacceptable?

18. What is the empirical probability that a copy will be acceptable?

19. Out of the next $1,000$ copies, how many should we expect to be acceptable?

An auditor examined 200 tax returns and found errors on 44 of them.

20. What percent of the tax returns contained errors?

21. How many of the next $1,000$ tax returns should we expect to contain errors?

22. What is the probability that a given tax return, chosen at random, will contain errors?

Answer

17. $\frac{8}{250} = 3.2\%$

18. $\frac{242}{250} = 96.8\%$

19. we should expect 968 copies to be acceptable

20. $\frac{44}{200} = 22\%$

21. we should expect 220 tax returns to have errors

22. $22\% = 0.22$

It was mentioned earlier in this module that independent events have no influence on each other. Some examples:

- Rolling two dice are independent events because the result of the first die does not affect the probability of what will happen with the second die.
- If we flip a coin ten times, each flip is independent of the previous flip because the coin doesn't remember how it landed before. The probability of heads or tails remains $\frac{1}{2}$ for each flip.
- Drawing marbles out of a bag are independent events only if we put the first marble back in the bag before drawing a second marble. If we draw two marbles at once, or we draw a second marble without replacing the first marble, these are

dependent events, which we are not studying in this course.

- Drawing two cards from a deck of 52 cards are independent events only if we put the first card back in the deck before drawing a second card. If we draw a second card without replacing the first card, these are *dependent events*; the probabilities change because there are only 51 cards available on the second draw.

If two events are independent, then the probability of both events happening can be found by multiplying the probability of each event happening separately.

$$\text{If } A \text{ and } B \text{ are independent events, then } P(A \text{ and } B) = P(A) \cdot P(B)\ .$$

Note: This can be extended to three or more events. Just multiply all of the probabilities together.

? Exercises 2.2.21.1

An auditor examined 200 tax returns and found errors on 44 of them.

23. What is the probability that the next two tax returns both contain errors?

24. What is the probability that the next three tax returns all contain errors?

25. What is the probability that the next tax return contains errors but the one after it does not?

26. What is the probability that the next tax return does not contain errors but the one after it does?

27. What is the probability that neither of the next two tax returns contain errors?

28. What is the probability that none of the next three tax returns contain errors?

29. What is the probability that at least one of the next three tax returns contain errors? (This one is tricky!)

Answer

23. $(0.22)^2 \approx 4.8\%$

24. $(0.22)^3 \approx 1.1\%$

25. $0.22 \cdot 0.78 \approx 17.2\%$

26. $0.78 \cdot 0.22 \approx 17.2\%$

27. $(0.78)^2 \approx 60.8\%$

28. $(0.78)^3 \approx 47.5\%$

29. $1-(0.78)^3 \approx 52.5\%$

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1. pdf file: https://www.transportation.gov/sites/dot.gov/files/2020-08/July%202020%20ATCR.pdf↩

2.2.22: Standard Deviation

This topic requires a leap of faith. It is one of the rare times when this textbook will say “don’t worry about *why* it’s true; just accept it.”

A normal distribution, often referred to as a bell curve, is symmetrical on the left and right, with the mean, median, and mode being the value in the center. There are lots of data values near the center, then fewer and fewer as the values get further from the center. A normal distribution describes the data in many real-world situations: heights of people, weights of people, errors in measurement, scores on standardized tests (IQ, SAT, ACT)…

One of the best ways to demonstrate the normal distribution is to drop balls through a board of evenly spaced pegs, as shown here.[1] Each time a ball hits a peg, it has a fifty-fifty chance of going left or right. For most balls, the number of lefts and rights are roughly equal, and the ball lands near the center. Only a few balls have an extremely lopsided number of lefts and rights, so there are not many balls at either end. As you can see, the distribution is not perfect, but it is approximated by the normal curve drawn on the glass.

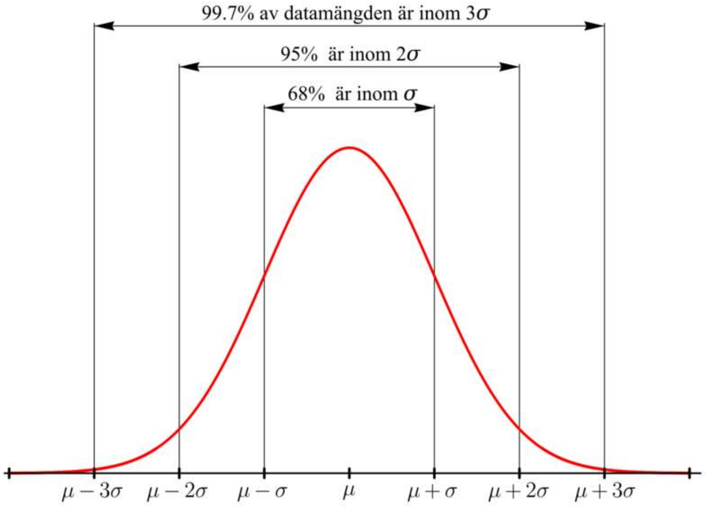
The standard deviation is a measure of the spread of the data: data with lots of numbers close to the mean has a smaller standard deviation, and data with numbers spaced further from the mean has a larger standard deviation. (In this textbook, you will be given the value of the standard deviation of the data and will never need to calculate it.) The standard deviation is a measuring stick for a particular set of data.

In a normal distribution…

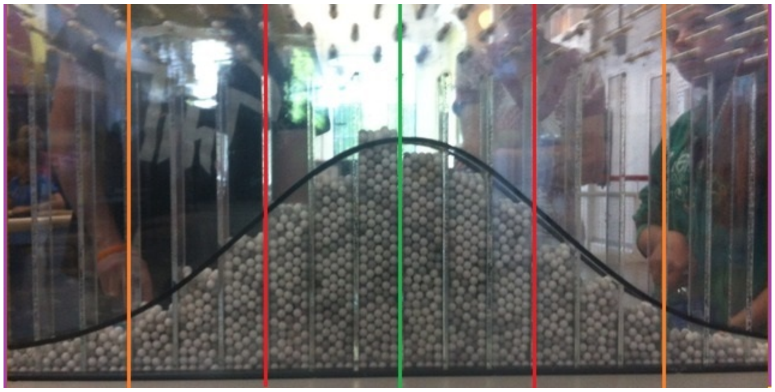
- roughly 68% of the numbers are within 1 standard deviation above or below the mean
- roughly 95% of the numbers are within 2 standard deviations above or below the mean
- roughly 99.7% of the numbers are within 3 standard deviations above or below the mean

This **68-95-99.7 rule** is called an *empirical rule* because it is based on observation rather than some formula. Nobody discovered a calculation to figure out the numbers 68%, 95%, and 99.7% until after the fact. Instead, statisticians looked at lots of different examples of normally distributed data and said “*Mon Dieu*, it appears that if you count up the data values that are within one standard deviation above or below the mean, you have about 68% of the data!” and so on.[2]

The following image is in Swedish, but you can probably decipher it because math is an international language.



Let’s go back to the ball-dropping experiment, and let’s assume that the standard deviation is three columns wide.[3] In the picture below, the **green** line marks the center of the distribution.



First, the two **red** lines are each three columns away from the center, which is **one standard deviation** above and below the **center**, so about **68%** of the balls will land between the red lines.

Next, the two **orange** lines are another three columns farther away from the center, which is six columns or **two standard deviations** above and below the **center**, so about **95%** of the balls will land between the orange lines.

And finally, the two **purple** lines are another three columns farther away from the center, which is nine columns or **three standard deviations** above and below the **center**, so about **99.7%** of the balls will land between the purple lines. We can expect that 997 out of $1,000$ balls will land between the purple lines, leaving only 3 out of $1,000$ landing beyond the purple lines on either end.

Here are Damian Lillard’s game results for points scored, in increasing order, for the 80 games he played in the 2018-19 NBA season.[4] This is broken up into eight rows of ten numbers each, and this is a total of $2,069$ points.

$$\begin{gathered}11,\ 13,\ 13,\ 13,\ 14,\ 14,\ 15,\ 15,\ 15,\ 16,\\ 16,\ 16,\ 17,\ 17,\ 17,\ 18,\ 18,\ 19,\ 19,\ 20,\\ 20,\ 20,\ 20,\ 20,\ 21,\ 21,\ 22,\ 22,\ 23,\ 23,\\ 23,\ 23,\ 24,\ 24,\ 24,\ 24,\ 24,\ 24,\ 24,\ 24,\\ 25,\ 25,\ 25,\ 26,\ 26,\ 26,\ 28,\ 28,\ 28,\ 29,\\ 29,\ 29,\ 29,\ 30,\ 30,\ 30,\ 30,\ 30,\ 31,\ 31,\\ 33,\ 33,\ 33,\ 33,\ 33,\ 33,\ 34,\ 34,\ 34,\ 35,\\ 36,\ 36,\ 37,\ 39,\ 40,\ 40,\ 41,\ 41,\ 42,\ 51\end{gathered}$$

? Exercises 2.2.22.1

This is a review of mean, median, and mode; you'll need to know the mean in order to complete the standard deviation exercises that follow.

1. What is the mean of the data? (Round to the nearest tenth.)

2. What is the median of the data?

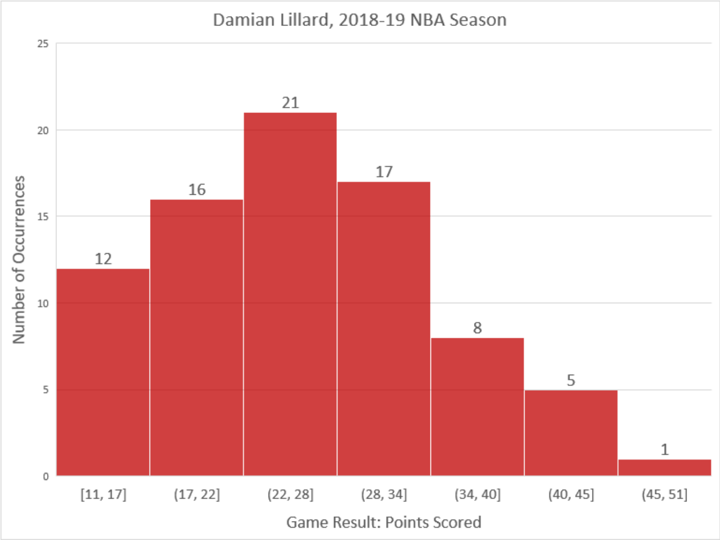
3. What is the mode of the data?

4. Do any of the mean, median, or mode seems misleading, or do all three seem to represent the data fairly well?

Answer

1. 25.9points
2. 24.5points
3. 24 points (which occurred eight times)
4. all three seem to represent the typical number of points scored; the mean is a bit high because there are no extremely low values but there are a few high values that pull the mean upwards.

Here is a histogram of the data, arbitrarily grouped in seven equally-spaced intervals. It shows that the data roughly follows a bell-shaped curve, somewhat truncated on the left and with an outlier on the right.



If we enter the data into a spreadsheet program such as Microsoft Excel or Google Sheets, we can quickly find that the standard deviation is 8.2 points.

Based on the empirical rule, we should expect approximately 68% of the results to be within 8.2 points above and below the mean.

? Exercises 2.2.22.1

5. Determine the range of points scored that are within one standard deviation of the mean.

6. How many of the 80 game results are within one standard deviation of the mean?

7. Is the previous answer close to 68% of the total number of game results?

Answer

5. $17.7$to 34.1points
6. 54 of the 80 game results

7. yes; $54 \div 80 = 67.5\%$

And we should expect approximately 95% of the results to be within $2 \cdot 8.2 = 16.4$ points above and below the mean.

? Exercises 2.2.22.1

8. Determine the range of points scored that are within two standard deviations of the mean.

9. How many of the 80 game results are within two standard deviations of the mean?

10. Is the previous answer close to 95% of the total number of game results?

Answer

8. 9.5 to 42.3 points

9. 79 of the 80 game results

10. sort of close but not really; $79 \div 80 = 98.75\%$

And we should expect approximately 99.7% of the results to be within $3 \cdot 8.2 = 24.6$ points above and below the mean.

? Exercises 2.2.22.1

11. Determine the range of points scored that are within three standard deviations of the mean.

12. How many of the 80 game results are within three standard deviations of the mean?

13. Is the previous answer close to 99.7% of the total number of game results?

Answer

11. 1.3 to 50.5 points

12. 79 of the 80 game results, again

13. yes, this is pretty close; $79 \div 80 = 98.75\%$

Notice that we could think about the standard deviations like a measurement error or tolerance: the mean ± 8.2, the mean ± 16.4, the mean $\pm 24.6 \ldots$

? Exercises 2.2.22.1

For U.S. females, the average height is around 63.5 inches (5 ft 3.5 in) and the standard deviation is 3 inches. Use the empirical rule to fill in the blanks.

14. About 68% of the women should be between _______ and _______ inches tall.

15. About 95% of the women should be between _______ and _______ inches tall.

16. About 99.7% of the women should be between _______ and _______ inches tall.

For U.S. males, the average height is around 69.5 inches (5 ft 9.5 in) and the standard deviation is 3 inches. Use the empirical rule to fill in the blanks.

17. About 68% of the men should be between _______ and _______ inches tall.

18. About 95% of the men should be between _______ and _______ inches tall.

19. About 99.7% of the men should be between _______ and _______ inches tall.

Answer

14. 60.5; 66.5

15. 57.5; 69.5

16. 54.5; 72.5

17. 66.5; 72.5

18. 63.5; 75.5

19. 60.5; 78.5

This graph at https://tall.life/height-percentile-calculator-age-country/ shows that, because the standard deviations are equal, the two bell curves have essentially the same shape but the women's graph is centered six inches below the men's.

? Exercises 2.2.22.1

Around 16% of U.S. males in their forties weigh less than 160 lb and 16% weigh more than 230 lb.[5] Assume a normal distribution.

20. What percent of U.S. males weigh between 160 lb and 230 lb?

21. What is the average weight? (Hint: think about symmetry.)

22. What is the standard deviation? (Hint: You have to work backwards to figure this out, but the math isn't complicated.)

23. Based on the empirical rule, about 95% of the men should weigh between _______ and _______ pounds.

Answer

20. 68% because $100\% - (16\% + 16\%) = 68\%$

21. 195 lb because this is halfway between 160 and 230 lb

22. 35 lb because $195 - 35$ lb and $195 + 35$ lb encompasses 68% of the data

23. 125; 265

If you are asked only one question about the empirical rule instead of three in a row (68%, 95%, 99.7%), you will most likely be asked about the 95%. This is related to the "95% confidence interval" that is often mentioned in relation to statistics. For example, the margin of error for a poll is usually close to two standard deviations.[6]

Let's finish up by comparing the performance of three NFL teams since the turn of the century.

The numbers of regular-season games won by the New England Patriots each NFL season from 2001-19:[7]

year	wins
2001	11
2002	9
2003	14
2004	14
2005	10
2006	12
2007	16
2008	11
2009	10
2010	14
2011	13
2012	12

year	wins
2013	12
2014	12
2015	12
2016	14
2017	13
2018	11
2019	12

? Exercises 2.2.22.1

For the Patriots, the mean number of wins is 12.2, and a spreadsheet tells us that the standard deviation is 1.7 wins.

24. There is a 95% chance of the Patriots winning between _______ and _______ games in a season.

25. In 2020, the Patriots won 7 games. Could you have predicted that based on the data? How many standard deviations from the mean is this number of wins?

Answer

24. 8.8; 15.6

25. You would not have predicted this from the data because it is more than two standard deviations below the mean, so there would be a roughly 2.5%chance of this happening randomly. In fact, $(12.2-7) \div 1.7$ is slightly larger than 3, so this is more than three standard deviations below the mean, making it even more unlikely. (You might have predicted that the Patriots would get worse when Tom Brady left them for Tampa Bay, but you wouldn't have predicted only 7 wins based on the previous nineteen years of data.)

The numbers of regular-season games won by the Buffalo Bills each NFL season from 2001-19:[8]

year	wins
2001	3
2002	8
2003	6
2004	9
2005	5
2006	7
2007	7
2008	7
2009	6
2010	4
2011	6
2012	6
2013	6
2014	9
2015	8

year	wins
2016	7
2017	9
2018	6
2019	10

? Exercises 2.2.22.1

For the Bills, the mean number of wins is 6.8, and a spreadsheet tells us that the standard deviation is 1.7 wins.

26. There is a 95% chance of the Bills winning between _______ and _______ games in a season.

27. In 2020, the Bills won 13 games. Could you have predicted that based on the data? How many standard deviations from the mean is this number of wins?

Answer

26. 3.4; 10.2

27. You would not predict this from the data because it is more than two standard deviations above the mean, so there would be a roughly 2.5% chance of this happening randomly. In fact, $(13-6.8) \div 1.7 \approx 3.6$, so this is more than three standard deviations above the mean, making it even more unlikely. This increased win total is partly due to external forces (i.e., the Patriots becoming weaker and losing two games to the Bills) but even 11 wins would have been a bold prediction, let alone 13.

The numbers of regular-season games won by the Denver Broncos each NFL season from 2001-19:[9]

year	wins
2001	8
2002	9
2003	10
2004	10
2005	13
2006	9
2007	7
2008	8
2009	8
2010	4
2011	8
2012	13
2013	13
2014	12
2015	12
2016	9
2017	5
2018	6

year	wins
2019	7

? Exercises 2.2.22.1

For the Broncos, the mean number of wins is 9.1, and a spreadsheet tells us that the standard deviation is 2.6 wins.

28. There is a 95% chance of the Broncos winning between _______ and _______ games in a season.

29. In 2020, the Broncos won 5 games. Could you have predicted that based on the data? How many standard deviations from the mean is this number of wins?

Answer

28. 3.9; 14.3

29. The trouble with making predictions about the Broncos is that their standard deviation is so large. You could choose any number between 4 and 14 wins and be within the 95% interval. $(9.1-5)\div 2.6\approx 1.6$, so this is around 1.6 standard deviations below the mean, which makes it not very unusual. Whereas the Patriots and Bills are more consistent, the Broncos' win totals fluctuate quite a bit and are therefore more unpredictable.

---

1. The Plinko game on *The Price Is Right* is the best-known example of this; here's a clip of Snoop Dogg helping a contestant win some money. ↩
2. Confession: This paragraph gives you the general idea of how these ideas developed but may not be perfectly historically accurate. ↩
3. I eyeballed it and it seemed like a reasonable assumption. ↩
4. Source: https://www.basketball-reference.com/players/l/lillada01/gamelog/2019↩
5. Source: https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=17&ved=2ahUKEwjm-d-whavhAhWCFXwKHQxMDz4QFjAQegQIARAC&url=https%3A%2F%2Fwww2.census.gov%2Flibrary%2Fpublications%2F2010%2Fcompendia%2Fstatab%2F130ed%2Ftables%2F11s0205.pdf&usg=AOvVaw1DFDbil78g-qXbIgK6JirW↩
6. Source: https://en.Wikipedia.org/wiki/Standard_deviation↩
7. Source: https://www.pro-football-reference.com/teams/nwe/index.htm ↩
8. Source: https://www.pro-football-reference.com/teams/buf/index.htm ↩
9. Source: https://www.pro-football-reference.com/teams/den/index.htm ↩

2.3: Rules of Exponents and Scientific Notation

Learning Objectives

- Review the rules of exponents.
- Review the definition of negative exponents and zero as an exponent.
- Work with numbers using scientific notation.

Review of the Rules of Exponents

In this section, we review the rules of exponents. Recall that if a factor is repeated multiple times, then the product can be written in exponential form x^n. The positive integer exponent n indicates the number of times the base x is repeated as a factor.

$$x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n\,times}$$

Consider the product of x^4 and x^6,

$$x^4 \cdot x^6 = \underbrace{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}_{10\ factors\ of\ x} = x^{10}$$

Expanding the expression using the definition produces multiple factors of the base which is quite cumbersome, particularly when n is large. For this reason, we have useful rules to help us simplify expressions with exponents. In this example, notice that we could obtain the same result by adding the exponents.

$$x^4 \cdot x^6 = x^{4+6} = x^{10} \quad \color{blue}{Product\ rule\ for\ exponents}$$

In general, this describes the **product rule for exponents**[1]. In other words, when multiplying two expressions with the same base we add the exponents. Compare this to raising a factor involving an exponent to a power, such as $(x^6)^4$.

$$\begin{aligned} (x^6)^4 &= \underbrace{x^6 \cdot x^6 \cdot x^6 \cdot x^6}_{4\ factors\ of\ x^6} \\ &= x^{6+6+6+6} \\ &= x^{24} \end{aligned}$$

Here we have 4 factors of x^6, which is equivalent to multiplying the exponents.

$$(x^6)^4 = x^{6 \cdot 4} = x^{24} \quad \color{blue}{Power\ rule\ for\ exponents}$$

This describes the **power rule for exponents**[2]. Now we consider raising grouped products to a power. For example,

$$\begin{aligned} (x^2y^3)^4 &= x^2y^3 \cdot x^2y^3 \cdot x^2y^3 \cdot x^2y^3 \\ &= x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot y^3 \cdot y^3 \cdot y^3 \cdot y^3 \quad \color{blue}{Commutative\ property} \\ &= x^{2+2+2+2} \cdot y^{3+3+3+3} \\ &= x^8y^{12} \end{aligned}$$

After expanding, we are left with four factors of the product x^2y^3. This is equivalent to raising each of the original grouped factors to the fourth power and applying the power rule.

$$(x^2y^3)^4 = (x^2)^4(y^3)^4 = x^8y^{12}$$

In general, this describes the use of the power rule for a product as well as the power rule for exponents. In summary, the rules of exponents streamline the process of working with algebraic expressions and will be used extensively as we move through our study of algebra. Given any positive integers m and n where $x, y \neq 0$ we have

Table 2.3.1

Product rule for exponents:	$x^m \cdot x^n = x^{m+n}$
Quotient rule for exponents:	$\frac{x^m}{x^n} = x^{m-n}$
Power rule for exponents:	$(x^m)^n = x^{m \cdot n}$

Power rule for a product:[3]	$(xy)^n = x^n y^n$
Power rule for a quotient:[4]	$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

These rules allow us to efficiently perform operations with exponents.

2.3.1 Example :

Simplify: $\frac{10^4 \cdot 10^{12}}{10^3}$

Answer

10^{13}

In the previous example, notice that we did not multiply the base 10 times itself. When applying the product rule, add the exponents and leave the base unchanged.

2.3.2 Example :

Simplify: $(x^5 \cdot x^4 \cdot x)^2$

Solution: Recall that the variable x is assumed to have an exponent of one, $x = x^1$.

Answer

x^{20}

The base could in fact be any algebraic expression.

2.3.3 Example :

Simplify: $(x+y)^9(x+y)^{13}$

Solution: Treat the expression $(x+y)$ as the base.

Answer

$(x+y)^{22}$

The commutative property of multiplication allows us to use the product rule for exponents to simplify factors of an algebraic expression.

2.3.4 Example :

Simplify: $-8x^5y \cdot 3x^7y^3$

Answer

$-24x^{12}y^4$

Division involves the quotient rule for exponents.

2.3.5 Example :

Simplify: $\frac{33x^7y^5(x-y)^{10}}{11x^6y(x-y)^3}$.

Answer

$3xy^4(x-y)^7$

The power rule for a quotient allows us to apply that exponent to the numerator and denominator. This rule requires that the denominator is nonzero and so we will make this assumption for the remainder of the section.

2.3.6 Example :

Simplify: $\left(\frac{-4a^2b}{c^4}\right)^3$

Answer

$-\frac{64a^6b^3}{c^{12}}$

Using the quotient rule for exponents, we can define what it means to have zero as an exponent. Consider the following calculation:

Twenty-five divided by twenty-five is clearly equal to one, and when the quotient rule for exponents is applied, we see that a zero exponent results. In general, given any nonzero real number x and integer n,

This leads us to the definition of **zero as an exponent**[5],

$$x^0 = 1 \qquad x \neq 0$$

It is important to note that 0^0 is indeterminate. If the base is negative, then the result is still positive one. In other words, any nonzero base raised to the zero power is defined to be equal to one. In the following examples assume all variables are nonzero.

2.3.7 Example :

Simplify:

a. $(-2x)^0$
b. $-2x^0$

Answer

a. Any nonzero quantity raised to the zero power is equal to 1.

b. In the example, $-2x^0$, the base is x, not $-2x$.

In general, given any nonzero real number x and integer n, the definition of **negative exponents**[6] is:

$$x^{-n} = \frac{1}{x^n} \qquad x \neq 0$$

An expression is completely simplified if it does not contain any negative exponents.

2.3.8 Example :

Simplify: $\left(-4x^2y\right)^{-2}$

Solution

Rewrite the entire quantity in the denominator with an exponent of 2 and then simplify further.

Answer

$\frac{1}{16x^4y^2}$

Sometimes negative exponents appear in the denominator.

2.3.9 Example :

Simplify: $\frac{x^{-3}}{y^{-4}}$

Answer

$\frac{y^4}{x^3}$

The previous example suggests a property of **quotients with negative exponents**[7]. Given any integers m and n where $x \neq 0$ and $y \neq 0$, then

$$\frac{x^{-n}}{y^{-m}} = \frac{\frac{1}{x^n}}{\frac{1}{y^m}} = \frac{1}{x^n} \cdot \frac{y^m}{1} = \frac{y^m}{x^n}$$

This leads us to the property

$$\frac{x^{-n}}{y^{-m}} = \frac{y^m}{x^n}$$

In other words, negative exponents in the numerator can be written as positive exponents in the denominator and negative exponents in the denominator can be written as positive exponents in the numerator.

2.3.10 Example :

Simplify: $\frac{-5x^{-3}y^3}{z^{-4}}$

Solution

Take care with the coefficient -5, recognize that this is the base and that the exponent is actually positive one: $-5 = (-5)^1$. Hence, the rules of negative exponents do not apply to this coefficient; leave it in the numerator.

Answer

$\frac{-5y^3z^4}{x^3}$

2.3.11 Exercise

Simplify: $\left(\frac{2x^{-2}y^3}{z}\right)^{-4}$

Answer

$\frac{x^8z^4}{16y^{12}}$

Scientific Notation

Real numbers expressed using **scientific notation**[8] have the form,

$$a \times 10^n$$

where n is an integer and $1 \leq a < 10$ .This form is particularly useful when the numbers are very large or very small. For example the number of atoms in a mole is 602200000000000000000000 $\frac{atoms}{mole}$ or Planck constant $\hbar$ which is 0.000000000000000000000000000000000663 $J \cdot sec$. It is cumbersome to write all the zeros in both of these cases. Scientific notation is an alternative, compact representation of these numbers. For scientific notation, the factor 10^n indicates the power of ten to multiply the coefficient by to convert back to decimal form:

$$9.46 \times 10^{15} = 9.46 \times 1,\overbrace{000,000,000,000,000}^{15\ zeros} = 9,460,000,000,000,000$$

This is equivalent to moving the decimal in the coefficient fifteen places to the right.

A negative exponent indicates that the number is very small:

$$2.5 \times 10^{-11} = 2.5 \times \frac{1}{10^{11}} = \frac{2.5}{\underbrace{100,000,000,000}_{11\ zeros}} = 0.000000000025$$

This is equivalent to moving the decimal in the coefficient eleven places to the left.

We will leave it to you to convert Planck constant and number of atoms per mole into scientific notation (you can check the internet for the correct answer).

$2.3.12$ Example :

Write 0.000003045 using scientific notation.

Answer

3.045×10^{-6}

Often we will need to perform operations when using numbers in scientific notation. All the rules of exponents developed so far also apply to numbers in scientific notation.

$2.3.13$ Example :

Multiply: $\left(4.36 \times 10^{-5}\right)\left(5.3 \times 10^{12}\right)$

Answer

2.3108×10^{8}

$2.3.14$ Example :

Divide: $\left(3.24 \times 10^{8}\right) \div \left(9.0 \times 10^{-3}\right)$

Answer

3.6×10^{10}

$2.3.15$ Example :

The speed of light is approximately 6.7×10^{8} miles per hour. Express this speed in miles per second.

Answer

The speed of light is approximately 1.9×10^{5} miles per second.

$2.3.16$ Example :

The Sun moves around the center of the galaxy in a nearly circular orbit. The distance from the center of our galaxy to the Sun is approximately $26,000$ light years. What is the circumference of the orbit of the Sun around the galaxy in meters?

Solution

One light-year measures 9.46×10^{15} meters. Therefore, multiply this by $26,000$ or 2.60×10^{4} to find the length of $26,000$ light years in meters.

$$\begin{aligned}\left(9.46 \times 10^{15}\right)\left(2.60 \times 10^{4}\right) &= 9.46 \cdot 2.60 \times 10^{15} \cdot 10^{4} \\ &\approx 24.6 \times 10^{19} \\ &= 2.46 \times 10^{1} \cdot 10^{19} \\ &= 2.46 \times 10^{20}\end{aligned}$$

The radius r of this very large circle is approximately 2.46×10^{20} meters. Use the formula $C=2\pi r$ to calculate the circumference of the orbit.

$$\begin{aligned} C&=2\pi r\\ &\approx 2(3.14)\left(2.46\times 10^{20}\right)\\ &=15.4\times 10^{20}\\ &=1.54\times 10^{1}\cdot 10^{20}\\ &=1.54\times 10^{21}\end{aligned}$$

Answer

The circumference of the Sun’s orbit is approximately 1.54×10^{21} meters.

$2.3.17$ Exercise

Divide: $\left(3.15\times 10^{-5}\right)\div\left(12\times 10^{-13}\right)$.

Answer

2.625×10^{7}

Key Takeaways

- When multiplying two quantities with the same base, add exponents: $x^{m}\cdot x^{n}=x^{m+n}$.
- When dividing two quantities with the same base, subtract exponents: $\frac{x^m}{x^n}=x^{m-n}$.
- When raising powers to powers, multiply exponents: $(x^m)^n=x^{m\cdot n}$.
- When a grouped quantity involving multiplication and division is raised to a power, apply that power to all of the factors in the numerator and the denominator: $(xy)^n=x^ny^n$ and $\left(\frac{x}{y}\right)^n=\frac{x^n}{y^n}$.
- Any nonzero quantity raised to the 0 power is defined to be equal to 1 : $x^0=1$.
- Expressions with negative exponents in the numerator can be rewritten as expressions with positive exponents in the denominator: $x^{-n}=\frac{1}{x^n}$.
- Expressions with negative exponents in the denominator can be rewritten as expressions with positive exponents in the numerator: $\frac{1}{x^{-m}}=x^m$.
- Take care to distinguish negative coefficients from negative exponents.
- Scientific notation is particularly useful when working with numbers that are very large or very small.

$2.3.3$ Exercise

Simplify. (Assume all variables represent nonzero numbers.)

1. $10^4\cdot 10^7$
2. $7^3\cdot 7^2$
3. $\dfrac{10^2\cdot 10^4}{10^5}$
4. $\dfrac{7^5\cdot 7^9}{7^2}$
5. $x^3\cdot x^2$
6. $y^5\cdot y^3$
7. $\dfrac{a^8\cdot a^6}{a^5}$
8. $\dfrac{b^4\cdot b^{10}}{b^8}$
9. $\dfrac{x^{2n}\cdot x^{3n}}{x^n}$
10. $\dfrac{x^{n}\cdot x^{8n}}{x^{3n}}$

11. $\left(x^{5}\right)^{3}$
12. $\left(y^{4}\right)^{3}$
13. $\left(x^{4}y^{5}\right)^{3}$
14. $\left(x^{7}y\right)^{5}$
15. $\left(x^{2}y^{3}z^{4}\right)^{4}$
16. $\left(xy^{2}z^{3}\right)^{2}$
17. $\left(-5x^{2}yz^{3}\right)^{2}$
18. $\left(-2xy^{3}z^{4}\right)^{5}$
19. $\left(x^{2}yz^{5}\right)^{n}$
20. $\left(xy^{2}z^{3}\right)^{2n}$
21. $\left(x\cdot x^{3}\cdot x^{2}\right)^{3}$
22. $\left(y^{2}\cdot y^{5}\cdot y\right)^{2}$
23. $\dfrac{a^{2}\cdot\left(a^{4}\right)^{2}}{a^{3}}$
24. $\dfrac{a\cdot a^{3}\cdot a^{2}}{\left(a^{2}\right)^{3}}$
25. $(2x+3)^{4}(2x+3)^{9}$
26. $(3y-1)^{7}(3y-1)^{2}$
27. $(a+b)^{3}(a+b)^{5}$
28. $(x-2y)^{7}(x-2y)^{3}$
29. $5x^{2}y\cdot 3xy^{2}$
30. $-10x^{3}y^{2}\cdot 2xy$
31. $-6x^{2}yz^{3}\cdot 3xyz^{4}$
32. $2xyz^{2}\left(-4x^{2}y^{2}z\right)$
33. $3x^{n}y^{2n}\cdot 5x^{2}y$
34. $8x^{5n}y^{n}\cdot 2x^{2n}y$
35. $\dfrac{40x^{5}y^{3}z}{4x^{2}y^{2}z}$
36. $\dfrac{8x^{2}y^{5}z^{3}}{16x^{2}yz}$
37. $\dfrac{24a^{8}b^{3}(a-5b)^{10}}{8a^{5}b^{3}(a-5b)^{2}}$
38. $\dfrac{175m^{9}n^{5}(m+n)^{7}}{25m^{8}n(m+n)^{3}}$
39. $\left(-2x^{4}y^{2}z\right)^{6}$
40. $\left(-3xy^{4}z^{7}\right)^{5}$
41. $\left(\dfrac{-3ab^{2}}{2c^{3}}\right)^{3}$
42. $\left(\dfrac{-10a^{3}b}{3c^{2}}\right)^{2}$
43. $\left(\dfrac{-2xy^{4}}{z^{3}}\right)^{4}$
44. $\left(\dfrac{-7x^{9}y}{z^{4}}\right)^{3}$
45. $\left(\dfrac{xy^{2}}{z^{3}}\right)^{n}$
46. $\left(\dfrac{2x^{2}y^{3}}{z}\right)^{n}$
47. $(-5x)^{0}$

48. $\left(3x^2y\right)^0$
49. $-5x^0$
50. $3x^2y^0$
51. $\left(-2a^2b^0c^3\right)^5$
52. $\left(-3a^4b^2c^0\right)^4$
53. $\frac{\left(9x^3y^2z^0\right)^2}{3xy^2}$
54. $\frac{\left(-5x^0y^5z\right)^3}{25y^2z^0}$
55. $-2x^{-3}$
56. $(-2x)^{-2}$
57. $a^4 \cdot a^{-5} \cdot a^2$
58. $b^{-8} \cdot b^3 \cdot b^4$
59. $\dfrac{a^8 \cdot a^{-3}}{a^{-6}}$
60. $\dfrac{b^{-10} \cdot b^4}{b^{-2}}$
61. $10x^{-3}y^2$
62. $-3x^{-5}y^{-2}$
63. $3x^{-2}y^2z^{-1}$
64. $-5x^{-4}y^{-2}z^2$
65. $\dfrac{25x^{-3}y^2}{5x^{-1}y^{-3}}$
66. $\dfrac{-9x^{-1}y^3z^{-5}}{3x^{-2}y^2z^{-1}}$
67. $\left(-5x^{-3}y^2z\right)^{-3}$
68. $\left(-7x^2y^{-5}z^{-2}\right)^{-2}$
69. $\left(\dfrac{2x^{-3}z}{y^2}\right)^{-5}$
70. $\left(\dfrac{5x^5z^{-2}}{2y^{-3}}\right)^{-3}$
71. $\left(\dfrac{12x^3y^2z}{2x^7yz^8}\right)^3$
72. $\left(\dfrac{150xy^8z^2}{90x^7y^2z}\right)^2$
73. $\left(\dfrac{-9a^{-3}b^4c^{-2}}{3a^3b^5c^{-7}}\right)^{-4}$
74. $\left(\dfrac{-15a^7b^5c^{-8}}{3a^{-6}b^2c^3}\right)^{-3}$

Answer

1. 10^{11}

3. 10

5. x^5

7. a^9

9. x^{4n}

11. x^{15}

13. $x^{12}y^{15}$

15. $x^8y^{12}z^{16}$

17. $25x^4y^2z^6$

19. $x^{2n}y^nz^{5n}$

21. x^{18}

23. a^7

25. $(2x+3)^{13}$

27. $(a+b)^8$

29. $15x^3y^3$

31. $-18x^3y^2z^7$

33. $15x^{n+2}y^{2n+1}$

35. $10x^3y$

37. $3a^3(a-5b)^8$

39. $64x^{24}y^{12}z^6$

41. $-\frac{27a^3b^6}{8c^9}$

43. $\frac{16x^4y^{16}}{z^{12}}$

45. $\frac{x^ny^{2n}}{z^{3n}}$

47. 1

49. -5

51. $-32a^{10}c^{15}$

53. $27x^5y^2$

55. $-\frac{2}{x^3}$

57. a

59. a^{11}

61. $\frac{10y^2}{x^3}$

63. $\frac{3y^2}{x^2z}$

65. $\frac{5y^5}{x^2}$

67. $-\frac{x^9}{125y^6z^3}$

69. $\frac{x^{15}y^{10}}{32z^5}$

71. $\frac{216y^3}{x^{12}z^{21}}$

73. $\frac{a^{24}b^4}{81c^{20}}$

2.3.4 Exercise

The value in dollars of a new mobile phone can be estimated by using the formula $V=210(2t+1)^{-1}$, where t is the number of years after purchase.

1. How much was the phone worth new?
2. How much will the phone be worth in 1 year?
3. How much will the phone be worth in 3 years?
4. How much will the phone be worth in 10 years?
5. How much will the phone be worth in 100 years?
6. According to the formula, will the phone ever be worthless? Explain.
7. The height of a particular right circular cone is equal to the square of the radius of the base, $h=r^2$. Find a formula for the volume in terms of r.
8. A sphere has a radius $r=3x^2$ .Find the volume in terms of x.

Answer

1. \$210

3. \$30

5. \$1.04

7. $V=\frac{1}{3}\pi r^4$

2.3.5 Exercise

Convert to a decimal number.

1. 5.2×10^8
2. 6.02×10^9
3. 1.02×10^{-6}
4. 7.44×10^{-5}

Answer

1. $520,000,000$

3. 0.00000102

2.3.6 Exercise

Rewrite using scientific notation.

1. $7,050,000$
2. $430,000,000,000$
3. 0.00005001
4. 0.000000231

Answer

1. 7.05×10^6

3. 5.001×10^{-5}

2.3.7 Exercise

Perform the operations.

1. $\left(1.2\times 10^9\right)\left(3\times 10^5\right)$
2. $\left(4.8\times 10^{-5}\right)\left(1.6\times 10^{20}\right)$

3. $\left(9.1 \times 10^{23}\right)\left(3 \times 10^{10}\right)$
4. $\left(5.5 \times 10^{12}\right)\left(7 \times 10^{-25}\right)$
5. $\frac{9.6 \times 10^{16}}{1.2 \times 10^{-4}}$
6. $\frac{4.8 \times 10^{-14}}{2.4 \times 10^{-6}}$
7. $\frac{4 \times 10^{-8}}{8 \times 10^{10}}$
8. $\frac{2.3 \times 10^{23}}{9.2 \times 10^{-3}}$
9. $987,000,000,000,000 \times 23,000,000$
10. $0.00000000024 \times 0.00000004$
11. $0.000000000522 \div 0.0000009$
12. $81,000,000,000 \div 0.0000648$
13. The population density of Earth refers to the number of people per square mile of land area. If the total land area on Earth is 5.751×10^{7} square miles and the population in 2007 was estimated to be 6.67×10^{9} people, then calculate the population density of Earth at that time.
14. In 2008 the population of New York City was estimated to be 8.364 million people. The total land area is 305 square miles. Calculate the population density of New York City.
15. The mass of Earth is 5.97×10^{24} kilograms and the mass of the Moon is 7.35×10^{22} kilograms. By what factor is the mass of Earth greater than the mass of the Moon?
16. The mass of the Sun is 1.99×10^{30} kilograms and the mass of Earth is 5.97×10^{24} kilograms. By what factor is the mass of the Sun greater than the mass of Earth? Express your answer in scientific notation.
17. The radius of the Sun is 4.322×10^{5} miles and the average distance from Earth to the Moon is 2.392×10^{5} miles. By what factor is the radius of the Sun larger than the average distance from Earth to the Moon?
18. One light year, 9.461×10^{15} meters, is the distance that light travels in a vacuum in one year. If the distance from our Sun to the nearest star, Proxima Centauri, is estimated to be 3.991×10^{16} meters, then calculate the number of years it would take light to travel that distance.
19. It is estimated that there are about 1 million ants per person on the planet. If the world population was estimated to be 6.67 billion people in 2007, then estimate the world ant population at that time.
20. The radius of the earth is 6.3×10^{6} meters and the radius of the sun is 7.0×10^{8} meters. By what factor is the radius of the Sun larger than the radius of the Earth?
21. A gigabyte is 1×10^{9} bytes and a megabyte is 1×10^{6} bytes. If the average song in the MP3 format consumes about 4.5 megabytes of storage, then how many songs will fit on a 4-gigabyte memory card?
22. Water weighs approximately 18 grams per mole. If one mole is about 6×10^{23} molecules, then approximate the weight of each molecule of water.

Answer

1. 3.6×10^{14}

3. 2.73×10^{34}

5. 8×10^{20}

7. 5×10^{-19}

9. 2.2701×10^{22}

11. 5.8×10^{-4}

13. About 116 people per square mile

15. 81.2

17. 1.807

19. $6.67 \times 10^{15} ants$

21. Approximately 889 songs

2.3.8 Exercise

1. Use numbers to show that $(x+y)^n \neq x^n + y^n$.
2. Why is 0^0 indeterminate?
3. Explain to a beginning algebra student why $2^2 \cdot 2^3 \neq 4^5$.
4. René Descartes (1637) established the usage of exponential form: a^2, a^3, and so on. Before this, how were exponents denoted?

Answer

1. Answer may vary

3. Answer may vary

---

[1] $x^m \cdot x^n = x^{m+n}$; the product of two expressions with the same base can be simplified by adding the exponents.

[2] $(x^m)^n = x^{mn}$; a power raised to a power can be simplified by multiplying the exponents.

[3] $(xy)^n = x^n y^n$; if a product is raised to a power, then apply that power to each factor in the product.

[4] $(xy)^n = x^n y^n$; if a quotient is raised to a power, then apply that power to the numerator and the denominator.

[5] $x^0 = 1$; any nonzero base raised to the 0 power is defined to be 1.

[6] $x^{-n} = \frac{1}{x^n}$, given any integer n, where x is nonzero.

[7] $\frac{x^{-n}}{y^{-m}} = \frac{y^m}{x^n}$, given any integers m and n, where $x \neq 0$ and $y \neq 0$.

[8] Real numbers expressed the form $a \times 10^n$, where n is an integer and $1 \leq a < 10$.

[9] $\frac{x^m}{x^n} = x^{m-n}$; the quotient of two expressions with the same base can be simplified by subtracting the exponents.

2.4: Calculator skills

Using the Scientific Calculator

Please be sure to obtain a calculator that has an exponent button on it. It will either say "exp" or have an "ee" key indicating this function. This key represents the function of 10^x. You do not need to enter a 10 for an exponent when using this particular function. Read the steps below and practice on your calculator.

Use of Calculator

To input powers of ten:

1. Locate the ee or exp button on your calculator. Do not use the carrot (looks like the ^ button).
2. Put the coefficient number in and then hit "ee" or "exp."
3. If you need to have a negative "exp," then locate the +/- button.
4. Lastly, input your power of ten. You do not need to put the actual number ten in for the exponent.

Table 2.4.3: *Examples*

Number	**How to Input in a calculator**
3.95×10^{15}	Put 3.95 in as the coefficient number. Hit the "ee" or "exp" button. Lastly, enter the 15 for the exponent. Your calculator should say $3.95e^{15}$.
9.26×10^{-4}	Put 9.26 in as the coefficient number. Hit the "ee" or "exp" button. Locate the small +/- button for negative (not minus function). Next, enter the number four. Your calculator should say $9.26e^{-4}$.

Suggested Calculators

This is an image of suggested calculators for Chem& 121. You will need a non-programmable, scientific calculator that does not have metric conversion functions. In this picture, you will see the correct way to input one of the numbers shown in the example above. Please take time to practice with your calculator before tests and quizzes.

Contributors and Attributions

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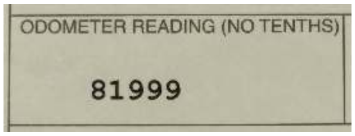
2.5: Precision and GPE

Morgan Chase



ODOMETER READING (NO TENTHS)

81999

When someone is selling a used car, its mileage might be listed as 80,000 miles or 82,000 miles because a buyer will want to know the approximate mileage but doesn't need an exact value. If you buy the car, however, you'll need to know the mileage to the nearest mile when you're completing the registration paperwork.

Precision

The ***precision*** of a number is the place value of the rightmost significant figure. For example, $82,000$ is precise to the thousands place, $81,999$ is precise to the ones place, and something like $81,999.2$ would be precise to the tenths place.

Exercises

Identify the precision (i.e., the place value of the rightmost significant figure) of each number.

1. $29,000$
2. $29,\bar{0}00$
3. $29,030$
4. 0.037
5. 0.0307
6. 0.03070

Precision-Based Rounding

We use ***precision-based rounding*** because we were rounding to a specified place value; for example, rounding to the nearest tenth. Let's practice this with overbars and trailing zeros.

1. Locate the **rounding digit** in the place to which you are rounding.
2. Look at the **test digit** directly to the right of the rounding digit.
3. If the test digit is 5 or greater, increase the rounding digit by 1 and drop all digits to its right. If the test digit is less than 5, keep the rounding digit the same and drop all digits to its right.

Remember, when the rounding digit of a whole number is a 9 that gets rounded up to a 0, we must write an overbar above that 0.

Also, when the rounding digit of a decimal number is a 9 that gets rounded up to a 0, we must include the 0 in that decimal place.

Exercises

Round each number to the indicated place value. Be sure to include an overbar or trailing zeros if necessary.

7. $81,999$ (thousands)
8. $81,999$ (hundreds)
9. $81,999$ (tens)
10. 0.5996 (tenths)
11. 0.5996 (hundredths)
12. 0.5996 (thousandths)

Precision when Adding and Subtracting

Suppose the attendance at a large event is estimated at $25,000$ people, but then you see 3 people leave. Is the new estimate $24,997$? No, because the original estimate was precise only to the nearest thousand. We can't start with an imprecise number and finish with a more precise number. If we estimated that 1,000 people had left, then we could revise our attendance estimate to 24,000 because this estimate maintains the same level of precision as our original estimate.

When ***adding or subtracting*** numbers with different levels of precision, the answer must be rounded to the same precision as the ***least*** precise of the original numbers.

Don’t round off the original numbers; do the necessary calculations first, then round the answer as your last step.

? Exercises

Add or subtract as indicated. Round to the appropriate level of precision.

13. Find the combined weight of four packages with the following weights: 9.7 lb, 13.0 lb, 10.5 lb, 6.1 lb.
14. Find the combined weight of four packages with the following weights: 9.7 lb, 13 lb, 10.5 lb, 6.1 lb.
15. While purchasing renter’s insurance, Chandra estimates the value of her insurable possessions at \$10,200. After selling some items valued at \$375, what would be the revised estimate?
16. Chandra knows that she has roughly \$840 in her checking account. After using her debit card to make two purchases of \$25.95 and \$16.38, how much would she have left in her account?

Before we move on, let’s circle back to multiplication and division again. If you are multiplying by an exact number, you can consider this a repeated addition. For example, suppose you measure the weight of an object to be 4.37 ounces and you want to know the weight of three of these objects; multiplying 4.37 times 3 is the same as adding $4.37+4.37+4.37=13.11$ ounces. The answer is still precise to the hundredths place. When working with an exact number, we can assume that it has infinitely many significant figures. (Treat exact numbers like royalty; their accuracy is perfect and it would be an insult to even question it.)

Greatest Possible Measurement Error (GPE)

Suppose you are weighing a dog with a scale that displays the weight rounded to the nearest pound. If the scale says Sir Barks-A-Lot weighs 23 pounds, he could weigh anywhere from 22.5 pounds to almost 23.5 pounds. The true weight could be as much as 0.5 pounds above or below the measured weight, which we could write as 23 ± 0.5.

Now suppose you are weighing Sir Barks-A-Lot with a scale that displays the weight rounded to the nearest tenth of a pound. If the scale says Sir Barks-A-Lot weighs 23.0 pounds, we now know that he could weigh anywhere from 22.95 pounds to almost 23.05 pounds. The true weight could be as much as 0.05 pounds above or below the measured weight, which we could write as 23.0 ± 0.05.

As we increase the level of precision in our measurement, we decrease the ***greatest possible measurement error*** or ***GPE***. The GPE is always one half the precision; if the precision is to the nearest tenth, 0.1, the GPE is half of one tenth, or five hundredths, 0.05. The GPE will always be a 5 in the place to the right of the place value of the number’s precision.

Another way to think about the GPE is that it gives the range of values that would round off to the number in question. 23 ± 0.5 tells us a lower value and an upper value. $23-0.5=22.5$ is the lowest weight that would round up to 23, and $23+0.5=23.5$ is the upper limit of the weights that would round down to 23. (Yes, 23.5 technically would round up to 24, but it is easier to just say 23.5 instead of 23.49.) Using inequalities, we could represent 23 ± 0.5 as the range of values $22.5 \leq \text{weight} < 23.5$.

? Exercises

The attendance at a Portland Thorns match is estimated to be $14,000$ people.

17. What is the precision of this estimate?
18. What is the greatest possible error in this estimate?

A roll of plastic sheeting is 0.00031 inches thick.

19. What is the precision of this measurement?
20. What is the greatest possible error in this measurement?

Plastic sheeting 0.00031 inches thick is referred to as 0.31 mil.

21. What is the precision of this measurement, in mils?
22. What is the greatest possible measurement error, in mils?

Summary of Accuracy, Precision, GPE

Here is a summary of the important terms we've discussed. It is easy to get them mixed up, but remembering that “precision” and “place value” both start with “p” can be helpful.

Significant figures: the digits in a number that we trust to be correct

Accuracy: the number of significant figures

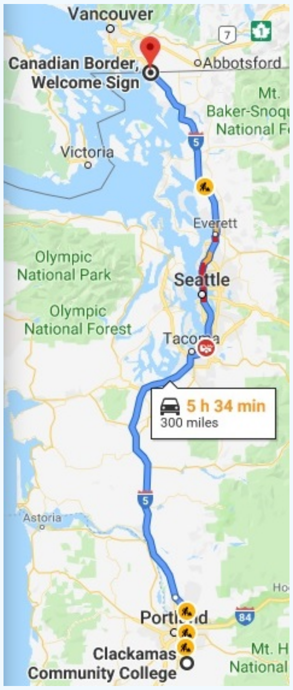
When multiplying or dividing, we use the accuracy to round the result.

Precision: the place value of the rightmost significant digit

Greatest possible measurement error (GPE): one half the precision

When adding or subtracting, we use the precision to round the result.

? Exercises



Google Maps says that the driving distance from CCC’s main campus to the Canadian border, rounded to the nearest mile, is 300 miles.

23. Write the distance with an overbar showing the correct precision.
24. What is the precision?
25. What is the GPE?

Google Maps says that the driving time, rounded to the nearest minute, is 5 hours and 34 minutes.

26. What is the precision of this estimate?
27. What is the GPE of this estimate?

Of course, the traffic conditions will change during the trip, making the estimate of 5 hours and 34 minutes unrealistically precise. Let’s assume that the drive will take 5.5 hours.

28. What is the *accuracy* of this estimate?
29. What is the *accuracy* of the distance?
30. Calculate the average speed of the vehicle, rounded appropriately.
31. Why did we need to consider the accuracy instead of the precision for this calculation?

Exercise Answers

2.6: Accuracy and Significant Figures

In the first few modules, we rarely concerned ourselves with rounding; we assumed that every number we were told was exact and we didn't have to worry about any measurement error. However, every measurement contains some error. A standard sheet of paper is 8.5 inches wide and 11 inches high, but it's possible that the actual measurements could be closer to 8.4999 and 11.0001 inches. Even if we measure something very carefully, with very sensitive instruments, we should assume that there could be some small measurement error.

Exact Values and Approximations

A number is an exact value if it is the result of counting or a definition.

A number is an approximation if it is the result of a measurement or of rounding.

Exercises 2.6.1

Identify each number as an exact value or an approximation.

1. An inch is $\dfrac{1}{12}$ of a foot.

2. This board is 78 inches long.

3. There are 14 students in class.

4. A car's tachometer reads $3,000$ rpm.

5. A right angle measures $90°$.

6. The angle of elevation of a ramp is $4°$.

Answer

1. exact value
2. approximation
3. exact value
4. approximation
5. exact value
6. approximation

Suppose a co-worker texts you that they will arrive in 20 minutes. It's hard to tell how precise this number is, because we often round off to the nearest 5 or 10 minutes. You might reasonably expect them to arrive anytime within the next 15 to 25 minutes. If your co-worker texts that they will arrive in 17 minutes, though, it is likely that their GPS told them that more precise number, and you could reasonably expect them to arrive within 16 to 18 minutes.

Accuracy and Significant Figures

Because measurements are inexact, we need to consider how accurate they are. This requires us to think about significant figures—often abbreviated "sig figs" in conversation—which are the digits in the measurement that we trust to be correct. The accuracy of a number is equal to the number of significant figures.[1] The following rules aren't particularly difficult to understand but they can take time to absorb and internalize, so we'll include lots of examples and exercises.

Figure $2.6.1$: African-American women were vital to NASA's success in the 1960s, as shown in the movie Hidden Figures.

Significant Figures

1. All nonzero digits are significant.
   Ex: $12,345$ has five sig figs, and 123.45 has five sig figs.
2. All zeros between other nonzero digits are significant.
   Ex: $10,045$ has five sig figs, and 100.45 has five sig figs.
3. Any zeros to the right of a decimal number are significant.
   Ex: 123 has three sig figs, but 123.00 has five sig figs.
4. Zeros on the left of a decimal number are NOT significant.
   Ex: 0.123 has three sig figs, and 0.00123 has three sig figs.
5. Zeros on the right of a whole number are NOT significant unless they are marked with an overbar.
   Ex: $12,300$ has three sig figs, but $12,30\overline{0}$ has five sig figs.

Another way to think about #4 and #5 above is that zeros that are merely showing the place value—where the decimal point belongs—are NOT significant.

? Exercises $2.6.1$

Determine the accuracy (i.e., the number of significant figures) of each number.

7. $63,400$

8. $63,040$

9. $63,004$

10. 0.085

11. 0.0805

12. 0.08050

Answer

7. three significant figures
8. four significant figures
9. five significant figures
10. two significant figures
11. three significant figures
12. four significant figures

As mentioned above, we use an ***overbar*** to indicate when a zero that looks insignificant is actually significant. For example, the precision[2] of $7,400$ is the hundreds place; if we rounded anything from $7,350$ to $7,449$ to the nearest hundred, we would write the result as $7,400$. An overbar shows that the number is more precise than it appears. If we rounded anything from $7,395$ to $7,404$ to the nearest ten, the result would be $7,400$, but it isn't clear anymore that the number was rounded to the tens place. Therefore, to show the level of precision, we write the result as $7,4\bar{0}0$. If we rounded anything from $7,399.5$ to $7,400.4$ to the nearest one, the result would be again $7,400$, and we again can't see how precise the rounded number really is. Therefore, to show that the number is precise to the ones place, we write the result as $7,40\bar{0}$.

Exercises 2.6.1

Determine the accuracy (i.e., the number of significant figures) of each number.

13. $8,000$

14. $8,\bar{0}00$

15. $8,0\bar{0}0$

16. $8,00\bar{0}$

Answer

13. one significant figure
14. two significant figures
15. three significant figures
16. four significant figures

Two things to remember: we don't put an overbar over a nonzero digit, and we don't need an overbar for any zeros on the right of a decimal number because those are already understood to be significant.

Accuracy-Based Rounding

As we saw in a previous module about decimals, it is often necessary to round a number. We often round to a certain place value, such as the nearest hundredth, but there is another way to round. ***Accuracy-based rounding*** considers the number of significant figures rather than the place value.

Accuracy-based rounding:

1. Locate the **rounding digit** to which you are rounding by counting from the left until you have the correct number of significant figures.
2. Look at the **test digit** directly to the right of the rounding digit.
3. If the test digit is 5 or greater, increase the rounding digit by 1 and drop all digits to its right. If the test digit is less than 5, keep the rounding digit the same and drop all digits to its right.

Exercises 2.6.1

Round each number so that it has the indicated number of significant figures.

17. $21,837$ (two sig figs)

18. $21,837$ (three sig figs)

19. $21,837$ (four sig figs)

20. 4.2782 (two sig figs)

21. 4.2782 (three sig figs)

22. 4.2782 (four sig figs)

Answer

17. $22,000$

18. $21,800$

19. $21,840$

20. 4.3

21. 4.28

22. 4.278

When the rounding digit of a whole number is a 9 that gets rounded up to a 0, we must write an overbar above that 0.

Similarly, when the rounding digit of a decimal number is a 9 that gets rounded up to a 0, we must include the 0 in that decimal place.

Exercises 2.6.1

Round each number so that it has the indicated number of significant figures. Be sure to include trailing zeros or an overbar if necessary.

23. $13,997$(two sig figs)

24. $13,997$(three sig figs)

25. $13,997$(four sig figs)

26. 2.5996(two sig figs)

27. 2.5996(three sig figs)

28. 2.5996(four sig figs)

Mt. Everest, Lohtse, and Nupse in the early morning

The mountain we know as Mt. Everest is called *Sagarmatha* in Nepal and *Chomolungma* in Tibet. On December 8, 2020, it was jointly announced by Nepal and China that the summit has an elevation of $29,031.69$ft, replacing the previously accepted elevation of $29,029$ ft.[3]

29. Round $29,031.69$ft to two sig figs.

30. Round $29,031.69$ft to three sig figs.

31. Round $29,031.69$ft to four sig figs.

32. Round $29,031.69$ft to five sig figs.

33. Round $29,031.69$ft to six sig figs.

Answer

23. $14,000$

24. $14,\bar{0}00$

25. $14,0\bar{0}0$

26. 2.6

27. 2.60

28. 2.600

29. $29,000$ ft

30. $29,\bar{0}00$ ft

31. $29,030$ ft

32. $29,032$ ft

33. $29,031.7$ ft

Accuracy when Multiplying and Dividing

Suppose you needed to square the number $3\frac{1}{3}$. You could rewrite $3\frac{1}{3}$ as the improper fraction $\frac{10}{3}$ and then figure out that $(\frac{10}{3})^2 = \frac{100}{9}$, which equals the repeating decimal $11.111\ldots$

Because most people prefer decimals to fractions, we might instead round $3\frac{1}{3}$ to 3.3 and find that $3.3^2 = 10.89$. However, this is not accurate because $11.111\ldots$ rounded to the nearest hundredth should be 11.11. The answer 10.89 looks very accurate, but it is a false accuracy because there is round-off error involved. If we round our answer 10.89 to the nearest tenth, we would get 10.9, which is still not accurate because $11.111\ldots$ rounded to the nearest tenth should be 11.1. If we round our answer 10.89 to the nearest whole number, we would get 11, which is accurate because $11.111\ldots$ rounded to the nearest whole number is indeed 11. It turns out that **we should be focusing on the number of significant figures rather than the place value**; because 3.3 has only two sig figs, our answer must be rounded to two sig figs.

Suppose instead that we round $3\frac{1}{3}$ to 3.33 and find that $3.33^2 = 11.0889$. Again, this is not accurate because $11.111\ldots$ rounded to the nearest ten-thousandth should be 11.1111. If we round 11.0889 to the nearest thousandth, we would get 11.089, which is still not accurate because $11.111\ldots$ rounded to the nearest thousandth should be 11.111. If we round 11.0889 to the nearest hundredth, we would get 11.09, which is still not accurate because $11.111\ldots$ rounded to the nearest hundredth should be 11.11. Only when we round to the nearest tenth do we get an accurate result: 11.0889 rounded to the nearest tenth is 11.1, which is accurate because $11.111\ldots$ rounded to the nearest tenth is indeed 11.1. As above, we need to focus on the number of significant figures rather than the place value; because 3.33 has only three sig figs, our answer must be rounded to three sig figs.

When ***multiplying or dividing*** numbers, the answer must be rounded to the same number of significant figures as the ***least*** accurate of the original numbers.

Don't round off the original numbers; do the necessary calculations first, then round the answer as your last step.

? Exercises 2.6.1

Use a calculator to multiply or divide as indicated. Then round to the appropriate level of accuracy.

34. $8.75 \cdot 12.25$

35. $355.12 \cdot 1.8$

36. $77.3 \div 5.375$

37. $53.2 \div 4.5$

38. Suppose you are filling a 5-gallon can of gasoline. The gasoline costs $2.579 per gallon, and you estimate that you will buy 5.0 gallons. How much should you expect to spend?

Answer

34. 107

35. 640

36. 14.4

37. 12

38. \$ 12.90

---

1. The terms "significant digits" and "significant figures" are used interchangeably. ↵
2. Precision is different from accuracy, as we'll learn in the next module, but it is mentioned here because it can be difficult to explain one without the other. ↵
3. https://www.washingtonpost.com/world/asia_pacific/mount-everest-height-nepal-china/2020/12/08/a7b3ad1e-389a-11eb-aad9-8959227280c4_story.html ↵

2.7: Significant Figures - Writing Numbers to Reflect Precision

Learning Objectives

- Identify the number of significant figures in a reported value.

The **significant figures** in a measurement consist of all the certain digits in that measurement plus one uncertain or estimated digit. In the ruler illustration below, the bottom ruler gave a length with 2 significant figures, while the top ruler gave a length with 3 significant figures. In a correctly reported measurement, the final digit is significant but not certain. Insignificant digits are not reported. With either ruler, it would not be possible to report the length at 2.553 cm as there is no possible way that the thousandths digit could be estimated. The 3 is not significant and would not be reported.

Ruler A and Ruler B measurements

Figure $2.7.1$: Measurement with two different rulers.

Measurement Uncertainty

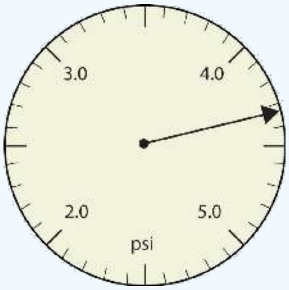
Some error or **uncertainty** always exists in any measurement. The amount of uncertainty depends both upon the skill of the measurer and upon the quality of the measuring tool. While some balances are capable of measuring masses only to the nearest 0.1 g, other highly sensitive balances are capable of measuring to the nearest 0.001 g or even better. Many measuring tools such as rulers and graduated cylinders have small lines which need to be carefully read in order to make a measurement. Figure $2.7.1$ shows two rulers making the same measurement of an object (indicated by the blue arrow).

With either ruler, it is clear that the length of the object is between 2 and 3 cm. The bottom ruler contains no millimeter markings. With that ruler, the tenths digit can be estimated and the length may be reported as 2.5 cm. However, another person may judge that the measurement is 2.4 cm or perhaps 2.6 cm. While the 2 is known for certain, the value of the tenths digit is uncertain.

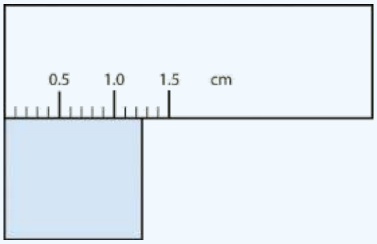
The top ruler contains marks for tenths of a centimeter (millimeters). Now the same object may be measured as 2.55 cm. The measurer is capable of estimating the hundredths digit because he can be certain that the tenths digit is a 5. Again, another measurer may report the length to be 2.54 cm or 2.56 cm. In this case, there are two certain digits (the 2 and the 5), with the hundredths digit being uncertain. Clearly, the top ruler is a superior ruler for measuring lengths as precisely as possible.

$2.7.1$ Example : Reporting Measurements to the Proper number of Significant Figures

Use each diagram to report a measurement to the proper number of significant figures.



a.



b.

Solutions

Explanation	Answer

	Explanation	Answer
a.	The arrow is between 4.0 and 5.0, so the measurement is at least 4.0. The arrow is between the third and fourth small tick marks, so it’s at least 0.3. We will have to estimate the last place. It looks like about one-third of the way across the space, so let us estimate the hundredths place as 3. The symbol psi stands for “pounds per square inch” and is a unit of pressure, like air in a tire. The measurement is reported to three significant figures.	4.33 psi
b.	The rectangle is at least 1.0 cm wide but certainly not 2.0 cm wide, so the first significant digit is 1. The rectangle’s width is past the second tick mark but not the third; if each tick mark represents 0.1, then the rectangle is at least 0.2 in the next significant digit. We have to estimate the next place because there are no markings to guide us. It appears to be about halfway between 0.2 and 0.3, so we will estimate the next place to be a 5. Thus, the measured width of the rectangle is 1.25 cm. The measurement is reported to three significant figures.	1.25 cm

2.7.1 Exercise

What would be the reported width of this rectangle?

Answer

1.25 cm

When you look at a reported measurement, it is necessary to be able to count the number of significant figures. The table below details the rules for determining the number of significant figures in a reported measurement. For the examples in the table, assume that the quantities are correctly reported values of a measured quantity.

Table 2.7.1: Significant Figure Rules

Rule	Examples
1. All nonzero digits in a measurement are significant.	237 has three significant figures. 1.897 has four significant figures.

Rule	Examples
2. Zeros that appear between other nonzero digits (middle zeros) are always significant.	39,004 has five significant figures. 5.02 has three significant figures.
3. Zeros that appear in front of all of the nonzero digits are called leading zeros. Leading zeros are never significant.	0.008 has one significant figure. 0.000416 has three significant figures.
4. Zeros that appear after all nonzero digits are called trailing zeros. A number with trailing zeros that lacks a decimal point may or may not be significant. **Use scientific notation to indicate the appropriate number of significant figures.**	1400 is ambiguous. ○ 1.4×10^3 has two significant figures. ○ 1.40×10^3 three significant figures. ○ 1.400×10^3 has four significant figures.
5. Trailing zeros in a number with a decimal point are significant. This is true whether the zeros occur before or after the decimal point.	620.0 has four significant figures. 19.000 has five significant figures.

Exact Numbers

Integers obtained either by counting objects or from definitions are exact numbers, which are considered to have infinitely many significant figures. If we have counted four objects, for example, then the number 4 has an infinite number of significant figures (i.e., it represents 4.000…). Similarly, 1 foot (ft) is defined to contain 12 inches (in), so the number 12 in the following equation has infinitely many significant figures:

2.7.2 Example

Give the number of significant figures in each. Identify the rule for each.

a. 5.87
b. 0.031
c. 52.90
d. 00.2001
e. 500
f. 6 atoms

Solution

	Explanation	Answer
a	All three numbers are significant (rule 1).	5.87, three significant figures
b	The leading zeros are not significant (rule 3). The 3 and the 1 are significant (rule 1).	0.031, two significant figures
c	The 5, the 2 and the 9 are significant (rule 1). The trailing zero is also significant (rule 5).	52.90, four significant figures
d	The leading zeros are not significant (rule 3). The 2 and the 1 are significant (rule 1) and the middle zeros are also significant (rule 2).	00.2001, four significant figures
e	The number is ambiguous. It could have one, two or three significant figures.	500, ambiguous
f	The 6 is a counting number. A counting number is an exact number.	6, infinite

2.7.2 Exercise

Give the number of significant figures in each.

a. 36.7 m
b. 0.006606 s
c. 2,002 kg
d. 306,490,000 people
e. 3,800 g

Answer a

three significant figures

Answer b

four significant figures

Answer c

four significant figures

Answer d

infinite (exact number)

Answer e

Ambiguous, could be two, three or four significant figures.

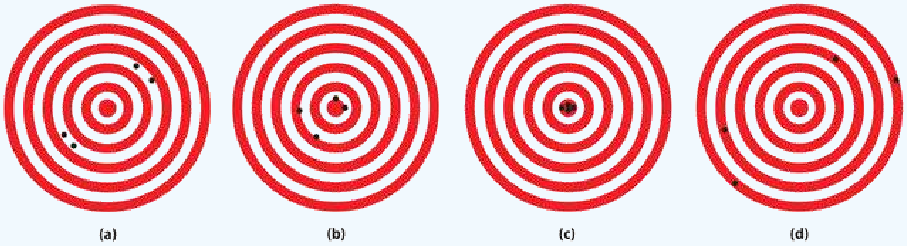
Accuracy and Precision

Measurements may be accurate, meaning that the measured value is the same as the true value; they may be precise, meaning that multiple measurements give nearly identical values (i.e., reproducible results); they may be both accurate and precise; or they may be neither accurate nor precise. The goal of scientists is to obtain measured values that are both accurate and precise. The video below demonstrates the concepts of accuracy and precision.

Video 2.7.1: Difference between precision and accuracy.

2.7.3 Example

The following archery targets show marks that represent the results of four sets of measurements.



Which target shows

- a. a precise, but inaccurate set of measurements?
- b. a set of measurements that is both precise and accurate?
- c. a set of measurements that is neither precise nor accurate?

Solution

- a. Set a is precise, but inaccurate
- b. Set c is both precise and accurate
- c. Set d is neither precise nor accurate

Summary

Uncertainty exists in all measurements. The degree of uncertainty is affected in part by the quality of the measuring tool. Significant figures give an indication of the certainty of a measurement. Rules allow decisions to be made about how many digits to use in any given situation.

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2.8: Measurement

Learning Objectives

- Express quantities properly using a number and a unit.

A coffee maker’s instructions tell you to fill the coffeepot with 4 cups of water and use 3 scoops of coffee. When you follow these instructions, you are measuring. When you visit a doctor’s office, a nurse checks your temperature, height, weight, and perhaps blood pressure (Figure $2.8.1$); the nurse is also measuring.

Figure $2.8.1$: Measuring Blood Pressure. A nurse or a doctor measuring a patient’s blood pressure is taking a measurement. (GNU Free Documentation License; Pia von Lützau via Wikipedia).

Chemists measure the properties of matter using a variety of devices or measuring tools, many of which are similar to those used in everyday life. Rulers are used to measure length, balances (scales) are used to measure mass (weight), and graduated cylinders or pipettes are used to measure volume. Measurements made using these devices are expressed as quantities. A **quantity** is an amount of something and consists of a **number** and a **unit**. The number tells us how many (or how much), and the unit tells us what the scale of measurement is. For example, when a distance is reported as “5.2 kilometers,” we know that the quantity has been expressed in units of kilometers and that the number of kilometers is 5.2.

$$\underbrace{5.2}_{\text{number}}\ \underbrace{\text{kilometers}}_{\text{unit}}$$

If you ask a friend how far he or she walks from home to school, and the friend answers “12” without specifying a unit, you do not know whether your friend walks—for example, 12 miles, 12 kilometers, 12 furlongs, or 12 yards.

Without units, a number can be meaningless, confusing, or possibly life threatening. Suppose a doctor prescribes phenobarbital to control a patient’s seizures and states a dosage of “100” without specifying units. Not only will this be confusing to the medical professional giving the dose, but the consequences can be dire: 100 mg given three times per day can be effective as an anticonvulsant, but a single dose of 100 g is more than 10 times the lethal amount.

> *Both a number and a unit must be included to express a quantity properly.*

To understand chemistry, we need a clear understanding of the units chemists work with and the rules they follow for expressing numbers. The next two sections examine the rules for expressing numbers.

✓ Exercise $2.8.1$

Identify the ***number*** and the ***unit*** in each quantity.

a. one dozen eggs
b. 2.54 centimeters
c. a box of pencils
d. 88 meters per second

Answer a

The number is one, and the unit is dozen.

Answer b

The number is 2.54, and the unit is centimeter.

Answer c

The number 1 is implied because the quantity is only *a* box. The unit is box of pencils.

Answer d

The number is 88, and the unit is meters per second. Note that in this case the unit is actually a combination of two units: meters and seconds.

? Exercise 2.8.2

Identify the ***number*** and the ***unit*** in each quantity.

a. 99 bottles of soda
b. 60 miles per hour
c. 32 fluid ounces
d. 98.6 degrees Fahrenheit

Answer a

The number is 99, and the unit is bottles of soda.

Answer b

The number is 60, and the unit is miles per hour.

Answer c

The number 32, and the unit is fluid ounces

Answer d

The number is 98.6, and the unit is degrees Fahrenheit

? Exercise 2.8.2

What are the two necessary parts of a quantity?

Answer

The two necessary parts are the number and the unit.

Key Takeaway

- Identify a quantity properly with a number and a unit.

1.3: Measurements

2.8.1: Systems of Measurement

Learning Objectives

- State and understand the uses of the three systems of measurement used in chemistry.

The units that people use change, based on where the person lives and the purpose of using each unit. The **metric system** is used by the citizens of nearly all countries, as shown below in Figure 2.8.1.1 Its units, such as meters, liters, and grams, are based on multiples of 10, making metric conversions very simple. The United States has legally adopted the metric system for measurements, but does not use it in everyday practice. Instead, citizens living in the United States use the **U.S. system** of units, which is occasionally referred to as the **English system** of units. U.S. units, such as inches, cups, and ounces, are not used in science because of the difficulty in converting from one unit to another.

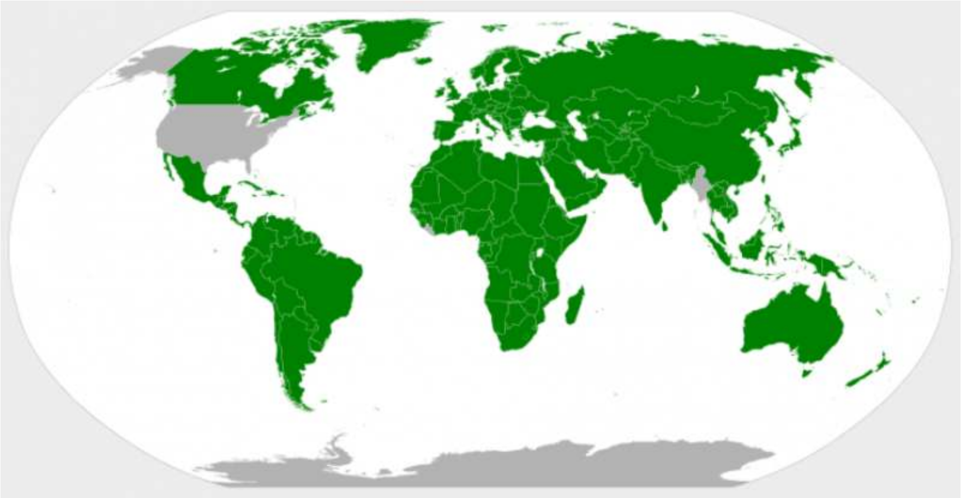


Figure 2.8.1.1: Global map. Countries colored in green have adopted the metric system, and those shaded in gray use the U.S. system of units.

How long is a yard? The answer depends on when the question was asked. Today, the definition of a "yard" is standardized. However, at one time the value of a "yard" was arbitrarily defined as the distance from the tip of the king's nose to the end of his outstretched hand, which created a significant problem: When a new king was crowned, the definition of the yard changed.

How hot is a cup of coffee? A citizen in the United States measures temperatures in degrees Fahrenheit, but a Canadian citizen would take a temperature measurement in degrees Celsius. This difference in units can cause confusion, since most individuals do not have an intrinsic understanding of units that they do not typically use in their everyday lives. Misunderstandings can be amplified when individuals are discussing science. The **International System of Units** is a system of measurement based on the metric system and has been adopted by scientists as their official system of measurement. The acronym **SI** is commonly used to refer to this system and stands for the French term, *Le Système International d'Unités*. While the metric and SI systems share many common units, there are some key differences, generally based on the unit's versatility.

2.8.2: The US Measurement System

Robin the cat has the spirit of ’76.

You may use a calculator throughout this module.

This system used to be called the English system, but now the U.S. has the dubious honor of being associated with the system that uses inches, feet, miles, ounces, pounds, cups, gallons, etc. To convert from one unit to another, we often have to perform messy calculations like dividing by 16 or multiplying by $5,280$.

We could solve these unit conversions using proportions, but there is another method than is more versatile, especially when a conversion requires more than one step. This method goes by various names, such as *dimensional analysis* or the *factor label method*. The basic idea is to begin with the measurement you know, then multiply it by a conversion ratio that will cancel the units you don’t want and replace it with the units you do want.

It’s okay if you don’t have the conversion ratios memorized; just be sure to have them available. If you discover other conversion ratios that aren’t provided here, go ahead and write them down!

U.S. System: Measurements of Length

1 foot = 12 inches

1 yard = 3 feet

1 mile = $5,280$ feet

Exercises 2.8.2.1

1. How many inches are in 4.5 feet?

2. How many feet make up 18 yards?

3. 1 yard is equal to how many inches?

4. 1 mile is equivalent to how many yards?

5. How many feet is 176 inches?

6. 45 feet is what length in yards?

7. Convert $10,560$ feet into miles.

8. How many yards are the same as $1,080$ inches?

Answer

1. 54 in
2. 54 ft
3. 36 in
4. $1,760$ yd

5. $14\frac{2}{3}$ ft or 14 ft 8 in

6. 15 yd

7. 2 mi

8. 30 yd

Notice that Exercises 3 & 4 give us two more conversion ratios that we could add to our list.

U.S. System: Measurements of Weight or Mass

1 pound = 16 ounces

1 ton = $2,000$ pounds

Exercises 2.8.2.1

9. How many ounces are in 2.5 pounds?

10. How many pounds are equivalent to 1.2 tons?

11. Convert 300 ounces to pounds.

12. 1 ton is equivalent to what number of ounces?

Answer

9. 40 oz

10. $2,400$ lb

11. 18.75 lb

12. $32,000$ oz

U.S. System: Measurements of Volume or Capacity

1 cup = 8 fluid ounces

1 pint = 2 cups

1 quart = 2 pints

1 gallon = 4 quarts

There are plenty of other conversions that could be provided, such as the number of fluid ounces in a gallon, but let's keep the list relatively short.

Exercises 2.8.2.1

13. How many fluid ounces are in 6 cups?

14. How many pints are in 3.5 quarts?

15. 1 gallon is equal to how many pints?

16. How many cups equal 1.25 quarts?

17. Convert 20 cups into gallons.

18. How many fluid ounces are in one half gallon?

Answer

13. 48 fl oz

14. 7 pt

15. 8 pt

16. 5 c

17. 1.25 gal

18. 64 fl oz

U.S. System: Using Mixed Units of Measurement

Measurements are frequently given with mixed units, such as a person's height being given as 5 ft 7 in instead of 67 in, or a newborn baby's weight being given as 8 lb 3 oz instead of 131 oz. This can sometimes make the calculations more complicated, but if you can convert between improper fractions and mixed numbers, you can handle this.

❓ Exercises 2.8.2.1

19. A bag of apples weighs 55 ounces. What is its weight in pounds and ounces?

20. A carton of orange juice contains 59 fluid ounces. Determine its volume in cups and fluid ounces.

21. A hallway is 182 inches long. Give its length in feet and inches.

22. The maximum loaded weight of a Ford F-150 pickup truck is 8,500 lb. Convert this weight into tons and pounds.

Answer

19. 3 lb 7 oz

20. 7 c 3 fl oz

21. 15 ft 2 in

22. 4 t 500 lb

We'll finish up this module by adding and subtracting with mixed units. Again, it may help to think of them as mixed numbers, with a whole number part and a fractional part.

❓ Exercises 2.8.2.1

Comet weighs 8 lb 7 oz and Fred weighs 11 lb 9 oz.

23. Comet and Fred are being put into a cat carrier together. What is their combined weight?

24. How much heavier is Fred than Comet?

Two tables are 5 ft 3 in long and 3 ft 10 in long.

25. If the two tables are placed end to end, what is their combined length?

26. What is the difference in length between the two tables?

Answer

23. 20 lb or 20 lb 0 oz combined

24. 3 lb 2 oz heavier

25. 9 ft 1 in combined

26. 1 ft 5 in longer

2.8.3: Units of Measure

The Metric System

The **metric system** is an internationally agreed-upon measurement system based on decimals or powers of 10. Scientists use a refined version called the **International System of Units** (abbreviated **SI**). In biology, you will often find a need to describe measurements of length, volume, mass, time, temperature or amount of substance.

International System of Units

850 Output 1

Waveform: Sine

Sweep Type: Off

Frequency: 69.5 Hz

Phase Shift: 0 °

Amplitude: 5 V

Offset and Limits

Voltage Offset: 0 V

Voltage Limit: 5 V

Current Limit: 0.55 A

Metric Units

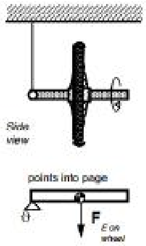
- length: meter (**m**)
- volume: liter (**L**)
- mass: gram (**g**)
- time: second (**s**)
- temperature: Celsius (°**C**)
    - Kelvin (**K**) is a unit of thermodynamic temperature and is the SI unit. The Kelvin scale in the same as the Celsius or centigrade scale but offset by 273.16
    - Biology uses Celsius predominantly because of the range in which organisms live.
- amount of substance: mole (**mol**)
    - A mole is a number representing 6.022×10^{23} of something
    - Just as a pair of shoes equals 2 shoes, a mole of shoes is 6.022×10^{23} shoes
    - Just as a dozen eggs equals 12 eggs, a mole of eggs is 6.022×10^{23} eggs

Strategy for Conversions

1. What unit is being asked for?
    - 500ml = ____L → liters
2. What unit are you starting from?
    - 500ml = ____L → milliliters
3. Which unit is larger? By how much is that unit larger?
    - Liters are the larger unit. Liters are 1,000X (10^3) greater than milliliters.
4. Which direction are we moving?
    - Since we are moving to a larger unit, our value will be smaller. In this case, the value is smaller by 1,000X
    - In other words, the value is 1/1000 or 0.001 the value.
    - So what is the answer?

Factoring Out

Using the idea of factors of ten, you can assess the difference between the two units and cancel out the original unit algebraically to reach the desired final unit.

- 500ml=_____L
    - 

or,
        - which states 1000 milliliter in every 1 liter
    -
        - pay attention to the units and how we've canceled out the ml in the numerator of 500ml and in the denominator in the conversion of 1L in 1000ml

Additional Resources

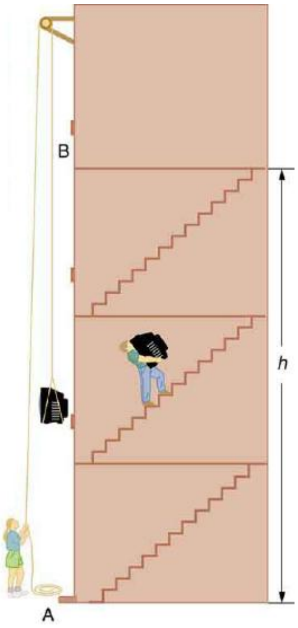
Accuracy and Precision

Accuracy refers to how closely a measured value agrees with the correct or target value.

Precision refers to how closely individual measurements agree with each other and reflects the repeatability in those measurements.

This illustrates accuracy. Measurements are on target.

This illustrates precision. Measurements are very close to each other and repeatable.



This illustrates Accuracy AND Precision. Each measurement is on target and also highly repeatable.

Instruments have a finite amount of accuracy and it is important to report measurements within that level of accuracy. **Significant figures**, report the number of digits that are known to some degree of confidence with the measuring device. With the increased sensitivity of the equipment, the number of significant figures increases.

Pipetting Basics

Types of Micropipettors

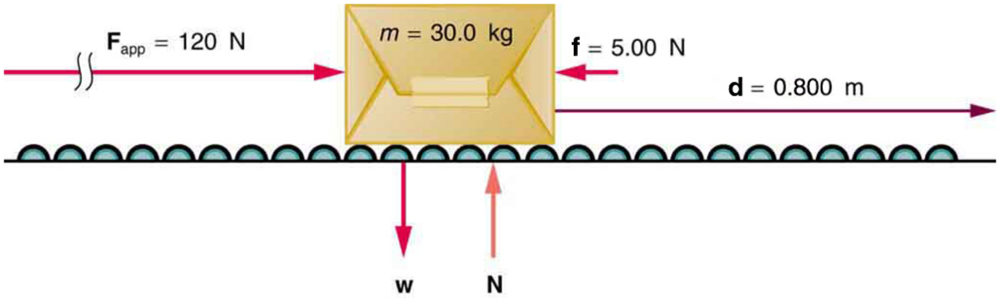
Pipettors are made by many different manufacturers and thus all do not look the same. Learning to correctly use one type of pipettor will provide you the knowledge to use others as they share the same method of distributing small volumes. This lab will illustrate the Rainin Pipetman® micropipettors.

The top of the plunger shows the pipettor size for the Pipetman models. Each pipettor has its own volume range and it is **CRITICAL** to use a pipettor only in its proper volume range. The “P-number” represents the maximum volume in µL that the pipettor can measure. Pipettors are more accurate in the *upper* part of their range. 20 µL should be measured with a P20 rather than with a P200. The four pipettor sizes (P10, P20, P200, P1000) used in our lab will measure from 1 µL – 1000 µL as shown below.

Correctly Adjusting the Pipettors

Gilson Pipetman pipetting ranges chart. Note that the P200 officially has a range from 50-200µl

Tutorial on Proper Usage



Pipetting sequence. Ensure the plunger is depressed outside of the liquid to displace air and avoid blowing bubbles into solution. Carefully draw the plunger up slowly and follow the liquid to avoid drawing air. Depress the plunger in destination tube to the first stop. Depress to the second stop if fluid remains in the tip to expel full volume.

https://www.youtube.com/watch?v=uEy_NGDfo_8

Rules For Use of the Micropipettors

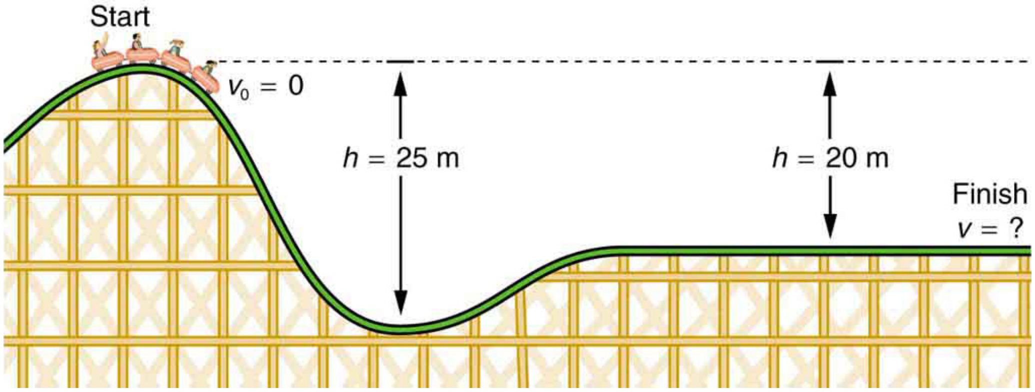
These are precision instruments which can easily be damaged. Treat them with respect and care. They are essential for your success in this course and shared amongst numerous students.

1. Never measure higher or lower than the range of the pipettor allows.

- An exception to this rule is the P200 labeled 50-200 µl.
    - While we have P100 pipettors for this range, they appear too similar to P20 that they are often confused.
    - Originally, P200 were labeled 20-200 µl and we know that the lower range is less precise on these.
2. Never turn the volume adjuster above or below this range indicated on the pipettor or you risk breaking the instrument.
3. Never allow liquid to get into the micropipettor.
    - This causes contamination.
    - This weakens the seal on the o-rings and can damage them.
4. Never use the micropipettor without a tip.
5. Never invert or lay down the micropipettor with liquid in the tip.
    - Liquids will roll into the piston this way.
6. Never let the plunger snap back when filling or ejecting liquid.
7. Never immerse the barrel in fluid.
    - This causes contamination.
8. Never set the micropipettor on the edge of the bench; this may result in the micropipettor falling or being knocked onto the floor.

Exercise: Pipetting Practice

1. Prepare seven dye mixtures as illustrated in the table below.
    1. One student mix samples in column A and a second mixes in column C.
    2. Column B is left empty and used if one student makes a mistake.
2. Each dye mixture prepared in the first well to reach a total volume of 45 µl.
3. Pipet 10 µl in triplicate from each well of the mixing plate into the center of the appropriate circles on the target card.




NASA
ERAST
AeroVironment Inc.
HELIOS

2.8.4: The Metric System

You will NOT need a calculator for this module.

The metric system was first implemented following the French Revolution; if we're overthrowing the monarchy, why should we use a unit of a "foot" that is based on the length of a king's foot?

The metric system was designed to be based on the natural world, and different units are related to each other by powers of 10 instead of weird numbers like 3, 12, 16, and $5,280\dots$ This makes converting between metric units incredibly simple.

kilo- (k)	hecta- (h)	deka- (da)	**[base unit]**	deci- (d)	centi- (c)	milli- (m)
$1,000$	100	10	1	0.1	0.01	0.001
$1,000$	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

Notice that because deka- and deci- both start with d, the abbreviation for deka- is da.

Metric System: Measurements of Length

The base unit of length is the meter, which is a bit longer than a yard (three feet). Because the prefix kilo- means one thousand, 1 kilometer is $1,000$ meters. (One kilometer is around six tenths of a mile.) Similarly, because the prefix centi- means one hundredth, 1 centimeter is $\frac{1}{100}$ of a meter, or 1 meter is 100 centimeters. (One centimeter is roughly the thickness of a pen.) And because the prefix milli- means one thousandth, 1 millimeter is $\frac{1}{1,000}$ of a meter, or 1 meter is $1,000$ millimeters. (One millimeter is roughly the thickness of a credit card.)

Exercises 2.8.4.1

From each of the four choices, choose the most reasonable measure.

1. The length of a car:
5 kilometers, 5 meters, 5 centimeters, 5 millimeters

2. The height of a notebook:
28 kilometers, 28 meters, 28 centimeters, 28 millimeters

3. The distance to the next town:
3.8 kilometers, 3.8 meters, 3.8 centimeters, 3.8 millimeters

4. An adult woman's height:
1.6 kilometers, 1.6 meters, 1.6 centimeters, 1.6 millimeters

5. An adult woman's height:
160 kilometers, 160 meters, 160 centimeters, 160 millimeters

6. The thickness of a pane of glass:
3 kilometers, 3 meters, 3 centimeters, 3 millimeters

Answer

1. 5 m
2. 28 cm
3. 3.8 km
4. 1.6 m
5. 160 cm
6. 3 mm

kilo- (km)	hecta- (hm)	deka- (dam)	**meter (m)**	deci- (dm)	centi- (cm)	milli- (mm)
$1,000$	100	10	1	0.1	0.01	0.001
$1,000$	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

To convert metric units, you can simply move the decimal point left or right the number of places indicated in the table above. No calculator required!

? Exercises 2.8.4.1

7. Convert 3.7 meters to centimeters.

8. Convert 3.7 meters to millimeters.

9. Convert 2.45 kilometers to meters.

10. Convert 2.45 kilometers to centimeters.

11. Convert 342 millimeters to meters.

12. Convert 342 millimeters to centimeters.

13. Convert 528 meters to kilometers.

14. Convert 45 centimeters to meters.

Answer

7. 370 cm
8. $3,700$ mm
9. $2,450$ m
10. $245,000$cm
11. 0.342m
12. 34.2cm
13. 0.528km
14. 0.45m

Metric System: Measurements of Weight or Mass

The base unit for mass is the gram, which is approximately the mass of a paper clip. A kilogram is $1,000$ grams; as we'll see in the next module, this is around 2.2 pounds. The active ingredients in medicines may be measured using the milligram, or possibly the microgram, which we will come back to in a future module. For now, we will focus on the prefixes between kilo- and milli-.

? Exercises 2.8.4.1

From each of the three choices, choose the most reasonable measure.

15. The mass of an apple:
100 kilograms, 100 grams, 100 milligrams

16. The mass of an adult man:
80 kilograms, 80 grams, 80 milligrams

17. The amount of active ingredient in a pain relief pill:
500 kilograms, 500 grams, 500 milligrams

18. The base vehicle weight of a GMC Sierra 1500:
$2,000$ kilograms, $2,000$ grams, $2,000$ milligrams

Answer

15. 100 g
16. 80 kg
17. 500 mg
18. $2,000$ kg

kilo- (kg)	hecta- (hg)	deka- (dag)	**gram (g)**	deci- (dg)	centi- (cg)	milli- (mg)
$1,000$	100	10	1	0.1	0.01	0.001
$1,000$	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

This table is identical to the previous table; the only difference is that the base unit "meter" has been replaced by "gram". This means that converting metric units of mass is exactly the same process as converting metric units of length; just move the decimal point!

? Exercises 2.8.4.1

19. Convert 0.813 grams to centigrams.

20. Convert 0.813 grams to milligrams.

21. Convert 1.25 kilograms to grams.

22. Convert 1.25 kilograms to milligrams.

23. Convert 960 milligrams to grams.

24. Convert 960 milligrams to centigrams.

25. Convert $1,350$ grams to dekagrams.

26. Convert 7.5 centigrams to grams.

Answer

19. 81.3cg

20. $813\,\mathrm{g}$

21. $1,250\,\mathrm{g}$

22. $1,250,000\mathrm{mg}$

23. $0.96\,\mathrm{g}$

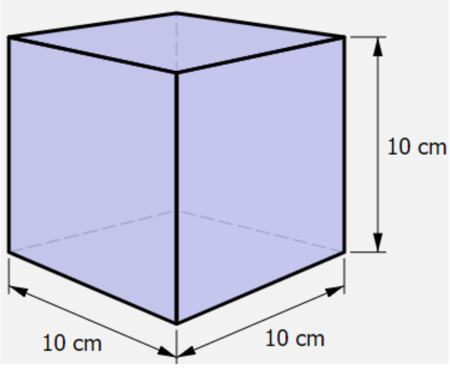
24. $96\ \mathrm{cg}$

25. $135\,\mathrm{dag}$

26. $0.075\,\mathrm{g}$

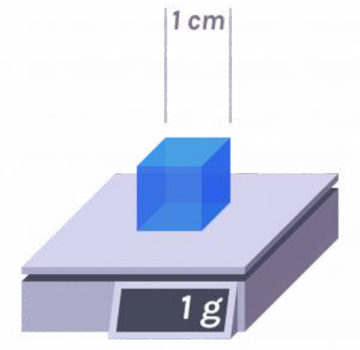
Metric System: Measurements of Volume or Capacity

The base unit of volume is the liter, which is slightly larger than one quart. The milliliter is also commonly used; of course, there are $1,000$ milliliters in 1 liter.



1 liter is equivalent to a cube with sides of 10 centimeters.

In case you were wondering, the units of volume, length, and mass are all connected; one cubic centimeter (a cube with each side equal to 1 cm) has the same volume as one milliliter, and one milliliter of water has a mass of one gram.



Exercises 2.8.4.1

From each of the two choices, choose the more reasonable measure.

27. The capacity of a car's gas tank: 50 liters, 50 milliliters

28. A dosage of liquid cough medicine: 30 liters, 30 milliliters

Answer

27. $50\,\mathrm{L}$

28. $30\,\mathrm{mL}$

kilo- (kL)	hecta- (hL)	deka- (daL)	**liter (L)**	deci- (dL)	centi- (cL)	milli- (mL)
$1,000$	100	10	1	0.1	0.01	0.001
$1,000$	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

Again, this table is identical to the previous tables; just move the decimal point left or right to convert the units.

? Exercises 2.8.4.1

29. Convert 2.8 liters to milliliters.

30. Convert 2.8 liters to deciliters.

31. Convert 15 dekaliters to liters.

32. Convert 0.75 deciliters to milliliters.

33. Convert 600 milliliters to centiliters.

34. Convert 600 milliliters to liters.

35. Convert 4.5 deciliters to liters.

36. Convert 550 centiliters to liters.

37. Flying on IcelandAir, you happen to notice that one mini bottle of booze is labeled 50 mL, but another mini bottle is labeled 5 cL. How do the two bottles compare in size?

38. How many 500-milliliter bottles of Coke[1] are equivalent to one 2-liter bottle?

39. The engine displacement of a Yamaha Majesty scooter is 125 cc (cubic centimeters), and the engine displacement of a Chevrolet Spark automobile is 1.4 L (liters). What is the approximate ratio of these engine displacements?

Answer

29. $2,800\text{mL}$

30. 28 dL

31. 150L

32. 75 mL

33. 60 cL

34. 0.6L

35. 0.45L

36. 5.5L

37. they are equal in size

38. 4 bottles

39. about 11 to 1

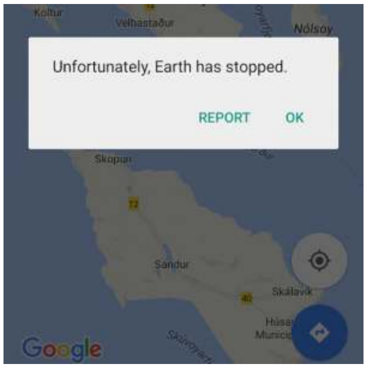
---

1. (Fun fact: in Spanish, a 500-milliliter bottle is called a *medio litro.*) ↵

2.8.5: Other Conversions

You may use a calculator in this module as needed.

Converting Measurements of Time



Unfortunately, Earth has stopped.

You probably know all of the necessary conversions for time. When we get to units of time larger than weeks, however, we encounter problems because not all months have the same number of days, a year is not exactly 52 weeks, and the time it takes for the Earth to orbit the Sun is not exactly 365 days. Therefore, it doesn't make sense to expect an exact answer to a question like "how many minutes are in one month?" We will have to use our best judgment in situations such as these.

$1 \text{ min} = 60 \text{ sec}$

$1 \text{ hr} = 60 \text{ min}$

$1 \text{ day (dy)} = 24 \text{ hr}$

$1 \text{ week (wk)} = 7 \text{ dy}$

$1 \text{ year (yr)} = 365 \text{ dy}$

Exercises 2.8.5.1

1. How many minutes is one standard 365-day year?[1]

2. Have you been alive for 1 billion seconds? Is this even possible?

Answer

1. $525,600 \text{min}$
2. this is roughly 31.7years, which is indeed possible

Converting Rates

Exercises 2.8.5.1

Usain Bolt holds the world record time for the 100-meter dash, 9.58 seconds.

3. What was his average speed in kilometers per hour?

4. What was his average speed in miles per hour?

Answer

3. 37.6km/hr
4. 23.3mi/hr

The more information we know, the more things we can figure out.

? Exercises 2.8.5.1

An F-15 fighter jet can reach a sustained top speed of roughly Mach 2.3; this is 2.3 times the speed of sound, which is 770 miles per hour.[2]

5. What is the jet's top speed in miles per hour?

6. At this speed, how many miles would the jet travel in one minute?

The jet's range at this speed before it runs out of fuel is around 600 miles.

7. If the jet flies 600 miles at top speed, for how many minutes will it fly?

The jet's maximum fuel capacity is 3, 475 gallons of fuel.

8. If the jet flies 600 miles and burns 3, 475 gallons of fuel, find the jet's fuel efficiency, in miles per gallon.

9. Rewrite the jet's fuel efficiency, in gallons per mile.

10. How many gallons of fuel does the jet consume in one minute?

Answer

5. 1, 770 mi/hr
6. 29.5 mi in 1 min
7. 20.3 min
8. 0.17 mi/gal
9. 5.8 gal/mi
10. 171 gal in 1 min

Measurement Prefixes: Larger

Now let's turn our attention to converting units based on their prefixes. We'll start with large units of measure.

tera- (T)	giga- (G)	mega- (M)	kilo- (k)	**[base unit]**
trillion	billion	million	thousand	one
1, 000, 000, 000, 000	1, 000, 000, 000	1, 000, 000	1, 000	1
10^{12}	10^{9}	10^{6}	10^{3}	10^{0}

Notice that the powers of these units are multiples of 3, just as with the engineering notation we saw in a previous module. Each prefix is 1, 000 times the next smaller prefix, so moving one place in the chart means moving the decimal point three places. Also notice that capitalization is important; megagram (which is also called a metric ton) is Mg with a capital M, but milligram is mg with a lowercase m.

Using computer memory as an example:

1 kilobyte = 1, 000 bytes

1 megabyte = 1, 000 kilobytes = 1, 000, 000bytes

1 gigabyte = $1,000$ megabytes = $1,000,000$kilobytes, etc.

1 terabyte = $1,000$ gigabytes = $1,000,000$megabytes, etc.

Note: There can be inconsistencies with different people's understanding of these prefixes with regards to computer memory, which is counted in powers of 2, not 10. Computer engineers originally defined 1 kilobyte as $1,024$ bytes because $2^{10} = 1,024$, which is very close to $1,000$. However, we will consider these prefixes to be powers of $1,000$, not $1,024$. There is an explanation at https://physics.nist.gov/cuu/Units/binary.html.

Exercises 2.8.5.1

11. A $5\frac{1}{4}$ inch floppy disk from the 1980s could store about 100 kB; a $3\frac{1}{2}$ inch floppy disk from the 1990s could store about 1.44 MB. By what factor was the storage capacity increased?

12. How many times greater is the storage capacity of a 2 TB hard drive than a 500 GB hard drive?

13. In an article describing small nuclear reactors that are designed to be retrofitted into coal plants, Dr. Jose Reyes of Oregon State University says "One module will produce 60 megawatts of electricity. That's enough for about 50 thousand homes."[3] How much electricity per home is this?

14. In the same article, Dr. Reyes says "a 60 megawatt module could produce about 60 million gallons of clean water per day using existing technologies in reverse osmosis." What is the rate of watts per gallon?

15. The destructive power of nuclear weapons is measured in kilotons (the equivalent of $1,000$ tons of TNT) or megatons (the equivalent of $1,000,000$ tons of TNT). The first nuclear device ever tested, the US's *Trinity*, was measured at roughly 20 kilotons on July 16, 1945. The largest thermonuclear weapon ever detonated, at 50 megatons, was the USSR's *Tsar Bomba*, on October 31, 1961.[4] (Video of *Tsar Bomba* was declassified almost 60 years later, in August 2020.) How many times more powerful was *Tsar Bomba* than *Trinity*?

Answer

11. the capacity increased by a factor of 14.4

12. 4 times greater

13. $1,200$ megawatts per home

14. 1 watt per gallon

15. $2,500$ times more powerful

Measurement Prefixes: Smaller

Now we'll go in the other direction and look at small units of measure.

[base unit]	milli- (m)	micro- (μ or mc)	nano- (n)	pico (p)
one	thousandth	millionth	billionth	trillionth
1	0.001	0.000001	0.000000001	0.000000000001
10^0	10^{-3}	10^{-6}	10^{-9}	10^{-12}

The symbol for micro- is the Greek letter μ (pronounced "myoo"). Because this character can be difficult to replicate, you may see the letter "u" standing in for "μ" in web-based or plaintext technical articles… or you may see the prefix "mc" instead.

Again, the powers are multiples of 3; each prefix gets smaller by a factor of $\frac{1}{1000}$. The negative exponents can sometime be complicated to work with, and it may help to think about things in reverse.

1 meter = 10^3 millimeters = 10^6 micrometers = 10^9 nanometers = 10^{12} picometers

1 second = 10^3 milliseconds = 10^6 microseconds = 10^9 nanoseconds = 10^{12} picoseconds

…and so on.

See https://physics.nist.gov/cuu/Units/prefixes.html for a list of more prefixes.

Exercises 2.8.5.1

16. An article about network latency compares the following latency times: "So a 10 Mbps link adds 0.4 milliseconds to the RTT, a 100 Mbps link 0.04 ms and a 1 Gbps link just 4 microseconds."[5] Rewrite these times so that they are all in terms of milliseconds, then rewrite them in terms of microseconds.

17. The wavelength of red light is around 700 nm. Infrared radiation has a wavelength of approximately 10 μm.[6] Find the ratio of these wavelengths.

18. Nuclear radiation is measured in units called Sieverts, but because this unit is too large to be practical when discussing people's exposure to radiation, milliSieverts and microSieverts are more commonly used. In 1986, workers cleaning up the Chernobyl disaster were exposed to an estimated dose of 250 mSv.[7] A typical chest x-ray exposes a person to 100 μSv.[8] How many chest x-rays' worth of radiation were the workers exposed to?

Answer

16. 0.4 ms, 0.04ms, 0.004ms; 400 μs, 40 μs, 4 μs the ratio of the wavelengths of red and infrared is 7 to 100;

17. the ratio of the wavelengths of infrared and red is around 14 to 1

18. this is equivalent to $2,500$ chest x-rays

---

1. If you're familiar with the musical *Rent*, then you already know the answer. ↩
2. My sources for the following set of questions are a combination of former students in the Air National Guard and people who sound like they know what they're talking about on the internet, particularly in this Quora discussion. ↩
3. https://www.kgw.com/article/news/local/oregon-company-get-approval-to-build-nuclear-power-plants/283-7b26b8cd-12d5-4116-928a-065731f7a0f6 ↩
4. https://en.Wikipedia.org/wiki/Nuclear_weapon_yield ↩
5. https://www.noction.com/blog/network-latency-effect-on-application-performance ↩
6. http://labman.phys.utk.edu/phys222core/modules/m6/The%20EM%20spectrum.html ↩
7. https://en.Wikipedia.org/wiki/Chernobyl_disaster ↩
8. https://www.cancer.org/treatment/understanding-your-diagnosis/tests/understanding-radiation-risk-from-imaging-tests.html ↩

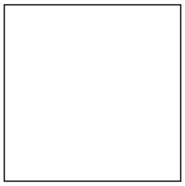
2.8.6: Converting Units of Area

You may use a calculator throughout this module.

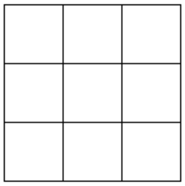
Converting between units of area requires us to be careful because square units behave differently than linear units.

U.S. System: Converting Measurements of Area

Consider a square yard; the area of a square with sides 1 yard long.



1 yard = 3 feet, so we can divide the square into three sections vertically and three sections horizontally to convert both dimensions of the square from yards to feet. This forms a 3 by 3 grid, which shows us visually that 1 square yard equals 9 square feet, not 3 square feet!



The linear conversion ratio of 1 to 3 means that that the conversion ratio for the areas is 1 to 3^2, or 1 to 9.

Here's another way to think about it without a diagram: $1 \text{ yd} = 3 \text{ ft}$, so $(1 \text{ yd})^2 = (3 \text{ ft})^2$. To remove the parentheses, we must square the number *and* square the units: $(3 \text{ ft})^2 = 3^2 \text{ ft}^2 = 9 \text{ ft}^2$.

More generally, we need to **square** the linear conversion factors when converting units of area. If the linear units have a ratio of 1 to n, the square units will have a ratio of 1 to n^2.

? Exercises 2.8.6.1

1. An acre is defined as the area of a 660 foot by 66 foot rectangle. (That's a furlong by a chain, if you were curious.) How many square feet are in 1 acre?

2. How many square yards are in 1 acre?

3. How many square inches equals 1 square foot?

Answer

1. $43,560 \text{ ft}^2$
2. $4,840 \text{yd}^2$
3. 144 in^2

It should be no surprise that this module will be full of conversion ratios. As always, if you discover other conversion ratios that aren't provided here, it would be a good idea to write them down so you can use them as needed.

- $1 \text{ ft}^2 = 144 \text{ in}^2$
- $1 \text{ yd}^2 = 9 \text{ ft}^2$
- $1 \text{ acre (ac)} = 43,560 \text{ ft}^2$
- $1 \text{ ac} = 4,840 \text{ yd}^2$
- $1 \text{ mi}^2 = 27,878,400 \text{ ft}^2$
- $1 \text{ mi}^2 = 3,097,600 \text{ yd}^2$

- $1 \text{ mi}^2 = 640 \text{ ac}$

An acre is defined as a unit of area; it would be wrong to say “acres squared” or put an exponent of 2 on the units.

Exercises 2.8.6.1

4. A hallway is 9 yards long and 2 yards wide. How many square feet of linoleum are needed to cover the hallway?

5. A proposed site for an elementary school is 600 feet by 600 feet. Find its area, in acres.

Answer

4. 162 ft^2

5. 8.3 ac

Metric System: Converting Measurements of Area

- $1 \text{ cm}^2 = 100 \text{ mm}^2$
- $1 \text{ m}^2 = 1,000,000 \text{ mm}^2$
- $1 \text{ m}^2 = 10,000 \text{ cm}^2$
- $1 \text{ hectare (ha)} = 10,000 \text{ m}^2$
- $1 \text{ km}^2 = 1,000,000 \text{ m}^2$
- $1 \text{ km}^2 = 100 \text{ ha}$

A hectare is defined as a square with sides 100 meters long. Dividing a square kilometer into ten rows and ten columns will make a 10 by 10 grid of 100 hectares. As with acres, it would be wrong to say “hectares squared” or put an exponent of 2 on the units.

Exercises 2.8.6.1

6. A hallway is 9 meters long and 2 meters wide. How many square centimeters of linoleum are needed to cover the hallway?

7. A proposed site for an elementary school is 200 meters by 200 meters. Find its area, in hectares.

Answer

6. $180,000 \text{ cm}^2$

7. 4 ha

Both Systems: Converting Measurements of Area

Converting between the U.S. and metric systems will involve messy decimal values. For example, because $1 \text{ in} = 2.54 \text{ cm}$, we can square both numbers and find that $(1 \text{ in})^2 = (2.54 \text{ cm})^2 = 6.4516 \text{ cm}^2$. The conversions are rounded to three or four significant digits in the table below.

- $1 \text{ in}^2 \approx 6.45 \text{ cm}^2 \leftrightarrow 1 \text{ cm}^2 \approx 0.155 \text{ in}^2$
- $1 \text{ in}^2 \approx 6.45 \text{ cm}^2 \leftrightarrow 1 \text{ cm}^2 \approx 0.155 \text{ in}^2$
- $1 \text{ yd}^2 \approx 0.836 \text{ m}^2 \leftrightarrow 1 \text{ m}^2 \approx 1.196 \text{ yd}^2$
- $1 \text{ mi}^2 \approx 2.59 \text{ km}^2 \leftrightarrow 1 \text{ km}^2 \approx 0.386 \text{ mi}^2$
- $1 \text{ ac} \approx 0.405 \text{ ha} \leftrightarrow 1 \text{ ha} \approx 2.47 \text{ ac}$

Exercises 2.8.6.1

8. The area of Portland is 145 mi^2. Convert this area to square kilometers.

9. How many hectares is a $5,000$ acre ranch?

10. A sheet of paper measures 8.5 inches by 11 inches. What is the area in square centimeters?

11. A soccer field is 100 meters long and 70 meters wide. What is its area in square feet?

Answer

8. 376 km^2

9. $2,000$ ha(to one sig fig) or $2,\bar{0}00$ ha(to two sig figs)

10. $6\bar{0}0 \text{ cm}^2$ (to two sig figs)

11. $75,300 \text{ ft}^2$

Areas of Similar Figures

Earlier in this module, it was stated that if the linear units have a ratio of 1 to n, the square units will have a ratio of 1 to n^2. This applies to similar figures as well.

If the linear dimensions of two similar figures have a ratio of 1 to n, then the areas will have a ratio of 1 to n^2. This is true for circles, similar triangles, similar rectangles, similar hexagons, you name it. We'll verify this in the following exercises.

? Exercises 2.8.6.1

A personal pizza has a 7-inch diameter. A medium pizza has a diameter twice that of a personal pizza.

12. Determine the area of the medium pizza.

13. Determine the area of the personal pizza.

14. What is the ratio of the areas of the two pizzas?

Right triangle ABC has legs 3 cm and 4 cm long. Right triangle DEF has legs triple the length of ABC's.

15. Determine the area of the larger triangle, DEF.

16. Determine the area of the smaller triangle, ABC.

17. What is the ratio of the areas of the two triangles?

Answer

12. 154 in^2

13. 38.5 in^2

14. 4 to 1

15. 54 cm^2

16. 6 cm^2

17. 9 to 1

2.8.7: Converting Units of Volume

You may use a calculator throughout this module.

Just as we saw with area, converting between units of volume requires us to be careful because cubic units behave differently than linear units.

Quantities of mulch, dirt, or gravel are often measured by the cubic yard. How many cubic feet are in one cubic yard?

> 1 yard = 3 feet, so we can divide the length into three sections, the width into three sections, and the height into three sections to convert all three dimensions of the cube from yards to feet. This forms a 3 by 3 by 3 cube, which shows us that 1 cubic yard equals 27 cubic feet. The linear conversion ratio of 1 to 3 means that that the conversion ratio for the volumes is 1 to 3^3, or 1 to 27.

Here's another way to think about it without a diagram: $1 \text{ yd} = 3 \text{ ft}$, so $(1 \text{ yd})^3 = (3 \text{ ft})^3$. To remove the parentheses, we must cube the number *and* cube the units: $(3 \text{ ft})^3 = 3^3 \text{ ft}^3 = 27 \text{ ft}^3$.

More generally, we need to **cube** the linear conversion factors when converting units of volume. If the linear units have a ratio of 1 to n, the cubic units will have a ratio of 1 to n^3.

? Exercises 2.8.7.1

1. Determine the number of cubic inches in 1 cubic foot.

2. Determine the number of cubic inches in 1 cubic yard.

3. Determine the number of cubic millimeters in 1 cubic centimeter.

4. Determine the number of cubic centimeters in 1 cubic meter.

Answer

1. $1,728 \text{ in}^3$
2. $46,656 \text{ in}^3$
3. $1,000 \text{ mm}^3$
4. $1,000,000 \text{ cm}^3$

U.S. System: Converting Measurements of Volume

$$1 \text{ ft}^3 = 1,728 \text{ in}^3$$

$$1 \text{ yd}^3 = 27 \text{ ft}^3$$

$$1 \text{ yd}^3 = 46,656 \text{ in}^3$$

? Exercises 2.8.7.1

5. True story: A friend at the National Guard base gave us three long wooden crates to use as raised planting beds. (The crates probably carried some kind of weapons or ammunition, but our friend wouldn't say.) Henry, who was taking geometry in high school, was asked to measure the crates and figure out how much soil we needed. The inside dimensions of each crate were 112 inches long, 14 inches wide, and 14 inches deep. We wanted to fill them most of the way full with soil, leaving about 4 inches empty at the top. How many cubic yards of soil did we need to order from the supplier?

6. True story, continued: I decided to check our answer and did a rough estimate by rounding each dimension to the nearest foot, then figuring out the volume from there. Did this give the same result?

Answer

5. the result is very close to 1 cubic yard: $(112 \text{ in} \cdot 14 \text{ in} \cdot 10 \text{ in}) \cdot 3 \text{ crates} = 47,040 \text{ in}^3 \approx 1.01 \text{ yd}^3$

6. this estimate is also 1 cubic yard: $(9 \text{ ft} \cdot 1 \text{ ft} \cdot 1 \text{ ft}) \cdot 3 \text{ crates} = 27 \text{ ft}^3 = 1 \text{ yd}^3$

We can convert between units of volume and liquid capacity. As you might expect, the numbers are messy in the U.S. system.

$$1 \text{ fl oz} \approx 1.805 \text{ in}^3 \leftrightarrow 1 \text{ in}^3 \approx 0.554 \text{ fl oz}$$

$$1 \text{ ft}^3 \approx 7.48 \text{ gal} \leftrightarrow 1 \text{ gal} \approx 0.1337 \text{ ft}^3$$

? Exercises 2.8.7.1

7. A wading pool has a diameter of roughly 5 feet and a depth of 6 inches. How many gallons of water are required to fill it about 80% of the way full?

8. A standard U.S. soda pop can has a diameter of $2\frac{1}{2}$ inches and a height of $4\frac{3}{4}$ inches. Verify that the can is able to hold 12 fluid ounces of liquid.

Answer

7. around 60 gallons

8. yes, the can is able to hold 12 fluid ounces; the can's volume is roughly 23.3cubic inches ≈ 12.9 fluid ounces.

Metric System: Converting Measurements of Volume

$$1 \text{ cm}^3 = 1 \text{ cc} = 1 \text{ mL}$$

$$1 \text{ cm}^3 = 1,000 \text{ mm}^3$$

$$1 \text{ m}^3 = 1,000,000 \text{ cm}^3$$

$$1 \text{ L} = 1,000 \text{ cm}^3$$

$$1 \text{ m}^3 = 1,000 \text{ L}$$

It's no surprise that the metric conversion ratios are all powers of 10.

? Exercise 2.8.7.1

9. A can of Perrier mineral water has a diameter of 5.6 cm and a height of 14.7 cm. Verify that the can is able to hold 330 milliliters of liquid.

Answer

yes, the can is able to hold 330 milliliters; the can's volume is roughly 362 cubic centimeters, which is equivalent to 362 milliliters.

Both Systems: Converting Measurements of Volume

Converting between the U.S. and metric systems will of course involve messy decimal values. For example, because $1\text{ in} = 2.54\text{ cm}$, we can cube both numbers and find that $1\text{ in}^3 = (2.54\text{ cm})^3 \approx 16.387\text{ cm}^3$. The conversions are rounded to three or four significant figures in the table below.

$$1\text{ in}^3 \approx 16.39\text{ cm}^3 \leftrightarrow 1\text{ cm}^3 \approx 0.0612\text{ in}^3$$
$$1\text{ ft}^3 \approx 0.0284\text{ m}^3 \leftrightarrow 1\text{ m}^3 \approx 35.29\text{ ft}^3$$
$$1\text{ yd}^3 \approx 0.7646\text{ m}^3 \leftrightarrow 1\text{ m}^3 \approx 1.308\text{ yd}^3$$

? Exercises 2.8.7.1

10. A “two yard” dumpster has a volume of 2 cubic yards. Convert this to cubic meters.

11. Convert 240 in^3 to cm^3.

12. Convert 500 cm^3 to in^3.

13. Convert $1,000\text{ ft}^3$ to m^3.

14. Convert 45 m^3 to yd^3.

Answer

10. 1.53 m^3

11. $3,930\text{ cm}^3$

12. 30.5 in^3

13. 28.3 m^3

14. 59 yd^3

Density

The density of a material is its weight per volume such as pounds per cubic foot, or mass per volume such as grams per cubic centimeter. Multiplying the volume of an object by its density will give its weight or mass.

? Exercises 2.8.7.1

15. The standard size of a gold bar in the U.S. Federal Reserve is 7 inches by $3\frac{5}{8}$ inches by $1\frac{3}{4}$ inches.[1] The density of gold is 0.698 pounds per per cubic inch. How much does one gold bar weigh?

16. A cylindrical iron bar has a diameter of 3.0 centimeters and a length of 20.0 centimeters. The density of iron is 7.87 grams per cubic centimeter. What is the bar’s mass, in kilograms?

Answer

15. 31 lb

16. 1.1 kg

Volumes of Similar Solids

Earlier in this module, it was stated that if the linear units have a ratio of 1 to n, the cubic units will have a ratio of 1 to n^3. This applies to similar solids as well.

If the linear dimensions of two similar solids have a ratio of 1 to n, then the volumes will have a ratio of 1 to n^3.

We'll verify this in the following exercises.

? Exercises 2.8.7.1

A table tennis (ping pong) ball has a diameter of 4 centimeters. A wiffle® ball has a diameter twice that of a table tennis ball.

17. Determine the volume of the wiffle® ball.

18. Determine the volume of the table tennis ball.

19. What is the ratio of the volumes of the two balls?

Rectangular solid A has dimensions 3 inches by 4 inches by 5 inches. Rectangular solid B has dimensions triple those of A's.

20. Determine the volume of the larger solid, B.

21. Determine the volume of the smaller solid, A.

22. What is the ratio of the volumes of the two solids?

Answer

17. 268 cm^3

18. 33.5 cm^3

19. 8 to 1

20. $1,620 \text{ in}^3$

21. 60 in^3

22. 27 to 1

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1. https://www.usmint.gov/about/mint-tours-facilities/fort-knox ↩

2.8.8: Converting Between Systems

You may use a calculator throughout this module.

Converting between the U.S. system and metric system is important in today's global economy; like it or not, the metric system is infiltrating our lives.

The numbers in these conversion ratios are usually difficult to work with, so we will use a calculator whenever necessary and pay attention to rounding. If you discover other conversion ratios that aren't provided here, write them down!

Converting Measurements of Length

You can use the conversion ratios in this table…

- $1 \text{ in} = 2.54 \text{ cm}$
- $1 \text{ ft} \approx 0.305 \text{ m}$
- $1 \text{ yd} \approx 0.914 \text{ m}$
- $1 \text{ mi} \approx 1.61 \text{ km}$

…or the equivalent conversion ratios in this table.

- $1 \text{ cm} \approx 0.394 \text{ in}$
- $1 \text{ m} \approx 3.28 \text{ ft}$
- $1 \text{ m} \approx 1.09 \text{ yd}$
- $1 \text{ km} \approx 0.621 \text{ mi}$

Okay, full disclosure: these conversion ratios aren't exactly equivalent. For example, if we reverse the conversion ratio 1 in $= 2.54$ cm[1], we discover that 1 cm is actually $\dfrac{1}{2.54} \approx 0.3937$ inches. All of these conversion have been rounded to three significant figures, which means that we may get a slightly different result depending on which version of the conversion ratio we use. As long as we round our answer to three or fewer significant figures, though, we should be all right.

Exercises 2.8.8.1

1. How many centimeters is 6 inches?

2. Convert 200 yards into meters.

3. 12 feet is equal to how many centimeters?

4. How many feet is 50 meters?

5. 15 meters is what length in yards?

6. Convert 75 centimeters into inches.

7. Is 21 kilometers equivalent to 13 miles? If not, what is the percent error?

8. Is 66 kilometers equivalent to 41 miles? If not, what is the percent error?

Answer

1. 15.2cm
2. $183\,\text{m}$
3. $366\,\text{cm}$
4. $164\,\text{ft}$
5. 16.4yd
6. $29.5\,\text{in}$
7. not exactly; the error is around 0.3%
8. they are essentially the same; the error is around 0.015%

Converting Measurements of Weight or Mass

You can use the conversion ratios in this table…

- 1 oz $\approx 28.35\,\text{g}$
- 1 lb $\approx 0.454\,\text{kg}$

…or the (almost) equivalent conversion ratios in this table.

- 1 g $\approx 0.0353\,\text{oz}$
- 1 kg $\approx 2.20\,\text{lb}$

You probably have noticed that some conversion problems involve multiplication but others involve division. This is based on which version of the conversion ratio you choose; any of these. If you prefer multiplying to dividing, see whether you can figure out a strategy for choosing your conversion ratio.

? Exercises 2.8.8.1

9. Convert 4 ounces into grams.

10. How many kilograms are equivalent to 120 pounds?

11. Convert 50 grams to ounces.

12. Convert 5 kilograms to pounds.

13. How many grams is a half pound of ground beef?

14. In around 2010, the National Collector's Mint (not affiliated with the U.S. Mint) ran a TV commercial selling an imitation $\$ 50$ gold coin modeled after the U.S. "buffalo" nickel. The commercial made the following claims. *This replica coin is coated in* 31 *milligrams of pure gold! And the price of gold keeps going up; gold is worth about* $\$ 1,000$ *per ounce! But you can order these fake coins for only* $\$ 19.95$ *apiece!* What is the approximate dollar value of the gold in one of these coins?

15. In 2020, the National Collector's Mint is still selling the imitation $\$ 50$ gold "buffalo" nickel for $\$ 19.95$. This version of the coin is coated in 14 milligrams of pure gold.[2] As of September 2, 2020, the price of gold is $\$ 1,940$ per ounce.[3] What is the approximate dollar value of the gold in one of these coins?

Answer

9. $113\,\text{g}$

10. 54.5kg

11. 1.76oz

12. 11.0lb

13. $227\,\text{g}$

14. about $\$ 1.09$

15. about $\$ 0.96$

Converting Measurements of Volume or Capacity

You can use the conversion ratios in this table…

- 1 fl oz ≈ 29.6 mL
- 1 qt ≈ 0.946 L
- 1 gal ≈ 3.79 L

…or the (almost) equivalent conversion ratios in this table. This table includes one extra entry.

- 1 mL ≈ 0.0338 fl oz
- 1 L ≈ 1.06 qt
- 1 L ≈ 0.264 gal
- 1 L ≈ 33.8 fl oz

? Exercises 2.8.8.1

16. A toilet uses 3.5 gallons of water per flush. How many liters is this?

17. How many milliliters of drink are in a 12-ounce can?

18. How many fluid ounces are in a 50 milliliter mini-sized bottle of alcohol?

19. What part of a gallon is in a 2-liter bottle of soda pop?

Answer

16. 13.3L

17. 355 mL

18. 1.69 fl oz

19. 0.53 gal

Converting Measurements: Extensions

Let’s finish up with some rates that require conversions.

? Exercises 2.8.8.1

Maxine is driving across Canada. Her car has a 14.2-gallon gas tank and gets an average of 26 miles per gallon.

20. Approximately how many kilometers—actually, the Canadian spelling is kilometres. Approximately how many kilometres can she travel on a full tank of gas?

21. Of course, Canada measures gas in liters. Actually, litres. Convert Maxine’s mileage rate, 26 miles per gallon, to kilometres per litre.

Answer

20. a bit less than 600 km

21. 11 km/L

---

1. An inch is defined to be exactly 2.54 cm, if you were curious. ↵
2. https://ncmint.com/2020-buffalo-tribute-proof/ ↵
3. https://markets.businessinsider.com/commodities/gold-price ↵

2.8.9: Dimensional Analysis

Learning Objectives

- To be introduced to the dimensional analysis and how it can be used to aid basic chemistry problem solving.
- To use dimensional analysis to identify whether an equation is set up correctly in a numerical calculation
- To use dimensional analysis to facilitate the conversion of units.

Dimensional analysis is amongst the most valuable tools physical scientists use. Simply put, it is the conversion between an amount in one unit to the corresponding amount in a desired unit using various conversion factors. This is valuable because certain measurements are more accurate or easier to find than others.

A Macroscopic Example: Party Planning

If you have every planned a party, you have used dimensional analysis. The amount of beer and munchies you will need depends on the number of people you expect. For example, if you are planning a Friday night party and expect 30 people you might estimate you need to go out and buy 120 bottles of sodas and 10 large pizza's. How did you arrive at these numbers? The following indicates the type of dimensional analysis solution to party problem:

$$(30\ \cancel{humans}) \times \left(\frac{4\text{ sodas}}{1\ \cancel{human}}\right) = 120\text{ sodas} \tag{2.8.9.1}$$

$$(30\ \cancel{humans}) \times \left(\frac{0.333\text{ pizzas}}{1\ \cancel{human}}\right) = 10\text{ pizzas} \tag{2.8.9.2}$$

Notice that the units that canceled out are lined out and only the desired units are left (discussed more below). Finally, in going to buy the soda, you perform another dimensional analysis: should you buy the sodas in six-packs or in cases?

$$(120\ sodas) \times \left(\frac{1\text{ six pack}}{6\ sodas}\right) = 20\text{ six packs} \tag{2.8.9.3}$$

$$(120\ sodas) \times \left(\frac{1\text{ case}}{24\ sodas}\right) = 5\text{ cases} \tag{2.8.9.4}$$

Realizing that carrying around 20 six packs is a real headache, you get 5 cases of soda instead.

In this party problem, we have used dimensional analysis in two different ways:

- In the first application (Equations 2.8.9.1 and Equation 2.8.9.2), dimensional analysis was used to calculate how much soda is needed need. This is based on knowing: (1) how much soda we need for one person and (2) how many people we expect; likewise for the pizza.
- In the second application (Equations 2.8.9.3 and 2.8.9.4), dimensional analysis was used to ***convert units*** (i.e. from individual sodas to the equivalent amount of six packs or cases)

Using Dimensional Analysis to Convert Units

Consider the conversion in Equation 2.8.9.3:

$$(120\ sodas) \times \left(\frac{1\text{ six pack}}{6\ sodas}\right) = 20\text{ six packs} \tag{2.8.9.5}$$

If we ignore the numbers for a moment, and just look at the units (i.e. ***dimensions***), we have:

$$\text{soda} \times \left(\frac{\text{six pack}}{\text{sodas}}\right)$$

We can treat the dimensions in a similar fashion as other numerical analyses (i.e. any number divided by itself is 1). Therefore:

$$\text{soda} \times \left(\frac{\text{six pack}}{\text{sodas}}\right) = \cancel{\text{soda}} \times \left(\frac{\text{six pack}}{\cancel{\text{sodas}}}\right)$$

So, the dimensions of the numerical answer will be "six packs".

How can we use dimensional analysis to be sure we have set up our equation correctly? Consider the following alternative way to set up the above unit conversion analysis:

$$120\ \cancel{\text{soda}} \times \left(\frac{6\text{ sodas}}{\text{six}\ \cancel{\text{pack}}}\right) = 720\ \frac{\text{sodas}^2}{1\text{ six pack}}$$

- While it is correct that there are 6 sodas in one six pack, the above equation yields a value of 720 with units of ***sodas^2/six pack***.
- These rather bizarre units indicate that the equation has been setup ***incorrectly*** (and as a consequence you will have a ton of extra soda at the party).

Using Dimensional Analysis in Calculations

In the above case it was relatively straightforward keeping track of units during the calculation. What if the calculation involves powers, etc? For example, the equation relating kinetic energy to mass and velocity is:

$$E_{kinetics} = \frac{1}{2}\text{mass} \times \text{velocity}^2 \tag{2.8.9.6}$$

An example of units of mass is kilograms (kg) and velocity might be in meters/second (m/s). What are the dimensions of $E_{kinetic}$?

$$(kg) \times \left(\frac{m}{s}\right)^2 = \frac{kg\ m^2}{s^2}$$

The $\frac{1}{2}$ factor in Equation 2.8.9.6 is neglected since pure numbers have no units. Since the velocity is squared in Equation 2.8.9.6, the ***dimensions*** associated with the numerical value of the velocity are also squared. We can double check this by knowing the the Joule (J) is a measure of energy, and as a composite unit can be decomposed thusly:

$$1\ J = kg\frac{m^2}{s^2}$$

Units of Pressure

Pressure (P) is a measure of the Force (F) per unit area (A):

$$P = \frac{F}{A}$$

Force, in turn, is a measure of the acceleration (a) on a mass (m):

$$F = m \times a$$

Thus, pressure (P) can be written as:

$$P = \frac{m \times a}{A}$$

What are the units of pressure from this relationship? (*Note: acceleration is the change in velocity per unit time*)

$$P = \frac{kg \times \frac{\cancel{m}}{s^2}}{m^{\cancel{2}}}$$

We can simplify this description of the units of Pressure by dividing numerator and denominator by m:

$$P = \frac{\frac{kg}{s^2}}{m} = \frac{kg}{m\ s^2}$$

In fact, these are the units of a the composite *Pascal* (*Pa*) unit and is the SI measure of pressure.

Performing Dimensional Analysis

The use of units in a calculation to ensure that we obtain the final proper units is called *dimensional analysis*. For example, if we observe experimentally that an object's potential energy is related to its mass, its height from the ground, and to a gravitational force, then when multiplied, the units of mass, height, and the force of gravity must give us units corresponding to those of energy.

Energy is typically measured in joules, calories, or electron volts (eV), defined by the following expressions:

- 1 J = 1 (kg·m^2)/s^2 = 1 coulomb·volt
- 1 cal = 4.184 J
- 1 eV = 1.602 × 10^{-19} J

Performing dimensional analysis begins with finding the appropriate conversion factors. Then, you simply multiply the values together such that the units cancel by having equal units in the numerator and the denominator. To understand this process, let us walk through a few examples.

✓ Example 2.8.9.1

Imagine that a chemist wants to measure out 0.214 mL of benzene, but lacks the equipment to accurately measure such a small volume. The chemist, however, is equipped with an analytical balance capable of measuring to $\pm 0.0001\ g$ Looking in a reference table, the chemist learns the density of benzene ($\rho = 0.8765\ g/mL$). How many grams of benzene should the chemist use?

Solution

$$0.214\ \cancel{mL} \left(\frac{0.8765\ g}{1\ \cancel{mL}} \right) = 0.187571\ g$$

Notice that the mL are being divided by mL, an equivalent unit. We can cancel these our, which results with the 0.187571 g. However, this is not our final answer, since this result has too many significant figures and must be rounded down to three significant digits. This is because 0.214 mL has three significant digits and the conversion factor had four significant digits. Since 5 is greater than or equal to 5, we must round the preceding 7 up to 8.

Hence, the chemist should weigh out 0.188 g of benzene to have 0.214 mL of benzene.

✓ Example 2.8.9.2

To illustrate the use of dimensional analysis to solve energy problems, let us calculate the kinetic energy in joules of a 320 g object traveling at 123 cm/s.

Solution

To obtain an answer in joules, we must convert grams to kilograms and centimeters to meters. Using Equation 2.8.9.6, the calculation may be set up as follows:

\[\begin{align*} KE&=\dfrac{1}{2}mv^2=\dfrac{1}{2}(g) \left(\dfrac{kg}{g}\right) \left[\left(\dfrac{cm}{s}\right)\left(\dfrac{m}{cm}\right) \right]^2 \

```
4pt] &= (\cancel{g})\left(\dfrac{kg}{\cancel{g}}\right) \left(\dfrac{\cancel{m^2}}{s^2}\right) \left(\dfrac{m^2}{\cancel{cm^2}}\right) = \dfrac{kg·m^2}{s^2} \\[4pt] &=\dfrac{1}{2}320\; \cancel{g} \left( \dfrac{1\; kg
```

Alternatively, the conversions may be carried out in a stepwise manner:

Step 1: convert g to kg

$$320\ \cancel{g} \left(\frac{1\ kg}{1000\ \cancel{g}} \right) = 0.320\ kg$$

Step 2: convert cm to m

$$123\ \cancel{cm} \left(\frac{1\ m}{100\ \cancel{cm}} \right) = 1.23\ m$$

Now the natural units for calculating joules is used to get final results

$$\begin{aligned} KE &= \frac{1}{2} 0.320\ kg (1.23\ ms)^2 \\ &= \frac{1}{2} 0.320\ kg \left(1.513 \frac{m^2}{s^2} \right) = 0.242\ \frac{kg \cdot m^2}{s^2} = 0.242\ J \end{aligned}$$

Of course, steps 1 and 2 can be done in the opposite order with no effect on the final results. However, this second method involves an additional step.

✓ Example 2.8.9.3

Now suppose you wish to report the number of kilocalories of energy contained in a 7.00 oz piece of chocolate in units of kilojoules per gram.

Solution

To obtain an answer in kilojoules, we must convert 7.00 oz to grams and kilocalories to kilojoules. Food reported to contain a value in Calories actually contains that same value in kilocalories. If the chocolate wrapper lists the caloric content as 120 Calories, the chocolate contains 120 kcal of energy. If we choose to use multiple steps to obtain our answer, we can begin with the conversion of kilocalories to kilojoules:

$$120\ \cancel{kcal}\left(\frac{1000\ \cancel{cal}}{\cancel{kcal}}\right)\left(\frac{4.184\ \cancel{J}}{1\ \cancel{cal}}\right)\left(\frac{1\ kJ}{1000\ \cancel{J}}\right)=502\ kJ$$

We next convert the 7.00 oz of chocolate to grams:

$$7.00\ \ \cancel{oz}\left(\frac{28.35\ g}{1\ \ \cancel{oz}}\right)=199\ g$$

The number of kilojoules per gram is therefore

$$\frac{502\ kJ}{199\ g}=2.52\ kJ/g$$

Alternatively, we could solve the problem in one step with all the conversions included:

$$\left(\frac{120\ \ \cancel{kcal}}{7.00\ \ \cancel{oz}}\right)\left(\frac{1000\ \ \cancel{cal}}{1\ \ \cancel{kcal}}\right)\left(\frac{4.184\ \ \cancel{J}}{1\ \ \cancel{cal}}\right)\left(\frac{1\ kJ}{1000\ \ \cancel{J}}\right)\left(\frac{1\ \ \cancel{oz}}{28.35\ g}\right)=2.53\ kJ/g$$

The discrepancy between the two answers is attributable to rounding to the correct number of significant figures for each step when carrying out the calculation in a stepwise manner. Recall that all digits in the calculator should be carried forward when carrying out a calculation using multiple steps. In this problem, we first converted kilocalories to kilojoules and then converted ounces to grams.

Converting Between Units: Converting Between Units, YouTube(opens in new window) [youtu.be]

Summary

Dimensional analysis is used in numerical calculations, and in converting units. It can help us identify whether an equation is set up correctly (i.e. the resulting units should be as expected). Units are treated similarly to the associated numerical values, i.e., if a variable in an equation is supposed to be squared, then the associated dimensions are squared, etc.

Contributors and Attributions

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2.9: Graphing

Learning Objectives

Goals:

- Become proficient with graphing in Google sheets by creating scatter-plot graphs and manipulating graph components with graphing techniques.
- Utilize this opportunity to become familiar working in collaborative documents that will be useful in many applications beyond chemistry.

By the end of this lab, students should be able to:

- **design**, **develop** and **generate** scatter-plot graphs.
- **examine** and **interpret** chemical graphs.
- **identify** the formula for the line of best fit based off of the R^2 value for various trend lines.
- **predict** future values of a measurement based on the appropriate line of best fit.

Prior knowledge:

Basics of Graphing in Chemistry

Background

A graph can be used to show the Relationship between two related values, the independent and the dependent variables. In this exercise we shall use graphing techniques to describe the temperature dependence of the solubility of aqueous sodium nitrate. In experiment 2b we will utilize measured values.

- Independent Variable: A measurable value that you can change during the experimental data collection process (Temperature)
- Dependent Variable: A measurable value which changes as a function of the independent variable during the data collection process (Solubility). The amount of salt which can dissolve in water depends on the temperature.

One of the objectives in graphing is to **make predictions** about values we have not yet measured by finding the mathematical relationship between the dependent and the independent variables, and relate this to our theoretical understanding. In so doing we derive a mathematical relationship where y is a function of x. In this statement y is the dependent variable and plotted on the ordinate (vertical) axis and x is the independent variable and plotted on the abscissa (horizontal) axis. Empirical data is collected through experimentation where x is changed and y is measured, with each data point being represented on a graph with the values (x,y). Without computational software a linear relationship can be determined and so we often mathematically manipulate our data to create linear relationships. In this course we shall look at linear, reciprocal, power and single exponential functions.

Note, a graph can explain any type of two variable relationships, even if the mathematical relationship is not known. Because the solubility of a salt is a very complex function, chemists use graphs (solubility curves) to express the temperature dependence of solubility of a salt and do not always fit the data to mathematical expressions. But since we need to know how to use these functions we will use solubility data to understand and plot linear, power and exponential functions.

Lets look at some data and fit it to different functions.

Linear Function:

$$Y = mX + b \tag{2.9.1}$$

Example: S= mT + b

Figure 2.9.1: Linear graph for solubility data of an unknown salt.

There are several things to note from the graph. First, from a visual perspective the linear function not only does not fit the data, but thee is a trend where it is above the line at low and high temperature and below the line in the middle. The R^2 value is not near one and most importantly, it predicts a negative solubility as the temperature approaches zero, which is nonsense. You can have a negative change in solubility, but you can never have negative solubility.

Power Function:

$$Y = aX^m \tag{2.9.2}$$

Example: $S = aT^m$, which can be converted to an equation of the form of a straight line (Y=mX+b) by converting to logs.

$$\begin{aligned} logS &= log(aT^m) \\ &= loga + logT^m \\ &= loga + mlogT \\ logS &= mlogT + loga \end{aligned} \tag{2.9.3}$$

In the following figure we used Google Sheets to convert the data to log data, and then made two plots, one using a power function fit to the raw data and the second using a linear fit with the logS vs. log T data. These two graphs are different ways of representing the same information. The slope of the second graph (m=0.983) is the m of $S=AT^m$ of the first graph and the y-intercept (-1.63)= lna from the second equation, so $a=10^{-1.63}$=0.023. There is one slight of hand going on here that the estute student might catch can that is that we removed the units from the log plots. Logs can not have units, they are in effect saying how many times you multiplied (or divided) the base by itself to get the number ($log_{10}100=2$ means 10 times itself 2 times equals 100). When we get to real systems we will need to transform values in exponents (or logs) to dimensionless entities). This could have been done here by dividing all solubilities by a standard solution with a numeric value of 1, and we will ignore this issue now, but logs do not have units.

Figure 2.9.2: Power fit on top and the Correlated logS/logT plot on bottom,

There are several things to notice in the above graphs. First, from the top graph the solubility is forced to zero as T goes to zero and this is nonsense, as that would occur no matter what scale you use. Now students might think a negative logS is an issue, but it is not, that just means the concentration is less than one molar. The R^2 value is also not close to one, and a student may notice that this power function looks like a linear line, and that is because the value of m is approaching 1 (when m=1 the power function is linear , S=aT +0)

Exponential Function

$$Y = ae^{mT} \tag{2.9.4}$$

Example $S = ae^{mT}$, which can be converted to a linear function of lnS to T (not lnT)

$$S = ae^{mT} \tag{2.9.5}$$

$$\begin{aligned} lnS &= ln(ae^{mT}) \\ &= lna + ln(e^{mT}) \\ &= lna + mT\ \cancel{ln(e)} \\ lnS &= mT + lna \end{aligned} \tag{2.9.6}$$

An exponential fit of the solubility to temperature gives the same information as a linear fit of the natural log of solubility to temperature, and so the slope of the bottom graph (m=0.025) is the value of m in the exponential function, and the y intercept of the bottom graph gives the pre-expontial, b=-1.39=lna, so $a = e^{-1.39}$.

Figure 2.9.4: Exponential plot of solubility as function of temperature. Note that unlike a power function, this plot does not have to go through the origin.

This looks like the best fit. The value of R^2=1 and this function makes the most sense in explaining the data. Also note that when the independent variable (T) equals zero you have e^0=1, and so the pre-exponential is the value of the solubility at T=0.

Graphing Video Tutorials

Graphing Exercise

Use the solubility data in Table 1.1 to construct seven graphs, six with Google Sheets and one by hand.

- Follow the formatting suggested in the above videos
  - axis correctly labeled
  - major **and** minor grid lines are shown
  - trendline and the equation for the line of best fit is shown clearly
- Create all 6 graphs on one Google sheet (not separate tabs or files for each graph)
- Make sure your data and graph are shown side by side
- Submit your *one* Google sheet to the Google classroom for this assignment by your sections' due date

test

Table 1. Solubility Data showing the maximum amount of various salts which can be dissolved in water at specified temperatures.

Temperature (^oC)	Solubility (g KI / 100g H_2O)	Solubility (g KNO_3 / 100g H_2O)
30	179	46
45	205	73
60	230	109
75	255	156
90	No data available	205

Interpreting data from the above table

Proceed to **Assignment A**. It will ask you some questions regarding the interpretation of the table of data above.

Linear Fit Graphs

Group Activity (assignment B)

From the data in Table 1, create the following two graphs:

Graph 1: Linear Fit of Temperature vs. solubility for KI

Graph 2: Linear Fit of Temperature vs. solubility of KNO_3

Things to keep in mind when making these graphs:

1. Label each graph with a title describing the graph and label all axes (don't just accept the automatic label)
2. Select the linear regression trend line (y=mx+b) and make sure that you select to show the equation for the line. Move the equation and adjust size so it is clearly visible

Note: a linear fit will only be a good fit (R^2 value close to 1) for one of these salts. Once you identify which one is a good fit, you have now created an equation that will allow you to predict the solubility of that salt at another temperature!

Power and Exponential Fit Graphs

Individual Activity (graphs 3-6 in assignment C)

For the salt that does not have a linear relationship between temperature and solubility, you will now make two more **graphs (3 & 4)**.

> Make sure you take the data that you need, copy the data (ctrl-C) and then paste it (ctrl-V) below to an empty part of the spreadsheet where you can make your next graphs.
>
> **Graph 3:** Power function trend line ($y=ax^b$) for the nonlinear graph
>
> **Graph 4:** Exponential trend line ($y=ae^{mx}$) fit for the nonlinear graph.
>
> - Write the appropriate equation on all graphs.
> - Determine which fit is best.

Graphing Logarithmic Data

Individual Activity (C)

For graphs 5 & 6 make another copy of the data you used for graphs 3 and 4. Paste it below your 4 graphs. You will now need to create a new column in the graph that calculates the log or ln of the values.

Graph 5: Find then make a plot of log S vs. log T plot (eq. 1.3 or 1.4) and run a linear fit on this graph. Be sure you can relate graph 5 to graph 3 as described in section 1.2 (figures 1.2 & 1.3).

- Show that the slope of a straight line in this graph is equal to the power (m) in graph 3.
- Mathematically show how the y intercept of this graph can be used to determine the pre-exponential term (A) in graph 3

Graph 6: Make a plot of ln S vs. T and run a linear fit on this graph. Be sure you can relate graph 6 to graph 4 as described in section 1.3 (figures 1.4 & 1.5)

- Show that the slope of a straight line in this graph is equal to the power (m) in graph 4.
- Mathematically show how the y intercept of this graph can be used to determine the pre-exponential term (A) in graph 4

Return to Assignment A and complete the second half of the questions.

TIPs:

- Use these as templates for future graphs by simply copying the graph, then changing the source data and labeling.
- Hence, you'll want to remember where you saved this file so for future labs you can come back and use the template you already created (saving you time)!

Assessment (self-check):

- Do you know the difference between a dependent and independent variable?
- Do you know where to place the dependent variable on graph? Independent variable?
- Can you title a graph properly?
- Can you label all graph components properly?
- Can you use Google sheets to convert data values to logarithm form?
- Do you know when to and when not to include units?
- Do you know how to add and evaluate a trend line (line of best fit) and correlation magnitude (R^2)?

Checklist for submitted report (Google Sheet):

- Appropriate Title of the Document, and at the top of the sheet (identifying it as your work).
- All 6 graphs are on one sheet.
- Beside each graph is the data used to create that graph.
- One image of a hand drawn graph at the bottom of your sheet, corresponding to data that was used to make graph 6.
- You must state the connection between Graphs 3 & 5 and Graphs 4 & 6. This can be described in a sentence or can be shown algebraically.

2.9.1: Graphing Data

Learning Objectives

- Read and interpret data from tables and pictographs.
- Read and interpret data from bar graphs and histograms.

Introduction

A nurse is collecting blood type data from her patients. When a new patient is checked in, the nurse does a simple finger-prick test to see whether the patient’s blood is type A, B, AB, or O. (These are the four possible blood types. Each one also carries a + or - to represent the RH factor, but for our purposes, let’s just track the type, not the + or -.) She tracks her results by creating a two-column table with the patient’s name and blood type.

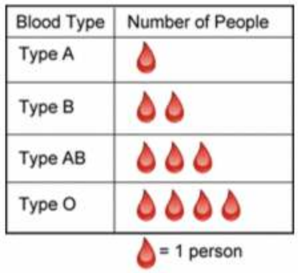
Name	Blood Type
Dominique	A
Ilya	O
Raul	AB
Madison	O
Philip	AB
Samuel	B
Josefine	O
Brett	O
Paula	B
Leticia	AB

The information in this table is an example of **data**, or information. In this case, the nurse has gathered a fair amount of data about her patients’ blood types. By analyzing the data, she can learn more about the range of patients that she serves.

Data helps us make many kinds of decisions. Organizing data into graphs can help us get a clear picture of a situation and can often help us make decisions based on the picture. So how do you take data and make a picture out of it? Let’s take a look.

Pictures of Data

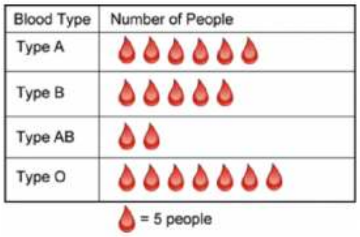
Let’s return to the data set used previously. If the nurse wanted to represent the data visually, she could use a **pictograph**. Pictographs represent data using images. This visual presentation helps illustrate that for the data in her table, Type O blood is the most common and Type A blood is the least common.



Blood Type	Number of People
Type A	💧
Type B	💧💧
Type AB	💧💧💧
Type O	💧💧💧💧

💧 = 1 person

Interested by the results of this small survey, the nurse continues to document the blood types of her patients until she has surveyed 100 people. She puts all of this data in a table, but she finds that it is hard for her to quickly identify what the data is telling her. She decides to make another pictograph using a different scale.



Blood Type	Number of People
Type A	💧💧💧💧💧💧
Type B	💧💧💧💧💧
Type AB	💧💧
Type O	💧💧💧💧💧💧💧

💧 = 5 people

To read this pictograph, all you need is the scale, which, in this case, is the number of people that each blood drop symbol represents. In this graph, each blood drop represents 5 people. There are six drops next to Type A, so $5 \cdot 6 = 30$ people had Type A blood. The table below shows the rest of the information.

Blood Type	Number of People
Type A	$6 \text{ drops } \cdot 5 \text{ people } = 30 \text{ people}$
Type B	$5 \text{ drops } \cdot 5 \text{ people } = 25 \text{ people}$
Type AB	$2 \text{ drops } \cdot 5 \text{ people } = 10 \text{ people}$
Type O	$7 \text{ drops } \cdot 5 \text{ people } = 35 \text{ people}$

✓ Example

The pictograph below shows the number of medals earned at an international competition. How many medals did Japan earn?

Country	Medals
Japan	**MMMMM**
Argentina	**M**
Germany	**MMMMMM**
Egypt	**MMMM**
	M = 4 medals

Solution

	Look at the scale of the pictograph. Each **M** represents 4 medals.
$5 \cdot 4 = 20$	Japan has **5M** symbols, so the total number of medals is $5 \cdot 4 = 20$.

Japan earned 20 medals.

? Exercise

Which table accurately represents the data shown in the pictograph below?

Employee	Hourly wage
Wayne	$ $ $ $ $
Sarah	$ $ $ $ $ $ $
Leigh	$ $ $ $
	$ = $4

A.

Employee	Hourly wage
Wayne	$5
Sarah	$7
Leigh	$4

B.

Employee	Hourly wage
Wayne	$20
Sarah	$22
Leigh	$19

C.

Employee	Hourly wage
Wayne	$10
Sarah	$14
Leigh	$8

D.

Employee	Hourly wage
Wayne	$20
Sarah	$28
Leigh	$16

Answer

A. You identified the correct number of symbols, but it looks like you forgot to factor in the scale. Each dollar sign represents $4, so multiply the number of dollar signs in a row by 4. The correct answer is Table D.
B. You correctly identified that Wayne earns $20, since the table indicates that he has 5 dollar signs, and each dollar sign represents $4. Sarah's row shows that she earns seven dollar sign symbols, so she earns $28, not $22. The correct answer is Table D.
C. You identified the correct number of symbols, but it looks like you used the wrong scale. Each dollar sign represents $4, not $2. Multiply the number of dollar signs in a row by 4. The correct answer is Table D.
D. Each dollar sign symbol represents $4, so if you multiply the number of dollar signs in a row by $4, you will find that Wayne earns $20, Sarah earns $28, and Leigh earns $16.

Bar Graphs

Representing data as pictures doesn't always make sense either. **Bar graphs** are an alternative (and popular) way to represent data sets, especially those with large amounts of data or which do not lend themselves well to individual symbols. In a bar graph, the number of items in a data category is represented by the height or length of bars.

As when reading pictographs, paying attention to the scale is essential: small differences in the height of two bars can sometimes represent thousands of dollars, for example!

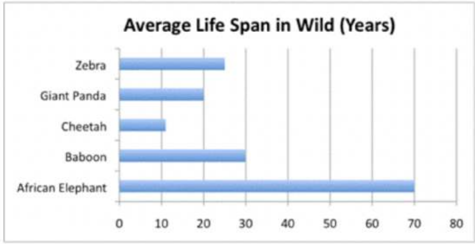
Let's consider an example. Here is some information about the average life span of five animals in the wild, presented in a table.

Source: National Geographic, accessed July 2011

Animal	Average Life Span in the Wild (Years)
Zebra	25

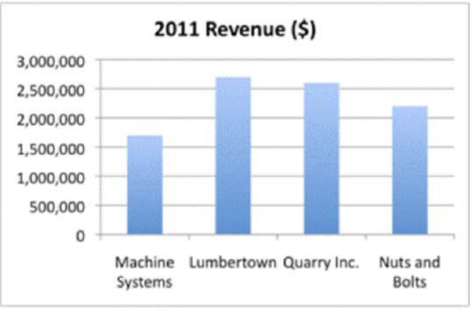
Giant Panda	20
Cheetah	11
Baboon	30
African Elephant	70

This data is fine in a table, but presenting it as a bar graph helps the viewer compare the different life spans more easily. Look at the bar graph below. In this example, the animals are listed on the left side of the graph (also called the **y-axis**), and the life span in years is listed on the bottom (the **x-axis**). The graph shows the information by the length of the bar associated with each animal name.



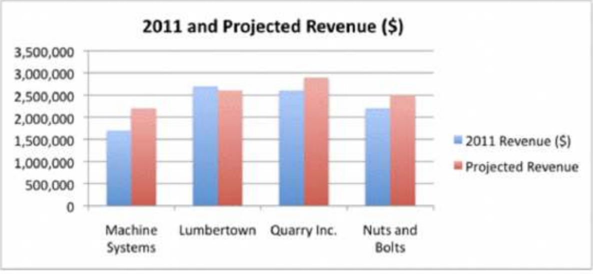
Bar graphs are generally used to compare quantities, not to determine exact quantities, especially when the scale is large, as in the next graph.

The bar graph below shows total revenue for four fictional companies in 2011. Notice that the scale, on the y-axis, is very large; each horizontal line represents an increase of $500,000. For this reason, it is difficult to tell exactly how much money each company made in 2011. However, comparing the bars is straightforward. Glancing at the data, you can tell that Lumbertown earned the most (a little over $2,500,000), while Machine Systems earned the least (about one million less, at just over $1,500,000).



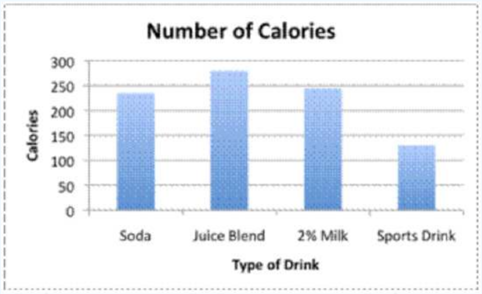
You can also use bar graphs to showcase multiple pieces of information about a specific situation. For example, let's show the next year's projected revenue for each company on the graph that you just looked at. You can leave the existing bars in the graph and just add four more.

The blue columns remain, but now they are accompanied by four new red columns that represent the projected revenue for these companies. Again, this data could be expressed in a table. With a bar graph, you gain ease of quick comparison, but lose the detail of the exact values. Looking at this graph tells you that while Lumbertown has the highest revenue for 2011, it is projected to decrease. Conversely, Machine Systems is projected to increase its revenue. Seeing data visually can help you understand the story that the data is telling about a situation.



Example

Use the graph to list the drinks from the most number of calories to the least number of calories (serving size: 16 ounces).



Solution

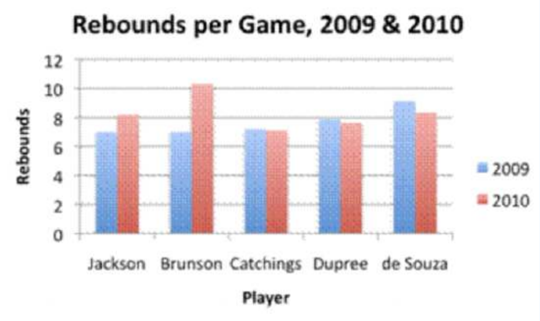
	The y-axis shows total calories, and the x-axis shows the drink. The taller the bar, the more calories the drink has.
$\text{Juice Blend } \approx 275$	The juice blend contains over 250 calories, so it has the most calories per serving.
$2\% \text{ Milk } \approx 245$ $\text{Soda } \approx 230$	Soda and 2% milk are both between 200 and 250 calories, but the bar for 2% milk is taller, so it must contain more calories.
$\text{Sports Drink } \approx 125$	Sports drink has the shortest bar; it contains about 125 calories.

From most to least number of calories per serving:

Juice Blend, 2% Milk, Soda, Sports Drink

Example

Based on the graph below, which player's rebounding increased the most from 2009 to 2010?



Source: WNBA.com, accessed July 2011

Solution

Jackson and Brunson	The y-axis shows rebounds per game, and the x-axis shows the player's name. A taller bar represents more rebounds per game by the player. This graph shows two sets of data: one for 2009, in blue, and one for 2010, in red. To compare the data from one year to the next, compare the heights of the two bars for each player. Two players had higher rebound averages in 2010 than they did in 2009. This is indicated by the red bar being taller than the blue bar. The other players' red bars are shorter, so their rebounds decreased. Comparing the sizes of the increases, you can tell that Brunson increased her per game rebounding *more* than Jackson did.

The player whose rebounding increased the most from 2009 to 2010 was Brunson.

Histograms

The data sets you have looked at so far have shown **categorical data**. Categorical data is data that can be separated into categories. For instance, data about eye color (brown, blue, green), blood type (O, A, B, AB), and the type of computer you use at work or home (PC or Mac) are all categorical.

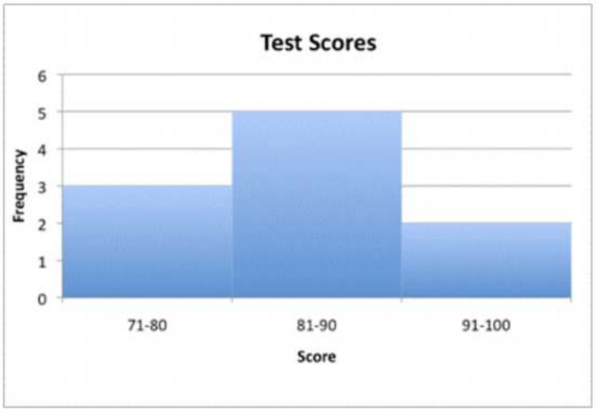
Quantitative data is sometimes called *numerical data*, because it is data that is represented by numbers. Quantitative data sets consist of quantities such as age (1, 2, 23, 34, 77...), test scores (90, 95, 100, 72...), and height (55 inches, 50 inches, 68 inches, ...). Notice that all of the data here are numbers.

Continuous quantitative data can be graphed using a **histogram**. A histogram resembles a bar graph, but instead of having categories along the axis, it has numbers listed in order and usually grouped in intervals (such as 0, to 10, 11 to 20, and so on). While the bars in bar graphs can have space between the bars, the bars of a histogram are usually touching, as the data is continuous.

Let's look at how you could display the set of test score data listed below.

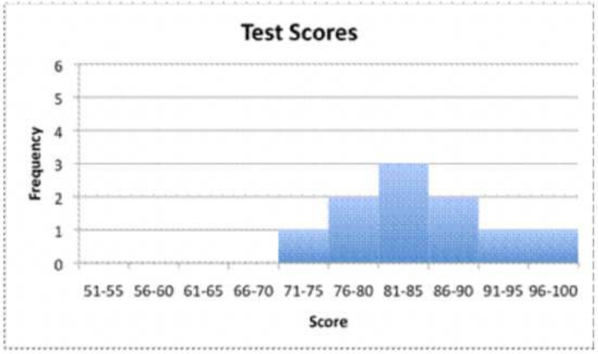
Name	Score (0 to 100)
Alex	81
Beatriz	73
Celia	79
Donnie	91
Erykah	87
Fred	79
Gigi	81
Helene	84
Irma	88
Joelle	96

Looking at the scores, you can tell that some people scored in the 70s, some scored in the 80s, and some scored in the 90s. You can organize the data in a histogram with the continuous data from 0 to 100, by using the intervals to 71 to 80, 81 to 90, and 91 to 100.



There are only three intervals for this histogram and ten data points. For a histogram to be meaningful, it should include 100 data points or more and 7 intervals or more. (For this reason, many histograms are created using a variety of technical tools.) The histogram then takes on interesting shapes that can provide a lot of information.

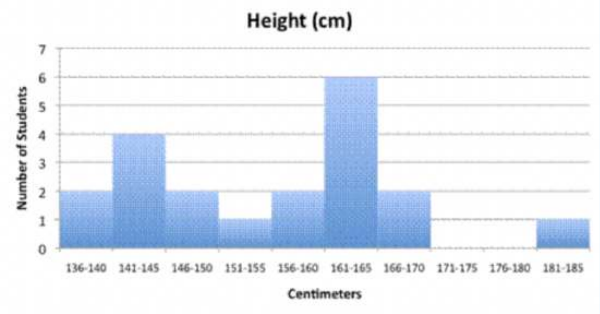
Even with only 10 data points, you find a much more meaningful histogram by using smaller intervals (such as 71 to 75, 76 to 80, 81 to 85, 86 to 90, 91 to 95, and 96 to 100).



This second graph shows a bit of a curve to the data. Nobody scored 70 or lower, and the data peaks in the 81 to 85 range. Increasing the number of intervals has an effect on the shape of the graph, and helps us see some trends that are in the data.

✓ Example

Based on the histogram below, how many students are between 151 and 180 centimeters tall?



Solution

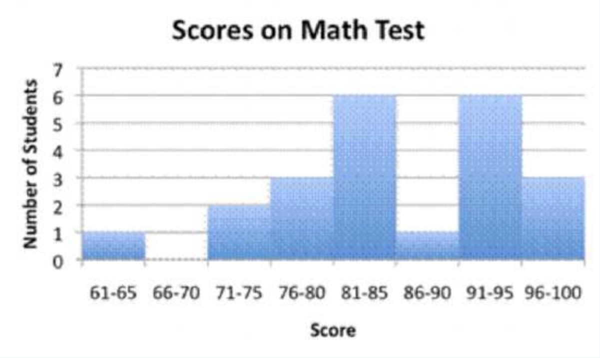
Number of students in each interval:	

$151 \text{ to } 155, \ 1$ $156 \text{ to } 160, \ 2$ $161 \text{ to } 165, \ 6$ $166 \text{ to } 170, \ 2$ $171 \text{ to } 175, \ 0$ $176 \text{ to } 180, \ 0$	Each interval in this histogram is 5 centimeters.
$1+2+6+2+0+0=11$	Add the number of students.

11 students are between 151 and 180 centimeters tall.

? Exercise

A teacher made this histogram to track scores on a recent math test. How many students scored in the 91 to 100 range?



A. 3
B. 6
C. 9
D. 22

Answer

A. Incorrect. Three students scored between 96 to 100 on the test, but if the teacher wants to find out how many students scored between 91 to 100, then she also has to consider the number of students who scored between 91 to 95. The correct answer is 9.
B. Incorrect. Six students scored between 91 to 95 on the test, but if the teacher wants to find out how many students scored between 91 to 100, then she also has to consider the number of students who scored between 96 to 100. The correct answer is 9.
C. Correct. Six students scored between 91 to 95 on the test, and 3 students scored between 96 to 100. The total number of students who scored between 91 to 100 is 9.
D. Incorrect. It looks like you calculated the total number of students who took the test. To find the number who scored between 91 to 100, look at the intervals for 91 to 95 and 96 to 100. The correct answer is 9.

Summary

Data is mathematical information. Mathematical data is often recorded in tables to organize, or spreadsheets to organize and sort. Graphs can help you see the data visually, which can help you to better understand the data. A pictograph is a graph that uses symbols to represent data. Bar graphs show the frequency of categorical data, using bars instead of symbols. Histograms are similar to bar graphs in that they are constructed out of a series of bars. However, histograms represent continuous quantitative (numerical) data, which can illustrate trends as well as other more advanced attributes.

2.9.2: Other Types of Graphs

Learning Objectives

- Read and interpret data from circle graphs (pie charts).
- Read and interpret data from simple line graphs.

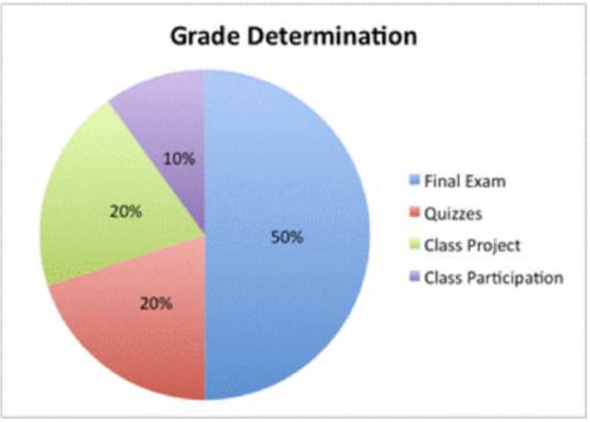
Introduction

Different graphs tell different stories. While a bar graph might be appropriate for comparing some types of data, there are a number of other types of graphs that can present data in a different way. You might see them in news stories or reports, so it's helpful to know how to read and interpret them.

Circle Graphs

Sometimes you will see categorical data presented in a **circle graph**, or pie chart. In these types of graphs, individual pieces of data are represented as sections of the circle (or "pieces of the pie"). Circle graphs are often used to show how a whole set of data is broken down into individual components.

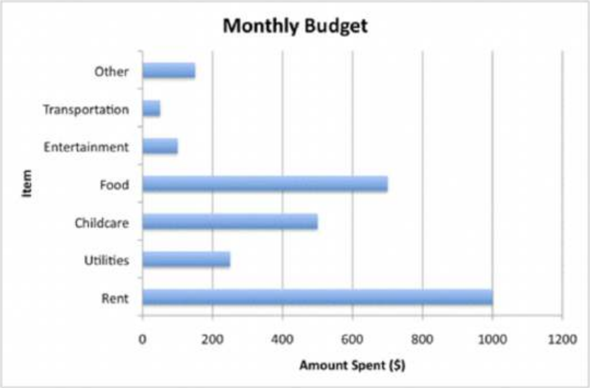
Here's an example. At the beginning of a semester, a teacher talks about how she will determine student grades. She says, "Half your grade will be based on the final exam and 20% will be determined by quizzes. A class project will also be worth 20% and class participation will count for 10%." In addition to telling the class this information, she could also create a circle graph.



This graph is useful because it relates each part (the final exam, the quizzes, the class project, and the class participation) to the whole. It is easy to see that students in this class had better study for the final exam!

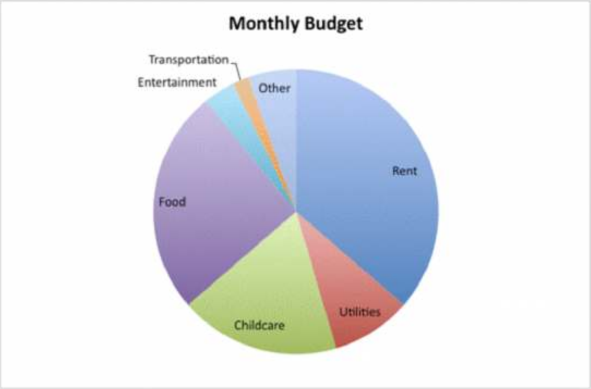
Because circle graphs relate individual parts and a whole, they are often used for budgets and other financial purposes. A sample family budget follows. It has been graphed two ways: first using a bar graph, and then using a circle graph. Each representation illustrates the information a little differently.

The bar graph shows the amounts of money spent on each item during one month. Using this data, you could figure out how much the family needs to earn every month to make this budget work.



The bar graph above focuses on the amount spent for each category. The circle graph below shows how each piece of the budget relates to the other pieces of the budget. This makes it easier to see where the greatest amounts of money are going, and how much of the whole budget these pieces take up. Rent and food are the greatest expenses here, with childcare also taking up a sizeable portion.

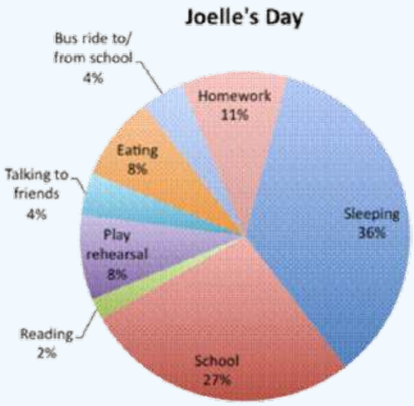
If you look closely at the circle graph, you can see that the sections for food, childcare, and utilities take up almost exactly half of the circle: this means that these three items represent half the budget! This kind of analysis is harder to do with bar graphs because each item is represented as its own entity, and is not part of a larger whole.



Circle graphs often show the relationship of each piece to the whole using percentages, as in the next example.

Example

The circle graph below shows how Joelle spent her day. Did she spend more time sleeping or doing school-related work (school, homework, and play rehearsal)?



Solution

Sleeping: 36% School-related: $27\% + 8\% + 11\% = 46\%$	Look at the circle graph. The section labeled “Sleeping” is a little larger than the section named “School” (and notice that the percentage of time sleeping is greater than the percentage of time at school!) “Homework” and “Play rehearsal” are both smaller, but when the percentages of time are added to “School,” they add up to a larger portion of the day.

Joelle spent more time doing school-related work.

? Exercise

The graph below shows data about how people in one company commute to work each day.

Which statement is true?

A. Everyone takes a car, bus, or train to work.
B. Taking the bus is more popular than walking or biking.
C. More people take the train than take the bus.
D. Telecommuting is the least popular method of commuting to work.

Answer

A. Incorrect. While most people commute by car, bus, or train, some commute by other means, as shown in the other sections of the graph. The correct answer is that taking the bus is more popular than walking or biking.
B. Correct. The graph shows that about one-fourth of the company takes the bus to work, but only a small portion of people walk or bike.
C. Incorrect. The “Train” section of the circle graph is smaller than the “Bus” section, so the reverse is correct: more people take the bus than take the train. The correct answer is that taking the bus is more popular than walking or biking.
D. Incorrect. The least popular method of transportation is walking, as it has the smallest slice of the pie. The correct answer is that taking the bus is more popular than walking or biking.

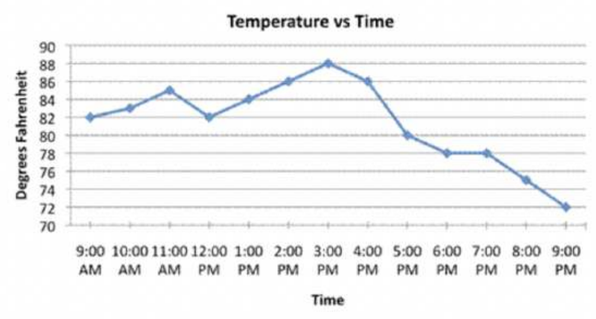
Line Graphs

Unlike circle graphs, **line graphs** are usually used to relate data over a period of time. In a line graph, the data is shown as individual points on a grid; a trend line connects all data points.

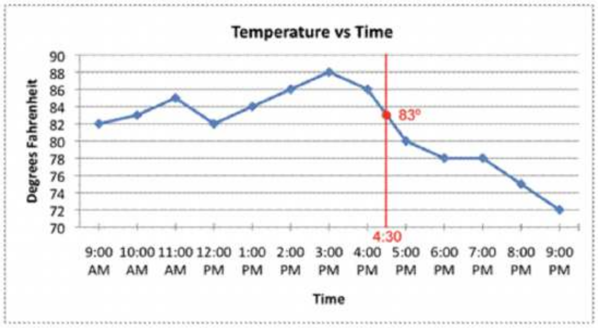
A typical use of a line graph involves the mapping of temperature over time. One example is provided below. Look at how the temperature is mapped on the y-axis and the time is mapped on the x-axis.

Each point on the grid shows a specific relationship between the temperature and the time. At 9AM, the temperature was 82^{o}. It rose to 83^{o} at 10 AM, and then again to 85^{o} at 11AM. It cooled off a bit by noon, as the temperature fell to 82^{o}. What happened the rest of the day?

The data on this graph shows that the temperature peaked at 88° at 3PM. By 9PM that evening, it was down to 72°.



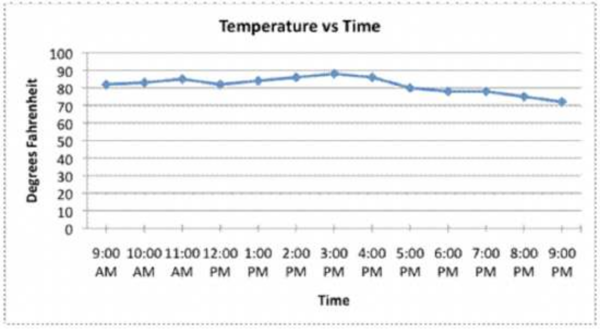
The line segments connecting each data point are important to consider, too. While this graph only provides data points for each hour, you could track the temperature each minute (or second!) if you wanted. The line segments connecting the data points indicate that the temperature vs. time relationship is continuous, meaning it can be read at any point. The line segments also provide an estimate for what the temperature would be if the temperature were measured at any point between two existing readings. For example, if you wanted to estimate the temperature at 4:30, you could find 4:30 on the x-axis and draw a vertical line that passes through the trend line; the place where it intersects the graph will be the temperature estimate at that time.



Note that this is just an estimate based on the data. There are many different possible temperature fluctuations between 4PM and 5PM. For example, the temperature could have held steady at 86° for most of the hour, and then dropped sharply to 80° just before 5PM. Alternatively, the temperature could have dropped to 76° due to a sudden storm, and then climbed back up to 80° once the storm passed. In either of these cases, our estimate of 83° would be incorrect! Based on the data, though, 83° seems like a reasonable prediction for 4:30PM.

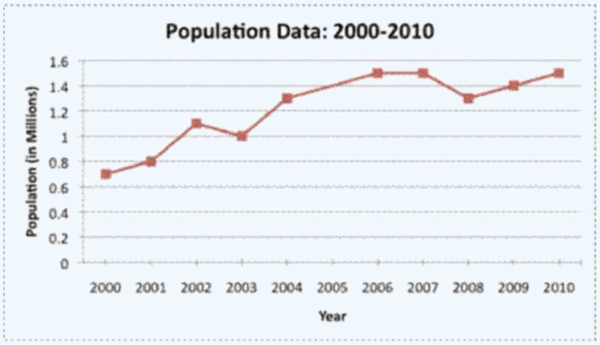
Finally, a quick word about the scale in this graph. Notice the y-axis, which is the vertical line where the Degrees Fahrenheit are listed. Notice that it starts at 70°, and then increases in increments of 2° each time. Since the scale is small and the graph begins at 70°, the temperature data looks pretty volatile, as if the temperature went from being warm to hot to very cold! Look at the same data set when plotted on a line graph that begins at 0° and has a scale of 10°.

As you can see, changing the scale of the graph can affect how a viewer perceives the data within the graph.



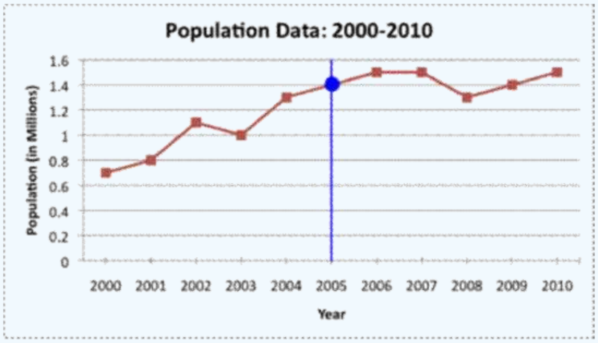
✓ Example

Population data for a fictional city is given below. Estimate the city's population in 2005.



Solution

Look at the line graph. The population starts at about 0.7 million (or 700,000) in 2000, rises to 0.8 million in 2001, and then again to 1.1 million in 2002. To find the population in 2005, find 2005 on the x-axis and draw a vertical line that intersects the trend line.



The lines intersect at 1.4, so million (or 1,400,000) would be a good estimate.

The population in 2005 was about 1.4 million.

Summary

Bar graphs are only the beginning of the story when it comes to visualizing data sets. Circle graphs show how a set of data is divided up into sections, and they help the viewer visualize how each section relates to the whole. By contrast, line graphs are usually used to relate continuous data over a period of time. A third type of graph, the stem-and-leaf plot, provides another way to organize quantitative data. Stem-and-leaf plots are useful for getting a quick picture of the smallest and largest values, clusters, and gaps of the data within a set.

2.10: End of Chapter Activity

Understanding NGSS for K-12 Education

The **Next Generation Science Standards (NGSS)** are a set of research-based, up-to-date K–12 science content standards. These standards provide a framework for science education that integrates three dimensions:

1. **Disciplinary Core Ideas (DCIs):** Key concepts in science that have broad importance within or across multiple science disciplines.
2. **Science and Engineering Practices (SEPs):** The skills necessary to engage in scientific inquiry and engineering design.
3. **Crosscutting Concepts (CCCs):** Concepts that bridge disciplinary boundaries, helping students to connect knowledge from various fields of science.

The goal of NGSS is to prepare students for college, career, and citizenship by focusing on a deeper understanding of content and developing key skills that students need to be informed and engaged in scientific and engineering practices.

Assignment: Develop a Lesson Plan for K-2 or 3-5 Grades

Students will develop a lesson plan to help K-2 or 3-5 students distinguish between mass, volume, and length through hands-on activities and observations using a balance scale, graduated cylinders, and rulers or measuring tapes.

2.11: End of Chapter Key Terms

Units, Measurement, Graphing, and Calculation Key Terms

1. **Measurement**: The process of obtaining the magnitude of a quantity relative to an agreed standard.
2. **Unit**: A standard quantity used to specify measurements.
3. **International System of Units (SI)**: The modern form of the metric system, used globally for scientific measurements, including units such as meter (m), kilogram (kg), second (s), ampere (A), kelvin (K), mole (mol), and candela (cd).
4. **Length**: A measure of the distance between two points; the SI unit is the meter (m).
5. **Mass**: A measure of the amount of matter in an object; the SI unit is the kilogram (kg).
6. **Time**: A measure of the duration of events; the SI unit is the second (s).
7. **Temperature**: A measure of the average kinetic energy of particles in a substance; the SI unit is the kelvin (K).
8. **Volume**: The amount of space occupied by an object or substance; commonly measured in liters (L) or cubic meters (m^3).
9. **Density**: The mass per unit volume of a substance; commonly measured in grams per cubic centimeter (g/cm^3) or kilograms per cubic meter (kg/m^3).
10. **Precision**: The degree to which repeated measurements under unchanged conditions show the same results.
11. **Accuracy**: The degree to which a measurement or calculation reflects the true value.
12. **Significant Figures**: The digits in a measurement that are known with certainty plus one final digit, which is somewhat uncertain or estimated.
13. **Scientific Notation**: A method of expressing numbers as a product of a coefficient and a power of 10, used to handle very large or very small numbers.
14. **Conversion Factor**: A ratio used to convert from one unit of measurement to another.
15. **Dimensional Analysis**: A technique for converting units of measurement by using conversion factors.
16. **Graph**: A visual representation of data that shows relationships between variables.
17. **Cartesian Coordinate System**: A coordinate system that specifies each point uniquely by a pair of numerical coordinates, typically (x, y).
18. **Origin**: The point (0, 0) in a Cartesian coordinate system where the x-axis and y-axis intersect.
19. **X-Axis**: The horizontal axis in a graph.
20. **Y-Axis**: The vertical axis in a graph.
21. **Data Point**: A specific value plotted on a graph representing an observed measurement.
22. **Line Graph**: A graph that uses points connected by lines to show how something changes in value (typically over time).
23. **Bar Graph**: A graph that uses bars to show comparisons among categories.
24. **Histogram**: A type of bar graph that shows the frequency of data within certain intervals.
25. **Pie Chart**: A circular chart divided into sectors, each representing a proportion of the whole.
26. **Scatter Plot**: A graph with points plotted to show a possible relationship between two sets of data.
27. **Slope**: A measure of the steepness of a line on a graph; calculated as the rise over the run ($\Delta y/\Delta x$).
28. **Intercept**: The point where a line crosses an axis on a graph.
29. **Trend Line**: A line indicating the general course or tendency of something, e.g., the relationship between variables in a scatter plot.
30. **Linear Relationship**: A relationship between two variables that can be graphically represented as a straight line.
31. **Nonlinear Relationship**: A relationship between two variables that does not graph as a straight line.
32. **Equation**: A mathematical statement that asserts the equality of two expressions.
33. **Variable**: A symbol used to represent a quantity that can change.
34. **Constant**: A value that does not change.
35. **Independent Variable**: The variable that is manipulated or controlled in an experiment.
36. **Dependent Variable**: The variable that is measured or observed in response to changes in the independent variable.
37. **Error**: The difference between a measured or calculated value and the true value.
38. **Percent Error**: The percent difference between the experimental value and the accepted value, calculated as Experimental Value−Accepted ValueAccepted Value×100%\frac{\text{Experimental Value} - \text{Accepted Value}}{\text{Accepted Value}} \times 100 \%Accepted ValueExperimental Value−Accepted Value×100%.
39. **Significant Figures Rules**: Rules that determine the number of significant figures in a calculated result, including rules for addition, subtraction, multiplication, and division.
40. **Standard Deviation**: A measure of the amount of variation or dispersion in a set of values.

41. **Uncertainty**: An estimate of the amount by which a measured or calculated value may differ from the true value.
42. **Calibration**: The process of adjusting a measuring instrument to match a known standard.

CHAPTER OVERVIEW

3: Atomic Theory and Periodic Table

3.1: Introduction and Learning Objectives

Chapter Learning Objectives

Learning Objectives

1. **Foundations of Atomic Theory:**
    - Understand the historical development of atomic theory, including key contributions from scientists such as Dalton, Thomson, Rutherford, and Bohr.
    - Explore the structure of the atom, including subatomic particles (protons, neutrons, and electrons) and their arrangement within the atom.
    - Analyze how the evolving understanding of atomic structure has impacted scientific thought and technological advancements.
2. **Understanding Electron Configuration and Chemical Properties:**
    - Comprehend the principles of electron configuration and how it determines the chemical properties and reactivity of elements.
    - Investigate the concepts of valence electrons, energy levels, and orbital shapes, and their roles in bonding and chemical reactions.
    - Apply the principles of electron configuration to predict the behavior of elements in chemical reactions.
3. **The Periodic Table: Organization and Trends:**
    - Understand the layout of the periodic table, including the arrangement of elements by atomic number, periods, and groups.
    - Examine periodic trends, such as atomic radius, ionization energy, electronegativity, and reactivity, and understand how these trends are related to the structure of atoms.
    - Analyze the classification of elements into metals, nonmetals, and metalloids, and explore the significance of this categorization.
4. **Chemical Bonding and Molecular Structure:**
    - Explore the types of chemical bonds (ionic, covalent, and metallic) and how they are related to the elements' positions on the periodic table.
    - Understand the concept of molecular structure, including the shapes of molecules and the VSEPR theory.
    - Investigate the role of chemical bonding in determining the physical and chemical properties of substances.
5. **Teaching Atomic Theory and the Periodic Table:**
    - Develop effective strategies for teaching the complex concepts of atomic theory and the periodic table in an engaging and understandable way.
    - Create interactive and hands-on activities that help students visualize atomic structures and understand periodic trends.
    - Utilize multimedia tools and resources to enhance learning and to demonstrate the practical applications of these concepts in everyday life and various scientific fields.
    - Address common misconceptions and challenges that students face when learning about atomic structure and the periodic table.
    - Emphasize the interdisciplinary nature of atomic theory and its relevance in fields such as chemistry, physics, biology, and environmental science.

Introduction to Chapter 4: Atomic Theory and the Periodic Table: A Scientific Overview

Atomic theory is a cornerstone of modern chemistry, providing a framework for understanding the structure, behavior, and interactions of matter. This chapter will take you on a journey through the historical development of atomic theory, exploring the contributions of pioneering scientists and the evolution of our understanding of atomic structure. We will also delve into the principles of electron configuration and chemical properties, the organization of the periodic table, and the nature of chemical bonding and molecular structure.

Foundations of Atomic Theory

Atomic theory has undergone significant transformations since its inception. The early ideas of Democritus, who proposed the existence of indivisible particles called atoms, laid the groundwork for future discoveries. In the 19th century, John Dalton formalized atomic theory, introducing the concept that atoms of different elements have distinct masses and combine in specific ratios to form compounds.

J.J. Thomson's discovery of the electron in 1897 challenged the notion of the atom as an indivisible particle. He proposed the "plum pudding" model, depicting the atom as a sphere of positive charge with embedded electrons. However, Ernest Rutherford's gold foil experiment in 1911 revealed the existence of a dense, positively charged nucleus, leading to the nuclear model of the atom.

Niels Bohr further refined atomic theory by introducing quantized electron orbits, explaining the stability of atoms and the emission spectra of elements. These foundational contributions have shaped our current understanding of atomic structure, consisting of protons, neutrons, and electrons.

Understanding Electron Configuration and Chemical Properties

Electron configuration describes the distribution of electrons in an atom's orbitals, which determines an element's chemical properties and reactivity. Electrons occupy energy levels, with the arrangement governed by the Pauli exclusion principle, Hund's rule, and the Aufbau principle. The outermost electrons, or valence electrons, play a crucial role in chemical bonding and reactions.

The periodic table organizes elements based on their electron configurations, allowing us to predict chemical behavior. Elements in the same group share similar valence electron configurations, resulting in analogous chemical properties. Understanding electron configuration helps explain periodic trends such as atomic radius, ionization energy, and electronegativity.

The Periodic Table: Organization and Trends

The periodic table is a powerful tool that categorizes elements by increasing atomic number, with rows called periods and columns called groups. This arrangement reflects periodic trends arising from the elements' electron configurations. Key trends include:

- **Atomic Radius:** Generally decreases across a period and increases down a group.
- **Ionization Energy:** Increases across a period and decreases down a group.
- **Electronegativity:** Increases across a period and decreases down a group.

Elements are also classified into metals, nonmetals, and metalloids based on their physical and chemical properties. Metals are typically malleable and conductive, nonmetals are diverse in state and properties, and metalloids exhibit intermediate characteristics.

Chemical Bonding and Molecular Structure

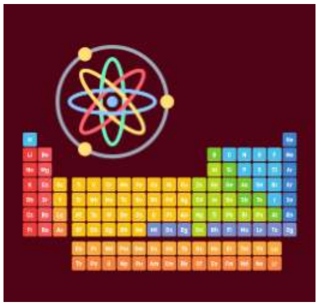
Chemical bonds form when atoms interact to achieve more stable electron configurations. The main types of chemical bonds are:

- **Ionic Bonds:** Formed by the transfer of electrons between a metal and a nonmetal, resulting in oppositely charged ions.
- **Covalent Bonds:** Involve the sharing of electron pairs between nonmetals.
- **Metallic Bonds:** Consist of a "sea" of delocalized electrons shared among metal atoms.

The Valence Shell Electron Pair Repulsion (VSEPR) theory explains the shapes of molecules based on the repulsion between electron pairs around the central atom. Molecular geometry affects the physical and chemical properties of substances.

K-12 Lens: Teaching Atomic Theory & The Periodic Table

For K-12 educators, teaching the concepts of atomic theory and the periodic table can be made engaging and comprehensible through a variety of strategies. Interactive activities, such as hands-on experiments and simulations to visualize atomic structures and periodic trends, can enhance understanding. Multimedia tools, including videos, animations, and interactive software, help illustrate complex concepts in an accessible manner. Relatable examples, like the chemical reactions in cooking or the materials used in technology, make learning more relevant. Addressing common misconceptions, such as the nature of electron orbits or the behavior of elements, through clear explanations and demonstrations is essential. Emphasizing the importance of atomic theory in fields like chemistry, physics, biology, and environmental science highlights its broad applicability and significance. By incorporating these strategies, educators can foster a deeper understanding of atomic theory and the periodic table, preparing students for further studies in science and technology.



SECTION OVERVIEW

3.2: Atoms and the Periodic Table

3.2.1: Basic Atomic Theory

Learning Objectives

- Give a short history of the concept of the atom.
- Describe the contributions of Democritus and Dalton to atomic theory.
- Summarize Dalton's atomic theory and explain its historical development.

Democritus and the Greek Philosophers

Around 2,500 years ago, early Greek philosophers believed the entire universe was a single, huge, entity. In other words, "everything was one." They believed that all objects, all matter, and all substances were connected as a single, big, unchangeable "thing." Democritus was one of the first people to propose the term "atoms." As an alternative to the beliefs of the Greek philosophers, he suggested that **atomos**, or atomon - tiny, indivisible, solid objects - make up all matter in the universe.

Figure 3.2.1.1: Democritus by Hendrick ter Brugghen, 1628. Democritus was known as the "laughing philosopher." It was a good thing he liked to laugh, because most other philosophers were laughing at his theories.

Early Greek philosophers tried to understand the nature of the world through reason and logic, not through experiment and observation. As a result, they had some very interesting ideas, but they felt no need to justify their ideas based on life experiences. In many ways, you can think of the Greek philosophers as being "all thought and no action." It's truly amazing how much they achieved using their minds, but because they never performed any experiments, they missed or rejected many discoveries that they could have made otherwise. Greek philosophers dismissed Democritus' theory entirely. Unfortunately, it took over two millennia before the theory of atomos (or "atoms," as they're known today) was fully accepted.

Video 3.2.1.1: Since ancient Greek times, philosophers and scientists have tried to figure out what an atom looks like. For a couple thousand years, humans could only speculate on the structure and other properties of the smallest unit of matter. It wasn't until the 1980s that chemists could see individual atoms. Bestselling author Sam Kean takes us through the nearly 2,400-year quest to see the atom in a new episode of the "Legends of Chemistry" series.

While it must be assumed that many more scientists, philosophers, and others studied composition of matter after Democritus, a major leap forward in our understanding of the composition of matter took place in the 1800's with the work of the British scientists John Dalton. He started teaching school at age twelve, and was primarily known as a teacher. In his twenties, he moved to the growing city of Manchester, where he was able to pursue some scientific studies. His work in several areas of science brought him a number of honors. When he died, over 40,000 people in Manchester marched at his funeral.

Figure $3.2.1.3$: British physicist and chemist John Dalton (1766-1844). Unlike the greek philosophers, John Dalton believed in both logical thinking and experimentation.

Dalton studied the weights of various elements and compounds. He noticed that matter always combined in fixed ratios based on weight, or volume in the case of gases. Chemical compounds always contain the same proportion of elements by mass, regardless of amount, which provided further support for Proust's law of definite proportions. Dalton also observed that there could be more than one combination of two elements.

From his experiments and observations, as well as the work from peers of his time, Dalton proposed a new theory of the atom (1803). This later became known as Dalton's **atomic theory**. The published (1808) tenets of this theory were as follows:

- All matter is composed of extremely small particles called atoms.
- Atoms of a given element are identical in size, mass, and other properties. Atoms of different elements differ in size, mass, and other properties.
- Atoms cannot be subdivided, created, or destroyed.
- Atoms of different elements can combine in simple whole number ratios to form chemical compounds.
- In chemical reactions, atoms are combined, separated, or rearranged.
-

Dalton's Eyes

As a scientist, John Dalton was also interested in colorblindness. Both he and his brother suffered from red/green colorblindness. At this time (1794), no other scientist had explored this medical issue. He devised theories (although not correct) as to why he could not differentiate between red and green. In addition, he named this medical condition "Daltonism." Upon his death (1844), he donated his eyes to a medical research facility in Manchester, England. Today, his eyes are still on display at the Museum of Science and Industry in Manchester, England.

Figure 3.2.1.3: Image taken from: https://c1.staticflickr.com/3/2823/1...37dd23af_b.jpg

The evidence for atoms is so great that few doubt their existence. In fact, individual atoms are now routinely observed with state-of-the art technologies. Moreover, atoms can even be used for making pretty images or as IBM research demonstrate in Video 3.2.1.2, control of individual atoms can be use used create animations. A Boy and His Atom is a 2012 stop-motion animated short film released by IBM Research. The movie tells the story of a boy and a wayward atom who meet and become friends. It depicts a boy playing with an atom that takes various forms. It was made by moving carbon monoxide molecules viewed with a scanning tunneling microscope, a device that magnifies them 100 million times. These molecules were moved to create images, which were then saved as individual frames to make the film.

Video 3.2.1.2: A Boy And His Atom: The World's Smallest Movie

Dalton was not completely correct

Dalton's atomic theory has been largely accepted by the scientific community, with the exception of three changes. We know now that:

1. An atom is composed of smaller particles (electrons, protons, and neutrons).
2. All atoms of an element are not identical. The existence of isotopes illustrates this phenomena.
3. Through the use of nuclear reactions, atoms of one element can be changed into atoms of another element.

The first two will be be discussed later in this chapter, while the last requires introducing "nuclear chemistry" and will be discussed in a different chapter.

References

1. https://pubs.acs.org/subscribe/archi...3chemchron.pdf

Contributors and Attributions

- Template:ContribCK12

3.2.2: Lecture 1 - Atomic Theory

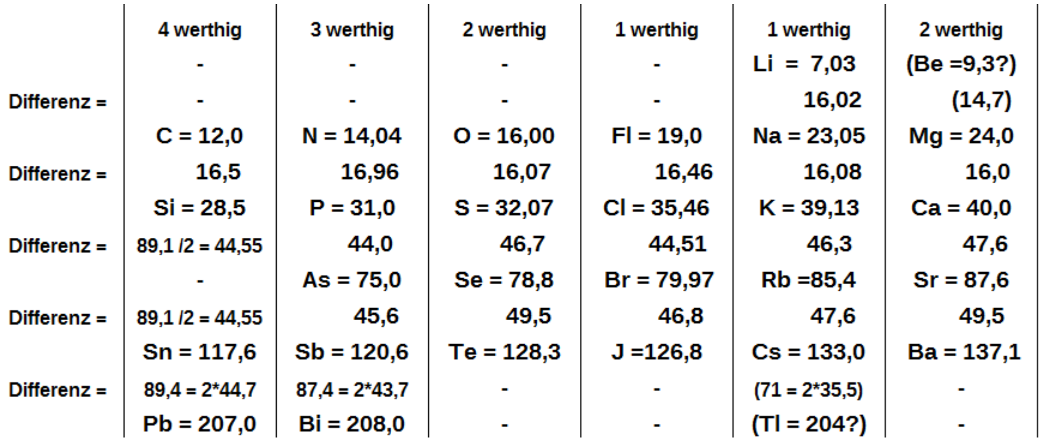
The lesson is based on section 2.1-2.2 in the OpenStax Chemistry 2e textbook. The lecture slides are provided in PowerPoint, Keynote, and pdf format.

3.2.3: Development of the Modern Periodic Table

Learning Objectives

- Appreciate how the modern periodic table was constructed.
- Explain how elements are organized into the periodic table.
- Describe how some characteristics of elements relate to their positions on the periodic table.
- Distinguish between the A (atomic mass) value and the Z (atomic number) value when viewing the periodic table.
- Compare and contrast Mendeleev's and Moseley's ordering of the periodic table.

In 1864, Julius Lothar Meyer, a German chemist, organized the elements by atomic mass and grouped them according to their chemical properties. Later that decade, Dmitri Mendeleev, a Russian chemist, organized all the known elements according to similar properties. He left gaps in his table for what he thought were undiscovered elements, and he made some bold predictions regarding the properties of those undiscovered elements. When elements were later discovered whose properties closely matched Mendeleev’s predictions, his version of the table gained favor in the scientific community. Because certain properties of the elements repeat on a regular basis throughout the table (that is, they are periodic), it became known as the periodic table.



	4 werthig	3 werthig	2 werthig	1 werthig	1 werthig	2 werthig
	-	-	-	-	Li = 7,03	(Be =9,3?)
Differenz =	-	-	-	-	16,02	(14,7)
	C = 12,0	N = 14,04	O = 16,00	Fl = 19,0	Na = 23,05	Mg = 24,0
Differenz =	16,5	16,96	16,07	16,46	16,08	16,0
	Si = 28,5	P = 31,0	S = 32,07	Cl = 35,46	K = 39,13	Ca = 40,0
Differenz =	89,1 /2 = 44,55	44,0	46,7	44,51	46,3	47,6
	-	As = 75,0	Se = 78,8	Br = 79,97	Rb =85,4	Sr = 87,6
Differenz =	89,1 /2 = 44,55	45,6	49,5	46,8	47,6	49,5
	Sn = 117,6	Sb = 120,6	Te = 128,3	J =126,8	Cs = 133,0	Ba = 137,1
Differenz =	89,4 = 2*44,7	87,4 = 2*43,7	-	-	(71 = 2*35,5)	-
	Pb = 207,0	Bi = 208,0	-	-	(Tl = 204?)	-

Figure 3.2.3.1: Copy of table on page 137 of (Julius) Lothar Meyer: "Die modernen Theorien der Chemie" (1864). (Copyright;By Kawarayaki - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/inde...curid=75552705)

Dimitri Mendeleev (1834–1907)

The father of the modern periodic table, Dmitri Mendeleev was born in Siberia. He was the youngest of fourteen living children. Shortly after Dmitri's birth, his father lost his sight. When Dmitri was thirteen, his father died suddenly. His mother was left to manage the family's glass factory to support the family. In 1848, their glass factory burned down leaving the family with little resources. Dmitri's mother chose to take her son to St. Petersburg, Russia. Here, he would study chemistry at his father's alma mater (Main Pedagogical Institute which is not called St. Petersburg University). After several illnesses and his mother's death, he would earn his doctorate in chemistry in 1865.

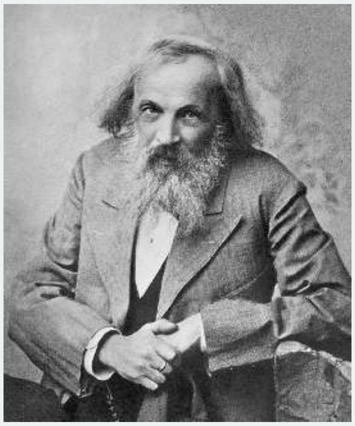


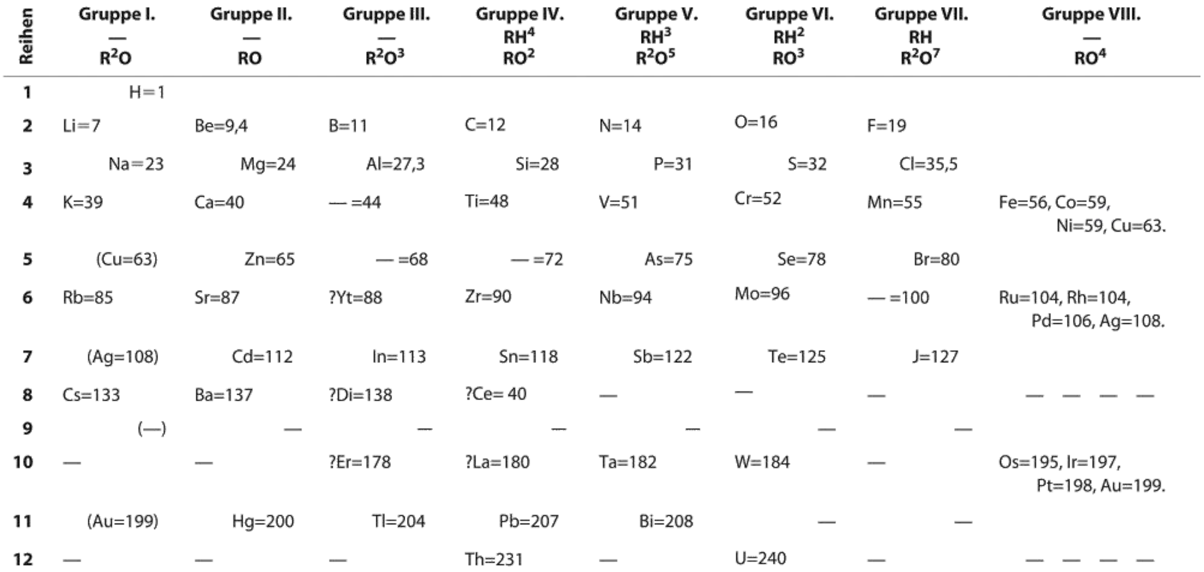
Figure 3.2.3.2: Dmitri Mendeleev, the father of the modern periodic table.

From 1867 to 1890, Dmitri served as a professor of general chemistry at St. Petersburg University. In his lifetime, he published more than 400 articles and books. Two of these works were noted in textbooks in general and organic chemistry.

As for his personal life, Dmitri Mendeleev married twice (his first marriage ended in a controversial divorce). Dmitri fathered a total of six children from both marriages. In 1907, he passed away from influenza complications. He was honored for his works by having element 101 (Mendelevium) named for him. Watch the video below to see how Mendeleev ordered his table of elements.

Mendeleev’s Periodic Table

Mendeleev, who first published his periodic table in 1869 (Figure 3.2.3.1), is usually credited with the origin of the modern periodic table. The key difference between his arrangement of the elements and that of Meyer and others is that Mendeleev did not assume that all the elements had been discovered (actually, only about two-thirds of the naturally occurring elements were known at the time). Instead, he deliberately left blanks in his table at atomic masses 44, 68, 72, and 100, in the expectation that elements with those atomic masses would be discovered. Those blanks correspond to the elements we now know as scandium, gallium, germanium, and technetium.



Reihen	Gruppe I. — R^2O	Gruppe II. — RO	Gruppe III. — R^2O^3	Gruppe IV. RH^4 RO^2	Gruppe V. RH^3 R^2O^5	Gruppe VI. RH^2 RO^3	Gruppe VII. RH R^2O^7	Gruppe VIII. — RO^4
1	H=1							
2	Li=7	Be=9,4	B=11	C=12	N=14	O=16	F=19	
3	Na=23	Mg=24	Al=27,3	Si=28	P=31	S=32	Cl=35,5	
4	K=39	Ca=40	— =44	Ti=48	V=51	Cr=52	Mn=55	Fe=56, Co=59, Ni=59, Cu=63.
5	(Cu=63)	Zn=65	— =68	— =72	As=75	Se=78	Br=80	
6	Rb=85	Sr=87	?Yt=88	Zr=90	Nb=94	Mo=96	— =100	Ru=104, Rh=104, Pd=106, Ag=108.
7	(Ag=108)	Cd=112	In=113	Sn=118	Sb=122	Te=125	J=127	
8	Cs=133	Ba=137	?Di=138	?Ce= 40	—	—	—	— — — —
9	(—)	—	—	—	—	—	—	
10	—	—	?Er=178	?La=180	Ta=182	W=184	—	Os=195, Ir=197, Pt=198, Au=199.
11	(Au=199)	Hg=200	Tl=204	Pb=207	Bi=208	—	—	
12	—	—	—	Th=231	—	U=240	—	— — — —

Figure 3.2.3.3: Mendeleev’s Periodic Table, as Published in the German Journal Annalen der Chemie und Pharmacie in 1872. The column headings “Reihen” and “Gruppe” are German for “row” and “group.” Formulas indicate the type of compounds formed by each group, with “R” standing for “any element” and superscripts used where we now use subscripts. Atomic masses are shown after equal signs and increase across each row from left to right.

The groups in Mendeleev's table are determined by how many oxygen or hydrogen atoms are needed to form compounds with each element. For example, in Group I (referred to as Gruppe 1), two atoms of each element (hydrogen, lithium, potassium) bond with one atom of oxygen to form compounds. In Group VII (Gruppe VI), one atom of fluorine, F, chlorine, Cl, and bromine, Br, reacts with one atom of hydrogen to form respective compounds. According to Mendeleev, elements in the same column of the periodic table produced similar types of compounds. This trend could not predict the types of hydrogen/oxygen-based compounds resulting from the middle section of the periodic table. Mendeleev had difficulty explaining the chemical nature of this part of the table now referred to as the transition metals.

The most convincing evidence in support of Mendeleev's arrangement of the elements was the discovery of two previously unknown elements whose properties closely corresponded with his predictions (Table 3.2.3.1). Two of the blanks Mendeleev had left in his original table were below aluminum and silicon, awaiting the discovery of two as-yet-unknown elements, eka-aluminum and eka-silicon (from the Sanskrit eka, meaning "one," as in "one beyond aluminum"). The observed properties of gallium and germanium matched those of eka-aluminum and eka-silicon so well that once they were discovered, Mendeleev's periodic table rapidly gained acceptance.

When the chemical properties of an element suggested that it might have been assigned the wrong place in earlier tables, Mendeleev carefully reexamined its atomic mass. He discovered, for example, that the atomic masses previously reported for beryllium, indium, and uranium were incorrect. The atomic mass of indium had originally been reported as 75.6, based on an assumed stoichiometry of InO for its oxide. If this atomic mass were correct, then indium would have to be placed in the middle of the nonmetals, between arsenic (atomic mass 75) and selenium (atomic mass 78). Because elemental indium is a silvery-white *metal*, however, Mendeleev postulated that the stoichiometry of its oxide was really In_2O_3 rather than InO. This would mean that indium's atomic mass was actually 113, placing the element between two other metals, cadmium, and tin.

Table 3.2.3.1: Comparison of the Properties Predicted by Mendeleev in 1869 for eka-Aluminum and eka-Silicon with the Properties of Gallium (Discovered in 1875) and Germanium (Discovered in 1886)

Property	***eka*-Aluminum (predicted)**	**Gallium (observed)**	***eka*-Silicon (predicted)**	**Germanium (observed)**
atomic mass	68	69.723	72	72.64
element	metal	metal	dirty-gray metal	gray-white metal
	low mp*	mp = 29.8°C	high mp	mp = 938°C
	$d = 5.9$ g/cm^3	$d = 5.91$ g/cm^3	$d = 5.5$ g/cm^3	$d = 5.323$ g/cm^3
oxide	E_2O_3	Ga_2O_3	EO_2	GeO_2
	$d = 5.5$ g/cm^3	$d = 6.0$ g/cm^3	$d = 4.7$ g/cm^3	$d = 4.25$ g/cm^3
chloride	ECl_3	$GaCl_3$	ECl_4	$GeCl_4$
	volatile	mp = 78°C bp* = 201°C	bp < 100°C	bp = 87°C

*mp = melting point; bp = boiling point.

One group of elements absent from Mendeleev's table is the noble gases, all of which were discovered more than 20 years later, between 1894 and 1898, by Sir William Ramsay (1852–1916; Nobel Prize in Chemistry 1904). Initially, Ramsay did not know where to place these elements in the periodic table. Argon, the first to be discovered, had an atomic mass of 40. This was greater than chlorine's and comparable to that of potassium, so Ramsay, using the same kind of reasoning as Mendeleev, decided to place the noble gases between the halogens and the alkali metals.

The Role of the Atomic Number in the Periodic Table

Despite its usefulness, Mendeleev's periodic table was based entirely on empirical observation supported by very little understanding. It was not until 1913, when a young British physicist, H. G. J. Moseley (1887–1915), while analyzing the frequencies of x-rays emitted by the elements, discovered that the underlying foundation of the order of the elements was by the *atomic number*, not the atomic mass. Moseley hypothesized that the placement of each element in his series corresponded to its atomic number Z, which is the number of positive charges (protons) in its nucleus. Argon, for example, although having an atomic mass greater than that of potassium (39.9 amu versus 39.1 amu, respectively), was placed *before* potassium in the periodic table. While analyzing the frequencies of the emitted x-rays, Moseley noticed that the atomic number of argon is 18, whereas that of potassium is 19, which indicated that they were indeed placed correctly. Moseley also noticed three gaps in his table of x-ray frequencies, so he predicted the existence of three unknown elements: technetium (Z = 43), discovered in 1937; promethium (Z = 61), discovered in 1945; and rhenium (Z = 75), discovered in 1925.

H. G. J. Moseley (1887–1915)

Moseley left his research work at the University of Oxford to join the British army as a telecommunications officer during World War I. He was killed during the Battle of Gallipoli in Turkey.

Figure 3.2.3.4:G.J. Moseley

Features of the Periodic Table

Elements that have similar chemical properties are grouped in columns called groups (or families). As well as being numbered, some of these groups have names—for example, *alkali metals* (the first column of elements), *alkaline earth metals* (the second column of elements), *halogens* (the next-to-last column of elements), and *noble gases* (the last column of elements). The word *halogen* comes from the Greek for “salt maker” because these elements combine with other elements to form a group of compounds called salts.

Each row of elements on the periodic table is called a **period**. Periods have different lengths; the first period has only 2 elements (hydrogen and helium), while the second and third periods have 8 elements each. The fourth and fifth periods have 18 elements each, and later periods are so long that a segment from each is removed and placed beneath the main body of the table.

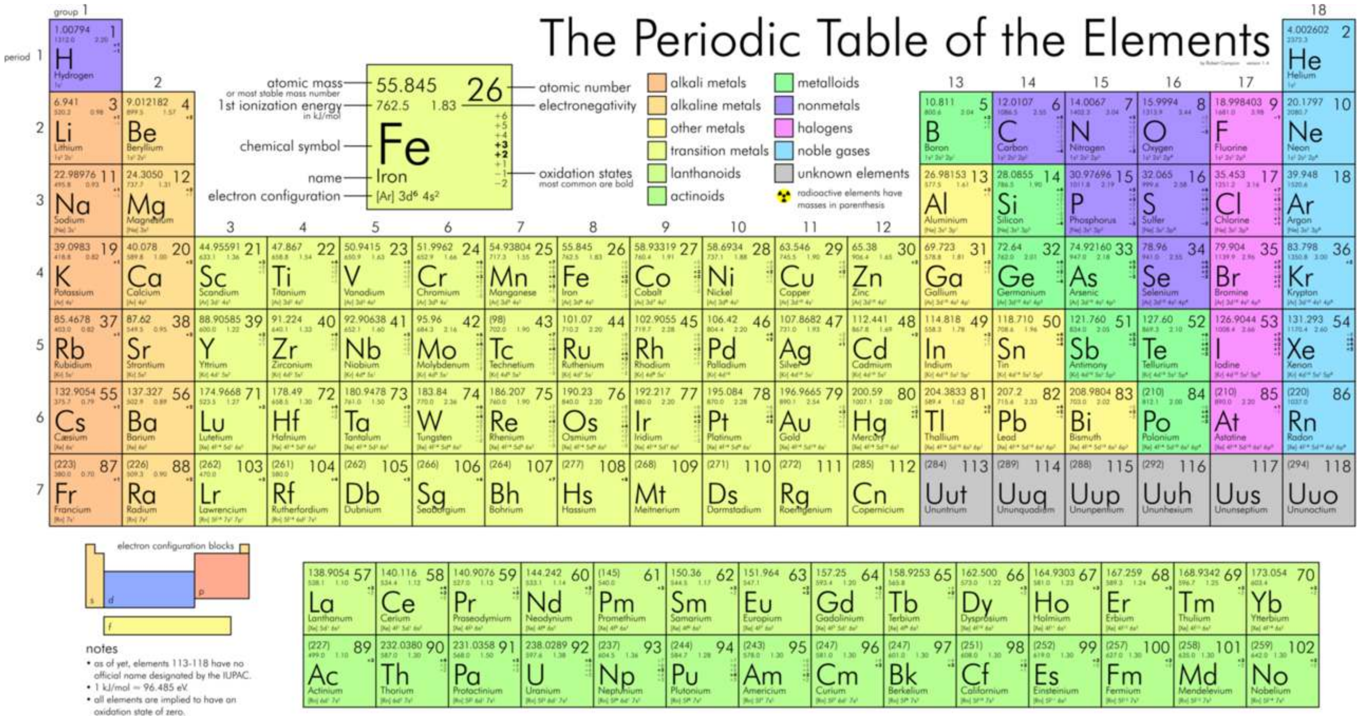


Figure $3.2.3.5$: Types of Elements. Elements are either metals, nonmetals, or metalloids. Each group is located in a different part of the periodic table. An interactive Periodic table can be found here. (Public Domain; 2012rc).

Certain elemental properties become apparent in a survey of the periodic table as a whole. Every element can be classified as either a metal, a nonmetal, or a metalloid (or semi-metal), as shown in Figure $3.2.3.5$. A **metal** is a substance that is shiny, typically (but not always) silvery in color, and an excellent conductor of electricity and heat. Metals are also malleable (they can be beaten into thin sheets) and ductile (they can be drawn into thin wires). A **nonmetal** is typically dull and a poor conductor of electricity and heat. Solid nonmetals are also very brittle. As shown in Figure $3.2.3.5$, metals occupy the left three-fourths of the periodic table, while nonmetals (except for hydrogen) are clustered in the upper right-hand corner of the periodic table. The elements with properties intermediate between those of metals and nonmetals are called **metalloids**. Elements adjacent to the bold line in the right-hand portion of the periodic table have semimetal properties.

✓ Example $3.2.3.1$

Based on its position in the periodic table, classify each element below as metal, a nonmetal, or a metalloid.

a. Se
b. Mg
c. Ge

Solution

a. In Figure $3.2.3.5$, selenium lies above and to the right of the diagonal line marking the boundary between metals and nonmetals, so it should be a nonmetal.
b. Magnesium lies to the left of the diagonal line marking the boundary between metals and nonmetals, so it should be metal.
c. Germanium lies within the diagonal line marking the boundary between metals and nonmetals, so it should be a metalloid.

The first two columns on the left and the last six columns on the right are called the main group elements. The ten-column block between these columns contains the **transition metals**. The two rows beneath the main body of the periodic table display the **inner transition metals**. The elements in these two rows are also referred to as, respectively, the **lanthanide metals** and the **actinide metals**.

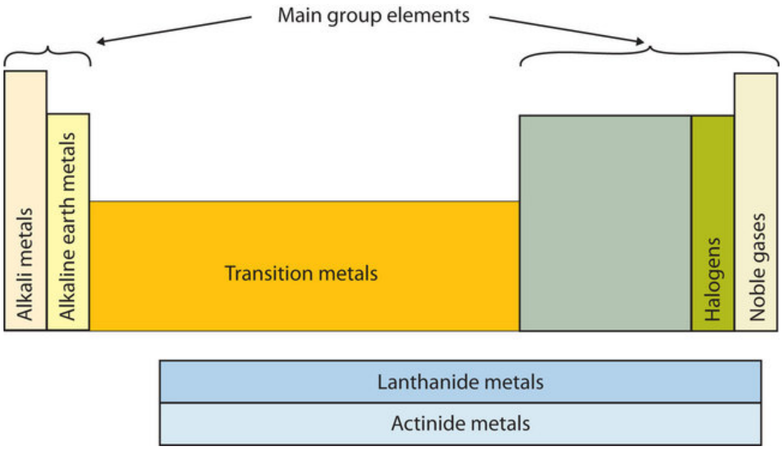


Figure 3.2.3.6: Some sections of the periodic table have specific names. Be sure to know the section names for quizzes and tests.

Descriptive Names

As previously noted, the periodic table is arranged so that elements with similar chemical behaviors are in the same group. Chemists often make general statements about the properties of the elements in a group using descriptive names with historical origins. To watch a video of your instructor presenting these groups and periods, access your moodle class chapter 3 documents.

Group 1: The Alkali Metals

The alkali metals are lithium, sodium, potassium, rubidium, cesium, and francium. Hydrogen is unique in that it is generally placed in Group 1, but it is not a metal. The compounds of the alkali metals are common in nature and daily life. One example is table salt (sodium chloride); lithium compounds are used in greases, in batteries, and as drugs to treat patients who exhibit manic-depressive, or bipolar, behavior. Although lithium, rubidium, and cesium are relatively rare in nature, and francium is so unstable and highly radioactive that it exists in only trace amounts, sodium and potassium are the seventh and eighth most abundant elements in Earth's crust, respectively.

Group 2: The Alkaline Earth Metals

The alkaline earth metals are beryllium, magnesium, calcium, strontium, barium, and radium. Beryllium, strontium, and barium are rare, and radium is unstable and highly radioactive. In contrast, calcium and magnesium are the fifth and sixth most abundant elements on Earth, respectively; they are found in huge deposits of limestone and other minerals.

Group 17: The Halogens

The halogens are fluorine, chlorine, bromine, iodine, and astatine. The name halogen is derived from the Greek words for "salt-forming," which reflects that all the halogens react readily with metals to form compounds, such as sodium chloride and calcium chloride (used in some areas as road salt). Compounds that contain fluorine ions (fluoride) are added to toothpaste and the water supply to prevent dental cavities. Fluorine is also found in Teflon coatings on kitchen utensils. Although chlorofluorocarbon propellants and refrigerants are believed to lead to the depletion of Earth's ozone layer and contain both fluorine and chlorine, the latter is responsible for the adverse effect on the ozone layer. Bromine and iodine are less abundant than chlorine, and astatine is so radioactive that it exists in only negligible amounts in nature.

Group 18: The Noble Gases

The noble gases are helium, neon, argon, krypton, xenon, and radon. Because noble gases are composed of only single atoms, they are called monatomic. At room temperature and pressure, they are unreactive gases. Because of their lack of reactivity, for many years they were called inert gases or rare gases. However, the first chemical compounds containing noble gases were prepared in 1962. Although the noble gases are relatively minor constituents of the atmosphere, natural gas contains substantial amounts of helium. Because of its low reactivity, argon is often used as an unreactive (inert) atmosphere for welding and in light bulbs. The red light emitted by neon in a gas discharge tube is used in neon lights.

✓ Example 3.2.3.2: Groups

Provide the family/group names and period numbers (horizontal values)of each element.

a. Li
b. Ar
c. Ra

Solution:

a. Lithium is an alkali metal. It is located in period two.
b. Argon is a noble gas. It is located in period three.
c. Radium is an alkaline metal. It is located in period seven.

✓ Example 3.2.3.3: Classification of Elements

Provide elemental names for the following combinations:

a. The alkali metal in period three.
b. The halogen in period two
c. A metalloid in period four
d. A transition metal in period three

Solution:

a. Sodium
b. Fluorine
c. Germanium or Arsenic
d. There are no transition metals in period three (gotcha!)

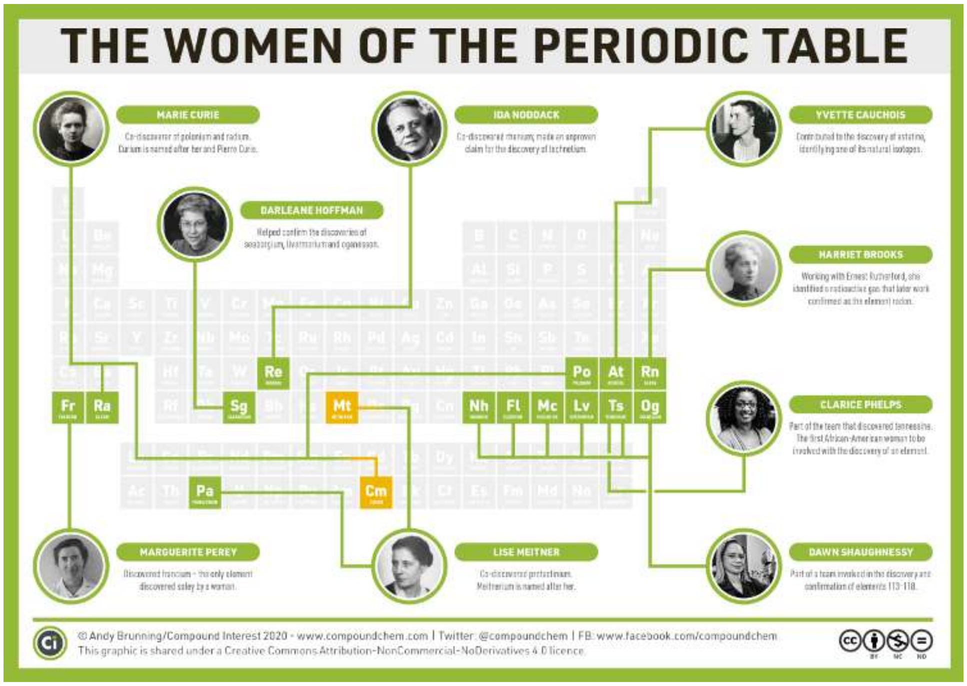


Figure 3.2.3.7: The Women of the Periodic Table Graphic by Andy Brunning.

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3.2.4: The Structure of the Atom

Learning Objectives

By the end of this section, you will be able to:

- Describe the structure of atoms and the components of nuclei
- Explain the behavior of electrons within atoms and how electrons interact with light to move among energy levels

The idea that matter is composed of tiny particles called atoms is at least 25 centuries old. It took until the twentieth century, however, for scientists to invent instruments that permitted them to probe inside an atom and find that it is not, as had been thought, hard and indivisible. Instead, the atom is a complex structure composed of still smaller particles.

Probing the Atom

The first of these smaller particles was discovered by British physicist James (J. J.) Thomson in 1897. Named the *electron*, this particle is negatively charged. (It is the flow of these particles that produces currents of electricity, whether in lightning bolts or in the wires leading to your lamp.) Because an atom in its normal state is electrically neutral, each electron in an atom must be balanced by the same amount of positive charge.

The next step was to determine where in the atom the positive and negative charges are located. In 1911, British physicist Ernest Rutherford devised an experiment that provided part of the answer to this question. He bombarded an extremely thin piece of gold foil, only about 400 atoms thick, with a beam of alpha particles (Figure 3.2.4.1). *Alpha particles* (α particles) are helium atoms that have lost their electrons and thus are positively charged. Most of these particles passed though the gold foil just as if it and the atoms in it were nearly empty space. About 1 in 8000 of the alpha particles, however, completely reversed direction and bounced backward from the foil. Rutherford wrote, “It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you.”

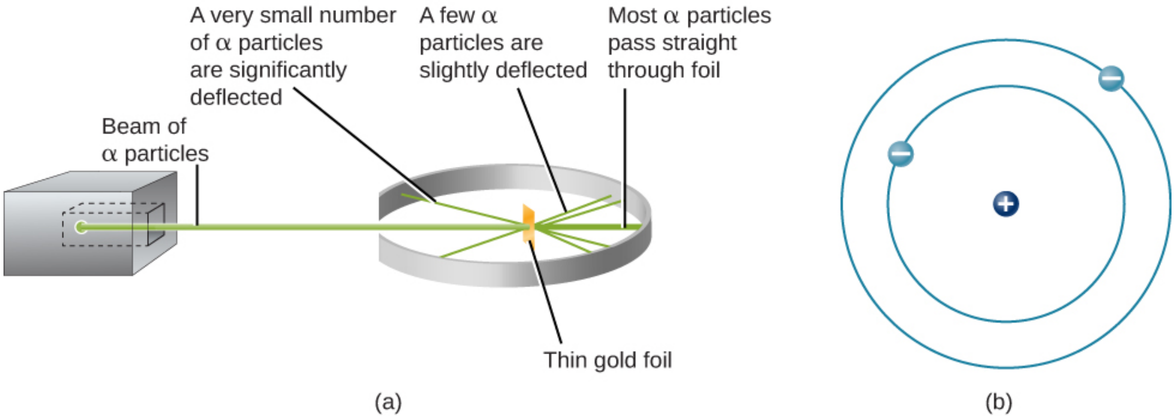

Figure 3.2.4.1 Rutherford’s Experiment. (a) When Rutherford allowed α particles from a radioactive source to strike a target of gold foil, he found that, although most of them went straight through, some rebounded back in the direction from which they came. (b) From this experiment, he concluded that the atom must be constructed like a miniature solar system, with the positive charge concentrated in the nucleus and the negative charge orbiting in the large volume around the nucleus. Note that this drawing is not to scale; the electron orbits are much larger relative to the size of the nucleus.

The only way to account for the particles that reversed direction when they hit the gold foil was to assume that nearly all of the mass, as well as all of the positive charge in each individual gold atom, is concentrated in a tiny center or **nucleus**. When a positively charged alpha particle strikes a nucleus, it reverses direction, much as a cue ball reverses direction when it strikes another billiard ball. Rutherford’s model placed the other type of charge—the negative electrons—in orbit around this nucleus.

Rutherford’s model required that the electrons be in motion. Positive and negative charges attract each other, so stationary electrons would fall into the positive nucleus. Also, because both the electrons and the nucleus are extremely small, most of the atom is empty, which is why nearly all of Rutherford’s particles were able to pass right through the gold foil without colliding with anything. Rutherford’s model was a very successful explanation of the experiments he conducted, although eventually scientists would discover that even the nucleus itself has structure.

The Atomic Nucleus

The simplest possible atom (and the most common one in the Sun and stars) is hydrogen. The nucleus of ordinary hydrogen contains a single proton. Moving around this proton is a single electron. The mass of an electron is nearly 2000 times smaller than the mass of a proton; the electron carries an amount of charge exactly equal to that of the proton but opposite in sign (Figure 3.2.4.2). Opposite charges attract each other, so it is an electromagnetic force that holds the proton and electron together, just as gravity is the force that keeps planets in orbit around the Sun.

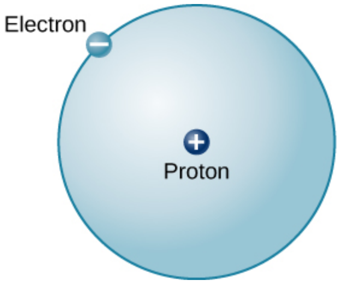

Figure 3.2.4.2 Hydrogen Atom. This is a schematic diagram of a hydrogen atom in its lowest energy state, also called the ground state. The proton and electron have equal but opposite charges, which exert an electromagnetic force that binds the hydrogen atom together. In the illustration, the size of the particles is exaggerated so that you can see them; they are not to scale. They are also shown much closer than they would actually be as it would take more than an entire page to show their actual distance to scale.

There are many other types of atoms in nature. Helium, for example, is the second-most abundant element in the Sun. Helium has two protons in its nucleus instead of the single proton that characterizes hydrogen. In addition, the helium nucleus contains two neutrons, particles with a mass comparable to that of the proton but with no electric charge. Moving around this nucleus are two electrons, so the total net charge of the helium atom is also zero (Figure 3.2.4.3).

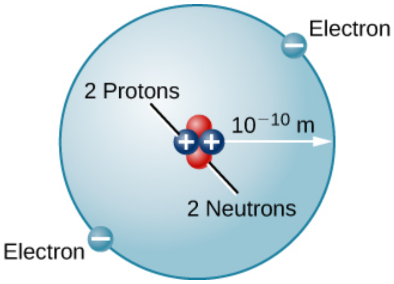

Figure 3.2.4.3 Helium Atom. Here we see a schematic diagram of a helium atom in its lowest energy state. Two protons are present in the nucleus of all helium atoms. In the most common variety of helium, the nucleus also contains two neutrons, which have nearly the same mass as the proton but carry no charge. Two electrons orbit the nucleus.

From this description of hydrogen and helium, perhaps you have guessed the pattern for building up all the elements (different types of atoms) that we find in the universe. The type of element is determined by the number of protons in the nucleus of the atom. For example, any atom with six protons is the element carbon, with eight protons is oxygen, with 26 is iron, and with 92 is uranium. On Earth, a typical atom has the same number of electrons as protons, and these electrons follow complex orbital patterns around the nucleus. Deep inside stars, however, it is so hot that the electrons get loose from the nucleus and (as we shall see) lead separate yet productive lives.

The ratio of neutrons to protons increases as the number of protons increases, but each element is unique. The number of neutrons is not necessarily the same for all atoms of a given element. For example, most hydrogen atoms contain no neutrons at all. There are, however, hydrogen atoms that contain one proton and one neutron, and others that contain one proton and two neutrons. The various types of hydrogen nuclei with different numbers of neutrons are called **isotopes** of hydrogen (Figure 3.2.4.4), and all other elements have isotopes as well. You can think of isotopes as siblings in the same element "family"—closely related but with different characteristics and behaviors.

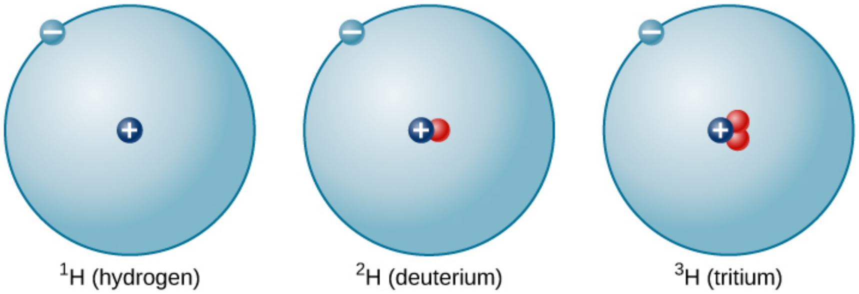

Figure 3.2.4.4 Isotopes of Hydrogen. A single proton in the nucleus defines the atom to be hydrogen, but there may be zero, one, or two neutrons. The most common isotope of hydrogen is the one with only a single proton and no neutrons.

> To explore the structure of atoms, go to the PhET Build an Atom website where you can add protons, neutrons, or electrons to a model and the name of the element you have created will appear. You can also see the net charge, the mass number, whether it is stable or unstable, and whether it is an ion or a neutral atom.

The Bohr Atom

Rutherford’s model for atoms has one serious problem. Maxwell’s theory of electromagnetic radiation says that when electrons change either speed or the direction of motion, they must emit energy. Orbiting electrons constantly change their direction of motion, so they should emit a constant stream of energy. Applying Maxwell’s theory to Rutherford’s model, all electrons should spiral into the nucleus of the atom as they lose energy, and this collapse should happen very quickly—in about 10^{-16} seconds.

It was Danish physicist Niels Bohr (1885–1962) who solved the mystery of how electrons remain in orbit. He was trying to develop a model of the atom that would also explain certain regularities observed in the spectrum of hydrogen. He suggested that the spectrum of hydrogen can be understood if we assume that orbits of only certain sizes are possible for the electron. Bohr further assumed that as long as the electron moves in only one of these allowed orbits, it radiates no energy: its energy would change only if it moved from one orbit to another.

This suggestion, in the words of science historian Abraham Pais, was “one of the most audacious hypotheses ever introduced in physics.” If something equivalent were at work in the everyday world, you might find that, as you went for a walk after astronomy class, nature permitted you to walk two steps per minute, five steps per minute, and 12 steps per minute, but no speeds in between. No matter how you tried to move your legs, only certain walking speeds would be permitted. To make things more bizarre, it would take no effort to walk at any one of the allowed speeds, but it would be difficult to change from one speed to another. Luckily, no such rules apply at the level of human behavior. But at the microscopic level of the atom, experiment after experiment has confirmed the validity of Bohr’s strange idea. Bohr’s suggestions became one of the foundations of the new (and much more sophisticated) model of the subatomic world called quantum mechanics.

In Bohr’s model, if the electron moves from one orbit to another closer to the atomic nucleus, it must give up some energy in the form of electromagnetic radiation. If the electron goes from an inner orbit to one farther from the nucleus, however, it requires some additional energy. One way to obtain the necessary energy is to absorb electromagnetic radiation that may be streaming past the atom from an outside source.

A key feature of Bohr’s model is that each of the permitted electron orbits around a given atom has a certain energy value; we therefore can think of each orbit as an **energy level**. To move from one orbit to another (which will have its own specific energy value) requires a change in the electron’s energy—a change determined by the difference between the two energy values. If the electron goes to a lower level, the energy difference will be given off; if the electron goes to a higher level, the energy difference must be obtained from somewhere else. Each jump (or transition) to a different level has a fixed and definite energy change associated with it.

A crude analogy for this situation might be life in a tower of luxury apartments where the rent is determined by the quality of the view. Such a building has certain, definite numbered levels or floors on which apartments are located. No one can live on floor 5.37 or 22.5. In addition, the rent gets higher as you go up to higher floors. If you want to exchange an apartment on the twentieth floor for one on the second floor, you will not owe as much rent. However, if you want to move from the third floor to the twenty-fifth floor, your rent will increase. In an atom, too, the “cheapest” place for an electron to live is the lowest possible level, and energy is required to move to a higher level.

Here we have one of the situations where it is easier to think of electromagnetic radiation as particles (photons) rather than as waves. As electrons move from one level to another, they give off or absorb little packets of energy. When an electron moves to a higher level, it absorbs a photon of just the right energy (provided one is available). When it moves to a lower level, it emits a photon with the exact amount of energy it no longer needs in its “lower-cost living situation.”

The photon and wave perspectives must be equivalent: light is light, no matter how we look at it. Thus, each photon carries a certain amount of energy that is proportional to the frequency (f) of the wave it represents. The value of its energy (E) is given by the formula

$$E = hf$$

where the constant of proportionality, h, is called Planck’s constant.

The constant is named for Max Planck, the German physicist who was one of the originators of the quantum theory (Figure 3.2.4.5). If metric units are used (that is, if energy is measured in joules and frequency in hertz), then Planck’s constant has the value $h = 6.626 \times 10^{-34}$ joule-seconds (J-s). Higher-energy photons correspond to higher-frequency waves (which have a shorter wavelength); lower-energy photons are waves of lower frequency.

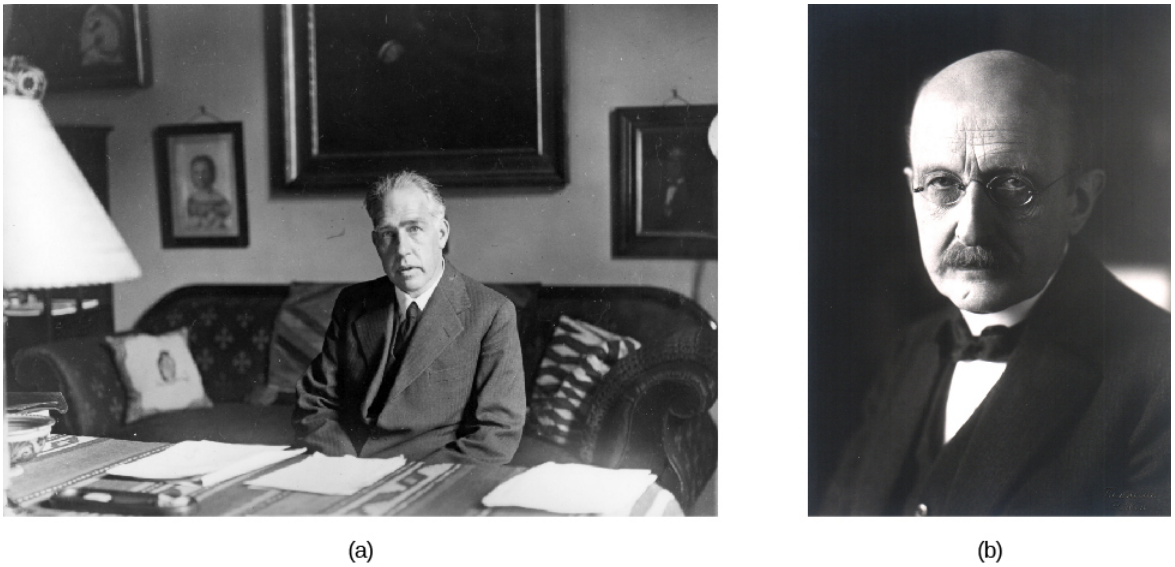

Figure 3.2.4.5 Niels Bohr (1885–1962) and Max Planck (1858–1947). (a) Bohr, shown at his desk in this 1935 photograph, and (b) Planck helped us understand the energy behavior of photons.

To take a specific example, consider a calcium atom inside the Sun’s atmosphere in which an electron jumps from a lower level to a higher level. To do this, it needs about 5 × 10^{-19} joules of energy, which it can conveniently obtain by absorbing a passing photon of that energy coming from deeper inside the Sun. This photon is equivalent to a wave of light whose frequency is about 7.5 × 10^{14} hertz and whose wavelength is about 3.9 × 10^{-7} meters (393 nanometers), in the deep violet part of the visible light spectrum. Although it may seem strange at first to switch from picturing light as a photon (or energy packet) to picturing it as a wave, such switching has become second nature to astronomers and can be a handy tool for doing calculations about spectra.

Example : The energy of a photon

Now that we know how to calculate the wavelength and frequency of a photon, we can use this information, along with Planck’s constant, to determine how much energy each photon carries. How much energy does a red photon of wavelength 630 nm have?

Solution

First, as we learned earlier, we can find the frequency of the photon:

$$f = \frac{c}{\lambda} = \frac{3 \times 10^{8} \text{ m/s}}{630 \times 10^{-9} \text{ m}} = 4.8 \times 10^{14} \text{ Hz}$$

Next, we can use Planck’s constant to determine the energy (remember that a Hz is the same as 1/s):

$$E = hf = \left(6.626 \times 10^{-34} \text{ J-s}\right)\left(4.8 \times 10^{14} (1/\text{s})\right) = 3.2 \times 10^{-19} \text{ J}$$

ise

What is the energy of a yellow photon with a frequency of 5.5×10^{14} Hz?

Answer

$$E = hf = \left(6.626 \times 10^{-34}\right)\left(5.5 \times 10^{14}\right) = 3.6 \times 10^{-19} \text{ J}$$

Key Concepts and Summary

Atoms consist of a nucleus containing one or more positively charged protons. All atoms except hydrogen can also contain one or more neutrons in the nucleus. Negatively charged electrons orbit the nucleus. The number of protons defines an element (hydrogen has one proton, helium has two, and so on) of the atom. Nuclei with the same number of protons but different numbers of neutrons are different isotopes of the same element. In the Bohr model of the atom, electrons on permitted orbits (or energy levels) don't give off any electromagnetic radiation. But when electrons go from lower levels to higher ones, they must absorb a photon of just the right energy, and when they go from higher levels to lower ones, they give off a photon of just the right energy. The energy of a photon is connected to the frequency of the electromagnetic wave it represents by Planck's formula, $E = hf$.

Glossary

energy level

a particular level, or amount, of energy possessed by an atom or ion above the energy it possesses in its least energetic state; also used to refer to the states of energy an electron can have in an atom

isotope

any of two or more forms of the same element whose atoms have the same number of protons but different numbers of neutrons

nucleus (of an atom)

the massive part of an atom, composed mostly of protons and neutrons, and about which the electrons revolve

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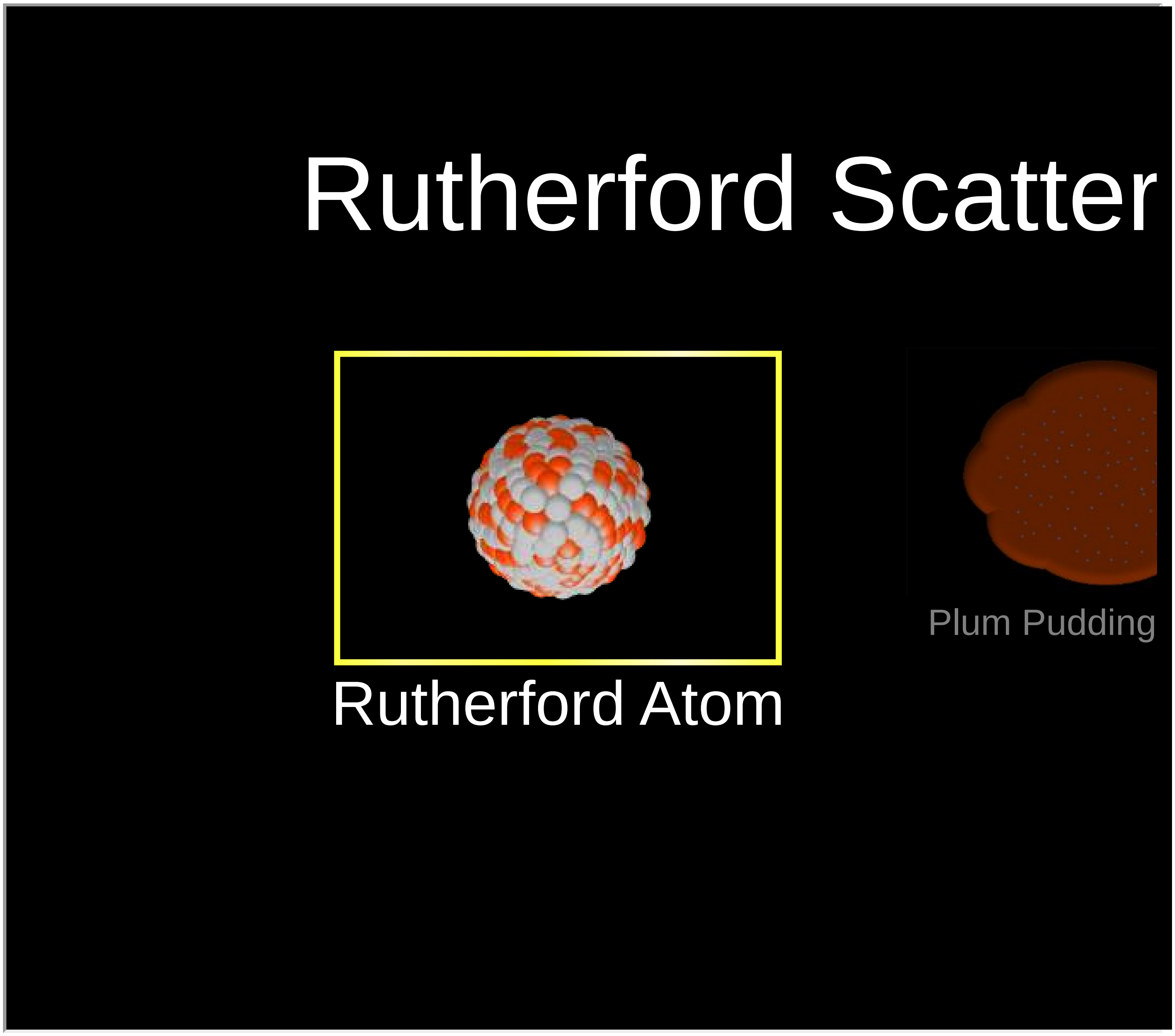
- **5.4: The Structure of the Atom** by OpenStax is licensed CC BY 4.0. Original source: https://openstax.org/details/books/astronomy.

3.2.5: PhET- Rutherford Scattering



Rutherford Scatter

Rutherford Atom

Plum Pudding

3.2.6: Subatomic Particles - Electrons, Protons, and Neutrons

Learning Objectives

- Know the basics of the experiments involving the discoveries of the three subatomic particles
- Memorize relative charge values and amu masses of the three subatomic particles.
- Describe the location of the proton, electron, and neutron inside the atomic model.

Identifying and Locating the Subatomic Particles

In 1897, the British physicist J. J. Thomson (1856–1940) proved that atoms were not the most basic form of matter. His experiments involved the use of cathode ray tubes (CRT). Under a vacuumed environment, these devices produce invisible rays that originate from a negative electrode (cathode) and continue to a positive electrode (anode). In these experiments, Thomson demonstrated that cathode rays could be deflected, or bent, by magnetic or electric fields, which indicated that cathode rays consist of charged particles . More importantly, by measuring the extent of the deflection of the cathode rays in magnetic or electric fields of various strengths, Thomson was able to calculate the mass-to-charge ratio of the particles. Because like charges repel each other and opposite charges attract, Thomson concluded that the particles had a net **negative charge**; these particles are now called **electrons**. Most relevant to the field of chemistry, Thomson found that the mass-to-charge ratio of cathode rays is independent of the nature of the metal electrodes or the gas, which suggested that electrons were fundamental components of all atoms.

Video 3.2.6.1: An instructor explains nature of a cathode ray tube and the discovery of the electron.

During the 1910's, experiments with x- rays led to this useful conclusion: the magnitude of the **positive charge** in the nucleus of every atom of a particular element is the same. In other words, all atoms of the same element have the same number of **protons**. Furthermore, different elements have a different number of protons in their nuclei, so the number of protons in the nucleus of an atom is characteristic of a particular element. This discovery was so important to our understanding of atoms that the number of protons in the nucleus of an atom is called the atomic number. These experiments were performed by Ernest Rutherford. He is credited with defining a central location of the atom which has now been named the nucleus.

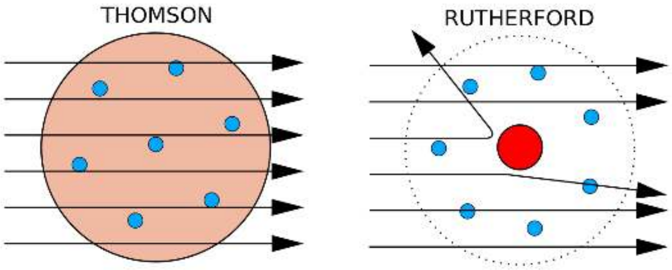


Figure 3.2.6.1: Expected results: alpha particles passing through the plum pudding model of the atom with negligible deflection. Bottom: Observed results: a small portion of the particles were deflected by the concentrated positive charge of the nucleus. (Public Domain).

Rutherford was aware that the protons were inside this region. One of Ernest Rutherford's colleagues, James Chadwick, performed and studied radioactive reactions of beryllium. Upon hitting a beryllium nucleus with alpha particles, a **neutron** particle was emitted. In 1932, James Chadwick announced the existence of a third subatomic particle, the neutron. This particle has a mass of 1 atomic mass unit, but **does not have any charge**. After this discovery, nuclear science and technology started immediately.

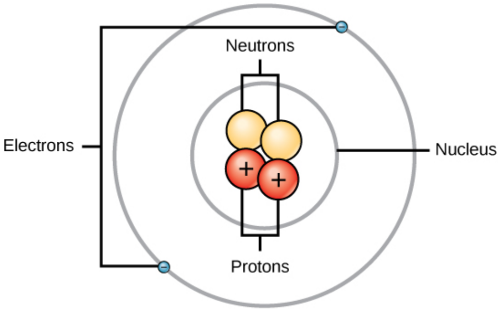


Figure 3.2.6.2: Elements, such as helium, depicted here, are made up of atoms. Atoms are made up of protons and neutrons located within the nucleus, with electrons in orbitals surrounding the nucleus.

Masses for the three subatomic particles can be expressed in amu (**atomic mass units**) or grams. For simplicity, we will use the amu unit for the three subatomics. Both neutrons and protons are assigned as having masses of 1 amu each. In contrast, the electron has a negligible mass of .0005 amu. This subatomic's mass is not represented in the decimal mass that is displayed on the periodic table. Only protons and neutrons contribute to an atom's mass value.

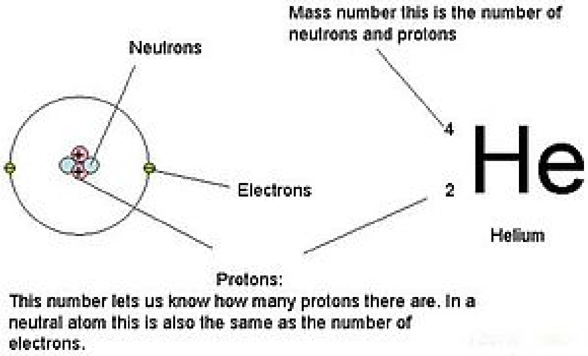


Figure 3.2.6.3: An explanation of the superscripts and subscripts seen in atomic number notation. (Copyright; en:User:Materialscientist)

For atoms (neutral species), the number of electrons is equivalent to the number of protons. As a result, all atoms have an overall charge of zero. For discussion of ions (atoms that gain or lose electrons), look to the latter part of this chapter. Atoms that differ in neutron and mass values from the table are called isotopes. These particular species can be seen in the following section of the text.

Table 3.2.6.1: Properties of Subatomic Particles

Subatomic particle	Symbol	Relative charge	Location	Amu mass
Proton	p	+1	inside the nucleus	1
Neutron	n	0	inside the nucleus	1
Electron	e^-	-1	outside the nucleus	0.0005 (~0)

✓ Example 3.2.6.1

Which is true?

a. The nucleus contains protons and electrons.
b. The mass of an electron is included in the mass of the atom
c. For an atom, the number of protons = number of electrons.

Solution

a. False, the nucleus contains the protons and the neutrons. Electrons are outside the core.
b. False, electrons do not contribute to the mass number of an atom.
c. True, all atoms are neutral and must have the same number of protons as electrons.

3.2.7: Atomic Mass and Atomic Number

Learning Outcomes

- Define atomic and mass numbers.
- Determine the number of protons, neutrons, and electrons in an atom.
- Identify the charge and relative mass of subatomic particles.
- Label the location of subatomic particles in the atom.
- Determine the mass of an atom based on its subatomic particles.
- Write A/Z and symbol-mass format for an atom.

Atoms are the fundamental building blocks of all matter and are composed of protons, neutrons, and electrons. Because atoms are electrically neutral, the number of positively charged protons must be equal to the number of negatively charged electrons. Since neutrons do not affect the charge, the number of neutrons is not dependent on the number of protons and will vary even among atoms of the same element.

Atomic Number

The **atomic number (represented by the letter Z)** *of an element is the number of protons in the nucleus of each atom of that element*. An atom can be classified as a particular element based solely on its atomic number. For example, any atom with an atomic number of 8 (its nucleus contains 8 protons) is an oxygen atom, and any atom with a different number of protons would be a different element. The periodic table (see figure below) displays all of the known elements and is arranged in order of increasing atomic number. In this table, an element's atomic number is indicated above the elemental symbol. Hydrogen, at the upper left of the table, has an atomic number of 1. Every hydrogen atom has one proton in its nucleus. Next on the table is helium, whose atoms have two protons in the nucleus. Lithium atoms have three protons, beryllium atoms have four, and so on.

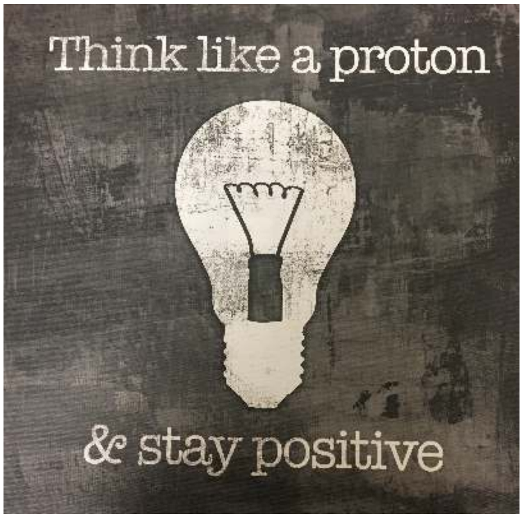
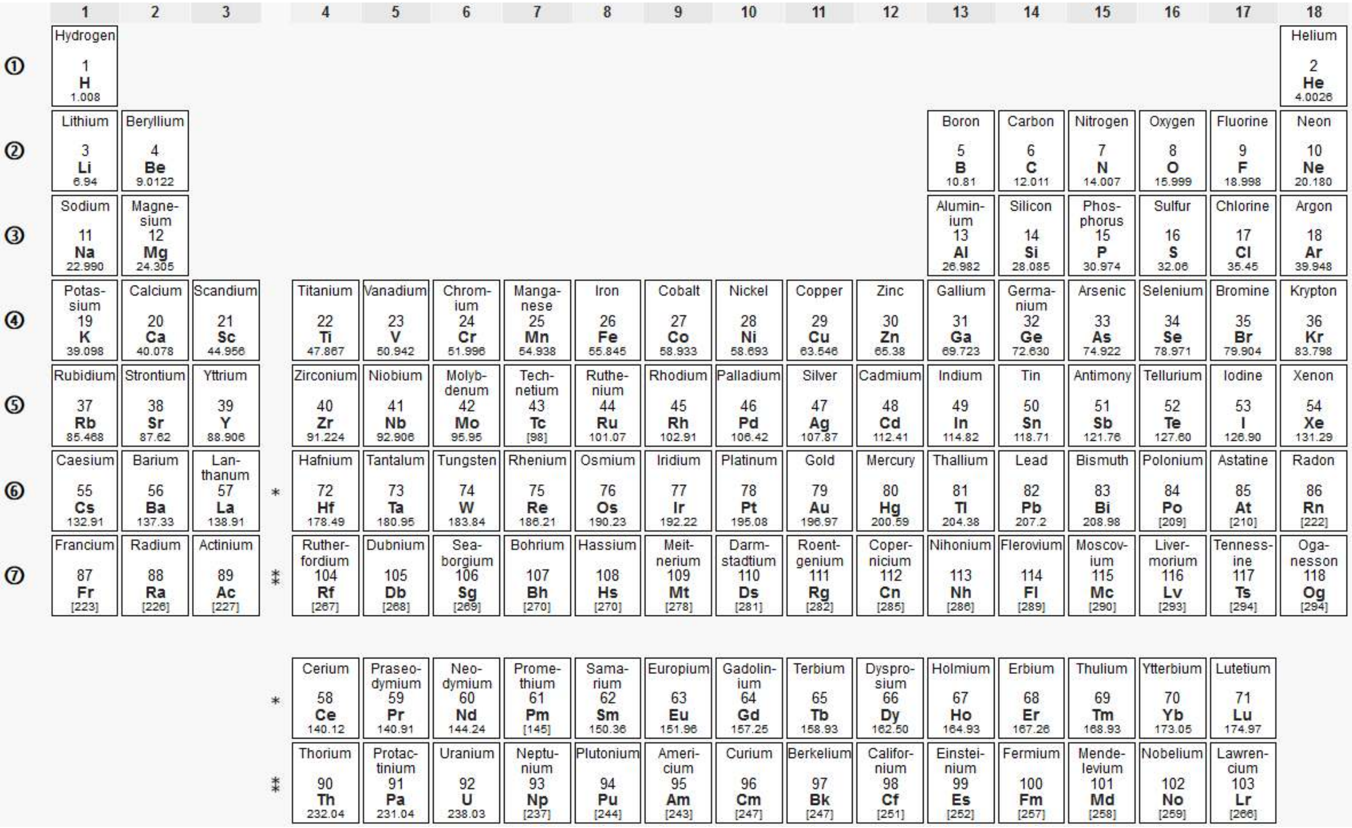


Figure 3.2.7.1: The social security number subatomic-the proton.

Since atoms are neutral, the number of electrons in an atom is equal to the number of protons. Hydrogen atoms all have one electron occupying the space outside of the nucleus. Helium, with two protons, will have two electrons. In the chemical classroom, the proton count will always be equivalent to an atom's atomic number. This value will not change unless the nucleus decays or is bombarded (nuclear physics).



	1	2	3		4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
①	Hydrogen 1 **H** 1.008																		Helium 2 **He** 4.0026
②	Lithium 3 **Li** 6.94	Beryllium 4 **Be** 9.0122												Boron 5 **B** 10.81	Carbon 6 **C** 12.011	Nitrogen 7 **N** 14.007	Oxygen 8 **O** 15.999	Fluorine 9 **F** 18.998	Neon 10 **Ne** 20.180
③	Sodium 11 **Na** 22.990	Magnesium 12 **Mg** 24.305												Aluminium 13 **Al** 26.982	Silicon 14 **Si** 28.085	Phosphorus 15 **P** 30.974	Sulfur 16 **S** 32.06	Chlorine 17 **Cl** 35.45	Argon 18 **Ar** 39.948
④	Potassium 19 **K** 39.098	Calcium 20 **Ca** 40.078	Scandium 21 **Sc** 44.956		Titanium 22 **Ti** 47.867	Vanadium 23 **V** 50.942	Chromium 24 **Cr** 51.996	Manganese 25 **Mn** 54.938	Iron 26 **Fe** 55.845	Cobalt 27 **Co** 58.933	Nickel 28 **Ni** 58.693	Copper 29 **Cu** 63.546	Zinc 30 **Zn** 65.38	Gallium 31 **Ga** 69.723	Germanium 32 **Ge** 72.630	Arsenic 33 **As** 74.922	Selenium 34 **Se** 78.971	Bromine 35 **Br** 79.904	Krypton 36 **Kr** 83.798
⑤	Rubidium 37 **Rb** 85.468	Strontium 38 **Sr** 87.62	Yttrium 39 **Y** 88.906		Zirconium 40 **Zr** 91.224	Niobium 41 **Nb** 92.906	Molybdenum 42 **Mo** 95.95	Technetium 43 **Tc** [98]	Ruthenium 44 **Ru** 101.07	Rhodium 45 **Rh** 102.91	Palladium 46 **Pd** 106.42	Silver 47 **Ag** 107.87	Cadmium 48 **Cd** 112.41	Indium 49 **In** 114.82	Tin 50 **Sn** 118.71	Antimony 51 **Sb** 121.76	Tellurium 52 **Te** 127.60	Iodine 53 **I** 126.90	Xenon 54 **Xe** 131.29
⑥	Caesium 55 **Cs** 132.91	Barium 56 **Ba** 137.33	Lanthanum 57 **La** 138.91	*	Hafnium 72 **Hf** 178.49	Tantalum 73 **Ta** 180.95	Tungsten 74 **W** 183.84	Rhenium 75 **Re** 186.21	Osmium 76 **Os** 190.23	Iridium 77 **Ir** 192.22	Platinum 78 **Pt** 195.08	Gold 79 **Au** 196.97	Mercury 80 **Hg** 200.59	Thallium 81 **Tl** 204.38	Lead 82 **Pb** 207.2	Bismuth 83 **Bi** 208.98	Polonium 84 **Po** [209]	Astatine 85 **At** [210]	Radon 86 **Rn** [222]
⑦	Francium 87 **Fr** [223]	Radium 88 **Ra** [226]	Actinium 89 **Ac** [227]	⁑	Rutherfordium 104 **Rf** [267]	Dubnium 105 **Db** [268]	Seaborgium 106 **Sg** [269]	Bohrium 107 **Bh** [270]	Hassium 108 **Hs** [270]	Meitnerium 109 **Mt** [278]	Darmstadtium 110 **Ds** [281]	Roentgenium 111 **Rg** [282]	Copernicium 112 **Cn** [285]	Nihonium 113 **Nh** [286]	Flerovium 114 **Fl** [289]	Moscovium 115 **Mc** [290]	Livermorium 116 **Lv** [293]	Tennessine 117 **Ts** [294]	Oganesson 118 **Og** [294]
				*	Cerium 58 **Ce** 140.12	Praseodymium 59 **Pr** 140.91	Neodymium 60 **Nd** 144.24	Promethium 61 **Pm** [145]	Samarium 62 **Sm** 150.36	Europium 63 **Eu** 151.96	Gadolinium 64 **Gd** 157.25	Terbium 65 **Tb** 158.93	Dysprosium 66 **Dy** 162.50	Holmium 67 **Ho** 164.93	Erbium 68 **Er** 167.26	Thulium 69 **Tm** 168.93	Ytterbium 70 **Yb** 173.05	Lutetium 71 **Lu** 174.97	
				⁑	Thorium 90 **Th** 232.04	Protactinium 91 **Pa** 231.04	Uranium 92 **U** 238.03	Neptunium 93 **Np** [237]	Plutonium 94 **Pu** [244]	Americium 95 **Am** [243]	Curium 96 **Cm** [247]	Berkelium 97 **Bk** [247]	Californium 98 **Cf** [251]	Einsteinium 99 **Es** [252]	Fermium 100 **Fm** [257]	Mendelevium 101 **Md** [258]	Nobelium 102 **No** [259]	Lawrencium 103 **Lr** [266]	

Figure 3.2.7.3: The periodic table of the elements. (CC BY-SA 4.0 International; DePiep via Wikipedia).

Mass Number

Experimental data showed that the vast majority of the mass of an atom is concentrated in its nucleus, which is composed of protons and neutrons. The **mass number (represented by the letter A)** *is defined as the total number of protons and neutrons in an atom*. Consider the table below, which shows data from the first six elements of the periodic table.

Table 3.2.7.1: Atoms of the First Six Elements

Name	Symbol	Atomic Number (Z)	Protons	Neutrons	Electrons	Mass Number (A) (rounded to two decimals)
hydrogen	H	1	1	0	1	1.01
helium	He	2	2	2	2	4.00
lithium	Li	3	3	4	3	6.94
beryllium	Be	4	4	5	4	9.01
boron	B	5	5	6	5	10.18
carbon	C	6	6	6	6	12.01

Consider the element helium. Its atomic number is 2, so it has two protons in its nucleus. Its nucleus also contains two neutrons. Since $2 + 2 = 4$, we know that the mass number of the helium atom is 4. Finally, the helium atom also contains two electrons, since the number of electrons must equal the number of protons. This example may lead you to believe that atoms have the same number of protons and neutrons, but a further examination of the table above will show that this is not the case. Lithium, for example, has three protons and four neutrons, giving it a mass number of 7.

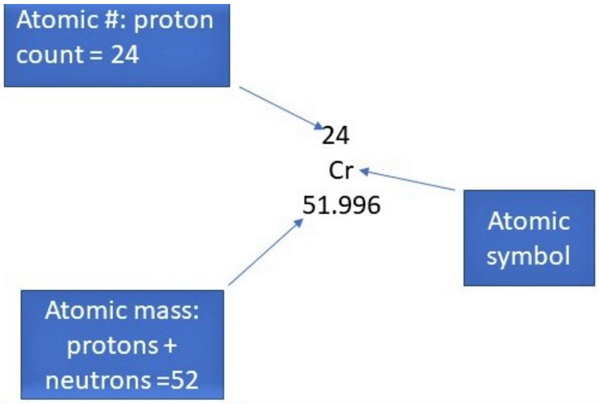


Figure 3.2.7.4: A/Z format and finding subatomics for elementChromium (Copyright; Elizabeth R. Gordon)

Knowing the mass number and the atomic number of an atom allows you to determine the number of neutrons present in that atom by subtraction.

$$\text{Number of neutrons} = \text{ rounded mass number} - \text{atomic number}$$

Atoms of the element chromium (Cr) have an atomic number of 24 and a mass number of 52. How many neutrons are in the nucleus of a chromium atom? To determine this, you would subtract as shown:

$$52 - 24 = 28 \text{ neutrons in a chromium atom}$$

The composition of any atom can be illustrated with a shorthand notation called A/Z format. Both the atomic number and mass are written to the left of the chemical symbol. The "A" value is written as a superscript while the "Z" value is written as a subscript. For an example of this notation, look to the chromium atom shown below:

$$^{52}_{24}\text{Cr}$$

Another way to refer to a specific atom is to write the mass number of the atom after the name, separated by a hyphen. Symbol-mass format for the above atom would be written as Cr-52. In this notation, the atomic number is not included. You will need to refer to a periodic table for proton values.

✓ Example 3.2.7.1

Calculate each of the three subatomic particles and give specific group or period names for each atom.

a. mercury
b. platinum
c. bromine

Solutions

a. Hg (transition metal)- has 80 electrons, 80 protons, and 121 neutrons
b. Pt (transition metal)- has 78 electrons, 78 protons, and 117 neutrons
c. Br (halogen)- has 35 electrons, 35 protons, and 45 neutrons

✓ Example 3.2.7.2

Write both A/Z and symbol-mass formats for the atoms in Example $3.2.7.1$.

Solutions

a. $^{201}_{80}\text{Hg}$ and Hg-201
b. $^{195}_{78}\text{Pt}$ and Pt-195
c. $^{80}_{35}\text{Br}$ and Br-80

✓ Example 3.2.7.3

Identify the elements based on the statements below.

a. Which element has 25 protons?
b. Which element has 0 neutrons?
c. Which element has 83 electrons?

Solutions

a. manganese

b. hydrogen

c. bismuth

Need More Practice?

- Turn to section 3.E of this OER and answer questions #1-#2, #4, and #8.

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3.2.8: Isotopes

Learning Objectives

- Calculate subatomic particles (protons, neutrons, and electrons) for any element by looking at the periodic table.
- Calculate subatomic particles (protons, neutrons, and electrons) for any element by looking at symbol-mass and A/Z format.
- Understand how isotopes differ in particles and mass.
- Identify the most abundant isotope when given specific values.
- Calculate the atomic mass of an element from the masses and relative percentages of the isotopes of the element.
- Define LEU/HEU with percentages and applications.

Isotopes are atoms of the same element that contain different numbers of neutrons. For these species, the number of electrons and protons remains constant. This difference in neutron amount affects the atomic mass (**A**) but not the atomic number (**Z**). In a chemical laboratory, isotopes of an element appear and react the same. For this reason, it is difficult to distinguish between an atom's isotopes. In contrast, nuclear scientists can identify and separate different types of atomic nuclei (Figure $3.2.8.1$). The technology required for this process is more sophisticated than what could be found in a typical chemical laboratory.

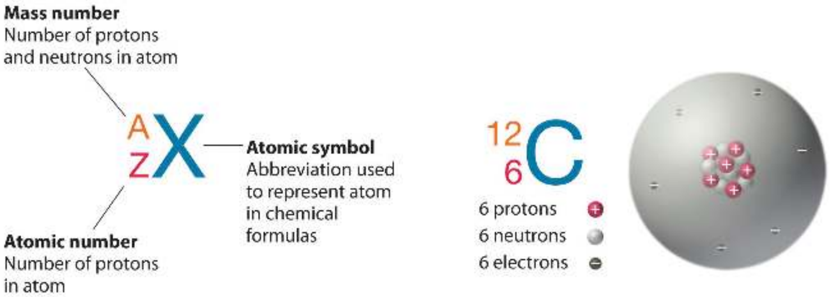

Figure $3.2.8.1$: Unlike protons, the number of neutrons is not absolutely fixed for most elements. Atoms that have the same number of protons, and hence the same atomic number, but different numbers of neutrons are called isotopes. All isotopes of an element have the same number of protons and electrons, which means they exhibit the same chemistry. The isotopes of an element differ only in their atomic mass, which is given by the mass number (A), the sum of the numbers of protons and neutrons. (CC BY-NS-SA; anonymous by request)

The element carbon (C) has an atomic number of 6, which means that all neutral carbon atoms contain 6 protons and 6 electrons. In a typical sample of carbon-containing material, 98.89% of the carbon atoms also contain 6 neutrons, so each has a mass number of 12. An isotope of any element can be uniquely represented as $^{A}_{Z}X$where X is the atomic symbol of the element. The isotope of carbon that has 6 neutrons is therefore $^{12}_{6}C$ The subscript indicating the atomic number is actually redundant because the atomic symbol already uniquely specifies *Z*. Consequently, it is more often written as ^{12}C, which is read as "carbon-12." Nevertheless, the value of Z is commonly included in the notation for *nuclear* reactions because these reactions involve changes in Z.

Most elements on the periodic table have at least two stable isotopes. For example, in addition to ^{12}C, a typical sample of carbon contains 1.11% $^{13}_{6}C$, with 7 neutrons and 6 protons, and a trace of $^{14}_{6}C$, with 8 neutrons and 6 protons. The nucleus of $^{14}_{6}C$ is not stable, however, but undergoes a slow radioactive decay that is the basis of the carbon-14 dating technique used in archaeology. Many elements other than carbon have more than one stable isotope; tin, for example, has 10 isotopes. There are about twenty elements that exist in only one isotopic form (sodium and fluorine are examples of these). Most scientists cannot tell you how many isotopic forms exist unless they consult an isotopic table.

An important series of isotopes is found with hydrogen atoms. Most hydrogen atoms have a nucleus with only a single proton. About 1 in 10,000 hydrogen nuclei, however, also has a neutron; this particular isotope is called *deuterium*. An extremely rare hydrogen isotope, *tritium*, has 1 proton and 2 neutrons in its nucleus. Figure $3.2.8.2$compares the three isotopes of hydrogen.

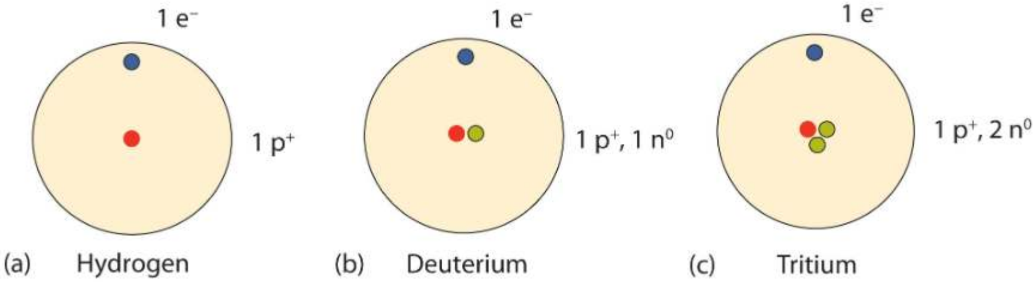

Figure 3.2.8.2: a). A small amount of hydrogen exists as the isotope deuterium, which has one proton and one neutron in its nucleus (b). A tiny amount of the hydrogen isotope tritium, with one proton and two neutrons in its nucleus, also exists on Earth (c). The nuclei and electrons are proportionately much smaller than depicted here.

There are currently over 3,500 isotopes known for all the elements. When scientists discuss individual isotopes, they need an efficient way to specify the number of neutrons in any particular nucleus. A/Z and symbol-mass formats (refer to Section 3.4) can be used to display periodic table information. When viewing either of these two notations, isotopic differences can be obtained.

> *The discovery of isotopes required a minor change in Dalton's atomic theory. Dalton thought that all atoms of the same element were exactly the same.*

Look at the **A/Z formats** for the three isotopes of hydrogen in Table $3.2.8.1$ Note how the atomic number (bottom value) remains the same while the atomic masses (top number) are varied. All isotopes of a particular element will vary in neutrons and mass. This variance in mass will be visible in the **symbol-mass format** of same isotopes as well.

Table 3.2.8.1

Common Name	A/Z formats	symbol-mass format	Expanded Name
Hydrogen	${}^{1}_{1}\mathrm{H}$	H-1	hydrogen-1
Deuterium	${}^{2}_{1}\mathrm{H}$	H-2	hydrogen-2
Tritium	${}^{3}_{1}\mathrm{H}$	H-3	hydrogen 3

Using a periodic table, A/Z or symbol-mass formats can be utilized to determine the number of subatomic particles (protons, neutrons, and electrons) contained inside an isotope. When given either format, these mass values should be used to calculate the number of neutrons in the nucleus.

✓ Exercise $3.2.8.1$

How many neutrons are in each atom?

a. ${}^{36}_{17}\mathrm{Cl}$
b. ${}^{58}_{26}\mathrm{Fe}$
c. C-14

Solution

a. In ${}^{36}_{17}\mathrm{Cl}$ there are 19 neutrons in this nucleus. To find this value, subtract A-Z or $36 - 17$.
b. In ${}^{58}_{26}\mathrm{Fe}$ there are 32 neutrons in this nucleus. Again, subtract A-Z or 58-26.
c. In this example, the C-14 represents symbol-mass format. Once the atomic number is located (look at periodic table), subtract 14-6. The final answer will be 8 neutrons.

? Exercise $3.2.8.2$

For the species below, translate the A/Z format to symbol-mass. Then, calculate the number of subatomics for each atom.

a. ${}^{197}_{79}\mathrm{Au}$
b. ${}^{23}_{11}\mathrm{Na}$
c. ${}^{239}_{94}\mathrm{Pu}$

Answer a

Au-197 contains 79 protons, 79 electrons, and 118 neutrons.

Answer b

Na-23 contains 11 protons, 11 electrons, and 12 neutrons.

Answer c

Pu-239 contains 94 protons, 94 electrons, and 145 neutrons.

Calculating Atomic Mass

The **atomic mass** of an element is the weighted mass of all the naturally presented isotopes. On the periodic table, it is assumed that this mass has units of amu (atomic mass unit) which can be abbreviated by using the letter u. To determine the most abundant isotopic form of an element, compare given isotopes to the weighted average on the periodic table. For example, the three hydrogen isotopes (shown above) are H-1, H-2, and H-3. The atomic mass or weighted average of hydrogen is around 1.008 amu (look again at the periodic table). Of the three hydrogen isotopes, H-1 is closest in mass to the weighted average; therefore, it is the most abundant. The other two isotopes of hydrogen are rare but are very exciting in the world of nuclear science.

✓ Example $3.2.8.2$

Identify the true statements:

a. Al-27 is more abundant than Al-25.
b. An appropriate isotope of bromine could be Br-35.
c. A chemist could easily distinguish between Cs-132 and Cs-133 by noting chemical and physical properties.
d. Most scientists know that calcium has 24 isotopes.

Answer a

This statement is true. Aluminum with a mass of 27 is closest to the mass on the periodic table. It would be more abundant than Al-25.

Answer b

This statement is false. An appropriate mass number of an isotope of bromine would be in around 80 amu (atomic mass units), not 35 (which is the atomic number).

Answer c

Chemists cannot distinguish between isotopes by looking at various properties. Most isotopes have similar solubilities, densities, and colors.

Answer d

Unless a scientist works heavily with a particular atom, he or she is not aware of the many forms that cannot exist. They would need a reference guide to know how many natural and artificial isotopes exist for a particular element.

You can calculate the atomic mass (or average mass) of an element provided you know the **relative abundances** (the fraction of an element that is a given isotope), the element's naturally occurring isotopes, and the masses of those different isotopes. We can calculate this by the following equation:

$$\text{Atomic mass} = (\%_1)(\text{mass}_1) + (\%_2)(\text{mass}_2) + \cdots \tag{3.2.8.1}$$

To confirm your answer, compare the calculated value to the weighted mass displayed on the periodic table.

> *An element's atomic mass can be calculated provided the relative abundances of the element's naturally occurring isotopes and the masses of those isotopes are known. If all the abundances are not provided, it is safe to assume that all numbers should add up to 100%.*

Example 3.2.8.3: Atomic mass of Boron

Boron has two naturally occurring isotopes. In a sample of boron, 20% of the atoms are B-10, which is an isotope of boron with 5 neutrons and mass of $10\ \text{amu}$. The other 80% of the atoms are B-11, which is an isotope of boron with 6 neutrons and a mass of $11\ \text{amu}$. What is the atomic mass of boron?

Solution

Boron has two isotopes. We will use the Equation 3.2.8.1:

$$\text{Atomic mass} = (\%_1)\,(\text{mass}_1) + (\%_2)\,(\text{mass}_2) + \cdots$$

Substitute these into the equation, and we get:

$$\begin{aligned}\text{Atomic mass } &= (0.20)\,(10) + (0.80)\,(11) \\ &= 10.8\ \text{amu}\end{aligned}$$

The mass of an average boron atom, and thus boron's atomic mass, is $10.8\ \text{amu}$

Example 3.2.8.4: Atomic mass of Neon

Neon has three naturally occurring isotopes. In a sample of neon, 90.92% of the atoms are Ne-20, which is an isotope of neon with 10 neutrons and a mass of $19.99\ \text{amu}$ Another 0.3% of the atoms are Ne-21, which is an isotope of neon with 11 neutrons and a mass of $20.99\ \text{amu}$ The final 8.85% of the atoms are Ne-22, which is an isotope of neon with 12 neutrons and a mass of $21.99\ \text{amu}$ What is the atomic mass of neon?

Solution

Neon has three isotopes. We will use the equation:

$$\text{Atomic mass} = (\%_1)\,(\text{mass}_1) + (\%_2)\,(\text{mass}_2) + \cdots$$

Substitute these into the equation, and we get:

$$\begin{aligned}\text{Atomic mass } &= (0.9092)\,(19.99) + (0.003)\,(20.99) + (0.0885)\,(21.99) \\ &= 20.18\ \text{amu}\end{aligned}$$

The mass of an average neon atom is $20.18\ \text{amu}$

Applications of Isotopes

During the Manhattan project, the majority of federal funding dedicated the separation of uranium isotopes. The two most common isotopes of uranium are U-238 and U-235. About 99.3% of uranium is of the U-238 variety, this form is not fissionable and will not work in a nuclear weapon or reaction. The remaining .7% is U-235 which is fissionable but first had to be separated from U-238. This separation process is called *enrichment*. During World War II, a nuclear facility was built in Oak Ridge, Tennessee to accomplish this project. At the time, the enrichment process only produced enough U-235 for one nuclear weapon. This fuel was placed inside the smaller of the two atomic bombs (Little Boy) dropped over Japan.

Figure 3.2.8.3: A billet of highly enriched uranium that was recovered from scrap processed at the Y-12 National Security Complex Plant. Original and unrotated.

Uranium is a natural element that can be found in several different countries. Countries that do not have natural uranium supplies would need to obtain it from one of the countries below. Most nuclear reactors that provide energy rely on U-235 as a source of fuel. Fortunately, reactors only need 2-5% U-235 for the production of megawatts or even gigawatts of power. If the purification process exceeds this level, then it is likely a country is focusing on making nuclear weapons. For example, Manhattan Project scientists enriched U-235 up to 90% in order to produce the Little Boy weapon.

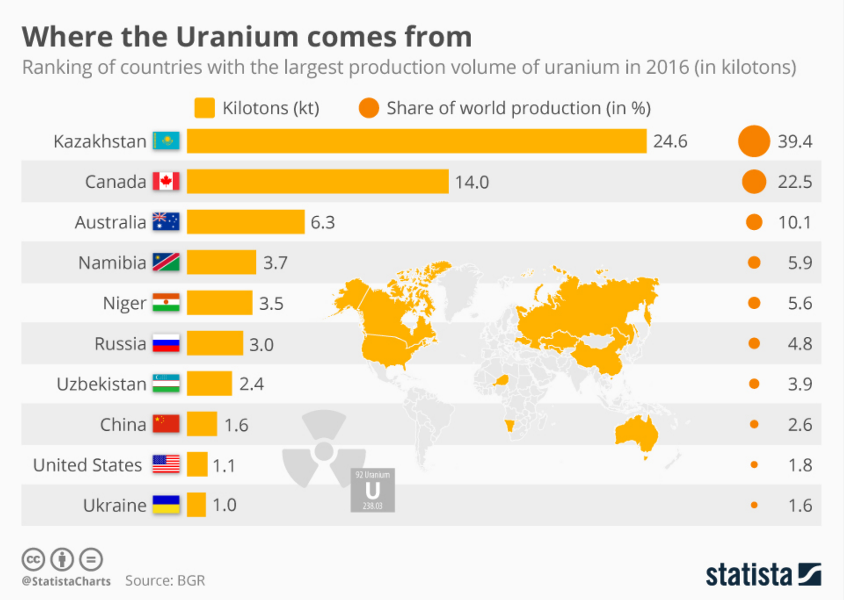

Figure 3.2.8.4: This infographic shows a ranking of countries with the biggest production volume of uranium in 2016 (in kilotons). (CC BY-ND; statista).

Abbreviations like HEU (highly enriched uranium) and LEU (low-enriched uranium) are used frequently by nuclear scientists and groups. HEU is defined as being over 20% pure U-235 and would not be used in most commercial nuclear reactors. This type of material is used to fuel larger submarines and aircraft carriers. If the purification of U-235 reaches 90%, then the HEU is further classified as being weapons-grade material. This type of U-235 could be used to make a nuclear weapon (fission or even fusion-based). As for LEU, its U-235 level would be below this 20% mark. LEU would be used for commercial nuclear reactors and smaller, nuclear-powered submarines. LEU is not pure enough to be used in a conventional nuclear weapon but could be used in a dirty bomb. This type of weapon uses conventional explosives like dynamite to spread nuclear material. Unlike a nuclear weapon, dirty bombs are not powerful enough to affect large groups of buildings or people. Unfortunately, the spread of nuclear material would cause massive chaos for a community and would result in casualties.

Need More Practice?

- Turn to Section 3.E of this OER and work problems #3, #6, #7, and #10.

Contributors and Attributions

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3.2.9: The Importance of Ions to a Chemist

Learning Objectives

- Identify how a charge affects the electron count of an element.
- Apply a provided charge when calculating the number of subatomic particles.
- Realize that cations are atoms that lose electrons resulting in an overall positive charge.
- Realize that anions are atoms that gain electrons resulting in an overall negative charge.
- Locate regions of the periodic table where cations and anions tend to form.
- Know that chemical energy corresponds to electrons, while nuclear energy results from changing the nucleus.
- Compare and contrast chemical and nuclear energy.

Chemists appreciate isotopes and use them in basic and applied research. However, they are more concerned with the movement of electrons. When an atom gains or loses electrons, it becomes a charged species or an ion. When this occurs, the nucleus is not altered. For atoms that lose electrons, an overall positive charge will result (**#protons** > **#electrons**). Atoms that form these types of ions are called cations. Metal atoms (located on the left side of the periodic table) always lose electrons to become cations.

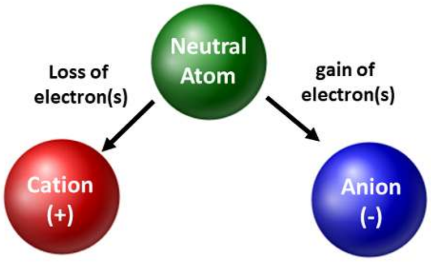
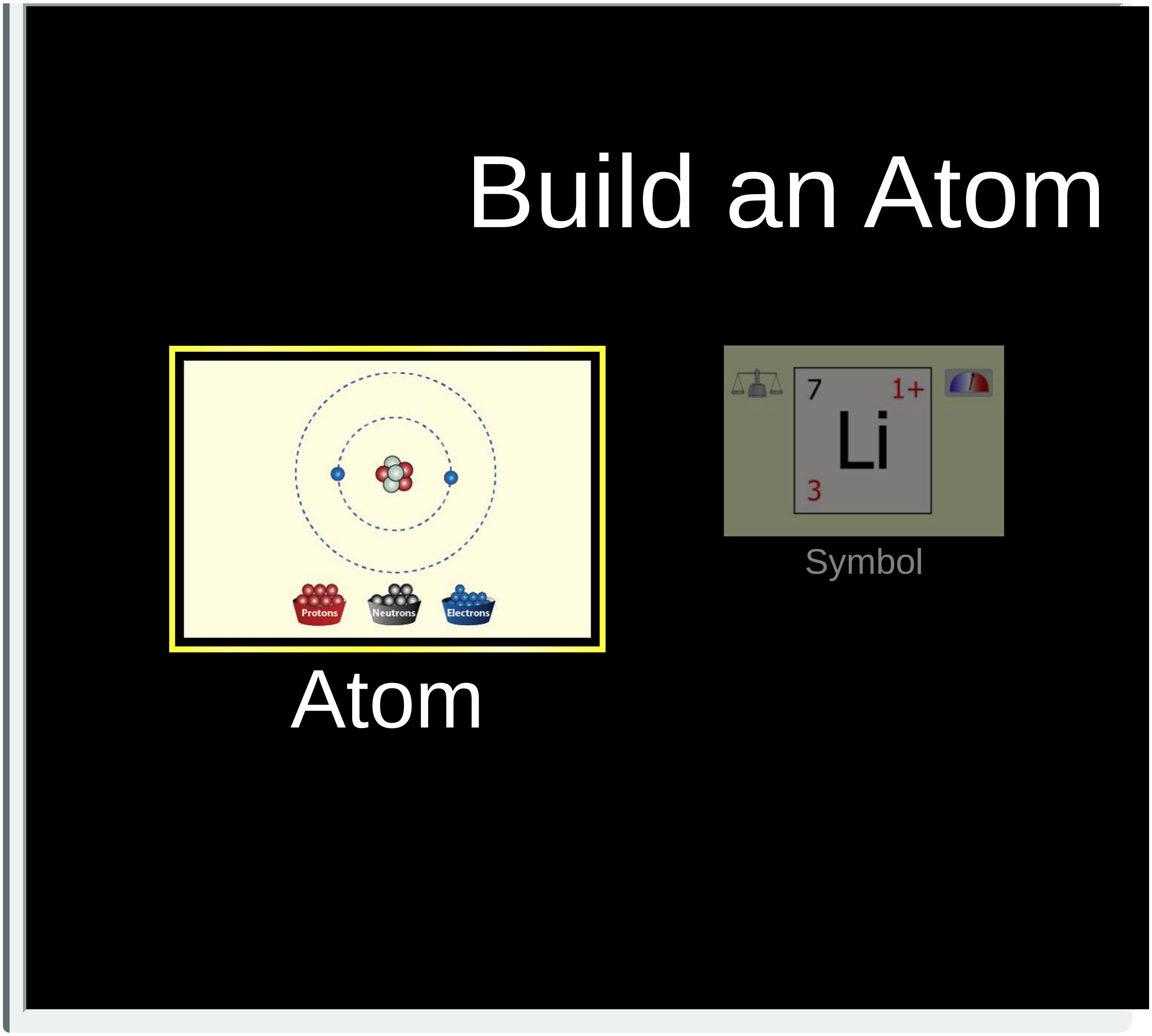


Figure 3.2.9.1: Neutral atoms become ions by either losing an electron (to form positively charged cations) or gaining electrons (to form negatively charged anions). (Public Domain; Delmar Larsen)

Unlike metal atoms, nonmetals will gain electrons to become anions. These types of ions have an overall negative charge (**#electrons** > **#protons**). With the exception of the noble gases, all atoms on the periodic table will lose or gain electrons to achieve electronic stability. Different types of bonding occur when atoms lose, gain, or share electrons. These types of atomic connections will be further discussed in chapter 4 of your textbook.

Interactive: Building an Atom

Build an atom out of protons, neutrons, and electrons, and see how the element, charge, and mass change. Then play a game to test your ideas!



Build an Atom

Atom

Symbol

At this point, you should be able to calculate all subatomic particles when given a specific ion charge. In addition, you should master classifications of ions (recognizing cations are positive and anions are negative). Memorizing charges of specific ions will be required when working on chapter 4 in the textbook. **For now, ion charges will be provided and should not be memorized**. Note and appreciate the ion charges for main group elements in the periodic table below. Lastly, chemists do not write a number for charges involving the number one (1). For these species, you will only see a + or - sign. Charges of higher magnitude will always have a number associated with the elemental symbol.

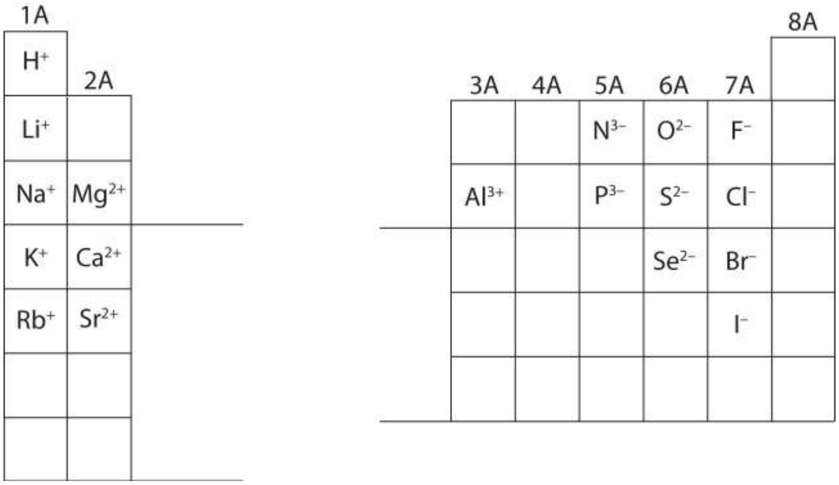


Figure 3.2.9.2: Periodic Table with ion charges of common main group elements. The charge that an atom acquires when it becomes an ion is related to the structure of the periodic table. Within a group (family) of elements, atoms form ions of a certain charge. Please note that hydrogen can adopt a + or - charge. When the word hydride is used, this means that hydrogen has a - charge.

✓ Example 3.2.9.1: IOnic Classification

Calculate the subatomic particles for the species below using the information in Figure 3.2.9.2 Label each as being an atom, cation, or anion. Refer to the periodic table for masses, atomic numbers, and specific ion charges.

a. Aluminum ion
b. Zirconium atom
c. Sulfur ion

Solutions

a. There are 13 protons, 14 neutrons, and 10 electrons in the aluminum ion. This ion is positively charged which means it has lost electrons and forms a cation.
b. There are 40 protons, 51 neutrons, and 40 electrons in the zirconium atom. This is an atom which has no overall charge.
c. There are 16 protons, 16 neutrons, and 18 electrons in the sulfur ion. This ion is negatively charged which means it has gained electrons and forms and anion.

✓ Example 3.2.9.2: A/Z and symbol mass notations

Write A/Z and symbol mass notations for the ions/atom in the previous question.

Solutions

a. $^{27}_{13}\text{Al}^{3+}$ and Al-27
b. $^{91}_{40}\text{Zr}$ and Zr-91
c. $^{32}_{16}\text{S}^{2-}$ and S-32

If an atom gains or loses electrons, this should be represented on the A/Z format. The symbol-mass format does not display ion charge. Remember, neutrons and protons are not affected by changes of electrons.

Need More Practice?

- Turn to Section 3.E of this OER and work problems #5 and #9.

Contributors

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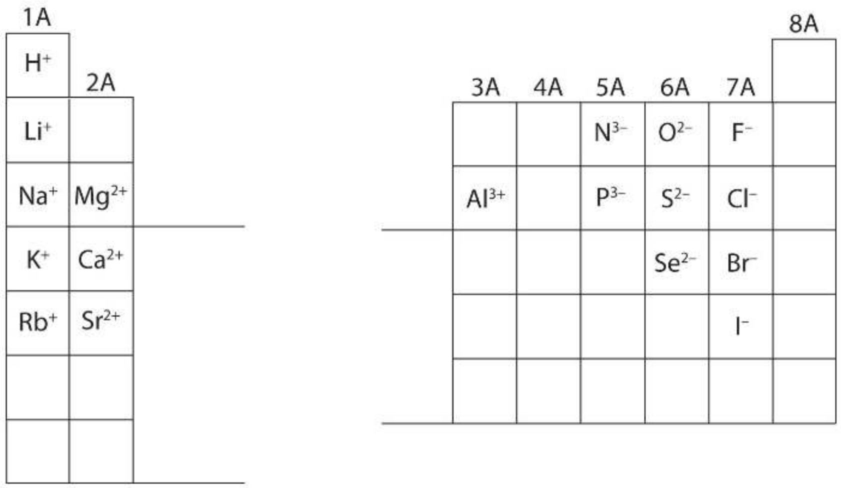
3.2.E: Atoms and the Periodic Table (Exercises)

Concept Review Exercises

1. Why is the atomic number so important to the identity of an atom?
2. What is the relationship between the number of protons and the number of electrons in an atom?
3. How do isotopes of an element differ from each other?
4. What is the mass number of an element?
5. Calculate the subatomic particles for
    a. Zn-67,
    b. radium,
    c. the aluminum cation,
    d. Mg-23, and
    e. the phosphorus anion.
6. Which element(s) has/have 32 neutrons: germanium, cobalt, or Mn-57?
7. Which natural isotope of platinum is most abundant?
8. Which element has 19 electrons?
9. Write A/Z format for the species in question #5.
10. Calculate the weighted average of nitrogen. N-14 has an exact mass of 14.003074u and is 99.63% abundant. N-15 has a mass of 15.000108u.

Solutions

1. The atomic number defines the identity of an element.
2. In an electrically neutral atom, the number of protons equals the number of electrons.
3. Isotopes have different numbers of neutrons in their nuclei.
4. The mass number is the sum of the numbers of protons and neutrons in the nucleus of an atom.
5. For ion charges, refer to the table for assistance:

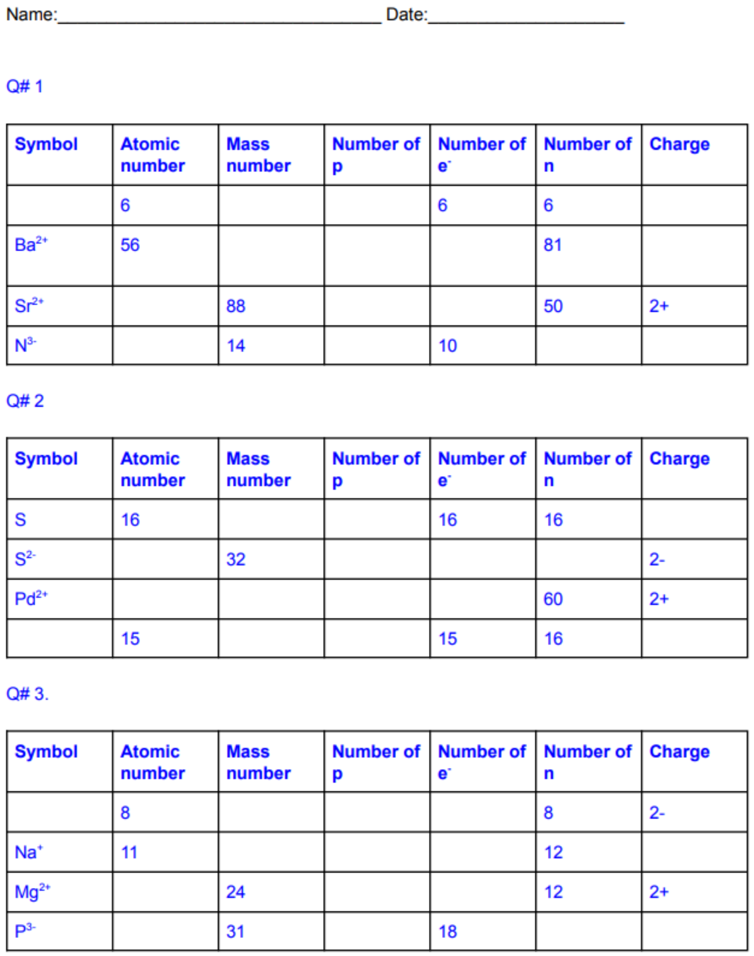


Periodic table with common group charges labeled.

Species	Protons	Electrons	Neutrons
Zn-67	30	30	37 (is isotope)
Radium	88	88	138
Aluminum cation	13	10 (look up charge, 3+)	14
Mg-23	12	12	11 (is isotope)
Phosphorus anion	15	18 (look up charge, 3-)	16

6. Both cobalt and Mn-57 have 32 neutrons. Germanium has 41 neutrons.
7. Pt-195 is closest to the weighted average (found on periodic table) and would be the most abundant.
8. If an ion charge is not given, locate the electrons of the element by looking to the atomic number. Potassium is the only atom of these three that would have 19 electrons.
9. $^{67}_{30}\mathrm{Zn}$, $^{226}_{88}\mathrm{Ra}$, $^{27}_{13}\mathrm{Al}^{3\,+}$, $^{23}_{12}\mathrm{Mg}$ and $^{31}_{15}\mathrm{P}^{3\,-}$
10. For this problem, you need to assume that all percentages add up to 100%. Since the problem provided 99.63%, you will need to subtract this given percentage from 100.00%.

$$\text{Atomic mass } = (0..9963)\,(14.003074) + (0.0037)\,(15.000108) = 14.01 \text{ amu} \tag{3.2.E.1}$$



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Q# 1

Symbol	Atomic number	Mass number	Number of p	Number of e$^-$	Number of n	Charge
	6			6	6	
Ba^{2+}	56				81	
Sr^{2+}		88			50	2+
N^{3-}		14		10		

Q# 2

Symbol	Atomic number	Mass number	Number of p	Number of e$^-$	Number of n	Charge
S	16			16	16	
S^{2-}		32				2-
Pd^{2+}					60	2+
	15			15	16	

Q# 3.

Symbol	Atomic number	Mass number	Number of p	Number of e$^-$	Number of n	Charge
	8				8	2-
Na^{+}	11				12	
Mg^{2+}		24			12	2+
P^{3-}		31		18		

3.3: Electronic Structure of Atoms (Electron Configurations)

Learning Objectives

- Derive the predicted ground-state electron configurations of atoms
- Identify and explain exceptions to predicted electron configurations for atoms and ions
- Relate electron configurations to element classifications in the periodic table

Having introduced the basics of atomic structure and quantum mechanics, we can use our understanding of quantum numbers to determine how atomic orbitals relate to one another. This allows us to determine which orbitals are occupied by electrons in each atom. The specific arrangement of electrons in orbitals of an atom determines many of the chemical properties of that atom.

3.3.1: Orbital Energies and Atomic Structure

The energy of atomic orbitals increases as the principal quantum number, n, increases. In any atom with two or more electrons, the repulsion between the electrons makes energies of subshells with different values of l differ so that the energy of the orbitals increases within a shell in the order *s* < *p* < *d* < *f*. Figure $3.3.1$ depicts how these two trends in increasing energy relate. The 1*s* orbital at the bottom of the diagram is the orbital with electrons of lowest energy. The energy increases as we move up to the 2*s* and then 2*p*, 3*s*, and 3*p* orbitals, showing that the increasing *n* value has more influence on energy than the increasing *l* value for small atoms. However, this pattern does not hold for larger atoms. The 3*d* orbital is higher in energy than the 4*s* orbital. Such overlaps continue to occur frequently as we move up the chart.

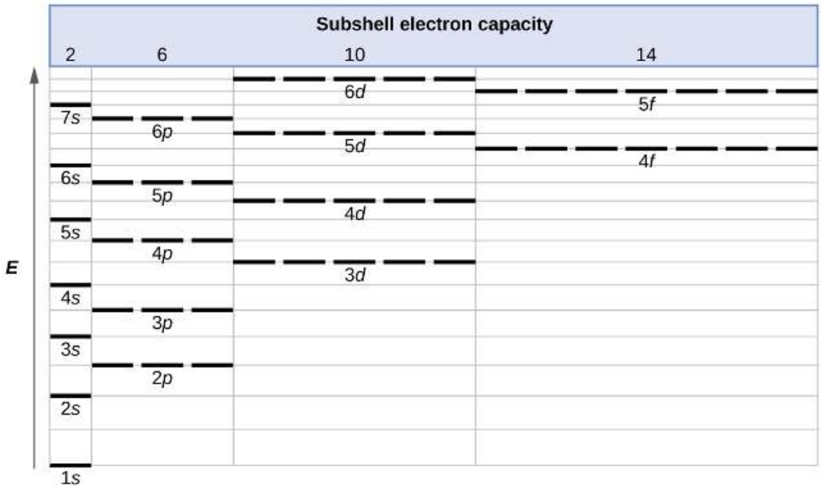

Figure $3.3.1$: Generalized energy-level diagram for atomic orbitals in an atom with two or more electrons (not to scale).

Electrons in successive atoms on the periodic table tend to fill low-energy orbitals first. Thus, many students find it confusing that, for example, the 5*p* orbitals fill immediately after the 4*d*, and immediately before the 6*s*. The filling order is based on observed experimental results, and has been confirmed by theoretical calculations. As the principal quantum number, *n*, increases, the size of the orbital increases and the electrons spend more time farther from the nucleus. Thus, the attraction to the nucleus is weaker and the energy associated with the orbital is higher (less stabilized). But this is not the only effect we have to take into account. Within each shell, as the value of *l* increases, the electrons are less penetrating (meaning there is less electron density found close to the nucleus), in the order *s* > *p* > *d* > *f*. Electrons that are closer to the nucleus slightly repel electrons that are farther out, offsetting the more dominant electron–nucleus attractions slightly (recall that all electrons have −1 charges, but nuclei have +*Z* charges). This phenomenon is called shielding and will be discussed in more detail in the next section. Electrons in orbitals that experience more shielding are less stabilized and thus higher in energy. For small orbitals (1*s* through 3*p*), the increase in energy due to *n* is more significant than the increase due to *l*; however, for larger orbitals the two trends are comparable and cannot be simply predicted. We will discuss methods for remembering the observed order.

The arrangement of electrons in the orbitals of an atom is called the electron configuration of the atom. We describe an electron configuration with a symbol that contains three pieces of information (Figure $3.3.2$):

1. The number of the principal quantum shell, *n*,

2. The letter that designates the orbital type (the subshell, *l*), and
3. A superscript number that designates the number of electrons in that particular subshell.

For example, the notation $2p^4$ (read "two–p–four") indicates four electrons in a *p* subshell (*l* = 1) with a principal quantum number (*n*) of 2. The notation $3d^8$ (read "three–d–eight") indicates eight electrons in the *d* subshell (i.e., *l* = 2) of the principal shell for which *n* = 3.

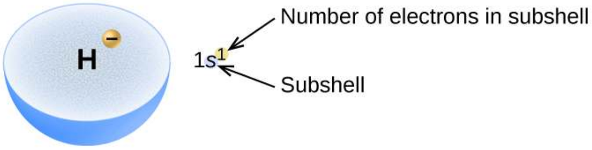

Figure 3.3.2: The diagram of an electron configuration specifies the subshell (n and l value, with letter symbol) and superscript number of electrons.

3.3.2: The Aufbau Principle

To determine the electron configuration for any particular atom, we can “build” the structures in the order of atomic numbers. Beginning with hydrogen, and continuing across the periods of the periodic table, we add one proton at a time to the nucleus and one electron to the proper subshell until we have described the electron configurations of all the elements. This procedure is called the Aufbau principle, from the German word *Aufbau* (“to build up”). Each added electron occupies the subshell of lowest energy available (in the order shown in Figure 3.3.3), subject to the limitations imposed by the allowed quantum numbers according to the Pauli exclusion principle. Electrons enter higher-energy subshells only after lower-energy subshells have been filled to capacity. Figure 3.3.3 illustrates the traditional way to remember the filling order for atomic orbitals.

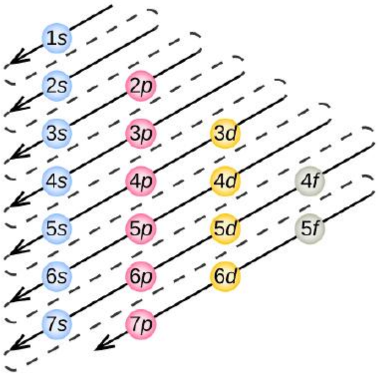

Figure 3.3.3: The arrow leads through each subshell in the appropriate filling order for electron configurations. This chart is straightforward to construct. Simply make a column for all the s orbitals with each n shell on a separate row. Repeat for p, d, and f. Be sure to only include orbitals allowed by the quantum numbers (no 1p or 2d, and so forth). Finally, draw diagonal lines from top to bottom as shown.

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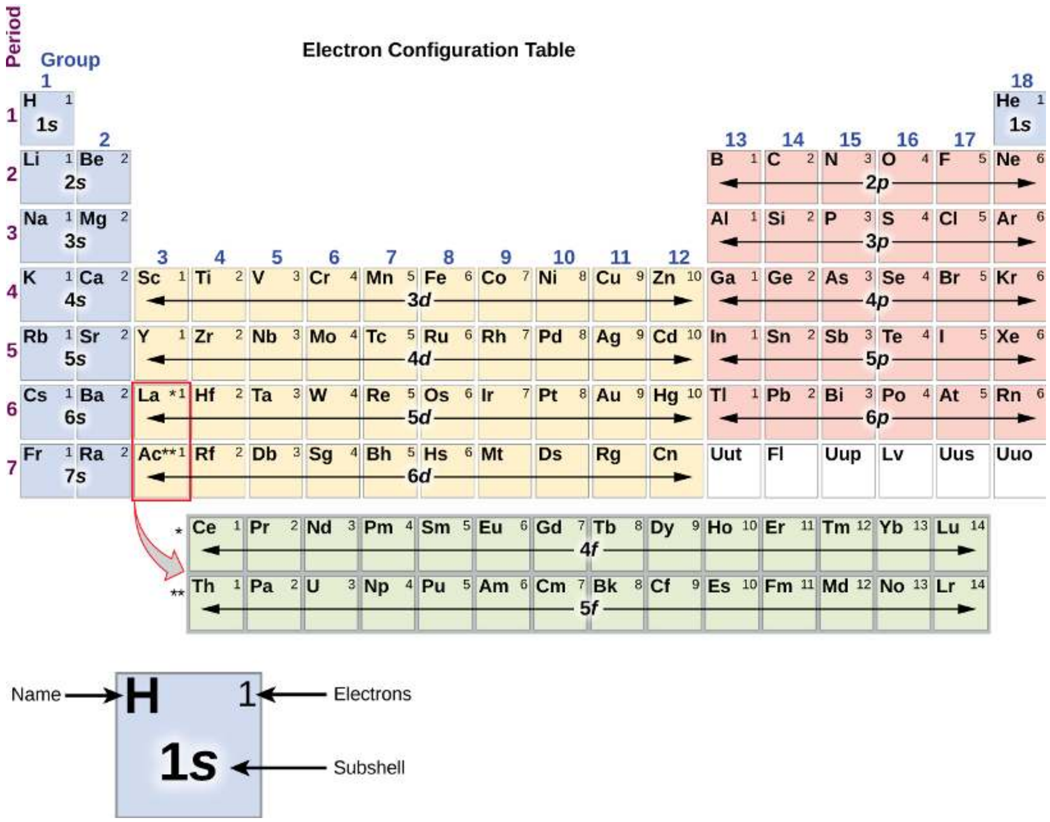
Since the arrangement of the periodic table is based on the electron configurations, Figure 3.3.4 provides an alternative method for determining the electron configuration. The filling order simply begins at hydrogen and includes each subshell as you proceed in increasing *Z* order. For example, after filling the 3*p* block up to Ar, we see the orbital will be 4s (K, Ca), followed by the 3*d* orbitals.



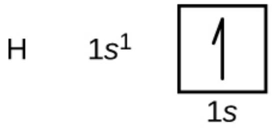
Electron Configuration Table

Period	Group 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	H 1																	He 1
	1s																	1s
2	Li 1	Be 2											B 1	C 2	N 3	O 4	F 5	Ne 6
	2s												2p					
3	Na 1	Mg 2											Al 1	Si 2	P 3	S 4	Cl 5	Ar 6
	3s												3p					
4	K 1	Ca 2	Sc 1	Ti 2	V 3	Cr 4	Mn 5	Fe 6	Co 7	Ni 8	Cu 9	Zn 10	Ga 1	Ge 2	As 3	Se 4	Br 5	Kr 6
	4s		3d										4p					
5	Rb 1	Sr 2	Y 1	Zr 2	Nb 3	Mo 4	Tc 5	Ru 6	Rh 7	Pd 8	Ag 9	Cd 10	In 1	Sn 2	Sb 3	Te 4	I 5	Xe 6
	5s		4d										5p					
6	Cs 1	Ba 2	La *1	Hf 2	Ta 3	W 4	Re 5	Os 6	Ir 7	Pt 8	Au 9	Hg 10	Tl 1	Pb 2	Bi 3	Po 4	At 5	Rn 6
	6s		5d										6p					
7	Fr 1	Ra 2	Ac**1	Rf 2	Db 3	Sg 4	Bh 5	Hs 6	Mt	Ds	Rg	Cn	Uut	Fl	Uup	Lv	Uus	Uuo
	7s		6d															

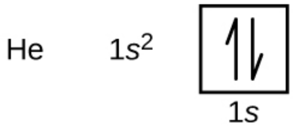
*	Ce 1	Pr 2	Nd 3	Pm 4	Sm 5	Eu 6	Gd 7	Tb 8	Dy 9	Ho 10	Er 11	Tm 12	Yb 13	Lu 14
	4f													
**	Th 1	Pa 2	U 3	Np 4	Pu 5	Am 6	Cm 7	Bk 8	Cf 9	Es 10	Fm 11	Md 12	No 13	Lr 14
	5f													

Figure 3.3.4: This periodic table shows the electron configuration for each subshell. By "building up" from hydrogen, this table can be used to determine the electron configuration for any atom on the periodic table.

We will now construct the ground-state electron configuration and orbital diagram for a selection of atoms in the first and second periods of the periodic table. Orbital diagrams are pictorial representations of the electron configuration, showing the individual orbitals and the pairing arrangement of electrons. We start with a single hydrogen atom (atomic number 1), which consists of one proton and one electron. Referring to either Figure 3.3.3 or 3.3.4, we would expect to find the electron in the 1*s* orbital. By convention, the $m_s = +\frac{1}{2}$ value is usually filled first. The electron configuration and the orbital diagram are:

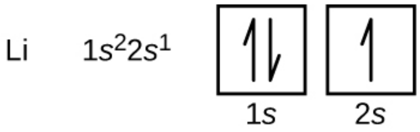


Following hydrogen is the noble gas helium, which has an atomic number of 2. The helium atom contains two protons and two electrons. The first electron has the same four quantum numbers as the hydrogen atom electron (n = 1, l = 0, m_l = 0, $m_s = +\frac{1}{2}$). The second electron also goes into the 1*s* orbital and fills that orbital. The second electron has the same n, l, and m_l quantum numbers, but must have the opposite spin quantum number, $m_s = -\frac{1}{2}$. This is in accord with the Pauli exclusion principle: No two electrons in the same atom can have the same set of four quantum numbers. For orbital diagrams, this means two arrows go in each box (representing two electrons in each orbital) and the arrows must point in opposite directions (representing paired spins). The electron configuration and orbital diagram of helium are:

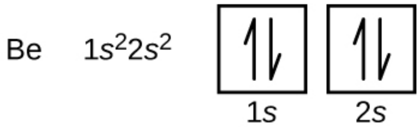


The n = 1 shell is completely filled in a helium atom.

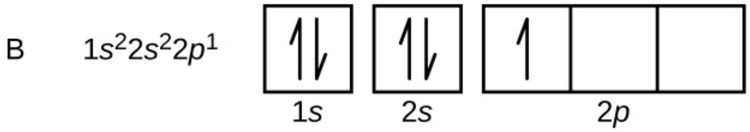
The next atom is the alkali metal lithium with an atomic number of 3. The first two electrons in lithium fill the 1*s* orbital and have the same sets of four quantum numbers as the two electrons in helium. The remaining electron must occupy the orbital of next lowest energy, the 2*s* orbital (Figure 3.3.3 or 3.3.4). Thus, the electron configuration and orbital diagram of lithium are:



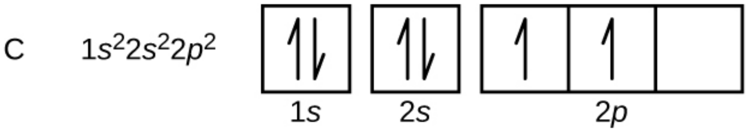
An atom of the alkaline earth metal beryllium, with an atomic number of 4, contains four protons in the nucleus and four electrons surrounding the nucleus. The fourth electron fills the remaining space in the 2*s* orbital.



An atom of boron (atomic number 5) contains five electrons. The n = 1 shell is filled with two electrons and three electrons will occupy the n = 2 shell. Because any *s* subshell can contain only two electrons, the fifth electron must occupy the next energy level, which will be a 2*p* orbital. There are three degenerate 2*p* orbitals (m_l = −1, 0, +1) and the electron can occupy any one of these *p* orbitals. When drawing orbital diagrams, we include empty boxes to depict any empty orbitals in the same subshell that we are filling.



Carbon (atomic number 6) has six electrons. Four of them fill the 1*s* and 2*s* orbitals. The remaining two electrons occupy the 2*p* subshell. We now have a choice of filling one of the 2*p* orbitals and pairing the electrons or of leaving the electrons unpaired in two different, but degenerate, *p* orbitals. The orbitals are filled as described by Hund's rule: the lowest-energy configuration for an atom with electrons within a set of degenerate orbitals is that having the maximum number of unpaired electrons. Thus, the two electrons in the carbon 2*p* orbitals have identical n, l, and m_s quantum numbers and differ in their m_l quantum number (in accord with the Pauli exclusion principle). The electron configuration and orbital diagram for carbon are:



Nitrogen (atomic number 7) fills the 1*s* and 2*s* subshells and has one electron in each of the three 2*p* orbitals, in accordance with Hund's rule. These three electrons have unpaired spins. Oxygen (atomic number 8) has a pair of electrons in any one of the 2*p* orbitals (the electrons have opposite spins) and a single electron in each of the other two. Fluorine (atomic number 9) has only one 2*p* orbital containing an unpaired electron. All of the electrons in the noble gas neon (atomic number 10) are paired, and all of the orbitals in the n = 1 and the n = 2 shells are filled. The electron configurations and orbital diagrams of these four elements are:

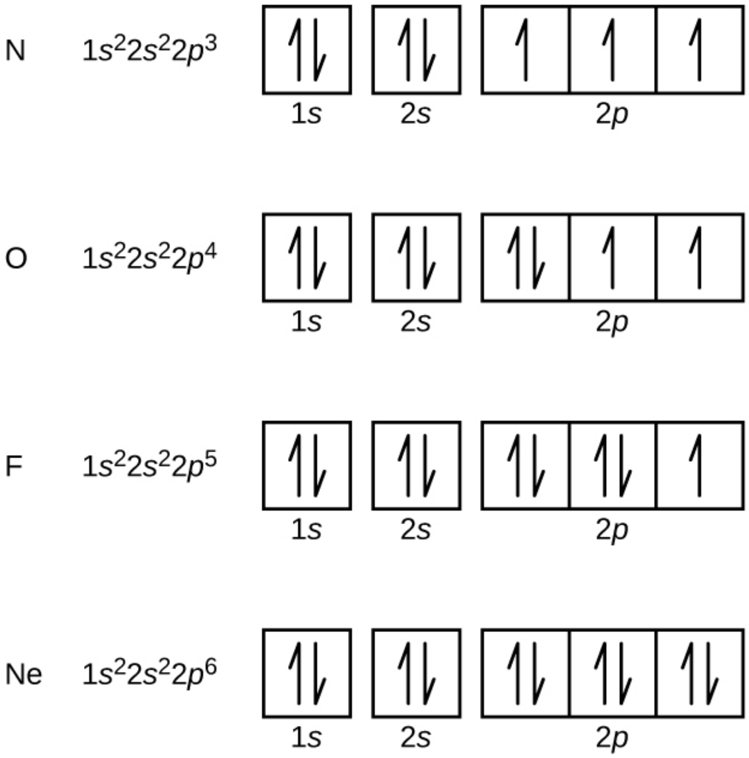


Figure 3.3.5: Since the core electron shells correspond to noble gas electron configurations, we can abbreviate electron configurations by writing the noble gas that matches the core electron configuration, along with the valence electrons in a condensed format. For our sodium example, the symbol [Ne] represents core electrons, ($1s^22s^22p^6$) and our abbreviated or condensed configuration is [Ne]$3s^1$.

The alkali metal sodium (atomic number 11) has one more electron than the neon atom. This electron must go into the lowest-energy subshell available, the 3*s* orbital, giving a $1s^22s^22p^63s^1$ configuration. The electrons occupying the outermost shell orbital(s) (highest value of *n*) are called valence electrons, and those occupying the inner shell orbitals are called core electrons (Figure \PageIndex5\PageIndex5). Since the core electron shells correspond to noble gas electron configurations, we can abbreviate electron configurations by writing the noble gas that matches the core electron configuration, along with the valence electrons in a condensed format. For our sodium example, the symbol [Ne] represents core electrons, ($1s^22s^22p^6$) and our abbreviated or condensed configuration is [Ne]$3s^1$.

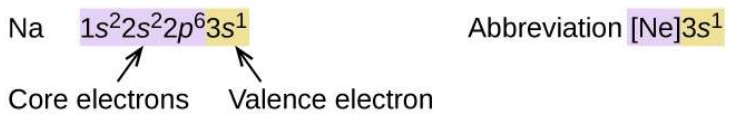


Figure 3.3.5: A core-abbreviated electron configuration (right) replaces the core electrons with the noble gas symbol whose configuration matches the core electron configuration of the other element.

Similarly, the abbreviated configuration of lithium can be represented as [He]$2s^1$, where [He] represents the configuration of the helium atom, which is identical to that of the filled inner shell of lithium. Writing the configurations in this way emphasizes the similarity of the configurations of lithium and sodium. Both atoms, which are in the alkali metal family, have only one electron in a valence *s* subshell outside a filled set of inner shells.

$$\mathrm{Li} : [\mathrm{He}]\, 2s^1$$
$$\mathrm{Na} : [\mathrm{Ne}]\, 3s^1$$

The alkaline earth metal magnesium (atomic number 12), with its 12 electrons in a [Ne]$3s^2$ configuration, is analogous to its family member beryllium, [He]$2s^2$. Both atoms have a filled *s* subshell outside their filled inner shells. Aluminum (atomic number 13), with 13 electrons and the electron configuration [Ne]$3s^23p^1$, is analogous to its family member boron, [He]$2s^22p^1$.

The electron configurations of silicon (14 electrons), phosphorus (15 electrons), sulfur (16 electrons), chlorine (17 electrons), and argon (18 electrons) are analogous in the electron configurations of their outer shells to their corresponding family members carbon, nitrogen, oxygen, fluorine, and neon, respectively, except that the principal quantum number of the outer shell of the heavier elements has increased by one to $n = 3$. Figure 3.3.6 shows the lowest energy, or ground-state, electron configuration for these elements as well as that for atoms of each of the known elements.

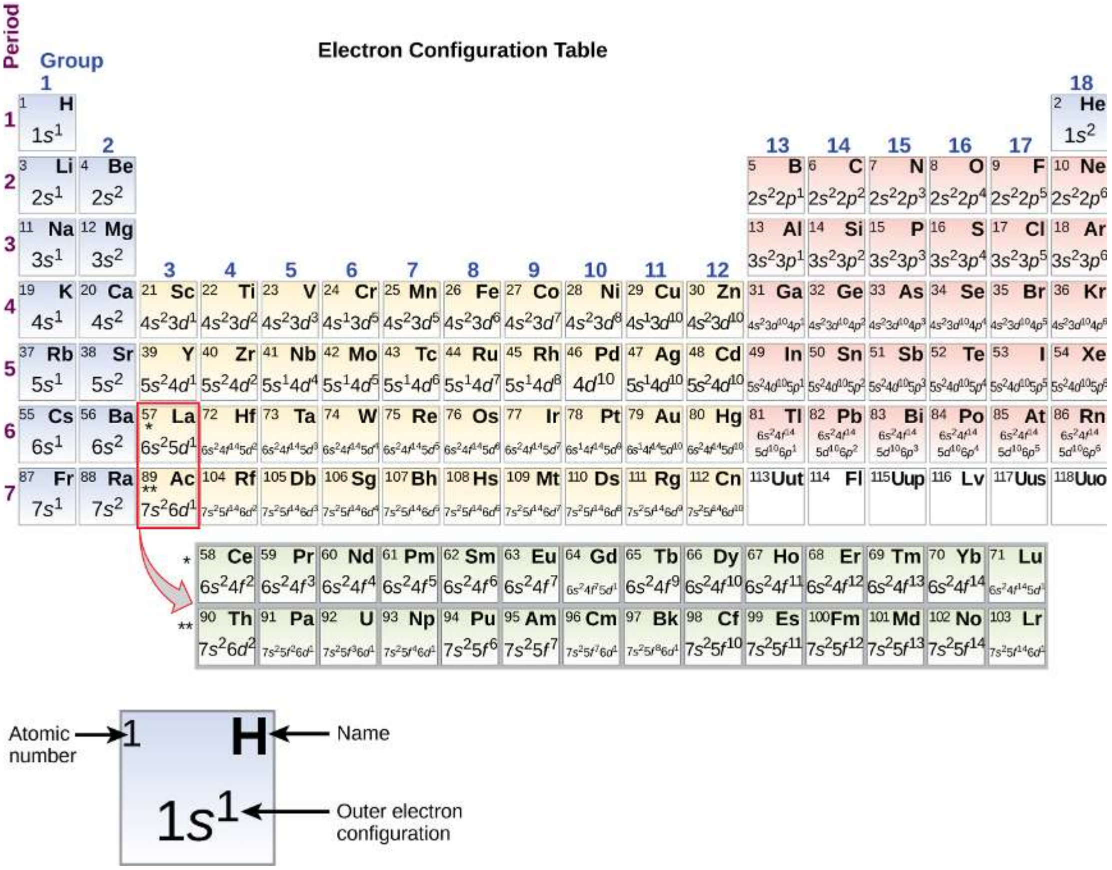


Figure 3.3.6: This version of the periodic table shows the outer-shell electron configuration of each element. Note that down each group, the configuration is often similar.

When we come to the next element in the periodic table, the alkali metal potassium (atomic number 19), we might expect that we would begin to add electrons to the 3*d* subshell. However, all available chemical and physical evidence indicates that potassium is like lithium and sodium, and that the next electron is not added to the 3*d* level but is, instead, added to the 4*s* level (Figure 3.3.3 or 3.3.4). As discussed previously, the 3*d* orbital with no radial nodes is higher in energy because it is less penetrating and more shielded from the nucleus than the 4*s*, which has three radial nodes. Thus, potassium has an electron configuration of $[Ar]4s^1$. Hence, potassium corresponds to Li and Na in its valence shell configuration. The next electron is added to complete the 4*s* subshell and calcium has an electron configuration of $[Ar]4s^2$. This gives calcium an outer-shell electron configuration corresponding to that of beryllium and magnesium.

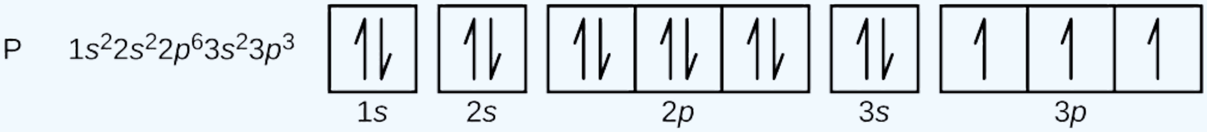
Beginning with the transition metal scandium (atomic number 21), additional electrons are added successively to the 3*d* subshell. This subshell is filled to its capacity with 10 electrons (remember that for $l = 2$ [*d* orbitals], there are $2l + 1 = 5$ values of m_l, meaning that there are five *d* orbitals that have a combined capacity of 10 electrons). The 4*p* subshell fills next. Note that for three series of elements, scandium (Sc) through copper (Cu), yttrium (Y) through silver (Ag), and lutetium (Lu) through gold (Au), a total of 10 *d* electrons are successively added to the $(n – 1)$ shell next to the *n* shell to bring that $(n – 1)$ shell from 8 to 18 electrons. For two series, lanthanum (La) through lutetium (Lu) and actinium (Ac) through lawrencium (Lr), 14 *f* electrons ($l = 3$, $2l + 1 = 7$ m_l values; thus, seven orbitals with a combined capacity of 14 electrons) are successively added to the $(n – 2)$ shell to bring that shell from 18 electrons to a total of 32 electrons.

✓ Example 3.3.1: Quantum Numbers and Electron Configurations

What is the electron configuration and orbital diagram for a phosphorus atom? What are the four quantum numbers for the last electron added?

Solution

The atomic number of phosphorus is 15. Thus, a phosphorus atom contains 15 electrons. The order of filling of the energy levels is 1*s*, 2*s*, 2*p*, 3*s*, 3*p*, 4*s*, . . . The 15 electrons of the phosphorus atom will fill up to the 3*p* orbital, which will contain three electrons:



The last electron added is a 3*p* electron. Therefore, *n* = 3 and, for a *p*-type orbital, *l* = 1. The m_l value could be –1, 0, or +1. The three *p* orbitals are degenerate, so any of these m_l values is correct. For unpaired electrons, convention assigns the value of $+\frac{1}{2}$ for the spin quantum number; thus, $m_s = +\frac{1}{2}$.

Exercise 3.3.1

Identify the atoms from the electron configurations given:

a. $[Ar]4s^2 3d^5$
b. $[Kr]5s^2 4d^{10} 5p^6$

Answer a

Mn

Answer b

Xe

The periodic table can be a powerful tool in predicting the electron configuration of an element. However, we do find exceptions to the order of filling of orbitals that are shown in Figure 3.3.3 or 3.3.4. For instance, the electron configurations of the transition metals chromium (Cr; atomic number 24) and copper (Cu; atomic number 29), among others, are not those we would expect. In general, such exceptions involve subshells with very similar energy, and small effects can lead to changes in the order of filling.

In the case of Cr and Cu, we find that half-filled and completely filled subshells apparently represent conditions of preferred stability. This stability is such that an electron shifts from the 4*s* into the 3*d* orbital to gain the extra stability of a half-filled 3*d* subshell (in Cr) or a filled 3*d* subshell (in Cu). Other exceptions also occur. For example, niobium (Nb, atomic number 41) is predicted to have the electron configuration $[Kr]5s^2 4d^3$. Experimentally, we observe that its ground-state electron configuration is actually $[Kr]5s^1 4d^4$. We can rationalize this observation by saying that the electron–electron repulsions experienced by pairing the electrons in the 5*s* orbital are larger than the gap in energy between the 5*s* and 4*d* orbitals. There is no simple method to predict the exceptions for atoms where the magnitude of the repulsions between electrons is greater than the small differences in energy between subshells.

3.3.3: Electron Configurations and the Periodic Table

As described earlier, the periodic table arranges atoms based on increasing atomic number so that elements with the same chemical properties recur periodically. When their electron configurations are added to the table (Figure 3.3.6), we also see a periodic recurrence of similar electron configurations in the outer shells of these elements. Because they are in the outer shells of an atom, valence electrons play the most important role in chemical reactions. The outer electrons have the highest energy of the electrons in an atom and are more easily lost or shared than the core electrons. Valence electrons are also the determining factor in some physical properties of the elements.

Elements in any one group (or column) have the same number of valence electrons; the alkali metals lithium and sodium each have only one valence electron, the alkaline earth metals beryllium and magnesium each have two, and the halogens fluorine and chlorine each have seven valence electrons. The similarity in chemical properties among elements of the same group occurs because they have the same number of valence electrons. It is the loss, gain, or sharing of valence electrons that defines how elements react.

It is important to remember that the periodic table was developed on the basis of the chemical behavior of the elements, well before any idea of their atomic structure was available. Now we can understand why the periodic table has the arrangement it has—the arrangement puts elements whose atoms have the same number of valence electrons in the same group. This arrangement is emphasized in Figure 3.3.6, which shows in periodic-table form the electron configuration of the last subshell to be filled by the Aufbau principle. The colored sections of Figure 3.3.6 show the three categories of elements classified by the orbitals being filled: main group, transition, and inner transition elements. These classifications determine which orbitals are counted in the valence shell, or highest energy level orbitals of an atom.

1. **Main group elements** (sometimes called **representative elements**) are those in which the last electron added enters an *s* or a *p* orbital in the outermost shell, shown in blue and red in Figure 3.3.6. This category includes all the nonmetallic elements, as well as many metals and the intermediate semimetallic elements. The valence electrons for main group elements are those with the highest *n* level. For example, gallium (Ga, atomic number 31) has the electron configuration $[\text{Ar}]\underline{4s^2}3d^{10}\underline{4p^1}$, which contains three valence electrons (underlined). The completely filled *d* orbitals count as core, not valence, electrons.
2. **Transition elements or transition metals**. These are metallic elements in which the last electron added enters a *d* orbital. The valence electrons (those added after the last noble gas configuration) in these elements include the *ns* and (*n* – 1) *d* electrons. The official IUPAC definition of transition elements specifies those with partially filled *d* orbitals. Thus, the elements with completely filled orbitals (Zn, Cd, Hg, as well as Cu, Ag, and Au in Figure 3.3.6) are not technically transition elements. However, the term is frequently used to refer to the entire *d* block (colored yellow in Figure 3.3.6), and we will adopt this usage in this textbook.
3. **Inner transition elements** are metallic elements in which the last electron added occupies an *f* orbital. They are shown in green in Figure 3.3.6. The valence shells of the inner transition elements consist of the (*n* – 2)*f*, the (*n* – 1)*d*, and the *ns* subshells. There are two inner transition series:
    1. The lanthanide series: lanthanide (La) through lutetium (Lu)
    2. The actinide series: actinide (Ac) through lawrencium (Lr)

Lanthanum and actinium, because of their similarities to the other members of the series, are included and used to name the series, even though they are transition metals with no *f* electrons.

3.3.4: Electron Configurations of Ions

We have seen that ions are formed when atoms gain or lose electrons. A cation (positively charged ion) forms when one or more electrons are removed from a parent atom. For main group elements, the electrons that were added last are the first electrons removed. For transition metals and inner transition metals, however, electrons in the *s* orbital are easier to remove than the *d* or *f* electrons, and so the highest *ns* electrons are lost, and then the (*n* – 1)*d* or (*n* – 2)*f* electrons are removed. An anion (negatively charged ion) forms when one or more electrons are added to a parent atom. The added electrons fill in the order predicted by the Aufbau principle.

✓ Example 3.3.2: Predicting Electron Configurations of Ions

What is the electron configuration and orbital diagram of:

a. Na$^+$
b. P^{3-}
c. Al^{2+}
d. Fe^{2+}
e. Sm^{3+}

Solution

First, write out the electron configuration for each parent atom. We have chosen to show the full, unabbreviated configurations to provide more practice for students who want it, but listing the core-abbreviated electron configurations is also acceptable.

Next, determine whether an electron is gained or lost. Remember electrons are negatively charged, so ions with a positive charge have *lost* an electron. For main group elements, the last orbital gains or loses the electron. For transition metals, the last *s* orbital loses an electron before the *d* orbitals.

a. Na: $1s^22s^22p^63s^1$. Sodium cation loses one electron, so Na$^+$: $1s^22s^22p^63s^1$ = Na$^+$: $1s^22s^22p^6$.

b. P: $1s^22s^22p^63s^23p^3$. Phosphorus trianion gains three electrons, so P^{3-}: $1s^22s^22p^63s^23p^6$.
c. Al: $1s^22s^22p^63s^23p^1$. Aluminum dication loses two electrons Al^{2+}: $1s^22s^22p^63s^23p^1$ = Al^{2+}: $1s^22s^22p^63s^1$.
d. Fe: $1s^22s^22p^63s^23p^64s^23d^6$. Iron(II) loses two electrons and, since it is a transition metal, they are removed from the 4*s* orbital Fe^{2+}: $1s^22s^22p^63s^23p^64s^23d^6 = 1s^22s^22p^63s^23p^63d^6$.
e. Sm: $1s^22s^22p^63s^23p^64s^23d^{10}4p^65s^24d^{10}5p^66s^24f^6$. Samarium trication loses three electrons. The first two will be lost from the 6*s* orbital, and the final one is removed from the 4*f* orbital. Sm^{3+}: $1s^22s^22p^63s^23p^64s^23d^{10}4p^65s^24d^{10}5p^66s^24f^6 = 1s^22s^22p^63s^23p^64s^23d^{10}4p^65s^24d^{10}5p^64f^5$.

Exercise 3.3.2

a. Which ion with a +2 charge has the electron configuration $1s^22s^22p^63s^23p^63d^{10}4s^24p^64d^5$?
b. Which ion with a +3 charge has this configuration?

Answer a

Tc^{2+}

Answer b

Ru^{3+}

Summary

The relative energy of the subshells determine the order in which atomic orbitals are filled (1*s*, 2*s*, 2*p*, 3*s*, 3*p*, 4*s*, 3*d*, 4*p*, and so on). Electron configurations and orbital diagrams can be determined by applying the Pauli exclusion principle (no two electrons can have the same set of four quantum numbers) and Hund's rule (whenever possible, electrons retain unpaired spins in degenerate orbitals).

Electrons in the outermost orbitals, called valence electrons, are responsible for most of the chemical behavior of elements. In the periodic table, elements with analogous valence electron configurations usually occur within the same group. There are some exceptions to the predicted filling order, particularly when half-filled or completely filled orbitals can be formed. The periodic table can be divided into three categories based on the orbital in which the last electron to be added is placed: main group elements (*s* and *p* orbitals), transition elements (*d* orbitals), and inner transition elements (*f* orbitals).

Glossary

Aufbau principle

procedure in which the electron configuration of the elements is determined by "building" them in order of atomic numbers, adding one proton to the nucleus and one electron to the proper subshell at a time

core electron

electron in an atom that occupies the orbitals of the inner shells

electron configuration

electronic structure of an atom in its ground state given as a listing of the orbitals occupied by the electrons

Hund's rule

every orbital in a subshell is singly occupied with one electron before any one orbital is doubly occupied, and all electrons in singly occupied orbitals have the same spin

orbital diagram

pictorial representation of the electron configuration showing each orbital as a box and each electron as an arrow

valence electrons

electrons in the outermost or valence shell (highest value of *n*) of a ground-state atom; determine how an element reacts

valence shell

outermost shell of electrons in a ground-state atom; for main group elements, the orbitals with the highest *n* level (*s* and *p* subshells) are in the valence shell, while for transition metals, the highest energy *s* and *d* subshells make up the valence shell and for inner transition elements, the highest *s*, *d*, and *f* subshells are included

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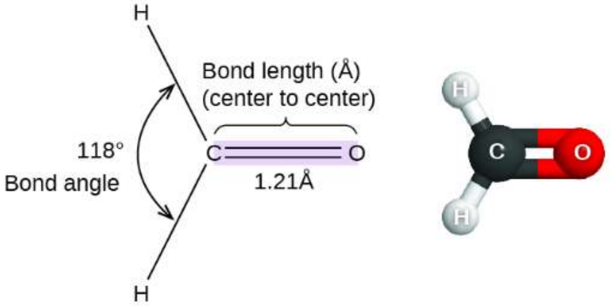
3.4: Molecular Structure and Polarity

Learning Objectives

By the end of this section, you will be able to:

- Predict the structures of small molecules using valence shell electron pair repulsion (VSEPR) theory
- Explain the concepts of polar covalent bonds and molecular polarity
- Assess the polarity of a molecule based on its bonding and structure

Thus far, we have used two-dimensional Lewis structures to represent molecules. However, molecular structure is actually three-dimensional, and it is important to be able to describe molecular bonds in terms of their distances, angles, and relative arrangements in space (Figure 3.4.1). A bond angle is the angle between any two bonds that include a common atom, usually measured in degrees. A bond distance (or bond length) is the distance between the nuclei of two bonded atoms along the straight line joining the nuclei. Bond distances are measured in Ångstroms (1 Å = 10^{-10} m) or picometers (1 pm = 10^{-12} m, 100 pm = 1 Å).



A pair of images are shown. The left image shows a carbon atom with three atoms bonded in a triangular arrangement around it. There are two hydrogen atoms bonded on the left side of the carbon and the angle between them is labeled, “118 degrees” and, “Bond angle.” The carbon is also double bonded to an oxygen atom. The double bond is shaded and there is a bracket which labels the bond, “Bond length (angstrom), (center to center),” and, “1.21 angstrom.” The right image shows a ball-and-stick model of the same elements. The hydrogen atoms are white, the carbon atom is black, and the oxygen atom is red.

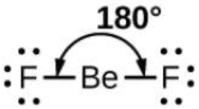
Figure 3.4.1: Bond distances (lengths) and angles are shown for the formaldehyde molecule, H_2CO.

VSEPR Theory

Valence shell electron-pair repulsion theory (VSEPR theory) enables us to predict the molecular structure, including approximate bond angles around a central atom, of a molecule from an examination of the number of bonds and lone electron pairs in its Lewis structure. The VSEPR model assumes that electron pairs in the valence shell of a central atom will adopt an arrangement that minimizes repulsions between these electron pairs by maximizing the distance between them. The electrons in the valence shell of a central atom form either bonding pairs of electrons, located primarily between bonded atoms, or lone pairs. The electrostatic repulsion of these electrons is reduced when the various regions of high electron density assume positions as far from each other as possible.

VSEPR theory predicts the arrangement of electron pairs around each central atom and, usually, the correct arrangement of atoms in a molecule. We should understand, however, that the theory only considers electron-pair repulsions. Other interactions, such as nuclear-nuclear repulsions and nuclear-electron attractions, are also involved in the final arrangement that atoms adopt in a particular molecular structure.

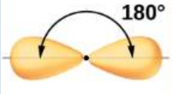
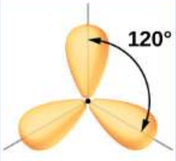
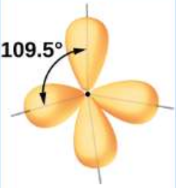
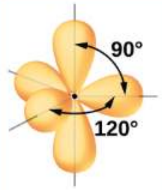
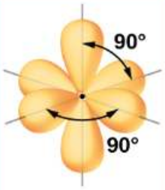
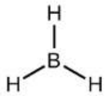
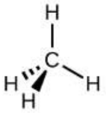
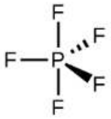
As a simple example of VSEPR theory, let us predict the structure of a gaseous BeF_2 molecule. The Lewis structure of BeF_2 (Figure 3.4.2) shows only two electron pairs around the central beryllium atom. With two bonds and no lone pairs of electrons on the central atom, the bonds are as far apart as possible, and the electrostatic repulsion between these regions of high electron density is reduced to a minimum when they are on opposite sides of the central atom. The bond angle is 180° (Figure 3.4.2).



A Lewis structure is shown. A fluorine atom with three lone pairs of electrons is single bonded to a beryllium atom which is single bonded to a fluorine atom with three lone pairs of electrons. The angle of the bonds between the two fluorine atoms and the beryllium atom is labeled, “180 degrees.”

Figure $3.4.2$: The BeF_2 molecule adopts a linear structure in which the two bonds are as far apart as possible, on opposite sides of the Be atom.

Figure $3.4.3$ illustrates this and other electron-pair geometries that minimize the repulsions among regions of high electron density (bonds and/or lone pairs). Two regions of electron density around a central atom in a molecule form a linear geometry; three regions form a trigonal planar geometry; four regions form a tetrahedral geometry; five regions form a trigonal bipyramidal geometry; and six regions form an octahedral geometry.

Number of regions	Two regions of high electron density (bonds and/or unshared pairs)	Three regions of high electron density (bonds and/or unshared pairs)	Four regions of high electron density (bonds and/or unshared pairs)	Five regions of high electron density (bonds and/or unshared pairs)	Six regions of high electron density (bonds and/or unshared pairs)
Spatial arrangement					
Line-dash-wedge notation	H—Be—H				
Electron region geometry	Linear; 180° angle	Trigonal planar; all angles 120°	Tetrahedral; all angles 109.5°	Trigonal bipyramidal; angles of 90° or 120° An attached atom may be equatorial (in the plane of the triangle) or axial (above or below the plane of the triangle).	Octahedral; all angles 90° or 180°

A table with four rows and six columns is shown. The header column contains the phrases, "Number of regions," "Spatial arrangement," "Wedge/dash Notation," and "Electron region Geometry." The first row reads: "Two regions of high electron density (bonds and/or unshared pairs)", "Three regions of high electron density (bonds and/or unshared pairs)," "Four regions of high electron density (bonds and/or unshared pairs)," "Five regions of high electron density (bonds and/or unshared pairs)," and "Six regions of high electron density (bonds and/or unshared pairs)." The second row shows diagrams of orbitals. The first image shows two oval-shaped orbs with an arrow indicating an angle of 180 degrees. The second image shows three oval-shaped orbs with an arrow indicating an angle of 120 degrees. The third image shows four oval-shaped orbs with an arrow indicating an angle of 109.5 degrees. The fourth image shows five oval-shaped orbs with an arrow indicating an angle of 90 and 120 degrees. The fifth image shows six oval-shaped orbs with an arrow indicating an angle of 90 degrees. The third row contains Lewis structures. The first structure shows a beryllium atom single bonded to two hydrogen atoms. The second structure shows a boron atom single bonded to three hydrogen atoms. The third structure shows a carbon atom single bonded to four hydrogen atoms. The fourth structure shows a phosphorus atom single bonded to five fluorine atoms. The fifth structure shows a sulfur atom single bonded to six fluorine atoms. The fourth row contains the phrases "Linear; 180 degree angle," Trigonal Planar; all angles 120 degrees," "Tetrahedral; all angles 109.5 degrees," "Trigonal bipyramidal; angles of 90 degrees and 120 degrees. An attached atom may be equatorial, (in the plane of the triangle), or axial, (above the plane of the triangle)," and "Octahedral; 90 degrees or 180 degrees."

Figure 3.4.3: The basic electron-pair geometries predicted by VSEPR theory maximize the space around any region of electron density (bonds or lone pairs).

Electron-pair Geometry versus Molecular Structure

It is important to note that electron-pair geometry around a central atom is *not* the same thing as its molecular structure. The electron-pair geometries shown in Figure 3.4.3 describe all regions where electrons are located, bonds as well as lone pairs. Molecular structure describes the location of the *atoms*, not the electrons.

We differentiate between these two situations by naming the geometry that includes *all* electron pairs the electron-pair geometry. The structure that includes only the placement of the atoms in the molecule is called the molecular structure. The electron-pair geometries will be the same as the molecular structures when there are no lone electron pairs around the central atom, but they will be different when there are lone pairs present on the central atom.

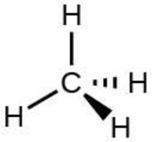


Figure 3.4.4: The molecular structure of the methane molecule, CH_4, is shown with a tetrahedral arrangement of the hydrogen atoms. VSEPR structures like this one are often drawn using the wedge and dash notation, in which solid lines represent bonds in the plane of the page, solid wedges represent bonds coming up out of the plane, and dashed lines represent bonds going down into the plane.

For example, the methane molecule, CH_4, which is the major component of natural gas, has four bonding pairs of electrons around the central carbon atom; the electron-pair geometry is tetrahedral, as is the molecular structure (Figure 3.4.4). On the other hand, the ammonia molecule, NH_3, also has four electron pairs associated with the nitrogen atom, and thus has a tetrahedral electron-pair geometry. One of these regions, however, is a lone pair, which is not included in the molecular structure, and this lone pair influences the shape of the molecule (Figure 3.4.5).

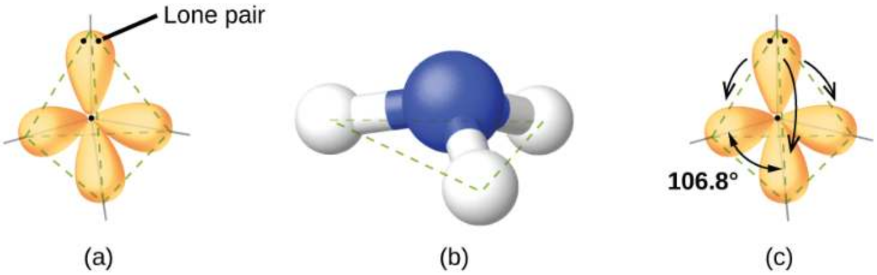


Figure 3.4.5: (a) The electron-pair geometry for the ammonia molecule is tetrahedral with one lone pair and three single bonds. (b) The trigonal pyramidal molecular structure is determined from the electron-pair geometry. (c) The actual bond angles deviate slightly from the idealized angles because the lone pair takes up a larger region of space than do the single bonds, causing the HNH angle to be slightly smaller than 109.5°.

As seen in Figure 3.4.5, small distortions from the ideal angles in Figure 3.4.3 can result from differences in repulsion between various regions of electron density. VSEPR theory predicts these distortions by establishing an order of repulsions and an order of the amount of space occupied by different kinds of electron pairs. The order of electron-pair repulsions from greatest to least repulsion is:

$$\text{lone pair-lone pair} > \text{lone pair-bonding pair} > \text{bonding pair-bonding pair}$$

This order of repulsions determines the amount of space occupied by different regions of electrons. A lone pair of electrons occupies a larger region of space than the electrons in a triple bond; in turn, electrons in a triple bond occupy more space than those in a double bond, and so on. The order of sizes from largest to smallest is:

$$\text{lone pair } > \text{ triple bond } > \text{ double bond } > \text{ single bond}$$

Consider formaldehyde, H_2CO, which is used as a preservative for biological and anatomical specimens (Figure 3.4.1). This molecule has regions of high electron density that consist of two single bonds and one double bond. The basic geometry is trigonal planar with 120° bond angles, but we see that the double bond causes slightly larger angles (121°), and the angle between the single bonds is slightly smaller (118°).

In the ammonia molecule, the three hydrogen atoms attached to the central nitrogen are not arranged in a flat, trigonal planar molecular structure, but rather in a three-dimensional trigonal pyramid (Figure 3.4.5) with the nitrogen atom at the apex and the three hydrogen atoms forming the base. The ideal bond angles in a trigonal pyramid are based on the tetrahedral electron pair geometry. Again, there are slight deviations from the ideal because lone pairs occupy larger regions of space than do bonding electrons. The H–N–H bond angles in NH_3 are slightly smaller than the 109.5° angle in a regular tetrahedron (Figure 3.4.3) because the lone pair-bonding pair repulsion is greater than the bonding pair-bonding pair repulsion (Figure 3.4.5). Figure 3.4.6 illustrates the ideal molecular structures, which are predicted based on the electron-pair geometries for various combinations of lone pairs and bonding pairs.

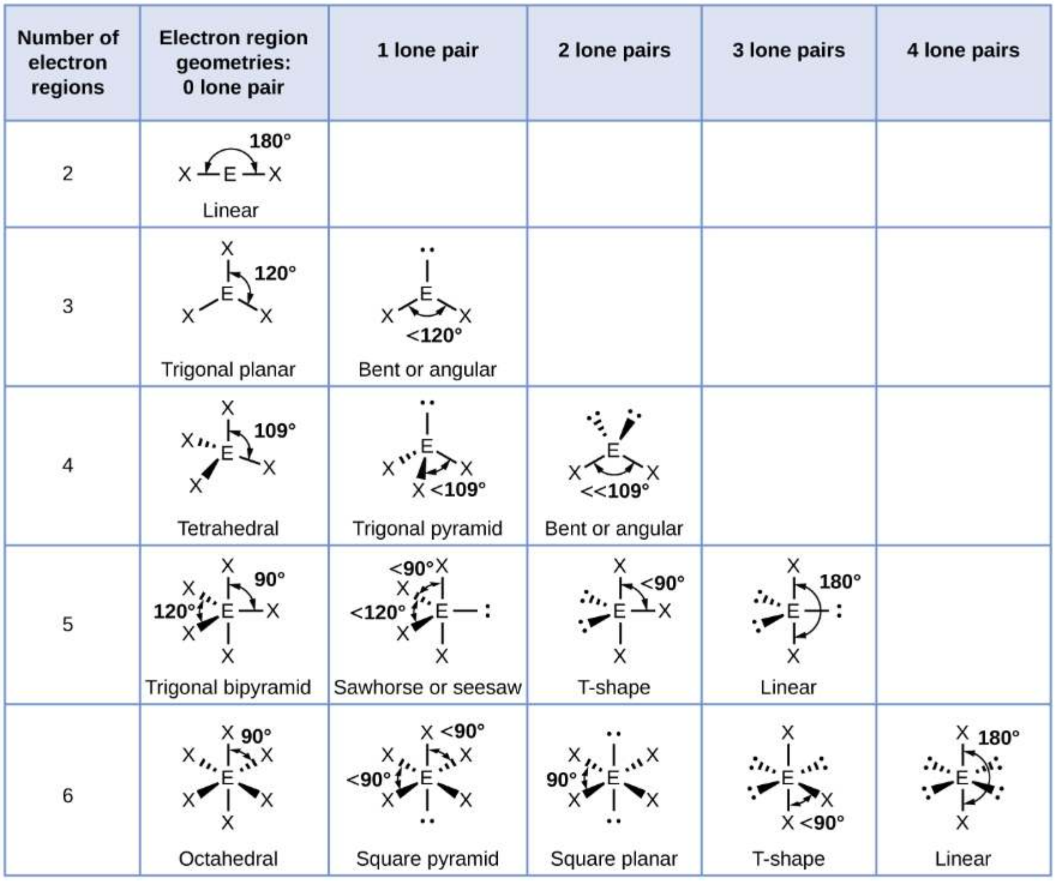


Figure 3.4.6: The molecular structures are identical to the electron-pair geometries when there are no lone pairs present (first column). For a particular number of electron pairs (row), the molecular structures for one or more lone pairs are determined based on modifications of the corresponding electron-pair geometry.

According to VSEPR theory, the terminal atom locations (Xs in Figure 3.4.6) are equivalent within the linear, trigonal planar, and tetrahedral electron-pair geometries (the first three rows of the table). It does not matter which X is replaced with a lone pair because the molecules can be rotated to convert positions. For trigonal bipyramidal electron-pair geometries, however, there are two distinct X positions, as shown in Figure 3.4.7 an axial position (if we hold a model of a trigonal bipyramid by the two axial positions, we have an axis around which we can rotate the model) and an equatorial position (three positions form an equator around the middle of the molecule). As shown in Figure 3.4.6, the axial position is surrounded by bond angles of 90°, whereas the equatorial position has more space available because of the 120° bond angles. In a trigonal bipyramidal electron-pair geometry, lone pairs always occupy equatorial positions because these more spacious positions can more easily accommodate the larger lone pairs.

Theoretically, we can come up with three possible arrangements for the three bonds and two lone pairs for the ClF_3 molecule (Figure 3.4.7). The stable structure is the one that puts the lone pairs in equatorial locations, giving a T-shaped molecular structure.

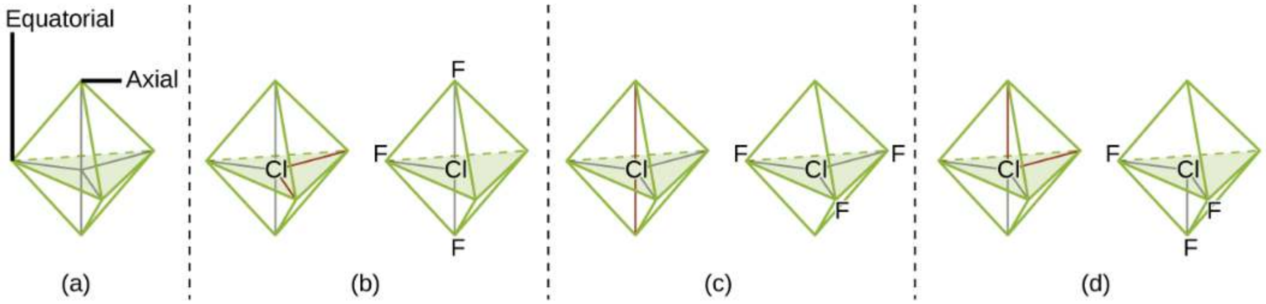


Figure 3.4.7: (a) In a trigonal bipyramid, the two axial positions are located directly across from one another, whereas the three equatorial positions are located in a triangular arrangement. (b–d) The two lone pairs (red lines) in ClF_3 have several possible arrangements, but the T-shaped molecular structure (b) is the one actually observed, consistent with the larger lone pairs both occupying equatorial positions.

When a central atom has two lone electron pairs and four bonding regions, we have an octahedral electron-pair geometry. The two lone pairs are on opposite sides of the octahedron (180° apart), giving a square planar molecular structure that minimizes lone pair-lone pair repulsions (Figure 3.4.6).

Predicting Electron Pair Geometry and Molecular Structure

The following procedure uses VSEPR theory to determine the electron pair geometries and the molecular structures:

1. Write the Lewis structure of the molecule or polyatomic ion.
2. Count the number of regions of electron density (lone pairs and bonds) around the central atom. A single, double, or triple bond counts as one region of electron density.
3. Identify the electron-pair geometry based on the number of regions of electron density: linear, trigonal planar, tetrahedral, trigonal bipyramidal, or octahedral (Figure 3.4.6, first column).
4. Use the number of lone pairs to determine the molecular structure (Figure 3.4.6). If more than one arrangement of lone pairs and chemical bonds is possible, choose the one that will minimize repulsions, remembering that lone pairs occupy more space than multiple bonds, which occupy more space than single bonds. In trigonal bipyramidal arrangements, repulsion is minimized when every lone pair is in an equatorial position. In an octahedral arrangement with two lone pairs, repulsion is minimized when the lone pairs are on opposite sides of the central atom.

The following examples illustrate the use of VSEPR theory to predict the molecular structure of molecules or ions that have no lone pairs of electrons. In this case, the molecular structure is identical to the electron pair geometry.

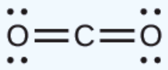
✓ Example 3.4.1: Predicting Electron-pair Geometry and Molecular Structure: CO_2 and BCl_3

Predict the electron-pair geometry and molecular structure for each of the following:

a. carbon dioxide, CO_2, a molecule produced by the combustion of fossil fuels
b. boron trichloride, BCl_3, an important industrial chemical

Solution

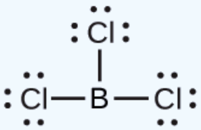
(a) We write the Lewis structure of CO_2 as:



$O=C=O$

This shows us two regions of high electron density around the carbon atom—each double bond counts as one region, and there are no lone pairs on the carbon atom. Using VSEPR theory, we predict that the two regions of electron density arrange themselves on opposite sides of the central atom with a bond angle of 180°. The electron-pair geometry and molecular structure are identical, and CO_2 molecules are linear.

(b) We write the Lewis structure of BCl_3 as:



Thus we see that BCl_3 contains three bonds, and there are no lone pairs of electrons on boron. The arrangement of three regions of high electron density gives a trigonal planar electron-pair geometry. The B–Cl bonds lie in a plane with 120° angles between them. BCl_3 also has a trigonal planar molecular structure (Figure $3.4.8$).

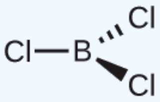


Figure $3.4.8$:

The electron-pair geometry and molecular structure of BCl_3 are both trigonal planar. Note that the VSEPR geometry indicates the correct bond angles (120°), unlike the Lewis structure shown above.

? Exercise $3.4.1$

Carbonate, is a common polyatomic ion found in various materials from eggshells to antacids. What are the electron-pair geometry and molecular structure of this polyatomic ion?

Answer

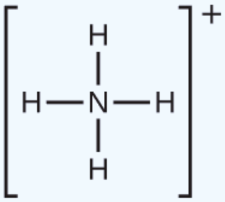
> The electron-pair geometry is trigonal planar and the molecular structure is trigonal planar. Due to resonance, all three C–O bonds are identical. Whether they are single, double, or an average of the two, each bond counts as one region of electron density.

✓ Example $3.4.2$: Predicting Electron-pair Geometry and Molecular Structure: Ammonium

Two of the top 50 chemicals produced in the United States, ammonium nitrate and ammonium sulfate, both used as fertilizers, contain the ammonium ion. Predict the electron-pair geometry and molecular structure of the cation.

Solution

We write the Lewis structure of as:



We can see that NH_4^+ contains four bonds from the nitrogen atom to hydrogen atoms and no lone pairs. We expect the four regions of high electron density to arrange themselves so that they point to the corners of a tetrahedron with the central nitrogen atom in the middle (Figure $3.4.6$). Therefore, the electron pair geometry of Figure $3.4.9$.

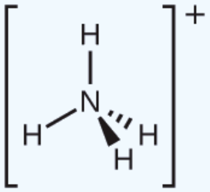


Figure $3.4.9$: The ammonium ion displays a tetrahedral electron-pair geometry as well as a tetrahedral molecular structure.

Exercise 3.4.2

Identify a molecule with trigonal bipyramidal molecular structure.

Answer

Any molecule with five electron pairs around the central atoms including no lone pairs will be trigonal bipyramidal. PF_5 is a common example.

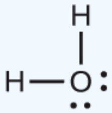
The next several examples illustrate the effect of lone pairs of electrons on molecular structure.

Example 3.4.3: Predicting Electron-pair Geometry and Molecular Structure: Lone Pairs on the Central Atom

Predict the electron-pair geometry and molecular structure of a water molecule.

Solution

The Lewis structure of H_2O indicates that there are four regions of high electron density around the oxygen atom: two lone pairs and two chemical bonds:



We predict that these four regions are arranged in a tetrahedral fashion (Figure 3.4.10), as indicated in Figure 3.4.6. Thus, the electron-pair geometry is tetrahedral and the molecular structure is bent with an angle slightly less than 109.5°. In fact, the bond angle is 104.5°.

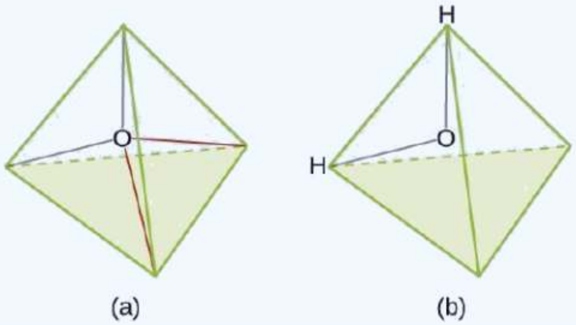


Figure 3.4.10: (a) H_2O has four regions of electron density around the central atom, so it has a tetrahedral electron-pair geometry. (b) Two of the electron regions are lone pairs, so the molecular structure is bent.

Exercise 3.4.3

The hydronium ion, H_3O^+, forms when acids are dissolved in water. Predict the electron-pair geometry and molecular structure of this cation.

Answer

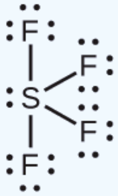
electron pair geometry: tetrahedral; molecular structure: trigonal pyramidal

Example 3.4.4: Predicting Electron-pair Geometry and Molecular Structure: SF_4

Sulfur tetrafluoride, SF_4, is extremely valuable for the preparation of fluorine-containing compounds used as herbicides (i.e., SF_4 is used as a fluorinating agent). Predict the electron-pair geometry and molecular structure of a SF_4 molecule.

Solution

The Lewis structure of SF_4 indicates five regions of electron density around the sulfur atom: one lone pair and four bonding pairs:



We expect these five regions to adopt a trigonal bipyramidal electron-pair geometry. To minimize lone pair repulsions, the lone pair occupies one of the equatorial positions. The molecular structure (Figure 3.4.11) is that of a seesaw (Figure 3.4.6).

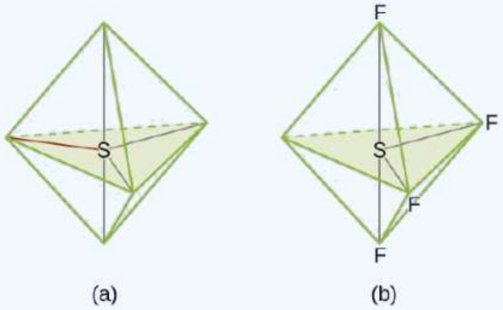


Figure 3.4.11: (a) SF4 has a trigonal bipyramidal arrangement of the five regions of electron density. (b) One of the regions is a lone pair, which results in a seesaw-shaped molecular structure.

? Exercise 3.4.4

Predict the electron pair geometry and molecular structure for molecules of XeF_2.

Answer

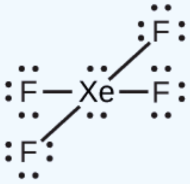
The electron-pair geometry is trigonal bipyramidal. The molecular structure is linear.

✓ Example 3.4.5: Predicting Electron-pair Geometry and Molecular Structure: XeF_4

Of all the noble gases, xenon is the most reactive, frequently reacting with elements such as oxygen and fluorine. Predict the electron-pair geometry and molecular structure of the XeF_4 molecule.

Solution

The Lewis structure of XeF_4 indicates six regions of high electron density around the xenon atom: two lone pairs and four bonds:



These six regions adopt an octahedral arrangement (Figure 3.4.6), which is the electron-pair geometry. To minimize repulsions, the lone pairs should be on opposite sides of the central atom (Figure 3.4.12). The five atoms are all in the same plane and have a square planar molecular structure.

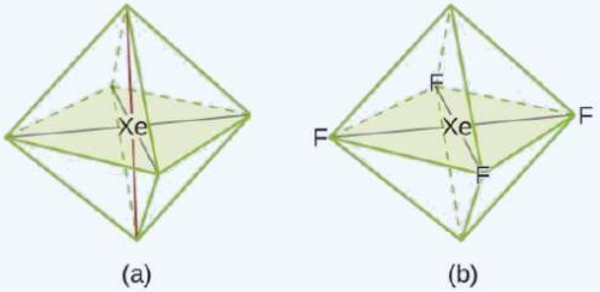


Figure 3.4.12: (a) XeF_4 adopts an octahedral arrangement with two lone pairs (red lines) and four bonds in the electron-pair geometry. (b) The molecular structure is square planar with the lone pairs directly across from one another.

? Exercise 3.4.5

In a certain molecule, the central atom has three lone pairs and two bonds. What will the electron pair geometry and molecular structure be?

Answer

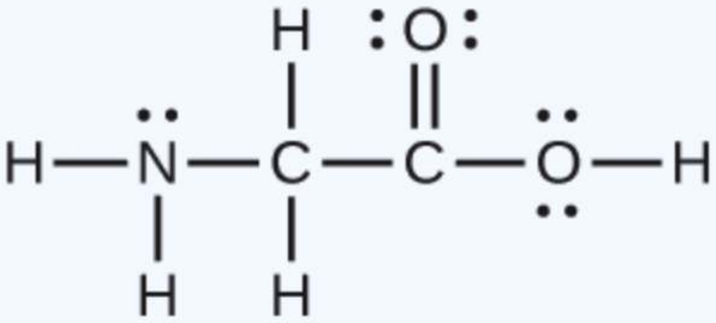
electron pair geometry: trigonal bipyramidal; molecular structure: linear

Molecular Structure for Multicenter Molecules

When a molecule or polyatomic ion has only one central atom, the molecular structure completely describes the shape of the molecule. Larger molecules do not have a single central atom, but are connected by a chain of interior atoms that each possess a “local” geometry. The way these local structures are oriented with respect to each other also influences the molecular shape, but such considerations are largely beyond the scope of this introductory discussion. For our purposes, we will only focus on determining the local structures.

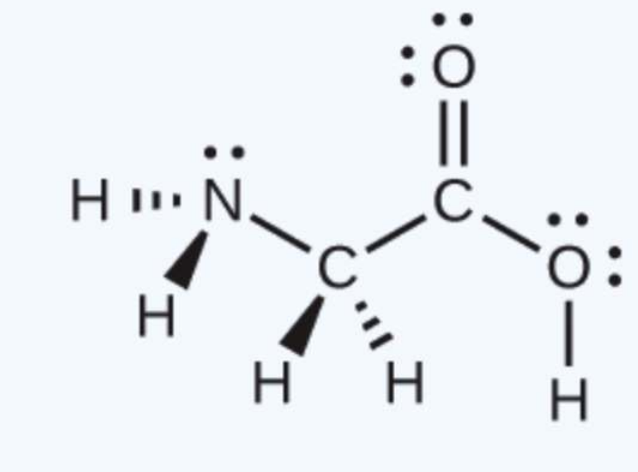
✓ Example 3.4.6: Predicting Structure in Multicenter Molecules

The Lewis structure for the simplest amino acid, glycine, $H_2NCH_2CO_2H$, is shown here. Predict the local geometry for the nitrogen atom, the two carbon atoms, and the oxygen atom with a hydrogen atom attached:



A Lewis structure depicts a nitrogen atom with one lone pair of electrons that is single bonded to two hydrogen atoms and a carbon atom, which is, in turn, single bonded to two hydrogen atoms and another carbon atom. This carbon atom is double bonded to an oxygen atom with two lone pairs of electrons and single bonded to an oxygen that has two lone pairs of electrons and a single bond to a hydrogen atom.

Solution



A Lewis structure depicts a nitrogen atom with one lone pair of electrons that is single bonded to two hydrogen atoms and a carbon atom. The atoms described are drawn with bonds that indicate a three-dimensional, tetrahedral shape around the nitrogen atom. The carbon is, in turn, single bonded to two hydrogen atoms and another carbon atom, and again, a tetrahedral, three dimensional configuration is indicated by the types of bonds. This second carbon atom is double bonded to an oxygen atom and single bonded to an oxygen that has two lone pairs of electrons and a single bond to a hydrogen atom.

Consider each central atom independently. The electron-pair geometries:

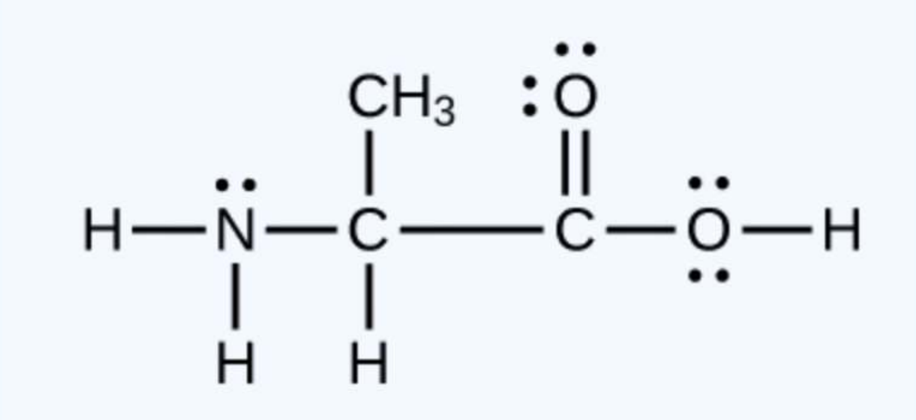
- nitrogen—four regions of electron density; tetrahedral
- carbon ($\underline{C}H_2$)—four regions of electron density; tetrahedral
- carbon ($\underline{C}O_2$)—three regions of electron density; trigonal planar
- oxygen ($\underline{O}H$)—four regions of electron density; tetrahedral

The local structures:

- nitrogen—three bonds, one lone pair; trigonal pyramidal
- carbon ($\underline{C}H_2$)—four bonds, no lone pairs; tetrahedral
- carbon ($\underline{C}O_2$)—three bonds (double bond counts as one bond), no lone pairs; trigonal planar
- oxygen ($\underline{O}H$)—two bonds, two lone pairs; bent (109°)

? Exercise 3.4.6

Another amino acid is alanine, which has the Lewis structure shown here. Predict the electron-pair geometry and local structure of the nitrogen atom, the three carbon atoms, and the oxygen atom with hydrogen attached:



A Lewis structure depicts a nitrogen atom with two lone pairs of electrons that is single bonded to two hydrogen atoms and a carbon atom, which is, in turn, single bonded to a hydrogen atom, a methyl group and another carbon atom. This carbon atom is single bonded to an oxygen atom with two lone pairs of electrons and single bonded to an oxygen that has two lone pairs of electrons and a single bond to a hydrogen atom.

Answer

> electron-pair geometries: nitrogen—tetrahedral; carbon ($\underline{C}H$)—tetrahedral; carbon ($\underline{C}H_3$)—tetrahedral; carbon ($\underline{C}O_2$)—trigonal planar; oxygen ($\underline{O}H$)—tetrahedral; local structures: nitrogen—trigonal pyramidal; carbon ($\underline{C}H$)—tetrahedral; carbon ($\underline{C}H_3$)—tetrahedral; carbon ($\underline{C}O_2$)—trigonal planar; oxygen ($\underline{O}H$)—bent (109°)

Link to Learning

The molecular shape simulator lets you build various molecules and practice naming their electron-pair geometries and molecular structures.

Example 3.4.7: Molecular Simulation

Using molecular shape simulator allows us to control whether bond angles and/or lone pairs are displayed by checking or unchecking the boxes under “Options” on the right. We can also use the “Name” checkboxes at bottom-left to display or hide the electron pair geometry (called “electron geometry” in the simulator) and/or molecular structure (called “molecular shape” in the simulator).

Build the molecule HCN in the simulator based on the following Lewis structure:

$$\mathrm{H{-}C{\equiv}N}$$

Click on each bond type or lone pair at right to add that group to the central atom. Once you have the complete molecule, rotate it to examine the predicted molecular structure. What molecular structure is this?

Solution

The molecular structure is linear.

Exercise 3.4.7

Build a more complex molecule in the simulator. Identify the electron-group geometry, molecular structure, and bond angles. Then try to find a chemical formula that would match the structure you have drawn.

Answer

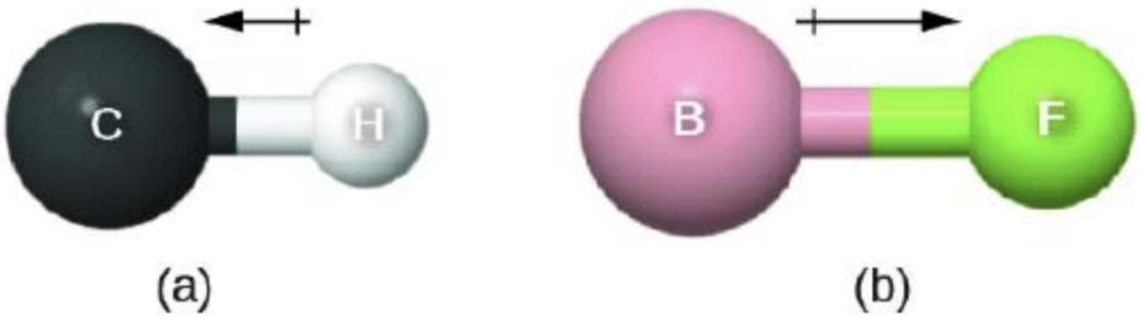
> Answers will vary. For example, an atom with four single bonds, a double bond, and a lone pair has an octahedral electron-group geometry and a square pyramidal molecular structure. $XeOF_4$ is a molecule that adopts this structure.

Molecular Polarity and Dipole Moment

As discussed previously, polar covalent bonds connect two atoms with differing electronegativities, leaving one atom with a partial positive charge (δ+) and the other atom with a partial negative charge (δ–), as the electrons are pulled toward the more electronegative atom. This separation of charge gives rise to a bond dipole moment. The magnitude of a bond dipole moment is represented by the Greek letter mu (μ) and is given by the formula shown here, where Q is the magnitude of the partial charges (determined by the electronegativity difference) and r is the distance between the charges:

$$\mu = Qr$$

This bond moment can be represented as a vector, a quantity having both direction and magnitude (Figure 3.4.13). Dipole vectors are shown as arrows pointing along the bond from the less electronegative atom toward the more electronegative atom. A small plus sign is drawn on the less electronegative end to indicate the partially positive end of the bond. The length of the arrow is proportional to the magnitude of the electronegativity difference between the two atoms.



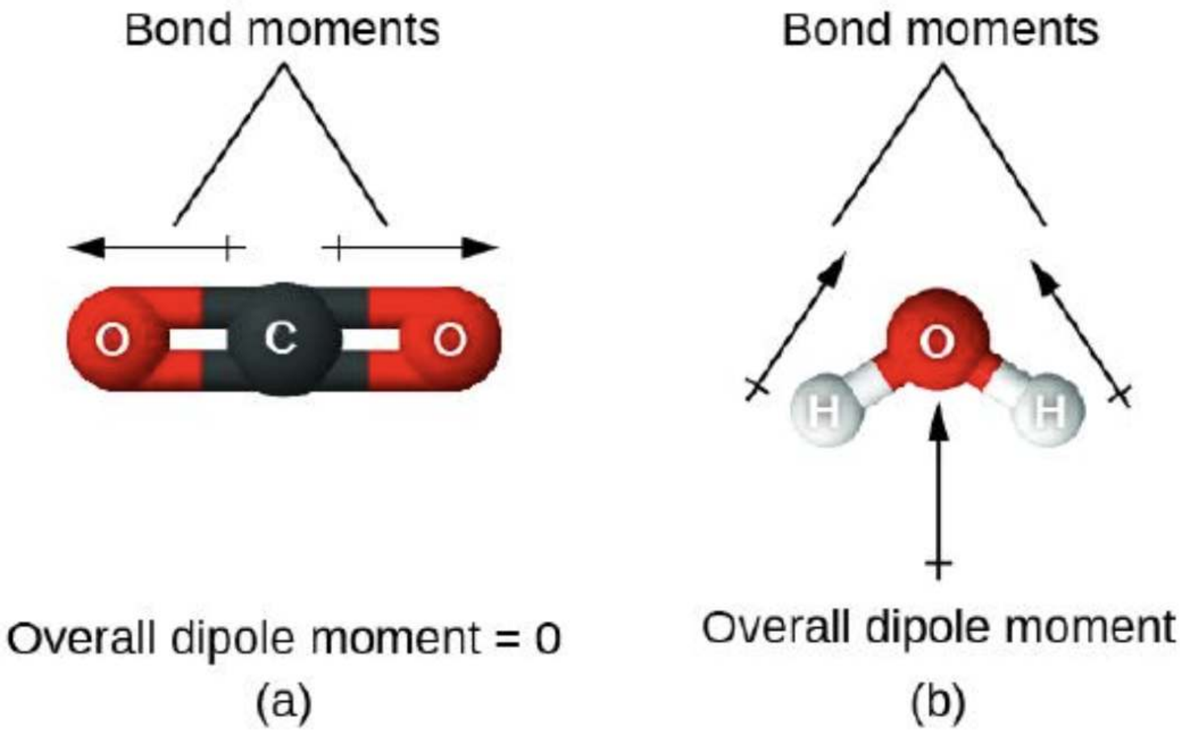
Two images are shown and labeled, “a” and “b.” Image a shows a large sphere labeled, “C,” a left-facing arrow with a crossed end, and a smaller sphere labeled “H.” Image b shows a large sphere labeled, “B,” a right-facing arrow with a crossed end, and a smaller sphere labeled “F.”

Figure 3.4.13: (a) There is a small difference in electronegativity between C and H, represented as a short vector. (b) The electronegativity difference between B and F is much larger, so the vector representing the bond moment is much longer.

A whole molecule may also have a separation of charge, depending on its molecular structure and the polarity of each of its bonds. If such a charge separation exists, the molecule is said to be a polar molecule (or dipole); otherwise the molecule is said to be nonpolar. The dipole moment measures the extent of net charge separation in the molecule as a whole. We determine the dipole moment by adding the bond moments in three-dimensional space, taking into account the molecular structure.

For diatomic molecules, there is only one bond, so its bond dipole moment determines the molecular polarity. Homonuclear diatomic molecules such as Br_2 and N_2 have no difference in electronegativity, so their dipole moment is zero. For heteronuclear molecules such as CO, there is a small dipole moment. For HF, there is a larger dipole moment because there is a larger difference in electronegativity.

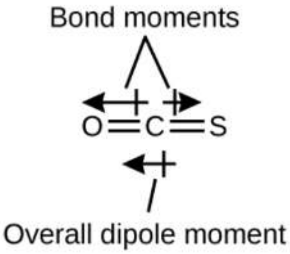
When a molecule contains more than one bond, the geometry must be taken into account. If the bonds in a molecule are arranged such that their bond moments cancel (vector sum equals zero), then the molecule is nonpolar. This is the situation in CO_2 (Figure 3.4.14). Each of the bonds is polar, but the molecule as a whole is nonpolar. From the Lewis structure, and using VSEPR theory, we determine that the CO_2 molecule is linear with polar C=O bonds on opposite sides of the carbon atom. The bond moments cancel because they are pointed in opposite directions. In the case of the water molecule (Figure 3.4.14), the Lewis structure again shows that there are two bonds to a central atom, and the electronegativity difference again shows that each of these bonds has a nonzero bond moment. In this case, however, the molecular structure is bent because of the lone pairs on O, and the two bond moments do not cancel. Therefore, water does have a net dipole moment and is a polar molecule (dipole).



Two images are shown and labeled, “a” and “b.” Image a shows a carbon atom bonded to two oxygen atoms in a ball-and-stick representation. Two arrows face away from the center of the molecule in opposite directions and are drawn horizontally like the molecule. These arrows are labeled, “Bond moments,” and the image is labeled, “Overall dipole moment equals 0.” Image b shows an oxygen atom bonded to two hydrogen atoms in a downward-facing v-shaped arrangement. An upward-facing, vertical arrow is drawn below the molecule while two upward and inward facing arrows are drawn above the molecule. The upper arrows are labeled, “Bond moments,” while the image is labeled, “Overall dipole moment.”

Figure 3.4.14: The overall dipole moment of a molecule depends on the individual bond dipole moments and how they are arranged. (a) Each CO bond has a bond dipole moment, but they point in opposite directions so that the net CO_2 molecule is nonpolar. (b) In contrast, water is polar because the OH bond moments do not cancel out.

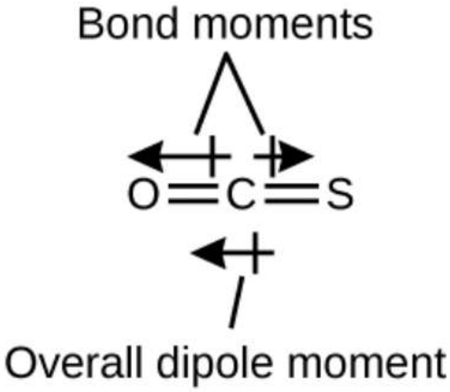
The OCS molecule has a structure similar to CO_2, but a sulfur atom has replaced one of the oxygen atoms. To determine if this molecule is polar, we draw the molecular structure. VSEPR theory predicts a linear molecule:



An image shows a carbon atom double bonded to a sulfur atom and an oxygen atom which are arranged in a horizontal plane. Two arrows face away from the center of the molecule in opposite directions and are drawn horizontally like the molecule. The left-facing arrow is larger than the right-facing arrow. These arrows are labeled, “Bond moments,” and a left-facing arrow below the molecule is labeled, “Overall dipole moment.”

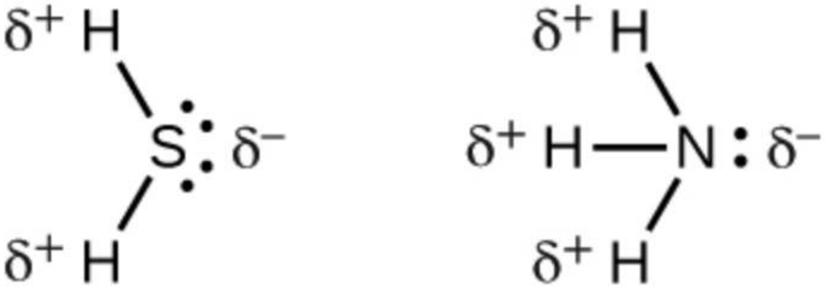
The C-O bond is considerably polar. Although C and S have very similar electronegativity values, S is slightly more electronegative than C, and so the C-S bond is just slightly polar. Because oxygen is more electronegative than sulfur, the oxygen end of the molecule is the negative end.

Chloromethane, CH_3Cl, is a tetrahedral molecule with three slightly polar C-H bonds and a more polar C-Cl bond. The relative electronegativities of the bonded atoms is H < C < Cl, and so the bond moments all point toward the Cl end of the molecule and sum to yield a considerable dipole moment (the molecules are relatively polar).



An image shows a carbon atom single bonded to three hydrogen atoms and a chlorine atom. There are arrows with crossed ends pointing from the hydrogen to the carbon near each bond, and one pointing from the carbon to the chlorine along that bond. The carbon and chlorine arrow is longer. This image uses dashes and wedges to give it a three-dimensional appearance.

For molecules of high symmetry such as BF_3 (trigonal planar), CH_4 (tetrahedral), PF_5 (trigonal bipyramidal), and SF_6 (octahedral), all the bonds are of identical polarity (same bond moment) and they are oriented in geometries that yield nonpolar molecules (dipole moment is zero). Molecules of less geometric symmetry, however, may be polar even when all bond moments are identical. For these molecules, the directions of the equal bond moments are such that they sum to give a nonzero dipole moment and a polar molecule. Examples of such molecules include hydrogen sulfide, H_2S (nonlinear), and ammonia, NH_3 (trigonal pyramidal).



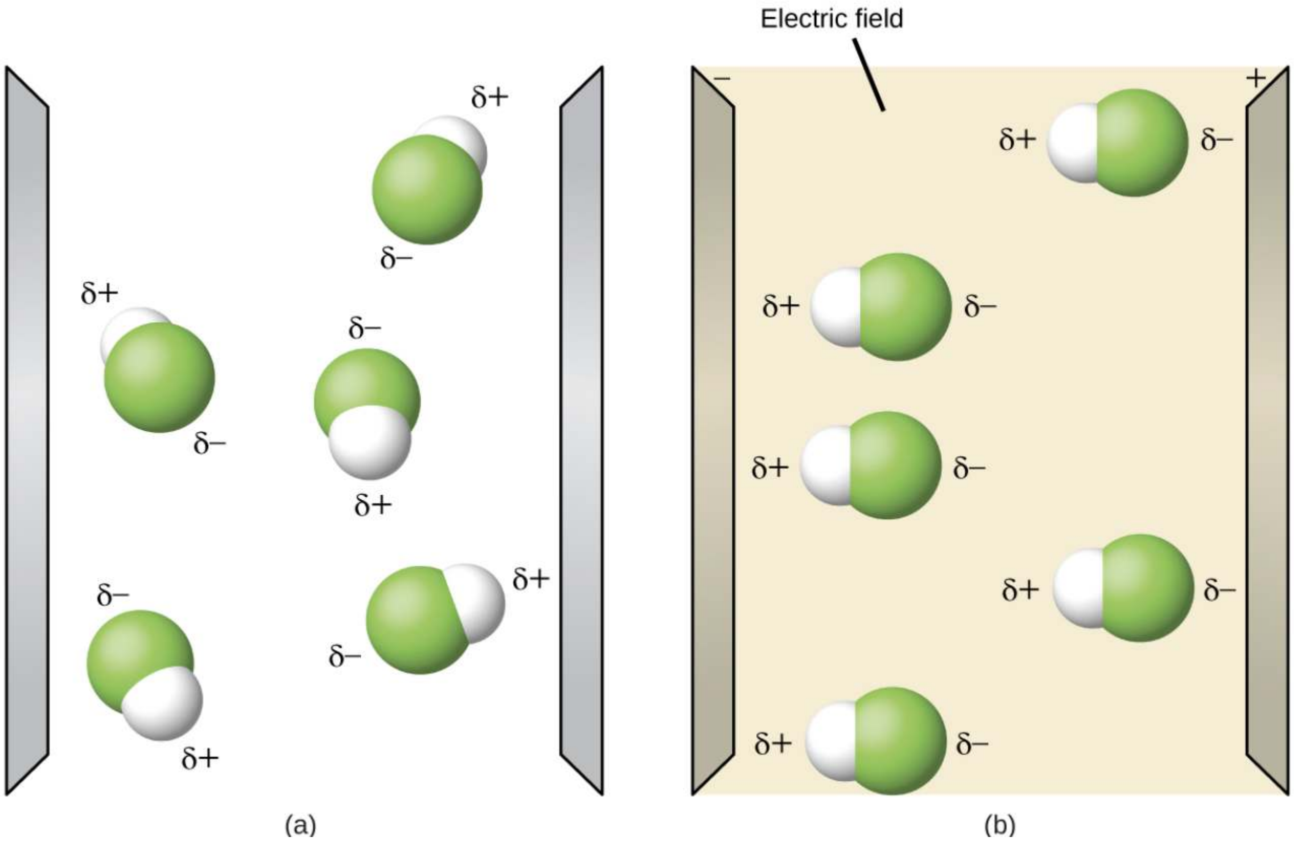
Two Lewis structures are shown. The left structure shows a sulfur atom with two lone pairs of electrons single bonded to two hydrogen atoms. Near the sulfur is a dipole symbol with a superscripted negative sign. Near each hydrogen is a dipole symbol with a superscripted positive sign. The right structure shows a nitrogen atom with one lone pair of electrons single bonded to three hydrogen atoms. Near the nitrogen is a dipole symbol with a superscripted negative sign. Near each hydrogen is a dipole symbol with a superscripted positive sign.

To summarize, to be polar, a molecule must:

1. Contain at least one polar covalent bond.
2. Have a molecular structure such that the sum of the vectors of each bond dipole moment does not cancel.

Properties of Polar Molecules

Polar molecules tend to align when placed in an electric field with the positive end of the molecule oriented toward the negative plate and the negative end toward the positive plate (Figure $3.4.15$). We can use an electrically charged object to attract polar molecules, but nonpolar molecules are not attracted. Also, polar solvents are better at dissolving polar substances, and nonpolar solvents are better at dissolving nonpolar substances.



Two diagrams are shown and labeled “a” and “b.” Diagram a shows two vertical, gray electrodes. There are five molecules in between. The molecules are separate from one another and are composed of a hydrogen atom bonded to a fluorine atom. The fluorine atom is labeled with a dipole symbol and a superscripted negative sign while the hydrogen atom is labeled with a dipole symbol and a superscripted positive sign. The molecules are randomly oriented in the space. The right diagram also shows two vertical gray electrodes, the left labeled as negative and the right labeled as positive. The space between is yellow. The same molecules are present, but this time they are all facing horizontally, with the hydrogen-end of each molecule facing toward the negative electrode.

Figure 3.4.15: (a) Molecules are always randomly distributed in the liquid state in the absence of an electric field. (b) When an electric field is applied, polar molecules like HF will align to the dipoles with the field direction.

Link to Learning

The molecule polarity simulation provides many ways to explore dipole moments of bonds and molecules.

Example 3.4.8: Polarity Simulations

Open the molecule polarity simulation and select the “Three Atoms” tab at the top. This should display a molecule ABC with three electronegativity adjustors. You can display or hide the bond moments, molecular dipoles, and partial charges at the right. Turning on the Electric Field will show whether the molecule moves when exposed to a field, similar to Figure 3.4.15.

Use the electronegativity controls to determine how the molecular dipole will look for the starting bent molecule if:

a. A and C are very electronegative and B is in the middle of the range.
b. A is very electronegative, and B and C are not.

Solution

a. Molecular dipole moment points immediately between A and C.
b. Molecular dipole moment points along the A–B bond, toward A.

Exercise 3.4.8

Determine the partial charges that will give the largest possible bond dipoles.

Answer

The largest bond moments will occur with the largest partial charges. The two solutions above represent how unevenly the electrons are shared in the bond. The bond moments will be maximized when the electronegativity difference is greatest. The controls for A and C should be set to one extreme, and B should be set to the opposite extreme. Although the magnitude of the bond moment will not change based on whether B is the most electronegative or the least, the direction of the bond moment will.

3.5: End of Chapter Activity

End of Chapter Activity: Creating a Lesson Plan on Atomic Theory and the Periodic Table with AI and Bloom's Taxonomy

Now that you have explored the fundamentals of atomic theory and the periodic table, it's time to put your knowledge into practice. Your task is to create a succinct lesson plan for high school students that introduces them to the basics of atomic theory and the periodic table. To help you with this, you will use AI tools and incorporate Bloom's Taxonomy to ensure a comprehensive learning experience. This lesson plan will go towards your digital notebook, a portfolio filled with lesson plans, activities, and labs for future use.

Activity Prompt:

Objective: Use AI and Bloom's Taxonomy to develop a lesson plan that effectively teaches high school students about atomic theory and the periodic table, including concepts such as the historical development of atomic theory, electron configuration, periodic trends, and chemical bonding.

Understanding the Concepts:

Knowledge (Remembering): Define key terms related to atomic theory, such as atom, proton, neutron, electron, electron configuration, periodic table, and chemical bonding.

Comprehension (Understanding): Explain these concepts in simple, age-appropriate language.

Planning the Lesson:

Application: Design an engaging activity or experiment that allows students to visualize and understand atomic structures and periodic trends. For example, use models or interactive simulations to demonstrate the structure of an atom and how elements are arranged on the periodic table.

Analysis: Use AI tools to create visual aids or interactive simulations that illustrate the concepts of atomic theory and the periodic table. For instance, create a simulation that shows the electron configuration of different elements and how this affects their chemical properties.

Deepening Understanding:

Synthesis (Creating): Ask students to predict the properties of unknown elements based on their position in the periodic table. For example, what properties would an element in group 2, period 3 exhibit?

Evaluation: Have students discuss and reflect on what they observed during the activities. Encourage them to think about how the structure of an atom influences its chemical behavior and the significance of periodic trends.

Using AI in the Classroom:

Explore AI tools like educational apps or platforms that provide interactive content for teaching atomic theory and the periodic table. Use these tools to create quizzes, flashcards, or interactive stories that reinforce the lesson's concepts.

Use AI to assess student understanding through formative assessments and provide instant feedback.

Deliverable:

Submit a detailed lesson plan that includes:

1. **A brief overview of the key concepts covered:** Outline the foundational concepts of atomic theory and the periodic table that will be taught.
2. **A description of the activities and experiments designed:** Detail the hands-on activities and experiments you will use to help students understand atomic structures and periodic trends.
3. **Examples of AI tools used and how they enhance the learning experience:** Describe the AI tools you plan to incorporate, such as simulations or interactive quizzes, and explain how they will help students grasp complex concepts.
4. **An explanation of how Bloom's Taxonomy was applied in the lesson plan to ensure a well-rounded educational experience:** Illustrate how each level of Bloom's Taxonomy (Remembering, Understanding, Applying, Analyzing, Creating, and Evaluating) is addressed in your lesson plan.

This activity will help you integrate modern technology and educational strategies to create an effective and engaging learning experience for high school students.

3.6: End of Chapter Key Terms

Atomic Theory and the Periodic Table Key Terms

1. **Atom**: The smallest unit of an element that retains the properties of that element, consisting of protons, neutrons, and electrons.
2. **Nucleus**: The dense central core of an atom, containing protons and neutrons.
3. **Proton**: A positively charged subatomic particle found in the nucleus of an atom.
4. **Neutron**: A neutral subatomic particle found in the nucleus of an atom.
5. **Electron**: A negatively charged subatomic particle that orbits the nucleus of an atom.
6. **Atomic Number (Z)**: The number of protons in the nucleus of an atom, which determines the element's identity.
7. **Mass Number (A)**: The total number of protons and neutrons in the nucleus of an atom.
8. **Isotope**: Atoms of the same element with the same number of protons but different numbers of neutrons.
9. **Atomic Mass Unit (amu)**: A unit of mass used to express atomic and molecular weights, approximately equal to the mass of a proton or neutron.
10. **Atomic Mass**: The weighted average mass of the isotopes of an element, typically expressed in atomic mass units (amu).
11. **Electron Cloud**: The region around the nucleus where electrons are likely to be found.
12. **Orbital**: A region of space around the nucleus where an electron is likely to be found, described by quantum numbers.
13. **Quantum Numbers**: Numbers that describe the properties of atomic orbitals and the properties of electrons in orbitals.
14. **Principal Quantum Number (n)**: Indicates the relative size and energy of atomic orbitals; larger nnn values correspond to higher energy levels.
15. **Angular Momentum Quantum Number (l)**: Describes the shape of an atomic orbital; values range from 0 to n−1n-1n−1.
16. **Magnetic Quantum Number (m_l)**: Describes the orientation of the orbital in space; values range from −l-l−l to +l+l+l.
17. **Spin Quantum Number (m_s)**: Describes the spin of an electron; can be +12+\frac{1}{2}+21 or −12-\frac{1}{2}−21.
18. **Electron Configuration**: The arrangement of electrons in an atom's orbitals, described by the distribution of electrons among energy levels and orbitals.
19. **Aufbau Principle**: States that electrons fill atomic orbitals of the lowest available energy levels before occupying higher levels.
20. **Pauli Exclusion Principle**: States that no two electrons in an atom can have the same set of four quantum numbers.
21. **Hund's Rule**: States that electrons will fill degenerate orbitals (orbitals of the same energy) singly before pairing up.
22. **Periodic Table**: A tabular arrangement of elements in order of increasing atomic number, with elements having similar properties placed in vertical columns (groups).
23. **Period**: A horizontal row in the periodic table, where elements have the same number of electron shells.
24. **Group (Family)**: A vertical column in the periodic table, where elements have similar chemical properties and the same number of valence electrons.
25. **Valence Electrons**: Electrons in the outermost shell of an atom, which determine the chemical properties and reactivity of the element.
26. **Alkali Metals**: Elements in Group 1 of the periodic table (excluding hydrogen), characterized by having one valence electron.
27. **Alkaline Earth Metals**: Elements in Group 2 of the periodic table, characterized by having two valence electrons.
28. **Transition Metals**: Elements in Groups 3-12 of the periodic table, characterized by having d electrons.
29. **Halogens**: Elements in Group 17 of the periodic table, characterized by having seven valence electrons.
30. **Noble Gases**: Elements in Group 18 of the periodic table, characterized by having a full valence shell of electrons, making them very stable and unreactive.
31. **Lanthanides**: A series of elements from atomic numbers 57 to 71, characterized by filling of 4f orbitals.
32. **Actinides**: A series of elements from atomic numbers 89 to 103, characterized by filling of 5f orbitals.
33. **Metals**: Elements that are typically shiny, conductive, malleable, and ductile.
34. **Nonmetals**: Elements that are typically not shiny, poor conductors, and brittle in solid form.
35. **Metalloids**: Elements with properties intermediate between metals and nonmetals.
36. **Atomic Radius**: The distance from the nucleus to the outermost electron shell of an atom.
37. **Ionization Energy**: The energy required to remove an electron from a gaseous atom or ion.
38. **Electron Affinity**: The energy change that occurs when an electron is added to a neutral atom in the gas phase.

39. **Electronegativity**: A measure of the ability of an atom in a chemical compound to attract electrons.
40. **Periodic Trends**: Patterns observed in the periodic table for properties such as atomic radius, ionization energy, electron affinity, and electronegativity.

CHAPTER OVERVIEW

4: Phases and Classification of Matter

4.1: Introduction and Learning Objectives

Chapter Learning Objectives

1. **Understanding the Classification of Matter:**
    - Grasp the basic concepts of matter and its classification into pure substances (elements and compounds) and mixtures (homogeneous and heterogeneous).
    - Learn to distinguish between elements, compounds, and mixtures based on their composition and properties.
    - Explore the methods of separating mixtures into their component substances, such as filtration, distillation, and chromatography.
2. **Phases of Matter and Phase Changes:**
    - Understand the four fundamental states of matter: solid, liquid, gas, and plasma, and the unique characteristics of each phase.
    - Examine the processes involved in phase changes, including melting, freezing, vaporization, condensation, sublimation, and deposition.
    - Analyze the energy changes associated with phase transitions and the concept of latent heat.
3. **Physical and Chemical Properties of Matter:**
    - Identify and differentiate between physical properties (such as density, boiling point, and melting point) and chemical properties (such as reactivity and flammability).
    - Understand how physical and chemical properties are used to identify and classify different types of matter.
    - Explore the concept of intensive and extensive properties and their significance in the study of matter.
4. **Intermolecular Forces and Their Effects on Matter:**
    - Investigate the different types of intermolecular forces (such as ionic, covalent, hydrogen bonding, dipole-dipole interactions, and Van der Waals forces) and their impact on the properties of substances.
    - Understand the role of intermolecular forces in determining the state of matter, solubility, and boiling/melting points of substances.
    - Explore real-world examples where intermolecular forces play a crucial role, such as in solutions, colloids, and biological structures.
5. **Teaching Strategies for Phases and Classification of Matter:**
    - Develop effective teaching methods for conveying the concepts of matter classification and phase changes to students, using practical examples and age-appropriate language.
    - Design classroom activities and experiments to help students understand and visualize the properties of different states of matter and the processes involved in phase changes.
    - Utilize multimedia tools and demonstrations to provide diverse learning experiences and reinforce the understanding of matter's properties and classification.
    - Address common misconceptions and challenges students face in learning about matter and its phases, and provide strategies for clarification.
    - Emphasize the interdisciplinary connections of these concepts, linking them to everyday life and various scientific and industrial applications.

Introduction to Phases & Classification of Matter a Scientific Overview

Matter, the substance of the universe, exists in various forms and can be classified based on its physical and chemical properties. This chapter will guide you through the fundamental concepts of matter classification, the different phases of matter, their properties, and the forces that govern their behavior. We will begin by exploring how matter is categorized into pure substances and mixtures, delve into the characteristics and transitions between different phases of matter, and investigate the physical and chemical properties that define various substances. Additionally, we will examine the role of intermolecular forces in determining the behavior of matter and provide insights on effective strategies for teaching these concepts to K-12 students.

Understanding the Classification of Matter

Matter can be broadly classified into pure substances and mixtures. Pure substances are further divided into elements, which consist of a single type of atom, and compounds, which are composed of two or more elements chemically bonded together. Mixtures, on the other hand, can be homogeneous (uniform composition throughout) or heterogeneous (distinct regions of different composition). Learning to distinguish between elements, compounds, and mixtures based on their composition and properties is crucial. Various methods, such as filtration, distillation, and chromatography, are used to separate mixtures into their component substances.

Phases of Matter and Phase Changes

Matter exists in four fundamental states: solid, liquid, gas, and plasma, each with unique characteristics. Solids have a fixed shape and volume, liquids have a fixed volume but take the shape of their container, gases have neither a fixed shape nor volume, and plasma is an ionized state of matter found in stars. Phase changes, including melting, freezing, vaporization, condensation, sublimation, and deposition, involve the transformation of matter from one state to another. These transitions are accompanied by energy changes, specifically latent heat, which is the energy absorbed or released during a phase change without changing the temperature of the substance.

Physical and Chemical Properties of Matter

The properties of matter are classified into physical and chemical properties. Physical properties, such as density, boiling point, and melting point, can be observed without changing the substance's chemical identity. Chemical properties, such as reactivity and flammability, describe how a substance interacts with other substances to form new compounds. Understanding these properties is essential for identifying and classifying different types of matter. Additionally, properties are categorized as intensive (independent of the amount of matter) or extensive (dependent on the amount of matter), which helps in the study and application of various materials.

Intermolecular Forces and Their Effects on Matter

Intermolecular forces, including ionic, covalent, hydrogen bonding, dipole-dipole interactions, and Van der Waals forces, play a significant role in determining the properties of substances. These forces influence the state of matter, solubility, and boiling/melting points of substances. For instance, hydrogen bonding is responsible for the high boiling point of water, and Van der Waals forces affect the behavior of gases. Real-world examples, such as the formation of solutions, colloids, and the structure of biological molecules, illustrate the importance of intermolecular forces in everyday life and various scientific fields.

Teaching Strategies for Phases and Classification of Matter

Teaching the concepts of matter classification and phase changes to K-12 students can be made effective and engaging through various strategies. Practical examples and age-appropriate language help convey these complex ideas. Classroom activities and experiments, such as observing the melting and freezing of ice or separating mixtures using filtration, help students understand and visualize the properties and transitions of different states of matter. Utilizing multimedia tools and demonstrations provides diverse learning experiences, reinforcing the understanding of matter's properties and classification. Addressing common misconceptions and challenges students face, such as confusing physical and chemical changes, is crucial. Emphasizing the interdisciplinary connections of these concepts links them to everyday life and various scientific and industrial applications, making the learning experience more relevant and impactful.

By incorporating these strategies, educators can foster a deeper understanding of the phases and classification of matter, preparing students for further studies in science and technology.

4.2: Classification and Properties of Matter

Learning Objectives

- Use physical and chemical properties, including phase, to describe matter.
- Distinguish between physical and chemical changes.
- Identify a sample of matter as an element, a compound, or a mixture (homogeneous/heterogeneous).
- Distinguish between organic and inorganic chemicals.
- Label all atoms within an organic or inorganic compound.
- Recognize physical methods used to separate mixtures.

Physical and Chemical Properties

The properties that chemists use to describe matter fall into two general categories. Physical properties are characteristics that **describe** matter. They include characteristics such as size, shape, color, and mass. Many of these properties can be quantitative in nature. For example, quantitative physical properties of water would be the boiling point (100 °C / 212 °F) and melting point (0°C / 32 °F).

Physical Properties

- Can be observed or measured without changing the matter's identity.

Examples of Physical Properties

Thermal conductivity (KAHN duhk TIV uh tee) is the rate at which a substance transfers heat. Plastic foam is a poor conductor.

State is the physical form in which a substance exists, such as a solid, liquid, or gas. Ice is water in the solid state.

Density is the mass per unit volume of a substance. Lead is very dense, so it makes a good sinker for a fishing line.

Solubility (SAHL yoo BIL uh tee) is the ability of a substance to dissolve in another substance. Flavored drink mix dissolves in water.

Ductility (duhk TIL uh tee) is the ability of a substance to be pulled into a wire. Copper is often used to make wiring because it is ductile.

Malleability (MAL ee uh BIL uh tee) is the ability of a substance to be rolled or pounded into thin sheets. Aluminum can be rolled into sheets to make foil.

Figure 4.2.1: Physical Properties. (Copyright;https://www.slideshare.net/cfoltz/ph...cal-properties)

Chemical properties are characteristics that describe how matter **changes** its chemical structure or composition. An example of a chemical property is flammability—a material's ability to burn—because burning (also known as combustion) changes the chemical composition of a material. Oxidation, rusting, decomposition, and inertness are chemical properties as well. Click on this video and record the physical and chemical properties of the element sodium.

Figure 4.2.2: Port Royal, South Carolina oxidation of iron. Image courtesy of Elizabeth R. Gordon.

Elements and Compounds

Any sample of matter that has the same physical and chemical properties throughout the sample is called a substance. There are two types of substances. A substance that cannot be broken down into chemically simpler components is an element. Aluminum, which is used in soda cans, is an element. A substance that can be broken down into chemically simpler components (because it has more than one element) is a compound (Figure $4.2.1$). Water is a compound composed of the elements hydrogen and oxygen. Today, there are 118 elements in the known universe. In contrast, scientists have identified tens of millions of different compounds to date.

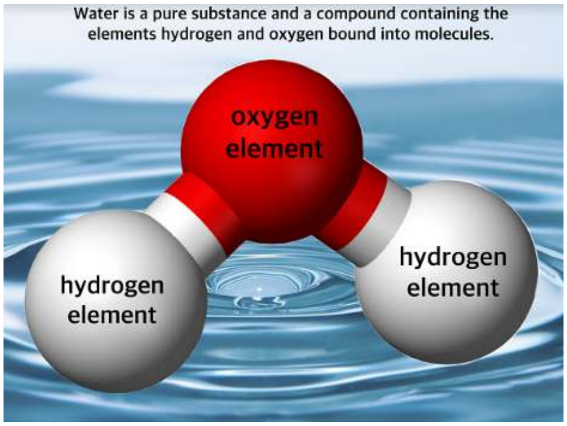


Figure $4.2.3$: To a chemist, the pure substance water is a compound (it contains two kinds of atoms bound to one another in just one kind of molecule). An environmentalist might consider water "pure" even if it contains the normal amount of dissolved oxygen and carbon dioxide, but no other "pollutants". To a chemist, water containing oxygen is no longer a pure substance, but a mixture. In pure water, the ratio of hydrogen to oxygen atoms is always 2:1.

> Sometimes the word *pure* is added to a substance, but this is not absolutely necessary. By definition, any single substance is pure.

The smallest part of an element that maintains the identity of that element is called an atom. Atoms are extremely tiny; to make a line 1 inch long, you would need 217 million iron atoms. The smallest part of a compound that maintains the identity of that compound is called a molecule. Molecules are composed of atoms that are attached together and behave as a unit. Scientists usually work with millions and millions of atoms and molecules at a time. When a scientist is working with large numbers of atoms or molecules at a time, the scientist is studying the macroscopic view of the universe.

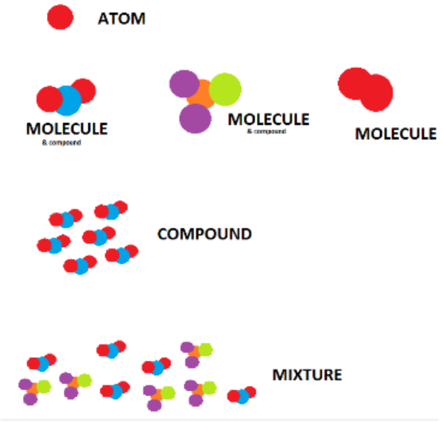


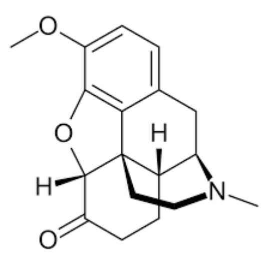
Figure $4.2.4$: Appreciating elements, molecules and compounds.

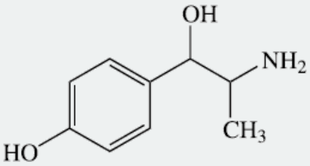
Inorganic and Organic Compounds

Compounds can be further classified as being inorganic or organic. Organic compounds contain carbon, hydrogen, and usually another element from the right side of the periodic table. Be careful when thinking about organic chemicals. In the real world, the term "organic" is typically used to describe products that have natural ingredients. In the world of science, the word "organic" is used to denote compounds that have carbon and hydrogen connections. Look at some of the structures below to get an appreciation of organic chemical structures.

Organic structures

The two molecules shown below are heavily used in the pharmaceutical industry. The compound on the left is known as hydrocodone. This is an opioid painkiller that is extremely addictive. It is regularly dispensed for surgical and chronic pain. The compound on the right is Adderall. This medication is used to treat ADHD and narcolepsy. Both of these medications are controlled by the DEA (Drug Enforcement Agency) and are illegal to possess unless you have a valid prescription.





Mixtures

A material composed of two or more substances is a mixture. In a mixture, the individual substances maintain their chemical identities. Many mixtures are obvious combinations of two or more substances, such as a mixture of sand and water. Such mixtures are called **heterogeneous mixtures**. In some mixtures, the components are so intimately combined that they act like a single substance (even though they are not). Mixtures with a consistent composition throughout are called **homogeneous mixtures** (or **solutions**). Sugar dissolved in water is an example of a solution. A metal alloy, such as steel, is an example of a solid solution. Air, a mixture of mainly nitrogen and oxygen, is a gaseous solution.

Exercise 4.2.1

How would a chemist categorize each example of matter?

a. saltwater
b. soil
c. water
d. oxygen

Solution

a. Saltwater acts as if it were a single substance even though it contains two substances—salt and water. Saltwater is a homogeneous mixture, or a solution.
b. Soil is composed of small pieces of a variety of materials, so it is a heterogeneous mixture.
c. Water is a substance; more specifically, because water is composed of hydrogen and oxygen, it is a compound.
d. Oxygen, a substance, is an element.

Exercise 4.2.2

How would a chemist categorize each example of matter?

a. coffee
b. hydrogen
c. an egg

Answer a

Coffee could be a heterogeneous or homogeneous mixture. If insoluble parts are able to be identified, then it would be classified as being heterogeneous. If not, then coffee would be a homogeneous mixture or a solution.

Answer b

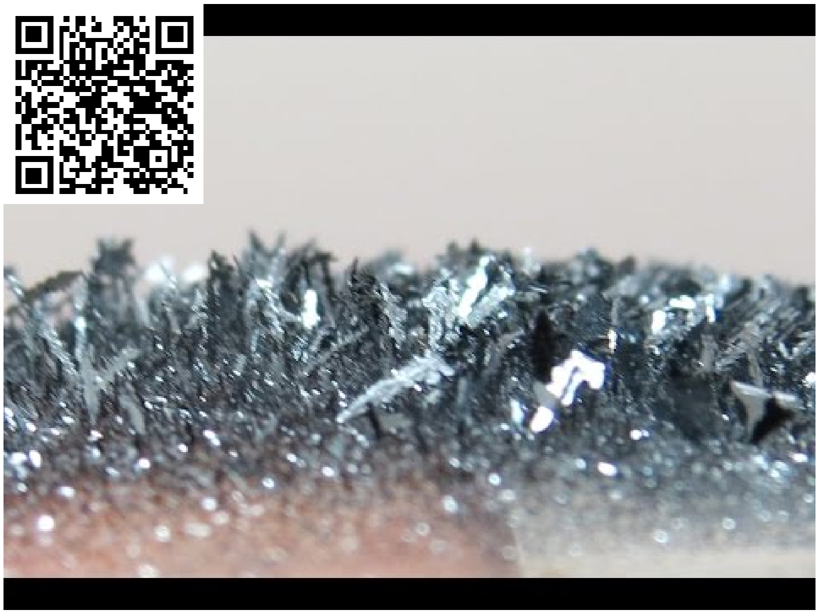
Hydrogen is an element. It can be located on the periodic table.

Answer c

An egg (assume raw) would be a heterogeneous mixture. The yolk and egg white are two different substances that can be easily seen.

Phases

Another way to classify matter is to describe it as a solid, a liquid, or a gas, which was done in the examples of solutions. These three descriptions, each implying that the matter has certain physical properties, represent the three phases of matter. A solid has a definite shape and a definite volume. Liquids ordinarily have a definite volume but not a definite shape; they take the shape of their containers. Gases have neither a definite shape nor a definite volume, and they expand to fill their containers. We encounter matter in each phase every day; in fact, we regularly encounter water in all three phases: ice (solid), water (liquid), and steam (gas).



Video4.2.35: Sublimation and deposition of Iodine.

We know from our experience with water that substances can change from one phase to another if the conditions are right. Typically, varying the temperature of a substance (and, less commonly, the pressure exerted on it) can cause a phase change, a physical process in which a substance goes from one phase to another (Figure 4.2.3). Phase changes have particular names depending on what phases are involved, as summarized in Table 4.2.1.

Table 4.2.1: Phase Changes

Change	Name
solid to liquid	melting, fusion
solid to gas	sublimation
liquid to gas	boiling, evaporation
liquid to solid	solidification, freezing
gas to liquid	condensation
gas to solid	deposition

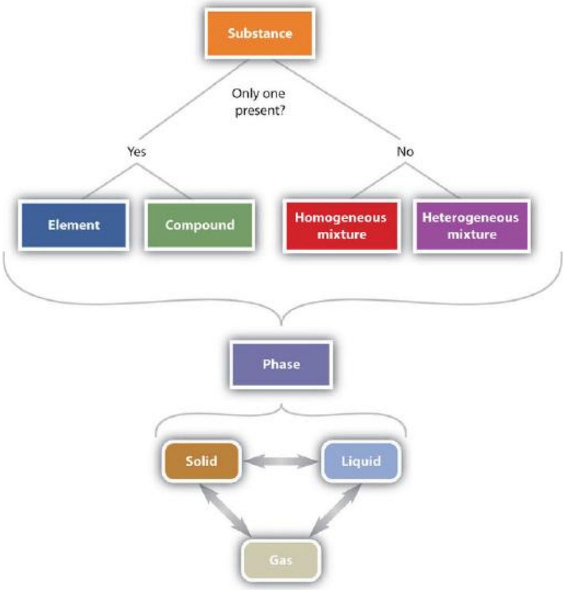


Figure 4.2.6: The Classification of Matter. Matter can be classified in a variety of ways, depending on its properties. (Cc BY-NC-SA; anonymous)

? Exercise 4.2.3

a. Classify each as being element, compound, heterogeneous mixture, or homogeneous mixture (solution): air at Furman University, Furman Lake water, copper wire, and Furman tap water.
b. What is the difference between a heterogeneous mixture and a homogeneous mixture? Give an example of each.
c. Which is/are physical change(s): burning of fuel, digestion of food, salt dissolving in water, copper conducting electricity, and water boiling.

Answer a

Hopefully, Furman air is a homogenous mixture (solution). If there is no burning or dust build-up in the area, this is safe to assume. Furman Lake is definitely a heterogeneous mixture (in some areas it can get a bit swampy). Copper wire is an element. There are very few elements in nature. Lastly, Furman tap water is usually a homogeneous mixture. Natural and man-made chemicals are in our tap water (trust me, you want the chlorine). Our toilets do use greywater which can look heterogeneous (ok for sewer but not to drink).

Answer b

A heterogeneous mixture is obviously a mixture, such as dirt; a homogeneous mixture behaves like a single substance, such as saltwater.

Answer c

The physical changes are salt dissolving water, copper conducting, and water boiling. If composition is not changed, then this is a physical change. If there is a method to physically separate the components (like boiling, evaporation, distillation, magnetism, or filtration), then a physical change has occurred.

Contributors and Attributions

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4.3: Pure Substances and Mixtures

Learning Objective

- Distinguish between pure substances and mixtures
- Identify mixtures as homogeneous or heterogeneous

Pure Substances

When we speak of a **pure substance**, we are speaking of something that contains only one kind of matter. This can either be one single element or one single compound, but every sample of this substance that you examine must contain exactly the same thing with a fixed, definite set of properties.

Pure Substance	Element or Compound?	Consists of:
Lead (Pb)	element	lead atoms
Oxygen gas (O_2)	element	oxygen molecules*
Water (H_2O)	compound	water molecules
Ammonia (NH_3)	compound	ammonia molecules

> *Note: pure oxygen gas consists of molecules but it is still considered an element, rather than a compound, as the molecules are made up of a single type of element. Compounds are made up of one or more element.

Mixtures

If we take two or more pure substances and mix them together, we refer to this as a **mixture.** Mixtures can always be separated again into component pure substances, because *bonding* among the atoms of the constituent substances does not occur in a mixture. Whereas a compound may have very different properties from the elements that compose it, in mixtures the substances keep their individual properties. For example sodium is a soft shiny metal and chlorine is a pungent green gas. These two elements can combine to form the compound, sodium chloride (table salt) which is a white, crystalline solid having *none* of the properties of either sodium or chlorine. If, however, you *mixed* table salt with ground pepper, you would still be able to see the individual grains of each of them and, if you were patient, you could take tweezers and carefully separate them back into pure salt and pure pepper.

Heterogeneous mixture

A **heterogeneous mixture** is a mixture in which the composition is not uniform throughout the mixture. Vegetable soup is a heterogeneous mixture. Any given spoonful of soup will contain varying amounts of the different vegetables and other components of the soup.

Homogeneous mixture/ Solution

A **homogeneous mixture** is combination of two or more substances that are so intimately mixed that the mixture behaves as a single substance. Another word for a homogeneous mixture is solution. Thus, a combination of salt and steel wool is a heterogeneous mixture because it is easy to see which particles of the matter are salt crystals and which are steel wool. On the other hand, if you take salt crystals and dissolve them in water, it is very difficult to tell that you have more than one substance present just by looking—even if you use a powerful microscope. The salt dissolved in water is a homogeneous mixture, or a solution (Figure 4.3.3).

***Figure 4.3.3:** Types of Mixtures © Thinkstock On the left, the combination of two substances is a heterogeneous mixture because the particles of the two components look different. On the right, the salt crystals have dissolved in the water so finely that you cannot tell that salt is present. The homogeneous mixture appears like a single substance.*

✓ Example 4.3.3

Identify the following combinations as heterogeneous mixtures or homogenous mixtures.

a. soda water (Carbon dioxide is dissolved in water.)
b. a mixture of iron metal filings and sulfur powder (Both iron and sulfur are elements.)

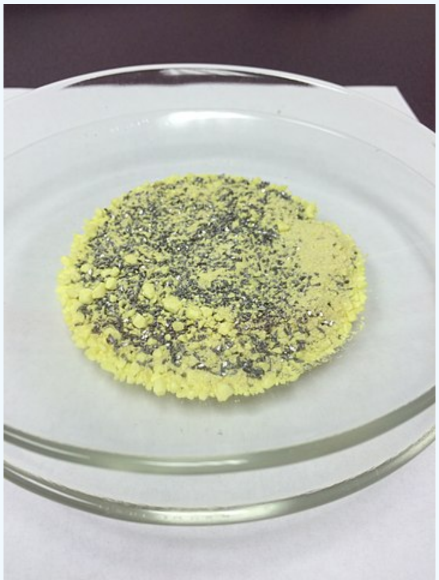

***Figure 4.3.4:** A mixture of iron filings and sulfur powder* (Asoult, Fe-S mixture 03, CC BY 4.0)

Solution

a. Because carbon dioxide is dissolved in water, we can infer from the behavior of salt crystals dissolved in water that carbon dioxide dissolved in water is (also) a homogeneous mixture.
b. Assuming that the iron and sulfur are simply mixed together, it should be easy to see what is iron and what is sulfur, so this is a heterogeneous mixture.

Exercise 4.3.3

Are the following combinations homogeneous mixtures or heterogeneous mixtures?

a. the human body
b. an amalgam, a combination of some other metals dissolved in a small amount of mercury

Answers

a. heterogeneous mixture
b. homogeneous mixture

Categorizing Matter

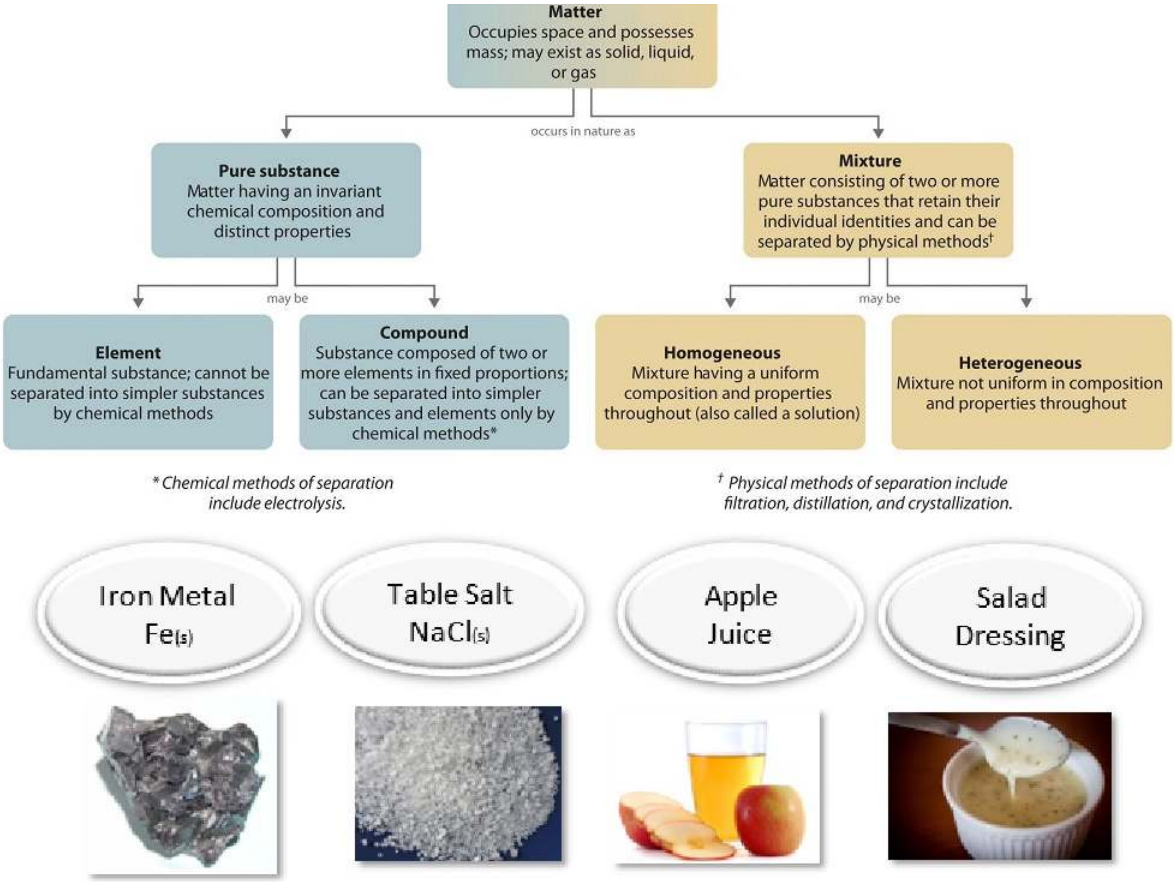

Figure 4.3.1: Relationships between the Types of Matter and the Methods Used to Separate Mixtures

Ordinary table salt is called sodium chloride. It is considered a **substance** because it has a uniform and definite composition. All samples of sodium chloride are chemically identical. Water is also a pure substance. Salt easily dissolves in water, but salt water cannot be classified as a substance because its composition can vary. You may dissolve a small amount of salt or a large amount into a given amount of water. A mixture is a physical blend of two or more components, each of which retains its own identity and properties in the **mixture**. Only the form of the salt is changed when it is dissolved into water. It retains its composition and properties.

Phase

A phase is any part of a sample that has a uniform composition and properties. By definition, a pure substance or a homogeneous mixture consists of a single phase. A heterogeneous mixture consists of two or more phases. When oil and water are combined, they do not mix evenly, but instead form two separate layers. Each of the layers is called a phase.

✓ Example 4.3.1

Identify each substance as a compound, an element, a heterogeneous mixture, or a homogeneous mixture (solution).

a. filtered tea
b. freshly squeezed orange juice
c. a compact disc
d. aluminum oxide, a white powder that contains a 2:3 ratio of aluminum and oxygen atoms
e. selenium

Given: a chemical substance

Asked for: its classification

Strategy:

A. Decide whether a substance is chemically pure. If it is pure, the substance is either an element or a compound. If a substance can be separated into its elements, it is a compound.
B. If a substance is not chemically pure, it is either a heterogeneous mixture or a homogeneous mixture. If its composition is uniform throughout, it is a homogeneous mixture.

Solution:

a. **A)** Tea is a solution of compounds in water, so it is not chemically pure. It is usually separated from tea leaves by filtration.
   B) Because the composition of the solution is uniform throughout, it is ***a homogeneous mixture***.
b. **A)** Orange juice contains particles of solid (pulp) as well as liquid; it is not chemically pure.
   B) Because its composition is not uniform throughout, orange juice is ***a heterogeneous mixture***.
c. **A)** A compact disc is a solid material that contains more than one element, with regions of different compositions visible along its edge. Hence a compact disc is not chemically pure.
   B) The regions of different composition indicate that a compact disc is ***a heterogeneous mixture.***
d. **A)** Aluminum oxide is a single, chemically ***pure compound***.
e. **A)** Selenium is one of the known ***elements***.

? Exercise 4.3.1

Identify each substance as a compound, an element, a heterogeneous mixture, or a homogeneous mixture (solution).

a. white wine
b. mercury
c. ranch-style salad dressing
d. table sugar (sucrose)

Answer a:

homogeneous mixture (solution)

Answer b:

element

Answer c:

heterogeneous mixture

Answer d:

compound

✓ Example 4.3.2

How would a chemist categorize each example of matter?

a. saltwater
b. soil
c. water
d. oxygen

Solution

a. Saltwater acts as if it were a single substance even though it contains two substances—salt and water. Saltwater is a homogeneous mixture, or a solution.
b. Soil is composed of small pieces of a variety of materials, so it is a heterogeneous mixture.
c. Water is a substance; more specifically, because water is composed of hydrogen and oxygen, it is a compound.
d. Oxygen, a substance, is an element.

? Exercise 4.3.2

How would a chemist categorize each example of matter?

a. coffee
b. hydrogen
c. an egg

Answer a:

a homogeneous mixture (solution), assume it's filtered coffee

Answer b:

element

Answer c:

heterogeneous mixture.

Summary

Matter can be classified into two broad categories: pure substances and mixtures. A pure substance is a form of matter that has a constant composition and properties that are constant throughout the sample. Mixtures are physical combinations of two or more elements and/or compounds. Mixtures can be classified as homogeneous or heterogeneous. Elements and compounds are both examples of pure substances. Compounds are substances that are made up of more than one type of atom. Elements are the simplest substances made up of only one type of atom.

Key Takeaways

- Pure substances are composed of a single element or compounds.
- Combinations of different substances are called mixtures.
- Homogeneous mixtures are mixtures of two or more compounds (or elements) that are **not** visually distinguishable from each other.
- Heterogeneous mixtures are mixtures of two or more compounds (or elements) that **are** visually distinguishable from one another.

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4.4: Separating Mixtures through Physcial Changes

There are many techniques that use physical changes to separate mixtures. Here are a number of common separation techniques:

Distillation

Distillation is an effective method to separate mixtures comprised of two or more pure liquids. Distillation is a purification process where one component of a liquid mixture is vaporized and then condensed and isolated. In simple distillation Figure 4.4.1, a mixture is heated and the most volatile component vaporizes at the lowest temperature. The vapor passes through a cooled tube (a condenser), where it condenses back into its liquid state. The condensate that is collected is called distillate.

Outside the chemistry lab, distillation is commonly used to increase the alcohol content of liquors such as vodka, whiskey, and brandy.

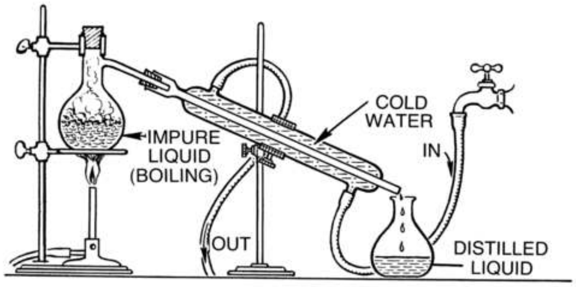


Figure 4.4.1 *Distillation apparatus.*

Evaporation

Evaporation is a technique used to separate out a soluble solid from a liquid. The method drives off the liquid components from the solid components. The process typically involves heating the mixture until no more liquid remains. Evaporation can be used, for example, to obtain table salt from sea water, Figure 4.4.2. The heat for the process comes from the sun.

Figure 4.4.2 *Once the sea water in these evaporation ponds has evaporated, the salt can be harvested.*

Filtration

Filtration is a separation method used to separate out pure substances in mixtures comprised of particles, some of which are large enough in size to be captured with a porous material. Particle size can vary considerably, given the type of mixture. For instance, stream water is a mixture that contains naturally occurring biological organisms like bacteria, viruses, and protozoans. Some water filters can filter out bacteria, the length of which is on the order of 1 micron. Other mixtures, like soil, have relatively large particle sizes, which can be filtered through something like a coffee filter.

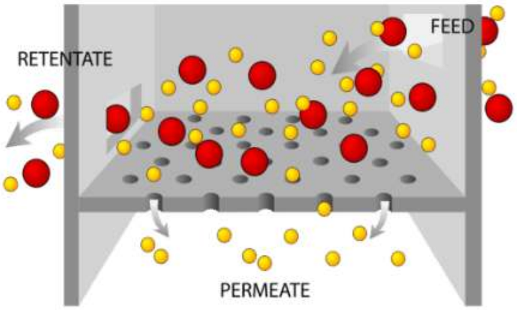


Figure 4.4.3 *Filtration.*

Chromatography

Chromatography is the separation of a liquid or gas mixture by passing it through a medium in which the components move at different rates. Thin-layer chromatography is perhaps the simplist form of chromatography. It uses a plate--a plastic or glass slide covered on one side with silica (essentially very fine, purified sand), alumina, or some other solid. The plate is placed so that the bottom of it is submerged in a solvent, and the solvent then moves upward on the plate. If a drop of a mixture is placed a little above the level of the solvent, different components of the mixture will be carried upward at different rates depending on how strongly they interact with the silica. Figure 4.4.4 shows separation of the chromophores (colored compounds) in a spinach leaf by thin layer chromatography.The pencil line at the bottom of the plate shows the original position of the mixture before the solvent started traveling up the plate.

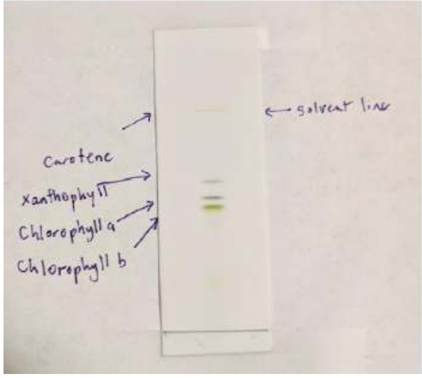


Figure 4.4.4 Separation of compounds in a spinch leaf. (photo credit: Heather Coleman)

4.5: Colligative Properties

Learning Objectives

- Express concentrations of solution components using mole fraction and molality
- Describe the effect of solute concentration on various solution properties (vapor pressure, boiling point, freezing point, and osmotic pressure)
- Perform calculations using the mathematical equations that describe these various colligative effects
- Describe the process of distillation and its practical applications
- Explain the process of osmosis and describe how it is applied industrially and in nature

The properties of a solution are different from those of either the pure solute(s) or solvent. Many solution properties are dependent upon the chemical identity of the solute. Compared to pure water, a solution of hydrogen chloride is more acidic, a solution of ammonia is more basic, a solution of sodium chloride is more dense, and a solution of sucrose is more viscous. There are a few solution properties, however, that depend *only* upon the total concentration of solute species, regardless of their identities. These colligative properties include vapor pressure lowering, boiling point elevation, freezing point depression, and osmotic pressure. This small set of properties is of central importance to many natural phenomena and technological applications, as will be described in this module.

Mole Fraction and Molality

Several units commonly used to express the concentrations of solution components were introduced in an earlier chapter of this text, each providing certain benefits for use in different applications. For example, molarity (M) is a convenient unit for use in stoichiometric calculations, since it is defined in terms of the molar amounts of solute species:

$$M=\frac{\text{mol solute}}{\text{L solution}} \tag{4.5.1}$$

Because solution volumes vary with temperature, molar concentrations will likewise vary. When expressed as molarity, the concentration of a solution with identical numbers of solute and solvent species will be different at different temperatures, due to the contraction/expansion of the solution. More appropriate for calculations involving many colligative properties are mole-based concentration units whose values are not dependent on temperature. Two such units are *mole fraction* (introduced in the previous chapter on gases) and *molality*.

The mole fraction, χ, of a component is the ratio of its molar amount to the total number of moles of all solution components:

$$\chi_{\text{A}}=\frac{\text{mol A}}{\text{total mol of all components}} \tag{4.5.2}$$

Molality is a concentration unit defined as the ratio of the numbers of moles of solute to the mass of the solvent in kilograms:

$$m=\frac{\text{mol solute}}{\text{kg solvent}} \tag{4.5.3}$$

Since these units are computed using only masses and molar amounts, they do not vary with temperature and, thus, are better suited for applications requiring temperature-independent concentrations, including several colligative properties, as will be described in this chapter module.

✓ Example 4.5.1: Calculating Mole Fraction and Molality

The antifreeze in most automobile radiators is a mixture of equal volumes of ethylene glycol and water, with minor amounts of other additives that prevent corrosion. What are the (a) mole fraction and (b) molality of ethylene glycol, $C_2H_4(OH)_2$, in a solution prepared from 2.22×10^3 g of ethylene glycol and 2.00×10^3 g of water (approximately 2 L of glycol and 2 L of water)?

Solution

(a) The mole fraction of ethylene glycol may be computed by first deriving molar amounts of both solution components and then substituting these amounts into the unit definition.

$$\text{mol } C_2H_4(OH)_2 = 2220 \text{ g} \times \frac{1 \text{ mol } C_2H_4(OH)_2}{62.07 \text{ g } C_2H_4(OH)_2} = 35.8 \text{ mol } C_2H_4(OH)_2$$

$$\text{mol } H_2O = 2000 \text{ g} \times \frac{1 \text{ mol } H_2O}{18.02 \text{ g } H_2O} = 111 \text{ mol } H_2O$$

$$\chi_{\text{ethylene glycol}} = \frac{35.8 \text{ mol } C_2H_4(OH)_2}{(35.8 + 111) \text{ mol total}} = 0.245$$

Notice that mole fraction is a dimensionless property, being the ratio of properties with identical units (moles).

(b) To find molality, we need to know the moles of the solute and the mass of the solvent (in kg).

First, use the given mass of ethylene glycol and its molar mass to find the moles of solute:

$$2220 \text{ g } C_2H_4(OH)_2 \left(\frac{\text{mol } C_2H_2(OH)_2}{62.07 \text{ g}}\right) = 35.8 \text{ mol } C_2H_4(OH)_2$$

Then, convert the mass of the water from grams to kilograms:

$$2000 \text{ g } H_2O \left(\frac{1 \text{ kg}}{1000 \text{ g}}\right) = 2 \text{ kg } H_2O$$

Finally, calculate molarity per its definition:

$$\begin{aligned}\text{molality} &= \frac{\text{mol solute}}{\text{kg solvent}}\\ \text{molality} &= \frac{35.8 \text{ mol } C_2H_4(OH)_2}{2 \text{ kg } H_2O}\\ \text{molality} &= 17.9\ m\end{aligned}$$

? Exercise 4.5.1

What are the mole fraction and molality of a solution that contains 0.850 g of ammonia, NH_3, dissolved in 125 g of water?

Answer

7.14×10^{-3}; 0.399 *m*

✓ Example 4.5.2: Converting Mole Fraction and Molal Concentrations

Calculate the mole fraction of solute and solvent in a 3.0 *m* solution of sodium chloride.

Solution

Converting from one concentration unit to another is accomplished by first comparing the two unit definitions. In this case, both units have the same numerator (moles of solute) but different denominators. The provided molal concentration may be written as:

$$\frac{3.0 \text{ mol NaCl}}{1.0 \text{ kg } H_2O}$$

The numerator for this solution's mole fraction is, therefore, 3.0 mol NaCl. The denominator may be computed by deriving the molar amount of water corresponding to 1.0 kg

$$1.0 \text{ kg } H_2O \left(\frac{1000 \text{ g}}{1 \text{ kg}}\right)\left(\frac{\text{mol } H_2O}{18.02 \text{ g}}\right) = 55 \text{ mol } H_2O$$

and then substituting these molar amounts into the definition for mole fraction.

$$\begin{aligned}
X_{\mathrm{H_2O}} &= \frac{\text{mol } \mathrm{H_2O}}{\text{mol NaCl} + \text{mol } \mathrm{H_2O}} \\
X_{\mathrm{H_2O}} &= \frac{55 \text{ mol } \mathrm{H_2O}}{3.0 \text{ mol NaCl} + 55 \text{ mol } \mathrm{H_2O}} \\
X_{\mathrm{H_2O}} &= 0.95 \\
X_{\mathrm{NaCl}} &= \frac{\text{mol NaCl}}{\text{mol NaCl} + \text{mol } \mathrm{H_2O}} \\
X_{\mathrm{NaCl}} &= \frac{3.0 \text{ mol NaCl}}{3.0 \text{ mol NaCl} + 55 \text{ mol } \mathrm{H_2O}} \\
X_{\mathrm{NaCl}} &= 0.052
\end{aligned}$$

? Exercise 4.5.2

The mole fraction of iodine, I_2, dissolved in dichloromethane, CH_2Cl_2, is 0.115. What is the molal concentration, *m*, of iodine in this solution?

Answer

1.50 *m*

Vapor Pressure Lowering

As described in the chapter on liquids and solids, the equilibrium vapor pressure of a liquid is the pressure exerted by its gaseous phase when vaporization and condensation are occurring at equal rates:

$$\text{liquid} \rightleftharpoons \text{gas} \tag{4.5.4}$$

Dissolving a nonvolatile substance in a volatile liquid results in a lowering of the liquid’s vapor pressure. This phenomenon can be rationalized by considering the effect of added solute molecules on the liquid's vaporization and condensation processes. To vaporize, solvent molecules must be present at the surface of the solution. The presence of solute decreases the surface area available to solvent molecules and thereby reduces the rate of solvent vaporization. Since the rate of condensation is unaffected by the presence of solute, the net result is that the vaporization-condensation equilibrium is achieved with fewer solvent molecules in the vapor phase (i.e., at a lower vapor pressure) (Figure 4.5.1). While this kinetic interpretation is useful, it does not account for several important aspects of the colligative nature of vapor pressure lowering. A more rigorous explanation involves the property of *entropy*, a topic of discussion in a later text chapter on thermodynamics. For purposes of understanding the lowering of a liquid's vapor pressure, it is adequate to note that the greater entropy of a solution in comparison to its separate solvent and solute serves to effectively stabilize the solvent molecules and hinder their vaporization. A lower vapor pressure results, and a correspondingly higher boiling point as described in the next section of this module.

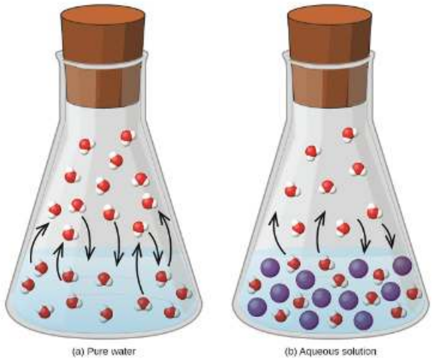


Figure 4.5.1: The presence of nonvolatile solutes lowers the vapor pressure of a solution by impeding the evaporation of solvent molecules.

This figure contains two images. Figure a is labeled “pure water.” It shows a beaker half-filled with liquid. In the liquid, eleven molecules are evenly dispersed in the liquid each consisting of one central red sphere and two slightly smaller white spheres are shown. Four molecules near the surface of the liquid have curved arrows drawn from them pointing to the space above the liquid in the beaker. Above the liquid, twelve molecules are shown, with arrows pointing from three of them into the liquid below. Figure b is labeled “Aqueous solution.” It is similar to figure a except that eleven blue spheres, slightly larger in size than the molecules, are dispersed evenly in the liquid. Only four curved arrows appear in this diagram with two from the molecules in the liquid pointing to the space above and two from molecules in the space above the liquid pointing into the liquid below.

The relationship between the vapor pressures of solution components and the concentrations of those components is described by Raoult’s law: *The partial pressure exerted by any component of an ideal solution is equal to the vapor pressure of the pure component multiplied by its mole fraction in the solution.*

$$P_\text{A} = X_\text{A} P^\circ_\text{A} \tag{4.5.5}$$

where P_A is the partial pressure exerted by component A in the solution, P°_A is the vapor pressure of pure A, and X_A is the mole fraction of A in the solution. (Mole fraction is a concentration unit introduced in the chapter on gases.)

Recalling that the total pressure of a gaseous mixture is equal to the sum of partial pressures for all its components (Dalton’s law of partial pressures), the total vapor pressure exerted by a solution containing *i* components is

$$P_\text{solution} = \sum_i P_i = \sum_i X_i P^\circ_i \tag{4.5.6}$$

A nonvolatile substance is one whose vapor pressure is negligible ($P° \approx 0$), and so the vapor pressure above a solution containing only nonvolatile solutes is due only to the solvent:

$$P_\text{solution} = X_\text{solvent} P^\circ_\text{solvent} \tag{4.5.7}$$

✓ Example 4.5.3: Calculation of a Vapor Pressure

Compute the vapor pressure of an ideal solution containing 92.1 g of glycerin, $C_3H_5(OH)_3$, and 184.4 g of ethanol, C_2H_5OH, at 40 °C. The vapor pressure of pure ethanol is 0.178 atm at 40 °C. Glycerin is essentially nonvolatile at this temperature.

Solution

Since the solvent is the only volatile component of this solution, its vapor pressure may be computed per Raoult’s law as:

$$P_\text{solution} = X_\text{solvent} P^\circ_\text{solvent}$$

First, calculate the molar amounts of each solution component using the provided mass data.

$$92.1\ \text{g}\ \cancel{C_3H_5(OH)_3} \times \frac{1\ \text{mol}\ C_3H_5(OH)_3}{92.094\ \text{g}\ \cancel{C_3H_5(OH)_3}} = 1.00\ \text{mol}\ C_3H_5(OH)_3$$

$$184.4\ \text{g}\ \cancel{C_2H_5OH} \times \frac{1\ \text{mol}\ C_2H_5OH}{46.069\ \text{g}\ \cancel{C_2H_5OH}} = 4.000\ \text{mol}\ C_2H_5OH$$

Next, calculate the mole fraction of the solvent (ethanol) and use Raoult’s law to compute the solution’s vapor pressure.

$$X_{C_2H_5OH} = \frac{4.000\ \text{mol}}{(1.00\ \text{mol} + 4.000\ \text{mol})} = 0.800$$

$$P_\text{solv} = X_\text{solv} P^\circ_\text{solv} = 0.800 \times 0.178\ \text{atm} = 0.142\ \text{atm}$$

? Exercise 4.5.3

A solution contains 5.00 g of urea, $CO(NH_2)_2$ (a nonvolatile solute) and 0.100 kg of water. If the vapor pressure of pure water at 25 °C is 23.7 torr, what is the vapor pressure of the solution?

Answer

23.4 torr

Elevation of the Boiling Point of a Solvent

As described in the chapter on liquids and solids, the *boiling point* of a liquid is the temperature at which its vapor pressure is equal to ambient atmospheric pressure. Since the vapor pressure of a solution is lowered due to the presence of nonvolatile solutes, it stands to reason that the solution's boiling point will subsequently be increased. Compared to pure solvent, a solution, therefore, will require a higher temperature to achieve any given vapor pressure, including one equivalent to that of the surrounding atmosphere. The increase in boiling point observed when nonvolatile solute is dissolved in a solvent, ΔT_b, is called boiling point elevation and is directly proportional to the molal concentration of solute species:

$$\Delta T_b = K_b m \tag{4.5.8}$$

where

- K_b is the boiling point elevation constant, or the *ebullioscopic constant* and
- m is the molal concentration (molality) of all solute species.

Boiling point elevation constants are characteristic properties that depend on the identity of the solvent. Values of K_b for several solvents are listed in Table $4.5.1$.

Table $4.5.1$: Boiling Point Elevation and Freezing Point Depression Constants for Several Solvents

Solvent	Boiling Point (°C at 1 atm)	K_b (Cm^{-1})	Freezing Point (°C at 1 atm)	K_f (Cm^{-1})
water	100.0	0.512	0.0	1.86
hydrogen acetate	118.1	3.07	16.6	3.9
benzene	80.1	2.53	5.5	5.12
chloroform	61.26	3.63	−63.5	4.68
nitrobenzene	210.9	5.24	5.67	8.1

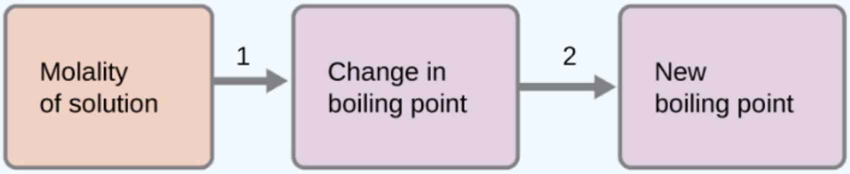
The extent to which the vapor pressure of a solvent is lowered and the boiling point is elevated depends on the total number of solute particles present in a given amount of solvent, not on the mass or size or chemical identities of the particles. A 1 *m* aqueous solution of sucrose (342 g/mol) and a 1 *m* aqueous solution of ethylene glycol (62 g/mol) will exhibit the same boiling point because each solution has one mole of solute particles (molecules) per kilogram of solvent.

✓ Example $4.5.4$: Calculating the Boiling Point of a Solution

What is the boiling point of a 0.33 *m* solution of a nonvolatile solute in benzene?

Solution

Use the equation relating boiling point elevation to solute molality to solve this problem in two steps.



1. $\Delta T_\text{b} = K_\text{b} m = 2.53\ ^\circ\text{C}\ m^{-1} \times 0.33\ m = 0.83\ ^\circ\text{C}$
   - *Add the boiling point elevation to the pure solvent's boiling point.*

$\text{Boiling temperature} = 80.1\ ^\circ\text{C} + 0.83\ ^\circ\text{C} = 80.9\ ^\circ\text{C}$

Exercise 4.5.4

What is the boiling point of the antifreeze described in Example 4.5.4?

Answer

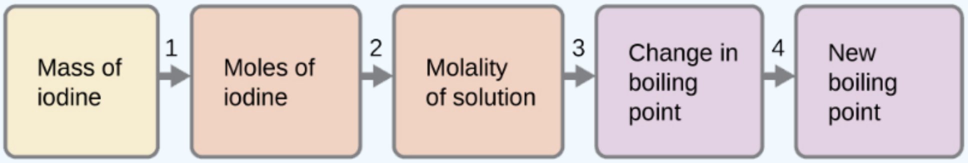
109.2 °C

Example 4.5.5: The Boiling Point of an Iodine Solution

Find the boiling point of a solution of 92.1 g of iodine, I_2, in 800.0 g of chloroform, $CHCl_3$, assuming that the iodine is nonvolatile and that the solution is ideal.

Solution

We can solve this problem using four steps.



1. *Convert from grams to moles of* I_2 *using the molar mass of* I_2 *in the unit conversion factor.*

Result: 0.363 mol

Determine the molality of the solution from the number of moles of solute and the mass of solvent, in kilograms.

Result: 0.454 *m*

Use the direct proportionality between the change in boiling point and molal concentration to determine how much the boiling point changes.

Result: 1.65 °C

Determine the new boiling point from the boiling point of the pure solvent and the change.

Result: 62.91 °C

Check each result as a self-assessment.

Exercise 4.5.5: glycerin:Water Solution

What is the boiling point of a solution of 1.0 g of glycerin, $C_3H_5(OH)_3$, in 47.8 g of water? Assume an ideal solution.

Answer

100.12 °C

Distillation of Solutions

Distillation is a technique for separating the components of mixtures that is widely applied in both in the laboratory and in industrial settings. It is used to refine petroleum, to isolate fermentation products, and to purify water. This separation technique involves the controlled heating of a sample mixture to selectively vaporize, condense, and collect one or more components of interest. A typical apparatus for laboratory-scale distillations is shown in Figure 4.5.2.

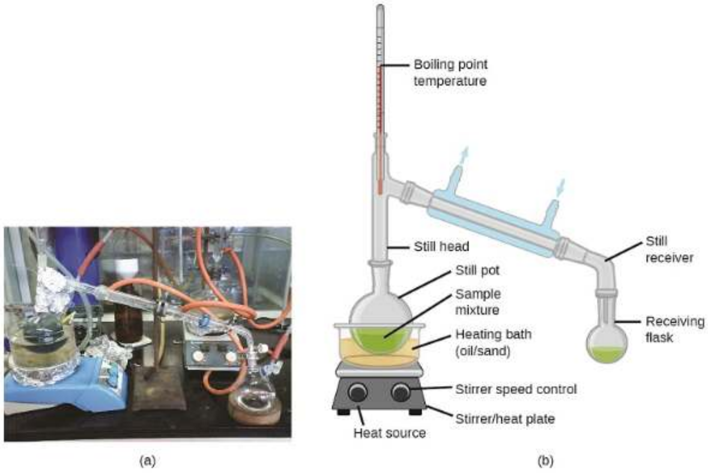


Figure 4.5.2: A typical laboratory distillation unit is shown in (a) a photograph and (b) a schematic diagram of the components. (credit a: modification of work by “Rifleman82”/Wikimedia commons; credit b: modification of work by “Slashme”/Wikimedia Commons) Figure a contains a photograph of a common laboratory distillation unit. Figure b provides a diagram labeling typical components of a laboratory distillation unit, including a stirrer/heat plate with heat and stirrer speed control, a heating bath of oil or sand, stirring means such as boiling chips, a still pot, a still head, a thermometer for boiling point temperature reading, a condenser with a cool water inlet and outlet, a still receiver with a vacuum or gas inlet, a receiving flask for holding distillate, and a cooling bath.

Oil refineries use large-scale *fractional distillation* to separate the components of crude oil. The crude oil is heated to high temperatures at the base of a tall *fractionating column*, vaporizing many of the components that rise within the column. As vaporized components reach adequately cool zones during their ascent, they condense and are collected. The collected liquids are simpler mixtures of hydrocarbons and other petroleum compounds that are of appropriate composition for various applications (e.g., diesel fuel, kerosene, gasoline), as depicted in Figure 4.5.3.

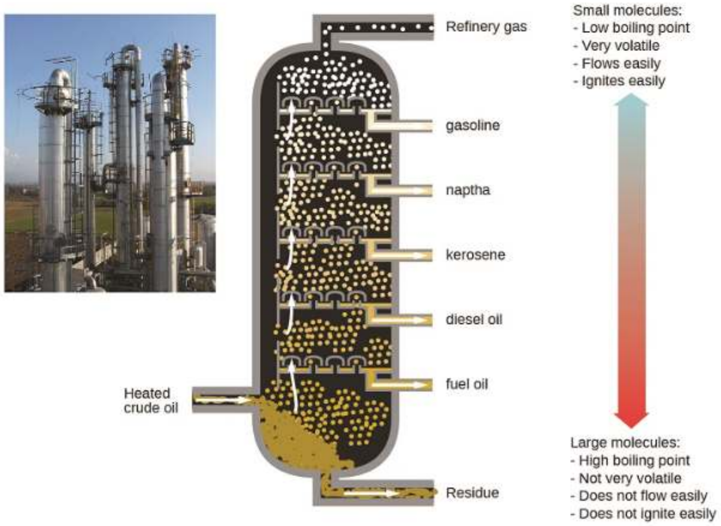


Figure 4.5.3: Crude oil is a complex mixture that is separated by large-scale fractional distillation to isolate various simpler mixtures. This figure contains a photo of a refinery, showing large columnar structures. A diagram of a fractional distillation column used in separating crude oil is also shown. Near the bottom of the column, an arrow pointing into the column shows a point of entry for heated crude oil. The column contains several layers at which different components are removed. At the very bottom, residue materials are removed as indicated by an arrow out of the column. At each successive level, different materials are removed proceeding from the bottom to the top of the column. The materials are fuel oil, followed by diesel oil, kerosene, naptha, gasoline, and refinery gas at the very top. To the right of the column diagram, a double sided arrow is shown that is blue at the top and gradually changes color to red moving downward. The blue top of the arrow is labeled, “small molecules: low boiling point, very

volatile, flows easily, ignites easily.” The red bottom of the arrow is labeled, “large molecules: high boiling point, not very volatile, does not flow easily, does not ignite easily.”

Depression of the Freezing Point of a Solvent

Solutions freeze at lower temperatures than pure liquids. This phenomenon is exploited in “de-icing” schemes that use salt (Figure $4.5.4$), calcium chloride, or urea to melt ice on roads and sidewalks, and in the use of ethylene glycol as an “antifreeze” in automobile radiators. Seawater freezes at a lower temperature than fresh water, and so the Arctic and Antarctic oceans remain unfrozen even at temperatures below 0 °C (as do the body fluids of fish and other cold-blooded sea animals that live in these oceans).

Figure $4.5.4$: Rock salt (NaCl), calcium chloride ($CaCl_2$), or a mixture of the two are used to melt ice. (credit: modification of work by Eddie Welker)

The decrease in freezing point of a dilute solution compared to that of the pure solvent, ΔT_f, is called the freezing point depression and is directly proportional to the molal concentration of the solute

$$\Delta T_\mathrm{f} = K_\mathrm{f} m \tag{4.5.9}$$

where

- m is the *molal* concentration of the solute in the solvent and
- K_f is called the freezing point depression constant (or *cryoscopic constant*).

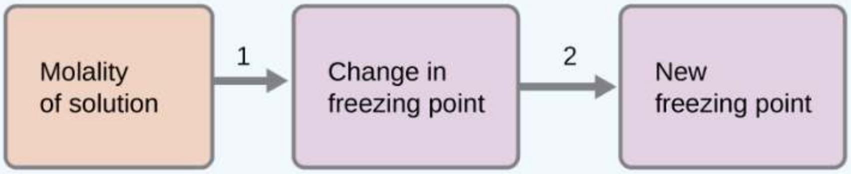
Just as for boiling point elevation constants, these are characteristic properties whose values depend on the chemical identity of the solvent. Values of K_f for several solvents are listed in Table $4.5.1$.

Example $4.5.5$: Calculation of the Freezing Point of a Solution

What is the freezing point of the 0.33 *m* solution of a nonvolatile nonelectrolyte solute in benzene described in Example $4.5.4$?

Solution

Use the equation relating freezing point depression to solute molality to solve this problem in two steps.



1. *Calculate the change in freezing point.* \[ΔT_\ce{f}=K_\ce{f}m=5.12\:°\ce C\:m^{−1}×0.33\:m=1.7\:°\ce C \nonumber \
2. *Subtract the freezing point change observed from the pure solvent’s freezing point.*

$$\text{Freezing Temperature} = 5.5\ ^\circ\text{C} - 1.7\ ^\circ\text{C} = 3.8\ ^\circ\text{C}$$

Exercise $4.5.6$

What is the freezing point of a 1.85 *m* solution of a nonvolatile nonelectrolyte solute in nitrobenzene?

Answer

−9.3 °C

Colligative Properties and De-Icing

Sodium chloride and its group 2 analogs calcium and magnesium chloride are often used to de-ice roadways and sidewalks, due to the fact that a solution of any one of these salts will have a freezing point lower than 0 °C, the freezing point of pure water. The group 2 metal salts are frequently mixed with the cheaper and more readily available sodium chloride (“rock salt”) for use on roads, since they tend to be somewhat less corrosive than the NaCl, and they provide a larger depression of the freezing point, since they dissociate to yield three particles per formula unit, rather than two particles like the sodium chloride.

Because these ionic compounds tend to hasten the corrosion of metal, they would not be a wise choice to use in antifreeze for the radiator in your car or to de-ice a plane prior to takeoff. For these applications, covalent compounds, such as ethylene or propylene glycol, are often used. The glycols used in radiator fluid not only lower the freezing point of the liquid, but they elevate the boiling point, making the fluid useful in both winter and summer. Heated glycols are often sprayed onto the surface of airplanes prior to takeoff in inclement weather in the winter to remove ice that has already formed and prevent the formation of more ice, which would be particularly dangerous if formed on the control surfaces of the aircraft (Video 4.5.1).

***Video 4.5.1**: Freezing point depression is exploited to remove ice from the control surfaces of aircraft.*

Phase Diagram for an Aqueous Solution of a Nonelectrolyte

The colligative effects on vapor pressure, boiling point, and freezing point described in the previous section are conveniently summarized by comparing the phase diagrams for a pure liquid and a solution derived from that liquid. Phase diagrams for water and an aqueous solution are shown in Figure 4.5.5.

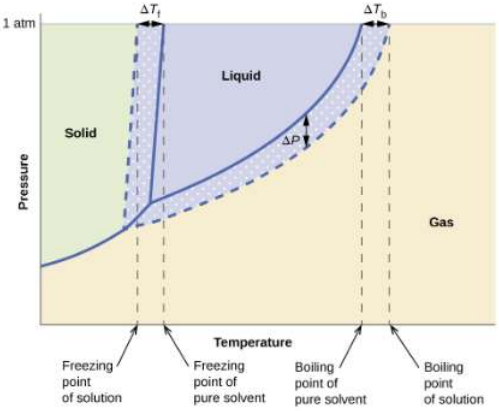


Figure 4.5.5: These phase diagrams show water (solid curves) and an aqueous solution of nonelectrolyte (dashed curves). This phase diagram indicates the pressure in atmospheres of water and a solution at various temperatures. The graph shows the freezing point of water and the freezing point of the solution, with the difference between these two values identified as delta T subscript f.

The graph shows the boiling point of water and the boiling point of the solution, with the difference between these two values identified as delta T subscript b. Similarly, the difference in the pressure of water and the solution at the boiling point of water is shown and identified as delta P. This difference in pressure is labeled vapor pressure lowering. The lower level of the vapor pressure curve for the solution as opposed to that of pure water shows vapor pressure lowering in the solution. Background colors on the diagram indicate the presence of water and the solution in the solid state to the left, liquid state in the central upper region, and gas to the right.

The liquid-vapor curve for the solution is located *beneath* the corresponding curve for the solvent, depicting the vapor pressure *lowering*, ΔP, that results from the dissolution of nonvolatile solute. Consequently, at any given pressure, the solution's boiling point is observed at a higher temperature than that for the pure solvent, reflecting the boiling point elevation, ΔT_b, associated with the presence of nonvolatile solute. The solid-liquid curve for the solution is displaced left of that for the pure solvent, representing the freezing point depression, ΔTf, that accompanies solution formation. Finally, notice that the solid-gas curves for the solvent and its solution are identical. This is the case for many solutions comprising liquid solvents and nonvolatile solutes. Just as for vaporization, when a solution of this sort is frozen, it is actually just the *solvent* molecules that undergo the liquid-to-solid transition, forming pure solid solvent that excludes solute species. The solid and gaseous phases, therefore, are composed solvent only, and so transitions between these phases are not subject to colligative effects.

Osmosis and Osmotic Pressure of Solutions

A number of natural and synthetic materials exhibit *selective permeation*, meaning that only molecules or ions of a certain size, shape, polarity, charge, and so forth, are capable of passing through (permeating) the material. Biological cell membranes provide elegant examples of selective permeation in nature, while dialysis tubing used to remove metabolic wastes from blood is a more simplistic technological example. Regardless of how they may be fabricated, these materials are generally referred to as semipermeable membranes.

Consider the apparatus illustrated in Figure $4.5.6$, in which samples of pure solvent and a solution are separated by a membrane that only solvent molecules may permeate. Solvent molecules will diffuse across the membrane in both directions. Since the concentration of *solvent* is greater in the pure solvent than the solution, these molecules will diffuse from the solvent side of the membrane to the solution side at a faster rate than they will in the reverse direction. The result is a net transfer of solvent molecules from the pure solvent to the solution. Diffusion-driven transfer of solvent molecules through a semipermeable membrane is a process known as osmosis.

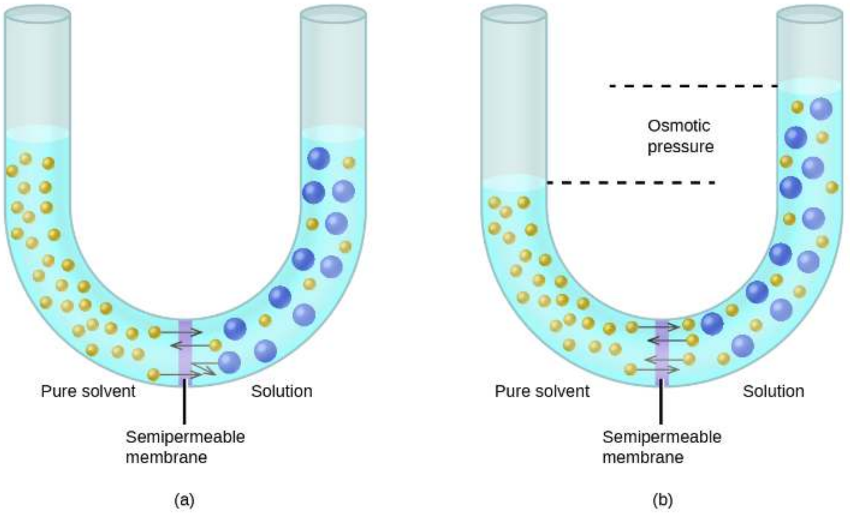


Figure $4.5.6$: Osmosis results in the transfer of solvent molecules from a sample of low (or zero) solute concentration to a sample of higher solute concentration. The figure shows two U shaped tubes with a semi permeable membrane placed at the base of the U. In figure a, pure solvent is present and indicated by small yellow spheres to the left of the membrane. To the right, a solution exists with larger blue spheres intermingled with some small yellow spheres. At the membrane, arrows pointing from three small yellow spheres on both sides of the membrane cross over the membrane. An arrow drawn from one of the large blue spheres does not cross the membrane, but rather is reflected back from the surface of the membrane. The levels of liquid in both sides of the U shaped tube are equal. In figure b, arrows again point from small yellow spheres across the semipermeable membrane from both sides. This diagram shows the level of liquid in the left, pure solvent, side to be significantly lower than the liquid level on the right. Dashed lines are drawn from these two liquid levels into the middle of the U-shaped tube and between them is the term osmotic pressure.

When osmosis is carried out in an apparatus like that shown in Figure $4.5.6$, the volume of the solution increases as it becomes diluted by accumulation of solvent. This causes the level of the solution to rise, increasing its hydrostatic pressure (due to the weight of the column of solution in the tube) and resulting in a faster transfer of solvent molecules back to the pure solvent side. When the pressure reaches a value that yields a reverse solvent transfer rate equal to the osmosis rate, bulk transfer of solvent ceases. This pressure is called the osmotic pressure (Π) of the solution. The osmotic pressure of a dilute solution is related to its solute molarity, *M*, and absolute temperature, *T*, according to the equation

$$\Pi = MRT \tag{4.5.10}$$

where R is the universal gas constant.

✓ Example $4.5.7$: Calculation of Osmotic Pressure

What is the osmotic pressure (atm) of a 0.30 *M* solution of glucose in water that is used for intravenous infusion at body temperature, 37 °C?

Solution

We can find the osmotic pressure, \(*Π*\), using Equation $4.5.10$, where *T* is on the Kelvin scale (310 K) and the value of *R* is expressed in appropriate units (0.08206 L atm/mol K).

$$\begin{aligned}\Pi &= MRT \\ &= 0.03 \text{ mol/L} \times 0.08206 \text{ L atm/mol K} \times 310 \text{ K} \\ &= 7.6 \text{ atm}\end{aligned}$$

? Exercise $4.5.7$

What is the osmotic pressure (atm) a solution with a volume of 0.750 L that contains 5.0 g of methanol, CH_3OH, in water at 37 °C?

Answer

5.3 atm

If a solution is placed in an apparatus like the one shown in Figure $4.5.7$, applying pressure greater than the osmotic pressure of the solution reverses the osmosis and pushes solvent molecules from the solution into the pure solvent. This technique of reverse osmosis is used for large-scale desalination of seawater and on smaller scales to produce high-purity tap water for drinking.

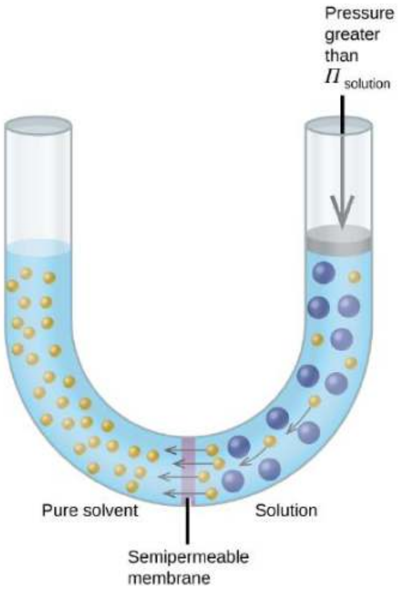


Figure 4.5.7: Applying a pressure greater than the osmotic pressure of a solution will reverse osmosis. Solvent molecules from the solution are pushed into the pure solvent. The figure shows a U shaped tube with a semi permeable membrane placed at the base of the U. Pure solvent is present and indicated by small yellow spheres to the left of the membrane. To the right, a solution exists with larger blue spheres intermingled with some small yellow spheres. At the membrane, arrows point from four small yellow spheres to the left of the membrane. On the right side of the U, there is a disk that is the same width of the tube and appears to block it. The disk is at the same level as the solution. An arrow points down from the top of the tube to the disk and is labeled, "Pressure greater than Π subscript solution."

Examples of osmosis are evident in many biological systems because cells are surrounded by semipermeable membranes. Carrots and celery that have become limp because they have lost water can be made crisp again by placing them in water. Water moves into the carrot or celery cells by osmosis. A cucumber placed in a concentrated salt solution loses water by osmosis and absorbs some salt to become a pickle. Osmosis can also affect animal cells. Solute concentrations are particularly important when solutions are injected into the body. Solutes in body cell fluids and blood serum give these solutions an osmotic pressure of approximately 7.7 atm. Solutions injected into the body must have the same osmotic pressure as blood serum; that is, they should be isotonic with blood serum. If a less concentrated solution, a hypotonic solution, is injected in sufficient quantity to dilute the blood serum, water from the diluted serum passes into the blood cells by osmosis, causing the cells to expand and rupture. This process is called hemolysis. When a more concentrated solution, a hypertonic solution, is injected, the cells lose water to the more concentrated solution, shrivel, and possibly die in a process called *crenation* (Figure 11.5.8).

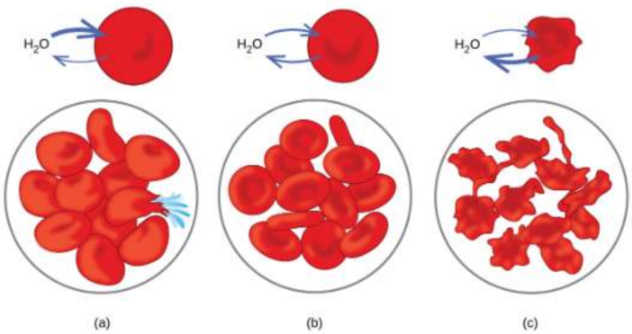


Figure 4.5.8: Red blood cell membranes are water permeable and will (a) swell and possibly rupture in a hypotonic solution; (b) maintain normal volume and shape in an isotonic solution; and (c) shrivel and possibly die in a hypertonic solution. (credit a/b/c: modifications of work by "LadyofHats"/Wikimedia commons) This figure shows three scenarios relating to red blood cell membranes. In a, H subscript 2 O has two arrows drawn from it pointing into a red disk. Beneath it in a circle are eleven similar disks with a bulging appearance, one of which appears to have burst with blue liquid erupting from it. In b, the image is similar except that rather than having two arrows pointing into the red disk, one points in and a second points out toward the H subscript 2

O. In the circle beneath, twelve of the red disks are present. In c, both arrows are drawn from a red shriveled disk toward the H subscript 2 O. In the circle below, twelve shriveled disks are shown.

Determination of Molar Masses

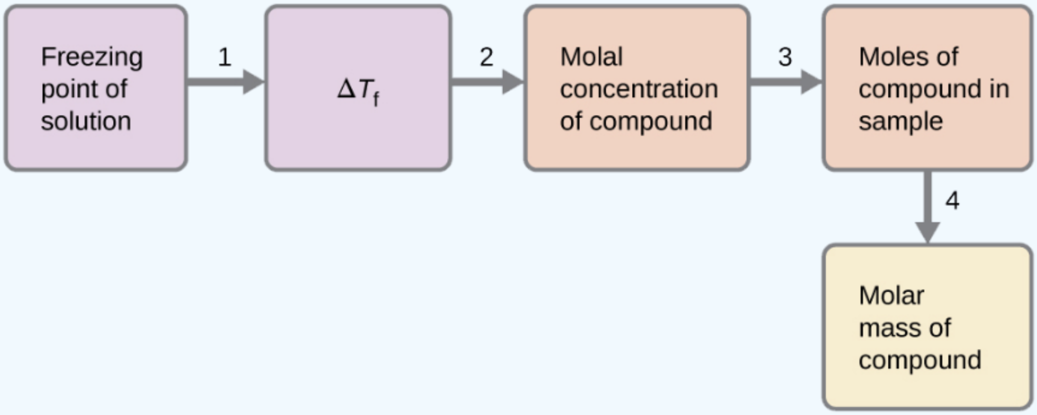
Osmotic pressure and changes in freezing point, boiling point, and vapor pressure are directly proportional to the concentration of solute present. Consequently, we can use a measurement of one of these properties to determine the molar mass of the solute from the measurements.

✓ Example 4.5.8: Determining Molar Mass from Freezing Point Depression

A solution of 4.00 g of a nonelectrolyte dissolved in 55.0 g of benzene is found to freeze at 2.32 °C. What is the molar mass of this compound?

Solution

We can solve this problem using the following steps.



1. *Determine the change in freezing point from the observed freezing point and the freezing point of pure benzene* (Table 11.5.1). $\Delta T_f = 5.5\ ^\circ C - 2.32\ ^\circ C = 3.2\ ^\circ C$
2. *Determine the molal concentration from* K_f*, the freezing point depression constant for benzene* (Table 11.5.1), *and* ΔT_f. \ (ΔT_\ce{f}=K_\ce{f}m\)$m = \dfrac{\Delta T_f}{K_f} = \dfrac{3.2\ ^\circ C}{5.12\ ^\circ C m^{-1}} = 0.63\ m$
3. *Determine the number of moles of compound in the solution from the molal concentration and the mass of solvent used to make the solution.* (\mathrm{Moles\: of\: solute=\dfrac{0.62\:mol\: solute}{1.00\cancel{kg\: solvent}}×0.0550\cancel{kg\: solvent}=0.035\:mol}\)
4. *Determine the molar mass from the mass of the solute and the number of moles in that mass.* \(\mathrm{Molar\: mass=\dfrac{4.00\:g}{0.034\:mol}=1.2×10^2\:g/mol}\)

? Exercise 4.5.8

A solution of 35.7 g of a nonelectrolyte in 220.0 g of chloroform has a boiling point of 64.5 °C. What is the molar mass of this compound?

Answer

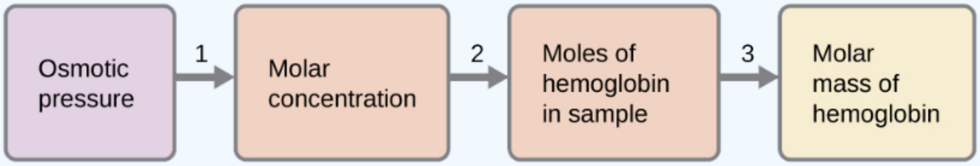
1.8 × 10^2 g/mol

✓ Example 4.5.9: Determination of a Molar Mass from Osmotic Pressure

A 0.500 L sample of an aqueous solution containing 10.0 g of hemoglobin has an osmotic pressure of 5.9 torr at 22 °C. What is the molar mass of hemoglobin?

Solution

Here is one set of steps that can be used to solve the problem:



1. [\Pi=\mathrm{\dfrac{5.9\:torr×1\:atm{760\:torr}=7.8×10^{−3}\:atm}\nonumber\ [\Pi=MRT\nonumber\] (M=\dfrac{Π{RT}=\mathrm{\dfrac{7.8×10^{−3}\:atm{(0.08206\:L\:atm/mol\:K(295\:K)}=3.2×10^{−4}\:M}\) (\mathrm{moles\: of\: hemoglobin=\dfrac{3.2×10^{−4}\:mol}{1\cancel{L\: solution}}×0.500\cancel{L\: solution}=1.6×10^{−4}\:mol}\)
2. *Determine the molar mass from the mass of hemoglobin and the number of moles in that mass*

$$\text{molar mass} = \frac{10.0\ \text{g}}{1.6 \times 10^{-4}\ \text{mol}} = 6.2 \times 10^{4}\ \text{g/mol}$$

? Exercise 4.5.9

What is the molar mass of a protein if a solution of 0.02 g of the protein in 25.0 mL of solution has an osmotic pressure of 0.56 torr at 25 °C?

Answer

2.7×10^{4} g/mol

Colligative Properties of Electrolytes

As noted previously in this module, the colligative properties of a solution depend only on the number, not on the kind, of solute species dissolved. For example, 1 mole of any nonelectrolyte dissolved in 1 kilogram of solvent produces the same lowering of the freezing point as does 1 mole of any other nonelectrolyte. However, 1 mole of sodium chloride (an electrolyte) forms *2 moles* of ions when dissolved in solution. Each individual ion produces the same effect on the freezing point as a single molecule does.

✓ Example 4.5.10: The Freezing Point of a Solution of an Electrolyte

The concentration of ions in seawater is approximately the same as that in a solution containing 4.2 g of NaCl dissolved in 125 g of water. Assume that each of the ions in the NaCl solution has the same effect on the freezing point of water as a nonelectrolyte molecule, and determine the freezing temperature the solution (which is approximately equal to the freezing temperature of seawater).

Solution

We can solve this problem using the following series of steps.

- Convert from grams to moles of NaCl using the molar mass of NaCl in the unit conversion factor. Result: 0.072 mol NaCl
- Determine the number of moles of ions present in the solution using the number of moles of ions in 1 mole of NaCl as the conversion factor (2 mol ions/1 mol NaCl). Result: 0.14 mol ions
- Determine the molality of the ions in the solution from the number of moles of ions and the mass of solvent, in kilograms. Result: 1.1 m
- Use the direct proportionality between the change in freezing point and molal concentration to determine how much the freezing point changes. Result: 2.0 °C
- Determine the new freezing point from the freezing point of the pure solvent and the change. Result: −2.0 °C

Check each result as a self-assessment.

? Exercise 4.5.10

Assume that each of the ions in calcium chloride, $CaCl_2$, has the same effect on the freezing point of water as a nonelectrolyte molecule. Calculate the freezing point of a solution of 0.724 g of $CaCl_2$ in 175 g of water.

Answer

−0.208 °C

Assuming complete dissociation, a 1.0 *m* aqueous solution of NaCl contains 2.0 mole of ions (1.0 mol Na^+ and 1.0 mol $Cl^−$) per each kilogram of water, and its freezing point depression is expected to be

$$\Delta T_\text{f} = 2.0 \text{ mol ions/kg water} \times 1.86\ ^\circ\text{C kg water/mol ion} = 3.7\ ^\circ\text{C}. \tag{4.5.11}$$

When this solution is actually prepared and its freezing point depression measured, however, a value of 3.4 °C is obtained. Similar discrepancies are observed for other ionic compounds, and the differences between the measured and expected colligative property values typically become more significant as solute concentrations increase. These observations suggest that the ions of sodium chloride (and other strong electrolytes) are not completely dissociated in solution.

To account for this and avoid the errors accompanying the assumption of total dissociation, an experimentally measured parameter named in honor of Nobel Prize-winning German chemist Jacobus Henricus van’t Hoff is used. The van’t Hoff factor (*i*) is defined as the ratio of solute particles in solution to the number of formula units dissolved:

$$i = \frac{\text{moles of particles in solution}}{\text{moles of formula units dissolved}} \tag{4.5.12}$$

Values for measured van’t Hoff factors for several solutes, along with predicted values assuming complete dissociation, are shown in Table $4.5.2$.

Table 4.5.2: Expected and Observed van’t Hoff Factors for Several 0.050 m Aqueous Electrolyte Solutions

Electrolyte	Particles in Solution	*i* (Predicted)	*i* (Measured)
HCl	H^+, $Cl^−$	2	1.9
NaCl	Na^+, $Cl^−$	2	1.9
$MgSO_4$	Mg^{2+}, $SO_4^{2−}$	2	1.3
$MgCl_2$	Mg^{2+}, $2Cl^−$	3	2.7
$FeCl_3$	Fe^{3+}, $3Cl^−$	4	3.4
glucose (a non-electrolyte)	$C_{12}H_{22}O_{11}$	1	1.0

In 1923, the chemists Peter Debye and Erich Hückel proposed a theory to explain the apparent incomplete ionization of strong electrolytes. They suggested that although interionic attraction in an aqueous solution is very greatly reduced by solvation of the ions and the insulating action of the polar solvent, it is not completely nullified. The residual attractions prevent the ions from behaving as totally independent particles (Figure $4.5.9$). In some cases, a positive and negative ion may actually touch, giving a solvated unit called an ion pair. Thus, the activity, or the effective concentration, of any particular kind of ion is less than that indicated by the actual concentration. Ions become more and more widely separated the more dilute the solution, and the residual interionic attractions become less and less. Thus, in extremely dilute solutions, the effective concentrations of the ions (their activities) are essentially equal to the actual concentrations. Note that the van’t Hoff factors for the electrolytes in Table $4.5.2$ are for 0.05 *m* solutions, at which concentration the value of *i* for NaCl is 1.9, as opposed to an ideal value of 2.

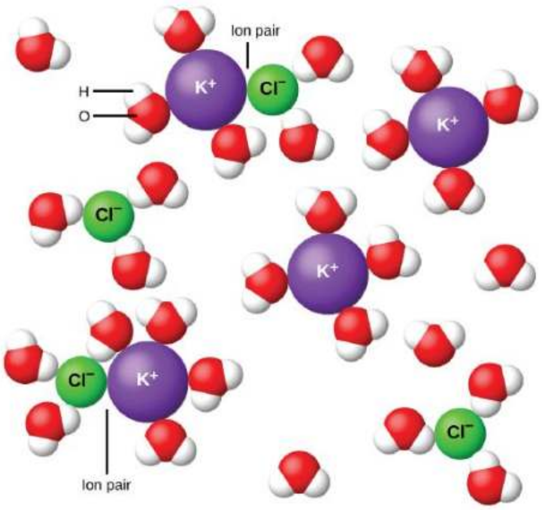


Figure 4.5.9: Ions become more and more widely separated the more dilute the solution, and the residual interionic attractions become less. The diagram shows four purple spheres labeled K superscript plus and four green spheres labeled C l superscript minus dispersed in H subscript 2 O as shown by clusters of single red spheres with two white spheres attached. Red spheres represent oxygen and white represent hydrogen. In two locations, the purple and green spheres are touching. In these two locations, the diagram is labeled ion pair. All red and green spheres are surrounded by the white and red H subscript 2 O clusters. The white spheres are attracted to the purple spheres and the red spheres are attracted to the green spheres.

Summary

Properties of a solution that depend only on the concentration of solute particles are called colligative properties. They include changes in the vapor pressure, boiling point, and freezing point of the solvent in the solution. The magnitudes of these properties depend only on the total concentration of solute particles in solution, not on the type of particles. The total concentration of solute particles in a solution also determines its osmotic pressure. This is the pressure that must be applied to the solution to prevent diffusion of molecules of pure solvent through a semipermeable membrane into the solution. Ionic compounds may not completely dissociate in solution due to activity effects, in which case observed colligative effects may be less than predicted.

Key Equations

- $\left(P_{\text{A}}=X_{\text{A}}P_{\text{A}}^{\circ}\right)$
- $P_{\text{solution}}=\sum_i P_i=\sum_i X_i P_i^{\circ}$
- $P_{\text{solution}}=X_{\text{solvent}}P_{\text{solvent}}^{\circ}$
- $\Delta T_{\text{b}} = K_{\text{b}}m$
- $\Delta T_{\text{f}} = K_{\text{f}}m$
- $\Pi = MRT$

Footnotes

1. A nonelectrolyte shown for comparison.

Glossary

boiling point elevation
elevation of the boiling point of a liquid by addition of a solute

boiling point elevation constant
the proportionality constant in the equation relating boiling point elevation to solute molality; also known as the ebullioscopic constant

colligative property
property of a solution that depends only on the concentration of a solute species

crenation

process whereby biological cells become shriveled due to loss of water by osmosis

freezing point depression

lowering of the freezing point of a liquid by addition of a solute

freezing point depression constant

(also, cryoscopic constant) proportionality constant in the equation relating freezing point depression to solute molality

hemolysis

rupture of red blood cells due to the accumulation of excess water by osmosis

hypertonic

of greater osmotic pressure

hypotonic

of less osmotic pressure

ion pair

solvated anion/cation pair held together by moderate electrostatic attraction

isotonic

of equal osmotic pressure

molality (m)

a concentration unit defined as the ratio of the numbers of moles of solute to the mass of the solvent in kilograms

osmosis

diffusion of solvent molecules through a semipermeable membrane

osmotic pressure (Π)

opposing pressure required to prevent bulk transfer of solvent molecules through a semipermeable membrane

Raoult's law

the partial pressure exerted by a solution component is equal to the product of the component's mole fraction in the solution and its equilibrium vapor pressure in the pure state

semipermeable membrane

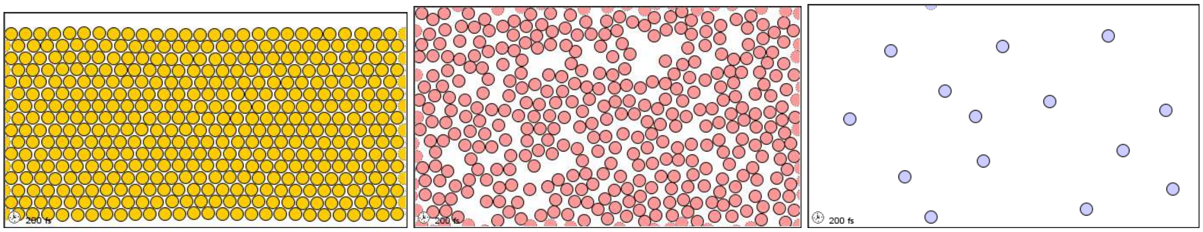
a membrane that selectively permits passage of certain ions or molecules

van't Hoff factor (i)

the ratio of the number of moles of particles in a solution to the number of moles of formula units dissolved in the solution

4.6: Phase Transitions

We have now looked at the physical properties which chemists use to define the solid, liquid, and gas phases. In a solid, atoms, ions or molecules, are locked into an organized, long range lattice structure, unable to move beyond an average position due to intermolecular forces. In a liquid, this structure breaks down, molecules can slip past each other, but they are still held together by attractive forces. In a gas, these attractive forces are overcome, and the substance expands to fill space, each particle having gained mobility to break free of the others. Below, all 3 phases are shown at the submicroscopic level in animations. Notice how the movement and freedom of molecules steadily increases as attractive forces decrease from solid to liquid to gas phase.



Substances can be transformed from one phase into another. Solids melt into liquids and liquids boil to form vapors at temperatures which depend on their molecular properties, so chemists are interested in these transitions between phases. We are all familiar with the changes in macroscopic properties that accompany these transitions. YouTube has time lapse movies of ice melting on a small scale, or of the more environmentally critical arctic ice melt from 1979 to 2007.

This is a familiar process. As the solid melts, the resulting liquid is able to flow and conforms to the shape of the container. Heat from a flame is needed to bring about this transition. On a microscopic level melting involves breaking the intermolecular interactions between molecules. This requires an increase in the potential energy of the molecules, and the necessary energy is supplied by the Bunsen burner. Melting (or freezing) can, in some cases, be caused by changing just the pressure.

Boiling is equally familiar. Under specific temperature and pressure conditions, liquids start to bubble, and are converted to a gaseous form. A YouTube video "Boiling water with ice" shows that water boils at low temperatures if the pressure is reduced. Heat energy is absorbed when a liquid boils because molecules which are held together by mutual attraction in the liquid are jostled free of each other as the gas is formed. Such a separation requires energy.

In general the energy needed differs from one liquid to another depending on the magnitude of the intermolecular forces. We can thus expect liquids with strong intermolecular forces to boil at higher temperatures. It should be noted as well, that because there is a distribution in the kinetic energies of molecules, an equilibrium between gas and liquid phase is established at temperatures other than the boiling point, and this behavior is another aspect of phase transitions that chemists study.

For phase transitions from solid to liquid, liquid to gas, or solid to gas, energy is required because they involve separation of particles which attract one another. Further, we can predict under which conditions of temperature and pressure such transitions will occur.

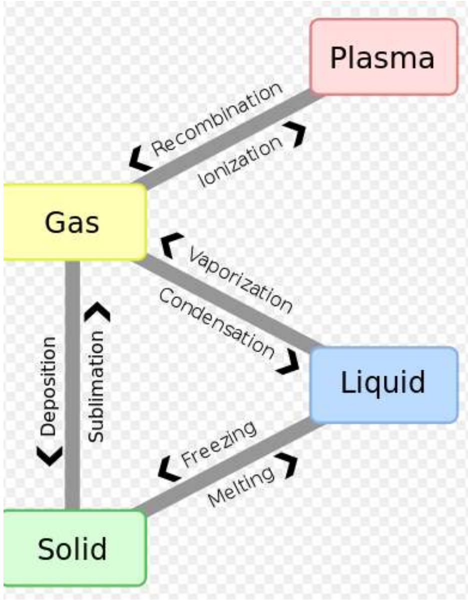


Figure 10.8.1 The figure above shows the transitions between phases and the corresponding names for each transition.

4.7: Phase Transitions

Learning Objectives

- Define phase transitions and phase transition temperatures
- Explain the relation between phase transition temperatures and intermolecular attractive forces
- Describe the processes represented by typical heating and cooling curves, and compute heat flows and enthalpy changes accompanying these processes

We witness and utilize changes of physical state, or phase transitions, in a great number of ways. As one example of global significance, consider the evaporation, condensation, freezing, and melting of water. These changes of state are essential aspects of our earth's water cycle as well as many other natural phenomena and technological processes of central importance to our lives. In this module, the essential aspects of phase transitions are explored.

Vaporization and Condensation

When a liquid vaporizes in a closed container, gas molecules cannot escape. As these gas phase molecules move randomly about, they will occasionally collide with the surface of the condensed phase, and in some cases, these collisions will result in the molecules re-entering the condensed phase. The change from the gas phase to the liquid is called condensation. When the rate of condensation becomes equal to the rate of vaporization, neither the amount of the liquid nor the amount of the vapor in the container changes. The vapor in the container is then said to be *in equilibrium* with the liquid. Keep in mind that this is not a static situation, as molecules are continually exchanged between the condensed and gaseous phases. Such is an example of a dynamic equilibrium, the status of a system in which reciprocal processes (for example, vaporization and condensation) occur at equal rates. The pressure exerted by the vapor in equilibrium with a liquid in a closed container at a given temperature is called the liquid's vapor pressure (or equilibrium vapor pressure). The area of the surface of the liquid in contact with a vapor and the size of the vessel have no effect on the vapor pressure, although they do affect the time required for the equilibrium to be reached. We can measure the vapor pressure of a liquid by placing a sample in a closed container, like that illustrated in Figure 4.7.1, and using a manometer to measure the increase in pressure that is due to the vapor in equilibrium with the condensed phase.

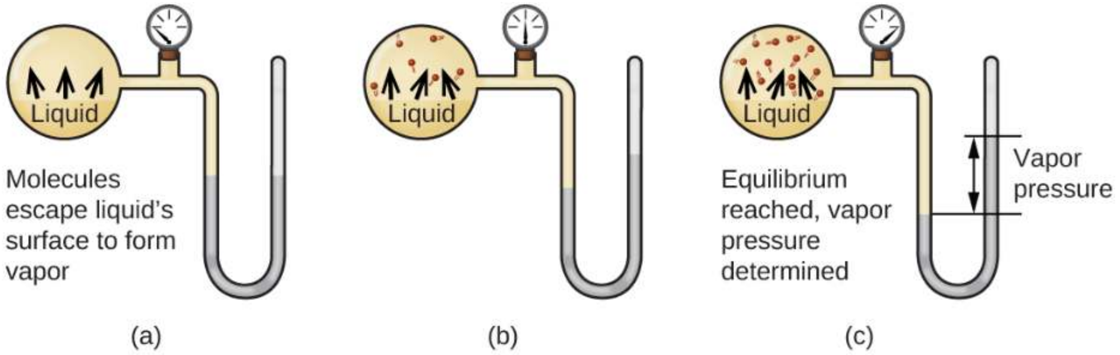
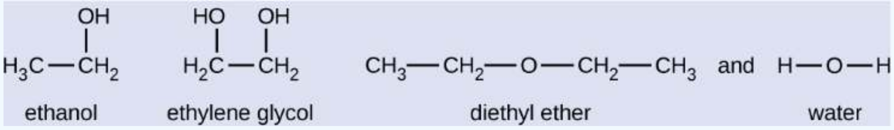


Figure 10.4.110.4.1: In a closed container, dynamic equilibrium is reached when (a) the rate of molecules escaping from the liquid to become the gas (b) increases and eventually (c) equals the rate of gas molecules entering the liquid. When this equilibrium is reached, the vapor pressure of the gas is constant, although the vaporization and condensation processes continue.

The chemical identities of the molecules in a liquid determine the types (and strengths) of intermolecular attractions possible; consequently, different substances will exhibit different equilibrium vapor pressures. Relatively strong intermolecular attractive forces will serve to impede vaporization as well as favoring "recapture" of gas-phase molecules when they collide with the liquid surface, resulting in a relatively low vapor pressure. Weak intermolecular attractions present less of a barrier to vaporization, and a reduced likelihood of gas recapture, yielding relatively high vapor pressures. The following example illustrates this dependence of vapor pressure on intermolecular attractive forces.

✓ Example 4.7.1: Explaining Vapor Pressure in Terms of IMFs

Given the shown structural formulas for these four compounds, explain their relative vapor pressures in terms of types and extents of IMFs:



Solution

Diethyl ether has a very small dipole and most of its intermolecular attractions are London forces. Although this molecule is the largest of the four under consideration, its IMFs are the weakest and, as a result, its molecules most readily escape from the liquid. It also has the highest vapor pressure. Due to its smaller size, ethanol exhibits weaker dispersion forces than diethyl ether. However, ethanol is capable of hydrogen bonding and, therefore, exhibits stronger overall IMFs, which means that fewer molecules escape from the liquid at any given temperature, and so ethanol has a lower vapor pressure than diethyl ether. Water is much smaller than either of the previous substances and exhibits weaker dispersion forces, but its extensive hydrogen bonding provides stronger intermolecular attractions, fewer molecules escaping the liquid, and a lower vapor pressure than for either diethyl ether or ethanol. Ethylene glycol has two −OH groups, so, like water, it exhibits extensive hydrogen bonding. It is much larger than water and thus experiences larger London forces. Its overall IMFs are the largest of these four substances, which means its vaporization rate will be the slowest and, consequently, its vapor pressure the lowest.

? Exercise 4.7.1

At 20 °C, the vapor pressures of several alcohols are given in this table. Explain these vapor pressures in terms of types and extents of IMFs for these alcohols:

At 20 °C, the vapor pressures of several alcohols

Compound	methanol CH_3OH	ethanol C_2H_5OH	propanol C_3H_7OH	butanol C_4H_9OH
Vapor Pressure at 20 °C	11.9 kPa	5.95 kPa	2.67 kPa	0.56 kPa

Answer

All these compounds exhibit hydrogen bonding; these strong IMFs are difficult for the molecules to overcome, so the vapor pressures are relatively low. As the size of molecule increases from methanol to butanol, dispersion forces increase, which means that the vapor pressures decrease as observed:

$$P_{methanol} > P_{ethanol} > P_{propanol} > P_{butanol}$$

As temperature increases, the vapor pressure of a liquid also increases due to the increased average KE of its molecules. Recall that at any given temperature, the molecules of a substance experience a range of kinetic energies, with a certain fraction of molecules having a sufficient energy to overcome IMF and escape the liquid (vaporize). At a higher temperature, a greater fraction of molecules have enough energy to escape from the liquid, as shown in Figure 4.7.2. The escape of more molecules per unit of time and the greater average speed of the molecules that escape both contribute to the higher vapor pressure.

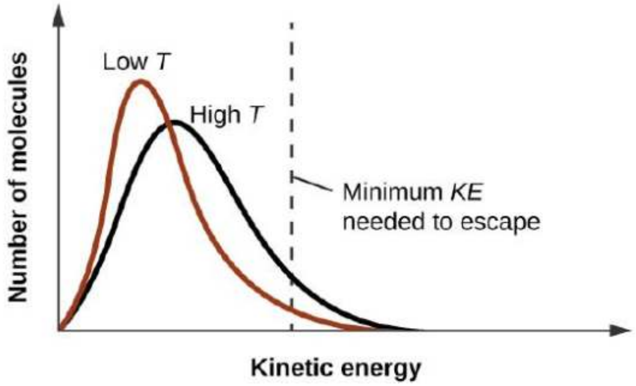


Figure 10.4.210.4.2: Temperature affects the distribution of kinetic energies for the molecules in a liquid. At the higher temperature, more molecules have the necessary kinetic energy, KE, to escape from the liquid into the gas phase.

Boiling Points

When the vapor pressure increases enough to equal the external atmospheric pressure, the liquid reaches its boiling point. The boiling point of a liquid is the temperature at which its equilibrium vapor pressure is equal to the pressure exerted on the liquid by its gaseous surroundings. For liquids in open containers, this pressure is that due to the earth's atmosphere. The normal boiling point of a liquid is defined as its boiling point when surrounding pressure is equal to 1 atm (101.3 kPa). Figure $4.7.3$ shows the variation in vapor pressure with temperature for several different substances. Considering the definition of boiling point, these curves may be seen as depicting the dependence of a liquid's boiling point on surrounding pressure.

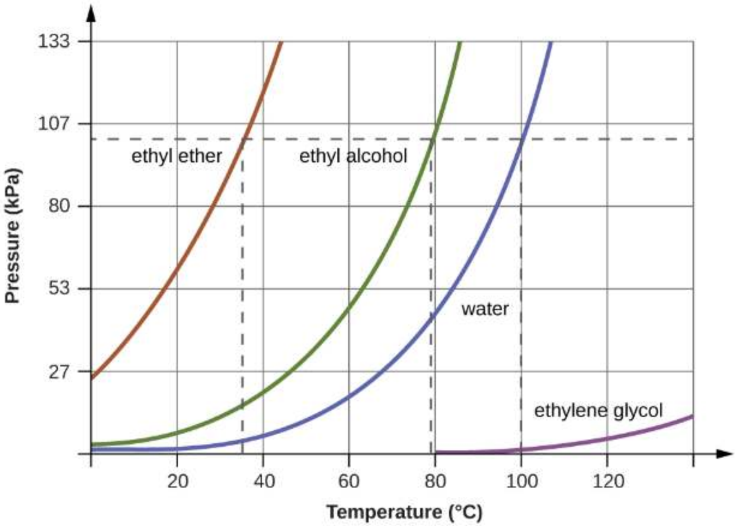


Figure 10.4.310.4.3: The boiling points of liquids are the temperatures at which their equilibrium vapor pressures equal the pressure of the surrounding atmosphere. Normal boiling points are those corresponding to a pressure of 1 atm (101.3 kPa.)

✓ Example $4.7.2$: A Boiling Point at Reduced Pressure

A typical atmospheric pressure in Leadville, Colorado (elevation 10,200 feet) is 68 kPa. Use the graph in Figure $4.7.3$ to determine the boiling point of water at this elevation.

Solution

The graph of the vapor pressure of water versus temperature in Figure $4.7.3$ indicates that the vapor pressure of water is 68 kPa at about 90 °C. Thus, at about 90 °C, the vapor pressure of water will equal the atmospheric pressure in Leadville, and water will boil.

? Exercise 4.7.2

The boiling point of ethyl ether was measured to be 10 °C at a base camp on the slopes of Mount Everest. Use Figure $4.7.3$ to determine the approximate atmospheric pressure at the camp.

Answer

Approximately 40 kPa (0.4 atm)

The quantitative relation between a substance's vapor pressure and its temperature is described by the Clausius-Clapeyron equation:

$$P = Ae^{-\Delta H_{vap}/RT} \tag{4.7.1}$$

where

- ΔH_{vap} is the enthalpy of vaporization for the liquid,
- R is the gas constant, and
- $\ln A$ is a constant whose value depends on the chemical identity of the substance.

Equation $4.7.1$ is often rearranged into logarithmic form to yield the linear equation:

$$\ln P = -\frac{\Delta H_{\text{vap}}}{RT} + \ln A \tag{4.7.2}$$

This linear equation may be expressed in a two-point format that is convenient for use in various computations, as demonstrated in the examples and exercises that follow. If at temperature T_1, the vapor pressure is P_1, and at temperature T_2, the vapor pressure is T_2, the corresponding linear equations are:

$$\ln P_1 = -\frac{\Delta H_{\text{vap}}}{RT_1} + \ln A$$

and

$$\ln P_2 = -\frac{\Delta H_{\text{vap}}}{RT_2} + \ln A \tag{4.7.3}$$

Since the constant, ln A, is the same, these two equations may be rearranged to isolate $\ln A$ and then set them equal to one another:

$$\ln P_1 + \frac{\Delta H_{\text{vap}}}{RT_1} = \ln P_2 + \frac{\Delta H_{\text{vap}}}{RT_2}$$

which can be combined into:

$$\ln\left(\frac{P_2}{P_1}\right) = \frac{\Delta H_{\text{vap}}}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right) \tag{4.7.4}$$

✓ Example 4.7.3: Estimating Enthalpy of Vaporization

Isooctane (2,2,4-trimethylpentane) has an octane rating of 100. It is used as one of the standards for the octane-rating system for gasoline. At 34.0 °C, the vapor pressure of isooctane is 10.0 kPa, and at 98.8 °C, its vapor pressure is 100.0 kPa. Use this information to estimate the enthalpy of vaporization for isooctane.

Solution

The enthalpy of vaporization, ΔH_{vap}, can be determined by using the Clausius-Clapeyron equation (Equation $4.7.4$):

$$\ln\left(\frac{P_2}{P_1}\right) = \frac{\Delta H_{\text{vap}}}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

Since we have two vapor pressure-temperature values

- $T_1 = 34.0^oC = 307.2\,K$
- $P_1 = 10.0\,kPa$ and

- $T_2 = 98.8^oC = 372.0\ K$
- $P_2 = 100\ kPa$

we can substitute them into this equation and solve for ΔH_{vap}. Rearranging the Clausius-Clapeyron equation and solving for ΔH_{vap} yields:

$$\begin{aligned}\Delta H_\text{vap} &= \dfrac{R \cdot \ln\left(\dfrac{P_2}{P_1}\right)}{\left(\dfrac{1}{T_1}-\dfrac{1}{T_2}\right)}\\ &= \dfrac{(8.3145\ J/mol \cdot K)\cdot \ln\left(\dfrac{100\ kPa}{10.0\ kPa}\right)}{\left(\dfrac{1}{307.2\ K}-\dfrac{1}{372.0\ K}\right)}\\ &= 33,800\ J/mol = 33.8\ kJ/mol\end{aligned}$$

Note that the pressure can be in any units, so long as they agree for both P values, but the temperature must be in kelvin for the Clausius-Clapeyron equation to be valid.

? Exercise 4.7.3

At 20.0 °C, the vapor pressure of ethanol is 5.95 kPa, and at 63.5 °C, its vapor pressure is 53.3 kPa. Use this information to estimate the enthalpy of vaporization for ethanol.

Answer

47,782 J/mol = 47.8 kJ/mol

✓ Example 4.7.4: Estimating Temperature (or Vapor Pressure)

For benzene (C_6H_6), the normal boiling point is 80.1 °C and the enthalpy of vaporization is 30.8 kJ/mol. What is the boiling point of benzene in Denver, where atmospheric pressure = 83.4 kPa?

Solution

If the temperature and vapor pressure are known at one point, along with the enthalpy of vaporization, $\Delta H_\text{vap,}$ then the temperature that corresponds to a different vapor pressure (or the vapor pressure that corresponds to a different temperature) can be determined by using the Clausius-Clapeyron equation (Equation 4.7.1) :

$$\ln\left(\dfrac{P_2}{P_1}\right) = \dfrac{\Delta H_\text{vap}}{R}\left(\dfrac{1}{T_1}-\dfrac{1}{T_2}\right)$$

Since the normal boiling point is the temperature at which the vapor pressure equals atmospheric pressure at sea level, we know one vapor pressure-temperature value (T_1 = 80.1 °C = 353.3 K, P_1 = 101.3 kPa, ΔH_{vap} = 30.8 kJ/mol) and want to find the temperature (T_2) that corresponds to vapor pressure P_2 = 83.4 kPa. We can substitute these values into the Clausius-Clapeyron equation and then solve for T_2. Rearranging the Clausius-Clapeyron equation and solving for T_2 yields:

$$T_2 = \left(\frac{-R\cdot\ln\left(\frac{P_2}{P_1}\right)}{\Delta H_{\text{vap}}} + \frac{1}{T_1}\right)^{-1}$$

$$= \left(\frac{-(8.3145\ \text{J/mol}\cdot\text{K})\cdot\ln\left(\frac{83.4\ \text{kPa}}{101.3\ \text{kPa}}\right)}{30,800\ \text{J/mol}} + \frac{1}{353.3\ \text{K}}\right)^{-1}$$

$$= 346.9\ \text{K or } 73.8^{\circ}\text{C}$$

? Exercise 4.7.4

For acetone $(\mathrm{CH_3})_2\mathrm{CO}$, the normal boiling point is 56.5 °C and the enthalpy of vaporization is 31.3 kJ/mol. What is the vapor pressure of acetone at 25.0 °C?

Answer

30.1 kPa

Enthalpy of Vaporization

Vaporization is an endothermic process. The cooling effect can be evident when you leave a swimming pool or a shower. When the water on your skin evaporates, it removes heat from your skin and causes you to feel cold. The energy change associated with the vaporization process is the enthalpy of vaporization, ΔH_{vap}. For example, the vaporization of water at standard temperature is represented by:

$$\mathrm{H_2O}(l) \longrightarrow \mathrm{H_2O}(g) \quad \Delta H_{\text{vap}} = 44.01\ \text{kJ/mol}$$

As described in the chapter on thermochemistry, the reverse of an endothermic process is exothermic. And so, the condensation of a gas releases heat:

$$\mathrm{H_2O}(g) \longrightarrow \mathrm{H_2O}(l) \quad \Delta H_{\text{con}} = -\Delta H_{\text{vap}} = -44.01\ \text{kJ/mol}$$

✓ Example 4.7.5: Using Enthalpy of Vaporization

One way our body is cooled is by evaporation of the water in sweat (Figure 4.7.4). In very hot climates, we can lose as much as 1.5 L of sweat per day. Although sweat is not pure water, we can get an approximate value of the amount of heat removed by evaporation by assuming that it is. How much heat is required to evaporate 1.5 L of water (1.5 kg) at T = 37 °C (normal body temperature); $\Delta H_{vap} = 43.46\ kJ/mol$ at 37 °C.

Figure 4.7.4: Evaporation of sweat helps cool the body. (credit: “Kullez”/Flickr)

Solution We start with the known volume of sweat (approximated as just water) and use the given information to convert to the amount of heat needed:

$$1.5\ \cancel{\text{L}} \times \frac{1000\ \cancel{\text{g}}}{1\ \cancel{\text{L}}} \times \frac{1\ \cancel{\text{mol}}}{18\ \cancel{\text{g}}} \times \frac{43.46\ \text{kJ}}{1\ \cancel{\text{mol}}} = 3.6 \times 10^3\ \text{kJ}$$

Thus, 3600 kJ of heat are removed by the evaporation of 1.5 L of water.

? Exercise 4.7.5: Boiling Ammonia

How much heat is required to evaporate 100.0 g of liquid ammonia, NH_3, at its boiling point if its enthalpy of vaporization is 4.8 kJ/mol?

Answer

28 kJ

Melting and Freezing

When we heat a crystalline solid, we increase the average energy of its atoms, molecules, or ions and the solid gets hotter. At some point, the added energy becomes large enough to partially overcome the forces holding the molecules or ions of the solid in their fixed positions, and the solid begins the process of transitioning to the liquid state, or melting. At this point, the temperature of the solid stops rising, despite the continual input of heat, and it remains constant until all of the solid is melted. Only after all of the solid has melted will continued heating increase the temperature of the liquid (Figure 4.7.5.

Figure 4.7.5: (a) This beaker of ice has a temperature of −12.0 °C. (b) After 10 minutes the ice has absorbed enough heat from the air to warm to 0 °C. A small amount has melted. (c) Thirty minutes later, the ice has absorbed more heat, but its temperature is still 0 °C. The ice melts without changing its temperature. (d) Only after all the ice has melted does the heat absorbed cause the temperature to increase to 22.2 °C. (credit: modification of work by Mark Ott).

If we stop heating during melting and place the mixture of solid and liquid in a perfectly insulated container so no heat can enter or escape, the solid and liquid phases remain in equilibrium. This is almost the situation with a mixture of ice and water in a very good thermos bottle; almost no heat gets in or out, and the mixture of solid ice and liquid water remains for hours. In a mixture of solid and liquid at equilibrium, the reciprocal processes of melting and freezing occur at equal rates, and the quantities of solid and liquid therefore remain constant. The temperature at which the solid and liquid phases of a given substance are in equilibrium is called the melting point of the solid or the freezing point of the liquid. Use of one term or the other is normally dictated by the direction of the phase transition being considered, for example, solid to liquid (melting) or liquid to solid (freezing).

The enthalpy of fusion and the melting point of a crystalline solid depend on the strength of the attractive forces between the units present in the crystal. Molecules with weak attractive forces form crystals with low melting points. Crystals consisting of particles with stronger attractive forces melt at higher temperatures.

The amount of heat required to change one mole of a substance from the solid state to the liquid state is the enthalpy of fusion, ΔHfus of the substance. The enthalpy of fusion of ice is 6.0 kJ/mol at 0 °C. Fusion (melting) is an endothermic process:

$$H_2O_{(s)} \rightarrow H_2O_{(l)} \quad \Delta H_{\text{fus}} = 6.01\ \text{kJ/mol} \tag{4.7.5}$$

The reciprocal process, freezing, is an exothermic process whose enthalpy change is −6.0 kJ/mol at 0 °C:

$$H_2O_{(l)} \rightarrow H_2O_{(s)} \quad \Delta H_{\text{frz}} = -\Delta H_{\text{fus}} = -6.01 \text{ kJ/mol} \tag{4.7.6}$$

Sublimation and Deposition

Some solids can transition directly into the gaseous state, bypassing the liquid state, via a process known as sublimation. At room temperature and standard pressure, a piece of dry ice (solid CO_2) sublimes, appearing to gradually disappear without ever forming any liquid. Snow and ice sublime at temperatures below the melting point of water, a slow process that may be accelerated by winds and the reduced atmospheric pressures at high altitudes. When solid iodine is warmed, the solid sublimes and a vivid purple vapor forms (Figure $4.7.6$). The reverse of sublimation is called deposition, a process in which gaseous substances condense directly into the solid state, bypassing the liquid state. The formation of frost is an example of deposition.

Figure $4.7.6$: Sublimation of solid iodine in the bottom of the tube produces a purple gas that subsequently deposits as solid iodine on the colder part of the tube above. (credit: modification of work by Mark Ott)

Like vaporization, the process of sublimation requires an input of energy to overcome intermolecular attractions. The enthalpy of sublimation, ΔHsub, is the energy required to convert one mole of a substance from the solid to the gaseous state. For example, the sublimation of carbon dioxide is represented by:

$$CO_2(s) \longrightarrow CO_2(g) \quad \Delta H_{\text{sub}} = 26.1 \text{ kJ/mol}$$

Likewise, the enthalpy change for the reverse process of deposition is equal in magnitude but opposite in sign to that for sublimation:

$$CO_2(g) \longrightarrow CO_2(s) \quad \Delta H_{\text{dep}} = -\Delta H_{\text{sub}} = -26.1 \text{ kJ/mol}$$

Consider the extent to which intermolecular attractions must be overcome to achieve a given phase transition. Converting a solid into a liquid requires that these attractions be only partially overcome; transition to the gaseous state requires that they be completely overcome. As a result, the enthalpy of fusion for a substance is less than its enthalpy of vaporization. This same logic can be used to derive an approximate relation between the enthalpies of all phase changes for a given substance. Though not an entirely accurate description, sublimation may be conveniently modeled as a sequential two-step process of melting followed by vaporization in order to apply Hess's Law.

$$\begin{array}{c} \text{solid} \longrightarrow \text{liquid} \quad \Delta H_{\text{fus}} \\ \underline{\text{liquid} \longrightarrow \text{gas} \quad \Delta H_{\text{vap}}} \\ \text{solid} \longrightarrow \text{gas} \quad \Delta H_{\text{sub}} = \Delta H_{\text{fus}} + \Delta H_{\text{vap}} \end{array}$$

Viewed in this manner, the enthalpy of sublimation for a substance may be estimated as the sum of its enthalpies of fusion and vaporization, as illustrated in Figure $4.7.7$. For example:

$$\begin{array}{lll} \text{solid} \longrightarrow \text{liquid} & & \Delta H_{\text{fus}} \\ \text{liquid} \longrightarrow \text{gas} & & \Delta H_{\text{vap}} \\ \hline \text{solid} \longrightarrow \text{gas} & \Delta H_{\text{sub}} & = \Delta H_{\text{fus}} + \Delta H_{\text{vap}} \end{array}$$

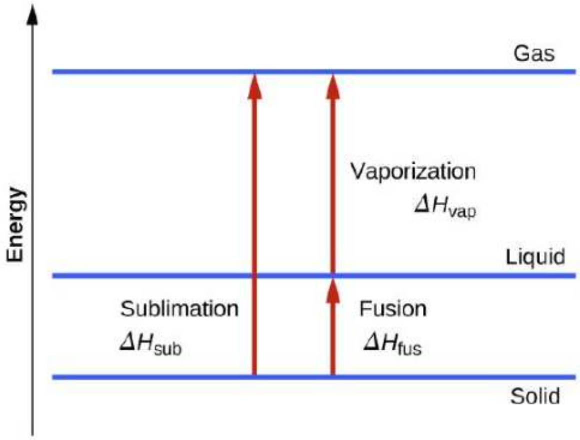


Figure $4.7.7$: For a given substance, the sum of its enthalpy of fusion and enthalpy of vaporization is approximately equal to its enthalpy of sublimation.

Heating and Cooling Curves

In the chapter on thermochemistry, the relation between the amount of heat absorbed or related by a substance, q, and its accompanying temperature change, ΔT, was introduced:

$$q = mc\Delta T$$

where m is the mass of the substance and c is its specific heat. The relation applies to matter being heated or cooled, but not undergoing a change in state. When a substance being heated or cooled reaches a temperature corresponding to one of its phase transitions, further gain or loss of heat is a result of diminishing or enhancing intermolecular attractions, instead of increasing or decreasing molecular kinetic energies. While a substance is undergoing a change in state, its temperature remains constant. Figure $4.7.8$ shows a typical heating curve.

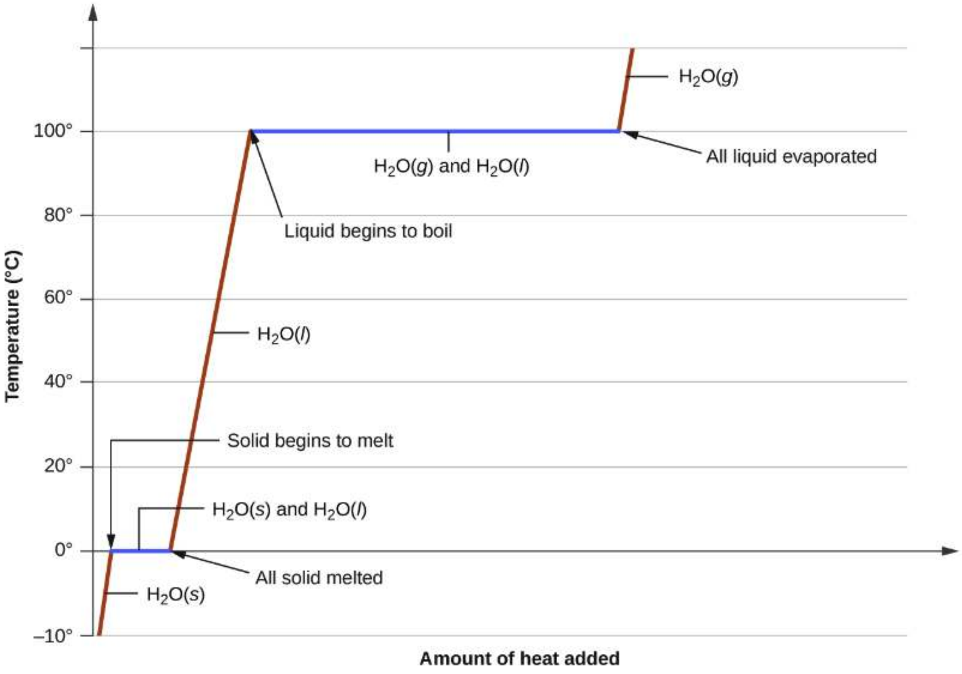


Figure $4.7.8$: A typical heating curve for a substance depicts changes in temperature that result as the substance absorbs increasing amounts of heat. Plateaus in the curve (regions of constant temperature) are exhibited when the substance undergoes phase transitions.

Consider the example of heating a pot of water to boiling. A stove burner will supply heat at a roughly constant rate; initially, this heat serves to increase the water’s temperature. When the water reaches its boiling point, the temperature remains constant despite the continued input of heat from the stove burner. This same temperature is maintained by the water as long as it is boiling. If the burner setting is increased to provide heat at a greater rate, the water temperature does not rise, but instead the boiling becomes more vigorous (rapid). This behavior is observed for other phase transitions as well: For example, temperature remains constant while the change of state is in progress.

Example 4.7.6: Total Heat Needed to Change Temperature and Phase for a Substance

How much heat is required to convert 135 g of ice at −15 °C into water vapor at 120 °C?

Solution

The transition described involves the following steps:

1. Heat ice from −15 °C to 0 °C
2. Melt ice
3. Heat water from 0 °C to 100 °C
4. Boil water
5. Heat steam from 100 °C to 120 °C

The heat needed to change the temperature of a given substance (with no change in phase) is: $q = m \times c \times \Delta T$ (see previous chapter on thermochemistry). The heat needed to induce a given change in phase is given by $q = n \times \Delta H$.

Using these equations with the appropriate values for specific heat of ice, water, and steam, and enthalpies of fusion and vaporization, we have:

$$\begin{aligned} q_{\text{total}} &= (m \cdot c \cdot \Delta T)_{\text{ice}} + n \cdot \Delta H_{\text{fus}} + (m \cdot c \cdot \Delta T)_{\text{water}} + n \cdot \Delta H_{\text{vap}} + (m \cdot c \cdot \Delta T)_{\text{steam}} \\ &= (135\ \text{g} \cdot 2.09\ \text{J/g} \cdot {}^\circ\text{C} \cdot 15^\circ\text{C}) + \left(135 \cdot \frac{1\ \text{mol}}{18.02\ \text{g}} \cdot 6.01\ \text{kJ/mol}\right) \\ &+ (135\ \text{g} \cdot 4.18\ \text{J/g} \cdot {}^\circ\text{C} \cdot 100^\circ\text{C}) + \left(135\ \text{g} \cdot \frac{1\ \text{mol}}{18.02\ \text{g}} \cdot 40.67\ \text{kJ/mol}\right) \\ &+ (135\ \text{g} \cdot 1.84\ \text{J/g} \cdot {}^\circ\text{C} \cdot 20^\circ\text{C}) \\ &= 4230\ \text{J} + 45.0\ \text{kJ} + 56,500\ \text{J} + 305\ \text{kJ} + 4970\ \text{J} \end{aligned}$$

Converting the quantities in J to kJ permits them to be summed, yielding the total heat required:

$$= 4.23\ \text{kJ} + 45.0\ \text{kJ} + 56.5\ \text{kJ} + 305\ \text{kJ} + 4.97\ \text{kJ} = 416\ \text{kJ}$$

Exercise 4.7.6

What is the total amount of heat released when 94.0 g water at 80.0 °C cools to form ice at −30.0 °C?

Answer

40.5 kJ

Summary

Phase transitions are processes that convert matter from one physical state into another. There are six phase transitions between the three phases of matter. Melting, vaporization, and sublimation are all endothermic processes, requiring an input of heat to overcome intermolecular attractions. The reciprocal transitions of freezing, condensation, and deposition are all exothermic processes, involving heat as intermolecular attractive forces are established or strengthened. The temperatures at which phase transitions occur are determined by the relative strengths of intermolecular attractions and are, therefore, dependent on the chemical identity of the substance.

Key Equations

- $P = Ae^{-\Delta H_{\text{vap}}/RT}$
- $\ln P = -\frac{\Delta H_{\text{vap}}}{RT} + \ln A$
- $\ln\left(\frac{P_2}{P_1}\right) = \frac{\Delta H_{\text{vap}}}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

Glossary

boiling point
temperature at which the vapor pressure of a liquid equals the pressure of the gas above it

Clausius-Clapeyron equation
mathematical relationship between the temperature, vapor pressure, and enthalpy of vaporization for a substance

condensation
change from a gaseous to a liquid state

deposition
change from a gaseous state directly to a solid state

dynamic equilibrium
state of a system in which reciprocal processes are occurring at equal rates

freezing
change from a liquid state to a solid state

freezing point
temperature at which the solid and liquid phases of a substance are in equilibrium; see also *melting point*

melting
change from a solid state to a liquid state

melting point
temperature at which the solid and liquid phases of a substance are in equilibrium; see also *freezing point*

normal boiling point
temperature at which a liquid’s vapor pressure equals 1 atm (760 torr)

sublimation
change from solid state directly to gaseous state

vapor pressure
(also, equilibrium vapor pressure) pressure exerted by a vapor in equilibrium with a solid or a liquid at a given temperature

vaporization
change from liquid state to gaseous state

4.8: Colloids

Learning Objectives

- Describe the composition and properties of colloidal dispersions
- List and explain several technological applications of colloids

As a child, you may have made suspensions such as mixtures of mud and water, flour and water, or a suspension of solid pigments in water, known as tempera paint. These suspensions are heterogeneous mixtures composed of relatively large particles that are visible (or that can be seen with a magnifying glass). They are cloudy, and the suspended particles settle out after mixing. On the other hand, when we make a solution, we prepare a homogeneous mixture in which no settling occurs and in which the dissolved species are molecules or ions. Solutions exhibit completely different behavior from suspensions. A solution may be colored, but it is transparent, the molecules or ions are invisible, and they do not settle out on standing. A group of mixtures called colloids (or colloidal dispersions) exhibit properties intermediate between those of suspensions and solutions (Figure $4.8.1$). The particles in a colloid are larger than most simple molecules; however, colloidal particles are small enough that they do not settle out upon standing.

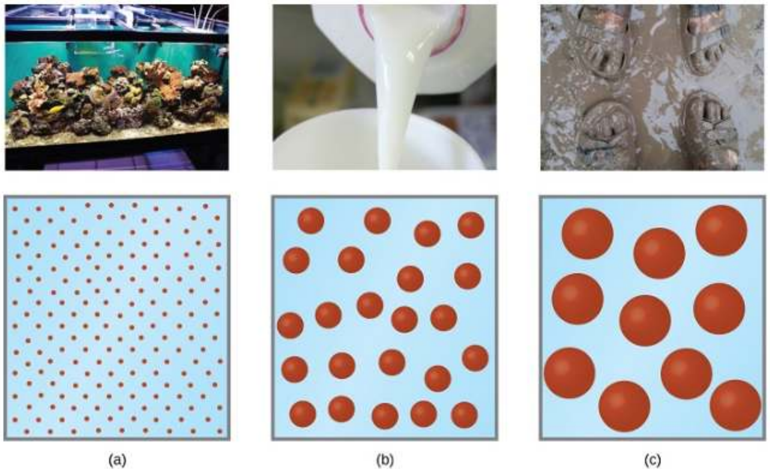


Figure $4.8.1$: (a) A solution is a homogeneous mixture that appears clear, such as the saltwater in this aquarium. (b) In a colloid, such as milk, the particles are much larger but remain dispersed and do not settle. (c) A suspension, such as mud, is a heterogeneous mixture of suspended particles that appears cloudy and in which the particles can settle. (credit a photo: modification of work by Adam Wimsatt; credit b photo: modification of work by Melissa Wiese; credit c photo: modification of work by Peter Burgess)

The particles in a colloid are large enough to scatter light, a phenomenon called the Tyndall effect. This can make colloidal mixtures appear cloudy or opaque, such as the searchlight beams shown in Figure $4.8.2$. Clouds are colloidal mixtures. They are composed of water droplets that are much larger than molecules, but that are small enough that they do not settle out.

Figure $4.8.2$: The paths of searchlight beams are made visible when light is scattered by colloidal-size particles in the air (fog, smoke, etc.). (credit: "Bahman"/Wikimedia Commons)

The term "colloid"—from the Greek words *kolla*, meaning "glue," and *eidos*, meaning "like"—was first used in 1861 by Thomas Graham to classify mixtures such as starch in water and gelatin. Many colloidal particles are aggregates of hundreds or thousands of molecules, but others (such as proteins and polymer molecules) consist of a single extremely large molecule. The protein and synthetic polymer molecules that form colloids may have molecular masses ranging from a few thousand to many million atomic mass units.

Analogous to the identification of solution components as "solute" and "solvent," the components of a colloid are likewise classified according to their relative amounts. The particulate component typically present in a relatively minor amount is called the dispersed phase and the substance or solution throughout which the particulate is dispersed is called the dispersion medium. Colloids may involve virtually any combination of physical states (gas in liquid, liquid in solid, solid in gas, etc.), as illustrated by the examples of colloidal systems given in Table $4.8.1$.

Table $4.8.2$: Examples of Colloidal Systems

Dispersed Phase	Dispersion Medium	Common Examples	Name
solid	gas	smoke, dust	—
solid	liquid	starch in water, some inks, paints, milk of magnesia	sol
solid	solid	some colored gems, some alloys	—
liquid	gas	clouds, fogs, mists, sprays	aerosol
liquid	liquid	milk, mayonnaise, butter	emulsion
liquid	solid	jellies, gels, pearl, opal (H_2O in SiO_2)	gel
gas	liquid	foams, whipped cream, beaten egg whites	foam
gas	solid	pumice, floating soaps	—

Preparation of Colloidal Systems

We can prepare a colloidal system by producing particles of colloidal dimensions and distributing these particles throughout a dispersion medium. Particles of colloidal size are formed by two methods:

1. Dispersion methods: that is, by breaking down larger particles. For example, paint pigments are produced by dispersing large particles by grinding in special mills.
2. Condensation methods: that is, growth from smaller units, such as molecules or ions. For example, clouds form when water molecules condense and form very small droplets.

A few solid substances, when brought into contact with water, disperse spontaneously and form colloidal systems. Gelatin, glue, starch, and dehydrated milk powder behave in this manner. The particles are already of colloidal size; the water simply disperses them. Powdered milk particles of colloidal size are produced by dehydrating milk spray. Some atomizers produce colloidal dispersions of a liquid in air.

We can prepare an emulsion by shaking together or blending two immiscible liquids. This breaks one liquid into droplets of colloidal size, which then disperse throughout the other liquid. Oil spills in the ocean may be difficult to clean up, partly because wave action can cause the oil and water to form an emulsion. In many emulsions, however, the dispersed phase tends to coalesce, form large drops, and separate. Therefore, emulsions are usually stabilized by an emulsifying agent, a substance that inhibits the coalescence of the dispersed liquid. For example, a little soap will stabilize an emulsion of kerosene in water. Milk is an emulsion of butterfat in water, with the protein casein as the emulsifying agent. Mayonnaise is an emulsion of oil in vinegar, with egg yolk components as the emulsifying agents.

Condensation methods form colloidal particles by aggregation of molecules or ions. If the particles grow beyond the colloidal size range, drops or precipitates form, and no colloidal system results. Clouds form when water molecules aggregate and form colloid-sized particles. If these water particles coalesce to form adequately large water drops of liquid water or crystals of solid water, they settle from the sky as rain, sleet, or snow. Many condensation methods involve chemical reactions. We can prepare a red colloidal suspension of iron(III) hydroxide by mixing a concentrated solution of iron(III) chloride with hot water:

$$\mathrm{Fe^{3+}}_{(aq)} + 3\mathrm{Cl^-}_{(aq)} + 6\mathrm{H_2O}_{(l)} \longrightarrow \mathrm{Fe(OH)}_{3(s)} + \mathrm{H_3O^+}_{(aq)} + 3\mathrm{Cl^-}_{(aq)} \tag{4.8.1}$$

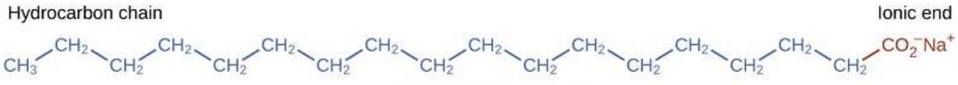
A colloidal gold sol results from the reduction of a very dilute solution of gold(III) chloride by a reducing agent such as formaldehyde, tin(II) chloride, or iron(II) sulfate:

$$\mathrm{Au^{3+}} + 3\,\mathrm{e^-} \rightarrow \mathrm{Au} \tag{4.8.2}$$

Some gold sols prepared in 1857 are still intact (the particles have not coalesced and settled), illustrating the long-term stability of many colloids.

Soaps and Detergents

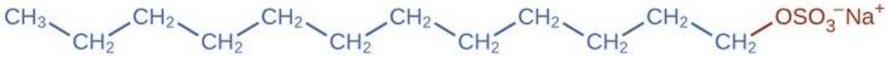
Pioneers made soap by boiling fats with a strongly basic solution made by leaching potassium carbonate, K_2CO_3, from wood ashes with hot water. Animal fats contain polyesters of fatty acids (long-chain carboxylic acids). When animal fats are treated with a base like potassium carbonate or sodium hydroxide, glycerol and salts of fatty acids such as palmitic, oleic, and stearic acid are formed. The salts of fatty acids are called soaps. The sodium salt of stearic acid, sodium stearate, has the formula $C_{17}H_{35}CO_2Na$ and contains an uncharged nonpolar hydrocarbon chain, the $C_{17}H_{35}-$ unit, and an ionic carboxylate group, the $-CO_2^-$ unit (Figure $4.8.3$).



sodium stearate (soap)

Figure $4.8.3$: Soaps contain a nonpolar hydrocarbon end (blue) and an ionic end (red). The ionic end is a carboxylate group. The length of the hydrocarbon end can vary from soap to soap.

*Deterg*ents (soap substitutes) also contain nonpolar hydrocarbon chains, such as $C_{12}H_{25}—$, and an ionic group, such as a sulfate $—OSO_3^-$, or a sulfonate $—SO_3^-$ (Figure $4.8.4$). Soaps form insoluble calcium and magnesium compounds in hard water; detergents form water-soluble products—a definite advantage for detergents.



sodium lauryl sulfate (detergent)

Figure 4.8.4: Detergents contain a nonpolar hydrocarbon end (blue) and an ionic end (red). The ionic end can be either a sulfate or a sulfonate. The length of the hydrocarbon end can vary from detergent to detergent.

The cleaning action of soaps and detergents can be explained in terms of the structures of the molecules involved. The hydrocarbon (nonpolar) end of a soap or detergent molecule dissolves in, or is attracted to, nonpolar substances such as oil, grease, or dirt particles. The ionic end is attracted by water (polar), illustrated in Figure 4.8.5. As a result, the soap or detergent molecules become oriented at the interface between the dirt particles and the water so they act as a kind of bridge between two different kinds of matter, nonpolar and polar. Molecules such as this are termed amphiphilic since they have both a hydrophobic (“water-fearing”) part and a hydrophilic (“water-loving”) part. As a consequence, dirt particles become suspended as colloidal particles and are readily washed away.

Figure 4.8.5: This diagrammatic cross section of an emulsified drop of oil in water shows how soap or detergent acts as an emulsifier.

Deepwater Horizon Oil Spill

The blowout of the Deepwater Horizon oil drilling rig on April 20, 2010, in the Gulf of Mexico near Mississippi began the largest marine oil spill in the history of the petroleum. In the 87 days following the blowout, an estimated 4.9 million barrels (210 million gallons) of oil flowed from the ruptured well 5000 feet below the water’s surface. The well was finally declared sealed on September 19, 2010.

Crude oil is immiscible with and less dense than water, so the spilled oil rose to the surface of the water. Floating booms, skimmer ships, and controlled burns were used to remove oil from the water’s surface in an attempt to protect beaches and wetlands along the Gulf coast. In addition to removal of the oil, attempts were also made to lessen its environmental impact by rendering it “soluble” (in the loose sense of the term) and thus allowing it to be diluted to hopefully less harmful levels by the vast volume of ocean water. This approach used 1.84 million gallons of the oil dispersant Corexit 9527, most of which was injected underwater at the site of the leak, with small amounts being sprayed on top of the spill. Corexit 9527 contains 2-butoxyethanol ($C_6H_{14}O_2$), an amphiphilic molecule whose polar and nonpolar ends are useful for emulsifying oil into small droplets, increasing the surface area of the oil and making it more available to marine bacteria for digestion (Figure 4.8.6). While this approach avoids many of the immediate hazards that bulk oil poses to marine and coastal ecosystems, it introduces the possibility of long-term effects resulting from the introduction of the complex and potential toxic components of petroleum into the ocean’s food chain. A number of organizations are involved in monitoring the extended impact of this oil spill, including the National Oceanic and Atmospheric Administration.

(a) (b) (c)

Figure 4.8.6: (a) This NASA satellite image shows the oil slick from the Deepwater Horizon spill. (b) A US Air Force plane sprays Corexit, a dispersant. (c) The molecular structure of 2-butoxyethanol is shown. (credit a: modification of work by “NASA, FT2, demis.nl”/Wikimedia Commons; credit b: modification of work by “NASA/MODIS Rapid Response Team”/Wikimedia Commons)

Electrical Properties of Colloidal Particles

Dispersed colloidal particles are often electrically charged. A colloidal particle of iron(III) hydroxide, for example, does not contain enough hydroxide ions to compensate exactly for the positive charges on the iron(III) ions. Thus, each individual colloidal particle bears a positive charge, and the colloidal dispersion consists of charged colloidal particles and some free hydroxide ions, which keep the dispersion electrically neutral. Most metal hydroxide colloids have positive charges, whereas most metals and metal sulfides form negatively charged dispersions. All colloidal particles in any one system have charges of the same sign. This helps keep them dispersed because particles containing like charges repel each other.

We can take advantage of the charge on colloidal particles to remove them from a variety of mixtures. If we place a colloidal dispersion in a container with charged electrodes, positively charged particles, such as iron(III) hydroxide particles, would move to the negative electrode. There, the colloidal particles lose their charge and coagulate as a precipitate.

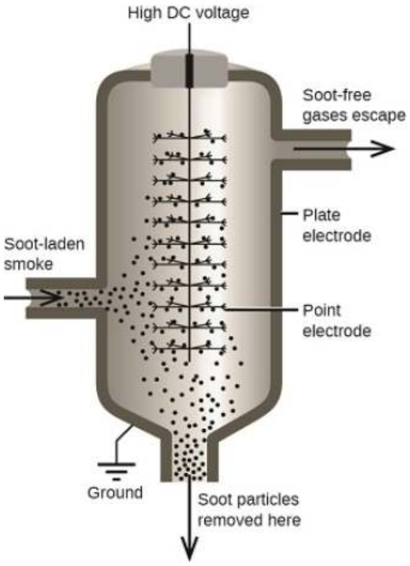


Figure 4.8.7: In a Cottrell precipitator, positively and negatively charged particles are attracted to highly charged electrodes, where they are neutralized and deposited as dust.

The carbon and dust particles in smoke are often colloidally dispersed and electrically charged. The charged particles are attracted to highly charged electrodes, where they are neutralized and deposited as dust (Figure 4.8.7. This is one of the important methods used to clean up the smoke from a variety of industrial processes. The process is also important in the recovery of valuable products from the smoke and flue dust of smelters, furnaces, and kilns. There are also ionic air filters designed for home use to improve indoor air quality.

Gels

When we make gelatin, such as Jell-O, we are making a type of colloid (Figure 4.8.8). Gelatin sets on cooling because the hot aqueous mixture of gelatin coagulates as it cools and the whole mass, including the liquid, sets to an extremely viscous body known as a gel, a colloid in which the dispersing medium is a solid and the dispersed phase is a liquid. It appears that the fibers of the dispersing medium form a complex three-dimensional network, the interstices being filled with the liquid medium or a dilute solution of the dispersing medium. Because the formation of a gel is accompanied by the taking up of water or some other solvent, the gel is said to be hydrated or solvated.

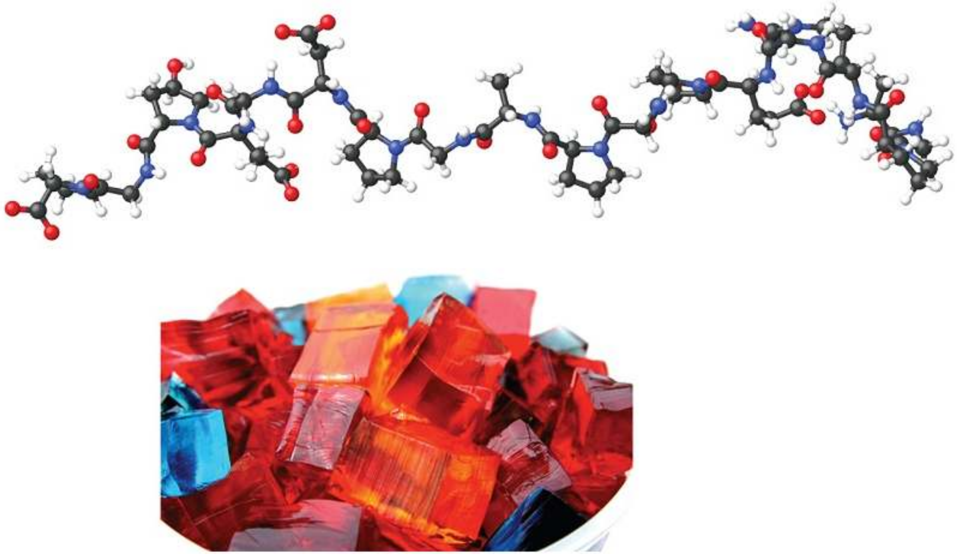


Figure 4.8.8: Gelatin desserts are colloids in which an aqueous solution of sweeteners and flavors is dispersed throughout a medium of solid proteins. (credit photo: modification of work by Steven Depolo).

Pectin, a carbohydrate from fruit juices, is a gel-forming substance important in jelly making. Silica gel, a colloidal dispersion of hydrated silicon dioxide, is formed when dilute hydrochloric acid is added to a dilute solution of sodium silicate. Canned Heat is a gel made by mixing alcohol and a saturated aqueous solution of calcium acetate.

Summary

Colloids are mixtures in which one or more substances are dispersed as relatively large solid particles or liquid droplets throughout a solid, liquid, or gaseous medium. The particles of a colloid remain dispersed and do not settle due to gravity, and they are often electrically charged. Colloids are widespread in nature and are involved in many technological applications.

Glossary

amphiphilic

molecules possessing both hydrophobic (nonpolar) and a hydrophilic (polar) parts

colloid

(also, colloidal dispersion) mixture in which relatively large solid or liquid particles are dispersed uniformly throughout a gas, liquid, or solid

dispersion medium

solid, liquid, or gas in which colloidal particles are dispersed

dispersed phase

substance present as relatively large solid or liquid particles in a colloid

emulsifying agent

amphiphilic substance used to stabilize the particles of some emulsions

emulsion
colloid formed from immiscible liquids

gel
colloidal dispersion of a liquid in a solid

Tyndall effect
scattering of visible light by a colloidal dispersion

4.9: Intermolecular Forces

Learning Objectives

- To describe the intermolecular forces in liquids.

The properties of liquids are intermediate between those of gases and solids, but are more similar to solids. In contrast to *intra*molecular forces, such as the covalent bonds that hold atoms together in molecules and polyatomic ions, *inter*molecular forces hold molecules together in a liquid or solid. Intermolecular forces are generally much weaker than covalent bonds. For example, it requires 927 kJ to overcome the intramolecular forces and break both O–H bonds in 1 mol of water, but it takes only about 41 kJ to overcome the intermolecular attractions and convert 1 mol of liquid water to water vapor at 100°C. (Despite this seemingly low value, the intermolecular forces in liquid water are among the strongest such forces known!) Given the large difference in the strengths of intra- and intermolecular forces, changes between the solid, liquid, and gaseous states almost invariably occur for molecular substances *without breaking covalent bonds*.

> *The properties of liquids are intermediate between those of gases and solids, but are more similar to solids.*

Intermolecular forces determine bulk properties, such as the melting points of solids and the boiling points of liquids. Liquids boil when the molecules have enough thermal energy to overcome the intermolecular attractive forces that hold them together, thereby forming bubbles of vapor within the liquid. Similarly, solids melt when the molecules acquire enough thermal energy to overcome the intermolecular forces that lock them into place in the solid.

Intermolecular forces are electrostatic in nature; that is, they arise from the interaction between positively and negatively charged species. Like covalent and ionic bonds, intermolecular interactions are the sum of both attractive and repulsive components. Because electrostatic interactions fall off rapidly with increasing distance between molecules, intermolecular interactions are most important for solids and liquids, where the molecules are close together. These interactions become important for gases only at very high pressures, where they are responsible for the observed deviations from the ideal gas law at high pressures.

In this section, we explicitly consider three kinds of intermolecular interactions. There are two additional types of electrostatic interaction that you are already familiar with: the ion–ion interactions that are responsible for ionic bonding, and the ion–dipole interactions that occur when ionic substances dissolve in a polar substance such as water. The first two are often described collectively as van der Waals forces.

Application: Geckos and Intermolecular Forces

Geckos have an amazing ability to adhere to most surfaces. They can quickly run up smooth walls and across ceilings that have no toe-holds, and they do this without having suction cups or a sticky substance on their toes. And while a gecko can lift its feet easily as it walks along a surface, if you attempt to pick it up, it sticks to the surface. How are geckos (as well as spiders and some other insects) able to do this? Although this phenomenon has been investigated for hundreds of years, scientists only recently uncovered the details of the process that allows geckos' feet to behave this way.

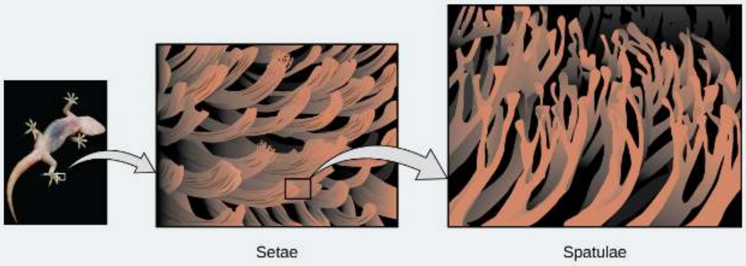


Figure 10.2.710.2.7: Geckos' toes contain large numbers of tiny hairs (setae), which branch into many triangular tips (spatulae). Geckos adhere to surfaces because of van der Waals attractions between the surface and a gecko's millions of spatulae. By changing how the spatulae contact the surface, geckos can turn their stickiness "on" and "off." (credit photo:

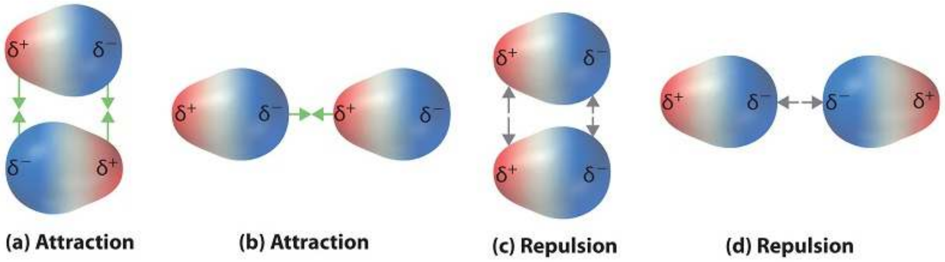
modification of work by “JC*+A!”/Flickr) Three figures are shown. The first is a photo of the bottom of a gecko’s foot. The second is bigger version which shows the setae. The third is a bigger version of the setae and shows the spatulae.

Geckos’ toes are covered with hundreds of thousands of tiny hairs known as *setae*, with each seta, in turn, branching into hundreds of tiny, flat, triangular tips called *spatulae*. The huge numbers of spatulae on its setae provide a gecko, shown in Figure 10.2.710.2.7, with a large total surface area for sticking to a surface. In 2000, Kellar Autumn, who leads a multi-institutional gecko research team, found that geckos adhered equally well to both polar silicon dioxide and nonpolar gallium arsenide. This proved that geckos stick to surfaces because of dispersion forces—weak intermolecular attractions arising from temporary, synchronized charge distributions between adjacent molecules. Although dispersion forces are very weak, the total attraction over millions of spatulae is large enough to support many times the gecko’s weight.

In 2014, two scientists developed a model to explain how geckos can rapidly transition from “sticky” to “non-sticky.” Alex Greaney and Congcong Hu at Oregon State University described how geckos can achieve this by changing the angle between their spatulae and the surface. Geckos’ feet, which are normally nonsticky, become sticky when a small shear force is applied. By curling and uncurling their toes, geckos can alternate between sticking and unsticking from a surface, and thus easily move across it. Further investigations may eventually lead to the development of better adhesives and other applications.

Dipole–Dipole Interactions

Polar covalent bonds behave as if the bonded atoms have localized fractional charges that are equal but opposite (i.e., the two bonded atoms generate a *dipole*). If the structure of a molecule is such that the individual bond dipoles do not cancel one another, then the molecule has a net dipole moment. Molecules with net dipole moments tend to align themselves so that the positive end of one dipole is near the negative end of another and vice versa, as shown in Figure 4.9.1*a*.



(a) Attraction (b) Attraction (c) Repulsion (d) Repulsion

Figure 4.9.1: Attractive and Repulsive Dipole–Dipole Interactions. (a and b) Molecular orientations in which the positive end of one dipole (δ^+) is near the negative end of another (δ^-) (and vice versa) produce attractive interactions. (c and d) Molecular orientations that juxtapose the positive or negative ends of the dipoles on adjacent molecules produce repulsive interactions. (CC BY-SA-NC; anonymous)

These arrangements are more stable than arrangements in which two positive or two negative ends are adjacent (Figure $4.9.1c$). Hence dipole–dipole interactions, such as those in Figure $4.9.1b$, are *attractive intermolecular interactions*, whereas those in Figure $4.9.1d$ are *repulsive intermolecular interactions*. Because molecules in a liquid move freely and continuously, molecules always experience both attractive and repulsive dipole–dipole interactions simultaneously, as shown in Figure $4.9.2$. On average, however, the attractive interactions dominate.

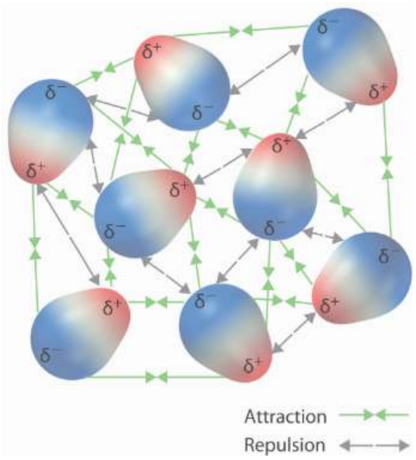


Figure $4.9.2$: Both attractive and repulsive dipole–dipole interactions occur in a liquid sample with many molecules. (CC BY-SA-NC; anonymous) The green arrows pointing towards each other represent attraction. The gray arrows pointing away from each other represent repulsion

Because each end of a dipole possesses only a fraction of the charge of an electron, dipole–dipole interactions are substantially weaker than the interactions between two ions, each of which has a charge of at least ±1, or between a dipole and an ion, in which one of the species has at least a full positive or negative charge. In addition, the attractive interaction between dipoles falls off much more rapidly with increasing distance than do the ion–ion interactions. Recall that the attractive energy between two ions is proportional to $1/r$, where r is the distance between the ions. Doubling the distance ($r \rightarrow 2r$) decreases the attractive energy by one-half. In contrast, the energy of the interaction of two dipoles is proportional to $1/r^3$, so doubling the distance between the dipoles decreases the strength of the interaction by 2^3, or 8-fold. Thus a substance such as HCl, which is partially held together by dipole–dipole interactions, is a gas at room temperature and 1 atm pressure. Conversely, NaCl, which is held together by interionic interactions, is a high-melting-point solid. Within a series of compounds of similar molar mass, the strength of the intermolecular interactions increases as the dipole moment of the molecules increases, as shown in Table $4.9.1$.

Table $4.9.1$: Relationships Between the Dipole Moment and the Boiling Point for Organic Compounds of Similar Molar Mass

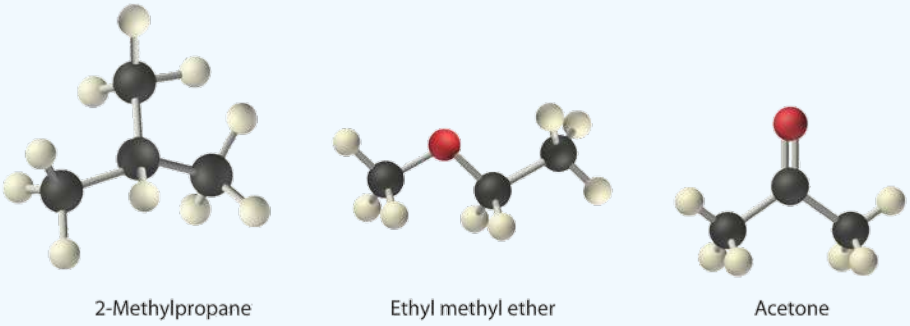
Compound	Molar Mass (g/mol)	Dipole Moment (D)	Boiling Point (K)
C_3H_6 (cyclopropane)	42	0	240
CH_3OCH_3 (dimethyl ether)	46	1.30	248
CH_3CN (acetonitrile)	41	3.9	355

> *The attractive energy between two ions is proportional to 1/r, whereas the attractive energy between two dipoles is proportional to 1/r6.*

Video Discussing Dipole Intermolecular Forces. Source: Dipole Intermolecular Force, YouTube(opens in new window) [youtu.be]

Example 4.9.1

Arrange ethyl methyl ether ($CH_3OCH_2CH_3$), 2-methylpropane [isobutane, $(CH_3)_2CHCH_3$], and acetone (CH_3COCH_3) in order of increasing boiling points. Their structures are as follows:



Given: compounds.

Asked for: order of increasing boiling points.

Strategy:

Compare the molar masses and the polarities of the compounds. Compounds with higher molar masses and that are polar will have the highest boiling points.

Solution:

The three compounds have essentially the same molar mass (58–60 g/mol), so we must look at differences in polarity to predict the strength of the intermolecular dipole–dipole interactions and thus the boiling points of the compounds.

The first compound, 2-methylpropane, contains only C–H bonds, which are not very polar because C and H have similar electronegativities. It should therefore have a very small (but nonzero) dipole moment and a very low boiling point.

Ethyl methyl ether has a structure similar to H_2O; it contains two polar C–O single bonds oriented at about a 109° angle to each other, in addition to relatively nonpolar C–H bonds. As a result, the C–O bond dipoles partially reinforce one another and generate a significant dipole moment that should give a moderately high boiling point.

Acetone contains a polar C=O double bond oriented at about 120° to two methyl groups with nonpolar C–H bonds. The C–O bond dipole therefore corresponds to the molecular dipole, which should result in both a rather large dipole moment and a high boiling point.

Thus we predict the following order of boiling points:

2-methylpropane < ethyl methyl ether < acetone

This result is in good agreement with the actual data: 2-methylpropane, boiling point = −11.7°C, and the dipole moment (μ) = 0.13 D; methyl ethyl ether, boiling point = 7.4°C and μ = 1.17 D; acetone, boiling point = 56.1°C and μ = 2.88 D.

? Exercise 4.9.1

Arrange carbon tetrafluoride (CF_4), ethyl methyl sulfide ($CH_3SC_2H_5$), dimethyl sulfoxide [$(CH_3)_2S{=}O$], and 2-methylbutane [isopentane, $(CH_3)_2CHCH_2CH_3$] in order of decreasing boiling points.

Answer

dimethyl sulfoxide (boiling point = 189.9°C) > ethyl methyl sulfide (boiling point = 67°C) > 2-methylbutane (boiling point = 27.8°C) > carbon tetrafluoride (boiling point = −128°C)

London Dispersion Forces

Thus far, we have considered only interactions between polar molecules. Other factors must be considered to explain why many nonpolar molecules, such as bromine, benzene, and hexane, are liquids at room temperature; why others, such as iodine and naphthalene, are solids. Even the noble gases can be liquefied or solidified at low temperatures, high pressures, or both (Table 4.9.2).

What kind of attractive forces can exist between nonpolar molecules or atoms? This question was answered by Fritz London (1900–1954), a German physicist who later worked in the United States. In 1930, London proposed that temporary fluctuations in the electron distributions within atoms and nonpolar molecules could result in the formation of short-lived instantaneous dipole moments, which produce attractive forces called London dispersion forces between otherwise nonpolar substances.

Table 4.9.2: Normal Melting and Boiling Points of Some Elements and Nonpolar Compounds

Substance	Molar Mass (g/mol)	Melting Point (°C)	Boiling Point (°C)
Ar	40	−189.4	−185.9
Xe	131	−111.8	−108.1
N_2	28	−210	−195.8
O_2	32	−218.8	−183.0
F_2	38	−219.7	−188.1
I_2	254	113.7	184.4
CH_4	16	−182.5	−161.5

Consider a pair of adjacent He atoms, for example. On average, the two electrons in each He atom are uniformly distributed around the nucleus. Because the electrons are in constant motion, however, their distribution in one atom is likely to be asymmetrical at any given instant, resulting in an instantaneous dipole moment. As shown in part (a) in Figure 4.9.3, the instantaneous dipole moment on one atom can interact with the electrons in an adjacent atom, pulling them toward the positive end of the instantaneous dipole or repelling them from the negative end. The net effect is that the first atom causes the temporary formation of a dipole, called an induced dipole, in the second. Interactions between these temporary dipoles cause atoms to be attracted to one another. These attractive interactions are weak and fall off rapidly with increasing distance. London was able to show with quantum mechanics that the attractive energy between molecules due to temporary dipole–induced dipole interactions falls off as $1/r^6$. Doubling the distance therefore decreases the attractive energy by 2^6, or 64-fold.

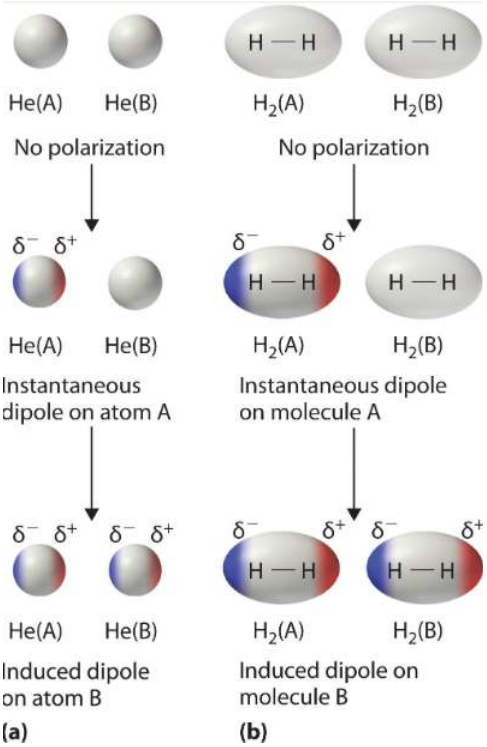


Figure 4.9.3: Instantaneous Dipole Moments. The formation of an instantaneous dipole moment on one He atom (a) or an H_2 molecule (b) results in the formation of an induced dipole on an adjacent atom or molecule.

Instantaneous dipole–induced dipole interactions between nonpolar molecules can produce intermolecular attractions just as they produce interatomic attractions in monatomic substances like Xe. This effect, illustrated for two H_2 molecules in part (b) in Figure 4.9.3, tends to become more pronounced as atomic and molecular masses increase (Table 4.9.2). For example, Xe boils at −108.1°C, whereas He boils at −269°C. The reason for this trend is that the strength of London dispersion forces is related to the ease with which the electron distribution in a given atom can be perturbed. In small atoms such as He, the two 1*s* electrons are held close to the nucleus in a very small volume, and electron–electron repulsions are strong enough to prevent significant asymmetry in their distribution. In larger atoms such as Xe, however, the outer electrons are much less strongly attracted to the nucleus because of filled intervening shells. As a result, it is relatively easy to temporarily deform the electron distribution to generate an instantaneous or induced dipole. The ease of deformation of the electron distribution in an atom or molecule is called its polarizability. Because the electron distribution is more easily perturbed in large, heavy species than in small, light species, we say that heavier substances tend to be much more *polarizable* than lighter ones.

> *For similar substances, London dispersion forces get stronger with increasing molecular size.*

The polarizability of a substance also determines how it interacts with ions and species that possess permanent dipoles. Thus, London dispersion forces are responsible for the general trend toward higher boiling points with increased molecular mass and greater surface area in a homologous series of compounds, such as the alkanes (part (a) in Figure 4.9.4). The strengths of London dispersion forces also depend significantly on molecular shape because shape determines how much of one molecule can interact with its neighboring molecules at any given time. For example, part (b) in Figure 4.9.4 shows 2,2-dimethylpropane (neopentane) and *n*-pentane, both of which have the empirical formula C_5H_{12}. Neopentane is almost spherical, with a small surface area for intermolecular interactions, whereas *n*-pentane has an extended conformation that enables it to come into close contact with other *n*-pentane molecules. As a result, the boiling point of neopentane (9.5°C) is more than 25°C lower than the boiling point of *n*-pentane (36.1°C).

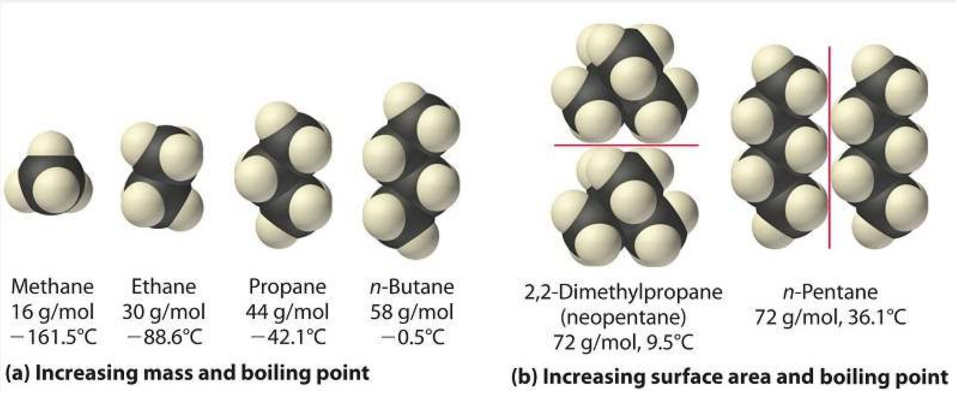


Figure 4.9.4: Mass and Surface Area Affect the Strength of London Dispersion Forces. (a) In this series of four simple alkanes, larger molecules have stronger London forces between them than smaller molecules do, and consequently have higher boiling points. (b) Linear *n*-pentane molecules have a larger surface area and stronger intermolecular forces than spherical neopentane molecules. As a result, neopentane is a gas at room temperature, whereas *n*-pentane is a volatile liquid.

All molecules, whether polar or nonpolar, are attracted to one another by London dispersion forces in addition to any other attractive forces that may be present. In general, however, dipole–dipole interactions in small polar molecules are significantly stronger than London dispersion forces, so the former predominate.

Video Discussing London/Dispersion Intermolecular Forces. Source: Dispersion Intermolecular Force, YouTube(opens in new window) [youtu.be]

✓ Example 4.9.2

Arrange *n*-butane, propane, 2-methylpropane [isobutene, (CH$_3$)$_2$CHCH$_3$], and *n*-pentane in order of increasing boiling points.

Given: compounds

Asked for: order of increasing boiling points

Strategy:

Determine the intermolecular forces in the compounds, and then arrange the compounds according to the strength of those forces. The substance with the weakest forces will have the lowest boiling point.

Solution:

The four compounds are alkanes and nonpolar, so London dispersion forces are the only important intermolecular forces. These forces are generally stronger with increasing molecular mass, so propane should have the lowest boiling point and *n*-pentane should have the highest, with the two butane isomers falling in between. Of the two butane isomers, 2-methylpropane is more compact, and *n*-butane has the more extended shape. Consequently, we expect intermolecular interactions for *n*-butane to be stronger due to its larger surface area, resulting in a higher boiling point. The overall order is thus as follows, with actual boiling points in parentheses: propane (−42.1°C) < 2-methylpropane (−11.7°C) < *n*-butane (−0.5°C) < *n*-pentane (36.1°C).

? Exercise $4.9.2$

Arrange GeH_4, $SiCl_4$, SiH_4, CH_4, and $GeCl_4$ in order of decreasing boiling points.

Answer

$GeCl_4$ (87°C) > $SiCl_4$ (57.6°C) > GeH_4 (−88.5°C) > SiH_4 (−111.8°C) > CH_4 (−161°C)

Hydrogen Bonds

Molecules with hydrogen atoms bonded to electronegative atoms such as O, N, and F (and to a much lesser extent, Cl and S) tend to exhibit unusually strong intermolecular interactions. These result in much higher boiling points than are observed for substances in which London dispersion forces dominate, as illustrated for the covalent hydrides of elements of groups 14–17 in Figure $4.9.5$. Methane and its heavier congeners in group 14 form a series whose boiling points increase smoothly with increasing molar mass. This is the expected trend in nonpolar molecules, for which London dispersion forces are the exclusive intermolecular forces. In contrast, the hydrides of the lightest members of groups 15–17 have boiling points that are more than 100°C greater than predicted on the basis of their molar masses. The effect is most dramatic for water: if we extend the straight line connecting the points for H_2Te and H_2Se to the line for period 2, we obtain an estimated boiling point of −130°C for water! Imagine the implications for life on Earth if water boiled at −130°C rather than 100°C.

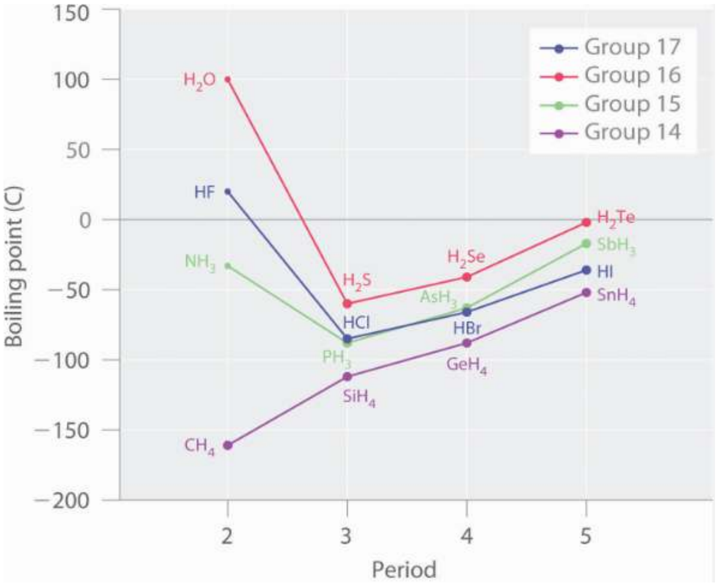


Figure $4.9.5$: The Effects of Hydrogen Bonding on Boiling Points. These plots of the boiling points of the covalent hydrides of the elements of groups 14–17 show that the boiling points of the lightest members of each series for which hydrogen bonding is possible (HF, NH_3, and H_2O) are anomalously high for compounds with such low molecular masses.

Why do strong intermolecular forces produce such anomalously high boiling points and other unusual properties, such as high enthalpies of vaporization and high melting points? The answer lies in the highly polar nature of the bonds between hydrogen and very electronegative elements such as O, N, and F. The large difference in electronegativity results in a large partial positive charge on hydrogen and a correspondingly large partial negative charge on the O, N, or F atom. Consequently, H–O, H–N, and H–F bonds have very large bond dipoles that can interact strongly with one another. Because a hydrogen atom is so small, these dipoles can also approach one another more closely than most other dipoles. The combination of large bond dipoles and short dipole–dipole distances results in very strong dipole–dipole interactions called hydrogen bonds, as shown for ice in Figure $4.9.6$. A hydrogen bond is usually indicated by a dotted line between the hydrogen atom attached to O, N, or F (the *hydrogen bond donor*) and the atom that has the lone pair of electrons (the *hydrogen bond acceptor*). Because each water molecule contains two hydrogen atoms and two lone pairs, a tetrahedral arrangement maximizes the number of hydrogen bonds that can be formed. In the structure of ice, each oxygen atom is surrounded by a distorted tetrahedron of hydrogen atoms that form bridges to the oxygen atoms of adjacent water molecules. The bridging hydrogen atoms are *not* equidistant from the two oxygen atoms they connect, however. Instead, each hydrogen atom is 101 pm from one oxygen and 174 pm from the other. In contrast, each oxygen atom is bonded to two H atoms at the shorter distance and two at the longer distance, corresponding to two O–H covalent bonds and two O⋯H hydrogen bonds from adjacent water molecules, respectively. The resulting open, cagelike structure of ice means that the solid is actually slightly less dense than the liquid, which explains why ice floats on water, rather than sinks.

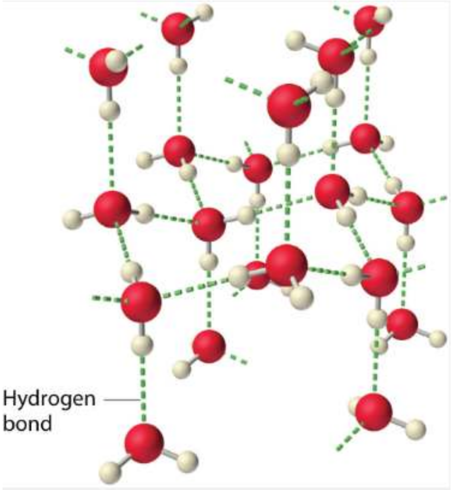


Figure 4.9.6: The Hydrogen-Bonded Structure of Ice

Each water molecule accepts two hydrogen bonds from two other water molecules and donates two hydrogen atoms to form hydrogen bonds with two more water molecules, producing an open, cage like structure. The structure of liquid water is very similar, but in the liquid, the hydrogen bonds are continually broken and formed because of rapid molecular motion.

> *Hydrogen bond formation requires both a hydrogen bond donor and a hydrogen bond acceptor.*

Because ice is less dense than liquid water, rivers, lakes, and oceans freeze from the top down. In fact, the ice forms a protective surface layer that insulates the rest of the water, allowing fish and other organisms to survive in the lower levels of a frozen lake or sea. If ice were denser than the liquid, the ice formed at the surface in cold weather would sink as fast as it formed. Bodies of water would freeze from the bottom up, which would be lethal for most aquatic creatures. The expansion of water when freezing also explains why automobile or boat engines must be protected by “antifreeze” and why unprotected pipes in houses break if they are allowed to freeze.

Video Discussing Hydrogen Bonding Intermolecular Forces. Source: Hydrogen Bonding Intermolecular Force, YouTube(opens in new window) [youtu.be]

✓ Example 4.9.3

Considering CH_3OH, C_2H_6, Xe, and $(CH_3)_3N$, which can form hydrogen bonds with themselves? Draw the hydrogen-bonded structures.

Given: compounds

Asked for: formation of hydrogen bonds and structure

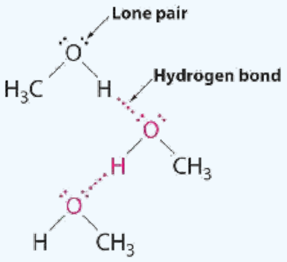
Strategy:

A. Identify the compounds with a hydrogen atom attached to O, N, or F. These are likely to be able to act as hydrogen bond donors.
B. Of the compounds that can act as hydrogen bond donors, identify those that also contain lone pairs of electrons, which allow them to be hydrogen bond acceptors. If a substance is both a hydrogen donor and a hydrogen bond acceptor, draw a structure showing the hydrogen bonding.

Solution:

A. Of the species listed, xenon (Xe), ethane (C_2H_6), and trimethylamine [$(CH_3)_3N$] do not contain a hydrogen atom attached to O, N, or F; hence they cannot act as hydrogen bond donors.

B. The one compound that can act as a hydrogen bond donor, methanol (CH_3OH), contains both a hydrogen atom attached to O (making it a hydrogen bond donor) and two lone pairs of electrons on O (making it a hydrogen bond acceptor); methanol can thus form hydrogen bonds by acting as either a hydrogen bond donor or a hydrogen bond acceptor. The hydrogen-bonded structure of methanol is as follows:



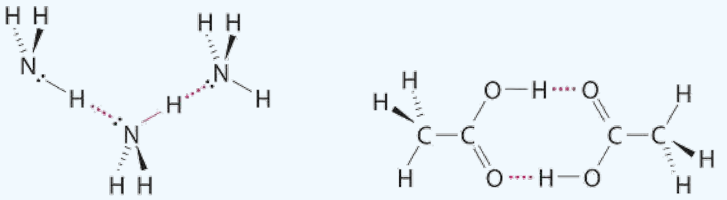
Hydrogen bonding in methanol

? Exercise 4.9.3

Considering CH_3CO_2H, $(CH_3)_3N$, NH_3, and CH_3F, which can form hydrogen bonds with themselves? Draw the hydrogen-bonded structures.

Answer

CH_3CO_2H and NH_3;



Hydrogen bonding in ammonia

Hydrogen bonding in acetic acid

Hydrogen bonding in ammonia between nitrogen and hydrogen. hydrogen bonding in acetic acid is between oxygen and hydrogen.

Although hydrogen bonds are significantly weaker than covalent bonds, with typical dissociation energies of only 15–25 kJ/mol, they have a significant influence on the physical properties of a compound. Compounds such as HF can form only two hydrogen bonds at a time as can, on average, pure liquid NH_3. Consequently, even though their molecular masses are similar to that of water, their boiling points are significantly lower than the boiling point of water, which forms *four* hydrogen bonds at a time.

✓ Example 4.9.4: Buckyballs

Arrange C_{60} (buckminsterfullerene, which has a cage structure), NaCl, He, Ar, and N_2O in order of increasing boiling points.

Given: compounds.

Asked for: order of increasing boiling points.

Strategy:

Identify the intermolecular forces in each compound and then arrange the compounds according to the strength of those forces. The substance with the weakest forces will have the lowest boiling point.

Solution

Electrostatic interactions are strongest for an ionic compound, so we expect NaCl to have the highest boiling point. To predict the relative boiling points of the other compounds, we must consider their polarity (for dipole–dipole interactions), their ability to form hydrogen bonds, and their molar mass (for London dispersion forces). Helium is nonpolar and by far the lightest, so it should have the lowest boiling point. Argon and N_2O have very similar molar masses (40 and 44 g/mol, respectively), but N_2O is polar while Ar is not. Consequently, N_2O should have a higher boiling point. A C_{60} molecule is nonpolar, but its molar mass is 720 g/mol, much greater than that of Ar or N_2O. Because the boiling points of nonpolar substances increase rapidly with molecular mass, C_{60} should boil at a higher temperature than the other nonionic substances. The predicted order is thus as follows, with actual boiling points in parentheses:

$$\text{He } (-269°C) < \text{Ar } (-185.7°C) < N_2O\ (-88.5°C) < C_{60}\ (>280°C) < \text{NaCl } (1465°C).$$

? Exercise 4.9.4

Arrange 2,4-dimethylheptane, Ne, CS_2, Cl_2, and KBr in order of decreasing boiling points.

Answer

KBr (1435°C) > 2,4-dimethylheptane (132.9°C) > CS_2 (46.6°C) > Cl_2 (−34.6°C) > Ne (−246°C)

✓ Example 4.9.5

Identify the most significant intermolecular force in each substance.

a. C_3H_8
b. CH_3OH
c. H_2S

Solution

a. Although C–H bonds are polar, they are only minimally polar. The most significant intermolecular force for this substance would be dispersion forces.
b. This molecule has an H atom bonded to an O atom, so it will experience hydrogen bonding.
c. Although this molecule does not experience hydrogen bonding, the Lewis electron dot diagram and VSEPR indicate that it is bent, so it has a permanent dipole. The most significant force in this substance is dipole-dipole interaction.

? Exercise 4.9.6

Identify the most significant intermolecular force in each substance.

a. HF
b. HCl

Answer a

hydrogen bonding

Answer b

dipole-dipole interactions

Summary

Intermolecular forces are electrostatic in nature and include van der Waals forces and hydrogen bonds. Molecules in liquids are held to other molecules by intermolecular interactions, which are weaker than the intramolecular interactions that hold the atoms together within molecules and polyatomic ions. Transitions between the solid and liquid, or the liquid and gas phases, are due to changes in intermolecular interactions, but do not affect intramolecular interactions. The three major types of intermolecular interactions are dipole–dipole interactions, London dispersion forces (these two are often referred to collectively as **van der Waals forces**), and hydrogen bonds. **Dipole–dipole interactions** arise from the electrostatic interactions of the positive and negative ends of molecules with permanent dipole moments; their strength is proportional to the magnitude of the dipole moment and to $1/r^3$, where r is the distance between dipoles. **London dispersion forces** are due to the formation of **instantaneous dipole moments** in polar or nonpolar molecules as a result of short-lived fluctuations of electron charge distribution, which in turn cause the temporary formation of an **induced dipole** in adjacent molecules; their energy falls off as $1/r^6$. Larger atoms tend to be more **polarizable** than smaller ones, because their outer electrons are less tightly bound and are therefore more easily perturbed. **Hydrogen bonds** are especially strong dipole–dipole interactions between molecules that have hydrogen bonded to a highly electronegative atom, such as O, N, or F. The resulting partially positively charged H atom on one molecule (the *hydrogen bond donor*) can interact strongly with a lone pair of electrons of a partially negatively charged O, N, or F atom on adjacent molecules (the *hydrogen bond acceptor*). Because of strong O⋯H hydrogen bonding between water molecules, water has an unusually high boiling point, and ice has an open, cage like structure that is less dense than liquid water.

4.10: Exercises

1.

In terms of their bulk properties, how do liquids and solids differ? How are they similar?

2.

In terms of the kinetic molecular theory, in what ways are liquids similar to solids? In what ways are liquids different from solids?

3.

In terms of the kinetic molecular theory, in what ways are liquids similar to gases? In what ways are liquids different from gases?

4.

Explain why liquids assume the shape of any container into which they are poured, whereas solids are rigid and retain their shape.

5.

What is the evidence that all neutral atoms and molecules exert attractive forces on each other?

6.

Open the PhET States of Matter Simulation to answer the following questions:

a. Select the Solid, Liquid, Gas tab. Explore by selecting different substances, heating and cooling the systems, and changing the state. What similarities do you notice between the four substances for each phase (solid, liquid, gas)? What differences do you notice?
b. For each substance, select each of the states and record the given temperatures. How do the given temperatures for each state correlate with the strengths of their intermolecular attractions? Explain.
c. Select the Interaction Potential tab, and use the default neon atoms. Move the Ne atom on the right and observe how the potential energy changes. Select the Total Force button, and move the Ne atom as before. When is the total force on each atom attractive and large enough to matter? Then select the Component Forces button, and move the Ne atom. When do the attractive (van der Waals) and repulsive (electron overlap) forces balance? How does this relate to the potential energy versus the distance between atoms graph? Explain.

7.

Define the following and give an example of each:

a. dispersion force
b. dipole-dipole attraction
c. hydrogen bond

8.

The types of intermolecular forces in a substance are identical whether it is a solid, a liquid, or a gas. Why then does a substance change phase from a gas to a liquid or to a solid?

9.

Why do the boiling points of the noble gases increase in the order He < Ne < Ar < Kr < Xe?

10.

Neon and HF have approximately the same molecular masses.

a. Explain why the boiling points of Neon and HF differ.
b. Compare the change in the boiling points of Ne, Ar, Kr, and Xe with the change of the boiling points of HF, HCl, HBr, and HI, and explain the difference between the changes with increasing atomic or molecular mass.

11.

Arrange each of the following sets of compounds in order of increasing boiling point temperature:

a. HCl, H_2O, SiH_4
b. F_2, Cl_2, Br_2
c. CH_4, C_2H_6, C_3H_8

d. O_2, NO, N_2

12.

The molecular mass of butanol, C_4H_9OH, is 74.14; that of ethylene glycol, $CH_2(OH)CH_2OH$, is 62.08, yet their boiling points are 117.2 °C and 174 °C, respectively. Explain the reason for the difference.

13.

On the basis of intermolecular attractions, explain the differences in the boiling points of *n*–butane (−1 °C) and chloroethane (12 °C), which have similar molar masses.

14.

On the basis of dipole moments and/or hydrogen bonding, explain in a qualitative way the differences in the boiling points of acetone (56.2 °C) and 1-propanol (97.4 °C), which have similar molar masses.

15.

The melting point of $H_2O(s)$ is 0 °C. Would you expect the melting point of $H_2S(s)$ to be −85 °C, 0 °C, or 185 °C? Explain your answer.

16.

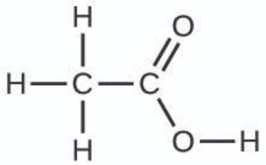
Silane (SiH_4), phosphine (PH_3), and hydrogen sulfide (H_2S) melt at −185 °C, −133 °C, and −85 °C, respectively. What does this suggest about the polar character and intermolecular attractions of the three compounds?

17.

Explain why a hydrogen bond between two water molecules is weaker than a hydrogen bond between two hydrogen fluoride molecules.

18.

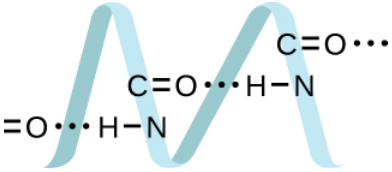
Under certain conditions, molecules of acetic acid, CH_3COOH, form “dimers,” pairs of acetic acid molecules held together by strong intermolecular attractions:



Draw a dimer of acetic acid, showing how two CH_3COOH molecules are held together, and stating the type of IMF that is responsible.

19.

Proteins are chains of amino acids that can form in a variety of arrangements, one of which is a helix. What kind of IMF is responsible for holding the protein strand in this shape? On the protein image, show the locations of the IMFs that hold the protein together:



20.

The density of liquid NH_3 is 0.64 g/mL; the density of gaseous NH_3 at STP is 0.0007 g/mL. Explain the difference between the densities of these two phases.

21.

Identify the intermolecular forces present in the following solids:

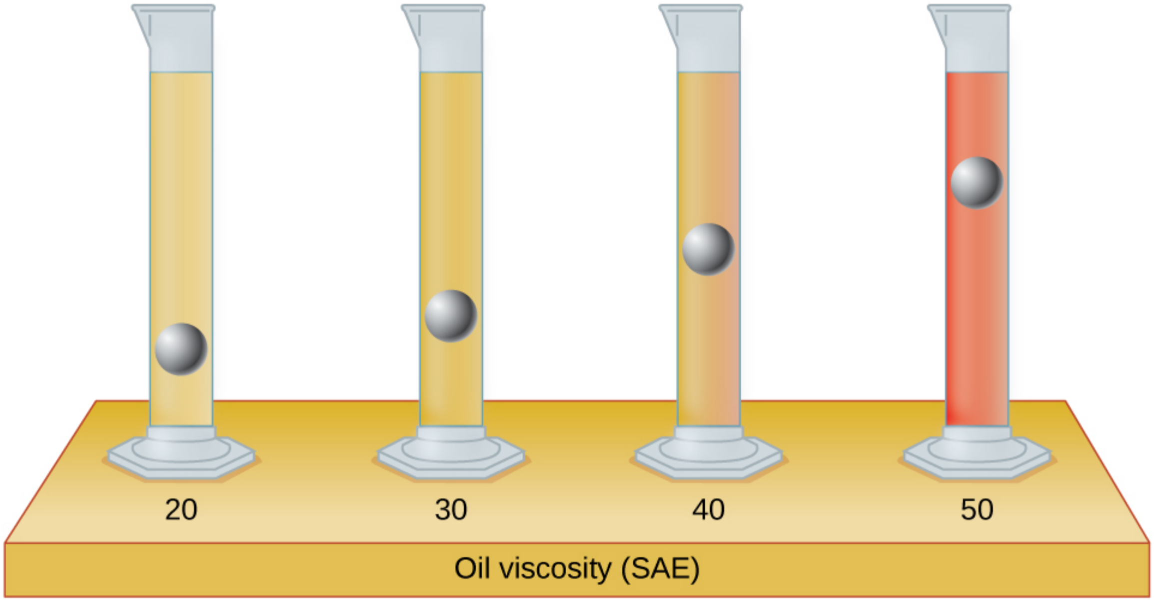
a. CH_3CH_2OH
b. $CH_3CH_2CH_3$

c. CH_3CH_2Cl

22.

The test tubes shown here contain equal amounts of the specified motor oils. Identical metal spheres were dropped at the same time into each of the tubes, and a brief moment later, the spheres had fallen to the heights indicated in the illustration.

Rank the motor oils in order of increasing viscosity, and explain your reasoning:



23.

Although steel is denser than water, a steel needle or paper clip placed carefully lengthwise on the surface of still water can be made to float. Explain at a molecular level how this is possible.

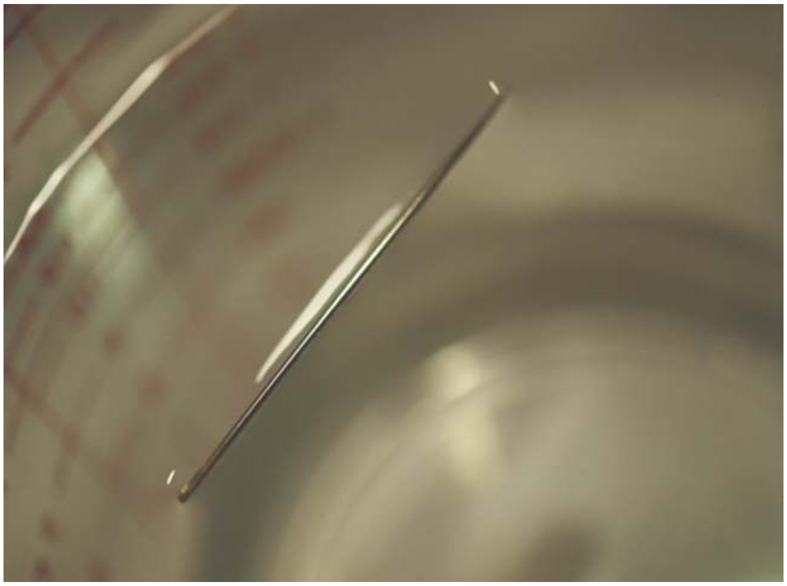
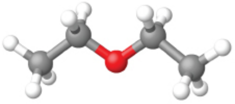
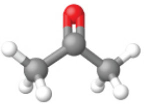
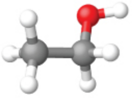
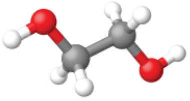


Figure 10.67 (credit: Cory Zanker)

24.

The surface tension and viscosity values for diethyl ether, acetone, ethanol, and ethylene glycol are shown here.

Compound	Molecule	Surface Tension (mN/m)	Viscosity (mPa s)
diethyl ether $C_2H_5OC_2H_5$		17	0.22
acetone CH_3COCH_3		23	0.31
ethanol C_2H_5OH		22	1.07
ethylene glycol $CH_2(OH)CH_2(OH)$		48	16.1

a. Explain their differences in viscosity in terms of the size and shape of their molecules and their IMFs.
b. Explain their differences in surface tension in terms of the size and shape of their molecules and their IMFs:

25.

You may have heard someone use the figure of speech “slower than molasses in winter” to describe a process that occurs slowly. Explain why this is an apt idiom, using concepts of molecular size and shape, molecular interactions, and the effect of changing temperature.

26.

It is often recommended that you let your car engine run idle to warm up before driving, especially on cold winter days. While the benefit of prolonged idling is dubious, it is certainly true that a warm engine is more fuel efficient than a cold one. Explain the reason for this.

27.

The surface tension and viscosity of water at several different temperatures are given in this table.

Water	Surface Tension (mN/m)	Viscosity (mPa s)
0 °C	75.6	1.79
20 °C	72.8	1.00
60 °C	66.2	0.47
100 °C	58.9	0.28

a. As temperature increases, what happens to the surface tension of water? Explain why this occurs, in terms of molecular interactions and the effect of changing temperature.
b. As temperature increases, what happens to the viscosity of water? Explain why this occurs, in terms of molecular interactions and the effect of changing temperature.

28.

At 25 °C, how high will water rise in a glass capillary tube with an inner diameter of 0.63 mm? Refer to Example 10.4 for the required information.

29.

Water rises in a glass capillary tube to a height of 17 cm. What is the diameter of the capillary tube?

30.

Heat is added to boiling water. Explain why the temperature of the boiling water does not change. What does change?

31.

Heat is added to ice at 0 °C. Explain why the temperature of the ice does not change. What does change?

32.

What feature characterizes the dynamic equilibrium between a liquid and its vapor in a closed container?

33.

Identify two common observations indicating some liquids have sufficient vapor pressures to noticeably evaporate?

34.

Identify two common observations indicating some solids, such as dry ice and mothballs, have vapor pressures sufficient to sublime?

35.

What is the relationship between the intermolecular forces in a liquid and its vapor pressure?

36.

What is the relationship between the intermolecular forces in a solid and its melting temperature?

37.

Why does spilled gasoline evaporate more rapidly on a hot day than on a cold day?

38.

Carbon tetrachloride, CCl_4, was once used as a dry cleaning solvent, but is no longer used because it is carcinogenic. At 57.8 °C, the vapor pressure of CCl_4 is 54.0 kPa, and its enthalpy of vaporization is 33.05 kJ/mol. Use this information to estimate the normal boiling point for CCl_4.

39.

When is the boiling point of a liquid equal to its normal boiling point?

40.

How does the boiling of a liquid differ from its evaporation?

41.

Use the information in Figure 10.24 to estimate the boiling point of water in Denver when the atmospheric pressure is 83.3 kPa.

42.

A syringe at a temperature of 20 °C is filled with liquid ether in such a way that there is no space for any vapor. If the temperature is kept constant and the plunger is withdrawn to create a volume that can be occupied by vapor, what would be the approximate pressure of the vapor produced?

43.

Explain the following observations:

a. It takes longer to cook an egg in Ft. Davis, Texas (altitude, 5000 feet above sea level) than it does in Boston (at sea level).
b. Perspiring is a mechanism for cooling the body.

44.

The enthalpy of vaporization of water is larger than its enthalpy of fusion. Explain why.

45.

Explain why the molar enthalpies of vaporization of the following substances increase in the order $CH_4 < C_2H_6 < C_3H_8$, even though the type of IMF (dispersion) is the same.

46.

Explain why the enthalpies of vaporization of the following substances increase in the order $CH_4 < NH_3 < H_2O$, even though all three substances have approximately the same molar mass.

47.

The enthalpy of vaporization of $CO_2(l)$ is 9.8 kJ/mol. Would you expect the enthalpy of vaporization of $CS_2(l)$ to be 28 kJ/mol, 9.8 kJ/mol, or −8.4 kJ/mol? Discuss the plausibility of each of these answers.

48.

The hydrogen fluoride molecule, HF, is more polar than a water molecule, H_2O (for example, has a greater dipole moment), yet the molar enthalpy of vaporization for liquid hydrogen fluoride is lesser than that for water. Explain.

49.

Ethyl chloride (boiling point, 13 °C) is used as a local anesthetic. When the liquid is sprayed on the skin, it cools the skin enough to freeze and numb it. Explain the cooling effect of liquid ethyl chloride.

50.

Which contains the compounds listed correctly in order of increasing boiling points?

a. $N_2 < CS_2 < H_2O < KCl$
b. $H_2O < N_2 < CS_2 < KCl$
c. $N_2 < KCl < CS_2 < H_2O$
d. $CS_2 < N_2 < KCl < H_2O$
e. $KCl < H_2O < CS_2 < N_2$

51.

How much heat is required to convert 422 g of liquid H_2O at 23.5 °C into steam at 150 °C?

52.

Evaporation of sweat requires energy and thus take excess heat away from the body. Some of the water that you drink may eventually be converted into sweat and evaporate. If you drink a 20-ounce bottle of water that had been in the refrigerator at 3.8 °C, how much heat is needed to convert all of that water into sweat and then to vapor? (Note: Your body temperature is 36.6 °C. For the purpose of solving this problem, assume that the thermal properties of sweat are the same as for water.)

53.

Titanium tetrachloride, $TiCl_4$, has a melting point of −23.2 °C and has a $\Delta H_{\text{fusion}} = 9.37$ kJ/mol.

a. How much energy is required to melt 263.1 g $TiCl_4$?
b. For $TiCl_4$, which will likely have the larger magnitude: ΔH_{fusion} or $\Delta H_{\text{vaporization}}$? Explain your reasoning.

54.

From the phase diagram for water (Figure 10.31), determine the state of water at:

a. (f) 60 °C and 50 kPa

55.

What phase changes will take place when water is subjected to varying pressure at a constant temperature of 0.005 °C? At 40 °C? At −40 °C?

56.

Pressure cookers allow food to cook faster because the higher pressure inside the pressure cooker increases the boiling temperature of water. A particular pressure cooker has a safety valve that is set to vent steam if the pressure exceeds 3.4 atm. What is the approximate maximum temperature that can be reached inside this pressure cooker? Explain your reasoning.

57.

From the phase diagram for carbon dioxide in Figure 10.34, determine the state of CO_2 at:

a. (f) −80 °C and 10 kPa

58.

Determine the phase changes that carbon dioxide undergoes as pressure is increased at a constant temperature of (a) −50 °C and (b) 50 °C. If the temperature is held at −40 °C? At 20 °C? (See the phase diagram in Figure 10.34.)

59.

Consider a cylinder containing a mixture of liquid carbon dioxide in equilibrium with gaseous carbon dioxide at an initial pressure of 65 atm and a temperature of 20 °C. Sketch a plot depicting the change in the cylinder pressure with time as gaseous carbon dioxide is released at constant temperature.

60.

Dry ice, $CO_2(s)$, does not melt at atmospheric pressure. It sublimes at a temperature of −78 °C. What is the lowest pressure at which $CO_2(s)$ will melt to give $CO_2(l)$? At approximately what temperature will this occur? (See Figure 10.34 for the phase diagram.)

61.

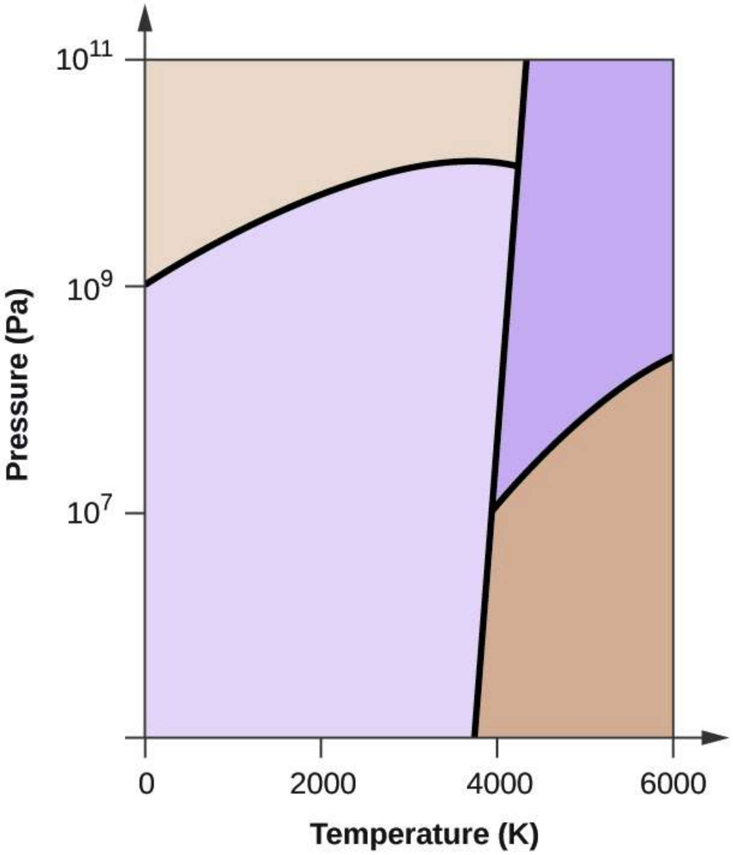
If a severe storm results in the loss of electricity, it may be necessary to use a clothesline to dry laundry. In many parts of the country in the dead of winter, the clothes will quickly freeze when they are hung on the line. If it does not snow, will they dry anyway? Explain your answer.

62.

Is it possible to liquefy nitrogen at room temperature (about 25 °C)? Is it possible to liquefy sulfur dioxide at room temperature? Explain your answers.

63.

Elemental carbon has one gas phase, one liquid phase, and two different solid phases, as shown in the phase diagram:



a. (f) If the temperature of a sample of carbon increases from 3000 K to 5000 K at a constant pressure of 10^6 Pa, which phase transition occurs, if any?

64.

What types of liquids typically form amorphous solids?

65.

At very low temperatures oxygen, O_2, freezes and forms a crystalline solid. Which best describes these crystals?

a. ionic
b. covalent network
c. metallic
d. amorphous
e. molecular crystals

66.

As it cools, olive oil slowly solidifies and forms a solid over a range of temperatures. Which best describes the solid?

a. ionic
b. covalent network
c. metallic
d. amorphous
e. molecular crystals

67.

Explain why ice, which is a crystalline solid, has a melting temperature of 0 °C, whereas butter, which is an amorphous solid, softens over a range of temperatures.

68.

Identify the type of crystalline solid (metallic, network covalent, ionic, or molecular) formed by each of the following substances:

a. (i) C_2H_5OH

69.

Identify the type of crystalline solid (metallic, network covalent, ionic, or molecular) formed by each of the following substances:

a. (i) K_3PO_4

70.

Classify each substance in the table as either a metallic, ionic, molecular, or covalent network solid:

Substance	Appearance	Melting Point	Electrical Conductivity	Solubility in Water
X	lustrous, malleable	1500 °C	high	insoluble
Y	soft, yellow	113 °C	none	insoluble
Z	hard, white	800 °C	only if melted/dissolved	soluble

71.

Classify each substance in the table as either a metallic, ionic, molecular, or covalent network solid:

Substance	Appearance	Melting Point	Electrical Conductivity	Solubility in Water
X	brittle, white	800 °C	only if melted/dissolved	soluble
Y	shiny, malleable	1100 °C	high	insoluble
Z	hard, colorless	3550 °C	none	insoluble

72.

Identify the following substances as ionic, metallic, covalent network, or molecular solids:

Substance A is malleable, ductile, conducts electricity well, and has a melting point of 1135 °C. Substance B is brittle, does not conduct electricity as a solid but does when molten, and has a melting point of 2072 °C. Substance C is very hard, does not conduct electricity, and has a melting point of 3440 °C. Substance D is soft, does not conduct electricity, and has a melting point of 185 °C.

73.

Substance A is shiny, conducts electricity well, and melts at 975 °C. Substance A is likely a(n):

a. ionic solid
b. metallic solid
c. molecular solid
d. covalent network solid

74.

Substance B is hard, does not conduct electricity, and melts at 1200 °C. Substance B is likely a(n):

a. ionic solid
b. metallic solid
c. molecular solid
d. covalent network solid

75.

Describe the crystal structure of iron, which crystallizes with two equivalent metal atoms in a cubic unit cell.

76.

Describe the crystal structure of Pt, which crystallizes with four equivalent metal atoms in a cubic unit cell.

77.

What is the coordination number of a chromium atom in the body-centered cubic structure of chromium?

78.

What is the coordination number of an aluminum atom in the face-centered cubic structure of aluminum?

79.

Cobalt metal crystallizes in a hexagonal closest packed structure. What is the coordination number of a cobalt atom?

80.

Nickel metal crystallizes in a cubic closest packed structure. What is the coordination number of a nickel atom?

81.

Tungsten crystallizes in a body-centered cubic unit cell with an edge length of 3.165 Å.

a. What is the atomic radius of tungsten in this structure?
b. Calculate the density of tungsten.

82.

Platinum (atomic radius = 1.38 Å) crystallizes in a cubic closely packed structure. Calculate the edge length of the face-centered cubic unit cell and the density of platinum.

83.

Barium crystallizes in a body-centered cubic unit cell with an edge length of 5.025 Å

a. What is the atomic radius of barium in this structure?
b. Calculate the density of barium.

84.

Aluminum (atomic radius = 1.43 Å) crystallizes in a cubic closely packed structure. Calculate the edge length of the face-centered cubic unit cell and the density of aluminum.

85.

The density of aluminum is 2.7 g/cm^3; that of silicon is 2.3 g/cm^3. Explain why Si has the lower density even though it has heavier atoms.

86.

The free space in a metal may be found by subtracting the volume of the atoms in a unit cell from the volume of the cell. Calculate the percentage of free space in each of the three cubic lattices if all atoms in each are of equal size and touch their nearest neighbors. Which of these structures represents the most efficient packing? That is, which packs with the least amount of unused space?

87.

Cadmium sulfide, sometimes used as a yellow pigment by artists, crystallizes with cadmium, occupying one-half of the tetrahedral holes in a closest packed array of sulfide ions. What is the formula of cadmium sulfide? Explain your answer.

88.

A compound of cadmium, tin, and phosphorus is used in the fabrication of some semiconductors. It crystallizes with cadmium occupying one-fourth of the tetrahedral holes and tin occupying one-fourth of the tetrahedral holes in a closest packed array of phosphide ions. What is the formula of the compound? Explain your answer.

89.

What is the formula of the magnetic oxide of cobalt, used in recording tapes, that crystallizes with cobalt atoms occupying one-eighth of the tetrahedral holes and one-half of the octahedral holes in a closely packed array of oxide ions?

90.

A compound containing zinc, aluminum, and sulfur crystallizes with a closest-packed array of sulfide ions. Zinc ions are found in one-eighth of the tetrahedral holes and aluminum ions in one-half of the octahedral holes. What is the empirical formula of the compound?

91.

A compound of thallium and iodine crystallizes in a simple cubic array of iodide ions with thallium ions in all of the cubic holes. What is the formula of this iodide? Explain your answer.

92.

Which of the following elements reacts with sulfur to form a solid in which the sulfur atoms form a closest-packed array with all of the octahedral holes occupied: Li, Na, Be, Ca, or Al?

93.

What is the percent by mass of titanium in rutile, a mineral that contains titanium and oxygen, if structure can be described as a closest packed array of oxide ions with titanium ions in one-half of the octahedral holes? What is the oxidation number of titanium?

94.

Explain why the chemically similar alkali metal chlorides NaCl and CsCl have different structures, whereas the chemically different NaCl and MnS have the same structure.

95.

As minerals were formed from the molten magma, different ions occupied the same cites in the crystals. Lithium often occurs along with magnesium in minerals despite the difference in the charge on their ions. Suggest an explanation.

96.

Rubidium iodide crystallizes with a cubic unit cell that contains iodide ions at the corners and a rubidium ion in the center. What is the formula of the compound?

97.

One of the various manganese oxides crystallizes with a cubic unit cell that contains manganese ions at the corners and in the center. Oxide ions are located at the center of each edge of the unit cell. What is the formula of the compound?

98.

NaH crystallizes with the same crystal structure as NaCl. The edge length of the cubic unit cell of NaH is 4.880 Å.

a. Calculate the ionic radius of H^-. (The ionic radius of Li^+ is 0.0.95 Å.)
b. Calculate the density of NaH.

99.

Thallium(I) iodide crystallizes with the same structure as CsCl. The edge length of the unit cell of TlI is 4.20 Å. Calculate the ionic radius of TI^+. (The ionic radius of I^- is 2.16 Å.)

100.

A cubic unit cell contains manganese ions at the corners and fluoride ions at the center of each edge.

a. What is the empirical formula of this compound? Explain your answer.
b. What is the coordination number of the Mn^{3+} ion?
c. Calculate the edge length of the unit cell if the radius of a Mn^{3+} ion is 0.65 A.
d. Calculate the density of the compound.

101.

What is the spacing between crystal planes that diffract X-rays with a wavelength of 1.541 nm at an angle θ of 15.55° (first order reflection)?

102.

A diffractometer using X-rays with a wavelength of 0.2287 nm produced first order diffraction peak for a crystal angle $\theta = 16.21°$. Determine the spacing between the diffracting planes in this crystal.

103.

A metal with spacing between planes equal to 0.4164 nm diffracts X-rays with a wavelength of 0.2879 nm. What is the diffraction angle for the first order diffraction peak?

104.

Gold crystallizes in a face-centered cubic unit cell. The second-order reflection (n = 2) of X-rays for the planes that make up the tops and bottoms of the unit cells is at $\theta = 22.20°$. The wavelength of the X-rays is 1.54 Å. What is the density of metallic gold?

105.

When an electron in an excited molybdenum atom falls from the L to the K shell, an X-ray is emitted. These X-rays are diffracted at an angle of 7.75° by planes with a separation of 2.64 Å. What is the difference in energy between the K shell and the L shell in molybdenum assuming a first order diffraction?

4.11: Exercises

1.

How do solutions differ from compounds? From other mixtures?

2.

Which of the principal characteristics of solutions are evident in the solutions of $K_2Cr_2O_7$ shown in Figure 11.2?

3.

When KNO_3 is dissolved in water, the resulting solution is significantly colder than the water was originally.

a. Is the dissolution of KNO_3 an endothermic or an exothermic process?
b. What conclusions can you draw about the intermolecular attractions involved in the process?
c. Is the resulting solution an ideal solution?

4.

Give an example of each of the following types of solutions:

a. a gas in a liquid
b. a gas in a gas
c. a solid in a solid

5.

Indicate the most important types of intermolecular attractions in each of the following solutions:

a. The solution in Figure 11.2.
b. $NO(l)$ in $CO(l)$
c. $Cl_2(g)$ in $Br_2(l)$
d. $HCl(g)$ in benzene $C_6H_6(l)$
e. Methanol $CH_3OH(l)$ in $H_2O(l)$

6.

Predict whether each of the following substances would be more soluble in water (polar solvent) or in a hydrocarbon such as heptane (C_7H_{16}, nonpolar solvent):

a. vegetable oil (nonpolar)
b. isopropyl alcohol (polar)
c. potassium bromide (ionic)

7.

Heat is released when some solutions form; heat is absorbed when other solutions form. Provide a molecular explanation for the difference between these two types of spontaneous processes.

8.

Solutions of hydrogen in palladium may be formed by exposing Pd metal to H_2 gas. The concentration of hydrogen in the palladium depends on the pressure of H_2 gas applied, but in a more complex fashion than can be described by Henry's law. Under certain conditions, 0.94 g of hydrogen gas is dissolved in 215 g of palladium metal (solution density = 10.8 g cm^3).

a. Determine the molarity of this solution.
b. Determine the molality of this solution.
c. Determine the percent by mass of hydrogen atoms in this solution.

9.

Explain why the ions Na^+ and Cl^- are strongly solvated in water but not in hexane, a solvent composed of nonpolar molecules.

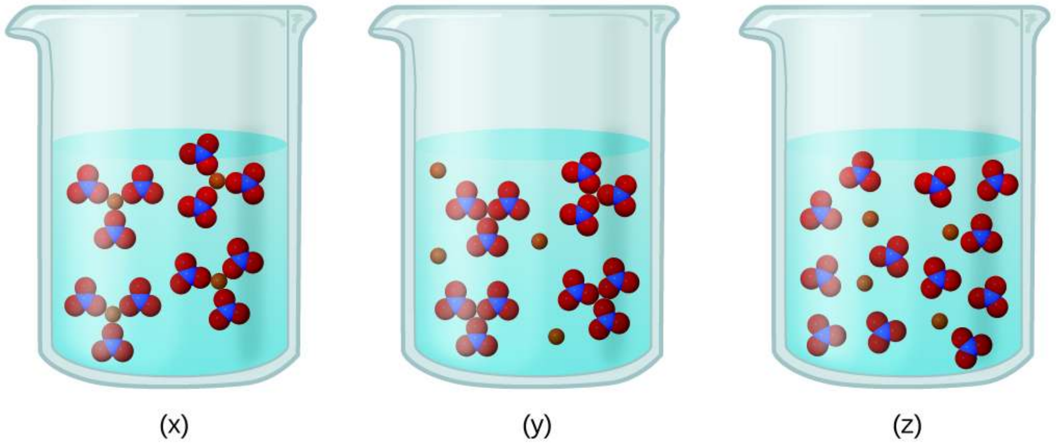
10.

Explain why solutions of HBr in benzene (a nonpolar solvent) are nonconductive, while solutions in water (a polar solvent) are conductive.

11.

Consider the solutions presented:

(a) Which of the following sketches best represents the ions in a solution of $Fe(NO_3)_3(aq)$?



(b) Write a balanced chemical equation showing the products of the dissolution of $Fe(NO_3)_3$.

12.

Compare the processes that occur when methanol (CH_3OH), hydrogen chloride (HCl), and sodium hydroxide (NaOH) dissolve in water. Write equations and prepare sketches showing the form in which each of these compounds is present in its respective solution.

13.

What is the expected electrical conductivity of the following solutions?

a. $NaOH(aq)$
b. $HCl(aq)$
c. $C_6H_{12}O_6(aq)$ (glucose)
d. $NH_3(aq)$

14.

Why are most *solid* ionic compounds electrically nonconductive, whereas aqueous solutions of ionic compounds are good conductors? Would you expect a *liquid* (molten) ionic compound to be electrically conductive or nonconductive? Explain.

15.

Indicate the most important type of intermolecular attraction responsible for solvation in each of the following solutions:

a. the solutions in Figure 11.7
b. methanol, CH_3OH, dissolved in ethanol, C_2H_5OH
c. methane, CH_4, dissolved in benzene, C_6H_6
d. the polar halocarbon CF_2Cl_2 dissolved in the polar halocarbon $CF_2ClCFCl_2$
e. $O_2(l)$ in $N_2(l)$

16.

Suppose you are presented with a clear solution of sodium thiosulfate, $Na_2S_2O_3$. How could you determine whether the solution is unsaturated, saturated, or supersaturated?

17.

Supersaturated solutions of most solids in water are prepared by cooling saturated solutions. Supersaturated solutions of most gases in water are prepared by heating saturated solutions. Explain the reasons for the difference in the two procedures.

18.

Suggest an explanation for the observations that ethanol, C_2H_5OH, is completely miscible with water and that ethanethiol, C_2H_5SH, is soluble only to the extent of 1.5 g per 100 mL of water.

19.

Calculate the percent by mass of KBr in a saturated solution of KBr in water at 10 °C. See Figure 11.16 for useful data, and report the computed percentage to one significant digit.

20.

Which of the following gases is expected to be most soluble in water? Explain your reasoning.

a. CH_4
b. CCl_4
c. $CHCl_3$

21.

At 0 °C and 1.00 atm, as much as 0.70 g of O_2 can dissolve in 1 L of water. At 0 °C and 4.00 atm, how many grams of O_2 dissolve in 1 L of water?

22.

Refer to Figure 11.10.

a. How did the concentration of dissolved CO_2 in the beverage change when the bottle was opened?
b. What caused this change?
c. Is the beverage unsaturated, saturated, or supersaturated with CO_2?

23.

The Henry's law constant for CO_2 is 3.4 \times 10^{−2} *M*/atm at 25 °C. Assuming ideal solution behavior, what pressure of carbon dioxide is needed to maintain a CO_2 concentration of 0.10 *M* in a can of lemon-lime soda?

24.

The Henry's law constant for O_2 is 1.3 \times 10^{−3} *M*/atm at 25 °C. Assuming ideal solution behavior, what mass of oxygen would be dissolved in a 40-L aquarium at 25 °C, assuming an atmospheric pressure of 1.00 atm, and that the partial pressure of O_2 is 0.21 atm?

25.

Assuming ideal solution behavior, how many liters of HCl gas, measured at 30.0 °C and 745 torr, are required to prepare 1.25 L of a 3.20-*M* solution of hydrochloric acid?

26.

Which is/are part of the macroscopic domain of solutions and which is/are part of the microscopic domain: boiling point elevation, Henry's law, hydrogen bond, ion-dipole attraction, molarity, nonelectrolyte, nonstoichiometric compound, osmosis, solvated ion?

27.

What is the microscopic explanation for the macroscopic behavior illustrated in Figure 11.14?

28.

Sketch a qualitative graph of the pressure versus time for water vapor above a sample of pure water and a sugar solution, as the liquids evaporate to half their original volume.

29.

A solution of potassium nitrate, an electrolyte, and a solution of glycerin ($C_3H_5(OH)_3$), a nonelectrolyte, both boil at 100.3 °C. What other physical properties of the two solutions are identical?

30.

What are the mole fractions of H_3PO_4 and water in a solution of 14.5 g of H_3PO_4 in 125 g of water?

a. Outline the steps necessary to answer the question.
b. Answer the question.

31.

What are the mole fractions of HNO_3 and water in a concentrated solution of nitric acid (68.0% HNO_3 by mass)?

a. Outline the steps necessary to answer the question.
b. Answer the question.

32.

Calculate the mole fraction of each solute and solvent:

a. 583 g of H_2SO_4 in 1.50 kg of water—the acid solution used in an automobile battery
b. 0.86 g of NaCl in 1.00 \times 10^{2} g of water—a solution of sodium chloride for intravenous injection
c. 46.85 g of codeine, $C_{18}H_{21}NO_3$, in 125.5 g of ethanol, C_2H_5OH
d. 25 g of I_2 in 125 g of ethanol, C_2H_5OH

33.

Calculate the mole fraction of each solute and solvent:

a. 0.710 kg of sodium carbonate (washing soda), Na_2CO_3, in 10.0 kg of water—a saturated solution at 0 °C
b. 125 g of NH_4NO_3 in 275 g of water—a mixture used to make an instant ice pack
c. 25 g of Cl_2 in 125 g of dichloromethane, CH_2Cl_2
d. 0.372 g of tetrahydropyridine, C_5H_9N, in 125 g of chloroform, $CHCl_3$

34.

Calculate the mole fractions of methanol, CH_3OH; ethanol, C_2H_5OH; and water in a solution that is 40% methanol, 40% ethanol, and 20% water by mass. (Assume the data are good to two significant figures.)

35.

What is the difference between a 1 *M* solution and a 1 *m* solution?

36.

What is the molality of phosphoric acid, H_3PO_4, in a solution of 14.5 g of H_3PO_4 in 125 g of water?

a. Outline the steps necessary to answer the question.
b. Answer the question.

37.

What is the molality of nitric acid in a concentrated solution of nitric acid (68.0% HNO_3 by mass)?

a. Outline the steps necessary to answer the question.
b. Answer the question.

38.

Calculate the molality of each of the following solutions:

a. 583 g of H_2SO_4 in 1.50 kg of water—the acid solution used in an automobile battery
b. 0.86 g of NaCl in 1.00 \times 10^{2} g of water—a solution of sodium chloride for intravenous injection
c. 46.85 g of codeine, $C_{18}H_{21}NO_3$, in 125.5 g of ethanol, C_2H_5OH
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39.

Calculate the molality of each of the following solutions:

a. 0.710 kg of sodium carbonate (washing soda), Na_2CO_3, in 10.0 kg of water—a saturated solution at 0°C
b. 125 g of NH_4NO_3 in 275 g of water—a mixture used to make an instant ice pack
c. 25 g of Cl_2 in 125 g of dichloromethane, CH_2Cl_2
d. 0.372 g of tetrahydropyridine, C_5H_9N, in 125 g of chloroform, $CHCl_3$

40.

The concentration of glucose, $C_6H_{12}O_6$, in normal spinal fluid is What is the molality of the solution?

41.

A 13.0% solution of K_2CO_3 by mass has a density of 1.09 g/cm^3. Calculate the molality of the solution.

42.

Why does 1 mol of sodium chloride depress the freezing point of 1 kg of water almost twice as much as 1 mol of glycerin?

43.

Assuming ideal solution behavior, what is the boiling point of a solution of 115.0 g of nonvolatile sucrose, $C_{12}H_{22}O_{11}$, in 350.0 g of water?

a. Outline the steps necessary to answer the question
b. Answer the question

44.

Assuming ideal solution behavior, what is the boiling point of a solution of 9.04 g of I_2 in 75.5 g of benzene, assuming the I_2 is nonvolatile?

a. Outline the steps necessary to answer the question.
b. Answer the question.

45.

Assuming ideal solution behavior, what is the freezing temperature of a solution of 115.0 g of sucrose, $C_{12}H_{22}O_{11}$, in 350.0 g of water?

a. Outline the steps necessary to answer the question.
b. Answer the question.

46.

Assuming ideal solution behavior, what is the freezing point of a solution of 9.04 g of I_2 in 75.5 g of benzene?

a. Outline the steps necessary to answer the following question.
b. Answer the question.

47.

Assuming ideal solution behavior, what is the osmotic pressure of an aqueous solution of 1.64 g of $Ca(NO_3)_2$ in water at 25 °C? The volume of the solution is 275 mL.

a. Outline the steps necessary to answer the question.
b. Answer the question.

48.

Assuming ideal solution behavior, what is osmotic pressure of a solution of bovine insulin (molar mass, 5700 g mol^{-1}) at 18 °C if 100.0 mL of the solution contains 0.103 g of the insulin?

a. Outline the steps necessary to answer the question.
b. Answer the question.

49.

Assuming ideal solution behavior, what is the molar mass of a solution of 5.00 g of a compound in 25.00 g of carbon tetrachloride (bp 76.8 °C; K_b = 5.02 °C/m) that boils at 81.5 °C at 1 atm?

a. Outline the steps necessary to answer the question.
b. Solve the problem.

50.

A sample of an organic compound (a nonelectrolyte) weighing 1.35 g lowered the freezing point of 10.0 g of benzene by 3.66 °C. Assuming ideal solution behavior, calculate the molar mass of the compound.

51.

A 1.0 m solution of HCl in benzene has a freezing point of 0.4 °C. Is HCl an electrolyte in benzene? Explain.

52.

A solution contains 5.00 g of urea, $CO(NH_2)_2$, a nonvolatile compound, dissolved in 0.100 kg of water. If the vapor pressure of pure water at 25 °C is 23.7 torr, what is the vapor pressure of the solution (assuming ideal solution behavior)?

53.

A 12.0-g sample of a nonelectrolyte is dissolved in 80.0 g of water. The solution freezes at −1.94 °C. Assuming ideal solution behavior, calculate the molar mass of the substance.

54.

Arrange the following solutions in order by their decreasing freezing points: 0.1 *m* Na_3PO_4, 0.1 *m* C_2H_5OH, 0.01 *m* CO_2, 0.15 *m* NaCl, and 0.2 *m* $CaCl_2$.

55.

Calculate the boiling point elevation of 0.100 kg of water containing 0.010 mol of NaCl, 0.020 mol of Na_2SO_4, and 0.030 mol of $MgCl_2$, assuming complete dissociation of these electrolytes and ideal solution behavior.

56.

How could you prepare a 3.08 *m* aqueous solution of glycerin, $C_3H_8O_3$? Assuming ideal solution behavior, what is the freezing point of this solution?

57.

A sample of sulfur weighing 0.210 g was dissolved in 17.8 g of carbon disulfide, CS_2 (K_b = 2.34 °C/*m*). If the boiling point elevation was 0.107 °C, what is the formula of a sulfur molecule in carbon disulfide (assuming ideal solution behavior)?

58.

In a significant experiment performed many years ago, 5.6977 g of cadmium iodide in 44.69 g of water raised the boiling point 0.181 °C. What does this suggest about the nature of a solution of CdI_2?

59.

Lysozyme is an enzyme that cleaves cell walls. A 0.100-L sample of a solution of lysozyme that contains 0.0750 g of the enzyme exhibits an osmotic pressure of 1.32 \times 10^{−3} atm at 25 °C. Assuming ideal solution behavior, what is the molar mass of lysozyme?

60.

The osmotic pressure of a solution containing 7.0 g of insulin per liter is 23 torr at 25 °C. Assuming ideal solution behavior, what is the molar mass of insulin?

61.

The osmotic pressure of human blood is 7.6 atm at 37 °C. What mass of glucose, $C_6H_{12}O_6$, is required to make 1.00 L of aqueous solution for intravenous feeding if the solution must have the same osmotic pressure as blood at body temperature, 37 °C (assuming ideal solution behavior)?

62.

Assuming ideal solution behavior, what is the freezing point of a solution of dibromobenzene, $C_6H_4Br_2$, in 0.250 kg of benzene, if the solution boils at 83.5 °C?

63.

Assuming ideal solution behavior, what is the boiling point of a solution of NaCl in water if the solution freezes at −0.93 °C?

64.

The sugar fructose contains 40.0% C, 6.7% H, and 53.3% O by mass. A solution of 11.7 g of fructose in 325 g of ethanol has a boiling point of 78.59 °C. The boiling point of ethanol is 78.35 °C, and K_b for ethanol is 1.20 °C/*m*. Assuming ideal solution behavior, what is the molecular formula of fructose?

65.

The vapor pressure of methanol, CH_3OH, is 94 torr at 20 °C. The vapor pressure of ethanol, C_2H_5OH, is 44 torr at the same temperature.

a. Calculate the mole fraction of methanol and of ethanol in a solution of 50.0 g of methanol and 50.0 g of ethanol.
b. Ethanol and methanol form a solution that behaves like an ideal solution. Calculate the vapor pressure of methanol and of ethanol above the solution at 20 °C.
c. Calculate the mole fraction of methanol and of ethanol in the vapor above the solution.

66.

The triple point of air-free water is defined as 273.16 K. Why is it important that the water be free of air?

67.

Meat can be classified as fresh (not frozen) even though it is stored at −1 °C. Why wouldn't meat freeze at this temperature?

68.

An organic compound has a composition of 93.46% C and 6.54% H by mass. A solution of 0.090 g of this compound in 1.10 g of camphor melts at 158.4 °C. The melting point of pure camphor is 178.4 °C. K_f for camphor is 37.7 °C/m. Assuming ideal solution behavior, what is the molecular formula of the solute? Show your calculations.

69.

A sample of $HgCl_2$ weighing 9.41 g is dissolved in 32.75 g of ethanol, C_2H_5OH (K_b = 1.20 °C/m). The boiling point elevation of the solution is 1.27 °C. Is $HgCl_2$ an electrolyte in ethanol? Show your calculations.

70.

A salt is known to be an alkali metal fluoride. A quick approximate determination of freezing point indicates that 4 g of the salt dissolved in 100 g of water produces a solution that freezes at about −1.4 °C. Assuming ideal solution behavior, what is the formula of the salt? Show your calculations.

71.

Identify the dispersed phase and the dispersion medium in each of the following colloidal systems: starch dispersion, smoke, fog, pearl, whipped cream, floating soap, jelly, milk, and ruby.

72.

Distinguish between dispersion methods and condensation methods for preparing colloidal systems.

73.

How do colloids differ from solutions with regard to dispersed particle size and homogeneity?

74.

Explain the cleansing action of soap.

75.

How can it be demonstrated that colloidal particles are electrically charged?

4.12: End of Chapter Activity

End of Chapter

Activity: Creating a Lesson Plan on Phases and Classification of Matter with AI and Bloom's Taxonomy

Now that you have explored the fundamentals of phases and classification of matter, it's time to put your knowledge into practice. Your task is to create a succinct lesson plan for 2nd graders that introduces them to the basics of matter. To help you with this, you will use AI tools and incorporate Bloom's Taxonomy to ensure a comprehensive learning experience. This lesson plan will go towards your digital notebook, a portfolio filled with lesson plans, activities, and labs for future use.

Activity Prompt:

> **Objective:** Use AI and Bloom's Taxonomy to develop a lesson plan that effectively teaches 2nd graders about the fundamentals of matter, including concepts such as the different states of matter, phase changes, and simple physical properties.

Understanding the Concepts:

> **Knowledge (Remembering):** Define key terms related to matter, such as solid, liquid, gas, and phase change.
>
> **Comprehension (Understanding):** Explain these concepts in simple, age-appropriate language.

Planning the Lesson:

> **Application:** Design an engaging activity or experiment that allows students to observe and understand the states of matter and phase changes. For example, use ice melting into water and then evaporating to demonstrate solid, liquid, and gas states.
>
> **Analysis:** Use AI tools to create visual aids or interactive simulations that illustrate the states of matter and phase changes. For instance, create a simple animation that shows water molecules in solid, liquid, and gas states.

Deepening Understanding:

> **Synthesis (Creating):** Ask students to predict what will happen to a substance when it is heated or cooled. For example, what happens to a popsicle left in the sun versus one placed in a freezer?
>
> **Evaluation:** Have students discuss and reflect on what they observed during the activities. Encourage them to think about why matter changes states and how these changes are part of everyday life.

Using AI in the Classroom:

Explore AI tools like educational apps or platforms that provide interactive content for teaching phases and classification of matter. Use these tools to create quizzes, flashcards, or interactive stories that reinforce the lesson's concepts.

Use AI to assess student understanding through formative assessments and provide instant feedback.

Deliverable:

Submit a detailed lesson plan that includes:

1. **A brief overview of the key concepts covered:** Outline the foundational concepts of matter that will be taught.
2. **A description of the activities and experiments designed:** Detail the hands-on activities and experiments you will use to help students understand states of matter and phase changes.
3. **Examples of AI tools used and how they enhance the learning experience:** Describe the AI tools you plan to incorporate, such as simulations or interactive quizzes, and explain how they will help students grasp complex concepts.
4. **An explanation of how Bloom's Taxonomy was applied in the lesson plan to ensure a well-rounded educational experience:** Illustrate how each level of Bloom's Taxonomy (Remembering, Understanding, Applying, Analyzing, Creating, and Evaluating) is addressed in your lesson plan.

This activity will help you integrate modern technology and educational strategies to create an effective and engaging learning experience for young students.

4.13: End of Chapter Key Terms

Phases and Classification of Matter Key Terms

1. **Matter**: Anything that has mass and occupies space.
2. **Element**: A pure substance that consists of only one type of atom, defined by its number of protons.
3. **Compound**: A substance formed from two or more elements chemically bonded together in a fixed ratio.
4. **Mixture**: A combination of two or more substances in which each substance retains its individual chemical properties.
5. **Homogeneous Mixture**: A mixture that has a uniform composition throughout, also known as a solution.
6. **Heterogeneous Mixture**: A mixture that does not have a uniform composition throughout, with distinct phases or parts.
7. **Pure Substance**: A material with a constant composition and distinct chemical properties, such as elements and compounds.
8. **Phase**: A distinct form of matter with uniform physical and chemical properties, such as solid, liquid, or gas.
9. **Solid**: A state of matter characterized by a definite shape and volume, with particles closely packed in a fixed arrangement.
10. **Liquid**: A state of matter characterized by a definite volume but no definite shape, with particles close together but able to move past one another.
11. **Gas**: A state of matter characterized by no definite shape or volume, with particles far apart and moving freely.
12. **Plasma**: A high-energy state of matter where atoms are ionized, found in stars and fluorescent lights.
13. **Bose-Einstein Condensate (BEC)**: A state of matter formed at temperatures close to absolute zero, where particles occupy the same space and quantum state.
14. **Phase Transition**: The transformation of matter from one phase to another, such as melting, freezing, boiling, or condensing.
15. **Melting**: The process of changing from a solid to a liquid by adding heat.
16. **Freezing**: The process of changing from a liquid to a solid by removing heat.
17. **Vaporization**: The process of changing from a liquid to a gas, including both evaporation and boiling.
18. **Condensation**: The process of changing from a gas to a liquid by removing heat.
19. **Sublimation**: The process of changing from a solid directly to a gas without passing through the liquid phase.
20. **Deposition**: The process of changing from a gas directly to a solid without passing through the liquid phase.
21. **Boiling Point**: The temperature at which a liquid changes to a gas throughout the liquid, occurring at a specific temperature and pressure.
22. **Melting Point**: The temperature at which a solid changes to a liquid.
23. **Freezing Point**: The temperature at which a liquid changes to a solid.
24. **Critical Point**: The temperature and pressure at which the liquid and gas phases of a substance become indistinguishable.
25. **Triple Point**: The temperature and pressure at which all three phases (solid, liquid, gas) of a substance coexist in equilibrium.
26. **Phase Diagram**: A graph showing the conditions of temperature and pressure under which a substance exists in different phases.
27. **Viscosity**: A measure of a liquid's resistance to flow.
28. **Density**: The mass of a substance per unit volume, often measured in grams per cubic centimeter (g/cm^3).
29. **Pressure**: The force exerted per unit area, often measured in pascals (Pa) or atmospheres (atm).
30. **Intermolecular Forces**: Forces of attraction or repulsion between molecules, affecting the physical properties of substances.
31. **Van der Waals Forces**: Weak intermolecular forces, including dispersion forces and dipole-dipole interactions.
32. **Hydrogen Bonding**: A strong type of dipole-dipole interaction between molecules containing hydrogen bonded to a highly electronegative atom (such as oxygen, nitrogen, or fluorine).
33. **Ionic Solid**: A solid consisting of ions held together by electrostatic forces (ionic bonds), typically with high melting points and electrical conductivity when molten.
34. **Molecular Solid**: A solid consisting of molecules held together by intermolecular forces, typically with lower melting points.
35. **Metallic Solid**: A solid consisting of metal atoms held together by a "sea" of shared electrons, characterized by electrical conductivity and malleability.
36. **Covalent Network Solid**: A solid consisting of atoms held together in large networks or chains by covalent bonds, typically with very high melting points.
37. **Amorphous Solid**: A solid in which atoms or molecules are not arranged in a regular, repeating pattern.
38. **Crystalline Solid**: A solid in which atoms or molecules are arranged in a regular, repeating pattern.
39. **Alloy**: A mixture of metals or a mixture of a metal and another element, designed to have specific properties.
40. **Solution**: A homogeneous mixture of two or more substances.
41. **Solvent**: The substance in which the solute is dissolved to form a solution.

42. **Solute**: The substance that is dissolved in a solvent to form a solution.

CHAPTER OVERVIEW

5: Density Mole and Molarity

5.1: Introduction and Chapter Objectives

Chapter Learning Objectives

Understanding the Concept of Density:

- Learn density as mass per unit volume.
- Calculate density and understand its significance in comparing substances and buoyancy.
- Explore how temperature and pressure affect density and its practical applications.

Introduction to Moles and Molar Mass:

- Understand the mole as a unit for measuring substance amount in chemistry.
- Calculate molar mass and convert between mass and moles.
- Use moles in balancing equations and stoichiometric calculations.

Basics of Molarity and Solution Concentration:

- Define molarity as moles of solute per liter of solution.
- Calculate molarity and prepare solutions of specific concentrations.
- Learn about solution dilution and related calculations.

Practical Applications of Density, Moles, and Molarity:

- Explore real-world uses in medicine, environmental science, and engineering.
- Understand their role in lab practices, solution preparation, and substance property determination.
- Emphasize the importance of accuracy in measurements and calculations.

Teaching Strategies for Density, Moles, and Molarity:

- Develop effective teaching methods with relevant examples and age-appropriate language.
- Design engaging activities and experiments for better visualization.
- Use multimedia tools to illustrate practical importance.
- Address common misconceptions and provide clarification strategies.
- Highlight interdisciplinary relevance and real-life applications.

Introduction to Density Mole and Molarity

Understanding the fundamental concepts of density, moles, and molarity is essential for grasping the intricacies of chemistry. These core principles not only form the basis for advanced chemical studies but also have broad applications across various scientific disciplines and everyday life.

Density, defined as mass per unit volume, is a crucial property that helps us compare different substances and understand phenomena such as buoyancy. By calculating density, we can predict how substances will interact in various environments. Furthermore, exploring how factors like temperature and pressure influence density provides deeper insights into material behavior under different conditions, enhancing our ability to manipulate and utilize materials effectively.

The concept of the mole serves as a cornerstone in chemistry for measuring the amount of substance. Understanding moles allows us to bridge the microscopic world of atoms and molecules with the macroscopic world we observe. By calculating molar mass, we can convert between mass and moles, facilitating the quantification of substances involved in chemical reactions. This understanding is crucial for balancing chemical equations and performing stoichiometric calculations, ensuring precise and accurate outcomes in both theoretical and practical applications.

Molarity, which measures the concentration of a solution in terms of moles of solute per liter of solution, is another vital concept. Learning to calculate molarity enables the preparation of solutions with specific concentrations, a common requirement in laboratory settings. Additionally, understanding the principles of dilution and the associated calculations is essential for adjusting solution concentrations to desired levels, a frequent necessity in various chemical processes.

These concepts extend beyond the classroom and laboratory, finding applications in diverse fields such as medicine, environmental science, and engineering. For instance, in medicine, understanding the concentration of solutions is critical for preparing accurate dosages of pharmaceuticals. In environmental science, calculating the density of pollutants helps in assessing their impact on ecosystems. Engineering applications often require precise measurements of material properties to ensure structural integrity and functionality.

In the context of K-12 education, introducing these fundamental chemical principles lays a strong foundation for future scientific learning. By developing effective teaching strategies that include relevant examples and age-appropriate language, educators can make these concepts accessible and engaging for younger students. Designing hands-on activities and experiments helps students visualize and understand abstract ideas, fostering a deeper appreciation for chemistry. Utilizing multimedia tools further enhances learning experiences by illustrating the practical importance of these concepts. Addressing common misconceptions and providing clear explanations ensure that students build a solid understanding. Emphasizing the interdisciplinary nature and real-life applications of density, moles, and molarity not only enriches the learning process but also highlights the relevance of chemistry in everyday life and various scientific fields.

5.2: Density

Learning Objectives

- Calculate density, mass, or volume when given 2 of these three variables.
- Identify what units are required for the density equation.
- Review metric conversions.
- Compare densities of different chemical substances.
- Classify a substance as being a heterogeneous or homogeneous mixture if solubility date is provided.
- Identify where a chemical would appear in water if solubility and density data are provided.
- Compare any chemical substance's density to the density of water (please memorize this value).

Density is a physical property that is defined as a substance’s mass divided by its volume:

$$\text{density} = \frac{\text{mass}}{\text{volume}} \tag{5.2.1}$$

$$d = \frac{m}{V} \tag{5.2.2}$$

Density is usually a measured property of a substance, so its numerical value affects the significant figures in a calculation. Notice that density is defined in terms of two dissimilar units, mass and volume. That means that density overall has *derived units*, just like velocity. Common units for density include g/mL, g/cm^3, g/L, kg/L, and even kg/m^3. Densities for some common substances are listed in Table $5.2.1$. **Memorize the density of water with its appropriate units**.

Table $5.2.1$: Densities of Some Common Substances

Substance	Density (g/mL or g/cm^3)
water	**1.0**
gold	19.3
mercury	13.6
air	0.0012
cork	0.22–0.26
aluminum	2.7
iron	7.87

Figure $5.2.1$: Separatory Funnel containing oil and colored water to display density differences. (CC BY-SA 3.0; PRHaney via Wikipedia).

Because of how it is defined, density can act as a conversion factor for switching between units of mass and volume. For example, suppose you have a sample of aluminum that has a volume of 7.88 cm^3. How can you determine what mass of aluminum you have without measuring it? You can use the volume to calculate it. If you multiply the given volume by the known density (Table 5.2.1), the volume units will cancel and leave you with mass units, telling you the mass of the sample:

Start with Equation 5.2.1

$$\text{density} = \frac{m}{V}$$

and insert the relevant numbers

$$\frac{2.7 g}{cm^3} = \frac{m}{7.88\, cm^3}$$

Cross multiplying both sides (right numerator x left denominator = left numerator x right denominator), we get the following expression with answer and appropriate unit.

$$7.88\ \cancel{cm^3} \times \frac{2.7\ g}{\cancel{cm^3}} = 21 g \text{ of aluminum}$$

Example 5.2.1: Mercury

What is the mass of 44.6 mL of mercury?

Solution

Use the density value for mercury from Table 5.2.1 and the definition of density (Equation 5.2.1)

$$density = \frac{mass}{volume} \Rightarrow d = \frac{m}{V}$$

$$\frac{13.6 g}{mL} = \frac{m}{44.6\, mL}$$

Remember to cross multiply here in order to isolate variable. Then, report answer with correct units.

$$44.6\ \cancel{mL} \times \frac{13.6\ g}{\cancel{mL}} = 607\ g$$

The mass of the mercury is 607 g.

Exercise 5.2.1

What is the mass of 25.0 cm^3 of iron?

Answer

Use the density value for iron from Table 5.2.1

$$density = \frac{mass}{volume} \Rightarrow d = \frac{m}{V}$$

$$\frac{7.87 g}{cm^3} = \frac{m}{25.0\, cm^3}$$

Cross multiplying both sides (right numerator x left denominator = left numerator x right denominator), we get the following expression with answer and appropriate unit.

$$25.0\ \cancel{cm^3} \times \frac{7.87\ g}{\cancel{cm^3}} = 197 g \text{ of iron}$$

Another way of looking at density (some students choose to perform calculations using this method)

Density can also be used as a conversion factor to convert mass to volume—but care must be taken. We have already demonstrated that the number that goes with density normally goes in the numerator when density is written as a fraction. Take the density of gold, for example:

$$d = 19.3\,g/mL = \frac{19.3\,g}{mL}$$

Although this was not previously pointed out, it can be assumed that there is a 1 in the denominator:

$$d = 19.3\,g/mL = \frac{19.3\,g}{mL}$$

That is, the density value tells us that we have 19.3 grams for every 1 milliliter of volume, and the 1 is an exact number. When we want to use density to convert from mass to volume, the numerator and denominator of density need to be switched—that is, we must take the *reciprocal* of the density. In so doing, we move not only the units but also the numbers:

$$\frac{19.3\ \cancel{g}}{mL} = \frac{45\ \cancel{g}}{V}$$

Cross multiplying denominators with numerators, we obtain the following algebraic equation.

$$19.3V = 45.9\,mL$$

then you will need to isolate the variable (volume)

$$V = \frac{45.9\,mL}{19.3}$$

After multiplication, the answer would be

$$V = 2.38\,mL$$

✓ Example 5.2.2: Wine Cork

A cork stopper from a bottle of wine has a mass of 3.78 g. If the density of cork is 0.22 g/mL, what is the volume of the cork? *Regardless of the method that is used, you should still be able to obtain the same (and correct) answer.*

Solution

To use density as a conversion factor, we need to take the reciprocal so that the mass unit of density is in the denominator. Taking the reciprocal, we find

$$\frac{0.22\ \cancel{g}}{mL} = \frac{3.78\ \cancel{g}}{V}$$

Cross multiplying denominators with numerators, we obtain the following algebraic equation.

$$0.22V = 3.78\,mL$$

then you will need to isolate the variable (volume)

$$V = \frac{3.78\,mL}{0.22}$$

so, the volume of the cork is 17.2 mL.

? Exercise 5.2.2

What is the volume of 3.78 g of gold?

Answer

Before attempting this question, be sure to obtain the density of gold in the table above. If you were to need this value on a quiz or a test, then it would be provided. Once you have this value, plug it into the density equation. Next, you will need to isolate the volume variable (basic algebra). The final answer should be 0.196 cm^3.

Care must be used with density as a conversion factor. Make sure the mass units are the same or the volume units are the same, before using density to convert to a different unit. Often, the unit of the given quantity must be first converted to the appropriate unit before applying density as a conversion factor.

Using Density in Environmental Applications

Along with solubility, density can help determine how a contaminant could affect an aquatic system. For example, imagine mercury has been spilled in Furman Lake. Looking at this element's density value and comparing it to liquid water, one could determine the location of the insoluble (you would be given solubility information) mercury layer. The more dense mercury layer would reside on the bottom of Furman Lake. If one were to take a cross-section of the lake, you could see that a heterogeneous mixture would result.

In contrast, spilling ethanol (density = 0.789g/mL) would result in the formation of a homogeneous mixture. Ethanol (grain alcohol) is soluble in water. This would make it miscible (mixable to form a solution) in water and one would not be able to denote separate layers. According to the density, an alcohol layer would remain on top, but would ultimately dissolve.

Applications

What difficulties would arise from the separation and removal of contaminants?

1. Hg in Furman Lake
2. Ethanol in Furman Lake
3. oil (less dense, insoluble) in Furman Lake

Watch this video and record your observations.

1. What component was different in the two types of beverages (mass or volume)?
2. How does the above-mentioned difference affect the density equation?
3. Which beverage is denser than water?

Need More Practice?

- Turn to Section 2.E of this OER and work problems #2 and #9.

Contributors and Attributions

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5.3: Lab 2 Density of sweet drinks

Lab 2 Density of sweet drinks

Objectives

- Explore density of solids, liquids and gases
- Explore how density is related to floating in water (buoyancy)
- Determine the density of a sweet drink (two choices, different colors)
- Evaluate sources of error in measurements and their effect on an experiment.

Safety and Notes

We will not generate any hazardous waste today. Once the sweet drinks enter the lab, they are no longer for human consumption, and we will discard them at the end of the week. In one task, we will heat up a metal strip to high temperature. Do not touch the strip until you cooled it down with cold water.

Introduction

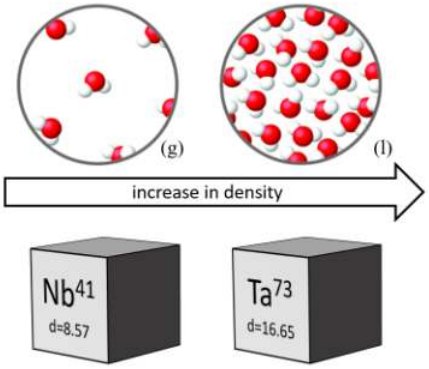
Density

The density of an object is defined as the ratio of its mass to its volume:

$$\text{Density } = \frac{\text{Mass}}{\text{Volume}}$$

For an example of density, consider the following: Imagine a brick that is made of Styrofoam. Imagine a second brick that is made of lead. Note that even though the bricks take up the same amount of space - that is, they have the same volume - there is a major difference in their mass. We would say that the lead is denser, that is it has more mass in the same volume.

How can we explain density at the atomic level? The closer atoms are to each other, the higher the density. For example, the density of liquid water is higher than that of water vapor (steam) because the water molecules are much closer together in the liquid (see image on the right). The heavier the atoms are, the higher the density. For example, the density of metallic tantalum is about twice that of niobium, even though they have similar chemical properties and are in the same group of the periodic table (which is why they are sometimes called twins).



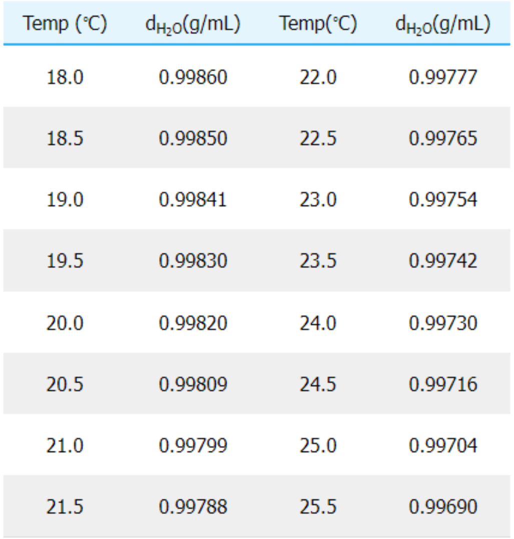
Density of (liquid) water

It is important to note that liquid water has a density of approximately 1.0 g/mL. So at room temperature, 50.0 mL of water has a mass of almost exactly 50.0 g.

Two factors have an effect on the density of water:

1. Temperature will have a small effect on the density. For water, density increases as temperature decreases. For example, a 10.000 mL volume of water will increase by 0.016 mL when the temperature is raised from 18˚C to 25˚C. See Table 1 for the density of water at different temperatures.

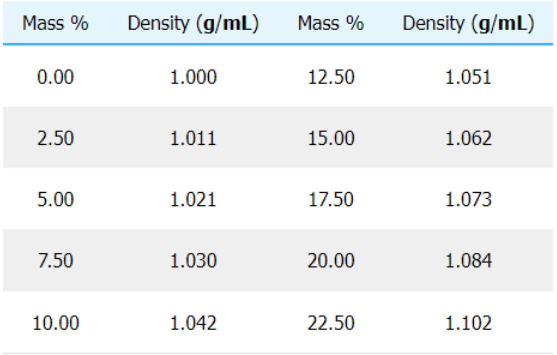
2. When solids are dissolved in the water, the solution will typically have a higher density than pure water. We might see this effect in today’s lab when we compare the density of sweet drinks (aqueous solutions) with the density of water.



Temp (°C)	d_{H_2O}(g/mL)	Temp(°C)	d_{H_2O}(g/mL)
18.0	0.99860	22.0	0.99777
18.5	0.99850	22.5	0.99765
19.0	0.99841	23.0	0.99754
19.5	0.99830	23.5	0.99742
20.0	0.99820	24.0	0.99730
20.5	0.99809	24.5	0.99716
21.0	0.99799	25.0	0.99704
21.5	0.99788	25.5	0.99690

Table 1. Density of water at different temperatures

A table of the theoretical values of density for sucrose (table sugar) solutions of various (*w*/*w*)% is included in Table 2 below.

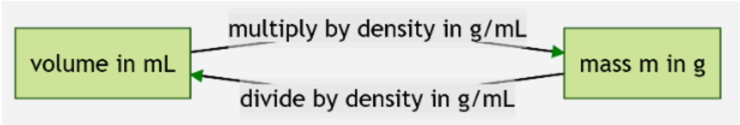


Mass %	Density (g/mL)	Mass %	Density (g/mL)
0.00	1.000	12.50	1.051
2.50	1.011	15.00	1.062
5.00	1.021	17.50	1.073
7.50	1.030	20.00	1.084
10.00	1.042	22.50	1.102

Table 2: Theoretical Density Values of Sucrose Solutions with Known Mass Percent

Calculating with density

If you know the density of a sample (in g/mL), you can determine the volume from the mass of the sample, and vice versa.



Buoyancy

The concept of buoyancy (a physics concept) is relevant to the interpretation of some of the tasks below. Buoyancy is an upward force of objects submerged in a liquid that explains why some objects sink and others float in a given liquid. The buoyancy is equal to the gravitational force on the displaced liquid, i.e. it depends on the volume of the floating object (or, if it is not fully submerged, the volume that is under the surface) and on the density of the liquid.

Objects that have a density less than water, that is, less than 1.0 g/mL, will float on the surface of the water, with more or less of the object submerged. Those that have a density greater than 1.0 g/mL will sink. Consider our two bricks again. The brick of Styrofoam will float if we toss it into water. The lead will sink "like a rock".

Relative error

In this experiment, we will discuss the relative error, expressed in percent, of our measurements. The equation for finding relative error (or percent error) is:

$$\text{Relative error } = \frac{\text{(Experimental Value-Theoretical Value)}}{\text{Theoretical Value}} \times 100\%$$

If we want to decide whether to use a 100 mL or a 10 mL graduated cylinder for our density measurements, the relative error will be the deciding factor. If the 100 mL cylinder has a tolerance of 0.4 mL and the 10 mL cylinder has a tolerance of 0.05 mL, which one has the smaller absolute volume error? Which one has the smaller relative error? Assuming you have a balance that has a relative error of 0.01%, which volume is better for a density determination?

Graphing Data

In one of the tasks, you will use a graphical method to determine density. It is recommended that you use a spreadsheet for this task (tutorial for Google sheets, MS Excel, IOS Numbers). Make a scatter plot of your data and fit a straight line to it. Then, jot down the formula of the line, given as y = m x + b. You might chose to set b (y-intercept) to zero, but you don’t have to. If you do, the formula is simply y = m x. In either case, m is the slope you need for the analysis.

Experimental Procedure

To have less crowding at the balances, you either start with task 1 or with task 2. Do task 3 only when you are done with the two other tasks. Record your observations and calculations in your lab notebook, and use the lab summary sheet to hand in your results and reflections.

Part 1: Density of sweet drink

The protocol below is a bit weird. We are filling a graduated cylinder (which is designed “to contain”) from a beaker, whose mass we monitor. We could have measured the mass of the graduated cylinder instead; however, the graduated cylinder has a higher mass (getting close to the maximum of the analytical balance) and an awkward shape for use in the balance.

1. Pour about 60 mL of sweet drink (there are two choices, with different colors) into a 100 mL beaker and measure the total mass (beaker + drink).
2. At your bench, pour 10 mL from the beaker into a 100 mL graduated cylinder. Go slow and steady, and use a transfer pipet for the last milliliter. Observe the meniscus carefully. Any leftover solution in the transfer pipet goes back into the beaker.
3. Measure the total mass of the sweet drink in the beaker again (the mass should be less because you removed 10 mL from it).
4. Repeat steps 2 and 3, adding drink to the same graduated cylinder (so it will be filled with 10 mL, 20 mL, 30 mL, 40 mL and finally 50 mL. Make sure the balance is zeroed before you place the beaker on the weighing dish.
5. Graph the removed volume against the removed mass. The x axis should be the volume in mL and the y axis the mass in g.
6. Fit a line to the data. The slope of the line will have units g/mL and is a measure of the density of the drink.
7. In a different way of analyzing the data, subtract the final mass from the initial mass and divide by the difference in volume to get the density.
8. When you are down, pour the drink into the appropriate one of the two provided containers (we will use it again, and monitor whether the density changes over the week due to evaporation or contamination).

Part 2: Density changes with temperature, pressure, and physical state

Walk through the exhibit (any order), explore the objects and answer the questions on the summary sheet

Part 3: Alternative density determination

Choose **one** of the alternative density determination methods below. Choice 1 can be done at your bench, the others at separate stations. Not all stations might be available.

1. Graduated cylinder

Taking a clean and dry graduated cylinder, measure the mass, and then fill to 10, 20, 30, 40, and 50 mL with the drink you used for task 1, measuring the mass at each step. Use the same graphical analysis as in task 1 to determine the density. When you are done, pour the drink into the appropriate one of the two provided containers (we will use it again, and monitor whether the density changes over the week due to evaporation or contamination).

2. Volumetric flask

Taking a clean and dry 50 mL volumetric flask, measure the mass, and then carefully fill to the line with the drink you used for task 1. If you have a drink that likes to make bubbles, a funnel will be helpful. Measure about 45 mL of the drink in a beaker, and slowly pour it into the funnel whose outlet is at the bottom of the volumetric flask. Use a transfer pipet for the last bit, making sure the liquid does not wet the volumetric flask above the line, and measure the mass. Calculate the density from your data and compare it with that from task 1. When you are done, pour the drink into the appropriate one of the two provided containers (we will use it again, and monitor whether the density changes over the week due to evaporation or contamination).

3. Potato and egg

See whether a potato and a hard-boiled egg float or sink in your drink. Each object has a volume of 60-80 mL, so choose a beaker that the object fits in, place it in the beaker, and fill with the sweet drink until submerged. If they float, estimate which fraction of the object is above the surface of the liquid. From your observations, estimate the density of the liquid. When you are down, rinse and dry the potato and the egg. Pour the drink into the appropriate one of the two provided containers (we will use it again, and monitor whether the density changes over the week due to evaporation or contamination).

4. Below balance measurement

Zero the balance and attach the “sinker” (a large marble on a string) to the hook, zeroing the balance. Submerge the sinker in pure water at room temperature and record the mass difference (which will be negative). Then, dry the sinker and submerge it in your sweet drink, recording the mass difference (which will be negative). The ratio of the mass differences is equal to the ratio of densities. Look up the density of pure water and calculate the density of your sweet drink. When you are done, pour the drink into the appropriate one of the two provided containers (we will use it again, and monitor whether the density changes over the week due to evaporation or contamination).

5. Above balance measurement

Place a beaker with water on the balance and zero the balance. Submerge a “sinker” in the water and record the mass. It represents the buoyancy of the sinker (and you can determine the volume of the sinker if you know the density of the water). Repeat with your sweet drink. If it has a higher density than water, the buoyancy will be larger. The ratio of the masses measured is equal to the ratio of densities. Look up the density of pure water and calculate the density of your sweet drink. When you are done, pour the drink into the appropriate one of the two provided containers (we will use it again, and monitor whether the density changes over the week due to evaporation or contamination).

Acknowledgement

Parts of this lab were adopted from Gerber-Morales's density lab.

5.4: Concentration of Solutions

Learning Objectives

- To describe the concentrations of solutions quantitatively

Many people have a qualitative idea of what is meant by *concentration*. Anyone who has made instant coffee or lemonade knows that too much powder gives a strongly flavored, highly concentrated drink, whereas too little results in a dilute solution that may be hard to distinguish from water. In chemistry, the concentration of a solution is the quantity of a **solute** that is contained in a particular quantity of **solvent** or solution. Knowing the concentration of solutes is important in controlling the stoichiometry of reactants for solution reactions. Chemists use many different methods to define concentrations, some of which are described in this section.

Molarity

The most common unit of concentration is *molarity*, which is also the most useful for calculations involving the stoichiometry of reactions in solution. The molarity (M) is defined as the number of moles of solute present in exactly 1 L of solution. It is, equivalently, the number of millimoles of solute present in exactly 1 mL of solution:

$$molarity = \frac{moles\ of\ solute}{liters\ of\ solution} = \frac{mmoles\ of\ solute}{milliliters\ of\ solution} \tag{5.4.1}$$

The units of molarity are therefore moles per liter of solution (mol/L), abbreviated as M. An aqueous solution that contains 1 mol (342 g) of sucrose in enough water to give a final volume of 1.00 L has a sucrose concentration of 1.00 mol/L or 1.00 M. In chemical notation, square brackets around the name or formula of the solute represent the molar concentration of a solute. Therefore,

$$[\text{sucrose}] = 1.00\ \text{M}$$

is read as “the concentration of sucrose is 1.00 molar.” The relationships between volume, molarity, and moles may be expressed as either

$$V_L M_{mol/L} = \cancel{L}\left(\frac{mol}{\cancel{L}}\right) = moles \tag{5.4.2}$$

or

$$V_{mL} M_{mmol/mL} = \cancel{mL}\left(\frac{mmol}{\cancel{mL}}\right) = mmoles \tag{5.4.3}$$

Figure $5.4.1$ illustrates the use of Equations $5.4.2$ and $5.4.3$.

(a) An amount of solute is weighed out on an analytical balance and then

(b) A portion of the solvent is added to the volumetric flask.

(c) The mixture is swirled until *all* of the solute is dissolved.

(d) Additional solvent is added up to the mark on the volumetric flask.

Figure $5.4.1$: Preparation of a Solution of Known Concentration Using a Solid Solute

Example 5.4.1: Calculating Moles from Concentration of NaOH

Calculate the number of moles of sodium hydroxide (NaOH) in 2.50 L of 0.100 M NaOH.

Given: identity of solute and volume and molarity of solution

Asked for: amount of solute in moles

Strategy:

Use either Equation 5.4.2 or Equation 5.4.3, depending on the units given in the problem.

Solution:

Because we are given the volume of the solution in liters and are asked for the number of moles of substance, Equation 5.4.2 is more useful:

$$moles\ NaOH = V_L M_{mol/L} = (2.50\ \cancel{L})\left(\frac{0.100\ mol}{\cancel{L}}\right) = 0.250\ mol\ NaOH$$

Exercise 5.4.1: Calculating Moles from Concentration of Alanine

Calculate the number of millimoles of alanine, a biologically important molecule, in 27.2 mL of 1.53 M alanine.

Answer

41.6 mmol

Calculations Involving Molarity (M): Calculations Involving Molarity (M), YouTube(opens in new window) [youtu.be]

Concentrations are also often reported on a mass-to-mass (m/m) basis or on a mass-to-volume (m/v) basis, particularly in clinical laboratories and engineering applications. A concentration expressed on an m/m basis is equal to the number of grams of solute per gram of solution; a concentration on an m/v basis is the number of grams of solute per milliliter of solution. Each measurement can be expressed as a percentage by multiplying the ratio by 100; the result is reported as percent m/m or percent m/v. The concentrations of very dilute solutions are often expressed in *parts per million* (*ppm*), which is grams of solute per 10^6 g of solution, or in *parts per billion* (*ppb*), which is grams of solute per 10^9 g of solution. For aqueous solutions at 20°C, 1 ppm corresponds to 1 μg per milliliter, and 1 ppb corresponds to 1 ng per milliliter. These concentrations and their units are summarized in Table 5.4.1.

Table 5.4.1: Common Units of Concentration

Concentration	Units
m/m	g of solute/g of solution
m/v	g of solute/mL of solution

Concentration	Units
ppm	g of solute/10^6 g of solution
	µg/mL
ppb	g of solute/10^9 g of solution
	ng/mL

The Preparation of Solutions

To prepare a solution that contains a specified concentration of a substance, it is necessary to dissolve the desired number of moles of solute in enough solvent to give the desired final volume of solution. Figure 5.4.1 illustrates this procedure for a solution of cobalt(II) chloride dihydrate in ethanol. Note that the volume of the *solvent* is not specified. Because the solute occupies space in the solution, the volume of the solvent needed is almost always *less* than the desired volume of solution. For example, if the desired volume were 1.00 L, it would be incorrect to add 1.00 L of water to 342 g of sucrose because that would produce more than 1.00 L of solution. As shown in Figure 5.4.2, for some substances this effect can be significant, especially for concentrated solutions.

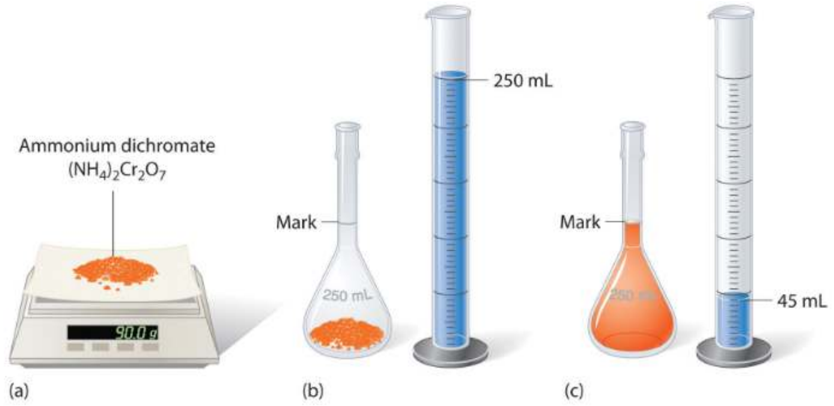


Figure 5.4.2: Preparation of 250 mL of a Solution of $(NH_4)_2Cr_2O_7$ in Water. The solute occupies space in the solution, so less than 250 mL of water are needed to make 250 mL of solution. 45 milliliters of water remain in the graduated cylinder even after addition to the mark of the volumetric flask.

✓ Example 5.4.2

The solution contains 10.0 g of cobalt(II) chloride dihydrate, $CoCl_2•2H_2O$, in enough ethanol to make exactly 500 mL of solution. What is the molar concentration of $\mathrm{CoCl_2 \bullet 2\,H_2O}$?

Given: mass of solute and volume of solution

Asked for: concentration (M)

Strategy:

To find the number of moles of $\mathrm{CoCl_2 \bullet 2\,H_2O}$, divide the mass of the compound by its molar mass. Calculate the molarity of the solution by dividing the number of moles of solute by the volume of the solution in liters.

Solution:

The molar mass of $CoCl_2•2H_2O$ is 165.87 g/mol. Therefore,

$$moles\ CoCl_2 \cdot 2H_2O = \left(\frac{10.0\ \cancel{g}}{165.87\ \cancel{g}/mol}\right) = 0.0603\ mol$$

The volume of the solution in liters is

$$volume = 500\ \cancel{mL}\left(\frac{1\ L}{1000\ \cancel{mL}}\right) = 0.500\ L$$

Molarity is the number of moles of solute per liter of solution, so the molarity of the solution is

$$molarity = \frac{0.0603\ mol}{0.500\ L} = 0.121\ M = CoCl_2 \cdot H_2O$$

Exercise 5.4.2

The solution shown in Figure 5.4.2 contains 90.0 g of $(NH_4)_2Cr_2O_7$ in enough water to give a final volume of exactly 250 mL. What is the molar concentration of ammonium dichromate?

Answer

$$(NH_4)_2Cr_2O_7 = 1.43\ M$$

To prepare a particular volume of a solution that contains a specified concentration of a solute, we first need to calculate the number of moles of solute in the desired volume of solution using the relationship shown in Equation 5.4.2. We then convert the number of moles of solute to the corresponding mass of solute needed. This procedure is illustrated in Example 5.4.3.

Example 5.4.3: D5W Solution

The so-called D5W solution used for the intravenous replacement of body fluids contains 0.310 M glucose. (D5W is an approximately 5% solution of dextrose [the medical name for glucose] in water.) Calculate the mass of glucose necessary to prepare a 500 mL pouch of D5W. Glucose has a molar mass of 180.16 g/mol.

Given: molarity, volume, and molar mass of solute

Asked for: mass of solute

Strategy:

A. Calculate the number of moles of glucose contained in the specified volume of solution by multiplying the volume of the solution by its molarity.
B. Obtain the mass of glucose needed by multiplying the number of moles of the compound by its molar mass.

Solution:

A We must first calculate the number of moles of glucose contained in 500 mL of a 0.310 M solution:

$$V_L M_{mol/L} = moles$$

$$500\ \cancel{mL}\left(\frac{1\ \cancel{L}}{1000\ \cancel{mL}}\right)\left(\frac{0.310\ mol\ glucose}{1\ \cancel{L}}\right) = 0.155\ mol\ glucose$$

B We then convert the number of moles of glucose to the required mass of glucose:

$$mass\ of\ glucose = 0.155\ \cancel{mol\ glucose}\left(\frac{180.16\ g\ glucose}{1\ \cancel{mol\ glucose}}\right) = 27.9\ g\ glucose$$

Exercise 5.4.3

Another solution commonly used for intravenous injections is normal saline, a 0.16 M solution of sodium chloride in water. Calculate the mass of sodium chloride needed to prepare 250 mL of normal saline solution.

Answer

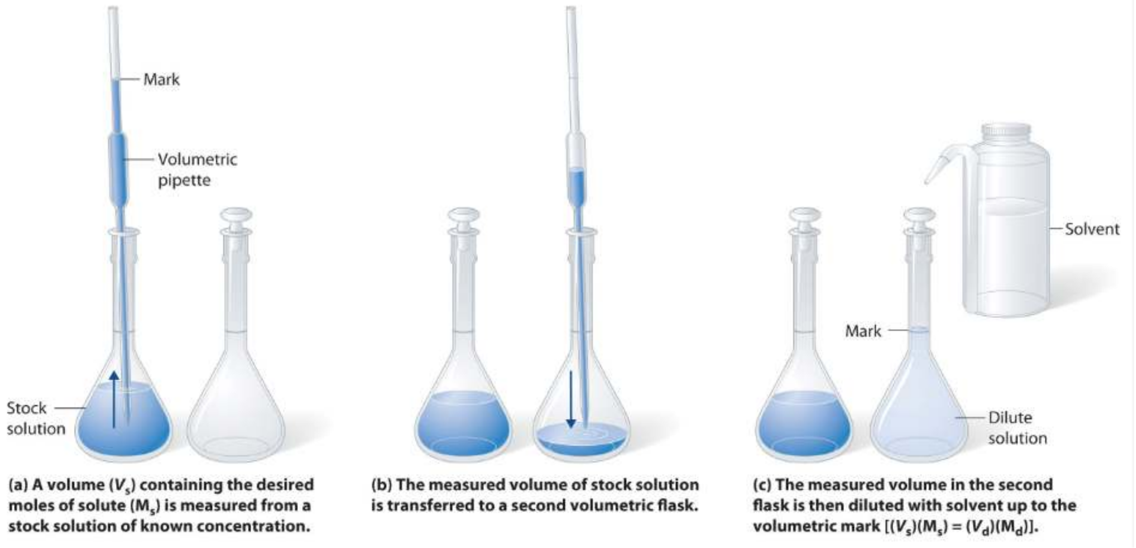
2.3 g NaCl

A solution of a desired concentration can also be prepared by diluting a small volume of a more concentrated solution with additional solvent. A stock solution is a commercially prepared solution of known concentration and is often used for this purpose. Diluting a stock solution is preferred because the alternative method, weighing out tiny amounts of solute, is difficult to carry out with a high degree of accuracy. Dilution is also used to prepare solutions from substances that are sold as concentrated aqueous solutions, such as strong acids.

The procedure for preparing a solution of known concentration from a stock solution is shown in Figure 5.4.3. It requires calculating the number of moles of solute desired in the final volume of the more dilute solution and then calculating the volume of the stock solution that contains this amount of solute. Remember that diluting a given quantity of stock solution with solvent does *not* change the number of moles of solute present. The relationship between the volume and concentration of the stock solution and the volume and concentration of the desired diluted solution is therefore

$$(V_s)(M_s) = moles\ of\ solute = (V_d)(M_d) \tag{5.4.4}$$

where the subscripts *s* and *d* indicate the stock and dilute solutions, respectively. Example 5.4.4 demonstrates the calculations involved in diluting a concentrated stock solution.



(a) A volume (V_s) containing the desired moles of solute (M_s) is measured from a stock solution of known concentration.

(b) The measured volume of stock solution is transferred to a second volumetric flask.

(c) The measured volume in the second flask is then diluted with solvent up to the volumetric mark [(V_s)(M_s) = (V_d)(M_d)].

Figure 5.4.3: Preparation of a Solution of Known Concentration by Diluting a Stock Solution. (a) A volume (V_s) containing the desired moles of solute (M_s) is measured from a stock solution of known concentration. (b) The measured volume of stock solution is transferred to a second volumetric flask. (c) The measured volume in the second flask is then diluted with solvent up to the volumetric mark [(V_s)(M_s) = (V_d)(M_d)].

✓ Example 5.4.4

What volume of a 3.00 M glucose stock solution is necessary to prepare 2500 mL of the D5W solution in Example 5.4.3?

Given: volume and molarity of dilute solution

Asked for: volume of stock solution

Strategy:

A. Calculate the number of moles of glucose contained in the indicated volume of dilute solution by multiplying the volume of the solution by its molarity.
B. To determine the volume of stock solution needed, divide the number of moles of glucose by the molarity of the stock solution.

Solution:

A The D5W solution in Example 4.5.3 was 0.310 M glucose. We begin by using Equation 4.5.4 to calculate the number of moles of glucose contained in 2500 mL of the solution:

$$moles\ glucose = 2500\ \cancel{mL}\left(\frac{1\ \cancel{L}}{1000\ \cancel{mL}}\right)\left(\frac{0.310\ mol\ glucose}{1\ \cancel{L}}\right) = 0.775\ mol\ glucose$$

B We must now determine the volume of the 3.00 M stock solution that contains this amount of glucose:

$$volume\ of\ stock\ soln = 0.775\ \ mol\ \cancel{glucose}\left(\frac{1\ L}{3.00\ \ mol\ \cancel{glucose}}\right) = 0.258\ L\ or\ 258\ mL$$

In determining the volume of stock solution that was needed, we had to divide the desired number of moles of glucose by the concentration of the stock solution to obtain the appropriate units. Also, the number of moles of solute in 258 mL of the stock solution is the same as the number of moles in 2500 mL of the more dilute solution; *only the amount of solvent has changed*. The answer we obtained makes sense: diluting the stock solution about tenfold increases its volume by about a factor of 10 (258 mL → 2500 mL). Consequently, the concentration of the solute must decrease by about a factor of 10, as it does (3.00 M → 0.310 M).

We could also have solved this problem in a single step by solving Equation 4.5.4 for V_s and substituting the appropriate values:

$$V_s = \frac{(V_d)(M_d)}{M_s} = \frac{(2.500\ L)(0.310\ \ \cancel{M})}{3.00\ \ \cancel{M}} = 0.258\ L$$

As we have noted, there is often more than one correct way to solve a problem.

? Exercise 5.4.4

What volume of a 5.0 M NaCl stock solution is necessary to prepare 500 mL of normal saline solution (0.16 M NaCl)?

Answer

16 mL

Ion Concentrations in Solution

In Example 5.4.2, the concentration of a solution containing 90.00 g of ammonium dichromate in a final volume of 250 mL were calculated to be 1.43 M. Let's consider in more detail exactly what that means. Ammonium dichromate is an ionic compound that contains two NH_4^+ ions and one $Cr_2O_7^{2-}$ ion per formula unit. Like other ionic compounds, it is a strong electrolyte that dissociates in aqueous solution to give hydrated NH_4^+ and $Cr_2O_7^{2-}$ ions:

$$(NH_4)_2Cr_2O_7(s) \xrightarrow{H_2O(l)} 2NH_4^+(aq) + Cr_2O_7^{2-}(aq) \tag{5.4.5}$$

Thus 1 mol of ammonium dichromate formula units dissolves in water to produce 1 mol of $Cr_2O_7^{2-}$ anions and 2 mol of NH_4^+ cations (see Figure 5.4.4).

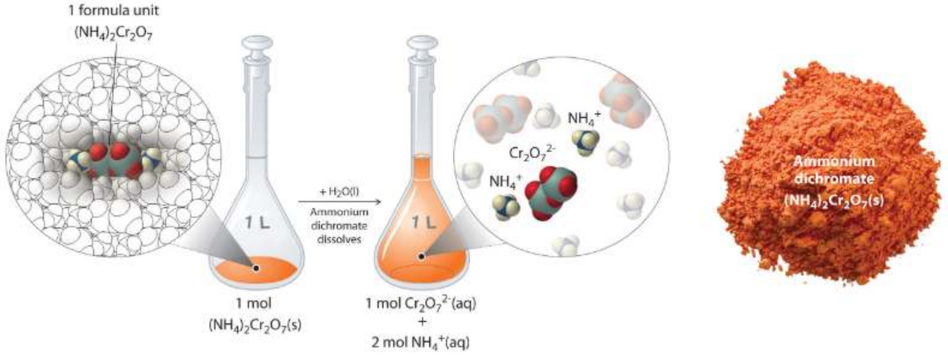


Figure 5.4.4: Dissolution of 1 mol of an Ionic Compound. In this case, dissolving 1 mol of $(NH_4)_2Cr_2O_7$ produces a solution that contains 1 mol of $Cr_2O_7^{2-}$ ions and 2 mol of NH_4^+ ions. (Water molecules are omitted from a molecular view of the solution for clarity.) 1 mol of ammonium dichromate is shown in a 1 liter volumetric flask. The resulting volumetric flask on the right contains 1 liter of solution after being dissolved with water. Powdered form of ammonium dichromate is also included in diagram.

When carrying out a chemical reaction using a solution of a salt such as ammonium dichromate, it is important to know the concentration of each ion present in the solution. If a solution contains 1.43 M $(NH_4)_2Cr_2O_7$, then the concentration of $Cr_2O_7^{2-}$

must also be 1.43 M because there is one $Cr_2O_7^{2-}$ ion per formula unit. However, there are two NH_4^+ ions per formula unit, so the concentration of NH_4^+ ions is 2 × 1.43 M = 2.86 M. Because each formula unit of $(NH_4)_2Cr_2O_7$ produces *three* ions when dissolved in water ($2NH_4^+ + 1Cr_2O_7^{2-}$), the *total* concentration of ions in the solution is 3 × 1.43 M = 4.29 M.

Concentration of Ions in Solution from a Soluble Salt: Concentration of Ions in Solution from a Soluble Salt, YouTube(opens in new window) [youtu.be]

Example 5.4.5

What are the concentrations of all species derived from the solutes in these aqueous solutions?

a. 0.21 M NaOH
b. 3.7 M $(CH_3)_2CHOH$
c. 0.032 M $In(NO_3)_3$

Given: molarity

Asked for: concentrations

Strategy:

A Classify each compound as either a strong electrolyte or a nonelectrolyte.

B If the compound is a nonelectrolyte, its concentration is the same as the molarity of the solution. If the compound is a strong electrolyte, determine the number of each ion contained in one formula unit. Find the concentration of each species by multiplying the number of each ion by the molarity of the solution.

Solution:

1. Sodium hydroxide is an ionic compound that is a strong electrolyte (and a strong base) in aqueous solution:

   $$NaOH(s) \xrightarrow{H_2O(l)} Na^+(aq) + OH^-(aq)$$

   B Because each formula unit of NaOH produces one Na^+ ion and one OH^- ion, the concentration of each ion is the same as the concentration of NaOH: $[Na^+]$ = 0.21 M and $[OH^-]$ = 0.21 M.

2. **A** The formula $(CH_3)_2CHOH$ represents 2-propanol (isopropyl alcohol) and contains the –OH group, so it is an alcohol. Recall from Section 4.1 that alcohols are covalent compounds that dissolve in water to give solutions of neutral molecules. Thus alcohols are nonelectrolytes.

   B The only solute species in solution is therefore $(CH_3)_2CHOH$ molecules, so $[(CH_3)_2CHOH]$ = 3.7 M.

3. **A** Indium nitrate is an ionic compound that contains In^{3+} ions and NO_3^- ions, so we expect it to behave like a strong electrolyte in aqueous solution:

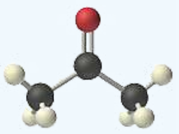
   $$In(NO_3)_3(s) \xrightarrow{H_2O(l)} In^{3+}(aq) + 3NO_3^-(aq)$$

   B One formula unit of $In(NO_3)_3$ produces one In^{3+} ion and three NO_3^- ions, so a 0.032 M $In(NO_3)_3$ solution contains 0.032 M In^{3+} and 3 × 0.032 M = 0.096 M NO_3^-—that is, $[In^{3+}]$ = 0.032 M and $[NO_3^-]$ = 0.096 M.

Exercise 5.4.5

What are the concentrations of all species derived from the solutes in these aqueous solutions?

a. 0.0012 M $Ba(OH)_2$
b. 0.17 M Na_2SO_4
c. 0.50 M $(CH_3)_2CO$, commonly known as acetone



Acetone
$(CH_3)_2CO$

Answer a

$$[Ba^{2+}] = 0.0012\ M;\ [OH^{-}] = 0.0024\ M$$

Answer b

$$[Na^{+}] = 0.34\ M;\ [SO_4^{2-}] = 0.17\ M$$

Answer c

$$[(CH_3)_2CO] = 0.50\ M$$

Summary

Solution concentrations are typically expressed as molarities and can be prepared by dissolving a known mass of solute in a solvent or diluting a stock solution.

- **definition of molarity:**

$$molarity = \frac{moles\ of\ solute}{liters\ of\ solution} = \frac{mmoles\ of\ solute}{milliliters\ of\ solution}$$

- **relationship among volume, molarity, and moles**:

$$V_L M_{mol/L} = \cancel{L}\left(\frac{mol}{\cancel{L}}\right) = moles$$

- **relationship between volume and concentration of stock and dilute solutions**:

$$(V_s)(M_s) = moles\ of\ solute = (V_d)(M_d)$$

The **concentration** of a substance is the quantity of solute present in a given quantity of solution. Concentrations are usually expressed in terms of **molarity**, defined as the number of moles of solute in 1 L of solution. Solutions of known concentration can be prepared either by dissolving a known mass of solute in a solvent and diluting to a desired final volume or by diluting the appropriate volume of a more concentrated solution (a **stock solution**) to the desired final volume.

Contributors and Attributions

- Modified by Joshua Halpern (Howard University)

5.5: Colligative Properties

Learning Objectives

- Express concentrations of solution components using mole fraction and molality
- Describe the effect of solute concentration on various solution properties (vapor pressure, boiling point, freezing point, and osmotic pressure)
- Perform calculations using the mathematical equations that describe these various colligative effects
- Describe the process of distillation and its practical applications
- Explain the process of osmosis and describe how it is applied industrially and in nature

The properties of a solution are different from those of either the pure solute(s) or solvent. Many solution properties are dependent upon the chemical identity of the solute. Compared to pure water, a solution of hydrogen chloride is more acidic, a solution of ammonia is more basic, a solution of sodium chloride is more dense, and a solution of sucrose is more viscous. There are a few solution properties, however, that depend *only* upon the total concentration of solute species, regardless of their identities. These colligative properties include vapor pressure lowering, boiling point elevation, freezing point depression, and osmotic pressure. This small set of properties is of central importance to many natural phenomena and technological applications, as will be described in this module.

Mole Fraction and Molality

Several units commonly used to express the concentrations of solution components were introduced in an earlier chapter of this text, each providing certain benefits for use in different applications. For example, molarity (M) is a convenient unit for use in stoichiometric calculations, since it is defined in terms of the molar amounts of solute species:

$$M = \frac{\text{mol solute}}{\text{L solution}} \tag{5.5.1}$$

Because solution volumes vary with temperature, molar concentrations will likewise vary. When expressed as molarity, the concentration of a solution with identical numbers of solute and solvent species will be different at different temperatures, due to the contraction/expansion of the solution. More appropriate for calculations involving many colligative properties are mole-based concentration units whose values are not dependent on temperature. Two such units are *mole fraction* (introduced in the previous chapter on gases) and *molality*.

The mole fraction, χ, of a component is the ratio of its molar amount to the total number of moles of all solution components:

$$\chi_\text{A} = \frac{\text{mol A}}{\text{total mol of all components}} \tag{5.5.2}$$

Molality is a concentration unit defined as the ratio of the numbers of moles of solute to the mass of the solvent in kilograms:

$$m = \frac{\text{mol solute}}{\text{kg solvent}} \tag{5.5.3}$$

Since these units are computed using only masses and molar amounts, they do not vary with temperature and, thus, are better suited for applications requiring temperature-independent concentrations, including several colligative properties, as will be described in this chapter module.

✓ Example 5.5.1: Calculating Mole Fraction and Molality

The antifreeze in most automobile radiators is a mixture of equal volumes of ethylene glycol and water, with minor amounts of other additives that prevent corrosion. What are the (a) mole fraction and (b) molality of ethylene glycol, $C_2H_4(OH)_2$, in a solution prepared from 2.22×10^3 g of ethylene glycol and 2.00×10^3 g of water (approximately 2 L of glycol and 2 L of water)?

Solution

(a) The mole fraction of ethylene glycol may be computed by first deriving molar amounts of both solution components and then substituting these amounts into the unit definition.

$$\text{mol } C_2H_4(OH)_2 = 2220 \text{ g} \times \frac{1 \text{ mol } C_2H_4(OH)_2}{62.07 \text{ g } C_2H_4(OH)_2} = 35.8 \text{ mol } C_2H_4(OH)_2$$

$$\text{mol } H_2O = 2000 \text{ g} \times \frac{1 \text{ mol } H_2O}{18.02 \text{ g } H_2O} = 111 \text{ mol } H_2O$$

$$\chi_{\text{ethylene glycol}} = \frac{35.8 \text{ mol } C_2H_4(OH)_2}{(35.8 + 111) \text{ mol total}} = 0.245$$

Notice that mole fraction is a dimensionless property, being the ratio of properties with identical units (moles).

(b) To find molality, we need to know the moles of the solute and the mass of the solvent (in kg).

First, use the given mass of ethylene glycol and its molar mass to find the moles of solute:

$$2220 \text{ g } C_2H_4(OH)_2 \left(\frac{\text{mol } C_2H_2(OH)_2}{62.07 \text{ g}} \right) = 35.8 \text{ mol } C_2H_4(OH)_2$$

Then, convert the mass of the water from grams to kilograms:

$$2000 \text{ g } H_2O \left(\frac{1 \text{ kg}}{1000 \text{ g}} \right) = 2 \text{ kg } H_2O$$

Finally, calculate molarity per its definition:

$$\begin{aligned} \text{molality} &= \frac{\text{mol solute}}{\text{kg solvent}} \\ \text{molality} &= \frac{35.8 \text{ mol } C_2H_4(OH)_2}{2 \text{ kg } H_2O} \\ \text{molality} &= 17.9\ m \end{aligned}$$

? Exercise 5.5.1

What are the mole fraction and molality of a solution that contains 0.850 g of ammonia, NH_3, dissolved in 125 g of water?

Answer

7.14×10^{-3}; 0.399 *m*

✓ Example 5.5.2: Converting Mole Fraction and Molal Concentrations

Calculate the mole fraction of solute and solvent in a 3.0 *m* solution of sodium chloride.

Solution

Converting from one concentration unit to another is accomplished by first comparing the two unit definitions. In this case, both units have the same numerator (moles of solute) but different denominators. The provided molal concentration may be written as:

$$\frac{3.0 \text{ mol NaCl}}{1.0 \text{ kg } H_2O}$$

The numerator for this solution's mole fraction is, therefore, 3.0 mol NaCl. The denominator may be computed by deriving the molar amount of water corresponding to 1.0 kg

$$1.0 \text{ kg } H_2O \left(\frac{1000 \text{ g}}{1 \text{ kg}} \right) \left(\frac{\text{mol } H_2O}{18.02 \text{ g}} \right) = 55 \text{ mol } H_2O$$

and then substituting these molar amounts into the definition for mole fraction.

$$\begin{aligned}
X_{\mathrm{H_2O}} &= \frac{\text{mol H}_2\text{O}}{\text{mol NaCl} + \text{mol H}_2\text{O}} \\
X_{\mathrm{H_2O}} &= \frac{55 \text{ mol H}_2\text{O}}{3.0 \text{ mol NaCl} + 55 \text{ mol H}_2\text{O}} \\
X_{\mathrm{H_2O}} &= 0.95 \\
X_{\mathrm{NaCl}} &= \frac{\text{mol NaCl}}{\text{mol NaCl} + \text{mol H}_2\text{O}} \\
X_{\mathrm{NaCl}} &= \frac{3.0 \text{ mol NaCl}}{3.0 \text{ mol NaCl} + 55 \text{ mol H}_2\text{O}} \\
X_{\mathrm{NaCl}} &= 0.052
\end{aligned}$$

? Exercise 5.5.2

The mole fraction of iodine, I_2, dissolved in dichloromethane, CH_2Cl_2, is 0.115. What is the molal concentration, *m*, of iodine in this solution?

Answer

1.50 *m*

Vapor Pressure Lowering

As described in the chapter on liquids and solids, the equilibrium vapor pressure of a liquid is the pressure exerted by its gaseous phase when vaporization and condensation are occurring at equal rates:

$$\text{liquid} \rightleftharpoons \text{gas} \tag{5.5.4}$$

Dissolving a nonvolatile substance in a volatile liquid results in a lowering of the liquid's vapor pressure. This phenomenon can be rationalized by considering the effect of added solute molecules on the liquid's vaporization and condensation processes. To vaporize, solvent molecules must be present at the surface of the solution. The presence of solute decreases the surface area available to solvent molecules and thereby reduces the rate of solvent vaporization. Since the rate of condensation is unaffected by the presence of solute, the net result is that the vaporization-condensation equilibrium is achieved with fewer solvent molecules in the vapor phase (i.e., at a lower vapor pressure) (Figure 5.5.1). While this kinetic interpretation is useful, it does not account for several important aspects of the colligative nature of vapor pressure lowering. A more rigorous explanation involves the property of *entropy*, a topic of discussion in a later text chapter on thermodynamics. For purposes of understanding the lowering of a liquid's vapor pressure, it is adequate to note that the greater entropy of a solution in comparison to its separate solvent and solute serves to effectively stabilize the solvent molecules and hinder their vaporization. A lower vapor pressure results, and a correspondingly higher boiling point as described in the next section of this module.

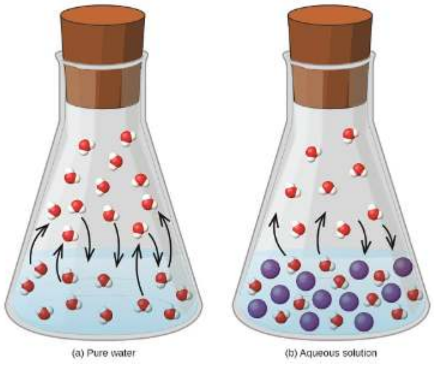


Figure 5.5.1: The presence of nonvolatile solutes lowers the vapor pressure of a solution by impeding the evaporation of solvent molecules.

This figure contains two images. Figure a is labeled "pure water." It shows a beaker half-filled with liquid. In the liquid, eleven molecules are evenly dispersed in the liquid each consisting of one central red sphere and two slightly smaller white spheres are shown. Four molecules near the surface of the liquid have curved arrows drawn from them pointing to the space above the liquid in the beaker. Above the liquid, twelve molecules are shown, with arrows pointing from three of them into the liquid below. Figure b is labeled “Aqueous solution.” It is similar to figure a except that eleven blue spheres, slightly larger in size than the molecules, are dispersed evenly in the liquid. Only four curved arrows appear in this diagram with two from the molecules in the liquid pointing to the space above and two from molecules in the space above the liquid pointing into the liquid below.

The relationship between the vapor pressures of solution components and the concentrations of those components is described by Raoult’s law: *The partial pressure exerted by any component of an ideal solution is equal to the vapor pressure of the pure component multiplied by its mole fraction in the solution.*

$$P_{\mathrm{A}} = X_{\mathrm{A}} P^{\circ}_{\mathrm{A}} \tag{5.5.5}$$

where P_{A} is the partial pressure exerted by component A in the solution, P°_{A} is the vapor pressure of pure A, and X_{A} is the mole fraction of A in the solution. (Mole fraction is a concentration unit introduced in the chapter on gases.)

Recalling that the total pressure of a gaseous mixture is equal to the sum of partial pressures for all its components (Dalton’s law of partial pressures), the total vapor pressure exerted by a solution containing *i* components is

$$P_{\text{solution}} = \sum_i P_i = \sum_i X_i P^{\circ}_i \tag{5.5.6}$$

A nonvolatile substance is one whose vapor pressure is negligible ($P° \approx 0$), and so the vapor pressure above a solution containing only nonvolatile solutes is due only to the solvent:

$$P_{\text{solution}} = X_{\text{solvent}} P^{\circ}_{\text{solvent}} \tag{5.5.7}$$

✓ Example 5.5.3: Calculation of a Vapor Pressure

Compute the vapor pressure of an ideal solution containing 92.1 g of glycerin, $C_3H_5(OH)_3$, and 184.4 g of ethanol, C_2H_5OH, at 40 °C. The vapor pressure of pure ethanol is 0.178 atm at 40 °C. Glycerin is essentially nonvolatile at this temperature.

Solution

Since the solvent is the only volatile component of this solution, its vapor pressure may be computed per Raoult’s law as:

$$P_{\text{solution}} = X_{\text{solvent}} P^{\circ}_{\text{solvent}}$$

First, calculate the molar amounts of each solution component using the provided mass data.

$$92.1 \text{ g } \cancel{\mathrm{C_3H_5(OH)_3}} \times \frac{1 \text{ mol } \mathrm{C_3H_5(OH)_3}}{92.094 \text{ g } \cancel{\mathrm{C_3H_5(OH)_3}}} = 1.00 \text{ mol } \mathrm{C_3H_5(OH)_3}$$

$$184.4 \text{ g } \cancel{\mathrm{C_2H_5OH}} \times \frac{1 \text{ mol } \mathrm{C_2H_5OH}}{46.069 \text{ g } \cancel{\mathrm{C_2H_5OH}}} = 4.000 \text{ mol } \mathrm{C_2H_5OH}$$

Next, calculate the mole fraction of the solvent (ethanol) and use Raoult’s law to compute the solution’s vapor pressure.

$$X_{\mathrm{C_2H_5OH}} = \frac{4.000 \text{ mol}}{(1.00 \text{ mol} + 4.000 \text{ mol})} = 0.800$$

$$P_{\text{solv}} = X_{\text{solv}} P^{\circ}_{\text{solv}} = 0.800 \times 0.178 \text{ atm} = 0.142 \text{ atm}$$

? Exercise 5.5.3

A solution contains 5.00 g of urea, $CO(NH_2)_2$ (a nonvolatile solute) and 0.100 kg of water. If the vapor pressure of pure water at 25 °C is 23.7 torr, what is the vapor pressure of the solution?

Answer

23.4 torr

Elevation of the Boiling Point of a Solvent

As described in the chapter on liquids and solids, the *boiling point* of a liquid is the temperature at which its vapor pressure is equal to ambient atmospheric pressure. Since the vapor pressure of a solution is lowered due to the presence of nonvolatile solutes, it stands to reason that the solution's boiling point will subsequently be increased. Compared to pure solvent, a solution, therefore, will require a higher temperature to achieve any given vapor pressure, including one equivalent to that of the surrounding atmosphere. The increase in boiling point observed when nonvolatile solute is dissolved in a solvent, ΔT_b, is called boiling point elevation and is directly proportional to the molal concentration of solute species:

$$\Delta T_b = K_b m \tag{5.5.8}$$

where

- K_b is the boiling point elevation constant, or the *ebullioscopic constant* and
- m is the molal concentration (molality) of all solute species.

Boiling point elevation constants are characteristic properties that depend on the identity of the solvent. Values of K_b for several solvents are listed in Table $5.5.1$.

Table $5.5.1$: Boiling Point Elevation and Freezing Point Depression Constants for Several Solvents

Solvent	Boiling Point (°C at 1 atm)	K_b (Cm^{-1})	Freezing Point (°C at 1 atm)	K_f (Cm^{-1})
water	100.0	0.512	0.0	1.86
hydrogen acetate	118.1	3.07	16.6	3.9
benzene	80.1	2.53	5.5	5.12
chloroform	61.26	3.63	−63.5	4.68
nitrobenzene	210.9	5.24	5.67	8.1

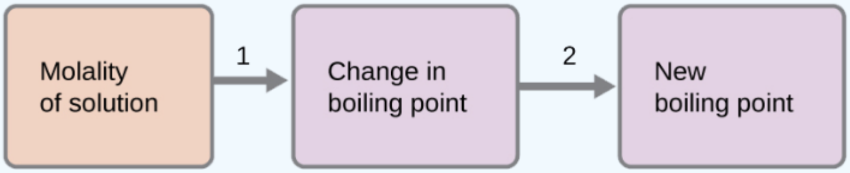
The extent to which the vapor pressure of a solvent is lowered and the boiling point is elevated depends on the total number of solute particles present in a given amount of solvent, not on the mass or size or chemical identities of the particles. A 1 *m* aqueous solution of sucrose (342 g/mol) and a 1 *m* aqueous solution of ethylene glycol (62 g/mol) will exhibit the same boiling point because each solution has one mole of solute particles (molecules) per kilogram of solvent.

Example $5.5.4$: Calculating the Boiling Point of a Solution

What is the boiling point of a 0.33 *m* solution of a nonvolatile solute in benzene?

Solution

Use the equation relating boiling point elevation to solute molality to solve this problem in two steps.



1. $\Delta T_\text{b} = K_\text{b} m = 2.53\ ^\circ\text{C}\ m^{-1} \times 0.33\ m = 0.83\ ^\circ\text{C}$

Add the boiling point elevation to the pure solvent's boiling point.

$\text{Boiling temperature} = 80.1\ ^\circ\text{C} + 0.83\ ^\circ\text{C} = 80.9\ ^\circ\text{C}$

? Exercise 5.5.4

What is the boiling point of the antifreeze described in Example 5.5.4?

Answer

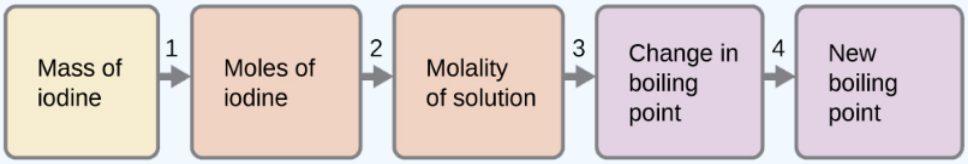
109.2 °C

✓ Example 5.5.5: The Boiling Point of an Iodine Solution

Find the boiling point of a solution of 92.1 g of iodine, I_2, in 800.0 g of chloroform, $CHCl_3$, assuming that the iodine is nonvolatile and that the solution is ideal.

Solution

We can solve this problem using four steps.



1. *Convert from grams to moles of I_2 using the molar mass of I_2 in the unit conversion factor.*

Result: 0.363 mol

Determine the molality of the solution from the number of moles of solute and the mass of solvent, in kilograms.

Result: 0.454 *m*

Use the direct proportionality between the change in boiling point and molal concentration to determine how much the boiling point changes.

Result: 1.65 °C

Determine the new boiling point from the boiling point of the pure solvent and the change.

Result: 62.91 °C

Check each result as a self-assessment.

? Exercise 5.5.5: glycerin:Water Solution

What is the boiling point of a solution of 1.0 g of glycerin, $C_3H_5(OH)_3$, in 47.8 g of water? Assume an ideal solution.

Answer

100.12 °C

Distillation of Solutions

Distillation is a technique for separating the components of mixtures that is widely applied in both in the laboratory and in industrial settings. It is used to refine petroleum, to isolate fermentation products, and to purify water. This separation technique involves the controlled heating of a sample mixture to selectively vaporize, condense, and collect one or more components of interest. A typical apparatus for laboratory-scale distillations is shown in Figure 5.5.2.

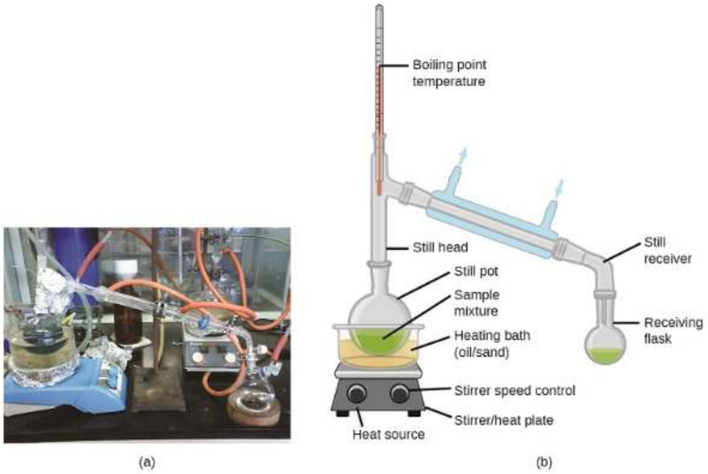


Figure 5.5.2: A typical laboratory distillation unit is shown in (a) a photograph and (b) a schematic diagram of the components. (credit a: modification of work by "Rifleman82"/Wikimedia commons; credit b: modification of work by "Slashme"/Wikimedia Commons)

Figure a contains a photograph of a common laboratory distillation unit. Figure b provides a diagram labeling typical components of a laboratory distillation unit, including a stirrer/heat plate with heat and stirrer speed control, a heating bath of oil or sand, stirring means such as boiling chips, a still pot, a still head, a thermometer for boiling point temperature reading, a condenser with a cool water inlet and outlet, a still receiver with a vacuum or gas inlet, a receiving flask for holding distillate, and a cooling bath.

Oil refineries use large-scale *fractional distillation* to separate the components of crude oil. The crude oil is heated to high temperatures at the base of a tall *fractionating column*, vaporizing many of the components that rise within the column. As vaporized components reach adequately cool zones during their ascent, they condense and are collected. The collected liquids are simpler mixtures of hydrocarbons and other petroleum compounds that are of appropriate composition for various applications (e.g., diesel fuel, kerosene, gasoline), as depicted in Figure 5.5.3.

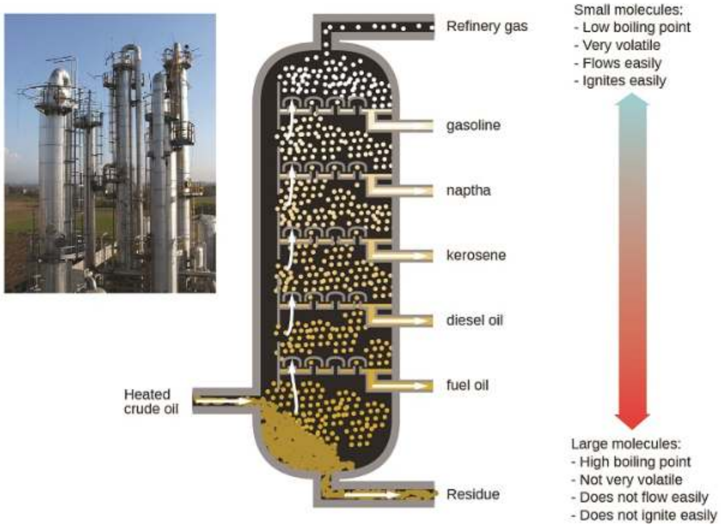


Figure 5.5.3: Crude oil is a complex mixture that is separated by large-scale fractional distillation to isolate various simpler mixtures.

This figure contains a photo of a refinery, showing large columnar structures. A diagram of a fractional distillation column used in separating crude oil is also shown. Near the bottom of the column, an arrow pointing into the column shows a point of entry for heated crude oil. The column contains several layers at which different components are removed. At the very bottom, residue materials are removed as indicated by an arrow out of the column. At each successive level, different materials are removed proceeding from the bottom to the top of the column. The materials are fuel oil, followed by diesel oil, kerosene, naptha, gasoline, and refinery gas at the very top. To the right of the column diagram, a double sided arrow is shown that is blue at the top and gradually changes color to red moving downward. The blue top of the arrow is labeled, “small molecules: low boiling point, very volatile, flows easily, ignites easily.” The red bottom of the arrow is labeled, “large molecules: high boiling point, not very volatile, does not flow easily, does not ignite easily.”

Depression of the Freezing Point of a Solvent

Solutions freeze at lower temperatures than pure liquids. This phenomenon is exploited in “de-icing” schemes that use salt (Figure 5.5.4), calcium chloride, or urea to melt ice on roads and sidewalks, and in the use of ethylene glycol as an “antifreeze” in automobile radiators. Seawater freezes at a lower temperature than fresh water, and so the Arctic and Antarctic oceans remain unfrozen even at temperatures below 0 °C (as do the body fluids of fish and other cold-blooded sea animals that live in these oceans).

Figure 5.5.4: Rock salt (NaCl), calcium chloride ($CaCl_2$), or a mixture of the two are used to melt ice. (credit: modification of work by Eddie Welker)

The decrease in freezing point of a dilute solution compared to that of the pure solvent, ΔT_f, is called the freezing point depression and is directly proportional to the molal concentration of the solute

$$\Delta T_\text{f} = K_\text{f} m \tag{5.5.9}$$

where

- m is the *molal* concentration of the solute in the solvent and
- K_f is called the freezing point depression constant (or *cryoscopic constant*).

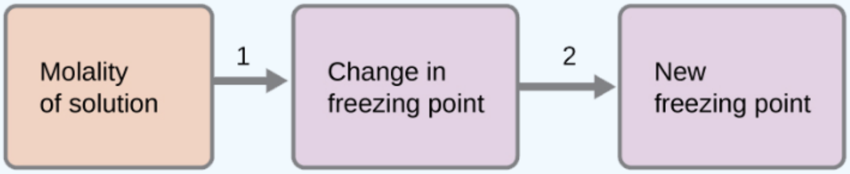
Just as for boiling point elevation constants, these are characteristic properties whose values depend on the chemical identity of the solvent. Values of K_f for several solvents are listed in Table 5.5.1.

✓ Example 5.5.5: Calculation of the Freezing Point of a Solution

What is the freezing point of the 0.33 *m* solution of a nonvolatile nonelectrolyte solute in benzene described in Example 5.5.4?

Solution

Use the equation relating freezing point depression to solute molality to solve this problem in two steps.



1. *Calculate the change in freezing point.*

$$\Delta T_\text{f} = K_\text{f} m = 5.12\ ^\circ\text{C}\ m^{-1} \times 0.33\ m = 1.7\ ^\circ\text{C}$$

2. *Subtract the freezing point change observed from the pure solvent’s freezing point.*

$$\text{Freezing Temperature} = 5.5\ ^\circ\text{C} - 1.7\ ^\circ\text{C} = 3.8\ ^\circ\text{C}$$

Exercise 5.5.6

What is the freezing point of a 1.85 *m* solution of a nonvolatile nonelectrolyte solute in nitrobenzene?

Answer

−9.3 °C

Colligative Properties and De-Icing

Sodium chloride and its group 2 analogs calcium and magnesium chloride are often used to de-ice roadways and sidewalks, due to the fact that a solution of any one of these salts will have a freezing point lower than 0 °C, the freezing point of pure water. The group 2 metal salts are frequently mixed with the cheaper and more readily available sodium chloride ("rock salt") for use on roads, since they tend to be somewhat less corrosive than the NaCl, and they provide a larger depression of the freezing point, since they dissociate to yield three particles per formula unit, rather than two particles like the sodium chloride.

Because these ionic compounds tend to hasten the corrosion of metal, they would not be a wise choice to use in antifreeze for the radiator in your car or to de-ice a plane prior to takeoff. For these applications, covalent compounds, such as ethylene or propylene glycol, are often used. The glycols used in radiator fluid not only lower the freezing point of the liquid, but they elevate the boiling point, making the fluid useful in both winter and summer. Heated glycols are often sprayed onto the surface of airplanes prior to takeoff in inclement weather in the winter to remove ice that has already formed and prevent the formation of more ice, which would be particularly dangerous if formed on the control surfaces of the aircraft (Video 5.5.1).

Phase Diagram for an Aqueous Solution of a Nonelectrolyte

The colligative effects on vapor pressure, boiling point, and freezing point described in the previous section are conveniently summarized by comparing the phase diagrams for a pure liquid and a solution derived from that liquid. Phase diagrams for water and an aqueous solution are shown in Figure 5.5.5.

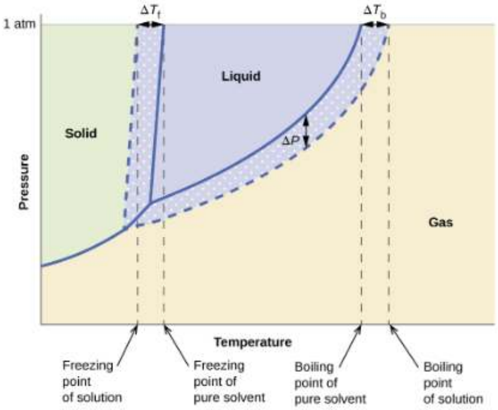


Figure 5.5.5: These phase diagrams show water (solid curves) and an aqueous solution of nonelectrolyte (dashed curves).

This phase diagram indicates the pressure in atmospheres of water and a solution at various temperatures. The graph shows the freezing point of water and the freezing point of the solution, with the difference between these two values identified as delta T subscript f. The graph shows the boiling point of water and the boiling point of the solution, with the difference between these two values identified as delta T subscript b. Similarly, the difference in the pressure of water and the solution at the boiling point of water is shown and identified as delta P. This difference in pressure is labeled vapor pressure lowering. The lower level of the vapor pressure curve for the solution as opposed to that of pure water shows vapor pressure lowering in the solution. Background colors on the diagram indicate the presence of water and the solution in the solid state to the left, liquid state in the central upper region, and gas to the right.

The liquid-vapor curve for the solution is located *beneath* the corresponding curve for the solvent, depicting the vapor pressure *lowering*, ΔP, that results from the dissolution of nonvolatile solute. Consequently, at any given pressure, the solution's boiling point is observed at a higher temperature than that for the pure solvent, reflecting the boiling point elevation, ΔT_b, associated with the presence of nonvolatile solute. The solid-liquid curve for the solution is displaced left of that for the pure solvent, representing the freezing point depression, ΔTf, that accompanies solution formation. Finally, notice that the solid-gas curves for the solvent and its solution are identical. This is the case for many solutions comprising liquid solvents and nonvolatile solutes. Just as for vaporization, when a solution of this sort is frozen, it is actually just the *solvent* molecules that undergo the liquid-to-solid transition, forming pure solid solvent that excludes solute species. The solid and gaseous phases, therefore, are composed solvent only, and so transitions between these phases are not subject to colligative effects.

Osmosis and Osmotic Pressure of Solutions

A number of natural and synthetic materials exhibit *selective permeation*, meaning that only molecules or ions of a certain size, shape, polarity, charge, and so forth, are capable of passing through (permeating) the material. Biological cell membranes provide elegant examples of selective permeation in nature, while dialysis tubing used to remove metabolic wastes from blood is a more simplistic technological example. Regardless of how they may be fabricated, these materials are generally referred to as semipermeable membranes.

Consider the apparatus illustrated in Figure 5.5.6, in which samples of pure solvent and a solution are separated by a membrane that only solvent molecules may permeate. Solvent molecules will diffuse across the membrane in both directions. Since the concentration of *solvent* is greater in the pure solvent than the solution, these molecules will diffuse from the solvent side of the membrane to the solution side at a faster rate than they will in the reverse direction. The result is a net transfer of solvent molecules from the pure solvent to the solution. Diffusion-driven transfer of solvent molecules through a semipermeable membrane is a process known as osmosis.

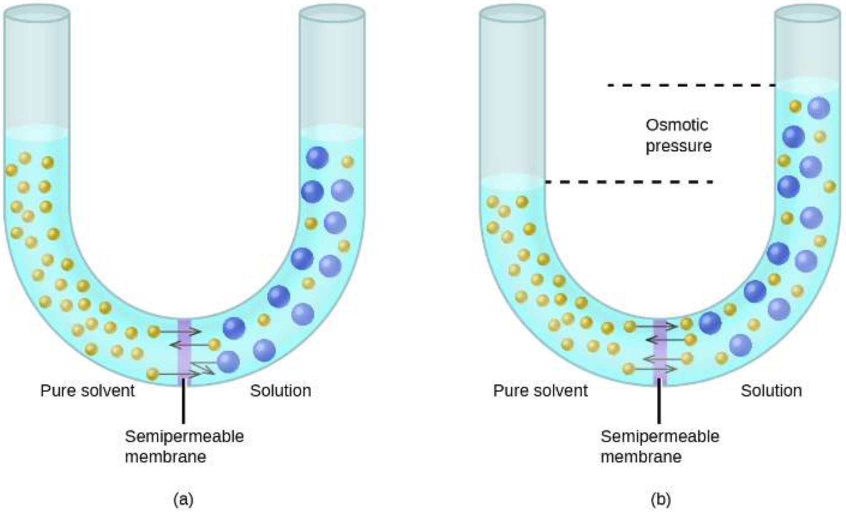


Figure 5.5.6: Osmosis results in the transfer of solvent molecules from a sample of low (or zero) solute concentration to a sample of higher solute concentration.

The figure shows two U shaped tubes with a semi permeable membrane placed at the base of the U. In figure a, pure solvent is present and indicated by small yellow spheres to the left of the membrane. To the right, a solution exists with larger blue spheres intermingled with some small yellow spheres. At the membrane, arrows pointing from three small yellow spheres on both sides of the membrane cross over the membrane. An arrow drawn from one of the large blue spheres does not cross the membrane, but rather is reflected back from the surface of the membrane. The levels of liquid in both sides of the U shaped tube are equal. In figure b, arrows again point from small yellow spheres across the semipermeable membrane from both sides. This diagram shows the level of liquid in the left, pure solvent, side to be significantly lower than the liquid level on the right. Dashed lines are drawn from these two liquid levels into the middle of the U-shaped tube and between them is the term osmotic pressure.

When osmosis is carried out in an apparatus like that shown in Figure $5.5.6$, the volume of the solution increases as it becomes diluted by accumulation of solvent. This causes the level of the solution to rise, increasing its hydrostatic pressure (due to the weight of the column of solution in the tube) and resulting in a faster transfer of solvent molecules back to the pure solvent side. When the pressure reaches a value that yields a reverse solvent transfer rate equal to the osmosis rate, bulk transfer of solvent ceases. This pressure is called the osmotic pressure (Π) of the solution. The osmotic pressure of a dilute solution is related to its solute molarity, *M*, and absolute temperature, *T*, according to the equation

$$\Pi = MRT \tag{5.5.10}$$

where R is the universal gas constant.

✓ Example $5.5.7$: Calculation of Osmotic Pressure

What is the osmotic pressure (atm) of a 0.30 *M* solution of glucose in water that is used for intravenous infusion at body temperature, 37 °C?

Solution

We can find the osmotic pressure, \(*Π*\), using Equation 5.5.10, where *T* is on the Kelvin scale (310 K) and the value of *R* is expressed in appropriate units (0.08206 L atm/mol K).

$$\begin{aligned} \Pi &= MRT \\ &= 0.03 \text{ mol/L} \times 0.08206 \text{ L atm/mol K} \times 310 \text{ K} \\ &= 7.6 \text{ atm} \end{aligned}$$

? Exercise $5.5.7$

What is the osmotic pressure (atm) a solution with a volume of 0.750 L that contains 5.0 g of methanol, CH_3OH, in water at 37 °C?

Answer

5.3 atm

If a solution is placed in an apparatus like the one shown in Figure $5.5.7$, applying pressure greater than the osmotic pressure of the solution reverses the osmosis and pushes solvent molecules from the solution into the pure solvent. This technique of reverse osmosis is used for large-scale desalination of seawater and on smaller scales to produce high-purity tap water for drinking.

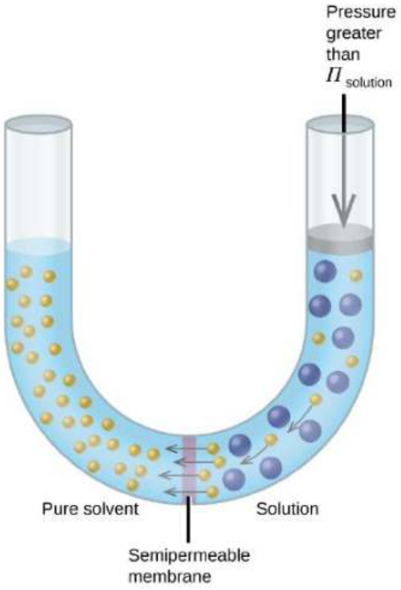


Figure 5.5.7: Applying a pressure greater than the osmotic pressure of a solution will reverse osmosis. Solvent molecules from the solution are pushed into the pure solvent.

The figure shows a U shaped tube with a semi permeable membrane placed at the base of the U. Pure solvent is present and indicated by small yellow spheres to the left of the membrane. To the right, a solution exists with larger blue spheres intermingled with some small yellow spheres. At the membrane, arrows point from four small yellow spheres to the left of the membrane. On the right side of the U, there is a disk that is the same width of the tube and appears to block it. The disk is at the same level as the solution. An arrow points down from the top of the tube to the disk and is labeled, “Pressure greater than Π subscript solution.”

Examples of osmosis are evident in many biological systems because cells are surrounded by semipermeable membranes. Carrots and celery that have become limp because they have lost water can be made crisp again by placing them in water. Water moves into the carrot or celery cells by osmosis. A cucumber placed in a concentrated salt solution loses water by osmosis and absorbs some salt to become a pickle. Osmosis can also affect animal cells. Solute concentrations are particularly important when solutions are injected into the body. Solutes in body cell fluids and blood serum give these solutions an osmotic pressure of approximately 7.7 atm. Solutions injected into the body must have the same osmotic pressure as blood serum; that is, they should be isotonic with blood serum. If a less concentrated solution, a hypotonic solution, is injected in sufficient quantity to dilute the blood serum, water from the diluted serum passes into the blood cells by osmosis, causing the cells to expand and rupture. This process is called hemolysis. When a more concentrated solution, a hypertonic solution, is injected, the cells lose water to the more concentrated solution, shrivel, and possibly die in a process called *crenation* (Figure 11.5.8).

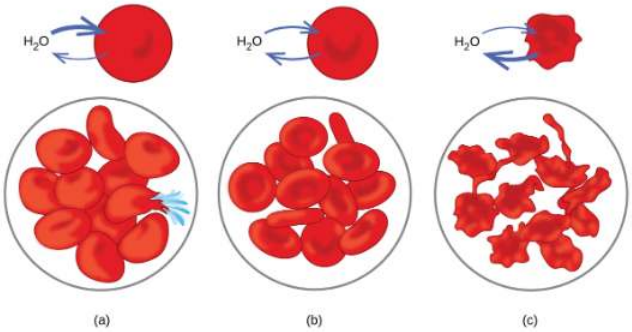


Figure 5.5.8: Red blood cell membranes are water permeable and will (a) swell and possibly rupture in a hypotonic solution; (b) maintain normal volume and shape in an isotonic solution; and (c) shrivel and possibly die in a hypertonic solution. (credit a/b/c: modifications of work by “LadyofHats”/Wikimedia commons)

This figure shows three scenarios relating to red blood cell membranes. In a, H subscript 2 O has two arrows drawn from it pointing into a red disk. Beneath it in a circle are eleven similar disks with a bulging appearance, one of which appears to have burst with blue liquid erupting from it. In b, the image is similar except that rather than having two arrows pointing into the red disk, one points in and a second points out toward the H subscript 2 O. In the circle beneath, twelve of the red disks are present. In c, both arrows are drawn from a red shriveled disk toward the H subscript 2 O. In the circle below, twelve shriveled disks are shown.

Determination of Molar Masses

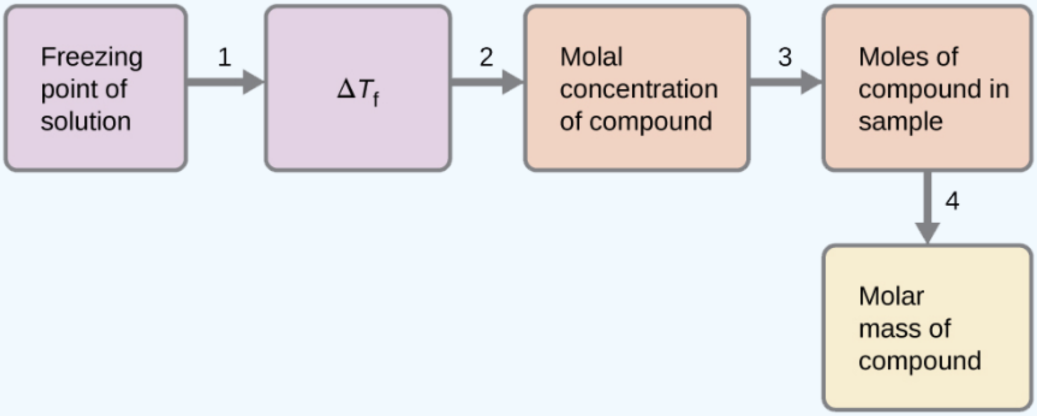
Osmotic pressure and changes in freezing point, boiling point, and vapor pressure are directly proportional to the concentration of solute present. Consequently, we can use a measurement of one of these properties to determine the molar mass of the solute from the measurements.

✓ Example 5.5.8: Determining Molar Mass from Freezing Point Depression

A solution of 4.00 g of a nonelectrolyte dissolved in 55.0 g of benzene is found to freeze at 2.32 °C. What is the molar mass of this compound?

Solution

We can solve this problem using the following steps.



1. *Determine the change in freezing point from the observed freezing point and the freezing point of pure benzene* (Table 11.5.1).

$\Delta T_\text{f} = 5.5\ ^\circ\text{C} - 2.32\ ^\circ\text{C} = 3.2\ ^\circ\text{C}$

2. *Determine the molal concentration from* K_f, *the freezing point depression constant for benzene* (Table 11.5.1), *and* ΔT_f.

\(ΔT_\ce{f}=K_\ce{f}m\)

$$m = \frac{\Delta T_\text{f}}{K_\text{f}} = \frac{3.2\ ^\circ\text{C}}{5.12\ ^\circ\text{C}m^{-1}} = 0.63\ m$$

3. Determine the number of moles of compound in the solution from the molal concentration and the mass of solvent used to make the solution.

$$\text{Moles of solute} = \frac{0.62\text{ mol solute}}{1.00\text{ kg }\cancel{\text{solvent}}} \times 0.0550\text{ kg }\cancel{\text{solvent}} = 0.035\text{ mol}$$

4. Determine the molar mass from the mass of the solute and the number of moles in that mass.

\(\mathrm{Molar\: mass=\dfrac{4.00\:g}{0.034\:mol}=1.2×10^2\:g/mol}\)

? Exercise 5.5.8

A solution of 35.7 g of a nonelectrolyte in 220.0 g of chloroform has a boiling point of 64.5 °C. What is the molar mass of this compound?

Answer

1.8×10^2 g/mol

Example 5.5.9: Determination of a Molar Mass from Osmotic Pressure

A 0.500 L sample of an aqueous solution containing 10.0 g of hemoglobin has an osmotic pressure of 5.9 torr at 22 °C. What is the molar mass of hemoglobin?

Solution

Here is one set of steps that can be used to solve the problem:

1.
$$\Pi = \frac{5.9 \text{ torr} \times 1 \text{ atm}}{760 \text{ torr}} = 7.8 \times 10^{-3} \text{ atm}$$

$$\Pi = MRT$$

$$M = \frac{\Pi}{RT} = \frac{7.8 \times 10^{-3} \text{ atm}}{(0.08206 \text{ L atm/mol K})(295 \text{ K})} = 3.2 \times 10^{-4} \text{ M}$$

1. $\text{moles of hemoglobin} = \frac{3.2 \times 10^{-4} \text{ mol}}{1 \text{ L } \cancel{\text{solution}}} \times 0.500 \text{ L } \cancel{\text{solution}} = 1.6 \times 10^{-4} \text{ mol}$
2. *Determine the molar mass from the mass of hemoglobin and the number of moles in that mass.*

$$\text{molar mass} = \frac{10.0 \text{ g}}{1.6 \times 10^{-4} \text{ mol}} = 6.2 \times 10^{4} \text{ g/mol}$$

Exercise 5.5.9

What is the molar mass of a protein if a solution of 0.02 g of the protein in 25.0 mL of solution has an osmotic pressure of 0.56 torr at 25 °C?

Answer

2.7×10^4 g/mol

Colligative Properties of Electrolytes

As noted previously in this module, the colligative properties of a solution depend only on the number, not on the kind, of solute species dissolved. For example, 1 mole of any nonelectrolyte dissolved in 1 kilogram of solvent produces the same lowering of the freezing point as does 1 mole of any other nonelectrolyte. However, 1 mole of sodium chloride (an electrolyte) forms *2 moles* of ions when dissolved in solution. Each individual ion produces the same effect on the freezing point as a single molecule does.

Example 5.5.10: The Freezing Point of a Solution of an Electrolyte

The concentration of ions in seawater is approximately the same as that in a solution containing 4.2 g of NaCl dissolved in 125 g of water. Assume that each of the ions in the NaCl solution has the same effect on the freezing point of water as a nonelectrolyte molecule, and determine the freezing temperature the solution (which is approximately equal to the freezing temperature of seawater).

Solution

We can solve this problem using the following series of steps.

- Convert from grams to moles of NaCl using the molar mass of NaCl in the unit conversion factor. Result: 0.072 mol NaCl
- Determine the number of moles of ions present in the solution using the number of moles of ions in 1 mole of NaCl as the conversion factor (2 mol ions/1 mol NaCl). Result: 0.14 mol ions
- Determine the molality of the ions in the solution from the number of moles of ions and the mass of solvent, in kilograms. Result: 1.1 m
- Use the direct proportionality between the change in freezing point and molal concentration to determine how much the freezing point changes. Result: 2.0 °C
- Determine the new freezing point from the freezing point of the pure solvent and the change. Result: −2.0 °C

Check each result as a self-assessment.

? Exercise 5.5.10

Assume that each of the ions in calcium chloride, $CaCl_2$, has the same effect on the freezing point of water as a nonelectrolyte molecule. Calculate the freezing point of a solution of 0.724 g of $CaCl_2$ in 175 g of water.

Answer

−0.208 °C

Assuming complete dissociation, a 1.0 *m* aqueous solution of NaCl contains 2.0 mole of ions (1.0 mol Na^+ and 1.0 mol Cl^-) per each kilogram of water, and its freezing point depression is expected to be

$$\Delta T_\mathrm{f} = 2.0 \text{ mol ions/kg water} \times 1.86\ ^\circ\text{C kg water/mol ion} = 3.7\ ^\circ\text{C}. \tag{5.5.11}$$

When this solution is actually prepared and its freezing point depression measured, however, a value of 3.4 °C is obtained. Similar discrepancies are observed for other ionic compounds, and the differences between the measured and expected colligative property values typically become more significant as solute concentrations increase. These observations suggest that the ions of sodium chloride (and other strong electrolytes) are not completely dissociated in solution.

To account for this and avoid the errors accompanying the assumption of total dissociation, an experimentally measured parameter named in honor of Nobel Prize-winning German chemist Jacobus Henricus van't Hoff is used. The van't Hoff factor (*i*) is defined as the ratio of solute particles in solution to the number of formula units dissolved:

$$i = \frac{\text{moles of particles in solution}}{\text{moles of formula units dissolved}} \tag{5.5.12}$$

Values for measured van't Hoff factors for several solutes, along with predicted values assuming complete dissociation, are shown in Table 5.5.2.

Table 5.5.2: Expected and Observed van't Hoff Factors for Several 0.050 m Aqueous Electrolyte Solutions

Electrolyte	Particles in Solution	*i* (Predicted)	*i* (Measured)
HCl	H^+, Cl^-	2	1.9
NaCl	Na^+, Cl^-	2	1.9
$MgSO_4$	Mg^{2+}, SO_4^{2-}	2	1.3
$MgCl_2$	Mg^{2+}, $2Cl^-$	3	2.7
$FeCl_3$	Fe^{3+}, $3Cl^-$	4	3.4
glucose (a non-electrolyte)	$C_{12}H_{22}O_{11}$	1	1.0

In 1923, the chemists Peter Debye and Erich Hückel proposed a theory to explain the apparent incomplete ionization of strong electrolytes. They suggested that although interionic attraction in an aqueous solution is very greatly reduced by solvation of the ions and the insulating action of the polar solvent, it is not completely nullified. The residual attractions prevent the ions from behaving as totally independent particles (Figure 5.5.9). In some cases, a positive and negative ion may actually touch, giving a solvated unit called an ion pair. Thus, the activity, or the effective concentration, of any particular kind of ion is less than that indicated by the actual concentration. Ions become more and more widely separated the more dilute the solution, and the residual interionic attractions become less and less. Thus, in extremely dilute solutions, the effective concentrations of the ions (their activities) are essentially equal to the actual concentrations. Note that the van't Hoff factors for the electrolytes in Table $5.5.2$ are for 0.05 *m* solutions, at which concentration the value of *i* for NaCl is 1.9, as opposed to an ideal value of 2.

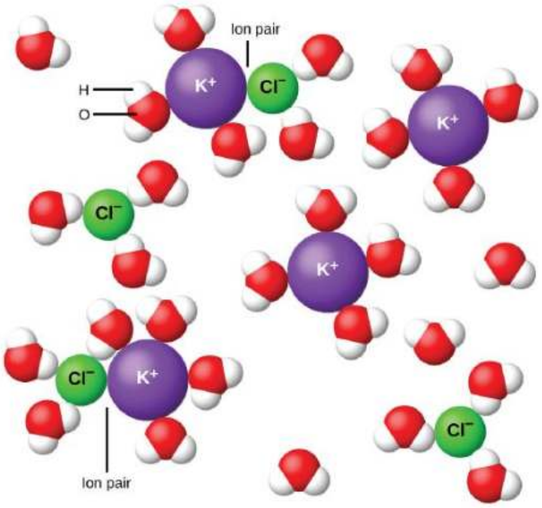


Figure $5.5.9$: Ions become more and more widely separated the more dilute the solution, and the residual interionic attractions become less.

The diagram shows four purple spheres labeled K superscript plus and four green spheres labeled C l superscript minus dispersed in H subscript 2 O as shown by clusters of single red spheres with two white spheres attached. Red spheres represent oxygen and white represent hydrogen. In two locations, the purple and green spheres are touching. In these two locations, the diagram is labeled ion pair. All red and green spheres are surrounded by the white and red H subscript 2 O clusters. The white spheres are attracted to the purple spheres and the red spheres are attracted to the green spheres.

Summary

Properties of a solution that depend only on the concentration of solute particles are called colligative properties. They include changes in the vapor pressure, boiling point, and freezing point of the solvent in the solution. The magnitudes of these properties depend only on the total concentration of solute particles in solution, not on the type of particles. The total concentration of solute particles in a solution also determines its osmotic pressure. This is the pressure that must be applied to the solution to prevent diffusion of molecules of pure solvent through a semipermeable membrane into the solution. Ionic compounds may not completely dissociate in solution due to activity effects, in which case observed colligative effects may be less than predicted.

Key Equations

- $\left(P_{\mathrm{A}}=X_{\mathrm{A}} P_{\mathrm{A}}^{\circ}\right)$
- $P_{\text{solution}}=\sum_i P_i=\sum_i X_i P_i^{\circ}$
- $P_{\text{solution}}=X_{\text{solvent}} P_{\text{solvent}}^{\circ}$
- $\Delta T_{\mathrm{b}} = K_{\mathrm{b}} m$
- $\Delta T_{\mathrm{f}} = K_{\mathrm{f}} m$
- $\Pi = MRT$

Footnotes

1. A nonelectrolyte shown for comparison.

Glossary

boiling point elevation

elevation of the boiling point of a liquid by addition of a solute

boiling point elevation constant

the proportionality constant in the equation relating boiling point elevation to solute molality; also known as the ebullioscopic constant

colligative property

property of a solution that depends only on the concentration of a solute species

crenation

process whereby biological cells become shriveled due to loss of water by osmosis

freezing point depression

lowering of the freezing point of a liquid by addition of a solute

freezing point depression constant

(also, cryoscopic constant) proportionality constant in the equation relating freezing point depression to solute molality

hemolysis

rupture of red blood cells due to the accumulation of excess water by osmosis

hypertonic

of greater osmotic pressure

hypotonic

of less osmotic pressure

ion pair

solvated anion/cation pair held together by moderate electrostatic attraction

isotonic

of equal osmotic pressure

molality (m)

a concentration unit defined as the ratio of the numbers of moles of solute to the mass of the solvent in kilograms

osmosis

diffusion of solvent molecules through a semipermeable membrane

osmotic pressure (Π)

opposing pressure required to prevent bulk transfer of solvent molecules through a semipermeable membrane

Raoult's law

the partial pressure exerted by a solution component is equal to the product of the component's mole fraction in the solution and its equilibrium vapor pressure in the pure state

semipermeable membrane

a membrane that selectively permits passage of certain ions or molecules

van't Hoff factor (i)

the ratio of the number of moles of particles in a solution to the number of moles of formula units dissolved in the solution

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5.6: Formula Mass and the Mole Concept

Learning Objectives

- Calculate formula masses for covalent and ionic compounds
- Define the amount unit mole and the related quantity Avogadro’s number
- Explain the relation between mass, moles, and numbers of atoms or molecules, and perform calculations deriving these quantities from one another

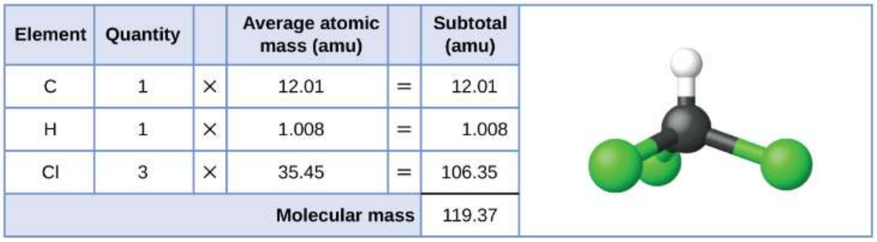
We can argue that modern chemical science began when scientists started exploring the quantitative as well as the qualitative aspects of chemistry. For example, Dalton’s atomic theory was an attempt to explain the results of measurements that allowed him to calculate the relative masses of elements combined in various compounds. Understanding the relationship between the masses of atoms and the chemical formulas of compounds allows us to quantitatively describe the composition of substances.

Formula Mass

In an earlier chapter, we described the development of the atomic mass unit, the concept of average atomic masses, and the use of chemical formulas to represent the elemental makeup of substances. These ideas can be extended to calculate the formula mass of a substance by summing the average atomic masses of all the atoms represented in the substance’s formula.

Formula Mass for Covalent Substances

For covalent substances, the formula represents the numbers and types of atoms composing a single molecule of the substance; therefore, the formula mass may be correctly referred to as a molecular mass. Consider chloroform ($CHCl_3$), a covalent compound once used as a surgical anesthetic and now primarily used in the production of tetrafluoroethylene, the building block for the “anti-stick” polymer, Teflon. The molecular formula of chloroform indicates that a single molecule contains one carbon atom, one hydrogen atom, and three chlorine atoms. The average molecular mass of a chloroform molecule is therefore equal to the sum of the average atomic masses of these atoms. Figure 5.6.1 outlines the calculations used to derive the molecular mass of chloroform, which is 119.37 amu.

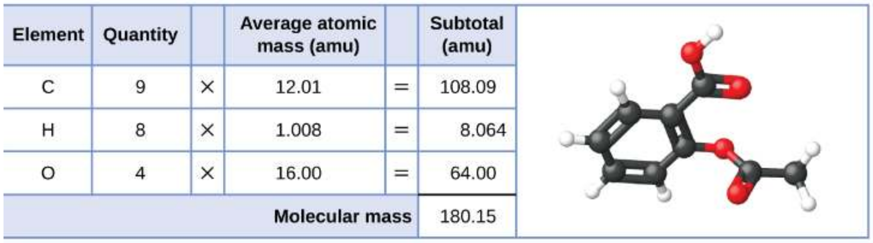


Element	Quantity		Average atomic mass (amu)		Subtotal (amu)
C	1	×	12.01	=	12.01
H	1	×	1.008	=	1.008
Cl	3	×	35.45	=	106.35
				Molecular mass	119.37

Figure 5.6.1: The average mass of a chloroform molecule, $CHCl_3$, is 119.37 amu, which is the sum of the average atomic masses of each of its constituent atoms. The model shows the molecular structure of chloroform.

A table and diagram are shown. The table is made up of six columns and five rows. The header row reads: “Element,” “Quantity,” a blank space, “Average atomic mass (a m u),” a blank space, and “Subtotal (a m u).” The first column contains the symbols “C,” “H,” “C l” and a blank, merged cell that runs the width of the first five columns. The second column contains the numbers “1,” “1,” and “3” as well as the merged cell. The third column contains the multiplication symbol in each cell except for the last, merged cell. The fourth column contains the numbers “12.01,” “1.008,” and “35.45” as well as the merged cell. The fifth column contains the symbol “=” in each cell except for the last, merged cell. The sixth column contains the values “12.01,” “1.008,” “106.35,” and “119.37.” There is a thick black line below the number 106.35. The merged cell under the first five columns reads “Molecular mass.” To the left of the table is a diagram of a molecule. Three green spheres are attached to a slightly smaller black sphere, which is also attached to a smaller white sphere. The green spheres lie beneath and to the sides of the black sphere while the white sphere is located straight up from the black sphere.

Likewise, the molecular mass of an aspirin molecule, $C_9H_8O_4$, is the sum of the atomic masses of nine carbon atoms, eight hydrogen atoms, and four oxygen atoms, which amounts to 180.15 amu (Figure 5.6.2).



Element	Quantity		Average atomic mass (amu)		Subtotal (amu)
C	9	×	12.01	=	108.09
H	8	×	1.008	=	8.064
O	4	×	16.00	=	64.00
				Molecular mass	180.15

Figure 5.6.2: The average mass of an aspirin molecule is 180.15 amu. The model shows the molecular structure of aspirin, $C_9H_8O_4$.

A table and diagram are shown.

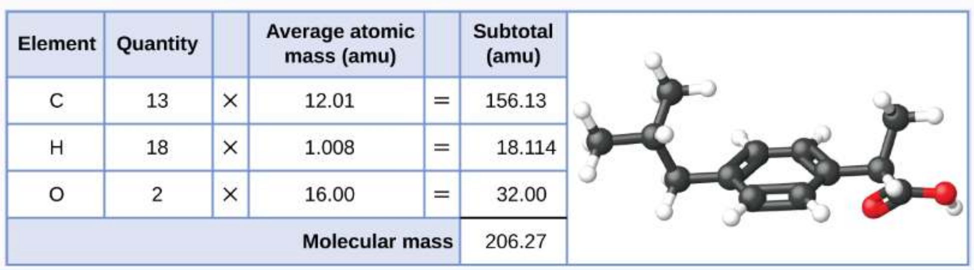
The table is made up of six columns and five rows. The header row reads: “Element,” “Quantity,” a blank space, “Average atomic mass (a m u),” a blank space, and “Subtotal (a m u).” The first column contains the symbols “C,” “H,” “O,” and a merged cell. The merged cell runs the length of the first five columns. The second column contains the numbers “9,” “8,” and “4” as well as the merged, cell. The third column contains the multiplication symbol in each cell except for the last, merged cell. The fourth column contains the numbers “12.01,” “1.008,” and “16.00” as well as the merged cell. The fifth column contains the symbol “=” in each cell except for the last, merged cell. The sixth column contains the values: “108.09,” “8.064,” “64.00,” and “180.15.” There is a thick black line below the number 64.00. The merged cell under the first five columns reads “Molecular mass.” To the left of the table is a diagram of a molecule. Six black spheres are located in a six-sided ring and connected by alternating double and single black bonds. Attached to each of the four black spheres is one smaller white sphere. Attached to the farthest right black sphere is a red sphere, connected to two more black spheres, all in a row. Attached to the last black sphere of that row are two more white spheres. Attached to the first black sphere of that row is another red sphere. A black sphere, attached to two red spheres and a white sphere is attached to the black sphere on the top right of the six-sided ring.

✓ Example 5.6.1: Computing Molecular Mass for a Covalent Compound

Ibuprofen, $C_{13}H_{18}O_2$, is a covalent compound and the active ingredient in several popular nonprescription pain medications, such as Advil and Motrin. What is the molecular mass (amu) for this compound?

Solution

Molecules of this compound are comprised of 13 carbon atoms, 18 hydrogen atoms, and 2 oxygen atoms. Following the approach described above, the average molecular mass for this compound is therefore:



Element	Quantity		Average atomic mass (amu)		Subtotal (amu)
C	13	×	12.01	=	156.13
H	18	×	1.008	=	18.114
O	2	×	16.00	=	32.00
				Molecular mass	206.27

? Exercise 5.6.1

Acetaminophen, $C_8H_9NO_2$, is a covalent compound and the active ingredient in several popular nonprescription pain medications, such as Tylenol. What is the molecular mass (amu) for this compound?

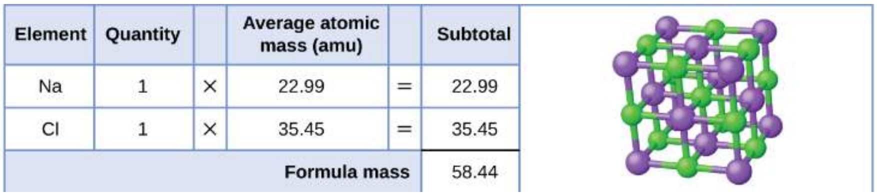
Answer

151.16 amu

Formula Mass for Ionic Compounds

Ionic compounds are composed of discrete cations and anions combined in ratios to yield electrically neutral bulk matter. The formula mass for an ionic compound is calculated in the same way as the formula mass for covalent compounds: by summing the average atomic masses of all the atoms in the compound's formula. Keep in mind, however, that the formula for an ionic compound does not represent the composition of a discrete molecule, so it may not correctly be referred to as the "molecular mass."

As an example, consider sodium chloride, NaCl, the chemical name for common table salt. Sodium chloride is an ionic compound composed of sodium cations, Na^+, and chloride anions, Cl^-, combined in a 1:1 ratio. The formula mass for this compound is computed as 58.44 amu (Figure 5.6.3).



Element	Quantity		Average atomic mass (amu)		Subtotal
Na	1	×	22.99	=	22.99
Cl	1	×	35.45	=	35.45
				Formula mass	58.44

Figure 5.6.3: Table salt, NaCl, contains an array of sodium and chloride ions combined in a 1:1 ratio. Its formula mass is 58.44 amu.

A table and diagram are shown. The table is made up of six columns and four rows. The header row reads: "Element," "Quantity," a blank space, "Average atomic mass (a m u)," a blank space and "Subtotal (a m u)." The first column contains the symbols "N a", "C l," and a merged cell. The merged cell runs the length of the first five columns. The second column contains the numbers "1" and "1" as well as the merged cell. The third column contains the multiplication symbol in each cell except for the last, merged cell. The fourth column contains the numbers "22.99" and "35.45" as well as the merged cell. The fifth column contains the symbol "=" in each cell except for the last, merged cell. The sixth column contains the values "22.99," "35.45," and "58.44." There is a thick black line below the number "35.45." The merged cell under the first five columns reads "Formula mass." To the left of the table is a diagram of a chemical structure. The diagram shows green and purple spheres placed in an alternating pattern, making up the corners of eight stacked cubes to form one larger cube. The green spheres are slightly smaller than the purple spheres.

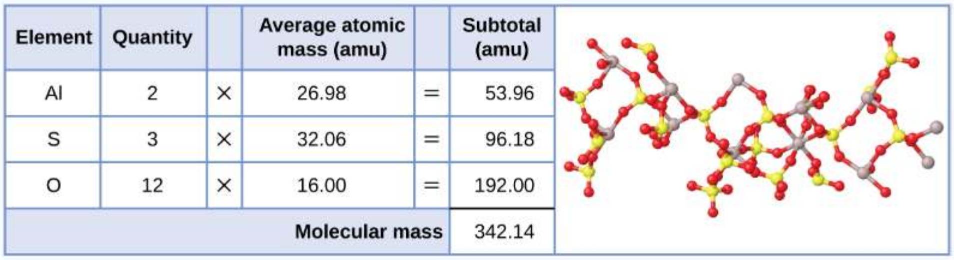
Note that the average masses of neutral sodium and chlorine atoms were used in this computation, rather than the masses for sodium cations and chlorine anions. This approach is perfectly acceptable when computing the formula mass of an ionic compound. Even though a sodium cation has a slightly smaller mass than a sodium atom (since it is missing an electron), this difference will be offset by the fact that a chloride anion is slightly more massive than a chloride atom (due to the extra electron). Moreover, the mass of an electron is negligibly small with respect to the mass of a typical atom. Even when calculating the mass of an isolated ion, the missing or additional electrons can generally be ignored, since their contribution to the overall mass is negligible, reflected only in the nonsignificant digits that will be lost when the computed mass is properly rounded. The few exceptions to this guideline are very light ions derived from elements with precisely known atomic masses.

✓ Example 5.6.2: Computing Formula Mass for an Ionic Compound

Aluminum sulfate, $Al_2(SO_4)_3$, is an ionic compound that is used in the manufacture of paper and in various water purification processes. What is the formula mass (amu) of this compound?

Solution

The formula for this compound indicates it contains Al^{3+} and SO_4^{2-} ions combined in a 2:3 ratio. For purposes of computing a formula mass, it is helpful to rewrite the formula in the simpler format, $Al_2S_3O_{12}$. Following the approach outlined above, the formula mass for this compound is calculated as follows:



Element	Quantity		Average atomic mass (amu)		Subtotal (amu)
Al	2	×	26.98	=	53.96
S	3	×	32.06	=	96.18
O	12	×	16.00	=	192.00
				Molecular mass	342.14

Exercise 5.6.2

Calcium phosphate, $Ca_3(PO_4)_2$, is an ionic compound and a common anti-caking agent added to food products. What is the formula mass (amu) of calcium phosphate?

Answer

310.18 amu

The Mole

The identity of a substance is defined not only by the types of atoms or ions it contains, but by the quantity of each type of atom or ion. For example, water, H_2O, and hydrogen peroxide, H_2O_2, are alike in that their respective molecules are composed of hydrogen and oxygen atoms. However, because a hydrogen peroxide molecule contains two oxygen atoms, as opposed to the water molecule, which has only one, the two substances exhibit very different properties. Today, we possess sophisticated instruments that allow the direct measurement of these defining microscopic traits; however, the same traits were originally derived from the measurement of macroscopic properties (the masses and volumes of bulk quantities of matter) using relatively simple tools (balances and volumetric glassware). This experimental approach required the introduction of a new unit for amount of substances, the *mole*, which remains indispensable in modern chemical science.

The mole is an amount unit similar to familiar units like pair, dozen, gross, etc. It provides a specific measure of *the number* of atoms or molecules in a bulk sample of matter. A mole is defined as the amount of substance containing the same number of discrete entities (such as atoms, molecules, and ions) as the number of atoms in a sample of pure ^{12}C weighing exactly 12 g. One Latin connotation for the word “mole” is “large mass” or “bulk,” which is consistent with its use as the name for this unit. The mole provides a link between an easily measured macroscopic property, bulk mass, and an extremely important fundamental property, number of atoms, molecules, and so forth.

The number of entities composing a mole has been experimentally determined to be $6.02214179 \times 10^{23}$, a fundamental constant named **Avogadro’s number** (N_A) or the Avogadro constant in honor of Italian scientist Amedeo Avogadro. This constant is properly reported with an explicit unit of “per mole,” a conveniently rounded version being $6.022 \times 10^{23}/\mathrm{mol}$.

Consistent with its definition as an amount unit, 1 mole of any element contains the same number of atoms as 1 mole of any other element. The masses of 1 mole of different elements, however, are different, since the masses of the individual atoms are drastically different. The molar mass of an element (or compound) is the mass in grams of 1 mole of that substance, a property expressed in units of grams per mole (g/mol) (Figure $5.6.4$).

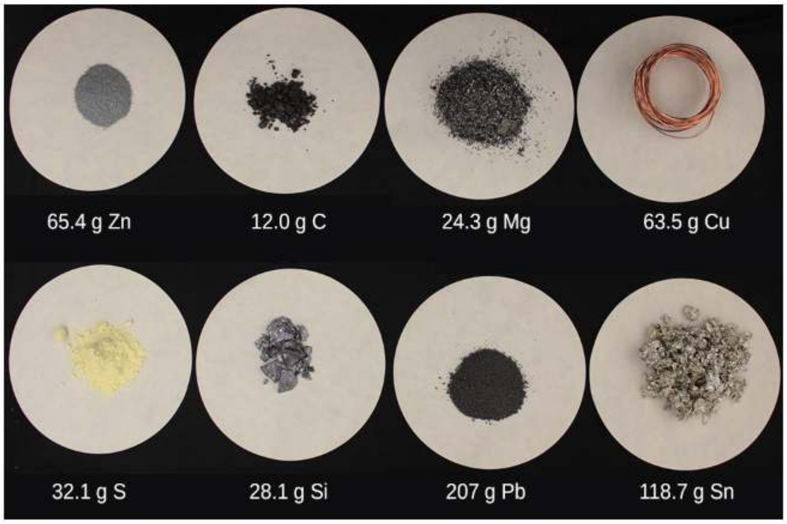


Figure $5.6.4$: Each sample contains 6.022×10^{23} atoms —1.00 mol of atoms. From left to right (top row): 65.4 g zinc, 12.0 g carbon, 24.3 g magnesium, and 63.5 g copper. From left to right (bottom row): 32.1 g sulfur, 28.1 g silicon, 207 g lead, and 118.7 g tin. (credit: modification of work by Mark Ott).

This figure contains eight different substances displayed on white circles. The amount of each substance is visibly different.

Because the definitions of both the mole and the atomic mass unit are based on the same reference substance, ^{12}C, the molar mass of any substance is numerically equivalent to its atomic or formula weight in amu. Per the amu definition, a single ^{12}C atom weighs 12 amu (its atomic mass is 12 amu). According to the definition of the mole, 12 g of ^{12}C contains 1 mole of ^{12}C atoms (its molar mass is 12 g/mol). This relationship holds for all elements, since their atomic masses are measured relative to that of the amu-reference substance, ^{12}C. Extending this principle, the molar mass of a compound in grams is likewise numerically equivalent to its formula mass in amu (Figure 5.6.5).

Figure 5.6.5: Each sample contains 6.022×10^{23} molecules or formula units—1.00 mol of the compound or element. Clock-wise from the upper left: 130.2 g of $C_8H_{17}OH$ (1-octanol, formula mass 130.2 amu), 454.4 g of HgI_2 (mercury(II) iodide, formula mass 454.4 amu), 32.0 g of CH_3OH (methanol, formula mass 32.0 amu) and 256.5 g of S_8 (sulfur, formula mass 256.5 amu). (credit: Sahar Atwa).

Table 5.6.1: Mass of one mole of elements

Element	Average Atomic Mass (amu)	Molar Mass (g/mol)	Atoms/Mole
C	12.01	12.01	6.022×10^{23}
H	1.008	1.008	6.022×10^{23}
O	16.00	16.00	6.022×10^{23}
Na	22.99	22.99	6.022×10^{23}
Cl	33.45	35.45	6.022×10^{23}

While atomic mass and molar mass are numerically equivalent, keep in mind that they are vastly different in terms of scale, as represented by the vast difference in the magnitudes of their respective units (amu versus g). To appreciate the enormity of the mole, consider a small drop of water after a rainfall. Although this represents just a tiny fraction of 1 mole of water (~18 g), it contains more water molecules than can be clearly imagined. If the molecules were distributed equally among the roughly seven billion people on earth, each person would receive more than 100 billion molecules.

Video 5.6.1: The mole is used in chemistry to represent 6.022×10^{23} of something, but it can be difficult to conceptualize such a large number. Watch this video and then complete the “Think” questions that follow. Explore more about the mole by reviewing the information under “Dig Deeper.”

The relationships between formula mass, the mole, and Avogadro’s number can be applied to compute various quantities that describe the composition of substances and compounds. For example, if we know the mass and chemical composition of a substance, we can determine the number of moles and calculate number of atoms or molecules in the sample. Likewise, if we know the number of moles of a substance, we can derive the number of atoms or molecules and calculate the substance’s mass.

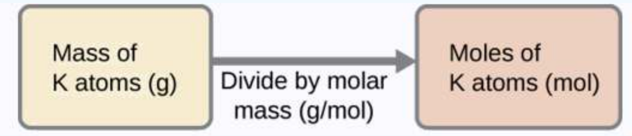
✓ Example 5.6.3: Deriving Moles from Grams for an Element

According to nutritional guidelines from the US Department of Agriculture, the estimated average requirement for dietary potassium is 4.7 g. What is the estimated average requirement of potassium in moles?

Solution

The mass of K is provided, and the corresponding amount of K in moles is requested. Referring to the periodic table, the atomic mass of K is 39.10 amu, and so its molar mass is 39.10 g/mol. The given mass of K (4.7 g) is a bit more than one-tenth the molar mass (39.10 g), so a reasonable "ballpark" estimate of the number of moles would be slightly greater than 0.1 mol.

The molar amount of a substance may be calculated by dividing its mass (g) by its molar mass (g/mol):



The factor-label method supports this mathematical approach since the unit "g" cancels and the answer has units of "mol:"

$$4.7\ \cancel{\text{g}}\,\text{K}\left(\frac{\text{mol K}}{39.10\ \cancel{\text{g}}}\right)=0.12\ \text{mol K}$$

The calculated magnitude (0.12 mol K) is consistent with our ballpark expectation, since it is a bit greater than 0.1 mol.

? Exercise 5.6.3: Beryllium

Beryllium is a light metal used to fabricate transparent X-ray windows for medical imaging instruments. How many moles of Be are in a thin-foil window weighing 3.24 g?

Answer

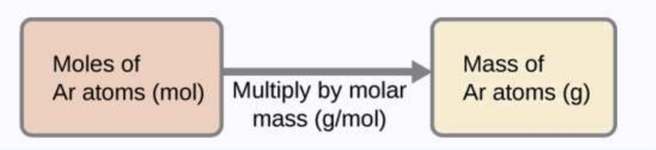
0.360 mol

✓ Example 5.6.4: Deriving Grams from Moles for an Element

A liter of air contains 9.2×10^{-4} mol argon. What is the mass of Ar in a liter of air?

Solution

The molar amount of Ar is provided and must be used to derive the corresponding mass in grams. Since the amount of Ar is less than 1 mole, the mass will be less than the mass of 1 mole of Ar, approximately 40 g. The molar amount in question is approximately one-one thousandth (~10^{-3}) of a mole, and so the corresponding mass should be roughly one-one thousandth of the molar mass (~0.04 g):



In this case, logic dictates (and the factor-label method supports) multiplying the provided amount (mol) by the molar mass (g/mol):

$$9.2\times10^{-4}\ \cancel{\text{mol}}\ \text{Ar}\left(\frac{39.95\ \text{g}}{\cancel{\text{mol}}\ \text{Ar}}\right)=0.037\ \text{g Ar}$$

The result is in agreement with our expectations, around 0.04 g Ar.

Exercise $5.6.4$

What is the mass of 2.561 mol of gold?

Answer

504.4 g

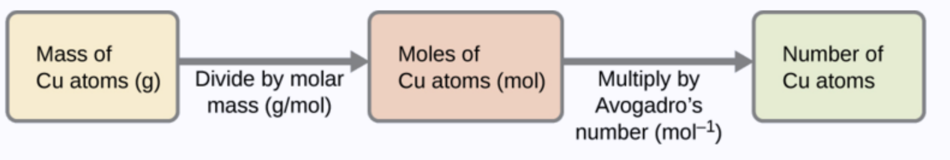
Example $5.6.6$: Deriving Number of Atoms from Mass for an Element

Copper is commonly used to fabricate electrical wire (Figure $5.6.6$). How many copper atoms are in 5.00 g of copper wire?

Figure $5.6.6$: Copper wire is composed of many, many atoms of Cu. (credit: Emilian Robert Vicol)

Solution

The number of Cu atoms in the wire may be conveniently derived from its mass by a two-step computation: first calculating the molar amount of Cu, and then using Avogadro's number (N_A) to convert this molar amount to number of Cu atoms:



Considering that the provided sample mass (5.00 g) is a little less than one-tenth the mass of 1 mole of Cu (~64 g), a reasonable estimate for the number of atoms in the sample would be on the order of one-tenth N_A, or approximately 10^{22} Cu atoms. Carrying out the two-step computation yields:

$$5.00\ \cancel{\text{g}}\ \text{Cu}\left(\frac{\cancel{\text{mol}}\ \text{Cu}}{63.55\ \cancel{\text{g}}}\right)\left(\frac{6.022\times10^{23}\ \text{atoms}}{\cancel{\text{mol}}}\right)=4.74\times10^{22}\ \text{atoms of copper}$$

The factor-label method yields the desired cancellation of units, and the computed result is on the order of 10^{22} as expected.

Exercise $5.6.6$

A prospector panning for gold in a river collects 15.00 g of pure gold. How many Au atoms are in this quantity of gold?

Answer

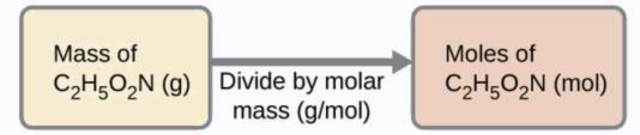
$4.586\times10^{22}\ Au\,\text{atoms}$

Example $5.6.7$: Deriving Moles from Grams for a Compound

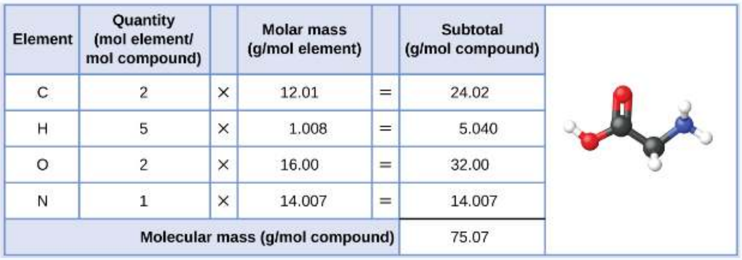
Our bodies synthesize protein from amino acids. One of these amino acids is glycine, which has the molecular formula $C_2H_5O_2N$. How many moles of glycine molecules are contained in 28.35 g of glycine?

Solution

We can derive the number of moles of a compound from its mass following the same procedure we used for an element in Example 5.6.6:



The molar mass of glycine is required for this calculation, and it is computed in the same fashion as its molecular mass. One mole of glycine, $C_2H_5O_2N$, contains 2 moles of carbon, 5 moles of hydrogen, 2 moles of oxygen, and 1 mole of nitrogen:



Element	Quantity (mol element/ mol compound)		Molar mass (g/mol element)		Subtotal (g/mol compound)
C	2	×	12.01	=	24.02
H	5	×	1.008	=	5.040
O	2	×	16.00	=	32.00
N	1	×	14.007	=	14.007
Molecular mass (g/mol compound)					75.07

A table is shown that is made up of six columns and six rows. The header row reads: “Element,” “Quantity (mol element / mol compound,” a blank space, “Molar mass (g / mol element),” a blank space, and “Subtotal (a m u).” The first column contains the symbols “C,” “H,” “O,” “N,” and a merged cell. The merged cell runs the width of the first five columns. The second column contains the numbers “2,” “5,” “2,” and “1” as well as the merged cell. The third column contains the multiplication symbol in each cell except for the last, merged cell. The fourth column contains the numbers “12.01,” “1.008,” “16.00,” and “14.007” as well as the merged cell. The fifth column contains the symbol “=” in each cell except for the last, merged cell. The sixth column contains the values “24.02,” “5.040,” “32.00,” “14.007,” and “75.07.” There is a thick black line under the number 14.007. The merged cell under the first five columns reads “Molar mass (g / mol compound). There is a ball-and-stick drawing to the right of this table. It shows a black sphere that forms a double bond with a slightly smaller red sphere, a single bond with another red sphere, and a single bond with another black sphere. The red sphere that forms a single bond with the black sphere also forms a single bond with a smaller, white sphere. The second black sphere forms a single bond with a smaller, white sphere and a smaller blue sphere. The blue sphere forms a single bond with two smaller, white spheres each.

The provided mass of glycine (~28 g) is a bit more than one-third the molar mass (~75 g/mol), so we would expect the computed result to be a bit greater than one-third of a mole (~0.33 mol). Dividing the compound’s mass by its molar mass yields:

$$28.35\ \cancel{g}\ \text{glycine}\left(\frac{\text{mol glycine}}{75.07\ \cancel{g}}\right)=0.378\ \text{mol glycine}$$

This result is consistent with our rough estimate.

? Exercise 5.6.7

How many moles of sucrose, $C_{12}H_{22}O_{11}$, are in a 25-g sample of sucrose?

Answer

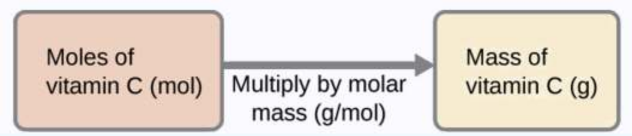
0.073 mol

✓ Example 5.6.8: Deriving Grams from Moles for a Compound

Vitamin C is a covalent compound with the molecular formula $C_6H_8O_6$. The recommended daily dietary allowance of vitamin C for children aged 4–8 years is 1.42×10^{-4} mol. What is the mass of this allowance in grams?

Solution

As for elements, the mass of a compound can be derived from its molar amount as shown:



The molar mass for this compound is computed to be 176.124 g/mol. The given number of moles is a very small fraction of a mole (~10^{-4} or one-ten thousandth); therefore, we would expect the corresponding mass to be about one-ten thousandth of the molar mass (~0.02 g). Performing the calculation, we get:

$$1.42 \times 10^{-4} \ \cancel{\text{mol}} \text{ vitamin C} \left(\frac{176.124 \text{ g}}{\cancel{\text{mol}} \text{ vitamin C}} \right) = 0.0250 \text{ g vitamin C}$$

This is consistent with the anticipated result.

? Exercise 5.6.8

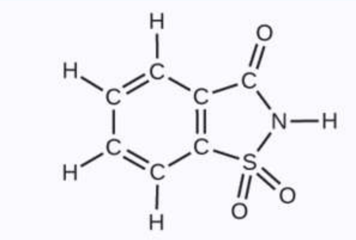
What is the mass of 0.443 mol of hydrazine, N_2H_4?

Answer

14.2 g

✓ Example 5.6.9: Deriving the Number of Molecules from the Compound Mass

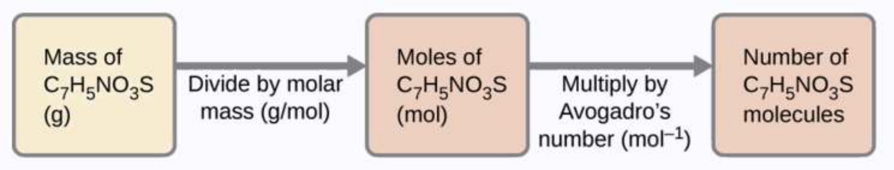
A packet of an artificial sweetener contains 40.0 mg of saccharin ($C_7H_5NO_3S$), which has the structural formula:



Given that saccharin has a molar mass of 183.18 g/mol, how many saccharin molecules are in a 40.0-mg (0.0400-g) sample of saccharin? How many carbon atoms are in the same sample?

Solution

The number of molecules in a given mass of compound is computed by first deriving the number of moles, as demonstrated in Example 5.6.8, and then multiplying by Avogadro's number:



Using the provided mass and molar mass for saccharin yields:

$$0.0400 \ \cancel{\text{g}} \ C_7H_5NO_3S \left(\frac{\cancel{\text{mol}} \ C_7H_5NO_3S}{183.18 \ \cancel{\text{g}} \ C_7H_5NO_3S} \right) \left(\frac{6.022 \times 10^{23} \ C_7H_5NO_3S \text{ molecules}}{1 \ \cancel{\text{mol}} \ C_7H_5NO_3S} \right)$$
$$= 1.31 \times 10^{20} \ C_7H_5NO_3S \text{ molecules}$$

The compound's formula shows that each molecule contains seven carbon atoms, and so the number of C atoms in the provided sample is:

$$1.31 \times 10^{20}\ \mathrm{C_7H_5NO_3S\ molecules} \left(\frac{7\ \mathrm{C\ atoms}}{1\ \mathrm{C_7H_5NO_3S\ molecule}} \right) = 9.20 \times 10^{21}\ \mathrm{C\ atoms}$$

? Exercise 5.6.9

How many C_4H_{10} molecules are contained in 9.213 g of this compound? How many hydrogen atoms?

Answer

- 9.545×10^{22} molecules C_4H_{10}
- 9.545×10^{23} atoms H

Summary

The formula mass of a substance is the sum of the average atomic masses of each atom represented in the chemical formula and is expressed in atomic mass units. The formula mass of a covalent compound is also called the molecular mass. A convenient amount unit for expressing very large numbers of atoms or molecules is the mole. Experimental measurements have determined the number of entities composing 1 mole of substance to be 6.022×10^{23}, a quantity called Avogadro's number. The mass in grams of 1 mole of substance is its molar mass. Due to the use of the same reference substance in defining the atomic mass unit and the mole, the formula mass (amu) and molar mass (g/mol) for any substance are numerically equivalent (for example, one H_2O molecule weighs approximately18 amu and 1 mole of H_2O molecules weighs approximately 18 g).

Footnotes

1. 1 Omiatek, Donna M., Amanda J. Bressler, Ann-Sofie Cans, Anne M. Andrews, Michael L. Heien, and Andrew G. Ewing. "The Real Catecholamine Content of Secretory Vesicles in the CNS Revealed by Electrochemical Cytometry." *Scientific Report* 3 (2013): 1447, accessed January 14, 2015, doi:10.1038/srep01447.

Glossary

Avogadro's number (N_A)

experimentally determined value of the number of entities comprising 1 mole of substance, equal to 6.022×10^{23} mol^{-1}

formula mass

sum of the average masses for all atoms represented in a chemical formula; for covalent compounds, this is also the molecular mass

mole

amount of substance containing the same number of atoms, molecules, ions, or other entities as the number of atoms in exactly 12 grams of ^{12}C

molar mass

mass in grams of 1 mole of a substance

5.7: Determining Empirical and Molecular Formulas

Learning Objectives

- Compute the percent composition of a compound
- Determine the empirical formula of a compound
- Determine the molecular formula of a compound

In the previous section, we discussed the relationship between the bulk mass of a substance and the number of atoms or molecules it contains (moles). Given the chemical formula of the substance, we were able to determine the amount of the substance (moles) from its mass, and vice versa. But what if the chemical formula of a substance is unknown? In this section, we will explore how to apply these very same principles in order to derive the chemical formulas of unknown substances from experimental mass measurements.

Percent Composition

The elemental makeup of a compound defines its chemical identity, and chemical formulas are the most succinct way of representing this elemental makeup. When a compound's formula is unknown, measuring the mass of each of its constituent elements is often the first step in the process of determining the formula experimentally. The results of these measurements permit the calculation of the compound's percent composition, defined as the percentage by mass of each element in the compound. For example, consider a gaseous compound composed solely of carbon and hydrogen. The percent composition of this compound could be represented as follows:

$$\%\text{H} = \frac{\text{mass H}}{\text{mass compound}} \times 100\%$$

$$\%\text{C} = \frac{\text{mass C}}{\text{mass compound}} \times 100\%$$

If analysis of a 10.0-g sample of this gas showed it to contain 2.5 g H and 7.5 g C, the percent composition would be calculated to be 25% H and 75% C:

$$\%\text{H} = \frac{2.5 \text{ g H}}{10.0 \text{ g compound}} \times 100\% = 25\%$$

$$\%\text{C} = \frac{7.5 \text{ g C}}{10.0 \text{ g compound}} \times 100\% = 75\%$$

Example 5.7.1: Calculation of Percent Composition

Analysis of a 12.04-g sample of a liquid compound composed of carbon, hydrogen, and nitrogen showed it to contain 7.34 g C, 1.85 g H, and 2.85 g N. What is the percent composition of this compound?

Solution

To calculate percent composition, we divide the experimentally derived mass of each element by the overall mass of the compound, and then convert to a percentage:

$$\%\text{C} = \frac{7.34 \text{ g C}}{12.04 \text{ g compound}} \times 100\% = 61.0\%$$

$$\%\text{H} = \frac{1.85 \text{ g H}}{12.04 \text{ g compound}} \times 100\% = 15.4\%$$

$$\%\text{N} = \frac{2.85 \text{ g N}}{12.04 \text{ g compound}} \times 100\% = 23.7\%$$

The analysis results indicate that the compound is 61.0% C, 15.4% H, and 23.7% N by mass.

? Exercise 5.7.1

A 24.81-g sample of a gaseous compound containing only carbon, oxygen, and chlorine is determined to contain 3.01 g C, 4.00 g O, and 17.81 g Cl. What is this compound's percent composition?

Answer

12.1% C, 16.1% O, 71.8% Cl

Determining Percent Composition from Formula Mass

Percent composition is also useful for evaluating the relative abundance of a given element in different compounds of known formulas. As one example, consider the common nitrogen-containing fertilizers ammonia (NH_3), ammonium nitrate (NH_4NO_3), and urea (CH_4N_2O). The element nitrogen is the active ingredient for agricultural purposes, so the mass percentage of nitrogen in the compound is a practical and economic concern for consumers choosing among these fertilizers. For these sorts of applications, the percent composition of a compound is easily derived from its formula mass and the atomic masses of its constituent elements. A molecule of NH_3 contains one N atom weighing 14.01 amu and three H atoms weighing a total of (3 × 1.008 amu) = 3.024 amu. The formula mass of ammonia is therefore (14.01 amu + 3.024 amu) = 17.03 amu, and its percent composition is:

$$\%\text{N} = \frac{14.01\ \text{amu N}}{17.03\ \text{amu NH}_3} \times 100\% = 82.27\%$$

$$\%\text{H} = \frac{3.024\ \text{amu N}}{17.03\ \text{amu NH}_3} \times 100\% = 17.76\%$$

This same approach may be taken considering a pair of molecules, a dozen molecules, or a mole of molecules, etc. The latter amount is most convenient and would simply involve the use of molar masses instead of atomic and formula masses, as demonstrated Example 5.7.2. As long as we know the chemical formula of the substance in question, we can easily derive percent composition from the formula mass or molar mass.

✓ Example 5.7.2: Determining Percent Composition from a Molecular Formula

Aspirin is a compound with the molecular formula $C_9H_8O_4$. What is its percent composition?

Solution

To calculate the percent composition, we need to know the masses of C, H, and O in a known mass of $C_9H_8O_4$. It is convenient to consider 1 mol of $C_9H_8O_4$ and use its molar mass (180.159 g/mole, determined from the chemical formula) to calculate the percentages of each of its elements:

$$\%\text{C} = \frac{9\ \text{mol C} \times \text{molar mass C}}{\text{molar mass C}_9\text{H}_{18}\text{O}_4} \times 100 = \frac{9 \times 12.01\ \text{g/mol}}{} 180.159\ \text{g/mol} \times 100 = \frac{108.09\ \text{g/mol}}{180.159\ \text{g/mol}} \times 100$$
$$\%\text{C} = 60.00\ \%\ \text{C}$$

$$\%\text{H} = \frac{8\ \text{mol H} \times \text{molar mass H}}{\text{molar mass C}_9\text{H}_{18}\text{O}_4} \times 100 = \frac{8 \times 1.008\ \text{g/mol}}{} 180.159\ \text{g/mol} \times 100 = \frac{8.064\ \text{g/mol}}{180.159\ \text{g/mol}} \times 100$$
$$\%\text{H} = 4.476\ \%\ \text{H}$$

$$\%\text{O} = \frac{4\ \text{mol O} \times \text{molar mass O}}{\text{molar mass C}_9\text{H}_{18}\text{O}_4} \times 100 = \frac{4 \times 16.00\ \text{g/mol}}{} 180.159\ \text{g/mol} \times 100 = \frac{64.00\ \text{g/mol}}{180.159\ \text{g/mol}} \times 100$$
$$\%\text{O} = 35.52\%$$

Note that these percentages sum to equal 100.00% when appropriately rounded.

? Exercise 5.7.2

o three significant digits, what is the mass percentage of iron in the compound Fe_2O_3?

Answer

69.9% Fe

Determination of Empirical Formulas

As previously mentioned, the most common approach to determining a compound's chemical formula is to first measure the masses of its constituent elements. However, we must keep in mind that chemical formulas represent the relative *numbers*, not masses, of atoms in the substance. Therefore, any experimentally derived data involving mass must be used to derive the corresponding numbers of atoms in the compound. To accomplish this, we can use molar masses to convert the mass of each element to a number of moles. We then consider the moles of each element relative to each other, converting these numbers into a whole-number ratio that can be used to derive the empirical formula of the substance. Consider a sample of compound determined to contain 1.71 g C and 0.287 g H. The corresponding numbers of atoms (in moles) are:

$$1.71 \text{ g C} \times \frac{1 \text{ mol C}}{12.01 \text{ g C}} = 0.142 \text{ mol C}$$

$$0.287 \text{ g H} \times \frac{1 \text{ mol H}}{1.008 \text{ g H}} = 0.284 \text{ mol H}$$

Thus, we can accurately represent this compound with the formula $C_{0.142}H_{0.284}$. Of course, per accepted convention, formulas contain whole-number subscripts, which can be achieved by dividing each subscript by the smaller subscript:

$$C_{\frac{0.142}{0.142}}H_{\frac{0.284}{0.142}} \text{ or } CH_2$$

(Recall that subscripts of "1" are not written, but rather assumed if no other number is present.)

The empirical formula for this compound is thus CH_2. This may or not be the compound's *molecular formula* as well; however, we would need additional information to make that determination (as discussed later in this section).

Consider as another example a sample of compound determined to contain 5.31 g Cl and 8.40 g O. Following the same approach yields a tentative empirical formula of:

$$Cl_{0.150}O_{0.525} = Cl_{\frac{0.150}{0.150}}O_{\frac{0.525}{0.150}} = ClO_{3.5}$$

In this case, dividing by the smallest subscript still leaves us with a decimal subscript in the empirical formula. To convert this into a whole number, we must multiply each of the subscripts by two, retaining the same atom ratio and yielding Cl_2O_7 as the final empirical formula.

Procedure

In summary, empirical formulas are derived from experimentally measured element masses by:

1. Deriving the number of moles of each element from its mass
2. Dividing each element's molar amount by the smallest molar amount to yield subscripts for a tentative empirical formula
3. Multiplying all coefficients by an integer, if necessary, to ensure that the smallest whole-number ratio of subscripts is obtained

Figure $5.7.1$ outlines this procedure in flow chart fashion for a substance containing elements A and X.

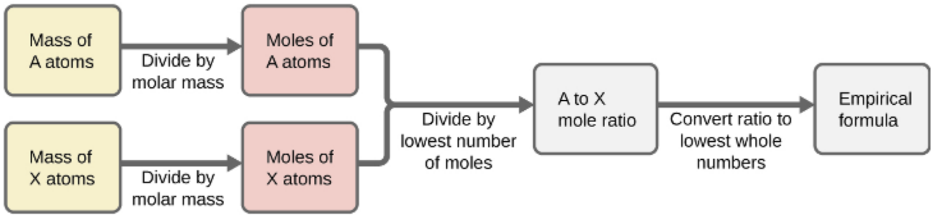


Figure 5.7.1: The empirical formula of a compound can be derived from the masses of all elements in the sample.

A flow chart is shown that is composed of six boxes, two of which are connected together by a right facing arrow and located above two more that are also connected by a right-facing arrow. These two rows of boxes are connected vertically by a line that leads to a right-facing arrow and the last two boxes, connected by a final right facing arrow. The first two upper boxes have the phrases, "Mass of A atoms" and "Moles of A atoms" respectively, while the arrow that connects them has the phrase, "Divide by molar mass," written below it. The second two bottom boxes have the phrases, "Mass of X atoms" and "Moles of X atoms" respectively, while the arrow that connects them has the phrase, "Divide by molar mass" written below it. The arrow that connects the upper and lower boxes to the last two boxes has the phrase "Divide by lowest number of moles" written below it. The last two boxes have the phrases, "A to X mole ratio" and "Empirical formula" respectively, while the arrow that connects them has the phrase, "Convert ratio to lowest whole numbers" written below it.

✓ Example 5.7.3: Determining an Empirical Formula from Masses of Elements

A sample of the black mineral hematite (Figure 5.7.2), an oxide of iron found in many iron ores, contains 34.97 g of iron and 15.03 g of oxygen. What is the empirical formula of hematite?

Figure 5.7.2: Hematite is an iron oxide that is used in jewelry. (credit: Mauro Cateb)

Solution

For this problem, we are given the mass in grams of each element. Begin by finding the moles of each:

$$34.97 \text{ g Fe} \left(\frac{\text{mol Fe}}{55.85 \text{ g}}\right) = 0.6261 \text{ mol Fe}$$

$$15.03 \text{ g O} \left(\frac{\text{mol O}}{16.00 \text{ g}}\right) = 0.9394 \text{ mol O}$$

Next, derive the iron-to-oxygen molar ratio by dividing by the lesser number of moles:

$$\frac{0.6261}{0.6261} = 1.000 \text{ mol Fe}$$

$$\frac{0.9394}{0.6261} = 1.500 \text{ mol O}$$

The ratio is 1.000 mol of iron to 1.500 mol of oxygen ($Fe_1O_{1.5}$). Finally, multiply the ratio by two to get the smallest possible whole number subscripts while still maintaining the correct iron-to-oxygen ratio:

$$2(\text{Fe}_1\text{O}_{1.5}) = \text{Fe}_2\text{O}_3$$

The empirical formula is Fe_2O_3.

Exercise 5.7.3

What is the empirical formula of a compound if a sample contains 0.130 g of nitrogen and 0.370 g of oxygen?

Answer

N_2O_5

Video 5.7.1: Additional worked examples illustrating the derivation of empirical formulas are presented in the brief video clip.

Deriving Empirical Formulas from Percent Composition

Finally, with regard to deriving empirical formulas, consider instances in which a compound’s percent composition is available rather than the absolute masses of the compound’s constituent elements. In such cases, the percent composition can be used to calculate the masses of elements present in any convenient mass of compound; these masses can then be used to derive the empirical formula in the usual fashion.

Example 5.7.4: Determining an Empirical Formula from Percent Composition

The bacterial fermentation of grain to produce ethanol forms a gas with a percent composition of 27.29% C and 72.71% O (Figure 5.7.3). What is the empirical formula for this gas?

Figure 5.7.3: An oxide of carbon is removed from these fermentation tanks through the large copper pipes at the top. (credit: “Dual Freq”/Wikimedia Commons)

A picture is shown of four copper-colored industrial containers with a large pipe connecting to the top of each one.

Solution

Since the scale for percentages is 100, it is most convenient to calculate the mass of elements present in a sample weighing 100 g. The calculation is “most convenient” because, per the definition for percent composition, the mass of a given element in grams is numerically equivalent to the element’s mass percentage. This numerical equivalence results from the definition of the “percentage” unit, whose name is derived from the Latin phrase *per centum* meaning “by the hundred.” Considering this definition, the mass percentages provided may be more conveniently expressed as fractions:

$$27.29\,\%\,\text{C} = \frac{27.29 \text{ g C}}{100 \text{ g compound}}$$

$$72.71\,\%\,\text{O} = \frac{72.71 \text{ g O}}{100 \text{ g compound}}$$

The molar amounts of carbon and hydrogen in a 100-g sample are calculated by dividing each element’s mass by its molar mass:

$$27.29 \text{ g C}\left(\frac{\text{mol C}}{12.01 \text{ g}}\right) = 2.272 \text{ mol C}$$

$$72.71 \text{ g O}\left(\frac{\text{mol O}}{16.00 \text{ g}}\right) = 4.544 \text{ mol O}$$

Coefficients for the tentative empirical formula are derived by dividing each molar amount by the lesser of the two:

$$\frac{2.272 \text{ mol C}}{2.272} = 1$$
$$\frac{4.544 \text{ mol O}}{2.272} = 2$$

Since the resulting ratio is one carbon to two oxygen atoms, the empirical formula is CO_2.

Exercise 5.7.4

What is the empirical formula of a compound containing 40.0% C, 6.71% H, and 53.28% O?

Answer

CH_2O

Derivation of Molecular Formulas

Recall that empirical formulas are symbols representing the *relative* numbers of a compound’s elements. Determining the *absolute* numbers of atoms that compose a single molecule of a covalent compound requires knowledge of both its empirical formula and its molecular mass or molar mass. These quantities may be determined experimentally by various measurement techniques. Molecular mass, for example, is often derived from the mass spectrum of the compound (see discussion of this technique in the previous chapter on atoms and molecules). Molar mass can be measured by a number of experimental methods, many of which will be introduced in later chapters of this text.

Molecular formulas are derived by comparing the compound’s molecular or molar mass to its empirical formula mass. As the name suggests, an empirical formula mass is the sum of the average atomic masses of all the atoms represented in an empirical formula. If we know the molecular (or molar) mass of the substance, we can divide this by the empirical formula mass in order to identify the number of empirical formula units per molecule, which we designate as *n*:

$$\frac{\text{molecular or molar mass}\left(\text{amu or }\frac{\text{g}}{\text{mol}}\right)}{\text{empirical formula mass}\left(\text{amu or }\frac{\text{g}}{\text{mol}}\right)} = n\,\text{formula units/molecule}$$

The molecular formula is then obtained by multiplying each subscript in the empirical formula by *n*, as shown by the generic empirical formula A_xB_y:

$$(\text{A}_\text{x}\text{B}_\text{y})_\text{n} = \text{A}_\text{nx}\text{B}_\text{nx}$$

For example, consider a covalent compound whose empirical formula is determined to be CH_2O. The empirical formula mass for this compound is approximately 30 amu (the sum of 12 amu for one C atom, 2 amu for two H atoms, and 16 amu for one O atom). If the compound’s molecular mass is determined to be 180 amu, this indicates that molecules of this compound contain six times the number of atoms represented in the empirical formula:

$$\frac{180\ \text{amu/molecule}}{30\ \dfrac{\text{amu}}{\text{formula unit}}} = 6\ \text{formula units/molecule}$$

Molecules of this compound are then represented by molecular formulas whose subscripts are six times greater than those in the empirical formula:

$$(\mathrm{CH_2O})_6 = \mathrm{C_6H_{12}O_6}$$

Note that this same approach may be used when the molar mass (g/mol) instead of the molecular mass (amu) is used. In this case, we are merely considering one mole of empirical formula units and molecules, as opposed to single units and molecules.

✓ Example 5.7.5: Determination of the Molecular Formula for Nicotine

Nicotine, an alkaloid in the nightshade family of plants that is mainly responsible for the addictive nature of cigarettes, contains 74.02% C, 8.710% H, and 17.27% N. If 40.57 g of nicotine contains 0.2500 mol nicotine, what is the molecular formula?

Solution

Determining the molecular formula from the provided data will require comparison of the compound's empirical formula mass to its molar mass. As the first step, use the percent composition to derive the compound's empirical formula. Assuming a convenient, a 100-g sample of nicotine yields the following molar amounts of its elements:

$$(74.02\ \text{g C})\left(\frac{1\ \text{mol C}}{12.01\ \text{g C}}\right) = 6.163\ \text{mol C} \tag{5.7.1}$$

$$(8.710\ \text{g H})\left(\frac{1\ \text{mol H}}{1.01\ \text{g H}}\right) = 8.624\ \text{mol H} \tag{5.7.2}$$

$$(17.27\ \text{g N})\left(\frac{1\ \text{mol N}}{14.01\ \text{g N}}\right) = 1.233\ \text{mol N} \tag{5.7.3}$$

Next, we calculate the molar ratios of these elements relative to the least abundant element, N.

$$6.163\ \text{mol C}/\ 1.233\ \text{mol N} = 5$$

$$8.264\ \text{mol H}/\ 1.233\ \text{mol N} = 7$$

$$1.233\ \text{mol N}/\ 1.233\ \text{mol N} = 1$$

$$1.233/1.233 = 1.000\ \text{mol N}$$

$$6.163/1.233 = 4.998\ \text{mol C}$$

$$8.624/1.233 = 6.994\ \text{mol H}$$

The C-to-N and H-to-N molar ratios are adequately close to whole numbers, and so the empirical formula is $\mathrm{C_5H_7N}$. The empirical formula mass for this compound is therefore 81.13 amu/formula unit, or 81.13 g/mol formula unit.

We calculate the molar mass for nicotine from the given mass and molar amount of compound:

$$\frac{40.57\ \text{g nicotine}}{0.2500\ \text{mol nicotine}} = \frac{162.3\ \text{g}}{\text{mol}}$$

Comparing the molar mass and empirical formula mass indicates that each nicotine molecule contains two formula units:

$$\frac{162.3\ \text{g/mol}}{81.13\ \dfrac{\text{g}}{\text{formula unit}}} = 2\ \text{formula units/molecule}$$

Thus, we can derive the molecular formula for nicotine from the empirical formula by multiplying each subscript by two:

$$(\mathrm{C_5H_7N})_2 = \mathrm{C_{10}H_{14}N_2}$$

? Exercise 5.7.5

What is the molecular formula of a compound with a percent composition of 49.47% C, 5.201% H, 28.84% N, and 16.48% O, and a molecular mass of 194.2 amu?

Answer

$C_8H_{10}N_4O_2$

Summary

The chemical identity of a substance is defined by the types and relative numbers of atoms composing its fundamental entities (molecules in the case of covalent compounds, ions in the case of ionic compounds). A compound's percent composition provides the mass percentage of each element in the compound, and it is often experimentally determined and used to derive the compound's empirical formula. The empirical formula mass of a covalent compound may be compared to the compound's molecular or molar mass to derive a molecular formula.

Key Equations

- $\%\mathrm{X} = \dfrac{\text{mass X}}{\text{mass compound}} \times 100\%$
- $\dfrac{\text{molecular or molar mass}\left(\text{amu or }\dfrac{\text{g}}{\text{mol}}\right)}{\text{empirical formula mass}\left(\text{amu or }\dfrac{\text{g}}{\text{mol}}\right)} = n\,\text{formula units/molecule}$
- $(A_xB_y)_n = A_{nx}B_{ny}$

Glossary

percent composition

percentage by mass of the various elements in a compound

empirical formula mass

sum of average atomic masses for all atoms represented in an empirical formula

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5.8: Mole Calculations in Chemical Reactions

Learning Objectives

- Balance a chemical equation in terms of moles.
- Use the balanced equation to construct conversion factors in terms of moles.
- Calculate moles of one substance from moles of another substance using a balanced chemical equation.

Consider this balanced chemical equation:

$$2\,H_2 + O_2 \rightarrow 2\,H_2O$$

We interpret this as "two molecules of hydrogen react with one molecule of oxygen to make two molecules of water." The chemical equation is balanced as long as the coefficients are in the ratio 2:1:2. For instance, this chemical equation is also balanced:

$$100\,H_2 + 50\,O_2 \rightarrow 100\,H_2O$$

This equation is not conventional—because convention says that we use the lowest ratio of coefficients—but it is balanced. So is this chemical equation:

$$5,000\,H_2 + 2,500\,O_2 \rightarrow 5,000\,H_2O$$

Again, this is not conventional, but it is still balanced. Suppose we use a much larger number:

$$12.044 \times 10^{23} H_2 + 6.022 \times 10^{23} O_2 \rightarrow 12.044 \times 10^{23} H_2O$$

These coefficients are also in the ratio of 2:1:2. But these numbers are related to the number of things in a mole: the first and last numbers are two times Avogadro's number, while the second number is Avogadro's number. That means that the first and last numbers represent 2 mol, while the middle number is just 1 mol. Well, why not just use the number of moles in balancing the chemical equation?

$$2\,H_2 + O_2 \rightarrow 2\,H_2O$$

is the same balanced chemical equation we started with! What this means is that chemical equations are not just balanced in terms of molecules; *they are also balanced in terms of moles*. We can just as easily read this chemical equation as "two moles of hydrogen react with one mole of oxygen to make two moles of water." All balanced chemical reactions are balanced in terms of moles.

Example 5.8.1

Interpret this balanced chemical equation in terms of moles.

$$P_4 + 5\,O_2 \rightarrow P_4O_{10}$$

Solution

The coefficients represent the number of moles that react, not just molecules. We would speak of this equation as "one mole of molecular phosphorus reacts with five moles of elemental oxygen to make one mole of tetraphosphorus decoxide."

Exercise 5.8.1

Interpret this balanced chemical equation in terms of moles.

$$N_2 + 3\,H_2 \rightarrow 2\,NH_3$$

Answer

One mole of elemental nitrogen reacts with three moles of elemental hydrogen to produce two moles of ammonia.

In Section 4.1, we stated that a chemical equation is simply a recipe for a chemical reaction. As such, chemical equations also give us equivalents—equivalents between the reactants and the products. However, now we understand that *these equivalents are expressed in terms of moles*. Consider the chemical equation

$$2\,H_2 + O_2 \rightarrow 2\,H_2O$$

This chemical reaction gives us the following equivalents:

$$2 \text{ mol } H_2 \Leftrightarrow 1 \text{ mol } O_2 \Leftrightarrow 2 \text{ mol } H_2O$$

Any two of these quantities can be used to construct a conversion factor that lets us relate the number of moles of one substance to an equivalent number of moles of another substance. If, for example, we want to know how many moles of oxygen will react with 17.6 mol of hydrogen, we construct a conversion factor between 2 mol of H_2 and 1 mol of O_2 and use it to convert from moles of one substance to moles of another:

$$17.6\ mol\ \cancel{H_2} \times \frac{1\,mol\,O_2}{2\ mol\ \cancel{H_2}} = 8.80\,mol\,O_2$$

Note how the mol H_2 unit cancels, and mol O_2 is the new unit introduced. This is an example of a **mole-mole calculation,** when you start with moles of one substance and convert to moles of another substance by using the balanced chemical equation. The example may seem simple because the numbers are small, but numbers won't always be so simple!

✓ Example 5.8.2

For the balanced chemical equation

$$2\,C_4H_{10}(g) + 13\,O_2 \rightarrow 8\,CO_2(g) + 10\,H_2O(\ell)$$

if 154 mol of O_2 are reacted, how many moles of CO_2 are produced?

Solution

We are relating an amount of oxygen to an amount of carbon dioxide, so we need the equivalence between these two substances. According to the balanced chemical equation, the equivalence is

$$13 \text{ mol } O_2 \Leftrightarrow 8 \text{ mol } CO_2$$

We can use this equivalence to construct the proper conversion factor. We start with what we are given and apply the conversion factor:

$$154\ mol\ \cancel{O_2} \times \frac{8\,mol\,CO_2}{13\ mol\ \cancel{O_2}} = 94.8\,mol\,CO_2$$

The mol O_2 unit is in the denominator of the conversion factor so it cancels. Both the 8 and the 13 are exact numbers, so they do not contribute to the number of significant figures in the final answer.

? Exercise 5.8.2

Using the above equation, how many moles of H_2O are produced when 154 mol of O_2 react?

Answer

118 mol

It is important to reiterate that balanced chemical equations are balanced in terms of *moles*. Not grams, kilograms, or liters—but moles. Any stoichiometry problem will likely need to work through the mole unit at some point, especially if you are working with a balanced chemical reaction.

Summary

Balanced chemical reactions are balanced in terms of moles. A balanced chemical reaction gives equivalents in moles that allow stoichiometry calculations to be performed.

5.9: Mole-Mass and Mass-Mass Calculations

Learning Objectives

- From a given number of moles of a substance, calculate the mass of another substance involved using the balanced chemical equation.
- From a given mass of a substance, calculate the moles of another substance involved using the balanced chemical equation.
- From a given mass of a substance, calculate the mass of another substance involved using the balanced chemical equation.

Mole-mole calculations are not the only type of calculations that can be performed using balanced chemical equations. Recall that the molar mass can be determined from a chemical formula and used as a conversion factor. We can add that conversion factor as another step in a calculation to make a **mole-mass calculation**, where we start with a given number of moles of a substance and calculate the mass of another substance involved in the chemical equation, or vice versa.

For example, suppose we have the balanced chemical equation

$$2Al + 3Cl_2 \rightarrow 2AlCl_3$$

Suppose we know we have 123.2 g of Cl_2. How can we determine how many moles of $AlCl_3$ we will get when the reaction is complete? First and foremost, *chemical equations are not balanced in terms of grams; they are balanced in terms of moles*. So to use the balanced chemical equation to relate an amount of Cl_2 to an amount of $AlCl_3$, we need to convert the given amount of Cl_2 into moles. We know how to do this by simply using the molar mass of Cl_2 as a conversion factor. The molar mass of Cl_2 (which we get from the atomic mass of Cl from the periodic table) is 70.90 g/mol. We must invert this fraction so that the units cancel properly:

$$123.2\ g\cancel{Cl_2} \times \frac{1\,mol\,Cl_2}{70.90\ g\cancel{Cl_2}} = 1.738\ mol\,Cl_2$$

Now that we have the quantity in moles, we can use the balanced chemical equation to construct a conversion factor that relates the number of moles of Cl_2 to the number of moles of $AlCl_3$. The numbers in the conversion factor come from the coefficients in the balanced chemical equation:

$$\frac{2\,mol\,AlCl_3}{3\,mol\,Cl_2}$$

Using this conversion factor with the molar quantity we calculated above, we get

$$1.738\ mol\,\cancel{Cl_2} \times \frac{2\,mol\,AlCl_3}{3\ mol\,\cancel{Cl_2}} = 1.159\,mol\,AlCl_3$$

So, we will get 1.159 mol of $AlCl_3$ if we react 123.2 g of Cl_2.

In this last example, we did the calculation in two steps. However, it is mathematically equivalent to perform the two calculations sequentially on one line:

$$123.2\ g\cancel{Cl_2} \times \frac{1\,mol\,Cl_2}{70.90\ g\cancel{Cl_2}} \times \frac{2\,mol\,AlCl_3}{3\ mol\,\cancel{Cl_2}} = 1.159\,mol\,AlCl_3$$

The units still cancel appropriately, and we get the same numerical answer in the end. Sometimes the answer may be slightly different from doing it one step at a time because of rounding of the intermediate answers, but the final answers should be effectively the same.

✓ Example 5.9.1

How many moles of HCl will be produced when 249 g of $AlCl_3$ are reacted according to this chemical equation?

$$2AlCl_3 + 3H_2O(\ell) \rightarrow Al_2O_3 + 6HCl(g)$$

Solution

We will do this in two steps: convert the mass of $AlCl_3$ to moles and then use the balanced chemical equation to find the number of moles of HCl formed. The molar mass of $AlCl_3$ is 133.33 g/mol, which we have to invert to get the appropriate conversion factor:

$$249\ \ \cancel{g\,AlCl_3} \times \frac{1\,mol\,AlCl_3}{133.33\ \ \cancel{g\,AlCl_3}} = 1.87\,mol\,AlCl_3$$

Now we can use this quantity to determine the number of moles of HCl that will form. From the balanced chemical equation, we construct a conversion factor between the number of moles of $AlCl_3$ and the number of moles of HCl:

$$frac6\,mol\,HCl2\ \ mol\,\cancel{AlCl_3}$$

Applying this conversion factor to the quantity of $AlCl_3$, we get

$$1.87\ \ mol\,\cancel{AlCl_3} \times \frac{6\,mol\,HCl}{2\ \ mol\,\cancel{AlCl_3}} = 5.61\,mol\,HCl$$

Alternatively, we could have done this in one line:

$$249\ \ \cancel{g\,AlCl_3} \times \frac{1\ \ mol\,\cancel{AlCl_3}}{133.33\ \ \cancel{g\,AlCl_3}} \times \frac{6\,mol\,HCl}{2\ \ mol\,\cancel{AlCl_3}} = 5.60\,mol\,HCl$$

The last digit in our final answer is slightly different because of rounding differences, but the answer is essentially the same.

? Exercise 5.9.1

How many moles of Al_2O_3 will be produced when 23.9 g of H_2O are reacted according to this chemical equation?

$$2AlCl_3 + 3H_2O(\ell) \rightarrow Al_2O_3 + 6HCl(g)$$

Answer

0.442 mol

A variation of the mole-mass calculation is to start with an amount in moles and then determine an amount of another substance in grams. The steps are the same but are performed in reverse order.

✓ Example 5.9.2

How many grams of NH_3 will be produced when 33.9 mol of H_2 are reacted according to this chemical equation?

$$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$$

Solution

The conversions are the same, but they are applied in a different order. Start by using the balanced chemical equation to convert to moles of another substance and then use its molar mass to determine the mass of the final substance. In two steps, we have

$$33.9\ mol\,\cancel{H_2} \times \frac{2\,mol\,NH_3}{3\ mol\,\cancel{H_2}} = 22.6\,mol\,NH_3$$

Now, using the molar mass of NH_3, which is 17.03 g/mol, we get

$$22.6\ mol\ \cancel{NH_3} \times \frac{17.03\ g\ NH_3}{1\ mol\ \cancel{NH_3}} = 385\ g\ NH_3$$

Exercise 5.9.2

How many grams of N_2 are needed to produce 2.17 mol of NH_3 when reacted according to this chemical equation?

$$\mathrm{N_2(g) + 3\,H_2(g) \rightarrow 2\,NH_3(g)}$$

Answer

30.4 g (Note: here we go from a product to a reactant, showing that mole-mass problems can begin and end with any substance in the chemical equation.)

It should be a trivial task now to extend the calculations to **mass-mass calculations**, in which we start with a mass of some substance and end with the mass of another substance in the chemical reaction. For this type of calculation, the molar masses of two different substances must be used—be sure to keep track of which is which. Again, however, it is important to emphasize that before the balanced chemical reaction is used, the mass quantity must first be converted to moles. Then the coefficients of the balanced chemical reaction can be used to convert to moles of another substance, which can then be converted to a mass.

For example, let us determine the number of grams of SO_3 that can be produced by the reaction of 45.3 g of SO_2 and O_2:

$$2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$$

First, we convert the given amount, 45.3 g of SO_2, to moles of SO_2 using its molar mass (64.06 g/mol):

$$45.3\ g\ \cancel{SO_2} \times \frac{1\ mol\ SO_2}{64.06\ g\ \cancel{SO_2}} = 0.707\ mol\ SO_2$$

Second, we use the balanced chemical reaction to convert from moles of SO_2 to moles of SO_3:

$$0.707\ mol\ \cancel{SO_2} \times \frac{2\ mol\ SO_3}{2\ mol\ \cancel{SO_2}} = 0.707\ mol\ SO_3$$

Finally, we use the molar mass of SO_3 (80.06 g/mol) to convert to the mass of SO_3:

$$0.707\ mol\ \cancel{SO_3} \times \frac{80.06\ g\ SO_3}{1\ mol\ \cancel{SO_3}} = 56.6\ g\ SO_3$$

We can also perform all three steps sequentially, writing them on one line as

$$45.3\ g\ \cancel{SO_2} \times \frac{1\ mol\ SO_2}{64.06\ g\ \cancel{SO_2}} \times \frac{2\ mol\ SO_3}{2\ mol\ \cancel{SO_2}} \times \frac{80.06\ g\ SO_3}{1\ mol\ \cancel{SO_3}} = 56.6\ g\ SO_3$$

We get the same answer. Note how the initial and all the intermediate units cancel, leaving grams of SO_3, which is what we are looking for, as our final answer.

Example 5.9.3

What mass of Mg will be produced when 86.4 g of K are reacted?

$$\mathrm{MgCl_2(s) + 2\,K(s) \rightarrow Mg(s) + 2\,KCl(s)}$$

Solution

We will simply follow the steps

mass K → mol K → mol Mg → mass Mg

In addition to the balanced chemical equation, we need the molar masses of K (39.09 g/mol) and Mg (24.31 g/mol). In one line,

$$86.4\ g\,\cancel{K} \times \frac{1\ mol\ K}{39.09\ g\,\cancel{K}} \times \frac{1\ mol\ \cancel{Mg}}{2\ mol\ \cancel{K}} \times \frac{24.31\ g\,Mg}{1\ mol\ \cancel{Mg}} = 26.87\ g\,Mg$$

Exercise 5.9.3

What mass of H_2 will be produced when 122 g of Zn are reacted?

$$\mathrm{Zn(s)} + 2\,\mathrm{HCl(aq)} \rightarrow \mathrm{ZnCl_2(aq)} + \mathrm{H_2(g)}$$

Answer

3.77 g

Summary

- Mole quantities of one substance can be related to mass quantities using a balanced chemical equation.
- Mass quantities of one substance can be related to mass quantities using a balanced chemical equation.
- In all cases, quantities of a substance must be converted to moles before the balanced chemical equation can be used to convert to moles of another substance.

5.10: Molarity

Learning Objectives

- Describe the fundamental properties of solutions
- Calculate solution concentrations using molarity
- Perform dilution calculations using the dilution equation

In preceding sections, we focused on the composition of substances: samples of matter that contain only one type of element or compound. However, mixtures—samples of matter containing two or more substances physically combined—are more commonly encountered in nature than are pure substances. Similar to a pure substance, the relative composition of a mixture plays an important role in determining its properties. The relative amount of oxygen in a planet's atmosphere determines its ability to sustain aerobic life. The relative amounts of iron, carbon, nickel, and other elements in steel (a mixture known as an "alloy") determine its physical strength and resistance to corrosion. The relative amount of the active ingredient in a medicine determines its effectiveness in achieving the desired pharmacological effect. The relative amount of sugar in a beverage determines its sweetness (Figure $5.10.1$). In this section, we will describe one of the most common ways in which the relative compositions of mixtures may be quantified.

Figure $5.10.1$: Sugar is one of many components in the complex mixture known as coffee. The amount of sugar in a given amount of coffee is an important determinant of the beverage's sweetness. (credit: Jane Whitney)

Solutions

We have previously defined solutions as homogeneous mixtures, meaning that the composition of the mixture (and therefore its properties) is uniform throughout its entire volume. Solutions occur frequently in nature and have also been implemented in many forms of manmade technology. We will explore a more thorough treatment of solution properties in the chapter on solutions and colloids, but here we will introduce some of the basic properties of solutions.

The relative amount of a given solution component is known as its concentration. Often, though not always, a solution contains one component with a concentration that is significantly greater than that of all other components. This component is called the solvent and may be viewed as the medium in which the other components are dispersed, or dissolved. Solutions in which water is the solvent are, of course, very common on our planet. A solution in which water is the solvent is called an aqueous solution.

A solute is a component of a solution that is typically present at a much lower concentration than the solvent. Solute concentrations are often described with qualitative terms such as dilute (of relatively low concentration) and concentrated (of relatively high concentration).

Concentrations may be quantitatively assessed using a wide variety of measurement units, each convenient for particular applications. Molarity (M) is a useful concentration unit for many applications in chemistry. Molarity is defined as the number of moles of solute in exactly 1 liter (1 L) of the solution:

$$M=\frac{\text{mol solute}}{\text{L solution}} \tag{5.10.1}$$

Example 5.10.1: Calculating Molar Concentrations

A 355-mL soft drink sample contains 0.133 mol of sucrose (table sugar). What is the molar concentration of sucrose in the beverage?

Solution

Since the molar amount of solute and the volume of solution are both given, the molarity can be calculated using the definition of molarity. Per this definition, the solution volume must be converted from mL to L:

$$\begin{aligned} M &= \frac{mol\ solute}{L\ solution} \\ &= \frac{0.133\ mol}{355\ mL \times \frac{1\ L}{1000\ mL}} \\ &= 0.375\ M \end{aligned}$$

Exercise 5.10.1

A teaspoon of table sugar contains about 0.01 mol sucrose. What is the molarity of sucrose if a teaspoon of sugar has been dissolved in a cup of tea with a volume of 200 mL?

Answer

0.05 M

Example 5.10.2: Deriving Moles and Volumes from Molar Concentrations

How much sugar (mol) is contained in a modest sip (~10 mL) of the soft drink from Example 5.10.1?

Solution

In this case, we can rearrange the definition of molarity to isolate the quantity sought, moles of sugar. We then substitute the value for molarity that we derived in Example 3.4.2, 0.375 *M*:

$$M = \frac{\text{mol solute}}{\text{L solution}} \tag{5.10.2}$$

$$\begin{aligned} \text{mol solute } &= \text{M} \times \text{L solution} \\ \text{mol solute } &= 0.375\ \frac{\text{mol sugar}}{\text{L}} \times \left(10\ \text{mL} \times \frac{1\ \text{L}}{1000\ \text{mL}}\right) \qquad = 0.004\ \text{mol sugar} \end{aligned}$$

Exercise 5.10.2

What volume (mL) of the sweetened tea described in Example 5.10.1 contains the same amount of sugar (mol) as 10 mL of the soft drink in this example?

Answer

80 mL

Example 5.10.3: Calculating Molar Concentrations from the Mass of Solute

Distilled white vinegar (Figure 5.10.2) is a solution of acetic acid, CH_3CO_2H, in water. A 0.500-L vinegar solution contains 25.2 g of acetic acid. What is the concentration of the acetic acid solution in units of molarity?

Figure $5.10.3$: Distilled white vinegar is a solution of acetic acid in water. A label on a container is shown.

The label has a picture of a salad with the words “Distilled White Vinegar,” and, “Reduced with water to 5% acidity,” written above it.

Solution

As in previous examples, the definition of molarity is the primary equation used to calculate the quantity sought. In this case, the mass of solute is provided instead of its molar amount, so we must use the solute’s molar mass to obtain the amount of solute in moles:

$$M = \frac{\text{mol solute}}{\text{L solution}} = \frac{25.2 \text{ g } \mathrm{CH_3CO_2H} \times \frac{1 \text{ mol } \mathrm{CH_3CO_2H}}{60.052 \text{ g } \mathrm{CH_3CO_2H}}}{0.500 \text{ L solution}} = 0.839\ M \tag{5.10.3}$$

$$M = \frac{0.839 \text{ mol solute}}{1.00 \text{ L solution}}$$

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$$M = \frac{\text{mol solute}}{\text{L solution}} = 0.839\ M \tag{5.10.4}$$

? Exercise $5.10.3$

Calculate the molarity of 6.52 g of $CoCl_2$ (128.9 g/mol) dissolved in an aqueous solution with a total volume of 75.0 mL.

Answer

0.674 M

✓ Example $5.10.4$: Determining the Mass of Solute in a Given Volume of Solution

How many grams of NaCl are contained in 0.250 L of a 5.30-*M* solution?

Solution

The volume and molarity of the solution are specified, so the amount (mol) of solute is easily computed as demonstrated in Example $5.10.3$:

$$M = \frac{\text{mol solute}}{\text{L solution}} \tag{5.10.5}$$

$$\text{mol solute} = M \times \text{L solution} \tag{5.10.6}$$

$$\text{mol solute} = 5.30\,\frac{\text{mol NaCl}}{\text{L}} \times 0.250\text{ L} = 1.325\text{ mol NaCl} \tag{5.10.7}$$

Finally, this molar amount is used to derive the mass of NaCl:

$$1.325\text{ mol NaCl} \times \frac{58.44\text{ g NaCl}}{\text{mol NaCl}} = 77.4\text{ g NaCl} \tag{5.10.8}$$

? Exercise 5.10.4

How many grams of $CaCl_2$ (110.98 g/mol) are contained in 250.0 mL of a 0.200-M solution of calcium chloride?

Answer

5.55 g $CaCl_2$

When performing calculations stepwise, as in Example 5.10.3, it is important to refrain from rounding any intermediate calculation results, which can lead to rounding errors in the final result. In Example 5.10.4, the molar amount of NaCl computed in the first step, 1.325 mol, would be properly rounded to 1.32 mol if it were to be reported; however, although the last digit (5) is not significant, it must be retained as a guard digit in the intermediate calculation. If we had not retained this guard digit, the final calculation for the mass of NaCl would have been 77.1 g, a difference of 0.3 g.

In addition to retaining a guard digit for intermediate calculations, we can also avoid rounding errors by performing computations in a single step (Example 5.10.5). This eliminates intermediate steps so that only the final result is rounded.

✓ Example 5.10.5: Determining the Volume of Solution

In Example 5.10.3, we found the typical concentration of vinegar to be 0.839 *M*. What volume of vinegar contains 75.6 g of acetic acid?

Solution

First, use the molar mass to calculate moles of acetic acid from the given mass:

$$\text{g solute} \times \frac{\text{mol solute}}{\text{g solute}} = \text{mol solute} \tag{5.10.9}$$

Then, use the molarity of the solution to calculate the volume of solution containing this molar amount of solute:

$$\text{mol solute} \times \frac{\text{L solution}}{\text{mol solute}} = \text{L solution} \tag{5.10.10}$$

Combining these two steps into one yields:

$$\text{g solute} \times \frac{\text{mol solute}}{\text{g solute}} \times \frac{\text{L solution}}{\text{mol solute}} = \text{L solution} \tag{5.10.11}$$

$$75.6\text{ g }CH_3CO_2H\left(\frac{\text{mol }CH_3CO_2H}{60.05\text{ g}}\right)\left(\frac{\text{L solution}}{0.839\text{ mol }CH_3CO_2H}\right) = 1.50\text{ L solution} \tag{5.10.12}$$

? Exercise 5.10.5:

What volume of a 1.50-M KBr solution contains 66.0 g KBr?

Answer

0.370 L

Dilution of Solutions

Dilution is the process whereby the concentration of a solution is lessened by the addition of solvent. For example, we might say that a glass of iced tea becomes increasingly diluted as the ice melts. The water from the melting ice increases the volume of the solvent (water) and the overall volume of the solution (iced tea), thereby reducing the relative concentrations of the solutes that give the beverage its taste (Figure $5.10.2$).

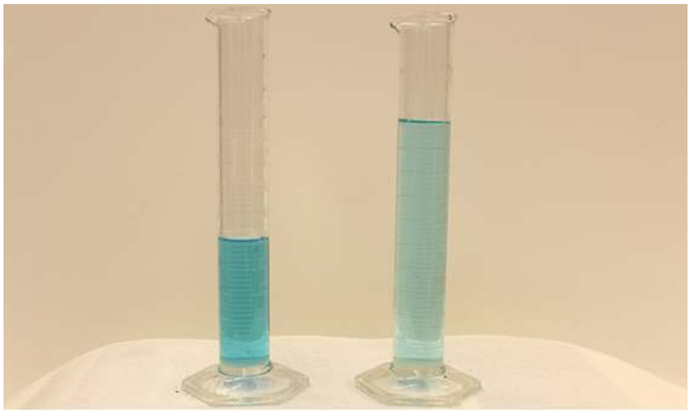

Figure $5.10.2$: Both solutions contain the same mass of copper nitrate. The solution on the right is more dilute because the copper nitrate is dissolved in more solvent. (credit: Mark Ott). This figure shows two graduated cylinders side-by-side. The first has about half as much blue liquid as the second. The blue liquid is darker in the first cylinder than in the second.

Dilution is also a common means of preparing solutions of a desired concentration. By adding solvent to a measured portion of a more concentrated *stock solution*, we can achieve a particular concentration. For example, commercial pesticides are typically sold as solutions in which the active ingredients are far more concentrated than is appropriate for their application. Before they can be used on crops, the pesticides must be diluted. This is also a very common practice for the preparation of a number of common laboratory reagents (Figure $5.10.3$).

A simple mathematical relationship can be used to relate the volumes and concentrations of a solution before and after the dilution process. According to the definition of molarity, the molar amount of solute in a solution is equal to the product of the solution's molarity and its volume in liters:

$$n = ML$$

Expressions like these may be written for a solution before and after it is diluted:

$$n_1 = M_1 L_1$$

$$n_2 = M_2 L_2$$

where the subscripts "1" and "2" refer to the solution before and after the dilution, respectively. Since the dilution process *does not change the amount of solute in the solution,*$n_1 = n_2$. Thus, these two equations may be set equal to one another:

$$M_1 L_1 = M_2 L_2$$

This relation is commonly referred to as the dilution equation. Although we derived this equation using molarity as the unit of concentration and liters as the unit of volume, other units of concentration and volume may be used, so long as the units properly cancel per the factor-label method. Reflecting this versatility, the dilution equation is often written in the more general form:

$$C_1 V_1 = C_2 V_2$$

where C and V are concentration and volume, respectively.

✓ Example $5.10.6$: Determining the Concentration of a Diluted Solution

If 0.850 L of a 5.00-*M* solution of copper nitrate, $Cu(NO_3)_2$, is diluted to a volume of 1.80 L by the addition of water, what is the molarity of the diluted solution?

Solution

We are given the volume and concentration of a stock solution, V_1 and C_1, and the volume of the resultant diluted solution, V_2. We need to find the concentration of the diluted solution, C_2. We thus rearrange the dilution equation in order to isolate C_2:

$$C_1V_1 = C_2V_2$$

$$C_2 = \frac{C_1V_1}{V_2}$$

Since the stock solution is being diluted by more than two-fold (volume is increased from 0.85 L to 1.80 L), we would expect the diluted solution's concentration to be less than one-half 5 *M*. We will compare this ballpark estimate to the calculated result to check for any gross errors in computation (for example, such as an improper substitution of the given quantities). Substituting the given values for the terms on the right side of this equation yields:

$$C_2 = \frac{0.850\ \text{L} \times 5.00\ \frac{\text{mol}}{\text{L}}}{1.80\ \text{L}} = 2.36\ M$$

This result compares well to our ballpark estimate (it's a bit less than one-half the stock concentration, 5 *M*).

? Exercise 5.10.6

What is the concentration of the solution that results from diluting 25.0 mL of a 2.04-M solution of CH3OH to 500.0 mL?

Answer

0.102 M CH_3OH

✓ Example 5.10.7: Volume of a Diluted Solution

What volume of 0.12 *M* HBr can be prepared from 11 mL (0.011 L) of 0.45 *M* HBr?

Solution

We are given the volume and concentration of a stock solution, V_1 and C_1, and the concentration of the resultant diluted solution, C_2. We need to find the volume of the diluted solution, V_2. We thus rearrange the dilution equation in order to isolate V_2:

$$C_1V_1 = C_2V_2$$

$$V_2 = \frac{C_1V_1}{C_2}$$

Since the diluted concentration (0.12 *M*) is slightly more than one-fourth the original concentration (0.45 *M*), we would expect the volume of the diluted solution to be roughly four times the original volume, or around 44 mL. Substituting the given values and solving for the unknown volume yields:

$$V_2 = \frac{(0.45\ M)(0.011\ \text{L})}{(0.12\ M)}$$

$$V_2 = 0.041\ \text{L}$$

The volume of the 0.12-*M* solution is 0.041 L (41 mL). The result is reasonable and compares well with our rough estimate.

? Exercise 5.10.7

A laboratory experiment calls for 0.125 M HNO_3. What volume of 0.125 M HNO_3 can be prepared from 0.250 L of 1.88 M HNO_3?

Answer

3.76 L

✓ Example 5.10.8: Volume of a Concentrated Solution Needed for Dilution

What volume of 1.59 *M* KOH is required to prepare 5.00 L of 0.100 *M* KOH?

Solution

We are given the concentration of a stock solution, C_1, and the volume and concentration of the resultant diluted solution, V_2 and C_2. We need to find the volume of the stock solution, V_1. We thus rearrange the dilution equation in order to isolate V_1:

$$C_1V_1 = C_2V_2$$

$$V_1 = \frac{C_2V_2}{C_1}$$

Since the concentration of the diluted solution 0.100 *M* is roughly one-sixteenth that of the stock solution (1.59 *M*), we would expect the volume of the stock solution to be about one-sixteenth that of the diluted solution, or around 0.3 liters. Substituting the given values and solving for the unknown volume yields:

$$V_1 = \frac{(0.100\ M)(5.00\ \mathrm{L})}{1.59\ M}$$

$$V_1 = 0.314\ \mathrm{L}$$

Thus, we would need 0.314 L of the 1.59-*M* solution to prepare the desired solution. This result is consistent with our rough estimate.

? Exercise 5.10.8

What volume of a 0.575-M solution of glucose, $C_6H_{12}O_6$, can be prepared from 50.00 mL of a 3.00-M glucose solution?

Answer

0.261

Summary

Solutions are homogeneous mixtures. Many solutions contain one component, called the solvent, in which other components, called solutes, are dissolved. An aqueous solution is one for which the solvent is water. The concentration of a solution is a measure of the relative amount of solute in a given amount of solution. Concentrations may be measured using various units, with one very useful unit being molarity, defined as the number of moles of solute per liter of solution. The solute concentration of a solution may be decreased by adding solvent, a process referred to as dilution. The dilution equation is a simple relation between concentrations and volumes of a solution before and after dilution.

Key Equations

- $M = \frac{\text{mol solute}}{\text{L solution}}$
- $C_1V_1 = C_2V_2$

Glossary

aqueous solution

solution for which water is the solvent

concentrated

qualitative term for a solution containing solute at a relatively high concentration

concentration
quantitative measure of the relative amounts of solute and solvent present in a solution

dilute
qualitative term for a solution containing solute at a relatively low concentration

dilution
process of adding solvent to a solution in order to lower the concentration of solutes

dissolved
describes the process by which solute components are dispersed in a solvent

molarity (*M*)
unit of concentration, defined as the number of moles of solute dissolved in 1 liter of solution

solute
solution component present in a concentration less than that of the solvent

solvent
solution component present in a concentration that is higher relative to other components

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5.11: Composition of Substances and Solutions (Exercises)

3.1: Formula Mass and the Mole Concept

What is the total mass (amu) of carbon in each of the following molecules?

1. (a) CH_4
2. (b) $CHCl_3$
3. (c) $C_{12}H_{10}O_6$
4. (d) $CH_3CH_2CH_2CH_2CH_3$

(a) 12.01 amu; (b) 12.01 amu; (c) 144.12 amu; (d) 60.05 amu

What is the total mass of hydrogen in each of the molecules?

1. (a) CH_4
2. (b) $CHCl_3$
3. (c) $C_{12}H_{10}O_6$
4. (d) $CH_3CH_2CH_2CH_2CH_3$

Calculate the molecular or formula mass of each of the following:

(a) P_4

(b) H_2O

(c) $Ca(NO_3)_2$

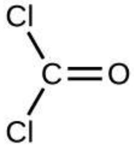
(d) CH_3CO_2H (acetic acid)

(e) $C_{12}H_{22}O_{11}$ (sucrose, cane sugar).

(a) 123.896 amu; (b) 18.015 amu; (c) 164.086 amu; (d) 60.052 amu; (e) 342.297 amu

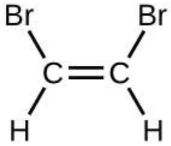
Determine the molecular mass of the following compounds:

(a)

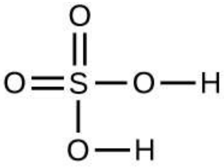


(b)

(c)

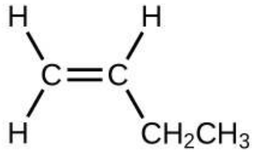


(d)

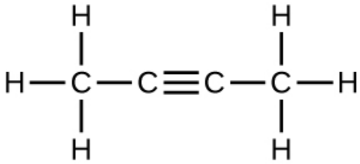


Determine the molecular mass of the following compounds:

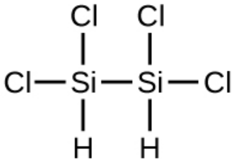
(a)



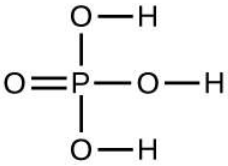
(b)



(c)



(d)

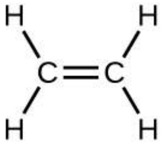


1. (a) 56.107 amu;
2. (b) 54.091 amu;
3. (c) 199.9976 amu;
4. (d) 97.9950 amu

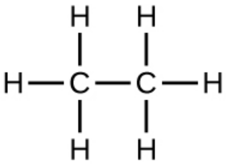
Which molecule has a molecular mass of 28.05 amu?

(a)

(b)



(c)



Write a sentence that describes how to determine the number of moles of a compound in a known mass of the compound if we know its molecular formula.

Use the molecular formula to find the molar mass; to obtain the number of moles, divide the mass of compound by the molar mass of the compound expressed in grams.

Compare 1 mole of H_2, 1 mole of O_2, and 1 mole of F_2.

1. (a) Which has the largest number of molecules? Explain why.
2. (b) Which has the greatest mass? Explain why.

Which contains the greatest mass of oxygen: 0.75 mol of ethanol (C_2H_5OH), 0.60 mol of formic acid (HCO_2H), or 1.0 mol of water (H_2O)? Explain why.

Formic acid. Its formula has twice as many oxygen atoms as the other two compounds (one each). Therefore, 0.60 mol of formic acid would be equivalent to 1.20 mol of a compound containing a single oxygen atom.

Which contains the greatest number of moles of oxygen atoms: 1 mol of ethanol (C_2H_5OH), 1 mol of formic acid (HCO_2H), or 1 mol of water (H_2O)? Explain why.

How are the molecular mass and the molar mass of a compound similar and how are they different?

The two masses have the same numerical value, but the units are different: The molecular mass is the mass of 1 molecule while the molar mass is the mass of 6.022×10^{23} molecules.

Calculate the molar mass of each of the following compounds:

1. (a) hydrogen fluoride, HF
2. (b) ammonia, NH_3
3. (c) nitric acid, HNO_3
4. (d) silver sulfate, Ag_2SO_4
5. (e) boric acid, $B(OH)_3$

Calculate the molar mass of each of the following:

1. (a) S_8
2. (b) C_5H_{12}
3. (c) $Sc_2(SO_4)_3$
4. (d) CH_3COCH_3 (acetone)
5. (e) $C_6H_{12}O_6$ (glucose)

(a) 256.528 g/mol; (b) 72.150 g mol^{-1}; (c) 378.103 g mol^{-1}; (d) 58.080 g mol^{-1}; (e) 180.158 g mol^{-1}

Calculate the empirical or molecular formula mass and the molar mass of each of the following minerals:

1. (a) limestone, $CaCO_3$
2. (b) halite, $NaCl$
3. (c) beryl, $Be_3Al_2Si_6O_{18}$
4. (d) malachite, $Cu_2(OH)_2CO_3$
5. (e) turquoise, $CuAl_6(PO_4)_4(OH)_8(H_2O)_4$

Calculate the molar mass of each of the following:

1. (a) the anesthetic halothane, $C_2HBrClF_3$
2. (b) the herbicide paraquat, $C_{12}H_{14}N_2Cl_2$
3. (c) caffeine, $C_8H_{10}N_4O_2$
4. (d) urea, $CO(NH_2)_2$
5. (e) a typical soap, $C_{17}H_{35}CO_2Na$

(a) 197.382 g mol^{-1}; (b) 257.163 g mol^{-1}; (c) 194.193 g mol^{-1}; (d) 60.056 g mol^{-1}; (e) 306.464 g mol^{-1}

Determine the number of moles of compound and the number of moles of each type of atom in each of the following:

1. (a) 25.0 g of propylene, C_3H_6
2. (b) 3.06×10^{-3} g of the amino acid glycine, $C_2H_5NO_2$
3. (c) 25 lb of the herbicide Treflan, $C_{13}H_{16}N_2O_4F$ (1 lb = 454 g)
4. (d) 0.125 kg of the insecticide Paris Green, $Cu_4(AsO_3)_2(CH_3CO_2)_2$

5. (e) 325 mg of aspirin, $C_6H_4(CO_2H)(CO_2CH_3)$

Determine the mass of each of the following:

1. (a) 0.0146 mol KOH
2. (b) 10.2 mol ethane, C_2H_6
3. (c) 1.6×10^{-3} mol Na_2SO_4
4. (d) 6.854×10^3 mol glucose, $C_6H_{12}O_6$
5. (e) 2.86 mol $Co(NH_3)_6Cl_3$

1. (a) 0.819 g;
2. (b) 307 g;
3. (c) 0.23 g;
4. (d) 1.235×10^6 g (1235 kg);
5. (e) 765 g

Determine the number of moles of the compound and determine the number of moles of each type of atom in each of the following:

1. (a) 2.12 g of potassium bromide, KBr
2. (b) 0.1488 g of phosphoric acid, H_3PO_4
3. (c) 23 kg of calcium carbonate, $CaCO_3$
4. (d) 78.452 g of aluminum sulfate, $Al_2(SO_4)_3$
5. (e) 0.1250 mg of caffeine, $C_8H_{10}N_4O_2$

Determine the mass of each of the following:

1. (a) 2.345 mol LiCl
2. (b) 0.0872 mol acetylene, C_2H_2
3. (c) 3.3×10^{-2} mol Na_2CO_3
4. (d) 1.23×10^3 mol fructose, $C_6H_{12}O_6$
5. (e) 0.5758 mol $FeSO_4(H_2O)_7$

1. (a) 99.41 g;
2. (b) 2.27 g;
3. (c) 3.5 g;
4. (d) 222 kg;
5. (e) 160.1 g

The approximate minimum daily dietary requirement of the amino acid leucine, $C_6H_{13}NO_2$, is 1.1 g. What is this requirement in moles?

Determine the mass in grams of each of the following:

1. (a) 0.600 mol of oxygen atoms
2. (b) 0.600 mol of oxygen molecules, O_2
3. (c) 0.600 mol of ozone molecules, O_3

(a) 9.60 g; (b) 19.2 g; (c) 28.8 g

A 55-kg woman has 7.5×10^{-3} mol of hemoglobin (molar mass = 64,456 g/mol) in her blood. How many hemoglobin molecules is this? What is this quantity in grams?

Determine the number of atoms and the mass of zirconium, silicon, and oxygen found in 0.3384 mol of zircon, $ZrSiO_4$, a semiprecious stone.

zirconium: 2.038×10^{23} atoms; 30.87 g; silicon: 2.038×10^{23} atoms; 9.504 g; oxygen: 8.151×10^{23} atoms; 21.66 g

Determine which of the following contains the greatest mass of hydrogen: 1 mol of CH_4, 0.6 mol of C_6H_6, or 0.4 mol of C_3H_8.

Determine which of the following contains the greatest mass of aluminum: 122 g of $AlPO_4$, 266 g of Al_2Cl_6, or 225 g of Al_2S_3.

$AlPO_4$: 1.000 mol

Al_2Cl_6: 1.994 mol

Al_2S_3: 3.00 mol

Diamond is one form of elemental carbon. An engagement ring contains a diamond weighing 1.25 carats (1 carat = 200 mg). How many atoms are present in the diamond?

The Cullinan diamond was the largest natural diamond ever found (January 25, 1905). It weighed 3104 carats (1 carat = 200 mg). How many carbon atoms were present in the stone?

3.113×10^{25} C atoms

One 55-gram serving of a particular cereal supplies 270 mg of sodium, 11% of the recommended daily allowance. How many moles and atoms of sodium are in the recommended daily allowance?

A certain nut crunch cereal contains 11.0 grams of sugar (sucrose, $C_{12}H_{22}O_{11}$) per serving size of 60.0 grams. How many servings of this cereal must be eaten to consume 0.0278 moles of sugar?

0.865 servings, or about 1 serving.

A tube of toothpaste contains 0.76 g of sodium monofluorophosphate (Na_2PO_3F) in 100 mL.

a. What mass of fluorine atoms in mg was present?
b. How many fluorine atoms were present?

Which of the following represents the least number of molecules?

a. 20.0 g of H_2O (18.02 g/mol)
b. 77.0 g of CH_4 (16.06 g/mol)
c. 68.0 g of CaH_2 (42.09 g/mol)
d. 100.0 g of N_2O (44.02 g/mol)
e. 84.0 g of HF (20.01 g/mol)

20.0 g H_2O represents the least number of molecules since it has the least number of moles.

3.2: Determining Empirical and Molecular Formulas

What information do we need to determine the molecular formula of a compound from the empirical formula?

Calculate the following to four significant figures:

a. (a) the percent composition of ammonia, NH_3
b. (b) the percent composition of photographic “hypo,” $Na_2S_2O_3$
c. (c) the percent of calcium ion in $Ca_3(PO_4)_2$

(a) % N = 82.24%

% H = 17.76%;

(b) % Na = 29.08%

% S = 40.56%

% O = 30.36%;

(c) % Ca^{2+} = 38.76%

Determine the following to four significant figures:

a. the percent composition of hydrazoic acid, HN_3
b. the percent composition of TNT, $C_6H_2(CH_3)(NO_2)_3$
c. the percent of SO_4^{2-} in $Al_2(SO_4)_3$

Determine the percent ammonia, NH_3, in $Co(NH_3)_6Cl_3$, to three significant figures.

% NH_3 = 38.2%

Determine the percent water in $CuSO_4{\cdot}5H_2O$ to three significant figures.

Determine the empirical formulas for compounds with the following percent compositions:

(a)15.8% carbon and 84.2% sulfur

(b) 40.0% carbon, 6.7% hydrogen, and 53.3% oxygen

(a) CS_2

(b) CH_2O

Determine the empirical formulas for compounds with the following percent compositions:

(a) 43.6% phosphorus and 56.4% oxygen

(b) 28.7% K, 1.5% H, 22.8% P, and 47.0% O

A compound of carbon and hydrogen contains 92.3% C and has a molar mass of 78.1 g/mol. What is its molecular formula?

C_6H_6

Dichloroethane, a compound that is often used for dry cleaning, contains carbon, hydrogen, and chlorine. It has a molar mass of 99 g/mol. Analysis of a sample shows that it contains 24.3% carbon and 4.1% hydrogen. What is its molecular formula?

Determine the empirical and molecular formula for chrysotile asbestos. Chrysotile has the following percent composition: 28.03% Mg, 21.60% Si, 1.16% H, and 49.21% O. The molar mass for chrysotile is 520.8 g/mol.

$Mg_3Si_2H_3O_8$ (empirical formula), $Mg_6Si_4H_6O_{16}$ (molecular formula)

Polymers are large molecules composed of simple units repeated many times. Thus, they often have relatively simple empirical formulas. Calculate the empirical formulas of the following polymers:

a. Lucite (Plexiglas); 59.9% C, 8.06% H, 32.0% O
b. Saran; 24.8% C, 2.0% H, 73.1% Cl
c. polyethylene; 86% C, 14% H
d. polystyrene; 92.3% C, 7.7% H
e. Orlon; 67.9% C, 5.70% H, 26.4% N

A major textile dye manufacturer developed a new yellow dye. The dye has a percent composition of 75.95% C, 17.72% N, and 6.33% H by mass with a molar mass of about 240 g/mol. Determine the molecular formula of the dye.

$C_{15}H_{15}N_3$

3.3: Molarity

Questions

Explain what changes and what stays the same when 1.00 L of a solution of NaCl is diluted to 1.80 L.

What information do we need to calculate the molarity of a sulfuric acid solution?

We need to know the number of moles of sulfuric acid dissolved in the solution and the volume of the solution.

What does it mean when we say that a 200-mL sample and a 400-mL sample of a solution of salt have the same molarity? In what ways are the two samples identical? In what ways are these two samples different?

Determine the molarity for each of the following solutions:

a. 0.444 mol of $CoCl_2$ in 0.654 L of solution
b. 98.0 g of phosphoric acid, H_3PO_4, in 1.00 L of solution
c. 0.2074 g of calcium hydroxide, $Ca(OH)_2$, in 40.00 mL of solution
d. 10.5 kg of $Na_2SO_4{\cdot}10H_2O$ in 18.60 L of solution
e. 7.0×10^{-3} mol of I_2 in 100.0 mL of solution
f. 1.8×10^4 mg of HCl in 0.075 L of solution

a. (a) 0.679 *M*;
b. (b) 1.00 *M*;
c. (c) 0.06998 *M*;
d. (d) 1.75 *M*;
e. (e) 0.070 *M*;

f. (f) 6.6 M

Determine the molarity of each of the following solutions:

a. 1.457 mol KCl in 1.500 L of solution
b. 0.515 g of H_2SO_4 in 1.00 L of solution
c. 20.54 g of $Al(NO_3)_3$ in 1575 mL of solution
d. 2.76 kg of $CuSO_4{\cdot}5H_2O$ in 1.45 L of solution
e. 0.005653 mol of Br_2 in 10.00 mL of solution
f. 0.000889 g of glycine, $C_2H_5NO_2$, in 1.05 mL of solution

Answers:

a.) 0.9713 M

b.) $5.25\text{x}10^{-3}$ M

c.) $6.122\text{x}10^{-2}$ M

d.) 7.62 M

e.) 0.5653 M

f.) $1.13\text{x}10^{-2}$ M

Consider this question: What is the mass of the solute in 0.500 L of 0.30 M glucose, $C_6H_{12}O_6$, used for intravenous injection?

(a) Outline the steps necessary to answer the question.

(b) Answer the question.

(a) determine the number of moles of glucose in 0.500 L of solution; determine the molar mass of glucose; determine the mass of glucose from the number of moles and its molar mass; (b) 27 g

Consider this question: What is the mass of solute in 200.0 L of a 1.556-M solution of KBr?

1. (a) Outline the steps necessary to answer the question.
2. (b) Answer the question.

Answer:

(a)

1. Calculate to moles of KBr by multiplying the Molarity by the amount of solution (200.0 L)
2. Find the Molar Mass of KBr and convert moles of solute to grams

(b)

$$\frac{1.556\ moles\ \text{KBr}}{1\ \cancel{L}} \times 200.0\ \cancel{L} = 311.2\ moles\ \text{KBr}$$

$$311.2\ \cancel{moles}\ \text{KBr} \times \frac{119.0\ g\ \text{KBr}}{1\ \cancel{mole}\ \text{KBr}} = 37,030\ g$$

37,030g; 37.03 kg

Calculate the number of moles and the mass of the solute in each of the following solutions:

1. (a) 2.00 L of 18.5 M H_2SO_4, concentrated sulfuric acid
2. (b) 100.0 mL of 3.8×10^{-5} M NaCN, the minimum lethal concentration of sodium cyanide in blood serum
3. (c) 5.50 L of 13.3 M H_2CO, the formaldehyde used to “fix” tissue samples
4. (d) 325 mL of 1.8×10^{-6} M $FeSO_4$, the minimum concentration of iron sulfate detectable by taste in drinking water

(a) 37.0 mol H_2SO_4;

3.63×10^3 g H_2SO_4;

(b) 3.8×10^{-6} mol NaCN;

1.9×10^{-4} g NaCN;

(c) 73.2 mol H_2CO;

2.20 kg H_2CO;

(d) 5.9×10^{-7} mol $FeSO_4$;

8.9×10^{-5} g $FeSO_4$

Calculate the number of moles and the mass of the solute in each of the following solutions:

a. 325 mL of 8.23×10^{-5} *M* KI, a source of iodine in the diet
b. 75.0 mL of 2.2×10^{-5} *M* H_2SO_4, a sample of acid rain
c. 0.2500 L of 0.1135 *M* K_2CrO_4, an analytical reagent used in iron assays
d. 10.5 L of 3.716 *M* $(NH_4)_2SO_4$, a liquid fertilizer

Answers:

a. 2.67×10^{-5} moles KI; 4.44×10^{-3}g KI

b. 1.7×10^{-6} moles H_2SO_4 ; 1.6×10^{-4} g H_2SO_4

c. 2.838×10^{-2} moles K_2CrO_4 ; 5.510g K_2CrO_4

d. 39.0 moles $(NH_4)_2SO_4$; 5,160 g $(NH_4)_2SO_4$

Consider this question: What is the molarity of $KMnO_4$ in a solution of 0.0908 g of $KMnO_4$ in 0.500 L of solution?

1. (a) Outline the steps necessary to answer the question.
2. (b) Answer the question.

(a) Determine the molar mass of $KMnO_4$; determine the number of moles of $KMnO_4$ in the solution; from the number of moles and the volume of solution, determine the molarity; (b) 1.15×10^{-3} *M*

Consider this question: What is the molarity of HCl if 35.23 mL of a solution of HCl contain 0.3366 g of HCl?

1. (a) Outline the steps necessary to answer the question.
2. (b) Answer the question.

Answer:

(a)

1. Convert g of HCl to moles of HCl and convert mL of solution to L of solution
2. Divide moles of HCl by L of solution

(b)

$$0.3366\ \cancel{g}\ \mathrm{HCl} \times \frac{1\ mole\ \mathrm{HCl}}{36.46\ \cancel{g}\ \mathrm{HCl}} = 9.232 \times 10^{-3}\ moles\ \mathrm{HCl}$$

$$35.23\ mL = 0.03523\ L$$

$$\frac{9.232 \times 10^{-3}\ moles\ \mathrm{HCl}}{0.03523\ L} = 0.2621\ M\ \mathrm{HCl}$$

0.2621 M ;

Calculate the molarity of each of the following solutions:

(a) 0.195 g of cholesterol, $C_{27}H_{46}O$, in 0.100 L of serum, the average concentration of cholesterol in human serum

(b) 4.25 g of NH_3 in 0.500 L of solution, the concentration of NH_3 in household ammonia

(c) 1.49 kg of isopropyl alcohol, C_3H_7OH, in 2.50 L of solution, the concentration of isopropyl alcohol in rubbing alcohol

(d) 0.029 g of I_2 in 0.100 L of solution, the solubility of I_2 in water at 20 °C

(a) 5.04×10^{-3} *M*;

(b) 0.499 *M*;

(c) 9.92 *M*;

(d) 1.1×10^{-3} *M*

Calculate the molarity of each of the following solutions:

a. 293 g HCl in 666 mL of solution, a concentrated HCl solution
b. 2.026 g $FeCl_3$ in 0.1250 L of a solution used as an unknown in general chemistry laboratories
c. 0.001 mg Cd^{2+} in 0.100 L, the maximum permissible concentration of cadmium in drinking water
d. 0.0079 g $C_7H_5SNO_3$ in one ounce (29.6 mL), the concentration of saccharin in a diet soft drink.

There is about 1.0 g of calcium, as Ca^{2+}, in 1.0 L of milk. What is the molarity of Ca^{2+} in milk?

0.025 *M*

What volume of a 1.00-*M* $Fe(NO_3)_3$ solution can be diluted to prepare 1.00 L of a solution with a concentration of 0.250 *M*?

If 0.1718 L of a 0.3556-*M* C_3H_7OH solution is diluted to a concentration of 0.1222 *M*, what is the volume of the resulting solution?

0.5000 L

If 4.12 L of a 0.850 *M*-H_3PO_4 solution is be diluted to a volume of 10.00 L, what is the concentration the resulting solution?

What volume of a 0.33-*M* $C_{12}H_{22}O_{11}$ solution can be diluted to prepare 25 mL of a solution with a concentration of 0.025 *M*?

1.9 mL

What is the concentration of the NaCl solution that results when 0.150 L of a 0.556-*M* solution is allowed to evaporate until the volume is reduced to 0.105 L?

What is the molarity of the diluted solution when each of the following solutions is diluted to the given final volume?

a. (a) 1.00 L of a 0.250-*M* solution of $Fe(NO_3)_3$ is diluted to a final volume of 2.00 L
b. (b) 0.5000 L of a 0.1222-*M* solution of C_3H_7OH is diluted to a final volume of 1.250 L
c. (c) 2.35 L of a 0.350-*M* solution of H_3PO_4 is diluted to a final volume of 4.00 L
d. (d) 22.50 mL of a 0.025-*M* solution of $C_{12}H_{22}O_{11}$ is diluted to 100.0 mL

1. (a) 0.125 *M*;
2. (b) 0.04888 *M*;
3. (c) 0.206 *M*;
4. (e) 0.0056 *M*

What is the final concentration of the solution produced when 225.5 mL of a 0.09988-*M* solution of Na_2CO_3 is allowed to evaporate until the solution volume is reduced to 45.00 mL?

A 2.00-L bottle of a solution of concentrated HCl was purchased for the general chemistry laboratory. The solution contained 868.8 g of HCl. What is the molarity of the solution?

11.9 *M*

An experiment in a general chemistry laboratory calls for a 2.00-*M* solution of HCl. How many mL of 11.9 *M* HCl would be required to make 250 mL of 2.00 *M* HCl?

What volume of a 0.20-*M* K_2SO_4 solution contains 57 g of K_2SO_4?

1.6 L

The US Environmental Protection Agency (EPA) places limits on the quantities of toxic substances that may be discharged into the sewer system. Limits have been established for a variety of substances, including hexavalent chromium, which is limited to 0.50 mg/L. If an industry is discharging hexavalent chromium as potassium dichromate ($K_2Cr_2O_7$), what is the maximum permissible molarity of that substance?

3.4: Other Units for Solution Concentrations

Questions

1. Consider this question: What mass of a concentrated solution of nitric acid (68.0% HNO3 by mass) is needed to prepare 400.0 g of a 10.0% solution of HNO3 by mass?
    a. Outline the steps necessary to answer the question.

   b. Answer the question.
2. What mass of a 4.00% NaOH solution by mass contains 15.0 g of NaOH?
3. What mass of solid NaOH (97.0% NaOH by mass) is required to prepare 1.00 L of a 10.0% solution of NaOH by mass? The density of the 10.0% solution is 1.109 g/mL.
4. What mass of HCl is contained in 45.0 mL of an aqueous HCl solution that has a density of 1.19 g cm–3 and contains 37.21% HCl by mass?
5. The hardness of water (hardness count) is usually expressed in parts per million (by mass) of $\mathrm{CaCO_3}$, which is equivalent to milligrams of $\mathrm{CaCO_3}$ per liter of water. What is the molar concentration of Ca^{2+} ions in a water sample with a hardness count of 175 mg $CaCO_3$/L?
6. The level of mercury in a stream was suspected to be above the minimum considered safe (1 part per billion by weight). An analysis indicated that the concentration was 0.68 parts per billion. Assume a density of 1.0 g/mL and calculate the molarity of mercury in the stream.
7. In Canada and the United Kingdom, devices that measure blood glucose levels provide a reading in millimoles per liter. If a measurement of 5.3 mM is observed, what is the concentration of glucose ($C_6H_{12}O_6$) in mg/dL?
8. A throat spray is 1.40% by mass phenol, $\mathrm{C_6H_5OH}$, in water. If the solution has a density of 0.9956 g/mL, calculate the molarity of the solution.
9. Copper(I) iodide (CuI) is often added to table salt as a dietary source of iodine. How many moles of CuI are contained in 1.00 lb (454 g) of table salt containing 0.0100% CuI by mass?
10. A cough syrup contains 5.0% ethyl alcohol, C_2H_5OH, by mass. If the density of the solution is 0.9928 g/mL, determine the molarity of the alcohol in the cough syrup.
11. D5W is a solution used as an intravenous fluid. It is a 5.0% by mass solution of dextrose ($\mathrm{C_6H_{12}O_6}$) in water. If the density of D5W is 1.029 g/mL, calculate the molarity of dextrose in the solution.
12. Find the molarity of a 40.0% by mass aqueous solution of sulfuric acid, $\mathrm{H_2SO_4}$, for which the density is 1.3057 g/mL.

Solutions

1

- (a) The dilution equation can be used, appropriately modified to accommodate mass-based concentration units:

$$\%\,\mathrm{mass}_1 \times \mathrm{mass}_1 = \%\,\mathrm{mass}_2 \times \mathrm{mass}_2 \tag{5.11.1}$$

  This equation can be rearranged to isolate mass_1 and the given quantities substituted into this equation.
- (b) 58.8 g

3. $114\ \mathrm{g}$5. $1.75 \times 10^{-3} M$ 7 $95\ \mathrm{mg/dL}$9 $2.38 \times 10^{-4}\ \mathrm{mol}$ 11 $0.29\mathrm{mol}$

5.12: Other Units for Solution Concentrations

Learning Objectives

- Define the concentration units of mass percentage, volume percentage, mass-volume percentage, parts-per-million (ppm), and parts-per-billion (ppb)
- Perform computations relating a solution’s concentration and its components’ volumes and/or masses using these units

In the previous section, we introduced molarity, a very useful measurement unit for evaluating the concentration of solutions. However, molarity is only one measure of concentration. In this section, we will introduce some other units of concentration that are commonly used in various applications, either for convenience or by convention.

Mass Percentage

Earlier in this chapter, we introduced percent composition as a measure of the relative amount of a given element in a compound. Percentages are also commonly used to express the composition of mixtures, including solutions. The mass percentage of a solution component is defined as the ratio of the component’s mass to the solution’s mass, expressed as a percentage:

$$\text{mass percentage} = \frac{\text{mass of component}}{\text{mass of solution}} \times 100\% \tag{5.12.1}$$

We are generally most interested in the mass percentages of solutes, but it is also possible to compute the mass percentage of solvent.

Mass percentage is also referred to by similar names such as *percent mass*, *percent weight*, *weight/weight percent*, and other variations on this theme. The most common symbol for mass percentage is simply the percent sign, %, although more detailed symbols are often used including %mass, %weight, and (w/w)%. Use of these more detailed symbols can prevent confusion of mass percentages with other types of percentages, such as volume percentages (to be discussed later in this section).

Mass percentages are popular concentration units for consumer products. The label of a typical liquid bleach bottle (Figure $5.12.1$) cites the concentration of its active ingredient, sodium hypochlorite ($NaOCl$), as being 7.4%. A 100.0-g sample of bleach would therefore contain 7.4 g of $NaOCl$.

Figure $5.12.1$: Liquid bleach is an aqueous solution of sodium hypochlorite (NaOCl). This brand has a concentration of 7.4% NaOCl by mass. The sides of two cylindrical containers are shown. Each container’s label is partially visible. The left container’s label reads “Bleach.” The right label contains more information about the product including the phrase, “Contains: Sodium hypochlorite 7.4 %.”

Example $5.12.1$: Calculation of Percent by Mass

A 5.0-g sample of spinal fluid contains 3.75 mg (0.00375 g) of glucose. What is the percent by mass of glucose in spinal fluid?

Solution

The spinal fluid sample contains roughly 4 mg of glucose in 5000 mg of fluid, so the mass fraction of glucose should be a bit less than one part in 1000, or about 0.1%. Substituting the given masses into the equation defining mass percentage yields:

$$\%\ \text{glucose} = \frac{3.75\ \text{mg glucose} \times \frac{1\ \text{g}}{1000\ \text{mg}}}{5.0\ \text{g spinal fluid}} = 0.075\%$$

The computed mass percentage agrees with our rough estimate (it’s a bit less than 0.1%).

> Note that while any mass unit may be used to compute a mass percentage (mg, g, kg, oz, and so on), the same unit must be used for both the solute and the solution so that the mass units cancel, yielding a dimensionless ratio. In this case, we converted the units of solute in the numerator from mg to g to match the units in the denominator. We could just as easily have converted the denominator from g to mg instead. As long as identical mass units are used for both solute and solution, the computed mass percentage will be correct.

? Exercise 5.12.1

A bottle of a tile cleanser contains 135 g of HCl and 775 g of water. What is the percent by mass of HCl in this cleanser?

Answer

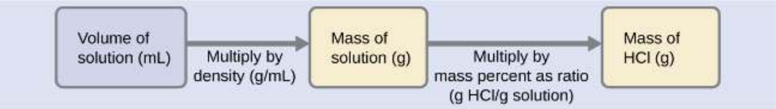
14.8%

✓ Example 5.12.2: Calculations using Mass Percentage

“Concentrated” hydrochloric acid is an aqueous solution of 37.2% HCl that is commonly used as a laboratory reagent. The density of this solution is 1.19 g/mL. What mass of HCl is contained in 0.500 L of this solution?

Solution

The HCl concentration is near 40%, so a 100-g portion of this solution would contain about 40 g of HCl. Since the solution density isn’t greatly different from that of water (1 g/mL), a reasonable estimate of the HCl mass in 500 g (0.5 L) of the solution is about five times greater than that in a 100 g portion, or $5 \times 40 = 200\ \text{g}$. To derive the mass of solute in a solution from its mass percentage, we need to know the corresponding mass of the solution. Using the solution density given, we can convert the solution’s volume to mass, and then use the given mass percentage to calculate the solute mass. This mathematical approach is outlined in this flowchart:



A diagram of three boxes connected by a right-facing arrow in between each is shown. The box on the left contains the phrase, “Volume of solution (m L),” the middle box reads, “Mass of solution (g),” while the one on the right contains the phrase, “Mass of H C l (g).” There is a phrase under the left arrow that says, “Multiply by density (g / m L)” and under the right arrow it states, “Multiply by mass percent as ratio (g H C l / g solution).”

For proper unit cancellation, the 0.500-L volume is converted into 500 mL, and the mass percentage is expressed as a ratio, 37.2 g HCl/g solution:

$$500\ \text{mL solution} \left(\frac{1.19\ \text{g solution}}{\text{mL solution}}\right)\left(\frac{37.2\ \text{g HCl}}{100\ \text{g solution}}\right) = 221\ \text{g HCl}$$

This mass of HCl is consistent with our rough estimate of approximately 200 g.

? Exercise 5.12.2

What volume of concentrated HCl solution contains 125 g of HCl?

Answer

282 mL

Volume Percentage

Liquid volumes over a wide range of magnitudes are conveniently measured using common and relatively inexpensive laboratory equipment. The concentration of a solution formed by dissolving a liquid solute in a liquid solvent is therefore often expressed as a volume percentage, %vol or (v/v)%:

$$\text{volume percentage} = \frac{\text{volume solute}}{\text{volume solution}} \times 100\% \tag{5.12.2}$$

✓ Example 5.12.3: Calculations using Volume Percentage

Rubbing alcohol (isopropanol) is usually sold as a 70%vol aqueous solution. If the density of isopropyl alcohol is 0.785 g/mL, how many grams of isopropyl alcohol are present in a 355 mL bottle of rubbing alcohol?

Solution

Per the definition of volume percentage, the isopropanol volume is 70% of the total solution volume. Multiplying the isopropanol volume by its density yields the requested mass:

$$355 \text{ mL solution}\left(\frac{70 \text{ mL isopropyl alcohol}}{100 \text{ mL solution}}\right)\left(\frac{0.785 \text{ g isopropyl alcohol}}{1 \text{ mL isopropyl alcohol}}\right) = 195 \text{ g isopropyl alcohol}$$

? Exercise 5.12.3

Wine is approximately 12% ethanol (CH_3CH_2OH) by volume. Ethanol has a molar mass of 46.06 g/mol and a density 0.789 g/mL. How many moles of ethanol are present in a 750-mL bottle of wine?

Answer

1.5 mol ethanol

Mass-Volume Percentage

"Mixed" percentage units, derived from the mass of solute and the volume of solution, are popular for certain biochemical and medical applications. A mass-volume percent is a ratio of a solute's mass to the solution's volume expressed as a percentage. The specific units used for solute mass and solution volume may vary, depending on the solution. For example, physiological saline solution, used to prepare intravenous fluids, has a concentration of 0.9% mass/volume (m/v), indicating that the composition is 0.9 g of solute per 100 mL of solution. The concentration of glucose in blood (commonly referred to as "blood sugar") is also typically expressed in terms of a mass-volume ratio. Though not expressed explicitly as a percentage, its concentration is usually given in milligrams of glucose per deciliter (100 mL) of blood (Figure 5.12.2).

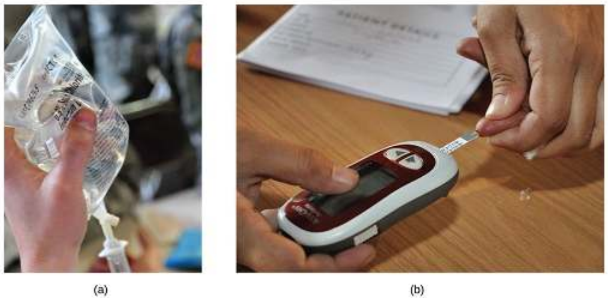


Figure 5.12.2: "Mixed" mass-volume units are commonly encountered in medical settings. (a) The NaCl concentration of physiological saline is 0.9% (m/v). (b) This device measures glucose levels in a sample of blood. The normal range for glucose concentration in blood (fasting) is around 70–100 mg/dL. (credit a: modification of work by "The National Guard"/Flickr; credit b: modification of work by Biswarup Ganguly). Two pictures are shown labeled a and b. Picture a depicts a clear, colorless solution in a plastic bag being held in a person's hand. Picture b shows a person's hand holding a detection meter with a digital readout screen while another hand holds someone's finger up to the end of the meter. The meter is pressed to the drop of blood that is at the end of the person's finger.

Parts per Million and Parts per Billion

Very low solute concentrations are often expressed using appropriately small units such as parts per million (ppm) or parts per billion (ppb). Like percentage ("part per hundred") units, ppm and ppb may be defined in terms of masses, volumes, or mixed mass-volume units. There are also ppm and ppb units defined with respect to numbers of atoms and molecules.

The mass-based definitions of ppm and ppb are given here:

$$\text{ppm} = \frac{\text{mass solute}}{\text{mass solution}} \times 10^6 \text{ ppm} \tag{5.12.3}$$

$$\text{ppb} = \frac{\text{mass solute}}{\text{mass solution}} \times 10^9 \text{ ppb} \tag{5.12.4}$$

Both ppm and ppb are convenient units for reporting the concentrations of pollutants and other trace contaminants in water. Concentrations of these contaminants are typically very low in treated and natural waters, and their levels cannot exceed relatively low concentration thresholds without causing adverse effects on health and wildlife. For example, the EPA has identified the maximum safe level of fluoride ion in tap water to be 4 ppm. Inline water filters are designed to reduce the concentration of fluoride and several other trace-level contaminants in tap water (Figure $5.12.3$).

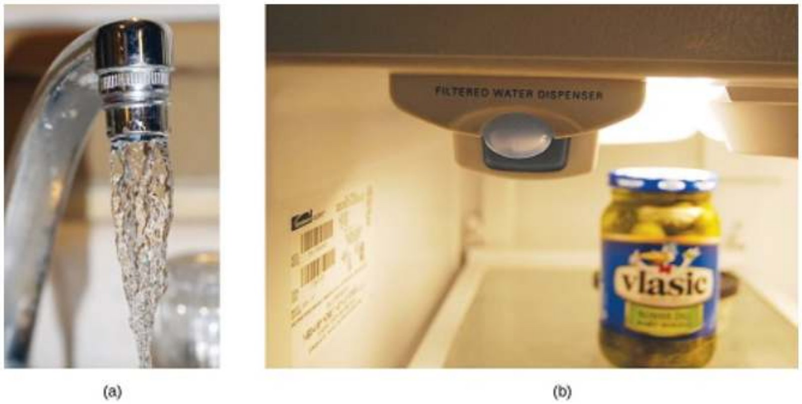


Figure $5.12.3$: (a) In some areas, trace-level concentrations of contaminants can render unfiltered tap water unsafe for drinking and cooking. (b) Inline water filters reduce the concentration of solutes in tap water. (credit a: modification of work by Jenn Durfey; credit b: modification of work by "vastateparkstaff"/Wikimedia commons). Two pictures are shown labeled a and b. Picture a depicts a clear, colorless solution in a plastic bag being held in a person's hand. Picture b shows a person's hand holding a detection meter with a digital readout screen while another hand holds someone's finger up to the end of the meter. The meter is pressed to the drop of blood that is at the end of the person's finger.

✓ Example $5.12.4$: Parts per Million and Parts per Billion Concentrations

According to the EPA, when the concentration of lead in tap water reaches 15 ppb, certain remedial actions must be taken. What is this concentration in ppm? At this concentration, what mass of lead (μg) would be contained in a typical glass of water (300 mL)?

Solution

The definitions of the ppm and ppb units may be used to convert the given concentration from ppb to ppm. Comparing these two unit definitions shows that ppm is 1000 times greater than ppb (1 ppm = 10^3 ppb). Thus:

$$15 \ \cancel{\text{ppb}} \times \frac{1 \text{ ppm}}{10^3 \ \cancel{\text{ppb}}} = 0.015 \text{ ppm}$$

The definition of the ppb unit may be used to calculate the requested mass if the mass of the solution is provided. However, only the volume of solution (300 mL) is given, so we must use the density to derive the corresponding mass. We can assume the density of tap water to be roughly the same as that of pure water (~1.00 g/mL), since the concentrations of any dissolved substances should not be very large. Rearranging the equation defining the ppb unit and substituting the given quantities yields:

$$\text{ppb} = \frac{\text{mass solute}}{\text{mass solution}} \times 10^9 \text{ ppb}$$

$$\text{mass solute} = \frac{\text{ppb} \times \text{mass solution}}{10^9 \text{ ppb}}$$

$$\text{mass solute} = \frac{15 \text{ ppb} \times 300 \text{ mL} \times \frac{1.00 \text{ g}}{\text{mL}}}{10^9 \text{ ppb}} = 4.5 \times 10^{-6} \text{ g}$$

Finally, convert this mass to the requested unit of micrograms:

$$4.5 \times 10^{-6} \text{ g} \times \frac{1 \ \mu\text{g}}{10^{-6} \text{ g}} = 4.5 \ \mu\text{g}$$

? Exercise 5.12.4

A 50.0-g sample of industrial wastewater was determined to contain 0.48 mg of mercury. Express the mercury concentration of the wastewater in ppm and ppb units.

Answer

9.6 ppm, 9600 ppb

Summary

In addition to molarity, a number of other solution concentration units are used in various applications. Percentage concentrations based on the solution components’ masses, volumes, or both are useful for expressing relatively high concentrations, whereas lower concentrations are conveniently expressed using ppm or ppb units. These units are popular in environmental, medical, and other fields where mole-based units such as molarity are not as commonly used.

Glossary

mass percentage
ratio of solute-to-solution mass expressed as a percentage

mass-volume percent
ratio of solute mass to solution volume, expressed as a percentage

parts per billion (ppb)
ratio of solute-to-solution mass multiplied by 10^9

parts per million (ppm)
ratio of solute-to-solution mass multiplied by 10^6

volume percentage
ratio of solute-to-solution volume expressed as a percentage

5.13: End of Chapter Key Terms

Density, Mole, and Molarity Key Terms

1. **Density**: The mass of a substance per unit volume, often measured in grams per cubic centimeter (g/cm^3) or kilograms per cubic meter (kg/m^3).
2. **Mass**: The amount of matter in an object, usually measured in grams (g) or kilograms (kg).
3. **Volume**: The amount of space occupied by an object or substance, often measured in liters (L), cubic meters (m^3), or cubic centimeters (cm^3).
4. **Specific Gravity**: The ratio of the density of a substance to the density of a reference substance, typically water.
5. **Archimedes' Principle**: A principle stating that an object submerged in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the object.
6. **Buoyancy**: The upward force exerted by a fluid on a submerged or partially submerged object.
7. **Mole (mol)**: The SI unit for the amount of substance, defined as the amount of substance that contains as many entities (atoms, molecules, ions, etc.) as there are atoms in 12 grams of carbon-12 (approximately 6.022×10236.022 \times 10^{23}6.022×1023 entities).
8. **Avogadro's Number**: The number of atoms, molecules, or particles in one mole of a substance, 6.022×10236.022 \times 10^{23}6.022×1023.
9. **Molar Mass**: The mass of one mole of a substance, usually expressed in grams per mole (g/mol).
10. **Molar Volume**: The volume occupied by one mole of a substance, typically measured for gases at standard temperature and pressure (STP).
11. **Standard Temperature and Pressure (STP)**: Standard conditions for measuring gases, defined as a temperature of 0°C (273.15 K) and a pressure of 1 atm.
12. **Concentration**: The amount of a substance in a given volume of solution, typically expressed as molarity (M).
13. **Molarity (M)**: A unit of concentration, defined as the number of moles of solute per liter of solution.
14. **Molality (m)**: A unit of concentration, defined as the number of moles of solute per kilogram of solvent.
15. **Parts per Million (ppm)**: A unit of concentration, defined as the number of parts of solute per million parts of solution, often used for very dilute solutions.
16. **Dilution**: The process of reducing the concentration of a solute in solution, usually by adding more solvent.
17. **Stock Solution**: A concentrated solution that can be diluted to a lower concentration for actual use.
18. **Dilution Formula**: The equation C1V1=C2V2C_1V_1 = C_2V_2C1V1=C2V2, where C1C_1C1 and V1V_1V1 are the concentration and volume of the stock solution, and C2C_2C2 and V2V_2V2 are the concentration and volume of the diluted solution.
19. **Equivalent Weight**: The mass of a substance that will react with or supply one mole of hydrogen ions (H^+) or hydroxide ions (OH^-) in an acid-base reaction, or one mole of electrons in a redox reaction.
20. **Normality (N)**: A unit of concentration, defined as the number of equivalents of solute per liter of solution.
21. **Percent Composition**: The percentage by mass of each element in a compound.
22. **Empirical Formula**: The simplest whole-number ratio of atoms of each element in a compound.
23. **Molecular Formula**: The actual number of atoms of each element in a molecule of a compound.
24. **Density Formula**: The equation ρ=mV\rho = \frac{m}{V}ρ=Vm, where ρ\rhoρ is density, mmm is mass, and VVV is volume.
25. **Ideal Gas Law**: The equation PV=nRTPV = nRTPV=nRT, where PPP is pressure, VVV is volume, nnn is moles of gas, RRR is the gas constant, and TTT is temperature in Kelvin.
26. **Gas Constant (R)**: A constant used in the ideal gas law, typically 8.314 J/mol\cdotpK8.314 \, \text{J/mol·K}8.314J/mol\cdotpK or 0.0821 L\cdotpatm/mol\cdotpK0.0821 \, \text{L·atm/mol·K}0.0821L\cdotpatm/mol\cdotpK.
27. **Concentration Units**: Various units used to express concentration, including molarity (M), molality (m), normality (N), and parts per million (ppm).
28. **Stoichiometry**: The calculation of reactants and products in chemical reactions using the balanced chemical equation.
29. **Limiting Reactant**: The reactant that is completely consumed in a chemical reaction, limiting the amount of product formed.
30. **Excess Reactant**: The reactant that is not completely consumed in a chemical reaction and remains after the reaction is complete.

31. **Theoretical Yield**: The maximum amount of product that can be produced in a chemical reaction based on the amount of limiting reactant.
32. **Actual Yield**: The amount of product actually produced in a chemical reaction.
33. **Percent Yield**: The ratio of the actual yield to the theoretical yield, multiplied by 100 to give a percentage.
34. **Solution Preparation**: The process of making a solution of a specific concentration by dissolving a known amount of solute in a specific volume of solvent.
35. **Serial Dilution**: A stepwise dilution of a substance in solution, often used to create a series of solutions with decreasing concentrations.

CHAPTER OVERVIEW

6: Physical and Chemical Reactions

6.1: Introduction and Learning Objectives

Chapter learning objectives

Differentiating Between Physical and Chemical Changes:

- Define physical changes and chemical changes.
- Identify indicators of chemical reactions (energy changes, color changes, gas production, precipitates).
- Explore everyday and laboratory examples of physical and chemical changes.

The Law of Conservation of Mass in Chemical Reactions:

- Understand the conservation of mass in chemical reactions.
- Apply this law to balance chemical equations and grasp stoichiometry.
- Investigate real-world applications in environmental science, engineering, and materials science.

Types and Characteristics of Chemical Reactions:

- Classify synthesis, decomposition, single replacement, double replacement, and combustion reactions.
- Understand conditions and energy changes for different reactions.
- Explore practical examples in industrial, environmental, and biological contexts.

Factors Affecting the Rate of Chemical Reactions:

- Identify factors influencing reaction rates (temperature, reactant concentration, surface area, catalysts).
- Understand activation energy and reaction mechanisms.
- Explore practical implications in food preservation, pharmaceuticals, and industrial manufacturing.

Teaching Strategies for Physical and Chemical Reactions:

- Develop effective teaching methods with demonstrations and real-world examples.
- Design interactive activities for observing and analyzing changes.
- Use multimedia tools and simulation software for complex concepts.
- Address misconceptions and emphasize the interdisciplinary relevance of these concepts.

Introduction

Understanding the differences between physical and chemical reactions is fundamental to the study of chemistry. These core concepts not only underpin much of the discipline but also have practical applications in a wide range of scientific and everyday contexts.

Physical changes involve alterations in the state or appearance of a substance without changing its chemical composition. Examples include melting, freezing, and dissolving. In contrast, chemical changes result in the formation of new substances with different properties, often indicated by energy changes, color shifts, gas production, or the formation of precipitates. Recognizing these indicators helps in distinguishing between physical and chemical reactions, a skill crucial for both laboratory work and real-world applications.

The Law of Conservation of Mass is a pivotal principle in understanding chemical reactions. This law states that the total mass of reactants equals the total mass of products, emphasizing that matter is neither created nor destroyed in a chemical reaction. This concept is essential for balancing chemical equations and understanding stoichiometry. Real-world applications of this law are seen in environmental science, engineering, and materials science, where precise calculations of reactant and product masses are vital for processes and innovations.

Chemical reactions can be classified into several types: synthesis, decomposition, single replacement, double replacement, and combustion. Each type has distinct characteristics and occurs under specific conditions, often involving energy changes. For instance, synthesis reactions combine elements or compounds to form more complex substances, while decomposition reactions break down compounds into simpler components. Understanding these types and their associated energy changes is critical for applications in industrial processes, environmental management, and biological systems.

The rate of chemical reactions is influenced by various factors, including temperature, concentration of reactants, surface area, and the presence of catalysts. These factors affect the activation energy required for reactions to occur and the overall reaction mechanisms. Practical implications of reaction rates are significant in areas such as food preservation, pharmaceutical development, and industrial manufacturing, where controlling the speed of reactions is crucial for efficiency and effectiveness.

In the context of K-12 education, teaching the concepts of physical and chemical reactions can be both engaging and enlightening. Effective teaching strategies include demonstrations and experiments that allow students to observe and analyze changes firsthand. Interactive activities, multimedia tools, and simulation software can further illustrate complex concepts and reaction mechanisms. Addressing common misconceptions and challenges helps students grasp the distinctions and intricacies of these reactions. By highlighting the interdisciplinary nature and real-life applications of physical and chemical reactions, educators can inspire students and underscore the relevance of chemistry in various scientific, environmental, and industrial contexts.

6.2: Physical and Chemical Changes

Learning Outcomes

- Distinguish between physical and chemical changes.
- Give examples of physical and chemical changes.

Physical Changes

As an ice cube melts, its shape changes as it acquires the ability to flow. However, its composition does not change. **Melting** is an example of a **physical change**. A physical change is a change to a sample of matter in which some properties of the material change, but the identity of the matter does not. Physical changes can further be classified as reversible or irreversible. The melted ice cube may be refrozen, so melting is a reversible physical change. Physical changes that involve a change of state are all reversible. Other changes of state include **vaporization** (liquid to gas), **freezing** (liquid to solid), and **condensation** (gas to liquid). Dissolving is also a reversible physical change. When salt is dissolved into water, the salt is said to have entered the aqueous state. The salt may be regained by boiling off the water, leaving the salt behind.

Figure 10.4.1: *Melting ice in the Beaufort Sea.*

When a piece of wood is ground into sawdust, that change is irreversible since the sawdust could not be reconstituted into the same piece of wood that it was before. Cutting the grass or pulverizing a rock would be other irreversible physical changes. Firewood also represents an irreversible physical change since the pieces cannot be put back together to form the tree.

Figure 10.4.2: *Firewood being cut is a physical change because the composition doesn't change when being cut.*

Chemical Changes

When exposed to air, an object made of iron will eventually begin to rust (see figure below).

Figure 10.4.3: *Rust (iron oxide) forms on an unprotected iron surface.*

As the rust forms on the surface of the iron, it flakes off to expose more iron, which will continue to rust. Rust is clearly a substance that is different from iron. Rusting is an example of a chemical change.

A **chemical property** *describes the ability of a substance to undergo a specific chemical change*. A chemical property of iron is that it is capable of combining with oxygen to form iron oxide, the chemical name of rust. A more general term for rusting and other similar processes is corrosion. Other terms that are commonly used in descriptions of chemical changes are *burn*, *rot*, *explode*, and *ferment*. Chemical properties are very useful as a way of identifying substances. However, unlike physical properties, chemical properties can only be observed as the substance is in the process of being changed into a different substance.

A chemical change is also called a chemical reaction. A **chemical reaction** *is a process that occurs when one or more substances are changed into one or more new substances*. Zinc (Zn) is a silver-gray element that can be ground into a powder. If zinc is mixed at room temperature with powdered sulfur (S), a bright yellow element, the result will simply be a mixture of zinc and sulfur. No chemical reaction occurs. However, if energy is provided to the mixture in the form of heat, the zinc will chemically react with the sulfur to form the compound zinc sulfide (ZnS). Pictured below are the substances involved in this reaction.

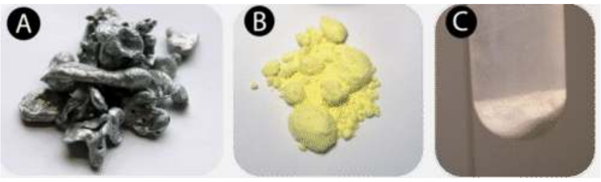


Figure 10.4.4: *Zinc (A) and sulfur (B) are two elements that undergo a chemical reaction when heated to form the the compound zinc sulfide (C).*

The reaction between zinc and sulfur can be depicted in something called a chemical equation. In words, we could write the reaction as:

$$\text{zinc} + \text{sulfur} \rightarrow \text{zinc sulfide} \tag{6.2.1}$$

A more convenient way to express a chemical reaction is to use the symbols and formulas of the substances involved:

$$\mathrm{Zn} + \mathrm{S} \rightarrow \mathrm{ZnS} \tag{6.2.2}$$

The substance(s) to the left of the arrow in a chemical equation are called reactants. A **reactant** *is a substance that is present at the start of a chemical reaction.* The substance(s) to the right of the arrow are called products. A **product** *is a substance that is present at the end of a chemical reaction*. In the equation above, zinc and sulfur are the reactants that chemically combine to form zinc sulfide as a product.

Recognizing Chemical Reactions

How can you tell if a chemical reaction is taking place? Certain visual clues indicate that a chemical reaction is likely (but not necessarily) occurring, including the following examples:

1. A change of color occurs during the reaction.
2. A gas is produced during the reaction.
3. A solid product, called a precipitate, is produced in the reaction.
4. A visible transfer of energy occurs in the form of light as a result of the reaction.

When zinc reacts with hydrochloric acid, the reaction bubbles vigorously as hydrogen gas is produced (see figure below). The production of a gas is also an indication that a chemical reaction may be occurring.

Figure 10.4.5: *Zinc reacts with hydrochloric acid to produce bubbles of hydrogen gas.*

When a colorless solution of lead (II) nitrate is added to a colorless solution of potassium iodide, a yellow solid called a precipitate is instantly produced (see figure below). A **precipitate** *is a solid product that forms from a reaction and settles out of a liquid mixture*. The formation of a precipitate may also indicate the occurrence of a chemical reaction.

$$\mathrm{Pb(NO_3)_2}\,(aq) + \mathrm{2KI}\,(aq) \rightarrow \mathrm{PbI_2}\,(s) + \mathrm{2KNO_3}\,(aq) \tag{6.2.3}$$

Figure 10.4.6: *A yellow precipitate of solid lead (II) iodide forms immediately when solutions of lead (II) nitrate and potassium iodide are mixed.*

Contributors and Attributions

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6.3: Evidence of a Chemical Reaction

Learning Objectives

- Identify the evidence for chemical reactions.

In a chemical change, new substances are formed. In order for this to occur, the chemical bonds of the substances break, and the atoms that compose them separate and rearrange themselves into new substances with new chemical bonds. When this process occurs, we call it a chemical reaction. A **chemical reaction** is the process in which one or more substances are changed into one or more new substances.

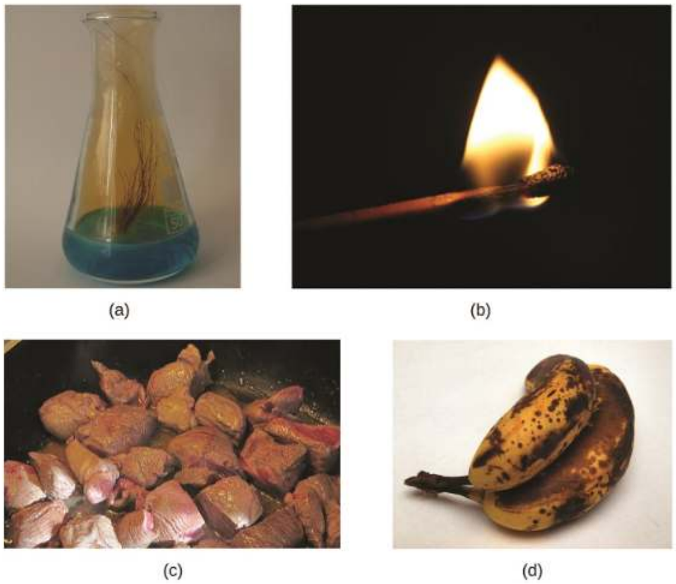

Figure $6.3.1$: (a) Copper and nitric acid undergo a chemical change to form copper nitrate and brown, gaseous nitrogen dioxide. (b) During the combustion of a match, cellulose in the match and oxygen from the air undergo a chemical change to form carbon dioxide and water vapor. (c) Cooking red meat causes a number of chemical changes, including the oxidation of iron in myoglobin that results in the familiar red-to-brown color change. (d) A banana turning brown is a chemical change as new, darker (and less tasty) substances form. (Credit b: modification of work by Jeff Turner; credit c: modification of work by Gloria Cabada-Leman; credit d: modification of work by Roberto Verzo.)

To identify a chemical reaction, we look for a **chemical change**. A chemical change always produces one or more types of matter that differ from the matter present before the change. The formation of rust is a chemical change because rust is a different kind of matter than the iron, oxygen, and water present before the rust formed. The explosion of nitroglycerin is a chemical change because the gases produced are very different kinds of matter from the original substance. Other examples of chemical changes include: reactions that are performed in a lab (such as copper reacting with nitric acid), all forms of combustion (burning), and food being cooked, digested, or rotting (Figure $6.3.1$).

Video*6.3.1*: Evidence of a Chemical Reaction

Example 6.3.1: Evidence of a Chemical Reaction

Which of the following is a chemical reaction?

a. Freezing liquid mercury.
b. Adding yellow to blue to make green.
c. Cutting a piece of paper into two pieces.
d. Dropping a sliced orange into a vat of sodium dydroxide.
e. Filling a balloon with natural air.

Solution

A, B, C, & E involve only physical changes. A sliced orange has acid (citric acid) that can react with sodium hydroxide, so the answer is D.

Exercise 6.3.1

Which of the following is a chemical reaction?

a. Painting a wall blue.
b. A bicycle rusting.
c. Ice cream melting.
d. Scratching a key across a desk.
e. Making a sand castle.

Answer

B

Example 6.3.2: Evidence of a Chemical Reaction

Which of the following is not a chemical reaction?

a. Shattering glass with a baseball.
b. Corroding metal.
c. Fireworks exploding.
d. Lighting a match.
e. Baking a cake.

Solution

Shattering glass with a baseball results in glass broken into many pieces but no chemical change happens, so the answer is A.

? Exercise 6.3.2

Which of the following is NOT a chemical reaction?

a. Frying an egg.
b. Slicing carrots.
c. A Macbook falling out of a window.
d. Creating ATP in the human body.
e. Dropping a fizzy tablet into a glass of water.

Answer

B and C

Summary

Chemical reactions can be identified via a wide range of different observable factors including change in color, energy change (temperature change or light produced), gas production, formation of precipitate and change in properties.

6.4: Law of Conservation of Mass

Figure 6.4.1 (Credit: Douglas Heriot; Source: http://www.flickr.com/photos/dhdesign/3843848623/(opens in new window); License: CC by 2.0(opens in new window))

Have you ever lost a screw?

The following situation happens all too often. You have taken apart a piece of equipment to clean it up. When you put the equipment back together, somehow you have an extra screw or two. Or you find out that a screw is missing that was a part of the original equipment. In either case, you know something is wrong. You expect to end up with the same amount of material that you started with, not with more or less than what you had originally.

Law of Conservation of Mass

By the late 1700s, chemists accepted the definition of an element as a substance that cannot be broken down into a simpler substance by ordinary chemical means. It was also clear that elements combine with one another to form more complex substances called compounds. The chemical and physical properties of these compounds are different than the properties of the elements from which they were formed. There were questions about the details of these processes.

In the 1790s, a greater emphasis began to be placed on the quantitative analysis of chemical reactions. Accurate and reproducible measurements of the masses of reacting elements and the compounds they form led to the formulation of several basic **laws**. One of these is called the law of **conservation of mass**, which states that during a chemical reaction, the total mass of the **products** must be equal to the total mass of the **reactants**. In other words, mass cannot be created or destroyed during a chemical reaction, but is always conserved.

As an example, consider the reaction between silver nitrate and sodium chloride. These two compounds will dissolve in water to form silver chloride and sodium nitrate. The silver chloride does not dissolve in water, so it forms a solid that we can filter off. When we evaporate the water, we can recover the sodium nitrate formed. If we react 58.5 grams of sodium chloride with 169.9 grams of silver nitrate, we start with 228.4 grams of materials. After the reaction is complete and the materials separated, we find that we have formed 143.4 grams of silver chloride and 85.0 grams of sodium nitrate, giving us a total mass of 228.4 grams for the products. So, the total mass of reactants equals the total mass of products, a proof of the law of conservation of mass.

Summary

- The law of conservation of mass states that, during a chemical reaction, the total mass of the products must be equal to the total mass of the reactants.

Review

1. The law of conservation of mass states that, during a chemical reaction, the total ______ of the products must be equal to the total ______ of the reactants.
2. Describe an example of the law of conservation of mass.

6.5: Writing and Balancing Chemical Equations

Learning Objectives

- Derive chemical equations from narrative descriptions of chemical reactions.
- Write and balance chemical equations in molecular, total ionic, and net ionic formats.

The preceding chapter introduced the use of element symbols to represent individual atoms. When atoms gain or lose electrons to yield ions, or combine with other atoms to form molecules, their symbols are modified or combined to generate chemical formulas that appropriately represent these species. Extending this symbolism to represent both the identities and the relative quantities of substances undergoing a chemical (or physical) change involves writing and balancing a chemical equation. Consider as an example the reaction between one methane molecule (CH_4) and two diatomic oxygen molecules (O_2) to produce one carbon dioxide molecule (CO_2) and two water molecules (H_2O). The chemical equation representing this process is provided in the upper half of Figure 6.5.1, with space-filling molecular models shown in the lower half of the figure.

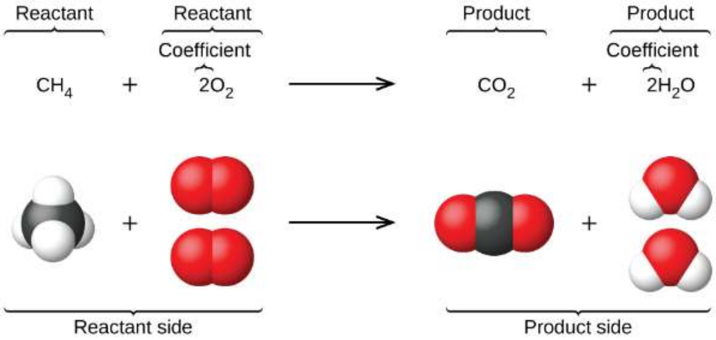

Figure 6.5.1: The reaction between methane and oxygen to yield carbon dioxide and water (shown at bottom) may be represented by a chemical equation using formulas (top). This figure shows a balanced chemical equation followed below by a representation of the equation using space-filling models. The equation reads C H subscript 4 plus 2 O subscript 2 arrow C O subscript 2 plus 2 H subscript 2 O. Under the C H subscript 4, the molecule is shown with a central black sphere, representing a C atom, to which 4 smaller white spheres, representing H atoms, are distributed evenly around. All four H atoms are bonded to the central black C atom. This is followed by a plus sign. Under the 2 O subscript 2, two molecules are shown. The molecules are each composed of two red spheres bonded together. The red spheres represent O atoms. To the right of an arrow and under the C O subscript 2, appears a single molecule with a black central sphere with two red spheres bonded to the left and right. Following a plus sign and under the 2 H subscript 2 O, are two molecules, each with a central red sphere and two smaller white spheres attached to the lower right and lower left sides of the central red sphere. Note that in space filling models of molecules, spheres appear slightly compressed in regions where there is a bond between two atoms.

This example illustrates the fundamental aspects of any chemical equation:

1. The substances undergoing reaction are called reactants, and their formulas are placed on the left side of the equation.
2. The substances generated by the reaction are called products, and their formulas are placed on the right sight of the equation.
3. Plus signs (+) separate individual reactant and product formulas, and an arrow ($\longrightarrow$) separates the reactant and product (left and right) sides of the equation.
4. The relative numbers of reactant and product species are represented by coefficients (numbers placed immediately to the left of each formula). A coefficient of 1 is typically omitted.

It is common practice to use the smallest possible whole-number coefficients in a chemical equation, as is done in this example. Realize, however, that these coefficients represent the *relative* numbers of reactants and products, and, therefore, they may be correctly interpreted as ratios. Methane and oxygen react to yield carbon dioxide and water in a 1:2:1:2 ratio. This ratio is satisfied if the numbers of these molecules are, respectively, 1-2-1-2, or 2-4-2-4, or 3-6-3-6, and so on (Figure 6.5.2). Likewise, these coefficients may be interpreted with regard to any amount (number) unit, and so this equation may be correctly read in many ways, including:

- *One* methane molecule and *two* oxygen molecules react to yield *one* carbon dioxide molecule and *two* water molecules.
- *One dozen* methane molecules and *two dozen* oxygen molecules react to yield *one dozen* carbon dioxide molecules and *two dozen* water molecules.
- *One mole* of methane molecules and *2 moles* of oxygen molecules react to yield *1 mole* of carbon dioxide molecules and *2 moles* of water molecules.

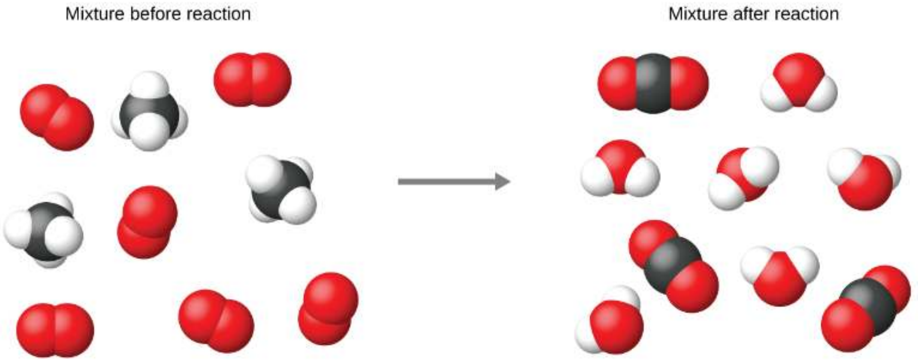

Figure $6.5.2$: Regardless of the absolute number of molecules involved, the ratios between numbers of molecules of each species that react (the reactants) and molecules of each species that form (the products) are the same and are given in the chemical equation.

Balancing Equations

When a chemical equation is balanced it means that equal numbers of atoms for each element involved in the reaction are represented on the reactant and product sides. This is a requirement the equation must satisfy to be consistent with the law of conservation of matter. It may be confirmed by simply summing the numbers of atoms on either side of the arrow and comparing these sums to ensure they are equal. Note that the number of atoms for a given element is calculated by multiplying the coefficient of any formula containing that element by the element's subscript in the formula. If an element appears in more than one formula on a given side of the equation, the number of atoms represented in each must be computed and then added together. For example, both product species in the example reaction, $\mathrm{CO_2}$ and $\mathrm{H_2O}$, contain the element oxygen, and so the number of oxygen atoms on the product side of the equation is

$$\left(1\ \mathrm{CO_2}\ \cancel{\text{molecule}} \times \frac{2\ \mathrm{O\ atoms}}{\mathrm{CO_2}\ \cancel{\text{molecule}}}\right) + \left(2\ \mathrm{H_2O}\ \cancel{\text{molecule}} \times \frac{1\ \mathrm{O\ atom}}{\mathrm{H_2O}\ \cancel{\text{molecule}}}\right) = 4\ \mathrm{O\ atoms}$$

The equation for the reaction between methane and oxygen to yield carbon dioxide and water is confirmed to be balanced per this approach, as shown here:

$$\mathrm{CH_4 + 2\,O_2 \rightarrow CO_2 + 2\,H_2O}$$

Equation for the reaction

$$\mathrm{CH_4 + 2\,O_2 \rightarrow CO_2 + 2\,H_2O}$$

Element	Reactants	Products	Balanced?
C	1 × 1 = 1	1 × 1 = 1	1 = 1, yes
H	4 × 1 = 4	2 × 2 = 4	4 = 4, yes
O	2 × 2 = 4	(1 × 2) + (2 × 1) = 4	4 = 4, yes

A balanced chemical equation often may be derived from a qualitative description of some chemical reaction by a fairly simple approach known as balancing by inspection. Consider as an example the decomposition of water to yield molecular hydrogen and oxygen. This process is represented qualitatively by an *unbalanced* chemical equation:

$$\mathrm{H_2O \rightarrow H_2 + O_2} \qquad \text{(unbalanced)}$$

Comparing the number of H and O atoms on either side of this equation confirms its imbalance:

Comparisons between H and O atoms

Element	Reactants	Products	Balanced?
H	1 × 2 = 2	1 × 2 = 2	2 = 2, yes
O	1 × 1 = 1	1 × 2 = 2	1 ≠ 2, no

The numbers of H atoms on the reactant and product sides of the equation are equal, but the numbers of O atoms are not. To achieve balance, the *coefficients* of the equation may be changed as needed. Keep in mind, of course, that the *formula subscripts* define, in part, the identity of the substance, and so these cannot be changed without altering the qualitative meaning of the equation. For example, changing the reactant formula from H_2O to H_2O_2 would yield balance in the number of atoms, but doing so also changes the reactant's identity (it's now hydrogen peroxide and not water). The O atom balance may be achieved by changing the coefficient for H_2O to 2.

$$2\,H_2O \rightarrow H_2 + O_2 \qquad \text{(unbalanced)}$$

O atom balance may be achieved by changing the coefficient for H2O to 2

Element	Reactants	Products	Balanced?
H	**2** × 2 = 4	1 × 2 = 2	4 ≠ 2, no
O	2 × 1 = 2	1 × 2 = 2	2 = 2, yes

The H atom balance was upset by this change, but it is easily reestablished by changing the coefficient for the H_2 product to 2.

$$2\,H_2O \rightarrow 2\,H_2 + O_2 \qquad \text{(balanced)}$$

H atom balance upset but easily reestablished by changing the coefficient for the H2 product to 2.

Element	Reactants	Products	Balanced?
H	2 × 2 = 4	**2** × 2 = 2	4 = 4, yes
O	2 × 1 = 2	1 × 2 = 2	2 = 2, yes

These coefficients yield equal numbers of both H and O atoms on the reactant and product sides, and the balanced equation is, therefore:

$$2\,H_2O \rightarrow 2\,H_2 + O_2$$

✓ Example 6.5.1: Balancing Chemical Equations

Write a balanced equation for the reaction of molecular nitrogen (N_2) and oxygen (O_2) to form dinitrogen pentoxide.

Solution

First, write the unbalanced equation.

$$N_2 + O_2 \rightarrow N_2O_5 \qquad \text{(unbalanced)}$$

Next, count the number of each type of atom present in the unbalanced equation.

Unbalanced Equation

Element	Reactants	Products	Balanced?
N	1 × 2 = 2	1 × 2 = 2	2 = 2, yes
O	1 × 2 = 2	1 × 5 = 5	2 ≠ 5, no

Though nitrogen is balanced, changes in coefficients are needed to balance the number of oxygen atoms. To balance the number of oxygen atoms, a reasonable first attempt would be to change the coefficients for the O_2 and N_2O_5 to integers that will yield 10 O atoms (the least common multiple for the O atom subscripts in these two formulas).

$$N_2 + 5\,O_2 \rightarrow 2\,N_2O_5 \qquad \text{(unbalanced)}$$

$$N_2 + 5\,O_2 \rightarrow 2\,N_2O_5 \qquad \text{(unbalanced)}$$

Element	Reactants	Products	Balanced?
N	1 × 2 = 2	**2** × 2 = 4	2 ≠ 4, no
O	**5** × 2 = 10	**2** × 5 = 10	10 = 10, yes

The N atom balance has been upset by this change; it is restored by changing the coefficient for the reactant N_2 to 2.

$$2\,N_2 + 5\,O_2 \rightarrow 2\,N_2O_5$$

N atom balance upset but restored by changing the coefficient for the reactant N2 to 2.

Element	Reactants	Products	Balanced?
N	**2** × 2 = 4	2 × 2 = 4	4 = 4, yes
O	5 × 2 = 10	2 × 5 = 10	10 = 10, yes

The numbers of N and O atoms on either side of the equation are now equal, and so the equation is balanced.

? Exercise 6.5.1

Write a balanced equation for the decomposition of ammonium nitrate to form molecular nitrogen, molecular oxygen, and water. (Hint: Balance oxygen last, since it is present in more than one molecule on the right side of the equation.)

Answer

$$2\,NH_4NO_3 \rightarrow 2\,N_2 + O_2 + 4\,H_2O$$

Balancing Reactions Which Contain Polyatomics: Balancing Reactions Which Contain Polyatomics(opens in new window) [youtu.be]

It is sometimes convenient to use fractions instead of integers as intermediate coefficients in the process of balancing a chemical equation. When balance is achieved, all the equation's coefficients may then be multiplied by a whole number to convert the fractional coefficients to integers without upsetting the atom balance. For example, consider the reaction of ethane (C_2H_6) with oxygen to yield H_2O and CO_2, represented by the unbalanced equation:

$$C_2H_6 + O_2 \rightarrow H_2O + CO_2 \qquad \text{(unbalanced)}$$

Following the usual inspection approach, one might first balance C and H atoms by changing the coefficients for the two product species, as shown:

$$C_2H_6 + O_2 \rightarrow 3\,H_2O + 2\,CO_2 \qquad \text{(unbalanced)}$$

This results in seven O atoms on the product side of the equation, an odd number—no integer coefficient can be used with the O_2 reactant to yield an odd number, so a fractional coefficient, $\frac{7}{2}$, is used instead to yield a provisional balanced equation:

$$C_2H_6 + \frac{7}{2}O_2 \rightarrow 3\,H_2O + 2\,CO_2$$

A conventional balanced equation with integer-only coefficients is derived by multiplying each coefficient by 2:

$$2\,C_2H_6 + 7\,O_2 \rightarrow 6\,H_2O + 4\,CO_2$$

Finally with regard to balanced equations, recall that convention dictates use of the *smallest whole-number coefficients*. Although the equation for the reaction between molecular nitrogen and molecular hydrogen to produce ammonia is, indeed, balanced,

$$3\,N_2 + 9\,H_2 \rightarrow 6\,NH_3$$

the coefficients are not the smallest possible integers representing the relative numbers of reactant and product molecules. Dividing each coefficient by the greatest common factor, 3, gives the preferred equation:

$$N_2 + 3\,H_2 \rightarrow 2\,NH_3$$

Phet Simulation

Use this interactive tutorial for additional practice balancing equations.

Additional Information in Chemical Equations

The physical states of reactants and products in chemical equations very often are indicated with a parenthetical abbreviation following the formulas. Common abbreviations include *s* for solids, *l* for liquids, *g* for gases, and *aq* for substances dissolved in water (*aqueous solutions*, as introduced in the preceding chapter). These notations are illustrated in the example equation here:

$$2\,Na(s) + 2\,H_2O(l) \rightarrow 2\,NaOH(aq) + H_2(g)$$

This equation represents the reaction that takes place when sodium metal is placed in water. The solid sodium reacts with liquid water to produce molecular hydrogen gas and the ionic compound sodium hydroxide (a solid in pure form, but readily dissolved in water).

Special conditions necessary for a reaction are sometimes designated by writing a word or symbol above or below the equation’s arrow. For example, a reaction carried out by heating may be indicated by the uppercase Greek letter delta (Δ) over the arrow.

$$CaCO_3(s) \xrightarrow{\Delta} CaO(s) + CO_2(g)$$

Other examples of these special conditions will be encountered in more depth in later chapters.

Equations for Ionic Reactions

Given the abundance of water on earth, it stands to reason that a great many chemical reactions take place in aqueous media. When ions are involved in these reactions, the chemical equations may be written with various levels of detail appropriate to their intended use. To illustrate this, consider a reaction between ionic compounds taking place in an aqueous solution. When aqueous solutions of $CaCl_2$ and $AgNO_3$ are mixed, a reaction takes place producing aqueous $Ca(NO_3)_2$ and solid $AgCl$:

$$CaCl_2(aq) + 2\,AgNO_3(aq) \rightarrow Ca(NO_3)_2(aq) + 2\,AgCl(s)$$

This balanced equation, derived in the usual fashion, is called a molecular equation because it doesn’t explicitly represent the ionic species that are present in solution. When ionic compounds dissolve in water, they may *dissociate* into their constituent ions, which are subsequently dispersed homogenously throughout the resulting solution (a thorough discussion of this important process is provided in the chapter on solutions). Ionic compounds dissolved in water are, therefore, more realistically represented as dissociated ions, in this case:

$$CaCl_2(aq) \rightarrow Ca^{2+}(aq) + 2\,Cl^-(aq)$$

$$2\,AgNO_3(aq) \rightarrow 2\,Ag^+(aq) + 2\,NO_3^-(aq)$$

$$Ca(NO_3)_2(aq) \rightarrow Ca^{2+}(aq) + 2\,NO_3^-(aq)$$

Unlike these three ionic compounds, AgCl does not dissolve in water to a significant extent, as signified by its physical state notation, *(s)*.

Explicitly representing all dissolved ions results in a complete ionic equation. In this particular case, the formulas for the dissolved ionic compounds are replaced by formulas for their dissociated ions:

$$\mathrm{Ca^{2+}}(aq)+2\,\mathrm{Cl^-}(aq)+2\,\mathrm{Ag^+}(aq)+2\,\mathrm{NO_3^-}(aq)\rightarrow \mathrm{Ca^{2+}}(aq)+2\,\mathrm{NO_3^-}(aq)+2\,\mathrm{Ag\,Cl}(s)$$

Examining this equation shows that two chemical species are present in identical form on both sides of the arrow, $\mathrm{Ca^{2\,+}(aq)}$ and $\mathrm{NO_3^-}(aq)$. These spectator ions—ions whose presence is required to maintain charge neutrality—are neither chemically nor physically changed by the process, and so they may be eliminated from the equation to yield a more succinct representation called a net ionic equation:

$$\mathrm{Ca^{2+}}\cancel{(aq)}+2\,\mathrm{Cl^-}(aq)+2\,\mathrm{Ag^+}(aq)+\ 2\,\mathrm{NO_3^-}\cancel{(aq)}\rightarrow\ \mathrm{Ca^{2+}}\cancel{(aq)}+\ 2\,\mathrm{NO_3^-}\cancel{(aq)}+2\,\mathrm{AgCl}(s)$$

$$2\,\mathrm{Cl^-}(aq)+2\,\mathrm{Ag^+}(aq)\rightarrow 2\,\mathrm{AgCl}(s)$$

Following the convention of using the smallest possible integers as coefficients, this equation is then written:

$$\mathrm{Cl^-}(aq)+\mathrm{Ag^+}(aq)\rightarrow \mathrm{AgCl}(s)$$

This net ionic equation indicates that solid silver chloride may be produced from dissolved chloride and silver(I) ions, regardless of the source of these ions. These molecular and complete ionic equations provide additional information, namely, the ionic compounds used as sources of $\mathrm{Cl^-}$ and $\mathrm{Ag^+}$.

✓ Example 6.5.2: Molecular and Ionic Equations

When carbon dioxide is dissolved in an aqueous solution of sodium hydroxide, the mixture reacts to yield aqueous sodium carbonate and liquid water. Write balanced molecular, complete ionic, and net ionic equations for this process.

Solution

Begin by identifying formulas for the reactants and products and arranging them properly in chemical equation form:

$$\mathrm{CO_2(aq)+NaOH(aq)\rightarrow Na_2CO_3(aq)+H_2O(l)}\qquad\text{(unbalanced)}$$

Balance is achieved easily in this case by changing the coefficient for NaOH to 2, resulting in the molecular equation for this reaction:

$$\mathrm{CO_2(aq)}{}^{+}_{2}\mathrm{NaOH(aq)\rightarrow Na_2CO_3(aq)+H_2O}(l)$$

The two dissolved ionic compounds, NaOH and $\mathrm{Na_2CO_3}$, can be represented as dissociated ions to yield the complete ionic equation:

$$\mathrm{CO_2(aq)+2\,Na^+(aq)+2\,OH^-(aq)\rightarrow 2\,Na^+(aq)+CO_3^{2\,-}(aq)+H_2O(l)}$$

Finally, identify the spectator ion(s), in this case $\mathrm{Na^+}(aq)$, and remove it from each side of the equation to generate the net ionic equation:

$$\mathrm{CO_2}(aq)+\ 2\,\mathrm{Na^+}\cancel{(aq)}+2\,\mathrm{OH^-}(aq)\ \rightarrow\ 2\,\mathrm{Na^+}\cancel{(aq)}+\mathrm{CO_3^{2-}}(aq)+\mathrm{H_2O}(l)$$

$$\mathrm{CO_2}(aq)+2\,\mathrm{OH^-}(aq)\ \rightarrow \mathrm{CO_3^{2-}}(aq)+\mathrm{H_2O}(l)$$

? Exercise 6.5.2

Diatomic chlorine and sodium hydroxide (lye) are commodity chemicals produced in large quantities, along with diatomic hydrogen, via the electrolysis of brine, according to the following unbalanced equation:

$$\mathrm{NaCl(aq)+H_2O(l)}\xrightarrow{electricity}\mathrm{NaOH(aq)+H_2(g)+Cl_2(g)}$$

Write balanced molecular, complete ionic, and net ionic equations for this process.

Answer

Balanced molecular equation:

$$\mathrm{2\,NaCl(aq)+2\,H_2O(l)\longrightarrow 2\,NaOH(aq)+H_2(g)+Cl_2(g)}$$

Balanced ionic equation:

$$2\,\text{Na}^{+}(aq) + 2\,\text{Cl}^{-}(aq) + 2\,\text{H}_2\text{O}(l) \longrightarrow 2\,\text{Na}^{+}(aq) + 2\,\text{OH}^{-}(aq) + \text{H}_2(g) + \text{Cl}_2(g)$$

Balanced net ionic equation:

$$2\,\text{Cl}^{-}(aq) + 2\,\text{H}_2\text{O}(l) \longrightarrow 2\,\text{OH}^{-}(aq) + \text{H}_2(g) + \text{Cl}_2(g)$$

Key Concepts and Summary

Chemical equations are symbolic representations of chemical and physical changes. Formulas for the substances undergoing the change (reactants) and substances generated by the change (products) are separated by an arrow and preceded by integer coefficients indicating their relative numbers. Balanced equations are those whose coefficients result in equal numbers of atoms for each element in the reactants and products. Chemical reactions in aqueous solution that involve ionic reactants or products may be represented more realistically by complete ionic equations and, more succinctly, by net ionic equations.

Glossary

balanced equation

chemical equation with equal numbers of atoms for each element in the reactant and product

chemical equation

symbolic representation of a chemical reaction

coefficient

number placed in front of symbols or formulas in a chemical equation to indicate their relative amount

complete ionic equation

chemical equation in which all dissolved ionic reactants and products, including spectator ions, are explicitly represented by formulas for their dissociated ions

molecular equation

chemical equation in which all reactants and products are represented as neutral substances

net ionic equation

chemical equation in which only those dissolved ionic reactants and products that undergo a chemical or physical change are represented (excludes spectator ions)

product

substance formed by a chemical or physical change; shown on the right side of the arrow in a chemical equation

reactant

substance undergoing a chemical or physical change; shown on the left side of the arrow in a chemical equation

spectator ion

ion that does not undergo a chemical or physical change during a reaction, but its presence is required to maintain charge neutrality

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6.6: Types of Chemical Reactions

Learning Outcomes

- Classify a reaction as combination, decomposition, single-replacement, double-replacement, or combustion.
- Predict the products and balance a combustion reaction.

Many chemical reactions can be classified as one of five basic types. Having a thorough understanding of these types of reactions will be useful for predicting the products of an unknown reaction. The five basic types of chemical reactions are combination, decomposition, single-replacement, double-replacement, and combustion. Analyzing the reactants and products of a given reaction will allow you to place it into one of these categories. Some reactions will fit into more than one category.

Combination Reactions

A **combination reaction**, also known as a **synthesis reaction**, *is a reaction in which two or more substances combine to form a single new substance.* Combination reactions can also be called synthesis reactions .The general form of a combination reaction is:

$$\mathrm{A+B\rightarrow AB} \tag{6.6.1}$$

One combination reaction is two elements combining to form a compound. Solid sodium metal reacts with chlorine gas to product solid sodium chloride.

$$\mathrm{2Na}\,(s)+\mathrm{Cl_2}\,(g)\rightarrow \mathrm{2NaCl}\,(s) \tag{6.6.2}$$

Notice that in order to write and balance the equation correctly, it is important to remember the seven elements that exist in nature as diatomic molecules (H_2, N_2, O_2, F_2, Cl_2, Br_2, and I_2).

One sort of combination reaction that occurs frequently is the reaction of an element with oxygen to form an oxide. Metals and nonmetals both react readily with oxygen under most conditions. Magnesium reacts rapidly and dramatically when ignited, combining with oxygen from the air to produce a fine powder of magnesium oxide.

$$\mathrm{2Mg}\,(s)+\mathrm{O_2}\,(g)\rightarrow \mathrm{2MgO}\,(s) \tag{6.6.3}$$

Decomposition Reactions

A **decomposition reaction** *is a reaction in which a compound breaks down into two or more simpler substances.* The general form of a decomposition reaction is:

$$\mathrm{AB\rightarrow A+B} \tag{6.6.4}$$

Most decomposition reactions require an input of energy in the form of heat, light, or electricity.

Binary compounds are compounds composed of just two elements. The simplest kind of decomposition reaction is when a binary compound decomposes into its elements. Mercury (II) oxide, a red solid, decomposes when heated to produce mercury and oxygen gas.

$$\mathrm{2HgO}\,(s)\rightarrow \mathrm{2Hg}\,(l)+\mathrm{O_2}\,(g) \tag{6.6.5}$$

A reaction is also considered to be a decomposition reaction even when one or more of the products is still a compound. A metal carbonate decomposes into a metal oxide and carbon dioxide gas. For example, calcium carbonate decomposes into calcium oxide and carbon dioxide.

$$\mathrm{CaCO_3}\,(s)\rightarrow \mathrm{CaO}\,(s)+\mathrm{CO_2}\,(g) \tag{6.6.6}$$

Metal hydroxides decompose on heating to yield metal oxides and water. Sodium hydroxide decomposes to produce sodium oxide and water.

$$\mathrm{2NaOH}\,(s)\rightarrow \mathrm{Na_2O}\,(s)+\mathrm{H_2O}\,(g) \tag{6.6.7}$$

Single-Replacement Reactions

A **single-replacement reaction** *is a reaction in which one element replaces a similar element in a compound*. The general form of a single-replacement (also called single-displacement) reaction is:

$$\mathrm{A} + \mathrm{BC} \rightarrow \mathrm{AC} + \mathrm{B} \tag{6.6.8}$$

In this general reaction, element A is a metal and replaces element B, also a metal, in the compound. When the element that is doing the replacing is a nonmetal, it must replace another nonmetal in a compound, and the general equation becomes:

$$\mathrm{Y} + \mathrm{XZ} \rightarrow \mathrm{XY} + \mathrm{Z} \tag{6.6.9}$$

Y is a nonmetal and replaces the nonmetal Z in the compound with X.

Magnesium is a more reactive metal than copper. When a strip of magnesium metal is placed in an aqueous solution of copper (II) nitrate, it replaces the copper. The products of the reaction are aqueous magnesium nitrate and solid copper metal.

$$\mathrm{Mg}\,(s) + \mathrm{Cu(NO_3)_2}\,(aq) \rightarrow \mathrm{Mg(NO_3)_2}\,(aq) + \mathrm{Cu}\,(s) \tag{6.6.10}$$

Many metals react easily with acids, and, when they do so, one of the products of the reaction is hydrogen gas. Zinc reacts with hydrochloric acid to produce aqueous zinc chloride and hydrogen (see figure below).

$$\mathrm{Zn}\,(s) + 2\mathrm{HCl}\,(aq) \rightarrow \mathrm{ZnCl_2}\,(aq) + \mathrm{H_2}\,(g) \tag{6.6.11}$$

Figure 10.6.1: *Zinc metal reacts with hydrochloric acid to give off hydrogen gas in a single-replacement reaction.*

Figure 10.6.2: *Pictured here is about 3 pounds of sodium metal reacting with water. Sodium metal reacts vigorously when dropped into a container of water, giving off hydrogen gas. A large piece of sodium will often generate so much heat that the hydrogen will*

ignite.

Double-Replacement Reactions

A **double-replacement reaction** *is a reaction in which the positive and negative ions of two ionic compounds exchange places to form two new compounds*. The general form of a double-replacement (also called double-displacement) reaction is:

$$\mathrm{AB} + \mathrm{CD} \rightarrow \mathrm{AD} + \mathrm{CB} \tag{6.6.12}$$

In this reaction, A and C are positively-charged cations, while B and D are negatively-charged anions. Double-replacement reactions generally occur between substances in aqueous solution. In order for a reaction to occur, one of the products is usually a solid precipitate, a gas, or a molecular compound such as water.

A precipitate forms in a double-replacement reaction when the cations from one of the reactants combine with the anions from the other reactant to form an insoluble ionic compound. When aqueous solutions of potassium iodide and lead (II) nitrate are mixed, the following reaction occurs.

$$2\mathrm{KI}\,(aq) + \mathrm{Pb(NO_3)_2}\,(aq) \rightarrow 2\mathrm{KNO_3}\,(aq) + \mathrm{PbI_2}\,(s) \tag{6.6.13}$$

Figure 10.6.3: *When a few drops of lead (II) nitrate are added to a solution of potassium iodide, a yellow precipitate of lead (II) iodide immediately forms in a double-replacement reaction.*

Combustion Reactions

A **combustion reaction** *is a reaction in which a substance reacts with oxygen gas, releasing energy in the form of light and heat.* Combustion reactions must involve $\mathrm{O_2}$ as one reactant. The combustion of hydrogen gas produces water vapor (see figure below).

$$2\mathrm{H_2}\,(g) + \mathrm{O_2}\,(g) + 2\mathrm{H_2O}\,(g) \tag{6.6.14}$$

Notice that this reaction also qualifies as a combination reaction.

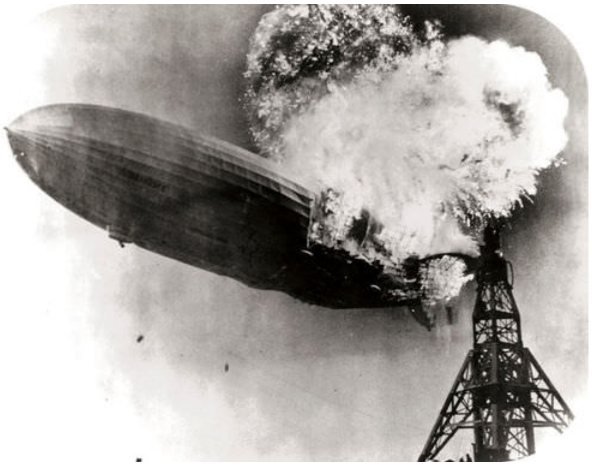

Figure 10.6.4: *The* Hindenburg *was a hydrogen-filled airship that suffered an accident upon its attempted landing in New Jersey in 1937. The hydrogen immediately combusted in a huge fireball, destroying the airship and killing 36 people. The chemical reaction was a simple one: hydrogen combining with oxygen to produce water.*

Many combustion reactions occur with a hydrocarbon, a compound made up solely of carbon and hydrogen. The products of the combustion of hydrocarbons are always carbon dioxide and water. Many hydrocarbons are used as fuel because their combustion releases very large amount of heat energy. Propane $\left(\mathrm{C_3H_8}\right)$ is a gaseous hydrocarbon that is commonly used as the fuel source in gas grills.

$$\mathrm{C_3H_8}\,(g) + 5\mathrm{O_2}\,(g) \rightarrow 3\mathrm{CO_2}\,(g) + 4\mathrm{H_2O}\,(g) \tag{6.6.15}$$

✓ Example 10.6.1

Ethanol can be used as a fuel source in an alcohol lamp.

- The formula for ethanol is $\mathrm{C_2H_5OH}$.

Write the balanced equation for the combustion of ethanol.

Solution:

Step 1: Plan the problem.

Ethanol and oxygen are the reactants. As with a hydrocarbon, the products of the combustion of an alcohol are carbon dioxide and water.

Step 2: Solve.

Write the skeleton equations: $\mathrm{C_2H_5OH}\,(l) + \mathrm{O_2}\,(g) \rightarrow \mathrm{CO_2}\,(g) + \mathrm{H_2O}\,(g)$

Balance the equation.

$$\mathrm{C_2H_5OH}\,(l) + 3\mathrm{O_2}\,(g) \rightarrow 2\mathrm{CO_2}\,(g) + 3\mathrm{H_2O}\,(g) \tag{6.6.16}$$

Step 3: Think about your result.

Combustion reactions must have oxygen as a reactant. Note that the water that is produced is in the gas state rather that the liquid state because of the high temperatures that accompany a combustion reaction.

Supplemental Resources

- Simulation of the synthesis of water: http://www.dit.ncssm/edu/core/Chapte...Synthesis.html
- View the synthesis of calcium oxide at http://www.youtube.com/watch?v-dszSKIM5rqk
- View the reaction between copper and chlorine gas at http://www.youtube.com/watch?v-edLpxdERQZc
- Watch the decomposition of hydrogen peroxide at http://www.youtube.com/watch?v=oX5FyaqNx54
- Watch the decomposition of potassium chlorate at http://www.youtube.com/watch?v=svRIg_kzE68
- A video experiment of hydrogen replacement by calcium can be seen at http://www.youtube.com/watch?v=hjB96do_fRw
- A video experiment of magnesium metal reacting with hydrochloric acid can be viewed at http://www.youtube.com/watch?v=OBdgeJFzSec
- Watch an animation of a double-replacement reaction at http://www.dlt.ncssm.edu/core/Chapte...cidToBase.html
- A video experiment in which a precipitate is formed by reacting sodium chloride with silver nitrate can be seen at http://www.youtube.com/watch?v=eFF3El4mwok
- A video experiment of the double-replacement reaction between copper (II) sulfate and sodium sulfide can be viewed at http://www.youtube.com/watch?v=KkKBDcFfZWo
- A video of the double-replacement reaction between sodium sulfate and barium chloride can be viewed at http://www.youtube.com/watch?v=XaMyfjYLhxU
- View an in-depth explanation of the combustion reaction that occurs when you strike a match at http://www.pbs.org/wgbh/nova/cigarette/onfire.html
- View an exciting video demonstration that confirms the importance of oxygen in a combustion reaction at http://education.jlab.org/frost/life_candle.html

- View another video demonstration that confirms the importance of oxygen in a combustion reaction at http://education.jlab.org/frost/combustion.html
- Reaction Identification Practice: http://www.sciencegeek.net/Chemistry...tification.htm

Contributors

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6.7: Real-World Examples of Chemical Reactions and Their Types

Real-World Examples of Chemical Reactions and Their Types

1. Synthesis Reaction: Formation of Ammonia

Scenario: The industrial production of ammonia (NH_3) through the Haber process is a crucial chemical reaction used to manufacture fertilizers.

Chemical Reaction:

$$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$$

Explanation: In this synthesis reaction, nitrogen gas (N_2) combines with hydrogen gas (H_2) to form ammonia (NH_3). This process is critical for producing fertilizers that support global agriculture.

2. Decomposition Reaction: Decomposition of Hydrogen Peroxide

Scenario: Hydrogen peroxide (H_2O_2) is commonly used as a disinfectant and bleach. It decomposes over time or when catalyzed by an enzyme called catalase.

Chemical Reaction:

$$2H_2O_2 \rightarrow 2H_2O + O_2$$

Explanation: In this decomposition reaction, hydrogen peroxide breaks down into water (H_2O) and oxygen gas (O_2). This reaction is utilized in disinfecting wounds, where the oxygen released helps kill bacteria.

3. Single Replacement Reaction: Zinc and Hydrochloric Acid

Scenario: Zinc (Zn) is often used in galvanizing processes to protect iron from rusting. When zinc reacts with hydrochloric acid (HCl), it demonstrates a single replacement reaction.

Chemical Reaction:.

$$Zn + 2HCl \rightarrow ZnCl_2 + H_2$$

Explanation: In this single replacement reaction, zinc displaces hydrogen in hydrochloric acid, forming zinc chloride ($ZnCl_2$) and releasing hydrogen gas (H_2). This reaction can be observed in laboratory demonstrations involving metal-acid reactions.

4. Double Replacement Reaction: Reaction Between Sodium Chloride and Silver Nitrate

Scenario: A classic laboratory experiment involves mixing sodium chloride (NaCl) and silver nitrate ($AgNO_3$) to observe the formation of a precipitate.

Chemical Reaction:

$$NaCl + AgNO_3 \rightarrow NaNO_3 + AgCl$$

Explanation: In this double replacement reaction, the cations and anions of the reactants exchange places, forming sodium nitrate ($NaNO_3$) and silver chloride (AgCl). The silver chloride forms a white precipitate, demonstrating the reaction visually.

5. Combustion Reaction: Burning of Propane

Scenario: Propane (C_3H_8) is commonly used as a fuel for heating, cooking, and in portable stoves.

Chemical Reaction:

$$C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O + Energy$$

Explanation: In this combustion reaction, propane reacts with oxygen (O_2) to produce carbon dioxide (CO_2), water (H_2O), and energy in the form of heat and light. This exothermic reaction is essential for everyday heating and cooking applications.

Summary

These real-world examples illustrate the different types of chemical reactions: synthesis, decomposition, single replacement, double replacement, and combustion. Each type of reaction plays a vital role in various industrial, laboratory, and everyday processes, showcasing the diverse and impactful nature of chemical transformations.

6.8: Factors Affecting Reaction Rates

Learning Objectives

By the end of this section, you will be able to:

- Describe the effects of chemical nature, physical state, temperature, concentration, and catalysis on reaction rates

The rates at which reactants are consumed and products are formed during chemical reactions vary greatly. Five factors typically affecting the rates of chemical reactions will be explored in this section: the chemical nature of the reacting substances, the state of subdivision (one large lump versus many small particles) of the reactants, the temperature of the reactants, the concentration of the reactants, and the presence of a catalyst.

The Chemical Nature of the Reacting Substances

The rate of a reaction depends on the nature of the participating substances. Reactions that appear similar may have different rates under the same conditions, depending on the identity of the reactants. For example, when small pieces of the metals iron and sodium are exposed to air, the sodium reacts completely with air overnight, whereas the iron is barely affected. The active metals calcium and sodium both react with water to form hydrogen gas and a base. Yet calcium reacts at a moderate rate, whereas sodium reacts so rapidly that the reaction is almost explosive.

The Chemical Nature of Reacting Substances: Factors Affecting Reaction Rates

The rate of a chemical reaction is influenced by various factors, one of which is the chemical nature of the reacting substances. This refers to the intrinsic properties of the reactants, including their molecular structure, bond energies, and reactivity. Here's a real-world example to illustrate how the nature of the reacting substances affects reaction rates.

"Rusting" by lamdogjunkie is licensed under CC BY 2.0.

Real-World Example: Rusting of Iron

> **Scenario:** The rusting of iron is a common example of a chemical reaction influenced by the nature of the reacting substances. Rusting occurs when iron reacts with oxygen and water to form iron oxide (rust).

Chemical Reaction:

$$4Fe + 3O_2 + 6H_2O \rightarrow 4Fe(OH)_3$$

Factors Influencing the Reaction Rate:

1. **Reactivity of the Substances:**
    - **Iron (Fe):** Iron is a moderately reactive metal, meaning it readily participates in chemical reactions, particularly oxidation.
    - **Oxygen (O_2):** Oxygen is a highly reactive non-metal, making it a strong oxidizing agent that readily reacts with metals.
2. **Surface Area:**
    - The larger the surface area of the iron, the faster the reaction. Finely divided iron (like iron filings) rusts more quickly than a solid iron bar because more iron atoms are exposed to react with oxygen and water.
3. **Presence of Water and Electrolytes:**
    - Water is essential for rusting as it facilitates the transfer of electrons. Electrolytes (like salt in seawater) increase the rate of rusting by enhancing the conductivity of water, allowing faster electron transfer.
4. **Bond Energies:**
    - The bond energies in iron and oxygen molecules influence the reaction rate. The process of breaking and forming bonds (between iron and oxygen atoms) requires energy. Lower bond energies in reactants can lead to faster reactions since less energy is needed to initiate the reaction.

The rusting of iron demonstrates how the chemical nature of reacting substances affects reaction rates. Iron's moderate reactivity, combined with oxygen's strong oxidizing properties, significantly influences the rate at which rust forms. Additionally, factors like surface area and the presence of water and electrolytes further affect the speed of this reaction. Understanding these factors helps in predicting and controlling the rates of various chemical reactions in real-world scenarios.

The Physical States of the Reactants

A chemical reaction between two or more substances requires intimate contact between the reactants. When reactants are in different physical states, or phases (solid, liquid, gaseous, dissolved), the reaction takes place only at the interface between the phases. Consider the heterogeneous reaction between a solid phase and either a liquid or gaseous phase. Compared with the reaction rate for large solid particles, the rate for smaller particles will be greater because the surface area in contact with the other reactant phase is greater. For example, large pieces of iron react more slowly with acids than they do with finely divided iron powder (Figure 6.8.1). Large pieces of wood smolder, smaller pieces burn rapidly, and saw dust burns explosively.

Figure 6.8.1: (a) Iron powder reacts rapidly with dilute hydrochloric acid and produces bubbles of hydrogen gas: $2Fe(s)$ + $6HCl(aq)$ $2FeCl_3(aq)$ + $3H_2(g)$. (b) An iron nail reacts more slowly because the surface area exposed to the acid is much less.

Link to Learning

Watch this video to see the reaction of cesium with water in slow motion and a discussion of how the state of reactants and particle size affect reaction rates.

Temperature of the Reactants

Chemical reactions typically occur faster at higher temperatures. Food can spoil quickly when left on the kitchen counter. However, the lower temperature inside of a refrigerator slows that process so that the same food remains fresh for days. Gas burners, hot plates, and ovens are often used in the laboratory to increase the speed of reactions that proceed slowly at ordinary temperatures. For many chemical processes, reaction rates are approximately doubled when the temperature is raised by 10 °C.

Concentrations of the Reactants

The rates of many reactions depend on the concentrations of the reactants. Rates usually increase when the concentration of one or more of the reactants increases. For example, calcium carbonate ($CaCO_3$) deteriorates as a result of its reaction with the pollutant sulfur dioxide. The rate of this reaction depends on the amount of sulfur dioxide in the air (Figure $6.8.2$). An acidic oxide, sulfur dioxide combines with water vapor in the air to produce sulfurous acid in the following reaction:

$$\mathrm{SO_2(g) + H_2O(g) \longrightarrow H_2SO_3(aq)}$$

Calcium carbonate reacts with sulfurous acid as follows:

$$\mathrm{CaCO_3(s) + H_2SO_3(aq) \longrightarrow CaSO_3(aq) + CO_2(g) + H_2O(l)}$$

In a polluted atmosphere where the concentration of sulfur dioxide is high, calcium carbonate deteriorates more rapidly than in less polluted air. Similarly, phosphorus burns much more rapidly in an atmosphere of pure oxygen than in air, which is only about 20% oxygen.

Figure $6.8.2$: Statues made from carbonate compounds such as limestone and marble typically weather slowly over time due to the actions of water, and thermal expansion and contraction. However, pollutants like sulfur dioxide can accelerate weathering. As the concentration of air pollutants increases, deterioration of limestone occurs more rapidly. (credit: James P Fisher III)

Link to Learning

Phosphorus burns rapidly in air, but it will burn even more rapidly if the concentration of oxygen is higher. Watch this video to see an example.

The Presence of a Catalyst

Relatively dilute aqueous solutions of hydrogen peroxide, $\mathrm{H_2O_2}$, are commonly used as topical antiseptics. Hydrogen peroxide decomposes to yield water and oxygen gas according to the equation:

$$\mathrm{2\,H_2O_2(aq) \longrightarrow 2\,H_2O(l) + O_2(g)}$$

Under typical conditions, this decomposition occurs very slowly. When dilute $\mathrm{H_2O_2(aq)}$ is poured onto an open wound, however, the reaction occurs rapidly and the solution foams because of the vigorous production of oxygen gas. This dramatic difference is caused by the presence of substances within the wound's exposed tissues that accelerate the decomposition process. Substances that function to increase the rate of a reaction are called **catalysts**, a topic treated in greater detail later in this chapter.

Link to Learning

Chemical reactions occur when molecules collide with each other and undergo a chemical transformation. Before physically performing a reaction in a laboratory, scientists can use molecular modeling simulations to predict how the parameters discussed earlier will influence the rate of a reaction. Use the PhET Reactions & Rates interactive to explore how temperature, concentration, and the nature of the reactants affect reaction rates.

6.9: End of Chapter Activity

End of Chapter Activity: Creating a Lesson Plan on Physical and Chemical Reactions with AI and Bloom's Taxonomy

Now that you have explored the fundamentals of physical and chemical reactions, it's time to put your knowledge into practice. Your task is to create a succinct lesson plan for 5th graders that introduces them to the basics of these concepts. To help you with this, you will use AI tools and incorporate Bloom's Taxonomy to ensure a comprehensive learning experience. This lesson plan will go towards your digital notebook, a portfolio filled with lesson plans, activities, and labs for future use.

Activity Prompt:

Objective: Use AI and Bloom's Taxonomy to develop a lesson plan that effectively teaches 5th graders about the fundamentals of physical and chemical reactions, including concepts such as changes in state, indicators of chemical reactions, and the distinction between physical and chemical changes.

Understanding the Concepts:

Knowledge (Remembering): Define key terms related to physical and chemical reactions, such as physical change, chemical change, reactants, and products.

Comprehension (Understanding): Explain these concepts in simple, age-appropriate language, emphasizing the differences between physical and chemical changes.

Planning the Lesson:

Application: Design an engaging activity that allows students to observe physical and chemical changes. For example, have students dissolve salt in water (physical change) and observe vinegar reacting with baking soda (chemical change).

Analysis: Use AI tools to create visual aids or interactive simulations that illustrate the process of physical and chemical changes. For instance, create a simple animation showing molecules rearranging during a chemical reaction.

Deepening Understanding:

Synthesis (Creating): Ask students to predict the outcomes of various experiments. For example, ask what they think will happen if they mix different household substances (like baking soda and vinegar).

Evaluation: Have students discuss and reflect on their observations. Encourage them to think about why some changes are reversible (physical) and others are not (chemical).

Using AI in the Classroom:

Explore AI tools like educational apps or platforms that provide interactive content for teaching about physical and chemical reactions. Use these tools to create quizzes, flashcards, or interactive stories that reinforce the lesson's concepts.

Use AI to assess student understanding through formative assessments and provide instant feedback.

Deliverable:

Submit a detailed lesson plan that includes:

1. **A brief overview of the key concepts covered:** Outline the foundational concepts of physical and chemical reactions that will be taught.
2. **A description of the activities and experiments designed:** Detail the hands-on activities and experiments you will use to help students understand these concepts.
3. **Examples of AI tools used and how they enhance the learning experience:** Describe the AI tools you plan to incorporate, such as simulations or interactive quizzes, and explain how they will help students grasp complex concepts.
4. **An explanation of how Bloom's Taxonomy was applied in the lesson plan to ensure a well-rounded educational experience:** Illustrate how each level of Bloom's Taxonomy (Remembering, Understanding, Applying, Analyzing, Creating, and Evaluating) is addressed in your lesson plan.

This activity will help you integrate modern technology and educational strategies to create an effective and engaging learning experience for young students.

Example Lesson Plan:

Grade: 5th Grade
Topic: Physical and Chemical Reactions
Duration: 1 Week

Overview:
Students will learn the differences between physical and chemical changes, identify signs of chemical reactions, and conduct experiments to observe these changes.

Day 1: Introduction to Physical and Chemical Reactions

Objective: Define physical and chemical changes and provide examples.

- **Remembering:** Define key terms (physical change, chemical change, reactants, products).
- **Understanding:** Explain the differences between physical and chemical changes using examples from everyday life (e.g., melting ice, rusting iron).

Activity:
Watch a video (created using AI tools) explaining physical and chemical changes with real-life examples.

Day 2: Observing Physical Changes

Objective: Observe and describe physical changes.

- **Applying:** Conduct an experiment to observe a physical change (e.g., dissolving salt in water).

Activity:
Students dissolve various substances (salt, sugar, sand) in water and observe which ones dissolve and which don't.

Day 3: Observing Chemical Changes

Objective: Identify and observe chemical changes.

- **Applying:** Conduct an experiment to observe a chemical change (e.g., vinegar reacting with baking soda).

Activity:
Students mix vinegar and baking soda and record their observations (e.g., gas production, temperature change).

Day 4: Analysis and Synthesis

Objective: Analyze the experiments and predict outcomes.

- **Analyzing:** Use AI tools to create visual aids showing molecular changes during chemical reactions.
- **Creating:** Predict what will happen if different substances are mixed.

Activity:
Students use an AI simulation to visualize the molecular changes in chemical reactions and predict the outcomes of mixing different household substances.

Day 5: Reflection and Evaluation

Objective: Reflect on and evaluate the experiments.

- **Evaluating:** Discuss and reflect on the experiments, focusing on why some changes were physical and others were chemical.

Activity:
Students discuss their findings in groups and present their conclusions to the class, using visual aids created with AI tools.

By incorporating these strategies and activities, educators can effectively teach 5th graders about physical and chemical reactions, helping them understand and differentiate these fundamental scientific concepts.

6.10: End of Chapter Key Terms

Physical and Chemical Reactions Key Terms

1. **Physical Change**: A change in which the form of matter is altered but one substance is not transformed into another, such as changes in state (solid, liquid, gas), shape, or size.
2. **Chemical Change**: A change that results in the formation of one or more new substances with different chemical properties and compositions, often involving a chemical reaction.
3. **Chemical Reaction**: A process in which substances (reactants) undergo chemical changes to form new substances (products), involving the breaking and forming of chemical bonds.
4. **Reactants**: The starting materials in a chemical reaction that undergo change to form products.
5. **Products**: The new substances formed as a result of a chemical reaction.
6. **Law of Conservation of Mass**: A principle stating that mass is neither created nor destroyed in a chemical reaction; the mass of the reactants equals the mass of the products.
7. **Exothermic Reaction**: A chemical reaction that releases energy to the surroundings, usually in the form of heat.
8. **Endothermic Reaction**: A chemical reaction that absorbs energy from the surroundings, usually in the form of heat.
9. **Activation Energy**: The minimum amount of energy required to initiate a chemical reaction.
10. **Catalyst**: A substance that increases the rate of a chemical reaction without being consumed in the process, by lowering the activation energy.
11. **Inhibitor**: A substance that decreases the rate of a chemical reaction or prevents it from occurring.
12. **Chemical Equilibrium**: A state in which the forward and reverse reactions occur at equal rates, resulting in no net change in the concentrations of reactants and products.
13. **Reversible Reaction**: A chemical reaction in which the products can react to reform the reactants.
14. **Irreversible Reaction**: A chemical reaction in which the products cannot easily reform the reactants.
15. **Precipitate**: An insoluble solid that forms and separates from a solution during a chemical reaction.
16. **Synthesis Reaction**: A type of chemical reaction in which two or more substances combine to form a more complex product ($A + B \rightarrow AB$).
17. **Decomposition Reaction**: A type of chemical reaction in which a single compound breaks down into two or more simpler substances ($AB \rightarrow A + B$).
18. **Single Replacement Reaction**: A type of chemical reaction in which one element replaces another element in a compound ($A + BC \rightarrow AC + B$).
19. **Double Replacement Reaction**: A type of chemical reaction in which the ions of two compounds exchange places in an aqueous solution to form two new compounds ($AB + CD \rightarrow AD + CB$).
20. **Combustion Reaction**: A chemical reaction in which a substance combines with oxygen, releasing energy in the form of heat and light, and producing carbon dioxide and water (usually).
21. **Oxidation**: The process in which a substance loses electrons during a chemical reaction.
22. **Reduction**: The process in which a substance gains electrons during a chemical reaction.
23. **Redox Reaction**: A chemical reaction involving the transfer of electrons from one substance to another, comprising both oxidation and reduction processes.
24. **Chemical Bond**: The attractive force that holds atoms or ions together in a compound.
25. **Covalent Bond**: A chemical bond formed by the sharing of electron pairs between atoms.
26. **Ionic Bond**: A chemical bond formed by the electrostatic attraction between oppositely charged ions.
27. **Bond Energy**: The amount of energy required to break one mole of bonds in a substance.
28. **Reaction Rate**: The speed at which a chemical reaction occurs, often measured as the change in concentration of reactants or products per unit time.
29. **Concentration**: The amount of a substance in a given volume, typically expressed in moles per liter (Molarity, M).
30. **Temperature**: A measure of the average kinetic energy of particles in a substance, affecting the reaction rate.
31. **Pressure**: The force exerted per unit area, influencing the reaction rate of gases.
32. **Surface Area**: The total area of the exposed surface of a solid, affecting the reaction rate.
33. **Equilibrium Constant (K)**: A numerical value that expresses the ratio of the concentrations of products to reactants at equilibrium for a reversible reaction.
34. **Le Chatelier's Principle**: A principle stating that if a dynamic equilibrium is disturbed by changing the conditions, the position of equilibrium will shift to counteract the change.

35. **Reaction Mechanism**: The step-by-step sequence of elementary reactions by which overall chemical change occurs.
36. **Intermediate**: A species that appears in some steps of a reaction mechanism but not in the net equation.
37. **Transition State**: A high-energy state during a chemical reaction where old bonds are breaking and new bonds are forming.
38. **Chemical Kinetics**: The study of the rates of chemical processes and the factors that affect them.
39. **Energy Profile Diagram**: A graphical representation of the energy changes that occur during a chemical reaction.

CHAPTER OVERVIEW

7: Solutions Acids and Bases pH

7.1: Introduction and Learning Objectives

Chapter Learning Objectives

Fundamentals of Solutions and Solubility:

- Define solutions as homogeneous mixtures.
- Explore factors affecting solubility: temperature, pressure, and solute-solvent nature.
- Analyze dissolving processes and solution formation at the molecular level, including saturated, unsaturated, and supersaturated solutions.

Acids, Bases, and the pH Scale:

- Understand acids and bases through Arrhenius, Brønsted-Lowry, and Lewis theories.
- Learn the pH scale and its relation to hydrogen ion concentration.
- Use indicators and pH meters to determine solution pH.

Properties and Behavior of Acids and Bases:

- Investigate general properties of acids (sour taste, metal reactivity) and bases (bitter taste, slippery feel).
- Differentiate between strong and weak acids and bases based on ionization in solutions.
- Examine the role of acids and bases in chemical reactions, including neutralization forming salts and water.

Applications of Solutions, Acids, and Bases:

- Explore practical applications in daily life, industry, and the environment.
- Discuss the importance of pH understanding in agriculture, medicine, and environmental science.
- Analyze the role of acids and bases in household products, industrial processes, and biological systems.

Teaching Strategies for Solutions, Acids, and Bases:

- Develop effective methods for teaching solutions, acids, bases, and pH engagingly.
- Design experiments and activities to demonstrate properties and real-world uses of pH.
- Use multimedia tools to enhance understanding.
- Address common misconceptions and learning challenges.
- Emphasize interdisciplinary relevance and practical applications.

Introduction to Solutions Acids Bases & pH

Understanding solutions, acids, bases, and pH is crucial for grasping the fundamental principles of chemistry, forming the bedrock of many chemical processes and having wide-ranging applications in both scientific research and everyday life. Solutions are homogeneous mixtures where solutes are uniformly distributed within solvents, and their solubility is influenced by factors such as temperature, pressure, and the nature of the solute and solvent. At the molecular level, analyzing the dissolving process helps us comprehend the formation of saturated, unsaturated, and supersaturated solutions. Acids and bases, explained through Arrhenius, Brønsted-Lowry, and Lewis theories, play a pivotal role in chemical reactions, with the pH scale measuring their acidity or alkalinity. The properties of acids (sour taste, reactivity with metals) and bases (bitter taste, slippery feel) highlight their behavior, with their strength determined by their degree of ionization in solutions.

These concepts are integral to many practical applications, from industrial processes and environmental management to biological systems. For instance, understanding pH is crucial in agriculture, medicine, and everyday products like cleaning agents and food items. Educators can employ effective teaching strategies to make these topics accessible and engaging, using demonstrations, hands-on experiments, and multimedia tools to enhance understanding. Addressing common misconceptions and highlighting the interdisciplinary and practical applications of these concepts can inspire students and underscore their relevance. In K-12 education, age-appropriate language, interactive activities, and relatable examples can spark interest in young learners, making these fundamental ideas more tangible and memorable, and fostering a deeper appreciation for chemistry.

7.2: The Dissolution Process

Learning Objectives

- Describe the basic properties of solutions and how they form.
- Predict whether a given mixture will yield a solution based on molecular properties of its components.
- Explain why some solutions either produce or absorb heat when they form.

An earlier chapter of this text introduced *solutions*, defined as homogeneous mixtures of two or more substances. Often, one component of a solution is present at a significantly greater concentration, in which case it is called the solvent. The other components of the solution present in relatively lesser concentrations are called solutes. Sugar is a covalent solid composed of sucrose molecules, $C_{12}H_{22}O_{11}$. When this compound dissolves in water, its molecules become uniformly distributed among the molecules of water:

$$C_{12}H_{22}O_{11}(s) \longrightarrow C_{12}H_{22}O_{11}(aq) \tag{7.2.1}$$

The subscript “aq” in the equation signifies that the sucrose molecules are solutes and are therefore *individually dispersed* throughout the *aqueous solution* (water is the solvent). Although sucrose molecules are heavier than water molecules, they remain dispersed throughout the solution; gravity does not cause them to “settle out” over time.

Potassium dichromate, $K_2Cr_2O_7$, is an ionic compound composed of colorless potassium ions, K^+, and orange dichromate ions, $Cr_2O_7^{2-}$. When a small amount of solid potassium dichromate is added to water, the compound dissolves and dissociates to yield potassium ions and dichromate ions uniformly distributed throughout the mixture (Figure $7.2.1$), as indicated in this equation:

$$K_2Cr_2O_7(s) \longrightarrow 2\,K^+(aq) + Cr_2O_7^{2-}(aq) \tag{7.2.2}$$

As with the mixture of sugar and water, this mixture is also an aqueous solution. Its solutes, potassium and dichromate ions, remain individually dispersed among the solvent (water) molecules.

Figure $7.2.1$: When potassium dichromate ($K_2Cr_2O_7$) is mixed with water, it forms a homogeneous orange solution. (credit: modification of work by Mark Ott). The first photo shows a small mound of an orange crystalline solid. There is a right-facing arrow. The second photo shows a translucent, colorless liquid in a clear container. There is a right-facing arrow. The third photo shows a translucent orange liquid in a clear, covered container.

Water is used so often as a solvent that the word solution has come to imply an aqueous solution to many people. However, almost any gas, liquid, or solid can act as a solvent. Many alloys are solid solutions of one metal dissolved in another; for example, US five-cent coins contain nickel dissolved in copper. Air is a gaseous solution, a homogeneous mixture of nitrogen, oxygen, and several other gases. Oxygen (a gas), alcohol (a liquid), and sugar (a solid) all dissolve in water (a liquid) to form liquid solutions. Table $7.2.1$ gives examples of several different solutions and the phases of the solutes and solvents.

Table $7.2.1$: Different Types of Solutions

Solution	Solute	Solvent
air	$O_2(g)$	$N_2(g)$
soft drinks	$CO_2(g)$	$H_2O(l)$
hydrogen in palladium	$H_2(g)$	$Pd(s)$
rubbing alcohol	$H_2O(l)$	$C_3H_8O(l)$ (2-propanol)

Solution	Solute	Solvent
saltwater	NaCl(*s*)	$H_2O(l)$
brass	Zn(*s*)	Cu(*s*)

Solutions exhibit these defining traits:

- They are homogeneous; that is, after a solution is mixed, it has the same composition at all points throughout (its composition is uniform).
- The physical state of a solution—solid, liquid, or gas—is typically the same as that of the solvent, as demonstrated by the examples in Table $7.2.1$.
- The components of a solution are dispersed on a molecular scale; that is, they consist of a mixture of separated molecules, atoms, and/or ions.
- The dissolved solute in a solution will not settle out or separate from the solvent.
- The composition of a solution, or the concentrations of its components, can be varied continuously, within limits.

The Formation of Solutions

The formation of a solution is an example of a spontaneous process, a process that occurs under specified conditions without the requirement of energy from some external source. Sometimes we stir a mixture to speed up the dissolution process, but this is not necessary; a homogeneous solution would form if we waited long enough. The topic of spontaneity is critically important to the study of chemical thermodynamics and is treated more thoroughly in a later chapter of this text. For purposes of this chapter's discussion, it will suffice to consider two criteria that *favor*, but do not guarantee, the spontaneous formation of a solution:

1. a decrease in the internal energy of the system (an exothermic change, as discussed in the previous chapter on thermochemistry)
2. an increase in the disorder in the system (which indicates an increase in the *entropy* of the system, as you will learn about in the later chapter on thermodynamics)

In the process of dissolution, an internal energy change often, but not always, occurs as heat is absorbed or evolved. An increase in disorder always results when a solution forms.

When the strengths of the intermolecular forces of attraction between solute and solvent species in a solution are no different than those present in the separated components, the solution is formed with no accompanying energy change. Such a solution is called an ideal solution. A mixture of ideal gases (or gases such as helium and argon, which closely approach ideal behavior) is an example of an ideal solution, since the entities comprising these gases experience no significant intermolecular attractions.

When containers of helium and argon are connected, the gases spontaneously mix due to diffusion and form a solution (Figure $7.2.2$). The formation of this solution clearly involves an increase in disorder, since the helium and argon atoms occupy a volume twice as large as that which each occupied before mixing.

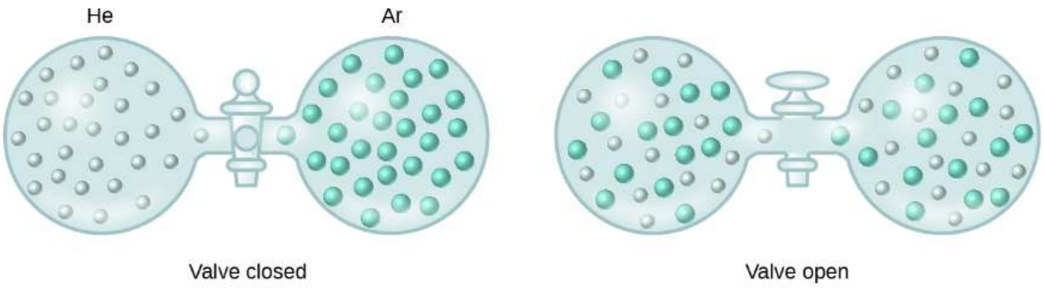

Figure $7.2.2$: Samples of helium and argon spontaneously mix to give a solution in which the disorder of the atoms of the two gases is increased. Two figures are shown. The first contains two spherical containers joined by a closed stopcock. The container to the left is labeled H e. It holds about thirty evenly dispersed, small, light blue spheres. The container on the right is labeled A r and contains about thirty slightly larger blue-green spheres. The second, similar figure has an open stopcock between the two spherical containers. The light blue and green spheres are evenly dispersed and present in both containers.

Ideal solutions may also form when structurally similar liquids are mixed. For example, mixtures of the alcohols methanol (CH_3OH) and ethanol (C_2H_5OH) form ideal solutions, as do mixtures of the hydrocarbons pentane, C_5H_{12}, and hexane, C_6H_{14}. Placing methanol and ethanol, or pentane and hexane, in the bulbs shown in Figure 7.2.2 will result in the same diffusion and subsequent mixing of these liquids as is observed for the He and Ar gases (although at a much slower rate), yielding solutions with no significant change in energy. Unlike a mixture of gases, however, the components of these liquid-liquid solutions do, indeed, experience intermolecular attractive forces. But since the molecules of the two substances being mixed are structurally very similar, the intermolecular attractive forces between like and unlike molecules are essentially the same, and the dissolution process, therefore, does not entail any appreciable increase or decrease in energy. These examples illustrate how diffusion alone can provide the driving force required to cause the spontaneous formation of a solution. In some cases, however, the relative magnitudes of intermolecular forces of attraction between solute and solvent species may prevent dissolution.

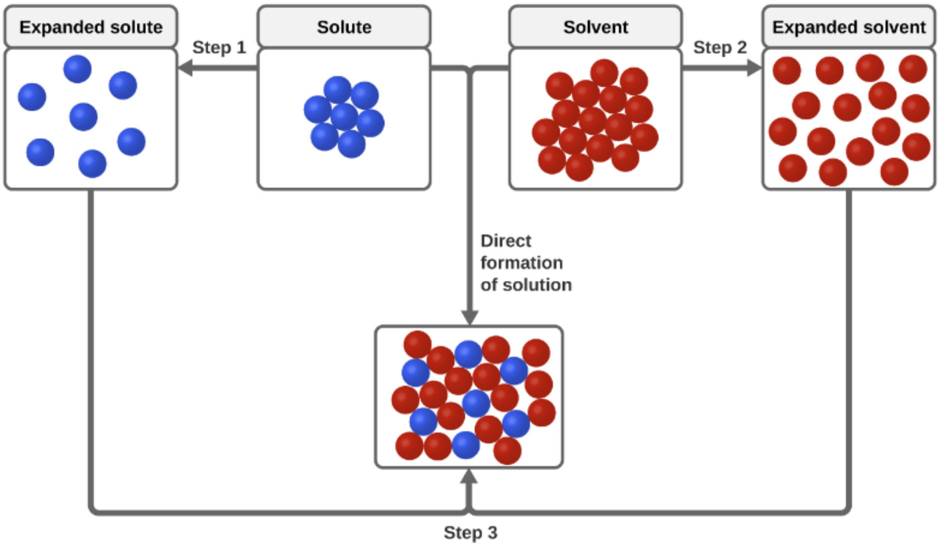

Figure 7.2.3: This schematic representation of dissolution shows a stepwise process involving the endothermic separation of solute and solvent species (Steps 1 and 2) and exothermic solvation (Step 3). The top, central region of the figure shows solute particles as seven blue spheres and solvent particles as 16 red spheres in separate, labeled boxes. The particles in these boxes are touching. An arrow labeled “Step 1” points left of the solute box, and shows the blue spheres no longer touching in another box labeled “expanded solute.” An arrow labeled “Step 2” points right from the solvent box and shows the red spheres no longer touching in another box labeled “expanded solvent.” Arrows proceed from the bottom of the expanded solute and expanded solvent boxes and join at the bottom of the figure where a step 3 label is shown. The joined arrows point to a box just above in which the red and blue spheres are mixed together and touching. The solute and solvent boxes are joined by another arrow labeled “direct formation of solution” which points downward at the center of the figure. This arrow also points to the box containing mixed red and blue spheres near the bottom of the figure.

Three types of intermolecular attractive forces are relevant to the dissolution process: solute-solute, solvent-solvent, and solute-solvent. As illustrated in Figure 7.2.3, the formation of a solution may be viewed as a stepwise process in which energy is consumed to overcome solute-solute and solvent-solvent attractions (endothermic processes) and released when solute-solvent attractions are established (an exothermic process referred to as solvation). The relative magnitudes of the energy changes associated with these stepwise processes determine whether the dissolution process overall will release or absorb energy. In some cases, solutions do not form because the energy required to separate solute and solvent species is so much greater than the energy released by solvation.

Figure 7.2.4: A mixture of nonpolar cooking oil and polar water does not yield a solution. (credit: Gautam Dogra).

For example, cooking oils and water will not mix to any appreciable extent to yield solutions (Figure 7.2.4). Hydrogen bonding is the dominant intermolecular attractive force present in liquid water; the nonpolar hydrocarbon molecules of cooking oils are not capable of hydrogen bonding, instead being held together by dispersion forces. Forming an oil-water solution would require overcoming the very strong hydrogen bonding in water, as well as the significantly strong dispersion forces between the relatively large oil molecules. And, since the polar water molecules and nonpolar oil molecules would not experience very strong intermolecular attraction, very little energy would be released by solvation.

On the other hand, a mixture of ethanol and water will mix in any proportions to yield a solution. In this case, both substances are capable of hydrogen bonding, and so the solvation process is sufficiently exothermic to compensate for the endothermic separations of solute and solvent molecules.

Figure 7.2.5: An instant cold pack gets cold when certain salts, such as ammonium nitrate, dissolve in water—an endothermic process. This figure shows a single use instant cold pack with labels indicating an inner bag of water surrounded by white particulate ammonium nitrate.

As noted at the beginning of this module, spontaneous solution formation is favored, but not guaranteed, by exothermic dissolution processes. While many soluble compounds do, indeed, dissolve with the release of heat, some dissolve endothermically. Ammonium nitrate (NH_4NO_3) is one such example and is used to make instant cold packs for treating injuries like the one pictured in Figure 7.2.5. A thin-walled plastic bag of water is sealed inside a larger bag with solid NH_4NO_3. When the smaller bag is broken, a solution of NH_4NO_3 forms, absorbing heat from the surroundings (the injured area to which the pack is applied) and providing a cold compress that decreases swelling. Endothermic dissolutions such as this one require a greater energy input to separate the solute species than is recovered when the solutes are solvated, but they are spontaneous nonetheless due to the increase in disorder that accompanies formation of the solution.

Video $\mathbf{7.2.1}$: *Watch this brief video illustrating endothermic and exothermic dissolution processes.*

Summary

A solution forms when two or more substances combine physically to yield a mixture that is homogeneous at the molecular level. The solvent is the most concentrated component and determines the physical state of the solution. The solutes are the other components typically present at concentrations less than that of the solvent. Solutions may form endothermically or exothermically, depending upon the relative magnitudes of solute and solvent intermolecular attractive forces. Ideal solutions form with no appreciable change in energy.

Footnotes

1. If bubbles of gas are observed within the liquid, the mixture is not homogeneous and, thus, not a solution.

Glossary

alloy

solid mixture of a metallic element and one or more additional elements

ideal solution

solution that forms with no accompanying energy change

solvation

exothermic process in which intermolecular attractive forces between the solute and solvent in a solution are established

spontaneous process

physical or chemical change that occurs without the addition of energy from an external source

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7.3: Electrolytes

Learning Objectives

- Define and give examples of electrolytes
- Distinguish between the physical and chemical changes that accompany dissolution of ionic and covalent electrolytes
- Relate electrolyte strength to solute-solvent attractive forces

When some substances are dissolved in water, they undergo either a physical or a chemical change that yields ions in solution. These substances constitute an important class of compounds called electrolytes. Substances that do not yield ions when dissolved are called nonelectrolytes. If the physical or chemical process that generates the ions is essentially 100% efficient (all of the dissolved compound yields ions), then the substance is known as a strong electrolyte. If only a relatively small fraction of the dissolved substance undergoes the ion-producing process, it is called a weak electrolyte.

Substances may be identified as strong, weak, or nonelectrolytes by measuring the electrical conductance of an aqueous solution containing the substance. To conduct electricity, a substance must contain freely mobile, charged species. Most familiar is the conduction of electricity through metallic wires, in which case the mobile, charged entities are electrons. Solutions may also conduct electricity if they contain dissolved ions, with conductivity increasing as ion concentration increases. Applying a voltage to electrodes immersed in a solution permits assessment of the relative concentration of dissolved ions, either quantitatively, by measuring the electrical current flow, or qualitatively, by observing the brightness of a light bulb included in the circuit (Figure $7.3.1$).

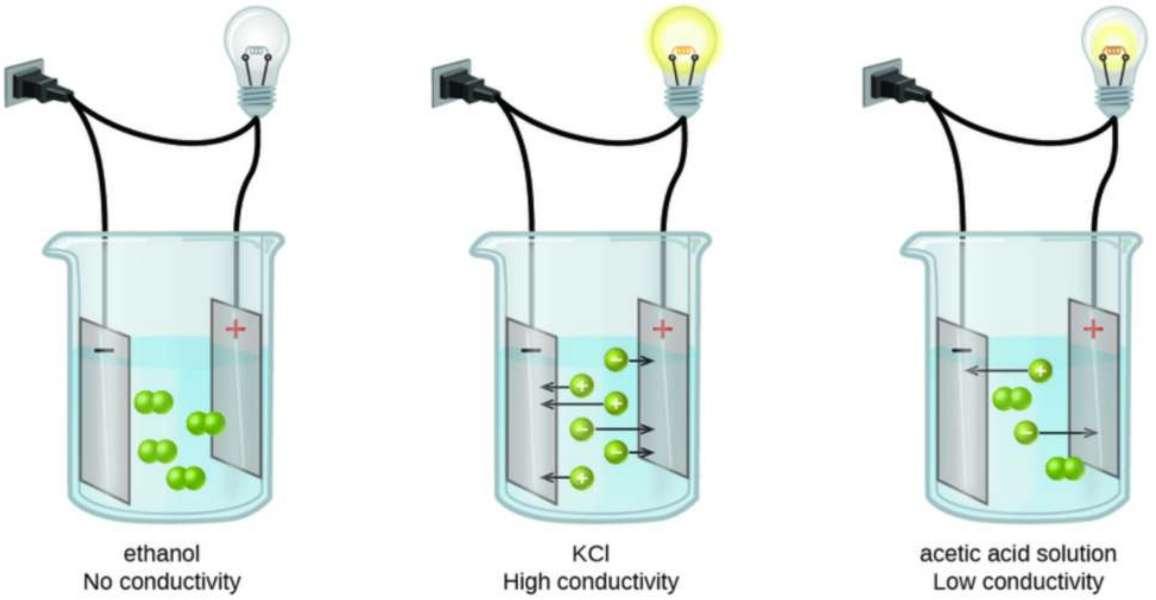


Figure $7.3.1$: Solutions of nonelectrolytes such as ethanol do not contain dissolved ions and cannot conduct electricity. Solutions of electrolytes contain ions that permit the passage of electricity. The conductivity of an electrolyte solution is related to the strength of the electrolyte.

This diagram shows three separate beakers. Each has a wire plugged into a wall outlet. In each case, the wire leads from the wall to the beaker and is split resulting in two ends. One end leads to a light bulb and continues on to a rectangle labeled with a plus sign. The other end leads to a rectangle labeled with a minus sign. The rectangles are in a solution. In the first beaker, labeled “Ethanol No Conductivity,” four pairs of linked small green spheres suspended in the solution between the rectangles. In the second beaker, labeled “K C l Strong Conductivity,” six individual green spheres, three labeled plus and three labeled minus are suspended in the solution. Each of the six spheres has an arrow extending from it pointing to the rectangle labeled with the opposite sign. In the third beaker, labeled “Acetic acid solution Weak conductivity,” two pairs of joined green spheres and two individual spheres, one labeled plus and one labeled minus are shown suspended between the two rectangles. The plus labeled sphere has an arrow pointing to the rectangle labeled minus and the minus labeled sphere has an arrow pointing to the rectangle labeled plus.

Ionic Electrolytes

Water and other polar molecules are attracted to ions, as shown in Figure 7.3.2. The electrostatic attraction between an ion and a molecule with a dipole is called an ion-dipole attraction. These attractions play an important role in the dissolution of ionic compounds in water.

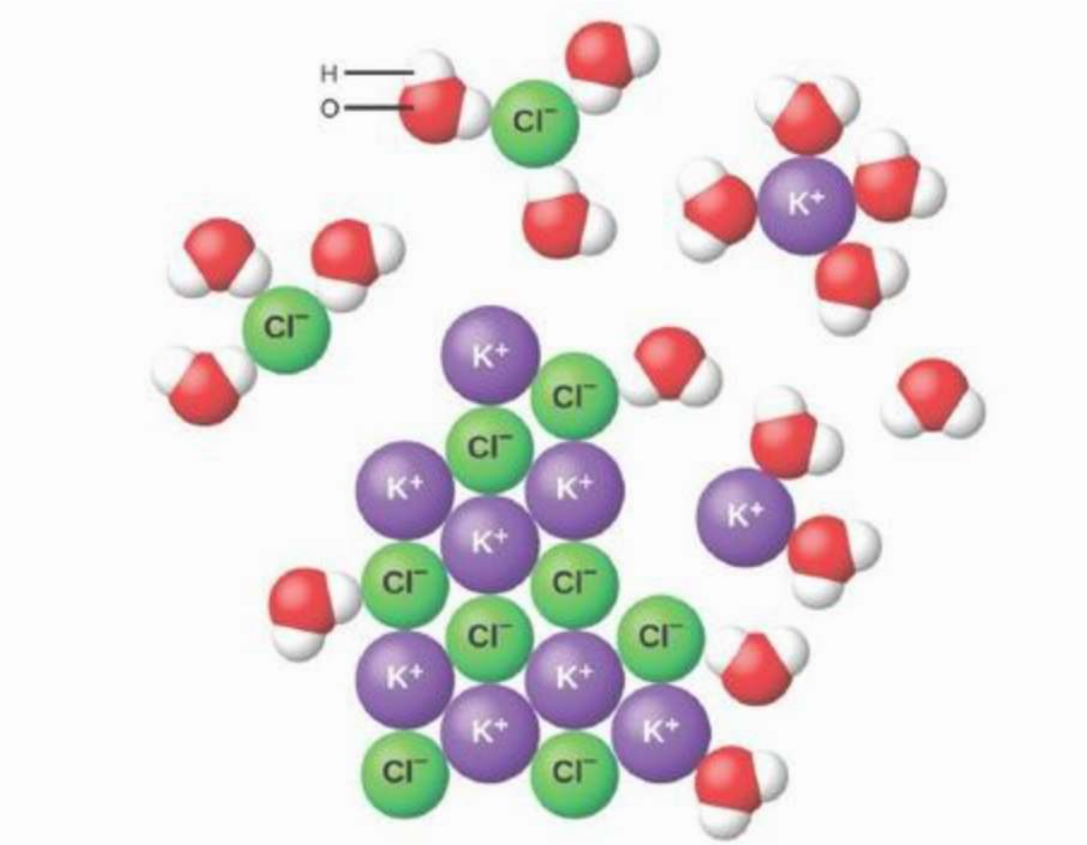


Figure 7.3.2: As potassium chloride (KCl) dissolves in water, the ions are hydrated. The polar water molecules are attracted by the charges on the K^+ and Cl^- ions. Water molecules in front of and behind the ions are not shown.

The diagram shows eight purple spheres labeled K superscript plus and eight green spheres labeled C l superscript minus mixed and touching near the center of the diagram. Outside of this cluster of spheres are seventeen clusters of three spheres, which include one red and two white spheres. A red sphere in one of these clusters is labeled O. A white sphere is labeled H. Two of the green C l superscript minus spheres are surrounded by three of the red and white clusters, with the red spheres closer to the green spheres than the white spheres. One of the K superscript plus purple spheres is surrounded by four of the red and white clusters. The white spheres of these clusters are closest to the purple spheres.

When ionic compounds dissolve in water, the ions in the solid separate and disperse uniformly throughout the solution because water molecules surround and solvate the ions, reducing the strong electrostatic forces between them. This process represents a physical change known as dissociation. Under most conditions, ionic compounds will dissociate nearly completely when dissolved, and so they are classified as strong electrolytes.

Let us consider what happens at the microscopic level when we add solid KCl to water. Ion-dipole forces attract the positive (hydrogen) end of the polar water molecules to the negative chloride ions at the surface of the solid, and they attract the negative (oxygen) ends to the positive potassium ions. The water molecules penetrate between individual K^+ and Cl^- ions and surround them, reducing the strong interionic forces that bind the ions together and letting them move off into solution as solvated ions, as Figure shows. The reduction of the electrostatic attraction permits the independent motion of each hydrated ion in a dilute solution, resulting in an increase in the disorder of the system as the ions change from their fixed and ordered positions in the crystal to mobile and much more disordered states in solution. This increased disorder is responsible for the dissolution of many ionic compounds, including KCl, which dissolve with absorption of heat.

In other cases, the electrostatic attractions between the ions in a crystal are so large, or the ion-dipole attractive forces between the ions and water molecules are so weak, that the increase in disorder cannot compensate for the energy required to separate the ions, and the crystal is insoluble. Such is the case for compounds such as calcium carbonate (limestone), calcium phosphate (the inorganic component of bone), and iron oxide (rust).

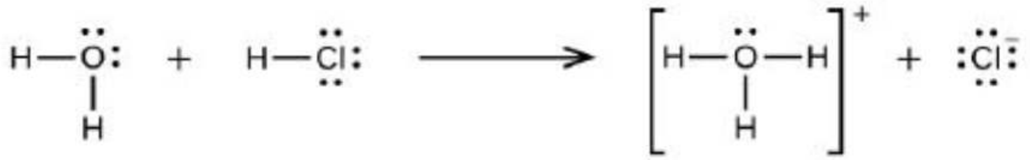
Covalent Electrolytes

Pure water is an extremely poor conductor of electricity because it is only very slightly ionized—only about two out of every 1 billion molecules ionize at 25 °C. Water ionizes when one molecule of water gives up a proton to another molecule of water, yielding hydronium and hydroxide ions.

$$\mathrm{H_2O(l)^+H_2O(l) \rightleftharpoons H_3O^+(aq) + OH^-(aq)} \tag{7.3.1}$$

In some cases, we find that solutions prepared from covalent compounds conduct electricity because the solute molecules react chemically with the solvent to produce ions. For example, pure hydrogen chloride is a gas consisting of covalent HCl molecules. This gas contains no ions. However, when we dissolve hydrogen chloride in water, we find that the solution is a very good conductor. The water molecules play an essential part in forming ions: Solutions of hydrogen chloride in many other solvents, such as benzene, do not conduct electricity and do not contain ions.

Hydrogen chloride is an *acid*, and so its molecules react with water, transferring H^+ ions to form hydronium ions (H_3O^+) and chloride ions (Cl^-):



A chemical equation is shown. To the left, two hydrogen atoms are linked, each with a single dash to a central oxygen atom to the left and below the oxygen symbol, which has two pairs of dots, above and to the right of the atom. A plus sign is shown to the right, then a hydrogen atom linked to the left side of chlorine atom by a single dash with three pairs of dots, above, to the right, and below the element symbol. An arrow points to the products which are three hydrogen atoms linked by single dashes to a central oxygen atom shown in brackets with superscript plus. The oxygen atom has a single pair of dots above the element symbol. This is followed by a plus and C l superscript minus. This symbol is surrounded by four pairs of dots, above and below and to the left and right of the element symbol.

This reaction is essentially 100% complete for HCl (i.e., it is a *strong acid* and, consequently, a strong electrolyte). Likewise, weak acids and bases that only react partially generate relatively low concentrations of ions when dissolved in water and are classified as weak electrolytes. The reader may wish to review the discussion of strong and weak acids provided in the earlier chapter of this text on reaction classes and stoichiometry.

Summary

Substances that dissolve in water to yield ions are called electrolytes. Electrolytes may be covalent compounds that chemically react with water to produce ions (for example, acids and bases), or they may be ionic compounds that dissociate to yield their constituent cations and anions, when dissolved. Dissolution of an ionic compound is facilitated by ion-dipole attractions between the ions of the compound and the polar water molecules. Soluble ionic substances and strong acids ionize completely and are strong electrolytes, while weak acids and bases ionize to only a small extent and are weak electrolytes. Nonelectrolytes are substances that do not produce ions when dissolved in water.

Glossary

dissociation

physical process accompanying the dissolution of an ionic compound in which the compound’s constituent ions are solvated and dispersed throughout the solution

electrolyte
substance that produces ions when dissolved in water

ion-dipole attraction
electrostatic attraction between an ion and a polar molecule

nonelectrolyte
substance that does not produce ions when dissolved in water

strong electrolyte
substance that dissociates or ionizes completely when dissolved in water

weak electrolyte
substance that ionizes only partially when dissolved in water

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7.4: Fundamentals of Solutions and Solubility

Fundamentals of Solutions and Solubility

Welcome to the world of solutions and solubility! In this introduction, we'll explore the fascinating ways substances mix, dissolve, and interact to form solutions. These concepts are foundational in chemistry and have practical applications in everyday life, from making a cup of coffee to developing pharmaceuticals.

What is a Solution?

A solution is a homogeneous mixture where one substance (the solute) is uniformly dispersed in another substance (the solvent). The most common example is saltwater, where salt (solute) dissolves in water (solvent).

Key Points:

- **Homogeneous**: The mixture has a uniform composition throughout.
- **Solute**: The substance that is dissolved.
- **Solvent**: The substance that does the dissolving.

Types of Solutions

Solutions can exist in different phases:

- **Gas in Gas**: Air (oxygen in nitrogen).
- **Liquid in Liquid**: Vinegar (acetic acid in water).
- **Solid in Liquid**: Sugar in tea.
- **Solid in Solid**: Alloys like brass (zinc in copper).

Solubility

Solubility is the ability of a solute to dissolve in a solvent at a specific temperature and pressure. It determines how much solute can be dissolved to form a solution.

Factors Affecting Solubility:

1. **Nature of Solute and Solvent**: "Like dissolves like" – polar solutes dissolve in polar solvents, and non-polar solutes dissolve in non-polar solvents.
    - **Example**: Salt (polar) dissolves in water (polar), but not in oil (non-polar).
2. **Temperature**: Solubility can increase or decrease with temperature.
    - **Example**: Sugar dissolves better in hot water than in cold water.
3. **Pressure**: Mainly affects gases; higher pressure increases gas solubility in liquids.
    - **Example**: Carbon dioxide is more soluble in soda under high pressure.

Concentration of Solutions

Concentration describes how much solute is present in a given quantity of solvent or solution. Common units include:

- **Molarity (M)**: Moles of solute per liter of solution.
- **Percent Composition**: Percentage of solute in the solution.
    - **Example**: 10% saline solution contains 10 grams of salt in 100 grams of solution.

Saturation

A solution can be:

- **Unsaturated**: More solute can be dissolved.
- **Saturated**: Maximum amount of solute is dissolved.
- **Supersaturated**: Contains more dissolved solute than normally possible at a given temperature; unstable and can precipitate.

Real-Life Applications

Understanding solutions and solubility is crucial in various fields:

- **Medicine**: Dissolving drugs for effective delivery.
- **Environmental Science**: Pollutant behavior in water.
- **Cooking**: Achieving perfect flavor balance.

By grasping these fundamentals, you'll see how chemistry is at play in numerous aspects of daily life and technological advancements. Let's dive deeper into the wonders of solutions and their behavior!

7.5: Solubility

Learning Objectives

- Describe the effects of temperature and pressure on solubility
- State Henry's law and use it in calculations involving the solubility of a gas in a liquid
- Explain the degrees of solubility possible for liquid-liquid solutions

Imagine adding a small amount of salt to a glass of water, stirring until all the salt has dissolved, and then adding a bit more. You can repeat this process until the salt concentration of the solution reaches its natural limit, a limit determined primarily by the relative strengths of the solute-solute, solute-solvent, and solvent-solvent attractive forces discussed in the previous two modules of this chapter. You can be certain that you have reached this limit because, no matter how long you stir the solution, undissolved salt remains. The concentration of salt in the solution at this point is known as its solubility.

The solubility of a solute in a particular solvent is the maximum concentration that may be achieved under given conditions when the dissolution process is at equilibrium. Referring to the example of salt in water:

$$\mathrm{NaCl}(s) \rightleftharpoons \mathrm{Na^+}(aq) + \mathrm{Cl^-}(aq) \tag{7.5.1}$$

When a solute's concentration is equal to its solubility, the solution is said to be saturated with that solute. If the solute's concentration is less than its solubility, the solution is said to be unsaturated. A solution that contains a relatively low concentration of solute is called dilute, and one with a relatively high concentration is called concentrated.

If we add more salt to a saturated solution of salt, we see it fall to the bottom and no more seems to dissolve. In fact, the added salt does dissolve, as represented by the forward direction of the dissolution equation. Accompanying this process, dissolved salt will precipitate, as depicted by the reverse direction of the equation. The system is said to be at equilibrium when these two reciprocal processes are occurring at equal rates, and so the amount of undissolved and dissolved salt remains constant. Support for the simultaneous occurrence of the dissolution and precipitation processes is provided by noting that the number and sizes of the undissolved salt crystals will change over time, though their combined mass will remain the same.

Video $7.5.1$: Watch this impressive video showing the precipitation of sodium acetate from a supersaturated solution.

Solutions may be prepared in which a solute concentration exceeds its solubility. Such solutions are said to be supersaturated, and they are interesting examples of nonequilibrium states. For example, the carbonated beverage in an open container that has not yet "gone flat" is supersaturated with carbon dioxide gas; given time, the CO_2 concentration will decrease until it reaches its equilibrium value.

7.5.1: Solutions of Gases in Liquids

In an earlier module of this chapter, the effect of intermolecular attractive forces on solution formation was discussed. The chemical structures of the solute and solvent dictate the types of forces possible and, consequently, are important factors in determining solubility. For example, under similar conditions, the water solubility of oxygen is approximately three times greater than that of helium, but 100 times less than the solubility of chloromethane, $CHCl_3$. Considering the role of the solvent's chemical structure, note that the solubility of oxygen in the liquid hydrocarbon hexane, C_6H_{14}, is approximately 20 times greater than it is in water.

Other factors also affect the solubility of a given substance in a given solvent. Temperature is one such factor, with gas solubility typically decreasing as temperature increases (Figure 7.5.1). This is one of the major impacts resulting from the thermal pollution of natural bodies of water.

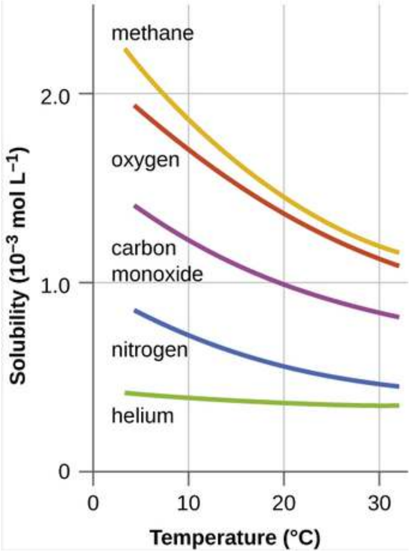


Figure 7.5.1: The solubilities of these gases in water decrease as the temperature increases. All solubilities were measured with a constant pressure of 101.3 kPa (1 atm) of gas above the solutions.

When the temperature of a river, lake, or stream is raised abnormally high, usually due to the discharge of hot water from some industrial process, the solubility of oxygen in the water is decreased. Decreased levels of dissolved oxygen may have serious consequences for the health of the water's ecosystems and, in severe cases, can result in large-scale fish kills (Figure 7.5.2).

Figure 7.5.2: (a) The small bubbles of air in this glass of chilled water formed when the water warmed to room temperature and the solubility of its dissolved air decreased. (b) The decreased solubility of oxygen in natural waters subjected to thermal pollution can result in large-scale fish kills. (credit a: modification of work by Liz West; credit b: modification of work by U.S. Fish and Wildlife Service)

The solubility of a gaseous solute is also affected by the partial pressure of solute in the gas to which the solution is exposed. Gas solubility increases as the pressure of the gas increases. Carbonated beverages provide a nice illustration of this relationship. The carbonation process involves exposing the beverage to a relatively high pressure of carbon dioxide gas and then sealing the beverage container, thus saturating the beverage with CO_2 at this pressure. When the beverage container is opened, a familiar hiss is heard as the carbon dioxide gas pressure is released, and some of the dissolved carbon dioxide is typically seen leaving solution in the form of small bubbles (Figure 7.5.3). At this point, the beverage is supersaturated with carbon dioxide and, with time, the dissolved carbon dioxide concentration will decrease to its equilibrium value and the beverage will become "flat."

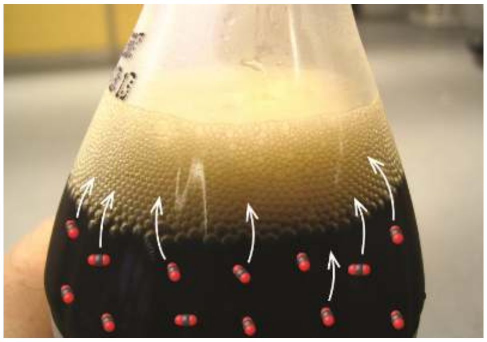


Figure 7.5.3: Opening the bottle of carbonated beverage reduces the pressure of the gaseous carbon dioxide above the beverage. The solubility of CO_2 is thus lowered, and some dissolved carbon dioxide may be seen leaving the solution as small gas bubbles. (credit: modification of work by Derrick Coetzee) A dark brown liquid is shown in a clear, colorless container. A thick layer of beige bubbles appear at the surface of the liquid. In the liquid, thirteen small clusters of single black spheres with two red spheres attached to the left and right are shown. Red spheres represent oxygen atoms and black represent carbon atoms. Seven white arrows point upward in the container from these clusters to the bubble layer at the top of the liquid.

For many gaseous solutes, the relation between solubility, C_g, and partial pressure, P_g, is a proportional one:

$$C_\mathrm{g} = kP_\mathrm{g}$$

where k is a proportionality constant that depends on the identities of the gaseous solute and solvent, and on the solution temperature. This is a mathematical statement of Henry's law: The quantity of an ideal gas that dissolves in a definite volume of liquid is directly proportional to the pressure of the gas.

✓ Example 7.5.1: Application of Henry's Law

At 20 °C, the concentration of dissolved oxygen in water exposed to gaseous oxygen at a partial pressure of 101.3 kPa (760 torr) is 1.38×10^{-3} mol L^{-1}. Use Henry's law to determine the solubility of oxygen when its partial pressure is 20.7 kPa (155 torr), the approximate pressure of oxygen in earth's atmosphere.

Solution

According to Henry's law, for an ideal solution the solubility, C_g, of a gas (1.38×10^{-3} mol L^{-1}, in this case) is directly proportional to the pressure, P_g, of the undissolved gas above the solution (101.3 kPa, or 760 torr, in this case). Because we know both C_g and P_g, we can rearrange this expression to solve for k.

$$\begin{aligned} C_\mathrm{g} &= kP_\mathrm{g} \\ k &= \frac{C_\mathrm{g}}{P_\mathrm{g}} \\ &= \frac{1.38 \times 10^{-3}\ \mathrm{mol\ L^{-1}}}{101.3\ \mathrm{kPa}} \\ &= 1.36 \times 10^{-5}\ \mathrm{mol\ L^{-1}\ kPa^{-1}} \\ &\ (1.82 \times 10^{-6}\ \mathrm{mol\ L^{-1}\ torr^{-1}}) \end{aligned}$$

Now we can use k to find the solubility at the lower pressure.

$$\begin{aligned} C_\mathrm{g} &= kP_\mathrm{g} \\ &1.36 \times 10^{-5}\ \mathrm{mol\ L^{-1}\ kPa^{-1}} \times 20.7\ \mathrm{kPa} \\ &(\text{or } 1.82 \times 10^{-6}\ \mathrm{mol\ L^{-1}\ torr^{-1}} \times 155\ \mathrm{torr}) \\ &= 2.82 \times 10^{-4}\ \mathrm{mol\ L^{-1}} \end{aligned}$$

Note that various units may be used to express the quantities involved in these sorts of computations. Any combination of units that yield to the constraints of dimensional analysis are acceptable.

? Exercise 7.5.1

Exposing a 100.0 mL sample of water at 0 °C to an atmosphere containing a gaseous solute at 20.26 kPa (152 torr) resulted in the dissolution of 1.45×10^{-3} g of the solute. Use Henry’s law to determine the solubility of this gaseous solute when its pressure is 101.3 kPa (760 torr).

Answer

7.25×10^{-3} g in 100.0 mL or 0.0725 g/L

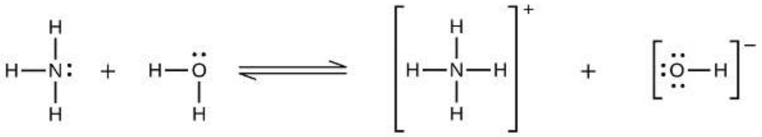
Case Study: Decompression Sickness (“The Bends”)

Decompression sickness (DCS), or “the bends,” is an effect of the increased pressure of the air inhaled by scuba divers when swimming underwater at considerable depths. In addition to the pressure exerted by the atmosphere, divers are subjected to additional pressure due to the water above them, experiencing an increase of approximately 1 atm for each 10 m of depth. Therefore, the air inhaled by a diver while submerged contains gases at the corresponding higher ambient pressure, and the concentrations of the gases dissolved in the diver’s blood are proportionally higher per Henry’s law.

Figure 7.5.4: (a) US Navy divers undergo training in a recompression chamber. (b) Divers receive hyperbaric oxygen therapy. Two photos are shown. The first shows two people seated in a steel chamber on benches that run length of the chamber on each side. The chamber has a couple of small circular windows and an open hatch-type door. One of the two people is giving a thumbs up gesture. The second image provides a view through a small, circular window. Inside the two people can be seen with masks over their mouths and noses. The people appear to be reading.

As the diver ascends to the surface of the water, the ambient pressure decreases and the dissolved gases becomes less soluble. If the ascent is too rapid, the gases escaping from the diver’s blood may form bubbles that can cause a variety of symptoms ranging from rashes and joint pain to paralysis and death. To avoid DCS, divers must ascend from depths at relatively slow speeds (10 or 20 m/min) or otherwise make several decompression stops, pausing for several minutes at given depths during the ascent. When these preventive measures are unsuccessful, divers with DCS are often provided hyperbaric oxygen therapy in pressurized vessels called decompression (or recompression) chambers (Figure 7.5.4).

Deviations from Henry’s law are observed when a chemical reaction takes place between the gaseous solute and the solvent. Thus, for example, the solubility of ammonia in water does not increase as rapidly with increasing pressure as predicted by the law because ammonia, being a base, reacts to some extent with water to form ammonium ions and hydroxide ions.



$$NH_3 + H_2O \rightleftharpoons [NH_4]^+ + [OH]^-$$

Gases can form **supersaturated solutions**. If a solution of a gas in a liquid is prepared either at low temperature or under pressure (or both), then as the solution warms or as the gas pressure is reduced, the solution may become supersaturated. In 1986, more than 1700 people in Cameroon were killed when a cloud of gas, almost certainly carbon dioxide, bubbled from Lake Nyos (Figure 7.5.5), a deep lake in a volcanic crater. The water at the bottom of Lake Nyos is saturated with carbon dioxide by volcanic activity beneath the lake. It is believed that the lake underwent a turnover due to gradual heating from below the lake, and the warmer, less-dense water saturated with carbon dioxide reached the surface. Consequently, tremendous quantities of dissolved CO_2 were released, and the colorless gas, which is denser than air, flowed down the valley below the lake and suffocated humans and animals living in the valley.

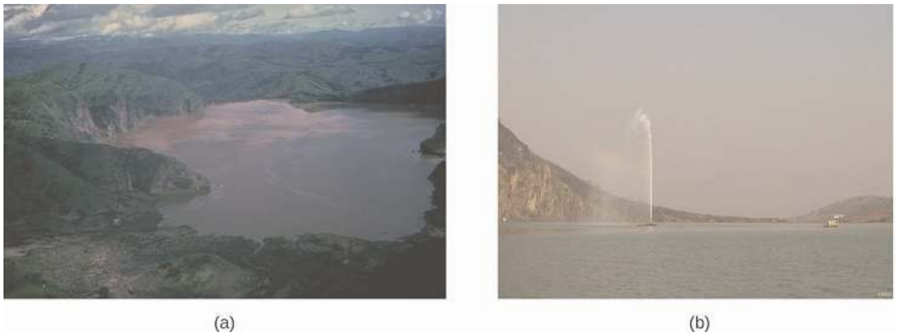

(a) (b)

Figure 7.5.5: (a) It is believed that the 1986 disaster that killed more than 1700 people near Lake Nyos in Cameroon resulted when a large volume of carbon dioxide gas was released from the lake. (b) A CO_2 vent has since been installed to help outgas the lake in a slow, controlled fashion and prevent a similar catastrophe from happening in the future. (credit a: modification of work by Jack Lockwood; credit b: modification of work by Bill Evans) Two photos are shown. The first is an aerial view of a lake surrounded by green hills. The second shows a large body of water with a fountain sending liquid up into the air several yards or meters above the surface of the water.

7.5.2: Solutions of Liquids in Liquids

We know that some liquids mix with each other in all proportions; in other words, they have infinite mutual solubility and are said to be miscible. Ethanol, sulfuric acid, and ethylene glycol (popular for use as antifreeze, pictured in Figure 7.5.6) are examples of liquids that are completely miscible with water. Two-cycle motor oil is miscible with gasoline.

Figure 7.5.6: Water and antifreeze are miscible; mixtures of the two are homogeneous in all proportions. (credit: “dno1967”/Wikimedia commons)

Liquids that mix with water in all proportions are usually polar substances or substances that form hydrogen bonds. For such liquids, the dipole-dipole attractions (or hydrogen bonding) of the solute molecules with the solvent molecules are at least as strong as those between molecules in the pure solute or in the pure solvent. Hence, the two kinds of molecules mix easily. Likewise, nonpolar liquids are miscible with each other because there is no appreciable difference in the strengths of solute-solute, solvent-solvent, and solute-solvent intermolecular attractions. The solubility of polar molecules in polar solvents and of nonpolar molecules in nonpolar solvents is, again, an illustration of the chemical axiom “like dissolves like.”

Two liquids that do not mix to an appreciable extent are called immiscible. Layers are formed when we pour immiscible liquids into the same container. Gasoline, oil (Figure 7.5.7), benzene, carbon tetrachloride, some paints, and many other nonpolar liquids are immiscible with water. The attraction between the molecules of such nonpolar liquids and polar water molecules is ineffectively weak. The only strong attractions in such a mixture are between the water molecules, so they effectively squeeze out the molecules of the nonpolar liquid. The distinction between immiscibility and miscibility is really one of degrees, so that miscible liquids are of infinite mutual solubility, while liquids said to be immiscible are of very low (though not zero) mutual solubility.

Figure 7.5.7: Water and oil are immiscible. Mixtures of these two substances will form two separate layers with the less dense oil floating on top of the water. (credit: "Yortw"/Flickr) This is a photo of a clear, colorless martini glass containing a golden colored liquid layer resting on top of a clear, colorless liquid.

Two liquids, such as bromine and water, that are of moderate mutual solubility are said to be partially miscible. Two partially miscible liquids usually form two layers when mixed. In the case of the bromine and water mixture, the upper layer is water, saturated with bromine, and the lower layer is bromine saturated with water. Since bromine is nonpolar, and, thus, not very soluble in water, the water layer is only slightly discolored by the bright orange bromine dissolved in it. Since the solubility of water in bromine is very low, there is no noticeable effect on the dark color of the bromine layer (Figure 7.5.8).

Figure 7.5.8: Bromine (the deep orange liquid on the left) and water (the clear liquid in the middle) are partially miscible. The top layer in the mixture on the right is a saturated solution of bromine in water; the bottom layer is a saturated solution of water in bromine. (credit: Paul Flowers) This figure shows three test tubes. The first test tube holds a dark orange-brown substance. The second test tube holds a clear substance. The amount of substance in both test tubes is the same. The third test tube holds a dark orange-brown substance on the bottom with a lighter orange substance on top. The amount of substance in the third test tube is almost double of the first two.

7.5.3: Solutions of Solids in Liquids

The dependence of solubility on temperature for a number of inorganic solids in water is shown by the solubility curves in Figure 7.5.9. Reviewing these data indicate a general trend of increasing solubility with temperature, although there are exceptions, as illustrated by the ionic compound cerium sulfate.

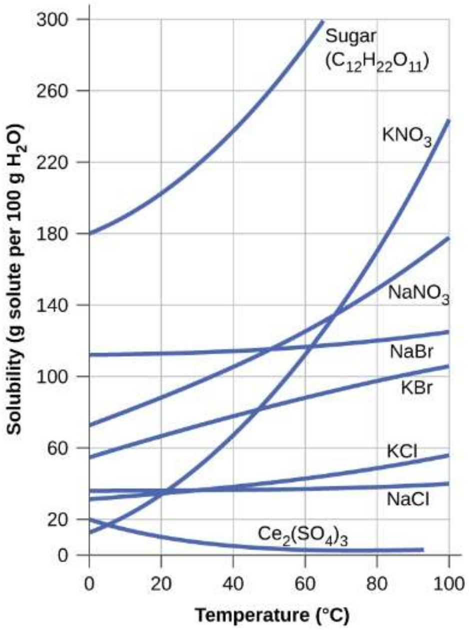


Figure 7.5.9: This graph shows how the solubility of several solids changes with temperature. This shows a graph of the solubility of sugar C subscript 12 H subscript 22 O subscript 11, K N O subscript 3, N a N O subscript 3, N a B r, K B r, N a subscript 2 S O subscript 4, K C l, and C e subscript 2 left parenthesis S O subscript 4 right parenthesis subscript 3 in g solute per 100 g H subscript 2 O at temperatures ranging from 0 degrees Celsius to 100 degrees Celsius. At 0 degrees Celsius, solubilities are approximately 180 for sugar C subscript 12 H subscript 22 O subscript 11, 115 for K N O subscript 3, 75 for N a N O subscript 3, 115 for N a B r, 55 for K B r, 7 for N a subscript 2 S O subscript 4, 25 for K C l, and 20 for C e subscript 2 left parenthesis S O subscript 4 right parenthesis subscript 3. At 0 degrees Celsius, solubilities are approximately 180 for sugar C subscript 12 H subscript 22 O subscript 11, 115 for K N O subscript 3, 75 for N a N O subscript 3, 115 for N a B r, 55 for K B r, 7 for N a subscript 2 S O subscript 4, 25 for K C l, and 20 for C e subscript 2 left parenthesis S O subscript 4 right parenthesis subscript 3. At 100 degrees Celsius, sugar C subscript 12 H subscript 22 O subscript 11 has exceeded the upper limit of solubility indicated on the graph, 240 for K N O subscript 3, 178 for N a N O subscript 3, 123 for N a B r, 105 for K B r, 52 for N a subscript 2 S O subscript 4, 58 for K C l, and the graph for C e subscript 2 left parenthesis S O subscript 4 right parenthesis subscript 3 stops at about 92 degrees Celsius where the solubility is nearly zero. The graph for N a subscript 2 S O subscript 4 is shown in red. All others substances are shown in blue. The solubility of this substance increases until about 30 degrees Celsius and declines beyond that point with increasing temperature.

The temperature dependence of solubility can be exploited to prepare supersaturated solutions of certain compounds. A solution may be saturated with the compound at an elevated temperature (where the solute is more soluble) and subsequently cooled to a lower temperature without precipitating the solute. The resultant solution contains solute at a concentration greater than its equilibrium solubility at the lower temperature (i.e., it is supersaturated) and is relatively stable. Precipitation of the excess solute can be initiated by adding a seed crystal (see the video in the Link to Learning earlier in this module) or by mechanically agitating the solution. Some hand warmers, such as the one pictured in Figure 7.5.10, take advantage of this behavior.

Figure 7.5.10: This hand warmer produces heat when the sodium acetate in a supersaturated solution precipitates. Precipitation of the solute is initiated by a mechanical shockwave generated when the flexible metal disk within the solution is “clicked.” (credit: modification of work by “Velela”/Wikimedia Commons) Three photos of hand warmers are shown side by side with an arrow pointing from the first photo to the second, and another arrow pointing from the second photo to the third. The first packet contains a clear colorless liquid and a small metal disc can be seen. In the second packet, the disc can’t be seen and a dispersion of white liquid is beginning. In the third packet, all of the liquid is white.

Video $7.5.2$: This video shows the crystallization process occurring in a hand warmer.

Summary

The extent to which one substance will dissolve in another is determined by several factors, including the types and relative strengths of intermolecular attractive forces that may exist between the substances' atoms, ions, or molecules. This tendency to dissolve is quantified as substance's solubility, its maximum concentration in a solution at equilibrium under specified conditions. A saturated solution contains solute at a concentration equal to its solubility. A supersaturated solution is one in which a solute's concentration exceeds its solubility—a nonequilibrium (unstable) condition that will result in solute precipitation when the solution is appropriately perturbed. Miscible liquids are soluble in all proportions, and immiscible liquids exhibit very low mutual solubility. Solubilities for gaseous solutes decrease with increasing temperature, while those for most, but not all, solid solutes increase with temperature. The concentration of a gaseous solute in a solution is proportional to the partial pressure of the gas to which the solution is exposed, a relation known as Henry's law.

7.5.4: Key Equations

- $C_{\mathrm{g}} = kP_{\mathrm{g}}$

Glossary

Henry's law

law stating the proportional relationship between the concentration of dissolved gas in a solution and the partial pressure of the gas in contact with the solution

immiscible

of negligible mutual solubility; typically refers to liquid substances

miscible

mutually soluble in all proportions; typically refers to liquid substances

partially miscible

of moderate mutual solubility; typically refers to liquid substances

saturated

of concentration equal to solubility; containing the maximum concentration of solute possible for a given temperature and pressure

solubility

extent to which a solute may be dissolved in water, or any solvent

supersaturated

of concentration that exceeds solubility; a nonequilibrium state

unsaturated

of concentration less than solubility

7.6: Arrhenius Acids and Bases

Learning Outcomes

- Understand that all Arrhenius Acids produce H^+ or protons in water.
- Understand that all Arrhenius Bases produce OH^- or hydroxide in water.
- Write formulas for binary and ternary Arrhenius Acids.
- Write formulas for Arrhenius Bases.
- Appreciate the pH scale.
- Know applications of selected Arrhenius Acids and Bases.
- Describe the properties of acids and bases.
- Describe how to neutralize an acid and a base.
- Know what chemical is used to absorb neutralized acid/base mixtures.

pH Scale

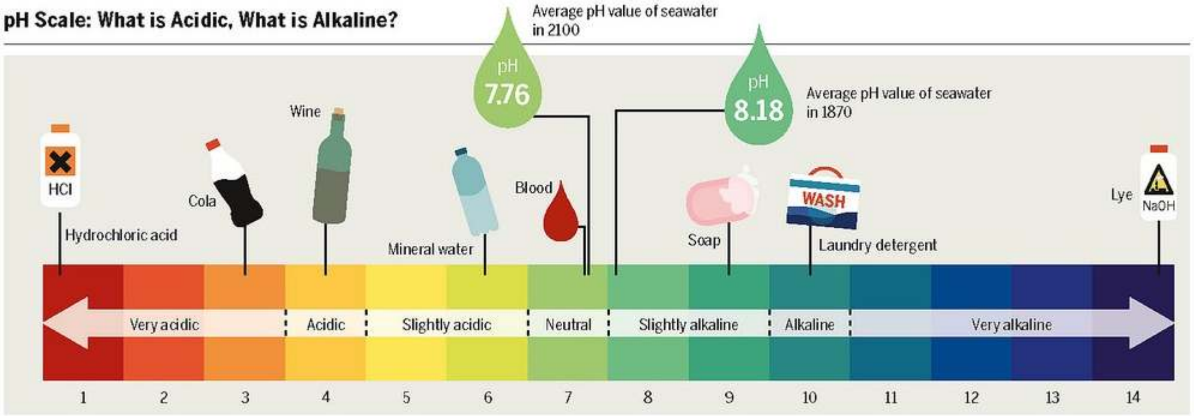


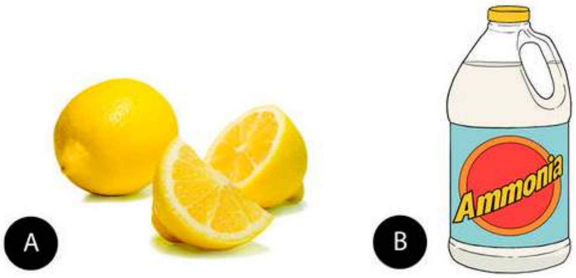
Figure 7.6.0: Average pH value of seawater in 1870 and 2100 (estimated). The difference may seem small, but the decline in the pH value from 1970 to 2100 would mean a 170 % increase in the acidity. Much small changes already post problems fro many sea creatures. pH value of various substances (Hydrochloric acid, Cola, Sopa, Laundry detergent, and more). (CC BY-SA 2.0; Generic; Heinrich-Böll-Stiftung).

Arrhenius Definitions

The Swedish chemist Svante Arrhenius (1859 - 1927) was the first to propose a theory to explain the observed behavior of acids and bases. Because of their ability to conduct a current, he knew that both acids and bases contained ions in solution. An **Arrhenius acid** *is a compound which ionizes to yield hydrogen ions* $\left(\mathrm{H}^{+}\right)$ *in aqueous solution*. An **Arrhenius base** *is a compound which ionizes to yield hydroxide ions* $\left(\mathrm{OH}^{-}\right)$ *in aqueous solution.*

Acid Properties

Acids are very common in some of the foods that we eat. Citrus fruits such as oranges and lemons contain citric acid and ascorbic acid, which is better known as vitamin C (see figure below). All carbonated beverages contain carbonic acid. This chemical easily decomposes to produce bubbly carbon dioxide and water. Acids can be commonly found in many places. For example, your stomach uses gastric acid (hydrochloric) to dissolve food. Vinegar is a mixture of water and acetic acid. This solution preserves condiments like ketchup and barbeque sauce.



Acids are a distinct class of compounds because of the properties of their aqueous solutions. Those properties are outlined below.

1. Aqueous solutions of acids are electrolytes, meaning that they conduct an electrical current. Some acids are strong electrolytes because they ionize completely in water. Other acids are weak electrolytes which partially ionize when dissolved in water.
2. Acids have a sour taste. Lemons, vinegar, and sour candies all contain acids.
3. Acids change the color of certain acid-base indicators .Two common indicators are litmus and phenolphthalein. Blue litmus turns red in the presence of an acid, while phenolphthalein remains colorless.
4. Acids react with some metals to yield hydrogen gas.
5. Acids (pH < 7.0) react with bases (pH > 7.0) to produce a salt and water. When equal moles of an acid and a base are combined, the acid is neutralized by the base. The resulting mixture will have a more neutral pH.

Acids in your world

Formula writing and Applications of Arrhenius Acids

If the acid name begins with the prefix " hydro" then it is a binary acid. These types of acids involve anions from the periodic table. For " hydro" acids, perform the steps below:

1. Start the formula with a proton (H^+) and refer to the periodic table for the anion species.
2. Once charges from step #1 have been obtained, cross the charges down diagonally and remove signs (+/-)
3. Acid formulas will need to display the state of matter (aq) beside the chemical formula.

If the acid name does not begin with the prefix "hydro," then it is a polyatomic or ternary acid.

1. Start the fomula with a proton (H^+) and note the ending of the acid name. Acids that have "ic" endings will use the "ate" polyatomic ion. In contrast, acids that have "ous" endings will use the "ite" polyatomic ion.
2. Cross the proton and polyatomic charges down and remove signs (+/-).
3. Acid formulas will need to display the state of matter (aq) beside the chemical formula.

Table 7.6.1: Common Acids

Acid Name	Formula	Real-World Applications
hydrochloric acid	HCl	Contained in stomach acid and liquid toilet bowl cleaner

Acid Name	Formula	Real-World Applications
nitric acid	HNO_3	Contained in chemical bombs and fertilizers.
sulfuric acid	H_2SO_4	Wet batter acid, chemical bombs, and fertilizers.
phosphoric acid	H_3PO_4	Preservative in dark-colored colas and fertilizer components.
acetic acid	CH_3COOH	Active ingredient of vinegar solution
hypochlorous acid	$HClO$	Acid is used to sanitize water.
carbonic acid	H_2CO_3	Acid contained in all carbonated beverages

A monoprotic acid is an acid that contains only one ionizable hydrogen. Hydrochloric acid and acetic acid are monoprotic acids. A polyprotic acid is an acid that contains multiple ionizable hydrogens. Most common polyprotic acids are either diprotic (such as H_2SO_4), or triprotic (such as H_3PO_4).

Bases Properties

Bases have properties that mostly contrast with those of acids.

1. Arrhenius bases that are soluble in water can conduct electricity.
2. Bases often have a bitter taste and are found in foods less frequently than acids. Many bases, like soaps, are slippery to the touch.
3. Bases also change the color of indicators. Red litmus turns blue in the presence of a base (see figure below), while phenolphthalein turns pink.
4. Some bases react with metals to produce hydrogen gas.
5. Acids ($pH < 7.0$) react with bases ($pH > 7.0$) to produce a salt and water. When equal moles of an acid and a base are combined, the acid is neutralized by the base. The resulting mixture will have a more neutral pH.

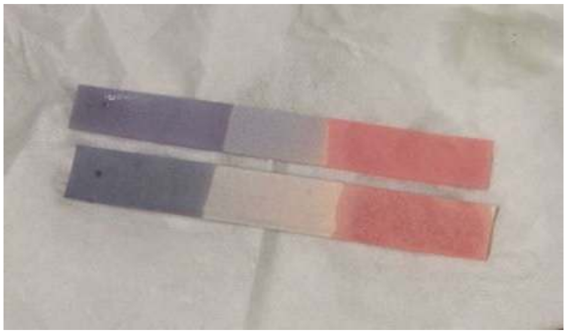


Figure $7.6.2$: *Litmus paper has be treated with the plant dye called litmus. It turns red in the presence of an acid and blue in the presence of a base.*

Applications of Arrhenius Bases

Bases are ionic compounds which yield the hydroxide ion $\left(OH^-\right)$ upon dissociating in water. The table below lists several of the more common bases.

Table $7.6.2$: Common Bases

Base Name	Formula	Real-World Applications
Sodium hydroxide	$NaOH$	Drano and Lye
Potassium hydroxide	KOH	Oven cleaner
Magnesium hydroxide	$Mg(OH)_2$	Milk of Magnesia (laxative)

Base Name	Formula	Real-World Applications
Calcium hydroxide	$Ca(OH)_2$	Chemical hair removal
Ammonium hydroxide	NH_4OH	ammonia (cleaner), do not mix with bleach

All of the bases listed in the table are solids at room temperature. Upon dissolving in water, each dissociates into a metal cation and the hydroxide ion.

$$\mathrm{NaOH}\,(s) \overset{\mathrm{H_2O}}{\rightarrow} \mathrm{Na^+}\,(aq) + \mathrm{OH^-}\,(aq)$$

Sodium hydroxide is a very caustic substance also known as lye. Lye is used as a rigorous cleaner and is an ingredient in the manufacture of soaps. Care must be taken with strong bases like sodium hydroxide, as exposure can lead to severe burns (see figure below).

Figure $7.6.4$: This foot has severe burns due to prolonged contact with a solution of sodium hydroxide, also known as lye.

Brønsted-Lowry Acids and Bases

The Arrhenius definition of acids and bases is somewhat limited. There are some compounds whose properties suggest that they are either acidic or basic, but which do not qualify according to the Arrhenius definition. An example is ammonia $\left(NH_3\right)$. An aqueous solution of ammonia turns litmus blue, reacts with acids, and displays various other properties that are common for bases. However, it does not contain the hydroxide ion. In 1923, a broader definition of acids and bases was independently proposed by the Danish chemist Johannes Brønsted (1879 - 1947) and the English chemist Thomas Lowry (1874 - 1936). A **Brønsted-Lowry acid** *is a molecule or ion that donates a hydrogen ion in a reaction.* A **Brønsted-Lowry base** *is a molecule or ion that accepts a hydrogen ion in a reaction*. Because the most common isotope of hydrogen consists of a single proton and a single electron, a hydrogen ion (in which the single electron has been removed) is commonly referred to as a proton. As a result, acids and bases are often called proton donors and proton acceptors, respectively, according to the Brønsted-Lowry definition. All substances that are categorized as acids and bases under the Arrhenius definition are also defined as such under the Brønsted-Lowry definition. The new definition, however, includes some substances that are left out according to the Arrhenius definition.

Acid-Base Neutralization Reactions

Recall the pH of both acids (below 7) and bases (above 7). If an acid and a base are mixed, the resulting mixture will have more of a neutral pH. The products of this reaction will be water and an ionic compound. In the laboratory, neutralization reactions are used before storage or disposal of acids and bases. For example, acids are neutralized by using ionic compounds that have base-like properties. One commonly used chemical to neutralize acids is sodium bicarbonate (baking soda). As for bases, acetic acid (common acid in vinegar) is used to neutralize the OH^- component. If a large amount of acid or base is spilled in the laboratory, it is neutralized and then absorbed into kitty litter before being properly disposed of.

Contributors and Attributions

- Template:ContribCK12
- Template:ContribSoult

7.7: Brønsted-Lowry Acids and Bases

The Arrhenius concept of acids and bases was a significant contribution to the scientific understanding of acids and bases. It replaced and expanded Lavoisier's original idea that all acids contained oxygen. However, the Arrhenius theory also had its shortcomings. It did not take into account the role of the solvent. In addition, this concept did not deal with acid-base behavior in solvents such as benzene, where there could be no ionization. So, formation of a new theory was imperative, which built on the findings of Arrhenius, but also went beyond them.

Brønsted-Lowry Acids and Bases

The Arrhenius definition of acids and bases is somewhat limited. There are some compounds whose properties suggest that they are either acidic or basic, but which do not qualify according to the Arrhenius definition. An example is ammonia $\left(\mathrm{NH_3}\right)$. Its aqueous solution turns litmus blue, it reacts with acids, and displays all the other properties of a base. However, it does not contain the hydroxide ion. In 1923, a broader definition of acids and bases was independently proposed by Danish chemist Johannes Brønsted (1879-1947) and English chemist Thomas Lowry (1874-1936). A **Brønsted-Lowry acid** is a molecule or ion that donates a hydrogen ion in a reaction. A **Brønsted-Lowry base** is a molecule or ion that accepts a hydrogen ion in a reaction. A hydrogen ion is commonly referred to as a proton, and so acids and bases are proton donors and proton acceptors, respectively, according to the Brønsted-Lowry definition. All substances that are categorized as acids and bases under the Arrhenius definition are also defined as such under the Brønsted-Lowry definition. The new definition, however, includes some substances that are left out according to the Arrhenius definition.

What kind of molecule would qualify as a Brønsted-Lowry base? These molecules need to be able to accept a hydrogen ion (or proton). Two possibilities come to mind: an anion that can form a neutral compound with a proton, or a molecule in which one or more atoms has lone-pair electrons. The most obvious anion is the Arrhenius base $\mathrm{OH^-}$. This ion can form a water molecule with a proton by accepting the proton. The acetate anion $\mathrm{CH_3COO^-}$ is another anion that can combine with a proton to form acetic acid. Lone-pair electron groups would include the nitrogen atom (see figure below). The two electrons at the top of the nitrogen atom are not connected in any type of bond, but they do interact readily with a bare proton.

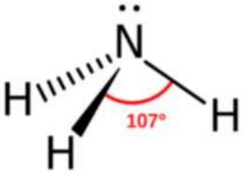


Figure $7.7.1$: *Lone pair electrons on nitrogen.* (CC BY-NC; CK-12)

Oxygen is another atom with lone pair electrons that can function as a Brønsted-Lowry base.

Figure $7.7.2$: Electrons in an oxygen atom. (CC BY-NC; CK-12)

The two single electrons (to the left and bottom of the atom) can form single covalent bonds with other atoms, while the two pairs of double electrons (top and right) are available to interact with a hydrogen ion.

Summary

- A Brønsted-Lowry acid is a molecule or ion that donates a hydrogen ion in a reaction.
- A Brønsted-Lowry base is a molecule or ion that accepts a hydrogen ion in a reaction.

7.8: Introduction to Lewis Acids and Bases

Introduction to Lewis Acids and Bases:

Acids and bases are an important part of chemistry. One of the most applicable theories is the Lewis acid/base motif that extends the definition of an acid and base beyond H^+ and OH^- ions as described by Brønsted-Lowry acids and bases.

The Brønsted acid-base theory has been used throughout the history of acid and base chemistry. However, this theory is very restrictive and focuses primarily on acids and bases acting as proton donors and acceptors. Sometimes conditions arise where the theory does not necessarily fit, such as in solids and gases. In 1923, G.N. Lewis from UC Berkeley proposed an alternate theory to describe acids and bases. His theory gave a generalized explanation of acids and bases based on structure and bonding. Through the use of the Lewis definition of acids and bases, chemists are now able to predict a wider variety of acid-base reactions. Lewis' theory used electrons instead of proton transfer and specifically stated that an acid is a species that accepts an electron pair while a base donates an electron pair.

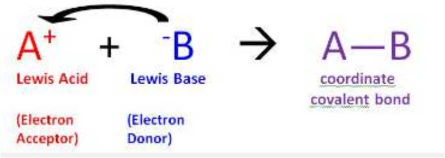


Figure 11: A Lewis Base (B) donates it electrons to a Lewis Acid (A) resulting in a coordinate covalently bonded compound, also known as an adduct.

The reaction of a Lewis acid and a Lewis base will produce a coordinate covalent bond (Figure 11). A coordinate covalent bond is just a type of covalent bond in which one reactant gives it electron pair to another reactant. In this case the lewis base donates its electrons to the Lewis acid. When they do react this way the resulting product is called an addition compound, or more commonly an adduct.

- Lewis Acid: a species that accepts an electron pair (i.e., an electrophile) and will have vacant orbitals
- Lewis Base: a species that donates an electron pair (i.e., a nucleophile) and will have lone-pair electrons

Additional Attributions:

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7.9: Lewis Acids and Bases

Learning Objectives

Make sure you thoroughly understand the following essential ideas which have been presented above. It is especially important that you know the precise meanings of all the highlighted terms in the context of this topic.

- Write the equation for the proton transfer reaction involving a Brønsted-Lowry acid or base, and show how it can be interpreted as an electron-pair transfer reaction, clearly identifying the donor and acceptor.
- Give an example of a *Lewis acid-base reaction* that does not involve protons.
- Write equations illustrating the behavior of a given non-aqueous acid-base system.

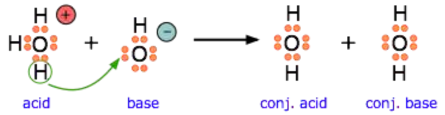
The Brønsted-Lowry proton donor-acceptor concept has been one of the most successful theories of Chemistry. But as with any such theory, it is fair to ask if this is not just a special case of a more general theory that could encompass an even broader range of chemical science. In 1916, G.N. Lewis of the University of California proposed that the *electron pair* is the dominant actor in acid-base chemistry. The Lewis theory did not become very well known until about 1923 (the same year that Brønsted and Lowry published their work), but since then it has been recognized as a very powerful tool for describing chemical reactions of widely different kinds and is widely used in organic and inorganic chemistry. According to Lewis,

- An *acid* is a substance that **accepts** a pair of electrons, and in doing so, forms a covalent bond with the entity that supplies the electrons.
- A *base* is a substance that **donates** an unshared pair of electrons to a recipient species with which the electrons can be shared.

> *In modern chemistry, electron donors are often referred to as nucleophiles, while acceptors are electrophiles.*

Proton-Transfer Reactions Involve Electron-Pair Transfer

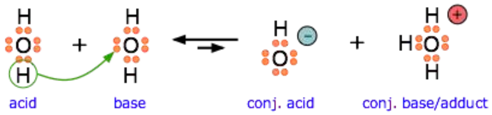
Just as any Arrhenius acid is also a Brønsted acid, any Brønsted acid is also a Lewis acid, so the various acid-base concepts are all "upward compatible". Although we do not really need to think about electron-pair transfers when we deal with ordinary aqueous-solution acid-base reactions, it is important to understand that it is the opportunity for electron-pair sharing that enables proton transfer to take place.



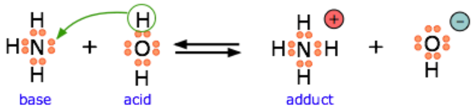
This equation for a simple acid-base neutralization shows how the Brønsted and Lewis definitions are really just different views of the same process. Take special note of the following points:

- The arrow shows the movement of a proton from the hydronium ion to the hydroxide ion.
- Note carefully that the electron-pairs themselves do not move; they remain attached to their central atoms. The electron pair on the base is "donated" to the acceptor (the proton) only in the sense that it ends up being *shared* with the acceptor, rather than being the exclusive property of the oxygen atom in the hydroxide ion.
- Although the hydronium ion is the nominal Lewis acid here, it does not itself accept an electron pair, but acts merely as the source of the proton that coordinates with the Lewis base.

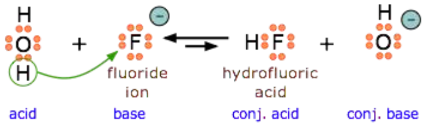
The point about the electron-pair remaining on the donor species is especially important to bear in mind. For one thing, it distinguishes a *Lewis acid-base reaction* from an *oxidation-reduction reaction*, in which a physical transfer of one or more electrons from donor to acceptor does occur. The product of a Lewis acid-base reaction is known formally as an "adduct" or "complex", although we do not ordinarily use these terms for simple proton-transfer reactions such as the one in the above example. Here, the proton combines with the hydroxide ion to form the "adduct" H_2O. The following examples illustrate these points for some other proton-transfer reactions that you should already be familiar with.



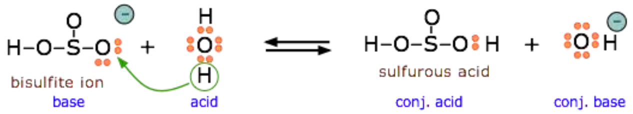
Another example, showing the autoprotolysis of water. Note that the conjugate base is also the adduct.



Ammonia is both a Brønsted and a Lewis base, owing to the unshared electron pair on the nitrogen. The reverse of this reaction represents the *hydrolysis* of the ammonium ion.



Because HF is a weak acid, fluoride salts behave as bases in aqueous solution. As a Lewis base, F⁻ accepts a proton from water, which is transformed into a hydroxide ion.



The bisulfite ion is *amphiprotic* and can act as an electron donor or acceptor.

Acid-base Reactions without Transferring Protons

The major utility of the Lewis definition is that it extends the concept of acids and bases beyond the realm of proton transfer reactions. The classic example is the reaction of boron trifluoride with ammonia to form an *adduct*:

$$BF_3 + NH_3 \rightarrow F_3B{-}NH_3 \tag{7.9.1}$$

One of the most commonly-encountered kinds of Lewis acid-base reactions occurs when electron-donating ligands form coordination complexes with transition-metal ions.

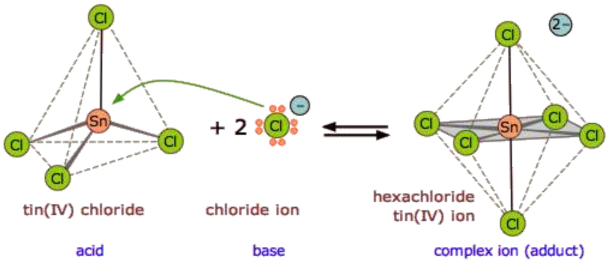


Figure 7.9.1: The tin atom in $SnCl_4$ can expand its valence shell by utilizing a pair of d-orbitals, changing its hybridization from sp^3 to sp^3d^2.

? Exercise 7.9.1

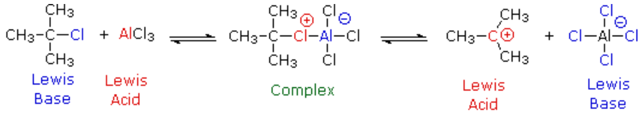
Here are several more examples of Lewis acid-base reactions that **cannot** be accommodated within the Brønsted or Arrhenius models. Identify the Lewis acid and Lewis base in each reaction.

a. $Al(OH)_3 + OH^- \rightarrow Al(OH)_4{}^-$
b. $SnS_2 + S^{2-} \rightarrow SnS_3^{2-}$

> c. $\mathrm{Cd(CN)_2 + 2\,CN- \rightarrow Cd(CN)_4^{2+}}$
> d. $\mathrm{AgCl + 2\,NH_3 \rightarrow Ag(NH_3)_2^{+} + Cl-}$
> e. $\mathrm{Fe^{2+} + NO \rightarrow Fe(NO)^{2+}}$
> f. $\mathrm{Ni^{2+} + 6\,NH_3 \rightarrow Ni(NH_3)_5^{2+}}$

Although organic chemistry is beyond the scope of these lessons, it is instructive to see how electron donors and acceptors play a role in chemical reactions. The following two diagrams show the mechanisms of two common types of reactions initiated by simple inorganic Lewis acids:

1



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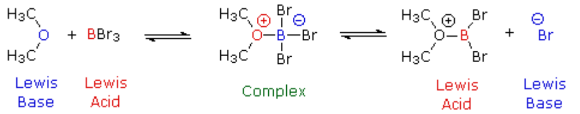


In each case, the species labeled "Complex" is an intermediate that decomposes into the products, which are conjugates of the original acid and base pairs. The electric charges indicated in the complexes are formal charges, but those in the products are "real".

In reaction 1, the incomplete octet of the aluminum atom in $\mathrm{AlCl_3}$ serves as a better electron acceptor to the chlorine atom than does the isobutyl part of the base. In reaction 2, the pair of non-bonding electrons on the dimethyl ether coordinates with the electron-deficient boron atom, leading to a complex that breaks down by releasing a bromide ion.

Non-aqueous Protonic Acid-Base Systems

We ordinarily think of Brønsted-Lowry acid-base reactions as taking place in aqueous solutions, but this need not always be the case. A more general view encompasses a variety of acid-base *solvent systems*, of which the *water system* is only one (Table 7.9.1). Each of these has as its basis an amphiprotic solvent (one capable of undergoing autoprotolysis), in parallel with the familiar case of water.

The ammonia system is one of the most common non-aqueous system in Chemistry. Liquid ammonia boils at –33° C, and can conveniently be maintained as a liquid by cooling with dry ice (–77° C). It is a good solvent for substances that also dissolve in water, such as ionic salts and organic compounds since it is capable of forming hydrogen bonds. However, many other familiar substances can also serve as the basis of protonic solvent systems as Table 7.9.1 indicates:

Table 7.9.1: Popular Solvent systems

solvent	autoprotolysis reaction	pK_{ap}
water	$2\,H_2O \rightarrow H_3O^+ + OH^-$	14
ammonia	$2\,NH_3 \rightarrow NH_4^+ + NH_2^-$	33
acetic acid	$2\,CH_3COOH \rightarrow CH_3COOH_2^+ + CH_3COO^-$	13
ethanol	$2\,C_2H_5OH \rightarrow C_2H_5OH_2^+ + C_2H_5O^-$	19
hydrogen peroxide	$2\,HO\text{-}OH \rightarrow HO\text{-}OH_2^+ + HO\text{-}O^-$	13
hydrofluoric acid	$2\,HF \rightarrow H_2F^+ + F^-$	10
sulfuric acid	$2\,H_2SO_4 \rightarrow H_3SO_4^+ + HSO_{4^-}$	3.5

One use of nonaqueous acid-base systems is to examine the relative strengths of the strong acids and bases, whose strengths are "leveled" by the fact that they are all totally converted into H_3O^+ or OH^- ions in water. By studying them in appropriate non-aqueous solvents which are poorer acceptors or donors of protons, their relative strengths can be determined.

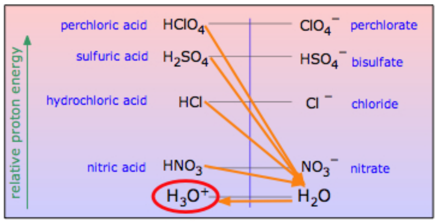


Figure $7.9.2$ Use of non-aqueous solvents allows the study of strong acids that are hindered by the "leveling" of the solvent.

7.10: Acid-Base Properties of Salt Solutions

Learning Objectives

- To recognize salts that will produce acidic, basic, or neutral solutions in water
- To understand the Lewis acidity of small, highly-charged metal ions in water

The chemical reaction between hydrochloric acid (HCl) and calcium hydroxide ($Ca(OH)_2$) to produce calcium chloride ($CaCl_2$) and water (H_2O) is an example of an acid-base neutralization reaction. Here is the balanced chemical equation for this reaction:

$$2\ \mathrm{HCl} + \mathrm{Ca(OH)_2} \rightarrow \mathrm{CaCl_2} + 2\ \mathrm{H_2O}$$

This reaction shows that two moles of hydrochloric acid react with one mole of calcium hydroxide to produce one mole of calcium chloride and two moles of water.

- All acids get neutralized with bases to produce salt solution and water molecules!
- Any base can be neutralized with acid, to produce salt solution and water molecules!

This phenomenon is called **Acid Based Neutralization.**

Here are some simple explanations and examples of the physical properties of acids and bases:

Physical Properties of Acids

1. **Taste**:
    - Acids have a sour taste.
    - **Example**: Citrus fruits like lemons, limes, and oranges contain citric acid, which gives them their sour flavor.
2. **Smell**:
    - Some acids have a sharp, pungent smell.
    - **Example**: Vinegar, which contains acetic acid, has a distinctive strong smell.
3. **pH Level**:
    - Acids have a pH level less than 7.
    - **Example**: Lemon juice has a pH around 2-3, making it a strong acid among common household substances.
4. **Reactivity with Metals**:
    - Acids react with certain metals to produce hydrogen gas.
    - **Example**: Hydrochloric acid in the lab reacts with zinc to produce hydrogen gas and zinc chloride.

Physical Properties of Bases

1. **Taste**:
    - Bases have a bitter taste.
    - **Example**: Baking soda (sodium bicarbonate) has a slightly bitter taste, though it's usually experienced in small quantities in food.
2. **Feel**:
    - Bases feel slippery or soapy to the touch.
    - **Example**: Many cleaning products, like household ammonia, feel slippery because they contain bases.
3. **pH Level**:
    - Bases have a pH level greater than 7.
    - **Example**: Household ammonia, used in many cleaners, has a pH around 11-12, making it a strong base.
4. **Conductivity**:
    - Bases can conduct electricity when dissolved in water.
    - **Example**: Lye (sodium hydroxide), used in oven cleaners, conducts electricity when dissolved in water.

Simple Examples:

1. **Citrus Fruits** (Acidic):
    - Contain citric acid.
    - Sour taste, pH around 2-3.
    - Example: Lemon, lime, orange.
2. **Vinegar** (Acidic):
    - Contains acetic acid.
    - Sour taste, strong smell, pH around 2.4.
    - Example: Used in salad dressings and for cleaning.
3. **Baking Soda** (Basic):
    - Contains sodium bicarbonate.
    - Slightly bitter taste, pH around 9.
    - Example: Used in baking and as a cleaning agent.
4. **Household Ammonia** (Basic):
    - Contains ammonium hydroxide.
    - Slippery feel, strong smell, pH around 11.
    - Example: Used in window cleaners and other household cleaners.

These examples help illustrate the everyday presence of acids and bases and their physical properties.

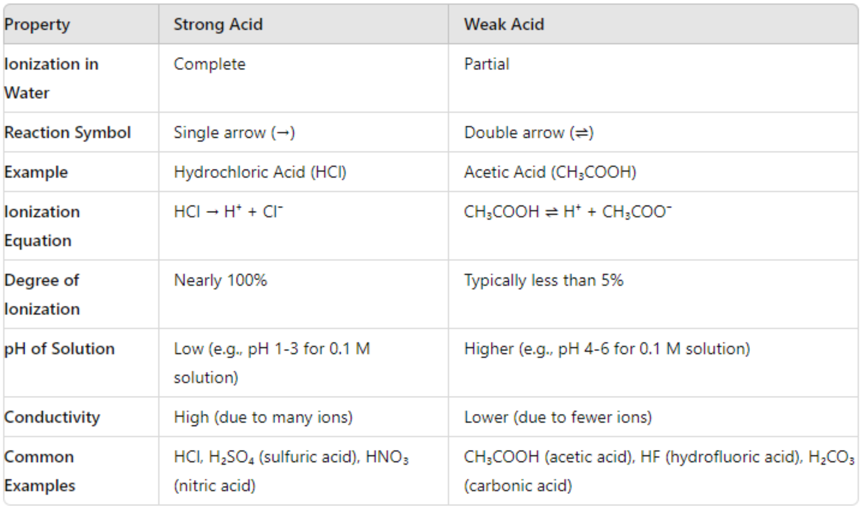
7.11: Concept of Strong and Weak Acids and Bases

Overview of Strong and Weak Acids

Acids can be classified as strong or weak based on their ability to ionize (dissociate) in water.

- **Strong Acids**: Completely ionize in solution. Nearly all molecules dissociate to produce hydrogen ions (H^+).
- **Weak Acids**: Partially ionize in solution. Only a small fraction of molecules dissociate to produce hydrogen ions (H^+).

Comparison Chart Acids



Property	Strong Acid	Weak Acid
Ionization in Water	Complete	Partial
Reaction Symbol	Single arrow (→)	Double arrow (⇌)
Example	Hydrochloric Acid (HCl)	Acetic Acid (CH_3COOH)
Ionization Equation	$HCl \rightarrow H^+ + Cl^-$	$CH_3COOH \rightleftharpoons H^+ + CH_3COO^-$
Degree of Ionization	Nearly 100%	Typically less than 5%
pH of Solution	Low (e.g., pH 1-3 for 0.1 M solution)	Higher (e.g., pH 4-6 for 0.1 M solution)
Conductivity	High (due to many ions)	Lower (due to fewer ions)
Common Examples	HCl, H_2SO_4 (sulfuric acid), HNO_3 (nitric acid)	CH_3COOH (acetic acid), HF (hydrofluoric acid), H_2CO_3 (carbonic acid)

Explanation:

Ionization in Water:

Strong Acids: They dissociate completely in water, meaning that every molecule of the acid releases an H^+ ion.

Example: Hydrochloric acid (HCl) dissociates as follows:

$$\mathrm{HCl} \rightarrow \mathrm{H}^+ + \mathrm{Cl}^-$$

Weak Acids: They only partially dissociate in water, meaning that only some of the acid molecules release H^+ ions.

Example: Acetic acid (CH_3COOH) dissociates as follows:

$$\mathrm{CH_3COOH} \rightleftharpoons \mathrm{H}^+ + \mathrm{CH_3COO}^-$$

Reaction Symbols:

- **Strong Acids**: Represented with a single arrow (→) to indicate complete dissociation.
- **Weak Acids**: Represented with a double arrow (⇌) to indicate an equilibrium between dissociated and undissociated forms.

Degree of Ionization:

- **Strong Acids**: Nearly 100% ionization means that almost all acid molecules donate protons.
- **Weak Acids**: Typically, less than 5% of the acid molecules ionize, leaving most of the acid molecules undissociated in solution.

pH of Solution:

- **Strong Acids**: Result in a low pH (very acidic).
- **Weak Acids**: Result in a higher pH compared to strong acids of the same concentration.

Conductivity:

- **Strong Acids**: High conductivity due to a large number of ions in solution.
- **Weak Acids**: Lower conductivity due to fewer ions in solution.

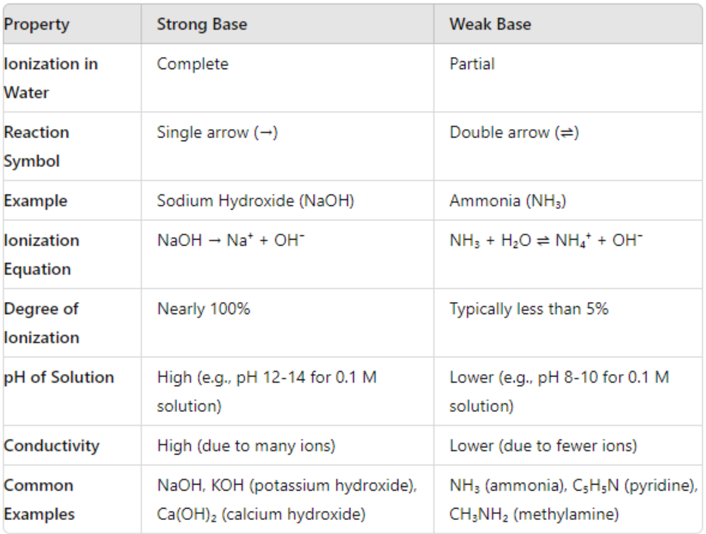
By comparing these properties, students can better understand the behavior of strong and weak acids in aqueous solutions.

Overview of Strong and Weak Bases

Bases can be classified as strong or weak based on their ability to ionize (dissociate) in water.

- **Strong Bases**: Completely ionize in solution. Nearly all molecules dissociate to produce hydroxide ions (OH^-).
- **Weak Bases**: Partially ionize in solution. Only a small fraction of molecules dissociate to produce hydroxide ions (OH^-).

Comparison Chart



Property	Strong Base	Weak Base
Ionization in Water	Complete	Partial
Reaction Symbol	Single arrow (→)	Double arrow (⇌)
Example	Sodium Hydroxide (NaOH)	Ammonia (NH_3)
Ionization Equation	$NaOH \rightarrow Na^+ + OH^-$	$NH_3 + H_2O \rightleftharpoons NH_4^+ + OH^-$
Degree of Ionization	Nearly 100%	Typically less than 5%
pH of Solution	High (e.g., pH 12-14 for 0.1 M solution)	Lower (e.g., pH 8-10 for 0.1 M solution)
Conductivity	High (due to many ions)	Lower (due to fewer ions)
Common Examples	NaOH, KOH (potassium hydroxide), $Ca(OH)_2$ (calcium hydroxide)	NH_3 (ammonia), C_5H_5N (pyridine), CH_3NH_2 (methylamine)

Explanation:

Ionization in Water:

- **Strong Bases**: They dissociate completely in water, meaning that every molecule of the base releases an OH^- ion.
  - Example: Sodium hydroxide (NaOH) dissociates as follows:

$$\mathrm{NaOH \rightarrow Na^+ + OH^-}$$

- **Weak Bases**: They only partially dissociate in water, meaning that only some of the base molecules release OH^- ions.
  - Example: Ammonia (NH_3) reacts with water as follows:

$$\mathrm{NH_3 + H_2O \rightleftharpoons NH_4^+ + OH^-}$$

Reaction Symbols:

- **Strong Bases**: Represented with a single arrow (→) to indicate complete dissociation.
- **Weak Bases**: Represented with a double arrow (⇌) to indicate an equilibrium between dissociated and undissociated forms.

Degree of Ionization:

- **Strong Bases**: Nearly 100% ionization means that almost all base molecules release hydroxide ions.
- **Weak Bases**: Typically, less than 5% of the base molecules ionize, leaving most of the base molecules undissociated in solution.

pH of Solution:

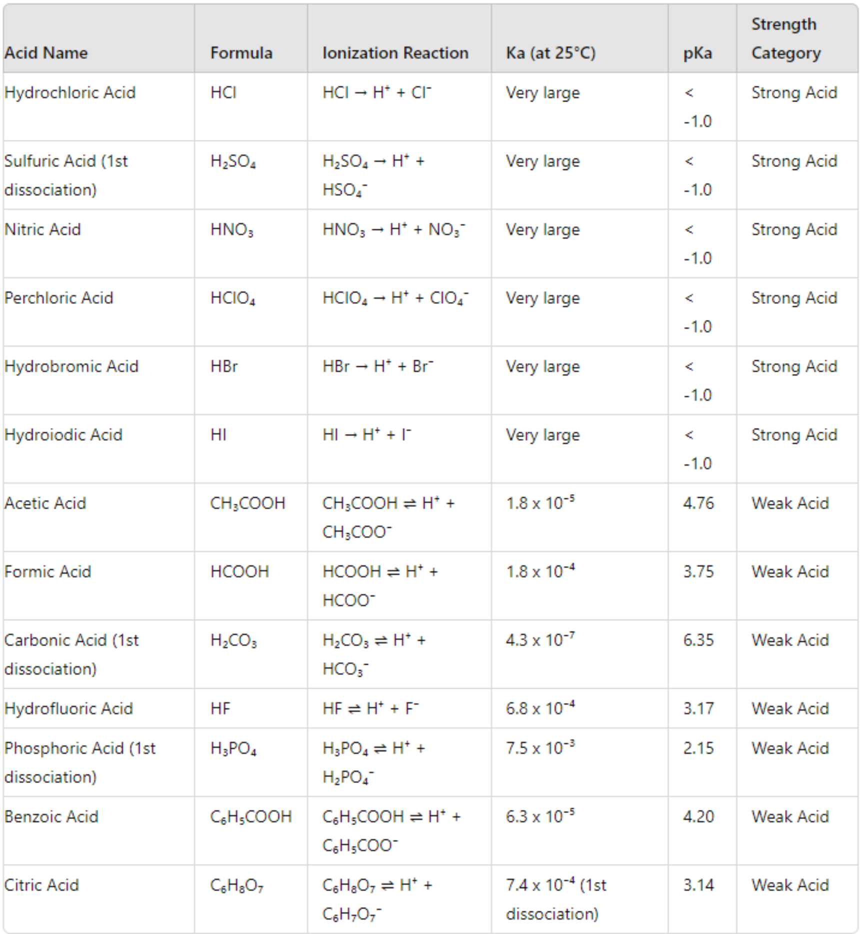
- **Strong Bases**: Result in a high pH (very basic).
- **Weak Bases**: Result in a lower pH compared to strong bases of the same concentration.

Conductivity:

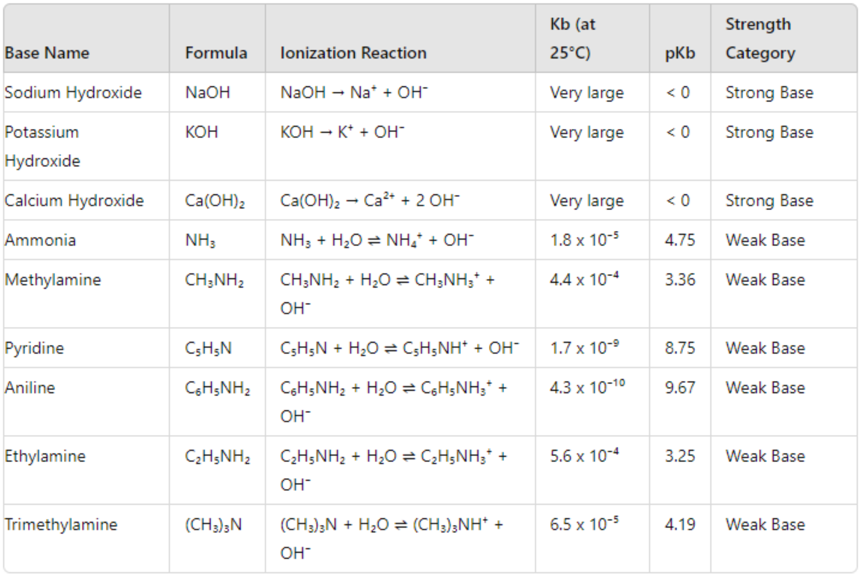
- **Strong Bases**: High conductivity due to a large number of ions in solution.
- **Weak Bases**: Lower conductivity due to fewer ions in solution.

By comparing these properties, students can better understand the behavior of strong and weak bases in aqueous solutions.

Relative Strength of Acids and Bases in context of Ionization Reactions



Acid Name	Formula	Ionization Reaction	Ka (at 25°C)	pKa	Strength Category
Hydrochloric Acid	HCl	$HCl \rightarrow H^+ + Cl^-$	Very large	< -1.0	Strong Acid
Sulfuric Acid (1st dissociation)	H_2SO_4	$H_2SO_4 \rightarrow H^+ + HSO_4^-$	Very large	< -1.0	Strong Acid
Nitric Acid	HNO_3	$HNO_3 \rightarrow H^+ + NO_3^-$	Very large	< -1.0	Strong Acid
Perchloric Acid	$HClO_4$	$HClO_4 \rightarrow H^+ + ClO_4^-$	Very large	< -1.0	Strong Acid
Hydrobromic Acid	HBr	$HBr \rightarrow H^+ + Br^-$	Very large	< -1.0	Strong Acid
Hydroiodic Acid	HI	$HI \rightarrow H^+ + I^-$	Very large	< -1.0	Strong Acid
Acetic Acid	CH_3COOH	$CH_3COOH \rightleftharpoons H^+ + CH_3COO^-$	1.8×10^{-5}	4.76	Weak Acid
Formic Acid	$HCOOH$	$HCOOH \rightleftharpoons H^+ + HCOO^-$	1.8×10^{-4}	3.75	Weak Acid
Carbonic Acid (1st dissociation)	H_2CO_3	$H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$	4.3×10^{-7}	6.35	Weak Acid
Hydrofluoric Acid	HF	$HF \rightleftharpoons H^+ + F^-$	6.8×10^{-4}	3.17	Weak Acid
Phosphoric Acid (1st dissociation)	H_3PO_4	$H_3PO_4 \rightleftharpoons H^+ + H_2PO_4^-$	7.5×10^{-3}	2.15	Weak Acid
Benzoic Acid	C_6H_5COOH	$C_6H_5COOH \rightleftharpoons H^+ + C_6H_5COO^-$	6.3×10^{-5}	4.20	Weak Acid
Citric Acid	$C_6H_8O_7$	$C_6H_8O_7 \rightleftharpoons H^+ + C_6H_7O_7^-$	7.4×10^{-4} (1st dissociation)	3.14	Weak Acid



Base Name	Formula	Ionization Reaction	Kb (at 25°C)	pKb	Strength Category
Sodium Hydroxide	$NaOH$	$NaOH \rightarrow Na^+ + OH^-$	Very large	< 0	Strong Base
Potassium Hydroxide	KOH	$KOH \rightarrow K^+ + OH^-$	Very large	< 0	Strong Base
Calcium Hydroxide	$Ca(OH)_2$	$Ca(OH)_2 \rightarrow Ca^{2+} + 2\ OH^-$	Very large	< 0	Strong Base
Ammonia	NH_3	$NH_3 + H_2O \rightleftharpoons NH_4^+ + OH^-$	1.8×10^{-5}	4.75	Weak Base
Methylamine	CH_3NH_2	$CH_3NH_2 + H_2O \rightleftharpoons CH_3NH_3^+ + OH^-$	4.4×10^{-4}	3.36	Weak Base
Pyridine	C_5H_5N	$C_5H_5N + H_2O \rightleftharpoons C_5H_5NH^+ + OH^-$	1.7×10^{-9}	8.75	Weak Base
Aniline	$C_6H_5NH_2$	$C_6H_5NH_2 + H_2O \rightleftharpoons C_6H_5NH_3^+ + OH^-$	4.3×10^{-10}	9.67	Weak Base
Ethylamine	$C_2H_5NH_2$	$C_2H_5NH_2 + H_2O \rightleftharpoons C_2H_5NH_3^+ + OH^-$	5.6×10^{-4}	3.25	Weak Base
Trimethylamine	$(CH_3)_3N$	$(CH_3)_3N + H_2O \rightleftharpoons (CH_3)_3NH^+ + OH^-$	6.5×10^{-5}	4.19	Weak Base

Notes

- **Ka and Kb Values**: The ionization constants (Ka for acids and Kb for bases) measure the extent of ionization. Higher Ka or Kb values indicate stronger acids or bases, respectively.
- **pKa and pKb Values**: The pKa and pKb are the negative logarithms of Ka and Kb, respectively. Lower pKa or pKb values indicate stronger acids or bases.

This table helps students visualize and understand the relative strengths of acids and bases through their ionization reactions and properties.

7.12: Relative Strengths of Acids and Bases

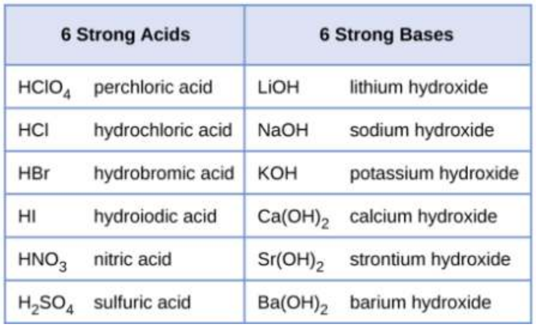
Learning Objectives

By the end of this section, you will be able to:

- Assess the relative strengths of acids and bases according to their ionization constants
- Rationalize trends in acid–base strength in relation to molecular structure
- Carry out equilibrium calculations for weak acid–base systems

Acid and Base Ionization Constants

The relative strength of an acid or base is the extent to which it ionizes when dissolved in water. If the ionization reaction is essentially complete, the acid or base is termed *strong*; if relatively little ionization occurs, the acid or base is weak. As will be evident throughout the remainder of this chapter, there are many more weak acids and bases than strong ones. The most common strong acids and bases are listed in Figure $7.12.1$.



6 Strong Acids		6 Strong Bases	
$HClO_4$	perchloric acid	LiOH	lithium hydroxide
HCl	hydrochloric acid	NaOH	sodium hydroxide
HBr	hydrobromic acid	KOH	potassium hydroxide
HI	hydroiodic acid	$Ca(OH)_2$	calcium hydroxide
HNO_3	nitric acid	$Sr(OH)_2$	strontium hydroxide
H_2SO_4	sulfuric acid	$Ba(OH)_2$	barium hydroxide

This table has seven rows and two columns. The first row is a header row, and it labels each column, “6 Strong Acids,” and, “6 Strong Bases.” Under the “6 Strong Acids” column are the following: H C l O subscript 4 perchloric acid; H C l hydrochloric acid; H B r hydrobromic acid; H I hydroiodic acid; H N O subscript 3 nitric acid; H subscript 2 S O subscript 4 sulfuric acid. Under the “6 Strong Bases” column are the following: L i O H lithium hydroxide; N a O H sodium hydroxide; K O H potassium hydroxide; C a (O H) subscript 2 calcium hydroxide; S r (O H) subscript 2 strontium hydroxide; B a (O H) subscript 2 barium hydroxide.

Figure $7.12.1$: Some of the common strong acids and bases are listed here.

The relative strengths of acids may be quantified by measuring their equilibrium constants in aqueous solutions. In solutions of the same concentration, stronger acids ionize to a greater extent, and so yield higher concentrations of hydronium ions than do weaker acids. The equilibrium constant for an acid is called the acid-ionization constant, K_a. For the reaction of an acid HA:

$$\mathrm{HA(aq)+H_2O(l) \rightleftharpoons H_3O^+(aq)+A^-(aq)}$$

the acid ionization constant is written

$$K_a=\frac{[\mathrm{H_3O^+}][\mathrm{A^-}]}{[\mathrm{HA}]}$$

where the concentrations are those at equilibrium. Although water is a reactant in the reaction, it is the solvent as well, so we do not include $[\mathrm{H_2O}]$ in the equation.

Note

Equilibrium constant expressions are actually ratios of activities, and the value of K is determined at the limit of infinite dilution of the solutes. In these very dilute solutions, the activity of the solvent has a value of unity (1) and the activity of each solute can be approximated by its molar concentration.

The larger the K_a of an acid, the larger the concentration of H_3O^+ and A^- relative to the concentration of the nonionized acid, HA, in an equilibrium mixture, and the stronger the acid. An acid is classified as "strong" when it undergoes complete ionization, in which case the concentration of HA is zero and the acid ionization constant is immeasurably large ($K_a \approx \infty$). Acids that are partially ionized are called "weak," and their acid ionization constants may be experimentally measured. A table of ionization constants for weak acids is provided in Appendix H.

To illustrate this idea, three acid ionization equations and K_a values are shown below. The ionization constants increase from first to last of the listed equations, indicating the relative acid strength increases in the order:

$$\mathrm{CH_3CO_2H} < \mathrm{HNO_2} < \mathrm{HSO_4^-}$$

as demonstrated with the equations below:

$$\mathrm{CH_3CO_2H(aq) + H_2O(l) \rightleftharpoons H_3O^+(aq) + CH_3CO_2^-(aq)} \qquad K_a = 1.8 \times 10^{-5}$$

$$\mathrm{HNO_2(aq) + H_2O(l) \rightleftharpoons H_3O^+(aq) + NO_2^-(aq)} \qquad K_a = 4.6 \times 10^{-4}$$

$$\mathrm{HSO_4^-(aq) + H_2O(aq) \rightleftharpoons H_3O^+(aq) + SO_4^{2-}(aq)} \qquad K_a = 1.2 \times 10^{-2}$$

Another measure of the strength of an acid is its percent ionization. The percent ionization of a weak acid is defined in terms of the composition of an equilibrium mixture:

$$\%\ \text{ionization} = \frac{[\mathrm{H_3O^+}]_{eq}}{[\mathrm{HA}]_0} \times 100$$

where the numerator is equivalent to the concentration of the acid's conjugate base (per stoichiometry, $[A^-] = [H_3O^+]$). Unlike the K_a value, the percent ionization of a weak acid varies with the initial concentration of acid, typically decreasing as concentration increases. Equilibrium calculations of the sort described later in this chapter can be used to confirm this behavior.

✓ Example 7.12.1: Calculation of Percent Ionization from pH

Calculate the percent ionization of a 0.125-*M* solution of nitrous acid (a weak acid), with a pH of 2.09.

Solution

The percent ionization for an acid is:

$$\frac{[\mathrm{H_3O^+}]_{eq}}{[\mathrm{HNO_2}]_0} \times 100$$

Converting the provided pH to hydronium ion molarity yields

$$[\mathrm{H_3O^+}] = 10^{-2.09} = 0.0081M$$

Substituting this value and the provided initial acid concentration into the percent ionization equation gives

$$\frac{8.1 \times 10^{-3}}{0.125} \times 100 = 6.5\%$$

(Recall the provided pH value of 2.09 is logarithmic, and so it contains just two significant digits, limiting the certainty of the computed percent ionization.)

? Exercise 7.12.1

Calculate the percent ionization of a 0.10-*M* solution of acetic acid with a pH of 2.89.

Answer

1.3% ionized

Link to Learning

View the simulation of strong and weak acids and bases at the molecular level.

Just as for acids, the relative strength of a base is reflected in the magnitude of its base-ionization constant (K_b) in aqueous solutions. In solutions of the same concentration, stronger bases ionize to a greater extent, and so yield higher hydroxide ion concentrations than do weaker bases. A stronger base has a larger ionization constant than does a weaker base. For the reaction of a base, B:

$$\mathrm{B(aq) + H_2O(l) \rightleftharpoons HB^+(aq) + OH^-(aq)},$$

the ionization constant is written as

$$K_b = \frac{[\mathrm{HB^+}]\,[\mathrm{OH^-}]}{[\mathrm{B}]}$$

Inspection of the data for three weak bases presented below shows the base strength increases in the order:

$$\mathrm{NO_2^- < CH_2CO_2^- < NH_3}$$

$$\begin{aligned} \mathrm{NO_2^-(aq) + H_2O(l)} &\rightleftharpoons \mathrm{HNO_2(aq) + OH^-(aq)} & K_b &= 2.17\times 10^{-11} \\ \mathrm{CH_3CO_2^-(aq) + H_2O(l)} &\rightleftharpoons \mathrm{CH_3CO_2H(aq) + OH^-(aq)} & K_b &= 5.6\times 10^{-10} \\ \mathrm{NH_3(aq) + H_2O(l)} &\rightleftharpoons \mathrm{NH_4^+(aq) + OH^-(aq)} & K_b &= 1.8\times 10^{-5} \end{aligned}$$

A table of ionization constants for weak bases appears in Appendix I. As for acids, the relative strength of a base is also reflected in its percent ionization, computed as

$$\%\text{ ionization } = \left(\frac{[\mathrm{OH^-}]_{eq}}{[\mathrm{B}]_0}\right)\times 100\%$$

but will vary depending on the base ionization constant and the initial concentration of the solution.

Relative Strengths of Conjugate Acid-Base Pairs

Brønsted-Lowry acid-base chemistry is the transfer of protons; thus, logic suggests a relation between the relative strengths of conjugate acid-base pairs. The strength of an acid or base is quantified in its ionization constant, K_a or K_b, which represents the extent of the acid or base ionization reaction. For the conjugate acid-base pair $\mathrm{HA/A^-}$, ionization equilibrium equations and ionization constant expressions are

$$\mathrm{HA(aq) + H_2O(l) \rightleftharpoons H_3O^+(aq) + A^-(aq)}$$

with

$$K_a = \frac{[\mathrm{H_3O^+}][\mathrm{A^-}]}{[\mathrm{HA}]}$$

and

$$\mathrm{A^-(aq) + H_2O(l) \rightleftharpoons OH^-(aq) + HA(aq)}$$

with

$$K_b = \frac{[\mathrm{HA}][\mathrm{OH-}]}{[\mathrm{A-}]}$$

Adding these two chemical equations yields the equation for the autoionization for water:

$$\begin{aligned} \mathrm{HA}\cancel{\mathrm{(aq)}} + \mathrm{H_2O(l)} + \mathrm{A^-}\cancel{\mathrm{(aq)}} + \mathrm{H_2O(l)} &\rightleftharpoons \mathrm{H_3O^+(aq)} + \mathrm{A^-}\cancel{\mathrm{(aq)}} + \mathrm{OH^-(aq)} + \mathrm{HA}\cancel{\mathrm{(aq)}} \\ \mathrm{2\,H_2O(l)} &\rightleftharpoons \mathrm{H_3O^+(aq) + OH^-(aq)} \end{aligned}$$

As discussed in another chapter on equilibrium, the equilibrium constant for a summed reaction is equal to the mathematical product of the equilibrium constants for the added reactions, and so

$$K_a \times K_b = \frac{[\mathrm{H_3O^+}][\mathrm{A^-}]}{[\mathrm{HA}]} \times \frac{[\mathrm{HA}][\mathrm{OH^-}]}{[\mathrm{A^-}]} = [\mathrm{H_3O^+}][\mathrm{OH^-}] = K_w$$

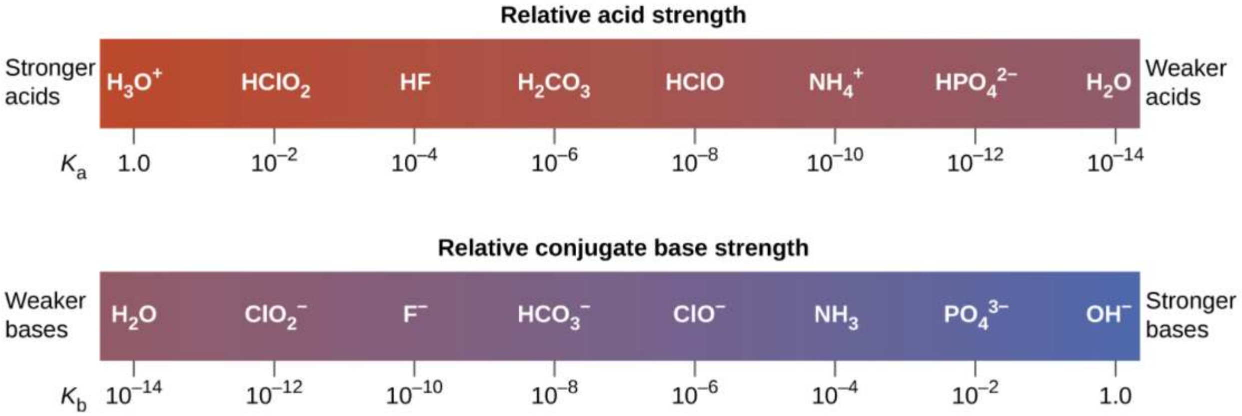
This equation states the relation between ionization constants for any conjugate acid-base pair, namely, their mathematical product is equal to the ion product of water, K_w. By rearranging this equation, a reciprocal relation between the strengths of a conjugate acid-base pair becomes evident:

$$K_a = \frac{K_w}{K_b}$$

or

$$K_b = \frac{K_w}{K_a}$$

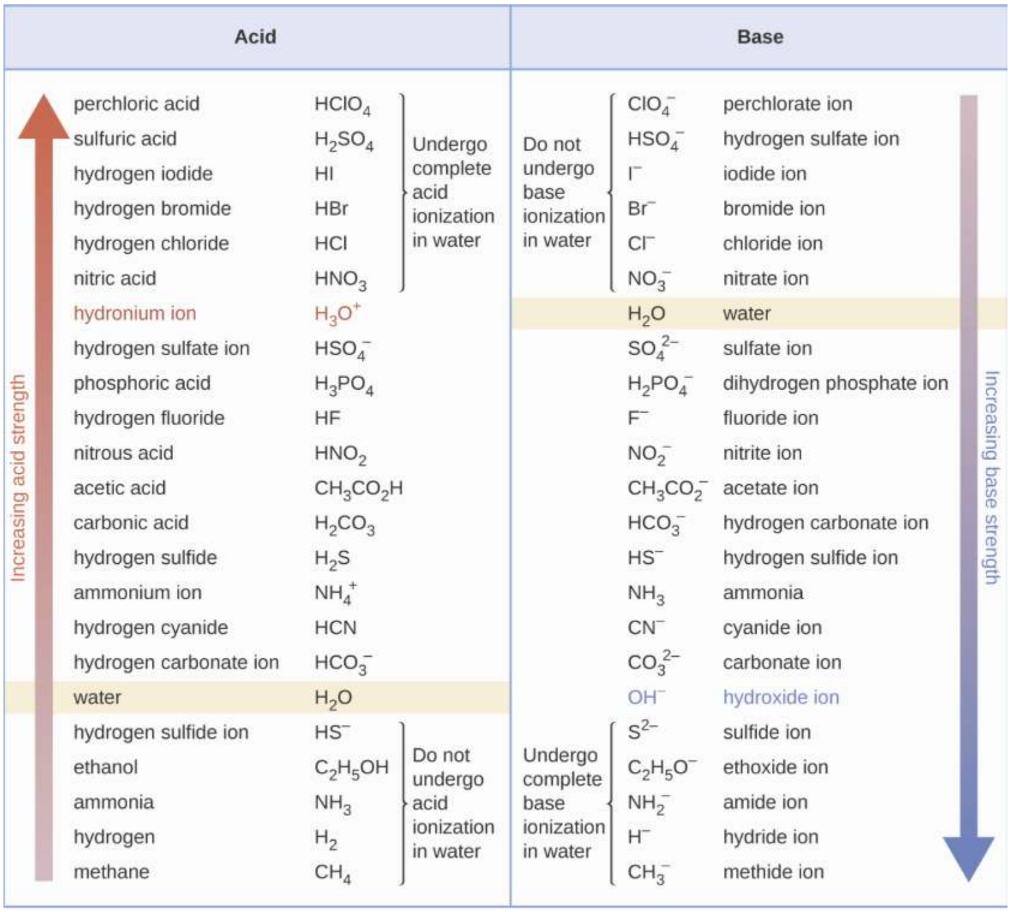
The inverse proportional relation between K_a and K_b means the stronger the acid or base, the weaker its **conjugate partner**. Figure $7.12.2$ illustrates this relation for several conjugate acid-base pairs.



The diagram shows two horizontal bars. The first, labeled, “Relative acid strength,” at the top is red on the left and gradually changes to purple on the right. The red end at the left is labeled, “Stronger acids.” The purple end at the right is labeled, “Weaker acids.” Just outside the bar to the lower left is the label, “K subscript a.” The bar is marked off in increments with a specific acid listed above each increment. The first mark is at 1.0 with H subscript 3 O superscript positive sign. The second is ten raised to the negative two with H C l O subscript 2. The third is ten raised to the negative 4 with H F. The fourth is ten raised to the negative 6 with H subscript 2 C O subscript 3. The fifth is ten raised to a negative 8 with C H subscript 3 C O O H. The sixth is ten raised to the negative ten with N H subscript 4 superscript positive sign. The seventh is ten raised to a negative 12 with H P O subscript 4 superscript 2 negative sign. The eighth is ten raised to the negative 14 with H subscript 2 O. Similarly the second bar, which is labeled “Relative conjugate base strength,” is purple at the left end and gradually becomes blue at the right end. Outside the bar to the left is the label, “Weaker bases.” Outside the bar to the right is the label, “Stronger bases.” Below and to the left of the bar is the label, “K subscript b.” The bar is similarly marked at increments with bases listed above each increment. The first is at ten raised to the negative 14 with H subscript 2 O above it. The second is ten raised to the negative 12 C l O subscript 2 superscript negative sign. The third is ten raised to the negative ten with F superscript negative sign. The fourth is ten raised to a negative eight with H C O subscript 3 superscript negative sign. The fifth is ten raised to the negative 6 with C H subscript 3 C O O superscript negative sign. The sixth is ten raised to the negative 4 with N H subscript 3. The seventh is ten raised to the negative 2 with P O subscript 4 superscript three negative sign. The eighth is 1.0 with O H superscript negative sign.

Figure $7.12.2$: Relative strengths of several conjugate acid-base pairs are shown.

The listing of conjugate acid–base pairs shown in Figure $7.12.3$ is arranged to show the relative strength of each species as compared with water, whose entries are highlighted in each of the table’s columns. In the acid column, those species listed below water are weaker acids than water. These species do not undergo acid ionization in water; they are not Bronsted-Lowry acids. All the species listed above water are stronger acids, transferring protons to water to some extent when dissolved in an aqueous solution to generate hydronium ions. Species above water but below hydronium ion are *weak acids*, undergoing partial acid ionization, wheres those above hydronium ion are *strong acids* that are completely ionized in aqueous solution.



Acid			Base		
perchloric acid	$HClO_4$	Undergo complete acid ionization in water	Do not undergo base ionization in water	ClO_4^-	perchlorate ion
sulfuric acid	H_2SO_4			HSO_4^-	hydrogen sulfate ion
hydrogen iodide	HI			I^-	iodide ion
hydrogen bromide	HBr			Br^-	bromide ion
hydrogen chloride	HCl			Cl^-	chloride ion
nitric acid	HNO_3			NO_3^-	nitrate ion
hydronium ion	H_3O^+			H_2O	water
hydrogen sulfate ion	HSO_4^-			SO_4^{2-}	sulfate ion
phosphoric acid	H_3PO_4			$H_2PO_4^-$	dihydrogen phosphate ion
hydrogen fluoride	HF			F^-	fluoride ion
nitrous acid	HNO_2			NO_2^-	nitrite ion
acetic acid	CH_3CO_2H			$CH_3CO_2^-$	acetate ion
carbonic acid	H_2CO_3			HCO_3^-	hydrogen carbonate ion
hydrogen sulfide	H_2S			HS^-	hydrogen sulfide ion
ammonium ion	NH_4^+			NH_3	ammonia
hydrogen cyanide	HCN			CN^-	cyanide ion
hydrogen carbonate ion	HCO_3^-			CO_3^{2-}	carbonate ion
water	H_2O			OH^-	hydroxide ion
hydrogen sulfide ion	HS^-	Do not undergo acid ionization in water	Undergo complete base ionization in water	S^{2-}	sulfide ion
ethanol	C_2H_5OH			$C_2H_5O^-$	ethoxide ion
ammonia	NH_3			NH_2^-	amide ion
hydrogen	H_2			H^-	hydride ion
methane	CH_4			CH_3^-	methide ion

This figure includes a table separated into a left half which is labeled "Acids" and a right half labeled "Bases." A red arrow points up the left side, which is labeled "Increasing acid strength." Similarly, a blue arrow points downward along the right side, which is labeled "Increasing base strength." Names of acids and bases are listed next to each arrow toward the center of the table, followed by chemical formulas. Acids listed top to bottom are sulfuric acid, H subscript 2 S O subscript 4, hydrogen iodide, H I, hydrogen bromide, H B r, hydrogen chloride, H C l, nitric acid, H N O subscript 3, hydronium ion (in pink text) H subscript 3 O superscript plus, hydrogen sulfate ion, H S O subscript 4 superscript negative, phosphoric acid, H subscript 3 P O subscript 4, hydrogen fluoride, H F, nitrous acid, H N O subscript 2, acetic acid, C H subscript 3 C O subscript 2 H, carbonic acid H subscript 2 C O subscript 3, hydrogen sulfide, H subscript 2 S, ammonium ion, N H subscript 4 superscript +, hydrogen cyanide, H C N, hydrogen carbonate ion, H C O subscript 3 superscript negative, water (shaded in beige) H subscript 2 O, hydrogen sulfide ion, H S superscript negative, ethanol, C subscript 2 H subscript 5 O H, ammonia, N H subscript 3, hydrogen, H subscript 2, methane, and C H subscript 4. The acids at the top of the listing from sulfuric acid through nitric acid are grouped with a bracket to the right labeled "Undergo complete acid ionization in water." Similarly, the acids at the bottom from hydrogen sulfide ion through methane are grouped with a bracket and labeled, "Do not undergo acid ionization in water." The right half of the figure lists bases and formulas. From top to bottom the bases listed are hydrogen sulfate ion, H S O subscript 4 superscript negative, iodide ion, I superscript negative, bromide ion, B r superscript negative, chloride ion, C l superscript negative, nitrate ion, N O subscript 3 superscript negative, water (shaded in beige), H subscript 2 O, sulfate ion, S O subscript 4 superscript 2 negative, dihydrogen phosphate ion, H subscript 2 P O subscript 4 superscript negative, fluoride ion, F superscript negative, nitrite ion, N O subscript 2 superscript negative, acetate ion, C H subscript 3 C O subscript 2 superscript negative, hydrogen carbonate ion, H C O subscript 3 superscript negative, hydrogen sulfide ion, H S superscript negative, ammonia, N H subscript 3, cyanide ion, C N superscript negative, carbonate ion, C O subscript 3 superscript 2 negative, hydroxide ion (in blue), O H superscript negative, sulfide ion, S superscript 2 negative, ethoxide ion, C subscript 2 H subscript 5 O superscript negative, amide ion N H subscript 2 superscript negative, hydride ion, H superscript negative, and methide ion C H subscript 3 superscript negative. The bases at the top, from *perchlorate ion through nitrate ion are group with a bracket which is labeled "Do not undergo base ionization in water." Similarly, the lower 5 in the listing, from sulfide ion through methide ion are grouped and labeled "Undergo complete base ionization in water."*

Figure $7.12.3$: This figure shows strengths of conjugate acid-base pairs relative to the strength of water as the reference substance.

If all these strong acids are completely ionized in water, why does the column indicate they vary in strength, with nitric acid being the weakest and perchloric acid the strongest? Notice that the sole acid species present in an aqueous solution of any strong acid is $H_3O^+(aq)$, meaning that hydronium ion is the strongest acid that may exist in water; any stronger acid will react completely with water to generate hydronium ions. This limit on the acid strength of solutes in a solution is called a **leveling effect**. To measure the differences in acid strength for "strong" acids, the acids must be dissolved in a solvent that is *less basic* than water. In such solvents, the acids will be "weak," and so any differences in the extent of their ionization can be determined. For example, the binary hydrogen halides HCl, HBr, and HI are strong acids in water but weak acids in ethanol (strength increasing HCl < HBr < HI).

The right column of Figure $7.12.3$ lists a number of substances in order of increasing base strength from top to bottom. Following the same logic as for the left column, species listed above water are weaker bases and so they don't undergo base ionization when dissolved in water. Species listed between water and its conjugate base, hydroxide ion, are weak bases that partially ionize. Species listed below hydroxide ion are strong bases that completely ionize in water to yield hydroxide ions (i.e., they are *leveled* to hydroxide). A comparison of the acid and base columns in this table supports the reciprocal relation between the strengths of conjugate acid-base pairs. For example, the conjugate bases of the strong acids (top of table) are all of negligible strength. A strong acid exhibits an immeasurably large K_a, and so its conjugate base will exhibit a K_b that is essentially zero:

- strong acid: $K_a \approx \infty$
- conjugate base: $K_b = K_w/K_a = K_w/\infty \approx 0$

A similar approach can be used to support the observation that conjugate acids of strong bases ($K_b \approx \infty$) are of negligible strength ($K_a \approx 0$).

✓ Example 7.12.2: Calculating Ionization Constants for Conjugate Acid-Base Pairs

Use the K_b for the nitrite ion, NO_2^-, to calculate the K_a for its conjugate acid.

Solution

K_b for NO_2^- is given in this section as 2.17×10^{-11}.

The conjugate acid of NO_2^- is HNO_2; K_a for HNO_2 can be calculated using the relationship:

$$K_a \times K_b = 1.0 \times 10^{-14} = K_w$$

Solving for K_a yields

$$K_a = \frac{K_w}{K_b} = \frac{1.0 \times 10^{-14}}{2.17 \times 10^{-11}} = 4.6 \times 10^{-4}$$

This answer can be verified by finding the Ka for HNO_2 in Appendix H.

? Exercise 7.12.2

Determine the relative acid strengths of $NH4^+$ and HCN by comparing their ionization constants. The ionization constant of HCN is given in Appendix H as 4.9×10^{-10}. The ionization constant of $NH4^+$ is not listed, but the ionization constant of its conjugate base, NH_3, is listed as 1.8×10^{-5}.

Answer

NH_4^+ is the slightly stronger acid (K_a for NH_4^+ is 5.6×10^{-10}).

Acid-Base Equilibrium Calculations

The chapter on chemical equilibria introduced several types of equilibrium calculations and the various mathematical strategies that are helpful in performing them. These strategies are generally useful for equilibrium systems regardless of chemical reaction class, and so they may be effectively applied to acid-base equilibrium problems. This section presents several example exercises involving equilibrium calculations for acid-base systems.

This image shows two bottles containing clear colorless solutions. Each bottle contains a single p H indicator strip. The strip in the bottle on the left is red, and a similar red strip is placed on a filter paper circle in front of the bottle on surface on which the bottles are resting. Similarly, the second bottle on the right contains and orange strip and an orange strip is placed in front of it on a filter paper circle. Between the two bottles is a pack of p Hydrion papers with a p H color scale on its cover.

This photo shows two glass containers filled with a transparent liquid. In between the containers is a p H strip indicator guide. There are p H strips placed in front of each glass container. The liquid in the container on the left appears to have a p H of 10 or 11. The liquid in the container on the right appears to have a p H of about 13 or 14.

Litmus paper in action

Example 7.12.3: Determination of K_a from Equilibrium Concentrations

Acetic acid is the principal ingredient in vinegar (Figure $7.12.4$) that provides its sour taste. At equilibrium, a solution contains $[CH_3CO_2H] = 0.0787\ M$: and $H_3O^+] = [CH_3CO_2^-] = 0.00118\ M$. What is the value of K_a for acetic acid?

Figure $7.12.4$: Vinegar contains acetic acid, a weak acid. (credit: modification of work by “HomeSpot HQ”/Flickr)

Solution

The relevant equilibrium equation and its equilibrium constant expression are shown below. Substitution of the provided equilibrium concentrations permits a straightforward calculation of the Ka for acetic acid.

$$\mathrm{CH_3CO_2H(aq) + H_2O(l) \rightleftharpoons H_3O^+(aq) + CH_3CO_2^-(aq)}$$

$$K_a = \frac{[\mathrm{H_3O^+}]\,[\mathrm{CH_3CO_2^-}]}{[\mathrm{CH_3CO_2H}]} = \frac{(0.00118)(0.00118)}{0.0787} = 1.77 \times 10^{-5}$$

? Exercise 7.12.3

The $\mathrm{HSO_4^-}$ ion, weak acid used in some household cleansers:

$$\mathrm{HSO_4^-(aq) + H_2O(l) \rightleftharpoons H_3O^+(aq) + SO_4^{2-}(aq)}$$

What is the acid ionization constant for this weak acid if an equilibrium mixture has the following composition:

$$[\mathrm{H_3O^+}] = 0.027\ M;\ [\mathrm{HSO_4^-}] = 0.29\ M;\ [\mathrm{HSO_4^-}] = 0.29\ M;\ \text{and}\ [\mathrm{SO_4^{2-}}] = 0.13\ M?$$

Answer

K_a for $\mathrm{HSO_4^-} = 1.2 \times 10^{-2}$

✓ Example 7.12.4: Determination of K_b from Equilibrium Concentrations

Caffeine, $\mathrm{C_8H_{10}N_4O_2}$ is a weak base. What is the value of K_b for caffeine if a solution at equilibrium has $[\mathrm{C_8H_{10}N_4O_2}] = 0.050$ M, $[\mathrm{C_8H_{10}N_4O_2H^+}] = 5.0 \times 10^{-3}$ M, and $[\mathrm{OH^-}] = 2.5 \times 10^{-3}$ M?

Solution

The relevant equilibrium equation and its equilibrium constant expression are shown below. Substitution of the provided equilibrium concentrations permits a straightforward calculation of the K_b for caffeine.

$$\mathrm{C_8H_{10}N_4O_2(aq) + H_2O(l) \rightleftharpoons C_8H_{10}N_4O_2H^+(aq) + OH^-(aq)}$$

$$\begin{aligned} K_b &= [\mathrm{C_8H_{10}N_4O_2H^+}][\mathrm{OH^-}][\mathrm{C_8H_{10}N_4O_2}] \\ &= (5.0 \times 10^{-3})(2.5 \times 10^{-3})(0.050) \\ &= 2.5 \times 10^{-4} \end{aligned}$$

? Exercise 7.12.4

What is the equilibrium constant for the ionization of the $\mathrm{HPO_4^{2-}}$ ion, a weak base

$$\mathrm{HPO_4^{2-}(aq) + H_2O(l) \rightleftharpoons H_2PO_4^-(aq) + OH^-(aq)}$$

if the composition of an equilibrium mixture is as follows: \(\ce{[OH^{-}}] =

1.3 \times 10^{−6} M}\); $\mathrm{H_2PO_4^-}] = 0.042\ M$; and $[\mathrm{HPO_4^{2-}}] = 0.341\ M$?

Answer

K_b for $\mathrm{HPO_4^{2-}}$ is 1.6×10^{-7}

✓ Example 7.12.5: Determination of K_a or K_b from pH

The pH of a 0.0516-M solution of nitrous acid, $\mathrm{HNO_2}$, is 2.34. What is its K_a?

$$\mathrm{HNO_2(aq) + H_2O(l) \rightleftharpoons H_3O^+(aq) + NO_2^-(aq)}$$

Solution

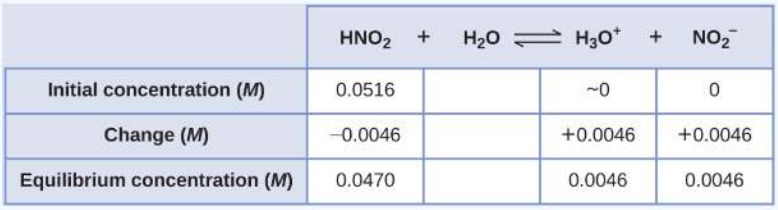
The nitrous acid concentration provided is a *formal* concentration, one that does not account for any chemical equilibria that may be established in solution. Such concentrations are treated as “initial” values for equilibrium calculations using the ICE table approach. Notice the initial value of hydronium ion is listed as *approximately* zero because a small concentration of $\mathrm{H_3O^+}$

is present ($1 \times \times 10^{-7}$ M) due to the autoprotolysis of water. In many cases, such as all the ones presented in this chapter, this concentration is much less than that generated by ionization of the acid (or base) in question and may be neglected.

The pH provided is a logarithmic measure of the hydronium ion concentration resulting from the acid ionization of the nitrous acid, and so it represents an “equilibrium” value for the ICE table:

$$\left[H_3O^+\right] = 10^{-2.34} = 0.0046\ \mathrm{M}$$

The ICE table for this system is then



	HNO_2 +	H_2O ⇌	H_3O^+ +	NO_2^-
Initial concentration (*M*)	0.0516		~0	0
Change (*M*)	−0.0046		+0.0046	+0.0046
Equilibrium concentration (*M*)	0.0470		0.0046	0.0046

This table has two main columns and four rows. The first row for the first column does not have a heading and then has the following in the first column: Initial concentration (M), Change (M), Equilibrium concentration (M). The second column has the header of “H N O subscript 2 plus sign H subscript 2 O equilibrium sign H subscript 3 O superscript positive sign plus sign N O subscript 2 superscript negative sign.” Under the second column is a subgroup of four columns and three rows. The first column has the following: 0.0516, negative 0.0046, 0.0470. The second column is blank in all three rows. The third column has the following: approximately 0, positive 0.0046, 0.0046. The fourth column has the following: 0, positive 0.0046, 0.0046.

Finally, calculate the value of the equilibrium constant using the data in the table:

$$K_a = \frac{\left[H_3O^+\right]\left[NO_2^-\right]}{\left[HNO_2\right]} = \frac{(0.0046)(0.0046)}{(0.0470)} = 4.6 \times 10^{-4}$$

? Exercise 7.12.5

The pH of a solution of household ammonia, a 0.950-M solution of NH_3, is 11.612. What is K_b for NH_3.

Answer

$K_b = 1.8 \times 10^{-5}$

✓ Example 7.12.6: Calculating Equilibrium Concentrations in a Weak Acid Solution

Formic acid, HCO_2H, is one irritant that causes the body’s reaction to some ant bites and stings (Figure 7.12.5).

Figure 7.12.5: The pain of some ant bites and stings is caused by formic acid. (credit: John Tann)

What is the concentration of hydronium ion and the pH of a 0.534-*M* solution of formic acid?

$$HCO_2H(aq) + H_2O(l) \rightleftharpoons H_3O^+(aq) + HCO_2^-(aq) \quad K_a = 1.8 \times 10^{-4}$$

Solution

The ICE table for this system is

	HCO_2H +	H_2O ⇌	H_3O^+ +	HCO_2^-
Initial concentration (*M*)	0.534		~0	0
Change (*M*)			+*x*	+*x*
Equilibrium concentration (*M*)	0.534 −*x*		*x*	*x*

Substituting the equilibrium concentration terms into the K_a expression gives

$$\begin{aligned}K_a &= 1.8 \times 10^{-4} = \frac{[H_3O^+][HCO_2^-]}{[HCO_2H]} \\ &= \frac{(x)(x)}{0.534 - x} = 1.8 \times 10^{-4}\end{aligned}$$

The relatively large initial concentration and small equilibrium constant permits the simplifying assumption that *x* will be much lesser than 0.534, and so the equation becomes

$$K_a = 1.8 \times 10^{-4} = \frac{x^2}{0.534}$$

Solving the equation for x yields

$$\begin{aligned}x^2 &= 0.534 \times (1.8 \times 10^{-4}) = 9.6 \times 10^{-5} \\ x &= \sqrt{9.6 \times 10^{-5}} \\ &= 9.8 \times 10^{-3} M\end{aligned}$$

To check the assumption that *x* is small compared to 0.534, its relative magnitude can be estimated:

$$\frac{x}{0.534} = \frac{9.8 \times 10^{-3}}{0.534} = 1.8 \times 10^{-2} (1.8\% \text{ of } 0.534)$$

Because *x* is less than 5% of the initial concentration, the assumption is valid.

As defined in the ICE table, *x* is equal to the equilibrium concentration of hydronium ion:

$$x = [H_3O^+] = 0.0098 \text{ M}$$

Finally, the pH is calculated to be

$$\text{pH} = -\log[H_3O^+] = -\log(0.0098) = 2.01$$

? Exercise 7.12.6

Only a small fraction of a weak acid ionizes in aqueous solution. What is the percent ionization of a 0.100-*M* solution of acetic acid, CH_3CO_2H?

$$CH_3CO_2H(aq) + H_2O(l) \rightleftharpoons H_3O^+(aq) + CH_3CO_2^-(aq) \quad K_a = 1.8 \times 10^{-5}$$

Answer

percent ionization = 1.3%

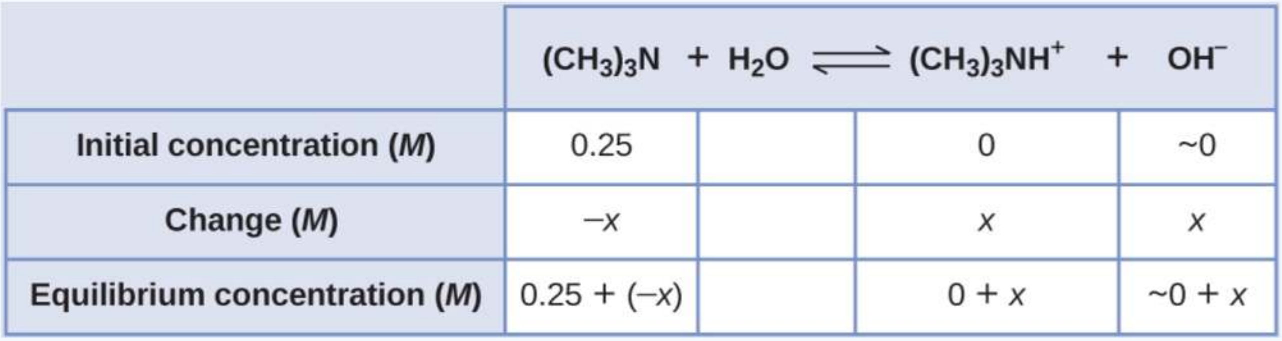
✓ Example 7.12.7: Calculating Equilibrium Concentrations in a Weak Base Solution

Find the concentration of hydroxide ion, the pOH, and the pH of a 0.25-*M* solution of trimethylamine, a weak base:

$$(CH_3)_3N(aq) + H_2O(l) \rightleftharpoons (CH_3)_3NH^+(aq) + OH^-(aq) \quad K_b = 6.3 \times 10^{-5}$$

Solution

The ICE table for this system is



	$(CH_3)_3N$ +	H_2O $\rightleftharpoons$	$(CH_3)_3NH^+$ +	OH^-
Initial concentration (*M*)	0.25		0	~0
Change (*M*)	−*x*		*x*	*x*
Equilibrium concentration (*M*)	0.25 + (−*x*)		0 + *x*	~0 + *x*

Substituting the equilibrium concentration terms into the K_b expression gives

$$K_b = \frac{\left[(CH_3)_3NH^+\right]\left[OH^-\right]}{\left[(CH_3)_3N\right]} = \frac{(x)(x)}{0.25 - x} = 6.3 \times 10^{-5}$$

Assuming $x \ll 0.25$ and solving for x yields

$$x = 4.0 \times 10^{-3}\ \mathrm{M}$$

This value is less than 5% of the initial concentration (0.25), so the assumption is justified. As defined in the ICE table, x is equal to the equilibrium concentration of hydroxide ion:

$$\begin{aligned}\left[OH^-\right] &= \sim 0 + x \\ &= x = 4.0 \times 10^{-3} M\end{aligned}$$

The pOH is calculated to be

$$\mathrm{pOH} = -\log\left(4.0 \times 10^{-3}\right) = 2.40$$

Using the relation introduced in the previous section of this chapter:

$$\mathrm{pH} + \mathrm{pOH} = pK_w = 14.00$$

permits the computation of pH:

$$\mathrm{pH} = 14.00 - \mathrm{pOH} = 14.00 - 2.40 = 11.60$$

? Exercise 7.12.7

Calculate the hydroxide ion concentration and the percent ionization of a 0.0325-*M* solution of ammonia, a weak base with a K_b of 1.76×10^{-5}.

Answer

$$7.56 \times 10^{-4}\ \mathrm{M},\ 2.33\%$$

In some cases, the strength of the weak acid or base and its formal (initial) concentration result in an appreciable ionization. Though the ICE strategy remains effective for these systems, the algebra is a bit more involved because the simplifying assumption that *x* is negligible cannot be made. Calculations of this sort are demonstrated in Example 7.12.8 below.

✓ Example 7.12.8: Calculating Equilibrium Concentrations without Simplifying Assumptions

Sodium bisulfate, NaHSO4, is used in some household cleansers as a source of the $\mathrm{HSO_4^-}$ ion, a weak acid. What is the pH of a 0.50-M solution of $\mathrm{HSO_4^-}$?

$$\mathrm{HSO_4^-(aq) + H_2O(l) \rightleftharpoons H_3O^+(aq) + SO_4^{2-}(aq)} \quad K_a = 1.2 \times 10^{-2}$$

Solution

The ICE table for this system is

	HSO_4^- +	H_2O ⇌	H_3O^+ +	SO_4^{2-}
Initial concentration (*M*)	0.50		~0	0
Change (*M*)	$-x$		$+x$	$+x$
Equilibrium concentration (*M*)	$0.50 - x$		x	x

Substituting the equilibrium concentration terms into the K_a expression gives

$$K_a = 1.2 \times 10^{-2} = \frac{[\mathrm{H_3O^+}][\mathrm{SO_4^{2-}}]}{[\mathrm{HSO_4^-}]} = \frac{(x)(x)}{0.50 - x}$$

If the assumption that $x \ll 0.5$ is made, simplifying and solving the above equation yields

$$x = 0.077 \text{ M}$$

This value of x is clearly not significantly less than 0.50 *M*; rather, it is approximately 15% of the initial concentration: When we check the assumption, we calculate:

$$\frac{x}{[\mathrm{HSO_4^-}]_i}$$

$$\frac{x}{0.50} = \frac{7.7 \times 10^{-2}}{0.50} = 0.15(15\%)$$

Because the simplifying assumption is not valid for this system, the equilibrium constant expression is solved as follows:

$$K_a = 1.2 \times 10^{-2} = \frac{(x)(x)}{0.50 - x}$$

Rearranging this equation yields

$$6.0 \times 10^{-3} - 1.2 \times 10^{-2} x = x^2$$

Writing the equation in quadratic form gives

$$x^2 + 1.2 \times 10^{-2} x - 6.0 \times 10^{-3} = 0$$

Solving for the two roots of this quadratic equation results in a negative value that may be discarded as physically irrelevant and a positive value equal to *x*. As defined in the ICE table, x is equal to the hydronium concentration.

$$x = [H3O^+] = 0.072 \text{ M}$$

$$pH = -\log[H_3O^+] = -\log(0.072) = 1.14$$

Exercise 7.12.8

Calculate the pH in a 0.010-*M* solution of caffeine, a weak base:

$$C_8H10\,N_4O_2(aq) + H_2O(l) \rightleftharpoons C_8H10\,N_4O_2H^+(aq) + OH^-(aq) \quad K_b = 2.5 \times 10^{-4}$$

Answer

pH = 11.16

Effect of Molecular Structure on Acid-Base Strength

Binary Acids and Bases

In the absence of any leveling effect, the acid strength of binary compounds of hydrogen with nonmetals (A) increases as the H-A bond strength decreases down a group in the periodic table. For group 17, the order of increasing acidity is

$$HF < HCl < HBr < HI.$$

Likewise, for group 16, the order of increasing acid strength is

$$H_2O < H_2S < H_2Se < H_2Te.$$

Across a row in the periodic table, the acid strength of binary hydrogen compounds increases with increasing electronegativity of the nonmetal atom because the polarity of the H-A bond increases. Thus, the order of increasing acidity (for removal of one proton) across the second row is $CH_4 < NH_3 < H_2O < HF$; across the third row, it is $SiH_4 < PH_3 < H_2S < HCl$ (see Figure 7.12.6).

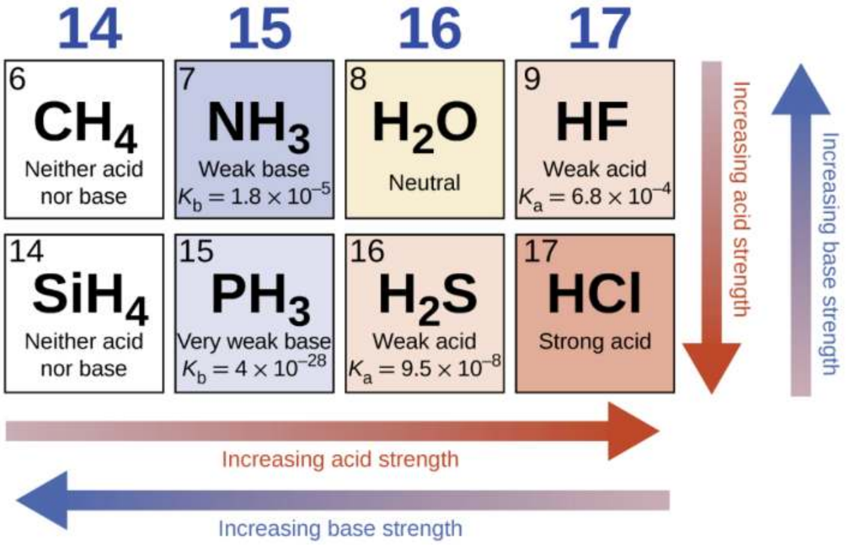
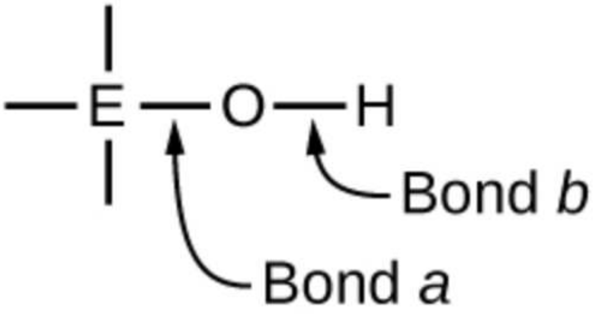


Figure 7.12.6: The figure shows trends in the strengths of binary acids and bases.

Ternary Acids and Bases

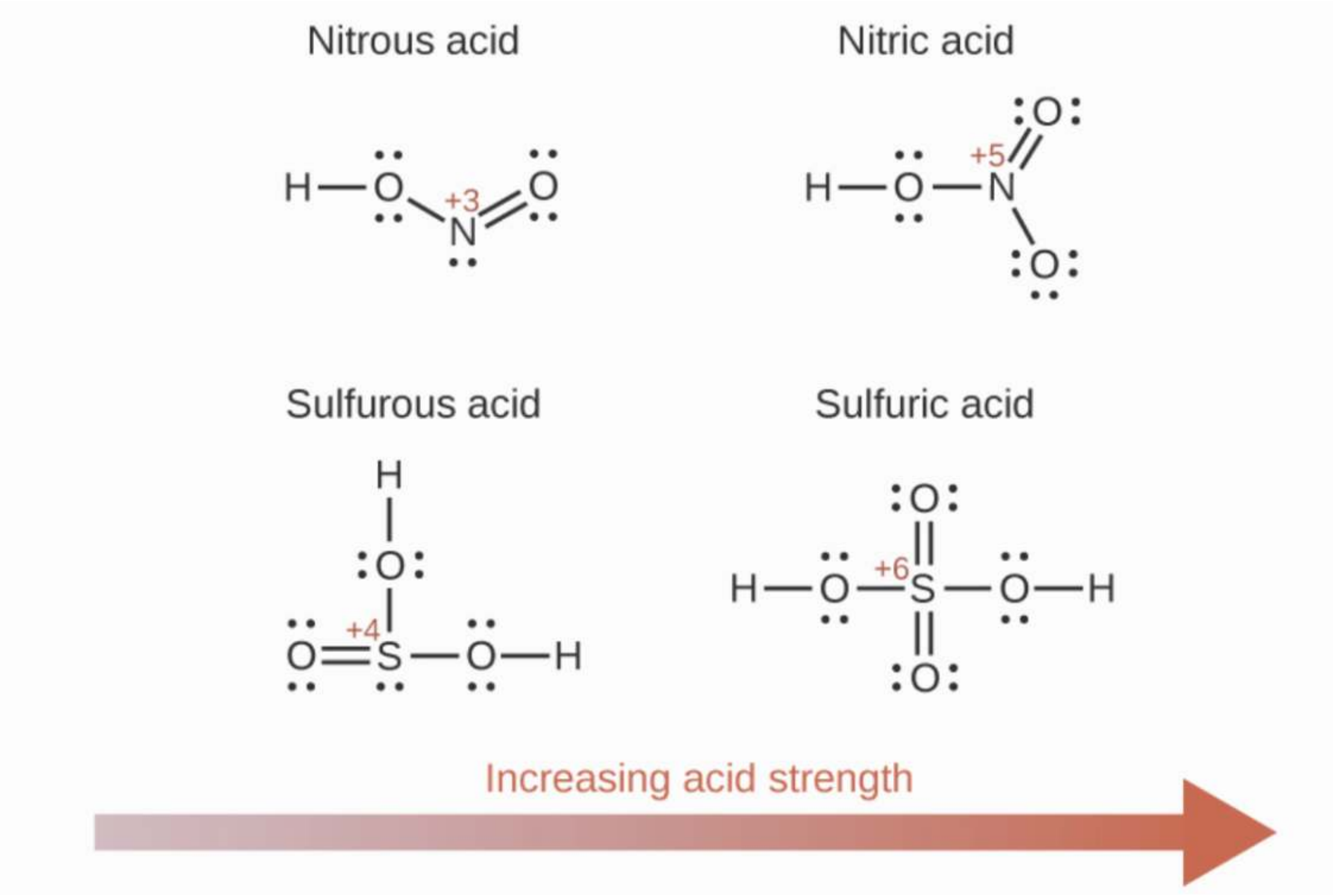
Ternary compounds composed of hydrogen, oxygen, and some third element (“E”) may be structured as depicted in the image below. In these compounds, the central E atom is bonded to one or more O atoms, and at least one of the O atoms is also bonded to an H atom, corresponding to the general molecular formula $O_mE(OH)_n$. These compounds may be acidic, basic, or amphoteric depending on the properties of the central E atom. Examples of such compounds include sulfuric acid, $O_2S(OH)_2$, sulfurous acid, $OS(OH)_2$, nitric acid, O_2NOH, perchloric acid, O_3ClOH, aluminum hydroxide, $Al(OH)_3$, calcium hydroxide, $Ca(OH)_2$, and potassium hydroxide, KOH:



If the central atom, E, has a low electronegativity, its attraction for electrons is low. Little tendency exists for the central atom to form a strong covalent bond with the oxygen atom, and bond *a* between the element and oxygen is more readily broken than bond *b* between oxygen and hydrogen. Hence bond *a* is ionic, hydroxide ions are released to the solution, and the material behaves as a base—this is the case with $Ca(OH)_2$ and KOH. Lower electronegativity is characteristic of the more metallic elements; hence, the metallic elements form ionic hydroxides that are by definition basic compounds.

If, on the other hand, the atom E has a relatively high electronegativity, it strongly attracts the electrons it shares with the oxygen atom, making bond *a* relatively strongly covalent. The oxygen-hydrogen bond, bond *b*, is thereby weakened because electrons are displaced toward E. Bond *b* is polar and readily releases hydrogen ions to the solution, so the material behaves as an acid. High electronegativities are characteristic of the more nonmetallic elements. Thus, nonmetallic elements form covalent compounds containing acidic −OH groups that are called oxyacids.

Increasing the oxidation number of the central atom E also increases the acidity of an oxyacid because this increases the attraction of E for the electrons it shares with oxygen and thereby weakens the O-H bond. Sulfuric acid, H_2SO_4, or $O_2S(OH)_2$ (with a sulfur oxidation number of +6), is more acidic than sulfurous acid, H_2SO_3, or $OS(OH)_2$ (with a sulfur oxidation number of +4). Likewise nitric acid, HNO_3, or O_2NOH (N oxidation number = +5), is more acidic than nitrous acid, HNO_2, or ONOH (N oxidation number = +3). In each of these pairs, the oxidation number of the central atom is larger for the stronger acid (Figure $7.12.7$).



A diagram is shown that includes four structural formulas for acids. A red, right pointing arrow is placed beneath the structures which is labeled "Increasing acid strength." At the top left, the structure of Nitrous acid is provided. It includes an H atom to which an O atom with two unshared electron pairs is connected with a single bond to the right. A single bond extends to the right and slightly below to a N atom with one unshared electron pair. A double bond extends up and to the right from this N atom to an O atom which has two unshared electron pairs. To the upper right is a structure for Nitric acid. This structure differs from the previous structure in that the N atom is directly to the right of the first O atom and a second O atom with three unshared electron pairs is connected with a single bond below and to the right of the N atom which has no unshared electron pairs. At the lower left, an O atom with two unshared electron pairs is double bonded to its right to an S atom with a single unshared electron pair. An O atom with two unshared electron pairs is bonded above and an H atom is single bonded to this O atom. To the right of the S atom is a single bond to another O atom with two unshared electron pairs to which an H atom is single bonded. This structure is labeled "Sulfurous acid." A similar structure which is labeled "Sulfuric acid" is placed in the lower right region of the figure. This structure differs in that an H atom is single bonded to the left of the first O atom, leaving it with two unshared electron pairs and a fourth O atom with two unshared electron pairs is double bonded beneath the S atom, leaving it with no unshared electron pairs.

Figure $7.12.7$: As the oxidation number of the central atom E increases, the acidity also increases.

Hydroxy compounds of elements with intermediate electronegativities and relatively high oxidation numbers (for example, elements near the diagonal line separating the metals from the nonmetals in the periodic table) are usually amphoteric. This means that the hydroxy compounds act as acids when they react with strong bases and as bases when they react with strong acids. The amphoterism of aluminum hydroxide, which commonly exists as the hydrate $\mathrm{Al(H_2O)_3(OH)_3}$, is reflected in its solubility in both strong acids and strong bases. In strong bases, the relatively insoluble hydrated aluminum hydroxide, $\mathrm{Al(H_2O)_3(OH)_3}$, is converted into the soluble ion, $\mathrm{[Al(H_2O)_2(OH)_4]^-}$, by reaction with hydroxide ion:

$$\mathrm{Al(H_2O)_3(OH)_3(aq) + OH^-(aq) \rightleftharpoons H_2O(l) + [Al(H_2O)_2(OH)_4]^-(aq)}$$

In this reaction, a proton is transferred from one of the aluminum-bound H2O molecules to a hydroxide ion in solution. The $\mathrm{Al(H_2O)_3(OH)_3}$ compound thus acts as an acid under these conditions. On the other hand, when dissolved in strong acids, it is converted to the soluble ion $[\mathrm{Al(H_2O)_6}]^{3+}$ by reaction with hydronium ion:

$$3\,\mathrm{H_3O^+(aq)} + \mathrm{Al(H_2O)_3(OH)_3(aq)} \rightleftharpoons \mathrm{Al(H_2O)_6^{3+}(aq)} + 3\,\mathrm{H_2O(l)}$$

In this case, protons are transferred from hydronium ions in solution to $\mathrm{Al(H_2O)_3(OH)_3}$, and the compound functions as a base.

7.13: End of Chapter Activity

End of Chapter Activity: Creating a Lesson Plan on Solutions, Acids, Bases, and pH with AI and Bloom's Taxonomy

Now that you have explored the fundamentals of solutions, acids, bases, and pH, it's time to put your knowledge into practice. Your task is to create a succinct lesson plan for 9th graders that introduces them to the basics of these concepts. To help you with this, you will use AI tools and incorporate Bloom's Taxonomy to ensure a comprehensive learning experience. This lesson plan will go towards your digital notebook, a portfolio filled with lesson plans, activities, and labs for future use.

Activity Prompt:

Objective: Use AI and Bloom's Taxonomy to develop a lesson plan that effectively teaches 9th graders about the fundamentals of solutions, acids, bases, and pH, including concepts such as solubility, properties of acids and bases, and pH scale.

Understanding the Concepts:

Knowledge (Remembering): Define key terms related to solutions, acids, bases, and pH, such as solute, solvent, acid, base, and pH scale.

Comprehension (Understanding): Explain these concepts in simple, age-appropriate language, focusing on their importance and real-life applications.

Planning the Lesson:

Application: Design an engaging activity or experiment that allows students to observe and understand solutions and the properties of acids and bases. For example, use common household items to create simple acid-base reactions and measure pH with pH paper or a digital pH meter.

Analysis: Use AI tools to create visual aids or interactive simulations that illustrate how solutes dissolve in solvents, how acids and bases react, and how the pH scale measures acidity and alkalinity. For instance, create a simulation showing the dissociation of acids and bases in water and the resulting pH changes.

Deepening Understanding:

Synthesis (Creating): Ask students to predict the outcome of mixing various solutions and to hypothesize the pH of different household substances. For example, predict whether lemon juice or soap solution will be more acidic or basic.

Evaluation: Have students discuss and reflect on what they observed during the activities. Encourage them to think about how the concepts of solutions, acids, bases, and pH are applied in everyday life, such as in cooking, cleaning, and industry.

Using AI in the Classroom:

Explore AI tools like educational apps or platforms that provide interactive content for teaching about solutions, acids, bases, and pH. Use these tools to create quizzes, flashcards, or interactive stories that reinforce the lesson's concepts.

Use AI to assess student understanding through formative assessments and provide instant feedback.

Deliverable:

Submit a detailed lesson plan that includes:

1. **A brief overview of the key concepts covered:** Outline the foundational concepts of solutions, acids, bases, and pH that will be taught.
2. **A description of the activities and experiments designed:** Detail the hands-on activities and experiments you will use to help students understand these concepts.
3. **Examples of AI tools used and how they enhance the learning experience:** Describe the AI tools you plan to incorporate, such as simulations or interactive quizzes, and explain how they will help students grasp complex concepts.
4. **An explanation of how Bloom's Taxonomy was applied in the lesson plan to ensure a well-rounded educational experience:** Illustrate how each level of Bloom's Taxonomy (Remembering, Understanding, Applying, Analyzing, Creating, and Evaluating) is addressed in your lesson plan.

This activity will help you integrate modern technology and educational strategies to create an effective and engaging learning experience for high school students.

7.14: End of Chapter Key Terms

Solutions, Acids, Bases, and pH

1. **Solution**: A homogeneous mixture composed of two or more substances, where one substance (solute) is dissolved in another (solvent).
2. **Solute**: The substance that is dissolved in a solution.
3. **Solvent**: The substance that dissolves the solute to form a solution; usually the component in greater quantity.
4. **Concentration**: The amount of solute present in a given quantity of solvent or solution; commonly expressed in molarity (M), which is moles of solute per liter of solution.
5. **Molarity (M)**: A unit of concentration, defined as the number of moles of solute per liter of solution.
6. **Solubility**: The maximum amount of solute that can dissolve in a given amount of solvent at a specific temperature and pressure.
7. **Saturated Solution**: A solution that contains the maximum amount of dissolved solute at a given temperature and pressure.
8. **Unsaturated Solution**: A solution that contains less than the maximum amount of dissolved solute at a given temperature and pressure.
9. **Supersaturated Solution**: A solution that contains more dissolved solute than it would under normal circumstances, typically achieved by changing the temperature or pressure.
10. **Dilution**: The process of reducing the concentration of a solute in solution, usually by adding more solvent.
11. **Acid**: A substance that donates hydrogen ions (H^+) in an aqueous solution; characterized by a sour taste and the ability to turn blue litmus paper red.
12. **Base**: A substance that accepts hydrogen ions (H^+) or donates hydroxide ions (OH^-) in an aqueous solution; characterized by a bitter taste and slippery feel, and the ability to turn red litmus paper blue.
13. **pH Scale**: A logarithmic scale used to measure the acidity or basicity of a solution, ranging from 0 to 14, where 7 is neutral, values below 7 are acidic, and values above 7 are basic.
14. **pH**: The negative logarithm of the hydrogen ion concentration in a solution, $pH = -\log[H^+]$.
15. **pOH**: The negative logarithm of the hydroxide ion concentration in a solution, $pOH = -\log[OH^-]$.
16. **pKa**: The negative logarithm of the acid dissociation constant (Ka), a measure of the strength of an acid; lower pKa values indicate stronger acids.
17. **pKb**: The negative logarithm of the base dissociation constant (Kb), a measure of the strength of a base; lower pKb values indicate stronger bases.
18. **Buffer**: A solution that resists changes in pH upon the addition of small amounts of an acid or base, typically composed of a weak acid and its conjugate base or a weak base and its conjugate acid.
19. **Acid-Base Titration**: A laboratory technique used to determine the concentration of an acid or base by reacting it with a standard solution of known concentration.
20. **Equivalence Point**: The point in a titration at which the amount of titrant added is exactly enough to completely neutralize the analyte solution.
21. **Indicator**: A substance that changes color at a specific pH range, used to determine the endpoint of a titration.
22. **Strong Acid**: An acid that completely ionizes in aqueous solution (e.g., hydrochloric acid, HCl).
23. **Weak Acid**: An acid that partially ionizes in aqueous solution (e.g., acetic acid, CH_3COOH).
24. **Strong Base**: A base that completely dissociates in aqueous solution (e.g., sodium hydroxide, $NaOH$).
25. **Weak Base**: A base that partially ionizes or dissociates in aqueous solution (e.g., ammonia, NH_3).
26. **Acid Dissociation Constant (Ka)**: A quantitative measure of the strength of an acid in solution, given by the equilibrium constant for the dissociation of the acid into its conjugate base and a hydrogen ion.
27. **Base Dissociation Constant (Kb)**: A quantitative measure of the strength of a base in solution, given by the equilibrium constant for the dissociation of the base into its conjugate acid and a hydroxide ion.
28. **Neutralization Reaction**: A chemical reaction in which an acid and a base react to form water and a salt.
29. **Salt**: An ionic compound formed from the neutralization reaction of an acid and a base.
30. **Le Chatelier’s Principle**: A principle stating that if a dynamic equilibrium is disturbed by changing the conditions, the position of equilibrium moves to counteract the change.
31. **Hydronium Ion (H_3O^+)**: A water molecule with an extra proton, representing the form in which hydrogen ions exist in aqueous solution.

32. **Autoionization of Water**: The process by which water molecules dissociate into hydrogen ions (H^+) and hydroxide ions (OH^-), expressed as H2O⇌H++OH−H_2O \rightleftharpoons H^+ + OH^-H2O⇌H++OH−.
33. **Ion Product of Water (Kw)**: The product of the concentrations of hydrogen ions and hydroxide ions in water, equal to 1.0×10−141.0 \times 10^{-14}1.0×10−14 at 25°C.
34. **Standard Solution**: A solution of known concentration used in titrations.
35. **Molar Mass**: The mass of one mole of a substance, usually expressed in grams per mole (g/mol).

CHAPTER OVERVIEW

8: Energy Physics and Chemistry

8.1: Introduction and Learning Objectives

Chapter Learning Objectives

Understanding the Basic Concepts of Energy:

- Define and understand forms of energy: kinetic, potential, thermal, chemical, and nuclear.
- Explore energy conservation and its principle in closed systems.
- Differentiate between renewable and non-renewable energy sources and their environmental and societal impacts.

Thermodynamics in Chemistry and Physics:

- Study the laws of thermodynamics and their applications in energy transfer, entropy, and process spontaneity.
- Understand these laws in chemical reactions, phase changes, heat engines, and refrigerators.
- Analyze real-world examples like metabolic processes and industrial systems.

Energy in Chemical Reactions and Bonding:

- Examine energy roles in exothermic and endothermic reactions and reaction enthalpies.
- Understand energy changes in bond formation and breaking, affecting molecular reactivity and stability.
- Apply energy concepts to predict reaction outcomes, rates, and equilibrium.

Physical Concepts of Work, Power, and Energy Transfer:

- Define work and power and their relationship to energy.
- Understand energy transfer mechanisms: work by forces, heat transfer, and radiation.
- Investigate applications in machines, electrical systems, and daily phenomena.

Teaching Strategies for Energy Concepts in Science Education:

- Develop engaging teaching methods using real-world examples and demonstrations.
- Design activities and experiments to apply energy principles.
- Utilize multimedia and technology tools for interactive simulations and visual aids.
- Address common misconceptions and challenges in learning energy concepts.
- Emphasize interdisciplinary nature and relevance in solving scientific and technological challenges.

Introduction to Energy

Energy is a fundamental concept that bridges the disciplines of physics and chemistry, playing a crucial role in understanding natural phenomena and technological processes. This chapter delves into the various forms of energy—kinetic, potential, thermal, chemical, and nuclear—providing a comprehensive overview of how energy is defined and understood. We explore the principle of energy conservation in closed systems and examine the distinctions between renewable and non-renewable energy sources, highlighting their environmental and societal impacts.

In the realm of thermodynamics, we study the laws that govern energy transfer, entropy, and the spontaneity of processes. These principles are vital in both chemical reactions and physical processes such as phase changes, heat engines, and refrigeration systems. Real-world examples, including metabolic processes and industrial applications, illustrate the practical importance of these laws.

The chapter also investigates the role of energy in chemical reactions, focusing on exothermic and endothermic reactions and the concept of reaction enthalpies. Understanding the energy changes associated with bond formation and breaking is essential for predicting reaction outcomes, rates, and equilibrium. Additionally, we cover the physical concepts of work, power, and energy transfer mechanisms, including work done by forces, heat transfer, and radiation, with practical applications in machines, electrical systems, and everyday phenomena.

For educators, effective teaching strategies are crucial for conveying these complex energy concepts. Engaging methods using real-world examples and practical demonstrations can make these ideas more accessible to students. Designing activities and experiments helps students apply the principles of energy in various scenarios. Multimedia and technology tools, such as interactive simulations and visual aids, enhance the teaching and learning experience. Addressing common misconceptions and challenges ensures a solid understanding of energy concepts. By emphasizing the interdisciplinary nature and relevance of these topics, educators can inspire students and highlight the importance of energy in solving current and future scientific and technological challenges.

In K-12 education, introducing these principles with age-appropriate language and relatable examples can ignite curiosity and understanding in young learners. Interactive activities and hands-on experiments allow students to see the real-world implications of energy concepts, making them more tangible and memorable. Utilizing multimedia resources caters to diverse learning styles, ensuring that all students grasp these foundational ideas. Highlighting how energy concepts are interconnected with everyday life and various scientific fields can foster a deeper appreciation for the subject, preparing students for advanced studies in physics and chemistry.

8.2: The Basics of Energy

Learning Objectives

- Define energy, distinguish types of energy, and describe the nature of energy changes that accompany chemical and physical changes
- Distinguish the related properties of heat, thermal energy, and temperature.

Chemical changes and their accompanying changes in energy are important parts of our everyday world (Figure 8.2.1). The macronutrients in food (proteins, fats, and carbohydrates) undergo metabolic reactions that provide the energy to keep our bodies functioning. We burn a variety of fuels (gasoline, natural gas, coal) to produce energy for transportation, heating, and the generation of electricity. Industrial chemical reactions use enormous amounts of energy to produce raw materials (such as iron and aluminum). Energy is then used to manufacture those raw materials into useful products, such as cars, skyscrapers, and bridges.

Figure 8.2.1: The energy involved in chemical changes is important to our daily lives: (a) A cheeseburger for lunch provides the energy you need to get through the rest of the day; (b) the combustion of gasoline provides the energy that moves your car (and you) between home, work, and school; and (c) coke, a processed form of coal, provides the energy needed to convert iron ore into iron, which is essential for making many of the products we use daily. (credit a: modification of work by "Pink Sherbet Photography"/Flickr; credit b: modification of work by Jeffery Turner).

Over 90% of the energy we use comes originally from the sun. Every day, the sun provides the Earth with almost 10,000 times the amount of energy necessary to meet all of the world's energy needs for that day. Our challenge is to find ways to convert and store incoming solar energy so that it can be used in reactions or chemical processes that are both convenient and nonpolluting. Plants and many bacteria capture solar energy through photosynthesis. We release the energy stored in plants when we burn wood or plant products such as ethanol. We also use this energy to fuel our bodies by eating food that comes directly from plants or from animals that get their energy by eating plants. Burning coal and petroleum also releases stored solar energy: These fuels are fossilized plant and animal matter.

Energy is usually defined as the capability to do **work (w) or** supply heat. For example, a billiard ball can collide with a second ball, changing the direction or speed of motion of the latter. In such a process the motion of the first ball would also be altered. We would say that one billiard ball did work on (transferred energy to) the other.

Kinetic Energy

Figure $8.2.2$ Cartoon of a boy riding a bike downhill. The words "kinetic energy" and "due to motion" are also on the picture.

Image source: Smart Learning for All

Energy due to motion is called kinetic energy and is represented by E_k. For an object moving in a straight line, the kinetic energy is one-half the product of the mass and the square of the speed:

$$E_k = \frac{1}{2} m u^2 \tag{8.2.1}$$

where

- m = mass of the object
- u = speed of object

If the two billiard balls mentioned above were studied in outer space, where friction due to their collisions with air molecules or the surface of a pool table would be negligible, careful measurements would reveal that their total kinetic energy would be the same before and after they collided. This is an example of the **law of conservation of energy**, which states that *energy cannot be created or destroyed* under the usual conditions of everyday life. Whenever there appears to be a decrease in energy somewhere, there is a corresponding increase somewhere else.

✓ Example $8.2.1$: Kinetic Energy

Calculate the kinetic energy of a Volkswagen Beetle of mass 844 kg (1860 lb) which is moving at 13.4 m s^{-1} (30 miles per hour).

Solution:

$$E_k = \frac{1}{2} m u^2 = \frac{1}{2} \times 844 \text{ kg} \times (13.4 \text{ m s}^{-1})^2 = 7.58 \times 10^4 \text{ kg m}^2 \text{ s}^{-2}$$

In other words the units for energy are derived from the SI base units kilogram for mass, meter for length, and second for time. A quantity of heat or any other form of energy may be expressed in kilogram meter squared per second squared. In honor of Joule’s pioneering work this derived unit 1 kg m^2 s^{-2} called the **joule**, abbreviated J. The Volkswagen in question could do nearly 76 000 J of work on anything it happened to run into.

Potential Energy

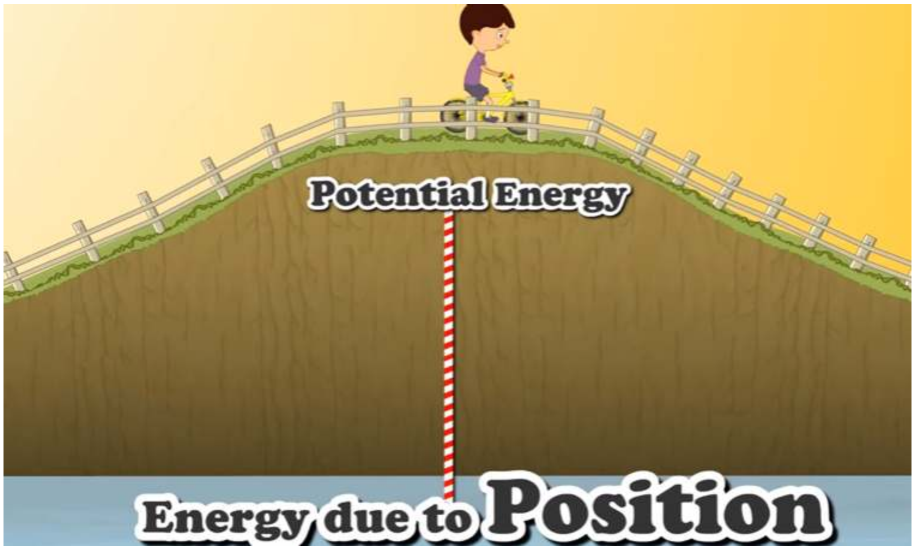


Figure 8.2.3Boy on a bike at the peak of a hill. The words "potential energy" and "energy due to position" are also in the picture.

Image source: Smart Learning for All

Potential Energy is energy that is stored by rising in height, or by other means. It frequently comes from separating things that attract, like rising birds are being separated from the Earth that attracts them, or by pulling magnets apart, or pulling an electrostatically charged balloon from an oppositely charged object to which it has clung. Potential Energy is abbreviated E_P and gravitational potential energy is calculated as follows:

$$E_P = mgh \tag{8.2.2}$$

where

- m = mass of the object in kg
- g = gravitational constant, 9.8 m s^2
- h = height in m

Notice that E_P has the same units, kg m^2 s^{-2} or ***Joule*** as kinetic energy.

Example 8.2.2: Kinetic Energy Application

How high would the VW weighing 844 kg and moving at 30 mph need to rise (vertically) on a hill to come to a complete stop, if none of the stopping power came from friction?

Solution:

The car's kinetic energy is 7.58×10^4 kg m^2 s^{-2}(from EXAMPLE 8.2.1), so all of this would have to be converted to E_P. Then we could calculate the vertical height:

$$E_P = mgh = 7.58 \times 10^4 \text{ kg m}^2 \text{ s}^{-2} = 844 \text{ kg} \times 9.8\text{ms}^{-2} \times h$$
$$h = 9.2 \text{ m}$$

Even when there is a great deal of friction, the law of conservation of energy still applies. If you put a milkshake on a mixer and leave it there for 10 min, you will have a warm, rather unappetizing drink. The whirling mixer blades do work on (transfer energy to) the milkshake, raising its temperature. The same effect could be produced by heating the milkshake, a fact which suggests that heating also involves a transfer of energy. The first careful experiments to determine how much work was equivalent to a given quantity of heat were done by the English physicist James Joule (1818 to 1889) in the 1840s. In an experiment very similar to our milkshake example, Joule connected falling weights through a pulley system to a paddle wheel immersed in an insulated container of water. This allowed him to compare the temperature rise, which resulted from the work done by the weights, with that which resulted from heating. Units with which to measure energy may be derived from the SI base units of Table 1 from The International System of Units (SI)(opens in new window) by using Eq. 8.2.1.

Another unit of energy still widely used by chemists is the **calorie**. The calorie used to be defined as the energy needed to raise the temperature of one gram of water from 14.5°C to 15.5°C but now it is defined as exactly 4.184 J.

Like matter, energy comes in different types and is conserved. One scheme classifies energy into two types: potential energy, the energy an object has because of its relative position, composition, or condition, and kinetic energy, the energy that an object possesses because of its motion. Water at the top of a waterfall or dam has potential energy because of its position; when it flows downward through generators, it has kinetic energy that can be used to do work and produce electricity in a hydroelectric plant (Figure 8.2.4). A battery has potential energy because the chemicals within it can produce **electricity**, another form of energy, that can do work.

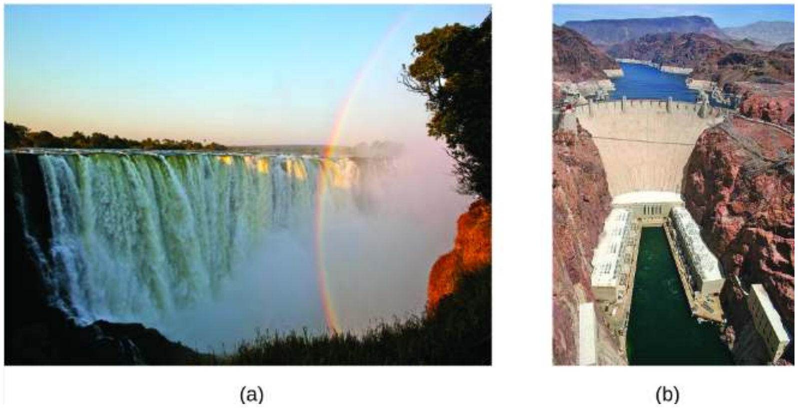


Figure 8.2.4: (a) Water that is higher in elevation, for example, at the top of Victoria Falls, has a higher potential energy than water at a lower elevation. As the water falls, some of its potential energy is converted into kinetic energy. (b) If the water flows through generators at the bottom of a dam, such as the Hoover Dam shown here, its kinetic energy is converted into electrical energy. (credit a: modification of work by Steve Jurvetson; credit b: modification of work by “curimedia”/Wikimedia commons).

Two pictures are shown and labeled a and b. Picture a shows a large waterfall with water falling from a high elevation at the top of the falls to a lower elevation. The second picture is a view looking down into the Hoover Dam. Water is shown behind the high wall of the dam on one side and at the base of the dam on the other.

Energy can be converted from one form into another, but all of the energy present before a change occurs always exists in some form after the change is completed. This observation is expressed in the law of conservation of energy: during a chemical or physical change, energy can be neither created nor destroyed, although it can be changed in form. (This is also one version of the first law of thermodynamics, as you will learn later.)

When one substance is converted into another, there is always an associated conversion of one form of energy into another. Heat is usually released or absorbed, but sometimes the conversion involves light, electrical energy, or some other form of energy. For example, chemical energy (a type of potential energy) is stored in the molecules that compose gasoline. When gasoline is combusted within the cylinders of a car’s engine, the rapidly expanding gaseous products of this chemical reaction generate mechanical energy (a type of kinetic energy) when they move the cylinders’ pistons.

According to the law of conservation of matter (seen in an earlier chapter), there is no detectable change in the total amount of matter during a chemical change. When chemical reactions occur, the energy changes are relatively modest and the mass changes are too small to measure, so the laws of conservation of matter and energy hold well. However, in nuclear reactions, the energy changes are much larger (by factors of a million or so), the mass changes are measurable, and matter-energy conversions are significant. This will be examined in more detail in a later chapter on nuclear chemistry. To encompass both chemical and nuclear changes, we combine these laws into one statement: The total quantity of matter and energy in the universe is fixed.

Thermal Energy, Temperature, and Heat

Thermal energy is kinetic energy associated with the random motion of atoms and molecules. Temperature is a quantitative measure of “hot” or “cold.” When the atoms and molecules in an object are moving or vibrating quickly, they have a higher average kinetic energy (KE), and we say that the object is “hot.” When the atoms and molecules are moving slowly, they have lower KE, and we say that the object is “cold” (Figure 8.2.5). Assuming that no chemical reaction or phase change (such as melting or vaporizing) occurs, increasing the amount of thermal energy in a sample of matter will cause its temperature to increase.

And, assuming that no chemical reaction or phase change (such as condensation or freezing) occurs, decreasing the amount of thermal energy in a sample of matter will cause its temperature to decrease.

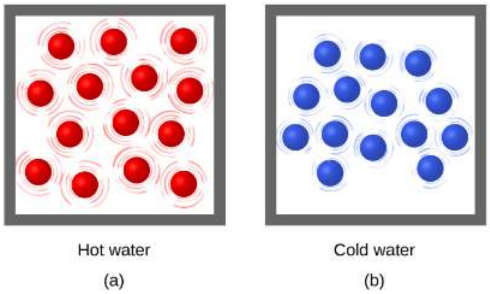
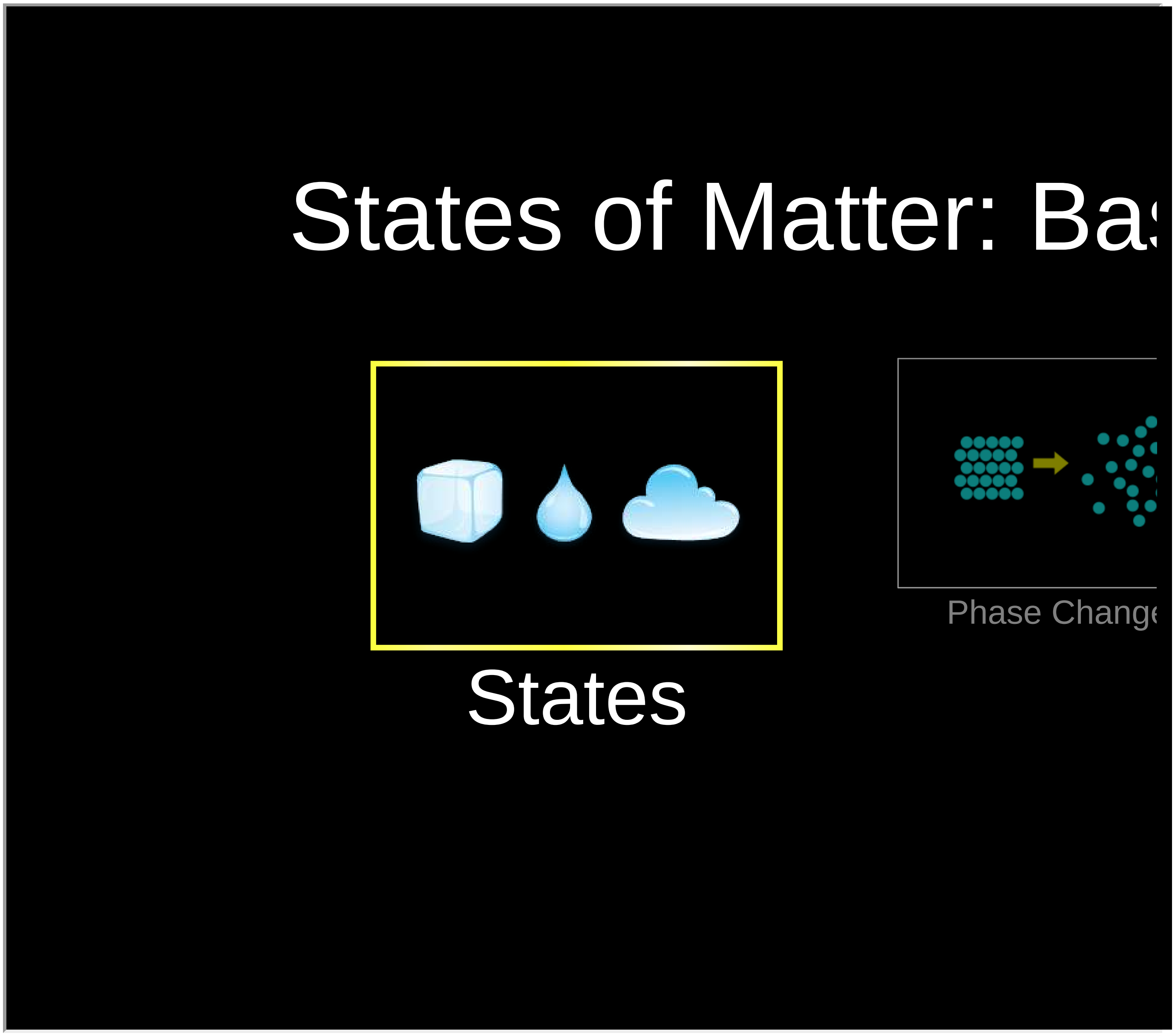
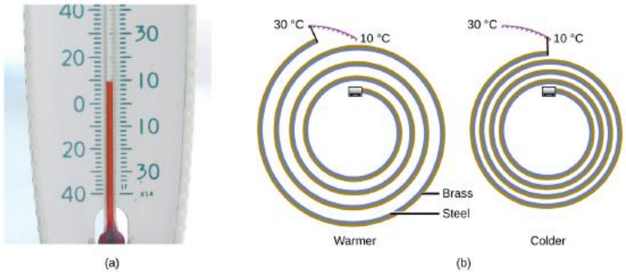


Figure $8.2.5$: (a) The molecules in a sample of hot water move more rapidly than (b) those in a sample of cold water.Two molecular drawings are shown and labeled a and b. Drawing a is a box containing fourteen red spheres that are surrounded by lines indicating that the particles are moving rapidly. This drawing has a label that reads “Hot water.” Drawing b depicts another box of equal size that also contains fourteen spheres, but these are blue. They are all surrounded by smaller lines that depict some particle motion, but not as much as in drawing a. This drawing has a label that reads “Cold water.”



Most substances expand as their temperature increases and contract as their temperature decreases. This property can be used to measure temperature changes, as shown in Figure 8.2.6. The operation of many thermometers depends on the expansion and contraction of substances in response to temperature changes.



A picture labeled a is shown as well as a pair of drawings labeled b. Picture a shows the lower portion of an alcohol thermometer. The thermometer has a printed scale to the left of the tube in the center that reads from negative forty degrees at the bottom to forty degrees at the top. It also has a scale printed to the right of the tube that reads from negative thirty degrees at the bottom to thirty five degrees at the top. On both scales, the volume of the alcohol in the tube reads between nine and ten degrees. The two images labeled b both depict a metal strip coiled into a spiral and composed of brass and steel. The left coil, which is loosely coiled, is labeled along its upper edge with the 30 degrees C and 10 degrees C. The end of the coil is near the 30 degrees C label. The right hand coil is much more tightly wound and the end is near the 10 degree C label.

Figure 8.2.6: (a) In an alcohol or mercury thermometer, the liquid (dyed red for visibility) expands when heated and contracts when cooled, much more so than the glass tube that contains the liquid. (b) In a bimetallic thermometer, two different metals (such as brass and steel) form a two-layered strip. When heated or cooled, one of the metals (brass) expands or contracts more than the other metal (steel), causing the strip to coil or uncoil. Both types of thermometers have a calibrated scale that indicates the temperature. (credit a: modification of work by “dwstucke”/Flickr). (c) The demonstration allows one to view the effects of heating and cooling a coiled bimetallic strip.A bimetallic coil from a thermometer reacts to the heat from a lighter, by uncoiling and then coiling back up when the lighter is removed. Animation used with permission from Hustvedt (via Wikipedia).

Heat (q) is the transfer of thermal energy between two bodies at different temperatures. Heat flow (a redundant term, but one commonly used) increases the thermal energy of one body and decreases the thermal energy of the other. Suppose we initially have a high temperature (and high thermal energy) substance (H) and a low temperature (and low thermal energy) substance (L). The atoms and molecules in H have a higher average KE than those in L. If we place substance H in contact with substance L, the thermal energy will flow spontaneously from substance H to substance L. The temperature of substance H will decrease, as will the average KE of its molecules; the temperature of substance L will increase, along with the average KE of its molecules. Heat flow will continue until the two substances are at the same temperature (Figure 8.2.7).\

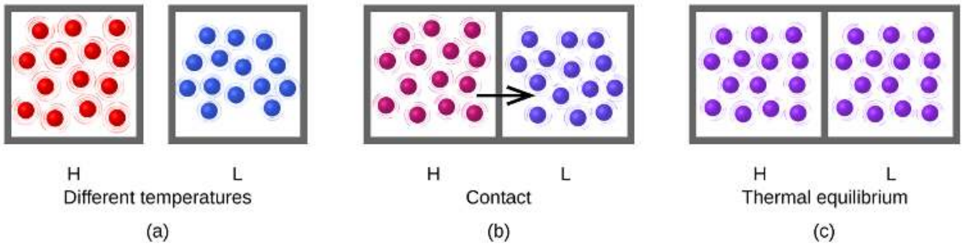


Figure 8.2.7: (a) Substances H and L are initially at different temperatures, and their atoms have different average kinetic energies. (b) When they are put into contact with each other, collisions between the molecules result in the transfer of kinetic (thermal) energy from the hotter to the cooler matter. (c) The two objects reach "thermal equilibrium" when both substances are at the same temperature, and their molecules have the same average kinetic energy.

Three drawings are shown and labeled a, b, and c, respectively. The first drawing labeled a depicts two boxes, with a space in between and the pair is captioned "Different temperatures." The left hand box is labeled H and holds fourteen well-spaced red spheres with lines drawn around them to indicate rapid motion. The right hand box is labeled L and depicts fourteen blue spheres that are closer together than the red spheres and have smaller lines around them showing less particle motion. The second drawing labeled b depicts two boxes that are touching one another. The left box is labeled H and contains fourteen maroon spheres that are spaced evenly apart. There are tiny lines around each sphere depicting particle movement. The right box is labeled L and holds fourteen purple spheres that are slightly closer together than the maroon spheres. There are also tiny lines around each sphere depicting particle movement. A black arrow points from the left box to the right box and the pair of diagrams is captioned "Contact." The third drawing labeled c, is labeled "Thermal equilibrium." There are two boxes shown in contact with one another. Both boxes contain fourteen purple spheres with small lines around them depicting moderate movement. The left box is labeled H and the right box is labeled L.

Matter undergoing chemical reactions and physical changes can release or absorb heat. A change that releases heat is called an exothermic process. For example, the combustion reaction that occurs when using an oxyacetylene torch is an exothermic process—this process also releases energy in the form of light as evidenced by the torch's flame (Figure 8.2.8*a*). A reaction or change that absorbs heat is an endothermic process. A cold pack used to treat muscle strains provides an example of an endothermic process. When the substances in the cold pack (water and a salt like ammonium nitrate) are brought together, the resulting process absorbs heat, leading to the sensation of cold.

Figure 8.2.8: (a) An oxyacetylene torch produces heat by the combustion of acetylene in oxygen. The energy released by this exothermic reaction heats and then melts the metal being cut. The sparks are tiny bits of the molten metal flying away. (b) A cold pack uses an endothermic process to create the sensation of cold. (credit a: modification of work by "Skatebiker"/Wikimedia commons).

Two pictures are shown and labeled a and b. Picture a shows a metal railroad tie being cut with the flame of an acetylene torch. Picture b shows a chemical cold pack containing ammonium nitrate.

Units of Energy

Energy is measured in terms of its ability to perform work or to transfer heat. **Mechanical work** (w) is done when a **force** f displaces an object by a **distance** *d*:

$$w = f \times d \tag{8.2.3}$$

The basic unit of energy is the ***joule***. One joule is the amount of work done when a force of 1 newton acts over a distance of 1 m; thus 1 J = 1 N-m. The newton is the amount of force required to accelerate a 1-kg mass by 1 m/sec^2, so the basic dimensions of the joule are kg m^2 s^{-2}. Historically, energy was measured in units of calories (cal). A calorie is the amount of energy required to raise one gram of water by 1 degree C (1 kelvin). However, this quantity depends on the atmospheric pressure and the starting temperature of the water. The ease of measurement of energy changes in calories has meant that the calorie is still frequently used. The **Calorie** (with a capital C), or large calorie, commonly used in quantifying food energy content, is a kilocalorie. Another common unit of measurement of energy is the *BTU* (British thermal unit) which is defined in terms of the heating effect on water. Because of the many forms that energy can take, there are a correspondingly large number of units in which it can be expressed, a few of which are summarized below.

Effect of units of energy	Joule equivalents
1 calorie will raise the temperature of 1 g of water by 1 C°. The "dietary" calorie is actually 1 kcal. An average young adult expends about 1800 kcal per day just to stay alive. (you should know this definition)	1 cal = 4.184 J
1 BTU (British Thermal Unit) will raise the temperature of 1 lb of water by 1F°.	1 BTU = 1055 J
The electron-volt is even tinier: 1 eV is the work required to move a unit electric charge (1 C) through a potential difference of 1 volt.	1 J = 6.24 × 1018 eV
The watt is a unit of power, which measures the rate of energy flow in J sec^{-1}. Thus the watt-hour is a unit of energy. An average human consumes energy at a rate of about 100 watts; the brain alone runs at about 5 watts.	1 J = 2.78 × 10^{-4}watt-hr 1 w-h = 3.6 kJ
The liter-atmosphere is a variant of force-displacement work associated with volume changes in gases.	1 L-atm = 101.325 J
The huge quantities of energy consumed by cities and countries are expressed in quads; the therm is a similar but smaller unit.	1 quad = 1015 Btu = 1.05 × 10^{18} J
If the object is to obliterate cities or countries with nuclear weapons, the energy unit of choice is the ton of TNT equivalent.	1 ton of TNT = 4.184 GJ (by definition)

Summary

Video* $8.2.1$*: *A video summary of Energy and Chemistry.*

Energy is the capacity to do work (applying a force to move matter). Kinetic energy (KE) is the energy of motion; potential energy is energy due to relative position, composition, or condition. When energy is converted from one form into another, energy is neither created nor destroyed (law of conservation of energy or first law of thermodynamics). Matter has thermal energy due to the KE of its molecules and temperature that corresponds to the average KE of its molecules. Heat is energy that is transferred between objects at different temperatures; it flows from a high to a low temperature. Chemical and physical processes can absorb heat (endothermic) or release heat (exothermic). The SI unit of energy, heat, and work is the joule (J). Specific heat and heat capacity are measures of the energy needed to change the temperature of a substance or object. The amount of heat absorbed or released by a substance depends directly on the type of substance, its mass, and the temperature change it undergoes.

Key Equations

- $q = c \times m \times \Delta T = c \times m \times (T_{\text{final}} - T_{\text{initial}})$
- $E_k = \frac{1}{2} m u^2$
- $E_P = mgh$
- $w = f \times d$

Glossary

calorie (cal)

unit of heat or other energy; the amount of energy required to raise 1 gram of water by 1 degree Celsius; 1 cal is defined as 4.184 J

endothermic process

chemical reaction or physical change that absorbs heat

energy

capacity to supply heat or do work

exothermic process

chemical reaction or physical change that releases heat

joule (J)

SI unit of energy; 1 joule is the kinetic energy of an object with a mass of 2 kilograms moving with a velocity of 1 meter per second, 1 J = 1 kg m^2/s and 4.184 J = 1 cal

kinetic energy

energy of a moving body, in joules, equal to $\frac{1}{2} m v^2$ (where *m* = mass and *v* = velocity)

potential energy

energy of a particle or system of particles derived from relative position, composition, or condition

temperature

intensive property of matter that is a quantitative measure of “hotness” and “coldness”

thermal energy

kinetic energy associated with the random motion of atoms and molecules

thermochemistry

study of measuring the amount of heat absorbed or released during a chemical reaction or a physical change

work (*w*)

energy transfer due to changes in external, macroscopic variables such as pressure and volume; or causing matter to move against an opposing force

Contributors

- Template:ContribOpenStax
- Adelaide E. Clark, Oregon Institute of Technology
- Template:contribLower
- Crash Course Chemistry: Crash Course is a division of Complexly and videos are free to stream for educational purposes.

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8.2.1: Practice Problems- The Basics of Energy

PROBLEM 8.2.1.1

A burning match and a bonfire may have the same temperature, yet you would not sit around a burning match on a fall evening to stay warm. Why not?

Answer

The temperature of 1 gram of burning wood is approximately the same for both a match and a bonfire. This is an intensive property and depends on the material (wood). However, the overall amount of produced heat depends on the amount of material; this is an extensive property. The amount of wood in a bonfire is much greater than that in a match; the total amount of produced heat is also much greater, which is why we can sit around a bonfire to stay warm, but a match would not provide enough heat to keep us from getting cold.

PROBLEM 8.2.1.2

Explain the difference between heat capacity and specific heat of a substance.

Answer

Heat capacity refers to the heat required to raise the temperature of the mass of the substance 1 degree; specific heat refers to the heat required to raise the temperature of 1 gram of the substance 1 degree. Thus, heat capacity is an extensive property, and specific heat is an intensive one.

PROBLEM 8.2.1.3

How much heat, in joules and in calories, must be added to a 75.0–g iron block with a specific heat of 0.449 J/g °C to increase its temperature from 25 °C to its melting temperature of 1535 °C?

Answer

50,800 J

12,200 cal

Click here to see a video of the solution

PROBLEM 8.2.1.4

How much heat, in joules and in calories, is required to heat a 28.4-g (1-oz) ice cube from −23.0 °C to −1.0 °C?

Answer

1310 J

313 cal

PROBLEM 8.2.1.5

How much would the temperature of 275 g of water increase if 36.5 kJ of heat were added?

Answer

31.7° C

Click here to see a video of the solution

PROBLEM 8.2.1.6

If 14.5 kJ of heat were added to 485 g of liquid water, how much would its temperature increase?

Answer

7.15 °C

PROBLEM 8.2.1.7

A piece of unknown substance weighs 44.7 g and requires 2110 J to increase its temperature from 23.2 °C to 89.6 °C.

a. What is the specific heat of the substance?
b. If it is one of the substances found in Table 8.1.1, what is its likely identity?

Answer a

$$C = \frac{0.711\ J}{g\ ^{\circ}C}$$

Answer b

Silicon

Click here to see a video of the solution

? PROBLEM 8.2.1.8

A piece of unknown solid substance weighs 437.2 g, and requires 8460 J to increase its temperature from 19.3 °C to 68.9 °C.

a. What is the specific heat of the substance?
b. If it is one of the substances found in Table 8.1.1, what is its likely identity?

Answer a

$$C = \frac{0.390\ J}{g\ ^{\circ}C}$$

Answer b

Copper

? PROBLEM 8.2.1.9

An aluminum kettle weighs 1.05 kg.

a. What is the heat capacity of the kettle (Table 8.1.1)?
b. How much heat is required to increase the temperature of this kettle from 23.0 °C to 99.0 °C?
c. How much heat (in kJ) is required to heat this kettle from 23.0 °C to 99.0 °C if it contains 1.25 L of water (density of 0.997 g/mL and a specific heat of 4.184 J/g °C)?

Answer a

$$C = \frac{0.897\ J}{g\ ^{\circ}C}$$

Answer b

71580 J

Answer c

467.86 kJ

Click here to see a video of the solution

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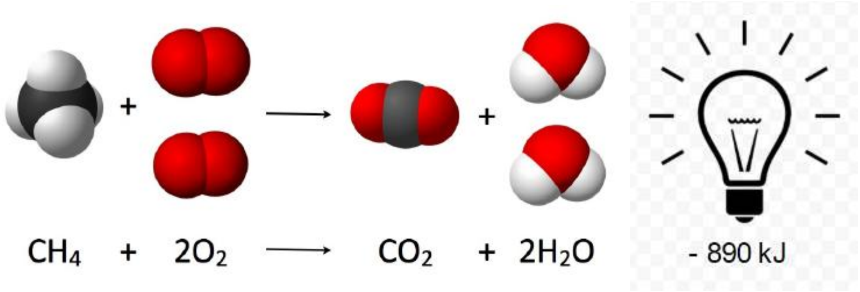
8.3: Thermochemical Equations

Energy changes which accompany chemical reactions are almost always expressed by **thermochemical equations**, such as

$$\begin{gathered}CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O(l)\ (25^\circ C,\ 1\ \text{atm pressure}) \\ \Delta H_m = -890\text{kJ}\end{gathered} \tag{8.3.1}$$

which is displayed on the atomic level below. To get an idea of what this reaction looks like on the macroscopic level, check out the flames on the far right.



Here the ΔH_m (delta *H* subscript m) tells us whether heat energy is released or absorbed when the reaction occurs as written, and also enables us to find the actual quantity of energy involved. By convention, if ΔH_m is *positive*, heat is *absorbed* by the reaction; i.e., it is **endothermic**. More commonly, ΔH_m is *negative* as in Eq. 8.3.1, indicating that heat energy is *released* rather than absorbed by the reaction, and that the reaction is **exothermic**. This convention as to whether ΔH_m is positive or negative looks at the heat change in terms of the matter actually involved in the reaction rather than its surroundings. In the reaction in Eq. 8.3.1, the C, H, and O atoms have collectively lost energy and it is this loss which is indicated by a negative value of ΔH_m.

It is important to notice that ΔH_m is the energy for the reaction as written. In the case of Equation 8.3.1, that represents the formation of 1 mol of carbon dioxide and 2 mol of water. The quantity of heat released or absorbed by a reaction is proportional to the amount of each substance consumed or produced by the reaction. Thus Eq. 8.3.1 tells us that 890.4 kJ of heat energy is given off *for every mole of* CH_4 which is consumed. Alternatively, it tells us that 890.4 kJ is released *for every 2 moles of* H_2O produced. Seen in this way, ΔH_m is a conversion factor enabling us to calculate the heat absorbed or released when a given amount of substance is consumed or produced. If q is the quantity of heat absorbed or released and n is the amount of substance involved, then

$$\Delta H_\text{m} = \frac{q}{n}$$

✓ Example 8.3.1 : Heat Energy

How much heat energy is obtained when 1 kg of ethane gas, C_2H_6, is burned in oxygen according to the equation:

$$2C_2H_6(g) + 7O_2(g) \rightarrow 4CO_2(g) + 6H_2O(l)$$

$$\text{with } \Delta H_m = -3120\text{ kJ.}$$

Solution

The mass of C_2H_6 is easily converted to the amount of C_2H_6 from which the heat energy q is easily calculated by means of Eq. (2). The value of ΔH_m is –3120 kJ per per 2 mol C_2H_6. The road map is

$$m_{\text{C}_2\text{H}_6} \xrightarrow{M} n_{\text{C}_2\text{H}_6} \xrightarrow{\Delta H_m} q$$

so that

$$\begin{aligned} q &= 1 \times 10^3\text{ g C}_2\text{H}_6 \times \frac{1\text{ mol C}_2\text{H}_6}{30.07\text{ g C}_2\text{H}_6} \times \frac{-3120\text{ kJ}}{2\text{ mol C}_2\text{H}_6} \\ &= -51\,879\text{ kJ} = -51.88\text{ MJ}\end{aligned}$$

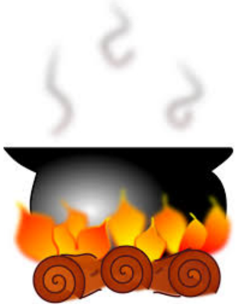
> *By convention, a negative value of q corresponds to a release of heat energy by the matter involved in the reaction.*

The quantity ΔH_m is referred to as an **enthalpy change for the reaction**. In this context the symbol Δ (delta) signifies change in" while H is the symbol for the quantity being changed, namely the enthalpy. We will deal with the enthalpy in some detail in Chap. 15. For the moment we can think of it as a property of matter which increases when matter absorbs energy and decreases when matter releases energy.

It is important to realize that the value of ΔH_m given in thermochemical equations like 8.3.1 or ??? depends on the physical state of both the reactants and the products. Thus, if water were obtained as a gas instead of a liquid in the reaction in Eq. 8.3.1, the value of ΔH_m would be different from -890.4 kJ. It is also necessary to specify both the temperature and pressure since the value of ΔH_m depends very slightly on these variables. If these are not specified they usually refer to 25°C and to normal atmospheric pressure.

Two more characteristics of thermochemical equations arise from the law of conservation of energy. The first is that *writing an equation in the reverse direction changes the sign of the enthalpy change.* For example,

$$\mathrm{H_2O}(l) \rightarrow \mathrm{H_2O}(g)$$
$$\Delta \mathrm{H}_m = 44 \text{ kJ}$$



In the image above, the flames input energy into the water, giving it the energy necessary to transition to the gas phase. Since flames provide the energy for the phase transition, this is an endothermic reaction (energy is absorbed).

Tells us that when a mole of liquid water vaporizes, 44 kJ of heat is absorbed. This corresponds to the fact that heat is absorbed from your skin when perspiration evaporates, and you cool off. Condensation of 1 mol of water vapor, on the other hand, gives off exactly the same quantity of heat.

$$\mathrm{H_2O}(g) \rightarrow \mathrm{H_2O}(l)$$
$$\Delta \mathrm{H}_m = -44 \text{kJ}$$

It's counterintuitive, but the common summer occurrence seen above is actually exothermic. Since the reaction isn't highly exothermic (like the combustion of CH_4), we find it hard to associate with a release of energy. Thermodynamics allows us to better understand on a micro level energy changes like this one.

To see why this must be true, suppose that ΔH_m [Eq. (4a)] = 44 kJ while ΔH_m [Eq. (4b)] = –50.0 kJ. If we took 1 mol of liquid water and allowed it to evaporate, 44 kJ would be absorbed. We could then condense the water vapor, and 50.0 kJ would be given off. We could again have 1 mol of liquid water at 25°C but we would also have 6 kJ of heat which had been created from nowhere! This would violate the law of conservation of energy. The only way the problem can he avoided is for ΔH_m of the reverse reaction to be equal in magnitude but opposite in sign from ΔH_m of the forward reaction. That is,

$$\Delta \mathrm{H}_m \text{forward} = -\Delta \mathrm{H}_m \text{reverse}$$

8.3.1: Biology- Weight of Food and Energy Production

As we've seen, the diet of Eagles (along with all other animals including us) includes certain masses of food. For eagles, it's 250-550 g/day. We also saw that this provides the energy for all the day's activities, and food that's left after energy production goes to weight gain. How is the mass of food related to the energy produced? The first step in answering this question is as simple one, and involves writing an overall "thermochemical equation" for the metabolism of sugar, which turns out to be the same as the equation for the combustion of sugar. A thorough answer to this question requires us to consider other factors, which we'll take up later.

> **Thermochemical equations** are used to relate energy changes to the chemical reactions that produce them. For example, we've already seen in Metabolism of Dietary Sugar that sugar is metabolized according to the equation[1]:

$C_6H_{12}O_6(s) + 6\ O_2(g) \rightarrow 6\ CO_2(g) + 6\ H_2O(l)$ (25º, 1 Atm) $\Delta H_m = –2808$ kJ (1) Here the sign of ΔH_m (delta *H* subscript m) tells us whether heat energy is released or absorbed when the reaction occurs and the value enables us to find the actual quantity of energy involved. By convention, if ΔH_m is *positive*, heat is *absorbed* by the reaction; i.e., it is endothermic. More commonly, ΔH_m is *negative* as in Eq. (1), indicating that heat energy is *released* rather than absorbed by the reaction, and that the reaction is **exothermic**. This convention as to whether ΔH_m is positive or negative looks at the heat change in terms of the matter actually involved in the reaction rather than its surroundings. In the reaction in Eq. (1), the C, H, and O atoms have collectively lost energy and it is this loss which is indicated by a negative value of ΔH_m.

It is important to notice that ΔH_m is the energy change for the equation as written. This is necessary because the quantity of heat released or absorbed by a reaction is proportional to the amount of each substance consumed or produced by the reaction. Thus Eq. (1) tells us that 2805 kJ of heat energy is given off *for every mole of* $C_6H_{12}O_6$ *which is consumed. Alternatively, it tells us that 2808 kJ is released* for every 6 mole of H_2O *produced, i.e., 468 kJ is produced for every mol* H_2O*.* $\Delta \mathrm{H_m}$ for Equation (1) also tells us that 2808 kJ of heat is released when 6 mol of carbon dioxide is produced, or 6 mol of oxygen is consumed. Seen in this way, $\Delta \mathrm{H_m}$ is a conversion factor enabling us to calculate the heat absorbed when a given amount of substance is consumed or produced. If q *is the quantity of heat absorbed and* n *is the amount of substance involved, then*

$$\Delta H_{\mathrm{m}} = \frac{q}{n} \quad (2)$$

EXAMPLE 1 How much heat energy is obtained if we assume that the eagle's diet of 250-550 g includes 350 g of glucose, $C_6H_{12}O_6$, which is burned in oxygen according to the equation:

$$C_6H_{12}O_6(s) + 6\ O_2(g) \rightarrow 6\ CO_2(g) + 6\ H_2O(l)$$

$$\Delta H_m = –2808 \text{ kJ} \quad (3)$$

Solution

The mass of $C_6H_{12}O_6$ is easily converted to the amount of $C_6H_{12}O_6$ from which the heat energy *q* is easily calculated by means of Eq. (2). The value of ΔH_m is –2805 kJ per mole of $C_6H_{12}O_6$,

$$m_{\mathrm{C_6H_{12}O_6}} \xrightarrow{M} n_{\mathrm{C_6H_{12}O_6}} \xrightarrow{\Delta H_m} q$$

so that

$$q = 350 \text{ g } \mathrm{C_6H_{12}O_6} \times \frac{1 \text{ mol } \mathrm{C_6H_{12}O_6}}{180.16 \text{ g } \mathrm{C_6H_{12}O_6}} \times \frac{-2808 \text{ kJ}}{\text{mol } \mathrm{C_6H_{12}O_6}} = -5\ 455 \text{ kJ}$$

Note: By convention a negative value of *q* corresponds to a release of heat energy by the matter involved in the reaction. The quantity ΔH_m is the **enthalpy change for the reaction equation as written**. In this context the symbol Δ (delta) signifies change in” while *H* is the symbol for the quantity being changed, namely the enthalpy. We will deal with the enthalpy in some detail in Enthalpy For the moment we can think of it as a property of matter which increases when matter absorbs energy and decreases when matter releases energy.

It is important to realize that the value of ΔH_m given in thermochemical equations like (1) or (3) depends on the physical state of both the reactants and the products. Thus, if water were obtained as a gas instead of a liquid in the reaction in Eq. (1), the value of ΔH_m would be different from -2808 kJ. It is also necessary to specify both the temperature and pressure since the value of ΔH_m depends very slightly on these variables. If these are not specified [as in Eq. (3)] they usually refer to 25°C and to normal atmospheric pressure.

Two more characteristics of thermochemical equations arise from the law of conservation of energy. The first is that *writing an equation in the reverse direction changes the sign of the enthalpy change.* For example,

$H_2O(l)$ → $H_2O(g)$ ΔH_m = 44 kJ (4*a*) tells us that when a mole of liquid water vaporizes, 44 kJ of heat is absorbed. This corresponds to the fact that heat is absorbed from your skin when perspiration evaporates, and you cool off. Condensation of 1 mol of water vapor, on the other hand, gives off exactly the same quantity of heat. $H_2O(g)$ → $H_2O(l)$ ΔH_m = –44 kJ (4*b*) To see why this must be true, suppose that ΔH_m [Eq. (4a)] = 44 kJ while ΔH_m [Eq. (4b)] = –50.0 kJ. If we took 1 mol of liquid water and allowed it to evaporate, 44 kJ would be absorbed. We could then condense the water vapor, and 50.0 kJ would be given off. We could again have 1 mol of liquid water at 25°C but we would also have 6 kJ of heat which had been created from nowhere! This would violate the law of conservation of energy. The only way the problem can he avoided is for ΔH_m of the reverse reaction to be equal in magnitude but opposite in sign from ΔH_m of the forward reaction. That is, ΔH_m forward = –ΔH_m reverse

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Contributors

- Template:ContribChemPRIME

8.3.2: Environment- Gas

You may have noticed "E85" gasoline selling for \$2.39/gallon for FlexFuel vehicles while regular gas was selling for \$2.79. Is E85 use advantageous if you have a FlexFuel vehicle capable of using either E85 (85% ethanol) or regular gas? Note that many states sell regular gas that is E10 (10% ethanol) for standard (non-FlexFuel) vehicles.

Ethanol for fuel is not a new idea

 85% Ethanol at a modern pump	 The model T was first designed to run on ethanol[1]

There are really (at least) two questions here:

1. Is it a good deal for the consumer (is energy from ethanol economical)?
2. Is it a good deal for the environment (is the ***Energy Balance*** favorable)?

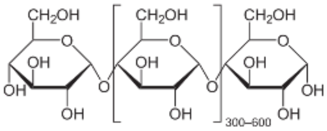
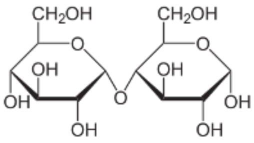
The Energy Balance

The energy balance is the ratio of the energy produced by 1 kg of the fuel (i.e. ethanol), to the energy necessary to produce it (cultivation of plants, fermentation, transportation, irrigation, etc.). The energy balance for ethanol in the US is only 1.3 to 1.6 (we get 1.3-1.6 J out for 1 J energy input), while in Brazil it's 8.3 to 10.2. [2][3]

The energy balance for oil is about 5: Today, about 5 barrels of oil extracted for every 1 barrel of oil is consumed in the process (a century ago, when oil was more plentiful the ratio was 50:1).[4]

The reason for the poor energy balance for fuel ethanol in the US is that we use corn. We extract the cornstarch, and then hydrolyze it at high temperatures (over 90 ^{o}C and ferment the resulting sugars to give the ethanol (simultaneous saccharification and fermentation, SSF). More energy efficient processes are being developed. [5] Starch is a polymer of glucose composed of amylose (shown below) and amylopectin, a similar polymer with more branching. The chemical reactions that occur are first, hydrolysis of starches to maltose ($C_{12}H_{22}O_{11}$):

$$[C_6H_{10}O_5]_n + n/2\ H_2O \rightarrow n/2\ C_{12}H_{22}O_{11} \quad (1)$$

 Amylose	 Maltose

The complex sugar maltose is further hydrolyzed (sometimes in the same step) to glucose ($C_6H_{12}O_6$):

$$C_{12}H_{22}O_{11} + H_2O \rightarrow 2\ C_6H_{12}O_6 \quad (2)$$

Finally, during ethanol fermentation, glucose is decomposed into ethanol (C_2H_5OH) and carbon dioxide.

$$C_6H_{12}O_6 \rightarrow 2\ C_2H_5OH + 2\ CO_2 + \text{heat} \quad (3)$$

Switchgrass requires similar processing, but has the potential for a higher energy balance. In Brazil, sugarcane is used in ethanol production, eliminating the high energy cost reaction (1), because sugarcane produces sucrose, a sugar similar to maltose. In all cases, a large energy expenditure is necessary to separate the ethanol from the water in which the reactions take place, and other impurities.

Sources of Ethanol

	 ↑ Sugarcane ← Switchgrass	 Corn

But even in Brazil, rising sugar costs led to a rebound in reliance on petroleum, and E100 capable vehicles declined sharply in the late 1980s[6]. Nonetheless, the existence of ethanol pumps at a large percentage of gas stations has helped sustain the use of other blends. Clearly, the economics and history of fuel use should be studied carefully when investing in future fuel development.

The Cost Advantage

So which is a better deal: filling your 10-gallon tank with E85 costing \$2.69/gallon or with gasoline costing \$2.99/gallon?[7].

Energy changes which accompany chemical reactions are almost always expressed by **thermochemical equations**. For example, during combustion ethanol reacts with oxygen to produce carbon dioxide, water vapor, and heat according to the thermochemical equation:

$$C_2H_5OH + 3\ O_2 \rightarrow 2\ CO_2 + 3\ H_2O\ (l)\ (4)\ \Delta H_m = -1367\ \text{kJ}$$ [8]

The quantity ΔH_m is the **enthalpy change for the reaction equation as written**. In this context the symbol Δ (delta) signifies change in" while *H* is the symbol for the quantity being changed, namely the enthalpy. We will deal with the enthalpy in some detail in Chap. 15. For the moment we can think of it as a property of matter which increases when matter absorbs energy and decreases when matter releases energy. Here the ΔH_m (delta *H* subscript m) tells us whether heat energy is released or absorbed when the reaction occurs and also enables us to find the actual quantity of energy involved. By convention, if ΔH_m is *positive*, heat is *absorbed* by the reaction; i.e., it is **endothermic**. More commonly, ΔH_m is *negative* as in Eq. (3), indicating that heat energy is *released* rather than absorbed by the reaction, and that the reaction is **exothermic**. This convention as to whether ΔH_m is positive or negative looks at the heat change in terms of the matter actually involved in the reaction rather than its surroundings. In the reaction in Eq. (4), stronger bonds have formed, leading to a decrease in potential energy, and it is this decrease which is indicated by a negative value of ΔH_m.

It is important to notice that the quantity of heat released or absorbed by a reaction is proportional to the amount of each substance consumed or produced by the reaction. Thus Eq. (4) tells us that 1367 kJ of heat energy is given off *for every mole* of C_2H_5OH which is consumed. Alternatively, it tells us that 1367 kJ is released *for every 3 moles of* H_2O produced, or every 2 mol of carbon dioxide produced, or every 3 mol of oxygen consumed. Seen in this way, ΔH_m is a conversion factor enabling us to calculate the heat absorbed when a given amount of substance is consumed or produced. If *q* is the quantity of heat absorbed and *n* is the amount of substance involved, then

$\Delta H_\mathrm{m} = \frac{q}{n}$ (5) Equation (4) represents the standard heat of combustion, which is related to the "higher heating value" of a fuel.[9] Because liquid water is produced in Equation (4) and the heat (1367 kJ/mol) is given for a theoretical reaction occurring at 25 °C, it is only an approximation of the heat produced in an actual combustion, where the reaction takes place at a high temperature and produces water vapor. A better estimate may be the Lower Heating Value (LHV) which is adjusted by adding the heat of vaporization of water, and heat required to raise the temperature of reactants to the combustion temperature and products to 150 °C (an arbitrarily chosen standard).[10][11] We'll use the LHV for ethanol, -1330 kJ/mol,[12] and abbreviate ethanol (C_2H_5OH) as EtOH to calculate the heat in 1 gallon of ethanol:

$$V_{\mathrm{EtOH}} \xrightarrow{M} m_{\mathrm{EtOH}} \xrightarrow{M} n_{\mathrm{EtOH}} \xrightarrow{\Delta H_m} q$$

so that

$$q = 1 \text{ gallon EtOH} \times \frac{3.79 \text{ L}}{1 \text{ gallon}} \times \frac{1000 \text{ mL}}{1 \text{ L}} \times \frac{0.789 \text{ g}}{\text{mL}} \times \frac{1 \text{ mol EtOH}}{46.07 \text{ g EtOH}} \times \frac{-1330 \text{ kJ}}{\text{mol EtOH}}$$

$$= -86\ 330 \text{ kJ} = -86.33 \text{ MJ (estimate)}$$

We can calculate the energy value for octane, which represents gasoline fairly well, in a similar way. The thermochemical equation for the combustion of octane (C_8H_{18}) is:

C_8H_{18} + 25/2 O_2 → 8 CO_2 + 9 H_2O (l) (6) ΔH ~ -5430 kJ/mol[13]

Again, we'll use the LHV for octane (-5064 kJ/mol[14]) in our calculation:

$$q = 1 \text{ gallon octane} \times \frac{3.79 \text{ L}}{1 \text{ gallon}} \times \frac{1000 \text{ mL}}{1 \text{ L}} \times \frac{0.737 \text{ g}}{\text{mL}} \times \frac{1 \text{ mol octane}}{114.23 \text{ g octane}} \times \frac{-5\ 064 \text{ kJ}}{\text{mol octane}}$$

$$= -123\ 800 \text{ kJ} = -123.80 \text{ MJ (estimate)}$$

So 1 gallon of gasoline has (123,800 / 86,330) or 1.4 times as much heating value, but costs only \$2.79 / \$2.39 or 1.2 times as much. It's the better buy. If the Energy Balance may only be about 1.3 in the US, it actually is a net loss to burn ethanol, *at least at the current price and availability of petroleum*. As the Energy Balance and availability of petroleum decreases, we had better develop more energy efficient means of ethanol production.

Indeed, based on EPA tests for all 2006 E85 models, the average fuel economy for E85 vehicles was 25.56% lower than unleaded gasoline.[15][16]

The Lower Heat Value (LHV)

It is important to realize that the value of ΔH_m given in thermochemical equations like (4) or (6) depends on the physical state of both the reactants and the products. Thus, if water were obtained as a gas instead of a liquid in the reaction in Eq. (4), the value of ΔH_m would be different from -1367 kJ. It is also necessary to specify both the temperature and pressure since the value of ΔH_m depends very slightly on these variables. If these are not specified [as in Eq. (3)] they usually refer to 25°C and to normal atmospheric pressure.

Two more characteristics of thermochemical equations arise from the law of conservation of energy. The first is that *writing an equation in the reverse direction changes the sign of the enthalpy change*. For example,

$H_2O(l)$ → $H_2O(g)$ ΔH_m = 44 kJ (7*a*) tells us that when a mole of liquid water vaporizes, 44 kJ of heat is absorbed. This corresponds to the fact that heat is absorbed from your skin when perspiration evaporates, and you cool off. Condensation of 1 mol of water vapor, on the other hand, gives off exactly the same quantity of heat. $H_2O(g)$ → $H_2O(l)$ ΔH_m = –44 kJ (7*b*) To see why this must be true, suppose that ΔH_m [Eq. (7a)] = 44 kJ while ΔH_m [Eq. (7b)] = –50.0 kJ. If we took 1 mol of liquid water and allowed it to evaporate, 44 kJ would be absorbed. We could then condense the water vapor, and 50.0 kJ would be given off. We could again have 1 mol of liquid water at 25°C but we would also have 6 kJ of heat which had been created from nowhere! This would violate the law of conservation of energy. The only way the problem can he avoided is for ΔH_m of the reverse reaction to be equal in magnitude but opposite in sign from ΔH_m of the forward reaction. That is,

$$\Delta H_m \text{ forward} = -\Delta H_m \text{ reverse}$$

Since Reaction (4) produces 3 mol of $H_2O(l)$, it would produce 3 mol x 44 kJ/mol = 132 kJ less energy if the water were produced as the vapor, because the heat released in condensation to the liquid would not be included. The enthalpy change would then be -1367 + 132 kJ or -1235 kJ. This is still different from the LHV because of the heat required to change the temperature of the reactants and products from the standard temperature (25 ^{o}C) to the combustion temperature.

FlexFuel Facts

In 2007, only 3.3% of American cars were FlexFuel and only 1% of the filling stations provided FlexFuel, but the percentage has been increasing rapidly. World ethanol production for transport fuel tripled between 2000 and 2007 from 17 billion to more than 52 billion litres, with 89% produced in Brazil and the US.[17] The price differential shows large changes (it was about 30% in 2007).[18] The National Alcohol Program in Brazil, a world leader in using ethanol fuel, mandated decreased reliance on petroleum after the first oil crisis in 1973, and by 1979 several automakers provided cars that ran on pure ethanol (E100). After reaching more than

4 million cars and light trucks running on pure ethanol by the late 1980s,[3] the use of E100-only vehicles sharply declined after increases in sugar prices produced shortages of ethanol fuel. The emphasis has been on E85 (85% ethanol) vehicles in the US, because engines won't start reliably on E100 below about 60 ^{o}F. Even E85 cannot be used below about 30 ^{o}F, and in northern states E70 is delivered by E85 pumps without changing the label.[19]

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8.3.3: Foods- Energy from Fats and Sugars

Energy from Fats & Sugars

Earlier we discussed the nature of fats and mentioned that fats typically provide 9 Cal/g of food energy, while sugars provide about 4 Cal/g. So in order to store the energy in 10 lb of fat, your body would need to store 22.5 lb of carbohydrates or sugars; but it's more extreme than that. Because sugars carry about their own weight of associated water in the body, 67.5 lb (31 kg) of hydrated glycogen has the energy equivalent of 10 lb (5 kg) of fat![1]

The food energy in various food types is given approximately in the following table [2], and you can find the fat content (as well as all other nutritional information) about nearly all foods in the United States Department of Agriculture's Bulletin #8 which has a searchable USDA Nutrient database. In the database, fats are list under "lipids"/"Fatty Acids" and then under "saturated" and "18:0", indicating the number of carbon atoms (18) in the fatty acid, and the number of double bonds (0) (see Example 1).

Food component	Energy Density	
	kJ/g	kcal/g
Fat	37	9
Ethanol (drinking alcohol)	29	7
Proteins	17	4
Carbohydrates	17	4
Sorbitol, sugar alcohol sweeteners)	10	2.4
dietary Fiber	8	2

These caloric values are measured in a "bomb calorimeter" like the one in the Figure.

A bomb Calorimeter

A 1-2 g sample of food is sealed in a heavy walled steel cylinder (about 4" in diameter and 7" high), shown in the center of the Figure, which is then filled with pure oxygen at 30-40 atmospheres pressure, and immersed in a few liters of water. The sample is ignited electrically, and the heat released is determined by measuring the temperature increase of water that surrounds the "bomb".

Let's investigate the basis for these caloric values in terms of the chemical reactions that provide the energy. Energy changes which accompany chemical reactions are almost always expressed by **thermochemical equations**. The combustion of stearic acid, which is the main component of saturated fats, is written:

$$C_{18}H_{36}O_2\ (s) + 24\ O_2(g) \rightarrow 18\ CO_2(g) + 18\ H_2O(l)\ (25°C,\ 1\ \text{atm pressure})$$

$$\Delta H_m = -11\ 407\ \text{kJ}^{[3]}\ (1)$$

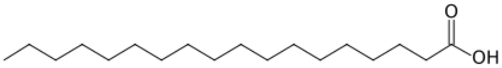
Here the ΔH_m (delta *H* subscript m) tells us whether heat energy is released or absorbed when the reaction occurs and also enables us to find the actual quantity of energy involved. By convention, if ΔH_m is *positive*, heat is *absorbed* by the reaction; i.e., it is

endothermic. More commonly, Δ*H*$_m$ is *negative* as in Eq. (1), indicating that heat energy is *released* rather than absorbed by the reaction, and that the reaction is **exothermic**. This convention as to whether Δ*H*$_m$ is positive or negative looks at the heat change in terms of the matter actually involved in the reaction rather than its surroundings. In the reaction in Eq. (1), the C, H, and O atoms have collectively lost energy and it is this loss which is indicated by a negative value of Δ*H*$_m$.

It is important to notice that Δ*H*$_m$ is the enthalpy for the *reaction as written.* The quantity of heat released or absorbed by a reaction is proportional to the amount of each substance consumed or produced by the reaction. Thus Eq. (1) tells us that 890.4 kJ of heat energy is given off *for every mole of* $C_{18}H_{36}O_2$ which is consumed. Alternatively, it tells us that 11407 kJ is released *for every 18 mole of* H_2O produced, or for every 18 mol of carbon dioxide produced, or 24 mol of oxygen consumed. Seen in this way, Δ*H*$_m$ is a conversion factor enabling us to calculate the heat absorbed when a given amount of substance is consumed or produced. If *q* is the quantity of heat absorbed and *n* is the amount of substance involved, then

$$\Delta H_\mathrm{m} = \frac{q}{n} \quad (2)$$

EXAMPLE 1



Stearic Acid, 18:0

In the molecular model, each bend in the structure is occupied by a carbon atom, and each carbon atom has 4 bonds; missing bonds are to hydrogen atoms, which are not shown.

a. How much heat energy is obtained when 1 g of $C_{18}H_{36}O_2$, is burned in oxygen according to the equation above? The molar mass of steric acid is 284.48 g/mol.

b. What is the caloric value of 1 g of stearic acid, given that Δ*H*$_m$ = –11407 kJ for equation (1)?

Solution

a. The mass of $C_{18}H_{36}O_2$ is easily converted to the amount of $C_{18}H_{36}O_2$ from which the heat energy *q* is easily calculated by means of Eq. (2). The value of Δ*H*$_m$ is –11407 kJ per mole of $C_{18}H_{36}O_2$. The road map is

$$m_{\mathrm{C_{18}H_{36}O_2}} \xrightarrow{M} n_{\mathrm{C_{18}H_{36}O_2}} \xrightarrow{\Delta H_m} q \quad \text{so that } q = 1.0\text{ g }\mathrm{C_{18}H_{36}O_2} \times \frac{1\text{ mol }\mathrm{C_{18}H_{36}O_2}}{284.48\text{ g }\mathrm{C_{18}H_{36}O_2}} \times \frac{-11407\text{ kJ}}{1\text{ mol }\mathrm{C_{182}H_{36}O_2}} = \text{-40.09 kJ}$$

b. $\text{-40.09 kJ} \times \frac{1\text{ kcal}}{4.184\text{ kJ}} \times \frac{1\text{ Cal}}{1\text{ kcal}} = -9.84\text{ Cal}$

Note: By convention a negative value of *q* corresponds to a release of heat energy by the matter involved in the reaction.

This is close to the estimated 9 Cal/g for fats. We saw earlier that most fats are triglycerides, that is, they would have 3 fatty acid substituents (like stearic acid) attached to a glycerol "backbone" in a fat like glyceryl tristearate ("stearin") ($C_{57}H_{110}O_6$, M = 891.48). Stearin has a heat of combustion of -35 663 kJ/mol, so 1 g produces (35 663 kJ/mol) / (891.48 g/mol) x (1 Cal / 4.184 kJ) = 9.57 Cal. When energy is required by our body, triglycerides are converted free fatty acids, and transported by serum albumin in the blood to cells where energy is required. Serum albumin is necessary because the solubility of fatty acids is low in water-based blood.[4]

In comparison, sucrose ($C_{12}H_{22}O_{11}$) has a molar mass of 342.3 g/mol and a heat of combustion of -5645 kJ/mol, so it produces 16.49 kJ/g or 3.94 Cal/g, very close to the estimated value, by the combustion:

$$C_{12}H_{22}O_{11}\,(s) + 12\ O_2(g) \rightarrow 12\ CO_2(g) + 11\ H_2O(l)\ (25°\text{C, 1 atm pressure})$$

$$\Delta H_m = –5\,645\text{ kJ mol}^{-1}$$[5]

The quantity Δ*H*$_m$ is the **enthalpy change for the reaction as written**. In this context the symbol Δ (delta) signifies change in" while *H* is the symbol for the quantity being changed, namely the enthalpy. We will deal with the enthalpy in some detail in Chap. 15. For the moment we can think of it as a property of matter which increases when matter absorbs energy and decreases when matter releases energy.

It is important to realize that the value of Δ*H*$_m$ given in thermochemical equations like (1) depends on the physical state of both the reactants and the products. Thus, if water were obtained as a liquid instead of a gas in the reaction in Eq. (1), the value of Δ*H*$_m$ would be different from -890.4 kJ. It is also necessary to specify both the temperature and pressure since the value of Δ*H*$_m$ depends very slightly on these variables. If these are not specified [as in Eq. (3)] they usually refer to 25°C and to normal atmospheric pressure.

Two more characteristics of thermochemical equations arise from the law of conservation of energy. The first is that *writing an equation in the reverse direction changes the sign of the enthalpy change.* For example,

$H_2O(l) \rightarrow H_2O(g)$ $\Delta H_m = 44$ kJ (3*a*) tells us that when a mole of liquid water vaporizes, 44 kJ of heat is absorbed. This corresponds to the fact that heat is absorbed from your skin when perspiration evaporates, and you cool off. Condensation of 1 mol of water vapor, on the other hand, gives off exactly the same quantity of heat. $H_2O(g) \rightarrow H_2O(l)$ $\Delta H_m = –44$ kJ (3*b*) To see why this must be true, suppose that ΔH_m [Eq. (4a)] = 44 kJ while ΔH_m [Eq. (4b)] = –50.0 kJ. If we took 1 mol of liquid water and allowed it to evaporate, 44 kJ would be absorbed. We could then condense the water vapor, and 50.0 kJ would be given off. We could again have 1 mol of liquid water at 25°C but we would also have 6 kJ of heat which had been created from nowhere! This would violate the law of conservation of energy. The only way the problem can he avoided is for ΔH_m of the reverse reaction to be equal in magnitude but opposite in sign from ΔH_m of the forward reaction. That is, ΔH_m forward = $–\Delta H_m$ reverse

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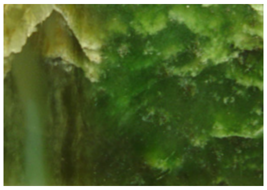
8.3.4: Geology- Heat Engine at Lost City

The Earth's mantle is composed largely of *ultramafic rock* which has high Fe or Mg content, and low silicon content. This rock may be converted to *"serpentinites"* by a process (logically) called *serpentinization"*, which is an ***exothermic*** process, releasing a lot of heat energy-- about 660,000,000 joules of heat per cubic meter of rock according to the NOAA (National Oceanic and Atmospheric Administration). This is enough energy to raise the temperature of the rock by 260°C (550°F)[1]. The "Lost City", located 20 km west of the Mid-Atlantic Ridge, is a hydrothermal vent field characterized by carbonate edifices that tower 60 m above the ocean floor and extreme conditions found nowhere else in the marine environment[2]

Serpentinites at Lost City. Photo taken with the robotic vehicle Hercules. The wreckfish is ~ 1 m in length[3]

Serpentine itself is often chrysotile, $Mg_3Si_2O_5(OH)_4$. The name "serpentine" derives from the Latin *serpentinus* ("serpent rock") because the mineral is often greenish with a smooth to scaly surface[4].

 Serpentine	 Polished Serpentine Sample

Serpentine marbles are used in architecture and jewelry, but other serpentines include asbestos (which is a lung cancer risk when mined) and another 20 varieties of hydrous magnesium/iron phyllosilicates. Serpentinites often are toxic to plants because they may contain significant levels of nickel, chromium, and cobalt. They are often mixed, and thus treated collectively as a group called "serpentinites."

In the case of the "Lost City" of hydrothermal vents in the mid-Atlantic, serpentinization is a greater source of energy than the radioactivity of the Earth's core . Radioactivity normally accounts for about 80% of the internal heat of the Earth [5]. But ***Thermochemical reactions*** like serpentinization, which produce or consume significant amounts of heat, are an inextricable part of all geological processes.

Thermochemical Equations

Since serpentinites are mixtures, several equations can be used to describe their reactions. While natural minerals may have indefinite compositions like olivine, $(Fe,Mg)_2SiO_4$, we always look at equations for specific reactions in order to associate definite energies with them. These are called **thermochemical equations**. For example, serpentinization may involve:

Fayalite + water → magnetite + aqueous silica + hydrogen

$$3\ Fe_2SiO_4 + 2\ H_2O \rightarrow 2\ Fe_3O_4 + 3\ SiO_2 + 3\ H_2 \quad \Delta H_m = 40.1\text{ kJ}$$ [6] (25°C, 1 atm pressure) (1)

Or: Forsterite + aqueous silica → serpentine (crysotile)

$$3\ Mg_2SiO_4 + SiO_2 + 4\ H_2O \rightarrow 2\ Mg_3Si_2O_5(OH)_4 \quad \Delta H_m = -179.7\text{ kJ}$$ [7] (25°C, 1 atm pressure) (2)

Or: Forsterite + water → serpentine (chrysotile) + brucite

$3\ Mg_2SiO_4 + 3\ H_2O \rightarrow 2\ Mg_3Si_2O_5(OH)_4 + Mg(OH)_2$ ΔH_m = -2267.2 kJ[8] (25°C, 1 atm pressure) (3)

Here the ΔH_m (delta *H* subscript m) tells us whether heat energy is released or absorbed when the reaction occurs and also enables us to find the actual quantity of energy involved. By convention, if ΔH_m is *positive*, as in Equation (1), heat is *absorbed* by the reaction; i.e., it is **endothermic**. More commonly, ΔH_m is *negative* as in Eq. (2), indicating that heat energy is *released* rather than absorbed by the reaction, and that the reaction is **exothermic**. This convention as to whether ΔH_m is positive or negative looks at the heat change in terms of the matter actually involved in the reaction rather than its surroundings. In the reaction in Eq. (2), the strength of the bonds increases as products are formed, so the products are lower in potential energy, and the lost energy is indicated by a negative value of ΔH_m.

The values are calculated, as we'll see later, from standard tabulated values found in databases developed especially for geologists[9] [10] [11] [12]

It is important to notice that ΔH_m is the energy for the reaction as written. The quantity of heat released or absorbed by a reaction is proportional to the amount of each substance consumed or produced by the reaction. Thus Eq. (2) tells us that 179.7 kJ of heat energy is given off *for every mole* of SiO_2 which is consumed, or for every 3 mol of Mg_2SiO_4 consumed. Alternatively, it tells us that 179.7 kJ is released *for every 2 moles of* $Mg_3Si_2O_5(OH)_4$ produced. Seen in this way, ΔH_m is a conversion factor enabling us to calculate the heat absorbed when a given amount of substance is consumed or produced. If *q* is the quantity of heat absorbed and *n* is the amount of substance involved, then:

$$\Delta H_{\mathrm{m}} = \frac{q}{n} \quad (4)$$

EXAMPLE 1

How much heat energy is obtained when 1 kg of the serpentine chrysotile, $Mg_3Si_2O_5(OH)_4$, is formed according to Equation (2)?

Solution

The mass of $Mg_3Si_2O_5(OH)_4$ is easily converted to the amount of $Mg_3Si_2O_5(OH)_4$ from which the heat energy *q* is easily calculated by means of Eq. (4). The value of ΔH_m is –179.7 kJ per 2 moles of $Mg_3Si_2O_5(OH)_4$. The road map is:

$$m_{\mathrm{Mg_2Si_2O_5OH_4}} \xrightarrow{M} n_{\mathrm{Mg_2Si_2O_5OH_4}} \xrightarrow{\Delta H_m} q \quad \text{so that} \quad q = 1 \times 10^3\ \mathrm{g\ Mg_2Si_2O_5(OH)_4} \times \frac{1\ \mathrm{mol\ Mg_2Si_2O_5(OH)_4}}{277.112\ \mathrm{g\ Si_2O_5(OH)_4}} \times \frac{-179.7 kJ}{2\ \mathrm{mol\ Mg_2Si_2O_5(OH)_4}} = q = -324\ \mathrm{kJ}$$

Note: By convention a negative value of *q* corresponds to a release of heat energy by the matter involved in the reaction. The quantity ΔH_m is the **enthalpy change as the reaction proceeds as written**. In this context the symbol Δ (delta) signifies change in" while *H* is the symbol for the quantity being changed, namely the enthalpy. We will deal with the enthalpy in some detail in Chap. 15. For the moment we can think of it as a property of matter which increases when matter absorbs energy and decreases when matter releases energy.

It is important to realize that the value of ΔH_m given in thermochemical equations like (1), (2) or (3) depends on the physical state of both the reactants and the products. Thus, if water were present as a gas instead of a liquid in the reaction in Eq. (1), the value of ΔH_m would be different from 40.1 kJ. These reactions may occur under conditions where water may be supercritical (above 374°C) and yet a different value would be obtained. It is also necessary to specify both the temperature and pressure since the value of ΔH_m depends very slightly on these variables. If these are not specified [as in Eq. (3)] they usually refer to 25°C and to normal atmospheric pressure. Since geochemical processes like those above normally occur at hundreds of atmospheres (hundreds of bars) pressure and elevated temperatures, geologists adjust the standard enthalpies to give values appropriate for the conditions. Although the adjustments are not difficult, computer programs exist to do the work.

Forward and Reverse Thermochemical Equations

Another characteristic of thermochemical equations arise from the law of conservation of energy. The first is that *writing an equation in the reverse direction changes the sign of the enthalpy change.*

Calcite/Aragonite

For example[13], the conversion of the two forms of calcium carbonate

$CaCO_3 \Leftrightarrow CaCO_3$ ΔH_m = -0.17 kJ (5)

calcite aragonite

Therefore, the forward direction (calcite to aragonite) is exothermic, releasing heat as a more stable crystal lattice forms. Logically, in the reverse direction, disrupting the stable lattice of aragonite must require energy, so the conversion of aragonite to calcite endothermic:

$CaCO_3 \Leftrightarrow CaCO_3$ ΔH_m = +0.17 kJ (6)

aragonite calcite

Water/Ice

Melting or freezing water can release or absorb significant amounts of heat:

$$H_2O(l) \rightarrow H_2O(g) \quad \Delta H_m = 44 \text{ kJ} \quad (7)$$

tells us that when a mole of liquid water vaporizes, 44 kJ of heat is absorbed. This corresponds to the fact that heat is absorbed from your skin when perspiration evaporates, and you cool off. Condensation of 1 mol of water vapor, on the other hand, gives off exactly the same quantity of heat.

$$H_2O(g) \rightarrow H_2O(l) \quad \Delta H_m = –44 \text{ kJ} \quad (8)$$

To see why this must be true, suppose that ΔH_m [Eq. (7)] = 44 kJ mol while ΔH_m [Eq. (8)] = –50.0 kJ. If we took 1 mol of liquid water and allowed it to evaporate, 44 kJ would be absorbed. We could then condense the water vapor, and 50.0 kJ would be given off. We could again have 1 mol of liquid water at 25°C but we would also have 6 kJ of heat which had been created from nowhere! This would violate the law of conservation of energy. The only way the problem can he avoided is for ΔH_m of the reverse reaction to be equal in magnitude but opposite in sign from ΔH_m of the forward reaction. That is, ΔH_m forward = $-\Delta H_m$ reverse

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SECTION OVERVIEW

8.4: Work and Energy

8.4.1: Potential Energy- Gravity and Springs

We've seen how energy can exist because of the movement of an object. As it turns out, energy can also exist within an object because it has the potential to have some sort of movement. There are multiple examples of this, but for now we will look at two: gravity and springs. Later in this chapter we will revisit the idea of energy and other ways in which we might categorize it.

8.4.1.1: Spring Potential Energy

Learning Objectives

- Explain the work done in deforming a spring.
- Describe the potential energy stored in a deformed spring.

Hooke's Law, $F=-kx$, describes force exerted by a spring being deformed. Here, F is the restoring force, x is the displacement from equilibrium or **deformation**, and k is a constant related to the difficulty in deforming the system. The minus sign indicates the restoring force is in the direction opposite to the displacement.

In order to produce a deformation, work must be done. That is, a force must be exerted through a distance, whether you pluck a guitar string or compress a car spring. If the only result is deformation, and no work goes into thermal, sound, or kinetic energy, then all the work is initially stored in the deformed object as some form of potential energy. The potential energy stored in a spring is $\mathrm{PE_{el}}=\frac{1}{2}kx^2$. Here, we generalize the idea to elastic potential energy for a deformation of any system that can be described by Hooke’s law. Hence,

$$\mathrm{PE_{el}}=\frac{1}{2}kx^2,$$

where $\mathrm{PE_{el}}$ is the **elastic potential energy** stored in any deformed system that obeys Hooke’s law and has a displacement x from equilibrium and a force constant k.

Connecting Ideas

Hooke's Law is the same equation that was used to develop the motion of waves. If we think about the oscillating motion of a compressed spring after the depressing force is released, this illustrates the reason for this connection.

It is possible to find the work done in deforming a system in order to find the energy stored. This work is performed by an applied force F_{app}. The applied force is exactly opposite to the restoring force (action-reaction), and so $F_{\mathrm{app}}=kx$. Figure 8.4.1.1.1 shows a graph of the applied force versus deformation x for a system that can be described by Hooke’s law. Work done on the system is force multiplied by distance, which equals the area under the curve or $(1/2)kx^2$ (Method A in the figure). Another way to determine the work is to note that the force increases linearly from 0 to kx, so that the average force is $(1/2)kx$, the distance moved is x, and thus $W=F_{\mathrm{app}}d=[(1/2)kx](x)=(1/2)kx^2$ (Method B in the figure).

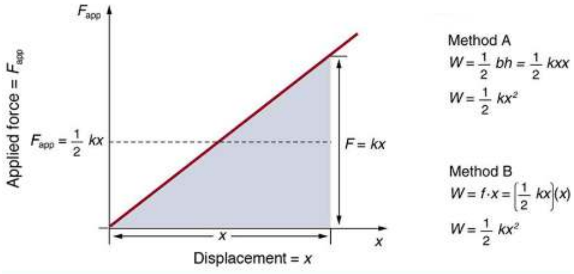

Figure 8.4.1.1.1: A graph of applied force versus distance for the deformation of a system that can be described by Hooke’s law is displayed. The work done on the system equals the area under the graph or the area of the triangle, which is half its base multiplied by its height, or $W=(1/2)kx^2$.

8.4.1.1.1 Example : Calculating Stored Energy: A Tranquilizer Gun Spring

We can use a toy gun’s spring mechanism to ask and answer two simple questions: (a) How much energy is stored in the spring of a tranquilizer gun that has a force constant of 50.0 N/m and is compressed 0.150 m? (b) If you neglect friction and the mass of the spring, at what speed will a 2.00-g projectile be ejected from the gun?

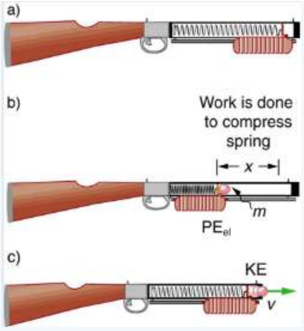

Figure 8.4.1.1.2 (a) In this image of the gun, the spring is uncompressed before being cocked. (b) The spring has been compressed a distance x, and the projectile is in place. (c) When released, the spring converts elastic potential energy $\mathrm{PE_{el}}$ into kinetic energy.

Strategy for a

(a): The energy stored in the spring can be found directly from elastic potential energy equation, because k and x are given.

Solution for a

Entering the given values for k and x yields

$$\mathrm{PE_{el}} = \frac{1}{2}kx^2 = \frac{1}{2}(50.0\ \mathrm{N/m})(0.150\ \mathrm{m})^2 = 0.563\ \mathrm{N \cdot m} = 0.563\ \mathrm{J}$$

Strategy for b

Because there is no friction, the potential energy is converted entirely into kinetic energy. The expression for kinetic energy can be solved for the projectile’s speed.

Solution for b

1. Identify known quantities:

$$\mathrm{KE_f} = \mathrm{PE_{el}} \text{ or } 1/2mv^2 = (1/2)kx^2 = \mathrm{PE_{el}} = 0.563\ \mathrm{J}$$

2. Solve for v:

$$v = \left[\frac{2\mathrm{PE_{el}}}{m}\right]^{1/2} = \left[\frac{2(0.563\ \mathrm{J})}{0.002\ \mathrm{kg}}\right]^{1/2} = 23.7(\mathrm{J/kg})^{1/2}$$

3. Convert units: 23.7m/s

Discussion

(a) and (b): This projectile speed is impressive for a tranquilizer gun (more than 80 km/h). The numbers in this problem seem reasonable. The force needed to compress the spring is small enough for an adult to manage, and the energy imparted to the dart is small enough to limit the damage it might do. Yet, the speed of the dart is great enough for it to travel an acceptable distance.

8.4.1.1.1 Exercise

Envision holding the end of a ruler with one hand and deforming it with the other. When you let go, you can see the oscillations of the ruler. In what way could you modify this simple experiment to increase the rigidity of the system?

Answer

You could hold the ruler at its midpoint so that the part of the ruler that oscillates is half as long as in the original experiment.

8.4.1.1.2 Exercise

If you apply a deforming force on an object and let it come to equilibrium, what happened to the work you did on the system?

Answer

It was stored in the object as potential energy.

Section Summary

- Hooke's law describes force exerted by a spring being deformed,

  $$F=-kx,$$

  where F is the restoring force, x is the displacement from equilibrium or deformation, and k is the force constant of the system.
- Elastic potential energy $\mathrm{PE}_{\mathrm{el}}$ stored in the deformation of a system that can be described by Hooke's law is given by

  $$\mathrm{PE}_{\mathrm{el}}=(1/2)kx^2.$$

Glossary

deformation

displacement from equilibrium

elastic potential energy

potential energy stored as a result of deformation of an elastic object, such as the stretching of a spring

---

8.4.2: Forms of Energy

Learning Objectives

- Describe some of the many forms of energy.
- Categorize forms of energy as kinetic or potential.

Earlier in this chapter we introduced the concept of kinetic and potential energy as ways of understanding particular types of mechanical energy. We also introduced the law of conservation of energy and stated that it applied to more than just the mechanical energy that we had considered so far. We will now explore some of these other types of energy and how they fit within the context of kinetic and potential energy.

Some of the Many Forms of Energy

Here are some of the many forms of energy. You probably have heard of some of these before; many of these will be covered in later chapters, but let us detail a few here. **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry **chemical energy** that can be transferred to a system through oxidation and other methods that can result in the kinetic energy of a moving car or the potential energy of climbing a mountain. Chemical fuel can also produce electrical energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as **radiant energy**, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. **Nuclear energy** comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power plants, and into the energy of the heat transfer and blast in nuclear bombs. Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called **thermal energy**, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

Transformation of Energy

As you have probably gathered from reading about these different forms of energy, the transformation of energy from one form into others is happening all the time. The chemical energy in food is converted into thermal energy through metabolism; light energy (a form of radiant energy) is converted into chemical energy through photosynthesis. In a larger example, the chemical energy contained in coal is converted into thermal energy as it burns to turn water into steam in a boiler. This thermal energy in the steam in turn is converted to mechanical energy as it spins a turbine, which is connected to a generator to produce electrical energy. That electrical energy moves through wires to homes and businesses where it is used to perform a variety of tasks which all require energy. (In all of these examples, not all of the initial energy is converted into the forms mentioned. This important point is discussed later in this text.)

Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell (see Figure $8.4.2.2$) produces electricity, which in turn can be used to run an electric motor. Energy is converted from the primary source of solar energy into electrical energy and then into mechanical energy.

Figure $8.4.2.2$: Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar-power aircraft. (credit: NASA)

Categorizing Energy

As you read through the different forms of energy, were there aspects of it that seemed similar to the ways we have previously discussed energy in this text? There were some forms of energy that seemed to fit the pattern of storage, while others seemed to fit the pattern of movement. These are also the patterns we saw when analyzing mechanical energy in more detail earlier in the chapter. We referred to energy as either being kinetic if it was moving or potential if it was stored. We can do the same for these other forms of energy we are now introducing. We will not be able to get into many of the details on these other forms of energy just yet, but we can hopefully see these trends.

Potential Energy: the energy of position and storage

In our introduction to potential energy we looked at springs and gravity as places where energy was stored. We can very clearly see the ways in which the energy stored in these situations might be converted into mechanical energy. Now we will examine some of the other forms of energy mentioned and how some of them might fit into the category of stored energy.

One way of understanding potential energy is to think of it as the position of an object somewhere within a field of force. We had earlier established that the force of gravity varies based on the distance between objects. As an object moves to different positions within a force field the energy stored will change. Likewise, we can think of the force fields associated with **electromagnetism.** As objects move to different locations within these fields they will consequently store different amounts of energy. It turns out that this property of electromagnetism is very important to our ability in the modern world to have energy available to us on demand by the literal flip of a switch. This is a concept we will explore in greater detail later in this text.

If you read through the portions of this section on the forms of energy and transformation of energy, you will see **chemical energy** described as "being transferred," "producing," or "converted into." This language suggests that this chemical energy is stored in a way similar to the ways the potential energy in springs is stored. We will discuss chemical bonding in greater detail later in this text, but for now we can understand chemical bonds as a location between atoms where energy is stored. When chemical bonds are broken the energy stored in those bonds is released in some form. New bonds might form that store energy in a different way and by a different amount than the ways in which that energy was stored previously. We might imagine an elaborate contraption of springs that is rearranged from an initial situation where some of those springs are storing energy to a final situation where different springs are storing energy. This is actually very similar to how some chemists understand the changes that occur during a chemical reaction. The difference is that we can't see those changes going on at the molecular level the way we can see changes in springs at the macrosopic level.

You may have also noticed a mention of converting mass itself into energy. How is that possible if both mass and energy are conserved? It turns out that mass itself is a form of potential energy. This is far from intuitive and was described by Einstein in one of the most famous equations in science: $E=mc^2$. We will exploring this equation in more detail when we consider the atomic nucleus later in this text.

Kinetic Energy: the energy of movement

Just as we read through our forms of energy looking for examples of position or storage as clues for types of potential energy, we can also read through our forms of energy for examples of movement for types of kinetic energy. Although we did not include an example of it in the list above, we have previously looked at waves in this text. Waves, of course, involve movement. So any energy transferred through a wave would be a form of kinetic energy and might be referred to as **wave energy**.

When we discussed energy coming to Earth from the Sun in the form of **radiant energy**, we can clearly see that movement is involved. Radiant energy does not have to come from the Sun, in fact radiant energy comes from everything. But it is always a form of energy that is only possible when there is movement. We will learn more about radiant energy in this text when we explore the concept of heat, and even more when we discuss electromagnetic radiation. We will also learn that **nuclear energy** is a very specific type of radiant energy.

We described **thermal energy** as a form of random motion. As such, it could be classified as a form of kinetic energy. Thermal energy is the random movement of the atoms that make up a substance. As we will see later in this text, there is a connection between thermal energy and what we measure as temperature. Thermal energy is also where much of the energy ends up that is "lost" to friction when we try to do work. It is often the missing piece when conservative forces are added together and it might appear that energy into and out of a process do not add up. But the energy is still there. In fact, that energy is still capable of doing some amount of work. We shall explore this idea in more detail later in this text.

When we discussed "electrical energy moving through wires" you might have recognized that it is also a form of kinetic energy. However, we had earlier discussed electromagnetic energy as being a form of potential energy. Is there a conflict here? Just as we can have mechanical energy exist as kinetic energy or potential energy, we can also have both kinetic and potential energy related to elecotromagnetism. The movement of an electrical charge through a wire demonstrates how **electrical energy** is a form of kinetic energy.

Section Summary

- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.
- Potential energy involves storage of energy, sometimes due to position.
- Kinetic energy involves movement, sometimes of matter.

Glossary

Kinetic energy
energy due to movement

potential energy
energy due to position, shape, or configuration

electrical energy

the energy carried by a flow of charge

chemical energy
the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction

radiant energy
the energy carried by electromagnetic waves

nuclear energy
energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

thermal energy
the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

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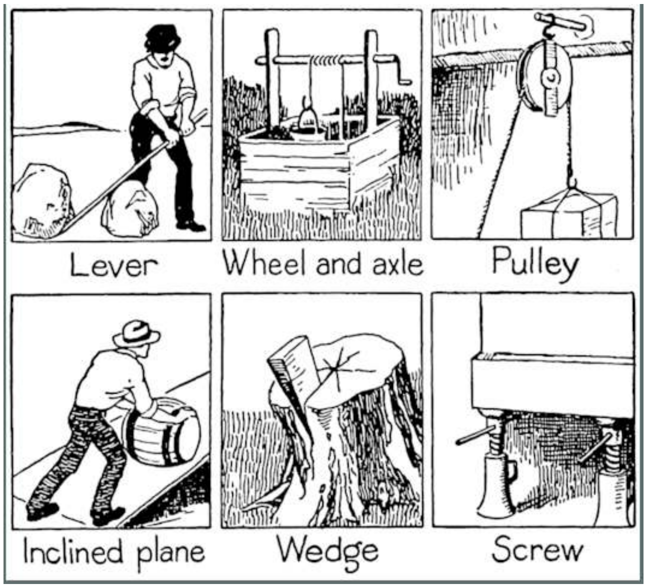
8.4.3: Simple Machines

Learning Objectives

- Identify the six simple machines.
- Understand how a simple machine can change the application of force based on the principle of conservation of energy

Long before the concept of energy was understood, the principles of conservation of energy were being used to develop simple machines. There is no documentation of the first machine ever used by humans or our ancestors, but it was probably a found object used in prehistoric times: perhaps a rock or stick used to pry open something to get at a source of food. What we refer to as the six classical simple machines were defined during the Renaissance period. They are the lever, the wheel and axle, the pulley, the inclined plane, the wedge, and the screw.



By Foster, Ellsworth D., ed; Hughes, James L. (James Laughlin) - The American educator; completely remodelled and rewritten from original text of the New practical reference library, with new plans and additional material (1919), Vol. 5, pg. 2290., Public Domain, [commons.wikimedia.org]

All of the simple machines function on the principle of conservation of energy, related to the work-energy theorem we discussed previously in this chapter. When considering simple machines, we typically ignore energy lost to friction even though we know that it exists. When we do that we can consider the relationship between the output of the machine and the applied force. The ratio of output to applied force is sometimes called the **mechanical advantage**. For example, a roadside service technician might use a hand or foot operated jack to push the body of a car up in order to change the tire. At no point in this process does the roadside technician apply a force with their own body capable of lifting the mass of a car in Earth's gravity. But the total energy the roadside technician puts into the machine is approximately equal to the total energy the machine expends to lift the car. The jack is an example of the lever, one of the six simple machines.

The relationship between the input and output of the machine is illustrated by the following equation:

$$W_{\text{in}} = W_{\text{out}}$$

.

Work is conserved because energy is conserved.

The energy that is conserved in simple machines relates to the force that enters and leaves the machine. As such, we can rewrite this equation to show it in terms of force and distance:

$$(Fd)_{\text{in}} = (Fd)_{\text{out}}$$

If the force that leaves the machine is greater than the force that enters the machine (as in our earlier example of the hand operated jack), then the distance over which that force operates must be proportionally smaller. For the hand operated jack, the distance the operator of the jack moves their hand in order to operate it is much greater than the distance through which the jack moves against the force of gravity on the car. We can say that the machine is multiplying the force output. It does this by increasing the distance over which the input force operates.

✓ Example 8.4.3.1

Suppose a technician using a hand operated jack needs to lift one end of a car by a quarter of a meter. In order to do so, they must move a hand operated lever 200 times through a distance of 0.10 meters each time. What is the average force they are depressing the lever with over those 200 times in order to lift the car? Assume the average mass of the portion of the car being lifted is 700 kg.

Solution

This is an example of using the formulation of conservation of mechanical energy for simple machines that we introduced in this section:

$$(Fd)_{\text{in}} = (Fd)_{\text{out}}$$

We can start by listing known values and determining other values which are part of this equation from the given information.

$$(d)_{\text{out}} = 0.25 \text{ m}$$

Because the lever has to be moved 200 times, we get the total distance by multiplying this by the length of the lever:

$$(d)_{\text{in}} = 200(0.10 \text{ m}) = 20 \text{ m}$$

We can assume that lifting this car is going to involve working against the Earth's gravity, so we find that force as follows:

$$(F)_{\text{out}} = (700 \text{ kg})(10 \cdot \text{m}^2/\text{s}^2) = 7000 \text{ N}$$

We can then solve for the force being added using some algebra.

$$(F)_{\text{in}} = \frac{(Fd)_{\text{out}}}{(d)_{\text{in}}}$$

And then if we put in the numbers we have already calculated we get:

$$(F)_{\text{in}} = \frac{(7000 \text{ N})(0.25 \text{ m})}{(20 \text{ m})} = 87.5 \text{ N}$$

This is considerably less than the 7000 N the half car is exerting on the Earth due to the gravitational force.

Another way in which machines alter the force is through changing the direction of the force. Work and energy are scalar functions, but force is still a vector. The direction of the vector might be much different due to the interaction with the simple machine. Simple machines might change the direction of the force without changing its magnitude. But many times they do both.

Section Summary

- There are six simple machines, many of which have been used since before recorded history.
- Simple machines function based on the conservation of energy.
- The direction and/or magnitude of a force might be changed by a simple machine.

8.4.4: Power

Learning Objectives

- Understand the relationship among power, energy, and work done.
- Examine power consumption and calculations of the cost of energy consumed.

What is Power?

Power—the word conjures up many images: a professional football player muscling aside his opponent, a dragster roaring away from the starting line, a volcano blowing its lava into the atmosphere, or a rocket blasting off, as in Figure 8.4.4.1.

Figure 8.4.4.1: This powerful rocket on the Space Shuttle *Endeavor* did work and consumed energy at a very high rate. (credit: NASA)

These images of power have in common the rapid performance of work, consistent with the scientific definition of **power** (**P**) as the rate at which work is done.

Definition: POWER

Power is the rate at which work is done.

$$P=\frac{W}{t}$$

The SI unit for power is the **watt** (**W**), where 1 watt equals 1 joule/second (1 W=1 J/s).

Because work is energy transfer, power is also the rate at which energy is expended. A 60-W light bulb, for example, expends 60 J of energy per second. Great power means a large amount of work or energy developed in a short time. For example, when a powerful car accelerates rapidly, it does a large amount of work and consumes a large amount of fuel in a short time.

Calculating Power from Energy

8.4.4.1 Example : Calculating the Power to Climb Stairs

What is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s? (See Figure 8.4.4.2)

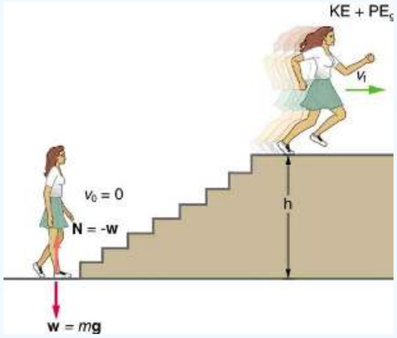


Figure 8.4.4.2: When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.

Strategy and Concept

The work going into mechanical energy is **W= KE + PE**. At the bottom of the stairs, we take both **KE** and $\mathrm{PE_g}$ as initially zero; thus, $W=\mathrm{KE_f}+\mathrm{PE_g}=\frac{1}{2}mv_\mathrm{f}^2+mgh$, where h is the vertical height of the stairs. Because all terms are given, we can calculate **W** and then divide it by time to get power.

Solution

Substituting the expression for **W** into the definition of power given in the previous equation, $P=W/t$ yields

$$P=\frac{W}{t}=\frac{\frac{1}{2}mv_\mathrm{f}^2+mgh}{t}.$$

Entering known values yields

$$\begin{aligned}P&=\frac{0.5(60.0\text{ kg})(2.00\text{ m/s})^2+(60.0\text{ kg})\left(9.80\text{ m/s}^2\right)(3.00\text{ m})}{3.50\text{ s}}\\&=\frac{120\text{ J}+1764\text{ J}}{3.50\text{ s}}\\&=538\text{ W}.\end{aligned}$$

Discussion

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

It is impressive that this woman's useful power output is slightly less than 1 **horsepower** (1 hp=746 W)! People can generate more than a horsepower with their leg muscles for short periods of time by rapidly converting available blood sugar and oxygen into work output. (A horse can put out 1 hp for hours on end.) Once oxygen is depleted, power output decreases and the person begins to breathe rapidly to obtain oxygen to metabolize more food—this is known as the *aerobic* stage of exercise. If the woman climbed the stairs slowly, then her power output would be much less, although the amount of work done would be the same.

MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—MEASURE YOUR POWER RATING

Determine your own power rating by measuring the time it takes you to climb a flight of stairs. We will ignore the gain in kinetic energy, as the above example showed that it was a small portion of the energy gain. Do not expect that your output will be more than about 0.5 hp.

Examples of Power

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. (See Table 8.4.4.1 for some examples.) Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter ($\mathrm{kW/m^2}$). A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40% of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1000 megawatts (1 megawatt (MW) is 10^6 W) of electric power. But the power plant consumes chemical energy at a rate of about 2500 MW, creating heat transfer to the surroundings at a rate of 1500 MW. (See Figure 8.4.4.3) Later in this text we will look at the efficiency of energy transfers in greater detail and learn why they happen the way that they do.

Figure 8.4.4.3: Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings. The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like. (credit: Kleinolive, Wikimedia Commons)

Table 8.4.4.1: Power Output or Consumption

Object or Phenomenon	Power in Watts
Supernova (at peak)	5×10^{37}
Milky Way galaxy	10^{37}
Crab Nebula pulsar	10^{28}
The Sun	4×10^{26}
Volcanic eruption (maximum)	4×10^{15}
Lightning bolt	2×10^{12}
Nuclear power plant (total electric and heat transfer)	3×10^{9}
Aircraft carrier (total useful and heat transfer)	10^{8}
Dragster (total useful and heat transfer)	2×10^{6}
Car (total useful and heat transfer)	8×10^{4}
Football player (total useful and heat transfer)	5×10^{3}
Clothes dryer	4×10^{3}
Person at rest (all heat transfer)	100
Typical incandescent light bulb (total useful and heat transfer)	60
Heart, person at rest (total useful and heat transfer)	8
Electric clock	3
Pocket calculator	10^{-3}

Power and Energy Consumption

We usually have to pay for the energy we use. It is interesting and easy to estimate the cost of energy for an electrical appliance if its power consumption rate and time used are known. The higher the power consumption rate and the longer the appliance is used, the greater the cost of that appliance. The power consumption rate is $P = W/t = E/t$, where $\boldsymbol{E}$ is the energy supplied by the electricity company. So the energy consumed over a time t is

$$E = Pt.$$

Electricity bills state the energy used in units of **kilowatt-hours** ($\mathrm{kW \cdot h}$), which is the product of power in kilowatts and time in hours. This unit is convenient because electrical power consumption at the kilowatt level for hours at a time is typical.

8.4.4.2 Example : Calculating Energy Costs

What is the cost of running a 0.200-kW computer 6.00 h per day for 30.0 d if the cost of electricity is \$0.120 per $\mathrm{kW \cdot h}$?

Strategy

Cost is based on energy consumed; thus, we must find $\boldsymbol{E}$ from $E = Pt$ and then calculate the cost. Because electrical energy is expressed in $\mathrm{kW \cdot h}$, at the start of a problem such as this it is convenient to convert the units into kW and hours.

Solution

The energy consumed in $\mathrm{kW \cdot h}$ is

$$E = Pt = (0.200\ \mathrm{kW})(6.00\ \mathrm{h/d})(30.0\ \mathrm{d}) = 36.0\ \mathrm{kW \cdot h},$$

and the cost is simply given by

$$\text{cost} = (36.0\ \mathrm{kW \cdot h})(\$0.120 \text{ per } \mathrm{kW \cdot h}) = \$4.32 \text{ per month.}$$

Discussion

The cost of using the computer in this example is neither exorbitant nor negligible. It is clear that the cost is a combination of power and time. When both are high, such as for an air conditioner in the summer, the cost is high.

The motivation to save energy has become more compelling with its ever-increasing price. Armed with the knowledge that energy consumed is the product of power and time, you can estimate costs for yourself and make the necessary value judgments about where to save energy. Either power or time must be reduced. It is most cost-effective to limit the use of high-power devices that normally operate for long periods of time, such as water heaters and air conditioners. This would not include relatively high power devices like toasters, because they are on only a few minutes per day. It would also not include electric clocks, in spite of their 24-hour-per-day usage, because they are very low power devices. It is sometimes possible to use devices that have greater efficiencies—that is, devices that consume less power to accomplish the same task. One example is the compact fluorescent light bulb, which produces over four times more light per watt of power consumed than its incandescent cousin.

Modern civilization depends on energy, but current levels of energy consumption and production are not sustainable. The link between global warming and fossil fuel use (with its concomitant production of carbon dioxide), has made reduction in energy use as well as a shift to non-fossil fuels of the utmost importance. Even though energy in an isolated system is a conserved quantity, the final result of most energy transformations is waste heat transfer to the environment, which is no longer useful for doing work. As we will discuss in more detail later, the potential for energy to produce useful work has been "degraded" in the energy transformation.

Section Summary

- Power is the rate at which work is done, or in equation form, for the average power $\boldsymbol{P}$ for work $\boldsymbol{W}$ done over a time t, $P = W/t$.
- The SI unit for power is the watt (W), where $1\ \mathrm{W} = 1\ \mathrm{J/s}$.
- The power of many devices such as electric motors is also often expressed in horsepower (hp), where $1\mathrm{hp} = 746\ \mathrm{W}$.

Glossary

power

the rate at which work is done

watt

(W) SI unit of power, with $1\ \mathrm{W} = 1\ \mathrm{J/s}$

horsepower

an older non-SI unit of power, with $1\ \mathrm{hp} = 746\ \mathrm{W}$

kilowatt-hour

$(\mathrm{kW \cdot h})$ unit used primarily for electrical energy provided by electric utility companies

8.4.5: Energy and Momentum

Learning Objectives

- Understand the differences between energy and momentum.

Energy and momentum have many similarities as well as many differences. These similarities and differences were confounding for the scientists of the 1700s and 1800s who were trying to formulate ideas about how the movements of masses followed basic repeatable principles.

Momentum and energy can both be formulated around ideas of mass and velocity. However, the formulation of energy is based on mass times velocity squared while the formulation of momentum is based on mass times velocity. Also, energy is not always based on movement while momentum is always based on movement. We have already seen how potential energy can be calculated for objects which are not moving. As you can probably imagine, there is no component of movement involved in most of the other forms of energy we have not considered in great detail yet.

The other similarity between momentum and energy is that they are both conserved. This is an important similarity, as there are very few properties in nature that are conserved. However, these two are conserved for different reasons and in different ways. Momentum is a vector quantity, while energy is a scalar quantity (an important type of scalar quantity called a state function, as we will learn later). As a vector quantity, momentum could cancel out during a collision. For example, if two cars of equal mass were moving at the same speed directly towards each other we might expect them to stop moving when they collide. In this case the momentum of the system defined as both cars is conserved because there overall momentum adds up to zero both before and after the collision. However, the energy does not cancel out. The energy of motion has been converted into potential energy as the bumpers are compressed and into thermal energy as they warm up. It does not cancel out as momentum does.

Section Summary

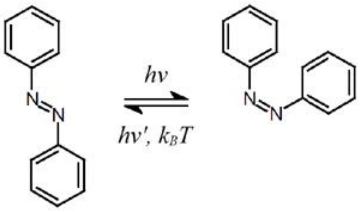
- Momentum is based on mass and velocity, and energy can be formulated to also be based on mass and velocity.
- Momentum and energy are both conserved.
- Momentum is a vector property, and energy is a scalar property.
- Momentum and energy are different properties, although they have sometimes been confused with each other.

8.5: Chemistry in Physics

Chemistry has been defined as the science that is concerned with the composition, properties, and structure of matter and with the ways in which substances can change from one form to another. But this definition is too broad to be useful. Chemistry isn't the only science that deals with the composition and transformations of matter. Matter is also composed of quarks, which transform by annihilation, of metals which transform by annealing, and of stars which may transform by supernovae. These are normally considered the subject of Physics. Chemists are unique because they understand or explain everything, even the subjects studied by physicists, in terms of the properties of just over 100 kinds of atoms found in all matter, and the amazing variety of molecules that are created by forming and breaking bonds between atoms. ***So chemistry is defined by its approach, not its subject matter***. Chemistry explains or understands any subject in terms of the properties of atoms and molecules.

Chemistry provides a unique perspective that complements Physics in many areas of contemporary research.

For example, nanotechnology techniques have been applied to building “molecular motors”, artificial muscles, and molecular electronics. The action of these devices results from a change in shape of an azobenzene molecule (molecular shape and bonding tendencies are chemical properties) when it absorbs light. The molecule is bonded to a metal surface, so that the mechanical energy might be harnessed [1] as shown here[2]



The mechanical work can then be measured and characterized by standard methods of physics. In contrast to chemical properties, physical properties do not require explanations involving atoms or molecules; Physical properties include the ductility, density, or hardness of the metal used to anchor the molecular motors. These properties are easily defined and measured without reference to atoms.

Chemistry and Physics often complement each other in the development of new materials. Materials are often complex mixtures which do not have atoms in definite ratios. Chemically pure substances, like sulfuric acid, H_2SO_4 have specific atom ratios (2 H : 1S : 4 O in this case). Most materials (like cement, for example, which contains sand, lime, water, and other substances) are not considered chemicals, but mixtures. But atomic properties still are used to understand bonding between atoms in materials.

Although the Romans brought the secret of glassworking to Britain in 55BC, we still don't know what makes glass rigid as it cools and hardens. Now researchers [3] are beginning to understand how a rigid atomic structure takes shape as a glass is cooled. Normally, we thought glasslike materials just become more viscous as they is cooled. There are myths that glass windows are thicker at the bottom due to glass flow, but panes were probably just mounted that way by glaziers because it was thought to be good technique. The relationship between atomic structure and properties of solids, whether the blades of Japanese swords [4], the blades of turbines, or boron nitride cutting wheels, is a matter of intense interest to physicists and engineers. Once more, properties like rigidity that are quantified by physical methods are explained through the chemist’s attention to bonding between atoms or molecules.

Japanese swords[5]

Important Terms: Chemistry

Chemical

- Chemical Property
- Chemical Change, Reaction

From ChemPRIME: 1.0: Prelude to Chemistry

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3. ↑ http://www.sciencedaily.com/releases/2009/02/090206081511.htm
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Contributors and Attributions

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8.6: Kinetic Energy and the Work-Energy Theorem

Learning Objectives

By the end of this section, you will be able to:

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

Work Transfers Energy

What happens to the work done on a system? Energy is transferred into the system, but in what form? Does it remain in the system or move on? The answers depend on the situation. For example, if the lawn mower in [link](a) is pushed just hard enough to keep it going at a constant speed, then energy put into the mower by the person is removed continuously by friction, and eventually leaves the system in the form of heat transfer. In contrast, work done on the briefcase by the person carrying it up stairs in [link](d) is stored in the briefcase-Earth system and can be recovered at any time, as shown in [link](e). In fact, the building of the pyramids in ancient Egypt is an example of storing energy in a system by doing work on the system. Some of the energy imparted to the stone blocks in lifting them during construction of the pyramids remains in the stone-Earth system and has the potential to do work.

In this section we begin the study of various types of work and forms of energy. We will find that some types of work leave the energy of a system constant, for example, whereas others change the system in some way, such as making it move. We will also develop definitions of important forms of energy, such as the energy of motion.

Net Work and the Work-Energy Theorem

We know from the study of Newton's laws in Dynamics: Force and Newton's Laws of Motion that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

Let us start by considering the total, or net, work done on a system. **Net work** is defined to be the sum of work done by all external forces—that is, net work is the work done by the net external force F_{net}. In equation form, this is $W_{net} = F_{net} d \cos\theta$, where θ is the angle between the force vector and the displacement vector. Figure (a) shows a graph of force versus displacement for the component of the force in the direction of the displacement—that is, an $F \cos\theta$ vs. d graph. In this case, $F \cos\theta$ is constant. You can see that the area under the graph is $F \cos\theta$, or the work done. Figure (b) shows a more general process where the force varies. The area under the curve is divided into strips, each having an average force $(F \cos\theta)_{i(ave)}$. The work done is e $(F \cos\theta)_{i(ave)} d_i$ for each strip, and the total work done is the sum of the W_i. Thus the total work done is the total area under the curve, a useful property to which we shall refer later.

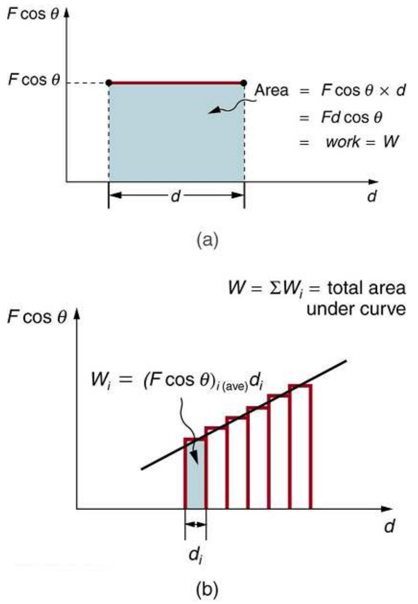

Figure 8.6.1: (a) A graph of $F\, cos\, \theta$ vs. d when $F\, cos\, \theta$ is constant. The area under the curve represents the work done by the force. (b) A graph of $F\, cos\, \theta$ vs. d in which the force varies. The work done for each interval is the area of each strip; thus, the total area under the curve equals the total work done.

Net work will be simpler to examine if we consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in Figure.

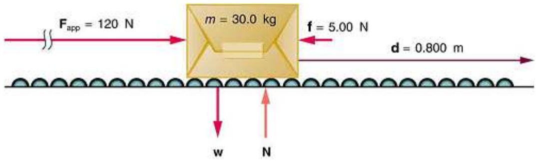

Figure 8.6.2: A package on a roller belt is pushed horizontally through a distance d.

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force F_{app} and the horizontal friction force f. Thus, as expected, the net force is parallel to the displacement, so that $\theta = 0$ and $cos\, \theta = 1$, and the net work is given by

$$W_{net} = F_{net} d. \tag{8.6.1}$$

The effect of the net force F_{net} is to accelerate the package from v_0 to v The kinetic energy of the package increases, indicating that the net work done on the system is positive. (See Example.) By using Newton's second law, and doing some algebra, we can reach an interesting conclusion. Substituting $F = ma$ from Newton's second law gives

$$W_{net} = mad. \tag{8.6.2}$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take $d = x - x_0$ and use the equation studied in Motion Equations for Constant Acceleration in One Dimension for the change in speed over a distance d if the acceleration has the constant value a, namely $v^2 = v_0^2 + 2ad$. (note that a appears in the expression for the net work). Solving for acceleration gives $a = \frac{v^2 - v_0^2}{2d}$. When a is substituted into the preceding expression for W_{net} we obtain

$$W_{net} = m \left(\frac{v^2 - v_0^2}{2d} \right) d. \tag{8.6.3}$$

The d cancels, and we rearrange this to obtain

$$W_{net} = \frac{1}{2} mv^2 - \frac{1}{2} mv_0^2. \tag{8.6.4}$$

This expression is called the **work-energy theorem,** and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity $\frac{1}{2}mv^2$. This quantity is our first example of a form of energy.

Work-Energy Theorem

The net work on a system equals the change in the quantity $\frac{1}{2}mv^2$.

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2. \tag{8.6.5}$$

The quantity $\frac{1}{2}mv^2$ in the work-energy theorem is defined to be the translational **kinetic energy** (KE) of a mass m moving at a speed v. (*Translational* kinetic energy is distinct from *rotational* kinetic energy, which is considered later.) In equation form, the translational kinetic energy,

$$KE = \frac{1}{2}mv^2, \tag{8.6.6}$$

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

We are aware that it takes energy to get an object, like a car or the package in Figure, up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 100 km/h has four times the kinetic energy it has at 50 km/h, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.

Example 8.6.1: Calculating the Kinetic Energy of a Package

Suppose a 30.0-kg package on the roller belt conveyor system in Figure 7.03.2 is moving at 0.500 m/s. What is its kinetic energy?

Strategy

Because the mass m and the speed v are given, the kinetic energy can be calculated from its definition as given in the equation $KE = \frac{1}{2}mv^2$.

Solution

The kinetic energy is given by

$$KE = \frac{1}{2}mv^2. \tag{8.6.7}$$

Entering known values gives

$$KE = 0.5(30.0\,kg)(0.500\,m/s)^2, \tag{8.6.8}$$

which yields

$$KE = 3.75\,kg \cdot m^2/s^2 = 3.75\,J \tag{8.6.9}$$

Discussion

Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

Example 8.6.2: Determining the Work to Accelerate a Package

Suppose that you push on the 30.0-kg package in Figure 7.03.2. with a constant force of 120 N through a distance of 0.800 m, and that the opposing friction force averages 5.00 N.

(a) Calculate the net work done on the package. (b) Solve the same problem as in part (a), this time by finding the work done by each force that contributes to the net force.

Strategy and Concept for (a)

This is a motion in one dimension problem, because the downward force (from the weight of the package) and the normal force have equal magnitude and opposite direction, so that they cancel in calculating the net force, while the applied force, friction, and the displacement are all horizontal. (See Figure 7.03.2.) As expected, the net work is the net force times distance.

Solution for (a)

The net force is the push force minus friction, or $F_{net} = 120\,N - 5.00\,N = 115\,N$. Thus the net work is

$$W_{net} = F_{net} d = (115\,N)(0.800\,m) \tag{8.6.10}$$

$$= 92.0\,N \cdot m = 92.0\,J \tag{8.6.11}$$

Discussion for (a)

This value is the net work done on the package. The person actually does more work than this, because friction opposes the motion. Friction does negative work and removes some of the energy the person expends and converts it to thermal energy. The net work equals the sum of the work done by each individual force.

Strategy and Concept for (b)

The forces acting on the package are gravity, the normal force, the force of friction, and the applied force. The normal force and force of gravity are each perpendicular to the displacement, and therefore do no work.

Solution for (b)

The applied force does work.

$$W_{app} = F_{app} d\,cos\,(0^o) = F_{app} d \tag{8.6.12}$$

$$= (120\,N)(0.800\,m) \tag{8.6.13}$$

$$= 96.0\,J \tag{8.6.14}$$

The friction force and displacement are in opposite directions, so that $\theta = 180^o$, and the work done by friction is

$$W_{fr} = F_{fr} d\,cos\,(180^o) \tag{8.6.15}$$

$$= -(5.00\,N)(0.800\,m) \tag{8.6.16}$$

$$= -4.00\,J \tag{8.6.17}$$

So the amounts of work done by gravity, by the normal force, by the applied force, and by friction are, respectively,

$$W_{gr} = 0, \tag{8.6.18}$$

$$W_N = 0, \tag{8.6.19}$$

$$W_{app} = 96.0\,J, \tag{8.6.20}$$

$$W_{fr} = -4\,J. \tag{8.6.21}$$

The total work done as the sum of the work done by each force is then seen to be

$$W_{total} = W_{gr} + W_N + W_{app} + W_{fr} = 92.0\,J. \tag{8.6.22}$$

Discussion for (b)

The calculated total work W_{total} as the sum of the work by each force agrees, as expected, with the work W_{net} done by the net force. The work done by a collection of forces acting on an object can be calculated by either approach.

✓ Example 8.6.3: Determining Speed from Work and Energy

Find the speed of the package in Figure 7.03.2. at the end of the push, using work and energy concepts.

Strategy

Here the work-energy theorem can be used, because we have just calculated the net work W_{net} and the initial kinetic energy, $\frac{1}{2}mv_0^2$ These calculations allow us to find the final kinetic energy, $\frac{1}{2}mv^2$ and thus the final speed v.

Solution

The work-energy theorem in equation form is

$$W_{net}=\frac{1}{2}mv^2-\frac{1}{2}mv_0^2. \tag{8.6.23}$$

Solving for $\frac{1}{2}mv^2$ gives

$$\frac{1}{2}mv^2=W_{net}+\frac{1}{2}mv_0^2 \tag{8.6.24}$$

Thus,

$$\frac{1}{2}mv^2=92.0\,J+3.75\,J=95.75\,J. \tag{8.6.25}$$

Solving for the final speed as requested and entering known values gives

$$v=\sqrt{\frac{2(95.75\,J)}{m}}=\sqrt{\frac{191.5\,kg\cdot m^2/s^2}{30.0\,kg}} \tag{8.6.26}$$

$$=2.53\,m/s \tag{8.6.27}$$

Discussion

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work done on the package. This means that the work indeed adds to the energy of the package.

✓ Example 8.6.4: Work and Energy Can Reveal Distance, Too

How far does the package in Figure 7.03.2. coast after the push, assuming friction remains constant? Use work and energy considerations.

Strategy

We know that once the person stops pushing, friction will bring the package to rest. In terms of energy, friction does negative work until it has removed all of the package’s kinetic energy. The work done by friction is the force of friction times the distance traveled times the cosine of the angle between the friction force and displacement; hence, this gives us a way of finding the distance traveled after the person stops pushing.

Solution

The normal force and force of gravity cancel in calculating the net force. The horizontal friction force is then the net force, and it acts opposite to the displacement, so $\theta=180^o$. To reduce the kinetic energy of the package to zero, the work W_{fr} by friction must be minus the kinetic energy that the package started with plus what the package accumulated due to the pushing. Thus $W_{fr}=-95.75\,J$. Furthermore, $W_{fr}=df'\,cos\,\theta=-Fd'$, where d' is the distance it takes to stop. Thus,

$$d'=-\frac{W_{fr}}{f}=\frac{-95.75\,J}{5.00\,N}, \tag{8.6.28}$$

and so

$$d'=19.2\,m \tag{8.6.29}$$

Discussion

> This is a reasonable distance for a package to coast on a relatively friction-free conveyor system. Note that the work done by friction is negative (the force is in the opposite direction of motion), so it removes the kinetic energy.

Some of the examples in this section can be solved without considering energy, but at the expense of missing out on gaining insights about what work and energy are doing in this situation. On the whole, solutions involving energy are generally shorter and easier than those using kinematics and dynamics alone.

Summary

- The net work W_{net} is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass m moving at speed v is $KE = \frac{1}{2}mv^2$.
- The work-energy theorem states that the net work W_{net} on a system changes its kinetic energy, $W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$.

Glossary

net work

work done by the net force, or vector sum of all the forces, acting on an object

work-energy theorem

the result, based on Newton’s laws, that the net work done on an object is equal to its change in kinetic energy

kinetic energy

the energy an object has by reason of its motion, equal to $\frac{1}{2}mv^2$ for the translational (i.e., non-rotational) motion of an object of mass m moving at speed v

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SECTION OVERVIEW

8.7: Thermal Physics

8.7.1: Introduction to Thermal Physics

Figure 8.7.1.1: The welder's gloves and helmet protect him from the electric arc that transfers enough thermal energy to melt the rod, spray sparks, and burn the retina of an unprotected eye. The thermal energy can be felt on exposed skin a few meters away, and its light can be seen for kilometers. (credit: Kevin S. O'Brien/U.S. Navy)

Heat is something familiar to each of us. We feel the warmth of the summer Sun, the chill of a clear summer night, the heat of coffee after a winter stroll, and the cooling effect of our sweat. Heat transfer is maintained by temperature differences. Manifestations of **heat transfer**—the movement of heat energy from one place or material to another—are apparent throughout the universe. Heat from beneath Earth's surface is brought to the surface in flows of incandescent lava. The Sun warms Earth's surface and is the source of much of the energy we find on it. Rising levels of atmospheric carbon dioxide threaten to trap more of the Sun's energy, perhaps fundamentally altering the ecosphere. In space, supernovas explode, briefly radiating more heat than an entire galaxy does.

What is heat? How do we define it? How is it related to temperature? What are heat's effects? How is it related to other forms of energy and to work? We will find that, in spite of the richness of the phenomena, there is a small set of underlying physical principles that unite the subjects and tie them to other fields.

Most notably, heat transfer can be used to do work. It can also be converted to any other form of energy. A car engine, for example, burns fuel for heat transfer into a gas. Work is done by the gas as it exerts a force through a distance, converting its energy into a variety of other forms—into the car's kinetic or gravitational potential energy; into electrical energy to run the spark plugs, radio, and lights; and back into stored energy in the car's battery. But most of the heat transfer produced from burning fuel in the engine does not do work on the gas-the engine is quite inefficient.

Basic physical laws govern how heat transfer for doing work takes place and place insurmountable limits onto its efficiency. This chapter will explore these laws as well as some applications and concepts associated with them. These topics are part of thermodynamics—the study of heat transfer and its relationship to doing work.

Figure 8.7.1.2: In a typical thermometer like this one, the alcohol, with a red dye, expands more rapidly than the glass containing it. When the thermometer's temperature increases, the liquid from the bulb is forced into the narrow tube, producing a large change in the length of the column for a small change in temperature. (credit: Chemical Engineer, Wikimedia Commons)

8.7.2: Temperature

Learning Objectives

- Define temperature.
- Convert temperatures between the Celsius, Fahrenheit, and Kelvin scales.
- Define thermal equilibrium.
- State the zeroth law of thermodynamics.

The concept of temperature has evolved from the common concepts of hot and cold. Human perception of what feels hot or cold is a relative one. For example, if you place one hand in hot water and the other in cold water, and then place both hands in tepid water, the tepid water will feel cool to the hand that was in hot water, and warm to the one that was in cold water. The scientific definition of temperature is less ambiguous than your senses of hot and cold. **Temperature** is operationally defined to be what we measure with a thermometer. (Many physical quantities are defined solely in terms of how they are measured. We shall see later how temperature is related to the kinetic energies of atoms and molecules, a more physical explanation.) Two accurate thermometers, one placed in hot water and the other in cold water, will show the hot water to have a higher temperature. If they are then placed in the tepid water, both will give identical readings (within measurement uncertainties). In this section, we discuss temperature, its measurement by thermometers, and its relationship to thermal equilibrium. Again, temperature is the quantity measured by a thermometer.

MISCONCEPTION ALERT: HUMAN PERCEPTION VS. REALITY

On a cold winter morning, the wood on a porch feels warmer than the metal of your bike. The wood and bicycle are in thermal equilibrium with the outside air, and are thus the same temperature. They *feel* different because of the difference in the way that they conduct heat away from your skin. The metal conducts heat away from your body faster than the wood does. This is just one example demonstrating that the human sense of hot and cold is not determined by temperature alone.

Another factor that affects our perception of temperature is humidity. Most people feel much hotter on hot, humid days than on hot, dry days. This is because on humid days, sweat does not evaporate from the skin as efficiently as it does on dry days. It is the evaporation of sweat (or water from a sprinkler or pool) that cools us off.

Any physical property that depends on temperature, and whose response to temperature is reproducible, can be used as the basis of a thermometer. Because many physical properties depend on temperature, the variety of thermometers is remarkable. For example, volume increases with temperature for most substances. This property is the basis for the common alcohol thermometer, the old mercury thermometer, and the bimetallic strip (Figure $8.7.2.1$). Other properties used to measure temperature include electrical resistance and color, as shown in Figure $8.7.2.2$, and the emission of infrared radiation, as shown in Figure $8.7.2.3$.

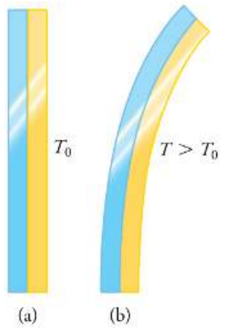

Figure $8.7.2.1$: The curvature of a bimetallic strip depends on temperature. (a) The strip is straight at the starting temperature, where its two components have the same length. (b) At a higher temperature, this strip bends to the right, because the metal on the left has expanded more than the metal on the right.

Figure 8.7.2.2: Each of the six squares on this plastic (liquid crystal) thermometer contains a film of a different heat-sensitive liquid crystal material. Below 95ºF, all six squares are black. When the plastic thermometer is exposed to temperature that increases to 95ºF, the first liquid crystal square changes color. When the temperature increases above 96.8ºF the second liquid crystal square also changes color, and so forth. (credit: Arkrishna, Wikimedia Commons)

Figure 8.7.2.3: Fireman Jason Ormand uses a pyrometer to check the temperature of an aircraft carrier's ventilation system. Infrared radiation (whose emission varies with temperature) from the vent is measured and a temperature readout is quickly produced. Infrared measurements are also frequently used as a measure of body temperature. These modern thermometers, placed in the ear canal, are more accurate than alcohol thermometers placed under the tongue or in the armpit. (credit: Lamel J. Hinton/U.S. Navy)

Temperature Scales

Thermometers are used to measure temperature according to well-defined scales of measurement, which use pre-defined reference points to help compare quantities. The three most common temperature scales are the Fahrenheit, Celsius, and Kelvin scales. A temperature scale can be created by identifying two easily reproducible temperatures. The freezing and boiling temperatures of water at standard atmospheric pressure are commonly used.

The **Celsius** scale (which replaced the slightly different *centigrade* scale) has the freezing point of water at 0ºC and the boiling point at 100ºC. Its unit is the **degree Celsius**(ºC). On the **Fahrenheit** scale (still the most frequently used in the United States), the freezing point of water is at 32ºF and the boiling point is at 212ºF. The unit of temperature on this scale is the **degree Fahrenheit**(ºF). Note that a temperature difference of one degree Celsius is greater than a temperature difference of one degree Fahrenheit. Only 100 Celsius degrees span the same range as 180 Fahrenheit degrees, thus one degree on the Celsius scale is 1.8 times larger than one degree on the Fahrenheit scale $180/100 = 9/5$.

The **Kelvin** scale is the temperature scale that is commonly used in science. It is an *absolute temperature* scale defined to have 0 K at the lowest possible temperature, called **absolute zero**. The official temperature unit on this scale is the *kelvin*, which is abbreviated K, and is not accompanied by a degree sign. The freezing and boiling points of water are 273.15 K and 373.15 K, respectively. Thus, the magnitude of temperature differences is the same in units of kelvins and degrees Celsius. Unlike other temperature scales, the Kelvin scale is an absolute scale. It is used extensively in scientific work because a number of physical quantities, such as the volume of an ideal gas, are directly related to absolute temperature. The kelvin is the SI unit used in scientific work.

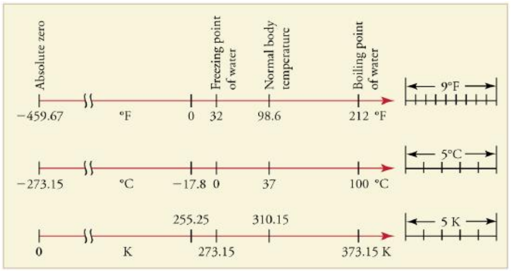

Figure 8.7.2.4: Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales, rounded to the nearest degree. The relative sizes of the scales are also shown.

The relationships between the three common temperature scales is shown in Figure 8.7.2.4

Temperature Ranges in the Universe

Figure 8.7.2.6 shows the wide range of temperatures found in the universe. Human beings have been known to survive with body temperatures within a small range, from 24ºC to 44ºC (75ºF to 111ºF). The average normal body temperature is usually given as 37.0ºC (98.6ºF), and variations in this temperature can indicate a medical condition: a fever, an infection, a tumor, or circulatory problems (see Figure 8.7.2.5).

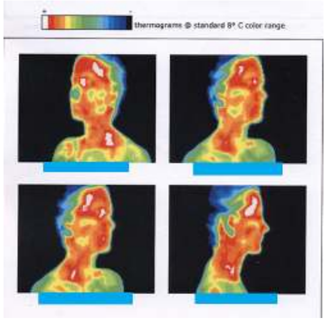

Figure 8.7.2.5: This image of radiation from a person's body (an infrared thermograph) shows the location of temperature abnormalities in the upper body. Dark blue corresponds to cold areas and red to white corresponds to hot areas. An elevated temperature might be an indication of malignant tissue (a cancerous tumor in the breast, for example), while a depressed temperature might be due to a decline in blood flow from a clot. In this case, the abnormalities are caused by a condition called hyperhidrosis. (credit: Porcelina81, Wikimedia Commons)

The lowest temperatures ever recorded have been measured during laboratory experiments: 4.5×10^{-10} K at the Massachusetts Institute of Technology (USA), and 1.0×10^{-10} K at Helsinki University of Technology (Finland). In comparison, the coldest recorded place on Earth's surface is Vostok, Antarctica at 183 K (–89ºC), and the coldest place (outside the lab) known in the universe is the Boomerang Nebula, with a temperature of 1 K.

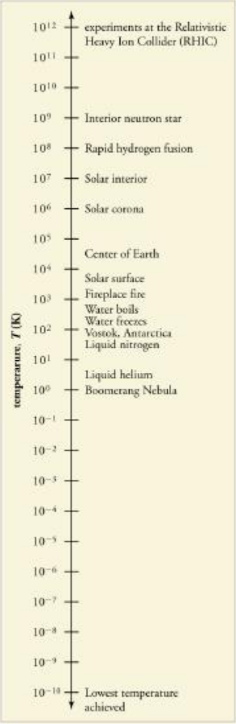

Figure 8.7.2.6: Each increment on this logarithmic scale indicates an increase by a factor of ten, and thus illustrates the tremendous range of temperatures in nature. Note that zero on a logarithmic scale would occur off the bottom of the page at infinity.

MAKING CONNECTIONS: ABSOLUTE ZERO

What is absolute zero? Absolute zero is the temperature at which all molecular motion has ceased. The concept of absolute zero arises from the behavior of gases. Figure 8.7.2.7 shows how the pressure of gases at a constant volume decreases as temperature decreases. Various scientists have noted that the pressures of gases extrapolate to zero at the same temperature, –273.15ºC. This extrapolation implies that there is a lowest temperature. This temperature is called *absolute zero*. Today we know that most gases first liquefy and then freeze, and it is not actually possible to reach absolute zero. The numerical value of absolute zero temperature is –273.15ºC or 0 K.

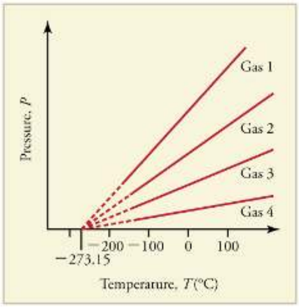

Figure 8.7.2.7: Graph of pressure versus temperature for various gases kept at a constant volume. Note that all of the graphs extrapolate to zero pressure at the same temperature.

Thermal Equilibrium and the Zeroth Law of Thermodynamics

Thermometers actually take their *own* temperature, not the temperature of the object they are measuring. This raises the question of how we can be certain that a thermometer measures the temperature of the object with which it is in contact. It is based on the fact that any two systems placed in *thermal contact* (meaning heat transfer can occur between them) will reach the same temperature. That is, heat will flow from the hotter object to the cooler one until they have exactly the same temperature. The objects are then in **thermal equilibrium**, and no further changes will occur. The systems interact and change because their temperatures differ, and the changes stop once their temperatures are the same. Thus, if enough time is allowed for this transfer of heat to run its course, the temperature a thermometer registers *does* represent the system with which it is in thermal equilibrium. Thermal equilibrium is established when two bodies are in contact with each other and can freely exchange energy.

Furthermore, experimentation has shown that if two systems, A and B, are in thermal equilibrium with each another, and B is in thermal equilibrium with a third system C, then A is also in thermal equilibrium with C. This conclusion may seem obvious, because all three have the same temperature, but it is basic to thermodynamics. It is called the **zeroth law of thermodynamics**.

THE ZEROTH LAW OF THERMODYNAMICS

If two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

This law was postulated in the 1930s, after the first and second laws of thermodynamics had been developed and named. It is called the *zeroth law* because it comes logically before the first and second laws. An example of this law in action is seen in babies in incubators: babies in incubators normally have very few clothes on, so to an observer they look as if they may not be warm enough. However, the temperature of the air, the cot, and the baby is the same, because they are in thermal equilibrium, which is accomplished by maintaining air temperature to keep the baby comfortable.

Exercise 8.7.2.1

Does the temperature of a body depend on its size?

Answer

No, the system can be divided into smaller parts each of which is at the same temperature. We say that the temperature is an *intensive* quantity. Intensive quantities are independent of size.

Section Summary

- Temperature is the quantity measured by a thermometer.
- Temperature is related to the average kinetic energy of atoms and molecules in a system.
- Absolute zero is the temperature at which there is no molecular motion.

- There are three main temperature scales: Celsius, Fahrenheit, and Kelvin.
- Temperatures on one scale can be converted to temperatures on another scale using the following equations:

$$T_{\circ}\mathrm{F} = \frac{9}{5}T_{\mathrm{C}}\mathrm{C} + 32$$
$$T_{\mathrm{o}}\mathrm{C} = \frac{5}{9}(T_{\mathrm{o}} - 32)$$
$$T_{\mathrm{K}} = T_{\mathrm{o}}\mathrm{C} + 273.15$$
$$T_{\mathrm{o}}\mathrm{C} = T_{\mathrm{K}} - 273.15$$

- Systems are in thermal equilibrium when they have the same temperature.
- Thermal equilibrium occurs when two bodies are in contact with each other and can freely exchange energy.
- The zeroth law of thermodynamics states that when two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

Glossary

temperature

the quantity measured by a thermometer

Celsius scale

temperature scale in which the freezing point of water is 0ºC and the boiling point of water is 100ºC

degree Celsius

unit on the Celsius temperature scale

Fahrenheit scale

temperature scale in which the freezing point of water is 32ºF and the boiling point of water is 212ºF

degree Fahrenheit

unit on the Fahrenheit temperature scale

Kelvin scale

temperature scale in which 0 K is the lowest possible temperature, representing absolute zero

absolute zero

the lowest possible temperature; the temperature at which all molecular motion ceases

thermal equilibrium

the condition in which heat no longer flows between two objects that are in contact; the two objects have the same temperature

zeroth law of thermodynamics

law that states that if two objects are in thermal equilibrium, and a third object is in thermal equilibrium with one of those objects, it is also in thermal equilibrium with the other object

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8.7.3: The Ideal Gas Law

Learning Objectives

- State and explain the ideal gas law using Boltzmann's constant
- Use the ideal gas law to calculate pressure change, temperature change, volume change, or the number of molecules in a given volume.

Figure $8.7.3.1$: The air inside this hot air balloon flying over Putrajaya, Malaysia, is hotter than the ambient air. As a result, the balloon experiences a buoyant force pushing it upward. (credit: Kevin Poh, Flickr)

In this section, we continue to explore the thermal behavior of gases. In particular, we examine the characteristics of atoms and molecules that compose gases. (Most gases, for example nitrogen, N_2, and oxygen, O_2, are composed of two or more atoms. We will primarily use the term "molecule" in discussing a gas because the term can also be applied to monatomic gases, such as helium.)

Gases are easily compressed. Gases expand and contract very rapidly with temperature changes. In addition, you will note that most gases expand at the *same* rate, or have the same β. This raises the question as to why gases should all act in nearly the same way, when liquids and solids have widely varying expansion rates.

The answer lies in the large separation of atoms and molecules in gases, compared to their sizes, as illustrated in Figure $8.7.3.2$ Because atoms and molecules have large separations, forces between them can be ignored, except when they collide with each other during collisions. The motion of atoms and molecules (at temperatures well above the boiling temperature) is fast, such that the gas occupies all of the accessible volume and the expansion of gases is rapid. In contrast, in liquids and solids, atoms and molecules are closer together and are quite sensitive to the forces between them.

Figure $8.7.3.2$: Atoms and molecules in a gas are typically widely separated, as shown. Because the forces between them are quite weak at these distances, the properties of a gas depend more on the number of atoms per unit volume and on temperature than on the type of atom.

To get some idea of how pressure, temperature, and volume of a gas are related to one another, consider what happens when you pump air into an initially deflated tire. The tire's volume first increases in direct proportion to the amount of air injected, without much increase in the tire pressure. Once the tire has expanded to nearly its full size, the walls limit volume expansion. If we continue to pump air into it, the pressure increases. The pressure will further increase when the car is driven and the tires move. Most manufacturers specify optimal tire pressure for cold tires. (See Figure $8.7.3.3$)

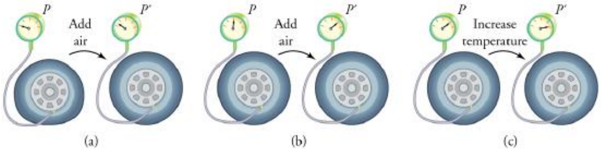

Figure 8.7.3.3: (a) When air is pumped into a deflated tire, its volume first increases without much increase in pressure. (b) When the tire is filled to a certain point, the tire walls resist further expansion and the pressure increases with more air. (c) Once the tire is inflated, its pressure increases with temperature.

At room temperatures, collisions between atoms and molecules can be ignored. In this case, the gas is called an ideal gas, in which case the relationship between the pressure, volume, and temperature is given by the equation of state called the ideal gas law.

IDEAL GAS LAW

The **ideal gas law** states that

$$PV = NkT,$$

where P is the absolute pressure of a gas, V is the volume it occupies, N is the number of atoms and molecules in the gas, and T is its absolute temperature. The constant k is called the **Boltzmann constant** in honor of Austrian physicist Ludwig Boltzmann (1844–1906) and has the value

$$k = 1.38 \times 10^{-23}\ \mathrm{J/K}.$$

The ideal gas law can be derived from basic principles, but was originally deduced from experimental measurements of Charles' law (that volume occupied by a gas is proportional to temperature at a fixed pressure) and from Boyle's law (that for a fixed temperature, the product PV is a constant). In the ideal gas model, the volume occupied by its atoms and molecules is a negligible fraction of V. The ideal gas law describes the behavior of real gases under most conditions. (Note, for example, that N is the total number of atoms and molecules, independent of the type of gas.)

Let us see how the ideal gas law is consistent with the behavior of filling the tire when it is pumped slowly and the temperature is constant. At first, the pressure P is essentially equal to atmospheric pressure, and the volume V increases in direct proportion to the number of atoms and molecules N put into the tire. Once the volume of the tire is constant, the equation $PV = NkT$ predicts that the pressure should increase in proportion to *the number N of atoms and molecules*.

Example 8.7.3.1: Calculating Pressure Changes Due to Temperature Changes: Tire Pressure

Suppose your bicycle tire is fully inflated, with an absolute pressure of $7.00 \times 10^5\ \mathrm{Pa}$ (a gauge pressure of just under $90.0\ \mathrm{lb/in^2}$) at a temperature of $18.0^\circ\mathrm{C}$. What is the pressure after its temperature has risen to 35.0ºC? Assume that there are no appreciable leaks or changes in volume.

Strategy

The pressure in the tire is changing only because of changes in temperature. First we need to identify what we know and what we want to know, and then identify an equation to solve for the unknown.

We know the initial pressure $P_0 = 7.00 \times 10^5\ \mathrm{Pa}$, the initial temperature $T_0 = 18.0^\circ\mathrm{C}$, and the final temperature $T_\mathrm{f} = 35.0^\circ\mathrm{C}$. We must find the final pressure P_f. How can we use the equation $PV = NkT$? At first, it may seem that not enough information is given, because the volume V and number of atoms N are not specified. What we can do is use the equation twice: $P_0V_0 = NkT_0$ and $P_\mathrm{f}V_\mathrm{f} = NkT_\mathrm{f}$. If we divide $P_\mathrm{f}V_\mathrm{f}$ by P_0V_0 we can come up with an equation that allows us to solve for Pf.

$$\frac{P_\mathrm{f}V_\mathrm{f}}{P_0V_0} = \frac{N_\mathrm{f}kT_\mathrm{f}}{N_0kT_0}$$

Since the volume is constant, V_f and V_0 are the same and they cancel out. The same is true for N_f and N_0, and k, which is a constant. Therefore,

$$\frac{P_\mathrm{f}}{P_0} = \frac{T_\mathrm{f}}{T_0}.$$

We can then rearrange this to solve for P_f:

$$P_{\mathrm{f}} = P_0 \frac{T_{\mathrm{f}}}{T_0},$$

where the temperature must be in units of kelvins, because T_0 and T_f are absolute temperatures.

Solution

1.Convert temperatures from Celsius to Kelvin.

$$\begin{aligned} T_0 &= (18.0+273)\mathrm{K} = 291\ \mathrm{K} \\ T_{\mathrm{f}} &= (35.0+273)\mathrm{K} = 308\ \mathrm{K} \end{aligned}$$

2. Substitute the known values into the equation.

$$P_{\mathrm{f}} = P_0 \frac{T_{\mathrm{f}}}{T_0} = 7.00 \times 10^5\ \mathrm{Pa} \left(\frac{308\ \mathrm{K}}{291\ \mathrm{K}}\right) = 7.41 \times 10^5\ \mathrm{Pa}$$

Discussion

The final temperature is about 6% greater than the original temperature, so the final pressure is about 6% greater as well. Note that *absolute* pressure and *absolute* temperature must be used in the ideal gas law.

MAKING CONNECTIONS: TAKE-HOME EXPERIMENT—REFRIGERATING A BALLOON

Inflate a balloon at room temperature. Leave the inflated balloon in the refrigerator overnight. What happens to the balloon, and why?

Example 8.7.3.2: Calculating the Number of Molecules in a Cubic Meter of Gas

How many molecules are in a typical object, such as gas in a tire or water in a drink? We can use the ideal gas law to give us an idea of how large N typically is.

Calculate the number of molecules in a cubic meter of gas at standard temperature and pressure (STP), which is defined to be 0ºC and atmospheric pressure.

Strategy

Because pressure, volume, and temperature are all specified, we can use the ideal gas law $PV = NkT$, to find N.

Solution

1. Identify the knowns.

$$\begin{aligned} T &= 0^{\circ}\mathrm{C} = 273\ \mathrm{K} \\ P &= 1.01 \times 10^5\ \mathrm{Pa} \\ V &= 1.00\ \mathrm{m}^3 \\ k &= 1.38 \times 10^{-23}\ \mathrm{J/K} \end{aligned}$$

2. Identify the unknown: number of molecules, N.

3. Rearrange the ideal gas law to solve for N.

$$\begin{aligned} PV &= NkT \\ N &= \frac{PV}{kT} \end{aligned}$$

4. Substitute the known values into the equation and solve for N.

$$N = \frac{PV}{kT} = \frac{\left(1.01 \times 10^5\ \mathrm{Pa}\right)\left(1.00\ \mathrm{m}^3\right)}{\left(1.38 \times 10^{-23}\ \mathrm{J/K}\right)(273\ \mathrm{K})} = 2.68 \times 10^{25}\ \text{molecules}$$

Discussion

> The calculated number, 2.68×10^{25}, is certainly very large. You might say that the volume of a cubic meter is also large ($1\ \mathrm{m}^3 = 1000$), but even in a small volume of $1\ \mathrm{cm}^3$, which is about size of a thimble ($1\ \mathrm{cm}^3 = 10^{-6}\ \mathrm{m}^3$), a gas at STP has 2.68×10^{19} molecules in it (still a very large number). Once again, note that N is the same for all types or mixtures of gases.

Section Summary

- The ideal gas law relates the pressure and volume of a gas to the number of gas molecules and the temperature of the gas.
- The ideal gas law can be written in terms of the number of molecules of gas:

  $$PV = NkT,$$

  where P is pressure, V is volume, T is temperature, N is number of molecules, and k is the Boltzmann constant

  $$k = 1.38 \times 10^{-23}\ \mathrm{J/K}.$$

- The ideal gas law is generally valid at temperatures well above the boiling temperature.

Glossary

ideal gas law

the physical law that relates the pressure and volume of a gas to the number of gas molecules or number of moles of gas and the temperature of the gas

Boltzmann constant

k , a physical constant that relates energy to temperature; $k=1.38\times10^{-23}$ J/K

8.7.4: Heat

Learning Objectives

- Define heat as transfer of energy.

We have defined work as force times distance and learned that work done on an object changes its kinetic energy. We have also seen that temperature is proportional to the (average) kinetic energy of atoms and molecules. We say that a thermal system has a certain internal energy: its internal energy is higher if the temperature is higher. If two objects at different temperatures are brought in contact with each other, energy is transferred from the hotter to the colder object until equilibrium is reached and the bodies reach thermal equilibrium (i.e., they are at the same temperature). No work is done by either object, because no force acts through a distance. The transfer of energy is caused by the temperature difference, and ceases once the temperatures are equal. These observations lead to the following definition of **heat**: Heat is the spontaneous transfer of energy due to a temperature difference.

Heat is often confused with temperature. For example, we may say the heat was unbearable, when we actually mean that the temperature was high. Heat is a form of energy, whereas temperature is not. The misconception arises because we are sensitive to the flow of heat, rather than the temperature.

Owing to the fact that heat is a form of energy, it has the SI unit of *joule* (J). The *calorie* (cal) is a common unit of energy, defined as the energy needed to change the temperature of 1.00 g of water by 1.00ºC —specifically, between 14.5ºC and 15.5ºC, since there is a slight temperature dependence. Perhaps the most common unit of heat is the **kilocalorie** (kcal), which is the energy needed to change the temperature of 1.00 kg of water by 1.00ºC. Since mass is most often specified in kilograms, kilocalorie is commonly used. Food calories (given the notation Cal, and sometimes called “big calorie”) are actually kilocalories (1kilocalorie=1000 calories), a fact not easily determined from package labeling.

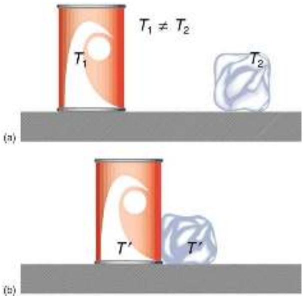

Figure 8.7.4.1: In figure (a) the soft drink and the ice have different temperatures, T_1 and T_2, and are not in thermal equilibrium. In figure (b), when the soft drink and ice are allowed to interact, energy is transferred until they reach the same temperature T', achieving equilibrium. Heat transfer occurs due to the difference in temperatures. In fact, since the soft drink and ice are both in contact with the surrounding air and bench, the equilibrium temperature will be the same for both.

Mechanical Equivalent of Heat

It is also possible to change the temperature of a substance by doing work. Work can transfer energy into or out of a system. This realization helped establish the fact that heat is a form of energy. James Prescott Joule (1818–1889) performed many experiments to establish the **mechanical equivalent of heat**—*the work needed to produce the same effects as heat transfer*. In terms of the units used for these two terms, the best modern value for this equivalence is

$$1.000 \text{ kcal} = 4186 \text{ J}.$$

We consider this equation as the conversion between two different units of energy.

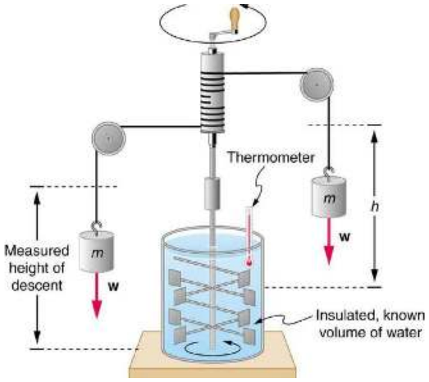

Figure 8.7.4.2: Schematic depiction of Joule's experiment that established the equivalence of heat and work.

The figure above shows one of Joule's most famous experimental setups for demonstrating the mechanical equivalent of heat. It demonstrated that work and heat can produce the same effects, and helped establish the principle of conservation of energy. Gravitational potential energy (PE) (work done by the gravitational force) is converted into kinetic energy (KE), and then randomized by viscosity and turbulence into increased average kinetic energy of atoms and molecules in the system, producing a temperature increase. His contributions to the field of thermodynamics were so significant that the SI unit of energy was named after him.

Heat added or removed from a system changes its internal energy and thus its temperature. Such a temperature increase is observed while cooking. However, adding heat does not necessarily increase the temperature. An example is melting of ice; that is, when a substance changes from one phase to another. Work done on the system or by the system can also change the internal energy of the system. Joule demonstrated that the temperature of a system can be increased by stirring. If an ice cube is rubbed against a rough surface, work is done by the frictional force. A system has a well-defined internal energy, but we cannot say that it has a certain "heat content" or "work content". We use the phrase "heat transfer" to emphasize its nature.

? Exercise 8.7.4.1

Two samples (A and B) of the same substance are kept in a lab. Someone adds 10 kilojoules (kJ) of heat to one sample, while 10 kJ of work is done on the other sample. How can you tell to which sample the heat was added?

Answer

Heat and work both change the internal energy of the substance. However, the properties of the sample only depend on the internal energy so that it is impossible to tell whether heat was added to sample A or B.

Summary

- Heat and work are the two distinct methods of energy transfer.
- Heat is energy transferred solely due to a temperature difference.
- Any energy unit can be used for heat transfer, and the most common are kilocalorie (kcal) and joule (J).
- Kilocalorie is defined to be the energy needed to change the temperature of 1.00 kg of water between 14.5ºC and 15.5ºC.
- The mechanical equivalent of this heat transfer is 1.00 kcal=4186 J.

Glossary

heat

the spontaneous transfer of energy due to a temperature difference

kilocalorie

$1 \text{ kilocalorie } = 1000 \text{ calories}$

mechanical equivalent of heat

the work needed to produce the same effects as heat transfer

8.7.5: Heat Transfer Methods

Learning Objectives

- Discuss the three different methods of heat transfer.

Equally as interesting as the effects of heat transfer on a system are the methods by which this occurs. Whenever there is a temperature difference, heat transfer occurs. Heat transfer may occur rapidly, such as through a cooking pan, or slowly, such as through the walls of a picnic ice chest. We can control rates of heat transfer by choosing materials (such as thick wool clothing for the winter), controlling air movement (such as the use of weather stripping around doors), or by choice of color (such as a white roof to reflect summer sunlight). So many processes involve heat transfer, so that it is hard to imagine a situation where no heat transfer occurs. Yet every process involving heat transfer takes place by only three methods:

1. **Conduction** is heat transfer through stationary matter by physical contact. (The matter is stationary on a macroscopic scale—we know there is thermal motion of the atoms and molecules at any temperature above absolute zero.) Heat transferred between the electric burner of a stove and the bottom of a pan is transferred by conduction.
2. **Convection** is the heat transfer by the macroscopic movement of a fluid. This type of transfer takes place in a forced-air furnace and in weather systems, for example.
3. Heat transfer by **radiation** occurs when microwaves, infrared radiation, visible light, or another form of electromagnetic radiation is emitted or absorbed. An obvious example is the warming of the Earth by the Sun. A less obvious example is thermal radiation from the human body.

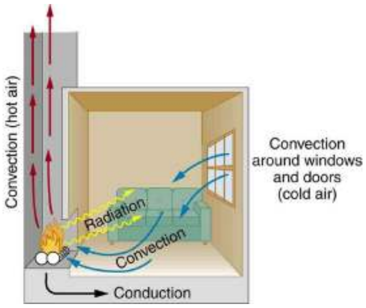

Figure $8.7.5.1$: In a fireplace, heat transfer occurs by all three methods: conduction, convection, and radiation. Radiation is responsible for most of the heat transferred into the room. Heat transfer also occurs through conduction into the room, but at a much slower rate. Heat transfer by convection also occurs through cold air entering the room around windows and hot air leaving the room by rising up the chimney.

Figure $8.7.5.1$ illustrates these three methods of heat transfer occurring in one system. While we skip detailed discussion of methods of heat transfer, the one thing all three methods-conduction, convection, and radiation-share is they are driven by temperature difference.

Exercise $8.7.5.1$

Name an example from daily life (different from the text) for each mechanism of heat transfer.

Answer

Conduction: Heat transfers into your hands as you hold a hot cup of coffee.

Convection: Heat transfers as the barista “steams” cold milk to make hot *cocoa*.

Radiation: Reheating a cold cup of coffee in a microwave oven.

Summary

- Heat is transferred by three different methods: conduction, convection, and radiation.

Glossary

conduction

heat transfer through stationary matter by physical contact

convection

heat transfer by the macroscopic movement of fluid

radiation

heat transfer which occurs when microwaves, infrared radiation, visible light, or other electromagnetic radiation is emitted or absorbed

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8.7.6: Temperature Change and Heat Capacity

Learning Objectives

- Observe heat transfer and change in temperature and mass.
- Calculate final temperature after heat transfer between two objects.

One of the major effects of heat transfer is temperature change: heating increases the temperature while cooling decreases it. We assume that there is no phase change and that no work is done on or by the system. Experiments show that the transferred heat depends on three factors—the change in temperature, the mass of the system, and the substance and phase of the substance.

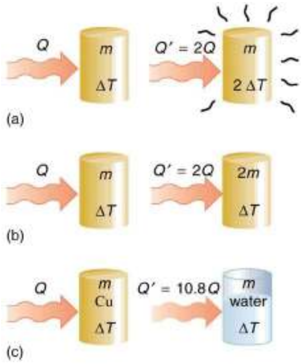


Figure 8.7.6.1: The heat Q transferred to cause a temperature change depends on the magnitude of the temperature change, the mass of the system, and the substance and phase involved. (a) The amount of heat transferred is directly proportional to the temperature change. To double the temperature change of a mass m, you need to add twice the heat. (b) The amount of heat transferred is also directly proportional to the mass. To cause an equivalent temperature change in a doubled mass, you need to add twice the heat. (c) The amount of heat transferred depends on the substance and its phase. If it takes an amount Q of heat to cause a temperature change ΔT in a given mass of copper, it will take 10.8 times that amount of heat to cause the equivalent temperature change in the same mass of water assuming no phase change in either substance.

The dependence on temperature change and mass are easily understood. Owing to the fact that the (average) kinetic energy of an atom or molecule is proportional to the absolute temperature, the internal energy of a system is proportional to the absolute temperature and the number of atoms or molecules. Owing to the fact that the transferred heat is equal to the change in the internal energy, the heat is proportional to the mass of the substance and the temperature change. The transferred heat also depends on the substance so that, for example, the heat necessary to raise the temperature is less for alcohol than for water. For the same substance, the transferred heat also depends on the phase (gas, liquid, or solid).

HEAT TRANSFER AND TEMPERATURE CHANGE

The quantitative relationship between heat transfer and temperature change contains all three factors:

$$Q = mc\Delta T,$$

where Q is the symbol for heat transfer, m is the mass of the substance, and ΔT is the change in temperature. The symbol c stands for **specific heat** and depends on the material and phase. The specific heat is the amount of heat necessary to change the temperature of 1.00 kg of mass by 1.00ºC. The specific heat c is a property of the substance; its SI unit is $\mathrm{J/(kg \cdot K)}$ or $\mathrm{J/\,(kg \cdot {}^\circ C)}$. Recall that the temperature change (ΔT) is the same in units of kelvin and degrees Celsius. If heat transfer is measured in kilocalories, then *the unit of specific heat* is $\mathrm{kcal/\,(kg \cdot {}^\circ C)}$.

Values of specific heat must generally be looked up in tables, because there is no simple way to calculate them. In general, the specific heat also depends on the temperature. Table 8.7.6.1 lists representative values of specific heat for various substances. Except for gases, the temperature and volume dependence of the specific heat of most substances is weak. We see from this table that the specific heat of water is five times that of glass and ten times that of iron, which means that it takes five times as much heat to raise the temperature of water the same amount as for glass and ten times as much heat to raise the temperature of water as for iron. In fact, water has one of the largest specific heats of any material, which is important for sustaining life on Earth.

✓ Example 8.7.6.1: Calculating the Required Heat: Heating Water in an Aluminum Pan

A 0.500 kg aluminum pan on a stove is used to heat 0.250 liters of water from $20.0^{\circ}\mathrm{C}$ to $80.0^{\circ}\mathrm{C}$. (a) How much heat is required? What percentage of the heat is used to raise the temperature of (b) the pan and (c) the water?

Strategy

The pan and the water are always at the same temperature. When you put the pan on the stove, the temperature of the water and the pan is increased by the same amount. We use the equation for the heat transfer for the given temperature change and mass of water and aluminum. The specific heat values for water and aluminum are given in Table 8.7.6.1.

Solution

Because water is in thermal contact with the aluminum, the pan and the water are at the same temperature.

1. Calculate the temperature difference:

$$\Delta T = T_{\mathrm{f}} - T_{\mathrm{i}} = 60.0^{\circ}\mathrm{C}.$$

2. Calculate the mass of water. Because the density of water is $1000\ \mathrm{kg/m^3}$, one liter of water has a mass of 1 kg, and the mass of 0.250 liters of water is $m_{\mathrm{w}} = 0.250\ \mathrm{kg}$.
3. Calculate the heat transferred to the water. Use the specific heat of water in Table 8.7.6.1:

$$Q_{\mathrm{w}} = m_{\mathrm{w}} c_{\mathrm{w}} \Delta T = (0.250\ \mathrm{kg})\,(4186\ \mathrm{J/kg^{\circ}C})\,(60.0^{\circ}\mathrm{C}) = 62.8\ \mathrm{kJ}.$$

4. Calculate the heat transferred to the aluminum. Use the specific heat for aluminum in Table 8.7.6.1:

$$Q_{\mathrm{Al}} = m_{\mathrm{Al}} c_{\mathrm{Al}} \Delta T = (0.500\ \mathrm{kg})\,(900\ \mathrm{J/kg^{\circ}C})\,(60.0^{\circ}\mathrm{C}) = 27.0 \times 10^4\ \mathrm{J} = 27.0\ \mathrm{kJ}.$$

5. Compare the percentage of heat going into the pan versus that going into the water. First, find the total transferred heat:

$$Q_{\mathrm{Total}} = Q_{\mathrm{W}} + Q_{\mathrm{Al}} = 62.8\ \mathrm{kJ} + 27.0\ \mathrm{kJ} = 89.8\ \mathrm{kJ}.$$

Thus, the amount of heat going into heating the pan is

$$\frac{27.0\ \mathrm{kJ}}{89.8\ \mathrm{kJ}} \times 100\% = 30.1\%,$$

and the amount going into heating the water is

$$\frac{62.8\ \mathrm{kJ}}{89.8\ \mathrm{kJ}} \times 100\% = 69.9\%.$$

Discussion

In this example, the heat transferred to the container is a significant fraction of the total transferred heat. Although the mass of the pan is twice that of the water, the specific heat of water is over four times greater than that of aluminum. Therefore, it takes a bit more than twice the heat to achieve the given temperature change for the water as compared to the aluminum pan.

Figure 8.7.6.2: The smoking brakes on this truck are a visible evidence of the mechanical equivalent of heat.

Example 8.7.6.2: Calculating the Temperature Increase from the Work Done on a Substance: Truck Brakes Overheat on Downhill Runs

Truck brakes used to control speed on a downhill run do work, converting gravitational potential energy into increased internal energy (higher temperature) of the brake material. This conversion prevents the gravitational potential energy from being converted into kinetic energy of the truck. The problem is that the mass of the truck is large compared with that of the brake material absorbing the energy, and the temperature increase may occur too fast for sufficient heat to transfer from the brakes to the environment.

Calculate the temperature increase of 100 kg of brake material with an average specific heat of $800\ \mathrm{J/kg \cdot {}^\circ C}$ if the material retains 10% of the energy from a 10,000-kg truck descending 75.0 m (in vertical displacement) at a constant speed.

Strategy

If the brakes are not applied, gravitational potential energy is converted into kinetic energy. When brakes are applied, gravitational potential energy is converted into internal energy of the brake material. We first calculate the gravitational potential energy (Mgh) that the entire truck loses in its descent and then find the temperature increase produced in the brake material alone.

Solution

1. Calculate the change in gravitational potential energy as the truck goes downhill

$$Mgh = (10,000\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right)(75.0\ \mathrm{m}) = 7.35 \times 10^6\ \mathrm{J}.$$

2. Calculate the temperature from the heat transferred using $Q = (0.10)Mgh$ (0.10 for 10% of energy being retained by brake material) and

$$\Delta T = \frac{Q}{mc},$$

where m is the mass of the brake material. Insert the values $m = 100\ \mathrm{kg}$ and $c = 800\ \mathrm{J/kg \cdot {}^\circ C}$ to find

$$\Delta T = \frac{\left(7.35 \times 10^5\ \mathrm{J}\right)}{(100\ \mathrm{kg})\left(800\ \mathrm{J/kg^\circ C}\right)} = 9.2^\circ\mathrm{C}.$$

Discussion

The same idea underlies the recent hybrid technology of cars, where mechanical energy (gravitational potential energy) is converted by the brakes into electrical energy (battery).

Table 8.7.6.1: Specific Heats[1] of Various Substances

Substances	Specific heat (c)	
Solids	J/kg·ºC	kcal/kg·ºC[2]
Aluminum	900	0.215
Asbestos	800	0.19
Concrete, granite (average)	840	0.20
Copper	387	0.0924
Glass	840	0.20
Gold	129	0.0308
Human body (average at 37 °C)	3500	0.83
Ice (average, -50°C to 0°C)	2090	0.50
Iron, steel	452	0.108
Lead	128	0.0305

Substances	Specific heat (c)	
Silver	235	0.0562
Wood	1700	0.4
Liquids		
Benzene	1740	0.415
Ethanol	2450	0.586
Glycerin	2410	0.576
Mercury	139	0.0333
Water (15.0 °C)	4186	1.000
Gases [3]		
Air (dry)	721 (1015)	0.172 (0.242)
Ammonia	1670 (2190)	0.399 (0.523)
Carbon dioxide	638 (833)	0.152 (0.199)
Nitrogen	739 (1040)	0.177 (0.248)
Oxygen	651 (913)	0.156 (0.218)
Steam (100°C)	1520 (2020)	0.363 (0.482)

Note that Example $8.7.6.2$ is an illustration of the mechanical equivalent of heat. Alternatively, the temperature increase could be produced by a blow torch instead of mechanically.

✓ Example $8.7.6.3$: Calculating the Final Temperature When Heat Is Transferred Between Two Bodies: Pouring Cold Water in a Hot Pan

Suppose you pour 0.250 kg of $20.0^{\circ}\mathrm{C}$ water (about a cup) into a 0.500-kg aluminum pan off the stove with a temperature of $150^{\circ}\mathrm{C}$. Assume that the pan is placed on an insulated pad and that a negligible amount of water boils off. What is the temperature when the water and pan reach thermal equilibrium a short time later?

Strategy

The pan is placed on an insulated pad so that little heat transfer occurs with the surroundings. Originally the pan and water are not in thermal equilibrium: the pan is at a higher temperature than the water. Heat transfer then restores thermal equilibrium once the water and pan are in contact. Because heat transfer between the pan and water takes place rapidly, the mass of evaporated water is negligible and the magnitude of the heat lost by the pan is equal to the heat gained by the water. The exchange of heat stops once a thermal equilibrium between the pan and the water is achieved. The heat exchange can be written as $|Q_{\text{hot}}| = Q_{\text{cold}}$.

Solution

1. Use the equation for heat transfer $Q = mc\Delta T$ to express the heat lost by the aluminum pan in terms of the mass of the pan, the specific heat of aluminum, the initial temperature of the pan, and the final temperature:
$$Q_{\text{hot}} = m_{\text{Al}} c_{\text{Al}} \left(T_{\text{f}} - 150^{\circ}\mathrm{C}\right).$$
2. Express the heat gained by the water in terms of the mass of the water, the specific heat of water, the initial temperature of the water and the final temperature:
$$Q_{\text{cold}} = m_{\text{W}} c_{\text{W}} \left(T_{\text{f}} - 20.0^{\circ}\mathrm{C}\right).$$
3. Note that $Q_{\text{hot}} < 0$ and $Q_{\text{cold}} > 0$ and that they must sum to zero because the heat lost by the hot pan must be the same as the heat gained by the cold water:

$$\begin{aligned} Q_{\text{cold}} + Q_{\text{hot}} &= 0, \\ Q_{\text{cold}} &= -\mathrm{Q}_{\text{hot}}\ , \\ m_{\mathrm{W}} c_{\mathrm{W}} \left(T_{\mathrm{f}} - 20.0^{\circ}\mathrm{C}\right) &= -\mathrm{m}_{\mathrm{Al}} c_{\mathrm{Al}} \left(T_{\mathrm{f}} - 150^{\circ}\mathrm{C}.\right) \end{aligned}$$

4. This an equation for the unknown final temperature, T_{f}
5. Bring all terms involving T_{f} on the left hand side and all other terms on the right hand side. Solve for T_{f},

$$T_{\mathrm{f}} = \frac{m_{\mathrm{Al}} c_{\mathrm{Al}} \left(150^{\circ}\mathrm{C}\right) + m_{\mathrm{W}} c_{\mathrm{W}} \left(20.0^{\circ}\mathrm{C}\right)}{m_{\mathrm{Al}} c_{\mathrm{Al}} + m_{\mathrm{W}} c_{\mathrm{W}}},$$

and insert the numerical values:

$$\begin{aligned} T_{\mathrm{f}} &= \frac{(0.500\ \mathrm{kg})\left(900\ \mathrm{J/kg^{\circ}C}\right)\left(150^{\circ}\mathrm{C}\right) + (0.250\ \mathrm{kg})\left(4186\ \mathrm{J/kg^{\circ}C}\right)\left(20.0^{\circ}\mathrm{C}\right)}{(0.500\ \mathrm{kg})\left(900\ \mathrm{J/kg^{\circ}C}\right) + (0.250\ \mathrm{kg})\left(4186\ \mathrm{J/kg^{\circ}C}\right)} \\ &= \frac{88430\ \mathrm{J}}{1496.5\ \mathrm{J/^{\circ}C}} \\ &= 59.1^{\circ}\mathrm{C}. \end{aligned}$$

Discussion

This is a typical *calorimetry* problem—two bodies at different temperatures are brought in contact with each other and exchange heat until a common temperature is reached. Why is the final temperature so much closer to 20.0ºC than 150ºC? The reason is that water has a greater specific heat than most common substances and thus undergoes a small temperature change for a given heat transfer. A large body of water, such as a lake, requires a large amount of heat to increase its temperature appreciably. This explains why the temperature of a lake stays relatively constant during a day even when the temperature change of the air is large. However, the water temperature does change over longer times (e.g., summer to winter).

TAKE-HOME EXPERIMENT: TEMPERATURE CHANGE OF LAND AND WATER

What heats faster, land or water?

To study differences in heat capacity:

- Place equal masses of dry sand (or soil) and water at the same temperature into two small jars. (The average density of soil or sand is about 1.6 times that of water, so you can achieve approximately equal masses by using 50% more water by volume.)
- Heat both (using an oven or a heat lamp) for the same amount of time.
- Record the final temperature of the two masses.
- Now bring both jars to the same temperature by heating for a longer period of time.
- Remove the jars from the heat source and measure their temperature every 5 minutes for about 30 minutes.

Which sample cools off the fastest? This activity replicates the phenomena responsible for land breezes and sea breezes.

? Exercise 8.7.6.1

If 25 kJ is necessary to raise the temperature of a block from 25ºC to 30ºC, how much heat is necessary to heat the block from 45ºC to 50ºC?

Answer

The heat transfer depends only on the temperature difference. Since the temperature differences are the same in both cases, the same 25 kJ is necessary in the second case.

Summary

- The transfer of heat Q that leads to a change ΔT in the temperature of a body with mass m is $Q = mc\Delta T$, where c is the specific heat of the material. This relationship can also be considered as the definition of specific heat.

Footnotes

- 1 The values for solids and liquids are at constant volume and at 25ºC, except as noted.
- 2 These values are identical in units of cal/g·ºC.
- 3 c_v at constant volume and at 20.0ºC, except as noted, and at 1.00 atm average pressure. Values in parentheses are c_p at a constant pressure of 1.00 atm.

Glossary

specific heat

the amount of heat necessary to change the temperature of 1.00 kg of a substance by 1.00 ºC

8.7.7: Phase Change and Latent Heat

Learning Objectives

- Describe phase changes.
- Explain the relationship between phase changes and heat transfer.

So far we have discussed temperature change due to heat transfer. No temperature change occurs from heat transfer if ice melts and becomes liquid water (i.e., during a phase change). For example, consider water dripping from icicles melting on a roof warmed by the Sun. Conversely, water freezes in an ice tray cooled by lower-temperature surroundings.

Figure 8.7.7.1: Heat from the air transfers to the ice causing it to melt. (credit: Mike Brand)

Energy is required to melt a solid because the cohesive bonds between the molecules in the solid must be broken apart such that, in the liquid, the molecules can move around at comparable kinetic energies; thus, there is no rise in temperature. Similarly, energy is needed to vaporize a liquid, because molecules in a liquid interact with each other via attractive forces. There is no temperature change until a phase change is complete. The temperature of a cup of soda initially at $0^{\circ}\mathrm{C}$ stays at $0^{\circ}\mathrm{C}$ until all the ice has melted. Conversely, energy is released during freezing and condensation, usually in the form of thermal energy. Work is done by cohesive forces when molecules are brought together. The corresponding energy must be given off (dissipated) to allow them to stay together Figure $8.7.7.2$

The energy involved in a phase change depends on two major factors: the number and strength of bonds or force pairs. The number of bonds is proportional to the number of molecules and thus to the mass of the sample. The strength of forces depends on the type of molecules. The heat Q required to change the phase of a sample of mass m is given by

$$Q=mL_{\mathrm{f}}\;(\mathrm{melting/freezing}),$$

$$Q=mL_{\mathrm{v}}\;(\mathrm{vaporization/condensation}),$$

where the latent heat of fusion, L_{f}, and latent heat of vaporization, L_{v}, are material constants that are determined experimentally. See (Table $8.7.7.1$).

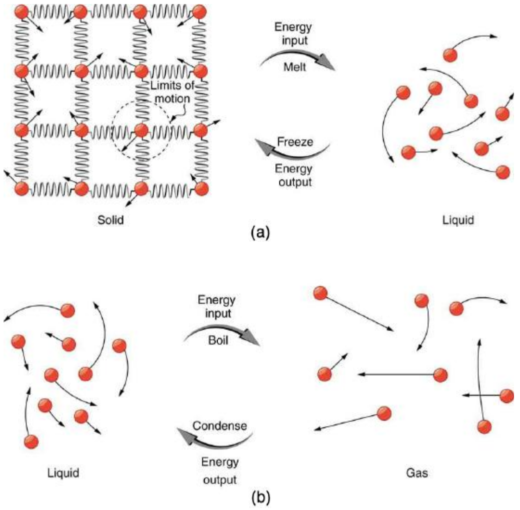


Figure 8.7.7.2: (a) Energy is required to partially overcome the attractive forces between molecules in a solid to form a liquid. That same energy must be removed for freezing to take place. (b) Molecules are separated by large distances when going from liquid to vapor, requiring significant energy to overcome molecular attraction. The same energy must be removed for condensation to take place. There is no temperature change until a phase change is complete.

Latent heat is measured in units of J/kg. Both L_f and L_v depend on the substance, particularly on the strength of its molecular forces as noted earlier. L_f and L_v are collectively called **latent heat coefficients**. They are *latent*, or hidden, because in phase changes, energy enters or leaves a system without causing a temperature change in the system; so, in effect, the energy is hidden. Table 8.7.7.1lists representative values of L_f and L_v, together with melting and boiling points.

The table shows that significant amounts of energy are involved in phase changes. Let us look, for example, at how much energy is needed to melt a kilogram of ice at 0°C to produce a kilogram of water at 0°C. Using the equation for a change in temperature and the value for water from Table 8.7.7.1, we find that $Q = mL_f = (1.0\text{ kg})(334\text{ kJ/kg}) = 334\text{ kJ}$ is the energy to melt a kilogram of ice. This is a lot of energy as it represents the same amount of energy needed to raise the temperature of 1 kg of liquid water from 0°C to 79.8ºC. Even more energy is required to vaporize water; it would take 2256 kJ to change 1 kg of liquid water at the normal boiling point (100°C at atmospheric pressure) to steam (water vapor). This example shows that the energy for a phase change is enormous compared to energy associated with temperature changes without a phase change.

Table 8.7.7.1: Heats of Fusion and Vaporization 1

Substance	Melting point (ºC)	L_f		Boiling point (°C)	L_v	
		kJ/kg	kcal/kg		kJ/kg	kcal/kg
Helium	−269.7	5.23	1.25	−268.9	20.9	4.99
Hydrogen	−259.3	58.6	14.0	−252.9	452	108
Nitrogen	−210.0	25.5	6.09	−195.8	201	48.0
Oxygen	−218.8	13.8	3.30	−183.0	213	50.9
Ethanol	−114	104	24.9	78.3	854	204
Ammonia	−75		108	−33.4	1370	327
Mercury	−38.9	11.8	2.82	357	272	65.0
Water	0.00	334	79.8	100.0	2256[2]	539[3]
Sulfur	119	38.1	9.10	444.6	326	77.9
Lead	327	24.5	5.85	1750	871	208
Antimony	631	165	39.4	1440	561	134
Aluminum	660	380	90	2450	11400	2720

Silver	961	88.3	21.1	2193	2336	558
Gold	1063	64.5	15.4	2660	1578	377
Copper	1083	134	32.0	2595	5069	1211
Uranium	1133	84	20	3900	1900	454
Tungsten	3410	184	44	5900	4810	1150

Phase changes can have a tremendous stabilizing effect even on temperatures that are not near the melting and boiling points, because evaporation and condensation (conversion of a gas into a liquid state) occur even at temperatures below the boiling point. Take, for example, the fact that air temperatures in humid climates rarely go above $35.0^{\circ}\mathrm{C}$, which is because most heat transfer goes into evaporating water into the air. Similarly, temperatures in humid weather rarely fall below the dew point because enormous heat is released when water vapor condenses.

We examine the effects of phase change more precisely by considering adding heat into a sample of ice at $-20^{\circ}\mathrm{C}$ (Figure $8.7.7.3$). The temperature of the ice rises linearly, absorbing heat at a constant rate of $0.50\ \mathrm{cal/g\cdot{}^{\circ}C}$ until it reaches $0^{\circ}\mathrm{C}$. Once at this temperature, the ice begins to melt until all the ice has melted, absorbing 79.8 cal/g of heat. The temperature remains constant at $0^{\circ}\mathrm{C}$ during this phase change. Once all the ice has melted, the temperature of the liquid water rises, absorbing heat at a new constant rate of $1.00\ \mathrm{cal/g\cdot{}^{\circ}C}$. At $100^{\circ}\mathrm{C}$, the water begins to boil and the temperature again remains constant while the water absorbs 539 cal/g of heat during this phase change. When all the liquid has become steam vapor, the temperature rises again, absorbing heat at a rate of $0.482\ \mathrm{cal/g\cdot{}^{\circ}C}$.

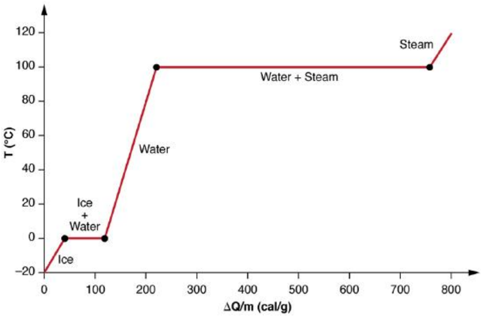


Figure $8.7.7.3$: A graph of temperature versus energy added. The system is constructed so that no vapor evaporates while ice warms to become liquid water, and so that, when vaporization occurs, the vapor remains in of the system. The long stretches of constant temperature values at $0^{\circ}\mathrm{C}$ and $100^{\circ}\mathrm{C}$ reflect the large latent heat of melting and vaporization, respectively.

Water can evaporate at temperatures below the boiling point. More energy is required than at the boiling point, because the kinetic energy of water molecules at temperatures below $100^{\circ}\mathrm{C}$ is less than that at $100^{\circ}\mathrm{C}$, hence less energy is available from random thermal motions. Take, for example, the fact that, at body temperature, perspiration from the skin requires a heat input of 2428 kJ/kg, which is about 10 percent higher than the latent heat of vaporization at $100^{\circ}\mathrm{C}$. This heat comes from the skin, and thus provides an effective cooling mechanism in hot weather. High humidity inhibits evaporation, so that body temperature might rise, leaving unevaporated sweat on your brow.

✓ Example $8.7.7.1$: Calculate Final Temperature from Phase Change: Cooling Soda with Ice Cubes

Three ice cubes are used to chill a soda at $20^{\circ}\mathrm{C}$ with $m_{\text{soda}} = 0.25\ \mathrm{kg}$. The ice is at $0^{\circ}\mathrm{C}$ and each ice cube has a mass of 6.0 g. Assume that the soda is kept in a foam container so that heat loss can be ignored. Assume the soda has the same heat capacity as water. Find the final temperature when all ice has melted.

Strategy

The ice cubes are at the melting temperature of $0^{\circ}\mathrm{C}$. Heat is transferred from the soda to the ice for melting. Melting of ice occurs in two steps: first the phase change occurs and solid (ice) transforms into liquid water at the melting temperature, then the temperature of this water rises. Melting yields water at $0^{\circ}\mathrm{C}$, so more heat is transferred from the soda to this water until the water plus soda system reaches thermal equilibrium,

$$Q_{\text{ice}} = -Q_{\text{soda}}.$$

The heat transferred to the ice is $Q_{\text{ice}} = m_{\text{ice}} L_{\text{f}} + m_{\text{ice}} c_{\text{W}} (T_{\text{f}} - 0^{\circ}\text{C})$. The heat given off by the soda is $Q_{\text{soda}} = m_{\text{soda}} c_{\text{W}} (T_{\text{f}} - 20^{\circ}\text{C})$. Since no heat is lost, $Q_{\text{ice}} = -Q_{\text{soda}}$, so that

$$m_{\text{ice}} L_{\text{f}} + m_{\text{ice}} c_{\text{W}} (T_{\text{f}} - 0^{\circ}\text{C}) = -m_{\text{soda}} c_{\text{W}} (T_{\text{f}} - 20^{\circ}\text{C}).$$

Bring all terms involving T_{f} on the left-hand-side and all other terms on the right-hand-side. Solve for the unknown quantity T_{f}:

$$T_{\text{f}} = \frac{m_{\text{soda}} c_{\text{W}} (20^{\circ}\text{C}) - m_{\text{ice}} L_{\text{f}}}{(m_{\text{soda}} + m_{\text{ice}}) c_{\text{W}}}.$$

Solution

1. Identify the known quantities. The mass of ice is $m_{\text{ice}} = 3 \times 6.0 \text{ g} = 0.018 \text{ kg}$ and the mass of soda is $m_{\text{soda}} = 0.25 \text{ kg}$.
2. Calculate the terms in the numerator:

$$m_{\text{soda}} c_{\text{W}} (20^{\circ}\text{C}) = (0.25 \text{ kg}) (4186 \text{ J/kg} \cdot {}^{\circ}\text{C}) (20^{\circ}\text{C}) = 20,930 \text{ J}$$

and

$$m_{\text{ice}} L_{\text{f}} = (0.018 \text{ kg})(334,000 \text{ J/kg}) = 6012 \text{ J}.$$

3. Calculate the denominator:

$$(m_{\text{soda}} + m_{\text{ice}}) c_{\text{W}} = (0.25 \text{ kg} + 0.018 \text{ kg}) (4186 \text{ K/} (\text{kg} \cdot {}^{\circ}\text{C}) = 1122 \text{ J/}^{\circ}\text{C}.$$

4. Calculate the final temperature:

$$T_{\text{f}} = \frac{20,930 \text{ J} - 6012 \text{ J}}{1122 \text{ J/}^{\circ}\text{C}} = 13^{\circ}\text{C}.$$

Discussion

This example illustrates the enormous energies involved during a phase change. The mass of ice is about 7 percent the mass of water but leads to a noticeable change in the temperature of soda. Although we assumed that the ice was at the freezing temperature, this is incorrect: the typical temperature is -6°C. However, this correction gives a final temperature that is essentially identical to the result we found. Can you explain why?

We have seen that vaporization requires heat transfer to a liquid from the surroundings, so that energy is released by the surroundings. Condensation is the reverse process, increasing the temperature of the surroundings. This increase may seem surprising, since we associate condensation with cold objects—the glass in the figure, for example. However, energy must be removed from the condensing molecules to make a vapor condense. The energy is exactly the same as that required to make the phase change in the other direction, from liquid to vapor, and so it can be calculated from $Q = mL_{\text{v}}$.

Figure 8.7.7.4: Condensation forms on this glass of iced tea because the temperature of the nearby air is reduced to below the dew point. The air cannot hold as much water as it did at room temperature, and so water condenses. Energy is released when the water condenses, speeding the melting of the ice in the glass. (credit: Jenny Downing)

REAL-WORLD APPLICATION

Energy is also released when a liquid freezes. This phenomenon is used by fruit growers in Florida to protect oranges when the temperature is close to the freezing point (0°C). Growers spray water on the plants in orchards so that the water freezes and heat is released to the growing oranges on the trees. This prevents the temperature inside the orange from dropping below freezing, which would damage the fruit.

Figure $8.7.7.5$: The ice on these trees released large amounts of energy when it froze, helping to prevent the temperature of the trees from dropping below 0°C. Water is intentionally sprayed on orchards to help prevent hard frosts. (credit: Hermann Hammer)

Sublimation is the transition from solid to vapor phase. You may have noticed that snow can disappear into thin air without a trace of liquid water, or the disappearance of ice cubes in a freezer. The reverse is also true: Frost can form on very cold windows without going through the liquid stage. A popular effect is the making of “smoke” from dry ice, which is solid carbon dioxide. Sublimation occurs because the equilibrium vapor pressure of solids is not zero. Certain air fresheners use the sublimation of a solid to inject a perfume into the room. Moth balls are a slightly toxic example of a phenol (an organic compound) that sublimates, while some solids, such as osmium tetroxide, are so toxic that they must be kept in sealed containers to prevent human exposure to their sublimation-produced vapors.

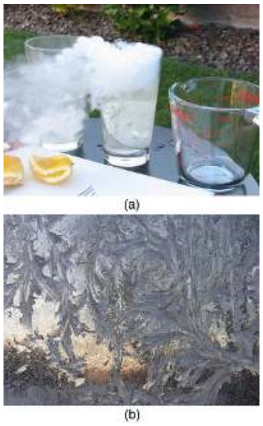


Figure $8.7.7.6$: Direct transitions between solid and vapor are common, sometimes useful, and even beautiful. (a) Dry ice sublimates directly to carbon dioxide gas. The visible vapor is made of water droplets. (credit: Windell Oskay) (b) Frost forms patterns on a very cold window, an example of a solid formed directly from a vapor. (credit: Liz West)

All phase transitions involve heat. In the case of direct solid-vapor transitions, the energy required is given by the equation $Q = mL_\text{s}$, where L_s is the **heat of sublimation**, which is the energy required to change 1.00 kg of a substance from the solid phase to the vapor phase. L_s is analogous to L_f and L_v, and its value depends on the substance. Sublimation requires energy input, so that dry ice is an effective coolant, whereas the reverse process (i.e., frosting) releases energy. The amount of energy required for sublimation is of the same order of magnitude as that for other phase transitions.

The material presented in this section and the preceding section allows us to calculate any number of effects related to temperature and phase change. In each case, it is necessary to identify which temperature and phase changes are taking place and then to apply the appropriate equation. Keep in mind that heat transfer and work can cause both temperature and phase changes.

? Exercise $8.7.7.1$

Why does snow remain on mountain slopes even when daytime temperatures are higher than the freezing temperature?

Answer

> Snow is formed from ice crystals and thus is the solid phase of water. Because enormous heat is necessary for phase changes, it takes a certain amount of time for this heat to be accumulated from the air, even if the air is above $0^{\circ}\mathrm{C}$. The warmer the air is, the faster this heat exchange occurs and the faster the snow melts.

Summary

- Most substances can exist either in solid, liquid, and gas forms, which are referred to as “phases.”
- Phase changes occur at fixed temperatures for a given substance at a given pressure, and these temperatures are called boiling and freezing (or melting) points.
- During phase changes, heat absorbed or released is given by:

$$Q = mL,$$

  where L is the latent heat coefficient.

Footnotes

- [1] Values quoted at the normal melting and boiling temperatures at standard atmospheric pressure (1 atm).
- [2] At $37.0^{\circ}\mathrm{C}$ (body temperature), the heat of vaporization L_{v} for water is 2430 kJ/kg or 580 kcal/kg
- [3] At $37.0^{\circ}\mathrm{C}$ (body temperature), the heat of vaporization L_{v} for water is 2430 kJ/kg or 580 kcal/kg

Glossary

heat of sublimation

the energy required to change a substance from the solid phase to the vapor phase

latent heat coefficient

a physical constant equal to the amount of heat transferred for every 1 kg of a substance during the change in phase of the substance

sublimation

the transition from the solid phase to the vapor phase

8.7.8: The First Law of Thermodynamics

Learning Objectives

- Define the first law of thermodynamics.
- Describe how conservation of energy relates to the first law of thermodynamics.
- Identify instances of the first law of thermodynamics working in everyday situations.

Figure 8.7.8.1: This boiling tea kettle represents energy in motion. The water in the kettle is turning to water vapor because heat is being transferred from the stove to the kettle. As the entire system gets hotter, work is done—from the evaporation of the water to the whistling of the kettle. (credit: Gina Hamilton)

If we are interested in how heat transfer is converted into doing work, then the conservation of energy principle is important. The first law of thermodynamics applies the conservation of energy principle to systems where heat transfer and doing work are the methods of transferring energy into and out of the system. The **first law of thermodynamics** states that the change in internal energy of a system equals the net heat transfer *into* the system minus the net work done *by* the system. In equation form, the first law of thermodynamics is

$$\Delta U = Q - W.$$

Here ΔU is the *change in internal energy* U of the system. Q is the *net heat transferred into the system*—that is, Q is the sum of all heat transfer into and out of the system. W is the *net work done by the system*—that is, W is the sum of all work done on or by the system. We use the following sign conventions: if Q is positive, then there is a net heat transfer into the system; if W is positive, then there is net work done by the system. So positive Q adds energy to the system and positive W takes energy from the system. Thus $\Delta U = Q - W$. Note also that if more heat transfer into the system occurs than work done, the difference is stored as internal energy. Heat engines are a good example of this—heat transfer into them takes place so that they can do work. (See Figure 8.7.8.2) We will now examine Q, W, and ΔU further.

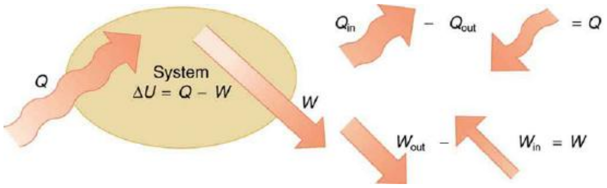


Figure 8.7.8.2: The first law of thermodynamics is the conservation-of-energy principle stated for a system where heat and work are the methods of transferring energy for a system in thermal equilibrium. Q represents the net heat transfer—it is the sum of all heat transfers into and out of the system. Q is positive for net heat transfer *into* the system. W is the total work done on and by the system. W is positive when more work is done *by* the system than on it. The change in the internal energy of the system, ΔU, is related to heat and work by the first law of thermodynamics, $\Delta U = Q - W$.

MAKING CONNECTIONS: LAW OF THERMODYNAMICS AND LAW OF CONSERVATION OF ENERGY

The first law of thermodynamics is actually the law of conservation of energy stated in a form most useful in thermodynamics. The first law gives the relationship between heat transfer, work done, and the change in internal energy of a system.

Heat Q and Work W

Heat transfer (Q) and doing work (W) are the two everyday means of bringing energy into or taking energy out of a system. The processes are quite different. Heat transfer, a less organized process, is driven by temperature differences. Work, a quite organized process, involves a macroscopic force exerted through a distance. Nevertheless, heat and work can produce identical results.For example, both can cause a temperature increase. Heat transfer into a system, such as when the Sun warms the air in a bicycle tire, can increase its temperature, and so can work done on the system, as when the bicyclist pumps air into the tire. Once the temperature increase has occurred, it is impossible to tell whether it was caused by heat transfer or by doing work. This uncertainty is an important point. Heat transfer and work are both energy in transit—neither is stored as such in a system. However, both can change the internal energy U of a system. Internal energy is a form of energy completely different from either heat or work.

Internal Energy U

We can think about the internal energy of a system in two different but consistent ways. The first is the atomic and molecular view, which examines the system on the atomic and molecular scale. The **internal energy** U of a system is the sum of the kinetic and potential energies of its atoms and molecules. Recall that kinetic plus potential energy is called mechanical energy. Thus internal energy is the sum of atomic and molecular mechanical energy. Because it is impossible to keep track of all individual atoms and molecules, we must deal with averages and distributions. A second way to view the internal energy of a system is in terms of its macroscopic characteristics, which are very similar to atomic and molecular average values.

Macroscopically, we define the change in internal energy ΔU to be that given by the first law of thermodynamics:

$$\Delta U = Q - W.$$

Many detailed experiments have verified that $\Delta U = Q - W$, where ΔU is the change in total kinetic and potential energy of all atoms and molecules in a system. It has also been determined experimentally that the internal energy U of a system depends only on the state of the system and *not how it reached that state.* More specifically, U is found to be a function of a few macroscopic quantities (pressure, volume, and temperature, for example), independent of past history such as whether there has been heat transfer or work done. This independence means that if we know the state of a system, we can calculate changes in its internal energy U from a few macroscopic variables.

MAKING CONNECTIONS: MACROSCOPIC AND MICROSCOPIC

In thermodynamics, we often use the macroscopic picture when making calculations of how a system behaves, while the atomic and molecular picture gives underlying explanations in terms of averages and distributions. We shall see this again in later sections of this chapter. For example, in the topic of entropy, calculations will be made using the atomic and molecular view.

Section Summary

- The first law of thermodynamics is given as $\Delta U = Q - W$, where ΔU is the change in internal energy of a system, Q is the net heat transfer (the sum of all heat transfer into and out of the system), and W is the net work done (the sum of all work done on or by the system).
- Both Q and W are energy in transit; only ΔU represents an independent quantity capable of being stored.
- The internal energy U of a system depends only on the state of the system and not how it reached that state.

Glossary

first law of thermodynamics

states that the change in internal energy of a system equals the net heat transfer *into* the system minus the net work done *by* the system

internal energy

the sum of the kinetic and potential energies of a system’s atoms and molecules

human metabolism

conversion of food into heat transfer, work, and stored fat

8.7.9: The First Law of Thermodynamics and Heat Engine Processes

Learning Objectives

- Describe the processes of a simple heat engine.
- Explain the differences among the simple thermodynamic processes—isobaric, isochoric, isothermal, and adiabatic.
- Explain the relationship between work done by a gas and change in its volume.

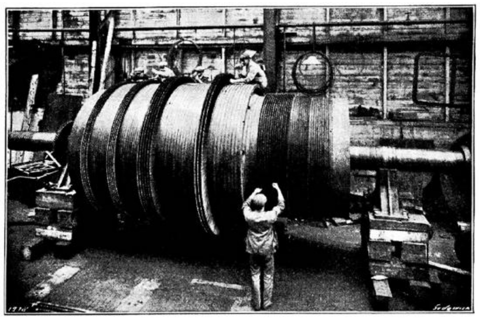


Figure $8.7.9.1$: Beginning with the Industrial Revolution, humans have harnessed power through the use of the first law of thermodynamics, before we even understood it completely. This photo, of a steam engine at the Turbinia Works, dates from 1911, a mere 61 years after the first explicit statement of the first law of thermodynamics by Rudolph Clausius. (credit: public domain; author unknown)

One of the most important things we can do with heat transfer is to use it to do work for us. Such a device is called a **heat engine**. Car engines and steam turbines that generate electricity are examples of heat engines. Figure $8.7.9.2$ shows schematically how the first law of thermodynamics applies to the typical heat engine.

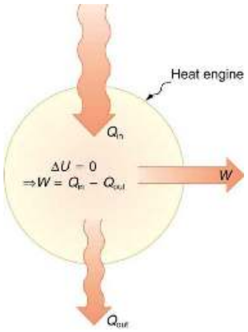


Figure $8.7.9.2$: Schematic representation of a heat engine, governed, of course, by the first law of thermodynamics (and other laws of thermodynamics we will discuss later).

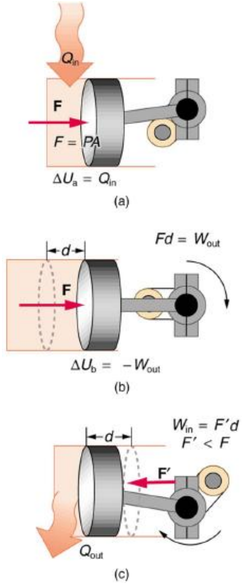


Figure 8.7.9.3: (a) Heat transfer to the gas in a cylinder increases the internal energy of the gas, creating higher pressure and temperature. (b) The force exerted on the movable cylinder does work as the gas expands. Gas pressure and temperature decrease when it expands, indicating that the gas's internal energy has been decreased by doing work. (c) Heat transfer to the environment further reduces pressure in the gas so that the piston can be more easily returned to its starting position.

The illustrations above show one of the ways in which heat transfer does work. Fuel combustion produces heat transfer to a gas in a cylinder, increasing the pressure of the gas and thereby the force it exerts on a movable piston. The gas does work on the outside world, as this force moves the piston through some distance. Heat transfer to the gas cylinder results in work being done. To repeat this process, the piston needs to be returned to its starting point. Heat transfer now occurs from the gas to the surroundings so that its pressure decreases, and a force is exerted by the surroundings to push the piston back through some distance. Variations of this process are employed daily in hundreds of millions of heat engines. Here, we consider some of the thermodynamic processes on which heat engines are based.

Work Done by a Gas

A process by which a gas does work on a piston at constant pressure is called an **isobaric process**. Since the pressure is constant, the force exerted is constant and the work done is given as

$$P\Delta V.$$

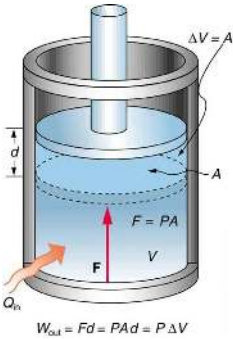


Figure 8.7.9.4: An isobaric expansion of a gas requires heat transfer to keep the pressure constant. Since pressure is constant, the work done is $P\Delta V$.

Recall from mechanics that work done by a force F on an object undergoing displacement d is

$$W = Fd.$$

See the symbols as shown in Figure 8.7.9.4 Now force is pressure times area ($F = PA$), and so

$$W = \text{PAd}.$$

Because the volume of a cylinder is its cross-sectional area A times its length d, we see that $Ad = \Delta V$, the change in volume; thus,

$$W = P\Delta V \text{ (isobaric process)}.$$

Note that if ΔV is positive, then W is positive, meaning that positive work is done by the gas on the outside world.

(Note that the pressure involved in this work that we've called P is the pressure of the gas *inside* the tank. If we call the pressure outside the tank P_{ext}, an expanding gas would be working *against* the external pressure; the work done would therefore be $W = -P_{\text{ext}}\,\Delta V$ (isobaric process). There are some—especially chemists—who use this definition of work, and not the definition based on internal pressure, as the basis of the First Law of Thermodynamics. This definition reverses the sign conventions for work, and results in a statement of the first law that becomes $\Delta U = Q + W$. In this textbook, we will use the *physics* convention of using work done by the system on the surrounding, not the other way around.)

This is the key lesson from the above derivation: *a gas expanding under pressure does work on its surrounding, and unless additional energy is added through heat transfer, the internal energy of the gas decreases*. We will examine the experimental results that come about as a consequence of this fact later.

Thermodynamic Processes

We introduced the **isobaric process** above in discussing work done by a gas. Isobaric process is an example of a **thermodynamic process**. A thermodynamic process describes a change that happens to a gas, which results in change in its pressure (P), volume (V), and/or temperature (T). An isobaric process is a thermodynamic process that takes place under constant pressure (so the volume and temperature of the gas may change in an isobaric process).

There are three more named thermodynamic processes. These processes are given special names because, like the isobaric process, they occur under some restrictions, which gives them their special properties, as described briefly below. These three additional named thermodynamic processes are: isochoric, isothermal, and adiabatic processes.

An **isochoric process** is a thermodynamic process in which no change in volume takes place. Because the work done by a gas is proportional to the change in volume, in an isochoric process, no work is done by (or on) the gas. Instead, in an isochoric process, a heat transfer takes place, and the energy from the heat transfer goes into increasing (or decreasing) the internal energy of the gas, increasing (or decreasing) its temperature.

An **isothermal process** is a thermodynamic process in which no change in temperature takes place. A gas expanding isothermally, for example, does work on the surrounding, but its internal energy (as represented by the temperature) does not change, because enough heat flows in to balance out the energy expended in doing work. This is consistent with the first law of thermodynamics (

$0 = \Delta U = Q - W$, because $Q = W$). An isothermal process occurs if a thermodynamic process in a gas occurs slowly enough so that the gas remains in thermal equilibrium with the surrounding at all times.

The adiabatic process is, in some sense, the opposite of an isothermal process. In an **adiabatic process**, no heat transfer takes place (that is, $Q = 0$). This may happen because the gas is well-insulated from the surrounding. It may also happen because the process occurs so quickly that no significant heat transfer can take place. In an adiabatic expansion, for example, the internal energy of the gas decreases, because of the work done by the gas in expansion. This is perhaps the clearest experimental evidence one can observe that it takes work for a gas to expand under pressure.

> Figure $8.7.9.5$illustrates these three processes on a plot of pressure and volume (a PV diagram). In an isothermal process, as the gas expands, the pressure decreases. This can be predicted from the ideal gas law ($PV = NkT$). Since the temperature is constant, if volume increases, the pressure must decrease, to keep PV constant. You can also see that in an adiabatic process, the pressure decreases with expanding volume more steeply than an isothermal process, because in an adiabatic process, the temperature is not constant, but it decreases. So with increasing V, the pressure decrease even more rapidly, so that PV actually decreases (for decreasing temperature).

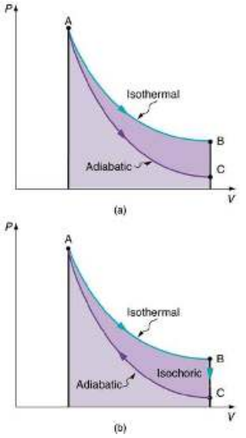


Figure $8.7.9.5$: (a) The upper curve is an isothermal process ($\Delta T = 0$), whereas the lower curve is an adiabatic process ($Q = 0$). Both start from the same point A, but the isothermal process does more work than the adiabatic because heat transfer into the gas takes place to keep its temperature constant. This keeps the pressure higher all along the isothermal path than along the adiabatic path, producing more work. The adiabatic path thus ends up with a lower pressure and temperature at point C, even though the final volume is the same as for the isothermal process. (b) The cycle ABCA produces a net work output.

Reversible Processes

Both isothermal and adiabatic processes such as shown in Figure $8.7.9.5$are reversible in principle. A **reversible process** is one in which both the system and its environment can return to exactly the states they were in by following the reverse path. The reverse isothermal and adiabatic paths are BA and CA, respectively. Real macroscopic processes are never exactly reversible. In the previous examples, our system is a gas (like that in Figure $8.7.9.4$), and its environment is the piston, cylinder, and the rest of the universe. If there are any energy-dissipating mechanisms, such as friction or turbulence, then heat transfer to the environment occurs for either direction of the piston. So, for example, if the path BA is followed and there is friction, then the gas will be returned to its original state but the environment will not—it will have been heated in both directions. Reversibility requires the direction of heat transfer to reverse for the reverse path. Since dissipative mechanisms cannot be completely eliminated, real processes cannot be reversible.

There must be reasons that real macroscopic processes cannot be reversible. We can imagine them going in reverse. For example, heat transfer occurs spontaneously from hot to cold and never spontaneously the reverse. Yet it would not violate the first law of thermodynamics for this to happen. In fact, all spontaneous processes, such as bubbles bursting, never go in reverse. There is a second thermodynamic law that forbids them from going in reverse. When we study this law, we will learn something about nature and also find that such a law limits the efficiency of heat engines. We will find that heat engines with the greatest possible theoretical efficiency would have to use reversible processes, and even they cannot convert all heat transfer into doing work. Table $8.7.9.1$summarizes the simpler thermodynamic processes and their definitions.

Table $8.7.9.1$: Summary of Simple Thermodynamic Processes

Isobaric	Constant pressure $W = P\Delta V$
Isochoric	Constant volume $W = 0$
Isothermal	Constant temperature $Q = W$
Adiabatic	No heat transfer $Q = 0$

Section Summary

- One of the important implications of the first law of thermodynamics is that machines can be harnessed to do work that humans previously did by hand or by external energy supplies such as running water or the heat of the Sun. A machine that uses heat transfer to do work is known as a heat engine.
- There are several simple processes, used by heat engines, that flow from the first law of thermodynamics. Among them are the isobaric, isochoric, isothermal and adiabatic processes.
- These processes differ from one another based on how they affect pressure, volume, temperature, and heat transfer.
- If the work done is performed on the outside environment, work (W) will be a positive value. If the work done is done to the heat engine system, work (W) will be a negative value.
- Some thermodynamic processes, including isothermal and adiabatic processes, are reversible in theory; that is, both the thermodynamic system and the environment can be returned to their initial states. However, because of loss of energy owing to the second law of thermodynamics, complete reversibility does not work in practice.

Glossary

first law of thermodynamics

states that the change in internal energy of a system equals the net heat transfer *into* the system minus the net work done *by* the system

internal energy

the sum of the kinetic and potential energies of a system’s atoms and molecules

human metabolism

conversion of food into heat transfer, work, and stored fat

8.7.10: Introduction to the Second Law of Thermodynamics- Heat Engines and Their Efficiency

Learning Objectives

- State the expressions of the second law of thermodynamics.
- Calculate the efficiency of a coal-fired electricity plant, using second law characteristics.
- Describe and define the Otto cycle.

Figure 8.7.10.1: These ice floes melt during the Arctic summer. Some of them refreeze in the winter, but the second law of thermodynamics predicts that it would be extremely unlikely for the water molecules contained in these particular floes to reform the distinctive alligator-like shape they formed when the picture was taken in the summer of 2009. (credit: Patrick Kelley, U.S. Coast Guard, U.S. Geological Survey)

The second law of thermodynamics deals with the direction taken by spontaneous processes. Many processes occur spontaneously in one direction only—that is, they are irreversible, under a given set of conditions. Here are some examples of irreversible processes seen in day-to-day life: a glass breaking, heat transferring from a hot object to a cold object, conversion of kinetic energy into thermal energy, and a puff of gas expanding from a corner of a vacuum chamber into the entire chamber. If you saw a video of glass un-breaking, or two objects in thermal contact where one gets hotter and the other gets colder, or a hot stationary object spontaneously cooling off and moving in some direction, or a gas in a chamber "regrouping" into a corner, you would say this is a video run backwards. This is because you intuitively recognize these are examples of **irreversible process**, and these processes in nature only take place in the forward direction, not reverse (See Figure $8.7.10.2$).

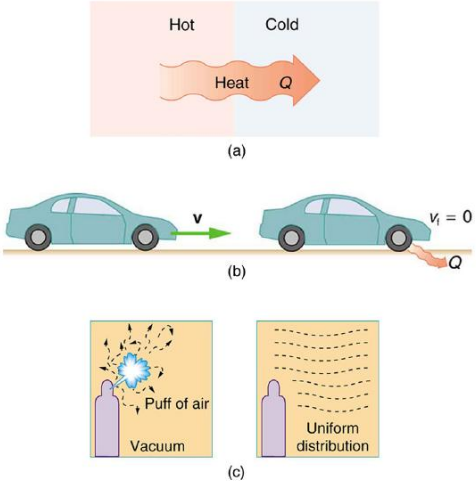


Figure 8.7.10.2: Examples of one-way processes in nature. (a) Heat transfer occurs spontaneously from hot to cold and not from cold to hot. (b) The brakes of this car convert its kinetic energy to heat transfer to the environment. The reverse process is impossible. (c) The burst of gas let into this vacuum chamber quickly expands to uniformly fill every part of the chamber. The random motions of the gas molecules will never return them to the corner.

The fact that certain processes never occur suggests that there is a law forbidding them to occur. The first law of thermodynamics would allow them to occur—none of those processes violate conservation of energy. The law that forbids these processes is called the second law of thermodynamics. We shall see that the second law can be stated in many ways that may seem different, but which in fact are equivalent. Like all natural laws, the second law of thermodynamics gives insights into nature, and its several statements imply that it is broadly applicable, fundamentally affecting many apparently disparate processes.

The already familiar direction of heat transfer from hot to cold is the basis of our first version of the **second law of thermodynamics**.

THE SECOND LAW OF THERMODYNAMICS (FIRST EXPRESSION)

> Heat transfer occurs spontaneously from higher- to lower-temperature bodies but never spontaneously in the reverse direction.

Another way of stating this: It is impossible for any process to have as its sole result heat transfer from a cooler to a hotter object.

Heat Engines

Now let us consider a device that uses heat transfer to do work. As noted in the previous section, such a device is called a heat engine, and one is shown schematically in Figure 8.7.10.3(b). Gasoline and diesel engines, jet engines, and steam turbines are all heat engines that do work by using part of the heat transfer from some source. Heat transfer from the hot object (or hot reservoir) is denoted as Q_h, while heat transfer into the cold object (or cold reservoir) is Q_c, and the work done by the engine is W. The temperatures of the hot and cold reservoirs are T_h and T_c, respectively.

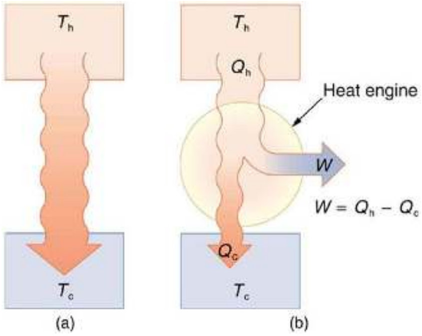


Figure 8.7.10.3: (a) Heat transfer occurs spontaneously from a hot object to a cold one, consistent with the second law of thermodynamics. (b) A heat engine, represented here by a circle, uses part of the heat transfer to do work. The hot and cold objects are called the hot and cold reservoirs. Q_h is the heat transfer out of the hot reservoir, W is the work output, and Q_c is the heat transfer into the cold reservoir.

Because the hot reservoir is heated externally, which is energy intensive, it is important that the work is done as efficiently as possible. In fact, we would like W to equal Q_h, and for there to be no heat transfer to the environment ($Q_\text{c} = 0$). Unfortunately, this is impossible. The **second law of thermodynamics** also states, with regard to using heat transfer to do work (the second expression of the second law):

THE SECOND LAW OF THERMODYNAMICS (SECOND EXPRESSION)

> It is impossible in any system for heat transfer from a reservoir to completely convert to work in a cyclical process in which the system returns to its initial state.

A **cyclical process** brings a system, such as the gas in a cylinder, back to its original state at the end of every cycle. Most heat engines, such as reciprocating piston engines and rotating turbines, use cyclical processes. The second law, just stated in its second form, clearly states that such engines cannot have perfect conversion of heat transfer into work done. Before going into the underlying reasons for the limits on converting heat transfer into work, we need to explore the relationships among W, Q_h, and Q_c, and to define the efficiency of a cyclical heat engine. As noted, a cyclical process brings the system back to its original condition at the end of every cycle. Such a system's internal energy U is the same at the beginning and end of every cycle—that is, $\Delta U = 0$. The first law of thermodynamics states that

$$\Delta U = Q - W,$$

where Q is the *net* heat transfer during the cycle ($Q = Q_\text{h} - Q_\text{c}$) and W is the net work done by the system. Since $\Delta U = 0$ for a complete cycle, we have

$$0 = Q - W,$$

so that

$$W = Q.$$

Thus the net work done by the system equals the net heat transfer into the system, or

$$W = Q_\mathrm{h} - Q_\mathrm{c} \text{ (cyclical process) },$$

just as shown schematically in Figure $8.7.10.3$(b). The problem is that in all processes, there is some heat transfer Q_c to the environment—and usually a very significant amount at that.

In the conversion of energy to work, we are always faced with the problem of getting less out than we put in. We define *conversion efficiency* Eff to be the ratio of useful work output to the energy input (or, in other words, the ratio of what we get to what we spend). In that spirit, we define the efficiency of a heat engine to be its net work output W divided by heat transfer to the engine Q_h; that is,

$$Eff = \frac{W}{Q_\mathrm{h}}.$$

Since $W = Q_\mathrm{h} - Q_\mathrm{c}$ in a cyclical process, we can also express this as

$$Eff = \frac{Q_\mathrm{h} - Q_\mathrm{c}}{Q_\mathrm{h}} = 1 - \frac{Q_\mathrm{c}}{Q_\mathrm{h}} \text{ (cyclical process) },$$

making it clear that an efficiency of 1, or 100%, is possible only if there is no heat transfer to the environment ($Q_\mathrm{c} = 0$). Note that all Qs are positive. The direction of heat transfer is indicated by a plus or minus sign. For example, Q_c is out of the system and so is preceded by a minus sign.

✓ Example $8.7.10.1$: Daily Work Done by a Coal-Fired Power Station, Its Efficiency and Carbon Dioxide Emissions

A coal-fired power station is a huge heat engine. It uses heat transfer from burning coal to do work to turn turbines, which are used to generate electricity. In a single day, a large coal power station has $2.50 \times 10^{14}\text{ J}$ of heat transfer from coal and $1.48 \times 10^{14}\text{ J}$ of heat transfer into the environment. (a) What is the work done by the power station? (b) What is the efficiency of the power station? (c) In the combustion process, the following chemical reaction occurs: $\mathrm{C} + \mathrm{O_2} \rightarrow \mathrm{CO_2}$. This implies that every 12 kg of coal puts 12 kg + 16 kg + 16 kg = 44 kg of carbon dioxide into the atmosphere. Assuming that 1 kg of coal can provide $2.5 \times 10^{6}\text{ J}$ of heat transfer upon combustion, how much $\mathrm{CO_2}$ is emitted per day by this power plant?

Strategy for (a)

We can use $W = Q_\mathrm{h} - Q_\mathrm{c}$ to find the work output W, assuming a cyclical process is used in the power station. In this process, water is boiled under pressure to form high-temperature steam, which is used to run steam turbine-generators, and then condensed back to water to start the cycle again.

Solution for (a)

Work output is given by:

$$W = Q_\mathrm{h} - Q_\mathrm{c}.$$

Substituting the given values:

$$\begin{aligned} W &= 2.50 \times 10^{14}\text{ J} - 1.48 \times 10^{14}\text{ J} \\ &= 1.02 \times 10^{14}\text{ J}. \end{aligned}$$

Strategy for (b)

The efficiency can be calculated with $Eff = \frac{W}{Q_\mathrm{h}}$ since Q_h is given and work W was found in the first part of this example.

Solution for (b)

Efficiency is given by: $Eff = \frac{W}{Q_h}$ since Q_h. The work W was just found to be 1.02×10^{14} J, and Q_h is given, so the efficiency is

$$\begin{aligned} Eff &= \frac{1.02 \times 10^{14} \text{ J}}{2.50 \times 10^{14} \text{ J}} \\ &= 0.408, \text{ or } 40.8\% \end{aligned}$$

3Strategy for (c)

The daily consumption of coal is calculated using the information that each day there is 2.50×10^{14} J of heat transfer from coal. In the combustion process, we have $C + O_2 \rightarrow CO_2$. So every 12 kg of coal puts 12 kg + 16 kg + 16 kg = 44 kg of CO_2 into the atmosphere.

Solution for (c)

The daily coal consumption is

$$\frac{2.50 \times 10^{14} \text{ J}}{2.50 \times 10^{6} \text{ J/kg}} = 1.0 \times 10^{8} \text{ kg}.$$

Assuming that the coal is pure and that all the coal goes toward producing carbon dioxide, the carbon dioxide produced per day is

$$1.0 \times 10^{8} \text{ kg coal } \times \frac{44 \text{ kg}CO_2}{12 \text{ kg coal}} = 3.7 \times 10^{8} \text{ kg}CO_2.$$

This is 370,000 metric tons of CO_2 produced every day.

Discussion

If all the work output is converted to electricity in a period of one day, the average power output is 1180 MW (this is left to you as an end-of-chapter problem). This value is about the size of a large-scale conventional power plant. The efficiency found is acceptably close to the value of 42% given for coal power stations. It means that fully 59.2% of the energy is heat transfer to the environment, which usually results in warming lakes, rivers, or the ocean near the power station, and is implicated in a warming planet generally. While the laws of thermodynamics limit the efficiency of such plants—including plants fired by nuclear fuel, oil, and natural gas—the heat transfer to the environment could be, and sometimes is, used for heating homes or for industrial processes. The generally low cost of energy has not made it economical to make better use of the waste heat transfer from most heat engines. Coal-fired power plants produce the greatest amount of CO_2 per unit energy output (compared to natural gas or oil), making coal the least efficient fossil fuel.

With the information given in Example 8.7.10.1, we can find characteristics such as the efficiency of a heat engine without any knowledge of how the heat engine operates, but looking further into the mechanism of the engine will give us greater insight. Figure 8.7.10.4 illustrates the operation of the common four-stroke gasoline engine. The four steps shown complete this heat engine’s cycle, bringing the gasoline-air mixture back to its original condition.

The **Otto cycle** shown in Figure 8.7.10.5(a) is used in four-stroke internal combustion engines, although in fact the true Otto cycle paths do not correspond exactly to the strokes of the engine.

The adiabatic process AB corresponds to the nearly adiabatic compression stroke of the gasoline engine. In both cases, work is done on the system (the gas mixture in the cylinder), increasing its temperature and pressure. Along path BC of the Otto cycle, heat transfer Q_h into the gas occurs at constant volume, causing a further increase in pressure and temperature. This process corresponds to burning fuel in an internal combustion engine, and takes place so rapidly that the volume is nearly constant. Path CD in the Otto cycle is an adiabatic expansion that does work on the outside world, just as the power stroke of an internal combustion engine does in its nearly adiabatic expansion. The work done by the system along path CD is greater than the work done on the system along path AB, because the pressure is greater, and so there is a net work output. Along path DA in the Otto cycle, heat transfer Q_c from the gas at constant volume reduces its temperature and pressure, returning it to its original state. In an internal combustion engine, this process corresponds to the exhaust of hot gases and the intake of an air-gasoline mixture at a considerably lower temperature. In both cases, heat transfer into the environment occurs along this final path.

The net work done by a cyclical process is the area inside the closed path on a PV diagram, such as that inside path ABCDA in Figure $8.7.10.5$. Note that in every imaginable cyclical process, it is absolutely necessary for heat transfer from the system to occur in order to get a net work output. In the Otto cycle, heat transfer occurs along path DA. If no heat transfer occurs, then the return path is the same, and the net work output is zero. The lower the temperature on the path AB, the less work has to be done to compress the gas. The area inside the closed path is then greater, and so the engine does more work and is thus more efficient. Similarly, the higher the temperature along path CD, the more work output there is. (See Figure $8.7.10.6$) So efficiency is related to the temperatures of the hot and cold reservoirs. In the next section, we shall see what the absolute limit to the efficiency of a heat engine is, and how it is related to temperature.

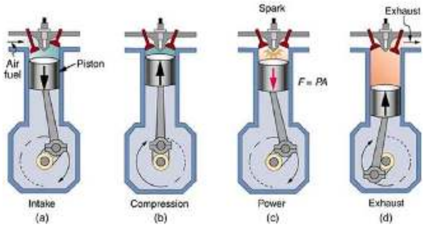


Figure 8.7.10.4: In the four-stroke internal combustion gasoline engine, heat transfer into work takes place in the cyclical process shown here. The piston is connected to a rotating crankshaft, which both takes work out of and does work on the gas in the cylinder. (a) Air is mixed with fuel during the intake stroke. (b) During the compression stroke, the air-fuel mixture is rapidly compressed in a nearly adiabatic process, as the piston rises with the valves closed. Work is done on the gas. (c) The power stroke has two distinct parts. First, the air-fuel mixture is ignited, converting chemical potential energy into thermal energy almost instantaneously, which leads to a great increase in pressure. Then the piston descends, and the gas does work by exerting a force through a distance in a nearly adiabatic process. (d) The exhaust stroke expels the hot gas to prepare the engine for another cycle, starting again with the intake stroke.

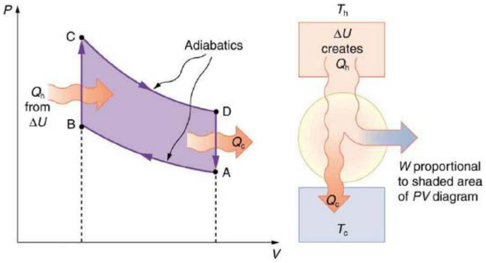


Figure 8.7.10.5: PV diagram for a simplified Otto cycle, analogous to that employed in an internal combustion engine. Point A corresponds to the start of the compression stroke of an internal combustion engine. Paths AB and CD are adiabatic and correspond to the compression and power strokes of an internal combustion engine, respectively. Paths BC and DA are isochoric and accomplish similar results to the ignition and exhaust-intake portions, respectively, of the internal combustion engine's cycle. Work is done on the gas along path AB, but more work is done by the gas along path CD, so that there is a net work output.

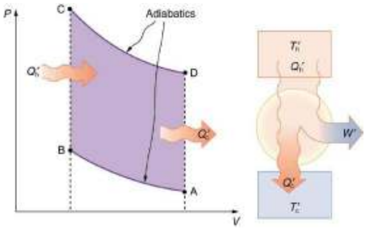


Figure 8.7.10.6: This Otto cycle produces a greater work output than the one in Figure 8.7.10.5, because the starting temperature of path CD is higher and the starting temperature of path AB is lower. The area inside the loop is greater, corresponding to greater net work output.

Section Summary

- The two expressions of the second law of thermodynamics are: (i) Heat transfer occurs spontaneously from higher- to lower-temperature bodies but never spontaneously in the reverse direction; and (ii) It is impossible in any system for heat transfer from a reservoir to completely convert to work in a cyclical process in which the system returns to its initial state. These expressions describe the direction of irreversible processes that occur in nature.
- Cyclical processes are processes that return to their original state at the end of every cycle.
- In a cyclical process, such as a heat engine, the net work done by the system equals the net heat transfer into the system, or $W=Q_{\mathrm{h}}-Q_{\mathrm{c}}$, where Q_{h} is the heat transfer from the hot object (hot reservoir), and Q_c is the heat transfer into the cold object (cold reservoir).

- Efficiency can be expressed as $Eff = \frac{W}{Q_\text{h}}$, the ratio of work output divided by the amount of energy input.
- The four-stroke gasoline engine is often explained in terms of the Otto cycle, which is a repeating sequence of processes that convert heat into work.

Glossary

irreversible process

a process which occurs in only one direction in nature; a process that cannot be exactly reversed

second law of thermodynamics

heat transfer flows from a hotter to a cooler object, never the reverse, and some heat energy in any process is lost to available work in a cyclical process

cyclical process

a process in which the path returns to its original state at the end of every cycle

Otto cycle

a thermodynamic cycle, consisting of a pair of adiabatic processes and a pair of isochoric processes, that converts heat into work, e.g., the four-stroke engine cycle of intake, compression, ignition, and exhaust

8.7.11: Carnot’s Perfect Heat Engine- The Second Law of Thermodynamics Restated

Learning Objectives

- Identify a Carnot cycle.
- Calculate maximum theoretical efficiency of a nuclear reactor.
- Explain how dissipative processes affect the ideal Carnot engine.

Figure 8.7.11.1: This novelty toy, known as the drinking bird, is an example of Carnot’s engine. It contains methylene chloride (mixed with a dye) in the abdomen, which boils at a very low temperature—about 100ºF. To operate, one gets the bird’s head wet. As the water evaporates, fluid moves up into the head, causing the bird to become top-heavy and dip forward back into the water. This cools down the methylene chloride in the head, and it moves back into the abdomen, causing the bird to become bottom heavy and tip up. Except for a very small input of energy—the original head-wetting—the bird becomes a perpetual motion machine of sorts. (credit: Arabesk.nl, Wikimedia Commons)

We know from the second law of thermodynamics that a heat engine cannot be 100% efficient, since there must always be some heat transfer Q_c to the environment, which is often called waste heat. How efficient, then, can a heat engine be? This question was answered at a theoretical level in 1824 by a young French engineer, Sadi Carnot (1796–1832), in his study of the then-emerging heat engine technology crucial to the Industrial Revolution. He devised a theoretical cycle, now called the **Carnot cycle**, which is the most efficient cyclical process possible. The second law of thermodynamics can be restated in terms of the Carnot cycle, and so what Carnot actually discovered was this fundamental law. Any heat engine employing the Carnot cycle is called a **Carnot engine**.

What is crucial to the Carnot cycle—and, in fact, defines it—is that only reversible processes are used. Irreversible processes involve dissipative factors, such as friction and turbulence. This increases heat transfer Q_c to the environment and reduces the efficiency of the engine. Obviously, then, reversible processes are superior.

CARNOT ENGINE

Stated in terms of reversible processes, the **second law of thermodynamics** has a third form:

> A Carnot engine operating between two given temperatures has the greatest possible efficiency of any heat engine operating between these two temperatures. Furthermore, all engines employing only reversible processes have this same maximum efficiency when operating between the same given temperatures.

Figure $8.7.11.2$shows the PV diagram for a Carnot cycle. The cycle comprises two isothermal and two adiabatic processes. Recall that both isothermal and adiabatic processes are, in principle, reversible.

Carnot also determined the efficiency of a perfect heat engine—that is, a Carnot engine. It is always true that the efficiency of a cyclical heat engine is given by:

$$Eff=\frac{Q_{\mathrm{h}}-Q_{\mathrm{c}}}{Q_{\mathrm{h}}}=1-\frac{Q_{\mathrm{c}}}{Q_{\mathrm{h}}}.$$

What Carnot found was that for a perfect heat engine, the ratio $Q_{\mathrm{c}}/Q_{\mathrm{h}}$ equals the ratio of the absolute temperatures of the heat reservoirs. That is, $Q_{\mathrm{c}}/Q_{\mathrm{h}}=T_{\mathrm{c}}/T_{\mathrm{h}}$ for a Carnot engine, so that the maximum or **Carnot efficiency** Eff_{C} is given by

$$Eff_{\mathrm{C}} = 1 - \frac{T_{\mathrm{c}}}{T_{\mathrm{h}}},$$

where T_{h} and T_{c} are in kelvins (or any other absolute temperature scale). No real heat engine can do as well as the Carnot efficiency—an actual efficiency of about 0.7 of this maximum is usually the best that can be accomplished. But the ideal Carnot engine, like the drinking bird above, while a fascinating novelty, has zero power. This makes it unrealistic for any applications.

Carnot's interesting result implies that 100% efficiency would be possible only if $T_{\mathrm{c}} = 0\,\mathrm{K}$ —that is, only if the cold reservoir were at absolute zero, a practical and theoretical impossibility. But the physical implication is this—the only way to have all heat transfer go into doing work is to remove *all* thermal energy, and this requires a cold reservoir at absolute zero.

It is also apparent that the greatest efficiencies are obtained when the ratio $T_{\mathrm{c}}/T_{\mathrm{h}}$ is as small as possible. Just as discussed for the Otto cycle in the previous section, this means that efficiency is greatest for the highest possible temperature of the hot reservoir and lowest possible temperature of the cold reservoir. (This setup increases the area inside the closed loop on the PV diagram; also, it seems reasonable that the greater the temperature difference, the easier it is to divert the heat transfer to work.) The actual reservoir temperatures of a heat engine are usually related to the type of heat source and the temperature of the environment into which heat transfer occurs. Consider the following example.

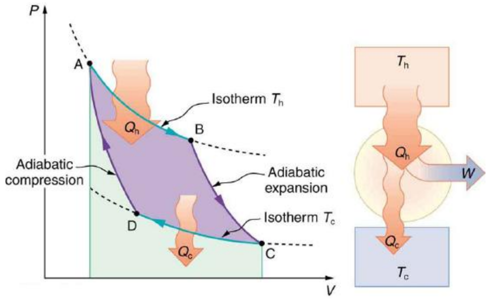


Figure $8.7.11.2$: PV diagram for a Carnot cycle, employing only reversible isothermal and adiabatic processes. Heat transfer Q_{h} occurs into the working substance during the isothermal path AB, which takes place at constant temperature T_{h}. Heat transfer Q_{c} occurs out of the working substance during the isothermal path CD, which takes place at constant temperature T_{c}. The net work output W equals the area inside the path ABCDA. Also shown is a schematic of a Carnot engine operating between hot and cold reservoirs at temperatures T_{h} and T_{c}. Any heat engine using reversible processes and operating between these two temperatures will have the same maximum efficiency as the Carnot engine.

✓ Example $8.7.11.1$: Maximum Theoretical Efficiency for a Nuclear Reactor

A nuclear power reactor has pressurized water at $300^{\circ}\mathrm{C}$. (Higher temperatures are theoretically possible but practically not, due to limitations with materials used in the reactor.) Heat transfer from this water is a complex process (see Figure $8.7.11$). Steam, produced in the steam generator, is used to drive the turbine-generators. Eventually the steam is condensed to water at $27^{\circ}\mathrm{C}$ and then heated again to start the cycle over. Calculate the maximum theoretical efficiency for a heat engine operating between these two temperatures.

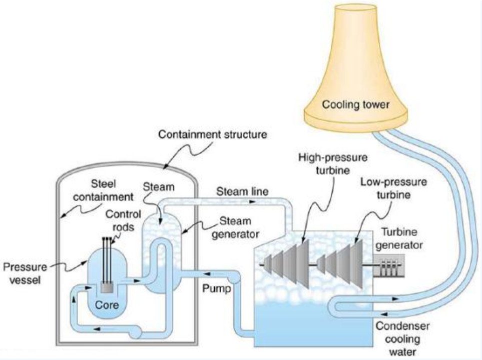


Figure 8.7.11.3: Schematic diagram of a pressurized water nuclear reactor and the steam turbines that convert work into electrical energy. Heat exchange is used to generate steam, in part to avoid contamination of the generators with radioactivity. Two turbines are used because this is less expensive than operating a single generator that produces the same amount of electrical energy. The steam is condensed to liquid before being returned to the heat exchanger, to keep exit steam pressure low and aid the flow of steam through the turbines (equivalent to using a lower-temperature cold reservoir). The considerable energy associated with condensation must be dissipated into the local environment; in this example, a cooling tower is used so there is no direct heat transfer to an aquatic environment. (Note that the water going to the cooling tower does not come into contact with the steam flowing over the turbines.)

Strategy

Since temperatures are given for the hot and cold reservoirs of this heat engine, $Eff_{\text{C}} = 1 - \frac{T_{\text{c}}}{T_{\text{h}}}$ can be used to calculate the Carnot (maximum theoretical) efficiency. Those temperatures must first be converted to kelvins.

Solution

The hot and cold reservoir temperatures are given as 300°C and 27.0°C, respectively. In kelvins, then, $T_{\text{h}} = 573 \text{ K}$ and $T_{\text{c}} = 300 \text{ K}$, so that the maximum efficiency is

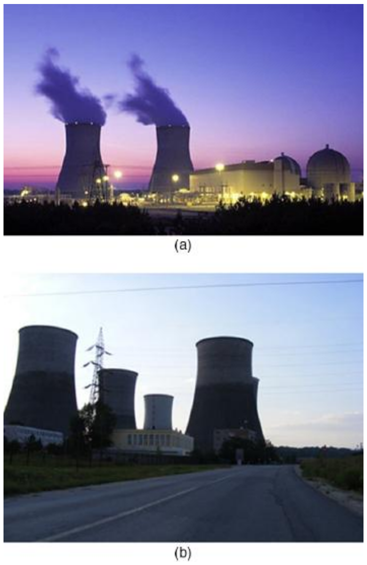
$$Eff_{\text{C}} = 1 - \frac{T_{\text{c}}}{T_{\text{h}}}.$$

Thus,

$$\begin{aligned} Eff_{\text{C}} &= 1 - \frac{300 \text{ K}}{573 \text{ K}} \\ &= 0.476, \text{ or } 47.6\%. \end{aligned}$$

Discussion

A typical nuclear power station’s actual efficiency is about 35%, a little better than 0.7 times the maximum possible value, a tribute to superior engineering. Electrical power stations fired by coal, oil, and natural gas have greater actual efficiencies (about 42%), because their boilers can reach higher temperatures and pressures. The cold reservoir temperature in any of these power stations is limited by the local environment. Figure 8.7.11.4shows (a) the exterior of a nuclear power station and (b) the exterior of a coal-fired power station. Both have cooling towers into which water from the condenser enters the tower near the top and is sprayed downward, cooled by evaporation.


(a)

(b)

Figure 8.7.11.4: (a) A nuclear power station (credit: BlatantWorld.com) and (b) a coal-fired power station. Both have cooling towers in which water evaporates into the environment, representing Q_c. The nuclear reactor, which supplies Q_h, is housed inside the dome-shaped containment buildings. (credit: Robert & Mihaela Vicol, publicphoto.org)

Since all real processes are irreversible, the actual efficiency of a heat engine can never be as great as that of a Carnot engine, as illustrated in Figure 8.7.11.5(a). Even with the best heat engine possible, there are always dissipative processes in peripheral equipment, such as electrical transformers or car transmissions. These further reduce the overall efficiency by converting some of the engine's work output back into heat transfer, as shown in Figure 8.7.11.5(b).

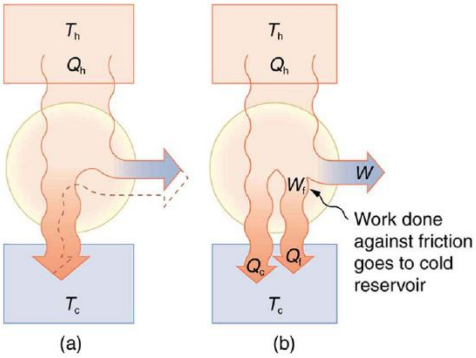


Figure 8.7.11.5: Real heat engines are less efficient than Carnot engines. (a) Real engines use irreversible processes, reducing the heat transfer to work. Solid lines represent the actual process; the dashed lines are what a Carnot engine would do between the same two reservoirs. (b) Friction and other dissipative processes in the output mechanisms of a heat engine convert some of its work output into heat transfer to the environment.

Section Summary

- The Carnot cycle is a theoretical cycle that is the most efficient cyclical process possible. Any engine using the Carnot cycle, which uses only reversible processes (adiabatic and isothermal), is known as a Carnot engine.
- Any engine that uses the Carnot cycle enjoys the maximum theoretical efficiency.
- While Carnot engines are ideal engines, in reality, no engine achieves Carnot's theoretical maximum efficiency, since dissipative processes, such as friction, play a role. Carnot cycles without heat loss may be possible at absolute zero, but this has never been seen in nature.

Glossary

Carnot cycle
a cyclical process that uses only reversible processes, the adiabatic and isothermal processes

Carnot engine
a heat engine that uses a Carnot cycle

Carnot efficiency
the maximum theoretical efficiency for a heat engine

8.7.12: Applications of Thermodynamics- Heat Pumps and Refrigerators

Learning Objectives

- Describe the use of heat engines in heat pumps and refrigerators.
- Demonstrate how a heat pump works to warm an interior space.
- Explain the differences between heat pumps and refrigerators.
- Calculate a heat pump's coefficient of performance.

Figure 8.7.12.1: Almost every home contains a refrigerator. Most people don't realize they are also sharing their homes with a heat pump. (credit: Id1337x, Wikimedia Commons)

Heat pumps, air conditioners, and refrigerators utilize heat transfer from cold to hot. They are heat engines run backward. We say backward, rather than reverse, because except for Carnot engines, all heat engines, though they can be run backward, cannot truly be reversed. Heat transfer occurs from a cold reservoir Q_c and into a hot one. This requires work input W, which is also converted to heat transfer. Thus the heat transfer to the hot reservoir is $Q_\mathrm{h} = Q_\mathrm{c} + W$. (Note that Q_h, Q_c, and W are positive, with their directions indicated on schematics rather than by sign.) A heat pump's mission is for heat transfer Q_h to occur into a warm environment, such as a home in the winter. The mission of air conditioners and refrigerators is for heat transfer Q_c to occur from a cool environment, such as chilling a room or keeping food at lower temperatures than the environment. (Actually, a heat pump can be used both to heat and cool a space. It is essentially an air conditioner and a heating unit all in one. In this section we will concentrate on its heating mode.)

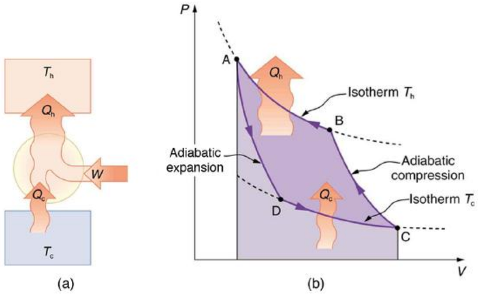


Figure 8.7.12.2: Heat pumps, air conditioners, and refrigerators are heat engines operated backward. The one shown here is based on a Carnot (reversible) engine. (a) Schematic diagram showing heat transfer from a cold reservoir to a warm reservoir with a heat pump. The directions of $\boldsymbol{W}$, Q_h, and Q_c are opposite what they would be in a heat engine. (b) PV diagram for a Carnot cycle similar to that in Figure $8.7.12.3$ but reversed, following path ADCBA. The area inside the loop is negative, meaning there is a net work input. There is heat transfer Q_c into the system from a cold reservoir along path DC, and heat transfer Q_h out of the system into a hot reservoir along path BA.

Heat Pumps

The great advantage of using a heat pump to keep your home warm, rather than just burning fuel, is that a heat pump supplies $Q_\mathrm{h} = Q_\mathrm{c} + W$. Heat transfer is from the outside air, even at a temperature below freezing, to the indoor space. You only pay for W, and you get an additional heat transfer of Q_c from the outside at no cost; in many cases, at least twice as much energy is transferred to the heated space as is used to run the heat pump. When you burn fuel to keep warm, you pay for all of it. The disadvantage is that the work input (required by the second law of thermodynamics) is sometimes more expensive than simply burning fuel, especially if the work is done by electrical energy.

The basic components of a heat pump in its heating mode are shown in Figure $8.7.12.3$ A working fluid such as a non-CFC refrigerant is used. In the outdoor coils (the evaporator), heat transfer Q_c occurs to the working fluid from the cold outdoor air, turning it into a gas.

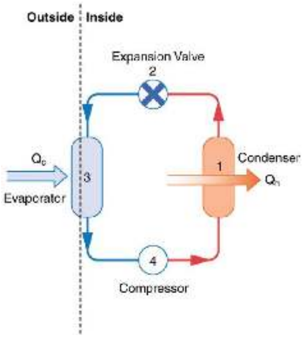


Figure 8.7.12.3: A simple heat pump has four basic components: (1) condenser, (2) expansion valve, (3) evaporator, and (4) compressor. In the heating mode, heat transfer Q_c occurs to the working fluid in the evaporator (3) from the colder outdoor air, turning it into a gas. The electrically driven compressor (4) increases the temperature and pressure of the gas and forces it into the condenser coils (1) inside the heated space. Because the temperature of the gas is higher than the temperature in the room, heat transfer from the gas to the room occurs as the gas condenses to a liquid. The working fluid is then cooled as it flows back through an expansion valve (2) to the outdoor evaporator coils.

The electrically driven compressor (work input W) raises the temperature and pressure of the gas and forces it into the condenser coils that are inside the heated space. Because the temperature of the gas is higher than the temperature inside the room, heat transfer to the room occurs and the gas condenses to a liquid. The liquid then flows back through a pressure-reducing valve to the outdoor evaporator coils, being cooled through expansion. (In a cooling cycle, the evaporator and condenser coils exchange roles and the flow direction of the fluid is reversed.)

The quality of a heat pump is judged by how much heat transfer Q_h occurs into the warm space compared with how much work input W is required. In the spirit of taking the ratio of what you get to what you spend, we define a **heat pump's coefficient of performance** (COP_hp) to be

$$COP_\mathrm{hp} = \frac{Q_\mathrm{h}}{W}.$$

Since the efficiency of a heat engine is $Eff = W/Q_\mathrm{h}$, we see that $COP_\mathrm{hp} = 1/Eff$, an important and interesting fact. First, since the efficiency of any heat engine is less than 1, it means that COP_hp is always greater than 1—that is, a heat pump always has more heat transfer Q_h than work put into it. Second, it means that heat pumps work best when temperature differences are small. The efficiency of a perfect, or Carnot, engine is $Eff_\mathrm{C} = 1-(T_\mathrm{c}/T_\mathrm{h})$; thus, the smaller the temperature difference, the smaller the efficiency and the greater the COP_hp (because $COP_\mathrm{hp} = 1/Eff$). In other words, heat pumps do not work as well in very cold climates as they do in more moderate climates.

Friction and other irreversible processes reduce heat engine efficiency, but they do *not* benefit the operation of a heat pump—instead, they reduce the work input by converting part of it to heat transfer back into the cold reservoir before it gets into the heat pump.

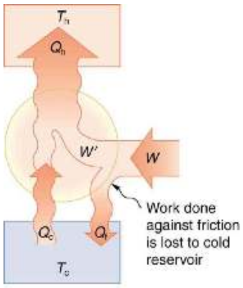


Figure 8.7.12.4: When a real heat engine is run backward, some of the intended work input (W) goes into heat transfer before it gets into the heat engine, thereby reducing its coefficient of performance COP_hp. In this figure, W' represents the portion of W that goes into the heat pump, while the remainder of W is lost in the form of frictional heat (Q_f) to the cold reservoir. If all of $\boldsymbol{W}$ had gone into the heat pump, then Q_h would have been greater. The best heat pump uses adiabatic and isothermal processes, since, in theory, there would be no dissipative processes to reduce the heat transfer to the hot reservoir.

✓ Example 8.7.12.1: The Best COP hp of a Heat Pump for Home Use

A heat pump used to warm a home must employ a cycle that produces a working fluid at temperatures greater than typical indoor temperature so that heat transfer to the inside can take place. Similarly, it must produce a working fluid at temperatures that are colder than the outdoor temperature so that heat transfer occurs from outside. Its hot and cold reservoir temperatures therefore cannot be too close, placing a limit on its COP_{hp}. (See Figure $8.7.12.5$) What is the best coefficient of performance possible for such a heat pump, if it has a hot reservoir temperature of 45.0°C and a cold reservoir temperature of -15.0°C?

Strategy

A Carnot engine reversed will give the best possible performance as a heat pump. As noted above, $COP_{\text{hp}} = 1/Eff$, so that we need to first calculate the Carnot efficiency to solve this problem.

Solution

Carnot efficiency in terms of absolute temperature is given by**:**

$$Eff_{\text{C}} = 1 - \frac{T_{\text{c}}}{T_{\text{h}}}.$$

The temperatures in kelvins are $T_{\text{h}} = 318\text{ K}$ and $T_{\text{c}} = 258\text{ K}$, so that

$$Eff_{\text{C}} = 1 - \frac{258\text{ K}}{318\text{ K}} = 0.1887.$$

Thus, from the discussion above,

$$COP_{\text{hp}} = \frac{1}{Eff} = \frac{1}{0.1887} = 5.30,$$

or

$$COP_{\text{hp}} = \frac{Q_{\text{h}}}{W} = 5.30,$$

so that

$$Q_{\text{h}} = 5.30\text{ W}.$$

Discussion

This result means that the heat transfer by the heat pump is 5.30 times as much as the work put into it. It would cost 5.30 times as much for the same heat transfer by an electric room heater as it does for that produced by this heat pump. This is not a violation of conservation of energy. Cold ambient air provides 4.3 J per 1 J of work from the electrical outlet.

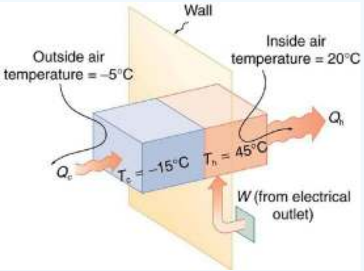


Figure 8.7.12.5: Heat transfer from the outside to the inside, along with work done to run the pump, takes place in the heat pump of the example above**.** Note that the cold temperature produced by the heat pump is lower than the outside temperature, so that heat transfer into the working fluid occurs. The pump’s compressor produces a temperature greater than the indoor temperature in order for heat transfer into the house to occur.

Real heat pumps do not perform quite as well as the ideal one in the previous example; their values of COP_{hp} range from about 2 to 4. This range means that the heat transfer Q_{h} from the heat pumps is 2 to 4 times as great as the work W put into them. Their economical feasibility is still limited, however, since W is usually supplied by electrical energy that costs more per joule than heat transfer by burning fuels like natural gas. Furthermore, the initial cost of a heat pump is greater than that of many furnaces, so that a heat pump must last longer for its cost to be recovered. Heat pumps are most likely to be economically superior where winter temperatures are mild, electricity is relatively cheap, and other fuels are relatively expensive. Also, since they can cool as well as heat a space, they have advantages where cooling in summer months is also desired. Thus some of the best locations for heat pumps are in warm summer climates with cool winters. Figure $8.7.12.6$ shows a heat pump, called a *"reverse cycle"* or *"split-system cooler"* in some countries.

Figure 8.7.12.6: In hot weather, heat transfer occurs from air inside the room to air outside, cooling the room. In cool weather, heat transfer occurs from air outside to air inside, warming the room. This switching is achieved by reversing the direction of flow of the working fluid.

Air Conditioners and Refrigerators

Air conditioners and refrigerators are designed to cool something down in a warm environment. As with heat pumps, work input is required for heat transfer from cold to hot, and this is expensive. The quality of air conditioners and refrigerators is judged by how much heat transfer Q_{c} occurs from a cold environment compared with how much work input W is required. What is considered the benefit in a heat pump is considered waste heat in a refrigerator. We thus define the **coefficient of performance** (COP_{ref}) of an air conditioner or refrigerator to be

$$COP_{\mathrm{ref}} = \frac{Q_{\mathrm{c}}}{W}.$$

Noting again that $Q_{\mathrm{h}} = Q_{\mathrm{c}} + W$, we can see that an air conditioner will have a lower coefficient of performance than a heat pump, because $COP_{\mathrm{hp}} = Q_{\mathrm{h}}/W$ and Q_{h} is greater than Q_{c}. In this module's Problems and Exercises, you will show that

$$COP_{\mathrm{ref}} = COP_{\mathrm{hp}} - 1$$

for a heat engine used as either an air conditioner or a heat pump operating between the same two temperatures. Real air conditioners and refrigerators typically do remarkably well, having values of COP_{ref} ranging from 2 to 6. These numbers are better than the COP_{hp} values for the heat pumps mentioned above, because the temperature differences are smaller, but they are less than those for Carnot engines operating between the same two temperatures.

A type of COP rating system called the "energy efficiency rating" (EER) has been developed. This rating is an example where non-SI units are still used and relevant to consumers. To make it easier for the consumer, Australia, Canada, New Zealand, and the U.S. use an Energy Star Rating out of 5 stars—the more stars, the more energy efficient the appliance. EERs are expressed in mixed units of British thermal units (Btu) per hour of heating or cooling divided by the power input in watts. Room air conditioners are readily available with EERs ranging from 6 to 12. Although not the same as the COPs just described, these EERs are good for comparison purposes—the greater the EER, the cheaper an air conditioner is to operate (but the higher its purchase price is likely to be).

The EER of an air conditioner or refrigerator can be expressed as

$$EER = \frac{Q_{\mathrm{c}}/t_1}{W/t_2},$$

where Q_{c} is the amount of heat transfer from a cold environment in British thermal units, t_1 is time in hours, W is the work input in joules, and t_2 is time in seconds.

PROBLEM-SOLVING STRATEGIES FOR THERMODYNAMICS

1. *Examine the situation to determine whether heat, work, or internal energy are involved.* Look for any system where the primary methods of transferring energy are heat and work. Heat engines, heat pumps, refrigerators, and air conditioners are examples of such systems.
2. *Identify the system of interest and draw a labeled diagram of the system showing energy flow.*

3. *Identify exactly what needs to be determined in the problem (identify the unknowns).* A written list is useful. Maximum efficiency means a Carnot engine is involved. Efficiency is not the same as the coefficient of performance.
4. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).* Be sure to distinguish heat transfer into a system from heat transfer out of the system, as well as work input from work output. In many situations, it is useful to determine the type of process, such as isothermal or adiabatic.
5. *Solve the appropriate equation for the quantity to be determined (the unknown).*
6. *Substitute the known quantities along with their units into the appropriate equation and obtain numerical solutions complete with units.*
7. *Check the answer to see if it is reasonable: Does it make sense?* For example, efficiency is always less than 1, whereas coefficients of performance are greater than 1.

Section Summary

- An artifact of the second law of thermodynamics is the ability to heat an interior space using a heat pump. Heat pumps compress cold ambient air and, in so doing, heat it to room temperature without violation of conservation principles.
- To calculate the heat pump’s coefficient of performance, use the equation $COP_{\text{hp}} = \frac{Q_{\text{h}}}{W}$.
- A refrigerator is a heat pump; it takes warm ambient air and expands it to chill it.

Glossary

heat pump

a machine that generates heat transfer from cold to hot

coefficient of performance

for a heat pump, it is the ratio of heat transfer at the output (the hot reservoir) to the work supplied; for a refrigerator or air conditioner, it is the ratio of heat transfer from the cold reservoir to the work supplied

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8.7.13: Entropy and the Second Law of Thermodynamics- Disorder and the Unavailability of Energy

Learning Objectives

- Define entropy and calculate the increase of entropy in a system with reversible and irreversible processes.
- Calculate the increasing disorder of a system.

Figure 8.7.13.1: The ice in this drink is slowly melting. Eventually the liquid will reach thermal equilibrium, as predicted by the second law of thermodynamics. (credit: Jon Sullivan, PDPhoto.org)

There is yet another way of expressing the second law of thermodynamics. This version relates to a concept called **entropy**. By examining it, we shall see that the directions associated with the second law—heat transfer from hot to cold, for example—are related to the tendency in nature for systems to become disordered and for less energy to be available for use as work. The entropy of a system can in fact be shown to be a measure of its disorder and of the unavailability of energy to do work.

MAKING CONNECTIONS: ENTROPY, ENERGY, AND WORK

Recall that the simple definition of energy is the ability to do work. Entropy is a measure of how much energy is not available to do work. Although all forms of energy are interconvertible, and all can be used to do work, it is not always possible, even in principle, to convert the entire available energy into work. That unavailable energy is of interest in thermodynamics, because the field of thermodynamics arose from efforts to convert heat to work.

We can see how entropy is defined by recalling our discussion of the Carnot engine. We noted that for a Carnot cycle, and hence for any reversible processes, $Q_c/Q_h = T_c/T_h$. Rearranging terms yields

$$\frac{Q_c}{T_c} = \frac{Q_h}{T_h}$$

for any reversible process. Q_c and Q_h are absolute values of the heat transfer at temperatures T_c and T_h, respectively. This ratio of Q/T is defined to be the **change in entropy** ΔS for a reversible process,

$$\Delta S = \left(\frac{Q}{T}\right)_{\text{rev}},$$

where Q is the heat transfer, which is positive for heat transfer into and negative for heat transfer out of, and T is the absolute temperature at which the reversible process takes place. The SI unit for entropy is joules per kelvin (J/K). If temperature changes during the process, then it is usually a good approximation (for small changes in temperature) to take T to be the average temperature, avoiding the need to use integral calculus to find ΔS.

The definition of ΔS is strictly valid only for reversible processes, such as used in a Carnot engine. However, we can find ΔS precisely even for real, irreversible processes. The reason is that the entropy S of a system, like internal energy U, depends only on the state of the system and not how it reached that condition. Entropy is a property of state. Thus the change in entropy ΔS of a system between state 1 and state 2 is the same no matter how the change occurs. We just need to find or imagine a reversible process that takes us from state 1 to state 2 and calculate ΔS for that process. That will be the change in entropy for any process going from state 1 to state 2. (See Figure 8.7.13.2)

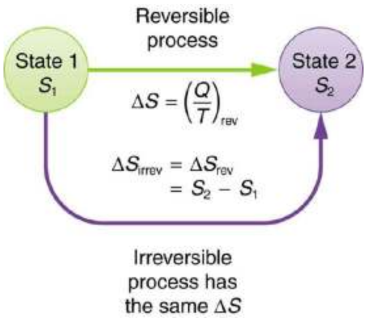

Figure 8.7.13.2: When a system goes from state 1 to state 2, its entropy changes by the same amount ΔS, whether a hypothetical reversible path is followed or a real irreversible path is taken.

Now let us take a look at the change in entropy of a Carnot engine and its heat reservoirs for one full cycle. The hot reservoir has a loss of entropy $\Delta S_{\text{h}} = -Q_{\text{h}}/T_{\text{h}}$, because heat transfer occurs out of it (remember that when heat transfers out, then Q has a negative sign). The cold reservoir has a gain of entropy $\Delta S_{\text{c}} = Q_{\text{c}}/T_{\text{c}}$, because heat transfer occurs into it. (We assume the reservoirs are sufficiently large that their temperatures are constant.) So the total change in entropy is

$$\Delta S_{\text{tot}} = \Delta S_{\text{h}} + \Delta S_{\text{c}}$$

Thus, since we know that $Q_{\text{h}}/T_{\text{h}} = Q_{\text{c}}/T_{\text{c}}$ for a Carnot engine,

$$\Delta S_{\text{tot}} = -\frac{Q_{\text{h}}}{T_{\text{h}}} + \frac{Q_{\text{c}}}{T_{\text{c}}} = 0.$$

This result, which has general validity, means that *the total change in entropy for a system in any reversible process is zero.*

The entropy of various parts of the system may change, but the total change is zero. Furthermore, the system does not affect the entropy of its surroundings, since heat transfer between them does not occur. Thus the reversible process changes neither the total entropy of the system nor the entropy of its surroundings. Sometimes this is stated as follows: *Reversible processes do not affect the total entropy of the universe.* Real processes are not reversible, though, and they do change total entropy. We can, however, use hypothetical reversible processes to determine the value of entropy in real, irreversible processes. The following example illustrates this point.

✓ Example 8.7.13.1: Entropy Increases in an Irreversible (Real) Process

Spontaneous heat transfer from hot to cold is an irreversible process. Calculate the total change in entropy if 4000 J of heat transfer occurs from a hot reservoir at $T_{\text{h}} = 600\text{ K } (327^\circ\text{C})$ to a cold reservoir at $T_{\text{c}} = 250\text{ K } (-23^\circ\text{C})$, assuming there is no temperature change in either reservoir. (See Figure 8.7.13.3)

Strategy

How can we calculate the change in entropy for an irreversible process when $\Delta S_{\text{tot}} = \Delta S_{\text{h}} + \Delta S_{\text{c}}$ is valid only for reversible processes? Remember that the total change in entropy of the hot and cold reservoirs will be the same whether a reversible or irreversible process is involved in heat transfer from hot to cold. So we can calculate the change in entropy of the hot reservoir for a hypothetical reversible process in which 4000 J of heat transfer occurs from it; then we do the same for a hypothetical reversible process in which 4000 J of heat transfer occurs to the cold reservoir. This produces the same changes in the hot and cold reservoirs that would occur if the heat transfer were allowed to occur irreversibly between them, and so it also produces the same changes in entropy.

Solution

We now calculate the two changes in entropy using $\Delta S_{\text{tot}} = \Delta S_{\text{h}} + \Delta S_{\text{c}}$. First, for the heat transfer from the hot reservoir,

$$\Delta S_{\text{h}} = \frac{-Q_{\text{h}}}{T_{\text{h}}} = \frac{-4000\text{ J}}{600\text{ K}} = -6.67\text{ J/K}.$$

And for the cold reservoir,

$$\Delta S_{\mathrm{c}}=\frac{Q_{\mathrm{c}}}{T_{\mathrm{c}}}=\frac{4000\ \mathrm{J}}{250\ \mathrm{K}}=16.0\ \mathrm{J/K}.$$

Thus the total is

$$\begin{aligned}\Delta S_{\mathrm{tot}} &=\Delta S_{\mathrm{h}}+\Delta S_{\mathrm{c}}\\ &=(-6.67+16.0)\mathrm{J/K}\\ &=9.33\ \mathrm{J/K}.\end{aligned}$$

Discussion

There is an *increase* in entropy for the system of two heat reservoirs undergoing this irreversible heat transfer. We will see that this means there is a loss of ability to do work with this transferred energy. Entropy has increased, and energy has become unavailable to do work.

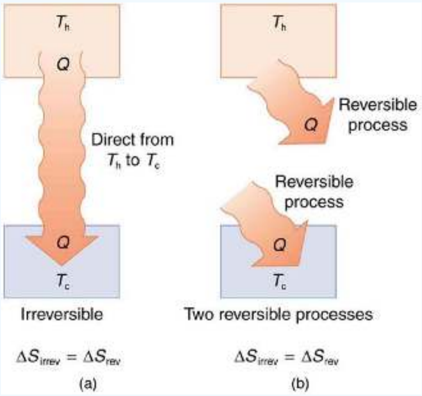

Figure 8.7.13.3: (a) Heat transfer from a hot object to a cold one is an irreversible process that produces an overall increase in entropy. (b) The same final state and, thus, the same change in entropy is achieved for the objects if reversible heat transfer processes occur between the two objects whose temperatures are the same as the temperatures of the corresponding objects in the irreversible process.

It is reasonable that entropy increases for heat transfer from hot to cold. Since the change in entropy is Q/T, there is a larger change at lower temperatures. The decrease in entropy of the hot object is therefore less than the increase in entropy of the cold object, producing an overall increase, just as in the previous example. This result is very general:

> *There is an increase in entropy for any system undergoing an irreversible process.*

With respect to entropy, there are only two possibilities: entropy is constant for a reversible process, and it increases for an irreversible process. There is a fourth version of **the second law of thermodynamics stated in terms of entropy**:

The total entropy of a system either increases or remains constant in any process; it never decreases.

For example, heat transfer cannot occur spontaneously from cold to hot, because entropy would decrease.

Entropy is very different from energy. Entropy is *not* conserved but increases in all real processes. Reversible processes (such as in Carnot engines) are the processes in which the most heat transfer to work takes place and are also the ones that keep entropy constant. Thus we are led to make a connection between entropy and the availability of energy to do work.

Order to Disorder

Entropy is related not only to the unavailability of energy to do work—it is also a measure of disorder. This notion was initially postulated by Ludwig Boltzmann in the 1800s. For example, melting a block of ice means taking a highly structured and orderly system of water molecules and converting it into a disorderly liquid in which molecules have no fixed positions. (See Figure $8.7.13.4$) There is a large increase in entropy in the process, as seen in the following example.

✓ Example 8.7.13.2: Entropy Associated with Disorder

Find the increase in entropy of 1.00 kg of ice originally at $0^{\circ}\mathrm{C}$ that is melted to form water at $0^{\circ}\mathrm{C}$.

Strategy

As before, the change in entropy can be calculated from the definition of ΔS once we find the energy Q needed to melt the ice.

Solution

The change in entropy is defined as:

$$\Delta S = \frac{Q}{T}.$$

Here Q is the heat transfer necessary to melt 1.00 kg of ice and is given by

$$Q = mL_{\mathrm{f}},$$

where m is the mass and L_{f} is the latent heat of fusion. $L_{\mathrm{f}} = 334\ \mathrm{kJ/kg}$ for water, so that

$$Q = (1.00\ \mathrm{kg})(334\ \mathrm{kJ/kg}) = 3.34 \times 10^5\ \mathrm{J}.$$

Now the change in entropy is positive, since heat transfer occurs into the ice to cause the phase change; thus,

$$\Delta S = \frac{Q}{T} = \frac{3.34 \times 10^5\ \mathrm{J}}{T}.$$

T is the melting temperature of ice. That is, $T = 0^\circ\mathrm{C} = 273\ \mathrm{K}$. So the change in entropy is

$$\begin{aligned}\Delta S &= \frac{3.34 \times 10^5\ \mathrm{J}}{273\ \mathrm{K}} \\ &= 1.22 \times 10^3\ \mathrm{J/K}.\end{aligned}$$

Discussion

This is a significant increase in entropy accompanying an increase in disorder.

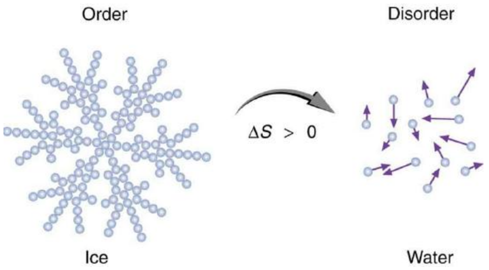

Figure 8.7.13.4: When ice melts, it becomes more disordered and less structured. The systematic arrangement of molecules in a crystal structure is replaced by a more random and less orderly movement of molecules without fixed locations or orientations. Its entropy increases because heat transfer occurs into it. Entropy is a measure of disorder.

In another easily imagined example, suppose we mix equal masses of water originally at two different temperatures, say $20.0^\circ\mathrm{C}$ and $40.0^\circ\mathrm{C}$. The result is water at an intermediate temperature of $30.0^\circ\mathrm{C}$. Three outcomes have resulted: entropy has increased, some energy has become unavailable to do work, and the system has become less orderly. Let us think about each of these results.

First, entropy has increased for the same reason that it did in the example above. Mixing the two bodies of water has the same effect as heat transfer from the hot one and the same heat transfer into the cold one. The mixing decreases the entropy of the hot water but increases the entropy of the cold water by a greater amount, producing an overall increase in entropy.

Second, once the two masses of water are mixed, there is only one temperature—you cannot run a heat engine with them. The energy that could have been used to run a heat engine is now unavailable to do work.

Third, the mixture is less orderly, or to use another term, less structured. Rather than having two masses at different temperatures and with different distributions of molecular speeds, we now have a single mass with a uniform temperature.

These three results—entropy, unavailability of energy, and disorder—are not only related but are in fact essentially equivalent.

Section Summary

- Entropy is the loss of energy available to do work.

- Another form of the second law of thermodynamics states that the total entropy of a system either increases or remains constant; it never decreases.
- Change of entropy is zero in a reversible process; it increases in an irreversible process.
- Entropy is also associated with the tendency toward disorder in an *isolated* system.

Glossary

entropy

a measurement of a system's disorder and its inability to do work in a system

change in entropy

the ratio of heat transfer to temperature Q/T

second law of thermodynamics stated in terms of entropy

the total entropy of a system either increases or remains constant; it never decreases

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8.7.14: Statistical Interpretation of Entropy and the Second Law of Thermodynamics- The Underlying Explanation

Learning Objectives

- Identify probabilities in entropy.
- Analyze statistical probabilities in entropic systems.

Figure 8.7.14.1: When you toss a coin a large number of times, heads and tails tend to come up in roughly equal numbers. Why doesn't heads come up 100, 90, or even 80% of the time? (credit: Jon Sullivan, PDPhoto.org)

The various ways of formulating the second law of thermodynamics tell what happens rather than why it happens. Why should heat transfer occur only from hot to cold? Why should energy become ever less available to do work? Why should the universe become increasingly disorderly? The answer is that it is a matter of overwhelming probability. Disorder is simply vastly more likely than order.

When you watch an emerging rain storm begin to wet the ground, you will notice that the drops fall in a disorganized manner both in time and in space. Some fall close together, some far apart, but they never fall in straight, orderly rows. It is not impossible for rain to fall in an orderly pattern, just highly unlikely, because there are many more disorderly ways than orderly ones. To illustrate this fact, we will examine some random processes, starting with coin tosses.

Coin Tosses

What are the possible outcomes of tossing 5 coins? Each coin can land either heads or tails. On the large scale, we are concerned only with the total heads and tails and not with the order in which heads and tails appear. The following possibilities exist:

Table 8.7.14.1: Possible Outcomes of 5 Coin Tosses

Combination of Heads and Tails	
5 heads	0 tails
4 heads	1 tails
3 heads	2 tails
2 heads	3 tails
1 heads	4 tails
0 heads	5 tails

These are what we call macrostates. A **macrostate** is an overall property of a system. It does not specify the details of the system, such as the order in which heads and tails occur or which coins are heads or tails.

Using this nomenclature, a system of 5 coins has the 6 possible macrostates just listed. Some macrostates are more likely to occur than others. For instance, there is only one way to get 5 heads, but there are several ways to get 3 heads and 2 tails, making the latter macrostate more probable. Table 8.7.14.2lists of all the ways in which 5 coins can be tossed, taking into account the order in which heads and tails occur. Each sequence is called a **microstate**—a detailed description of every element of a system.

Table 8.7.14.2: 5-Coin Toss

Combinations	Individual microstates	Number of microstates
5 heads, 0 tails	HHHHH	1
4 heads, 1 tail	HHHHT, HHHTH, HHTHH, HTHHH, THHHH	5
3 heads, 2 tails	HTHTH, THTHH, HTHHT, THHTH, THHHT HTHTH, THTHH, HTHHT, THHTH, THHHT	10
2 heads, 3 tails	TTTHH, TTHHT, THHTT, HHTTT, TTHTH, THTHT, HTHTT, THTTH, HTTHT, HTTTH	10
1 head, 4 tails	TTTTH, TTTHT, TTHTT, THTTT, HTTTT	5
0 heads, 5 tails	TTTTT	1
		Total: 32

The macrostate of 3 heads and 2 tails can be achieved in 10 ways and is thus 10 times more probable than the one having 5 heads. Not surprisingly, it is equally probable to have the reverse, 2 heads and 3 tails. Similarly, it is equally probable to get 5 tails as it is to get 5 heads. Note that all of these conclusions are based on the crucial assumption that each microstate is equally probable. With coin tosses, this requires that the coins not be asymmetric in a way that favors one side over the other, as with loaded dice. With any system, the assumption that all microstates are equally probable must be valid, or the analysis will be erroneous.

The two most orderly possibilities are 5 heads or 5 tails. (They are more structured than the others.) They are also the least likely, only 2 out of 32 possibilities. The most disorderly possibilities are 3 heads and 2 tails and its reverse. (They are the least structured.) The most disorderly possibilities are also the most likely, with 20 out of 32 possibilities for the 3 heads and 2 tails and its reverse. If we start with an orderly array like 5 heads and toss the coins, it is very likely that we will get a less orderly array as a result, since 30 out of the 32 possibilities are less orderly. So even if you start with an orderly state, there is a strong tendency to go from order to disorder, from low entropy to high entropy. The reverse can happen, but it is unlikely.

Table 8.7.14.3: 100-Coin Toss

Macrostate		Number of microstates
Heads	Tails	(W)
100	0	1
99	1	1.0×10^{2}
95	5	7.5×10^{7}
90	10	1.7×10^{13}
75	25	2.4×10^{23}
60	40	1.4×10^{28}
55	45	6.1×10^{28}
51	49	9.9×10^{28}
50	50	1.0×10^{29}
49	51	9.9×10^{28}
45	55	6.1×10^{28}
40	60	1.4×10^{28}

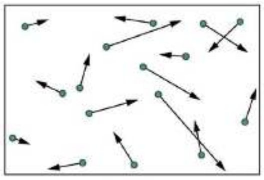
Macrostate		Number of microstates
Heads	Tails	(W)
25	75	2.4×10^{23}
10	90	1.7×10^{13}
5	95	7.5×10^{7}
1	99	1.0×10^{2}
0	100	1
		Total: 1.27×10^{30}

This result becomes dramatic for larger systems. Consider what happens if you have 100 coins instead of just 5. The most orderly arrangements (most structured) are 100 heads or 100 tails. The least orderly (least structured) is that of 50 heads and 50 tails. There is only 1 way (1 microstate) to get the most orderly arrangement of 100 heads. There are 100 ways (100 microstates) to get the next most orderly arrangement of 99 heads and 1 tail (also 100 to get its reverse). And there are 1.0×10^{29} ways to get 50 heads and 50 tails, the least orderly arrangement. Table $8.7.14.3$ is an abbreviated list of the various macrostates and the number of microstates for each macrostate. The total number of microstates—the total number of different ways 100 coins can be tossed—is an impressively large $1.27\times10^{30.}$ Now, if we start with an orderly macrostate like 100 heads and toss the coins, there is a virtual certainty that we will get a less orderly macrostate. If we keep tossing the coins, it is possible, but exceedingly unlikely, that we will ever get back to the most orderly macrostate. If you tossed the coins once each second, you could expect to get either 100 heads or 100 tails once in 2×10^{22} years! This period is 1 trillion (10^{12}) times longer than the age of the universe, and so the chances are essentially zero. In contrast, there is an 8% chance of getting 50 heads, a 73% chance of getting from 45 to 55 heads, and a 96% chance of getting from 40 to 60 heads. Disorder is highly likely.

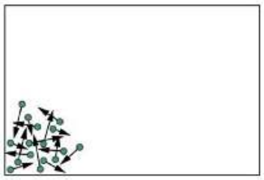
Disorder in a Gas

The fantastic growth in the odds favoring disorder that we see in going from 5 to 100 coins continues as the number of entities in the system increases. Let us now imagine applying this approach to perhaps a small sample of gas. Because counting microstates and macrostates involves statistics, this is called **statistical analysis**. The macrostates of a gas correspond to its macroscopic properties, such as volume, temperature, and pressure; and its microstates correspond to the detailed description of the positions and velocities of its atoms. Even a small amount of gas has a huge number of atoms: 1.0 cm^3 of an ideal gas at 1.0 atm and 0º C has 2.7×10^{19} atoms. So each macrostate has an immense number of microstates. In plain language, this means that there are an immense number of ways in which the atoms in a gas can be arranged, while still having the same pressure, temperature, and so on.

The most likely conditions (or macrostates) for a gas are those we see all the time—a random distribution of atoms in space with a Maxwell-Boltzmann distribution of speeds in random directions, as predicted by kinetic theory. This is the most disorderly and least structured condition we can imagine. In contrast, one type of very orderly and structured macrostate has all of the atoms in one corner of a container with identical velocities. There are very few ways to accomplish this (very few microstates corresponding to it), and so it is exceedingly unlikely ever to occur. (See Figure $8.7.14.2$(b).) Indeed, it is so unlikely that we have a law saying that it is impossible, which has never been observed to be violated—the second law of thermodynamics.



(a) Likely



(b) Highly unlikely

Figure 8.7.14.2: (a) The ordinary state of gas in a container is a disorderly, random distribution of atoms or molecules with a Maxwell-Boltzmann distribution of speeds. It is so unlikely that these atoms or molecules would ever end up in one corner of the container that it might as well be impossible. (b) With energy transfer, the gas can be forced into one corner and its entropy greatly reduced. But left alone, it will spontaneously increase its entropy and return to the normal conditions, because they are immensely more likely.

The disordered condition is one of high entropy, and the ordered one has low entropy. With a transfer of energy from another system, we could force all of the atoms into one corner and have a local decrease in entropy, but at the cost of an overall increase in entropy of the universe. If the atoms start out in one corner, they will quickly disperse and become uniformly distributed and will never return to the orderly original state (Figure $8.7.14.2$(b)). Entropy will increase. With such a large sample of atoms, it is possible—but unimaginably unlikely—for entropy to decrease. Disorder is vastly more likely than order.

The arguments that disorder and high entropy are the most probable states are quite convincing. The great Austrian physicist Ludwig Boltzmann (1844–1906)—who, along with Maxwell, made so many contributions to kinetic theory—proved that the entropy of a system in a given state (a macrostate) can be written as

$$S = k \ln W,$$

where $k = 1.38 \times 10^{-23}\ \mathrm{J/K}$ is Boltzmann’s constant, and $\ln W$ is the natural logarithm of the number of microstates W corresponding to the given macrostate. W is proportional to the probability that the macrostate will occur. Thus entropy is directly related to the probability of a state—the more likely the state, the greater its entropy. Boltzmann proved that this expression for S is equivalent to the definition $\Delta S = Q/T$, which we have used extensively.

Thus the second law of thermodynamics is explained on a very basic level: entropy either remains the same or increases in every process. This phenomenon is due to the extraordinarily small probability of a decrease, based on the extraordinarily larger number of microstates in systems with greater entropy. Entropy *can* decrease, but for any macroscopic system, this outcome is so unlikely that it will never be observed.

Example 8.7.14.1: Entropy Increases in a Coin Toss

Suppose you toss 100 coins starting with 60 heads and 40 tails, and you get the most likely result, 50 heads and 50 tails. What is the change in entropy?

Strategy

Noting that the number of microstates is labeled W in Table $8.7.14.3$ for the 100-coin toss, we can use $\Delta S = S_\mathrm{f} - S_\mathrm{i} = k \ln W_\mathrm{f} - k \ln W_\mathrm{i}$ to calculate the change in entropy.

Solution

The change in entropy is

$$\Delta S = S_\mathrm{f} - S_\mathrm{i} = k \ln W_\mathrm{f} - k \ln W_\mathrm{i},$$

where the subscript i stands for the initial 60 heads and 40 tails state, and the subscript f for the final 50 heads and 50 tails state. Substituting the values for W from Table $8.7.14.3$gives

$$\begin{aligned} \Delta S &= \left(1.38 \times 10^{-23}\ \mathrm{J/K}\right)\left[\ln\left(1.0 \times 10^{29}\right) - \ln\left(1.4 \times 10^{28}\right)\right] \\ &= 2.7 \times 10^{-23}\ \mathrm{J/K} \end{aligned}$$

Discussion

This increase in entropy means we have moved to a less orderly situation. It is not impossible for further tosses to produce the initial state of 60 heads and 40 tails, but it is less likely. There is about a 1 in 90 chance for that decrease in entropy (–2.7×10^{-23} J/K) to occur. If we calculate the decrease in entropy to move to the most orderly state, we get $\Delta S = -92 \times 10^{-23}\ \mathrm{J/K}$. There is about a **1 in 10^{30}** chance of this change occurring. So while very small decreases in entropy are unlikely, slightly greater decreases are impossibly unlikely. These probabilities imply, again, that for a macroscopic system, a decrease in entropy is impossible. For example, for heat transfer to occur spontaneously from 1.00 kg of 0ºC ice to its 0ºC environment, there would be a decrease in entropy of 1.22×10^{3} J/K. Given that a ΔS of $10^{-21}\ \mathrm{J/K}$ corresponds to about a 1 in 1030 chance, a decrease of this size (10^{3} J/K) is an *utter* impossibility. Even for a milligram of melted ice to spontaneously refreeze is impossible.

PROBLEM-SOLVING STRATEGIES FOR ENTROPY

1. *Examine the situation to determine if entropy is involved.*
2. *Identify the system of interest and draw a labeled diagram of the system showing energy flow.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns).* A written list is useful.
4. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).* You must carefully identify the heat transfer, if any, and the temperature at which the process takes place. It is also important to identify the initial and final states.
5. *Solve the appropriate equation for the quantity to be determined (the unknown).* Note that the change in entropy can be determined between any states by calculating it for a reversible process.
6. *Substitute the known value along with their units into the appropriate equation, and obtain numerical solutions complete with units.*
7. *To see if it is reasonable: Does it make sense?* For example, total entropy should increase for any real process or be constant for a reversible process. Disordered states should be more probable and have greater entropy than ordered states.

Section Summary

- Disorder is far more likely than order, which can be seen statistically.
- The entropy of a system in a given state (a macrostate) can be written as

$$S = k \ln W,$$

where $k=1.38\times10^{-23}$ J/K is Boltzmann’s constant, and lnW is the natural logarithm of the number of microstates W corresponding to the given macrostate.

Glossary

macrostate
an overall property of a system

microstate
each sequence within a larger macrostate

statistical analysis
using statistics to examine data, such as counting microstates and macrostates

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8.7.E: Thermal Physics (Exercises)

Conceptual Questions

9.1: Temperature

1. What does it mean to say that two systems are in thermal equilibrium?

2. Give an example of a physical property that varies with temperature and describe how it is used to measure temperature.

3. When a cold alcohol thermometer is placed in a hot liquid, the column of alcohol goes *down* slightly before going up. Explain why.

4. If you add boiling water to a cup at room temperature, what would you expect the final equilibrium temperature of the unit to be? You will need to include the surroundings as part of the system. Consider the zeroth law of thermodynamics.

9.2: The Ideal Gas Law

5. Under what circumstances would you expect a gas to behave significantly differently than predicted by the ideal gas law?

6. A constant-volume gas thermometer contains a fixed amount of gas. What property of the gas is measured to indicate its temperature?

9.3: Heat

7. How is heat transfer related to temperature?

8. Describe a situation in which heat transfer occurs. What are the resulting forms of energy?

9. When heat transfers into a system, is the energy stored as heat? Explain briefly.

9.4: Heat Transfer Methods

10. What are the main methods of heat transfer from the hot core of Earth to its surface? From Earth's surface to outer space?

11. When our bodies get too warm, they respond by sweating and increasing blood circulation to the surface to transfer thermal energy away from the core. What effect will this have on a person in a 40.0ºC hot tub?

12. Figure $8.7.E.1$ shows a cut-away drawing of a thermos bottle (also known as a Dewar flask), which is a device designed specifically to slow down all forms of heat transfer. Explain the functions of the various parts, such as the vacuum, the silvering of the walls, the thin-walled long glass neck, the rubber support, the air layer, and the stopper.

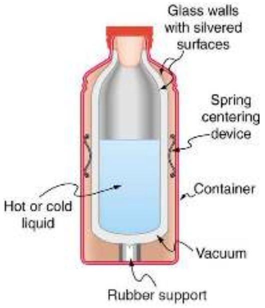

Figure $8.7.E.1$: The construction of a thermos bottle is designed to inhibit all methods of heat transfer.

9.5: Temperature Change and Heat Capacity

13. What three factors affect the heat transfer that is necessary to change an object's temperature?

14. The brakes in a car increase in temperature by ΔT when bringing the car to rest from a speed v. How much greater would ΔT be if the car initially had twice the speed? You may assume the car to stop sufficiently fast so that no heat transfers out of the brakes.

9.6: Phase Change and Latent Heat

15. Heat transfer can cause temperature and phase changes. What else can cause these changes?

16. How does the latent heat of fusion of water help slow the decrease of air temperatures, perhaps preventing temperatures from falling significantly below 0°C, in the vicinity of large bodies of water?

17. What is the temperature of ice right after it is formed by freezing water?

18. If you place 0°C ice into 0°C water in an insulated container, what will happen? Will some ice melt, will more water freeze, or will neither take place?

19. What effect does condensation on a glass of ice water have on the rate at which the ice melts? Will the condensation speed up the melting process or slow it down?

20. In very humid climates where there are numerous bodies of water, such as in Florida, it is unusual for temperatures to rise above about 35ºC(95ºF). In deserts, however, temperatures can rise far above this. Explain how the evaporation of water helps limit high temperatures in humid climates.

21. In winters, it is often warmer in San Francisco than in nearby Sacramento, 150 km inland. In summers, it is nearly always hotter in Sacramento. Explain how the bodies of water surrounding San Francisco moderate its extreme temperatures.

22. Putting a lid on a boiling pot greatly reduces the heat transfer necessary to keep it boiling. Explain why.

23. Freeze-dried foods have been dehydrated in a vacuum. During the process, the food freezes and must be heated to facilitate dehydration. Explain both how the vacuum speeds up dehydration and why the food freezes as a result.

24. When still air cools by radiating at night, it is unusual for temperatures to fall below the dew point. Explain why.

25. In a physics classroom demonstration, an instructor inflates a balloon by mouth and then cools it in liquid nitrogen. When cold, the shrunken balloon has a small amount of light blue liquid in it, as well as some snow-like crystals. As it warms up, the liquid boils, and part of the crystals sublimate, with some crystals lingering for awhile and then producing a liquid. Identify the blue liquid and the two solids in the cold balloon. Justify your identifications using data from Table 9.6.1.

9.7: The First Law of Thermodynamics

26. Describe the photo of the tea kettle at the beginning of this section in terms of heat transfer, work done, and internal energy. How is heat being transferred? What is the work done and what is doing it? How does the kettle maintain its internal energy?

27. The first law of thermodynamics and the conservation of energy are clearly related. How do they differ in the types of energy considered?

28. Heat transfer Q and work done W are always energy in transit, whereas internal energy U is energy stored in a system. Give an example of each type of energy, and state specifically how it is either in transit or resides in a system.

29. How do heat transfer and internal energy differ? In particular, which can be stored as such in a system and which cannot?

30. If you run down some stairs and stop, what happens to your kinetic energy and your initial gravitational potential energy?

31. Give an explanation of how food energy (calories) can be viewed as molecular potential energy (consistent with the atomic and molecular definition of internal energy).

32. Identify the type of energy transferred to your body in each of the following as either internal energy, heat transfer, or doing work:

(a) basking in sunlight;

(b) eating food;

(c) riding an elevator to a higher floor.

9.8: The First Law of Thermodynamics and Heat Engine Processes

33. A great deal of effort, time, and money has been spent in the quest for the so-called perpetual-motion machine, which is defined as a hypothetical machine that operates or produces useful work indefinitely and/or a hypothetical machine that produces more work or energy than it consumes. Explain, in terms of heat engines and the first law of thermodynamics, why or why not such a machine is likely to be constructed.

34. One method of converting heat transfer into doing work is for heat transfer into a gas to take place, which expands, doing work on a piston, as shown in the figure below.

(a) Is the heat transfer converted directly to work in an isobaric process, or does it go through another form first? Explain your answer.

(b) What about in an isothermal process?

(c) What about in an adiabatic process (where heat transfer occurred prior to the adiabatic process)?

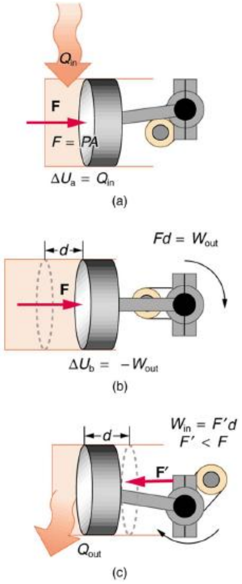

Figure $8.7.E.2$

35. Would the previous question make any sense for an isochoric process? Explain your answer.

36. We ordinarily say that $\Delta U = 0$ for an isothermal process. Does this assume no phase change takes place? Explain your answer.

37. The temperature of a rapidly expanding gas decreases. Explain why in terms of the first law of thermodynamics. (Hint: Consider whether the gas does work and whether heat transfer occurs rapidly into the gas through conduction.)

38. A real process may be nearly adiabatic if it occurs over a very short time. How does the short time span help the process to be adiabatic?

39. It is unlikely that a process can be isothermal unless it is a very slow process. Explain why. Is the same true for isobaric and isochoric processes? Explain your answer.

9.9: Introduction to the Second Law of Thermodynamics- Heat Engines and Their Efficiency

40. Imagine you are driving a car up Pike’s Peak in Colorado. To raise a car weighing 1000 kilograms a distance of 100 meters would require about a million joules. You could raise a car 12.5 kilometers with the energy in a gallon of gas. Driving up Pike's Peak (a mere 3000-meter climb) should consume a little less than a quart of gas. But other considerations have to be taken into account. Explain, in terms of efficiency, what factors may keep you from realizing your ideal energy use on this trip.

41. Is a temperature difference necessary to operate a heat engine? State why or why not.

42. Definitions of efficiency vary depending on how energy is being converted. Compare the definitions of efficiency for the human body and heat engines. How does the definition of efficiency in each relate to the type of energy being converted into doing work?

43. Why—other than the fact that the second law of thermodynamics says reversible engines are the most efficient—should heat engines employing reversible processes be more efficient than those employing irreversible processes? Consider that dissipative mechanisms are one cause of irreversibility.

9.10: Carnot’s Perfect Heat Engine- The Second Law of Thermodynamics Restated

44. Think about the drinking bird at the beginning of this section (Figure 9.10.1). Although the bird enjoys the theoretical maximum efficiency possible, if left to its own devices over time, the bird will cease “drinking.” What are some of the dissipative processes that might cause the bird’s motion to cease?

45. Can improved engineering and materials be employed in heat engines to reduce heat transfer into the environment? Can they eliminate heat transfer into the environment entirely?

46. Does the second law of thermodynamics alter the conservation of energy principle?

9.11: Applications of Thermodynamics- Heat Pumps and Refrigerators

47. Explain why heat pumps do not work as well in very cold climates as they do in milder ones. Is the same true of refrigerators?

48. In some Northern European nations, homes are being built without heating systems of any type. They are very well insulated and are kept warm by the body heat of the residents. However, when the residents are not at home, it is still warm in these houses. What is a possible explanation?

49. Why do refrigerators, air conditioners, and heat pumps operate most cost-effectively for cycles with a small difference between T_h and T_c? (Note that the temperatures of the cycle employed are crucial to its COP.)

50. Grocery store managers contend that there is *less* total energy consumption in the summer if the store is kept at a *low* temperature. Make arguments to support or refute this claim, taking into account that there are numerous refrigerators and freezers in the store.

51. Can you cool a kitchen by leaving the refrigerator door open?

9.12: Entropy and the Second Law of Thermodynamics- Disorder and the Unavailability of Energy

52. Does a gas become more orderly when it liquefies? Does its entropy change? If so, does the entropy increase or decrease? Explain your answer.

53. Explain how water’s entropy can decrease when it freezes without violating the second law of thermodynamics. Specifically, explain what happens to the entropy of its surroundings.

54. Is a uniform-temperature gas more or less orderly than one with several different temperatures? Which is more structured? In which can heat transfer result in work done without heat transfer from another system?

55. Give an example of a spontaneous process in which a system becomes less ordered and energy becomes less available to do work. What happens to the system’s entropy in this process?

56. What is the change in entropy in an adiabatic process? Does this imply that adiabatic processes are reversible? Can a process be precisely adiabatic for a macroscopic system?

57. Does the entropy of a star increase or decrease as it radiates? Does the entropy of the space into which it radiates (which has a temperature of about 3 K) increase or decrease? What does this do to the entropy of the universe?

58. Explain why a building made of bricks has smaller entropy than the same bricks in a disorganized pile. Do this by considering the number of ways that each could be formed (the number of microstates in each macrostate).

9.13: Statistical Interpretation of Entropy and the Second Law of Thermodynamics- The Underlying Explanation

59. Explain why a building made of bricks has smaller entropy than the same bricks in a disorganized pile. Do this by considering the number of ways that each could be formed (the number of microstates in each macrostate).

Problems & Exercises

9.1: Temperature

1. What is the Fahrenheit temperature of a person with a 39.0ºC fever?

Answer

102ºF

2. Frost damage to most plants occurs at temperatures of 28.0ºF or lower. What is this temperature on the Kelvin scale?

3. To conserve energy, room temperatures are kept at 68.0ºF in the winter and 78.0ºF in the summer. What are these temperatures on the Celsius scale?

Answer

20.0ºC and 25.6ºC

4. A tungsten light bulb filament may operate at 2900 K. What is its Fahrenheit temperature? What is this on the Celsius scale?

5. The surface temperature of the Sun is about 5750 K. What is this temperature on the Fahrenheit scale?

Answer

9890ºF

6. One of the hottest temperatures ever recorded on the surface of Earth was 134ºF in Death Valley, CA. What is this temperature in Celsius degrees? What is this temperature in Kelvin?

7. (a) Suppose a cold front blows into your locale and drops the temperature by 40.0 Fahrenheit degrees. How many degrees Celsius does the temperature decrease when there is a 40.0ºF decrease in temperature?

(b) Show that any change in temperature in Fahrenheit degrees is nine-fifths the change in Celsius degrees.

Answer

(a) 22.2ºC

(b)
$$\begin{aligned}\Delta T\,({}^\circ\mathrm{F}) &= T_2\,({}^\circ\mathrm{F}) - T_1\,({}^\circ\mathrm{F}) \\ &= \frac{9}{5}T_2\,({}^\circ\mathrm{C}) + 32.0^\circ - \left(\frac{9}{5}T_1\,({}^\circ\mathrm{C}) + 32.0^\circ\right) \\ &= \frac{9}{5}(T_2\,({}^\circ\mathrm{C}) - T_1\,({}^\circ\mathrm{C})) = \frac{9}{5}\Delta T\,({}^\circ\mathrm{C})\end{aligned}$$

8. (a) At what temperature do the Fahrenheit and Celsius scales have the same numerical value?

(b) At what temperature do the Fahrenheit and Kelvin scales have the same numerical value?

9.2: The Ideal Gas Law

9. The gauge pressure in your car tires is $2.50 \times 10^5\ \mathrm{N/m^2}$ at a temperature of 35.0ºC when you drive it onto a ferry boat to Alaska. What is their gauge pressure later, when their temperature has dropped to –40.0ºC?

Answer

1.62 atm

10. Convert an absolute pressure of $7.00 \times 10^5\ \mathrm{N/m^2}$ to gauge pressure in $\mathrm{lb/in^2}$. (This value was stated to be just less than $90.0\ \mathrm{lb/in^2}$ in Example 9.2.1. Is it?)

11. Suppose a gas-filled incandescent light bulb is manufactured so that the gas inside the bulb is at atmospheric pressure when the bulb has a temperature of 20.0ºC.

(a) Find the gauge pressure inside such a bulb when it is hot, assuming its average temperature is 60.0ºC (an approximation) and neglecting any change in volume due to thermal expansion or gas leaks.

(b) The actual final pressure for the light bulb will be less than calculated in part (a) because the glass bulb will expand. What will the actual final pressure be, taking this into account? Is this a negligible difference?

Answer

(a) 0.136 atm

(b) 0.135 atm. The difference between this value and the value from part (a) is negligible.

12. Large helium-filled balloons are used to lift scientific equipment to high altitudes. (a) What is the pressure inside such a balloon if it starts out at sea level with a temperature of 10.0ºC and rises to an altitude where its volume is twenty times the original volume and its temperature is –50.0ºC? (b) What is the gauge pressure? (Assume atmospheric pressure is constant.)

13. In the text, it was shown that $N/V = 2.68 \times 10^{25}\ \mathrm{m^{-3}}$ for gas at STP.

(a) Show that this quantity is equivalent to $N/V = 2.68 \times 10^{19}\ \mathrm{cm^{-3}}$, as stated.

(b) About how many atoms are there in one $\mu\mathrm{m}^3$ (a cubic micrometer) at STP?

(c) What does your answer to part (b) imply about the separation of atoms and molecules?

14. An airplane passenger has $100\ \mathrm{cm^3}$ of air in his stomach just before the plane takes off from a sea-level airport. What volume will the air have at cruising altitude if cabin pressure drops to $7.50 \times 10^4\ \mathrm{N/m^2}$?

15. An expensive vacuum system can achieve a pressure as low as $1.00 \times 10^{-7}\ \mathrm{N/m^2}$ at 20ºC. How many atoms are there in a cubic centimeter at this pressure and temperature?

16. The number density of gas atoms at a certain location in the space above our planet is about $1.00 \times 10^{11}\ \mathrm{m^{-3}}$, and the pressure is $2.75 \times 10^{-10}\ \mathrm{N/m^2}$ in this space. What is the temperature there?

Answer

−73.9ºC

17. A bicycle tire has a pressure of $7.00 \times 10^5\ \mathrm{N/m^2}$ at a temperature of 18.0ºC and contains 2.00 L of gas. What will its pressure be if you let out an amount of air that has a volume of 100 $\mathrm{cm^3}$ at atmospheric pressure? Assume tire temperature and volume remain constant.

18. A high-pressure gas cylinder contains 50.0 L of toxic gas at a pressure of $1.40 \times 10^7\ \mathrm{N/m^2}$ and a temperature of 25.0ºC. Its valve leaks after the cylinder is dropped. The cylinder is cooled to dry ice temperature (–78.5ºC) to reduce the leak rate and pressure so that it can be safely repaired.

(a) What is the final pressure in the tank, assuming a negligible amount of gas leaks while being cooled and that there is no phase change?

(b) What is the final pressure if one-tenth of the gas escapes?

(c) To what temperature must the tank be cooled to reduce the pressure to 1.00 atm (assuming the gas does not change phase and that there is no leakage during cooling)?

(d) Does cooling the tank appear to be a practical solution?

Answer

(a) $9.14 \times 10^6\ \mathrm{N/m^2}$

(b) $8.23 \times 10^6\ \mathrm{N/m^2}$

(c) 2.16 K

(d) No. The final temperature needed is much too low to be easily achieved for a large object.

19. (a) What is the gauge pressure in a 25.0ºC car tire containing 3.60 mol of gas in a 30.0 L volume?

(b) What will its gauge pressure be if you add 1.00 L of gas originally at atmospheric pressure and 25.0ºC? Assume the temperature returns to 25.0ºC and the volume remains constant.

9.10: Carnot's Perfect Heat Engine- The Second Law of Thermodynamics Restated

20. A certain gasoline engine has an efficiency of 30.0%. What would the hot reservoir temperature be for a Carnot engine having that efficiency, if it operates with a cold reservoir temperature of $200^\circ\mathrm{C}$?

Solution

403ºC

21. A gas-cooled nuclear reactor operates between hot and cold reservoir temperatures of $700^\circ\mathrm{C}$ and $27.0^\circ\mathrm{C}$.

(a) What is the maximum efficiency of a heat engine operating between these temperatures?

(b) Find the ratio of this efficiency to the Carnot efficiency of a standard nuclear reactor (found in Example 9.10.1).

22. (a) What is the hot reservoir temperature of a Carnot engine that has an efficiency of 42.0% and a cold reservoir temperature of $27.0^{\circ}\mathrm{C}$?

(b) What must the hot reservoir temperature be for a real heat engine that achieves 0.700 of the maximum efficiency, but still has an efficiency of 42.0% (and a cold reservoir at $27.0^{\circ}\mathrm{C}$)?

(c) Does your answer imply practical limits to the efficiency of car gasoline engines?

Solution

(a) 244ºC

(b) 477ºC

(c) Yes, since automobiles engines cannot get too hot without overheating, their efficiency is limited.

23. Steam locomotives have an efficiency of 17.0% and operate with a hot steam temperature of $425^{\circ}\mathrm{C}$.

(a) What would the cold reservoir temperature be if this were a Carnot engine?

(b) What would the maximum efficiency of this steam engine be if its cold reservoir temperature were $150^{\circ}\mathrm{C}$?

24. Practical steam engines utilize $450^{\circ}\mathrm{C}$ steam, which is later exhausted at $270^{\circ}\mathrm{C}$.

(a) What is the maximum efficiency that such a heat engine can have?

(b) Since $270^{\circ}\mathrm{C}$ steam is still quite hot, a second steam engine is sometimes operated using the exhaust of the first. What is the maximum efficiency of the second engine if its exhaust has a temperature of $150^{\circ}\mathrm{C}$?

(c) What is the overall efficiency of the two engines? (d) Show that this is the same efficiency as a single Carnot engine operating between $450^{\circ}\mathrm{C}$ and $150^{\circ}\mathrm{C}$. Explicitly show how you follow the steps in the Problem-Solving Strategies for Thermodynamics.

Solution

(a) $Eff_1 = 1 - \frac{T_{\mathrm{c},1}}{T_{\mathrm{h},1}} = 1 - \frac{543\ \mathrm{K}}{723\ \mathrm{K}} = 0.249 \text{ or } 24.9\%$

(b) $Eff_2 = 1 - \frac{423\ \mathrm{K}}{543\ \mathrm{K}} = 0.221 \text{ or } 22.1\%$

(c) $$Eff_1 = 1 - \frac{T_{\mathrm{c},1}}{T_{\mathrm{h},1}} \Rightarrow T_{\mathrm{c},1} = T_{\mathrm{h},1}\left(1, -, eff_1\right) \quad \text{similarly, } T_{\mathrm{c},2} = T_{\mathrm{h},2}\left(1 - Eff_2\right)$$

$$T_{\mathrm{c},2} = T_{\mathrm{h},1}\left(1 - Eff_1\right)\left(1 - Eff_2\right) \equiv T_{\mathrm{h},1}\left(1 - Eff_{\text{overall}}\right)$$

$$\text{using } T_{\mathrm{h},2} = T_{\mathrm{c},1} \text{ in above equation gives } \therefore \left(1 - Eff_{\text{overall}}\right) = \left(1 - Eff_1\right)\left(1 - Eff_2\right)$$

$$Eff_{\text{overall}} = 1 - (1 - 0.249)(1 - 0.221) = 41.5\%$$

(d) $Eff_{\text{overall}} = 1 - \frac{423\ \mathrm{K}}{723\ \mathrm{K}} = 0.415 \text{ or } 41.5\%$

25. A coal-fired electrical power station has an efficiency of 38%. The temperature of the steam leaving the boiler is $550^{\circ}\mathrm{C}$. What percentage of the maximum efficiency does this station obtain? (Assume the temperature of the environment is $20^{\circ}\mathrm{C}$.)

26. Would you be willing to financially back an inventor who is marketing a device that she claims has 25 kJ of heat transfer at 600 K, has heat transfer to the environment at 300 K, and does 12 kJ of work? Explain your answer.

Solution

The heat transfer to the cold reservoir is $Q_{\mathrm{c}} = Q_{\mathrm{h}} - W = 25\ \mathrm{kJ} - 12\ \mathrm{kJ} = 13\ \mathrm{kJ}$, so the efficiency is $Eff = 1 - \frac{Q_{\mathrm{c}}}{Q_{\mathrm{h}}} = 1 - \frac{13\ \mathrm{kJ}}{25\ \mathrm{kJ}} = 0.48$. The Carnot efficiency is $Eff_{\mathrm{C}} = 1 - \frac{T_{\mathrm{c}}}{T_{\mathrm{h}}} = 1 - \frac{300\ \mathrm{K}}{600\ \mathrm{K}} = 0.50$. The actual efficiency is 96% of the Carnot efficiency, which is much higher than the best-ever achieved of about 70%, so her scheme is likely to be fraudulent.

Unreasonable Results

27. (a) Suppose you want to design a steam engine that has heat transfer to the environment at 270ºC and has a Carnot efficiency of 0.800. What temperature of hot steam must you use?

(b) What is unreasonable about the temperature?

(c) Which premise is unreasonable?

Unreasonable Results

28. Calculate the cold reservoir temperature of a steam engine that uses hot steam at $450^{\circ}\mathrm{C}$ and has a Carnot efficiency of 0.700.

(b) What is unreasonable about the temperature?

(c) Which premise is unreasonable?

Solution

(a) –56.3ºC

(b) The temperature is too cold for the output of a steam engine (the local environment). It is below the freezing point of water.

(c) The assumed efficiency is too high.

9.11: Applications of Thermodynamics- Heat Pumps and Refrigerators

29. What is the coefficient of performance of an ideal heat pump that has heat transfer from a cold temperature of −25.0ºC to a hot temperature of 40.0ºC?

Answer

4.82

30. Suppose you have an ideal refrigerator that cools an environment at −20.0ºC and has heat transfer to another environment at 50.0ºC. What is its coefficient of performance?

31. What is the best coefficient of performance possible for a hypothetical refrigerator that could make liquid nitrogen at −200ºC and has heat transfer to the environment at 35.0ºC?

Answer

0.311

32. In a very mild winter climate, a heat pump has heat transfer from an environment at 5.00ºC to one at 35.0ºC. What is the best possible coefficient of performance for these temperatures? Explicitly show how you follow the steps in the Problem-Solving Strategies for Thermodynamics.

33. (a) What is the best coefficient of performance for a heat pump that has a hot reservoir temperature of 50.0ºC and a cold reservoir temperature of −20.0ºC?

(b) How much heat transfer occurs into the warm environment if $3.60 \times 10^{7}\ \mathrm{J}$ of work $(10.0\ \mathrm{kW} \cdot \mathrm{h})$ is put into it?

(c) If the cost of this work input is $10.0\ \mathrm{cents}\ /\mathrm{kW} \cdot \mathrm{h}$, how does its cost compare with the direct heat transfer achieved by burning natural gas at a cost of 85.0 cents per therm. (A therm is a common unit of energy for natural gas and equals $1.055 \times 10^{8}\ \mathrm{J}$.)

Answer

(a) 4.61

(b) $1.66 \times 10^{8}\ \mathrm{J}$ or $3.97 \times 10^{4}\mathrm{kcal}$

(c) To transfer $1.66 \times 10^{8}\ \mathrm{J}$, heat pump costs \$1.00, natural gas costs \$1.34.

34. (a) What is the best coefficient of performance for a refrigerator that cools an environment at −30.0ºC and has heat transfer to another environment at 45.0ºC?

(b) How much work in joules must be done for a heat transfer of 4186 kJ from the cold environment?

(c) What is the cost of doing this if the work costs 10.0 cents per $3.60 \times 10^{6}\ \mathrm{J}$ (a kilowatt-hour)?

(d) How many kJ of heat transfer occurs into the warm environment?

(e) Discuss what type of refrigerator might operate between these temperatures.

35. Suppose you want to operate an ideal refrigerator with a cold temperature of −10.0ºC, and you would like it to have a coefficient of performance of 7.00. What is the hot reservoir temperature for such a refrigerator?

Answer

27.6ºC

36. An ideal heat pump is being considered for use in heating an environment with a temperature of 22.0ºC. What is the cold reservoir temperature if the pump is to have a coefficient of performance of 12.0?

37. A 4-ton air conditioner removes 5.06×10^7 J (48,000 British thermal units) from a cold environment in 1.00 h.

(a) What energy input in joules is necessary to do this if the air conditioner has an energy efficiency rating (EER) of 12.0?

(b) What is the cost of doing this if the work costs 10.0 cents per 3.60×10^6 J (one kilowatt-hour)?

(c) Discuss whether this cost seems realistic. Note that the energy efficiency rating (EER) of an air conditioner or refrigerator is defined to be the number of British thermal units of heat transfer from a cold environment per hour divided by the watts of power input.

Answer

(a) 1.44×10^7 J

(b) 40 cents

(c) This cost seems quite realistic; it says that running an air conditioner all day would cost $9.59 (if it ran continuously).

38. Show that the coefficients of performance of refrigerators and heat pumps are related by $COP_{\text{ref}} = COP_{\text{hp}} - 1$.

39. Start with the definitions of the COPs and the conservation of energy relationship between Q_h, Q_c, and W.

9.13: Statistical Interpretation of Entropy and the Second Law of Thermodynamics- The Underlying Explanation

40. Using Table 9.13.3, verify the contention that if you toss 100 coins each second, you can expect to get 100 heads or 100 tails once in 2×10^{22} years; calculate the time to two-digit accuracy.

Solution

It should happen twice in every 1.27×10^{30} s or once in every 6.35×10^{29} s

$$\left(6.35 \times 10^{29} \text{ s}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{1 \text{ d}}{24 \text{ h}}\right)\left(\frac{1 \text{y}}{365.25 \text{ d}}\right)$$

$$= 2.0 \times 10^{22} \text{y}$$

41. What percent of the time will you get something in the range from 60 heads and 40 tails through 40 heads and 60 tails when tossing 100 coins? The total number of microstates in that range is 1.22×10^{30}. (Consult Table 9.13.3.)

42. (a) If tossing 100 coins, how many ways (microstates) are there to get the three most likely macrostates of 49 heads and 51 tails, 50 heads and 50 tails, and 51 heads and 49 tails?

(b) What percent of the total possibilities is this? (Consult Table 9.13.3.)

Solution

(a) 3.0×10^{29}

(b) 24%

43. (a) What is the change in entropy if you start with 100 coins in the 45 heads and 55 tails macrostate, toss them, and get 51 heads and 49 tails?

(b) What if you get 75 heads and 25 tails?

(c) How much more likely is 51 heads and 49 tails than 75 heads and 25 tails?

(d) Does either outcome violate the second law of thermodynamics?

44. (a) What is the change in entropy if you start with 10 coins in the 5 heads and 5 tails macrostate, toss them, and get 2 heads and 8 tails?

(b) How much more likely is 5 heads and 5 tails than 2 heads and 8 tails? (Take the ratio of the number of microstates to find out.)

(c) If you were betting on 2 heads and 8 tails would you accept odds of 252 to 45? Explain why or why not.

Solution

(a) -2.38×10^{-23} J/K

(b) 5.6 times more likely

(c) If you were betting on two heads and 8 tails, the odds of breaking even are 252 to 45, so on average you would break even. So, no, you wouldn't bet on odds of 252 to 45.

Table $8.7.E.1$: 10-Coin Toss

Macrostate			Number of Microstates (W)	
Heads	Tails			
10	0		1	
9	1		10	
8	2		45	
7	3		120	
6	4		210	
5	5		252	
4	6		210	
3	7		120	
2	8		45	
1	9		10	
0	10		1	
			Total: 1024	

45. (a) If you toss 10 coins, what percent of the time will you get the three most likely macrostates (6 heads and 4 tails, 5 heads and 5 tails, 4 heads and 6 tails)?

(b) You can realistically toss 10 coins and count the number of heads and tails about twice a minute. At that rate, how long will it take on average to get either 10 heads and 0 tails or 0 heads and 10 tails?

46. (a) Construct a table showing the macrostates and all of the individual microstates for tossing 6 coins. (Use Table $8.7.E.1$ as a guide.)

(b) How many macrostates are there?

(c) What is the total number of microstates?

(d) What percent chance is there of tossing 5 heads and 1 tail?

(e) How much more likely are you to toss 3 heads and 3 tails than 5 heads and 1 tail? (Take the ratio of the number of microstates to find out.)

Solution

(b) 7

(c) 64

(d) 9.38%

(e) 3.33 times more likely (20 to 6)

47. In an air conditioner, 12.65 MJ of heat transfer occurs from a cold environment in 1.00 h.

(a) What mass of ice melting would involve the same heat transfer?

(b) How many hours of operation would be equivalent to melting 900 kg of ice?

(c) If ice costs 20 cents per kg, do you think the air conditioner could be operated more cheaply than by simply using ice? Describe in detail how you evaluate the relative costs.

8.8: Conservation of Energy

Learning Objectives

By the end of this section, you will be able to:

- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.
- Define efficiency of an energy conversion process as the fraction left as useful energy or work, rather than being transformed, for example, into thermal energy.

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The law of conservation of energy can be stated as follows:

We have explored some forms of energy and some ways it can be transferred from one system to another. This exploration led to the definition of two major types of energy—mechanical energy $(KE+PE)$ and energy transferred via work done by nonconservative forces (W_{nc}) But energy takes *many* other forms, manifesting itself in *many* different ways, and we need to be able to deal with all of these before we can write an equation for the above general statement of the conservation of energy.

Other Forms of Energy than Mechanical Energy

At this point, we deal with all other forms of energy by lumping them into a single group called other energy (OE). Then we can state the conservation of energy in equation form as

$$KE_i+PE_i+W_{nc}+OE_i=KE_f+PE_f+OE_f. \tag{8.8.1}$$

All types of energy and work can be included in this very general statement of conservation of energy. Kinetic energy is KE, work done by a conservative force is represented by PE, work done by nonconservative forces is W_{nc} and all other energies are included as OE. This equation applies to all previous examples; in those situations OE was constant, and so it subtracted out and was not directly considered.

Usefulness of the Energy Conservation Principle

The fact that energy is conserved and has many forms makes it very important. You will find that energy is discussed in many contexts, because it is involved in all processes. It will also become apparent that many situations are best understood in terms of energy and that problems are often most easily conceptualized and solved by considering energy.

When does OE play a role? One example occurs when a person eats. Food is oxidized with the release of carbon dioxide, water, and energy. Some of this chemical energy is converted to kinetic energy when the person moves, to potential energy when the person changes altitude, and to thermal energy (another form of OE).

Some of the Many Forms of Energy

What are some other forms of energy? You can probably name a number of forms of energy not yet discussed. Many of these will be covered in later chapters, but let us detail a few here. **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry **chemical energy** that can be transferred to a system through oxidation. Chemical fuel can also produce electrical energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as **radiant energy**, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. **Nuclear energy** comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power plants, and into the energy of the heat transfer and blast in weapons. Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called **thermal energy**, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

Table gives the amount of energy stored, used, or released from various objects and in various phenomena. The range of energies and the variety of types and situations is impressive.

Problem-Solving Strategies for Energy

You will find the following problem-solving strategies useful whenever you deal with energy. The strategies help in organizing and reinforcing energy concepts. In fact, they are used in the examples presented in this chapter. The familiar general problem-solving strategies presented earlier—involving identifying physical principles, knowns, and unknowns, checking units, and so on —continue to be relevant here.

Step 1. Determine the system of interest and identify what information is given and what quantity is to be calculated. A sketch will help.

Step 2. Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step 3 or step 4.

Step 3. If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of energy is

$$KE_i + PE_i = KE_f + PE_f. \tag{8.8.2}$$

Step 4. If you know the potential energy for only some of the forces, possibly because some of them are nonconservative and do not have a potential energy, or if there are other energies that are not easily treated in terms of force and work, then the conservation of energy law in its most general form must be used.

$$KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f. \tag{8.8.3}$$

In most problems, one or more of the terms is zero, simplifying its solution. Do not calculate W_c, the work done by conservative forces; it is already incorporated in the PE terms.

Step 5. You have already identified the types of work and energy involved (in step 2). Before solving for the unknown, *eliminate terms wherever possible* to simplify the algebra. For example, choose $h = 0$ at either the initial or final point, so that PE_g is zero there. Then solve for the unknown in the customary manner.

Step 6. *Check the answer to see if it is reasonable*. Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on. Also check to see that the numerical value obtained is reasonable. For example, the final speed of a skateboarder who coasts down a 3-m-high ramp could reasonably be 20 km/h, but *not* 80 km/h.

Transformation of Energy

The transformation of energy from one form into others is happening all the time. The chemical energy in food is converted into thermal energy through metabolism; light energy is converted into chemical energy through photosynthesis. In a larger example, the chemical energy contained in coal is converted into thermal energy as it burns to turn water into steam in a boiler. This thermal energy in the steam in turn is converted to mechanical energy as it spins a turbine, which is connected to a generator to produce electrical energy. (In all of these examples, not all of the initial energy is converted into the forms mentioned. This important point is discussed later in this section.)

Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell (Figure 7.7.1) produces electricity, which in turn can be used to run an electric motor. Energy is converted from the primary source of solar energy into electrical energy and then into mechanical energy.

Figure 8.8.1: Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar-power aircraft. (credit: NASA)

Object/phenomenon	Energy in joules
Big Bang	10^{68}
Energy released in a supernova	10^{44}
Fusion of all the hydrogen in Earth’s oceans	10^{34}
Annual world energy use	4×10^{20}
Large fusion bomb (9 megaton)	3.8×10^{16}
1 kg hydrogen (fusion to helium)	6.4×10^{14}
1 kg uranium (nuclear fission)	8.0×10^{13}
Hiroshima-size fission bomb (10 kiloton)	4.2×10^{13}
90,000-ton aircraft carrier at 30 knots	1.1×10^{10}
1 barrel crude oil	5.9×10^{9}
1 ton TNT	4.2×10^{9}
1 gallon of gasoline	1.2×10^{8}
Daily home electricity use (developed countries)	$7n \times 10^{7}$
Daily adult food intake (recommended)	1.2×10^{7}
1000-kg car at 90 km/h	3.1×10^{5}
1 g fat (9.3 kcal)	3.9×10^{4}
ATP hydrolysis reaction	3.2×10^{4}
1 g carbohydrate (4.1 kcal)	1.7×10^{4}
1 g protein (4.1 kcal)	1.7×10^{4}
Tennis ball at 100 km/h	22
Mosquito $^{-2}$ g at $0.5\ \mathrm{m/s})$	1.3×10^{-6}
Single electron in a TV tube beam	4.0×10^{-15}
Energy to break one DNA strand	4.0×10^{-19}

Efficiency

Even though energy is conserved in an energy conversion process, the output of *useful energy* or work will be less than the energy input. The efficiency E_{ff} of an energy conversion process is defined as

$$Efficiency\,(E_{ff}) = \frac{useful\,energy\,or\,work\,output}{total\,energy\,input} = \frac{W_{out}}{E_{in}}. \tag{8.8.4}$$

Table lists some efficiencies of mechanical devices and human activities. In a coal-fired power plant, for example, about 40% of the chemical energy in the coal becomes useful electrical energy. The other 60% transforms into other (perhaps less useful) energy forms, such as thermal energy, which is then released to the environment through combustion gases and cooling towers.

Activity/device	Efficiency (%)
Cycling and climbing	20
Swimming, surface	2
Swimming, submerged	4
Shoveling	3
Weightlifting	9
Steam engine	17
Gasoline engine	30
Diesel engine	35
Nuclear power plant	35
Coal power plant	42
Electric motor	98
Compact fluorescent light	20
Gas heater (residential)	90
Solar cell	10

Efficiency of the Human Body and Mechanical Devices

PhET Explorations: Masses and Springs

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energies for each spring.

Figure 8.8.2: Masses and Springs

Summary

- The law of conservation of energy states that the total energy is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same.
- When all forms of energy are considered, conservation of energy is written in equation form as

$$KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f, \tag{8.8.5}$$

  where OE is all **other forms of energy** besides mechanical energy.
- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.

- Energy is often utilized to do work, but it is not possible to convert all the energy of a system to work.

The efficiency E_{ff} of a machine or human is defined to be $E_{ff} = \frac{W_{out}}{E_{in}}$, where W_{out} is useful work output and E_{in} s the energy consumed.

Glossary

law of conservation of energy
the general law that total energy is constant in any process; energy may change in form or be transferred from one system to another, but the total remains the same

electrical energy
the energy carried by a flow of charge

chemical energy
the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction

radiant energy
the energy carried by electromagnetic waves

nuclear energy
energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

thermal energy
the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

efficiency
a measure of the effectiveness of the input of energy to do work; useful energy or work divided by the total input of energy

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8.9: End of Chapter Activity

End of Chapter Activity: Creating a Lesson Plan on Energy in Physics and Chemistry with AI and Bloom's Taxonomy

Now that you have explored the fundamentals of energy in physics and chemistry, it's time to put your knowledge into practice. Your task is to create a succinct lesson plan for 6th graders that introduces them to the basics of energy, including its forms, transformations, and conservation. To help you with this, you will use AI tools and incorporate Bloom's Taxonomy to ensure a comprehensive learning experience. This lesson plan will go towards your digital notebook, a portfolio filled with lesson plans, activities, and labs for future use.

Activity Prompt:

> **Objective:** Use AI and Bloom's Taxonomy to develop a lesson plan that effectively teaches 6th graders about the fundamentals of energy, including concepts such as kinetic and potential energy, energy transformations, and the law of conservation of energy.

Understanding the Concepts:

> **Knowledge (Remembering):** Define key terms related to energy, such as kinetic energy, potential energy, energy transformation, and conservation of energy.
>
> **Comprehension (Understanding):** Explain these concepts in simple, age-appropriate language, focusing on their importance and real-life applications.

Planning the Lesson:

> **Application:** Design an engaging activity that allows students to observe and understand different forms of energy and energy transformations. For example, use a pendulum or a roller coaster model to demonstrate kinetic and potential energy.
>
> **Analysis:** Use AI tools to create visual aids or interactive simulations that illustrate how energy is transformed from one form to another. For instance, create a simulation that shows a roller coaster in motion, highlighting the conversion between kinetic and potential energy.

Deepening Understanding:

> **Synthesis (Creating):** Ask students to design their own simple machines that demonstrate energy transformations. For example, they could design a Rube Goldberg machine that incorporates different forms of energy.
>
> **Evaluation:** Have students discuss and reflect on their designs and the energy transformations involved. Encourage them to think about how energy conservation applies in their daily lives.

Using AI in the Classroom:

Explore AI tools like educational apps or platforms that provide interactive content for teaching about energy in physics and chemistry. Use these tools to create quizzes, flashcards, or interactive stories that reinforce the lesson's concepts.

Use AI to assess student understanding through formative assessments and provide instant feedback.

Deliverable:

Submit a detailed lesson plan that includes:

1. **A brief overview of the key concepts covered:** Outline the foundational concepts of energy that will be taught.
2. **A description of the activities and experiments designed:** Detail the hands-on activities and experiments you will use to help students understand these concepts.
3. **Examples of AI tools used and how they enhance the learning experience:** Describe the AI tools you plan to incorporate, such as simulations or interactive quizzes, and explain how they will help students grasp complex concepts.
4. **An explanation of how Bloom's Taxonomy was applied in the lesson plan to ensure a well-rounded educational experience:** Illustrate how each level of Bloom's Taxonomy (Remembering, Understanding, Applying, Analyzing, Creating, and Evaluating) is addressed in your lesson plan.

Additionally, include a creative project component where students create a multimedia presentation or a digital story that explains a real-world application of energy concepts, using AI tools to enhance their projects.

Example Lesson Plan:

Grade: 6th Grade
Topic: Energy in Physics and Chemistry
Duration: 1 Week

Overview:
Students will learn about different forms of energy, energy transformations, and the law of conservation of energy through engaging activities and creative projects.

Day 1: Introduction to Energy

Objective: Define different forms of energy and provide examples.

- **Remembering:** Define key terms (kinetic energy, potential energy, energy transformation, conservation of energy).
- **Understanding:** Explain the concepts using examples from everyday life (e.g., a moving car, a compressed spring).

Activity:
Watch a video (created using AI tools) explaining different forms of energy with real-life examples.

Day 2: Observing Energy Transformations

Objective: Observe and describe energy transformations.

- **Applying:** Conduct an experiment to observe energy transformations (e.g., a pendulum swing to demonstrate kinetic and potential energy).

Activity:
Students use pendulums to observe how potential energy is converted to kinetic energy and vice versa. They record their observations and discuss the energy changes they see.

Day 3: Energy Conservation

Objective: Understand the law of conservation of energy.

- **Analyzing:** Use AI tools to create visual aids showing how energy is conserved in various processes.

Activity:
Students use an AI simulation to visualize energy transformations in a roller coaster and identify points where potential and kinetic energy are at their maximum and minimum.

Day 4: Creative Project – Designing Simple Machines

Objective: Design a simple machine that demonstrates energy transformations.

- **Creating:** Students design and build a Rube Goldberg machine that incorporates different forms of energy.

Activity:
In groups, students design and build a simple Rube Goldberg machine using household items. They document each step, describing the energy transformations involved.

Day 5: Reflection and Multimedia Presentation

Objective: Reflect on and present their projects.

- **Evaluating:** Discuss and reflect on the energy transformations in their projects.
- **Creating:** Use AI tools to create a multimedia presentation or digital story explaining a real-world application of energy concepts.

Activity:
Students create a multimedia presentation or a digital story about a real-world application of energy, such as how renewable energy sources work. They use AI tools to enhance their presentation, such as adding animations or interactive elements.

By incorporating these strategies and activities, educators can effectively teach 6th graders about energy in physics and chemistry, helping them understand and appreciate the fundamental concepts and their applications in the real world.

8.10: End of Chapter Key Terms

Energy in Physics and Chemistry Key Terms

1. **Energy**: The capacity to do work or produce heat; exists in various forms such as kinetic, potential, thermal, chemical, and nuclear.
2. **Kinetic Energy**: The energy an object possesses due to its motion; calculated as 12mv2\frac{1}{2}mv^221mv2 where mmm is mass and vvv is velocity.
3. **Potential Energy**: The energy stored in an object due to its position or configuration, such as gravitational potential energy or elastic potential energy.
4. **Thermal Energy**: The total kinetic energy of particles in a substance; often associated with temperature.
5. **Chemical Energy**: The potential energy stored in chemical bonds between atoms and molecules.
6. **Nuclear Energy**: The energy stored in the nucleus of an atom; released during nuclear fission or fusion.
7. **Law of Conservation of Energy**: A principle stating that energy cannot be created or destroyed, only transformed from one form to another.
8. **Work**: The transfer of energy that occurs when a force is applied over a distance; calculated as W=Fdcosθ W = Fd \cos \thetaW=Fdcosθ where FFF is force, ddd is distance, and θ\thetaθ is the angle between the force and displacement vectors.
9. **Power**: The rate at which work is done or energy is transferred; measured in watts (W).
10. **Heat**: The transfer of thermal energy between substances of different temperatures.
11. **Temperature**: A measure of the average kinetic energy of particles in a substance.
12. **Specific Heat Capacity**: The amount of heat required to raise the temperature of one gram of a substance by one degree Celsius.
13. **Calorimetry**: The measurement of the amount of heat released or absorbed during a chemical reaction.
14. **Enthalpy (H)**: The total heat content of a system; changes in enthalpy (ΔH\Delta HΔH) are used to describe heat changes at constant pressure.
15. **Entropy (S)**: A measure of the disorder or randomness in a system; systems tend to move towards higher entropy.
16. **Free Energy (G)**: The energy in a system that can be used to do work; changes in free energy (ΔG\Delta GΔG) predict the spontaneity of a process.
17. **Endothermic Reaction**: A chemical reaction that absorbs energy from its surroundings, resulting in a decrease in temperature.
18. **Exothermic Reaction**: A chemical reaction that releases energy to its surroundings, resulting in an increase in temperature.
19. **Activation Energy**: The minimum energy required for a chemical reaction to occur.
20. **Catalyst**: A substance that increases the rate of a chemical reaction without being consumed in the process by lowering the activation energy.
21. **Bond Energy**: The amount of energy required to break one mole of bonds in a substance.
22. **First Law of Thermodynamics**: The principle of conservation of energy applied to thermodynamic systems, stating that the total energy of an isolated system is constant.
23. **Second Law of Thermodynamics**: States that the total entropy of an isolated system can never decrease over time; systems naturally progress towards thermodynamic equilibrium, the state of maximum entropy.
24. **Third Law of Thermodynamics**: States that the entropy of a perfect crystal approaches zero as the temperature approaches absolute zero.
25. **Gibbs Free Energy**: A thermodynamic quantity representing the amount of energy available to do work; ΔG=ΔH−TΔS\Delta G = \Delta H - T \Delta SΔG=ΔH−TΔS.
26. **Chemical Equilibrium**: A state in which the forward and reverse reactions occur at the same rate, resulting in no net change in the concentration of reactants and products.
27. **Electrochemical Cell**: A device that generates electrical energy from chemical reactions or facilitates chemical reactions through the introduction of electrical energy.
28. **Redox Reaction**: A chemical reaction involving the transfer of electrons from one substance to another, comprising oxidation and reduction processes.
29. **Oxidation**: The loss of electrons by a molecule, atom, or ion.
30. **Reduction**: The gain of electrons by a molecule, atom, or ion.

31. **Fuel Cell**: A device that converts the chemical energy of a fuel into electrical energy through a chemical reaction with oxygen or another oxidizing agent.
32. **Photovoltaic Cell**: A device that converts light energy into electrical energy using the photovoltaic effect.
33. **Battery**: A device consisting of one or more electrochemical cells that store chemical energy and convert it to electrical energy.
34. **Heat Engine**: A device that converts thermal energy into mechanical work.
35. **Carnot Cycle**: A theoretical cycle that represents the most efficient sequence of processes possible for a heat engine.
36. **Thermochemistry**: The study of the heat energy associated with chemical reactions and changes of state.

CHAPTER OVERVIEW

9: Motion

Content from this chapter is remixed from:

- Introduction to Physics by Park, Open Stax, Unit 1: Mechnaics I- Motion and Forces, Chapter 1 Kinematics https://phys.libretexts.org/Bookshel...%3A_Kinematics
- Thermal physics https://phys.libretexts.org/Bookshel...hermal_Physics
-

9.1: Chapter Objectives

Learning Objectives

1. **Fundamentals of Motion:**
    - Understand and define key concepts related to motion, including distance, displacement, speed, velocity, and acceleration.
    - Explore the differences between scalar and vector quantities and their significance in describing motion.
    - Analyze motion in one dimension, including uniform and non-uniform motion, and learn to represent motion through graphical methods such as distance-time and velocity-time graphs.
2. **Laws of Motion and Their Applications:**
    - Investigate Newton's Laws of Motion and their relevance in explaining the behavior of moving objects.
    - Understand the concepts of force, mass, and inertia, and how they relate to motion.
    - Apply Newton's Laws to solve problems related to everyday situations and simple mechanical systems.
3. **Circular Motion and Centripetal Force:**
    - Explore the characteristics of circular motion, including uniform and non-uniform circular motion.
    - Understand the concept of centripetal force and its role in keeping an object in circular motion.
    - Examine real-world examples of circular motion, such as satellites orbiting planets and vehicles turning on curved paths.
4. **Motion in Two and Three Dimensions:**
    - Investigate the principles of motion in two and three dimensions, including projectile motion and relative motion.
    - Understand how to analyze and resolve motion into its components using vector addition.
    - Explore practical applications of two and three-dimensional motion in sports, engineering, and nature.
5. **Teaching Motion in Physical Science Education:**
    - Develop effective teaching strategies for conveying the concepts of motion to students, using demonstrations, experiments, and real-world examples.
    - Design classroom activities that allow students to explore and understand the principles of motion through hands-on experience.
    - Utilize multimedia tools and simulation software to visually represent and enhance the understanding of complex motion scenarios.
    - Address common misconceptions and challenges students face when learning about motion.
    - Emphasize the importance of understanding motion in various fields of science and everyday life applications.

9.2: Introduction- Fundamentals of Motion- Scientific Overview

Introduction to the Fundamentals of Motion: A Scientific Overview

- *This chapter was authored and remixed by Kylee Ford from Coalinga College*

Motion is a fundamental concept in physics, describing the change in position of an object over time. Key concepts include distance (the total path covered by an object), displacement (the change in position of an object), speed (the rate at which an object covers distance), velocity (speed with a specified direction), and acceleration (the rate of change of velocity). Understanding these concepts requires distinguishing between scalar quantities, which have only magnitude (like distance and speed), and vector quantities, which have both magnitude and direction (like displacement and velocity). Analyzing motion often involves graphical methods, such as distance-time and velocity-time graphs, which help visualize how an object's position and velocity change over time.

Newton's Laws of Motion form the basis for understanding how forces influence motion. Newton's First Law states that an object at rest stays at rest, and an object in motion stays in motion, unless acted upon by an external force. The Second Law quantifies this relationship with the equation

$F=ma$ (force equals mass times acceleration),

and the Third Law states that for every action, there is an equal and opposite reaction. These laws explain everyday phenomena, from the motion of vehicles to the dynamics of sports.

Circular motion, a type of motion where an object moves along a circular path, introduces the concept of centripetal force, which keeps the object in its circular trajectory. Real-world examples include satellites orbiting planets and cars navigating curves. Motion in two and three dimensions, such as projectile motion, involves analyzing motion components using vector addition, crucial in fields like engineering and sports science.

Teaching Motion in K-6 Classrooms

For K-6 educators, teaching these principles can be engaging by using hands-on activities and relatable examples. Simple experiments, like rolling balls on different surfaces to illustrate friction and motion, help students grasp these concepts. Using multimedia tools and interactive simulations can also make learning about motion more accessible and fun for young students.

9.2.1: Scope of Physics

An antenna depends on the interaction of matter and energy to function, making it a perfect example of physics in action

Figure 9.2.1.1

Physics is more than calculating the momentum of billiard balls hitting each other or the friction acting on a speeding car’s tires. Physics includes the study of practically every form of matter and how it interacts with other matter and with energy in various forms. The image shows one of several large parabolic antennas that NASA physicists used for years to communicate with ships and devices completing solar system exploration missions.

Definition of Physics

What is **physics**? Physics is the branch of science that studies the physical world, including objects as small as subatomic particles and as large as galaxies. It studies the nature of matter and energy and how they interact. Physicists are inquisitive people who want to know the causes of what they see. How does the moon move? Why does the moon move? Why do the stars shine? Why do your hands get warm when you rub them together? Physicists, like all scientists, hope to find explanations that describe more than one phenomenon and offer a better understanding of how the universe works.

Common Misconceptions

People commonly believe that physics is all about solving word problems and memorizing equations. While it is true that many physics classes focus on the equations, it is important to remember that the purpose of physics is less about the problems and more about using equations, laws, and theories to understand the world we live in.

Summary

- Physics is the branch of science that studies matter and energy and how they interact.

Review

1. Give your own definition of physics.
2. What do you already know about physics? What do you think you know?
3. Physics is all around us, all the time. Give a few examples of physics you have experienced.

Explore More

Use the resource below to answer the questions that follow.

1. Why can’t Hadfield dip the washcloth in a bag full of water?
2. Pause the video at 1:55. What do you expect will happen as he wrings out the washcloth?
3. What does the water do? Why?

Additional Resources

Video:

Real World Application: Ollie Up

Real World Application: Teen Uses Science to Make Water Safe

Video:

Video:

- **1.1: Scope of Physics** by CK-12 Foundation is licensed CK-12. Original source: https://ck12.org.

9.2.2: Vectors, Scalars, and Coordinate Systems

Learning Objectives

- Define and distinguish between scalar and vector quantities.
- Assign a coordinate system for a scenario involving one-dimensional motion.

Figure 9.2.2.1: The motion of this Eclipse Concept jet can be described in terms of the distance it has traveled (a scalar quantity) or its displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. In this case, it may be convenient to choose motion toward the left as positive motion (it is the forward direction for the plane), although in many cases, the x-coordinate runs from left to right, with motion to the right as positive and motion to the left as negative. (credit: Armchair Aviator, Flickr)

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A **vector** is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

The direction of a vector in one-dimensional motion is given simply by a plus (+) or minus (−) sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector’s magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A **scalar** is any quantity that has a magnitude, but no direction. For example, a 20ºC temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person’s 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a −20ºC temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in Figure 9.2.2.1, it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.

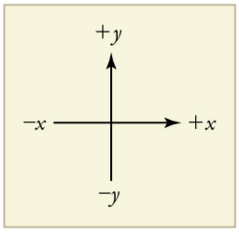

Figure 9.2.2.2: It is usually convenient to consider motion upward or to the right as positive (+) and motion downward or to the left as negative (−).

Exercise 9.2.2.1

A person’s speed can stay the same as he or she rounds a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

Answer

Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

Section Summary

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

Glossary

scalar

a quantity that is described by magnitude, but not direction

vector

a quantity that is described by both magnitude and direction

9.2.3: Resolving Vectors into Components

Resolving Vectors into Axial Components

We know that when two vectors are in the same dimension, they can be added arithmetically. Suppose we have two vectors that are on a north-south, east-west grid, as shown below. One of the methods we can use to add these vectors is to resolve each one into a pair of vectors that lay on the north-south and east-west axes.

Two vectors on a north-south, east-west grid, one originating between north and east, one originating between north and west

Figure 1.5.1

The two vectors we are to add is a force of 65 N at 30° north of east and a force of 35 N at 60° north of west.

Sum of two vectors on a north-south, east-west grid, creating a triangle for each vector

Figure 1.5.2

We can resolve each of the vectors into two components on the axes lines. Each vector is resolved into a component on the north-south axis and a component on the east-west axis.

Using trigonometry, we can resolve (break down) each of these vectors into a pair of vectors that lay on the axial lines (shown in red above).

The east-west component of the first vector is (65 N)(cos 30°) = (65 N)(0.866) = 56.3 N east

The north-south component of the first vector is (65 N)(sin 30°) = (65 N)(0.500) = 32.5 N north

The east-west component of the 2nd vector is (35 N)(cos 60°) = (35 N)(0.500) = 17.5 N west

The north-south component of the 2nd vector is (35 N)(sin 60°) = (35 N)(0.866) = 30.3 N north

Explore how to resolve vectors into axial components by hanging portraits of legendary physicists in the simulation below. The angle of each string determines how its tension force is distributed in both the horizontal and vertical dimensions. Under what conditions will the strings holding the portrait snap?

Summary

- Vectors can be resolved into component vectors that lie on the axes lines.

Review

1. A force of 150. N is exerted 22° north of east. Find the northward and eastward components of this force.
2. An automobile travels a displacement of 75 km 45° north of east. How far east does it travel and how far north does it travel?

Explore More

Use this resource to answer the questions that follow.

1. What does SohCahToa mean?
2. Why is SohCahToa relevant to resolving a vector into components?
3. Why is the sum of the components larger than the resultant vector?

Additional Resources

Real World Application: Physics Behind Darts

PLIX: Play, Learn, Interact, eXplore: Bow and Arrow

Video:

9.2.4: Vector Addition

Adding Vectors in Two Dimensions

In the following image, vectors A and B represent the two displacements of a person who walked 90. m east and then 50. m north. We want to add these two vectors to get the vector sum of the two movements.

Graph of two vectors, representing two displacements of a person who walked 90 mi east and then 50 mi north

Figure 1.6.1

The graphical process for adding vectors in two dimensions is to place the tail of the second vector on the arrow head of the first vector as shown above.

The sum of the two vectors is the vector that begins at the origin of the first vector and goes to the ending of the second vector, as shown below.

Two vectors representing two displacements of a person who walked 90 mi east and then 50 mi north, with the third vector connecting the two, creating a triangle indicating the sum

Figure 1.6.2

If we are using totally graphic means of adding these vectors, the magnitude of the sum can be determined by measuring the length of the sum vector and comparing it to the original standard. We then use a compass to measure the angle of the summation vector.

If we are using calculation, we first determine the inverse tangent of 50 units divided by 90 units and get the angle of 29° north of east. The length of the sum vector can then be determined mathematically by the Pythagorean theorem, a^2+b^2=c^2. In this case, the length of the hypotenuse would be the square root of (8100 + 2500), or 103 units.

If three or four vectors are to be added by graphical means, we would continue to place each new vector head to toe with the vectors to be added until all the vectors were in the coordinate system. The resultant, or sum, vector would be the vector from the origin of the first vector to the arrowhead of the last vector; the magnitude and direction of this sum vector would then be measured.

Mathematical Methods of Vector Addition

We can add vectors mathematically using trig functions, the law of cosines, or the Pythagorean theorem.

If the vectors to be added are at right angles to each other, such as the example above, we would assign them to the sides of a right triangle and calculate the sum as the hypotenuse of the right triangle. We would also calculate the direction of the sum vector by using an inverse sin or some other trig function.

Suppose, however, that we wish to add two vectors that are not at right angles to each other. Let’s consider the vectors in the following images.

Two vectors on a north-south, east-west grid, one originating between north and east, one originating between north and west

Figure 1.6.3

The two vectors we are to add are a force of 65 N at 30° north of east and a force of 35 N at 60° north of west.

We know that vectors in the same dimension can be added by regular arithmetic. Therefore, we can resolve each of these vectors into components that lay on the axes as pictured below. The **resolution of vectors** reduces each vector to a component on the north-south axis and a component on the east-west axis.

Sum of two vectors on a north-south, east-west grid, creating a triangle for each vector

Figure 1.6.4

After resolving each vector into two components, we can now mathematically determine the magnitude of the components. Once we have done that, we can add the components in the same direction arithmetically. This will give us two vectors that are perpendicular to each other and can be the legs of a right triangle.

The east-west component of the first vector is (65 N)(cos 30°) = (65 N)(0.866) = 56.3 N north

The north-south component of the first vector is (65 N)(sin 30°) = (65 N)(0.500) = 32.5 N north

The east-west component of the second vector is (35 N)(cos 60°) = (35 N)(0.500) = 17.5 N west

The north-south component of the second vector is (35 N)(sin 60°) = (35 N)(0.866) = 30.3 N north

The sum of the two east-west components is 56.3 N - 17.5 N = 38.8 N east

The sum of the two north-south components is 32.5 N + 30.3 N = 62.8 N north

We can now consider those two vectors to be the sides of a right triangle and find the length and direction of the hypotenuse using the Pythagorean Theorem and trig functions.

$$c=38.82+62.82=74\ \text{N}$$

$$\sin x=62.874 \text{ so } x=\sin{-1}0.84 \text{ so } x=58\circ$$

The direction of the sum vector is 74 N at 58° north of east.

Perpendicular vectors have no components in the other direction. For example, if a boat is floating down a river due south, and you are paddling the boat due east, the eastward vector has no component in the north-south direction and therefore, has no effect on the north-south motion. If the boat is floating down the river at 5 mph south and you paddle the boat eastward at 5 mph, the boat continues to float southward at 5 mph. The eastward motion has absolutely no effect on the southward motion. Perpendicular vectors have NO effect on each other.

Examples

A motorboat heads due east at 16 m/s across a river that flows due north at 9.0 m/s.

✓ Example 1.6.1

What is the resultant velocity of the boat?

Sketch:

Figure 1.6.5

Solution

Since the two motions are perpendicular to each other, they can be assigned to the legs of a right triangle and the hypotenuse (resultant) calculated.

$$c=a2+b2=(16\ \text{m/s})2+(9.0\ \text{m/s})2=18\ \text{m/s}$$

$$\sin\theta=9.018=0.500 \text{ and therefore } \theta=30\circ$$

The resultant is 18 m/s at 30° north of east.

✓ Example 1.6.2

If the river is 135 m wide, how long does it take the boat to reach the other side?

Solution

The boat is traveling across the river at 16 m/s due to the motor. The current is perpendicular and therefore has no effect on the speed across the river. The time required for the trip can be determined by dividing the distance by the velocity.

$$t=dv=135\ \text{m}16\ \text{m/s}=8.4\ \text{s}$$

✓ Example 1.6.3

The boat is traveling across the river for 8.4 seconds and therefore, it is also traveling downstream for 8.4 seconds. We can determine the distance downstream the boat will travel by multiplying the speed downstream by the time of the trip.

Solution

$d_{\text{downstream}}=(v_{\text{downstream}})(t)=(9.0\ \text{m/s})(8.4\ \text{s})=76\ \text{m}$

Use this PLIX Interactive to visualize how any vector can be broken down into separate x and y components:

Summary

- Vectors can be added mathematically using geometry and trigonometry.
- Vectors that are perpendicular to each other have no effect on each other.
- Vector addition can be accomplished by resolving the vectors to be added into components those vectors, and then completing the addition with the perpendicular components.

Review

1. A hiker walks 11 km due north from camp and then turns and walks 11 km due east.
    1. What is the total distance walked by the hiker?
    2. What is the displacement (on a straight line) of the hiker from the camp?
2. While flying due east at 33 m/s, an airplane is also being carried due north at 12 m/s by the wind. What is the plane’s resultant velocity?
3. Two students push a heavy crate across the floor. John pushes with a force of 185 N due east and Joan pushes with a force of 165 N at 30° north of east. What is the resultant force on the crate?
4. An airplane flying due north at 90. km/h is being blown due west at 50. km/h. What is the resultant velocity of the plane?

Explore More

Use this resource to answer the questions that follow.

1. What are the steps the teacher undertakes in order to calculate the resultant vector in this problem?
2. How does she find the components of the individual vectors?
3. How does she use the individual vector’s components to find the components of the resultant vector?
4. Once the components are determined, how does she find the overall resultant vector?

Additional Resources

Real World Application: Banked With No Friction

Video:

Video:

Video:

9.2.5: Graphical Methods of Vector Addition

This man shooting a basketball requires a subconscious understanding of vectors

Figure 9.2.5.1: Copy and Paste Caption here. (Copyright; author via source)

Successfully shooting a basketball requires a subconscious understanding of the vectors involved in how the basketball moves through the air. The vertical and horizontal vectors must be perfectly organized if the ball is to pass through the basket.

Graphical Methods Vector Addition

In physics, a quantity, such as mass, length, or speed that is completely specified by its magnitude and has no direction is called a **scalar**. A **vector**, on the other hand, is a quantity possessing both magnitude and direction. A vector quantity is typically represented by an arrow-tipped line segment. The length of the line, drawn to scale, represents the magnitude of the quantity. The direction of the arrow indicates the direction of the vector. Not only can vectors be represented graphically, but they can also be added graphically.

For one dimensional **vector addition**, the first vector is placed on a number line with the tail of the vector on the origin. The second vector is placed with its tail exactly on the arrow head of the first vector. The sum of the two vectors is the vector that begins at the origin and ends at the arrow head of the final added vector.

Consider the following two vectors.

The top number line shows a vector with magnitude 11 in the positive direction and the bottom number line shows a vector with magnitude -3

Figure 9.2.5.2

The red vector has a magnitude of 11 in the positive direction on the number line. The blue vector has a magnitude of -3, indicating 3 units in the negative direction on the number line. In order to add these two vectors, we place one of the vectors on a number line and then the second vector is placed on the same number line such that its origin is on the arrow head of the first vector.

One number line with one vector with magnitude 11 in the positive direction and another vector overlapping with magnitude -3

Figure 9.2.5.3

The sum of these two vectors is the vector that begins at the origin of the first vector (the red one) and ends at the arrow head of the second (blue) vector. So the sum of these two vectors is the purple vector, as shown below.

One number line with one vector with magnitude 11 in the positive direction and another vector overlapping with magnitude -3, showing the sum of the two vectors with a third with magnitude 8

Figure 9.2.5.4

The vector sum of the first two vectors is a vector that begins at the origin and has a magnitude of 8 units in the positive direction. If we were adding three or four vectors all in one dimension, we would continue to place them head to toe in sequence on the number line. The sum would be the vector that begins at the beginning of the first vector and goes to the ending of the final vector.

Adding Vectors in Two Dimensions

In the following image, vectors A and B represent the two displacements of a person who walked 90. m east and then 50. m north. We want to add these two vectors to get the vector sum of the two movements.

Graph of two vectors, representing two displacements of a person who walked 90 mi east and then 50 mi north

Figure 9.2.5.5: Copy and Paste Caption here. (Copyright; author via source)

The graphical process for adding vectors in two dimensions is to place the tail of the second vector on the arrow head of the first vector as shown above.

The sum of the two vectors is the vector that begins at the origin of the first vector and goes to the ending of the second vector, as shown below.

Two vectors representing two displacements of a person who walked 90 mi east and then 50 mi north, with the third vector connecting the two, creating a triangle indicating the sum

Figure 9.2.5.6: Copy and Paste Caption here. (Copyright; author via source)

If we are using totally graphic means of adding these vectors, the magnitude of the sum would be determined by measuring the length of the sum vector and comparing it to the original standard. We would also use a compass to measure the angle of the summation vector.

If we are using calculation means, we can divide 50. m by 90. m and determine inverse tangent of the dividend. The result of 29.05 indicates the angle of 29° north of east. The length of the sum vector can also be determined mathematically by the Pythagorean theorem, a2+b2=c2. In this case, the length of the hypotenuse would be the square root of (8100 + 2500) or 103 m.

If three or four vectors are to be added by graphical means, we would continue to place each new vector head to toe with the vectors to be added until all the vectors were in the coordinate system. The sum vector is the vector from the origin of the first vector to the arrowhead of the last vector. The magnitude and direction of the sum vector can be measured.

alt

Figure 9.2.5.7: Copy and Paste Caption here. (Copyright; author via source)

Have you ever used a phone app that provides directions or a navigation system in your car? These programs help you get from Point A to Point B by breaking it down into a series of left and right turns that exemplify many of the graphical methods of vector addition described above. The navigation systems in self-driving cars are even more advanced. Continue to practice vector addition by helping a driverless car get to its destination in the following simulation:

Summary

- Scalars are quantities, such as mass, length, or speed, that are completely specified by magnitude and have no direction.
- Vectors are quantities possessing both magnitude and direction and can be represented by an arrow; the direction of the arrow indicates the direction of the quantity and the length of the arrow is proportional to the magnitude.
- Vectors that are in one dimension can be added arithmetically.
- Vectors that are in two dimensions are added geometrically.
- When vectors are added graphically, graphs must be done to scale and answers are only as accurate as the graphing.

Review

1. On the following number line, add the vector 7.5 m/s and the vector -2.0 m/s.

A number line

Figure 9.2.5.8

2. On a sheet of graph paper, add a vector that is 4.0 km due east and a vector that is 3.0 km due north.

Explore More

Use this resource to answer the questions that follow.

1. What is a resultant?
2. What are the steps necessary to add vectors in two dimensions?

Additional Resources

PLIX: Play, Learn, Interact, eXplore: Submarine Target Practice

Real World Application: Drift

Real World Application: Threading the Needle

Video:

SECTION OVERVIEW

9.3: Motion in One-Dimension

9.3.1: Position and Displacement

Figure 2.1.1

In stockcar races, drivers often travel 500 miles or more, but their final displacement is only a few feet. Why?

Position, Distance, and Displacement

In order to study how something moves, we must know where it is. This location is an object's **position.** To visualize position for objects moving in a straight line, you can imagine the object is on a number line. It may be placed at any point on the number line in the positive numbers or the negative numbers. It is common to choose the original position of the object to be on the zero mark. In making the zero mark the reference point, you have chosen a frame of reference. The exact position of an object is the separation between the object and the reference point.

When an object moves, we often refer to the amount it moves as the **distance**. Distance does not need a reference point and does not need a direction. If an automobile moves 50 kilometers, the distance traveled is 50 kilometers regardless of the starting point or the direction of movement. If we wish to find the final position of the automobile, however, just having the distance traveled will not allow us to determine the final position. We need to know the starting point and the direction of the motion. The change in the position of the object is called its **displacement**. The displacement must include a direction because the final position may be either in the positive or negative direction along the number line from the initial position. The displacement is a vector quantity and vectors are discussed in the section "Vectors". This tells us that the displacement of an object is a **vector** (a quantity that has both magnitude and direction), rather than a **scalar** (a quantity that has only magnitude).

To return to the first image, these cars travel a distance of 500 miles over the course of the race. However, they are traveling in a circle, and the start and finish line are the same. Therefore, when the car finishes the race, it is in essentially the same position it was when it started. The car's total displacement is only a few feet.

Use the following simulation to further explore the concepts of position, distance and displacement:

Summary

- The length traveled by an object moving in any direction or even changing direction is called distance.
- The location of an object in a frame of reference is called position.
- For straight line motion, positions can be shown using a number line.
- The separation between original and final position is called displacement.

Review

1. Explain the difference between distance and displacement in your own words.
2. Suppose that John lives on a square block that is 180 yards per side, and in the evenings, he walks with his dog around the block for a little exercise.
    1. If John walks once around the block, what distance does he travel?
    2. If John walks once around the block, what is his final displacement?

3. Joanna's house is 8000 feet due west of her school. If her house is assigned the position of zero and her school is assigned the position of +8000, what would be Joanna's position if she walked 100 feet west of her house?

Explore More

Watch the video below to explore more on scalar and vector quantities and answer the questions below:

1. What is the difference between a scalar and a vector?
2. Provide an example of a scalar and an example of a vector.

Watch these two short videos to understand position, distance, and displacement, and answer the questions below:

1. What is position?
2. Can two objects be the same distance from a single point but be in different positions? Why or why not?
3. What is the difference between distance and displacement?
4. Does distance have direction? Does displacement have direction?

Additional Resources

Study Guide: Motion Study Guide

Real World Application: Driving Safely at Night

Video:

PLIX: Play, Learn, Interact, eXplore: Driverless Car

Other Sources: OpenStax College Physics Textbook https://openstax.org/books/college-p...1-displacement

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9.3.2: Time

Figure 9.3.2.1: The motion of these racing snails can be described by their speeds and their velocities. (credit: tobitasflickr, Flickr)

There is more to motion than distance and displacement. Questions such as, “How long does a foot race take?” and “What was the runner’s speed?” cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

Time

The most fundamental physical quantities are defined by how they are measured, including time. Every measurement of time involves observing a change in some physical quantity, whether it's a digital clock ticking, a heartbeat, or the position of the Sun in the sky. In physics, **time** is defined simply as change or the interval over which change occurs; it's impossible to perceive the passage of time without some form of change.

Time or change is calibrated by comparison with a standard, with the SI unit for time being the second (s). For instance, we might observe a pendulum completing one swing every 0.75 seconds and use this to measure time by counting its swings or connecting it to a clock mechanism. This not only allows us to measure time but also to establish a sequence of events. When considering time in relation to motion, we typically focus on elapsed time for a particular motion, such as the time it takes for an airplane passenger to move from one seat to another. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time Δt** is the difference between the ending time and beginning time,

$$\Delta t = t_\mathrm{f} - t_0,$$

where Δt is the change in time or elapsed time, t_f is the time at the end of the motion, and t_0 is the time at the beginning of the motion. (As usual, the delta symbol, Δ, means the change in the quantity that follows it.)

Section Summary

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is

$$\Delta t = t_\mathrm{f} - t_0,$$

  where t_f is the final time and t_0 is the initial time.

Glossary

time

change, or the interval over which change occurs

model

simplified description that contains only those elements necessary to describe the physics of a physical situation

elapsed time

the difference between the ending time and beginning time

- **1.4: Time, Velocity, and Speed** by OpenStax is licensed CC BY 4.0.

9.3.3: Average Velocity

Test Pilot Neil Armstrong is seen standing next to the X-15 ship after a research flight

Figure 2.2.1

Test Pilot Neil Armstrong (later to become a famous astronaut) is seen here next to the X-15 ship after a research flight. The servo-actuated ball nose, seen at Armstrong's right hand, provided accurate measurement of air speed at hypersonic speeds. The X-15 was flown over a period of nearly 10 years, and set the world's unofficial speed record of 4,250 mph.

Average Velocity

In ordinary language, the words *speed* and *velocity* both refer to how fast an object is moving, and are often used interchangeably. In physics, however, they are fundamentally different. **Speed** is the magnitude of an object's motion, with no regard for the direction. **Velocity**, on the other hand, includes direction. It is a vector, and thus must have a magnitude and a direction.

Average speed is calculated by dividing the total distance traveled by the time interval. For example, someone who takes 40 minutes to drive 20 miles north and then 20 miles south (to end up at the same place), has an average speed of 40 miles divided by 40 minutes, or 1 mile per minute (60 mph). Average velocity, however, involves total displacement, instead of distance. It is calculated by dividing the total displacement by the time interval. In this example, the driver's displacement is zero (since they start and end at the same point), which makes the average velocity zero mph.

Neither average speed nor average velocity implies a constant rate of motion. That is to say, an object might travel at 10 m/s for 10 s and then travel at 20 m/s for 5 s and then travel at 100 m/s for 5 s. This motion would cover a distance of 700 m in 20 s and the average speed would be 35 m/s. We would report the average speed during this 20 s interval to be 35 m/s and yet at no time during the interval was the speed necessarily 35 m/s.

Constant velocity is very different from average velocity. If an object traveled at 35 m/s for 20 s, it would travel the same distance in the same time as the previous example but in the second case, the object's velocity would always be 35 m/s.

Example

✓ Example 2.2.1

The position of a runner as a function of time is plotted as moving along the x-axis of a coordinate system. During a 3.00 s time interval, the runner's position changes from x_1=50.0 m to x_2=30.5 m. What was the runner's average velocity?

Solution

Displacement=30.5 m−50.0 m=−19.5 m (the object was traveling back toward zero)

Δt=3.00 s

v_{ave}=Δx/Δt=−19.5 m/3.00 s=−6.50 m/s

✓ Example 9.3.3.1

Maria travels by car from Coalinga to Fresno, and back in 2 hours and 40 minutes (160 minutes). The distance between the cities is approximately 70 miles. What is (a) the average velocity of the car, and (b) the average speed of the car in m/s?

Solution

(a) The average velocity of the car is zero (0 m/s) because $x_f = x_i$; the car ends up at the same place it starts.

(b) The average speed of the car is calculated below. Note that the car travels 70 miles one way and 70 miles back, for a total distance of 140 miles.

$$v = \frac{x}{t} = \frac{140 \text{ miles}}{160 \text{ minutes}} \times \frac{1609 \text{ meters}}{1 \text{ mile}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = 24 \text{ m/s}$$

Observe the differences between constant velocity and average velocity in the simulation below where two silly robots, Irwin and Ruthie, are racing to the elevator:

Another important aspect of velocity is that it is always measured relative to something. We usually measure how fast a person walks or how fast a car drives relative to the ground. In Astronomy, we often measure the motion of the Earth relative to the Sun. A reference frame is a fixed point and we measure directions relative to it.

Use the following simulation to explore what happens when two bikers approach each other at various velocities and be sure to click on different observers each time to observe their motion from different reference frames:

Summary

- Average speed is distance divided by time.
- Average velocity is displacement divided by time.

Review

1. On a one day vacation, Jane traveled 340 miles in 8.0 hours. What was her average speed?
2. An object on a number line moved from $x = 12$ m to $x = 124$ m and moved back to $x = 98$ m. The time interval for all the motion was 10 s. What was the average velocity of the object?
3. An object on a number line moved from x = 15 cm to x = 165 cm and then moved back to x = 25 cm, all in a time of 100 seconds.
   1. What was the average velocity of the object?
   2. What was the average speed of the object?
4. If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:
   1. Calculate typical car speeds in meters per second
   2. Estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
   3. Determine the speed of an ant, snail, or falling leaf

Explore More

Use this resource to answer the question that follows.

1. What is the main difference between average speed and average velocity?

Additional Resources

PLIX: Play, Learn, Interact, eXplore: Velocity

Video:

Real World Application: How Fast Does A Gravitational Wave Travel?

Study Guide: Motion Study Guide

9.3.4: Instantaneous Velocity

Figure 2.3.1

In a footrace such as the one shown here, the initial velocity of a runner is zero. The runner increases his velocity out of the starting blocks and his velocity continues to increase as the race proceeds. For the well-trained athlete, his highest velocity is maintained through the finish line.

Instantaneous Velocity

The **instantaneous velocity** of an object is the velocity of the object at a given moment. If the object is moving with constant velocity, then the instantaneous velocity at every moment, the average velocity, and the constant velocity are all the same.

Position vs Time Graphs

Consider a position versus time graph for an object starting at t=0 and x=0 that has a constant velocity of 80. m/s.

A graph of velocity as position versus time

Figure 2.3.2

The velocity of an object can be found from a position vs time graph. On a position vs time graph, the displacement is the vertical separation between two points and the time interval is the horizontal separation. The ratio of displacement to time interval is the average velocity. This ratio is also the slope of the line. Therefore, the slope of the straight line is the average velocity. For the motion pictured above,

$$\text{slope}=\text{rise}/\text{run}=\Delta d/\Delta t=400.\text{m}/5.0\text{s}=80.\text{m/s}$$

For accelerated motion (the velocity is constantly changing), the position vs time graph will be a curved line. The slope of the curved line at any point is the instantaneous velocity at that time. If we were using calculus, the slope of a curved line could be calculated. Without calculus, we approximate the instantaneous velocity at a particular point by laying a straight edge along the curved line and estimating the slope.

The position versus time graph shows accelerated motion as a curved line

Figure 2.3.3

In the image above, the red line is the position vs time graph and the blue line is an approximated slope for the line at t=2.5 seconds. The rise for this slope is approximately 170 m and the time interval (run) is 4.0 seconds. Therefore, the approximated slope is 43 m/s.

Swimming is another sport in which the velocity of the swimmer is constantly changing over time. In simulation below, adjust the sliders to change the swimmer's expertise in the Butterfly Stroke. Then, try to derive the instantaneous velocity of the swimmer at various points in the race using the position-time graph:

Summary

- The slope of a position versus time graph is the velocity.
- For constant velocity motion, the slope gives the constant velocity, the average velocity, and the instantaneous velocity at every point.

- For constant acceleration motion, the slope of the position versus time curve at a particular point gives the instantaneous velocity at that point.

Review

Draw a velocity versus time graph for an object whose constant velocity is 15 m/s and whose position starts at x=0 when t=0. Graph the motion for the first 5.0 seconds.

Use the graph below to answer the following questions:

A motion graphed on a position versus time graph

Figure 2.3.4

1. For the motion graphed in the position versus time graph shown above, what is the average velocity in the time interval 1 to 3 seconds?
2. For the motion graphed in the position versus time graph shown above, what is the average velocity in the time interval 3 to 4 seconds?
3. For the motion graphed in the position versus time graph shown above, what is the average velocity in the time interval 5 to 6 seconds?

Explore More

Use this resource to answer the questions that follow.

1. In the graph on the video, what is graphed on the vertical axis?
2. What is graphed on the horizontal axis?
3. What does the slope of this graph represent?

Additional Resources

PLIX: Play, Learn, Interact, eXplore: Changing Speeds

Real World Application: The Reality of Speeding

Video:

Study Guide: Motion Study Guide

9.3.5: Average Acceleration

A space shuttle, such as the Atlantis, must accelerate to 28,968 km/h to escape from Earth's orbit

Figure 2.4.1

To escape from Earth's orbit, space shuttles such as the Atlantis shown here must accelerate to 28,968 kilometers per hour, called the shuttle's *escape velocity*.

Average Acceleration

An object whose velocity is changing is said to be accelerating. **Average acceleration**, a, is defined as the rate of change of velocity, or the change in velocity per unit time. The units of acceleration are distance over time squared. A symbol with a bar over it is read as average – so a-bar is average acceleration.

Examples

Example 2.4.1

A car accelerates along a straight road from rest to +60.0 km/h in 5.00 s. What is the magnitude of its average acceleration?

Solution

The acceleration in this problem is read as kilometers per hour per second. In general, it is undesirable to have two different units for the same quantity in a unit expression. In this case, it is undesirable to have both hours and seconds. To eliminate this problem, we convert the hour units to seconds. Converting the original 60.0 km/h to m/s, gives 17.0 m/s.

$(17.0 \text{ m/s})/(5.00 \text{ s})=3.40 \text{ m/s}^2$

The acceleration is 3.40 m/s^2.

Example 2.4.2

An automobile is moving along a straight highway in the positive direction and the driver steps on the brakes. If the initial velocity is +15.0 m/s and 5.0 s is required to slow down to +5.0 m/s, what was the car’s acceleration?

Solution

$a=\Delta v/\Delta t=(-10.0 \text{ m/s})/(5.0 \text{ s})=-2.0 \text{ m/s}^2$

Note that an acceleration is merely a change in velocity. This change can be either positive or negative. A negative change, such as that in the example problem above, is sometimes called negative acceleration or deceleration.

Velocity-time graphs and acceleration-time graphs can depict the motion of any object and can also be used to derive the object’s average acceleration. Use the following PLIX Interactive to predict what the velocity-time graph and acceleration-time graph will look like to accurately describe Roger’s bike ride to school:

Summary

- Average acceleration is the rate of change of velocity, or the change in velocity per unit time.

Review

1. The velocity of a car increases from +2.0 m/s to +16.0 m/s in a time period of 3.5 s. What was the average acceleration?
2. If an automobile slows from +26 m/s to +18 m/s in a period of 4.0 s, what was the average acceleration?
3. If a runner increases his velocity from 0 m/s to +20 m/s in 2.0 s, what was his average acceleration?
4. If a runner decreases his velocity from +20 m/s to +10 m/s in 2.0 s, what was his average acceleration?

Additional Resources

Study Guide: Motion Study Guide

Real World Application: Falling From Space

Interactives: Irwin and Ruthie, Model Rocket

Video:

9.3.6: Uniform Acceleration

Figure 2.5.1

After jumping, this cliff diver undergoes effective free fall. Cliff diving is a sport in which athletes twist and flip on their way down. Even with the air resistance, these divers will accelerate the whole way down.

Constant Acceleration

Acceleration that does not change in time is called uniform or **constant acceleration.** The velocity at the beginning of the time interval is called initial velocity, v_i, and the velocity at the end of the time interval is called final velocity, vf. In a velocity versus time graph for uniform acceleration, the slope of the line is the acceleration. The equation that describes the curve is $v_f = v_i + at$.

✓ Example 2.5.1

If an automobile with a velocity of 4.0 m/s accelerates at a rate of 4.0 m/s^2 for 2.5 s, what is the final velocity?

Solution

$v_f = v_i + at = 4.0\text{ m/s} + (4.0\text{ m/s2})(2.5\text{ s}) = 4.0\text{ m/s} + 10.\text{ m/s} = 14\text{ m/s}$

✓ Example 2.5.2

If a cart slows from 22.0 m/s with an acceleration of -2.0 m/s^2, how long does it require to get to 4 m/s?

Solution

$t = (v_f - v_i)/a = (-18\text{ m/s} - 2.0\text{ m/s})/2 = 9.0\text{ s}$

Use the simulation below to further explore the uniform acceleration of a cliff diver. Observe the relationship between position, velocity, and acceleration as the diver falls under the influence of gravity. Begin by turning off the air resistance and see what happens when the diver falls as if he is in a vacuum. Then, if you are up for a challenge, turn the air resistance on and consider its effects on the diver.

Misconception Alert: Deceleration versus Negative Acceleration

Deceleration refers to the reduction in the speed or velocity of an object. It occurs when an object slows down, either due to the application of a force in the opposite direction of its motion or due to a change in direction. Deceleration can be caused by various factors involving force such as friction, air resistance, or the application of brakes.

Negative acceleration, on the other hand, is a more general term that refers to any *acceleration in the direction opposite to the direction of motion*. It can represent either a decrease in speed (deceleration) or a change in direction. Negative acceleration may or may not be deceleration, and deceleration may or may not be considered negative acceleration. For example, consider Figure 2.14.

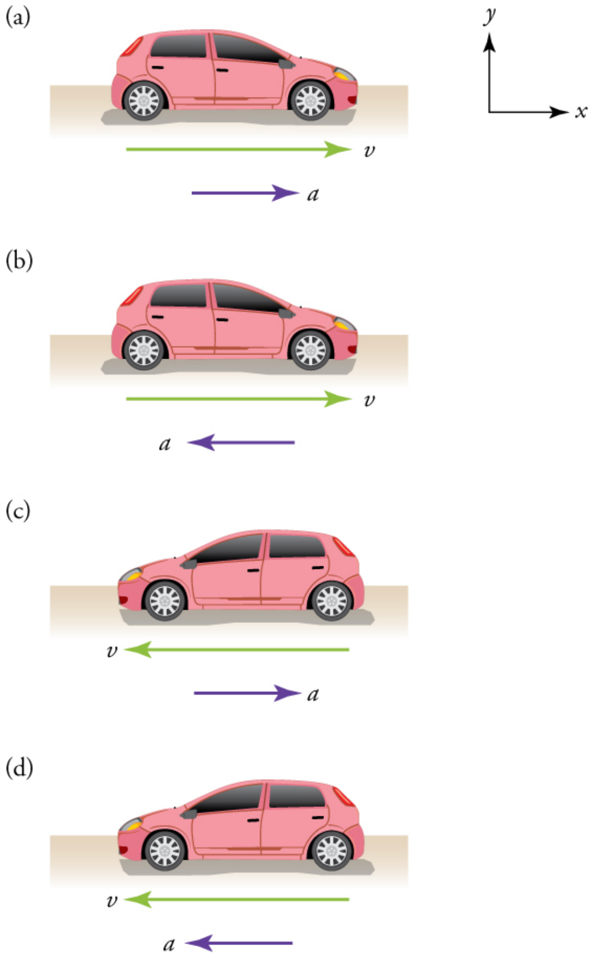

Figure 2.14

(a) This car is speeding up as it moves toward the right (positive x-direction), so it has positive acceleration in our coordinate system. (b) This car is slowing down as it moves toward the right, so it has negative acceleration in our coordinate system because the acceleration is toward the left (negative x-direction). The car is decelerating because the acceleration is opposite to its motion. (c) This car is moving toward the left but slowing down over time, so its acceleration is positive in our coordinate system because it is toward the right. The car is decelerating because the acceleration is opposite to its motion. (d) This car is speeding up as it moves toward the left, so it has negative acceleration because it is accelerating toward the left. The car is speeding up because the acceleration is in the same direction as its motion.

Summary

- Acceleration that does not change in time is uniform, or constant, acceleration.
- The equation relating initial velocity, final velocity, time, and acceleration is $v_f = v_i + at$.

Review

1. If an object has zero acceleration, does that mean it has zero velocity? Give an example.
2. If an object has zero velocity, does that mean it has zero acceleration? Give an example.
3. If the acceleration of a motorboat is 4.0 m/s^2, and the motorboat starts from rest, what is its velocity after 6.0 s?
4. The friction of the water on a boat produces an acceleration of -10. m/s^2. If the boat is traveling at 30. m/s and the motor is shut off, how long does it take the boat to slow down to 5.0 m/s?

Additional Resources

PLIX: Play, Learn, Interact, eXplore: Uniform Acceleration

Real World Application: Race to The Finish

Video:

Study Guide: Motion Study Guide

9.3.7: Displacement During Uniform Acceleration

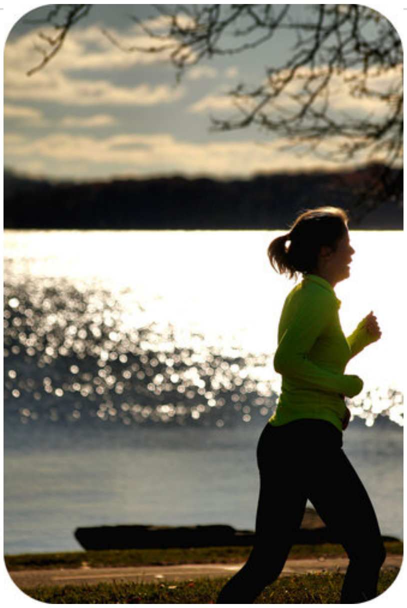

Figure 2.6.1

Long distance runners try to maintain constant velocity with very little acceleration or deceleration to conserve energy.

Displacement During Constant Acceleration

When acceleration is constant, there are three equations that relate displacement to two of the other three quantities we use to describe motion – time, velocity, and acceleration. These equations only work when acceleration is constant, but there are, fortunately, quite a few cases of motion where the acceleration is constant. One of the most common, if we ignore air resistance, are objects falling due to gravity.

When an object is moving with constant velocity, the displacement can be found by multiplying the velocity by the time interval, as shown in the equation below

$$d = vt$$

If the object is moving with constant acceleration, but not a constant velocity, we can use a derivation of this equation. Instead of using *v*, as velocity, we must calculate and use the average velocity using this equation:

$$v_{avg} = \frac{1}{2}(v_f + v_i)$$

The distance, then, for uniformly accelerating motion can be found by multiplying the average velocity by the time.

$$d = \frac{1}{2}(v_f + v_i)t \quad \textbf{(Equation 1)}$$

We know that the final velocity for constantly accelerated motion can be found by multiplying the acceleration times time and adding the result to the initial velocity, $v_f = v_i + at$.

The second equation that relates displacement, time, initial velocity, and final velocity is generated by substituting this equation into equation 1.

Start by distributing the 1/2 in equation 1 through:

$$d = \frac{1}{2}(v_f + v_i)t = \frac{1}{2}v_f t + \frac{1}{2}v_i t$$

We know that $v_f = v_i + at$. Therefore:

$$d = \frac{1}{2}v_i t + \frac{1}{2}(v_i + at)$$

$$d = \frac{1}{2}v_i t + \frac{1}{2}v_i t + \frac{1}{2}at^2$$

$$d = v_i t + \frac{1}{2}at^2 \quad \textbf{(Equation 2)}$$

The third equation is formed by combining $v_f = v_i + at$ and $d = \frac{1}{2}(v_f + v_i)t$. If we solve the first equation for t and then substitute into the second equation, we get

$$d = \frac{1}{2}(v_f + v_i)\frac{v_f - v_i}{a} = \frac{1}{2}\frac{(v_f^2 - v_i^2)}{a}$$

And solving for v_f^2 yields

$$v_f^2 = v_i^2 + 2ad \text{ } \textbf{(Equation 3)}$$

Keep in mind that these three equations are only valid when acceleration is constant. In many cases, the initial velocity can be set to zero and that simplifies the three equations considerably. When acceleration is constant and the initial velocity is zero, the equations can be simplified to:

$$d = \frac{1}{2}v_f t$$

$$d = \frac{1}{2}at^2 \text{ and}$$

$$v_f^2 = 2ad$$

✓ Example 2.6.1

Suppose a planner is designing an airport for small airplanes. Such planes must reach a speed of 56 m/s before takeoff and can accelerate at 12.0 m/s^2. What is the minimum length for the runway of this airport?

Solution

The acceleration in this problem is constant and the initial velocity of the airplane is zero. Therefore, we can use the equation vf2=2ad and solve for d.

d=(v$_f^2$)/2a=(56 m/s)^2/((2)(12.0 m/s^2))=130 m

✓ Example 2.6.2

How long does it take a car to travel 30.0 m if it accelerates from rest at a rate of 2.00 m/s^2?

Solution

The acceleration in this problem is constant and the initial velocity is zero, therefore, we can use d=(1/2)at^2 solved for t.

t=(2d/a)^(1/2)=((2)(30.0 m)/(2.00 m/s^2))^(1/2)=5.48 s

✓ Example 2.6.3

A baseball pitcher throws a fastball with a speed of 30.0 m/s. Assume the acceleration is uniform and the distance through which the ball is accelerated is 3.50 m. What is the acceleration?

Solution

Since the acceleration is uniform and the initial velocity is zero, we can use v$_f^2$=2ad solve for a.

a=v$_f^2$/2d=(30.0 m/s)^2/(2)(3.50 m)=900 (m^2/s^2)/7.00 m=129 m/s^2

Suppose we plot the velocity versus time graph for an object undergoing uniform acceleration. In this first case, we will assume the object started from rest.

If the object has a uniform acceleration of 6.0 m/s^2 and started from rest, then each succeeding second, the velocity will increase by 6.0 m/s. Here is the table of values and the graph.

Velocity versus time graph

Figure 2.6.2

In displacement versus time graphs, the slope of the line is the velocity of the object. In this case of a velocity versus time graph, the slope of the line is the acceleration. If you take any segment of this line and determine the Δy to Δx ratio, you will get 6.0 m/s^2 which we know to be the constant acceleration of this object.

We know from geometry that the area of a triangle is calculated by multiplying one-half the base times the height. The area under the curve in the image above is the area of the triangle shown below. The area of this triangle would be calculated by area=(1/2)(6.0 s)(36 m/s)=108 m.

Velocity versus time graph with the area under the line being equal to the area of a triangle

Figure 2.6.3

By going back to equation 2, we know that displacement=(1/2)at^2 .

Using this equation, we can determine that the displacement of this object in the first 6 seconds of travel is =(1/2)(6.0 m/s2)(6.0 s)^2=108 m.

It is not coincidental that this number is the same as the area of the triangle. In fact, the area underneath the curve in a velocity versus time graph is always equal to the displacement that occurs during that time interval.

Use the PLIX Interactive below to analyze the motion of Jane's dog, Sparky, and visualize how acceleration and displacement can be derived from a velocity-time graph:

MAKING CONNECTIONS: TAKE-HOME EXPERIMENT—BREAKING NEWS

We have been using SI units of meters per second squared to describe some examples of acceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking acceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration, a=v/t. While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the acceleration in miles per hour per second. Convert this to meters per second squared and compare with other accelerations mentioned in this chapter. Calculate the distance traveled in braking.

Summary

- There are three equations we can use when acceleration is constant to relate displacement to two of the other three quantities we use to describe motion – time, velocity, and acceleration:
  - $d=\frac{1}{2}(v_f+v_i)t$ (Equation 1)
  - $d=v_it+\frac{1}{2}at^2$ (Equation 2)
  - $v_f^2=v_i^2+2ad$ (Equation 3)
- When the initial velocity of the object is zero, these three equations become:
  - $d=\frac{1}{2}v_ft$ (Equation 1’)
  - $d=\frac{1}{2}at^2$ (Equation 2’)
  - $v_f^2=2ad$ (Equation 3’)
- The slope of a velocity versus time graph is the acceleration of the object.
- The area under the curve of a velocity versus time graph is the displacement that occurs during the given time interval.

Review

1. An airplane accelerates with a constant rate of 3.0 m/s^2 starting at a velocity of 21 m/s. If the distance traveled during this acceleration was 535 m, what is the final velocity?
2. An car is brought to rest in a distance of 484 m using a constant acceleration of -8.0 m/s^2. What was the velocity of the car when the acceleration first began?
3. An airplane starts from rest and accelerates at a constant 3.00 m/s^2 for 20.0 s. What is its displacement in this time?
4. A driver brings a car to a full stop in 2.0 s.
   1. If the car was initially traveling at 22 m/s, what was the acceleration?
   2. How far did the car travel during braking?

Explore More

Use this resource to answer the question that follows.

1. What does the area bounded by a velocity versus time graph represent?

Additional Resources

Study Guide: Motion Study Guide

Real World Application: Dangerous Pennies

Interactive Element: Model Rocket, Irwin 2D

PLIX: Play, Learn, Interact, eXplore: Constant Acceleration and Inertia

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9.3.8: Acceleration Due to Gravity

This Parachutist maximizes air resistance in order to limit the acceleration of the fall

Figure 2.7.1

In the absence of air resistance, all objects fall toward the Earth with the same acceleration. Parachutists, like the one from the U.S. Army Parachute Team shown above, make maximum use of air resistance in order to limit the acceleration of the fall.

Acceleration Due to Gravity

One of the most common examples of uniformly accelerated motion is that an object allowed to drop will fall vertically to the Earth due to gravity. In treating falling objects as uniformly accelerated motion, we must ignore air resistance. Galileo's original statement about the motion of falling objects is:

> *At a given location on the Earth and in the absence of air resistance, all objects fall with the same uniform acceleration.*

We call this **acceleration due to gravity** on the Earth and we give it the symbol g. The value of *g* is 9.81 m/s^2 in the downward direction. All of the equations involving constant acceleration can be used for falling bodies but we insert "-g" wherever "a" appeared and the value of g is always 9.81 m/s^2.

The equations for objects in free-fall are as follows:

$$v_f = v_i - gt$$

$$d = v_i t - \frac{1}{2}gt^2$$

$$v_f^2 = v_i^2 - 2(y_f - y_i)$$

In the absence of air resistance, is the cliff diver's acceleration 9.81 m/s^2 in the simulation below? How do you know? What information can you derive from the graphs?

Example: A rock is dropped from a tower 70.0 m tall. How far will the rock have fallen after 1.00 s, 2.00 s, and 3.00 s? Assume the displacement is negative downward.

Solution: We are looking for displacement and we have time and acceleration. Therefore, we can use $d = -\frac{1}{2}gt^2$.

Displacement after 1.00 s: $-(1/2)(9.81\mathrm{m/s^2})(1.00\mathrm{s})^2 = -4.91\mathrm{m}$

Displacement after 2.00 s: $-(1/2)(9.81\mathrm{m/s^2})(2.00\mathrm{s})^2 = -19.6\mathrm{m}$

Displacement after 3.00 s: $-(1/2)(9.81\mathrm{m/s^2})(3.00\mathrm{s})^2 = -44.1\mathrm{m}$

Example: A stone is dropped from the top of a cliff. It is hits the ground after 5.5 s. How tall is the cliff?

Solution:

$$d = v_i t - \frac{1}{2}gt^2 = (0\mathrm{m/s})(5.5\mathrm{s}) - \frac{1}{2}(9.81\mathrm{m/s^2})(5.5\mathrm{s})^2 = 150\mathrm{m}$$

Further explore the acceleration of a model rocket due to gravity in the simulation below. Can you use the graphs to determine when the rocket is speeding up or slowing down?

Summary

- At any given location on the Earth and in the absence of air resistance, all objects fall with the same uniform acceleration.
- We call this acceleration the acceleration due to gravity on the Earth and we give it the symbol g.
- The value of *g* is 9.81 m/s^2.
- The equations for objects in free-fall are as follows:

$$v_f = v_i - gt$$

$$d = v_i t - \frac{1}{2}gt^2$$

$$v_f^2 = v_i^2 - 2(y_f - y_i)$$

Review

1. A baseball is thrown vertically into the air with a speed of 24.7 m/s.
    1. How high does it go?

    2. How long does the round trip up and down require?
2. A salmon jumps up a waterfall 2.4 m high. With what minimum speed did the salmon leave the water below to reach the top?
3. A kangaroo jumps to a vertical height of 2.8 m. How long will it be in the air before returning to Earth?

Explore More

This video offers a discussion and demonstration of the acceleration due to gravity.

1. What is the gravitational acceleration given in the video? Why does it differ from that given in this text?
2. Why does the ball travel further in later time intervals than in the earlier ones?

Additional Resources

PLIX: Play, Learn, Interact, eXplore: Jumping Buses

Real World Application: Jumping Buses

Video:

Study Guide: Motion Study Guide

9.3.9: Position vs. Time Graphs

Figure 2.8.1

Drawing line graphs can help you understand motion. In this article, you'll learn how to draw position-time graphs and how they show velocity.

> **Q:** What's missing from the graph being drawn in the picture above?
>
> **A:** The x- and y-axes are missing.

Graphing Position and Time

The motion of an object can be represented by a position-time graph like Graph 1 in the **Figure** below. In this type of graph, the y-axis represents position relative to the starting point, and the x-axis represents time. A position-time graph shows how far an object has traveled from its starting position at any given time since it started moving.

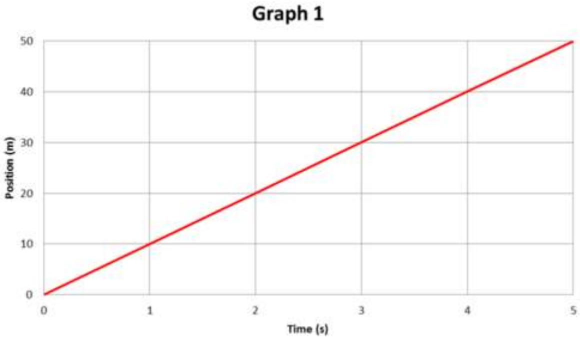

Figure 2.8.2

> **Q:** In the **Figure** above, what distance has the object traveled from the starting point by the time 5 seconds have elapsed?
>
> **A:** The object has traveled a distance of 50 meters.

Slope Equals Velocity

In a position-time graph, the velocity of the moving object is represented by the slope, or steepness, of the graph line. If the graph line is horizontal, like the line after time = 5 seconds in Graph 2 in the **Figure** below, then the slope is zero and so is the velocity. The position of the object is not changing. The steeper the line is, the greater the slope of the line is and the faster the object's motion is changing.

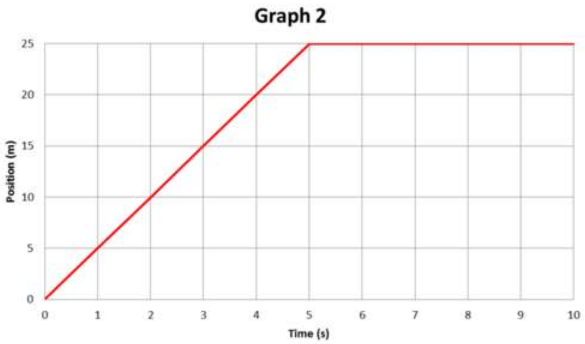

Figure 2.8.3

Calculating Average Velocity from a Position-Time Graph

It's easy to calculate the average velocity of a moving object from a position-time graph. **Average velocity** equals the change in position (represented by Δd) divided by the corresponding change in time (represented by Δt):

$$v_{avg} = \frac{\Delta d}{\Delta t}$$

For example, in Graph 2 in the **Figure** above, the average velocity between 0 seconds and 5 seconds is:

$$v_{avg} = \frac{\Delta d}{\Delta t} = \frac{(25\text{m}-0\text{m})}{(5\text{s}-0\text{s})} = \frac{25\text{m}}{5\text{s}} = 5\text{m/s}$$

Watch this two-part video series for more position vs. time graph examples.

Use the following PLIX Interactive to make a position-time graph for a runner who changes speed during their run:

Summary

- Motion can be represented by a position-time graph, which plots position relative to the starting point on the y-axis and time on the x-axis.
- The slope of a position-time graph represents velocity. The steeper the slope is, the faster the motion is changing.
- Average velocity can be calculated from a position-time graph as the change in position divided by the corresponding change in time.

Review

1. Describe how to make a position-time graph.
2. What is the slope of a line graph? What does the slope of a position-time graph represent?
3. Can a line on a position-time graph have a negative slope, that is, can it slope downward from left to right? Why or why not?
4. In Graph 1 in the **Figure** above, what is the object's average velocity?

Additional Resources

Study Guide: Motion Study Guide

Real World Application: The Bullet Drop

PLIX: Play, Learn, Interact, eXplore: Irwin and Ruthie

Video:

9.3.10: Velocity vs. Time Graphs

Figure 2.9.1

The sprinter in this image is just taking off from the starting blocks to run a short race down a straight track. She starts in a burst of speed and will pick up even more speed during the first few seconds of the race. She'll keep running at top speed until she crosses the finish line. Only then will she slow down. **Velocity** is a measure of both speed and direction of motion. A change in velocity is called acceleration. In the case of the sprinter, she accelerates as she runs down the track because her speed is changing even though her direction stays the same.

Plotting Velocity Against Time

The changing velocity of the sprinter—or of any other moving person or object—can be represented by a velocity-time graph like the one in the **Figure** below for the sprinter. A velocity-time graph shows how velocity changes over time. The sprinter's velocity increases for the first 4 seconds of the race, it remains constant for the next 3 seconds, and it decreases during the last 3 seconds after she crosses the finish line.

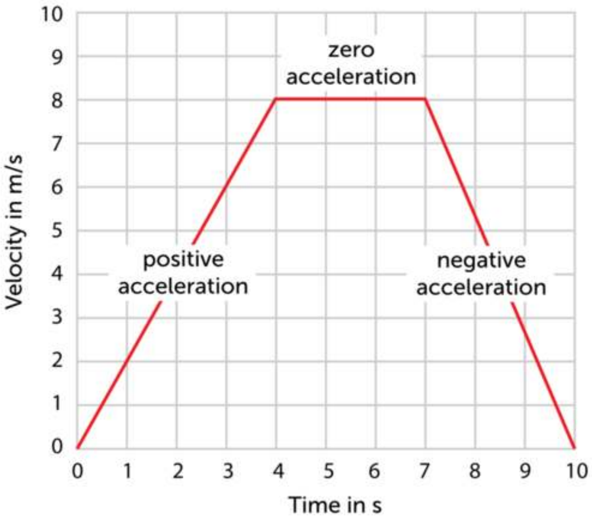

Figure 2.9.2

Acceleration and Slope

In a velocity-time graph, acceleration is represented by the slope, or steepness, of the graph line. If the line slopes upward, like the line between 0 and 4 seconds in the **Figure** above, velocity is increasing, so acceleration is positive. If the line is horizontal, as it is between 4 and 7 seconds, velocity is constant and acceleration is zero. If the line slopes downward, like the line between 7 and 10 seconds, velocity is decreasing and acceleration is negative. Negative acceleration is called deceleration.

> **Q:** Assume that another sprinter is running the same race. The other runner reaches a top velocity of 9 m/s by 4 seconds after the start of the race. How would the first 4 seconds of the velocity-time graph for this runner be different from the **Figure** above?
>
> **A:** The graph line for this runner during seconds 0–4 would be steeper (have a greater slope). This would show that acceleration is greater during this time period for the other sprinter.

Use the following PLIX Interactive to predict what the velocity-time graphs will look like for three baseballs accelerating at different rates:

Summary

- A velocity-time graph shows changes in velocity of a moving object over time.
- The slope of a velocity-time graph represents acceleration of the moving object.

Review

1. Describe a velocity-time graph. What does the slope of the graph line represents?
2. In the **Figure** above, the sprinter reaches a velocity of 2 m/s in just 1 second. At a constant rate of acceleration, how long does it take for her to double this velocity? What is her acceleration during this time period?
3. Create a velocity-time graph by plotting the data in the **Table** below.

Velocity (m/s)	**Time (s)**
10	1
30	2
50	3
40	4
40	5

Additional Resources

Video:

Real World Application: Speeding Up by Falling Down

Interactives: Irwin 2D, Butterfly Stroke

SECTION OVERVIEW

9.4: Motion in Two-Dimensions

9.4.1: Projectile Motion for an Object Launched Horizontally

Figure 4.2.1

The activity of bike jumping, like other sports that involve vector motions in perpendicular directions, requires more physical practice than mathematical analysis. The laws of physics apply to the activity, however, whether the biker is aware of them or not.

Projectile Motion for an Object Launched Horizontally

Objects that are launched into the air are called **projectiles**. The path followed by a projectile in motion is called a **trajectory**. The image of the snowboarder in the chapter introduction showed his trajectory; every object has a trajectory even when we cannot see it. **Projectile motion**, or the object's trajectory, is described in terms of position, velocity, and acceleration. Even with only an object's current location, velocity, and acceleration, we can calculate when and where the object will land. Our knowledge that perpendicular components of vectors do not affect each other allow us to easily analyze the motion of projectiles.

Diagram of two balls dropping

Figure 4.2.2

In the diagram, two balls (one red and one blue) are dropped at the same time. The red ball is released with no horizontal motion and the blue ball is dropped but also given a horizontal velocity of 10 m/s. As the balls fall to the floor, a photograph is taken every second so that in 5 seconds, we have 5 images of the two balls. Each vertical line on the diagram represents 5 m. Since the blue ball has a horizontal velocity of 10 m/s, you will see that for every second, the blue ball has moved horizontally 10 m. That is, in each second, the blue ball has increased its horizontal distance by 10 m. This horizontal motion is due to the ball's constant velocity.

The red ball was dropped straight down with no horizontal velocity and therefore, in each succeeding second, the red ball falls straight down with no horizontal motion. The increasing distances between seconds in the red ball's motion indicates that this motion is accelerating.

A very important point here is that the vertical motion of these two balls is identical. That is, they each fall exactly the same distance vertically in each succeeding second. The constant horizontal velocity of the blue ball has no effect on its accelerated vertical motion. Therefore, the vertical motion of the blue ball can be analyzed exactly the same as the vertical motion of the red ball.

Examples

Example 4.2.1

If an arrow is fired from a bow with a perfectly horizontal velocity of 60.0 m/s and the arrow was 2.00 m above the ground when the it was released, how far will the arrow fly horizontally before it strikes the ground?

Solution

This problem is solved by determining how long it takes the arrow to fall to the ground in exactly the same manner as if the arrow was dropped with no horizontal velocity. The time required for the arrow to fall to the ground will be the same time that the arrow flies horizontally at 60.0 m/s, so

$d=(1/2)at^2$ solved for $t=(2d/a)^{\wedge}(1/2)=((2)(2.00\text{ m})/9.80\text{ m/s}^2)^{\wedge}(1/2)=0.639$ s

The time required for the fall is multiplied by the horizontal velocity to get the horizontal distance.

dhorizontal=(vhorizontal)(time)=(60.0 m/s)(0.639 s)=38.3 m

✓ Example 4.2.2

A rock was thrown horizontally from a 100.0 m high cliff. It strikes the ground 90.0 m from the base of the cliff. At what speed was it thrown?

Solution

We can calculate how long it takes for a rock to free fall 100.0 m and then divide this time into the horizontal distance to get the horizontal velocity.

$t=(2d/a)^{\wedge}(1/2)=((2)(100.0\text{ m})/9.80\text{ m/s}^2)^{\wedge}(1/2)=4.52$ s

v=d/t=90.0 m/4.52 s=19.9 m/s

Use the following simulation to adjust the initial velocity and height of a launched arrow and try to predict whether or not the archer will hit her target:

Summary

- Perpendicular components of vectors do not influence each other.
- The horizontal motion of a projectile does not influence its free fall.

Review

1. If a bullet is fired from a high powered rifle at the exact time a duplicate bullet is dropped by hand near the barrel of the rifle, which bullet will hit the ground first?
    1. the one dropped straight down
    2. the one fired horizontally
    3. both will hit the ground at the same time
2. A cannon is fired from the edge of a small cliff. The height of the cliff is 80.0 m. The cannon ball is fired with a perfectly horizontal velocity of 80.0 m/s. How far will the cannon ball fly horizontally before it strikes the ground?
3. A cliff diver running 3.60 m/s dives out horizontally from the edge of a vertical cliff and reaches the water below 2.00 s later. How high is the cliff and how far from the base of the cliff did the diver hit the water?

Explore More

Use this resource to answer the questions that follow.

1. What was the problem the mythbusters had with the dropped bullet? Why was fixing this so important?
2. Why did they move the bullet being dropped to 360 ft away?
3. What was the final result?
4. What are some possible reasons that the bullets didn't hit the ground at exactly the same time?

Additional Resources

Interactive: Water Fountain

PLIX: Play, Learn, Interact, eXplore: Kicking a Gold Ball

Real World Application: Physics of Archery

Video: Horizontal Projectile Motion - Overview

Study Guide: Motion Study Guide

9.4.2: Projectile Motion for an Object Launched at an Angle

Understanding the physics before a human cannonball launch is very important

Figure 4.3.1

In the case of the human cannonball shown, all the vector and gravitational calculations must be worked out perfectly before the first practice session. With this activity, you cannot afford trial and error – the first miss might be the last trial.

Projectile Motion for an Object Launched at an Angle

When an object is projected from rest at an upward angle, its initial velocity can be resolved into two components. These two components operate independently of each other. The upward velocity undergoes constant downward acceleration which will result in it rising to a highest point and then falling backward to the ground. The horizontal motion is constant velocity motion and undergoes no changes due to gravity.The analysis of the motion involves dealing with the two motions independently.

Parabolic path of an object launched at an angle

Figure 4.3.2

Example 4.3.1

A cannon ball is fired with an initial velocity of 100.0 m/s at an angle of 45° above the horizontal. What maximum height will it reach and how far will it fly horizontally?

Solution

The first step in the analysis of this motion is to resolve the initial velocity into its vertical and horizontal components.

$$v_{i-\text{up}}=(100.0\ \text{m/s})(\sin 45\circ)=(100.0\ \text{m/s})(0.707)=70.7\ \text{m/s}$$

$$v_{i-\text{horizontal}}=(100.0\ \text{m/s})(\cos 45\circ)=(100.0\ \text{m/s})(0.707)=70.7\ \text{m/s}$$

We will deal with the vertical motion first. The vertical motion is symmetrical. As the object rises to its highest point and then falls back down, it will travel the same distance in each direction, and take the same amount of time. This is often hard to accept, but the amount of time the object takes to come to a stop at its highest point is the same amount of time it takes to return to where it was launched from. Similarly, the initial velocity upward will be the same magnitude (opposite in direction) as the final velocity when it returns to its original height. There are several ways we could approach the upward motion. We could calculate the time it would take gravity to bring the initial velocity to rest. Or, we could calculate the time it would take gravity to change the initial velocity from +70.7 m/s to -70.0 m/s. Yet another way would be to calculate the time it takes for the object to return to its original height.

$$v_f=v_i+at \text{ so } t=(v_f-v_i)/a$$

If we calculate the time required for the ball to rise up to its highest point and come to rest, the initial velocity is 70.7 m/s and the final velocity is 0 m/s. Since we have called the upward velocity positive, then the acceleration must be negative or -9.80 m/s^2.

$$t=(v_f-v_i)/a=(0\ \text{m/s}-70.7\ \text{m/s})/-9.80\ \text{m/s}^2=7.21\ \text{s}$$

Since this is the time required for the cannon ball to rise up to its highest point and come to rest, then the time required for the entire trip up and down would be double this value, or 14.42 s. The maximum height reached can be calculated by multiplying the time for the upward trip by the average vertical velocity. Since the object's velocity at the top is 0 m/s, the average upward velocity during the trip up is one-half the initial velocity.

$$v_{\text{up}-\text{ave}}=(12)(70.7\ \text{m/s})=35.3\ \text{m/s}$$

$$\text{height}=(v_{\text{up}-\text{ave}})(t_{\text{up}})=(35.3\ \text{m/s})(7.21\ \text{s})=255\ \text{m}$$

The horizontal distance traveled during the flight is calculated by multiplying the total time by the constant horizontal velocity.

$$d_{\text{horizontal}}=(14.42\ \text{s})(70.7\ \text{m/s})=1020\ \text{m}$$

Example 4.3.2

A golf ball was hit into the air with an initial velocity of 4.47 m/s at an angle of 66° above the horizontal. How high did the ball go and how far did it fly horizontally?

Solution

A golf ball was hit into the air with an initial velocity of 4.47 m/s at an angle of 66° above the horizontal. How high did the ball go and how far did it fly horizontally?

$$v_{i-up}=(4.47\text{ m/s})(\sin 66^\circ)=(4.47\text{ m/s})(0.913)=4.08\text{ m/s}$$

$$v_{i-hor}=(4.47\text{ m/s})(\cos 66^\circ)=(4.47\text{ m/s})(0.407)=1.82\text{ m/s}$$

$$t_{up}=(v_f-v_i)/a=(0\text{ m/s}-4.08\text{ m/s})/-9.80\text{ m/s}^2=0.416\text{ s}$$

$$v_{up-ave}=(12)(4.08\text{ m/s})=2.04\text{ m/s}$$

$$\text{height}=(v_{up-ave})(t_{up})=(2.04\text{ m/s})(0.416\text{ s})=0.849\text{ m}$$

$$t_{\text{total trip}}=(2)(0.416\text{ s})=0.832\text{ s}$$

$$d_{\text{horizontal}}=(0.832\text{ s})(1.82\text{ m/s})=1.51\text{ m}$$

Example 4.3.3

Suppose a cannon ball is fired downward from a 50.0 m high cliff at an angle of 45° with an initial velocity of 80.0 m/s. How far horizontally will it land from the base of the cliff?

Solution

Suppose a cannon ball is fired downward from a 50.0 m high cliff at an angle of 45° with an initial velocity of 80.0 m/s. How far horizontally will it land from the base of the cliff?

In this case, the initial vertical velocity is downward and the acceleration due to gravity will increase this downward velocity.

$$v_{i-down}=(80.0\text{ m/s})(\sin 45^\circ)=(80.0\text{ m/s})(0.707)=56.6\text{ m/s}$$

$$v_{i-hor}=(80.0\text{ m/s})(\cos 45^\circ)=(80.0\text{ m/s})(0.707)=56.6\text{ m/s}$$

$$d=v_{i-down}t+(1/2)at^2$$

$$50.0=56.6t+4.9t^2$$

Changing to standard quadratic form yields $4.9t^2+56.6t-50.0=0$

This equation can be solved with the quadratic formula. The quadratic formula will produce two possible solutions for t:

$$t=(-b+(b2-4ac){}^\wedge(1/2))/2a \text{ and } t=(-b-(b2-4ac){}^\wedge(1/2))/2a$$

$$t=(-56.6+((56.6)^2-(4)(4.9)(-50)){}^\wedge(1/2))/(2)(4.9)=0.816\text{ s}$$

The other solution to the quadratic formula is -12.375 *s*. Clearly, the cannon ball doesn't take -12 seconds to fly. Therefore, we take the positive answer. Using the quadratic formula will give you two answers; be careful to think about the answer you get - does it make sense?

To solve the problem, we plug the speed and time into the equation for distance:

$$d_{\text{horizontal}}=(0.816\text{ s})(56.6\text{ m/s})=46.2\text{ m}$$

In the following simulation, we can see that water from a fountain falls in a parabolic motion and lands at different positions based on its initial height, velocity and launch angle. Practice calculating where the water should land and then move the catcher to see if you are correct:

Summary

- To calculate projectile motion at an angle, first resolve the initial velocity into its horizontal and vertical components.
- Analysis of projectile motion involves dealing with two motions independently.

- Vertical components will always have the acceleration of gravity acting on them.
- Vertical motion is symmetrical - the distance and time are the same in the rise as in the fall; the final velocity will have the same magnitude as the initial velocity but in the opposite direction.
- Horizontal components will never be effected by gravity; it is constant velocity motion.

Review

1. A player kicks a football from ground level with a velocity of magnitude 27.0 m/s at an angle of 30.0° above the horizontal.
    1. Find the time the ball is in the air.
    2. Find the maximum height of the ball.
    3. Find the horizontal distance the ball travels.
2. A person standing on top of a 30.0 m high building throws a ball with an initial velocity of 20. m/s at an angle of 20.0° below horizontal. How far from the base of the building will the ball land?
3. An arrow is fired downward at an angle of 45 degrees from the top of a 200 m cliff with a velocity of 60.0 m/s.
    1. How long will it take the arrow to hit the ground?
    2. How far from the base of the cliff will the arrow land?

Explore More

Use this video to answer the questions that follow.

1. What is the cannon ball in this video?
2. What is used as the monkey in this video?
3. Does the velocity of the cannon ball matter, or will it hit the monkey at any velocity?

Additional Resources

Interactive: Archery

Real World Application: To Hit An Ace

Videos: Parabolic Projectile Motion - Overview, The Physics Behind a Curveball

Study Guide: Motion Study Guide

9.5: End of Chapter Activity

End of Chapter Activity: Creating a Lesson Plan with AI and Bloom's Taxonomy

Now that you have explored the fundamentals of motion, it's time to put your knowledge into practice. Your task is to create a succinct lesson plan for 3rd graders that introduces them to the basics of motion. To help you with this, you will use AI tools and incorporate Bloom's Taxonomy to ensure a comprehensive learning experience. This lesson plan will go towards your digital notebook, a portfolio filled with lesson plans, activities and labs for future use.

Example: Activity Prompt:

Objective: Use AI and Bloom's Taxonomy to develop a lesson plan that effectively teaches 3rd graders about the fundamentals of motion, including concepts such as distance, displacement, speed, and direction.

1. Understanding the Concepts:
    - Knowledge (Remembering): Define key terms related to motion, such as distance, displacement, speed, and direction.
    - Comprehension (Understanding): Explain these concepts in simple, age-appropriate language.
2. Planning the Lesson:
    - Application: Design an engaging activity or experiment that allows students to observe and measure motion. For example, use toy cars or balls to demonstrate how objects move and how to measure their movement.
    - Analysis: Use AI tools to create visual aids or interactive simulations that illustrate the concepts of motion. For instance, you can create a simple animation that shows a car moving along a track, highlighting changes in speed and direction.
3. Deepening Understanding:
    - Synthesis (Creating): Ask students to predict the outcome of different scenarios involving motion. For example, what happens if a toy car is pushed on a smooth versus a rough surface?
    - Evaluation: Have students discuss and reflect on what they observed during the activities. Encourage them to think about why objects move the way they do and how they can use this knowledge in everyday life.
4. Using AI in the Classroom:
    - Explore AI tools like educational apps or platforms that provide interactive content for teaching motion. Use these tools to create quizzes, flashcards, or interactive stories that reinforce the lesson's concepts.
    - Use AI to assess student understanding through formative assessments and provide instant feedback.

Deliverable: Submit a detailed lesson plan that includes:

- A brief overview of the key concepts covered.
- A description of the activities and experiments designed.
- Examples of AI tools used and how they enhance the learning experience.
- An explanation of how Bloom's Taxonomy was applied in the lesson plan to ensure a well-rounded educational experience.

This activity will help you integrate modern technology and educational strategies to create an effective and engaging learning experience for young students.

9.6: End of Chapter Key Terms

Definition: Motion

- **Motion**: The change in position of an object over time.
- **Displacement**: The change in position of an object from its initial point to its final point, taking into account direction; measured in meters (m).
- **Distance**: The total length of the path traveled by an object, regardless of direction; measured in meters (m).
- **Speed**: The rate at which an object covers distance; calculated as distance divided by time ($s = d/t$); measured in meters per second (m/s).
- **Velocity**: The rate at which an object changes its position, including direction; calculated as displacement divided by time ($v = \Delta x/\Delta t$); measured in meters per second (m/s).
- **Acceleration**: The rate of change of velocity of an object; calculated as change in velocity divided by time ($a = \Delta v/\Delta t$); measured in meters per second squared (m/s^2).
- **Uniform Motion**: Motion at a constant speed in a straight line.
- **Non-uniform Motion**: Motion that involves a change in speed or direction, or both.
- **Instantaneous Speed**: The speed of an object at a specific instant in time.
- **Average Speed**: The total distance traveled divided by the total time taken.
- **Instantaneous Velocity**: The velocity of an object at a specific instant in time.
- **Average Velocity**: The total displacement divided by the total time taken.
- **Uniform Acceleration**: Constant acceleration, such as the acceleration due to gravity.
- **Free Fall**: The motion of an object under the influence of gravitational force only.
- **Projectile Motion**: The motion of an object thrown or projected into the air, subject to only the acceleration of gravity.
- **Trajectory**: The path followed by a projectile or moving object.
- **Relative Motion**: The calculation of the motion of an object with regard to some other moving object.
- **Frame of Reference**: A coordinate system used to define and measure the position, orientation, and other properties of objects in it.
- **Linear Motion**: Motion in a straight line.
- **Rotational Motion**: Motion of an object about an internal axis.
- **Circular Motion**: Motion along a circular path or a circular orbit.
- **Centripetal Acceleration**: The acceleration directed toward the center of a circular path, necessary to maintain circular motion.
- **Centripetal Force**: The force directed toward the center of a circular path, necessary to maintain circular motion.
- **Tangential Velocity**: The linear speed of something moving along a circular path.
- **Kinematics**: The branch of mechanics that describes the motion of objects without considering the causes of motion.
- **Dynamics**: The branch of mechanics that deals with the forces and their effects on motion.
- **Newton’s Laws of Motion**: Three fundamental laws describing the relationship between the motion of an object and the forces acting on it.
- **Inertia**: The tendency of an object to resist changes in its state of motion.
- **Momentum**: The product of an object's mass and velocity ($p = mv$); a measure of how difficult it is to stop a moving object.
- **Impulse**: The change in momentum resulting from a force applied over a period of time ($J = Ft$).
- **Equilibrium**: The state in which the net force on an object is zero, resulting in no acceleration.
- **Translational Motion**: Movement that changes the position of an object without rotation.
- **Angular Motion**: Rotation around a central point or axis.
- **Harmonic Motion**: Motion that repeats in cycles, such as a pendulum or a mass on a spring.
- **Damped Harmonic Motion**: Oscillatory motion that gradually decreases in amplitude over time due to friction or other resistance.
- **Simple Harmonic Motion**: Oscillatory motion under a restoring force proportional to the displacement from an equilibrium position.
- **Uniform Circular Motion**: Motion in a circle at constant speed.

- **Non-uniform Circular Motion**: Motion in a circle with changing speed.
- **Velocity-Time Graph**: A graph that shows how velocity changes over time, where the slope represents acceleration.
- **Position-Time Graph**: A graph that shows how position changes over time, where the slope represents velocity.

CHAPTER OVERVIEW

10: Forces

10.1: Introduction and Learning Objectives

Chapters Learning Objectives

1. Fundamental Concepts of Forces:
    - Understand and define force as a vector quantity that causes an object to change its state of motion or shape.
    - Learn to identify different types of forces such as gravitational, electromagnetic, normal, frictional, and tension forces.
    - Explore the concept of net force and its significance in determining the overall effect on an object's motion.
2. Newton's Laws of Motion:
    - Investigate Newton's First Law of Motion (Law of Inertia) and its application in understanding motion and equilibrium of objects.
    - Examine Newton's Second Law of Motion and its formulation (F=ma), and learn to calculate the force acting on an object given its mass and acceleration.
    - Understand Newton's Third Law of Motion (action-reaction principle) and its implications in various physical situations.
3. Frictional Forces and Their Effects:
    - Explore the nature of frictional forces, including static and kinetic friction, and the factors that affect them.
    - Understand the role of friction in everyday life and its importance in various mechanical systems.
    - Analyze how frictional forces can be both beneficial (e.g., in transportation, machinery) and detrimental (e.g., in energy efficiency).
4. Gravitational Force and Universal Gravitation:
    - Study the concept of gravitational force and its universal nature, as described by Newton's Law of Universal Gravitation.
    - Understand the relationship between mass, distance, and gravitational force, and apply this understanding to celestial bodies and orbital motion.
    - Explore the implications of gravity in astronomical phenomena, space exploration, and everyday life.
5. Teaching Strategies for Forces in Physical Science:
    - Develop effective methods for teaching the principles of forces, using real-world examples, demonstrations, and experiments.
    - Design engaging and interactive classroom activities that help students visualize and understand the effects of different forces.
    - Utilize multimedia tools and simulations to demonstrate complex concepts and scenarios involving forces.
    - Address common misconceptions and challenges students face in learning about forces.
    - Emphasize the interdisciplinary nature of force concepts and their relevance in various scientific, engineering, and practical applications.

Introduction to Forces

Forces are fundamental to the study of physics, serving as the invisible agents that cause objects to move, change direction, or alter their shape. This chapter provides a comprehensive exploration of the basic concepts of forces, defining force as a vector quantity that influences an object's motion or shape. We will examine different types of forces such as gravitational, electromagnetic, normal, frictional, and tension forces, and understand the concept of net force, which determines the overall effect on an object's motion.

Newton's Laws of Motion are central to understanding the behavior of forces. Newton's First Law, the Law of Inertia, explains the conditions for motion and equilibrium. Newton's Second Law provides a mathematical framework (F=ma) for calculating the force acting on an object based on its mass and acceleration. Newton's Third Law, the action-reaction principle, highlights the reciprocal nature of forces in various physical interactions.

Frictional forces, including static and kinetic friction, play a crucial role in our daily lives and mechanical systems. This chapter explores the nature of these forces, the factors affecting them, and their dual role as both beneficial (enhancing traction and grip) and detrimental (causing wear and reducing energy efficiency).

Gravitational force, governed by Newton's Law of Universal Gravitation, is another key focus. Understanding the relationship between mass, distance, and gravitational force helps explain celestial phenomena, orbital motion, and the role of gravity in space exploration and everyday life.

Educators play a pivotal role in conveying these complex concepts to students. Effective teaching strategies involve using real-world examples, demonstrations, and experiments to make abstract ideas tangible. Designing engaging and interactive classroom activities can help students visualize and understand the effects of different forces. Multimedia tools and simulations are valuable resources for illustrating complex scenarios and addressing common misconceptions. Emphasizing the interdisciplinary nature of force concepts highlights their relevance in various scientific, engineering, and practical applications.

In K-12 education, introducing these principles with age-appropriate language and relatable examples can ignite curiosity and understanding in young learners. Interactive activities and hands-on experiments allow students to see the real-world implications of force concepts, making them more tangible and memorable. Utilizing multimedia resources caters to diverse learning styles, ensuring that all students grasp these foundational ideas. Highlighting how force concepts are interconnected with everyday life and various scientific fields can foster a deeper appreciation for the subject, preparing students for advanced studies in physics.

10.2: Weight

When astronauts experience weightlessness, their mass remains unchanged

Figure 3.1.1

Astronauts in training often fly in the KC-135 training aircraft to experience near-weightlessness. Three Japan Aerospace Exploration Agency astronauts—Akihiko Hoshide, Satoshi Furukawa, and Naoko Yamazaki—are shown here during such an exercise. Though they experience near-weightlessness, we can see that their mass has not changed. What is the relationship between mass and weight?

Mass and Weight

The **mass** of an object is defined as the amount of matter in the object. The amount of mass an object has does not change; a moon rock that has been returned to Earth has the same mass on the Earth's surface as it had on the moon. The amount of mass in an object is measured by comparing the object to known masses on an instrument called a balance.

The mass of an object is measured by comparing the object to known masses on a balance

Figure 3.1.2

Using the balance shown here, the object would be placed in one pan and known masses would be placed in the other pan until the pans were exactly balanced. When balanced, the mass of the object would be equal to the sum of the known masses in the other pan. A balance will work in any location; whether on the moon or on Earth, the moon rock mentioned earlier will have the same mass.

The **weight** of an object is the force pulling the object downward. On Earth, this would be the gravitational force of the Earth on the object. On the moon, this would be the gravitational force of the moon on the object. The gravitational force of the moon is one-sixth the magnitude of the gravitational force of the Earth; the weight of the moon rock on the moon will be one-sixth the weight of the moon rock on the Earth's surface.

The gravitational force is derived from Newton's Second Law, $F=ma$, where F is the force of gravity in Newtons, m is the mass of the object in kilograms, and a is the acceleration due to gravity on Earth, 9.81 m/s^2 . When the formula is used specifically to solve for the weight of an object, it appears as $W=mg$. Weight is always measured in force units Newtons, m is the mass of the object in kilograms, and g is the gravitational strength, or the acceleration due to gravity, of the planet in N/kg or $\mathrm{m/s^2}$ ($g_{\mathrm{Earth}} = 9.81\ \mathrm{m/s^2}$).

Gain access to an Astronaut Training Chamber in the simulation below and observe how the mass of an object on various planets stays the same but its weight (or force due to gravity) differs. Try to use the graph to derive the strength of the gravitational field on the moon, Mars and even Saturn's moon, Dione!

Examples

✓ Example 3.1.1

What is the weight of an object sitting on the Earth's surface if the mass of the object is 43.7 kg?

Solution

$\mathrm{W=mg=(43.7\ kg)(9.81\ m/s^2)=429\ N}$

✓ Example 3.1.2

What is the mass of an object whose weight sitting on the Earth is 2570 N?

Solution

$\mathrm{m=W/g=(2570\ N)(/9.81\ m/s^2)=262\ kg}$

Summary

- The mass of an object is measured in kilograms and is defined as the amount of matter in an object.
- Mass is determined by comparing an object to known masses on a balance.
- The weight of an object on the Earth is defined as the force acting on the object by the Earth's gravity.

- Weight is measured by a calibrated spring scale.
- The formula relating mass and weight is W=mg.

Review

1. The mass of an object on the Earth is 100 kg.
    1. What is the weight of the object on the Earth?
    2. What is the mass of the object on the moon?
    3. Assuming the acceleration due to gravity on the moon is exactly one-sixth of the acceleration due to gravity on Earth, what is the weight of the object on the moon?
2. A man standing on the Earth can exert the same force with his legs as when he is standing on the moon. We know that the mass of the man is the same on the Earth and the moon. We also know that F=ma is true on both the Earth and the moon. Will the man be able to jump higher on the moon than the Earth? Why or why not?

Explore More

Use the resource below to answer the questions that follow.

1. What is the difference between mass and weight?

Additional Resources

Study Guide: Newton's Laws Study Guide

Real World Application: The Weight of Gold

PLIX: Play, Learn, Interact, eXplore: The Weight of Gold

Videos:

10.3: Connecting Newton's First and Second Laws

Astronauts weigh less on the moon than on Earth because the force of gravity is weaker on the moon

Figure 3.3.1

This image is of Buzz Aldrin, one of the first men to walk on the moon. Apollo 11 was the spaceflight that landed the first humans, Neil Armstrong and Buzz Aldrin, on the moon on July 20, 1969. Armstrong became the first to step onto the lunar surface 6 hours later on July 21. As you probably already know, men weigh less on the moon than on Earth; this is because the force of gravity is less on the moon than on Earth.

What is a force?

Newton’s First and Second Laws of Motion

A **force** is a push or a pull on an object. When you place a book on a table, the book pushes downward on the table and the table pushes upward on the book. The two forces are equal and there is no resulting motion of the book. If, on the other hand, you hold the book in the air and let go, the force of gravity will pull the book to the ground.

If you slide a book across the floor or a table, the book will experience a frictional force, which acts in the opposite direction of the motion. This force will slow down the motion of the book and eventually bring it to rest. A smoother surface has a smaller force of friction, which will allow the book to slide further before coming to rest. If a perfectly smooth floor could be created, there would be no friction and the book would slide forever at constant speed.

Newton’s First Law of Motion states that an object at rest will stay at rest and an object in motion will remain in motion. It describes a phenomenon called **inertia**. Inertia is the tendency of an object to resist change in its state of motion. In the absence of any force, an object will continue to move at the same constant speed and in the same straight line. If the object is at rest, in the absence of any force, it will remain at rest. Newton’s First Law states that an object with no force acting on it moves with constant velocity. (The constant velocity could, of course, be 0 m/s.)

According to Newton’s First Law, is a force needed to keep a sled sliding on ice at a constant velocity? Use the simulation below to discover more:

Newton’s First Law is equivalent to saying that “if there is no net force on an object, there will be no acceleration.” In the absence of acceleration, an object will remain at rest or will move with constant velocity in a straight line. The acceleration of an object is the result of an unbalanced force. If an object undergoes two forces, the motion of the object is determined by the net force. The magnitude of the acceleration is directly proportional to the magnitude of the unbalanced force. The direction of the acceleration is the same direction as the direction of the unbalanced force. The magnitude of the acceleration is inversely proportional to the mass of the object; the more massive the object, the smaller the acceleration produced by the same force.

These relationships are stated in **Newton’s Second Law of Motion**: "the acceleration of an object is directly proportional to the net force on the object and inversely proportional to the mass of the object."

Newton’s Second Law can be summarized in an equation:

$a=F/m$ or more commonly, $F=ma$

According to Newton’s Second Law, a new force on an object causes it to accelerate. However, the larger the mass, the smaller the acceleration. We say that a more massive object has a greater inertia.

The units for force are defined by the equation for Newton’s Second Law. Suppose we wish to express the force that will give a 1.00 kg object an acceleration of 1.00 m/s^2.

$$\mathbf{F=ma=(1.00\ kg)(1.00\ m/s^2)=1.00\ kg{\cdot}m/s^2}$$

This unit is defined as 1.00 newton or 1.00 N.

$$\mathbf{kg{\cdot}m/s^2=newton}$$

What Forces are Acting on You?

A classic physics example of the applications of Newton’s Second Law is the acceleration we experience while riding in an elevator. How can you accelerate upward if your weight (or the force due to gravity) always remains the same? At what points are you speeding up in the elevator? Slowing down? Moving at a constant rate? What is the net force required for these accelerations to occur? Use the simulation below to explore these questions and gain a deeper understanding of Newton’s Second Law:

Examples

Example 3.3.1

What force is required to accelerate a 2000. kg car at 2.000 m/s^2?

Solution

$F=ma=(2000.\ \text{kg})(2.000\ \text{m/s}^2)=4000.\ \text{N}$

Example 3.3.2

A net force of 150 N is exerted on a rock. The rock has an acceleration of 20. m/s^2 due to this force. What is the mass of the rock?

Solution

$m=F/a=(150\ \text{N})(20.\ \text{m/s}^2)=7.5\ \text{kg}$

Example 3.3.3

A net force of 100. N is exerted on a ball. If the ball has a mass of 0.72 kg, what acceleration will it undergo?

Solution

$a=F/m=(100.\ \text{N})(0.72\ \text{kg})=140\ \text{m/s}^2$

Apply Newton’s Second Law in this simulation to determine the acceleration of an airboat with a given mass and a variety of forces acting on it:

Summary

- A force is a push or pull on an object.
- Newton’s First Law states that an object with no net force acting on it remains at rest or moves with constant velocity in a straight line.
- Newton’s Second Law states that the acceleration of an object is directly proportional to the net force on the object and inversely proportional to the mass of the object.
- Newton’s Second Law is expressed as an equation, F=ma.

Review

1. A car of mass 1200 kg traveling westward at 30. m/s is slowed to a stop in a distance of 50. m by the car’s brakes. What was the braking force?
2. Calculate the average force that must be exerted on a 0.145 kg baseball in order to give it an acceleration of 130 m/s^2.
3. After a rocket ship going from the Earth to the moon leaves the gravitational pull of the Earth, it can shut off its engine and the ship will continue on to the moon due to the gravitational pull of the moon.
    1. True
    2. False
4. If a space ship traveling at 1000 miles per hour enters an area free of gravitational forces, its engine must run at some minimum level in order to maintain the ships velocity.
    1. True
    2. False
5. Suppose a space ship traveling at 1000 miles per hour enters an area free of gravitational forces and free of air resistance. If the pilot wishes to slow the ship down, he can accomplish that by shutting off the engine for a while.
    1. True
    2. False

Explore More

Use the resource below to answer the questions that follow.

1. What is a Hero's Engine?
2. How does Newton's First Law of Motion have to do with the Hero's Engine?
3. Why does the yellow ball go further?

Additional Resources

PLIX: Play, Learn, Interact, eXplore: Newton's First and Second Laws, Bowling for Physics, Determining the Net Force, Hot Air Balloons, The Forces of Flying

Video:

Study Guide: Newton's Laws Study Guide

- **3.3: Connecting Newton's First and Second Laws** by CK-12 Foundation is licensed CK-12. Original source: https://flexbooks.ck12.org/cbook/ck-12-physics-flexbook-2.0.

10.4: Newton's Third Law

Newton's third law would prevent the proposed space elevator from falling to Earth

Figure 3.4.1

The image above is a NASA artist's conceptual illustration of a space elevator. It was imagined as a geo-stationary transfer station for passengers and cargo between Earth and space. This idea was not pursued beyond the initial discussion and evaluation stage, but the scientists involved believe the concept will become truly possible with the expected technological advances of the late 21st century.

Newton's Third Law of Motion

Where do forces come from? Observations suggest that a force applied to an object is always applied by another object. A hammer strikes a nail, a car pulls a trailer, and a person pushes a grocery cart. Newton realized that forces are not so one-sided. When the hammer exerts a force on the nail, the nail also exerts a force on the hammer—after all, the hammer comes to rest after the interaction. This led to **Newton's Third Law of Motion**, which states that whenever one object exerts a force on a second object, the second object also exerts a force on the first object, equal in magnitude and opposite in direction.

This law is sometimes paraphrased as: "For every action, there is an equal and opposite reaction." A very important point to remember is that the two forces are on different objects—never on the same object. It is frequently the case that one of the objects moves as a result of the force applied but the motion of the other object in the opposite direction is not apparent.

Consider the situation where an ice skater is standing at the edge of the skating rink holding on to the side rail. If the skater exerts a force on the rail, the rail is held in place with tremendous friction and therefore, will not move in any noticeable way. The skater, on the other hand, had little friction with the ice, and therefore will be accelerated in the direction opposite of her original push. This is the process people use to jump up into the air. The person's feet exert force on the ground and the ground exerts an equal and opposite force on the person's feet. The force on the feet is sufficient to raise the person off the ground. The force on the ground has little effect because the Earth is so large. One of the accelerations is visible but the other is not visible.

A case where the reaction motion due to the reaction force is visible is the case of a person throwing a heavy object out of a small boat, such as a kayak. The object is accelerated in one direction and the boat is accelerated in the opposite direction. In this case, both the motion of the object is visible and the motion of the boat in the opposite direction is also visible. Explore the resulting motion of two interacting objects as a result of Newton's Third Law in this Pirate Ship simulation below:

Rockets also work in this manner. It is a misconception that the rocket moves forward because the escaping gas pushes on the ground or the surrounding air to make the rocket go forward. Rockets work in outer space where there is no ground or surrounding air. The rocket exerts a force on the gases causing them to be expelled and the gases exert a force on the rocket causing it to be accelerated forward.

The applications of Newton's Third Law can also be explored in the classic example if a horse pulling a cart. How does the horse-cart system move if the cart pulls on the horse with the same exact force and in the opposite direction as the horse pulls on the cart? (Beware, there are many misconceptions related to this example! Always remember that Newton's Third Law applies to the same ***type*** of force acting on different objects).

Summary

- A force applied to an object is always applied by another object.
- Newton’s Third Law of Motion states, “Whenever one object exerts a force on a second object, the second object also exerts a force on the first object, equal in magnitude and opposite in direction.”

Review

1. What is wrong with the following statement: When you exert a force on a baseball, the equal and opposite force on the ball balances the original force and therefore, the ball will not accelerate in any direction.
2. When a bat strikes a ball, the force exerted can send the ball deep into the outfield. Where is the equal and opposite force in this case?
3. Suppose you wish to jump horizontally and in order for you to jump a distance of 4 feet horizontally, you must exert a force of 200 N. When you are standing on the ground, you have no trouble jumping 4 feet horizontally. If you are standing in a canoe, however, and you need to jump 4 feet to reach the pier, you will surely fall into the lake. Why is it that you cannot jump 4 feet out of a canoe when you can easily do this when on land?

Explore More

Use the resource below to answer the questions that follow.

1. What does the bottle rocket have to do with Newton’s Third Law of Motion?

Additional Resources

PLIX: Play, Learn, Interact, eXplore: Rocket Science, Newton's 3rd Law: Gravity of Neptune

Real World Application: Rebuilding the Bridge

Video:

Study Guide: Newton's Laws Study Guide

10.5: Friction

Matches on different tennis court surfaces feel like distinct experiences as a result of friction

Figure 3.2.1

Tennis is played on a variety of court surfaces: grass, clay, hardcourt, and even carpet. Players know that matches on different surfaces are distinctly different games. The ball bounces differently, and shoes slip on some surfaces but stick on others. These differences are a result of friction.

Friction

Friction is the force that resists motion. In most beginning physics classes, friction is ignored. Concepts can be understood and calculations made assuming friction to be nonexistent. Whenever physics intersects with the real world, however, friction must be taken into account. Friction exists between two touching surfaces because even the smoothest looking surface is quite rough on a microscopic scale.

Sliding the two blocks over each other will result in frictional force between the two surfaces

Figure 3.2.2

Every surface has microscopic bumps, lumps, and imperfections, emphasized as in the image above. If we try to slide the top block over the lower block, there will be numerous collisions as the bumps impact each other. The forward motion causes the collisions with bumps which then exert a force in opposite way the block is moving. The force of friction always opposes whatever motion is causing the friction.

The force of friction between these two blocks is related to two factors. The first factor is the roughness of the surfaces that are interacting, which is called the **coefficient of friction**, μ (Greek letter mu). The second factor is the magnitude of the force pushing the top block down onto the lower block. It is reasonable that the more forcefully the blocks are pushed together, the more difficult it will be for one to slide over the other. The force pushing these blocks together is the result of gravity acting on the top block and pressing it against the bottom block, which resists the weight with an equal and opposite force called the **normal force**. The force of friction can be calculated by

> $F_{friction}=\mu\times F_{normal}$ and the normal force will be equal to the force of gravity on the object, if the object is on a flat surface (one parallel to the ground).

This is an approximate but reasonably useful and accurate relationship. It is not exact because **μ** depends on a variety of factors, including whether the surface is wet or dry.

The frictional force we have been discussing is referred to as **kinetic** (or **sliding**) **friction**; it is involved when one surface is sliding over another. If you have ever tried to slide a heavy object across a rough surface, you may be aware that it is a great deal easier to keep an object sliding than it is to start the object sliding in the first place. When the object to slide is resting on a surface with no movement, the force of friction is called **static friction** and it is somewhat greater than kinetic friction. Surfaces that move against one another will have both a coefficient of static friction and a coefficient of kinetic friction, and the two values will not be the same. For example, the coefficient of kinetic friction for ice on ice is 0.03 whereas the coefficient of static friction for ice on ice is 0.10—more than three times as great.

How can we pinpoint the exact amount of force needed to overcome static friction? Use the simulation below to find out:

Examples

✓ Example 3.2.1

A box weighing 2000. N is sliding across a cement floor. The force pushing the box is 500. N, and the coefficient of kinetic friction between the box and the floor is 0.20. What is the acceleration of the box?

Solution

In this case, the box is sliding along the ground, so the normal force for the box is equal to its weight. Using the normal force and the coefficient of friction, we can find the frictional force. We can also find the mass of the box from its weight since we know the acceleration due to gravity. Then we can find the net force and the acceleration.

$$\mathrm{F_F = \mu F_N = (0.20)(2000.\ N) = 400.\ N}$$

$\mathrm{mass\ of\ box = weight/g = 2000.\ N/9.8\ m/s^2 = 204\ kg}$

$$\mathrm{F_{NET} = pushing\ force - frictional\ force = 500.\ N - 400.\ N = 100.\ N}$$

$$\mathrm{a = F_N/m = 100.\ N/204\ kg = 0.49\ m/s^2}$$

✓ Example 3.2.2

Two boxes are connected by a rope running over a pulley, as shown in the figure below. The coefficient of kinetic friction between box A and the table is 0.20. (Ignore the masses of the rope and the pulley and any friction in the pulley.) The mass of box A is 5.0 kg and the mass of box B is 2.0 kg. The entire system (both boxes) will move together with the same acceleration and velocity. Find the acceleration of the system.

Solution

A diagram of a system of two boxes connected by a rope running over a pulley

Figure 3.2.3

The force acting to move the system is the weight of box B, and the force resisting the movement is the force of friction between the table and box A. The mass of the system is the sum of the masses of both boxes. The acceleration of the system can be found by dividing the net force by the total mass.

$$\mathrm{F_N(box\ A) = mg = (5.0\ kg)(9.8\ m/s^2) = 49\ N}$$

$$\mathrm{F_{friction} = \mu F_N = (0.20)(49\ N) = 9.8\ N}$$

$$\mathrm{Weight\ of\ box\ B = mg = (2.0\ kg)(9.8\ m/s^2) = 19.6\ N}$$

$$\mathrm{F_{NET} = 19.6\ N - 9.8\ N = 9.8\ N}$$

$$\mathrm{a = F_{NET}/mass = 9.8\ N/7.0\ kg = 1.4\ m/s^2}$$

In the simulation below, you can adjust the type of shoe a runner wears to change the coefficient of static friction between the runner’s shoe and the track surface. Play around and observe how a greater static friction coefficient means the sprinter can push off with more force without the danger of slipping.

Summary

- Friction is caused by bodies sliding over rough surfaces.
- The degree of surface roughness is indicated by the coefficient of friction, μ.
- The force of friction is calculated by multiplying the coefficient of friction by the normal force.
- The frictional force always opposes motion.
- The net force is found by subtracting the frictional force from the applied force.

Review

1. A 52 N sled is pulled across a cement sidewalk at constant speed. A horizontal force of 36 N is exerted. What is the coefficient of kinetic friction between the sidewalk and the metal runners of the sled?
2. If the coefficient of kinetic friction between a 25 kg crate and the floor is 0.45, how much force is required to move the crate at a constant velocity across the floor?
3. A smooth wooden 40.0 N block is placed on a smooth wooden table. A force of 14.0 N is required to keep the block moving at constant velocity.
    1. What is the coefficient of kinetic friction between the block and the table top?
    2. If a 20.0 N brick is placed on top of the wooden block, what force will be required to keep the block and brick moving at constant velocity?

Explore More

Use the resource below to answer the following questions.

1. 1 lbf (pound-force) = 4.44 N. Given this information, how many newtons of force did it take to rip the two phonebooks apart?
2. Why do you think there is so much friction between the two phonebooks?

Resources

When you fall thousands of feet from the sky, it seems like something strange is happening with the laws of physics. Turns out, everything relies on a simple force called drag. Though it may seem skydivers are in free fall, air resistance allows skydivers to speed up, slow down, and even change direction.

Additional Resources

Study Guide: Newton's Laws Study Guide

Real World Application: Slippery When Wet

PLIX: Play, Learn, Interact, eXplore: Steel Friction

Video:

10.6: Normal Force and Tension

Learning Objectives

- Define normal force.
- Explain the relationship between normal force and apparent weight.
- Describe properties of tension force.
- Apply Newton's laws of motion to analyze forces.

Forces are given many names, such as push, pull, thrust, lift, weight, friction, and tension. Traditionally, forces have been grouped into several categories and given names relating to their source, how they are transmitted, or their effects. The most important of these categories are discussed in this section, together with some interesting applications. Further examples of forces are discussed later in this text.

Normal Force

> **Weight** (also called force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You definitely notice that you must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in Figure $10.6.1$(a). But how do inanimate objects like a table support the weight of a mass placed on them, such as shown in Figure $10.6.1$(b)? When the bag of dog food is placed on the table, the table actually sags slightly under the load. This would be noticeable if the load were placed on a card table, but even rigid objects deform when a force is applied to them. Unless the object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or trampoline or diving board). The greater the deformation, the greater the restoring force. So when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly, and the sag is slight so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.

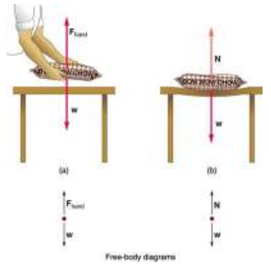

Figure 10.6.1: (a) The person holding the bag of dog food must supply an upward force $\boldsymbol{F}_{\text{hand}}$ equal in magnitude and opposite in direction to the weight of the food w. (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force **N** equal in magnitude and opposite in direction to the weight of the load.

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. The force supporting a load is perpendicular to the surface of contact between the load and its support, and this force is called a **normal force**, often indicated with symbol **N** (Please do not confuse this with the abbreviation for newton, the unit of force, N). The word *normal* means perpendicular to a surface. The normal force is not always equal to the object's weight, if there are other forces acting on the object, or if the object is accelerating, so that the net force is not zero.

Normal force also goes by another name, **apparent weight**. This is because all the forces you feel, especially your sensation of weight, are directly attributed to contact forces, such as the normal force. When NASA astronauts are weightlessly floating in the International Space Station, their actual weight (gravitational force on them by the Earth) is not zero, but their apparent weight (sensation of weight due to contact forces) is zero.

Tension

A **tension** is a force along the length of a medium, especially a force carried by a flexible medium, such as a rope or cable. The word “tension” comes from a Latin word meaning “to stretch.” Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called *tendons*. Any flexible connector, such as a string, rope, chain, wire, or cable, can exert pulls only parallel to its length; thus, a force carried by a flexible connector is a tension with direction parallel to the connector. It is important to understand that tension is a pull in a connector. In contrast, consider the phrase: “You can’t push a rope.” The tension force pulls outward along the two ends of a rope.

Consider a person holding a mass on a rope as shown in Figure $10.6.2$.

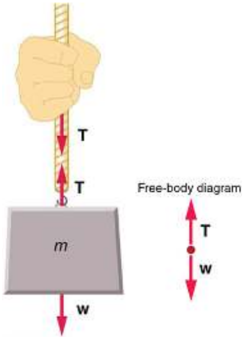

Figure $10.6.2$: When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force $\boldsymbol{T}$, that force must be parallel to the length of the rope, as shown. The pull such a flexible connector exerts is a tension. Note that the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). This is an example of Newton’s third law. The rope is the medium that carries the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.

Tension in the rope must equal the weight of the supported mass, as we can prove using Newton’s second law. If the 5.00-kg mass in the figure is stationary, then its acceleration is zero, and thus $\boldsymbol{F}_{\text{net }}=0$. The only external forces acting on the mass are its weight w and the tension $\boldsymbol{T}$ supplied by the rope. Thus,

$$F_{\text{net }}=T-w=0,$$

where $\boldsymbol{T}$ and w are the magnitudes of the tension and weight and their signs indicate direction, with up being positive here. Thus, just as you would expect, the tension equals the weight of the supported mass:

$$T=w=mg.$$

For a 5.00-kg mass, then (neglecting the mass of the rope) we see that

$$T=mg=(5.00\ \mathrm{kg})\left(9.80\ \mathrm{m}/\mathrm{s}^{2}\right)=49.0\ \mathrm{N}.$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a finger joint, or a bicycle brake cable. If there is no friction, the tension is transmitted undiminished. Only its direction changes, and it is always parallel to the flexible connector. This is illustrated in Figure $10.6.3$(a) and (b).

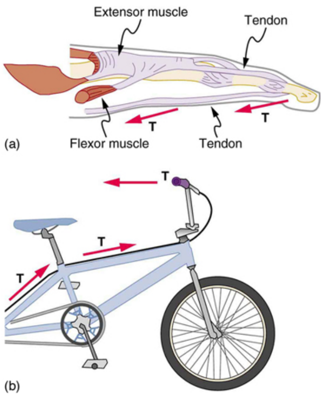

Figure $10.6.3$: (a) Tendons in the finger carry force $\boldsymbol{T}$ from the muscles to other parts of the finger, usually changing the force’s direction, but not its magnitude (the tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension $\boldsymbol{T}$ from the handlebars to the brake mechanism. Again, the direction but not the magnitude of $\boldsymbol{T}$ is changed.

Figure 10.6.4: Unless an infinite tension is exerted, any flexible connector—such as the chain at the bottom of the picture—will sag under its own weight, giving a characteristic curve when the weight is evenly distributed along the length, known as a catenary curve. Suspension bridges—such as the Golden Gate Bridge shown in this image—are essentially very heavy flexible connectors. The weight of the bridge is evenly distributed along the length of flexible connectors, usually cables, which take on the characteristic shape. (credit: Leaflet, Wikimedia Commons)

Normal force and tension are examples of forces that are determined not by a specific formula but by enforcing a constraint on motion state of a body. In the example of the bag on a table above, the normal force is equal to weight, because that is the value needed to ensure that the acceleration of the bag is zero. What happens if the body experiences a non-zero acceleration? We will consider example below of a person standing on a scale (which measures his apparent weight) while riding in an elevator.

Example $10.6.1$: What Does the Bathroom Scale Read in an Elevator?

Figure $10.6.5$ shows a 75.0-kg man (weight of about 165 lb.) standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate of $1.20\ \mathrm{m/s^2}$, and (b) if the elevator moves upward at a constant speed of 1 m/s.

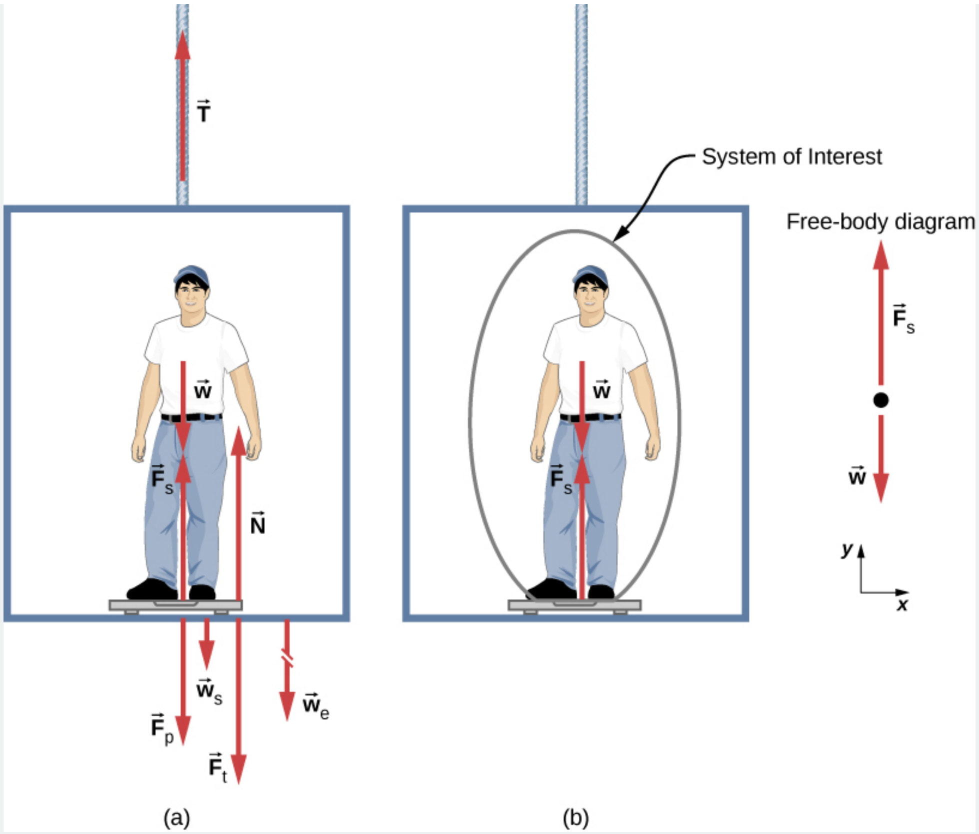

Figure 10.6.5: (a) The various forces acting when a person stands on a bathroom scale in an elevator. The arrows are approximately correct for when the elevator is accelerating upward—broken arrows represent forces too large to be drawn to scale. $\overrightarrow{\mathbf{T}}$ is the tension in the supporting cable, $\overrightarrow{\mathbf{w}}$ is the weight of the person, $\overrightarrow{\mathbf{w}}_{\mathrm{s}}$ is the weight of the scale, $\overrightarrow{\mathbf{w}}_{\mathrm{e}}$ is the weight of the elevator, $\overrightarrow{\mathbf{F}}_{\mathrm{s}}$ is the force of the scale on the person, $\overrightarrow{\mathbf{F}}_{\mathrm{P}}$ is the force of the person on the scale, $\overrightarrow{\mathbf{F}}_{\mathrm{t}}$ is the force of the scale on the floor of the elevator, and $\overrightarrow{\mathbf{N}}$ is the force of the floor upward on the scale. (b) The free-body diagram shows only the external forces acting *on* the designated system of interest—the person—and is the diagram we use for the solution of the problem.

Strategy

If the scale at rest is accurate, its reading equals $\overrightarrow{\mathbf{F}}_{\mathrm{P}}$, the magnitude of the force the person exerts downward on it. Figure 10.6.5(a) shows the numerous forces acting on the elevator, scale, and person. It makes this one-dimensional problem look much more formidable than if the person is chosen to be the system of interest and a free-body diagram is drawn, as in Figure 10.6.5(b). Analysis of the free-body diagram using Newton's laws can produce answers to both Figure 10.6.5(a) and (b) of this example, as well as some other questions that might arise. The only forces acting on the person are his weight $\overrightarrow{\mathbf{w}}$ and the upward force of the scale $\overrightarrow{\mathbf{F}}_{\mathrm{s}}$. According to Newton's third law, $\overrightarrow{\mathbf{F}}_{\mathrm{P}}$ and $\overrightarrow{\mathbf{F}}_{\mathrm{s}}$ are equal in magnitude and opposite in direction, so that we need to find F_s in order to find what the scale reads. We can do this, as usual, by applying Newton's second law,

$$\overrightarrow{\mathbf{F}}_{\mathrm{net}} = m\overrightarrow{\mathbf{a}}.$$

From the free-body diagram, we see that the net force $\overrightarrow{\mathbf{F}}_{\mathrm{net}}$ is sum of the scale force $\overrightarrow{\mathbf{F}}_{\mathrm{s}}$ and the weight $\overrightarrow{\mathbf{w}}$. Since the scale force and the weight are pointing in opposite directions, in terms of the magnitudes of the vectors,

$$F_{\mathrm{net}} = F_s - w = ma.$$

Solving for F_s gives us an equation with only one unknown:

$$F_{\mathrm{s}} = ma + w,$$

or, because $w = mg$, simply

$$F_s = ma + mg.$$

No assumptions were made about the acceleration, so this solution should be valid for a variety of accelerations in addition to those in this situation. (*Note:* We are considering the case when the elevator is accelerating upward. If the elevator is accelerating downward, Newton's second law becomes $F_s - w = -ma$.)

Solution

a. We have $a = 1.20\ \mathrm{m/s^2}$, so that

$$F_s = (75.0\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right) + (75.0\ \mathrm{kg})\left(1.20\ \mathrm{m/s^2}\right)$$

yielding

$$F_s = 825\ \mathrm{N}.$$

b. Now, what happens when the elevator reaches a constant upward velocity? Will the scale still read more than his weight? For any constant velocity—up, down, or stationary—acceleration is zero because $a = \frac{\Delta v}{\Delta t}$ and $\Delta v = 0$. Thus,

c. $$F_s = ma + mg = 0 + mg$$

or

$$F_s = (75.0\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right),$$

which gives

$$F_s = 735\ \mathrm{N}.$$

Significance

The scale reading in Figure 10.6.5(a) is about 185 lb. What would the scale have read if he were stationary? Since his acceleration would be zero, the force of the scale would be equal to his weight:

$$\begin{aligned} F_{\text{net}} &= ma = 0 = F_s - w \\ F_s &= w = mg \\ F_s &= (75.0\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right) = 735\ \mathrm{N}. \end{aligned}$$

Thus, the scale reading in the elevator is greater than his 735-N (165-lb.) weight. This means that the scale is pushing up on the person with a force greater than his weight, as it must in order to accelerate him upward. Clearly, the greater the acceleration of the elevator, the greater the scale reading, consistent with what you feel in rapidly accelerating versus slowly accelerating elevators. In Figure 10.6.5(b), the scale reading is 735 N, which equals the person's weight. This is the case whenever the elevator has a constant velocity—moving up, moving down, or stationary.

Section Summary

- When objects rest on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts perpendicular to and away from the surface. It is called a normal force, and it also goes by the name "apparent weight."
- *When objects rest on a non-accelerating horizontal surface*, the magnitude of the normal force is equal to the weight of the object:

$$N = mg.$$

- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension, $\boldsymbol{T}$. When a rope supports the weight of an object that is at rest, the tension in the rope is equal to the weight of the object:

$$T = mg.$$

Glossary

normal force

the force that a surface applies to an object to support the weight of the object; acts perpendicular to the surface on which the object rests

apparent weight

sensation of weight due to contact forces; usually the same as normal force

tension

the pulling force that acts along a medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force on the object due to the rope is called a tension force

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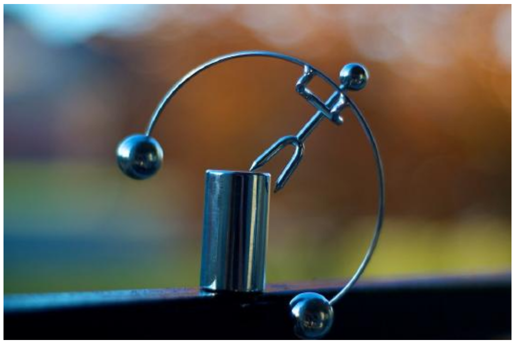
10.7: Gravitational Force and Inclined Planes

Figure 4.1.1

This photograph was taken by Apollo 8 crew member Bill Anders on December 24, 1968, showing the Earth rising above the lunar surface. Gravitational forces keep the moon orbiting the Earth.

Gravitational Force and Inclined Planes

Every object has a **center of gravity**. The center of gravity is the point at which the entire weight of a body may be considered to be concentrated; if supported at this point, the body would remain in equilibrium in any position. For example, if we were discussing a 12-inch ruler, the center of gravity for the ruler would be at the center of the 6-inch line. You could put your finger directly under the 6-inch line to hold the ruler and it would not fall either left or right. If you placed your finger underneath any other place on the ruler, it would fall off to one side or the other.



Figure

The force of gravity acting on an object is directed through this center of gravity and toward the center of the Earth. The object's weight, W, can be represented by a vector directed down (along the line the object would fall if it were dropped). When this object is resting on a level surface, its weight acts perpendicularly to the surface and will be equal to the **normal force**, which is the force keeping the object from falling through the table. The normal force is always perpendicular to the surface; when the surface is not level, the normal force will be equal to some subset of the weight. This is seen in the image below, which shows a box on an inclined plane.

alt

Figure 4.1.2

The weight of the box acts through the center of gravity and directly towards the center of the Earth. The weight vector in the sketch is red and labeled W. The normal force acts perpendicular to the surface of the inclined plane to keep the box from falling through the plane. The force of the box on the plane is equal to the normal force (as specified in Newton's Third Law). Since the normal force and the force of the box acting on the plane are the same, we can reference the force against the plane as the normal force. That force, FN, is purple in the image above. In addition, there is a force acting on the box parallel to the surface of the plane and pushing the box down the plane. This force is drawn in blue and is called the **parallel force**. The normal force and the parallel force add to give the weight.

The triangle of the black inclined plane and the yellow triangle are similar triangles; the corresponding sides are mutually perpendicular. Therefore, the angle at the top of the yellow triangle is also 30°. For a right triangle, if we know one other angle (the 30° angle) and one side (the weight), we can calculate the other two sides. Therefore, we can calculate the parallel force pushing the box down the incline and we could also calculate the acceleration of the box.

Examples

A 500. N box is resting on a plane inclined at 30° with the horizontal.

✓ Example 4.1.1

Find the normal force and the parallel force acting on this box.

Solution

(a) F_N=Wcos30°,=(500. N)(0.866)=433 N

$F_{parallel}$=Wsin30°=(500. N)(0.500)=250. N

Example 4.1.2

Find the acceleration of the box.

Solution

mass of the box=500. N/9.81 m/s^2=51.0 kg

a=$F_{parallel}$/mass=250. N/51.0 kg=4.90 m/s^2 (The direction is down the plane.)

Have you ever been in a car as it is driving around a banked turn? A banked turn is simply a corner of a road that is tilted at an angle and acts as an inclined plane. As a car drives around one of these corners, it experiences the force of friction, gravity and normal force acting on it. Use the PLIX Interactive below to explore how each of these forces affect what the car does in the turn:

Summary

- Each object has a center of gravity, which is the point at which the weight can be considered to be concentrated when balancing the object.
- The weight of an object is a vector that is directed toward the center of the Earth.
- When an object is placed on an inclined plane, its weight vector can be resolved into the normal force, which is equal to the force of the object perpendicular to the plane, and a parallel force, which pushes the object down the inclined plane.

Review

1. A car weighing 12,000 N is parked on a 36° slope.
    1. Find the force tending to cause the car to roll down the hill.
    2. Find the acceleration of the car rolling down the hill.

Additional Resources

Video: Inclined Planes - Overview

Real World Application: Mass Doesn't Matter

Interactive: Ramp And Piano

PLIX: Play, Learn, Interact, eXplore: Centripetal Force Problems: Banked Turns

Video:

10.8: End of Chapter Activity

End of Chapter Activity: Creating a Lesson Plan on Forces in Physics with AI and Bloom's Taxonomy

Now that you have explored the fundamentals of forces in physics, it's time to put your knowledge into practice. Your task is to create a succinct lesson plan for 4th graders that introduces them to the basics of forces, including concepts such as gravity, friction, and magnetism. To help you with this, you will use AI tools and incorporate Bloom's Taxonomy to ensure a comprehensive learning experience. This lesson plan will go towards your digital notebook, a portfolio filled with lesson plans, activities, and labs for future use.

Activity Prompt:

> **Objective:** Use AI and Bloom's Taxonomy to develop a lesson plan that effectively teaches 4th graders about the fundamentals of forces, including concepts such as gravity, friction, and magnetism.

Understanding the Concepts:

> **Knowledge (Remembering):** Define key terms related to forces, such as force, gravity, friction, and magnetism.
>
> **Comprehension (Understanding):** Explain these concepts in simple, age-appropriate language, focusing on their importance and real-life applications.

Planning the Lesson:

> **Application:** Design an engaging activity that allows students to observe and understand different types of forces. For example, use simple experiments to demonstrate gravity (dropping objects), friction (sliding objects on different surfaces), and magnetism (using magnets to move objects).
>
> **Analysis:** Use AI tools to create visual aids or interactive simulations that illustrate how forces act on objects. For instance, create a simulation that shows how different forces affect the motion of an object.

Deepening Understanding:

> **Synthesis (Creating):** Ask students to design their own simple experiments that demonstrate the effects of forces. For example, they could create a small obstacle course where they have to use different forces to move a ball through it.
>
> **Evaluation:** Have students discuss and reflect on their experiments and the forces involved. Encourage them to think about how forces interact in their everyday lives.

Using AI in the Classroom:

Explore AI tools like educational apps or platforms that provide interactive content for teaching about forces in physics. Use these tools to create quizzes, flashcards, or interactive stories that reinforce the lesson's concepts.

Use AI to assess student understanding through formative assessments and provide instant feedback.

Deliverable:

Submit a detailed lesson plan that includes:

1. **A brief overview of the key concepts covered:** Outline the foundational concepts of forces that will be taught.
2. **A description of the activities and experiments designed:** Detail the hands-on activities and experiments you will use to help students understand these concepts.
3. **Examples of AI tools used and how they enhance the learning experience:** Describe the AI tools you plan to incorporate, such as simulations or interactive quizzes, and explain how they will help students grasp complex concepts.
4. **An explanation of how Bloom's Taxonomy was applied in the lesson plan to ensure a well-rounded educational experience:** Illustrate how each level of Bloom's Taxonomy (Remembering, Understanding, Applying, Analyzing, Creating, and Evaluating) is addressed in your lesson plan.

Additionally, include a creative project component where students create a short animated video or a comic strip that explains a force-related concept, using AI tools to enhance their projects.

Example Lesson Plan:

Grade: 4th Grade
Topic: Forces in Physics
Duration: 1 Week

Overview:
Students will learn about different types of forces, including gravity, friction, and magnetism, through engaging activities and creative projects.

Day 1: Introduction to Forces

Objective: Define different types of forces and provide examples.

- **Remembering:** Define key terms (force, gravity, friction, magnetism).
- **Understanding:** Explain the concepts using examples from everyday life (e.g., gravity pulling objects down, friction slowing things down, magnets attracting metals).

Activity:
Watch a video (created using AI tools) explaining different types of forces with real-life examples.

Day 2: Observing Gravity

Objective: Observe and describe the effects of gravity.

- **Applying:** Conduct an experiment to observe the effects of gravity (e.g., dropping different objects and observing how they fall).

Activity:
Students drop various objects (e.g., balls, feathers) from the same height and record their observations, discussing why some fall faster than others.

Day 3: Exploring Friction

Objective: Understand the effects of friction on motion.

- **Applying:** Conduct an experiment to observe the effects of friction (e.g., sliding objects on different surfaces).

Activity:
Students slide objects (e.g., toy cars) on different surfaces (e.g., carpet, tile, sandpaper) and measure how far they travel. They discuss how friction affects the motion.

Day 4: Discovering Magnetism

Objective: Explore the effects of magnetism.

- **Applying:** Conduct an experiment to observe the effects of magnetism (e.g., using magnets to move objects).

Activity:
Students use magnets to move paper clips and other small metal objects. They explore how magnets attract and repel each other and create simple magnetic sculptures.

Day 5: Creative Project – Animated Video or Comic Strip

Objective: Create a multimedia presentation explaining a force-related concept.

- **Creating:** Students create a short animated video or a comic strip that explains a concept related to forces.

Activity:
In groups, students use AI tools to create an animated video or comic strip explaining a force-related concept (e.g., how gravity works, how friction affects motion). They present their projects to the class, using the animations or comic strips to illustrate their explanations.

By incorporating these strategies and activities, educators can effectively teach 4th graders about forces in physics, helping them understand and appreciate the fundamental concepts and their applications in the real world.

10.9: End of Chapter Key Terms

Definition: Forces

- **Force**: A vector quantity that causes an object to accelerate, change its velocity, or deform; measured in newtons (N).
- **Vector**: A quantity that has both magnitude and direction.
- **Magnitude**: The size or amount of a quantity, often referred to as the "strength" of a force.
- **Direction**: The line or course along which something is moving or pointing.
- **Contact Force**: A force that acts at the point of contact between two objects, such as friction, tension, and normal force.
- **Non-contact Force**: A force that acts over a distance without physical contact, such as gravitational, electromagnetic, and nuclear forces.
- **Gravity**: The force of attraction between two masses, typically noticeable between Earth and objects on it; measured in newtons (N).
- **Weight**: The force of gravity acting on an object’s mass, calculated as weight = mass × gravitational acceleration ($W = mg$).
- **Normal Force**: The support force exerted upon an object in contact with another stable object, acting perpendicular to the surface.
- **Friction**: The force that opposes the relative motion or tendency of such motion of two surfaces in contact.
- **Tension**: The pulling force transmitted along a string, rope, cable, or similar object.
- **Spring Force**: The restoring force exerted by a spring, proportional to the displacement of the spring from its equilibrium position (Hooke’s Law).
- **Air Resistance**: The force that opposes the motion of an object through air, a type of frictional force.
- **Applied Force**: A force that is applied to an object by a person or another object.
- **Net Force**: The overall force acting on an object when all the individual forces acting on the object are combined.
- **Balanced Forces**: Forces that are equal in magnitude and opposite in direction, resulting in no change in an object's motion.
- **Unbalanced Forces**: Forces that are not equal and opposite, causing a change in an object's motion.
- **Newton’s First Law of Motion (Law of Inertia)**: A principle stating that an object at rest will stay at rest, and an object in motion will stay in motion at constant velocity unless acted upon by a net external force.
- **Inertia**: The tendency of an object to resist changes in its state of motion.
- **Newton’s Second Law of Motion**: A principle stating that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass ($F = ma$).
- **Newton’s Third Law of Motion**: A principle stating that for every action, there is an equal and opposite reaction.
- **Centripetal Force**: A force that acts on an object moving in a circular path, directed toward the center of the circle.
- **Centrifugal Force**: An apparent force that acts outward on a body moving around a center, arising from the body's inertia.
- **Momentum**: The quantity of motion of a moving body, calculated as the product of its mass and velocity ($p = mv$).
- **Impulse**: The change in momentum resulting from a force applied over a period of time, calculated as impulse = force × time ($J = Ft$).
- **Torque**: A measure of the force that can cause an object to rotate about an axis, calculated as torque = force × lever arm distance ($\tau = rF \sin\theta$).
- **Equilibrium**: The state in which the net force on an object is zero, resulting in no acceleration.
- **Static Equilibrium**: The condition where an object is at rest and the net force acting on it is zero.
- **Dynamic Equilibrium**: The condition where an object is moving at constant velocity and the net force acting on it is zero.
- **Free-body Diagram**: A graphical representation used to visualize the forces acting on an object.
- **Elastic Force**: The force exerted by an elastic material when it is stretched or compressed.
- **Gravitational Force**: The attractive force that acts between any two masses.
- **Electromagnetic Force**: The force associated with electric and magnetic fields, responsible for electric and magnetic interactions.
- **Nuclear Force**: The strong attractive force that binds protons and neutrons together in the nucleus of an atom.

- **Action-at-a-Distance Force**: Forces that can act on an object without physical contact, such as gravitational, electromagnetic, and nuclear forces.

CHAPTER OVERVIEW

11: Electricity

11.1: Introduction and Learning Objectives

Chapter Learning Objectives

1. Understanding the Fundamentals of Electricity:
    - Grasp the basic concepts of electricity, including charge, voltage (electric potential), current, and resistance.
    - Explore the nature of electric charge and the laws governing the interaction of charged particles.
    - Understand the principles of electric circuits, including series and parallel circuits, and the role of batteries and power sources.
2. Ohm's Law and Electrical Circuits:
    - Investigate Ohm's Law and its application in determining the relationship between voltage, current, and resistance in an electric circuit.
    - Learn to calculate the total resistance in series and parallel circuits and understand the behavior of current and voltage in these circuits.
    - Explore the practical applications of Ohm’s Law in designing and troubleshooting electrical circuits.
3. Electromagnetism and its Applications:
    - Understand the principles of electromagnetism, including the magnetic effects of electric currents and the electric effects of moving magnets.
    - Examine the operation of electromagnetic devices such as solenoids, electromagnets, and transformers.
    - Investigate real-world applications of electromagnetism in technology, such as in motors, generators, and communication devices.
4. Safety and Practical Aspects of Electricity:
    - Learn about electrical safety measures, including insulation, grounding, and circuit protection devices like fuses and circuit breakers.
    - Understand the hazards associated with electricity, including electric shocks, short circuits, and fire risks, and how to mitigate them.
    - Discuss the importance of energy efficiency and sustainable practices in the use of electrical energy.
5. Teaching Electricity in the Classroom:
    - Develop effective teaching strategies for explaining the concepts of electricity to students, using demonstrations, lab activities, and real-life examples.
    - Design interactive experiments that help students understand and apply electrical principles in a safe and engaging manner.
    - Utilize multimedia tools and simulation software to visualize electric phenomena and enhance the learning experience.
    - Address common misconceptions and challenges students face when learning about electricity.
    - Emphasize the interdisciplinary connections of electricity with other areas of science and its relevance in everyday life and technological advancement.

Introduction to Electricity

Electricity is a cornerstone of modern physics and chemistry, integral to understanding both natural phenomena and technological innovations. This chapter covers the basic concepts of electricity, including charge, voltage (electric potential), current, and resistance. We explore the nature of electric charge and the fundamental laws governing the interaction of charged particles, as well as the principles of electric circuits, distinguishing between series and parallel circuits and the roles of batteries and power sources.

Ohm's Law is central to understanding electrical circuits, providing a relationship between voltage, current, and resistance. We delve into its applications, learning to calculate total resistance in series and parallel circuits, and examining the behavior of current and voltage within these systems. Practical applications of Ohm’s Law in designing and troubleshooting electrical circuits are also explored, emphasizing its significance in real-world scenarios.

The principles of electromagnetism, including the magnetic effects of electric currents and the electric effects of moving magnets, are crucial for understanding various technologies. This chapter examines electromagnetic devices such as solenoids, electromagnets, and transformers, and investigates their applications in motors, generators, and communication devices.

Safety is paramount when dealing with electricity. We discuss electrical safety measures such as insulation, grounding, and circuit protection devices like fuses and circuit breakers. Understanding the hazards associated with electricity, including electric shocks, short circuits, and fire risks, and how to mitigate them, is essential. The importance of energy efficiency and sustainable practices in the use of electrical energy is also highlighted.

Educators play a vital role in teaching these complex concepts. Effective strategies include using demonstrations, lab activities, and real-life examples to explain electricity. Designing interactive experiments helps students understand and apply electrical principles safely and engagingly. Multimedia tools and simulation software can visualize electric phenomena, enhancing the learning experience. Addressing common misconceptions and challenges ensures a solid grasp of electricity. Emphasizing interdisciplinary connections and the relevance of electricity in everyday life and technological advancement inspires students and underscores the importance of this fundamental concept.

In K-12 education, introducing these principles with age-appropriate language and relatable examples can ignite curiosity and understanding in young learners. Interactive activities and hands-on experiments allow students to see the real-world implications of electrical concepts, making them more tangible and memorable. Utilizing multimedia resources caters to diverse learning styles, ensuring that all students grasp these foundational ideas. Highlighting how electrical concepts are interconnected with everyday life and various scientific fields can foster a deeper appreciation for the subject, preparing students for advanced studies in physics and chemistry.

11.2: Introduction to Electricity

Figure 11.2.1: Static electricity from this plastic slide causes the child's hair to stand on end. The sliding motion stripped electrons away from the child's body, leaving an excess of positive charges, which repel each other along each strand of hair. (credit: Ken Bosma/Wikimedia Commons)

The image of American politician and scientist Benjamin Franklin (1706–1790) flying a kite in a thunderstorm is familiar to every schoolchild. (See Figure.) In this experiment, Franklin demonstrated a connection between lightning and **static electricity**. Sparks were drawn from a key hung on a kite string during an electrical storm. These sparks were like those produced by static electricity, such as the spark that jumps from your finger to a metal doorknob after you walk across a wool carpet. What Franklin demonstrated in his dangerous experiment was a connection between phenomena on two different scales: one the grand power of an electrical storm, the other an effect of more human proportions. Connections like this one reveal the underlying unity of the laws of nature, an aspect we humans find particularly appealing.

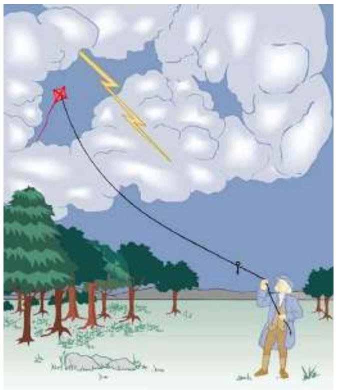


Figure 11.2.2: When Benjamin Franklin demonstrated that lightning was related to static electricity, he made a connection that is now part of the evidence that all directly experienced forces except the gravitational force are manifestations of the electromagnetic force.

Our understanding of electricity and the range of phenomena that are electrical in nature has vastly expanded since those early days. For example, atomic and molecular interactions, such as the forces of friction, cohesion, and adhesion, are now known to be manifestations of the **electromagnetic force**. Static electricity is just one aspect of the electromagnetic force, which also includes moving electricity and magnetism. Furthermore, All the macroscopic forces that we experience directly, such as the sensations of touch and the tension in a rope, are due to the electromagnetic force, one of the four fundamental forces in nature. The gravitational force, another fundamental force, is actually sensed through the electromagnetic interaction of molecules, such as between those in our feet and those on the top of a bathroom scale. (The other two fundamental forces, the strong nuclear force and the weak nuclear force, cannot be sensed on the human scale.)

In this chapter, we will develop the concepts of electric fields and electric potential (also known as *voltage*), which are used to describe electric force and electrical energy. Consider, for example, great amounts of electrical energy stored in batteries or transmitted cross-country through power lines, or consider electrical signals sent in our nervous systems at molecular levels, with *ions* crossing cell membranes and transferring information, or the role of electricity in our household appliances and devices, including your laptop and smartphones. We will look at the laws that help us describe electric circuits and begin to explore some of the many applications of electricity.

Glossary

static electricity

a buildup of electric charge on the surface of an object

electromagnetic force

one of the four fundamental forces of nature; the electromagnetic force consists of static electricity, moving electricity and magnetism

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SECTION OVERVIEW

11.3: Static Electricity

11.3.1: Electric Charge and Electric Force

Multiple lightning discharges from a single cloud

Figure 15.1.1

Lightning is the discharge of static electricity that has built up on clouds. Every year, the earth experiences an average of 25 million lightning strikes. Lightning bolts travel at speeds up to 60,000 miles per second, and can reach temperatures of 50,000°F, which is five times the temperature of the surface of the sun. The energy contained in a single lightning strike could light a 100 Watt light bulb 24 hours per day for 90 days.

Forces on Charged Objects

Electric charges exist within the atom. At the turn of the 20th century, J. J. Thomson and Ernest Rutherford determined that atoms contain very light-weight negatively charged particles called **electrons** and more massive, positively charged particles called **protons**. The protons are lodged in the **nucleus** of the atoms, along with the neutrally charged particles called **neutrons,** while the electrons surround the nucleus. When the number of electrons in the electron cloud and the number of protons in the nucleus are equal, the object is said to be **neutral**.

Changes to the nucleus of an atom require tremendous amounts of energy, so protons are not easily gained or lost by atoms. Electrons, on the other hand, are held fairly loosely and can often be removed quite easily. When an object loses some electrons, the remaining object is now positively charged because it has an excess of protons. The electrons may either remain free or may attach to another object. In that case, the extra electrons cause that object to become negatively charged. Atoms that have lost electrons and become positively charged are called **positive ions,** and atoms that have gained electrons and become negatively charged are called **negative ions**.

Electrons can be removed from some objects using friction, simply by rubbing one substance against another substance. There are many examples of objects becoming charged by friction, including a rubber comb through hair, and a balloon on a sweater. In both these instances, the electrons move from the second object to the first, causing the first object to become negatively charged and the second one positively charged. Friction between the tires on a moving car and the road cause the tires to become charged, and wind causes friction between clouds and air which causes clouds to become charged and can result in tremendous bolts of lightning.

Illustration of charged sweater and balloon

Figure 15.1.2

A common method of producing charge in the lab is to rub cat or rabbit fur against stiff rubber, producing a negative charge on the rubber rod. If you hold a rubber rod on one end and rub only the tip of the other end with a fur, you will find that only the tip becomes charged. The electrons you add to the tip of the rod remain where you put them instead of moving around on the rod. Rubber is an **insulator.** Insulators are substances that do not allow electrons to move through them. Glass, dry wood, most plastics, cloth, and dry air are common insulators. Materials that allow electrons to flow freely are called **conductors.** Metals have at least one electron that can move around freely, and all metals are conductors.

Forces are exerted on charged objects by other charged objects. You've probably heard the saying "opposites attract," which is true in regards to charged particles. Opposite charges attract each other, while like charges repulse each other. This can be seen in the image below. When two negatively charged objects are brought near each other, a repulsive force is produced. When two positively charged objects are brought near each other, a similar repulsive force is produced. When a negatively charged object is brought near a positively charged object, an attractive force is produced. Neutral objects have no influence on each other.

Attraction and repulsion of charged objects

Figure 15.1.3

Use the PLIX Interactive below to try to predict the attractive and repulsive forces exerted on one charged object by another charged object:

A laboratory instrument used to analyze and test for static charge is called an **electroscope.** Seen below, an electroscope consists of a metal knob connected by a metal stem to two very lightweight pieces of metal called leaves, shown in yellow. The leaves are enclosed in a box to eliminate stray air currents.

Uncharged electroscope

Figure 15.1.4

When a negatively charged object is brought near the knob of a neutral electroscope, the negative charge repels the electrons in the knob, and those electrons move down the stem into the leaves. Excess electrons flow from the rod into the ball, and then downwards making both leaves negatively charged. Since both leaves are negatively charged, they repel each other. When the rod is removed, the electroscope will remain charged because of the extra electrons added to it.

Electroscope that has been negatively charged by conduction

Figure 15.1.5

Conversely, if the rod is brought near the knob but doesn’t touch it, the electroscope will appear the same while the rod is near. That is, the negative charge in the rod repels the electrons in the ball, causing them to travel down to the leaves. The leaves will separate while the rod is nearby. No extra electrons were added to the electroscope, meaning that the electrons in the electroscope will redistribute when the negatively charged rod is taken away. The leaves return to neutral, and they stop repelling each other. If the rod touches the knob, the electroscope leaves are permanently charged but if the rod is brought near but does not touch the knob, the electroscope leaves are only temporarily charged.

If the leaves are permanently charged and the rod removed, the electroscope can then be used to determine the type of unknown charge on an object. If the electroscope has been permanently negatively charged, and a negatively charge object is brought near the knob, the leaves will separate even further, showing the new object has the same charge as the leaves. If a positively charged object is brought near a negatively charged electroscope, it will attract some of the excess electrons up the stem and out of the leaves, causing the leaves to come slightly together.

Similar to the results of a negatively charged rod, if a positively charged rod is brought near the knob of a neutral electroscope, it will attract some electrons up from the leaves onto the knob. That process causes both of the leaves to be positively charged (excess protons), and the leaves will diverge. If the positively charged rob is actually touched to the knob, the rob will remove some electrons and then when the rob is removed, the electroscope will remain positively charged. This is a permanent positive charge.

Electroscope that has been positively charged by induction

Figure 15.1.6

Charging an object by touching it with another charged object is called **charging by conduction.** By bringing a charged object into contact with an uncharged object, some electrons will migrate to even out the charge on both objects. Charging by conduction gives the previously uncharged object a permanent charge. An uncharged object can also be charged using a method called **charging by induction**. This process allows a change in charge without actually touching the charged and uncharged objects to each other. Imagine a negatively charged rod held near the knob, but not touching. If we place a finger on the knob, some of the electrons will escape into our body, instead of down the stem and into the leaves. When both our finger and the negatively charged rod are removed, the previously uncharged electroscope now has a slight positive charge. It was charged by induction. Notice that charging by induction causes the newly charged object to have the opposite charge as the originally charged object, while charging by conduction gives them both the same charge.

Summary

- Electric charges exist with the atom.
- Atoms contain light-weight, loosely held, negatively charged particles called electrons and heavier, tightly-held, positvely charged particles called protons.

- When the number of electrons and the number of protons are equal, the object is neutral.
- The loss of electrons gives an ion a positive charge, while the gain of electrons gives it a negative charge.
- Materials that allow electrons to flow freely are called conductors, while those that do not are called insulators.
- Opposite charges attract, and like charges repel.
- Charging an object by touching it with another charged object is called charging by conduction.

Review

1. How does friction generate static electricity?
    1. Friction heats the materials, thus causing electricity.
    2. Rubbing materials together displaces atoms, causing sparks to fly.
    3. Rubbing materials together can strip electrons off atoms, causing one material to become positive and the other to become negative.
    4. Rubbing materials together causes neutrons and electrons to trade places.
    5. None of the above.
2. What electrical charge does an electron have?
    1. A negative charge.
    2. A positive charge.
    3. A neutral charge.
    4. May be any of the above.
    5. None of the above.
3. What happens when opposite charges get close to each other?
    1. They repel each other.
    2. They attract each other.
    3. Nothing happens.
    4. They attract surrounding objects.
    5. They repel surrounding objects.
4. What is an electrical conductor?
    1. A material that allows electrons to travel through it freely.
    2. A material that doesn’t allow electrons to travel through it freely.
    3. A material that melts at low temperature.
    4. A material that creates free electrons.
    5. None of the above.
5. Which of the following is a good insulator of electricity?
    1. Copper
    2. Iron
    3. Rubber
    4. Salt water
    5. None of these.

Explore More

Use this resource to answer the questions that follow.

1. What will happen to Barbie's hair when the lady touches the Van de Graaff Generator?
2. Why do you think this happens to Barbie's hair?

Additional Resources

Study Guide: Electrostatics Study Guide

Video: Introduction to Electric Forces - Overview

Real World Application: Extreme Electrical Discharge

Interactive: lightning-rod

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11.3.2: Coulomb’s Law

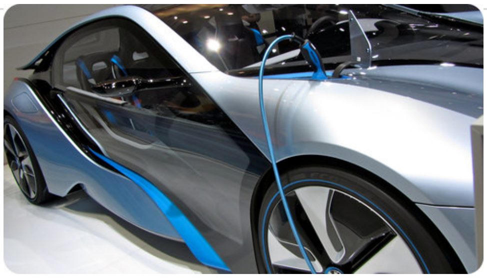

Figure 15.3.1

Electric cars are becoming more popular. One large advantage for electric cars is the low cost of operation, which may become an ever bigger advantage as gas prices climb. Energy costs for electric cars average about one-third of the cost for gasoline engine cars, but they can only travel about 200 miles per charge at this point. These cars run using the science of electrical charges and forces.

Coulomb’s Law

The questions regarding the relationship between the electrical force, the size of the charge, and the separation between the charges were solved by Charles Coulomb in 1785. He determined that electrical force between two charges is directly related to the size of the charges and inversely proportional to the distance between the charges. This is known as **Coulomb’s Law.**

$$F_e = \frac{kq_1q_2}{d^2}$$

In this equation, q_1 and q_2 are the two charges, d is the distance between the two charges, and k is a constant of proportionality. F_e is the **electric force**, which occurs as a result of interactions between two charged particles. For the purpose of calculating electric forces, we assume all charge is a **point charge**, in which the entire charge of the particle is located in a massless point.

The SI unit of charge is the coulomb, C, which is the charge of 6.25×1018 electrons. The charge on a single electron is 1.60×10^{-19} C. The charge on a single electron is known as the **elementary charge**. The charge on a proton is the same magnitude but opposite in sign. When the charges are measured in coulombs, the distance in meters, and the force in Newtons, the constant k is 9.0×10^{9} $\text{N}\cdot\text{m}^2/\text{C}^2$.

The electrical force, like all forces, is a vector quantity. If the two charges being considered are both positive or both negative, the sign of the electrical force is positive and this force is repulsive. If the two charges are opposite in sign, the force will have a negative sign and the force is attractive.

Examples

Example 15.3.1

Object A has a positive charge of $6.0 \times 10^{-6} C$. Object B has a positive charge of $3.0 \times 10^{-}6C$. If the distance between A and B is 0.030 m, what is the force on A?

Solution

\(F_e = \frac{kq_1q_2}{d^2} \) \(F_e=\frac{9.0×10^{9} N·m^{2}/C^{2} 6.0×10^{-6} C 3.0×10^{-6} C}{0.030 m^2} = 180 N \)

The positive sign of the force indicates the force is repulsive. This makes sense, because both objects have a positive charge.

✓ Example 15.3.2

In the sketch below, the charges are q1=10.0×10−6 C, q2=2.0×10−6 C, and q3=−6.0×10−6 C. Calculate the total force on q_2.

Example problem with three charged objects on a line

Figure 15.3.2

Solution

F_e=kq_1q_2/d^2=(9.0×109 N·m2/C2)(10.0×10−6 C)(2.0×10−6 C)(2.0 m)2=0.045 N (towards q_3)

F_e=kq_1q_3/d^2=(9.0×10^9 N·m^2/C^2)(2.0×10^{-6} C)(−6.0×10^{-6} C)/(4.0 m)^2=−0.007 N (towards q_3)

Since the two forces act in the same direction, their absolute values can be added together; the total force on q_2 is 0.052 N towards q_3.

Launch the Coulomb's Law simulation below. Start with adjusting the Configuration slider to a Hydrogen Atom, the Overlay slider to Electric Field and then Click to Draw the Electric Field. By simply clicking around the simulation screen, you can unveil the electric field vector at every point in space. These vector arrows are used to visualize the magnitude and direction of the electric force a positively charged object would feel if placed at that point. The longer the arrow, the greater the force. Have fun exploring:

Summary

- Coulomb determined that electrical force between two charges is directly related to the size of the charges and inversely proportional to the distance between the charges:

$$F_e = \frac{kq_1q_2}{d^2}$$

- The SI unit of charge is the coulomb, C, which is the charge of 6.25×10^{18} electrons.
- The charge on a single electron is 1.60×10^{-19} C and is known as the elementary charge.
- The electrical force is a vector quantity that is positive in repulsion and negative in attraction.

Review

1. Suppose that two point charges, each with a charge of +1.00 C, are separated by a distance of 1.0 m:
    1. Will the charges attract or repel?
    2. What is the magnitude of the force between them?
    3. If the distance between them is doubled, what does the force become?
2. What is the electrical force between two balloons, each having 5.00 C of charge, that are 0.300 m apart?
3. Two spheres are charged with the same charge of -0.0025 C and are separated by a distance of 8.00 m. What is the electrical force between them?
4. A red foam ball and a blue foam ball are 4.00 m apart. The blue ball has a charge of 0.000337 C and is attracting the red ball with a force of 626 N. What is the charge on the red ball?

Explore More

Use this resource to answer the questions that follow.

1. What happens when like charges are placed near each other?
2. What happens when opposite charged are placed near each other?
3. What happens to the force of attraction if the charges are placed closer together?

Additional Resources

Study Guide: Electrostatics Study Guide

Video: Coulomb's Law - Overview

Real World Application: Electrolytically Charged Sphere

PLIX: Play, Learn, Interact, eXplore: Charged Particles

11.3.3: Electric Fields

Plasma globes contain strong electrical fields

Figure 15.2.1

A plasma globe, such as the one pictured above, is filled with a mixture of noble gases and has a high-voltage electrode at the center. The swirling lines are electric discharge lines that connect from the inner electrode to the outer glass insulator. When a hand is placed on the surface of the globe, all the electric discharge travels directly to that hand.

The Electric Field

Coulomb’s Law gives us the formula to calculate the force exerted on a charge by another charge. On some occasions, however, a test charge suffers an electrical force with no apparent cause. That is, as observers, we cannot see or detect the original charge creating the electrical force. Michael Faraday dealt with this problem by developing the concept of an **electric field.** According to Faraday, a charge creates an electric field about it in all directions. If a second charge is placed at some point in the field, the second charge interacts with the field and experiences an electrical force. Thus, the interaction we observe is between the test charge and the field and a second particle at some distance is no longer necessary.

The strength of the electric field is determined point by point and can only be identified by the presence of test charge. When a positive test charge, q_t, is placed in an electric field, the field exerts a force on the charge. The field strength can be measured by dividing the force by the charge of the test charge. Electric field strength is given the symbol E and its unit is Newtons/coulomb.

$$E=F_{\text{on}q_t}/q_t$$

The test charge can be moved from location to location within the electric field until the entire electric field has been mapped in terms of **electric field intensity.**

✓ Example 11.3.3.1

A positive test charge of 2.0×10−5 C is placed in an electric field. The force on the test charge is 0.60 N. What is the electric field intensity at the location of the test charge?

Solution

$$E=F/q=0.60\text{ N}/2.0\times10^{-5}\text{ C}=3.0\times10^{4}\text{ N/C}$$

Launch the Hockey simulation below and try to use the electric field to help you score a goal:

Summary

- An electric field surrounds every charge and acts on other charges in the vicinity.
- The strength of the electric field is given by the symbol *E*, and has the unit of Newtons/coulomb.
- The equation for electric field intensity is E=F/q.

Review

1. The weight of a proton is 1.64×10^{-26} N. The charge on a proton is $+1.60\times10^{-19}$ C. If a proton is placed in a uniform electric field so that the electric force on the proton just balances its weight, what is the magnitude and direction of the field?
2. A negative charge of 2.0×10^{-8} C experiences a force of 0.060 N to the right in an electric field. What is the magnitude and direction of the field?
3. A positive charge of 5.0×10^{-4} C is in an electric field that exerts a force of 2.5×10^{-4} N on it. What is the magnitude of the electric field at the location of the charge?
4. If you determined the electric field intensity in a field using a test charge of 1.0×10^{-6} C and then repeated the process with a test charge of 2.0×10^{-6} C, would the forces on the charges be the same? Would you find the value for E?

Explore More

Use this resource to answer the questions that follow.

1. What does it mean when a force is called a non-contact force?
2. What symbol is used to represent electric field strength?
3. What is the relationship between the direction of the electric field and the direction of the electric force?

Additional Resources

Study Guide: Electrostatics Study Guide

Real World Application: Using Static, Homemade Static Electricity

Interactive: Touch Screen

Video:

11.3.4: Electric Field Lines

Learning Objectives

- Calculate the total force (magnitude and direction) exerted on a test charge from more than one charge
- Describe an electric field diagram of a positive point charge; of a negative point charge with twice the magnitude of positive charge
- Draw the electric field lines between two points of the same charge; between two points of opposite charge.

Drawings using lines to represent **electric fields** around charged objects are very useful in visualizing field strength and direction. Since the electric field has both magnitude and direction, it is a vector. Like all **vectors**, the electric field can be represented by an arrow that has length proportional to its magnitude and that points in the correct direction. (We have used arrows extensively to represent force vectors, for example.)

Figure 11.3.4.1 shows two pictorial representations of the same electric field created by a positive point charge Q. Figure 11.3.4.1(b) shows the standard representation using continuous lines. Figure 11.3.4.1(a) shows numerous individual arrows with each arrow representing the force on a test charge q.

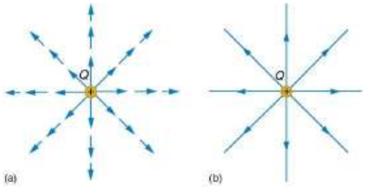

Figure 11.3.4.1: Two equivalent representations of the electric field due to a positive charge Q. (a) Arrows representing the electric field's magnitude and direction. (b) In the standard representation, the arrows are replaced by continuous field lines having the same direction at any point as the electric field. The closeness of the lines is directly related to the strength of the electric field. A test charge placed anywhere will feel a force in the direction of the field line; this force will have a strength proportional to the density of the lines (being greater near the charge, for example).

Note that the electric field is defined for a positive test charge q, so that the field lines point away from a positive charge and toward a negative charge. (See Figure 11.3.4.2) The electric field strength is exactly proportional to the number of field lines per unit area, since the magnitude of the electric field for a point charge is $E = k|Q|/r^2$ and area is proportional to r^2. This pictorial representation, in which field lines represent the direction and their closeness (that is, their areal density or the number of lines crossing a unit area) represents strength, is used for all fields: electrostatic, gravitational, magnetic, and others.

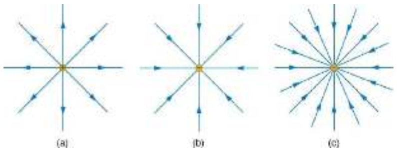

Figure 11.3.4.2: The electric field surrounding three different point charges. (a) A positive charge. (b) A negative charge of equal magnitude. (c) A larger negative charge.

In many situations, there are multiple charges. The total electric field created by multiple charges is the vector sum of the individual fields created by each charge.

Figure 11.3.4.3 shows how the electric field from two point charges can be drawn by finding the total field at representative points and drawing electric field lines consistent with those points. While the electric fields from multiple charges are more complex than those of single charges, some simple features are easily noticed.

For example, the field is weaker between like charges, as shown by the lines being farther apart in that region. (This is because the fields from each charge exert opposing forces on any charge placed between them.) (See Figure 11.3.4.3 and Figure 11.3.4.4(a).) Furthermore, at a great distance from two like charges, the field becomes identical to the field from a single, larger charge.

Figure 11.3.4.4(b) shows the electric field of two unlike charges. The field is stronger between the charges. In that region, the fields from each charge are in the same direction, and so their strengths add. The field of two unlike charges is weak at large distances, because the fields of the individual charges are in opposite directions and so their strengths subtract. At very large distances, the field of two unlike charges looks like that of a smaller single charge.

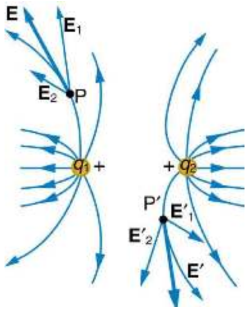

Figure 11.3.4.3: Two positive point charges q_1 and q_2 produce the resultant electric field shown. The field is calculated at representative points and then smooth field lines drawn following the rules outlined in the text.

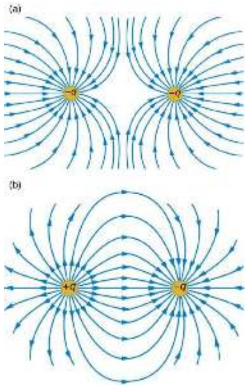

Figure 11.3.4.4: (a) Two negative charges produce the fields shown. It is very similar to the field produced by two positive charges, except that the directions are reversed. The field is clearly weaker between the charges. The individual forces on a test charge in that region are in opposite directions. (b) Two opposite charges produce the field shown, which is stronger in the region between the charges.

We use electric field lines to visualize and analyze electric fields (the lines are a pictorial tool, not a physical entity in themselves). The properties of electric field lines for any charge distribution can be summarized as follows:

1. Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
2. The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
3. The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
4. The direction of the electric field is tangent to the field line at any point in space.
5. Field lines can never cross.

The last property means that the field is unique at any point. The field line represents the direction of the field; so if they crossed, the field would have two directions at that location (an impossibility if the field is unique).

Section Summary

- Drawings of electric field lines are useful visual tools. The properties of electric field lines for any charge distribution are that:
- Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
- The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
- The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
- The direction of the electric field is tangent to the field line at any point in space.
- Field lines can never cross.

Glossary

electric field

a three-dimensional map of the electric force extended out into space from a point charge

electric field lines

a series of lines drawn from a point charge representing the magnitude and direction of force exerted by that charge

vector

a quantity with both magnitude and direction

vector addition

mathematical combination of two or more vectors, including their magnitudes, directions, and positions

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11.3.5: Electric Field- Concept of a Field Revisited

Learning Objectives

- Describe a force field and calculate the strength of an electric field due to a point charge.
- Calculate the force exerted on a test charge by an electric field.
- Explain the relationship between electrical force (F) on a test charge and electrical field strength (E).

Contact forces, such as between a baseball and a bat, are explained on the small scale by the interaction of the charges in atoms and molecules in close proximity. They interact through forces that include the **Coulomb force**. Action at a distance is a force between objects that are not close enough for their atoms to “touch.” That is, they are separated by more than a few atomic diameters.

For example, a charged rubber comb attracts neutral bits of paper from a distance via the Coulomb force. It is very useful to think of an object being surrounded in space by a **force field**. The force field carries the force to another object (called a test object) some distance away.

Concept of a Field

A field is a way of conceptualizing and mapping the force that surrounds any object and acts on another object at a distance without apparent physical connection. For example, the gravitational field surrounding the earth (and all other masses) represents the gravitational force that would be experienced if another mass were placed at a given point within the field.

In the same way, the Coulomb force field surrounding any charge extends throughout space. Using Coulomb’s law, $F = k|q_1 q_2|/r^2$, its magnitude is given by the equation $F = k|qQ|/r^2$, for a **point charge** (a particle having a charge Q) acting on a **test charge** q at a distance r (see Figure 11.3.5.1). Both the magnitude and direction of the Coulomb force field depend on Q and the test charge q.

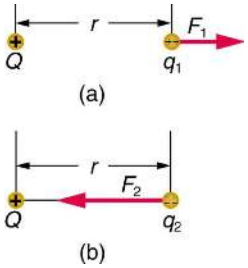

Figure 11.3.5.1: The Coulomb force field due to a positive charge Q is shown acting on two different charges. Both charges are the same distance from Q. (a) Since q_1 is positive, the force F_1 acting on it is repulsive. (b) The charge q_2 is negative and greater in magnitude than q_1, and so the force F_2 acting on it is attractive and stronger than F_1. The Coulomb force field is thus not unique at any point in space, because it depends on the test charges q_1 and q_2 as well as the charge Q.

To simplify things, we would prefer to have a field that depends only on Q and not on the test charge q. The electric field is defined in such a manner that it represents only the charge creating it and is unique at every point in space. Specifically, the electric field E is defined to be the ratio of the Coulomb force to the test charge:

$$\mathbf{E} = \frac{\mathbf{F}}{q},$$

where $\mathbf{F}$ is the electrostatic force (or Coulomb force) exerted on a positive test charge q. It is understood that $\mathbf{E}$ is in the same direction as $\mathbf{F}$. It is also assumed that q is so small that it does not alter the charge distribution creating the electric field. The units of electric field are newtons per coulomb (N/C). If the electric field is known, then the electrostatic force on any charge q is simply obtained by multiplying charge times electric field, or $\mathbf{F} = q\mathbf{E}$. Consider the electric field due to a point charge Q. According to Coulomb’s law, the force it exerts on a test charge q is $F = k|qQ|/r^2$. Thus the magnitude of the electric field, E, for a point charge is

$$E = \left|\frac{F}{q}\right| = k\left|\frac{qQ}{qr^2}\right| = k\frac{|Q|}{r^2}.$$

Since the test charge cancels, we see that

$$E=k\frac{|Q|}{r^2}.$$

The electric field is thus seen to depend only on the charge Q and the distance r; it is completely independent of the test charge q.

✓ Example 11.3.5.1: Calculating the Electric Field of a Point Charge

Calculate the strength and direction of the electric field E due to a point charge of 2.00 nC (nano-Coulombs) at a distance of 5.00 mm from the charge.

Strategy

We can find the electric field created by a point charge by using the equation $E=kQ/r^2$.

Solution

Here $Q=2.00\times10^{-9}$ C and $r=5.00\times10^{-3}$ m. Entering those values into the above equation gives

$$\begin{aligned} E &= k\frac{Q}{r^2} \\ &= \left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\times\frac{\left(2.00\times10^{-9}\mathrm{C}\right)}{\left(5.00\times10^{-3}\ \mathrm{m}\right)^2} \\ &= 7.19\times10^5\ \mathrm{N/C}. \end{aligned}$$

Discussion

This **electric field strength** is the same at any point 5.00 mm away from the charge Q that creates the field. It is positive, meaning that it has a direction pointing away from the charge Q.

✓ Example 11.3.5.2: Calculating the Force Exerted on a Point Charge by an Electric Field

What force does the electric field found in the previous example exert on a point charge of $-0.250\ \mu$C?

Strategy

Since we know the electric field strength and the charge in the field, the force on that charge can be calculated using the definition of electric field $\mathbf{E}=\mathbf{F}/q$ rearranged to $\mathbf{F}=q\mathbf{E}$.

Solution

The magnitude of the force on a charge $q=-0.250\ \mu$C exerted by a field of strength $E=7.20\times10^5$ N/C is thus,

$$\begin{aligned} F &= -qE \\ &= \left(0.250\times10^{-6}\mathrm{C}\right)\left(7.20\times10^5\ \mathrm{N/C}\right) \\ &= 0.180\ \mathrm{N}. \end{aligned}$$

Because q is negative, the force is directed opposite to the direction of the field.

Discussion

The force is attractive, as expected for unlike charges. (The field was created by a positive charge and here acts on a negative charge.) The charges in this example are typical of common static electricity, and the modest attractive force obtained is similar to forces experienced in static cling and similar situations.

Section Summary

- The electrostatic force field surrounding a charged object extends out into space in all directions.
- The electrostatic force exerted by a point charge on a test charge at a distance r depends on the charge of both charges, as well as the distance between the two.
- The electric field $\mathbf{E}$ is defined to be

$$\mathbf{E}=\frac{\mathbf{F}}{q},$$

where $\mathbf{F}$ is the Coulomb or electrostatic force exerted on a small positive test charge q. $\mathbf{E}$ has units of N/C.

- The magnitude of the electric field $\mathbf{E}$ created by a point charge Q is

$$\mathbf{E}=k\frac{|Q|}{r^2}.$$

where r is the distance from Q. The electric field $\mathbf{E}$ is a vector and fields due to multiple charges add like vectors.

Glossary

field

a map of the amount and direction of a force acting on other objects, extending out into space

point charge

A charged particle, designated Q, generating an electric field

test charge

A particle (designated q) with either a positive or negative charge set down within an electric field generated by a point charge

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11.3.6: Electric Potential and Potential Energy

Learning Objectives

- Define electric potential and electric potential energy.
- Describe the relationship between electric potential difference and electric field.
- Describe the relationship between electric potential and electrical potential energy.
- Explain electron volt and its usage in submicroscopic process.

When a free positive charge q is accelerated by an electric field, such as shown in Figure $11.3.6.1$, it is given kinetic energy. The process is analogous to an object being accelerated by a gravitational field. It is as if the charge is going down an electrical hill where its electric potential energy is converted to kinetic energy. Let us explore the work done on a charge q by the electric field in this process, so that we may develop a definition of electric potential energy.

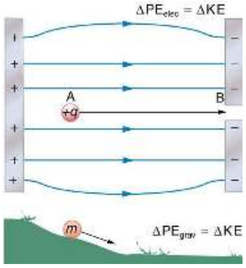

Figure 11.3.6.1: A charge accelerated by an electric field is analogous to a mass going down a hill. In both cases potential energy is converted to another form. Work is done by a force, but since this force is conservative, we can write $W = -\Delta\text{PE}$.

The electrostatic or Coulomb force is conservative, which means that the work done on q is independent of the path taken. This is exactly analogous to the gravitational force in the absence of dissipative forces such as friction. When a force is conservative, it is possible to define a potential energy associated with the force, and it is usually easier to deal with the potential energy (because it depends only on position) than to calculate the work done directly from force ($W = \langle \boldsymbol{F}, \boldsymbol{d} \rangle$, where d is displacement and F is the force).

We use the letters PE to denote electric potential energy, which has units of joules (J). The change in potential energy, $\triangle\text{PE}$, is crucial, since the work done by a conservative force is the negative of the change in potential energy; that is, $W = -\Delta\text{PE}$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative $\triangle\text{PE}$. There must be a minus sign in front of $\triangle\text{PE}$ to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Definition: POTENTIAL ENERGY

$W = -\Delta\text{PE}$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative ΔPE. There must be a minus sign in front of ΔPE to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Gravitational potential energy and electric potential energy are quite analogous. Potential energy accounts for work done by a conservative force and gives added insight regarding energy and energy transformation without the necessity of dealing with the force directly. It is much more common, for example, to use the concept of voltage (related to electric potential energy) than to deal with the electric field (related to Coulomb force) directly.

Given some conservative force F and displacement d under the force, the work done and the change in potential energy can be calculated as, $W = \langle \boldsymbol{F}, \boldsymbol{d} \rangle$ and $\Delta\text{PE} = -\text{W} = \langle -\mathbf{F}, \boldsymbol{d} \rangle$. For electric force, the force is given by the product of electric charge and the electric field, $\boldsymbol{F} = q\boldsymbol{E}$, where q is the charge experiencing the force and E is the electric field at the location of the charge. So the potential energy change due to work done by electric force is $\Delta\text{PE} = q(\langle -\mathbf{E}, \boldsymbol{d} \rangle)$. If we define change in **electric potential** V as $\Delta V = \langle -\mathbf{E}, \boldsymbol{d} \rangle$, then the electric potential energy PEPE is simply expressed in terms of electric potential, $\text{PE} = qV$, or,

$$V = \frac{\text{PE}}{q},$$

electric potential energy per charge.

Definition: ELECTRIC POTENTIAL

Electric potential is the electric potential energy per unit charge.

$$V = \frac{\text{PE}}{q}$$

With potential energy, the case often is that its value at a single point has no significant meaning but what is important is the difference in potential energy. From the difference in potential energy, we are able to calculate other quantities, such as change in kinetic energy (if no force other than the conservative force acts) or work needing to be done by other forces (if other forces act). So likewise, rather than the electric potential itself, we are often interested in difference in electric potential ΔV between two points, where,

$$\Delta V = V_{\text{B}} - V_{\text{A}} = \frac{\Delta \text{PE}}{q}.$$

The **potential difference** between points A and B, $V_{\text{B}} - V_{\text{A}}$, is thus defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$1 \text{ V} = 1 \frac{\text{J}}{\text{C}}$$

Definition: POTENTIAL DIFFERENCE

The potential difference between points A and B, $V_{\text{B}} - V_{\text{A}}$, is defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$1 \text{ V} = 1 \frac{\text{J}}{\text{C}}$$

The familiar term **voltage** is the common name for potential difference. Keep in mind that whenever a voltage is quoted, it is understood to be the potential difference between two points. For example, every battery has two terminals, and its voltage is the potential difference between them. More fundamentally, the point you choose to be zero volts is arbitrary. This is analogous to the fact that gravitational potential energy has an arbitrary zero, such as sea level or perhaps a lecture hall floor.

In summary, the relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = \frac{\Delta \text{PE}}{q} \text{ and } \Delta \text{PE} = q \Delta V.$$

POTENTIAL DIFFERENCE AND ELECTRICAL POTENTIAL ENERGY

The relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = \frac{\Delta \text{PE}}{q} \text{ and } \Delta \text{PE} = q \Delta V.$$

The second equation is equivalent to the first.

Voltage is not the same as energy. Voltage is the energy per unit charge. Thus a motorcycle battery and a car battery can both have the same voltage (more precisely, the same potential difference between battery terminals), yet one stores much more energy than the other since $\Delta \text{PE} = q \Delta V$. The car battery can move more charge than the motorcycle battery, although both are 12 V batteries.

Example 11.3.6.1: Calculating Energy

Suppose you have a 12.0 V motorcycle battery that can move 5000 C of charge, and a 12.0 V car battery that can move 60,000 C of charge. How much energy does each deliver? (Assume that the numerical value of each charge is accurate to three significant figures.)

Strategy

To say we have a 12.0 V battery means that its terminals have a 12.0 V potential difference. When such a battery moves charge, it puts the charge through a potential difference of 12.0 V, and the charge is given a change in potential energy equal to $\Delta \mathrm{PE} = q\Delta V$.

So to find the energy output, we multiply the charge moved by the potential difference.

Solution

For the motorcycle battery, $q = 5000\ \mathrm{C}$ and $\Delta V = 12.0\ \mathrm{V}$. The total energy delivered by the motorcycle battery is

$$\begin{aligned} \Delta \mathrm{PE}_{\text{cycle}} &= (5000\mathrm{C})(12.0\ \mathrm{V}) \\ &= (5000\mathrm{C})(12.0\ \mathrm{J/C}) \\ &= 6.00 \times 10^4\ \mathrm{J}. \end{aligned}$$

Similarly, for the car battery, $q = 60,000\ \mathrm{C}$ and

$$\begin{aligned} \Delta \mathrm{PE}_{\text{car}} &= (60,000\mathrm{C})(12.0\ \mathrm{V}) \\ &= 7.20 \times 10^5\ \mathrm{J}. \end{aligned}$$

Discussion

While voltage and energy are related, they are not the same thing. The voltages of the batteries are identical, but the energy supplied by each is quite different. Note also that as a battery is discharged, some of its energy is used internally and its terminal voltage drops, such as when headlights dim because of a low car battery. The energy supplied by the battery is still calculated as in this example, but not all of the energy is available for external use.

Note that the energies calculated in the previous example are absolute values. The change in potential energy for the battery is negative, since it loses energy. These batteries, like many electrical systems, actually move negative charge—electrons in particular. The batteries repel electrons from their negative terminals (A) through whatever circuitry is involved and attract them to their positive terminals (B) as shown in Figure 11.3.6.2 The change in potential is $\Delta V = V_{\mathrm{B}} - V_{\mathrm{A}} = +12\ \mathrm{V}$ and the charge q is negative, so that $\Delta \mathrm{PE} = q\Delta V$ is negative, meaning the potential energy of the battery has decreased when q has moved from A to B.

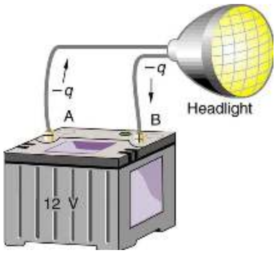

Figure 11.3.6.2: A battery moves negative charge from its negative terminal through a headlight to its positive terminal. Appropriate combinations of chemicals in the battery separate charges so that the negative terminal has an excess of negative charge, which is repelled by it and attracted to the excess positive charge on the other terminal. In terms of potential, the positive terminal is at a higher voltage than the negative. Inside the battery, both positive and negative charges move.

Example 11.3.6.2: How Many Electrons Move through a Headlight Each Second?

When a 12.0 V car battery runs a single 30.0 W headlight, how many electrons pass through it each second?

Strategy

To find the number of electrons, we must first find the charge that moved in 1.00 s. The charge moved is related to voltage and energy through the equation $\Delta \mathrm{PE} = q\Delta V$. A 30.0 W lamp uses 30.0 joules per second. Since the battery loses energy, we have $\Delta \mathrm{PE} = -30.0\ \mathrm{J}$ and, since the electrons are going from the negative terminal to the positive, we see that $\Delta V = +12.0\ \mathrm{V}$.

Solution

To find the charge q moved, we solve the equation $\Delta \mathrm{PE} = q\Delta V$:

$$q = \frac{\Delta \mathrm{PE}}{\Delta V}.$$

Entering the values for $\triangle \mathrm{PE}$ and ΔV, we get

$$q = \frac{-30.0\ \mathrm{J}}{+12.0\ \mathrm{V}} = \frac{-30.0\ \mathrm{J}}{+12.0\ \mathrm{J/C}} = -2.50\mathrm{C}.$$

The number of electrons $\mathrm{n_e}$ is the total charge divided by the charge per electron. That is,

$$\mathrm{n_e} = \frac{-2.50\mathrm{C}}{-1.60 \times 10^{-19}\mathrm{C/e^-}} = 1.56 \times 10^{19}\ \text{electrons}.$$

Discussion

This is a very large number. It is no wonder that we do not ordinarily observe individual electrons with so many being present in ordinary systems. In fact, electricity had been in use for many decades before it was determined that the moving charges in many circumstances were negative. Positive charge moving in the opposite direction of negative charge often produces identical effects; this makes it difficult to determine which is moving or whether both are moving.

The Electron Volt

The energy per electron is very small in macroscopic situations like that in the previous example—a tiny fraction of a joule. But on a submicroscopic scale, such energy per particle (electron, proton, or ion) can be of great importance. For example, even a tiny fraction of a joule can be great enough for these particles to destroy organic molecules and harm living tissue. The particle may do its damage by direct collision, or it may create harmful X-rays, which can also inflict damage. It is useful to have an energy unit related to submicroscopic effects. Figure $11.3.6.3$ shows a situation related to the definition of such an energy unit. An electron is accelerated between two charged metal plates as it might be in an old-model television tube or oscilloscope. The electron is given kinetic energy that is later converted to another form—light in the television tube, for example. (Note that downhill for the electron is uphill for a positive charge.) Since energy is related to voltage by $\Delta \mathrm{PE} = q\Delta V$, we can think of the joule as a coulomb-volt.

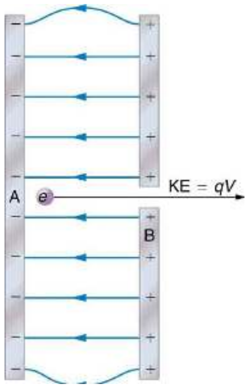

Figure 11.3.6.3: A typical electron gun accelerates electrons using a potential difference between two metal plates. The energy of the electron in electron volts is numerically the same as the voltage between the plates. For example, a 5000 V potential difference produces 5000 eV electrons.

Definition: ELECTRON VOLT

On the submicroscopic scale, it is more convenient to define an energy unit called the electron volt (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$\begin{aligned} 1\mathrm{eV} &= \left(1.60 \times 10^{-19}\mathrm{C}\right)(1\ \mathrm{V}) = \left(1.60 \times 10^{-19}\mathrm{C}\right)(1\ \mathrm{J/C}) \\ &= 1.60 \times 10^{-19}\ \mathrm{J}. \end{aligned}$$

An electron accelerated through a potential difference of 1 V is given an energy of 1 eV. It follows that an electron accelerated through 50 V is given 50 eV. A potential difference of 100,000 V (100 kV) will give an electron an energy of 100,000 eV (100 keV), and so on. Similarly, an ion with a double positive charge accelerated through 100 V will be given 200 eV of energy. These simple relationships between accelerating voltage and particle charges make the electron volt a simple and convenient energy unit in such circumstances.

CONNECTIONS: ENERGY UNITS

The electron volt (eV) is the most common energy unit for submicroscopic processes. This will be particularly noticeable in the chapters on modern physics. Energy is so important to so many subjects that there is a tendency to define a special energy unit for each major topic. There are, for example, calories for food energy, and kilowatt-hours for electrical energy.

The electron volt is commonly employed in submicroscopic processes—chemical valence energies and molecular and nuclear binding energies are among the quantities often expressed in electron volts. For example, about 5 eV of energy is required to break up certain organic molecules. If a proton is accelerated from rest through a potential difference of 30 kV, it is given an energy of 30 keV (30,000 eV) and it can break up as many as 6000 of these molecules ($30,000 \text{ eV} \div 5\text{eV per molecule } = 6000 \text{ molecules }$). Nuclear decay energies are on the order of 1 MeV (1,000,000 eV) per event and can, thus, produce significant biological damage.

Conservation of Energy

The total energy of a system is conserved if there is no net addition (or subtraction) of work or heat transfer. For conservative forces, such as the electrostatic force, conservation of energy states that mechanical energy is a constant.

Mechanical energy is the sum of the kinetic energy and potential energy of a system; that is, $\mathrm{KE}+\mathrm{PE}= \text{ constant }$. A loss of PE of a charged particle becomes an increase in its KE. Here PE is the electric potential energy. Conservation of energy is stated in equation form as

$$\mathrm{KE}+\mathrm{PE}= \text{ constant}$$

or

$$\mathrm{KE_i}+\mathrm{PE_i}=\mathrm{KE_f}+\mathrm{PE_f},$$

where i and f stand for initial and final conditions. As we have found many times before, considering energy can give us insights and facilitate problem solving.

Example 11.3.6.3: Electrical Potential Energy Converted to Kinetic Energy

Calculate the final speed of a free electron accelerated from rest through a potential difference of 100 V. (Assume that this numerical value is accurate to three significant figures.)

Strategy

We have a system with only conservative forces. Assuming the electron is accelerated in a vacuum, and neglecting the gravitational force (we will check on this assumption later), all of the electrical potential energy is converted into kinetic energy. We can identify the initial and final forms of energy to be $\mathrm{KE_i}=0, \mathrm{KE_f}=1/2mv^2, \mathrm{PE_i}=qV, \text{ and } \mathrm{PE_f}=0$.

Solution

Conservation of energy states that

$$\mathrm{KE_i}+\mathrm{PE_i}=\mathrm{KE_f}+\mathrm{PE_f}.$$

Entering the forms identified above, we obtain

$$qV=\frac{mv^2}{2}.$$

We solve this for v:

$$v=\sqrt{\frac{2qV}{m}}.$$

Entering values for q,V, and m gives

$$\begin{aligned}v&=\sqrt{\frac{2\left(-1.60\times10^{-19}\mathrm{C}\right)(-100\ \mathrm{J/C})}{9.11\times10^{-31}\ \mathrm{kg}}}\\&=5.93\times10^{6}\ \mathrm{m/s}.\end{aligned}$$

Discussion

Note that both the charge and the initial voltage are negative, as in Figure $11.3.6.3$ We know that electrostatic forces on small particles are generally very large compared with the gravitational force. The large final speed confirms that the gravitational force is indeed negligible here. The large speed also indicates how easy it is to accelerate electrons with small voltages because of their very small mass. Voltages much higher than the 100 V in this problem are typically used in electron guns.

Section Summary

- Electric potential is potential energy per unit charge.
- The potential difference between points A and B, $V_{\mathrm{B}}-V_{\mathrm{A}}$, defined to be the change in potential energy of a charge q moved from A to B, is equal to the change in potential energy divided by the charge, Potential difference is commonly called voltage, represented by the symbol ΔV.

$$.\ \Delta V=\frac{\Delta\mathrm{PE}}{q}\text{ and }\Delta\mathrm{PE}=q\Delta V$$

- An electron volt is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$\begin{aligned}1\mathrm{eV}&=\left(1.60\times10^{-19}\mathrm{C}\right)(1\ \mathrm{V})=\left(1.60\times10^{-19}\mathrm{C}\right)(1\ \mathrm{J/C})\\&=1.60\times10^{-19}\ \mathrm{J}.\end{aligned}$$

- Mechanical energy is the sum of the kinetic energy and potential energy of a system, that is, $\mathrm{KE}+\mathrm{PE}$ This sum is a constant.

Glossary

electric potential

potential energy per unit charge

potential difference (or voltage)

change in potential energy of a charge moved from one point to another, divided by the charge; units of potential difference are joules per coulomb, known as volt

electron volt

the energy given to a fundamental charge accelerated through a potential difference of one volt

mechanical energy

sum of the kinetic energy and potential energy of a system; this sum is a constant

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11.3.7: Conductors and Applications of Electrostatics

Learning Objectives

- List the three properties of a conductor in electrostatic equilibrium.
- Explain why the electric field is zero inside a conductor in electrostatic equilibrium.
- Describe how a lightning rod works.
- Name several real-world applications of the study of electrostatics.

Conductors contain **free charges** that move easily. When excess charge is placed on a conductor or the conductor is put into a static electric field, charges in the conductor quickly respond to reach a steady state called **electrostatic equilibrium**.

Figure 11.3.7.1 shows the effect of an electric field on free charges in a conductor. The free charges move until the field is perpendicular to the conductor's surface. There can be no component of the field parallel to the surface in electrostatic equilibrium, since, if there were, it would produce further movement of charge.

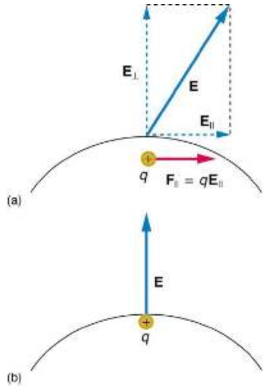

Figure 11.3.7.1: When an electric field $\mathbf{E}$ is applied to a conductor, free charges inside the conductor move until the field is perpendicular to the surface. (a) The electric field is a vector quantity, with both parallel and perpendicular components. The parallel component ($\mathbf{E}_\parallel$) exerts a force ($\mathbf{F}_\parallel$) on the free charge q, which moves the charge until $\mathbf{F}_\parallel = 0$. (b) The resulting field is perpendicular to the surface. The free charge has been brought to the conductor's surface, leaving electrostatic forces in equilibrium. A positive free charge is shown in the figure for the example, but the result holds similarly for a negative free charge.

A conductor placed in an **electric field** will be **polarized**. Figure 11.3.7.2 shows the result of placing a neutral conductor in an originally uniform electric field. The field becomes stronger near the conductor but entirely disappears inside it.

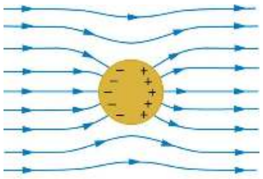

Figure 11.3.7.2: This illustration shows a spherical conductor in static equilibrium with an originally uniform electric field. Free charges move within the conductor, polarizing it, until the electric field lines are perpendicular to the surface. The field lines end on excess negative charge on one section of the surface and begin again on excess positive charge on the opposite side. No electric field exists inside the conductor, since free charges in the conductor would continue moving in response to any field until it was neutralized.

MISCONCEPTION ALERT: ELECTRIC FIELD INSIDE A CONDUCTOR

Excess charges placed on a spherical conductor repel and move until they are evenly distributed, as shown in Figure 11.3.7.3. Excess charge is forced to the surface until the field inside the conductor is zero. Outside the conductor, the field is exactly the same as if the conductor were replaced by a point charge at its center equal to the excess charge.

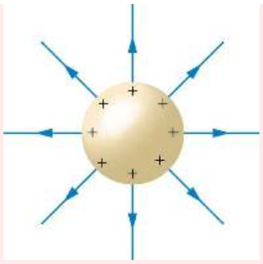

Figure 11.3.7.3: The mutual repulsion of excess positive charges on a spherical conductor distributes them uniformly on its surface. The resulting electric field is perpendicular to the surface and zero inside. Outside the conductor, the field is identical to that of a point charge at the center equal to the excess charge.

PROPERTIES OF A CONDUCTOR IN ELECTROSTATIC EQUILIBRIUM

1. The electric field is zero inside a conductor.
2. Just outside a conductor, the electric field lines are perpendicular to its surface, ending or beginning on charges on the surface.
3. Any excess charge resides entirely on the surface or surfaces of a conductor.

The properties of a conductor are consistent with the situations already discussed and can be used to analyze any conductor in electrostatic equilibrium. This can lead to some interesting new insights, such as described below.

How can a very uniform electric field be created? Consider a system of two metal plates with opposite charges on them, as shown in Figure 11.3.7.4 The properties of conductors in electrostatic equilibrium indicate that the electric field between the plates will be uniform in strength and direction. Except near the edges (which can be ignored when plates are close to each other), the excess charges distribute themselves uniformly, producing field lines that are uniformly spaced (hence uniform in strength) and perpendicular to the surfaces (hence uniform in direction, since the plates are flat).

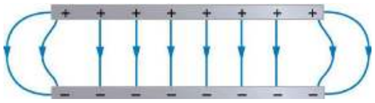

Figure 11.3.7.4: Two metal plates with equal, but opposite, excess charges. The field between them is uniform in strength and direction except near the edges. One use of such a field is to produce uniform acceleration of charges between the plates, such as in the electron gun of a TV tube.

In fact, this is a particular example of electric circuit elements called **capacitors**. This particular example is called a **parallel-plate capacitor** and can be used to analyze properties of capacitors in general, such as how many charges can be stored using two nearby conductors set at a certain electric potential difference (voltage) apart. Capacitors find wide use in electric circuits alongside the resistors and inductors that you will learn about later.

Electric Fields on Uneven Surfaces

So far we have considered excess charges on a smooth, symmetrical conductor surface. What happens if a conductor has sharp corners or is pointed? Excess charges on a nonuniform conductor become concentrated at the sharpest points. Additionally, excess charge may move on or off the conductor at the sharpest points. To see how and why this happens, consider the charged conductor in Figure 11.3.7.5 The electrostatic repulsion of like charges is most effective in moving them apart on the flattest surface, and so they become least concentrated there. The same effect is produced on a conductor by an externally applied electric field, as seen in Figure 11.3.7.5(c). Since the field lines must be perpendicular to the surface, more of them are concentrated on the most curved parts.

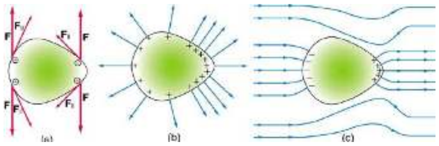

Figure 11.3.7.5: Excess charge on a nonuniform conductor becomes most concentrated at the location of greatest curvature. (a) The forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surface are different. It is $\mathbf{F}_{\parallel}$ that moves the charges apart once they have reached the surface. (b) $\mathbf{F}_{\parallel}$ is smallest at the more pointed end, the charges are left closer together, producing the electric field shown. (c) An uncharged conductor in an originally uniform electric field is polarized, with the most concentrated charge at its most pointed end.

Applications of Conductors

On a very sharply curved surface, such as shown in Figure $11.3.7.6$, the charges are so concentrated at the point that the resulting electric field can be great enough to remove them from the surface. This can be useful.

Lightning rods work best when they are most pointed. The large charges created in storm clouds induce an opposite charge on a building that can result in a lightning bolt hitting the building. The induced charge is bled away continually by a lightning rod, preventing the more dramatic lightning strike.

Of course, we sometimes wish to prevent the transfer of charge rather than to facilitate it. In that case, the conductor should be very smooth and have as large a radius of curvature as possible (see Figure $11.3.7.7$, which shows a Van de Graaff generator). Smooth surfaces are used on high-voltage transmission lines, for example, to avoid leakage of charge into the air.

Another device that makes use of some of these principles is a **Faraday cage**. This is a metal shield that encloses a volume. All electrical charges will reside on the outside surface of this shield, and there will be no electrical field inside. A Faraday cage is used to prohibit stray electrical fields in the environment from interfering with sensitive measurements, such as the electrical signals inside a nerve cell.

During electrical storms if you are driving a car, it is best to stay inside the car as its metal body acts as a Faraday cage with zero electrical field inside. If in the vicinity of a lightning strike, its effect is felt on the outside of the car and the inside is unaffected, provided you remain totally inside. This is also true if an active ("hot") electrical wire was broken (in a storm or an accident) and fell on your car.

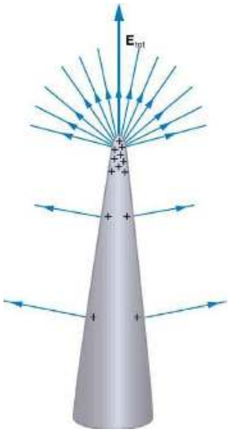

Figure 11.3.7.6: A very pointed conductor has a large charge concentration at the point. The electric field is very strong at the point and can exert a force large enough to transfer charge on or off the conductor. Lightning rods are used to prevent the buildup of large excess charges on structures and, thus, are pointed.

Figure 11.3.7.7: (a) A lightning rod is pointed to facilitate the transfer of charge. (credit: Romaine, Wikimedia Commons) (b) This Van de Graaff generator has a smooth surface with a large radius of curvature to prevent the transfer of charge and allow a large voltage to be generated. The mutual repulsion of like charges is evident in the person's hair while touching the metal sphere. (credit: Jon 'ShakataGaNai' Davis/Wikimedia Commons).

Below are additional applications of **electrostatics**, the study of electricity in electrostatic equilibrium, many of them using conductors for their properties just discussed above.

The Van de Graaff Generator

Van de Graaff generators (or Van de Graaffs) are not only spectacular devices used to demonstrate high voltage due to static electricity—they are also used for serious research. The first was built by Robert Van de Graaff in 1931 (based on original suggestions by Lord Kelvin) for use in nuclear physics research. Figure 11.3.7.8 shows a schematic of a large research version. Van de Graaffs utilize both smooth and pointed surfaces, and conductors and insulators to generate large static charges and, hence, large voltages.

A very large excess charge can be deposited on the sphere, because it moves quickly to the outer surface. Practical limits arise because the large electric fields polarize and eventually ionize surrounding materials, creating free charges that neutralize excess charge or allow it to escape. Nevertheless, voltages of 15 million volts are well within practical limits.

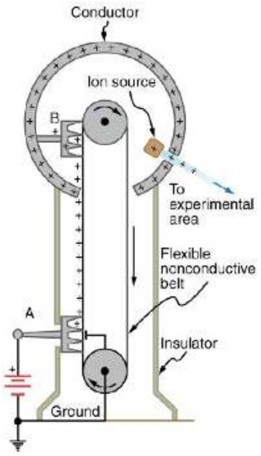

Figure 11.3.7.8: Schematic of Van de Graaff generator. A battery (A) supplies excess positive charge to a pointed conductor, the points of which spray the charge onto a moving insulating belt near the bottom. The pointed conductor (B) on top in the large sphere picks up the charge. (The induced electric field at the points is so large that it removes the charge from the belt.) This can be done because the charge does not remain inside the conducting sphere but moves to its outside surface. An ion source inside the sphere produces positive ions, which are accelerated away from the positive sphere to high velocities.

TAKE-HOME EXPERIMENT: ELECTROSTATICS AND HUMIDITY

Rub a comb through your hair and use it to lift pieces of paper. It may help to tear the pieces of paper rather than cut them neatly. Repeat the exercise in your bathroom after you have had a long shower and the air in the bathroom is moist. Is it easier to get electrostatic effects in dry or moist air? Why would torn paper be more attractive to the comb than cut paper? Explain your observations.

Xerography

Most copy machines use an electrostatic process called **xerography**—a word coined from the Greek words *xeros* for dry and *graphos* for writing. The heart of the process is shown in simplified form in Figure 11.3.7.9.

A selenium-coated aluminum drum is sprayed with positive charge from points on a device called a corotron. Selenium is a substance with an interesting property—it is a **photoconductor**. That is, selenium is an insulator when in the dark and a conductor when exposed to light.

In the first stage of the xerography process, the conducting aluminum drum is **grounded** so that a negative charge is induced under the thin layer of uniformly positively charged selenium. In the second stage, the surface of the drum is exposed to the image of whatever is to be copied. Where the image is light, the selenium becomes conducting, and the positive charge is neutralized. In dark areas, the positive charge remains, and so the image has been transferred to the drum.

The third stage takes a dry black powder, called toner, and sprays it with a negative charge so that it will be attracted to the positive regions of the drum. Next, a blank piece of paper is given a greater positive charge than on the drum so that it will pull the toner from the drum. Finally, the paper and electrostatically held toner are passed through heated pressure rollers, which melt and permanently adhere the toner within the fibers of the paper.

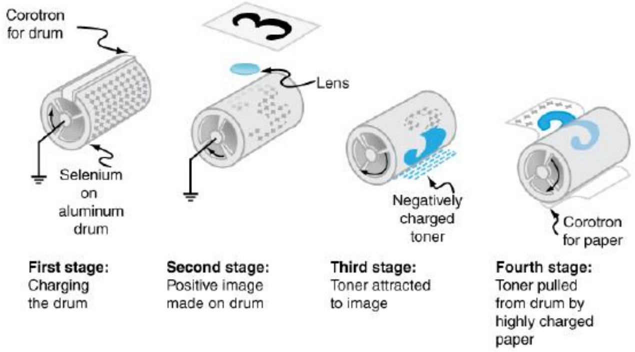

Figure 11.3.7.9: Xerography is a dry copying process based on electrostatics. The major steps in the process are the charging of the photoconducting drum, transfer of an image creating a positive charge duplicate, attraction of toner to the charged parts of the drum, and transfer of toner to the paper. Not shown are heat treatment of the paper and cleansing of the drum for the next copy.

Laser Printers

Laser printers use the xerographic process to make high-quality images on paper, employing a laser to produce an image on the photoconducting drum as shown in Figure 11.3.7.10. In its most common application, the laser printer receives output from a computer, and it can achieve high-quality output because of the precision with which laser light can be controlled. Many laser printers do significant information processing, such as making sophisticated letters or fonts, and may contain a computer more powerful than the one giving them the raw data to be printed.

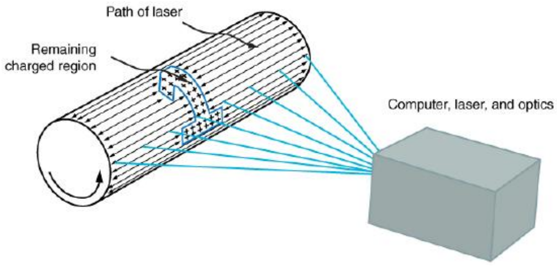

Figure 11.3.7.10: In a laser printer, a laser beam is scanned across a photoconducting drum, leaving a positive charge image. The other steps for charging the drum and transferring the image to paper are the same as in xerography. Laser light can be very precisely controlled, enabling laser printers to produce high-quality images.

Ink Jet Printers and Electrostatic Painting

The **ink jet printer**, commonly used to print computer-generated text and graphics, also employs electrostatics. A nozzle makes a fine spray of tiny ink droplets, which are then given an electrostatic charge. (See Figure 11.3.7.11)

Once charged, the droplets can be directed, using pairs of charged plates, with great precision to form letters and images on paper. Ink jet printers can produce color images by using a black jet and three other jets with primary colors, usually cyan, magenta, and yellow, much as a color television produces color. (This is more difficult with xerography, requiring multiple drums and toners.)

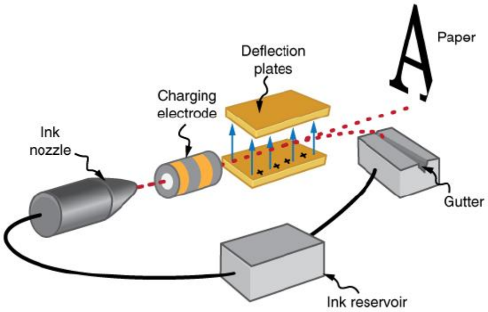

Figure 11.3.7.11: The nozzle of an ink-jet printer produces small ink droplets, which are sprayed with electrostatic charge. Various computer-driven devices are then used to direct the droplets to the correct positions on a page.

Electrostatic painting employs electrostatic charge to spray paint onto odd-shaped surfaces. Mutual repulsion of like charges causes the paint to fly away from its source. Surface tension forms drops, which are then attracted by unlike charges to the surface to be painted. Electrostatic painting can reach those hard-to-get at places, applying an even coat in a controlled manner. If the object is a conductor, the electric field is perpendicular to the surface, tending to bring the drops in perpendicularly. Corners and points on conductors will receive extra paint. Felt can similarly be applied.

Smoke Precipitators and Electrostatic Air Cleaning

Another important application of electrostatics is found in air cleaners, both large and small. The electrostatic part of the process places excess (usually positive) charge on smoke, dust, pollen, and other particles in the air and then passes the air through an oppositely charged grid that attracts and retains the charged particles. (See Figure 11.3.7.12)

Large **electrostatic precipitators** are used industrially to remove over 99% of the particles from stack gas emissions associated with the burning of coal and oil. Home precipitators, often in conjunction with the home heating and air conditioning system, are very effective in removing polluting particles, irritants, and allergens.

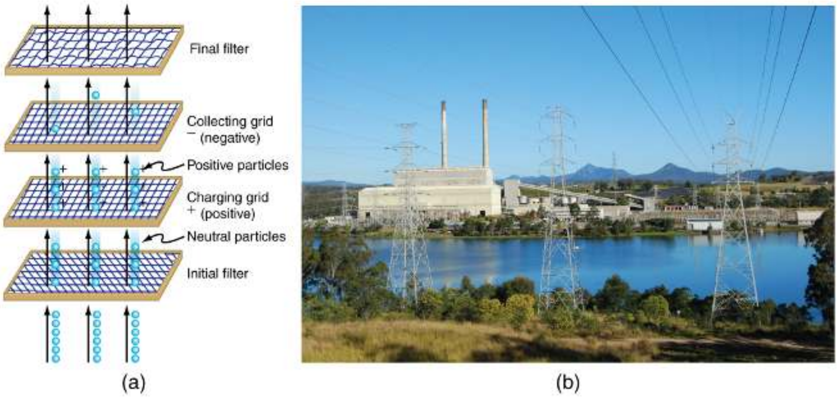

Figure 11.3.7.12: (a) Schematic of an electrostatic precipitator. Air is passed through grids of opposite charge. The first grid charges airborne particles, while the second attracts and collects them. (b) The dramatic effect of electrostatic precipitators is seen by the absence of smoke from this power plant. (credit: Cmdalgleish, Wikimedia Commons)

Section Summary

- The electrical forces around a conductor will cause free charges to move around inside the conductor until static equilibrium is reached.
- Any excess charge will collect along the surface of a conductor.
- Conductors with sharp corners or points will collect more charge at those points.
- A lightning rod is a conductor with sharply pointed ends that collect excess charge on the building caused by an electrical storm and allow it to dissipate back into the air.
- A Faraday cage acts like a shield around an object, preventing electric charge from penetrating inside.

- In addition to research using equipment such as a Van de Graaff generator, many practical applications of electrostatics exist, including photocopiers, laser printers, ink-jet printers and electrostatic air filters.

Glossary

conductor

an object with properties that allow charges to move about freely within it

free charge

an electrical charge (either positive or negative) which can move about separately from its base molecule

electrostatic equilibrium

an electrostatically balanced state in which all free electrical charges have stopped moving about

polarized

a state in which the positive and negative charges within an object have collected in separate locations

Faraday cage

a metal shield which prevents electric charge from penetrating its surface

capacitor

an arrangement of conductors designed to store charge using voltage difference

parallel-plate capacitor

an example of capacitor using arrangement of two parallel conducting plates placed near each other

Van de Graaff generator

a machine that produces a large amount of excess charge, used for experiments with high voltage

electrostatics

the study of electricity in electrostatic equilibrium

photoconductor

a substance that is an insulator until it is exposed to light, when it becomes a conductor

xerography

a dry copying process based on electrostatics

grounded

connected to the ground with a conductor, so that charge flows freely to and from the Earth to the grounded object

laser printer

uses a laser to create a photoconductive image on a drum, which attracts dry ink particles that are then rolled onto a sheet of paper to print a high-quality copy of the image

ink-jet printer

small ink droplets sprayed with an electric charge are controlled by electrostatic plates to create images on paper

electrostatic precipitators

filters that apply charges to particles in the air, then attract those charges to a filter, removing them from the airstream

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SECTION OVERVIEW

11.4: Electric Current and Resistance

11.4.1: Voltage (Electric Potential)

Figure 16.1.1

Often the star of science shows and museum installments, Tesla coils create magnificent displays of light and sound. They have low electrical current but incredibly large electric potentials.

The Electric Potential in a Uniform Field

Electric Potential

In the study of mechanics, the concept of energy, and the conservation thereof, was extremely useful. The same will be true for the study of electrical interactions. The work done moving a charged particle in an electric field can result in the particle gaining or losing both kinetic and potential energy.

Lifting an object in a gravitational field requires work and increases the object's potential energy. A similar situation occurs when you move two charged objects relative to each other. We already know that each object has an electric field surrounding it, which effects the other charge. If the two charged objects have the same charge, they repel each other. Moving these two objects closer to each other requires working against the repulsive force, which increases the potential energy of the system. Conversely, moving two like charges apart will decrease the potential energy. If the objects attract each other, the opposite situations occur: if you pull them apart, you do work against the force, which increases the potential energy of the system, but bringing attractive charges closer together decreases the potential energy.

It is often easy to think of the change in energy as a mountain or an inverted cone, depending on the situation. Imagine a positive point charge, with the corresponding electric field around it. If you are moving another positive point charge, the situation is like a mountain, with the original point charge at the peak. As the second charge moves towards the first, it must go up the mountain. This requires work, and increases the potential energy of the system. If, however, the second charge is a negative point charge, the two charges attract each other and the situation is like a cone: the second charge easily falls towards the first, decreasing the potential energy. It would then require work to get the second charge up and out of the cone, away from the first charge.

Uniform Electric Fields

As we know from Coulomb's law, the electric field around a point charge decreases as the distance from the point increases. However, it is possible to create a constant electric field between two large, flat conducting plates parallel to each other. If one of the plates is positively charged and the other negatively charged, the space between the plates will have a constant electric field except near the edges of the plates.

Diagram of a uniform electric field created from two uniformly charged plates

Figure 16.1.2

Voltage, which measures the electric potential difference across two points, based on the unit volt (V). To measure the voltage across some distance, it is necessary to pick a position to be the relative zero, because voltage is the change in potential difference. Any point in a system can be given the value of zero volts, but it is typically the point of a point charge or one plate in a uniform electric field as shown above. The voltage is commonly referred to as the **electric potential difference** and can be measured using a voltmeter.

The electrical potential difference between the two plates is expressed as V=Ed, the electric field strength times the distance between the plates. The units in this expression are Newtons/coulomb times meters, which gives the final units Joules/coulomb. Voltage is an expression of the amount of potential energy per unit charge. The work done moving a charge against the field can be calculated by multiplying the electric field potential by the charge, W=Vq.

Examples

Example 16.1.1

Two large parallel metal plates are 5.0 cm apart. The magnitude of the electric field between them is 800. N/C.

(a) What is the potential difference between the plates?

(b) What work is done when one electron is moved from the positive to the negative plate?

Solution

(a) V=Ed=(800. N/C)(0.050 m)=40. J/C=40. V

(b) $W=Vq=(40.0\ J/C)(1.6\times10^{-19}\ C)=6.4\times10^{-18}\ J$

✓ Example 16.1.2

A voltmeter measures the potential difference between two large parallel plates to be 50.0 volts. The plates are 3.0 cm apart. What is the magnitude of the electric field strength between the plates?

Solution

E=V/d=50.0 volts/0.030 m=1700 N/C

Have you ever wondered what causes lightning to strike? Launch the simulation below and adjust the Charge Separation slider to high, the Cloud Separation slider to low, and be sure to turn the Lightning Rod on (to protect the people in the building). Then, press play and see what happens:

Summary

- The work done moving a charged particle in an electric field can result in the particle gaining or losing both kinetic and potential energy.
- The difference in electric potential energy is measured with a voltmeter in units called volts.
- A constant electric field can be produced by placing two large flat conducting plates parallel to each other.
- The electrical potential difference in a uniform electric field is given as V=Ed.
- The work done moving a charge against the field can be found by W=Vq.

Review

1. Two large parallel plates are 0.00630 m apart and the voltage across them is 10.0 volts. What is the electric field strength between the plates?
2. The potential difference between points A and B in an electric field is 25.0 volts. How much work is required to transfer 10.0 coulombs of charge from A to B?
3. 10.0 J of work are required to transfer 2.00 coulombs of charge from point X to point Y in an electric field. What is the difference in potential between these two points?
4. The electric field between two parallel plates connected to a 45 V battery (which produces a 45 V difference in potential between the plates) is 1500 N/C. How far apart are the plates?
5. How much kinetic energy will an electron gain if it accelerates through a potential difference of 23,000 volts in a cathode ray tube?

Explore More

Use this resource to answer the questions that follow.

1. If you do work to move a charged object in an electric field, where does the work go?
2. Points A and B in an electric field have a difference in potential energy. This difference in electrical potential energy is called ___________________.

Additional Resources

Study Guide: Electrical Systems Study Guide

Videos: Electrical Potential - Overview

Real World Application: Faraday Cage

Interactive: Electric Analogies

11.4.2: Current

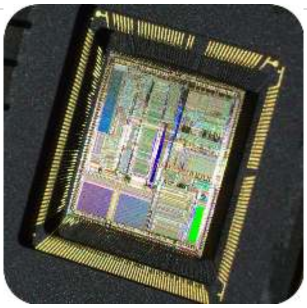

Figure 16.2.1

You've probably heard of vacuum tubes and large transistors, which were once common in electronics like televisions. These individual devices were mounted on large printed circuit boards. Then, in 1959, two researchers working independently developed the first integrated circuits. These circuits could combine several transistors and resistors into one circuit on one small chip of silicon. These chips, like the one pictured above, are used today in virtually every electrical device.

Electric Current and Circuits

Electric Current

Remember that it often requires work to force electrons into a specific location. If we have two conducting spheres and we have forced excess electrons onto one of the spheres by doing work on the electrons, then that sphere, and those electrons, will have a higher potential energy than those on the uncharged sphere. If the two spheres are touched together, electrons will flow from the sphere with excess electrons to the sphere with no excess electrons. That is, electrons will flow from the high potential energy position to the lower potential energy position. The flow will continue until the electrons on the two spheres have the same potential energy. A flow of charged particles such as this is called an **electric current.**

It is possible for an electric current to be either a flow of positively charged particles or negatively charged particles. In gases, both positive and negative ions can flow. The difficulty of freeing protons, however, makes it extremely rare to have an electric current of positive particles in solid conductors. Virtually all electric currents consist of the movement of electrons.

Common Misconceptions

It is easy to assume that current is the flow of positive charges. In fact, when the conventions of positive and negative charge were invented two centuries ago, it was assumed that positive charge flowed through a wire. In reality, however, we know now that the flow of positive charge is actually a flow of negative charge in the opposite direction. That is, when an electron moves from position A to position B, it is the same as a positive hole moving from B to A.

Today, even though we know it is not correct, we still use the historical convention of positive current flow when discussing the direction of a current. **Conventional current**, the current we commonly use and discuss, is the direction positive current would flow. When we want to speak of the direction of electron flow, we will specifically state that we are referring to electron flow.

Electric current flows from positions of higher potential energy to positions of lower potential energy. Electrons acquire higher potential energy from an electron pump that does work on the electrons, moving them from positions of lower *PE* to positions of higher *PE*. Electrons in galvanic cells (several cells together comprise a battery) have higher potential energy at one terminal of the battery that at the other. This difference in potential is related to chemical energy. When the two terminals of the battery are connected to each other via a conducting wire, the electric current will travel from the terminal with higher potential energy to that with lower potential energy. This setup is the most simple of **electric circuits**.

It can be helpful to think about an electric circuit like water flowing through a system. Use the Electric Analogies simulation below to help you visualize what is going on in an electric circuit. The pump and water tower represent the battery and the water wheel represents a resistor. You can adjust the slider to put these “resistors” in series and in parallel. Have fun exploring:

Electric Circuits

An electric circuit is any closed loop that goes from one battery terminal to the other and allows current to flow through it. A relatively simple circuit is shown in the image below. The charges move from the higher potential energy terminal on the battery, through the light bulb, through the switch, and back to the lower potential energy terminal on the battery.

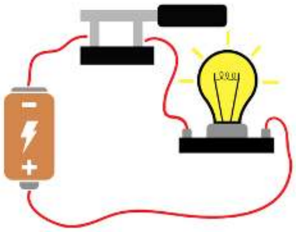

Figure 16.2.2

Other resistors include motors, which convert energy into kinetic energy, and heaters, which convert it into thermal energy. A circuit consists of a battery, or a charge pump, which increases the potential energy of the charges, and one or more devices that decrease the potential energy. As the potential energy is reduced, it is converted into some other form of energy. In the image above, the device that decreases the charges' potential energy is the light bulb; the excess energy is converted into light energy. Any device that reduces the potential energy of the charge flowing through it is said to have **resistance** because it resists the flow of charge.

The charges in the circuit can neither be created nor destroyed, nor can they pile up in one spot. The charged particles moving through the circuit move the same everywhere in the circuit. If one coulomb of charge leaves the charge pump, then one coulomb of charge moves through the light, and one coulomb of charge moves through the switch. The net change of energy through the circuit is zero. That is, the increase in potential energy through the charge pump is exactly equal to the potential drop through the light. If the generator (charge pump) does 120 J of work on each coulomb of charge that it transfers, then the light uses 120 J of energy as the charge passes through the light.

The electric current is measured in coulombs per second. A flow of one coulomb per second is called one **ampere,** A, of current.

$$1.00 \text{ Ampere}=1.00 \text{ coulomb}/1.00 \text{ second}$$

The energy carried by an electric current depends on the charge transferred and the potential difference across which it moves, $E=qV$. The voltage or potential difference is expressed in Joules/coulomb and multiplying this by the charge in coulombs yields energy in Joules.

Electrical power is a measure of the rate at which energy is transferred, and is expressed in watts, or Joules/second. Power can also be obtained by multiplying the voltage by the current:

$$\text{Power, } P=VI=(\text{Joules/coulomb})(\text{coulomb/second})=\text{Joules/second}=\text{watts}.$$

Examples

Example 16.2.1

What is the power delivered to a light bulb when the circuit has a voltage drop of 120 V and produces a current of 3.0 ampere?

Solution

$$P=VI=(120 \text{ J/C})(3.0 \text{ C/s})=360 \text{ J/s}=360 \text{ watts}$$

✓ Example 16.2.2

A 6.00 V battery delivers a 0.400 A current to an electric motor that is connected across the battery terminals.

1. What power is consumed by the motor?
2. How much electric energy is delivered in 500. seconds?

Solution

1. $P=VI=(6.00 \text{ V})(0.400 \text{ A})=2.4$ watts
2. $\text{Joules}=(\text{J/s})(\text{s})=(2.4 \text{ J/s})(500. \text{ s})=1200$ Joules

Launch the Flashlight simulation below to see how electricity flows in a simple circuit. Be sure to observe the Power vs Current graph to develop a deeper understanding of the relationship between these two variables. Can you adjust the sliders to maximize the current running through the circuit? Try it out:

Summary

- Electric current is the flow of electrons from the high potential energy position to the lower potential energy position.
- Current flow is the direction a positive current would be traveling, or the opposite direction that electrons actually flow.
- A closed loop containing current flow is called an electric circuit.
- Electric current is measured in coulombs per second, or amperes.
- Electric power is measured in joules per second, or watts.
- The energy carried by an electric current depends on the charge transferred and the potential difference across which it moves, E=qV.
- Power, P=VI=(Joules/coulomb)(coulomb/second)=Joules/second=watts.

Review

1. The current through a light bulb connected across the terminals of a 120 V outlet is 0.50 A. At what rate does the bulb convert electric energy to light?
2. A 12.0 V battery causes a current of 2.0 A to flow through a lamp. What is the power used by the lamp?
3. What current flows through a 100. W light bulb connected to a 120. V outlet?
4. The current through a motor is 210 A. If a battery keeps a 12.0 V potential difference across the motor, what electric energy is delivered to the motor in 10.0 s?

Explore More

Use this resource to answer the questions that follow.

1. What type of current is described in this video (electron or conventional)?
2. What drives the current through the circuit?
3. What inhibits the flow of current in the circuit?

Additional Resources

Study Guide: Electrical Systems Study Guide

Real World Application: Stunning Technology

Videos:

11.4.3: Ohm’s Law- Resistance and Simple Circuits

Learning Objectives

- Explain the origin of Ohm’s law.
- Calculate voltages, currents, or resistances with Ohm’s law.
- Explain what an ohmic material is.
- Describe a simple circuit.

What drives current? We can think of various devices—such as batteries, generators, wall outlets, and so on—which are necessary to maintain a current. All such devices create a potential difference and are loosely referred to as voltage sources. When a voltage source is connected to a conductor, it applies a potential difference V that creates an electric field. The electric field in turn exerts force on charges, causing current.

Ohm’s Law

The current that flows through most substances is directly proportional to the voltage V applied to it. The German physicist Georg Simon Ohm (1787–1854) was the first to demonstrate experimentally that the current in a metal wire is *directly proportional to the voltage applied*:

$$I \propto V.$$

This important relationship is known as **Ohm’s law**. It can be viewed as a cause-and-effect relationship, with voltage the cause and current the effect. This is an empirical law like that for friction—an experimentally observed phenomenon. Such a linear relationship doesn’t always occur.

Resistance and Simple Circuits

If voltage drives current, what impedes it? The electric property that impedes current (crudely similar to friction and air resistance) is called **resistance** R. Collisions of moving charges with atoms and molecules in a substance transfer energy to the substance and limit current. Resistance is defined as inversely proportional to current, or

$$I \propto \frac{1}{R}.$$

Thus, for example, current is cut in half if resistance doubles. Combining the relationships of current to voltage and current to resistance gives

$$I = \frac{V}{R}.$$

This relationship is also called Ohm’s law. Ohm’s law in this form really defines resistance for certain materials. Ohm’s law (like Hooke’s law) is not universally valid. The many substances for which Ohm’s law holds are called **ohmic**. These include good conductors like copper and aluminum, and some poor conductors under certain circumstances. Ohmic materials have a resistance R that is independent of voltage V and current I. An object that has simple resistance is called a *resistor*, even if its resistance is small. The unit for resistance is an **ohm** and is given the symbol Ω (upper case Greek omega). Rearranging $I = V/R$ gives $R = V/I$, and so the units of resistance are 1 ohm = 1 volt per ampere:

$$1\,\Omega = 1\frac{V}{A}.$$

Figure $11.4.3.1$shows the schematic for a simple circuit. A **simple circuit** has a single voltage source and a single resistor. The wires connecting the voltage source to the resistor can be assumed to have negligible resistance, or their resistance can be included in R.

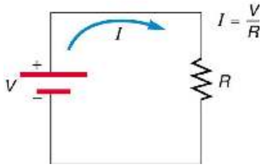

Figure 11.4.3.1: A simple electric circuit in which a closed path for current to flow is supplied by conductors (usually metal wires) connecting a load to the terminals of a battery, represented by the red parallel lines. The zigzag symbol represents the single resistor and includes any resistance in the connections to the voltage source.

✓ Example 11.4.3.1: Calculating Resistance: An Automobile Headlight

What is the resistance of an automobile headlight through which 2.50 A flows when 12.0 V is applied to it?

Strategy

We can rearrange Ohm's law as stated by $I = V/R$ and use it to find the resistance.

Solution

Rearranging $I = V/R$ and substituting known values gives

$$R = \frac{V}{I} = \frac{12.0\text{ V}}{2.50\text{ A}} = 4.80\Omega.$$

Discussion

This is a relatively small resistance, but it is larger than the cold resistance of the headlight. Resistance usually increases with temperature, and so the bulb has a lower resistance when it is first switched on and will draw considerably more current during its brief warm-up period.

Resistances range over many orders of magnitude. Some ceramic insulators, such as those used to support power lines, have resistances of $10^{12}\ \Omega$ or more. A dry person may have a hand-to-foot resistance of $10^5\ \Omega$, whereas the resistance of the human heart is about $10^3\ \Omega$. A meter-long piece of large-diameter copper wire may have a resistance of $10^{-5}\ \Omega$, and superconductors have no resistance at all (they are non-ohmic). Resistance is related to the shape of an object and the material of which it is composed.

Additional insight is gained by solving $I = V/R$ for V yielding

$$V = IR.$$

This expression for V can be interpreted as the *voltage drop across a resistor produced by the flow of current* I. The phrase IR *drop* is often used for this voltage. For instance, the headlight in Example 11.4.3.1 has an IR drop of 12.0 V. If voltage is measured at various points in a circuit, it will be seen to increase at the voltage source and decrease at the resistor. Voltage is similar to fluid pressure. The voltage source is like a pump, creating a pressure difference, causing current—the flow of charge. The resistor is like a pipe that reduces pressure and limits flow because of its resistance. Conservation of energy has important consequences here. The voltage source supplies energy (causing an electric field and a current), and the resistor converts it to another form (such as thermal energy). In a simple circuit (one with a single simple resistor), the voltage supplied by the source equals the voltage drop across the resistor, since $\text{PE} = q\Delta V$, and the same q flows through each. Thus the energy supplied by the voltage source and the energy converted by the resistor are equal. (See Figure 11.4.3.2)

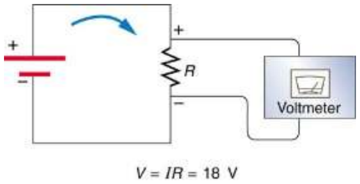

Figure 11.4.3.2: The voltage drop across a resistor in a simple circuit equals the voltage output of the battery.

MAKING CONNECTIONS: CONSERVATION OF ENERGY

In a simple electrical circuit, the sole resistor converts energy supplied by the source into another form. Conservation of energy is evidenced here by the fact that all of the energy supplied by the source is converted to another form by the resistor alone. We will find that conservation of energy has other important applications in circuits and is a powerful tool in circuit analysis.

Section Summary

- A simple circuit *is* one in which there is a single voltage source and a single resistance.
- One statement of Ohm's law gives the relationship between current I, voltage V, and resistance R in a simple circuit to be $I=\frac{V}{R}$
- Resistance has units of ohms (Ω), related to volts and amperes by $1\ \Omega=1\ \mathrm{V/A}$.
- There is a voltage or IR drop across a resistor, caused by the current flowing through it, given by $V=IR$.

Glossary

Ohm's law

an empirical relation stating that the current I is proportional to the potential difference V. It is often written as $I = V/R$, where R is the resistance

resistance

the electric property that impedes current; for ohmic materials, it is the ratio of voltage to current, $R = V/I$

ohm

the unit of resistance, given by $1\ \Omega=1\ \mathrm{V/A}$

ohmic

a type of a material for which Ohm's law is valid

simple circuit

a circuit with a single voltage source and a single resistor

11.4.4: Ohm's Law

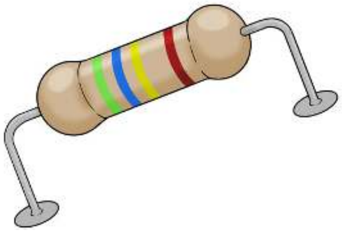

Figure 16.5.1

The bands of color on a resistor are a code that indicates the magnitude of the resistance of the resistor. There are four color bands identified by letter: A, B, C, and D, with a gap between the C and D bands so that you know which end is A. This particular resistor has a red A band, blue B band, green C band, and gold D band, but the bands can be different colors on different resistors. Based on the colors of the bands, it is possible to identify the type of resistor. the A and B bands represent significant digits; red is 2 and blue is 6. The C band indicates the multiplier, and green indicates 10^5. These three together indicate that this particular resistor is a 26,000 Ohm resistor. Finally, the D band indicates the tolerance, in this case 5%, as shown by the gold band. These terms will be explained over the course of this lesson.

Resistance and Ohm's Law

When a potential difference is placed across a metal wire, a large current will flow through the wire. If the same potential difference is placed across a glass rod, almost no current will flow. The property that determines how much current will flow is called the **resistance.** Resistance is measured by finding the ratio of potential difference, V, to current flow, I.

$$R=V/I$$

When given in the form V=IR, this formula is known as **Ohm's Law,** after the man that discovered the relationship. The units of resistance can be determined using the units of the other terms in the equation, namely that the potential difference is in volts (J/C) and current in amperes (C/s):

$$R=\text{volts/amperes}=(\text{joules/coulomb})/(\text{coulombs/second})=\text{joules}\cdot\text{seconds/coulombs}^2=\text{ohms}$$

The units for resistance have been given the name **ohms** and the abbreviation is the Greek letter omega, Ω. 1.00 Ω is the resistance that will allow 1.00 ampere of current to flow through the resistor when the potential difference is 1.00 volt. Most conductors have a constant resistance regardless of the potential difference; these are said to obey Ohm's Law.

There are two ways to control the current in a circuit. Since the current is directly proportional to the potential difference and inversely proportional to the resistance, you can increase the current in a circuit by increasing the potential or by decreasing the resistance.

✓ Example 16.5.1

A 50.0 V battery maintains current through a 20.0 Ω resistor. What is the current through the resistor?

Solution

$$I=V/R=50.0\text{ V}/20.0\ \Omega=2.50\text{ amps}$$

Launch the PLIX Interactive below to explore how Ohm's Law affects the current in a circuit:

Summary

- Resistance is the property that determines the amount of current flow through a particular material.
- V=IR is known as Ohm's Law.
- The unit for resistance is the ohm, and it has the abbreviation Ω.

Review

1. If the potential stays the same and the resistance decreases, what happens to the current?
    1. increase

    2. decrease
    3. stay the same
2. If the resistance stays the same and the potential increases, what happens to the current?
    1. increase
    2. decrease
    3. stay the same
3. How much current can be pushed through a 30.0 Ω resistor by a 12.0 V battery?
4. What voltage is required to push 4.00 A of current through a 32.0 Ω resistor?
5. If a 6.00 volt battery will produce 0.300 A of current in a circuit, what is the resistance in the circuit?

Explore More

Use this resource to answer the questions that follow.

1. What happens to current flow when voltage is increased?
2. What happens to current flow when resistance is increased?

Additional Resources

Study Guide: Electrical Systems Study Guide

Video: Basic Ohm's Law Problems - Overview

Real World Application: Guess The Efficiency, Batteries

Interactives: Dollhouse, Flashlight, Electric Analogies

11.4.5: Electric Power and Energy

Learning Objectives

- Calculate the power dissipated by a resistor and power supplied by a power supply.
- Calculate the cost of electricity under various circumstances.

Power in Electric Circuits

Power is associated by many people with electricity. Knowing that power is the rate of energy use or energy conversion, what is the expression for **electric power**? Power transmission lines might come to mind. We also think of lightbulbs in terms of their power ratings in watts. Let us compare a 25-W bulb with a 60-W bulb. (See Figure 11.4.5.1(a).) Since both operate on the same voltage, the 60-W bulb must draw more current to have a greater power rating. Thus the 60-W bulb's resistance must be lower than that of a 25-W bulb. If we increase voltage, we also increase power. For example, when a 25-W bulb that is designed to operate on 120 V is connected to 240 V, it briefly glows very brightly and then burns out. Precisely how are voltage, current, and resistance related to electric power?

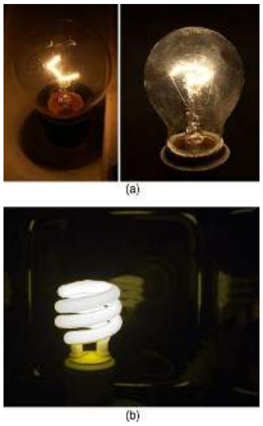

Figure 11.4.5.1: (a) Which of these lightbulbs, the 25-W bulb (upper left) or the 60-W bulb (upper right), has the higher resistance? Which draws more current? Which uses the most energy? Can you tell from the color that the 25-W filament is cooler? Is the brighter bulb a different color and if so why? (credits: Dickbauch, Wikimedia Commons; Greg Westfall, Flickr) (b) This compact fluorescent light (CFL) puts out the same intensity of light as the 60-W bulb, but at 1/4 to 1/10 the input power. (credit: dbgg1979, Flickr)

Electric energy depends on both the voltage involved and the charge moved. This is expressed most simply as $\mathrm{PE}=qV$, where q is the charge moved and V is the voltage (or more precisely, the potential difference the charge moves through). Power is the rate at which energy is moved, and so electric power is

$$P=\frac{PE}{t}=\frac{qV}{t}.$$

Recognizing that current is $I=q/t$ (note that $\Delta t=t$ here), the expression for power becomes

$$P=IV.$$

Electric power (P) is simply the product of current times voltage. Power has familiar units of watts. Since the SI unit for potential energy (PE) is the joule, power has units of joules per second, or watts. Thus, $1\ \mathrm{A}\cdot\mathrm{V}=1\ \mathrm{W}$. For example, cars often have one or more auxiliary power outlets with which you can charge a cell phone or other electronic devices. These outlets may be rated at 20 A, so that the circuit can deliver a maximum power $P=IV=(20\ \mathrm{A})(12\ \mathrm{V})=240\ \mathrm{W}$. In some applications, electric power may be expressed as volt-amperes or even kilovolt-amperes ($1\mathrm{kA}\cdot\mathrm{V}=1\ \mathrm{kW}$).

To see the relationship of power to resistance, we combine Ohm's law with $P=IV$. Substituting $I=V/R$ gives $P=(V/R)V=V^2/R$. Similarly, substituting $V=IR$ gives $P=I(IR)=I^2R$. Three expressions for electric power are listed together here for convenience:

$$P=IV$$

$$P = \frac{V^2}{R}$$

$$P = I^2 R.$$

Note that the first equation is always valid, whereas the other two can be used only for resistors. In a simple circuit, with one voltage source and a single resistor, the power supplied by the voltage source and that dissipated by the resistor are identical. (In more complicated circuits, P can be the power dissipated by a single device and not the total power in the circuit.)

Different insights can be gained from the three different expressions for electric power. For example, $P = V^2/R$ implies that the lower the resistance connected to a given voltage source, the greater the power delivered. Furthermore, since voltage is squared in $P = V^2/R$, the effect of applying a higher voltage is perhaps greater than expected. Thus, when the voltage is doubled to a 25-W bulb, its power nearly quadruples to about 100 W, burning it out. If the bulb's resistance remained constant, its power would be exactly 100 W, but at the higher temperature its resistance is higher, too.

✓ Example 11.4.5.1: Calculating Power Dissipation and Current

Consider the example given in "Ohm's Law: Resistance and Simple Circuits." Then find the power dissipated by the car headlight.

Strategy

For the headlight, we know voltage and current, so we can use $P = IV$ to find the power.

Solution

Entering the known values of current and voltage for the hot headlight, we obtain

$$P = IV = (2.50 \text{ A})(12.0 \text{ V}) = 30.0 \text{ W}.$$

Discussion

The 30 W dissipated by the hot headlight is typical.

The Cost of Electricity

The more electric appliances you use and the longer they are left on, the higher your electric bill. This familiar fact is based on the relationship between energy and power. You pay for the energy used. Since $P = E/t$, we see that

$$E = Pt$$

is the energy used by a device using power P for a time interval t. For example, the more lightbulbs burning, the greater P used; the longer they are on, the greater t is. The energy unit on electric bills is the kilowatt-hour ($\text{kW} \cdot \text{h}$), consistent with the relationship $E = Pt$. It is easy to estimate the cost of operating electric appliances if you have some idea of their power consumption rate in watts or kilowatts, the time they are on in hours, and the cost per kilowatt-hour for your electric utility. Kilowatt-hours, like all other specialized energy units such as food calories, can be converted to joules. You can prove to yourself that $1 \text{ kW} \cdot \text{h} = 3.6 \times 10^6 \text{ J}$.

The electrical energy (E) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This will not only reduce the cost, but it will also result in a reduced impact on the environment. Improvements to lighting are some of the fastest ways to reduce the electrical energy used in a home or business. About 20% of a home's use of energy goes to lighting, while the number for commercial establishments is closer to 40%. Fluorescent lights are about four times more efficient than incandescent lights—this is true for both the long tubes and the compact fluorescent lights (CFL). (See Figure 11.4.5.1(b).) Thus, a 60-W incandescent bulb can be replaced by a 15-W CFL, which has the same brightness and color. CFLs have a bent tube inside a globe or a spiral-shaped tube, all connected to a standard screw-in base that fits standard incandescent light sockets. (Original problems with color, flicker, shape, and high initial investment for CFLs have been addressed in recent years.) The heat transfer from these CFLs is less, and they last up to 10 times longer. The significance of an investment in such bulbs is addressed in the next example. New white LED lights (which are clusters of small LED bulbs) are even more efficient (twice that of CFLs) and last 5 times longer than CFLs. However, their cost is still high.

MAKING CONNECTIONS: ENERGY, POWER, AND TIME

The relationship $E=Pt$ is one that you will find useful in many different contexts. The energy your body uses in exercise is related to the power level and duration of your activity, for example. The amount of heating by a power source is related to the power level and time it is applied. Even the radiation dose of an X-ray image is related to the power and time of exposure.

✓ Example 11.4.5.2: Calculating the Cost Effectiveness of Compact Fluorescent Lights (CFL)

If the cost of electricity in your area is 12 cents per kWh, what is the total cost (capital plus operation) of using a 60-W incandescent bulb for 1000 hours (the lifetime of that bulb) if the bulb cost 25 cents? (b) If we replace this bulb with a compact fluorescent light that provides the same light output, but at one-quarter the wattage, and which costs \$1.50 but lasts 10 times longer (10,000 hours), what will that total cost be?

Strategy

To find the operating cost, we first find the energy used in kilowatt-hours and then multiply by the cost per kilowatt-hour.

Solution for (a)

The energy used in kilowatt-hours is found by entering the power and time into the expression for energy:

$$E=Pt=(60\text{ W})(1000\text{ h})=60,000\text{ W}\cdot\text{h}.$$

In kilowatt-hours, this is

$$E=60.0\text{ kW}\cdot\text{h}.$$

Now the electricity cost is

$$\text{cost}=(60.0\text{ kW}\cdot\text{h})(\$0.12/\text{kW}\cdot\text{h})=\$7.20.$$

The total cost will be \$7.20 for 1000 hours (about one-half year at 5 hours per day).

Solution for (b)

Since the CFL uses only 15 W and not 60 W, the electricity cost will be \$7.20/4 = \$1.80. The CFL will last 10 times longer than the incandescent, so that the investment cost will be 1/10 of the bulb cost for that time period of use, or 0.1(\$1.50) = \$0.15. Therefore, the total cost will be \$1.95 for 1000 hours.

Discussion

Therefore, it is much cheaper to use the CFLs, even though the initial investment is higher. The increased cost of labor that a business must include for replacing the incandescent bulbs more often has not been figured in here.

MAKING CONNECTIONS: TAKE-HOME EXPERIMENT—ELECTRICAL ENERGY USE INVENTORY

1) Make a list of the power ratings on a range of appliances in your home or room. Explain why something like a toaster has a higher rating than a digital clock. Estimate the energy consumed by these appliances in an average day (by estimating their time of use). Some appliances might only state the operating current. If the household voltage is 120 V, then use $P=IV$. 2) Check out the total wattage used in the rest rooms of your school's floor or building. (You might need to assume the long fluorescent lights in use are rated at 32 W.) Suppose that the building was closed all weekend and that these lights were left on from 6 p.m. Friday until 8 a.m. Monday. What would this oversight cost? How about for an entire year of weekends?

Section Summary

- Electric power P is the rate (in watts) that energy is supplied by a source or dissipated by a device.
- Three expressions for electrical power are

$$P=IV,$$

$$P=\frac{V^2}{R},$$

and

$$P = I^2 R.$$

- The energy used by a device with a power P over a time t is $E = Pt$.

Glossary

electric power

the rate at which electrical energy is supplied by a source or dissipated by a device; it is the product of current times voltage

---

11.4.6: Ammeters and Voltmeters

Figure 16.4.1

This photo is of the interior of the control room for a nuclear power plant. Many of the meters are reading information about the water temperature and the nuclear reaction that is occurring, but the majority of the meters are reading data about the electric energy being generated.

Ammeters and Voltmeters

Ammeters and **voltmeters** are cleverly designed for the way they are used. Ammeters measure the current of a circuit, and voltmeters measure the voltage drop across a resistor. It is important in the design and use of these meters that they don't change the circuit in such a way as to influence the readings. While both types of meters are technically resistors, they are specifically designed to make their readings without changing the circuit itself.

Diagram of a circuit with an ammeter and a voltmeter inserted

Figure 16.4.2

Ammeter

An ammeter measures the current traveling through the circuit. They are designed to be connected to the circuit in series, and have an extremely low resistance. If an ammeter were connected in parallel, all of the current would go through the ammeter and very little through any other resistor. As such, it is necessary for the ammeter to be connected in series with the resistors. This allows the ammeter to accurately measure the current flow without causing any disruptions. In the circuit sketched above, the ammeter is m2.

Voltmeter

In contrast, a voltmeter is designed to be connected to a circuit in parallel, and has a very high resistance. A voltmeter measures the voltage drop across a resistor, and does not need to have the current travel through it to do so. When a voltmeter is placed in parallel with a resistor, all the current continues to travel through the resistor, avoiding the very high resistance of the voltmeter. However, we know that the voltage drop across all resistors in parallel is the same, so connecting a voltmeter in parallel allows it to accurately measure the voltage drop. In the sketch, the voltmeter is m1.

A galvanometer is a device that uses the magnetic field generated by a flowing electric current to push a needle. When placed in parallel with a resistor, it can act as an ammeter and be used to measure the amount of current flowing through a circuit. When placed in series with a resistor, it can act as a voltmeter and measure the voltage across the device. Launch the Galvanometer simulation below to learn more:

Summary

- Ammeters measure the current through a resistor.
- Ammeters have low resistances and are placed in the circuit in series.
- Voltmeters measure the voltage drop across a resistor.
- Voltmeters have high resistances and are placed in the circuit in parallel.

Review

Practice problem for identifying correct and incorrect positions of voltmeters and ammeters

Figure 16.4.3

1. In the sketch at above, there are four positions available for the placement of meters. Which position(s) would be appropriate for placement of an ammeter?
    1. 1
    2. 3
    3. 4
    4. All of them.
    5. None of them.
2. Which position(s) would be appropriate for placement of a voltmeter?
    1. 1
    2. 2
    3. 3
    4. All of them.
    5. None of them.
3. Which position could hold an ammeter that would read the total current through the circuit?
    1. 1
    2. 2
    3. 3
    4. 4
    5. None of them.
4. Which position could hold a voltmeter that would read the total voltage drop through the circuit?
    1. 1
    2. 2
    3. 3 or 4
    4. All of them.
    5. None of them.

Explore More

Use this resource to answer the questions that follow.

1. What happens when the ammeter is connected in parallel with the lamp?
2. Why do the problems occur when the narrator places the ammeter in parallel with the lamp?

Additional Resources

Study Guide: Electrical Systems Study Guide

Videos:

11.4.7: Chemical and Solar Cells

Figure 16.3.1

Can you identify the two objects pictured above? You’ve probably used objects like these many times. The photos show a TV remote (left) and a calculator (right). Both of them run on electric current. Current requires a source of voltage, which is a difference in electric potential energy.

> **Q:** The source of voltage is different in the remote and the calculator. Do you know what source of voltage each device uses?
>
> **A:** The TV remote uses chemical cells as a source of voltage. The calculator uses a solar cell.

Chemical Cells

Chemical cells are found in batteries. They produce voltage by means of chemical reactions. Chemical cells have two electrodes, which are strips of different materials, such as zinc and carbon. The electrodes are suspended in an electrolyte. This is a substance that contains free ions, which can carry electric current. The electrolyte may be either a paste, in which case the cell is called a dry cell, or a liquid, in which case the cell is called a wet cell. Flashlight batteries contain dry cells. Car batteries contain wet cells.

The Figure below shows how a battery works. The diagram represents the simplest type of battery, one that contains a single chemical cell. Both dry and wet cells work the same basic way. The electrodes react chemically with the electrolyte, causing one electrode to give up electrons and the other electrode to accept electrons. Electrons flow through the electrolyte from the negative to positive electrode. The electrodes extend out of the battery for the attachment of wires that carry the current. The current can be used to power a light bulb or other electric device.

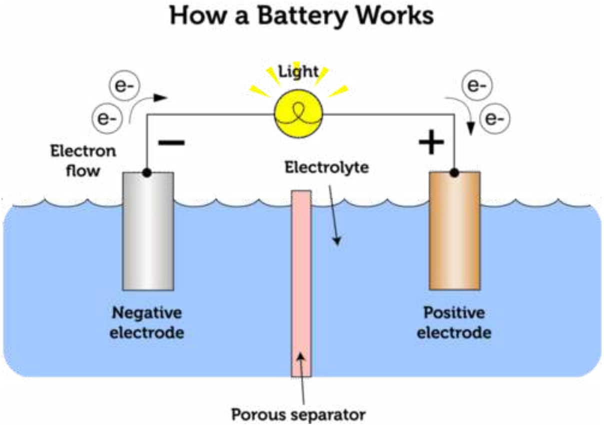

Figure 16.3.2

Solar Cells

Solar cells convert the energy in sunlight to electrical energy. Solar cells are also called photovoltaic (PV) cells because they use light (*photo-*) to produce voltage (*-voltaic*). Solar cells contain a material such as silicon that absorbs light energy. The energy knocks electrons loose so they can flow freely and produce a difference in electric potential energy, or voltage. The flow of electrons creates electric current. Solar cells have positive and negative contacts, like the terminals in chemical cells. If the contacts are connected with wire, current flows from the negative to positive contact. The Figure below shows how a solar cell works.

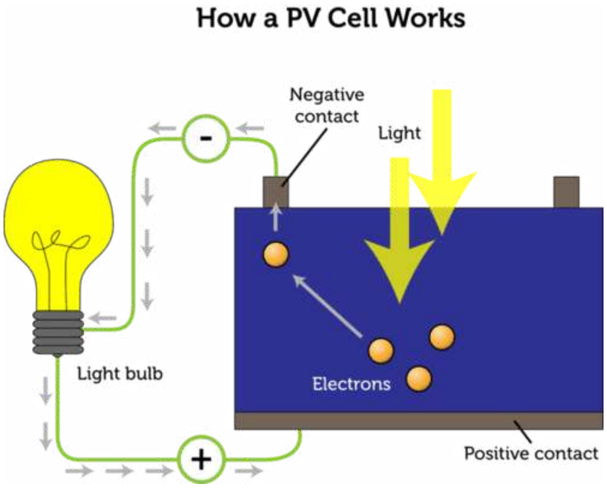

Figure 16.3.3

Summary

- Current requires a source of voltage, which is a difference in electric potential energy. Sources of voltage include chemical cells and solar cells.
- Chemical cells are found in batteries. They produce voltage by means of chemical reactions. They contain electrodes and an electrolyte, which may be a paste (dry cell) or a liquid (wet cell).
- Solar cells convert the energy in sunlight to electrical energy. They contain a material such as silicon that absorbs light energy and gives off electrons.

Review

1. What is voltage? How is it related to electric current?
2. How does a chemical cell produce current?
3. Explain how a solar cell works.

Additional Resources

Real World Application: Lithium Lasts Longer

Videos:

Study Guide: Electrical Systems Study Guide

SECTION OVERVIEW

11.5: Electric Circuits

11.5.1: Energy Transfer in Electric Circuits

Electrical transmission substation

Figure 17.1.1

Part of the electrical grid, an electrical transmission sub-station receives extremely high current levels, then passes the electrical energy on to as many as 200,000 homes. Approximately 5000 megawatt-hours of energy passes through this particular substation each year.

Energy Transfer in Electric Circuits

Electric power is the energy per unit time converted by an electric circuit into another form of energy. We already know that power through a circuit is equal to the voltage multiplied by the current in a circuit: $P=VI$. It is possible to determine the power dissipated in a single resistor if we combine this expression with Ohm's Law, $V=IR$. This becomes particularly useful in circuits with more than one resistor, to determine the power dissipated in each one. Combining these two equations, we get an expression for electric power that involves only the current and resistance in a circuit.

$$P=I^2R$$

The power dissipated in a resistor is proportional to the square of the current that passes through it and to its resistance.

Electrical energy itself can be expressed as the electrical power multiplied by time:

$$E=Pt$$

We can incorporate this equation to obtain an equation for electrical energy based on current, resistance, and time. The electrical energy across a resistor is determined to be the current squared multiplied by the resistance and the time.

$$E=I^2Rt$$

This equation holds true in ideal situations. However, devices used to convert electrical energy into other forms of energy are never 100% efficient. An electric motor is used to convert electrical energy into kinetic energy, but some of the electrical energy in this process is lost to thermal energy. When a lamp converts electrical energy into light energy, some electrical energy is lost to thermal energy.

✓ Example 17.1.1

A heater has a resistance of 25.0 Ω and operates on 120.0 V.

1. How much current is supplied to the resistance?
2. How many joules of energy is provided by the heater in 10.0 s?

Solution

1. $I=V/R=120.0\text{ V}/25.0\ \Omega=4.8\text{ A}$
2. $E=I^2Rt=(4.8\text{ A})^2(25.0\ \Omega)(10.0\text{ s})=5760$ joules

Think again about the power grid. When electricity is transmitted over long distances, some amount of energy is lost in overcoming the resistance in the transmission lines. We know the equation for the power dissipated is given by $P=I^2R$. The energy loss can be minimized by choosing the material with the least resistance for power lines, but changing the current also has significant effects. Consider a reduction of the current by a power of ten:

How much power is dissipated when a current of 10.0 A passes through a power line whose resistance is 1.00 Ω? $P=I^2R=(10.0\text{ A})^2(1.00\ \Omega)=100.$ Watts

How much power is dissipated when a current of 1.00 A passes through a power line whose resistance is 1.00 Ω? $P=I^2R=(1.00\text{ A})^2(1.00\ \Omega)=1.00$ Watts

The power loss is reduced tremendously by reducing the magnitude of the current through the resistance. Power companies must transmit the same amount of energy over the power lines but keep the power loss minimal. They do this by reducing the current. From the equation $P=VI$, we know that the voltage must be increased to keep the same power level.

The Kilowatt-Hour

Even though the companies that supply electrical energy are often called “power” companies, they are actually selling energy. Your electricity bill is based on energy, not power. The amount of energy provided by electric current can be calculated by multiplying the watts (J/s) by seconds to yield joules. The joule, however, is a very small unit of energy and using the joule to state the amount of energy used by a household would require a very large number. For that reason, electric companies measure their energy sales in a large number of joules called a **kilowatt hour** (kWh). A kilowatt hour is exactly as it sounds - the number of kilowatts (1,000 W) transferred per hour.

$$1.00 \text{ kilowatt hour}=(1000 \text{ J/s})(3600 \text{ s})=3.6\times10^6 \text{ J}$$

✓ Example 17.1.2

A color television uses about 2.0 A when operated on 120 V.

1. How much power does the set use?
2. If the TV is operated for 8.00 hours per day, how much energy in kWh does it use per day?
3. At $0.15 per kWh, what does it cost to run the TV for 30 days?

Solution

1. $P=VI=(120 \text{ V})(2.0 \text{ A})=240 \text{ W}$
2. $E=(240 \text{ J/s})(8 \text{ h})(3600 \text{ s/h})/3.6\times10^6 \text{ J/kWh}=1.92 \text{ kWh}$
3. Cost=(1.92 kWh)(30)($0.15)=$8.64

Launch the Dollhouse simulation below and click Show Power to observe the energy consumed by the dollhouse. Try to adjust the Potential slider and Device sliders to maximize the total power consumed by the dollhouse:

Summary

- Electric power is the energy per unit time converted by an electric circuit into another form of energy.
- The formula for electric power is $P=I^2R$.
- The electric energy transferred to a resistor in a time period is equal to the electric power multiplied by time, $E=Pt$, and can also be calculated using $E=I^2Rt$.
- Electric companies measure their energy sales in a large number of joules called a kilowatt hour (kWh) which is equivalent to 3.6×10^6 J.

Review

1. A 2-way light bulb for a 110. V lamp has filament that uses power at a rate of 50.0 W and another filament that uses power at a rate of 100. W. Find the resistance of these two filaments.
2. Find the power dissipation of a 1.5 A lamp operating on a 12 V battery.
3. A high voltage (4.0×10^5 V) power transmission line delivers electrical energy from a generating station to a substation at a rate of 1.5×10^9 W. What is the current in the lines?
4. A toaster oven indicates that it operates at 1500 W on a 110 V circuit. What is the resistance of the oven?

Explore More

Use this resource to answer the questions that follow.

1. What is the definition of electrical power?
2. What happens to the electrical energy that is not converted into work?

Additional Resources

Study Guide: Electrical Systems Study Guide

Interactive: Electric Analogies

Videos:

11.5.2: Controlling Current in Electric Circuits

Examples of devices that help protect users from electricity

Figure 17.2.1

The amount of electric current that passes through a circuit depends on the voltage pushing the current and the resistance opposing the flow of current. Circuits are designed for a specific voltage and resistance so that the current flow will be known. If the current is too large, the heat loss can damage the circuit, burn the resistors, or even burn the surrounding objects. Even with circuits designed to prevent such situations, accidents can happen, such as when wires touch each other unexpectedly. To prevent disasters, electrical circuits contain devices such as electrical fuses, circuit breakers, and surge protectors to control the amount of current in the circuit. Examples of these devices are shown above, and explained below.

Controlling Current in Electric Circuits

In the wiring of a building, the wires carrying the current in and out are different and never touch directly. The charge passing through the circuit always passes through an appliance (which acts as a resistor) or through another resistor, which limits the amount of current that can flow through a circuit. Appliances are designed to keep current at a relatively low level for safety purposes. The appropriate voltage and resistance in a circuit keeps the current in control and keeps the circuit safe. It is possible, however, for something to happen that causes the wire bringing the current in to come into contact with either the wire carrying the current out or the ground wire, thus causing what is called a **short circuit**. In a short circuit, some or all of the resistance is cut out of the circuit allowing the voltage to push a huge current through the wires.

For example, if a circuit has a potential difference of 100 volts and it is intended for the circuit to contain a 100 ohm resistance, then the wires carrying the current for this circuit will be designed for 1.0 amp. If that 100 ohm resistance is suddenly cut out of the circuit and only 0.10 ohm resistance remains, then the voltage will push 1000 amps of current through the circuit. This current overheats the wires and may damage the circuits or start a fire in the walls or the appliance. There are many possible causes of a short circuit, one possible cause could be something overheating, melting wires, and thereby fusing the circuit closed, bypassing the resistance. Another cause might be something damaging the insulation of a wire, allowing the incoming and grounds wires to touch. In any case, once the resistance is lost, the voltage pushes a huge amount of charge through the wires causing them to overheat.

There are multiple ways to prevent these situations from occurring. The three most common protective devices are fuses, circuit breakers, and surge protectors.

Fuses

An electrical **fuse** is a device that sacrifices itself to provide over-current protection in an electrical circuit.

Conducting wires have a small electrical resistance. The electrical resistance of a conducting wire is related to the type of metal used and inversely related to the cross-sectional area of the wire. The larger the diameter of a wire, the greater the cross-sectional area, and the lower the electrical resistance will be. When electrons flow through a wire, they repel each other and interfere with the passage of other electrons. If there are a very large number of electrons passing through a wire with small diameter, the electrons will experience a greater resistance than if the wire diameter were larger and the electrons had more space between them.

Close-up image of an electrical fuse

Figure 17.2.2

The image above is one type of electrical fuse. This fuse has a specially designed wire inside glass housing. The wire inside the glass is made of a particular metal and with a particular thickness to give the wire an exact resistance. This resistance will allow passage of normal current plus a marginal percentage more. If, however, the current exceeds the allowed value, the wire in the fuse will heat and melt, thus interrupting current flow through the circuit. The fuse wire is placed in the glass housing and a fuse mount to insulate the melting wire from its surroundings. Any unintended increase in current, such as short circuits, overloading, mismatched loads, or device failure will cause the fuse to burn out – protecting the other parts of the circuit from overheating and further damage.

This 'wire in glass' fuse is only one type of electrical fuse. Most fuses consist of a wire with small cross-sectional area compared to the circuit conductors, mounted between electrical terminals, and enclosed in a non-combustible housing. Regardless of the specific type of fuse, it is placed such that it carries all the current passing through the protected circuit. If the current flow gets too high, the fuse will melt. This destroys the fuse, but protects the remainder of the circuit.

Circuit Breakers

The problem with fuses is they only work once. Every time you blow a fuse, you have to replace it with a new one. Like a fuse, a **circuit breaker** opens the circuit if the current reaches unsafe levels. Unlike a fuse, however, a circuit breaker can be used over and over again. The basic circuit breaker consists of a simple **switch**, connected to an electromagnet. The diagram below shows a typical basic circuit breaker design.

Diagram of a circuit breaker

Figure 17.2.3

The red wire is the electrical circuit and is closed when the two contact points are connected. When the switch is in the on position, electricity can flow through the red circuit, through the electromagnet, and out into the rest of the circuit.

The electricity magnetizes the electromagnet. Increasing the current boosts the electromagnet's magnetic force, while decreasing the current reduces the magnetic force. When the current reaches an unsafe level, the electromagnet's magnetic field becomes strong enough to act on the soft iron bar, which is holding the left red wire in contact. When the soft iron bar is pulled down, the spring attached to the left red wire will lift the wire and break the circuit, causing the electricity to shut off. Once the circuit is broken, the electromagnet no longer emits a magnetic field, and the system can be reset by pushing the reset button. The reset button pushes the left red wire back in contact with the right red wire and also re-engages the soft iron bar that holds the contact in place.

More advanced circuit breakers use electronic components (semiconductor devices) to monitor current levels rather than simple electrical devices. These elements are a lot more precise, and they shut down the circuit more quickly, but they are also a lot more expensive. For this reason, most houses still use conventional electric circuit breakers.

When circuit breakers pop open due to excessive current, you do not have to replace the circuit breaker, you simply push a reset button. Of course, if the reason that the circuit breaker popped open is still present (like a short circuit), the circuit will simply pop open again when you reset it. When a circuit breaker pops open, you should determine the reason it opened and fix the problem before resetting the circuit breaker.

Surge Protection

A standard surge protector passes the electrical current along from the outlet to a number of electrical and electronic devices plugged into the power strip. If the voltage from the outlet **surges or spikes** (rises above the accepted level) the surge protector diverts the extra electricity into the outlet's grounding wire.

The most common type of surge protector contains a component called a **metal oxide varistor**, or **MOV**, which diverts the extra voltage. As you can see in the diagram below, an MOV forms a connection between the hot power line and the grounding line.

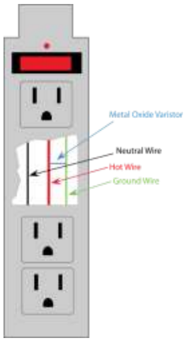

Figure 17.2.4

An MOV has three parts: a piece of metal oxide material in the middle and two semiconductors that join it to the power and grounding lines. These semiconductors have a variable resistance that is dependent on voltage. When voltage is below a certain level, the electrons in the semiconductors flow in such a way as to create a very high resistance. When the voltage exceeds that level, the electrons behave differently, creating a much lower resistance. When the voltage is low, an MOV does nothing. When voltage is too high, an MOV can conduct a lot of current to the ground line, thus eliminating the extra voltage.

As soon as the extra current is diverted into the MOV and to ground, the voltage in the hot line returns to a normal level, causing the MOV's resistance to shoot up again. In this way, the MOV only diverts the surge current, while allowing the standard current to continue powering whatever machines are connected to the surge protector. Essentially, the MOV acts as a pressure release valve, only opening when the voltage exceeds set values.

Have you ever wondered how the electricity in your house works? How can you turn the TV on without causing the lights in your living room to flicker or dim? How can the refrigerator stay on when the lights in the kitchen are off? Explore these questions and more in the Dollhouse simulation below:

Summary

- An electrical fuse is a device that sacrifices itself to provide over-current protection in an electrical circuit. If too much currentflows, the fuse wire rises to a higher temperature and melts, thus opening the circuit, but destroying the fuse.
- A circuit breaker opens a circuit as soon as the current climbs to unsafe levels, and can be used repeatedly.
- A standard surge protector passes a constant electrical current forward by diverting any extra electricity into a grounding wire.

Review

1. As a wire conductor increases in diameter, the resistance _________________.
    1. increases
    2. decreases
    3. stays the same
    4. changes color
2. The purpose of a fuse in an electrical circuit is to
    1. add another resistor
    2. increase the circuit voltage
    3. limit the maximum current in the circuit
    4. none of these
3. What is the purpose of a circuit breaker in an electrical circuit?
4. In some wire-in-glass fuses, the wire is not the same diameter all the way across the fuse. Many such fuses have one part of the wire that is considerably more narrow than the remainder of the wire. What is the purpose of this narrowing of the fuse wire?

Explore More

Use this resource to answer the questions that follow.

1. What happens to the fuse wire when the recommended current in a circuit is exceeded?
2. What is a common mistake made by homeowners when replacing burned fuses?

Additional Resources

Study Guide: Electrical Systems Study Guide

Real World Application: Circuit Breakers

Interactive: Flashlight

Videos:

11.5.3: Series Circuits

Figure 17.3.1

Resistors, including electrical appliances, have a particular current at which they operate most effectively and safely, and excessive current can cause irreparable damage. Therefore, it is important to limit the amount of current that may pass through a particular electrical circuit. There are a number of safety devices used in electrical circuits to limit the current: fuses, circuit breakers, and surge suppressors. When fuses, such as those shown above, are placed in an electrical circuit, all the current must pass through the wire in the fuse.

Series Circuits

Electrical circuits are often modeled by using water in a river. The potential energy of the water is the highest at the source of the river and decreases as the water flows down the river toward the end. When the water reaches the ocean, its potential energy has become zero. The circuit shown below has a similar situation. The current in this circuit is drawn in the direction of the electron flow. It starts at the battery on the left, where electrons leave the negative terminal and travel around the circuit. Since all of the current travels across each resistor, these resistors are said to be in **series**. A series circuit is one in which all of the current must pass through every resistor in the circuit. Returning to the water analogy, there is only one riverbed from the top of the mountain to the ocean.

Three resistors in series

Figure 17.3.2

Consider the series circuit sketched above. This circuit has a voltage drop for the entire circuit of 120 V and has three resistors connected in series. The current in this circuit is drawn in terms of electron flow. The electrons leave the potential difference source at the negative terminal and flow through the three resistors, starting with R3. Though they have a small amount of resistance, the resistance of the connecting wires is so small in relation to the resistors that we ignore it. Therefore, we say that there is no voltage drop when the current passes through the connecting wires. The voltage drops occur when the current passes through each of the resistors and the total voltage drop for the entire circuit is equal to the sum of the voltage drops through the three resistors.

$$V_T=V_1+V_2+V_3$$

The current through each of the resistors must be exactly the same because the current in a series circuit is the same everywhere. The current is moving in the entire circuit at the same time.

$$I_T=I_1=I_2=I_3$$

Since the current passes through each resistor, the total resistance in the circuit is equal to the sum of the resistors. In the circuit above, the total resistance is:

$$R_T=R_1+R_2+R_3=30\ \Omega+15\ \Omega+15\ \Omega=60\ \Omega$$

Therefore, the total current and the current through each resistor is

$$I=V/R=120\ V/60\ \Omega=2.0\ A.$$

The individual voltage drops can be calculated using the current through each resistor and each resistor's individual resistance.

$$V_1=I_1R_1=(2.0\ A)(30\ \Omega)=60\ V$$

$$V_2=I_2R_2=(2.0\ A)(15\ \Omega)=30\ V$$

$$V_3=I_3R_3=(2.0\ A)(15\ \Omega)=30\ V$$

✓ Example 17.3.1

Four 15 Ω resistors are connected in series with a 45 V battery. What is the current in the circuit?

Solution

$$RT=15\ \Omega+15\ \Omega+15\ \Omega+15\ \Omega=60\ \Omega$$

$$I=VR=45\ V60\ \Omega=0.75\ A$$

Adjust the the resistance provided by each light bulb in the circuit and observe what happens to the total resistance in the PLIX Interactive below:

Adjust the total voltage of the circuit and observe what happens to the individual voltage drops in the PLIX Interactive below:

Summary

- A series circuit is one in which all of the current must pass through every resistor in the circuit.
- $V_T=V_1+V_2+V_3$
- $I_T=I_1=I_2=I_3$
- $R_T=R_1+R_2+R_3$

Review

1. There are three 20.0 Ω resistors connected in series across a 120 V generator.
    1. What is the total resistance of the circuit?
    2. What is the current in the circuit?
    3. What is the voltage drop across one of the resistors?
2. A 5.00 Ω, a 10.0 Ω, and a 15.0 Ω resistor are connected in series across a 90.0 V battery.
    1. What is the equivalent resistance of the circuit?
    2. What is the current in the circuit?
    3. What is the voltage drop across the 5.00 Ω resistor?
3. A 5.00 Ω and a 10.0 Ω resistor are connected in series across an unknown voltage. The total current in the circuit is 3.00 A.
    1. What is the equivalent resistance of the circuit?
    2. What is the current through the 5.00 Ω resistor?
    3. What is the total voltage drop for the entire circuit?

Explore More

Use this resource to answer the questions that follow.

1. How do the voltage drops across the two light bulbs in the video relate to the total voltage drop for the entire circuit?
2. In the video, what was the assumed voltage drop for the connecting wires and the switch?
3. What was the current through the second light bulb as compared to the current through the first light bulb?

Additional Resources

Study Guide: Electrical Systems Study Guide

Real World Application: Lights in Series

Interactive: Dollhouse, Marquee Lights

Video:

11.5.4: Parallel Circuits

Photograph of a circuit board

Figure 17.4.1

Electrical circuits are everywhere: skyscrapers, jumbo jets, arcade games, lights, heating, security... Very few complex things work without electrical circuits. Since the late 1970's, electrical circuits have primarily looked like this. The circuits are formed by a thin layer of conducting material deposited on the surface of an insulating board. Individual components are soldered to the interconnecting circuits. Circuit boards are vastly more complicated than the series circuits previously discussed, but operate on many similar principles.

Parallel Circuits

Parallel circuits are circuits in which the charges leaving the potential source have different paths they can follow to get back to the source. In the sketch below, the current leaves the battery, passes through the orange switch, and then has three different paths available to complete the circuit. Each individual electron in this circuit passes through only one of the light bulbs. After the current passes through the switch, it divides into three pieces and each piece passes through one of the bulbs. The three pieces of current rejoin after the light bulbs and continue in the circuit to the potential source.

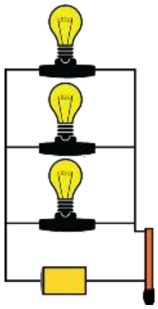

Figure 17.4.2

In the design of this parallel circuit, each resistor (light bulb) is connected across the battery as if the other two resistors were not present. Remember that the current going through each resistor goes through only the one resistor. Therefore, the voltage drop across each resistor must be equal to the total voltage drop though the circuit.

$$V_T=V_1=V_2=V_3$$

The total current passing through the circuit will be the sum of the individual currents passing through each resistor.

$$I_T=I_1+I_2+I_3$$

If we return to the analogy of a river, a parallel circuit is the same as the river breaking into three streams, which later rejoin to one river again. The amount of water flowing in the river is equal to the sum of the amounts of water flowing in the individual streams.

Ohm's Law applies to resistors in parallel, just as it did to resistors in a series. The current flowing through each resistor is equal to the total voltage drop divided by the resistance in that resistor.

$$I_1=V_T/R_1 \text{ and } I_2=V_T/R_2 \text{ and } I_3=V_T/R_3$$

$$\text{Since } I_T=I_1+I_2+I_3,$$

$$\text{then } I_T=V_T/R_1+V_T/R_2+V_T/R_3,$$

$$\text{and } V_T/R_T=V_T/R_1+V_T/R_2+V_T/R_3.$$

If we divide both sides of the final equation by V_T, we get the relationship between the total resistance of the circuit and the individual parallel resistances in the circuit. The total resistance is sometimes called the **equivalent resistance.**

$$1/R_T=1/R_1+1/R_2+1/R_3$$

Consider the parallel circuit sketched below.

The voltage drop for the entire circuit is 90. V. Therefore, the voltage drop in each of the resistors is also 90. V.

Three resistors in parallel

Figure 17.4.3

The current through each resistor can be found using the voltage drop and the resistance of that resistor:

$$I_1=V_T/R_1=90.\ \text{V}/60.\ \Omega=1.5\ \text{A}\ I_2=V_T/R_2=90.\ \text{V}/30.\ \Omega=3.0\ \text{A}\ I_3=V_T/R_3=90.\ \text{V}/30.\ \Omega=3.0\ \text{A}$$

The total current through the circuit would be the sum of the three currents in the individual resistors.

$$I_T=I_1+I_2+I_3=1.5\ \text{A}+3.0\ \text{A}+3.0\ \text{A}=7.5\ \text{A}$$

The equivalent resistance for this circuit is found using the equation above.

$$1/R_T=1/R_1+1/R_2+1/R_3=1/60.\ \Omega+1/30.\ \Omega+1/30.\ \Omega=160.\ \Omega+260.\ \Omega+260.\ \Omega=560.\ \Omega$$

$$R_T=60.\ \Omega/5=12\ \Omega$$

The equivalent resistance for the circuit could also be found by using the total voltage drop and the total current.

$$R_T=V_T/I_T=90.\ \Omega/7.5\ \text{A}=12\ \Omega$$

Use the PLIX below to observe the relationship between the individual voltage drops and the total voltage in a parallel circuit:

Use the PLIX Interactive below to observe the differences between parallel and series circuits. Adjust the number of resistors in the circuit and observe the difference in total resistance between a parallel and series circuit. Then, try to explain the different levels of bulb brightness observed in each circuit configuration:

Summary

- Parallel electrical circuits have multiple paths the current may take.
- $V_T=V_1=V_2=V_3$
- $I_T=I_1+I_2+I_3$
- $1/R_T=1/R_1+1/R_2+1/R_3$

Review

1. Three 15.0 Ω resistors are connected in parallel and placed across a 30.0 V potential difference.
    1. What is the equivalent resistance of the parallel circuit?
    2. What is the total current through the circuit?
    3. What is the current through a single branch of the circuit?
2. A 12.0 Ω and a 15.0 Ω resistor are connected in parallel and placed across a 30.0 V potential.
    1. What is the equivalent resistance of the parallel circuit?
    2. What is the total current through the circuit?
    3. What is the current through each branch of the circuit?
3. A 120.0 Ω resistor, a 60.0 Ω resistor, and a 40.0 Ω resistor are connected in parallel and placed across a potential difference of 12.0 V.
    1. What is the equivalent resistance of the parallel circuit?
    2. What is the total current through the circuit?
    3. What is the current through each branch of the circuit?

Explore More

Use this resource to answer the questions that follow.

1. Why do the light bulbs glow less brightly when connected across a 120 V source in a series circuit than when connected across the same 120 V source in a parallel circuit?
2. Why do the other bulbs go dark when one bulb is removed in the series circuit but the other bulbs do not go dark when one bulb is removed in the parallel circuit?

Additional Resources

Study Guide: Electrical Systems Study Guide

Real World Application: Lights in Parallel

Interactives: Dollhouse, Marquee Lights

Video:

11.5.5: Capacitors

Capacitors on a circuit board

Figure 17.6.1

The circuit boards found in your computer, phone, calculator, and pretty much every other electronic device you own often look much like the one shown above. Many circuit boards have capacitors, including this one. Capacitors can be used to smooth out electrical impulses or to turn constant electric currents into a series of impulses. Use the simulation below to begin your exploration of capacitors and discover how capacitors are used in a camera's flash:

The Capacitor

Sharing Charge

We already know that insulators are materials that do not allow electrons to flow through them easily. When you place excess electrons on an insulator, the electrons remain where you put them and do not move around. Conversely, conductors are materials that allow electrons to flow thorugh them freely. Since electrons repel each other, excess electrons on a conductor move to positions as far apart as possible. The difference can be seen in the image below, where the electrons on the insulator remain clumped near each other, while the electrons on the conductor have spread out to cover the whole surface.

Illustration of the distribution of charge in insulators and conductors

Figure 17.6.2

Consider a charged conductor and an uncharged conductor. When the charged conductor is touched to the uncharged conductor, as far as the electrons are concerned, it has become one large piece of conducting material. The electrons on the charged object will run onto the uncharged object until the density of the charge is evenly distributed over the entire surface of both objects. If the objects are the same size, the charge will be shared equally throughout. This method is occasionally used to divide a charge by half.

The earth is also a conductor. Touching a charged object to the earth is called **grounding**. When you touch a conductor to the earth, you allow the earth to share the charge. Since the earth is billions of times bigger than the object, the earth takes nearly all of the charge. The charged object that was grounded now has zero charge.

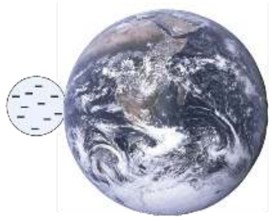

Figure 17.6.3

It is very easy to ground an object. All that is necessary is to touch a conducting wire to both the object and the earth. Electrical devices that run the risk of picking up a large static charge are grounded, meaning they are connected to the earth via such a conducting wire. Virtually all household appliances, especially washers and dryers, are grounded in this way to eliminate static charge. Similarly, large trucks, especially gasoline tankers, are grounded via a large chain hanging off the back to prevent sparks when fuel is being unloaded.

Spheres, whether hollow or filled, will always have the excess charge on the surface. In hollow spheres, the only place for an electron to exist is on the surface. Similarly, in a solid conducting sphere all the excess charge sits on the surface. This conclusion is a result of Gauss's Law, which tells us that the symmetry of the sphere and the fact that the electric field within the sphere is 0 forces the charge to the outside.

Capacitors Store Charge

Pieter Van Musschenbroek, a Dutch physician, invented a device in 1746 that could store electric charge. Though he named the device a Leyden jar, similar devices today are called **capacitors.** A typical capacitor consists of a pair of parallel plates of area Aseparated by a small distance d. The space between the two plates is most often filled with an insulator and frequently the plates are rolled into the form of a cylinder.

If voltage is applied to a capacitor, it quickly becomes charged. One of the parallel plates acquires a negative charge and the other an equal amount of positive charge. For a given capacitor, the amount of charge, Q, acquired by each plate is proportional to the potential difference, V.

$$Q=CV$$

Q is the charge in coulombs, V is the voltage in volts, and C is the particular capacitor's constant of proportionality. C is also called the **capacitance** of the capacitor. The capacitance is the voltage the capacitor can reach before it discharges, allowing the voltage across the capacitor to drop to zero and the current to cross the capacitor. The SI unit for capacitance, according to the equation above, will be coulombs/volt, and this unit has been given the name **farad** (F). Most capacitors have capacitances in the range of one picofarad (10^{-12} F) to one microfarad (10^{-6} F).

✓ Example 17.6.1

A sphere has a potential difference between it and the earth of 60.0 V when charged with 3.0×10^{-6} C. What is the capacitance?

Solution

$C=Q/V=3.0\times10^{-6}$ coulombs/60.0 volts$=5.0\times10^{-8}$ farads

Have you ever used an electronic device that can be controlled by human touch? These types of touch screens are really amazing and utilize capacitance to work. When you place your finger on the screen, you disrupt the electric field patterns that exist in the device and cause changes to its capacitive system, which the computer understands. Launch the Touch Screen simulation below to play around and learn more:

Summary

- Since electrons repel each other, when excess electrons are placed on a conductor, they will move to positions as far away from each other as possible.
- When a charged conductor is touched to an uncharged conductor, the electrons will migrate until the density of the charge becomes evenly distributed over the entire surface.
- Touching a charged object to the earth is called grounding.
- A charged conducting sphere will always have all the excess charge on its surface.
- A typical capacitor consists of a pair of parallel plates, separated by a small distance.
- Q=CV, where Q is the charge in coulombs, V is the voltage in volts, and C is the constant of proportionality, or capacitance.

Review

1. The two plates of a capacitor hold $+2.5\times10^{-3}$ C and -2.5×10^{-3} C of charge when the potential difference is 950 V. What is the capacitance?
2. The potential difference between two parallel wires in air is 120. V. They hold equal and opposite charges of 9.5×10^{-11} C. What is the capacitance of the two wires?
3. How much charge flows from a 12.0 V battery when it is connected to a 9.00 microfarad capacitor?

Explore More

Use this resource to answer the questions that follow.

1. What do capacitors do?
2. What are the units of capacitance?
3. What is the formula for capacitance?

Additional Resources

Study Guide: Electrical Systems Study Guide

Videos: Capacitance - Overview

Real World Application: Storing Static Electricity

Interactive: Flashing Neon Light

11.5.6: Electric Hazards and the Human Body

Learning Objectives

- Define thermal hazard, shock hazard, and short circuit.
- Explain what effects various levels of current have on the human body.

There are two known hazards of electricity—thermal and shock. A **thermal hazard** is one where excessive electric power causes undesired thermal effects, such as starting a fire in the wall of a house. A **shock hazard** occurs when electric current passes through a person. Shocks range in severity from painful, but otherwise harmless, to heart-stopping lethality. This section considers these hazards and the various factors affecting them in a quantitative manner.

Thermal Hazards

Electric power causes undesired heating effects whenever electric energy is converted to thermal energy at a rate faster than it can be safely dissipated. A classic example of this is the **short circuit**, a low-resistance path between terminals of a voltage source. An example of a short circuit is shown in Figure $11.5.6.1$. Insulation on wires leading to an appliance has worn through, allowing the two wires to come into contact. Such an undesired contact with a high voltage is called a *short*. Since the resistance of the short, r, is very small, the power dissipated in the short, $P = V^2/r$, is very large. For example, if V is 120 V and r is $0.100\ \Omega$, then the power is 144 kW, *much* greater than that used by a typical household appliance. Thermal energy delivered at this rate will very quickly raise the temperature of surrounding materials, melting or perhaps igniting them.

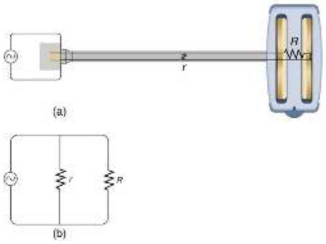

Figure 11.5.6.1: A short circuit is an undesired low-resistance path across a voltage source. (a) Worn insulation on the wires of a toaster allow them to come into contact with a low resistance r. Since $P = V^2/r$, thermal power is created so rapidly that the cord melts or burns. (b) A schematic of the short circuit.

One particularly insidious aspect of a short circuit is that its resistance may actually be decreased due to the increase in temperature. This can happen if the short creates ionization. These charged atoms and molecules are free to move and, thus, lower the resistance r. Since $P = V^2/r$, the power dissipated in the short rises, possibly causing more ionization, more power, and so on. High voltages, such as the 480-V AC used in some industrial applications, lend themselves to this hazard, because higher voltages create higher initial power production in a short.

Another serious, but less dramatic, thermal hazard occurs when wires supplying power to a user are overloaded with too great a current. As discussed in the previous section, the power dissipated in the supply wires is $P = I^2R_w$, where R_w is the resistance of the wires and I the current flowing through them. If either I or R_w is too large, the wires overheat. For example, a worn appliance cord (with some of its braided wires broken) may have $R_w = 2.00\ \Omega$ rather than the $0.100\ \Omega$ it should be. If 10.0 A of current passes through the cord, then $P = I^2R_w = 200\text{ W}$ is dissipated in the cord—much more than is safe. Similarly, if a wire with a $0.100 - \Omega$ resistance is meant to carry a few amps, but is instead carrying 100 A, it will severely overheat. The power dissipated in the wire will in that case be $P = 1000\text{ W}$. Fuses and circuit breakers are used to limit excessive currents. (See Figure $11.5.6.2$and Figure $11.5.6.3$) Each device opens the circuit automatically when a sustained current exceeds safe limits.

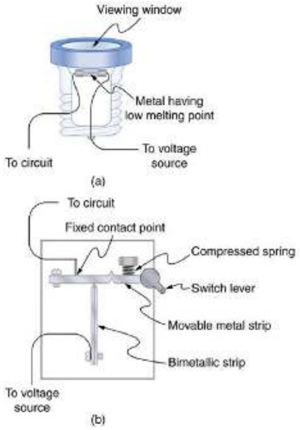

Figure 11.5.6.2: (a) A fuse has a metal strip with a low melting point that, when overheated by an excessive current, permanently breaks the connection of a circuit to a voltage source. (b) A circuit breaker is an automatic but restorable electric switch. The one shown here has a bimetallic strip that bends to the right and into the notch if overheated. The spring then forces the metal strip downward, breaking the electrical connection at the points.

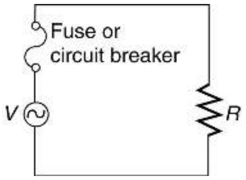

Figure 11.5.6.3: Schematic of a circuit with a fuse or circuit breaker in it. Fuses and circuit breakers act like automatic switches that open when sustained current exceeds desired limits.

Fuses and circuit breakers for typical household voltages and currents are relatively simple to produce, but those for large voltages and currents experience special problems. For example, when a circuit breaker tries to interrupt the flow of high-voltage electricity, a spark can jump across its points that ionizes the air in the gap and allows the current to continue flowing. Large circuit breakers found in power-distribution systems employ insulating gas and even use jets of gas to blow out such sparks. Here AC is safer than DC, since AC current goes through zero 120 times per second, giving a quick opportunity to extinguish these arcs.

Shock Hazards

Electrical currents through people produce tremendously varied effects. An electrical current can be used to block back pain. The possibility of using electrical current to stimulate muscle action in paralyzed limbs, perhaps allowing paraplegics to walk, is under study. TV dramatizations in which electrical shocks are used to bring a heart attack victim out of ventricular fibrillation (a massively irregular, often fatal, beating of the heart) are more than common. Yet most electrical shock fatalities occur because a current put the heart into fibrillation. A pacemaker uses electrical shocks to stimulate the heart to beat properly. Some fatal shocks do not produce burns, but warts can be safely burned off with electric current (though freezing using liquid nitrogen is now more common). Of course, there are consistent explanations for these disparate effects. The major factors upon which the effects of electrical shock depend are

1. The amount of current I
2. The path taken by the current
3. The duration of the shock
4. The frequency f of the current ($f=0$ for DC)

Table 11.5.6.1gives the effects of electrical shocks as a function of current for a typical accidental shock. The effects are for a shock that passes through the trunk of the body, has a duration of 1 s, and is caused by 60-Hz power.

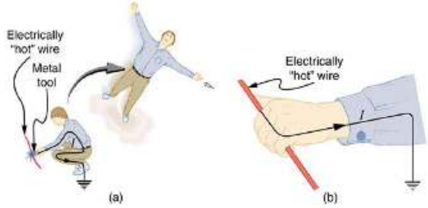

Figure 11.5.6.4: An electric current can cause muscular contractions with varying effects. (a) The victim is "thrown" backward by involuntary muscle contractions that extend the legs and torso. (b) The victim can't let go of the wire that is stimulating all the muscles in the hand. Those that close the fingers are stronger than those that open them.

Table 11.5.6.1: Effects of Electrical Shock as a Function of Current[1]

Current (mA)	Effect
1	Threshold of sensation
5	Maximum harmless current
10–20	Onset of sustained muscular contraction; cannot let go for duration of shock; contraction of chest muscles may stop breathing during shock
50	Onset of pain
100–300+	Ventricular fibrillation possible; often fatal
300	Onset of burns depending on concentration of current
6000 (6 A)	Onset of sustained ventricular contraction and respiratory paralysis; both cease when shock ends; heartbeat may return to normal; used to defibrillate the heart

Our bodies are relatively good conductors due to the water in our bodies. Given that larger currents will flow through sections with lower resistance (to be further discussed in the next chapter), electric currents preferentially flow through paths in the human body that have a minimum resistance in a direct path to earth. The earth is a natural electron sink. Wearing insulating shoes, a requirement in many professions, prohibits a pathway for electrons by providing a large resistance in that path. Whenever working with high-power tools (drills), or in risky situations, ensure that you do not provide a pathway for current flow (especially through the heart).

Very small currents pass harmlessly and unfelt through the body. This happens to you regularly without your knowledge. The threshold of sensation is only 1 mA and, although unpleasant, shocks are apparently harmless for currents less than 5 mA. A great number of safety rules take the 5-mA value for the maximum allowed shock. At 10 to 20 mA and above, the current can stimulate sustained muscular contractions much as regular nerve impulses do. People sometimes say they were knocked across the room by a shock, but what really happened was that certain muscles contracted, propelling them in a manner not of their own choosing. (See Figure 11.5.6.4(a).) More frightening, and potentially more dangerous, is the "can't let go" effect illustrated in Figure 11.5.6.4(b). The muscles that close the fingers are stronger than those that open them, so the hand closes involuntarily on the wire shocking it. This can prolong the shock indefinitely. It can also be a danger to a person trying to rescue the victim, because the rescuer's hand may close about the victim's wrist. Usually the best way to help the victim is to give the fist a hard knock/blow/jar with an insulator or to throw an insulator at the fist. Modern electric fences, used in animal enclosures, are now pulsed on and off to allow people who touch them to get free, rendering them less lethal than in the past.

Greater currents may affect the heart. Its electrical patterns can be disrupted, so that it beats irregularly and ineffectively in a condition called "ventricular fibrillation." This condition often lingers after the shock and is fatal due to a lack of blood circulation. The threshold for ventricular fibrillation is between 100 and 300 mA. At about 300 mA and above, the shock can cause burns, depending on the concentration of current—the more concentrated, the greater the likelihood of burns.

Very large currents cause the heart and diaphragm to contract for the duration of the shock. Both the heart and breathing stop. Interestingly, both often return to normal following the shock. The electrical patterns on the heart are completely erased in a manner that the heart can start afresh with normal beating, as opposed to the permanent disruption caused by smaller currents that can put the heart into ventricular fibrillation. The latter is something like scribbling on a blackboard, whereas the former completely erases it. TV dramatizations of electric shock used to bring a heart attack victim out of ventricular fibrillation also show large paddles. These are used to spread out current passed through the victim to reduce the likelihood of burns.

Current is the major factor determining shock severity (given that other conditions such as path, duration, and frequency are fixed, such as in the table and preceding discussion). A larger voltage is more hazardous, but since $I = V/R$, the severity of the shock depends on the combination of voltage and resistance. For example, a person with dry skin has a resistance of about $200\ \text{k}\Omega$. If he comes into contact with 120-V AC, a current $I = (120\ \text{V})/(200\ \text{k}\Omega) = 0.6\ \text{mA}$ passes harmlessly through him. The same person soaking wet may have a resistance of $10.0\ \text{k}\Omega$ and the same 120 V will produce a current of 12 mA—above the “can’t let go” threshold and potentially dangerous.

Most of the body’s resistance is in its dry skin. When wet, salts go into ion form, lowering the resistance significantly. The interior of the body has a much lower resistance than dry skin because of all the ionic solutions and fluids it contains. If skin resistance is bypassed, such as by an intravenous infusion, a catheter, or exposed pacemaker leads, a person is rendered **microshock sensitive**. In this condition, currents about 1/1000 those listed in Table $11.5.6.1$produce similar effects. During open-heart surgery, currents as small as $20\ \mu\text{A}$ can be used to still the heart. Stringent electrical safety requirements in hospitals, particularly in surgery and intensive care, are related to the doubly disadvantaged microshock-sensitive patient. The break in the skin has reduced his resistance, and so the same voltage causes a greater current, and a much smaller current has a greater effect.

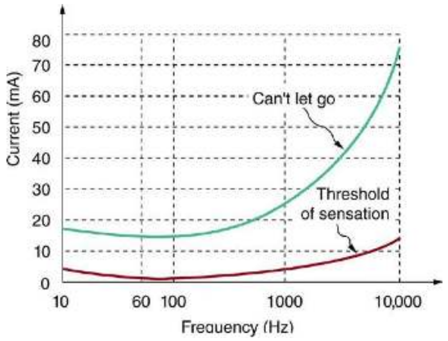

Figure 11.5.6.5: Graph of average values for the threshold of sensation and the “can’t let go” current as a function of frequency. The lower the value, the more sensitive the body is at that frequency.

Factors other than current that affect the severity of a shock are its path, duration, and AC frequency. Path has obvious consequences. For example, the heart is unaffected by an electric shock through the brain, such as may be used to treat manic depression. And it is a general truth that the longer the duration of a shock, the greater its effects. Figure $11.5.6.5$presents a graph that illustrates the effects of frequency on a shock. The curves show the minimum current for two different effects, as a function of frequency. The lower the current needed, the more sensitive the body is at that frequency. Ironically, the body is most sensitive to frequencies near the 50- or 60-Hz frequencies in common use. The body is slightly less sensitive for DC ($f = 0$), mildly confirming Edison’s claims that AC presents a greater hazard. At higher and higher frequencies, the body becomes progressively less sensitive to any effects that involve nerves. This is related to the maximum rates at which nerves can fire or be stimulated. At very high frequencies, electrical current travels only on the surface of a person. Thus a wart can be burned off with very high frequency current without causing the heart to stop. (Do not try this at home with 60-Hz AC!) Some of the spectacular demonstrations of electricity, in which high-voltage arcs are passed through the air and over people’s bodies, employ high frequencies and low currents. (See Figure $11.5.6.6$)

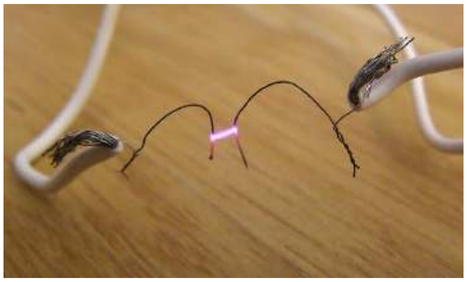

Figure 11.5.6.6: Is this electric arc dangerous? The answer depends on the AC frequency and the power involved. (credit: Khimich Alex, Wikimedia Commons)

Section Summary

- The two types of electric hazards are thermal (excessive power) and shock (current through a person).
- Shock severity is determined by current, path, duration, and AC frequency.
- Table 11.5.6.1lists shock hazards as a function of current.
- Figure 11.5.6.5graphs the threshold current for two hazards as a function of frequency.

Footnotes

- 1 For an average male shocked through trunk of body for 1 s by 60-Hz AC. Values for females are 60–80% of those listed.

Glossary

thermal hazard

a hazard in which electric current causes undesired thermal effects

shock hazard

when electric current passes through a person

short circuit

also known as a “short,” a low-resistance path between terminals of a voltage source

microshock sensitive

a condition in which a person’s skin resistance is bypassed, possibly by a medical procedure, rendering the person vulnerable to electrical shock at currents about 1/1000 the normally required level

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11.6: End of Chapter Activity

End of Chapter Activity: Creating a Lesson Plan on Electricity with AI and Bloom's Taxonomy

Now that you have explored the fundamentals of electricity, it's time to put your knowledge into practice. Teaching complex scientific concepts like electricity to 1st graders can be challenging, as it requires simplifying the material to make it accessible and engaging for young minds. Your task is to create a succinct lesson plan for 1st graders that introduces them to the basics of electricity. To help you with this, you will use AI tools and incorporate Bloom's Taxonomy to ensure a comprehensive learning experience. This lesson plan will go towards your digital notebook, a portfolio filled with lesson plans, activities, and labs for future use.

Activity Prompt:

Objective: Use AI and Bloom's Taxonomy to develop a lesson plan that effectively teaches 1st graders about the fundamentals of electricity, including concepts such as electric circuits, conductors and insulators, and the importance of electricity in daily life.

Understanding the Concepts:

> **Knowledge (Remembering):** Define key terms related to electricity, such as electric current, circuit, conductor, and insulator.
>
> **Comprehension (Understanding):** Explain these concepts in simple, age-appropriate language, using relatable examples and visuals.

Planning the Lesson:

> **Application:** Design an engaging activity that allows students to observe and understand basic concepts of electricity. For example, create a simple circuit using a battery, wires, and a small light bulb to demonstrate how electricity flows.
>
> **Analysis:** Use AI tools to create visual aids or interactive simulations that illustrate how electricity flows through a circuit. For instance, create a simple animation showing electrons moving through a wire to light up a bulb.

Deepening Understanding:

> **Synthesis (Creating):** Ask students to create their own simple circuits using safe, child-friendly materials. For example, they could use conductive playdough to build circuits that light up LED bulbs.
>
> **Evaluation:** Have students discuss and reflect on their creations and observations. Encourage them to think about how electricity is used in their daily lives and the importance of being safe around electricity.

Using AI in the Classroom:

Explore AI tools like educational apps or platforms that provide interactive content for teaching about electricity. Use these tools to create quizzes, flashcards, or interactive stories that reinforce the lesson's concepts.

Use AI to assess student understanding through formative assessments and provide instant feedback.

Deliverable:

Submit a detailed lesson plan that includes:

1. **A brief overview of the key concepts covered:** Outline the foundational concepts of electricity that will be taught.
2. **A description of the activities and experiments designed:** Detail the hands-on activities and experiments you will use to help students understand these concepts.
3. **Examples of AI tools used and how they enhance the learning experience:** Describe the AI tools you plan to incorporate, such as simulations or interactive quizzes, and explain how they will help students grasp complex concepts.
4. **An explanation of how Bloom's Taxonomy was applied in the lesson plan to ensure a well-rounded educational experience:** Illustrate how each level of Bloom's Taxonomy (Remembering, Understanding, Applying, Analyzing, Creating, and Evaluating) is addressed in your lesson plan.

Additionally, include a creative project component where students create a digital story or an interactive game using AI tools to explain a concept related to electricity.

Example Lesson Plan:

Grade: 1st Grade
Topic: Electricity
Duration: 1 Week

Overview:
Students will learn about basic concepts of electricity, including electric circuits, conductors, and insulators, through engaging activities and creative projects.

Day 1: Introduction to Electricity

Objective: Define basic concepts related to electricity and provide examples.

- **Remembering:** Define key terms (electric current, circuit, conductor, insulator).
- **Understanding:** Explain the concepts using examples from everyday life (e.g., turning on a light switch).

Activity:
Watch a short, animated video (created using AI tools) explaining what electricity is and how it works.

Day 2: Exploring Simple Circuits

Objective: Understand how a simple electric circuit works.

- **Applying:** Conduct an experiment to create a simple circuit using a battery, wires, and a small light bulb.

Activity:
Students use batteries, wires, and small light bulbs to build simple circuits. They observe how the bulb lights up when the circuit is complete.

Day 3: Conductors and Insulators

Objective: Identify materials that are conductors and insulators.

- **Applying:** Conduct an experiment to test different materials to see if they conduct electricity.

Activity:
Students test various materials (e.g., metal, plastic, wood) to see which ones can complete a circuit and light up a bulb. They classify the materials as conductors or insulators.

Day 4: Creative Project – Building with Conductive Playdough

Objective: Create simple circuits using conductive playdough.

- **Creating:** Students build their own circuits using conductive playdough and LED bulbs.

Activity:
In groups, students use conductive and insulating playdough to create fun shapes and circuits that light up LED bulbs. They document their creations and explain how the circuits work.

Day 5: Reflection and Digital Story

Objective: Reflect on what they have learned and create a digital story about electricity.

- **Evaluating:** Discuss and reflect on the experiments and activities.
- **Creating:** Use AI tools to create a digital story that explains a concept related to electricity.

Activity:
Students create a short digital story using AI tools, such as a simple animation or interactive book, that explains what they have learned about electricity. They can include drawings, photos of their projects, and voice recordings.

By incorporating these strategies and activities, educators can effectively teach 1st graders about electricity, helping them understand and appreciate the fundamental concepts in a fun and engaging way.

11.7: End of Chapter Key Terms

Definition: Electricity

- **Electric Charge**: A property of subatomic particles that causes them to experience a force when placed in an electric field; measured in coulombs (C).
- **Electric Current**: The flow of electric charge through a conductor; measured in amperes (A).
- **Voltage (Electric Potential)**: The difference in electric potential energy per unit charge between two points; measured in volts (V).
- **Resistance**: The opposition to the flow of electric current through a conductor; measured in ohms (Ω).
- **Conductor**: A material that allows the flow of electric charge, typically metals like copper and aluminum.
- **Insulator**: A material that resists the flow of electric charge, such as rubber, glass, and plastic.
- **Semiconductor**: A material with electrical conductivity between that of a conductor and an insulator, used in electronic devices.
- **Ohm's Law**: A law stating that the current (I) through a conductor between two points is directly proportional to the voltage (V) across the two points and inversely proportional to the resistance (R); expressed as $V = IR$.
- **Series Circuit**: An electric circuit in which components are connected end-to-end so that the current flows through each component in turn.
- **Parallel Circuit**: An electric circuit in which components are connected across common points or junctions, providing multiple paths for the current.
- **Direct Current (DC)**: Electric current that flows in one direction only.
- **Alternating Current (AC)**: Electric current that periodically reverses direction.
- **Power**: The rate at which electrical energy is transferred by an electric circuit; measured in watts (W).
- **Capacitor**: An electrical component used to store energy electrostatically in an electric field, consisting of two conductive plates separated by an insulating material.
- **Inductor**: An electrical component that stores energy in a magnetic field when electric current flows through it, typically a coil of wire.
- **Electric Field**: A field around a charged particle that exerts a force on other charged particles.
- **Electromotive Force (EMF)**: The energy provided per charge by an energy source, such as a battery or generator; measured in volts (V).
- **Coulomb's Law**: A law stating that the force between two charged objects is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.
- **Electric Potential Energy**: The energy a charged particle has due to its position in an electric field.
- **Circuit Breaker**: An automatically operated electrical switch designed to protect an electrical circuit from damage caused by overload or short circuit.
- **Fuse**: A safety device consisting of a strip of wire that melts and breaks an electric circuit if the current exceeds a safe level.
- **Grounding**: The process of connecting an electrical circuit to the Earth to ensure safety by preventing buildup of voltages that could cause electric shocks.
- **Short Circuit**: A low-resistance connection between two points in an electric circuit, allowing an excessive current to flow.
- **Superconductor**: A material that can conduct electricity without resistance when cooled to very low temperatures.
- **Electrolysis**: A process that uses an electric current to drive a non-spontaneous chemical reaction.
- **Kirchhoff's Laws**: Two rules regarding the conservation of current and voltage in electrical circuits: Kirchhoff's Current Law (KCL) states that the total current entering a junction equals the total current leaving; Kirchhoff's Voltage Law (KVL) states that the sum of the electrical potential differences around any closed circuit is zero.
- **Transformers**: Electrical devices that transfer electrical energy between two or more circuits through electromagnetic induction, used to increase or decrease voltage levels.
- **Resistor**: An electrical component that limits or regulates the flow of electrical current in an electronic circuit.
- **Electric Power**: The rate at which electrical energy is transferred by an electric circuit, typically expressed as the product of current and voltage ($P = IV$).

- **Electromagnetic Induction**: The production of an electromotive force across a conductor when it is exposed to a varying magnetic field.
- **Faraday’s Law**: A law stating that the induced electromotive force in any closed circuit is equal to the negative of the time rate of change of the magnetic flux through the circuit.
- **Lenz’s Law**: A law stating that the direction of an induced current is such that it will oppose the change in magnetic flux that produced it.
- **Electrical Conductivity**: The measure of a material’s ability to conduct an electric current.
- **Dielectric**: An insulating material that can be polarized by an electric field, increasing a capacitor's ability to store charge.
- **Electrostatic Force**: The force between two charges at rest, described by Coulomb’s Law.
- **Ampere**: The unit of electric current in the International System of Units (SI), equivalent to one coulomb per second.
- **Volt**: The unit of electric potential difference and electromotive force in the International System of Units (SI), equivalent to one joule per coulomb.
- **Ohm**: The unit of electrical resistance in the International System of Units (SI), equivalent to one volt per ampere.
- **Watt**: The unit of power in the International System of Units (SI), equivalent to one joule per second.

11.E: Electricity (Exercise)

Conceptual Questions

10.1: Static Electricity and Charge- Conservation of Charge

1. There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electricity?

2. Why do most objects tend to contain nearly equal numbers of positive and negative charges?

10.2: Coulomb's Law

3. Figure $11.E.1$ shows the charge distribution in a water molecule, which is called a polar molecule because it has an inherent separation of charge. Given water's polar character, explain what effect humidity has on removing excess charge from objects.

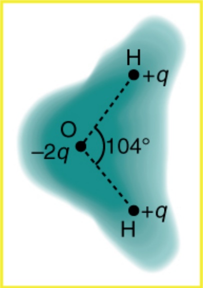


Figure $11.E.1$: Schematic representation of the outer electron cloud of a neutral water molecule. The electrons spend more time near the oxygen than the hydrogens, giving a permanent charge separation as shown. Water is thus a *polar molecule*. It is more easily affected by electrostatic forces than molecules with uniform charge distributions.

4. Using Figure $11.E.1$, explain, in terms of Coulomb's law, why a polar molecule (such as in Figure $11.E.1$) is attracted by both positive and negative charges.

5. Given the polar character of water molecules, explain how ions in the air form nucleation centers for rain droplets.

10.3: Electric Field- Concept of a Field Revisited

6. Why must the test charge q in the definition of the electric field be vanishingly small?

7. Are the direction and magnitude of the Coulomb force unique at a given point in space? What about the electric field?

10.4: Electric Field Lines

8. Compare and contrast the Coulomb force field and the electric field. To do this, make a list of five properties for the Coulomb force field analogous to the five properties listed for electric field lines. Compare each item in your list of Coulomb force field properties with those of the electric field—are they the same or different? (For example, electric field lines cannot cross. Is the same true for Coulomb field lines?)

10.5: Electric Potential and Potential Energy

9. Voltage is the common word for potential difference. Which term is more descriptive, voltage or potential difference?

10. If the voltage between two points is zero, can a test charge be moved between them with zero net work being done? Can this necessarily be done without exerting a force? Explain.

11. What is the relationship between voltage and energy? More precisely, what is the relationship between potential difference and electric potential energy?

12. Voltages are always measured between two points. Why?

13. How are units of volts and electron volts related? How do they differ?

10.6: Conductors and Applications of Electrostatics

14. Is the object in Figure $11.E.2$ a conductor or an insulator? Justify your answer.

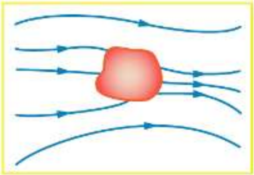


Figure $11.E.2$

15. If the electric field lines in the figure above were perpendicular to the object, would it necessarily be a conductor? Explain.

16. The discussion of the electric field between two parallel conducting plates, in this module states that edge effects are less important if the plates are close together. What does close mean? That is, is the actual plate separation crucial, or is the ratio of plate separation to plate area crucial?

17. Would the self-created electric field at the end of a pointed conductor, such as a lightning rod, remove positive or negative charge from the conductor? Would the same sign charge be removed from a neutral pointed conductor by the application of a similar externally created electric field? (The answers to both questions have implications for charge transfer utilizing points.)

18. Why is a golfer with a metal club over her shoulder vulnerable to lightning in an open fairway? Would she be any safer under a tree?

19. Can the belt of a Van de Graaff accelerator be a conductor? Explain.

20. Are you relatively safe from lightning inside an automobile? Give two reasons.

21. Discuss pros and cons of a lightning rod being grounded versus simply being attached to a building.

22. Using the symmetry of the arrangement, show that the net Coulomb force on the charge q at the center of the square below (Figure $11.E.3$) is zero if the charges on the four corners are exactly equal.

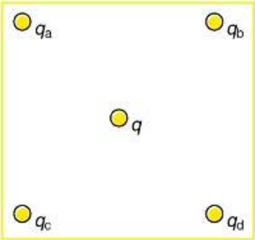


Figure $11.E.3$: Four point charges q_a, q_b, q_c, and q_d lie on the corners of a square and q is located at its center.

23. (a) Using the symmetry of the arrangement, show that the electric field at the center of the square in Figure $11.E.3$ is zero if the charges on the four corners are exactly equal.

(b) Show that this is also true for any combination of charges in which $q_a = q_d$ and $q_b = q_c$

24. (a) What is the direction of the total Coulomb force on q in Figure $11.E.3$ if q is negative, $q_a = q_c$ and both are negative, and $q_b = q_c$ and both are positive?

(b) What is the direction of the electric field at the center of the square in this situation?

25. Considering Figure $11.E.3$, suppose that $q_a = q_d$ and $q_b = q_c$. First show that q is in static equilibrium. (You may neglect the gravitational force.) Then discuss whether the equilibrium is stable or unstable, noting that this may depend on the signs of the charges and the direction of displacement of q from the center of the square.

26. If $q_a = 0$ in Figure $11.E.3$, under what conditions will there be no net Coulomb force on q?

27. In regions of low humidity, one develops a special "grip" when opening car doors, or touching metal door knobs. This involves placing as much of the hand on the device as possible, not just the ends of one's fingers. Discuss the induced charge and explain why this is done.

28. Tollbooth stations on roadways and bridges usually have a piece of wire stuck in the pavement before them that will touch a car as it approaches. Why is this done?

29. Suppose a woman carries an excess charge. To maintain her charged status can she be standing on ground wearing just any pair of shoes? How would you discharge her? What are the consequences if she simply walks away?

10.7: Current

30. Can a wire carry a current and still be neutral—that is, have a total charge of zero? Explain.

31. Car batteries are rated in ampere-hours ($A \cdot h$). To what physical quantity do ampere-hours correspond (voltage, charge, . . .), and what relationship do ampere-hours have to energy content?

32. Why are two conducting paths from a voltage source to an electrical device needed to operate the device?

33. In cars, one battery terminal is connected to the metal body. How does this allow a single wire to supply current to electrical devices rather than two wires?

34. Why isn't a bird sitting on a high-voltage power line electrocuted? Contrast this with the situation in which a large bird hits two wires simultaneously with its wings.

10.8: Ohm's Law- Resistance and Simple Circuits

35. The IR drop across a resistor means that there is a change in potential or voltage across the resistor. Is there any change in current as it passes through a resistor? Explain.

36. How is the IR drop in a resistor similar to the pressure drop in a fluid flowing through a pipe?

10.9: Electric Power and Energy

37. Why do incandescent lightbulbs grow dim late in their lives, particularly just before their filaments break?

38. The power dissipated in a resistor is given by $P = V^2/R$, which means power decreases if resistance increases. Yet this power is also given by $P = I^2R$, which means power increases if resistance increases. Explain why there is no contradiction here.

10.10: Resistors in Series and Parallel

39. A switch has a variable resistance that is nearly zero when closed and extremely large when open, and it is placed in series with the device it controls. Explain the effect the switch in Figure $11.E.4$ has on current when open and when closed.

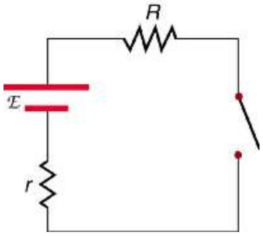


Figure $11.E.4$: A switch is ordinarily in series with a resistance and voltage source. Ideally, the switch has nearly zero resistance when closed but has an extremely large resistance when open. (Note that in this diagram, the script E represents the voltage (or electromotive force) of the battery.)

40. What is the voltage across the open switch in Figure $11.E.4$?

41. There is a voltage across an open switch, such as in Figure $11.E.4$. Why, then, is the power dissipated by the open switch small?

42. Why is the power dissipated by a closed switch, such as in Figure $11.E.4$, small?

43. A student in a physics lab mistakenly wired a light bulb, battery, and switch as shown in Figure $11.E.5$. Explain why the bulb is on when the switch is open, and off when the switch is closed. (Do not try this—it is hard on the battery!)

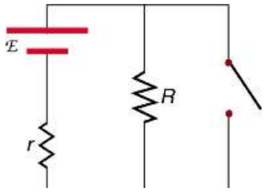


Figure 11.E.5: A wiring mistake put this switch in parallel with the device represented by R. (Note that in this diagram, the script E represents the voltage (or electromotive force) of the battery.)

44. Knowing that the severity of a shock depends on the magnitude of the current through your body, would you prefer to be in series or parallel with a resistance, such as the heating element of a toaster, if shocked by it? Explain.

45. Would your headlights dim when you start your car's engine if the wires in your automobile were superconductors? (Do not neglect the battery's internal resistance.) Explain.

46. Some strings of holiday lights are wired in series to save wiring costs. An old version utilized bulbs that break the electrical connection, like an open switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 40 identical bulbs, what is the normal operating voltage of each? Newer versions use bulbs that short circuit, like a closed switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 39 remaining identical bulbs, what is then the operating voltage of each?

47. If two household lightbulbs rated 60 W and 100 W are connected in series to household power, which will be brighter? Explain.

48. Suppose you are doing a physics lab that asks you to put a resistor into a circuit, but all the resistors supplied have a larger resistance than the requested value. How would you connect the available resistances to attempt to get the smaller value asked for?

49. Before World War II, some radios got power through a "resistance cord" that had a significant resistance. Such a resistance cord reduces the voltage to a desired level for the radio's tubes and the like, and it saves the expense of a transformer. Explain why resistance cords become warm and waste energy when the radio is on.

50. Some light bulbs have three power settings (not including zero), obtained from multiple filaments that are individually switched and wired in parallel. What is the minimum number of filaments needed for three power settings?

10.11: Electric Hazards and the Human Body

51. Using an ohmmeter, a student measures the resistance between various points on his body. He finds that the resistance between two points on the same finger is about the same as the resistance between two points on opposite hands—both are several hundred thousand ohms. Furthermore, the resistance decreases when more skin is brought into contact with the probes of the ohmmeter. Finally, there is a dramatic drop in resistance (to a few thousand ohms) when the skin is wet. Explain these observations and their implications regarding skin and internal resistance of the human body.

52. What are the two major hazards of electricity?

53. Why isn't a short circuit a shock hazard?

54. What determines the severity of a shock? Can you say that a certain voltage is hazardous without further information?

55. An electrified needle is used to burn off warts, with the circuit being completed by having the patient sit on a large butt plate. Why is this plate large?

56. Some surgery is performed with high-voltage electricity passing from a metal scalpel through the tissue being cut. Considering the nature of electric fields at the surface of conductors, why would you expect most of the current to flow from the sharp edge of the scalpel? Do you think high- or low-frequency AC is used?

57. Some devices often used in bathrooms, such as hairdryers, often have safety messages saying "Do not use when the bathtub or basin is full of water." Why is this so?

58. We are often advised to not flick electric switches with wet hands, dry your hand first. We are also advised to never throw water on an electric fire. Why is this so?

59. Before working on a power transmission line, linemen will touch the line with the back of the hand as a final check that the voltage is zero. Why the back of the hand?

60. Why is the resistance of wet skin so much smaller than dry, and why do blood and other bodily fluids have low resistances?

61. Could a person on intravenous infusion (an IV) be microshock sensitive?

62. In view of the small currents that cause shock hazards and the larger currents that circuit breakers and fuses interrupt, how do they play a role in preventing shock hazards?

Problems & Exercises

10.1: Static Electricity and Charge- Conservation of Charge

63. Common static electricity involves charges ranging from nanocoulombs to microcoulombs.

(a) How many electrons are needed to form a charge of –2.00 nC

(b) How many electrons must be removed from a neutral object to leave a net charge of $0.500\ \mu\text{C}$?

Solution

(a) 1.25×1010

(b) 3.13×1012

64. If 1.80×10^{20} electrons move through a pocket calculator during a full day's operation, how many coulombs of charge moved through it?

65. To start a car engine, the car battery moves 3.75×10^{21} electrons through the starter motor. How many coulombs of charge were moved?

Solution

-600 C

66. A certain lightning bolt moves 40.0 C of charge. How many fundamental units of charge $|q_e|$ is this?

10.2: Coulomb's Law

67. What is the repulsive force between two pith balls that are 8.00 cm apart and have equal charges of – 30.0 nC?

68. (a) How strong is the attractive force between a glass rod with a $0.700\ \mu\text{C}$ charge and a silk cloth with a $-0.600\ \mu\text{C}$ charge, which are 12.0 cm apart, using the approximation that they act like point charges?

(b) Discuss how the answer to this problem might be affected if the charges are distributed over some area and do not act like point charges.

Solution

(a) 0.263 N

(b) If the charges are distributed over some area, there will be a concentration of charge along the side closest to the oppositely charged object. This effect will increase the net force.

69. Two point charges exert a 5.00 N force on each other. What will the force become if the distance between them is increased by a factor of three?

70. Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased?

Solution

The separation decreased by a factor of 5.

71. How far apart must two point charges of 75.0 nC (typical of static electricity) be to have a force of 1.00 N between them?

72. If two equal charges each of 1 C each are separated in air by a distance of 1 km, what is the magnitude of the force acting between them? You will see that even at a distance as large as 1 km, the repulsive force is substantial because 1 C is a very significant amount of charge.

73. Bare free charges do not remain stationary when close together. To illustrate this, calculate the acceleration of two isolated protons separated by 2.00 nm (a typical distance between gas atoms).

Solution

$$\begin{aligned} F &= k\frac{|q_1 q_2|}{r^2} = ma \Rightarrow a = \frac{kq^2}{mr^2} \\ &= \frac{\left(9.00\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(1.60\times10^{-19}\ \mathrm{m}\right)^2}{\left(1.67\times10^{-27}\ \mathrm{kg}\right)\left(2.00\times10^{-9}\ \mathrm{m}\right)^2} \\ &= 3.45\times10^{16}\ \mathrm{m/s^2} \end{aligned}$$

74. (a) By what factor must you change the distance between two point charges to change the force between them by a factor of 10?

(b) Explain how the distance can either increase or decrease by this factor and still cause a factor of 10 change in the force.

Solution

(a) 3.2

(b) If the distance increases by 3.2, then the force will decrease by a factor of 10 ; if the distance decreases by 3.2, then the force will increase by a factor of 10. Either way, the force changes by a factor of 10.

75. Suppose you have a total charge q_{tot} that you can split in any manner. Once split, the separation distance is fixed. How do you split the charge to achieve the greatest force?

76. (a) Common transparent tape becomes charged when pulled from a dispenser. If one piece is placed above another, the repulsive force can be great enough to support the top piece's weight. Assuming equal point charges (only an approximation), calculate the magnitude of the charge if electrostatic force is great enough to support the weight of a 10.0 mg piece of tape held 1.00 cm above another.

(b) Discuss whether the magnitude of this charge is consistent with what is typical of static electricity.

Solution

(a) 1.04×10^{-9} C

(b) This charge is approximately 1 nC, which is consistent with the magnitude of charge typical for static electricity

77. (a) Find the ratio of the electrostatic to gravitational force between two electrons.

(b) What is this ratio for two protons?

(c) Why is the ratio different for electrons and protons?

78. At what distance is the electrostatic force between two protons equal to the weight of one proton?

79. A certain five cent coin contains 5.00 g of nickel. What fraction of the nickel atoms' electrons, removed and placed 1.00 m above it, would support the weight of this coin? The atomic mass of nickel is 58.7, and each nickel atom contains 28 electrons and 28 protons.

Solution

1.02×10^{-11}

80. (a) Two point charges totaling 8.00 μC exert a repulsive force of 0.150 N on one another when separated by 0.500 m. What is the charge on each?

(b) What is the charge on each if the force is attractive?

10.3: Electric Field- Concept of a Field Revisited

81. What is the magnitude and direction of an electric field that exerts a 2.00×10^{-5} N upward force on a $-1.75\ \mu\mathrm{C}$ charge?

82. What is the magnitude and direction of the force exerted on a $3.50\ \mu\mathrm{C}$ charge by a 250 N/C electric field that points due east?

Solution

8.75×10^{-4} N

83. Calculate the magnitude of the electric field 2.00 m from a point charge of 5.00 mC (such as found on the terminal of a Van de Graaff).

84. (a) What magnitude point charge creates a 10,000 N/C electric field at a distance of 0.250 m?

(b) How large is the field at 10.0 m?

Solution

(a) 6.94×10^{-8} C

(b) 6.25 N/C

85. Calculate the initial (from rest) acceleration of a proton in a 5.00×10^{6} N/C electric field (such as created by a research Van de Graaff). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

86. (a) Find the direction and magnitude of an electric field that exerts a 4.80×10^{-17} N westward force on an electron.

(b) What magnitude and direction force does this field exert on a proton?

Solution

(a) 300 N/C (east)

(b) 4.80×10^{-17} N (east)

10.4: Electric Field Lines

87. (a) Sketch the electric field lines near a point charge $+q$. (b) Do the same for a point charge $-3.00q$

88. Sketch the electric field lines a long distance from the charge distributions shown in Figure 10.4.4 (a) and (b)

89. Figure $11.E.6$ shows the electric field lines near two charges q_1 and q_2. What is the ratio of their magnitudes? (b) Sketch the electric field lines a long distance from the charges shown in the figure.

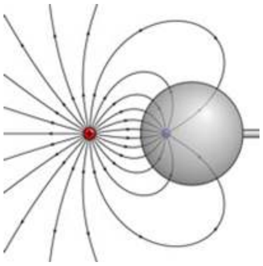


Figure $11.E.6$: The electric field near two charges.

90. Sketch the electric field lines in the vicinity of two opposite charges, where the negative charge is three times greater in magnitude than the positive. (See Figure $11.E.6$ for a similar situation).

10.5: Electric Potential and Potential Energy

91. Find the ratio of speeds of an electron and a negative hydrogen ion (one having an extra electron) accelerated through the same voltage, assuming non-relativistic final speeds. Take the mass of the hydrogen ion to be $1.67\times10^{-27}\ \mathrm{kg}$.

Solution

42.8

92. An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce X-rays. Non-relativistically, what would be the maximum speed of these electrons?

93. A bare helium nucleus has two positive charges and a mass of 6.64×10^{-27} kg

(a) Calculate its kinetic energy in joules at 2.00% of the speed of light.

(b) What is this in electron volts?

(c) What voltage would be needed to obtain this energy?

Integrated Concepts

94. Singly charged gas ions are accelerated from rest through a voltage of 13.0 V. At what temperature will the average kinetic energy of gas molecules be the same as that given these ions?

Solution

1.00×10^5 K

Integrated Concepts

95. The temperature near the center of the Sun is thought to be 15 million degrees Celsius $(1.5 \times 10^{7\circ}\mathrm{C})$. Through what voltage must a singly charged ion be accelerated to have the same energy as the average kinetic energy of ions at this temperature?

Integrated Concepts

96. (a) What is the average power output of a heart defibrillator that dissipates 400 J of energy in 10.0 ms?

(b) Considering the high-power output, why doesn't the defibrillator produce serious burns?

Solution

(a) 4×10^4 W

(b) A defibrillator does not cause serious burns because the skin conducts electricity well at high voltages, like those used in defibrillators. The gel used aids in the transfer of energy to the body, and the skin doesn't absorb the energy, but rather lets it pass through to the heart.

Integrated Concepts

97. A lightning bolt strikes a tree, moving 20.0 C of charge through a potential difference of 1.00×10^2 MV.

(a) What energy was dissipated?

(b) What mass of water could be raised from $15^\circ\mathrm{C}$ to the boiling point and then boiled by this energy?

(c) Discuss the damage that could be caused to the tree by the expansion of the boiling steam.

Integrated Concepts

98. A 12.0 V battery-operated bottle warmer heats 50.0 g of glass, 2.50×10^2 g of baby formula, and 2.00×10^2 g of aluminum from $20.0^\circ\mathrm{C}$ to $90.0^\circ\mathrm{C}$.

(a) How much charge is moved by the battery?

(b) How many electrons per second flow if it takes 5.00 min to warm the formula? (Hint: Assume that the specific heat of baby formula is about the same as the specific heat of water.)

Solution

(a) $7.40 \times 10^3\mathrm{C}$

(b) 1.54×10^{20} electrons per second

Integrated Concepts

99. A battery-operated car utilizes a 12.0 V system. Find the charge the batteries must be able to move in order to accelerate the 750 kg car from rest to 25.0 m/s, make it climb a 2.00×10^2 m high hill, and then cause it to travel at a constant 25.0 m/s by exerting a 5.00×10^2 N force for an hour.

Solution

$3.89 \times 10^6 \mathrm{C}$

Integrated Concepts

100. Fusion probability is greatly enhanced when appropriate nuclei are brought close together, but mutual Coulomb repulsion must be overcome. This can be done using the kinetic energy of high-temperature gas ions or by accelerating the nuclei toward one another.

(a) Calculate the potential energy of two singly charged nuclei separated by 1.00×10^{-12} m by finding the voltage of one at that distance and multiplying by the charge of the other.

(b) At what temperature will atoms of a gas have an average kinetic energy equal to this needed electrical potential energy?

10.7: Current

101. What is the current in milliamperes produced by the solar cells of a pocket calculator through which 4.00 C of charge passes in 4.00 h?

Solution

0.278 mA

102. A total of 600 C of charge passes through a flashlight in 0.500 h. What is the average current?

103. What is the current when a typical static charge of $0.250\ \mu\mathrm{C}$ moves from your finger to a metal doorknob in $1.00\ \mu$s??

Solution

0.250 A

104. Find the current when 2.00 nC jumps between your comb and hair over a $0.500 - \mu$s time interval.

105. A large lightning bolt had a 20,000-A current and moved 30.0 C of charge. What was its duration?

Solution

1.50ms

106. The 200-A current through a spark plug moves 0.300 mC of charge. How long does the spark last?

107. (a) A defibrillator passes 12.0 A of current through the torso of a person for 0.0100 s. How much charge moves?

(b) How many electrons pass through the wires connected to the patient? (See figure two problems earlier.)

Solution

(a) 0.120 C

(b) 7.50×10^{17} electrons

108. A clock battery wears out after moving 10,000 C of charge through the clock at a rate of 0.500 mA.

(a) How long did the clock run?

(b) How many electrons per second flowed?

109. The batteries of a submerged non-nuclear submarine supply 1000 A at full speed ahead. How long does it take to move Avogadro's number (6.02×10^{23}) of electrons at this rate?

Solution

96.3 s

110. Electron guns are used in X-ray tubes. The electrons are accelerated through a relatively large voltage and directed onto a metal target, producing X-rays.

(a) How many electrons per second strike the target if the current is 0.500 mA?

(b) What charge strikes the target in 0.750 s?

112. A large cyclotron directs a beam of He^{++} nuclei onto a target with a beam current of 0.250 mA.

(a) How many He^{++} nuclei per second is this?

(b) How long does it take for 1.00 C to strike the target?

(c) How long before 1.00 mol of He^{++} nuclei strike the target?

Solution

(a) $7.81\times 10^{14}\ \mathrm{He}^{++}\mathrm{nuclei/s}$

(b) $4.00\times 10^{3}\ \mathrm{s}$

(c) $7.71\times 10^{8}\ \mathrm{s}$

10.8: Ohm's Law- Resistance and Simple Circuits

113. What current flows through the bulb of a 3.00-V flashlight when its hot resistance is $3.60\ \Omega$?

Solution

0.833 A

114. Calculate the effective resistance of a pocket calculator that has a 1.35-V battery and through which 0.200 mA flows.

115. What is the effective resistance of a car's starter motor when 150 A flows through it as the car battery applies 11.0 V to the motor?

Solution

$$7.33\times 10^{-2}\ \Omega$$

116. How many volts are supplied to operate an indicator light on a DVD player that has a resistance of $140\ \Omega$, given that 25.0 mA passes through it?

117. (a) Find the voltage drop in an extension cord having a $0.0600-\Omega$ resistance and through which 5.00 A is flowing.

(b) A cheaper cord utilizes thinner wire and has a resistance of $0.300\ \Omega$. What is the voltage drop in it when 5.00 A flows?

(c) Why is the voltage to whatever appliance is being used reduced by this amount? What is the effect on the appliance?

Solution

(a) 0.300 V

(b) 1.50 V

(c) The voltage supplied to whatever appliance is being used is reduced because the total voltage drop from the wall to the final output of the appliance is fixed. Thus, if the voltage drop across the extension cord is large, the voltage drop across the appliance is significantly decreased, so the power output by the appliance can be significantly decreased, reducing the ability of the appliance to work properly.

118. A power transmission line is hung from metal towers with glass insulators having a resistance of $1.00\times 10^{9}\ \Omega$ What current flows through the insulator if the voltage is 200 kV? (Some high-voltage lines are DC.)

10.9: Electric Power and Energy

119. What is the power of a $1.00\ \times 10^{2}\mathrm{MV}$ lightning bolt having a current of $2.00\times 10^{4}\ \mathrm{A}$?

Solution

$2.00\times 10^{12}\ \mathrm{W}$

120. What power is supplied to the starter motor of a large truck that draws 250 A of current from a 24.0-V battery hookup?

121. A charge of 4.00 C of charge passes through a pocket calculator's solar cells in 4.00 h. What is the power output, given the calculator's voltage output is 3.00 V? (See Figure $11.E.7$.)

Figure $11.E.7$: The strip of solar cells just above the keys of this calculator convert light to electricity to supply its energy needs. (credit: Evan-Amos, Wikimedia Commons)

122. How many watts does a flashlight that has 6.00×10^2 C pass through it in 0.500 h use if its voltage is 3.00 V?

123. Find the power dissipated in each of these extension cords:

(a) an extension cord having a $0.0600 - \Omega$ resistance and through which 5.00 A is flowing;

(b) a cheaper cord utilizing thinner wire and with a resistance of $0.300\ \Omega$.

Solution

(a) 1.50 W

(b) 7.50 W

124. Verify that the units of a volt-ampere are watts, as implied by the equation $P = IV$.

125. Show that the units $1\ \mathrm{V}^2/\Omega = 1\ \mathrm{W}$, as implied by the equation $P = V^2/R$.

Solution

$$\frac{\mathrm{V}^2}{\Omega} = \frac{\mathrm{V}^2}{\mathrm{V/A}} = \mathrm{AV} = \left(\frac{\mathrm{C}}{\mathrm{s}}\right)\left(\frac{\mathrm{J}}{\mathrm{C}}\right) = \frac{\mathrm{J}}{\mathrm{s}} = 1\ \mathrm{W}$$

126. Show that the units $1\ \mathrm{A}^2 \cdot \Omega = 1\ \mathrm{W}$, as implied by the equation $P = I^2R$.

127. Verify the energy unit equivalence that $1\ \mathrm{kW \cdot h} = 3.60 \times 10^6\ \mathrm{J}$.

Solution

$$1\ \mathrm{kW \cdot h} = \left(\frac{1 \times 10^3\ \mathrm{J}}{1\ \mathrm{s}}\right)(1\ \mathrm{h})\left(\frac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right) = 3.60 \times 10^6\ \mathrm{J}$$

128. Electrons in an X-ray tube are accelerated through 1.00×10^2 kV and directed toward a target to produce X-rays. Calculate the power of the electron beam in this tube if it has a current of 15.0 mA.

129. An electric water heater consumes 5.00 kW for 2.00 h per day. What is the cost of running it for one year if electricity costs 12.0 cents $/\mathrm{kW \cdot h}$? See Figure $11.E.8$.

Figure $11.E.8$: On-demand electric hot water heater. Heat is supplied to water only when needed. (credit: aviddavid, Flickr)

Solution

$438/y

130. With a 1200-W toaster, how much electrical energy is needed to make a slice of toast (cooking time = 1 minute)? At 9.0 cents $/\mathrm{kW \cdot h}$, how much does this cost?

131. Some makes of older cars have 6.00-V electrical systems.

(a) What is the hot resistance of a 30.0-W headlight in such a car?

(b) What current flows through it?

132. Alkaline batteries have the advantage of putting out constant voltage until very nearly the end of their life. How long will an alkaline battery rated at $1.00 \mathrm{~A} \cdot \mathrm{h}$ and 1.58 V keep a 1.00-W flashlight bulb burning?

Solution

1.58 h

133. A cauterizer, used to stop bleeding in surgery, puts out 2.00 mA at 15.0 kV.

(a) What is its power output?

(b) What is the resistance of the path?

134. The average television is said to be on 6 hours per day. Estimate the yearly cost of electricity to operate 100 million TVs, assuming their power consumption averages 150 W and the cost of electricity averages $12.0 \text{ cents}/\mathrm{kW} \cdot \mathrm{h}$.

Solution

$3.94 billion/year

10.10: Resistors in Series and Parallel

Note: Data taken from figures can be assumed to be accurate to three significant digits.

135. (a) What is the resistance of ten $275-\Omega$ resistors connected in series?

(b) In parallel?

Solution

(a) $2.75 \mathrm{k\Omega}$

(b) $27.5 \ \Omega$

136. (a) What is the resistance of a $1.00 \times 10^2-\Omega$, a $2.50-\mathrm{k}\ \Omega$, and a $4.00-\mathrm{k}\ \Omega$ resistor connected in series?

(b) In parallel?

137. What are the largest and smallest resistances you can obtain by connecting a $36.0-\Omega$, a $50.0-\Omega$, and a $700-\Omega$ resistor together?

Solution

(a) $786 \ \Omega$

(b) $20.3 \ \Omega$

138. An 1800-W toaster, a 1400-W electric frying pan, and a 75-W lamp are plugged into the same outlet in a 15-A, 120-V circuit. (The three devices are in parallel when plugged into the same socket.).

(a) What current is drawn by each device?

(b) Will this combination blow the 15-A fuse?

139. Your car's 30.0-W headlight and 2.40-kW starter are ordinarily connected in parallel in a 12.0-V system. What power would one headlight and the starter consume if connected in series to a 12.0-V battery? (Neglect any other resistance in the circuit and any change in resistance in the two devices.)

Solution

$29.6 \mathrm{~W}$

140. (a) Given a 48.0-V battery and $24.0-\Omega$ and $96.0-\Omega$ resistors, find the current and power for each when connected in series.

(b) Repeat when the resistances are in parallel.

141. Referring to the example combining series and parallel circuits and Figure 10.10.5, calculate I_3 in the following two different ways: (a) from the known values of I and I_2; (b) using Ohm's law for R_3. In both parts explicitly show how you follow the steps in the Problem-Solving Strategies for Series and Parallel Resistors.

Solution

(a) 0.74 A

(b) 0.742 A

142. Referring to Figure: 10.10.5(a) Calculate P_3 and note how it compares with P_3 found in the first two example problems in this module. (b) Find the total power supplied by the source and compare it with the sum of the powers dissipated by the resistors.

143. Refer to Figure 10.10.6 and the discussion of lights dimming when a heavy appliance comes on. (a) Given the voltage source is 120 V, the wire resistance is $0.400\ \Omega$, and the bulb is nominally 75.0 W, what power will the bulb dissipate if a total of 15.0 A passes through the wires when the motor comes on? Assume negligible change in bulb resistance. (b) What power is consumed by the motor?

Solution

(a) 60.8 W

(b) 3.18 kW

144. A 240-kV power transmission line carrying $5.00 \times 10^2\ \mathrm{A}$ is hung from grounded metal towers by ceramic insulators, each having a $1.00 \times 10^9 - \Omega$ resistance. Figure $11.E.9$. (a) What is the resistance to ground of 100 of these insulators? (b) Calculate the power dissipated by 100 of them. (c) What fraction of the power carried by the line is this? Explicitly show how you follow the steps in the Problem-Solving Strategies for Series and Parallel Resistors.

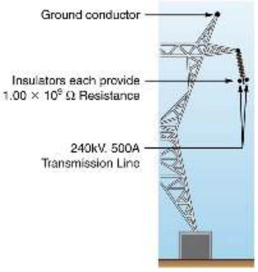


Figure $11.E.9$: High-voltage (240-kV) transmission line carrying $5.00 \times 10^2\ \mathrm{A}$ is hung from a grounded metal transmission tower. The row of ceramic insulators provide $1.00 \times 10^9\ \Omega$ of resistance each.

145. Show that if two resistors R_1 and R_2 are combined and one is much greater than the other $(R_1 >> R_2)$:

(a) Their series resistance is very nearly equal to the greater resistance R_1.

(b) Their parallel resistance is very nearly equal to smaller resistance R_2.

Solution

(a)
$$\begin{gathered} R_s = R_1 + R_2 \\ \Rightarrow R_s \approx R_1\ (R_1 >> R_2) \end{gathered}$$

(b) $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1+R_2}{R_1R_2}$,

so that

$$R_p = \frac{R_1R_2}{R_1+R_2} \approx \frac{R_1R_2}{R_1} = R_2\ (R_1 >> R_2)\ .$$

Unreasonable Results

146. Two resistors, one having a resistance of $145\ \Omega$, are connected in parallel to produce a total resistance of $150\ \Omega$.

(a) What is the value of the second resistance?

(b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Unreasonable Results

147. Two resistors, one having a resistance of $900\ \mathrm{k}\Omega$, are connected in series to produce a total resistance of $0.500\mathrm{M}\ \Omega$.

(a) What is the value of the second resistance?

(b) What is unreasonable about this result?

(c) Which assumptions are unreasonable or inconsistent?

Solution

(a) $-400\ \text{k}\Omega$

(b) Resistance cannot be negative.

(c) Series resistance is said to be less than one of the resistors, but it must be greater than any of the resistors.

10.11: Electric Hazards and the Human Body

148. (a) How much power is dissipated in a short circuit of 240-V AC through a resistance of $0.250\ \Omega$?

(b) What current flows?

Solution

(a) 230 kW

(b) 960 A

149. What voltage is involved in a 1.44-kW short circuit through a $0.100-\Omega$ resistance?

150. Find the current through a person and identify the likely effect on her if she touches a 120-V AC source:

(a) if she is standing on a rubber mat and offers a total resistance of $300\ \text{k}\Omega$;

(b) if she is standing barefoot on wet grass and has a resistance of only $4000\ \text{k}\Omega$.

Solution

(a) 0.400 mA, no effect

(b) 26.7 mA, muscular contraction for duration of the shock (can't let go)

151. While taking a bath, a person touches the metal case of a radio. The path through the person to the drainpipe and ground has a resistance of $4000\ \Omega$. What is the smallest voltage on the case of the radio that could cause ventricular fibrillation?

152. Foolishly trying to fish a burning piece of bread from a toaster with a metal butter knife, a man comes into contact with 120-V AC. He does not even feel it since, luckily, he is wearing rubber-soled shoes. What is the minimum resistance of the path the current follows through the person?

Solution

$1.20 \times 10^5\ \Omega$

153. (a) During surgery, a current as small as $20.0\mu\text{A}$ applied directly to the heart may cause ventricular fibrillation. If the resistance of the exposed heart is $300\ \Omega$, what is the smallest voltage that poses this danger?

(b) Does your answer imply that special electrical safety precautions are needed?

154. (a) What is the resistance of a 220-V AC short circuit that generates a peak power of 96.8 kW?

(b) What would the average power be if the voltage was 120 V AC?

Solution

(a) $1.00\ \Omega$

(b) 14.4 kW

155. A heart defibrillator passes 10.0 A through a patient’s torso for 5.00 ms in an attempt to restore normal beating.

(a) How much charge passed?

(b) What voltage was applied if 500 J of energy was dissipated?

(c) What was the path’s resistance?

(d) Find the temperature increase caused in the 8.00 kg of affected tissue.

Integrated Concepts

156. A short circuit in a 120-V appliance cord has a $0.500-\Omega$ resistance. Calculate the temperature rise of the 2.00 g of surrounding materials, assuming their specific heat capacity is $0.200 \mathrm{cal/g \cdot {}^\circ C}$and that it takes 0.0500 s for a circuit breaker to interrupt the current. Is this likely to be damaging?

Solution

Temperature increases $860^\circ\ \mathrm{C}$. It is very likely to be damaging.

Construct Your Own Problem

157. Consider a person working in an environment where electric currents might pass through her body. Construct a problem in which you calculate the resistance of insulation needed to protect the person from harm. Among the things to be considered are the voltage to which the person might be exposed, likely body resistance (dry, wet, …), and acceptable currents (safe but sensed, safe and unfelt, …).

CHAPTER OVERVIEW

12: Magnetism

12.1: Introduction and Learning Objectives

Chapter Learning Outcomes

1. **Understanding Basic Concepts of Magnetism:**
   - Define magnetism and describe the properties of magnets.
   - Differentiate between magnetic poles and understand their interactions (like poles repel, unlike poles attract).
2. **Exploring Magnetic Fields:**
   - Explain what a magnetic field is and how it is visualized.
   - Understand the concept of magnetic field lines and their significance.
   - Analyze the Earth's magnetic field and its effects.
3. **Studying Electromagnetism:**
   - Explore the relationship between electricity and magnetism.
   - Understand how electric currents create magnetic fields (Ampere's Law).
   - Examine practical applications of electromagnetism, such as in electromagnets, motors, and generators.
4. **Investigating Magnetic Materials:**
   - Identify and classify materials as ferromagnetic, paramagnetic, or diamagnetic.
   - Understand the properties and uses of different magnetic materials.
   - Explore real-world applications of magnetic materials in technology and industry.
5. **Teaching Magnetism in the Classroom:**
   - Develop effective strategies for teaching the principles of magnetism to students.
   - Design and conduct classroom experiments to demonstrate magnetic properties and behaviors.
   - Utilize multimedia resources to enhance student understanding of magnetism.
   - Address common misconceptions about magnetism and provide clear, accurate explanations.
   - Foster interdisciplinary learning by connecting magnetism concepts to real-life applications in various fields, such as engineering, medicine, and environmental science.

Introduction to Magnetism

Magnetism is a fundamental force of nature that plays a crucial role in both everyday life and advanced technology. At its core, magnetism arises from the motion of electric charges, resulting in the attractive and repulsive forces between objects. Understanding the basic concepts of magnetism involves exploring the properties of magnets, the behavior of magnetic poles, and the interactions between them. Magnetic fields, which are regions where magnetic forces can be detected, are visualized through field lines and are instrumental in explaining phenomena such as the Earth's magnetosphere.

The relationship between electricity and magnetism, known as electromagnetism, reveals how electric currents produce magnetic fields, a principle central to numerous technological applications including electromagnets, electric motors, and generators. Investigating different magnetic materials, such as ferromagnetic, paramagnetic, and diamagnetic substances, further illustrates the diverse properties and uses of magnetism in technology and industry. From household appliances to advanced medical imaging techniques, magnetism's applications are vast and varied.

For K-12 educators, making the principles of magnetism accessible and engaging requires a blend of theoretical knowledge and hands-on learning experiences. Classroom experiments demonstrating magnetic fields, the behavior of magnetic materials, and the principles of electromagnetism can make abstract concepts tangible. Utilizing multimedia resources, such as simulations and interactive activities, can further enhance student understanding and interest. By connecting these scientific theories to real-world applications in fields like engineering, medicine, and environmental science, educators can foster an appreciation for the pervasive influence of magnetism and inspire the next generation of scientists and engineers.

12.2: Magnet

Figure 18.1.1

Some countries are using powerful electromagnets to develop high-speed trains, called maglev, or magnetic levitation, trains. These trains use the repulsive force of magnets to float over a guide way, removing the friction of steel wheels and train tracks. Reducing this friction allows the trains to travel at much higher speeds.

Properties of Magnets

Any **magnet**, regardless of its shape, has two ends called poles where the magnetic effect is strongest. If a magnet is suspended by a fine thread, it is found that one pole of the magnet will always point toward the north. This fact has been made use of in navigation since the eleventh century. The pole of the magnet that seeks the north pole is called the north pole of the magnet, while the opposite side is the south pole.

It is a familiar fact that when two magnets are brought near one another, the magnets exert a force on each other. The magnetic force can be either attractive or repulsive. If two north poles or two south poles are brought near each other, the force will be repulsive. If a north pole is brought near a south pole, the force will be attractive.

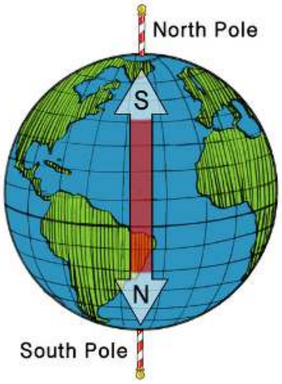


Figure 18.1.2

The Earth's geographic north pole (which is close to, but not exactly at the magnetic pole) attracts the north poles of magnets. We know, therefore, that this pole is actually the Earth's magnetic south pole. This can be seen in the image above; the geographic north and south poles are labeled with barber shop poles, and the Earth's **magnetic poles** are indicated with the double-headed arrow.

Only iron and few other materials such as cobalt, nickel, and gadolinium show strong magnetic effects. These materials are said to be **ferromagnetic**. Other materials show some slight magnetic effect but it is extremely small and can be detected only with delicate instruments.

Ferromagnetic Domains

Microscopic examination reveals that a magnet is actually made up of tiny regions known as **magnetic domains**, which are about one millimeter in length and width. Each domain acts like a tiny magnet with a north and south pole.

Magnetic domains before and after magnetization

Figure 18.1.3

If an object remains a magnet even when removed from the other magnetic field it is called a **permanent magnet**. If alignment is made in the presence of a permanent magnetic field, the object is a **temporary magnet.** If the material is not magnetized in the presence of another magnetic field because the domains are randomly organized so that the north and south poles do not line up and often cancel each other it is **non-magnetic**.

When a ferrous material is placed in a magnetic field, the domains line up with the magnetic field so that the north poles are all pointed in the same direction and the south poles are all pointed in the opposite direction. In this way, the ferrous material has become a magnet. In many cases, the domains will remain aligned only while the ferrous material is in a strong magnetic field; when the material is removed from the field, the domains return to their previous random organization and the ferrous material loses any magnetic properties. These temporary magnets have magnetic properties while in the field of another magnet but lose the magnetic properties when removed from the field.

The formation of temporary magnets allows a magnet to attract a non-magnetized piece of iron. You have most likely seen a magnet pick up a paper clip. The presence of the magnet aligns the domains in the iron paper clip and it becomes a temporary magnet. Whichever pole of the magnet is brought near the paper clip will induce magnetic properties in the paper clip that remain as long as the magnet is near.

Permanent magnets lose their magnetic properties when the domains are dislodged from their organized positions and returned to a random jumble. This can occur if the magnet is hammered on or if it is heated strongly.

Magnetic Fields

When we were dealing with electrical effects, it was very useful to speak of an electric field that surrounded an electric charge. In the same way, we can imagine a **magnetic field** surrounding a magnetic pole. The force that one magnet exerts on another can be described as the interaction between one magnet and the magnetic field of the other magnet. Magnetic field lines go from the north magnetic pole to the south magnetic pole. We define the magnetic field at any point as a vector (represented by the letter **B**) whose direction is from north to south magnetic poles.

Launch the Field Lines simulation below to help you visualize the invisible magnetic field that surrounds the Earth. You can also travel to different planets in the solar system and measure the properties of their magnetic fields. Can you determine which planet has the largest magnetic field? Which planet has no field? Then, try to develop a hypothesis about why this might be.

Summary

- Any magnet has two ends called poles where the magnetic effect is strongest.
- The magnetic pole found at the north geographical pole of the earth is a south magnetic pole.
- The force that one magnet exerts on another can be described as the interaction between one magnet and the magnetic field of the other magnet.
- Magnetic field lines go from the north magnetic pole to the south magnetic pole.

Review

1. The earth’s magnetic field
    1. has a north magnetic pole at exactly the same spot as the geographical north pole.
    2. is what causes compasses to work.
    3. is what causes electromagnets to work.
    4. all of these are true.
    5. none of these are true.
2. A material that can be permanently magnetized is generally said to be
    1. magnetic.
    2. electromagnetic.

    3. ferromagnetic.
    4. none of these are true.
3. The force between like magnetic poles will be
    1. repulsive.
    2. attractive.
    3. could be repulsive or attractive.
4. Why is a magnet able to attract a non-magnetic piece of iron?
5. If you had two iron rods and noticed that they attract each other, how could you determine if both were magnets or only one was a magnet?

Explore More

Use this resource to answer the question that follows.

1. In the video, one object rests on top of the magnetic field of another. Compare the friction between these two objects to the friction between a saucer and a table the saucer rests on.

Additional Resources

Study Guide: Magnetism Study Guide

Videos: Introduction to Magnetism - Overview

Real World Application: Levitating Trains

12.3: Magnetic Fields

Electromagnet lifting scrap metal

Figure 18.2.1

Powerful electromagnets are commonly used for industrial lifting. Here, a magnet is lifting scrap iron and loading it onto a railroad car for transporting to a scrap iron recovery plant. Other uses for lifting magnets include moving cars in a junk yard, lifting rolls of steel sheeting, and lifting large steel parts for various machines. Electromagnets are usually used for these jobs because they are magnets only when the electric current is on. The magnet will hold the iron object when the current is on and release it when the current is off.

Electric Currents and Magnetic Fields

Electricity and magnetism are inextricably linked. Under certain conditions, electric current causes a magnetic field. Under other conditions, a magnetic field can cause an electric current. A moving charged particle creates a magnetic field around it. Additionally, when a moving charged particle moves through a different magnetic field, the two magnetic fields will interact. The result is a force exerted on the moving charged particle.

Magnetic Field Around a Current Carrying Wire

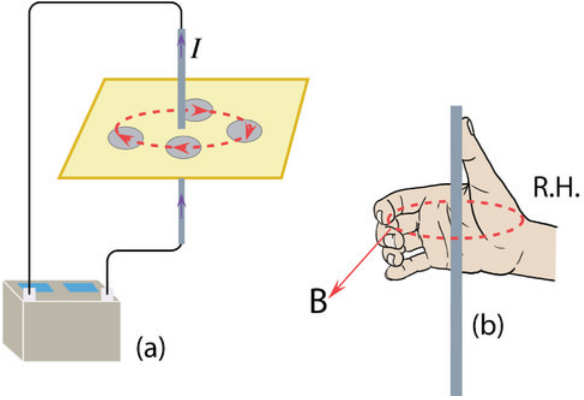


Figure 18.2.2

In sketch (a) above, a current is being pushed through a straight wire. Small compasses placed around the wire point in a circle, instead of all towards the north pole. This demonstrates the presence of a magnetic field around the wire. If the current is turned off, the compass points return to pointing north.

The current moving in a straight wire produces a circular magnetic field around the wire. When using conventional current, the direction of the magnetic field is determined by using the **right hand rule**. The rule says to curl your right hand around the wire such that your thumb points in the direction of the conventional current. Having done this, your fingers will curl around the wire in the direction of the magnetic field. Note that the right hand rule is for conventional current. If you are dealing with an electron flow current, the charges are flowing in the opposite direction, so you must use your left hand. That is, curl your left hand around the wire with your thumb pointing in the direction of the electron flow and your fingers will point in the direction of the magnetic field.

Charged Particles Moving Through a Magnetic Field

When a charged particle moves through a magnetic field at right angles to the field, the field exerts a force on the charged particle in a different direction.

Illustration of an electron moving through a magnetic field

Figure 18.2.3

In the case sketched above, an electron is moving downward through a magnetic field. The motion of the electron is perpendicular to the magnetic field. The force (F) exerted on the electron can be calculated by the equation,

$$F=Bqv$$

where v is the velocity of the particle in meters per second and q is the charge on the particle in coulombs. The term B represents the strength of the magnetic field in teslas. A tesla is equal to 1NewtonAmpere·meter and is a unit named after the Serbian physicist Nikolai Tesla. An ampere is equivalent to 1coulombsecond and you can see in the equations below that the units on the side of the equation cancel out to leave us with newtons, the unit of force.

$$F=Bqv$$

$$F=(\text{Newton/Amp}\cdot\text{meter})\times\text{coulomb}\times(\text{meter/second})$$

Replacing amperes with its unit definition:

$$F=(\text{Newton}\cdot\text{second/coulomb}\cdot\text{meter})\times\text{coulomb}\times(\text{meter/second})$$

Cancelling units:

$$F=(\text{Newton}\cdot\cancel{\text{second}}/\text{coulomb}\cdot\cancel{\text{meter}})\times\text{coulomb}\times(\cancel{\text{meter}}/\cancel{\text{second}})$$

$$F=\text{Newtons}$$

Again, we can determine the direction of the force acting upon the electron using a hand rule. Since the electron has a negative charge, the **left hand rule** is used. The fingers of the left hand are pointed in the direction of the magnetic field and the thumb points in the direction of the initial electron movement. The direction of the force acting on the electron is the direction the palm of the left hand faces. The direction of the magnetic field, the direction of the moving charge, and the direction of the force on the particle are all perpendicular to each other.

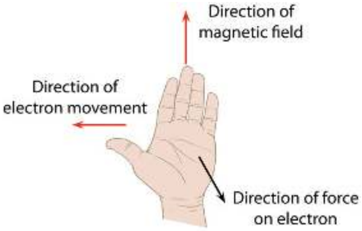


Figure 18.2.4

In most situations, a positive test charged is used, instead of an electron. In these circumstances, the **right hand rule** is used. The right hand rule is the same as the left hand rule; the thumb is the direction of initial charge movement, the fingers are the direction of the field, and the palm is the direction of the acting force.

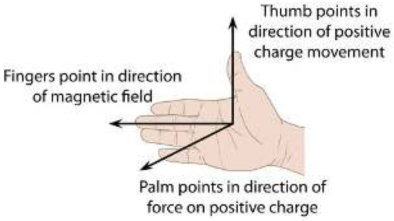


Figure 18.2.5

In dealing with the relationships that exist between magnetic fields and electric charges, there are both left hand and right hand rules that we use to indicate various directions – directions of fields, directions of currents, directions of motion. To avoid errors, it is absolutely vital to know and express whether the system we are observing is using conventional current or electron current. This allows us to use the appropriate rule.

Example 18.2.1

An electron traveling at 3.0×106 m/s passes through a 0.0400 N/amp·m uniform magnetic field. The electron is moving at right angles to the magnetic field. What force acts on the electron?

Solution

$$F=Bqv=(0.0400\ \text{N/amp}\cdot\text{m})(1.6\times10^{-19}\ \text{C})(3.0\times10^{6}\ \text{m/s})$$

$$=1.9\times10^{-14}\ \text{N}$$

When the current is traveling through a magnetic field while inside a wire, the magnetic force is still exerted but now it is calculated as the force on the wire rather than on the individual charges in the current.

The equation for the force on the wire is given as F=BIL, where *B* is the strength of the magnetic field, *I* is the current in amps and *L* is the length of the wire in and perpendicular to the field.

✓ Example 18.2.2

A wire 0.10 m long carries a current of 5.0 A. The wire is at right angles to a uniform magnetic field. The force the field exerts on the wire is 0.20 N. What is the magnitude of the magnetic field?

Solution

$$B=F/IL=0.20\text{ N}/(5.0\text{ A})(0.10\text{ m})=0.40\text{ N/A}\cdot\text{m}$$

In a particle accelerator, a strong magnetic field is used to exert a force on moving particles in a direction perpendicular to their motion. As a result, the particles begin to travel in a circle. Scientists can measure the radius of these circles and use them to distinguish the type of particle (electron, proton, neutrino, muon, etc). Different particles will have different masses and charges, and so will interact with the magnetic field in different ways. Use the Particle Tracks simulation below to learn more:

Summary

- A moving charged particle creates a magnetic field around it.
- Charge through a wire creates a magnetic field around it, the properties of which can be determined using a right hand rule.
- When a moving charged particle moves through another magnetic field, that field will exert a force on the moving charged particle that can be expressed using F=Bqv.
- The relationships between the moving charged particle, magnetic field, and the force exerted can be determined using the right hand rule if the particle is positive, or the left hand rule if it is negative.
- When the current is traveling through a magnetic field while inside a wire, the magnetic force is still exerted but now it is calculated as the force on the wire rather than on the individual charges in the current, calculated using F=BIL.

Review

1. Find the force on a 115 m long wire at right angles to a 5.0×10^{-5} N/A·m magnetic field, if the current through the wire is 400. A.
2. Find the force on an electron passing through a 0.50 T magnetic field if the velocity of the electron is 4.0×10^{6} m/s.
3. A stream of doubly ionized particles (charge=2+) moves at a velocity of 3.0×10^{4} m/s perpendicularly to a magnetic field of 0.0900 T. What is the magnitude of the force on the particles?
4. A wire 0.50 m long carrying a current of 8.0 A is at right angles to a 1.0 T magnetic field. What force acts on the wire?
5. Suppose a magnetic field exists with the north pole at the top of the computer monitor and the south pole at the bottom of the monitor screen. If a positively charged particle entered the field moving from your face to the other side of the monitor screen, which way would the path of the particle bend?
    1. left
    2. right
    3. up
    4. down
    5. none of these
6. Suppose the surface of your dining room table is a magnetic field with the north pole at the north edge and the south pole at the south edge. If an electron passes through this field from ceiling to floor, which way will the path of the electron bend?
    1. west
    2. east
    3. north
    4. south
    5. toward the ceiling

Explore More

Use this resource to answer the questions that follow.

1. What happens to the wire when the current begins to flow?
2. What difference would it make if the magnetic field were stronger?
3. What difference would it make if the battery were 3.0 V instead of 1.5 V?

Additional Resources

Study Guide: Magnetism Study Guide

Real World Application: Solar Attack

Videos:

- **18.2: Magnetic Fields** by CK-12 Foundation is licensed CK-12. Original source: https://flexbooks.ck12.org/cbook/ck-12-physics-flexbook-2.0.

12.4: Earth as a Magnet

Figure 18.3.1

Did you ever use a compass like the one in this picture? Even if you’ve never used a compass, you probably know that the needle of a compass always points north. That’s because a compass needle is magnetized, so it is attracted by a magnet.

> **Q:** What magnet attracts a compass needle?
>
> **A:** A compass needle is attracted by magnet Earth. It always points north because Earth acts as a giant magnet.

Earth’s Magnetic Poles

Imagine a huge bar magnet passing through Earth’s axis, as in the Figure below. This is a good representation of Earth as a magnet. Like a bar magnet, Earth has north and south magnetic poles. A **magnetic pole** is the north or south end of a magnet, where the magnet exerts the most force.

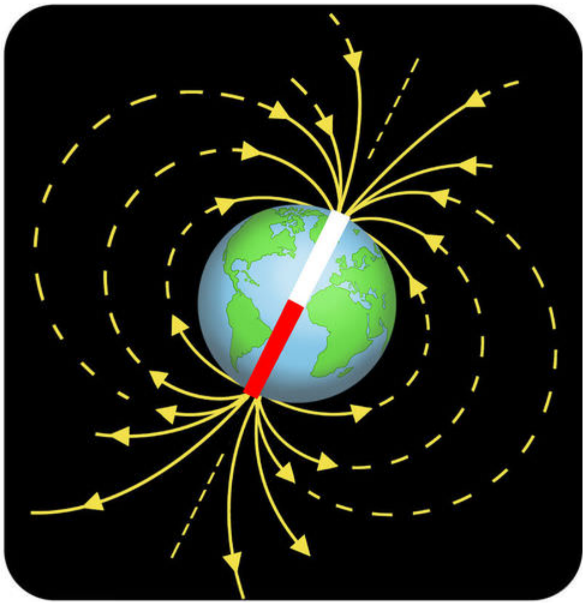


Figure 18.3.2

Two North Poles

Although the needle of a compass always points north, it doesn’t point to Earth’s north geographic pole. Find the north geographic pole in the Figure below. As you can see, it is located at 90° north latitude. Where does a compass needle point instead? It points to Earth’s north magnetic pole, which is located at about 80° north latitude. Earth also has two south poles: a south geographic pole and a south magnetic pole.

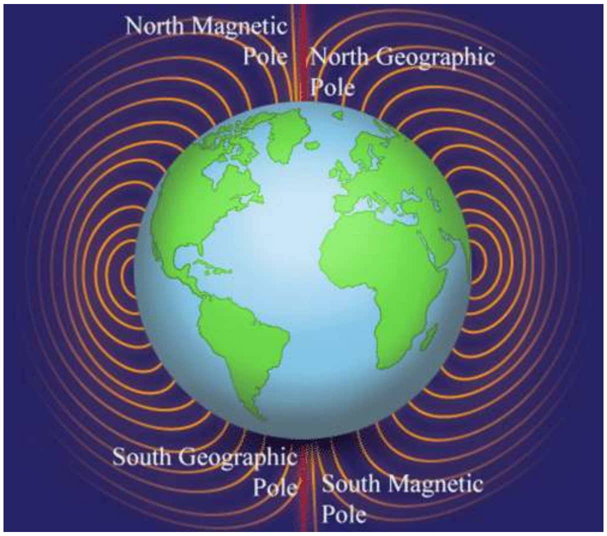


Figure 18.3.3

Q: The north end of a compass needle points toward Earth’s north magnetic pole. The like poles of two magnets repel each other, and the opposite poles attract. So why doesn’t the north end of a compass needle point to Earth’s south magnetic pole instead?

A: The answer may surprise you. The compass needle actually does point to the south pole of magnet Earth. However, it is called the north magnetic pole because it is close to the north geographic pole. This naming convention was adopted a long time ago to avoid confusion.

Earth’s Magnetic Field

Like all magnets, Earth has a magnetic field. Earth’s magnetic field is called the **magnetosphere**. You can see a model of the magnetosphere in the Figure below. It is a huge region that extends outward from Earth in all directions. Earth exerts magnetic force over the entire field, but the force is strongest at the poles, where lines of force converge.

Launch the PLIX Interactive below to learn more about how a compass utilizes the Earth’s magnetic field and observe what happens to a compass as you change positions on Earth:

Summary

- Earth acts as a giant magnet with magnetic poles and a magnetic field over which it exerts magnetic force.
- Earth has north and south magnetic poles like a bar magnet. Earth’s magnetic poles are not the same as the geographic poles.
- Earth’s magnetic field is called the magnetosphere. It is strongest at the poles.

Review

1. How does Earth act as a bar magnet?
2. The compass in a car shows that the car is moving north. Does this mean that the car is moving toward 90° north latitude? Why or why not?
3. Describe the magnetosphere.

Additional Resources

Study Guide: Magnetism Study Guide

Real World Application: Sly as a Fox, What Will Happen When Earth's Poles Flip?

Videos:

SECTION OVERVIEW

12.5: Electromagnetism

12.5.1: Electromagnet

Figure 19.1.1

One of the most famous electric car companies is Tesla, named after Nikola Tesla. These electric cars, and all others, require an electromagnet to run the engine.

Electromagnets

A long coil of wire consisting of many loops of wire and making a complete circuit is called a **solenoid**. The magnetic field within a solenoid can be quite large since it is the sum of the fields due to the current in each individual loop.

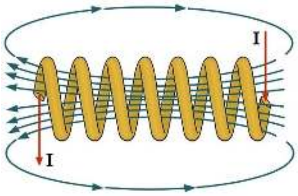


Figure 19.1.2

The magnetic field around the wire is determined by a hand rule. Since this description doesn't mention electron flow, we must assume that the current indicated by *I* is conventional current (positive). Therefore, we would use a right hand rule. We grasp a section of wire with our right hand pointing the thumb in the direction of the current flow and our fingers will curl around the wire in the direction of the magnetic field. Therefore, the field points down the cavity in these loops from right to left as shown in the sketch.

If a piece of iron is placed inside the coil of wire, the magnetic field is greatly increased because the domains of the iron are aligned by the magnetic field of the current. The resulting magnetic field is hundreds of time stronger than the field from the current alone. This arrangement is called an **electromagnet**. The picture below shows an electromagnet with an iron bar inside a coil.

Electromagnet formed by a wire wrapped around an iron core

Figure 19.1.3

Our knowledge of electromagnets developed from a series of observations. In 1820, Hans Oersted discovered that a current-carrying wire produced a magnetic field. Later in the same year, André-Marie Ampere discovered that a coil of wire acted like a permanent magnet and François Arago found that an iron bar could be magnetized by putting it inside of a coil of current-carrying wire. Finally, William Sturgeon found that leaving the iron bar inside the coil greatly increased the magnetic field.

Two major advantages of electromagnets are that they are extremely strong magnetic fields, and that the magnetic field can be turned on and off. When the current flows through the coil, it is a powerful magnet, but when the current is turned off, the magnetic field essentially disappears.

A telegraph is one example of a device that utilizes an electromagnet and its ability to be turned on and off to transmit information very long distances. When the electromagnet is on, it creates a click. When it is off, there is no click. We translate the language of clicks and no-clicks into alphabet characters, known as Morse Code. Launch the Telegraph simulation below and click on a letter in the Morse code chart. Then, press play to hear how the telegraph communicates this letter using electromagnets:

Electromagnets find use in many practical applications. Electromagnets are used to lift large masses of magnetic materials such as scrap iron, rolls of steel, and auto parts.

Electromagnet on a loading dock

Figure 19.1.4

The overhead portion of this machine (painted yellow) is a lifting electromagnet. It is lowered to the deck where steel pipe is stored and it picks up a length of pipe and moves it to another machine where it is set upright and lowered into an oil well drill hole.

Electromagnets are essential to the design of the electric generator and electric motor and are also employed in circuit breakers, television receivers, loudspeakers, electric dead bolts, car starters, clothes washers, atomic particle accelerators, and electromagnetic brakes and clutches. Electromagnets are commonly used as switches in electrical machines or even a simple doorbell. Launch the Doorbell simulation below to learn more:

Summary

- A solenoid is a long coil of wire consisting of many loops of wire that makes a complete circuit.
- An electromagnet is a piece of iron inside a solenoid.
- While the magnetic field of a solenoid may be quite large, an electromagnet has a significantly larger magnetic field.
- Electromagnets' magnetic fields can be easily turned off by just halting the current.

Review

1. Magnetism is always present when electric charges ___________.
2. What happens to the strength of an electromagnet if the number of loops of wire is increased?
3. What happens to the strength of an electromagnet if the current in the wire is increased?
4. Which direction does the magnetic field point in the solenoid sketched here?

Practice problem for determining the direction of the magnetic field of a solenoid

Figure 19.1.5

Explore More

Use this resource to answer the questions that follow.

1. What components are needed to make a homemade electromagnet?
2. What objects were attracted by the electromagnet in the video?

Additional Resources

Study Guide: Magnetism Study Guide

Real World Application: Lucky Discovery

Videos:

12.5.2: Electromotive Force

Hydroelectric power generator

Figure 19.2.1

Electrical generators convert mechanical energy into electrical energy. Every electrical generator needs some method for spinning the coil inside the magnetic field. Hydroelectric generators use water pressure to spin the coil while windmills, of course, use the wind to spin the coil. The image here is a combination of steam turbine and generator. The steam can be produced by burning coal or diesel fuel or by a nuclear reaction and the steam then turns the coil and generates electricity.

Electromotive Force

When an individual charge flies through a magnetic field, a force is exerted on the charge and the path of the charge bends. In the case shown in the sketch below, the charge is positive and the right hand rule shows us the force will be upward, perpendicular to both the field and the path of the charge.

Charge passing through a magnetic field

Figure 19.2.2

If a wire that is part of a complete circuit is moved through a magnetic field, the force on the individual electrons in the wire occurs in exactly the same manner. Since the electrons in the wire are negatively charged, the force would be in the opposite direction but otherwise the situation is the same. When the wire is pulled downward through the magnetic field, the force on the electrons cause them to move within the wire. Since the charges are negative, the left hand rule shows that the electrons would move as diagrammed in the sketch. (Point fingers in the direction of the magnetic field, point thumb in the direction of wire movement, and palm shows direction of electron flow.) No current will flow, of course, unless the section of wire is part of a complete circuit.

Diagram of a conducting rod being passed through a magnetic field

Figure 19.2.3

This process allows us to convert mechanical energy (the motion of the wire) into electrical energy (the current). This is the opposite of what happens in an electric motor where electrical energy is converted into mechanical energy.

In order to maintain a constant current flow, it is necessary to have a potential difference or voltage in the circuit. The voltage or potential difference is also frequently referred to **electromotive force**. The term electromotive force, like many historical terms, is a misnomer. Electromotive force is NOT a force, it is a potential difference or potential energy per unit charge and is measured in volts. The potential difference in the case of moving a wire through a magnetic field is produced by the work done on the charges by whatever is pushing the wire through the field.

The EMF (or voltage) depends on the magnetic field strength, B, the length of the wire in the magnetic field, l, and the velocityof the wire in the field.

$$\text{EMF}=Blv$$

This calculation is based on the wire moving perpendicularly through the field. If the wire moves an angle to the field, then only the component of the wire perpendicular to the field will generate EMF.

✓ Example 19.2.1

A 0.20 m piece of wire that is part of a complete circuit moves perpendicularly through a magnetic field whose magnetic induction is 0.0800 T. If the speed of the wire is 7.0 m/s, what EMF is **induced** in the wire?

Solution

$$\text{EMF}=Blv=(0.0800\text{ N/A}\cdot\text{m})(0.20\text{ m})(7.0\text{ m/s})=0.11\text{ N}\cdot\text{m/C}=0.11\text{ J/C}=0.11\text{ V}$$

Summary

- If a wire that is part of a complete circuit is moved through a magnetic field, the magnetic field exerts a force on the individual electrons in the wire, which causes a current to flow.
- The potential difference in the case of moving a wire through a magnetic field is produced by the work done on the charges by whatever is pushing the wire through the field.

- The EMF (or voltage) depends on the magnetic field strength, B, the length of the wire in the magnetic field, l, and the velocity of the wire in the field, $EMF=Blv$.

Review

1. Which of the following units are equivalent to those of EMF produced in a generator?
    1. T·m/s
    2. V·m2/s
    3. J/s
    4. A·Ω
    5. T·m
2. A straight wire 0.500 m long is moved straight up through a 0.400 T magnetic field pointed in the horizontal direction. The speed of the wire is 20.0 m/s.
    1. What EMF is induced in the wire?
    2. If the wire is part of a circuit with a total resistance of 6.00 Ω, what is the current in the circuit?
3. A straight wire, 25.0 m long, is mounted on an airplane flying at 125 m/s. The wire moves perpendicularly through earth’s magnetic field (B=5.00×10−5 T). What is the EMF induced in the wire?
4. A straight wire, 30.0 m long, moves at 2.00 m/s perpendicularly through a 1.00 T magnetic field.
    1. What is the induced EMF?
    2. If the total resistance of the circuit is 15.0 Ω, what is the current in the circuit?

Explore More

Use this resource to answer the questions that follow.

1. We have been discussing the process of generating electricity by moving a wire through a magnetic field. What happens if the wire is held steady and the magnetic field moves instead?
2. When a loop of wire is turned circularly in a magnetic field, what type of current is produced?

Additional Resources

Interactive: AC Transformer

Real World Application: Controlling Traffic In Real Time

Videos:

Study Guide: Magnetism Study Guide

12.5.3: Electric Motor

An electric vehicle and a hybrid vehicle

Figure 19.3.1

As gas prices continue to rise, electric cars and hybrids are becoming increasingly popular. These cars are certainly a part of our future. On the left in the image above is an all-electric vehicle, and on the right is a hybrid vehicle that uses gas part time and electricity part time.

Electric Motors

In an earlier concept, we described and calculated the force that a magnetic field exerts on a current carrying wire. Since you are familiar with Newton's third law of motion, you know that if the magnetic field exerts a force on the current carrying wire, then the current carrying wire also exerts a force of equal magnitude and opposite direction on the magnetic field.

Schematic for a motor prototype

Figure 19.3.2

In the sketch above, a circuit is connected to a battery, with one part of the circuit placed inside a magnetic field. When current runs through the circuit, a force will be exerted on the wire by the magnetic field, causing the wire to move. If we choose to consider electron current in this case, the electrons flow from the back of the sketch to the front while the magnetic field is directed upward. Using the left hand rule for this, we find that the force on the wire is to the right of the page. Had we chosen to consider the current to be conventional current, then the current would be flowing from the front of the sketch to the back and we would use the right hand rule. The force on the wire would, once again, be toward the right. This movement is harnessed in

Electrical motors change electrical energy into mechanical energy. The motor consists of an electrical circuit with part of the wires inside a magnetic field. This can be seen below. Positive charges move through the circuit in the direction of the light purple arrows. When the charges move up through the part of the coil that is right next to the north pole, the right hand rule tells us that the wire suffers the force, *F*, pushing the wire in the direction of the blue arrow, toward the back of the sketch. On the other side of the coil, where the charges are moving down through the field, the right hand rule shows the force would push this side of the coil toward the front. These two forces are working together, rotating the coil in the direction of the circular red arrow.

Schematic of a more advanced electric motor

Figure 19.3.3

Where the rotating coil (in grey) meets the wires attached to the power source (black), we find a split ring **commutator.** The coil turns, but the commutator and power source do not. As the coil turns, it moves off of the blue box connector and as it continues to turn, it connects to the other blue box connector. As the coil turns, it reverses its connections to the external circuit. Therefore, inside the coil, the current is always flowing in the same direction because the left side of the coil is always attached to the left side of the external circuit. This allows the coil, or **armature,** to continue to spin the same direction all the time.

In electrical motors, these coils often consist of not just one, but many wires, as can be seen here:

Cutaway of a real electric motor

Figure 19.3.4

Launch the Electric Motor simulation below to visualize how we convert electrical energy into mechanical energy to create a motor. When experimenting with the simulation, you can manipulate the size of the motor, the strength of the input current, the strength of the magnetic field, and the number of turns in the wire:

Summary

- When current runs through the circuit, a force will be exerted on the wire.
- An electrical motor changes electrical energy into mechanical energy.
- A split ring commutator keeps the current in the coil flowing in the same direction even though the coil changes sides every half turn.

Review

1. Which way will the wire be pushed when current passes through the wire?
    1. up

   2. down
   3. left
   4. right
   5. None of these.

Practice problem with a wire around a magnet

Figure 19.3.5

2. Which way will the coil spin when current passes through the wire?
   1. clockwise
   2. counterclockwise

Practice problem with a wire that between through two magnets

Figure 19.3.6

Explore More

Use this resource to answer the questions that follow.

1. Who first built an electric motor?
2. What size battery was used in the video motor?

Additional Resources

Study Guide: Magnetism Study Guide

Real World Application: The Rail Gun

Videos:

- **19.3: Electric Motor** by CK-12 Foundation is licensed CK-12. Original source: https://flexbooks.ck12.org/cbook/ck-12-physics-flexbook-2.0.

12.5.4: Generator

Figure 19.4.1

These large machines are electric generators. This particular row of generators is installed in a hydroelectric power station. The insides of these generators are coils of wire spinning in a magnetic field. The relative motion between the wire and the magnetic field is what generates electric current. In all generators, some mechanical energy is used to spin the coil of wire in the generator. In the case of hydroelectric power, the coil of wire is spun by water falling from higher PE to lower PE. Windmills and steam turbines are used in other types of power generators to spin the coil.

Electric Generators

Electric generators convert mechanical energy to electric energy. The generator consists of some number of wire loops wrapped around an iron core and placed in a strong magnetic field. The loops of wire and the iron core are called the **armature**. The armature is mounted so that it can rotate freely inside the magnetic field. Mechanical energy is used to spin the armature in the field so that the wire loops cut across the field and produce electric current. The EMF of this current is calculated by $EMF=Blv$.

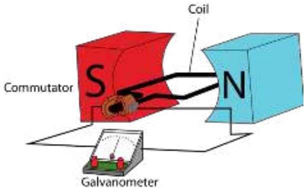


Figure 19.4.2

Consider the coil and magnetic field sketched above. When the right hand side of the coil moves up through the field, the left hand rule indicates that the electron flow will be from the front to the back in that side of the coil. The current generated will have the greatest EMF as the wire is cutting perpendicularly across the field. When the wire reaches the top of its arc, it is moving parallel to the field and therefore, not cutting across the field at all. The EMF at this point will be zero. As that same wire then cuts down through the field as it continues to spin, the left hand rule indicates that the electron flow will be from the back to the front in that side of the coil. In this second half of the arc, the direction of the electron flow has reversed. The magnitude of the EMF will reach maximum again as the wire cuts perpendicularly down through the field and the EMF will become zero again as the wire passes through the bottom of the arc. The current produced as the armature goes around will resemble a sine wave where the EMF reaches a maximum in one direction, then goes to zero, then goes to a maximum in the other direction. This type of current is called **alternating current**. By having more and more loops of wire on the armature, the crests and troughs overlap and fill in until a constant current is produced.

Diagram of a periodic wave

Figure 19.4.3

A **direct current** is one that always flows in the same direction rather than alternating back and forth. Batteries produce direct currents. A generator can also produce direct current by using a split ring commutator that changes external connections every half turn of the armature so that even though the current in the coil changes direction, every time the current in the coil changes direction, the external connection switches so that the external current always goes in the same direction.

Generators and motors are almost identical in construction but convert energy in opposite directions. Generators convert mechanical energy to electrical energy and motors convert electrical energy to mechanical.

Because of the alternating direction in alternating current, the average value is less than the power supplied by a direct current. In fact, the average power of an AC current is one-half its maximum power and one-half the power of an equivalent DC current. The effective current of an AC generator is 0.707 times its maximum current. The same is true for the effective voltage of an AC generator.

$$I_{eff}=0.707\ I_{max}$$

$$V_{eff}=0.707\ V_{max}$$

✓ Example 19.4.1

An AC generator develops a maximum voltage of 34.0 V and delivers a maximum current of 0.170 A.

1. What is the effective voltage of the generator?
2. What is the effective current delivered by the generator?
3. What is the resistance in the circuit?

Solution

1. $V_{eff}=0.707\ V_{max}=(0.707)(34.0\ V)=24.0\ V$
2. $I_{eff}=0.707\ I_{max}=(0.707)(0.17\ A)=0.120\ A$
3. $R=V/I=24.0\ V/0.120\ A=200.\ \Omega$

Summary

- Electric generators convert mechanical energy to electric energy.
- The generator consists of some number of wire loops wrapped around an iron core and placed in a strong magnetic field.
- The loops of wire and the iron core are called the armature.
- The armature is mounted so that it can rotate freely inside the magnetic field.
- Mechanical energy is used to spin the armature in the field so that the wire loops cut across the field and produce electric current.
- The current produced as the armature goes around will resemble a sine wave where the EMF reaches a maximum in one direction, then goes to zero, then goes to a maximum in the other direction. This type of current is called alternating current.
- A generator can also produce direct current by using a split ring commutator that changes external connections every half turn of the armature so that even though the current in the coil changes direction, every time the current in the coil changes direction, the external connection switches so that the external current always goes in the same direction.
- The effective current of an AC generator is 0.707 times its maximum current.
- The effective voltage of an AC generator is 0.707 times its maximum voltage.

Review

1. What three things are necessary to produce EMF mechanically?
    1. magnet, force lines, and magnetic field
    2. EMF, conductor, and magnetic field
    3. conducting wire, magnetic field, and relative motion
    4. conducting wire, electrical field, and relative motion
    5. none of these will produce EMF mechanically
2. Increasing which of the following will increase the output of a generator?
    1. EMF
    2. strength of the magnetic field
    3. resistance of the conductor
    4. load on the meter
    5. none of these
3. The current in the rotating coil of all generators is
    1. AC

    2. DC
    3. pulsating AC
    4. pulsating DC
4. A generator in a power plant develops a maximum voltage of 170. V.
    1. What is the effective voltage?
    2. A 60.0 W light bulb is placed across the generator. A maximum current of 0.70 A flows through the bulb. What effective current flows through the bulb?
    3. What is the resistance of the light bulb when it is working?
5. The effective voltage of a particular AC household outlet is 117 V.
    1. What is the maximum voltage across a lamp connected to the outlet?
    2. The effective current through the lamp is 5.50 A. What is the maximum current in the lamp?

Explore More

Use this resource to answer the questions that follow.

1. Which of the two generators in the video (an AC generator and a DC generator) involves a magnetic field?
2. Which of the two generators in the video involves a wire-wrapped armature?
3. What is the difference between the DC generator and the AC generator?

Additional Resources

Real World Application: Faraday Flashlight

Videos:

Study Guide: Magnetism Study Guide

12.5.5: Lenz's Law

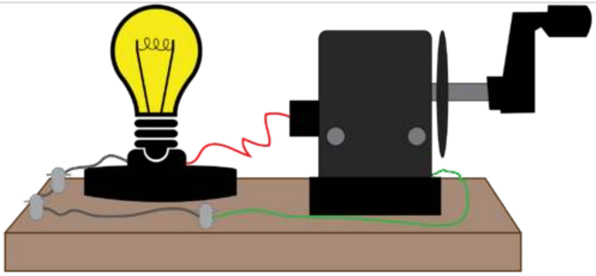


Figure 19.5.1

This is a diagram of a hand-cranked demonstration generator. Turning the hand crank spins a coil inside horseshoe magnets and generates an electric current that lights the bulb. In order to generate an electric current in a conductor, the conductor must be part of a complete circuit. If the light bulb is removed from this apparatus, the crank is very easy to turn. If the light bulb is absent, the circuit is incomplete and no current is generated. When the light bulb is place in the circuit, the circuit is complete and turning the crank will generate a current and light the bulb. When the crank is turned with bulb in the circuit, it is more difficult to turn the crank. A great deal more effort is needed to turn the crank while current is generated.

Lenz's Law

In the sketch below, moving the wire downward through the magnetic field causes electrons to flow in the wire as diagrammed. This is because relative motion between charged particles and magnetic fields produces a force on the charged particles . . . so the downward movement of the wire causes a force which moves the electrons, producing a current. When the electrons begin to flow, however, there is a second motion of the charged particles and this second motion will produce a second force on the particles.

Diagram of a wire being passed through a magnetic field

Figure 19.5.2

If we use the left hand rule on the electron flow, we point our fingers in the direction of the field and our thumb in the direction of electron movement and the palm of our hand indicates the direction of the force – UP !

This secondary force always opposes the first movement of the wire. **Lenz's law** states the induced current produces a magnetic field that opposes the motion that caused the induced current.

> *"An induced electromotive force generates a current that induces a counter magnetic field that opposes the magnetic field generating the current."*

Lenz's law also applies to electric motors. When an EMF is sent through a motor (aka, a current is passed through the armature of a motor) a force is produced that causes the armature to turn. This is electric energy converted to mechanical energy. The spin of the armature, however, now causes the motor to act like a generator and generate an EMF. The direction of the EMF is always the opposite of the original EMF that was sent through the motor. This second EMF is called **back-EMF**.

Summary

- Lenz's law states, "An induced electromotive force generates a current that induces a counter magnetic field that opposes the magnetic field generating the current."
- The spin of the armature, however, now causes the motor to act like a generator and generate an EMF. The direction of the EMF is always the opposite to the original EMF that was sent through the motor.

Review

1. Consider the sketch at below. Will a current be induced in the coil on the right when the magnet passes through?
    1. Yes
    2. No

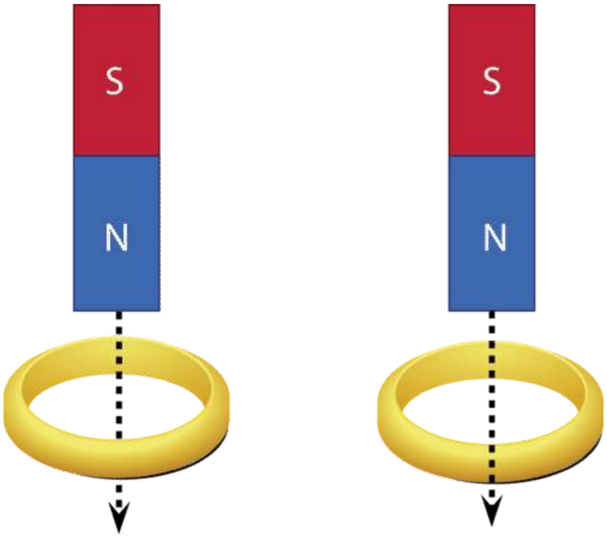


Figure 19.5.3

2. Will a current be induced in the coil on the left when the magnet passes through?
    1. Yes
    2. No
3. If a current is induced in the coil when the magnet passes through, which way will the current flow?
    1. clockwise
    2. counterclockwise
4. If both magnets are dropped though the loops shown at the same time, which magnet would reach the ground first?
    1. the one on the left
    2. the one on the right
    3. they would hit the ground together
5. Will a hand generator be more difficult to turn when it is generating current or when it is not generating current?
    1. generating
    2. not generating

Explore More

Use this resource to answer the questions that follow.

1. In general, how would the observation change if the aluminum tube were replaced with a copper tube?
2. In general, how would the observation change if the glass tube were replaced with a copper tube?

Additional Resources

Real World Application: Let's Play Pinball

Videos:

Study Guide: Magnetism Study Guide

12.5.6: Induced Voltage and Magnetic Flux

Learning Objectives

- Describe methods to produce an induced voltage with a magnetic field or magnet and a loop of wire.

The apparatus used by Faraday to demonstrate that magnetic fields can create currents is illustrated in Figure $12.5.6.1$. When the switch is closed, a magnetic field is produced in the coil on the top part of the iron ring and transmitted to the coil on the bottom part of the ring. The galvanometer is used to detect any current induced in the coil on the bottom. It was found that each time the switch is closed, the galvanometer detects a current in one direction in the coil on the bottom. Each time the switch is opened, the galvanometer detects a current in the opposite direction. Interestingly, if the switch remains closed or open for any length of time, there is no current through the galvanometer. *Closing and opening the switch* induces the current. It is the *change* in magnetic field that creates the current in the following way: *a changing magnetic field induces an electric field*, which results in the induced voltage. When this induced voltage occurs over a conducting path, as in this example, the induced voltage causes a current to flow. As a shorthand, we call the resulting current **induced current**; the changing magnetic field does not induce the current directly but through the induced voltage and an application of Ohm's law.

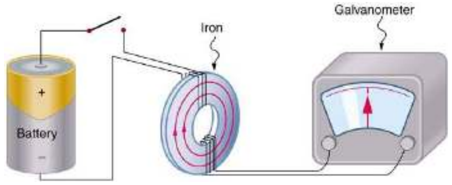


Figure $12.5.6.1$: Faraday's apparatus for demonstrating that a magnetic field can produce a current. A change in the field produced by the top coil induces an voltage and, hence, a current in the bottom coil. When the switch is opened and closed, the galvanometer registers currents in opposite directions. No current flows through the galvanometer when the switch remains closed or open.

An experiment easily performed and often done in physics labs is illustrated in Figure $12.5.6.2$. A voltage is induced in the coil when a bar magnet is pushed in and out of it. Voltages of opposite signs are produced by motion in opposite directions, and the voltages are also reversed by reversing poles. The same results are produced if the coil is moved rather than the magnet—it is the relative motion that is important. The faster the motion, the greater the voltage, and there is no voltage when the magnet is stationary relative to the coil.

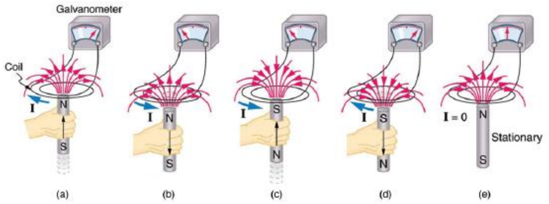


Figure $12.5.6.2$: Movement of a magnet relative to a coil produces voltage as shown. The same voltages are produced if the coil is moved relative to the magnet. The greater the speed, the greater the magnitude of the voltage, and the voltage is zero when there is no motion.

The method of inducing a voltage used in most electric generators is shown in Figure $12.5.6.3$. A coil is rotated in a magnetic field, producing an alternating voltage (and current), which depends on rotation rate and other factors that will be explored in later sections. Note that the generator is remarkably similar in construction to a motor.

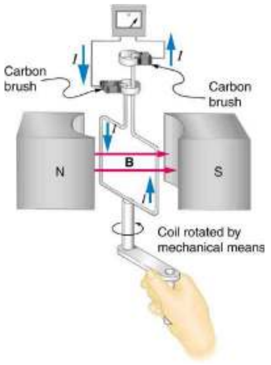


Figure 12.5.6.3: Rotation of a coil in a magnetic field produces a voltage. This is the basic construction of a generator, where work done to turn the coil is converted to electric energy. Note the generator is very similar in construction to a motor.

So we see that changing the magnitude or direction of a magnetic field produces a voltage. Experiments revealed that there is a crucial quantity called the **magnetic flux**, Φ, given by

$$\Phi = B_{\perp} A,$$

where B is the magnetic field strength over an area A, at an angle θ with the perpendicular to the area as shown in Figure $12.5.6.4$ **Any change in magnetic flux Φ induces a voltage.** This process is defined to be **electromagnetic induction**. Units of magnetic flux Φ are $\mathrm{T \cdot m^2}$.

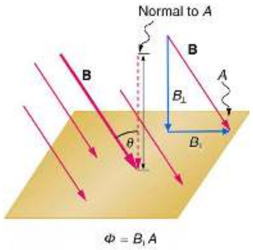


Figure 12.5.6.4: Magnetic flux Φ is related to the magnetic field and the area A over which it exists. Only the portion of the magnetic field that is perpendicular to the area ($B_{\perp}$) contributes to the flux. The flux $\Phi = B_{\perp} A$ is related to induction; any change in Φ induces a voltage.

All induction, including the examples given so far, arises from some change in magnetic flux Φ. For example, Faraday changed B and hence Φ when opening and closing the switch in his apparatus (shown in Figure $12.5.6.1$). This is also true for the bar magnet and coil shown in Figure $12.5.6.2$ When rotating the coil of a generator, the angle θ and, hence, Φ is changed. Just how great a voltage and what direction it takes depend on the change in Φ and how rapidly the change is made, as examined in the next section.

Section Summary

- The crucial quantity in induction is magnetic flux Φ, defined to be $\Phi = B_{\perp} A$, where $B_{\perp}$ is the magnetic field strength perpendicular to the area A.
- Units of magnetic flux Φ are $\mathrm{T \cdot m^2}$.
- Any change in magnetic flux Φ induces a voltage—the process is defined to be electromagnetic induction.

Glossary

induced current

the current created by a changing magnetic field through voltage induced over a conducting path

magnetic flux

the amount of magnetic field going through a particular area, calculated with $\Phi = B_{\perp} A$, where $B_{\perp}$ is the magnetic field strength perpendicular to the area A

electromagnetic induction

the process of inducing a voltage with a change in magnetic flux

12.5.7: Transformers

Figure 19.6.1

Power loss in long transmission lines is related to the magnitude of the current. Specifically, the power loss can be decreased by decreasing the magnitude of the current.

The amount of power passed through transmission lines can be calculated by multiplying voltage by current. The same power can be transmitted using a very high voltage and a very low current as with a low voltage and high current.

Since power companies do not wish to waste power as it is transmitted to homes and businesses, they deliberately 'step up' the voltage and reduce the current before transmitting the power over extended distances. That type of power transmits well without great loss of energy but it cannot be used in household appliances. It becomes necessary to convert it back ('step down') to low voltage and high current for household use. That is the job of electrical transformers – those big gray barrels you see on power poles.

Transformers

When we move a wire through a magnetic field, a force is exerted on the charges in the wire and a current is induced. Essentially the same thing happens if we hold the wire steady and move the magnetic field by moving the magnet. Yet a third way of causing relative motion between the charges in a wire and a magnetic field is to expand or contract the field through the wire.

When a current begins to flow in a wire, a circular magnetic field forms around the wire. Within the first fractional second when the current begins to flow, the magnetic field expands outward from the wire. If a second wire is placed nearby, the expanding field will pass through the second wire and induce a brief current in the wire.

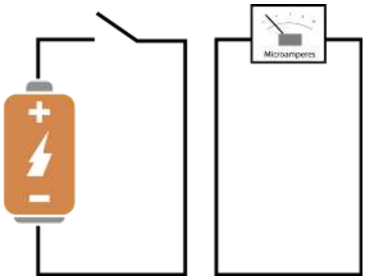


Figure 19.6.2

Consider the sketch above. When the knife switch is closed, current begins to flow in the first circuit and therefore, a magnetic field expands outward around the wire. When the magnetic field expands outward from the wire on the right side, it will pass through the wire in the second circuit. This relative motion between wire and field induces a current in the second circuit. The magnetic field expands outward for only a very short period of time and therefore, only a short jolt of current is induced in the second circuit. You can leave the knife switch closed and the current will continue to flow in the first circuit but no current is induced in the second circuit because the field is constant and therefore there is no relative motion between the field and the wire in the second circuit. When the knife switch is opened, the current in the first circuit ceases to flow and the magnetic field collapses back through the wire to zero. As the magnetic field collapses, it passes through the wire and once again we have relative motion between the wire in the second circuit and the magnetic field. Therefore, we once again have a short jolt of current induced in the second circuit. This second jolt of induced current will be flowing in the opposite direction of the first induced current. We can produce an alternating current in the second circuit simply by closing and opening the knife switch continuously in the first circuit.

Obviously, a transformer would have little use in the case of DC current because current is only induced in the second circuitwhen the first circuit is started or stopped. With AC current, however, since the current changes direction 60 times per second, the magnetic field would constantly be expanding and contracting through the second wire.

A **transformer** is a device used to increase or decrease alternating current voltages. They do this with essentially no loss of energy. A transformer has two coils, electrically insulated from each other as shown in the sketch. One coil is called the **primary coil** and the other is called the **secondary coil**. When the primary coil is connected to a source of AC voltage, the changing current creates a varying magnetic field. The varying magnetic field induces a varying EMF in the secondary coil. The EMF induced in the secondary coil is called the secondary voltage and is proportional to the primary voltage. The secondary voltage also depends on the ratio of turns on the secondary coil to turns on the primary coil.

Illustrations of step-up and step-down transformers

Figure 19.6.3

(secondary voltage/primary voltage)=(number of turns on secondary/number of turns on primary)

$$V_S/V_P=N_S/N_P$$

If the secondary voltage is larger than the primary voltage, the transformer is called a **step-up transformer**. If the voltage out of the transformer is smaller than the voltage in, then the transformer is called a **step-down transformer**.

In an ideal transformer, the electric power put into the primary equals the electric power delivered by the secondary.

$$V_P/I_P=V_S/I_S$$

The current that flows in the primary depends on how much current is required by the secondary circuit.

$$I_S/I_P=V_P/V_S=N_P/N_S$$

✓ Example 19.6.1

A particular step-up transformer has 200 turns on the primary coil and 3000 turns on the secondary coil.

1. If the voltage on the primary coil is 90.0 V, what is the voltage on the secondary coil?
2. If the current in the secondary circuit is 2.00 A, what is the current in the primary coil?
3. What is the power in the primary circuit?
4. What is the power in the secondary circuit?

Solution

1. $V_P/V_S=N_P/N_S$ $90.0\text{ V}/V_S=200/3000$ $V_S=(90.0\text{ V})(3000)/(200)=1350\text{ V}$
2. $I_S/I_P=N_P/N_S$ $2.00\text{ A}/I_P=200/3000$ $I_P=30.0\text{ A}$
3. $P_P=V_P/I_P=(90.0\text{ V})/(30.0\text{ A})=2700\text{ W}$
4. $P_S=V_S/I_S=(1350\text{ V})/(2.00\text{ A})=2700\text{ W}$

Launch the AC Transformer simulation below to visualize how we use a transformer to “step down” the high voltage on a residential power line:

Summary

- When a current begins to flow in a wire, a circular magnetic field forms around the wire.
- Within the first fractional second when the current begins to flow, the magnetic field expands outward from the wire.
- If a second wire is placed nearby, the expanding field will pass through the second wire and induce a brief current in the wire.
- A transformer is a device used to increase or decrease alternating current voltages.
- A transformer has two coils, electrically insulated from each other. One coil is called the primary coil and the other is called the secondary coil.
- The varying magnetic field induces a varying EMF in the secondary coil.
- The EMF induced in the secondary coil is called the secondary voltage and is proportional to the primary voltage. The secondary voltage also depends on the ratio of turns on the secondary coil to turns on the primary coil.

(secondary voltage/primary voltage)=(number of turns on secondary/number of turns on primary)

Review

1. A step-down transformer has 7500 turns on its primary and 125 turns on its secondary. The voltage across the primary is 7200 V.
    1. What is the voltage across the secondary?
    2. The current in the secondary is 36 A. What current flows in the primary?
2. The secondary of a step-down transformer has 500 turns. The primary has 15,000 turns.
    1. The EMF of the primary is 3600 V. What is the EMF of the secondary?
    2. The current in the primary is 3.0 A. What is the current in the secondary?
3. An ideal step-up transformer's primary coil has 500 turns and its secondary coil has 15,000 turns. The primary EMF is 120 V.
    1. Calculate the EMF of the secondary.
    2. If the secondary current is 3.0 A, what is the primary current?
    3. What power is drawn by the primary?

Explore More

Use this resource to answer the questions that follow.

1. What type of transformer is used at the power station where the electric power is generated?
2. What type of transformer is used at power sub-stations?
3. What type of transformer is used inside cell phone chargers?

Additional Resources

Study Guide: Magnetism Study Guide

Real World Application: Transforming Your Life

Videos:

12.6: End of Chapter Activity

End of Chapter Activity: Creating a Lesson Plan on Magnetism with AI and Bloom's Taxonomy

Now that you have explored the fundamentals of magnetism, it's time to put your knowledge into practice. Your task is to create a succinct lesson plan for 11th graders that introduces them to the advanced concepts of magnetism, including magnetic fields, electromagnetic induction, and applications of magnetism. To help you with this, you will use AI tools and incorporate Bloom's Taxonomy to ensure a comprehensive learning experience. This lesson plan will go towards your digital notebook, a portfolio filled with lesson plans, activities, and labs for future use.

Activity Prompt:

> **Objective:** Use AI and Bloom's Taxonomy to develop a lesson plan that effectively teaches 11th graders about the fundamentals and applications of magnetism, including concepts such as magnetic fields, electromagnetic induction, and technological applications of magnetism.

Understanding the Concepts:

> **Knowledge (Remembering):** Define key terms related to magnetism, such as magnetic field, magnetic flux, electromagnetism, and induction.
>
> **Comprehension (Understanding):** Explain these concepts in detail, using diagrams and real-world examples to illustrate them.

Planning the Lesson:

> **Application:** Design an engaging lab activity that allows students to observe and understand magnetic fields and electromagnetic induction. For example, use a magnetic field sensor to map the magnetic field around a magnet or create an electromagnet using a coil and a battery.
>
> **Analysis:** Use AI tools to create visual aids or interactive simulations that illustrate the behavior of magnetic fields and the principles of electromagnetic induction. For instance, create a simulation that shows how a changing magnetic field induces an electric current in a conductor.

Deepening Understanding:

> **Synthesis (Creating):** Ask students to design their own experiments or projects that demonstrate principles of magnetism. For example, they could design a simple electric generator or a magnetic levitation system.
>
> **Evaluation:** Have students discuss and reflect on their experiments and the principles they demonstrate. Encourage them to think critically about the applications of magnetism in modern technology and industry.

Using AI in the Classroom:

Explore AI tools like educational apps or platforms that provide interactive content for teaching about magnetism. Use these tools to create quizzes, flashcards, or interactive stories that reinforce the lesson's concepts.

Use AI to assess student understanding through formative assessments and provide instant feedback.

Deliverable:

Submit a detailed lesson plan that includes:

1. **A brief overview of the key concepts covered:** Outline the foundational concepts of magnetism that will be taught.
2. **A description of the activities and experiments designed:** Detail the hands-on activities and experiments you will use to help students understand these concepts.
3. **Examples of AI tools used and how they enhance the learning experience:** Describe the AI tools you plan to incorporate, such as simulations or interactive quizzes, and explain how they will help students grasp complex concepts.
4. **An explanation of how Bloom's Taxonomy was applied in the lesson plan to ensure a well-rounded educational experience:** Illustrate how each level of Bloom's Taxonomy (Remembering, Understanding, Applying, Analyzing, Creating, and Evaluating) is addressed in your lesson plan.

Additionally, include a creative project component where students create a digital presentation or a video tutorial that explains a concept related to magnetism, using AI tools to enhance their projects.

Example Lesson Plan:

Grade: 11th Grade
Topic: Magnetism
Duration: 1 Week

Overview:
Students will learn about the advanced concepts of magnetism, including magnetic fields, electromagnetic induction, and applications of magnetism, through engaging activities and creative projects.

Day 1: Introduction to Magnetism

Objective: Define basic and advanced concepts related to magnetism and provide examples.

- **Remembering:** Define key terms (magnetic field, magnetic flux, electromagnetism, induction).
- **Understanding:** Explain the concepts using examples from everyday life and technological applications (e.g., MRI machines, electric motors).

Activity:
Watch a detailed video (created using AI tools) explaining what magnetism is, how magnetic fields work, and the principles of electromagnetic induction.

Day 2: Exploring Magnetic Fields

Objective: Understand the nature of magnetic fields and how to map them.

- **Applying:** Conduct a lab activity to map the magnetic field around a bar magnet using a magnetic field sensor or iron filings.

Activity:
Students use magnetic field sensors or sprinkle iron filings around a bar magnet to visualize and map the magnetic field lines. They document their findings and discuss the shape and direction of the field lines.

Day 3: Electromagnetic Induction

Objective: Understand the principles of electromagnetic induction and its applications.

- **Applying:** Conduct an experiment to observe electromagnetic induction (e.g., moving a magnet through a coil to induce a current).

Activity:
Students create an electromagnet using a coil of wire and a battery. They then move a magnet through a coil connected to a galvanometer to observe and measure the induced current. They discuss the relationship between the movement of the magnet and the induced current.

Day 4: Creative Project – Designing Magnetic Applications

Objective: Design a project that demonstrates principles of magnetism.

- **Creating:** Students design and build a project that applies principles of magnetism, such as a simple electric generator or a magnetic levitation device.

Activity:
In groups, students brainstorm, design, and build a project that uses magnetism. They document each step, explain the principles involved, and present their project to the class. Examples could include a homemade electric generator or a magnetic levitation setup.

Day 5: Reflection and Digital Presentation

Objective: Reflect on what they have learned and create a digital presentation about magnetism.

- **Evaluating:** Discuss and reflect on the experiments and activities.

- **Creating:** Use AI tools to create a digital presentation or video tutorial explaining a concept related to magnetism.

Activity:
Students create a digital presentation or a video tutorial using AI tools, such as an animation or an interactive slideshow, that explains a concept they have learned about magnetism. They present their projects to the class, using their digital presentations to enhance their explanations.

By incorporating these strategies and activities, educators can effectively teach 11th graders about magnetism, helping them understand and appreciate the advanced concepts and their applications in the real world.

12.7: End of Chapter Key Terms

Definition: Magnetism

- **Magnetism**: A physical phenomenon produced by the motion of electric charges, resulting in attractive and repulsive forces between objects.
- **Magnet**: An object that produces a magnetic field, with a north and south pole.
- **Magnetic Field**: A field around a magnet where magnetic forces can be detected, represented by field lines.
- **Magnetic Pole**: One of two ends of a magnet where the magnetic force is strongest; includes the north pole and south pole.
- **Magnetic Flux**: A measure of the amount of magnetic field passing through a given area, measured in Weber (Wb).
- **Magnetic Flux Density**: The amount of magnetic flux through a unit area perpendicular to the direction of magnetic flow, measured in Tesla (T).
- **Electromagnet**: A magnet created by electric current flowing through a coil of wire, often wrapped around a ferromagnetic core.
- **Ferromagnetic Material**: A material, such as iron, cobalt, or nickel, that can be magnetized because of its high magnetic permeability.
- **Permanent Magnet**: A magnet that retains its magnetic properties for a long period of time without the need for external power.
- **Temporary Magnet**: A magnet that behaves like a permanent magnet when in the presence of a magnetic field but loses its magnetism when the field is removed.
- **Domain**: A region within a ferromagnetic material where the magnetic moments of atoms are aligned in the same direction.
- **Curie Temperature**: The temperature above which a ferromagnetic material loses its permanent magnetism and becomes paramagnetic.
- **Paramagnetic Material**: A material that is weakly attracted by a magnetic field and does not retain magnetic properties when the external field is removed.
- **Diamagnetic Material**: A material that is weakly repelled by a magnetic field and does not retain magnetic properties.
- **Magnetic Permeability**: A measure of how easily a material can become magnetized, indicating the ability to support the formation of a magnetic field within itself.
- **Gauss's Law for Magnetism**: A fundamental law stating that the net magnetic flux through a closed surface is zero, indicating that magnetic monopoles do not exist.
- **Magnetization**: The process of aligning the magnetic moments of a material to produce a magnetic field.
- **Hysteresis**: The lag between changes in the magnetization of a material and changes in the external magnetic field, often represented by a hysteresis loop.
- **Electromagnetic Induction**: The process of generating an electric current by changing the magnetic field within a coil of wire.
- **Faraday's Law of Induction**: A law stating that the induced electromotive force (EMF) in a coil is proportional to the rate of change of magnetic flux through the coil.
- **Lenz's Law**: A law stating that the direction of an induced current is such that it opposes the change in magnetic flux that caused it.
- **Inductor**: A passive electronic component that stores energy in its magnetic field, typically a coil of wire.
- **Transformer**: A device that uses electromagnetic induction to increase or decrease the voltage of alternating current (AC) in a circuit.
- **Magnetic Declination**: The angle between geographic north and the direction a compass needle points, varying with location.
- **Magnetic Inclination**: The angle between the horizontal plane and the Earth's magnetic field lines, varying with latitude.
- **Geomagnetic Field**: The magnetic field that extends from the Earth's interior into space, protecting the planet from solar wind and cosmic radiation.

- **Magnetosphere**: The region around Earth dominated by its magnetic field, which deflects charged particles from the solar wind.
- **Solenoid**: A coil of wire that generates a magnetic field when an electric current passes through it, often used to create uniform magnetic fields.
- **Toroid**: A donut-shaped coil of wire that generates a magnetic field within the core of the coil when an electric current passes through it.
- **Hall Effect**: The production of a voltage difference across an electrical conductor when a magnetic field is applied perpendicular to the current.
- **Magnetic Levitation**: The suspension of an object in the air using magnetic forces to counteract gravitational forces.
- **Magnetoresistance**: The change in electrical resistance of a material in response to an applied magnetic field.
- **Magnetic Resonance Imaging (MRI)**: A medical imaging technique that uses strong magnetic fields and radio waves to generate detailed images of the body.
- **Spintronics**: A field of technology that exploits the intrinsic spin of electrons and their associated magnetic moment, in addition to their charge.
- **Magnetic Recording**: The process of storing data by magnetizing a medium, used in devices like hard drives and magnetic tapes.

CHAPTER OVERVIEW

13: Transverse and Longitudinal Waves

13.1: Introduction and Learning Objectives

The concept of a wave is a central part of much of physics, and by extension this includes chemistry, geology, engineering, astronomy, biology and more. Each discipline is interested in the behavior of particular types of waves, and each discipline uses waves to describe different aspects of the objects being investigated.

Most of the ways that we gather information involve the production and/or detection of waves and whether we use our eyes, cameras or telescopes to look at objects doesn't matter. All these systems gather information from electromagnetic waves. So do our RADAR systems, radio and cell phones. Our microwave ovens also utilize electromagnetic waves to heat our food. Our ears gather information from pressure waves in the surroundings, and our seismographs examine the results of pressure waves traveling through the earth.

In the end, wave behavior is usually used to describe the process by which energy moves through different materials. We use language that describes the outcomes, i.e. "That sound is loud", or "That light is bright", or "That was a small earthquake", and most people will understand in a general way what you mean when you say these things. This chapter will make some fundamental definitions that can be applied to any system that involves waves and will give us a common language to describe waves of all kinds. Examining the relationship between materials and the types of waves that can be created will help us to better understand the universe.

Although the idea of a wave is ephemeral, the study of waves is readily available to everyone. The ability to link the abstract explanation to a concrete example is useful at all levels of education. Whether water waves in a ripple tank, light waves through lenses or sound waves from instruments, there are numerous physical examples of waves that help students appreciate the utility and importance of understanding the fundamentals of wave behavior.

The chapter begins with a discussion of oscillations, then moves on to classification of waves and general behaviors. Types of waves and the relationship between the wave and the medium are explored. Specific applications of waves such as light (vision) and sound (hearing) are discussed in later chapters.

Chapter Learning Objectives

1. Differentiating Longitudinal and Transverse Waves:
    - Define and distinguish between longitudinal and transverse waves.
    - Identify natural and technological examples of each type (e.g., sound waves, electromagnetic waves).
2. Exploring the Properties of Longitudinal Waves:
    - Investigate properties like compression, rarefaction, wavelength, frequency, amplitude, and speed.
    - Understand sound wave production and propagation through different mediums.
    - Analyze applications in acoustics and communication.
3. Studying the Properties of Transverse Waves:
    - Explore properties like crest, trough, wavelength, frequency, amplitude, and speed.
    - Examine generation and propagation with examples from light waves and mechanical waves.
    - Discuss applications in optics, telecommunications, and seismic studies.
4. Wave Phenomena in Longitudinal and Transverse Waves:
    - Understand reflection, refraction, interference, and diffraction in both wave types.
    - Examine polarization in transverse waves.
    - Explore scientific and technological uses of these phenomena.
5. Teaching Longitudinal and Transverse Waves in the Classroom:
    - Develop strategies to teach wave concepts using age-appropriate examples.
    - Conduct experiments and demonstrations to reinforce understanding.
    - Use multimedia resources to visualize wave motion.
    - Address misconceptions and provide clear explanations.

- Connect wave concepts to real-life applications in various fields.

13.2: Simple Harmonic Motion and Oscillations

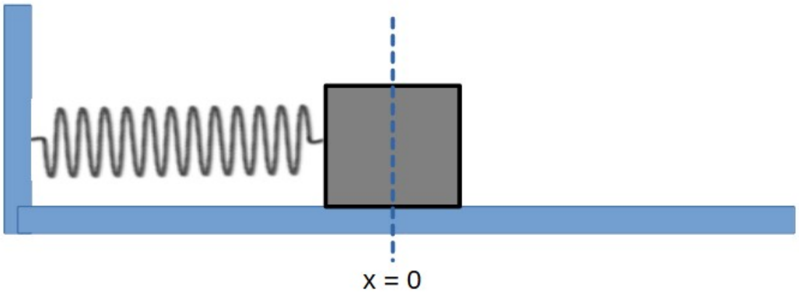
To understand the properties of waves, we start by examining oscillations. An oscillation is a type of repetitive, periodic behavior. This is a fairly wide definition and includes a huge number of systems. In fact, oscillations are an important part of biology, chemistry, economics, ecology, engineering, geology, music and more. The focus for now will be on a specific type of oscillation called *simple harmonic oscillation*. This is also referred to as simple harmonic motion (SHM).

Even though the 'simple' part of simple harmonic motion requires that we ignore things like friction or air resistance, we can still learn a lot about real-world systems that behave in a similar way. A pendulum swinging back and forth, and a mass attached to a spring are two examples of oscillations that are nearly the same as the simplified system. Before we start, some general definitions are useful.

Oscillation Fundamentals

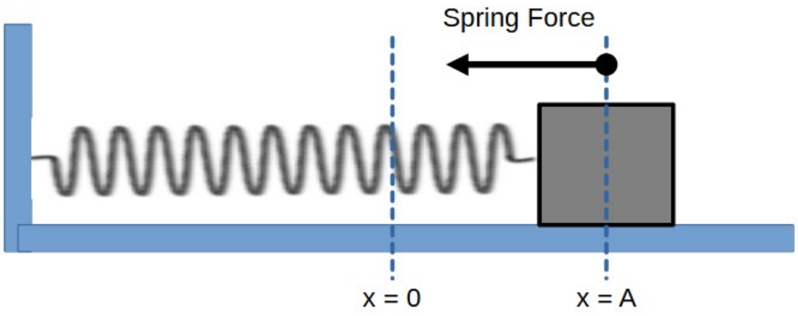
- Oscillation or Cycle: A process during which the system returns to its initial conditions.
- Amplitude: The maximum displacement from the equilibrium position. This is typically measured in meters and is symbolized by A.
- Period: The length of time the system takes to complete one cycle. This has units of seconds per cycle and is represented by T.
- Frequency: How often the system completes an oscillation. This has units of cycles per second, also called Hertz, written as f. $f = \frac{1}{T}$

To help build some intuition about oscillating systems imagine a flat, horizontal surface that is frictionless. A stiff spring is attached to a wall, and a block is attached to the other end. The block is in equilibrium, meaning that any forces acting on the block are balanced. The surface supports the block and cancels the effect of gravity. The spring is relaxed, meaning it isn't stretched or compressed. The block is at some distance from the wall. We'll label the position of the block as x, and set x = 0.



Mass-spring system in equilibrium, by Claude Mona, under CC BY NC

Pulling the block to the right would move the block in the positive direction and stretch the spring. We know that spring forces act to try and keep the spring the same length, so stretching the spring to the right means that the spring will pull to the left. If we let go of the block at point x = A, the block will start moving to the left.



Mass-spring system (net spring force), by Claude Mona, under CC BY NC

If we pushed the block to the left, the spring would compress and push to the right. In either case, the spring force acts on the block to try and return the block to the equilibrium position, x = 0. A force that acts this way is called a *restoring force*, and restoring forces are needed for systems to oscillate. The spring force (restoring force) depends on the properties of the spring and how far the spring has been stretched from equilibrium:

$$\mathrm{F}_{spring} = -k\Delta x$$

Δx is a measure of how far the block has been moved from equilibrium. In the picture above, this would be Δx = A. How far the block is moved from equilibrium is also the definition of *amplitude*. In this scenario, the amplitude of motion is A.

The 'k' is called the spring constant, and describes the 'stiffness' of the spring. A large k value means that it requires a large force to stretch or compress the spring. The minus sign helps us to remember that the spring is a restoring force that acts to return the block to the equilibrium position. The spring constant typically has units of Newtons per meter.

The stiffer the spring is, the larger the force that acts on the block. Larger forces lead to larger accelerations. Larger accelerations lead to larger speeds. Larger speeds mean shorter times to travel from one point to another. So, we imagine that large k values lead to short periods.

For any particular spring force, the larger the mass of the block, the less it will accelerate. Lower accelerations lead to slower speeds and that should mean longer travel times. Here we see that large mass values lead to longer periods.

Finally, the further the spring is stretched (larger amplitude), the larger the force the spring produces. We see that larger forces lead to larger speeds, but the block also has to travel a greater distance because the spring was stretched further. Larger speeds mean shorter travel times, but greater distances mean longer travel times. At this point, it isn't exactly clear if these effects cancel each other out. Maybe the amplitude has an effect on the period and maybe it doesn't.

✓ Example 13.2.1

Spring 1 requires a force of 54 Newtons to achieve a 14 centimeter stretch. Spring 2 stretches 18 centimeters when a 62 Newton force is applied. Which one stretches the furthest when a 9 Newton force is applied to it?

Solution

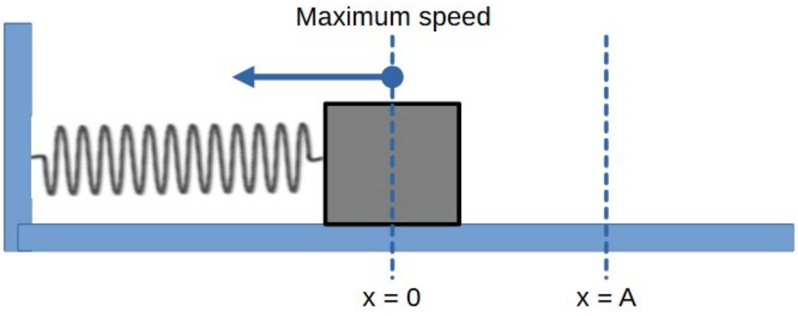
The spring constant provides information about the 'stretchiness' of the spring. Spring 1: $\mathrm{k}_1 = \frac{54 \text{ Newtons}}{0.14 \text{ meters}} = 385.7\ \frac{\mathrm{N}}{\mathrm{m}}$. Spring 2: $\mathrm{k}_2 = \frac{62 \text{ Newtons}}{0.18 \text{ meters}} = 344.4\ \frac{\mathrm{N}}{\mathrm{m}}$.

Since spring 2 has the lower spring constant, it will stretch more for any applied force. You could also take the reciprocal of k and calculate the amount of stretch per Newton of force applied. For spring 1, this is 2.59 millimeters per Newton and spring 2 stretches 2.9 millimeters per Newton.

13.2.1: Anatomy of an Oscillation

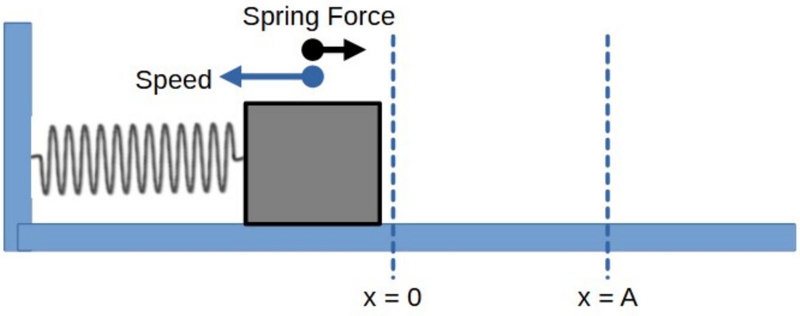
Anatomy of an Oscillation

At time = 0 seconds, the block is released at x = A. It has zero velocity, maximum acceleration and maximum displacement. The spring is at maximum stretch length. As the spring pulls the block towards x = 0, the block's velocity starts to increase. As the spring contracts, the spring force decreases so the acceleration decreases. But, even though the acceleration is decreasing, the block's velocity is still increasing. At some later time, the block has reached x = 0. The spring force becomes zero, so the acceleration becomes zero. The block reaches maximum speed, traveling to the left.



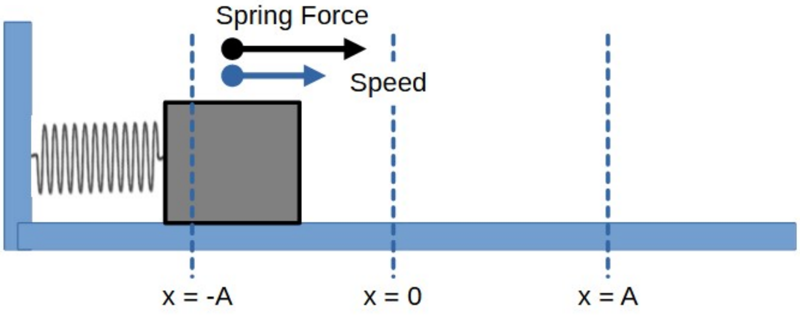
Mass-spring system (maximum speed), by Claude Mona, under CC BY NC

As the block crosses the equilibrium position moving to the left, it starts to compress the spring. The spring force now acts to the right, meaning the acceleration is in the opposite direction that the block is traveling. This causes the block to slow down as it keeps moving to the left.



Mass-spring system (slowing down), by Claude Mona, under CC BY NC

Eventually the block stops moving. This happens at position x = -A. At this instant the block is half-way through the cycle, so t = $\frac{1}{2}$T. It experiences a maximum spring force to the right, which means maximum acceleration to the right. Now the block starts moving to the right, speeding up as the spring force and acceleration decrease. It returns to x = 0, moving with maximum speed, still moving to the right.



Mass-spring system (reversing direction), by Claude Mona, under CC BY NC

From x = 0 to x = A, the block slows down, because the spring force is to the left and the block is still moving to the right. The block momentarily stops moving when it reaches x = A. This completes one cycle. The block has returned to the same location it started from, T seconds after it was initially released.

13.2.2: Characteristics of Oscillations

Characteristics of Oscillations with Simple Harmonic Motion

The following characteristics are true for any oscillating system that can be described as simple harmonic.

- Acceleration is maximum when amplitude is maximum.
- Velocity is zero when acceleration and amplitude are maximum,
- Velocity is maximum when amplitude and displacement are zero.

For an object that starts with maximum amplitude, the relationship between position and period is as follows:

- $t = 0$, object is at maximum amplitude ($x = A$).
- $t = \frac{T}{4}$, object reaches equilibrium position ($x = 0$), headed towards $x = -A$.
- $t = \frac{T}{2}$, objects reaches maximum amplitude on other side of equilibrium position ($x = -A$).
- $t = \frac{3T}{4}$, object reaches equilibrium position ($x = 0$), headed towards $x = A$.
- $t = T$, object is back at maximum amplitude ($x = A$).

Mathematics of Simple Harmonic Motion

To describe an object oscillating back and forth between two endpoints, we need a function that also oscillates between two extremes. A sin or a cosine function has limits of +1 and -1, making it an ideal choice to describe these systems. Deriving the results means using differential equations, but the following relationships emerge for an object that starts at rest at maximum amplitude.

- Amplitude changes over time: $x = A \cos(2\pi \frac{t}{T}) = A \cos(2\pi f t)$.
- Velocity changes over time: $v = -(2\pi \frac{t}{T})\, A \sin(2\pi \frac{t}{T}) = -(2\pi f)\, A \sin(2\pi f t)$.
- Acceleration changes over time: $x = [(2\pi \frac{t}{T})]^2 A \cos(2\pi \frac{t}{T}) = -[(2\pi f)]^2 A \cos(2\pi f t)$.

We saw in our study of energy that kinetic energy depends on the velocity of the object ($K = \frac{1}{2} mv^2$), but now the velocity of the object depends on the frequency of the object, so $K = \frac{1}{2} mv^2$ becomes $K = \frac{1}{2} m [-(2\pi f) A \sin(2\pi f t)]^2$.

The maximum kinetic energy of an oscillating mass happens when sin = 1 or -1, so $K_{max} = \frac{1}{2} m(4\pi^2 f^2)A^2 = 2m\pi^2 f^2 A^2$. The fact that the kinetic energy depends on the oscillation frequency will prove to be important in our study of electromagnetic energy.

✓ Example 13.2.2.1

An oscillating mass on a spring has an amplitude of 2 meters and a period of 10 seconds. It is released from rest at $t = 0$ seconds. Calculate the following:

1. The position of the object when $t = 6$ seconds.
2. The velocity of the object when $t = 9$ seconds.
3. The acceleration of the object when $t = 20$ seconds.

Solution

Since period and frequency are inversely related, we can calculate f. $f = 1$ cycle / 10 seconds $= 0.1 \frac{\text{cycle}}{\text{second}}$

1. $x = 2$ meters $\cos(2\pi(0.1)(6)) = -1.62$ meters. This negative value means it is to the left of the equilibrium position.
2. $v = -(2\pi(0.1)(9))$ 2 meters $\sin(2\pi(0.1)(9)) = 3.32$ meters per second. The positive value means it is moving to the right.
3. $a = -[(2\pi(0.1)(20))]^2 A \cos(2\pi(0.1)(20)) = -157.9$ meters per second^2. The negative value means it is accelerating to the left.

From the solution to the differential equation that describes the system we can also find how the spring and the mass interact to produce the particular period of motion:

$$T = 2\pi \sqrt{M/k}$$

From this we can see that some of our intuition was correct. Larger masses lead to longer periods, as expected. Stiffer springs lead to shorter periods, again as expected. Since there is no amplitude in the equation for the period, the period of a simple harmonic system doesn't depend on the amplitude.

13.3: Waves and Oscillations

We've all seen ripples in water produced when raindrops fall on a lake or pond. We've seen surfers ride on ocean waves. We have heard about sound waves. We've waved goodbyes to friends and family, though that last one probably isn't the kind of wave we're talking about in this chapter. What 'is' a wave?

A wave is an outcome. It is the result of an interaction between two parts of a system. A wave is something that happens when a source (someone or something) adds energy to a medium (material) that is governed by restoring forces. When both these things are present, a wave can result.

We know that restoring forces cause oscillations, so a wave is the oscillations of the atoms of the substance. Why do the atoms oscillate? Something pulled them away from their equilibrium position and started the motion. We know that atoms are connected to each other by chemical bonds so when one atom starts to oscillate, it causes its neighbors to begin oscillating as well. The transfer of energy from one atom to another causes the oscillation to 'move' through the material. The movement of the energy through the oscillating material is what we call a 'wave'. Waves can move from one medium to the next, like when an earthquake causes ripples in a swimming pool. Waves can reflect off an interface between two different mediums, like when light moves from the air to water and reflects off the water surface.

There are only two types of waves, mechanical and electromagnetic. Electromagnetic waves are the topic of a later chapter, so we will focus our attention on mechanical waves. Mechanical waves travel through a material (medium) of some kind. Mechanical waves can travel through solids, liquids or gases. When you wiggle the end of a garden hose, or pluck a guitar string you've created a mechanical wave. When you hear the sound of a guitar, or when an earthquake shakes the walls of your house you are experiencing the transfer of mechanical wave energy from one medium to another.

Types of Mechanical Waves

There are only two types of mechanical waves, transverse and longitudinal. These terms refer to the way that the atoms of the material move as the energy passes through the material. Mechanical waves share a common set of variables used to describe their behavior and there are only a few small differences between the types.

Because there are many points of similarity between oscillations and waves, there is a significant overlap between the terms used to describe them. This can sometimes become a point of confusion because there are two aspects to describing mechanical waves. One is the description of the energy moving through the material (the 'wave' itself) and one is a description of how the atoms of the material behave as the wave passes through it. Although these ideas are related, they are not the same.

13.3.1: Transverse Waves

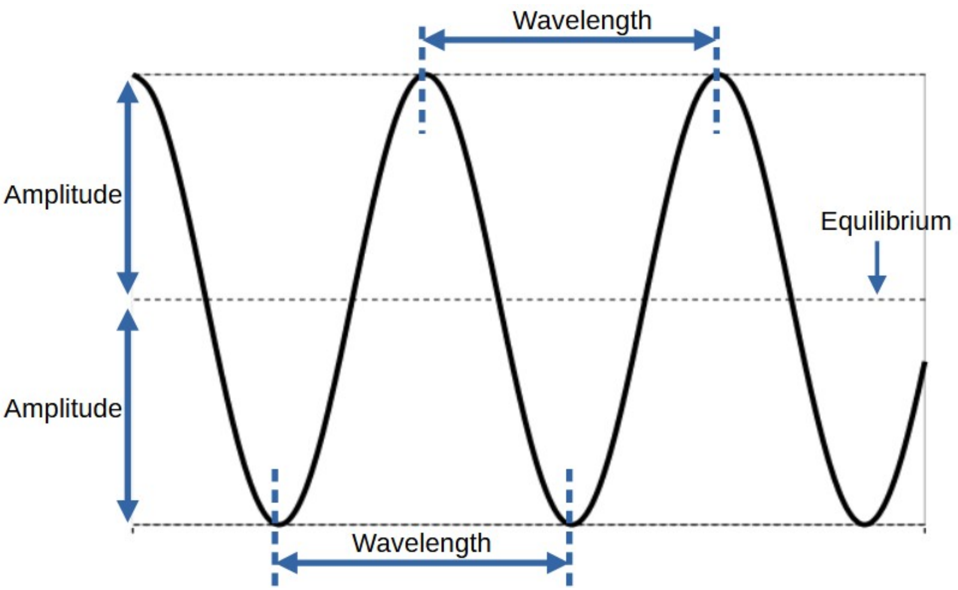
Transverse Waves

A familiar example of a transverse wave occurs when you toss a rock into a lake or pond. When the rock hits the surface of the water, ripples are produced. The ripples are formed by the energy of the falling rock being shared by the water molecules near the impact point. As those molecules transfer the energy to molecules located further away, the ripples expand outward from the impact point. Although the energy spreads out horizontally through the water, the water molecules do not move horizontally. As the energy is transferred from one water molecule to the next, the surface of the water bounces up and down, each molecule acting like a simple harmonic oscillator.

A way to help visualize the difference between the wave speed and the speed of the water molecules is to imagine a line of people standing side by side. The person (water molecule) on the far left end is handed a ball (energy), and they pass the ball to the next person in line. Each person passes the ball to the next person until the ball has traveled from one end of the line to the other. None of the people move very much. They might lean to the left to receive the ball and then lean to the right to pass it to their neighbor, but they do not run with the ball. The ball definitely travels along the line with an average velocity, and even though each person leans left and right they do not change position, which gives them an average velocity of zero.

"Stones-Pebbles-Water" *by geralt via* *Pixabay* under CCO

Imagine a ripple moving through the water. Ahead of the ripple, the water is flat and undisturbed. There, the water surface is in a state of equilibrium. As the energy passes through the water, it produces a series of crests and troughs. The crests and troughs are created by the water surface rising and falling in a simple harmonic oscillation. If you took a picture of the water surface as the energy passes through, it would look something like this:



Parts of a wave, by Claude Mona, under CC BY NC

In a transverse wave, the atoms of the medium move perpendicular to the direction that the energy is moving through the medium. When the wave has passed, the atoms of the medium have returned to their equilibrium positions. All transverse waves share certain characteristics.

- Waves have an amplitude (A). The amplitude is how far from equilibrium the atoms of the material have moved as the wave energy passes through the system. The 'height' of the crests is the same as the 'depth' of the troughs.
- Waves have a wavelength (λ). The wavelength is set by how far the energy travels in one period. The wavelength is different from the amplitude. The wavelength measures the distance between two successive crests (or troughs) as the wave travels through the medium.
- Waves have a velocity (v_{wave}). The wave velocity is a measure of how long it takes for the energy to pass from one atom to another. This is different than the velocity of the moving atoms.

The distance between two crests is defined to be the wavelength. We know that the time between two maximum amplitude positions in a simple harmonic system is defined to be the period. The energy traveled a distance λ in a time T. The ratio of displacement over time is the definition of velocity, so the wave speed is defined as $v_{wave} = \frac{\lambda}{T}$. Since period and frequency are related then $v_{wave} = \lambda f$. This is true for all simple harmonic waves.

Another example of a transverse wave can be seen when two people hold the ends of a rope and one person rapidly wiggles their end of the rope up and down. A series of 'bumps' is produced and these 'bumps' travel along the rope, away from the wiggling end. Although the energy is moving horizontally along the rope, the rope is not moving horizontally. Instead, the rope travels up and down, with each section of the rope acting as a simple harmonic oscillator. Because of this, there is a wave speed which describes how quickly the 'bumps' move horizontally along the rope, but there is also the 'rope speed' which talks about how quickly the end of the rope is moving up and down.

✓ Example 13.3.1.1

A certain transverse wave has a wave velocity of 1580 meters per second. If the wavelength is 1.35 meters, what is the frequency of the wave?

Solution

Starting with $v_{wave} = \lambda f$, you can solve for $f = \frac{v_{wave}}{\lambda} = \frac{1580 \text{ meters/second}}{1.35 \text{ meters}} = 1170.4$ Hertz.

13.3.1: Transverse Waves is shared under a CC BY-NC-SA license and was authored, remixed, and/or curated by Claude Mona.

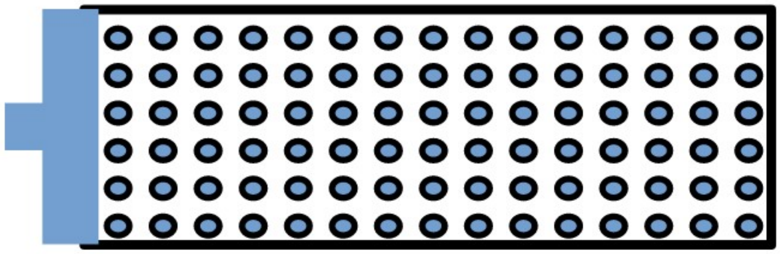
13.3.2: Longitudinal Waves

Longitudinal Waves

These waves are a little more difficult to visualize, though the fundamental ideas are the same as for transverse waves. Amplitude is still the displacement from the equilibrium, wavelength is still the distance between two identical points on the wave, and period is still the time for one cycle to complete. The wave speed is still written as the product of wavelength and frequency. The difference lies in the behavior of the atoms. In longitudinal waves, the atoms of the medium move parallel to the direction that the energy is transported.

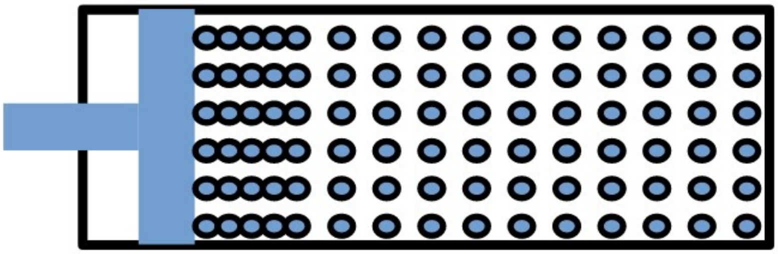
The atoms have some sort of average spacing when they are in equilibrium. A force can act on them to cause them to move, pushing them closer together or pulling them apart than the equilibrium spacing. The regions where the atoms are forced closer together are called *compressions* and the places where they are pulled further apart are called *rarefactions*. Because of this atomic behavior, these types of waves are also called compression waves.

The image shows the top view of a cylindrical 'tube' of air. There is a moveable piston at the left end of the cylinder. Initially, the piston is stationary and the density of the gas is 'normal' so the gas atoms have a certain average spacing.



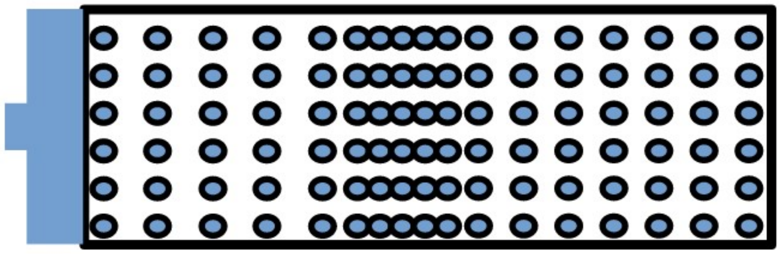
Ideal gas in equilibrium, by Claude Mona, under CC BY NC

When the piston is moved to the right, it exerts a force on the nearest atoms, causing them to move closer together and making them move away from the piston. These atoms collide with the next nearest atoms, causing them to move and so on. Although each atom doesn't move very far, the compression region travels away from the piston.



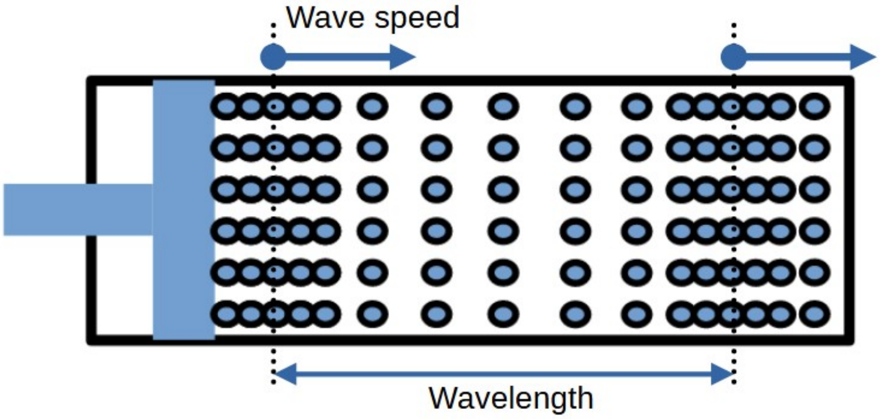
Ideal gas under compression, by Claude Mona, under CC BY NC

When the piston is retracted, it drags the nearest atoms 'backwards' pulling them further apart and forming a rarefaction. The region of rarefaction also moves along the tube.



Ideal gas rarefaction, by Claude Mona, under CC BY NC

When the piston compresses the gas for the second time it has returned to its initial position, completing one cycle. The first compression region has traveled some distance during this time, so the distance between the first and second compressions defines the wavelength.



Compression wave, by Claude Mona, under CC BY NC

You can easily create compression waves with a long coiled spring like a slinky. Take a slinky and stretch it along a horizontal surface. Notice how far apart the coils are when the slinky is in an equilibrium state. Have someone hold one end of the slinky stationary while you rhythmically push and pull on the other end. After a little while you'll notice that there are sections of the slinky where the coils are closer together and sections where the coils are farther apart than when the slinky was stationary. The locations of the compressed areas travel along the slinky.

You can find many youtube videos and virtual physics demonstrations (phet.colorado.edu/en/simulations/waves-intro) illustrating these types of compression waves. The side to side shaking that happens in an earthquake is the result of a longitudinal wave, as is the sound wave produced by the movement of a speaker cone or the vibration of a guitar string. The points of compression and rarefaction will be linked to high and low air pressure regions in sound waves.

13.3.3: Wave Speeds in Materials

Wave Speed

While it is true that all wave energy travels through the medium at a constant speed given by $v_{wave} = \lambda$ f, the same type of wave will have different speeds in different materials (mediums). The properties of the material are important in determining how quickly energy is passed from one atom to the next. Solids produce the fastest wave speeds and gases transmit energy the slowest. Even among solids, there are differences in wave speeds. The more rigid a solid is, the faster the wave travels through it, and 'squishier' objects produce slower wave speeds. Materials with a larger atomic or molecular mass tend to transmit energy more slowly, since the inertia of those atoms prevents rapid changes from taking place.

This should make sense, given our understanding of how masses and springs oscillate. We saw that the period of motion of a mass on a spring was larger for larger masses. That means it will take longer for the mass to complete one cycle, and it is the movement of one atom that transfer the wave energy to the next atom. If the atoms are moving more slowly, then it will take longer for them to transfer their energy to the next atom. Larger periods mean smaller frequencies and those give slower wave speeds.

We also saw that the mass-spring period decreased as the spring constant increased. Larger spring constants means stiffer springs that generate larger forces. Larger forces should produce a greater effect on the movement of the atoms, so we would expect the atoms to transfer their energy more rapidly. Rapid energy transfer between atoms causes wave speed to increase. Smaller periods give larger frequencies and that translates to higher wave speeds.

13.4: End of Chapter Activity

End of Chapter Activity: Creating a Lesson Plan on Longitudinal and Transverse Waves with AI and Bloom's Taxonomy

Now that you have explored the fundamentals of longitudinal and transverse waves, it's time to put your knowledge into practice. Teaching complex scientific concepts like wave properties to 6th graders can be challenging, as it requires simplifying the material to make it accessible and engaging for young minds. Your task is to create a succinct lesson plan for 6th graders that introduces them to the basics of longitudinal and transverse waves. To help you with this, you will use AI tools and incorporate Bloom's Taxonomy to ensure a comprehensive learning experience. This lesson plan will go towards your digital notebook, a portfolio filled with lesson plans, activities, and labs for future use.

Activity Prompt:

> **Objective:** Use AI and Bloom's Taxonomy to develop a lesson plan that effectively teaches 6th graders about the fundamentals of longitudinal and transverse waves, including concepts such as wave properties, wave behavior, and real-life applications.

Understanding the Concepts:

> **Knowledge (Remembering):** Define key terms related to waves, such as wavelength, frequency, amplitude, longitudinal wave, and transverse wave.
>
> **Comprehension (Understanding):** Explain these concepts in simple, age-appropriate language, using relatable examples and visuals.

Planning the Lesson:

Application: Design an engaging activity that allows students to observe and understand different types of waves. For example, use a slinky to demonstrate longitudinal waves and a rope to demonstrate transverse waves.

Analysis: Use AI tools to create visual aids or interactive simulations that illustrate the behavior of longitudinal and transverse waves. For instance, create a simulation showing how particles move in each type of wave.

Deepening Understanding:

> **Synthesis (Creating):** Ask students to design their own simple experiments that demonstrate wave properties. For example, they could create waves in a water tank or use sound waves to visualize longitudinal waves.
>
> **Evaluation:** Have students discuss and reflect on their experiments and the wave properties they observed. Encourage them to think about how waves are present in their everyday lives.

Using AI in the Classroom:

Explore AI tools like educational apps or platforms that provide interactive content for teaching about waves. Use these tools to create quizzes, flashcards, or interactive stories that reinforce the lesson's concepts.

Use AI to assess student understanding through formative assessments and provide instant feedback.

Deliverable:

Submit a detailed lesson plan that includes:

1. **A brief overview of the key concepts covered:** Outline the foundational concepts of longitudinal and transverse waves that will be taught.
2. **A description of the activities and experiments designed:** Detail the hands-on activities and experiments you will use to help students understand these concepts.
3. **Examples of AI tools used and how they enhance the learning experience:** Describe the AI tools you plan to incorporate, such as simulations or interactive quizzes, and explain how they will help students grasp complex concepts.
4. **An explanation of how Bloom's Taxonomy was applied in the lesson plan to ensure a well-rounded educational experience:** Illustrate how each level of Bloom's Taxonomy (Remembering, Understanding, Applying, Analyzing, Creating, and Evaluating) is addressed in your lesson plan.

Additionally, include a creative project component where students create a digital animation or a simple stop-motion video that explains a concept related to waves, using AI tools to enhance their projects.

Example Lesson Plan:

Grade: 6th Grade
Topic: Longitudinal and Transverse Waves
Duration: 1 Week

Overview:
Students will learn about the basics of longitudinal and transverse waves, including wave properties and behavior, through engaging activities and creative projects.

Day 1: Introduction to Waves

Objective: Define basic concepts related to waves and provide examples.

- **Remembering:** Define key terms (wavelength, frequency, amplitude, longitudinal wave, transverse wave).
- **Understanding:** Explain the concepts using examples from everyday life (e.g., sound waves, light waves).

Activity:
Watch a short, animated video (created using AI tools) explaining what waves are, how they move, and the difference between longitudinal and transverse waves.

Day 2: Exploring Longitudinal Waves

Objective: Understand how longitudinal waves move and behave.

- **Applying:** Conduct an activity using a slinky to demonstrate longitudinal waves.

Activity:
Students use slinkies to create longitudinal waves by compressing and releasing them. They observe the movement of the coils and relate it to how sound waves travel through the air.

Day 3: Exploring Transverse Waves

Objective: Understand how transverse waves move and behave.

- **Applying:** Conduct an activity using a rope to demonstrate transverse waves.

Activity:
Students create transverse waves by moving a rope up and down. They observe the motion of the waves and discuss the properties such as crest, trough, wavelength, and amplitude.

Day 4: Creative Project – Designing Wave Experiments

Objective: Design and conduct experiments to demonstrate wave properties.

- **Creating:** Students design their own simple experiments to visualize wave properties.

Activity:
In groups, students design experiments such as creating waves in a water tank to observe wave behavior or using sound waves to visualize longitudinal waves. They document their experiments and explain the wave properties they observed.

Day 5: Reflection and Digital Animation

Objective: Reflect on what they have learned and create a digital animation or stop-motion video about waves.

- **Evaluating:** Discuss and reflect on the experiments and activities.
- **Creating:** Use AI tools to create a digital animation or stop-motion video that explains a concept related to waves.

Activity:
Students create a short digital animation or stop-motion video using AI tools that explains what they have learned about waves. They can include drawings, photos of their experiments, and voice recordings. They present their projects to the class, using the animations or videos to enhance their explanations.

By incorporating these strategies and activities, educators can effectively teach 6th graders about longitudinal and transverse waves, helping them understand and appreciate the fundamental concepts in a fun and engaging way.

13.5: End of Chapter Key Terms

Definition: Longitudinal and Transverse Waves

- **Wave**: A disturbance that transfers energy through matter or space, characterized by its wavelength, frequency, and amplitude.
- **Longitudinal Wave**: A wave in which the particle displacement is parallel to the direction of wave propagation, such as sound waves in air.
- **Compression**: The region in a longitudinal wave where particles are closest together.
- **Rarefaction**: The region in a longitudinal wave where particles are furthest apart.
- **Transverse Wave**: A wave in which the particle displacement is perpendicular to the direction of wave propagation, such as waves on a string or electromagnetic waves.
- **Crest**: The highest point of a wave in a transverse wave.
- **Trough**: The lowest point of a wave in a transverse wave.
- **Wavelength**: The distance between successive crests, troughs, compressions, or rarefactions in a wave.
- **Frequency**: The number of waves that pass a given point per second measured in cycles per second, called Hertz (Hz).
- **Amplitude**: The maximum displacement of particles from their rest position, indicating the wave's energy or intensity.
- **Period**: The time it takes for one complete wave cycle to pass a given point, the inverse of frequency.
- **Wave Speed**: The speed at which a wave travels through a medium, calculated as the product of wavelength and frequency.
- **Medium**: The substance or material through which a wave travels.
- **Mechanical Wave**: A wave that requires a medium to travel, such as sound waves or water waves.
- **Displacement**: The distance and direction from the equilibrium position to a point on the wave.
- **Propagation**: The movement or transmission of waves through a medium or space.
- **Wave Equation**: A mathematical formula that describes the propagation of waves through a medium, often expressed as $v_{wave} = \lambda f$, where f is frequency, and λ is wavelength.

CHAPTER OVERVIEW

14: Property of Sound, Doppler Effect and Interferences

14.1: Introduction and Learning Objectives

Chapter Learning Objectives

1. **Fundamentals of Sound and Its Properties:**
    - Understand sound as a mechanical wave and its propagation through solids, liquids, and gases.
    - Explore sound properties: speed, frequency, wavelength, and amplitude, and their relation to pitch and loudness.
    - Analyze how different mediums affect sound speed and quality.
2. **Exploring the Doppler Effect:**
    - Explain the Doppler effect and its impact on sound frequency and wavelength due to relative motion.
    - Discuss applications of the Doppler effect in radar, sonar, ultrasound, and astronomy.
    - Investigate its role in understanding star movement and universe expansion.
3. **Understanding Sound Interference and Wave Superposition:**
    - Explore constructive and destructive interference in sound waves.
    - Understand superposition and its effect on resultant sound wave patterns.
    - Examine practical examples like noise cancellation and architectural acoustics.
4. **Sound Intensity and the Human Ear:**
    - Investigate measurement of sound intensity and its correlation with wave energy.
    - Understand the range of human hearing and effects of different sound intensities.
    - Discuss the physiological process of hearing and sound wave translation into brain signals.
5. **Teaching Concepts of Sound in Education:**
    - Develop effective teaching methodologies for sound, the Doppler effect, and interference.
    - Create engaging demonstrations and experiments to illustrate sound properties.
    - Utilize multimedia tools and simulations to enhance student understanding.
    - Address common misconceptions about sound and provide accurate explanations.
    - Emphasize the interdisciplinary nature of sound science, linking it to music, engineering, medicine, and environmental studies.

Introduction to the Properties of Sound: Doppler Effect, Interference, and Sound Intensity

In the study of physical science, understanding the properties of sound provides foundational knowledge essential for exploring how mechanical waves propagate through various mediums, such as solids, liquids, and gases. Sound is characterized by its speed, frequency, wavelength, and amplitude, all of which influence our perception of pitch and loudness. The behavior of sound waves changes depending on the medium they travel through, affecting their speed and quality.

The Doppler Effect is a crucial phenomenon that describes how the frequency and wavelength of sound waves change when there is relative motion between the source and the observer. This effect has significant applications in radar, sonar, medical ultrasound, and astronomy, helping us understand movements within our universe, such as star movements and the expansion of the universe.

Sound interference and wave superposition are key concepts that illustrate how overlapping sound waves interact, resulting in constructive or destructive interference. This understanding is applied in technologies such as noise cancellation and in the design of acoustically optimized architectural spaces. Additionally, the intensity of sound, which correlates with wave energy, is crucial for understanding the human hearing range and the physiological processes involved in how we perceive sound.

For K-12 educators, effectively conveying these scientific theories involves creating engaging and comprehensible lessons. By developing interactive demonstrations and experiments, utilizing multimedia tools, and addressing common misconceptions, educators can make complex concepts accessible. Linking sound science to interdisciplinary applications, such as music, engineering, medicine, and environmental studies, can help students appreciate the relevance and impact of these principles in their everyday lives and future careers.

14.2: Prelude to Sound

Sound is an example of a mechanical wave, specifically, a pressure wave: Sound waves travel through the air and other media as oscillations of molecules. Normal human hearing encompasses an impressive range of frequencies from 20 Hz to 20 kHz. Sounds below 20 Hz are called infrasound, whereas those above 20 kHz are called ultrasound. Some animals, like the bat shown in Figure 17.1, can hear sounds in the ultrasonic range.

Figure 14.2.1: Hearing is an important human sense that can detect frequencies of sound, ranging between 20 Hz and 20 kHz. However, other species have very different ranges of hearing. Bats, for example, emit clicks in ultrasound, using frequencies beyond 20 kHz. They can detect nearby insects by hearing the echo of these ultrasonic clicks. Ultrasound is important in several human applications, including probing the interior structures of human bodies, Earth, and the Sun. Ultrasound is also useful in industry for nondestructive testing. (credit: modification of work by Angell Williams)

Many of the concepts covered in Waves also have applications in the study of sound. For example, when a sound wave encounters an interface between two media with different wave speeds, reflection and transmission of the wave occur.

Ultrasound has many uses in science, engineering, and medicine. Ultrasound is used for nondestructive testing in engineering, such as testing the thickness of coating on metal. In medicine, sound waves are far less destructive than X-rays and can be used to image the fetus in a mother’s womb without danger to the fetus or the mother. Later in this chapter, we discuss the Doppler effect, which can be used to determine the velocity of blood in the arteries or wind speed in weather systems.

14.3: Sound Waves

Figure 12.1.1

Crack! Crash! Thud! That’s what you’d hear if you were in the forest when this old tree cracked and came crashing down to the ground. But what if there was nobody there to hear the tree fall? Would it still make these sounds? This is an old riddle. To answer the riddle correctly, you need to know the scientific definition of sound.

Defining Sound

In science, **sound** is defined as the transfer of energy from a vibrating object in waves that travel through matter. Most people commonly use the term sound to mean what they hear when sound waves enter their ears. The tree above generated sound waves when it fell to the ground, so it made sound according to the scientific definition. But the sound wasn’t detected by a person’s ears if there was nobody in the forest. So the answer to the riddle is both yes and no!

How Sound Waves Begin

All sound waves begin with vibrating matter. Look at the first guitar string on the left in the Figure below. Plucking the string makes it vibrate. The diagram below the figure shows the wave generated by the vibrating string. The moving string repeatedly pushes against the air particles next to it, which causes the air particles to vibrate. The vibrations spread through the air in all directions away from the guitar string as longitudinal waves. In longitudinal waves, particles of the medium vibrate back and forth parallel to the direction that the waves travel.

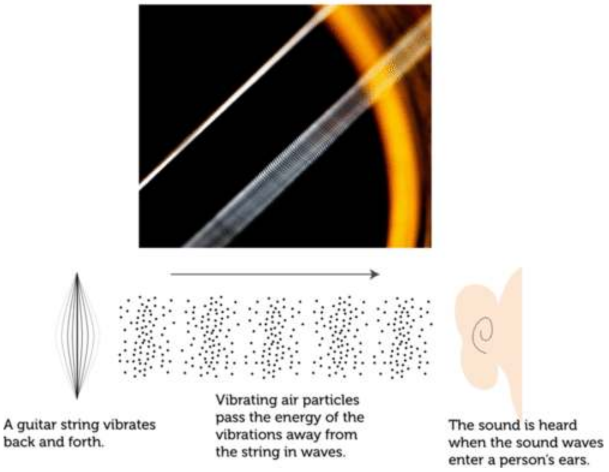


Figure 12.1.2

Q: If there were no air particles to carry the vibrations away from the guitar string, how would sound reach the ear?

A: It wouldn’t unless the vibrations were carried by another medium. Sound waves are mechanical waves, so they can travel only though matter and not through empty space.

What about the sound waves created by other instruments? Why does the same musical note on different instruments, such as a guitar and violin, sound different? Begin your exploration of sound by exploring the different types of sound waves produced by string instruments in the Violin simulation below:

A pan flute is another example of a musical instrument that depends on the vibration of air particles. Use the Pan Flute simulation below to visualize how the movement of invisible air molecules inside the tubes produces musical notes:

A Ticking Clock

The fact that sound cannot travel through empty space was first demonstrated in the 1600s by a scientist named Robert Boyle. Boyle placed a ticking clock in a sealed glass jar. The clock could be heard ticking through the air and glass of the jar. Then Boyle pumped the air out of the jar. The clock was still ticking, but the ticking sound could no longer be heard. That’s because the sound couldn’t travel away from the clock without air particles to pass the sound energy along.

Sound Waves and Matter

Most of the sounds we hear reach our ears through the air, but sounds can also travel through liquids and solids. If you swim underwater—or even submerge your ears in bathwater—any sounds you hear have traveled to your ears through the water. Some solids, including glass and metals, are very good at transmitting sounds. Foam rubber and heavy fabrics, on the other hand, tend to muffle sounds. They absorb rather than pass on the sound energy.

Q: How can you tell that sounds travel through solids?

A: One way is that you can hear loud outdoor sounds such as sirens through closed windows and doors. You can also hear sounds through the inside walls of a house. For example, if you put your ear against a wall, you may be able to eavesdrop on a conversation in the next room—not that you would, of course.

Summary

- In science, sound is defined as the transfer of energy from a vibrating object in waves that travel through matter.
- All sound waves begin with vibrating matter. The vibrations generate longitudinal waves that travel through matter in all directions.
- Most sounds we hear travel through air, but sounds can also travel through liquids and solids.

Review

1. How is sound defined in science? How does this definition differ from the common meaning of the word?
2. Hitting a drum, as shown in the Figure below, generates sound waves. Create a diagram to show how the sound waves begin and how they reach a person’s ears.

Figure 12.1.3

3. How do you think earplugs work?

Additional Resources

Study Guide:Waves Study Guide

Real World Application: Telephone Magic

Videos:

14.4: Speed of Sound

Figure 12.2.1

Has this ever happened to you? You see a flash of lightning on the horizon, but several seconds pass before you hear the rumble of thunder. The reason? The speed of light is much faster than the speed of sound.

What Is the Speed of Sound?

The **speed of sound** is the distance that sound waves travel in a given amount of time. You'll often see the speed of sound given as 343 meters per second (m/s). But that's just the speed of sound under a certain set of conditions, specifically, through dry air at 20°C (or 68°F). The speed of sound may be very different through other matter or at other temperatures.

Speed of Sound in Different Media

Sound waves are mechanical waves, and mechanical waves can only travel through matter. The matter through which the waves travel is called the medium (plural, media). The Table below gives the speed of sound in several different media. Generally, sound waves travel most quickly through solids, followed by liquids, and then by gases. Particles of matter are closest together in solids and farthest apart in gases. When particles are closer together, they can more quickly pass the energy of vibrations to nearby particles.

Medium (20 °C)	Speed of Sound Waves (m/s)
Dry Air	343
Water	1437
Wood	3850
Glass	4540
Aluminum	6320

> **Q:** The table gives the speed of sound in dry air. Do you think that sound travels more or less quickly through air that contains water vapor? (Hint: Compare the speed of sound in water and air in the table.)
>
> **A:** Sound travels at a higher speed through water than air, so it travels more quickly through air that contains water vapor than it does through dry air.

Temperature and Speed of Sound

The speed of sound also depends on the temperature of the medium. For a given medium, sound has a slower speed at lower temperatures. You can compare the speed of sound in dry air at different temperatures in the following Table below. At a lower temperature, particles of the medium are moving more slowly, so it takes them longer to transfer the energy of the sound waves.

Temperature of Air	Speed of Sound Waves (m/s)
0 °C	331
20 °C	343
100 °C	386

> **Q:** What do you think the speed of sound might be in dry air at a temperature of -20 °C?
>
> **A:** For each 1 degree Celsius that temperature decreases, the speed of sound decreases by 0.6 m/s. So sound travels through dry, -20 °C air at a speed of 319 m/s.

Can you calculate the speed of sound in air and the wave speed on the string in the Violin simulation below? Use the Wavelength vs Frequency graph on the top left to analyze the waves produced by the violin. The product of the wavelength and frequency can be used to determine the speed of sound in air and wave speed on the violin string:

Summary

- The speed of sound is the distance that sound waves travel in a given amount of time. The speed of sound in dry air at 20 °C is 343 meters per second.
- Generally, sound waves travel most quickly through solids, followed by liquids, and then by gases.
- For a given medium, sound waves travel more slowly at lower temperatures.

Review

1. What is the speed of sound in dry air at 20 °C?
2. Describe variation in the speed of sound through various media.
3. Explain how temperature affects the speed of sound.

Additional Resources

Real World Application: Tracking the Storm

Videos:

Study Guide: Waves Study Guide

14.5: Intensity and Loudness of Sound

Figure 12.3.1

A friend whispers to you in a voice so soft that she has to lean very close so you can hear what she's saying. Later that day, your friend shouts to you from across the gymnasium. Now her voice is loud enough for you to hear her clearly even though she's several meters away. Obviously, sounds can vary in loudness.

It's All About Energy

Loudness refers to how loud or soft a sound seems to a listener. The loudness of sound is determined, in turn, by the intensity of the sound waves. **Intensity** is a measure of the amount of energy in sound waves. The unit of intensity is the **decibel (dB)**.

Decibel Levels

The Figure below shows decibel levels of several different sounds. As decibel levels get higher, sound waves have greater intensity and sounds are louder. For every 10-decibel increase in the intensity of sound, loudness is 10 times greater. Therefore, a 30-decibel "quiet" room is 10 times louder than a 20-decibel whisper, and a 40-decibel light rainfall is 100 times louder than the whisper. High-decibel sounds are dangerous. They can damage the ears and cause loss of hearing.

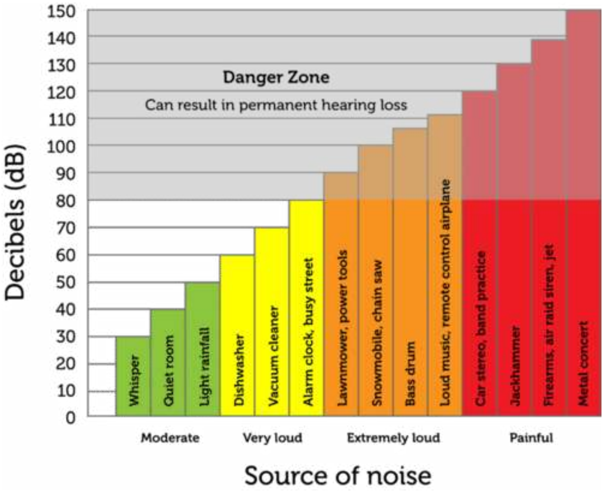


Figure 12.3.2

Q: How much louder than a 20-decibel whisper is the 60-decibel sound of a vacuum cleaner?

A: The vacuum cleaner is 10,000 times louder than the whisper!

Amplitude and Distance

The intensity of sound waves determines the loudness of sounds, but what determines intensity? Intensity results from two factors: the amplitude of the sound waves and how far they have traveled from the source of the sound.

- **Amplitude** is a measure of the size of sound waves. It depends on the amount of energy that started the waves. Greater amplitude waves have more energy and greater intensity, so they sound louder.
- As sound waves travel farther from their source, the more spread out their energy becomes. You can see how this works in the Figure below. As distance from the sound source increases, the area covered by the sound waves increases. The same amount of energy is spread over a greater area, so the intensity and loudness of the sound is less. This explains why even loud sounds fade away as you move farther from the source.

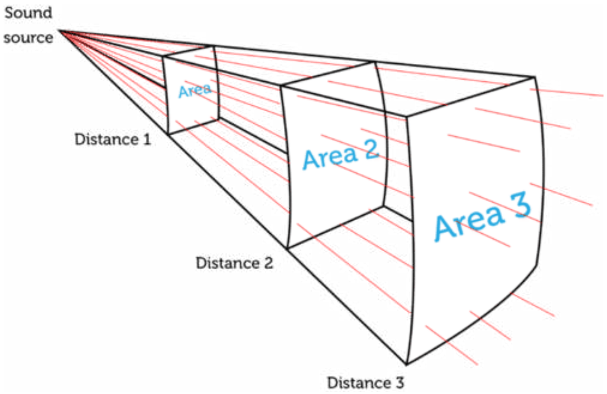


Figure 12.3.3

This diagram represents just a small section of the total area of sound waves spreading out from a source. Sound waves actually travel away from the source in **all directions.**

Q: Why can low-amplitude sounds like whispers be heard only over short distances?

A: The sound waves already have so little energy that spreading them out over a wider area quickly reduces their intensity below the level of hearing.

The spectrum of sound varies for different musical instruments, which is why they all sound different playing the same musical note. Observe the Amplitude vs Frequency graph, known as the Fourier Transform, on the top right graph in the Pan Flute simulation below to learn more:

Summary

- Loudness refers to how loud or soft a sound seems to a listener. The loudness of sound is determined, in turn, by the intensity, or amount of energy, in sound waves. The unit of intensity is the decibel (dB).
- As decibel levels get higher, sound waves have greater intensity and sounds are louder. For every 10-decibel increase in the intensity of sound, loudness is 10 times greater.
- Intensity of sound results from two factors: the amplitude of the sound waves and how far they have traveled from the source of the sound.

Review

1. Define loudness and intensity of sound. How are the two concepts related?
2. What is the unit of intensity of sound?
3. At what decibel level do sounds start to become harmful to the ears and hearing?
4. Relate amplitude and distance to the intensity and loudness of sound.

Additional Resources

Study Guide: Waves Study Guide

Real World Application: Playing with Trash

Video: Characteristics of Sound Waves - Overview

14.6: Frequency and Pitch of Sound

Figure 12.4.1

A marching band passes you as it parades down the street. You heard it coming from several blocks away. Now that the different instruments have finally reached you, their distinctive sounds can be heard. The tiny piccolos trill their bird-like high notes, and the big tubas rumble out their booming bass notes. Clearly, some sounds are higher or lower than others.

High or Low

How high or low a sound seems to a listener is its **pitch**. Pitch, in turn, depends on the frequency of sound waves. **Wave frequency** is the number of waves that pass a fixed point in a given amount of time. High-pitched sounds, like the sounds of the piccolo in the Figure below, have high-frequency waves. Low-pitched sounds, like the sounds of the tuba Figure below, have low-frequency waves.

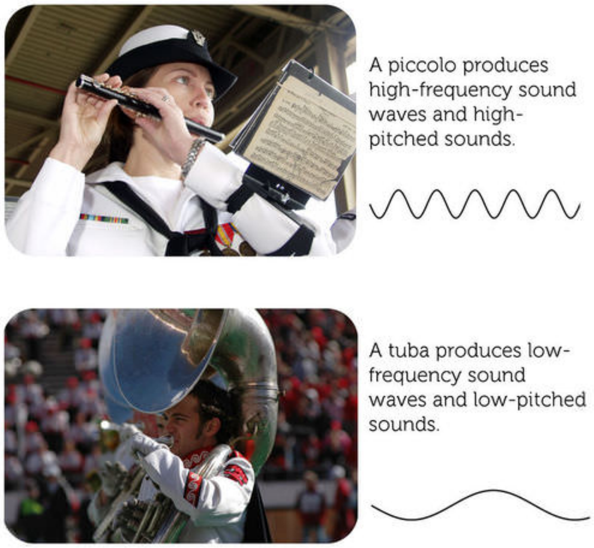


Figure 12.4.2

Can You Hear It?

The frequency of sound waves is measured in hertz (Hz), or the number of waves that pass a fixed point in a second. Human beings can normally hear sounds with a frequency between about 20 Hz and 20,000 Hz. Sounds with frequencies below 20 hertz are called **infrasound**. Infrasound is too low-pitched for humans to hear. Sounds with frequencies above 20,000 hertz are called **ultrasound**. Ultrasound is too high-pitched for humans to hear.

Some other animals can hear sounds in the ultrasound range. For example, dogs can hear sounds with frequencies as high as 50,000 Hz. You may have seen special whistles that dogs—but not people—can hear. The whistles produce sounds with frequencies too high for the human ear to detect. Other animals can hear even higher-frequency sounds. Bats, like the one pictured in the Figure below, can hear sounds with frequencies higher than 100,000 Hz!

Figure 12.4.3

> **Q:** Bats use ultrasound to navigate in the dark. Can you explain how?
>
> **A:** Bats send out ultrasound waves, which reflect back from objects ahead of them. They sense the reflected sound waves and use the information to detect objects they can't see in the dark. This is how they avoid flying into walls and trees and also how they find flying insects to eat.

Use the Violin simulation below to play different musical notes and observe the graph of Frequency vs Amplitude produced by the violin strings vibrating at different tensions:

Summary

- How high or low a sound seems to a listener is its pitch. Pitch, in turn, depends on the frequency of sound waves.
- High-frequency sound waves produce high-pitched sounds, and low-frequency sound waves produce low-pitched sounds.
- Infrasound has wave frequencies too low for humans to hear. Ultrasound has wave frequencies too high for humans to hear.

Review

1. What is the pitch of sound?
2. How is the pitch of sound related to the frequency of sound waves?
3. Define infrasound and ultrasound.

Additional Resources

Study Guide: Waves Study Guide

Real World Application: Tune-up Time

Videos:

14.7: Doppler Effect and Sonic Booms

Learning Objectives

By the end of this section, you will be able to:

- Define Doppler effect, Doppler shift, and sonic boom.
- Calculate the frequency of a sound heard by someone observing Doppler shift.
- Describe the sounds produced by objects moving faster than the speed of sound.

The characteristic sound of a motorcycle buzzing by is an example of the **Doppler effect**. The high-pitch scream shifts dramatically to a lower-pitch roar as the motorcycle passes by a stationary observer. The closer the motorcycle brushes by, the more abrupt the shift. The faster the motorcycle moves, the greater the shift. We also hear this characteristic shift in frequency for passing race cars, airplanes, and trains. It is so familiar that it is used to imply motion and children often mimic it in play.

The Doppler effect is an alteration in the observed frequency of a sound due to motion of either the source or the observer. Although less familiar, this effect is easily noticed for a stationary source and moving observer. For example, if you ride a train past a stationary warning bell, you will hear the bell's frequency shift from high to low as you pass by. The actual change in frequency due to relative motion of source and observer is called a **Doppler shift**. The Doppler effect and Doppler shift are named for the Austrian physicist and mathematician Christian Johann Doppler (1803–1853), who did experiments with both moving sources and moving observers. Doppler, for example, had musicians play on a moving open train car and also play standing next to the train tracks as a train passed by. Their music was observed both on and off the train, and changes in frequency were measured.

What causes the Doppler shift? Figure 14.7.1, Figure 14.7.2, and Figure 14.7.3 compare sound waves emitted by stationary and moving sources in a stationary air mass. Each disturbance spreads out spherically from the point where the sound was emitted. If the source is stationary, then all of the spheres representing the air compressions in the sound wave centered on the same point, and the stationary observers on either side see the same wavelength and frequency as emitted by the source, as in Figure 14.7.1. If the source is moving, as in Figure 14.7.2, then the situation is different. Each compression of the air moves out in a sphere from the point where it was emitted, but the point of emission moves. This moving emission point causes the air compressions to be closer together on one side and farther apart on the other. Thus, the wavelength is shorter in the direction the source is moving (on the right in Figure 14.7.2), and longer in the opposite direction (on the left in Figure 14.7.2). Finally, if the observers move, as in Figure 14.7.3, the frequency at which they receive the compressions changes. The observer moving toward the source receives them at a higher frequency, and the person moving away from the source receives them at a lower frequency.

Figure 14.7.1: Sounds emitted by a source spread out in spherical waves. Because the source, observers, and air are stationary, the wavelength and frequency are the same in all directions and to all observers.

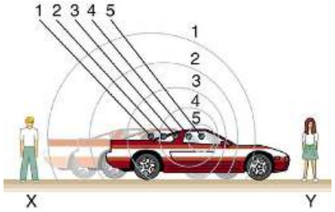


Figure 14.7.2: Sounds emitted by a source moving to the right spread out from the points at which they were emitted. The wavelength is reduced and, consequently, the frequency is increased in the direction of motion, so that the observer on the right hears a higher-pitch sound. The opposite is true for the observer on the left, where the wavelength is increased and the frequency is reduced.

Figure 14.7.3: The same effect is produced when the observers move relative to the source. Motion toward the source increases frequency as the observer on the right passes through more wave crests than she would if stationary. Motion away from the source decreases frequency as the observer on the left passes through fewer wave crests than he would if stationary.

We know that wavelength and frequency are related by $v_w = f\lambda$, where v_w is the fixed speed of sound. The sound moves in a medium and has the same speed v_w in that medium whether the source is moving or not. Thus f multiplied by λ is a constant. Because the observer on the right in Figure $14.7.2$ receives a shorter wavelength, the frequency she receives must be higher. Similarly, the observer on the left receives a longer wavelength, and hence he hears a lower frequency. The same thing happens in Figure $14.7.3$. A higher frequency is received by the observer moving toward the source, and a lower frequency is received by an observer moving away from the source. In general, then, relative motion of source and observer toward one another increases the received frequency. Relative motion apart decreases frequency. The greater the relative speed is, the greater the effect.

Doppler Effect

The Doppler effect occurs not only for sound but for any wave when there is relative motion between the observer and the source. There are Doppler shifts in the frequency of sound, light, and water waves, for example. Doppler shifts can be used to determine velocity, such as when ultrasound is reflected from blood in a medical diagnostic. The recession of galaxies is determined by the shift in the frequencies of light received from them and has implied much about the origins of the universe. Modern physics has been profoundly affected by observations of Doppler shifts.

For a stationary observer and a moving source, the frequency f_{obs} received by the observer can be shown to be

$$f_{obs} = f_s \left(\frac{v_w}{v_w \pm v_s} \right), \tag{14.7.1}$$

where f_s is the frequency of the source, v_s\) is the speed of the source along a line joining the source and observer, and v_s is the speed of sound. The minus sign is used for motion toward the observer and the plus sign for motion away from the observer, producing the appropriate shifts up and down in frequency. Note that the greater the speed of the source, the greater the effect. Similarly, for a stationary source and moving observer, the frequency received by the observer f_{obs} is given by

$$f_{obs} = f_s \left(\frac{v_w \pm v_{obs}}{v_w} \right), \tag{14.7.2}$$

where v_{obs} is the speed of the observer along a line joining the source and observer. Here the plus sign is for motion toward the source, and the minus is for motion away from the source.

Example 14.7.1: Calculate Doppler Shift: A Train Horn

Suppose a train that has a 150-Hz horn is moving at 35.0 m/s in still air on a day when the speed of sound is 340 m/s.

(a) What frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes?

(b) What frequency is observed by the train’s engineer traveling on the train?

Strategy

To find the observed frequency in (a), $f_{obs} = f_s \left(\frac{v_s}{v_w \pm v_s} \right)$, must be used because the source is moving. The minus sign is used for the approaching train, and the plus sign for the receding train. In (b), there are two Doppler shifts—one for a moving source and the other for a moving observer.

Solution for (a)

(1) Enter known values into $f_{obs} = f_s \left(\frac{v_w}{v_w - v_s} \right)$.

$$f_{obs} = f_s\left(\frac{v_w}{v_w - v_s}\right) = (150\,Hz)\left(\frac{340\,m/s}{340\,m/s - 35.0\,m/s}\right) \tag{14.7.3}$$

(2) Calculate the frequency observed by a stationary person as the train approaches.

$$f_{obs} = (150\,Hz)(1.11) = 167\,Hz \tag{14.7.4}$$

(3) Use the same equation with the plus sign to find the frequency heard by a stationary person as the train recedes.

$$f_{obs} = f_s\left(\frac{v_w}{v_w - v_s}\right) = (150\,Hz)\left(\frac{340\,m/s}{340n\,m/s + 35.0\,m/s}\right) \tag{14.7.5}$$

(4) Calculate the second frequency.

$$f_{obs} = (150\,Hz)(0.907) = 136\,Hz \tag{14.7.6}$$

Discussion on (a)

The numbers calculated are valid when the train is far enough away that the motion is nearly along the line joining train and observer. In both cases, the shift is significant and easily noticed. Note that the shift is 17.0 Hz for motion toward and 14.0 Hz for motion away. The shifts are not symmetric.

Solution for (b)

(1) Identify knowns:

- It seems reasonable that the engineer would receive the same frequency as emitted by the horn, because the relative velocity between them is zero.
- Relative to the medium (air), the speeds are $v_s = v_{obs} = 35.0\,m/s$.
- The first Doppler shift is for the moving observer; the second is for the moving source.

(2) Use the following equation

$$f_{obs} = \left[f_s\left(\frac{v_w \pm v_{obs}}{v_w}\right)\right]\left(\frac{v_w}{v_w \pm v_s}\right). \tag{14.7.7}$$

The quantity in the square brackets is the Doppler-shifted frequency due to a moving observer. The factor on the right is the effect of the moving source.

(3) Because the train engineer is moving in the direction toward the horn, we must use the plus sign for v_{obs}; however, because the horn is also moving in the direction away from the engineer, we also use the plus sign for v_s. But the train is carrying both the engineer and the horn at the same velocity, so $v_s = v_{obs}$. As a result, everything but f_s cancels, yielding

$$f_{obs} = f_s. \tag{14.7.8}$$

Discussion for (b)

We may expect that there is no change in frequency when source and observer move together because it fits your experience. For example, there is no Doppler shift in the frequency of conversations between driver and passenger on a motorcycle. People talking when a wind moves the air between them also observe no Doppler shift in their conversation. The crucial point is that source and observer are not moving relative to each other.

Sonic Booms to Bow Wakes

What happens to the sound produced by a moving source, such as a jet airplane, that approaches or even exceeds the speed of sound? The answer to this question applies not only to sound but to all other waves as well.

Suppose a jet airplane is coming nearly straight at you, emitting a sound of frequency f_s. The greater the plane's speed v_s, the greater the Doppler shift and the greater the value observed for f_{obs}. Now, as v_s approaches the speed of sound, f_{obs} approaches infinity, because the denominator in $f_{obs} = f_s\left(\frac{v_w}{v_w \pm v_s}\right)$ approaches zero. At the speed of sound, this result means that in front of the source, each successive wave is superimposed on the previous one because the source moves forward at the speed of sound. The observer gets them all at the same instant, and so the frequency is infinite. (Before airplanes exceeded the speed of sound, some people argued it would be impossible because such constructive superposition would produce pressures great enough to destroy the airplane.) If the source exceeds the speed of sound, no sound is received by the observer until the source has passed, so that the sounds from the approaching source are mixed with those from it when receding. This mixing appears messy, but something interesting happens—a sonic boom is created. (See Figure 14.7.4)

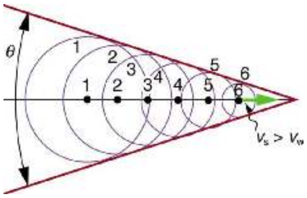


Figure 14.7.4: Sound waves from a source that moves faster than the speed of sound spread spherically from the point where they are emitted, but the source moves ahead of each. Constructive interference along the lines shown (actually a cone in three dimensions) creates a shock wave called a sonic boom. The faster the speed of the source, the smaller the angle θ.

There is constructive interference along the lines shown (a cone in three dimensions) from similar sound waves arriving there simultaneously. This superposition forms a disturbance called a **sonic boom**, a constructive interference of sound created by an object moving faster than sound. Inside the cone, the interference is mostly destructive, and so the sound intensity there is much less than on the shock wave. An aircraft creates two sonic booms, one from its nose and one from its tail. (See Figure 14.7.5) During television coverage of space shuttle landings, two distinct booms could often be heard. These were separated by exactly the time it would take the shuttle to pass by a point. Observers on the ground often do not see the aircraft creating the sonic boom, because it has passed by before the shock wave reaches them, as seen in Figure 14.7.5. If the aircraft flies close by at low altitude, pressures in the sonic boom can be destructive and break windows as well as rattle nerves. Because of how destructive sonic booms can be, supersonic flights are banned over populated areas of the United States.

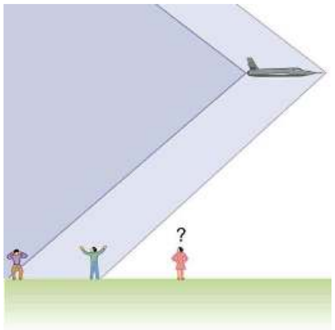


Figure 14.7.5: Two sonic booms, created by the nose and tail of an aircraft, are observed on the ground after the plane has passed by.

Sonic booms are one example of a broader phenomenon called bow wakes. A bow wake, such as the one in Figure 14.7.6, is created when the wave source moves faster than the wave propagation speed. Water waves spread out in circles from the point where created, and the bow wake is the familiar V-shaped wake trailing the source. A more exotic bow wake is created when a subatomic particle travels through a medium faster than the speed of light travels in that medium. (In a vacuum, the maximum speed of light will be $3.99 \times 10^8\ m/s$; in the medium of water, the speed of light is closer to $0.75c$. If the particle creates light in its passage, that light spreads on a cone with an angle indicative of the speed of the particle, as illustrated in Figure 14.7.7. Such a bow wake is called Cerenkov radiation and is commonly observed in particle physics.

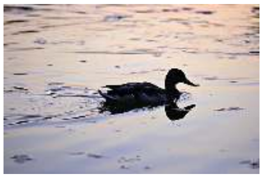


Figure 14.7.6: Bow wake created by a duck. Constructive interference produces the rather structured wake, while there is relatively little wave action inside the wake, where interference is mostly destructive. (credit: Horia Varlan, Flickr)

Figure 14.7.7: The blue glow in this research reactor pool is Cerenkov radiation caused by subatomic particles traveling faster than the speed of light in water. (credit: U.S. Nuclear Regulatory Commission)

Doppler shifts and sonic booms are interesting sound phenomena that occur in all types of waves. They can be of considerable use. For example, the Doppler shift in ultrasound can be used to measure blood velocity, while police use the Doppler shift in radar (a microwave) to measure car velocities. In meteorology, the Doppler shift is used to track the motion of storm clouds; such "Doppler Radar" can give velocity and direction and rain or snow potential of imposing weather fronts. In astronomy, we can examine the light emitted from distant galaxies and determine their speed relative to ours. As galaxies move away from us, their light is shifted to a lower frequency, and so to a longer wavelength—the so-called red shift. Such information from galaxies far, far away has allowed us to estimate the age of the universe (from the Big Bang) as about 14 billion years.

Exercise 14.7.1

Why did scientist Christian Doppler observe musicians both on a moving train and also from a stationary point not on the train?

Answer

Doppler needed to compare the perception of sound when the observer is stationary and the sound source moves, as well as when the sound source and the observer are both in motion.

Exercise 14.7.2

Describe a situation in your life when you might rely on the Doppler shift to help you either while driving a car or walking near traffic.

Answer

If I am driving and I hear Doppler shift in an ambulance siren, I would be able to tell when it was getting closer and also if it has passed by. This would help me to know whether I needed to pull over and let the ambulance through.

Summary

- The Doppler effect is an alteration in the observed frequency of a sound due to motion of either the source or the observer.
- The actual change in frequency is called the Doppler shift.
- A sonic boom is constructive interference of sound created by an object moving faster than sound.
- A sonic boom is a type of bow wake created when any wave source moves faster than the wave propagation speed.
- For a stationary observer and a moving source, the observed frequency f_{obs} is: $f_{obs} = f_s \left(\frac{v_w}{v_w \pm v_s} \right)$, where f_{obs} is the frequency of the source, v_s is the speed of the source, and v_w is the speed of sound. The minus sign is used for motion toward the observer and the plus sign for motion away.
- For a stationary source and moving observer, the observed frequency is: $f_{obs} = f_s \left(\frac{v_w \pm v_{obs}}{v_w} \right)$, where v_{obs} is the speed of the observer.

Glossary

Doppler effect

an alteration in the observed frequency of a sound due to motion of either the source or the observer

Doppler shift

the actual change in frequency due to relative motion of source and observer

sonic boom

a constructive interference of sound created by an object moving faster than sound

bow wake

V-shaped disturbance created when the wave source moves faster than the wave propagation speed

Contributors and Attributions

{{template.ContribOpenStaxCollege()}}

14.8: Ultrasound

Figure 12.6.1

What could this smiling dolphin and yawning bat possibly have in common? It’s not just that both of them are mammals. Both of them also use ultrasound to find things in the dark.

What Is Ultrasound?

Ultrasound is sound that has a wave frequency higher than the human ear can detect. It includes all sound with wave frequencies higher than 20,000 waves per second, or 20,000 hertz (Hz). Although we can’t hear ultrasound, it is very useful to humans and some other animals. Uses of ultrasound include echolocation, sonar, and ultrasonography.

Echolocation

Animals such as bats and dolphins send out ultrasound waves and use their echoes, or reflected waves, to identify the locations of objects they cannot see. This is called echolocation. Animals use echolocation to find prey and avoid running into objects in the dark. You can see in the Figure below how a bat uses echolocation to find insect prey.

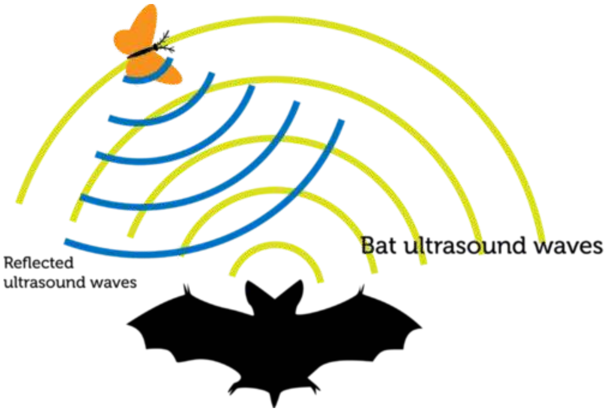


Figure 12.6.2

Sonar

Sonar uses ultrasound in a way that is similar to echolocation. **Sonar** stands for sound navigation and ranging. It is used to locate underwater objects such as submarines. That’s how the ship pictured in the Figure below is using it. A sonar device is both a sender and a receiver. It sends out ultrasound waves and detects the waves after they reflect from underwater objects.

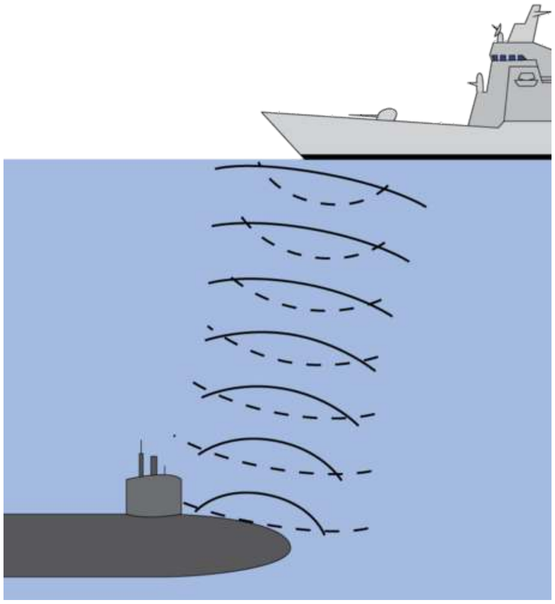


Figure 12.6.3

The distance to underwater objects can be calculated from the known speed of sound in water and the time it takes for the sound waves to travel to the object. The equation for distance traveled when speed and time are known is:

Distance = Speed × Time

Consider the ship and submarine pictured in the Figure above. If an ultrasound wave travels from the ship to the submarine and back again in 2 seconds, what is the distance from the ship to the submarine? The sound wave travels from the ship to the submarine in just 1 second, or half the time it takes to make the round trip. The speed of sound waves through ocean water is 1437 m/s. Therefore, the distance from the ship to the submarine is:

Distance = 1437 m/s × 1 s = 1437 m

> **Q:** Now assume that the sonar device on the ship sends an ultrasound wave to the bottom of the water. If the sound wave is reflected back to the device in 4 seconds, how deep is the water?
>
> **A:** The time it takes the wave to reach the bottom is 2 seconds. So the distance from the ship to the bottom of the water is:

Distance = 1437 m/s × 2 s = 2874 m

Use the PLIX Interactive below to simulate how echolocation can be used to map the seafloor:

Ultrasonography

Another use of ultrasound is to “see” inside the human body. This use of ultrasound is called ultrasonography. Harmless ultrasound waves are sent inside the body, and the reflected waves are used to create an image on a screen. This technology is used to examine internal organs and unborn babies without risk to the patient. You can see a doctor using ultrasound in the Figure below.

Figure 12.6.4

Summary

- Ultrasound is sound that has a wave frequency higher than the human ear can detect. It includes all sounds with wave frequencies higher than 20,000 waves per second, or 20,000 hertz (Hz).
- Animals such as bats and dolphins send out ultrasound waves and use their echoes to identify the locations of objects they cannot see. This is called echolocation.
- Sonar stands for sound navigation and ranging. It is used to locate underwater objects such as submarines.
- Ultrasonography is the use of reflected ultrasound waves to “see” inside the body.

Review

1. Define ultrasound.
2. Explain how animals use echolocation to find things in the dark.
3. What does sonar stand for? How is it used to locate underwater objects?
4. A sonar device on a ship sends ultrasound waves under the water to locate a sunken ship. It takes the ultrasound wave 0.6 seconds to travel from the device to the sunken ship and back again. How far below the surface is the sunken ship?
5. How is ultrasonography like sonar?

Additional Resources

Study Guide: Waves Study Guide

Real World Application: Shipwreck!, Echolocation

PLIX: Play, Learn, Interact, eXplore: Seafloor: Sonar Boats

Video:

14.9: End of Chapter Activity

End of Chapter Activity: Creating a Lesson Plan on the Properties of Sound, Doppler Effect, and Interference with AI and Bloom's Taxonomy

Now that you have explored the properties of sound, the Doppler effect, and interference, it's time to put your knowledge into practice. Your task is to create a succinct lesson plan for 12th graders that introduces them to these advanced concepts. To help you with this, you will use AI tools and incorporate Bloom's Taxonomy to ensure a comprehensive learning experience. This lesson plan will go towards your digital notebook, a portfolio filled with lesson plans, activities, and labs for future use.

Activity Prompt:

Objective: Use AI and Bloom's Taxonomy to develop a lesson plan that effectively teaches 12th graders about the properties of sound, the Doppler effect, and interference, including concepts such as sound waves, frequency, amplitude, wave interference, and real-life applications.

Understanding the Concepts:

> **Knowledge (Remembering):** Define key terms related to sound, such as sound waves, frequency, amplitude, Doppler effect, constructive interference, and destructive interference.
>
> **Comprehension (Understanding):** Explain these concepts in detail, using diagrams, real-world examples, and mathematical formulas where appropriate.

Planning the Lesson:

> **Application:** Design an engaging lab activity that allows students to observe and understand the properties of sound and the Doppler effect. For example, use tuning forks to demonstrate sound waves and their properties, or use a moving sound source to illustrate the Doppler effect.
>
> **Analysis:** Use AI tools to create visual aids or interactive simulations that illustrate the behavior of sound waves, the Doppler effect, and wave interference. For instance, create a simulation showing how the frequency of a sound changes as the source moves relative to the observer.

Deepening Understanding:

> **Synthesis (Creating):** Ask students to design their own experiments or projects that demonstrate principles of sound, the Doppler effect, or wave interference. For example, they could design an experiment to measure the speed of sound using echoes or create a demonstration of constructive and destructive interference.
>
> **Evaluation:** Have students discuss and reflect on their experiments and the principles they demonstrate. Encourage them to think critically about the applications of sound, the Doppler effect, and interference in modern technology and industry.

Using AI in the Classroom:

Explore AI tools like educational apps or platforms that provide interactive content for teaching about sound, the Doppler effect, and interference. Use these tools to create quizzes, flashcards, or interactive stories that reinforce the lesson's concepts.

Use AI to assess student understanding through formative assessments and provide instant feedback.

Deliverable:

Submit a detailed lesson plan that includes:

1. **A brief overview of the key concepts covered:** Outline the foundational concepts of sound, the Doppler effect, and interference that will be taught.
2. **A description of the activities and experiments designed:** Detail the hands-on activities and experiments you will use to help students understand these concepts.
3. **Examples of AI tools used and how they enhance the learning experience:** Describe the AI tools you plan to incorporate, such as simulations or interactive quizzes, and explain how they will help students grasp complex concepts.
4. **An explanation of how Bloom's Taxonomy was applied in the lesson plan to ensure a well-rounded educational experience:** Illustrate how each level of Bloom's Taxonomy (Remembering, Understanding, Applying, Analyzing,

Creating, and Evaluating) is addressed in your lesson plan.

Additionally, include a creative project component where students create a digital presentation or a video tutorial that explains a concept related to sound, the Doppler effect, or interference, using AI tools to enhance their projects.

Example Lesson Plan:

Grade: 12th Grade
Topic: Properties of Sound, Doppler Effect, and Interference
Duration: 1 Week

Overview:
Students will learn about the advanced concepts of sound, the Doppler effect, and interference, including their properties, behaviors, and applications, through engaging activities and creative projects.

Day 1: Introduction to Sound

Objective: Define basic and advanced concepts related to sound and provide examples.

- **Remembering:** Define key terms (sound waves, frequency, amplitude).
- **Understanding:** Explain the concepts using examples from everyday life and technological applications (e.g., musical instruments, sound engineering).

Activity:
Watch a detailed video (created using AI tools) explaining what sound is, how sound waves work, and the properties of sound such as frequency and amplitude.

Day 2: Exploring the Doppler Effect

Objective: Understand the Doppler effect and its applications.

- **Applying:** Conduct a lab activity to observe the Doppler effect (e.g., using a moving sound source and a stationary observer).

Activity:
Students use a sound source (like a smartphone with a tone generator app) attached to a moving object (e.g., a toy car) to observe the Doppler effect. They measure and record the change in frequency as the sound source moves towards and away from the observer.

Day 3: Understanding Interference

Objective: Understand wave interference and its types.

- **Applying:** Conduct an experiment to observe constructive and destructive interference (e.g., using two sound sources emitting the same frequency).

Activity:
Students use two speakers emitting the same frequency and move around the room to observe points of constructive and destructive interference. They map the locations where the sound is loudest and quietest and discuss the wave interference patterns.

Day 4: Creative Project – Designing Sound Experiments

Objective: Design and conduct experiments to demonstrate principles of sound, the Doppler effect, or interference.

- **Creating:** Students design their own experiments to visualize sound properties or the Doppler effect.

Activity:
In groups, students design experiments such as measuring the speed of sound using echoes or creating a demonstration of wave interference using water waves in a tank. They document each step, explain the principles involved, and present their projects to the class.

Day 5: Reflection and Digital Presentation

Objective: Reflect on what they have learned and create a digital presentation about sound, the Doppler effect, or interference.

- **Evaluating:** Discuss and reflect on the experiments and activities.

- **Creating:** Use AI tools to create a digital presentation or video tutorial explaining a concept related to sound.

Activity:
Students create a digital presentation or a video tutorial using AI tools, such as an animation or an interactive slideshow, that explains a concept they have learned about sound. They present their projects to the class, using their digital presentations to enhance their explanations.

By incorporating these strategies and activities, educators can effectively teach 12th graders about the properties of sound, the Doppler effect, and interference, helping them understand and appreciate the advanced concepts and their applications in the real world.

14.10: End of Chapter Key Terms

Definition: Property of Sound, Doppler Effect & Interferences

- **Sound**: the transfer of energy from a vibrating object in waves that travel through matter.
- **Speed of Sound**: The speed at which sound waves travel through a medium, influenced by the medium's properties (e.g., temperature, density).
- **Sound Wave**: A mechanical wave that propagates through a medium (such as air, water, or solids) by the vibration of particles.
- **Loudness**: The perceived intensity or volume of a sound, related to the amplitude of the sound wave.
- **Intensity**: measure of the amount of energy in sound waves.
- **Decibel (dB)**: A logarithmic unit used to measure the intensity of sound, representing the ratio of a particular sound pressure to a reference level.
- **Amplitude**: The maximum displacement of particles in the medium from their rest position, related to the loudness of the sound.
- **Frequency**: The number of vibrations or cycles per second of a sound wave, measured in hertz (Hz), determining the pitch of the sound.
- **Pitch**: The perceived frequency of a sound; higher frequencies are heard as higher pitches, and lower frequencies as lower pitches.
- **Wavelength**: The distance between successive crests or troughs of a sound wave, determining its spatial period.
- **Ultrasound**: Sound waves with frequencies above the upper limit of human hearing (>20,000 Hz), used in medical imaging and industrial applications.
- **Infrasound**: Sound waves with frequencies below the lower limit of human hearing (<20 Hz), often produced by natural phenomena like earthquakes.
- **Doppler Effect**: The change in frequency or wavelength of a sound wave as the source and observer move relative to each other, resulting in a perceived change in pitch.
- **Redshift (Sound)**: The decrease in frequency (and increase in wavelength) of a sound wave as the source moves away from the observer.
- **Blueshift (Sound)**: The increase in frequency (and decrease in wavelength) of a sound wave as the source moves toward the observer.
- **Echolocation**: The use of reflected sound waves to determine the location of objects, used by animals such as bats and dolphins.
- **Sonar**: A technique that uses sound waves to detect and locate objects underwater.

CHAPTER OVERVIEW

15: Electromagnetic Radiation

15.1: Introduction and Learning Objectives

Chapter Learning Objectives

1. **Understanding Electromagnetic Waves and the Electromagnetic Spectrum:**
    - Understand the relationship between the source charge, the medium and the electromagnetic wave.
    - Identify types of radiation across the electromagnetic spectrum by wavelength and frequency.
    - Discuss applications and implications in fields like medicine, communication, and astronomy.
2. **The Electromagnetic Wave Properties:**
    - Understand and apply the inverse relationship between wavelength and frequency.
    - Understand the origins of radiation pressure and wave intensity.
    - Analyze how changes in wavelength and frequency affect radiation energy.
    - Understand how material affects the behavior of electromagnetic waves.
3. **Color as a Manifestation of Visible Light:**
    - Explore visible light within the electromagnetic spectrum and its relation to color perception.
    - Understand the correlation between wavelength and perceived color.
    - Examine how the human eye perceives color and the roles of specialized retinal structures.
    - Learn how color can be related to temperature in astronomical objects.
4. **Exploring the Particle Model of Light**
    - Understand the implications of the ultraviolet catastrophe.
    - Explore Planck's Hypothesis and how it changed the nature of physics.
    - Analyze the photon model and how it relates to a new view of electromagnetic waves.
    - Describe how the photoelectric effect provided evidence for quantization.
    - Understand the process by which lasers produce light.
5. **Exploring the Particle-Wave Duality**
    - Be able to describe the implications of the deBroglie hypothesis.
    - Discuss the properties of wave-like and particle-like systems.
    - Discuss the evidence for such a duality

Introduction to Electromagnetic Radiation: Wavelength, Frequency, and Color

Electromagnetic radiation is a cornerstone of physical science, encompassing a wide range of waves, from radio waves to gamma rays, each characterized by its wavelength and frequency. These properties define the behavior and applications of electromagnetic waves. The wavelength, the distance between successive wave peaks, and the frequency, the number of wave cycles per second, are inversely related; as one increases, the other decreases. This relationship is critical for understanding the electromagnetic spectrum, where different types of radiation have unique wavelengths and frequencies. Visible light, a small portion of this spectrum, is what we perceive as color, with each hue corresponding to a specific wavelength range. For example, red light has the longest wavelength, while violet has the shortest. This interplay of wavelength and frequency not only explains the vivid array of colors we see but also underpins many technological applications, from medical imaging to communication systems.

Understanding how electromagnetic waves interact with matter provides insight into various natural and technological phenomena. For instance, the concept of blackbody radiation explains how objects emit light based on their temperature, a principle used in astronomy to determine the properties of stars. Additionally, the principles of wavelength and frequency are crucial in designing optical instruments, calibrating display screens, and ensuring accurate color reproduction in digital media.

For K-12 educators, making these complex scientific theories accessible and engaging is vital. Teaching strategies might include hands-on experiments, such as using prisms to demonstrate light dispersion or creating models to visualize wave properties. Interactive activities, multimedia resources, and real-world examples can help students grasp the significance of electromagnetic radiation in their everyday lives. By integrating these concepts into the classroom, educators can inspire curiosity and a deeper understanding of the physical world, preparing students for further exploration in science and technology.

15.2: Electromagnetic Waves Overview

In an earlier chapter we described transverse and longitudinal mechanical waves and how those waves depended on the material or environment in which the energy propagated (traveled). We also explored how electrical and magnetic fields could be described. We saw that electrical charges created electrical fields, and moving electrical charges (currents) created magnetic fields. Discovering and investigating these fundamental aspects of the universe was the result of decades of investigation by physicists including Gauss, Coulomb, Ampere and Faraday.

A Scottish physicist named James Clerk Maxwell was able to use the work of these pioneers to develop an overarching model, describing the ways that electric and magnetic fields are related to each other. Maxwell's Equations involve mathematics that go far beyond the scope of this textbook, but the ideas they represent are simple, yet profound.

- Electric fields can be visualized by arrows (field lines) that point away from positive charges and point towards negative charges. Electric fields determine the strength and direction of an electric force acting on an object with net charge. The electric field strength is also related to the environment. For the vacuum of space or the atmosphere of earth, the electric constant is written as ϵ_0. Different environments have different values of ϵ.
- Magnetic fields have no beginning or end, appearing as 'loops' of magnetic field. Magnetic fields determine the strength and direction of a magnetic force acting on a moving object with net charge. The magnetic field strength is also related to the environment. μ_0 is used for magnetic fields in a similar way that ϵ_0 is used for electric fields.
- A changing magnetic field creates a changing electric field.
- A changing electric field creates a changing magnetic field.

Maxwell was able to show that electrical and magnetic fields are not separate 'things'. Instead, electrical and magnetic fields are two aspects of a bigger 'thing' called the electromagnetic field. Likewise, electrical and magnetic forces are two aspects of the electromagnetic force. The idea that what appeared to be two different things were actually one had a profound effect on the philosophy of physics, and is what drives the current attempts to unify the four known forces of the universe, namely: Gravitational Force, Electromagnetic Force, Weak (Nuclear) Force, and Strong (Nuclear) Force.

Oscillating Charges Make Waves

An electric force can cause an electric charge to move. That moving electric charge will generate a changing electric field, and that changing electric field will generate a changing magnetic field. But, a changing magnetic field generates a changing electric field, and so on. The net effect is that if we 'wiggle' (oscillate) a charged object, we create a chain reaction of electric and magnetic fields that move away from the wiggling charge.

We've seen that electric and magnetic fields can exert forces, and that means that electric and magnetic fields can do work on charged objects. As we defined earlier, work is the transfer of energy between objects. So, we wiggle a charge at location A and some time later a charge at location B will wiggle. This is consistent with our description of wave behavior. Further reinforcing our ideas of wave-like behavior is the fact that the frequency at which we wiggle the charge at point A is the same frequency that the charge at point B will wiggle.

Using what we know about the mathematical description of electric and magnetic fields, we arrive at a wave equation that describes the way that electromagnetic forces can do work, how fast the energy will move from one point to another and how the amount of charge used will affect how much energy is transferred. It can be shown that the following relationship is true for electromagnetic energy transfers (electromagnetic waves) on earth and in space:

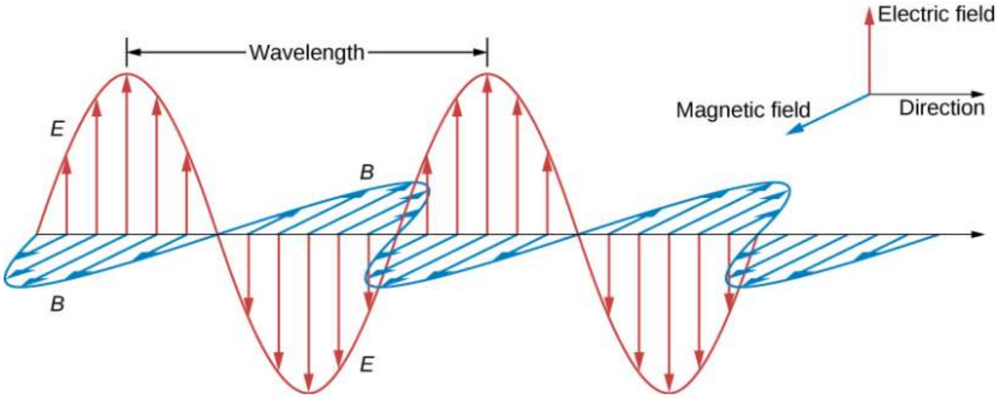
$$\mathrm{v} = \frac{1}{\sqrt{(\epsilon_0)(\mu_0)}}$$

If we insert the known values for ϵ_0 and μ_0, we get a wave speed of 3 x 10^8 meters per second, the speed of light.

Maxwell was able to show that what we call light is a traveling electromagnetic wave. Beyond that, he was able to show that all electromagnetic waves travel at the speed of light. Radio waves, microwaves, gamma rays, etc. are all forms of electromagnetic radiation and they all travel at the same speed.

15.2.1: Electromagnetic Wave Properties

If we wiggle a charge, we would find electromagnetic waves heading off in all directions. Lets call the charge that creates the waves, the *source charge*. The simplest description of an electromagnetic wave involves an electric field that oscillates along one direction and a magnetic field that oscillates perpendicular to it. The energy is carried in a direction that is perpendicular to both the electric and the magnetic fields. Note that this makes the electromagnetic wave a transverse wave, since the electric field (amplitude) is perpendicular to the direction that the wave travels. A way to visualize these waves is shown below:



Plane wave image taken from OpenStax *University Physics vol. 2* and is licensed under CC-BY

Because electromagnetic energy can be described as a wave, it has all the properties and behaviors of a wave:

- Amplitude: The magnitude of the electric field (E) is used to describe the amplitude of the wave.
- Wavelength: The distance separating adjacent locations (λ) where the electric field is maximum.
- Frequency: The frequency (f) at which the source charge oscillates.
- Wave Speed: $v_{wave} = \lambda$ f
- Wave Interference: Overlapping waves can reinforce or cancel each other, just as in sound waves. Purely Constructive Interference results when two waves overlap perfectly, resulting in an increase of electric field amplitude (E + E = 2E). Purely destructive interference results from waves overlapping in a way that decreases the electric field to zero (E + (-E) = 0).

Electromagnetic waves are also described by their *intensity*. Intensity is a measure of how much energy the wave delivers to a particular area each second. Since energy changes per second is the definition of power, Intensity has units of power per unit of area, $I = \frac{\text{Watts}}{\text{meters}^2}$.

Electromagnetic waves also exert forces on, and transfer momentum to, charged objects. Atoms contain positive and negative charges, so all matter experiences a net force from interacting with an electromagnetic wave. Since a force per unit of area is defined as a pressure, electromagnetic waves exert a pressure on objects. This is called *radiation pressure*. The radiation pressure is twice as large for an object that reflects light when compared to an object that absorbs light. This is reminiscent of the forces exerted in elastic and inelastic collisions between masses.

The maximum radiation pressure for objects that reflects all light is written as $P_{rad} = 2\,\frac{I}{c}$, while the radiation pressure for an object that absorbs all light is $P_{rad} = \frac{I}{c}$

We can calculate the intensity of the wave by the following relationship:

$$I = \frac{1}{2}\, c\, \epsilon_0\, E^2$$

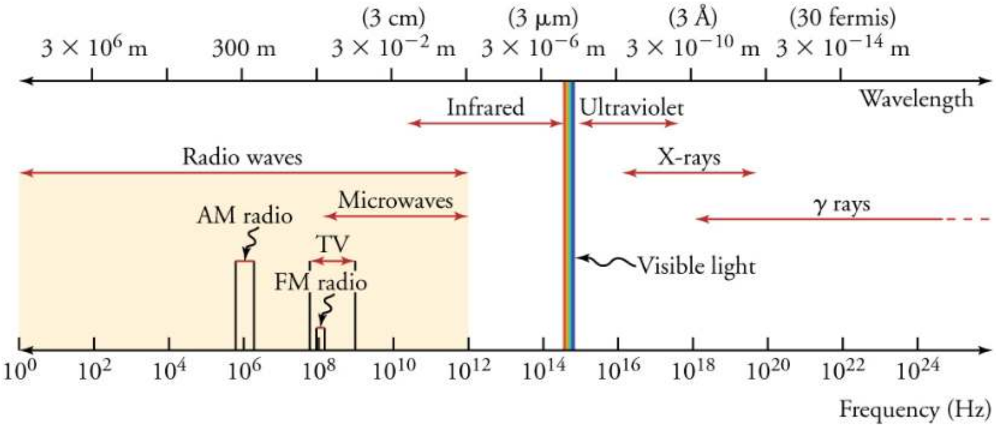
Just as we saw in our discussion of mechanical waves, the wave energy has to move through something. In the case of electromagnetic waves, the energy moves through space, and space has certain properties that determine how electric and magnetic fields behave. We've seen that the wave speed in space or on Earth is 3×10^8 meters per second. That is because ϵ_0 and μ_0 have certain values. As mentioned above, these values change in different materials and as a result, the speed of light through space is the largest it can be. In different materials, the electromagnetic wave travels more slowly.

Because light is a wave and obeys the wave speed relationship, light can be composed of a vast number of frequencies and wavelengths. It turns out that even in the same material, waves with different frequencies travel at slightly different speeds. This has an impact on the development of camera lenses and telescopes, and it is the difference of wave speeds in different materials that underlies all of optics and the basis for sight.

Being able to manipulate the speed at which electromagnetic energy travels through materials might open the door to improved data flow in optical networks, like fiber optic cables used to transport huge amounts of data. In 1998, a joint research project between Harvard University and the Rowland Institute for Science successfully slowed the speed of light to about 17 meters per second. In 2005, IBM successfully manufactured a microchip that can slow the speed of light. This continues to be an area of investigation since using light to transmit information instead of electrical impulses is faster and more energy efficient.

15.3: Electromagnetic Spectrum

As discussed, the fact that wave speed can be written as the product of frequency and wavelength means that there are two ways to describe the electromagnetic spectrum. Below is an image showing the wavelength and corresponding frequencies for various types of electromagnetic radiation. Note the tiny slice of the entire spectrum that encompasses all visible light.



EMR spectrum taken from OpenStax *Physics* and is licensed under CC-BY

Wavelengths just longer than red light are called *infrared* and it is these waves that we experience as heat when standing in the sunshine, or near a fire. Wavelengths shorter than violet are called *ultraviolet* and are responsible for skin damage. Sunscreens are designed to absorb and reflect these damaging wavelengths.

Although the most common way of describing light is in terms of the wavelength, this can be misleading. Recall that the frequency of the oscillations is set by the source. Since the frequency is set by the source and the wave speed is set by the medium, that means the wavelengths change as the wave speed changes. Because of this, the standard wavelengths used to describe light assume the light is traveling in vacuum.

✓ Example 15.3.1

The frequency of a microwave is 10^{10} Hertz. The speed of light in air is 3 x 10^8 meters per second. The speed of light in water is about 2.25 x 10^8 meters per second. Calculate the wavelength of microwaves in water and air.

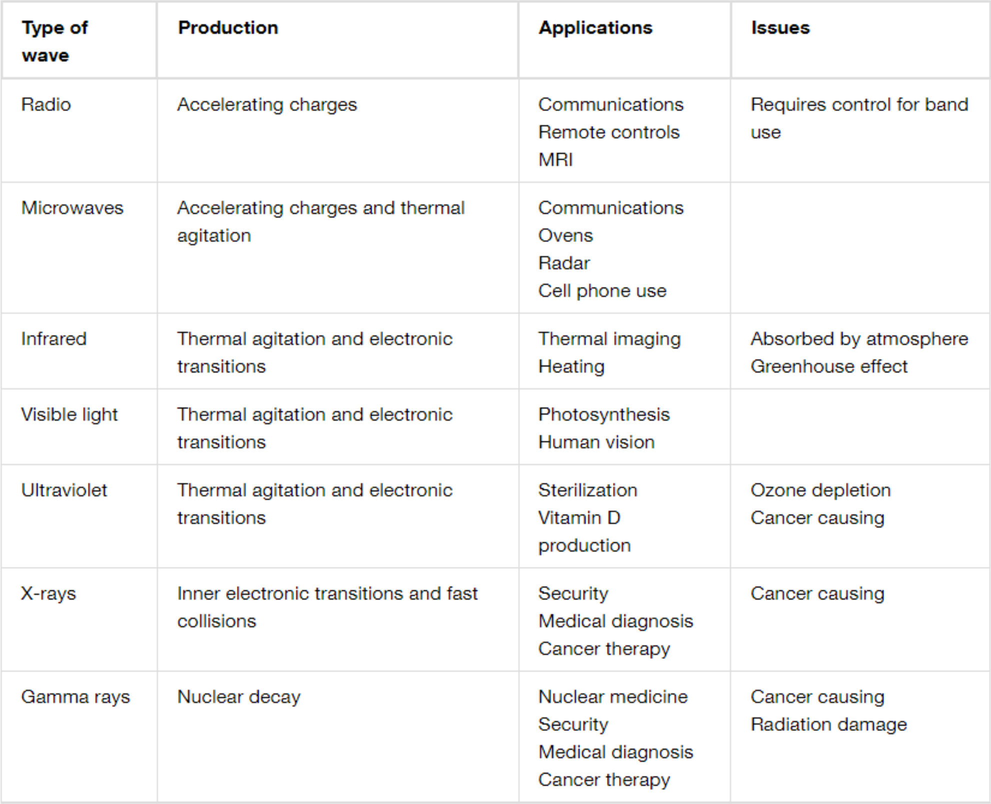
Solution

$$v_{\text{wave}} = \lambda f \text{, so } \lambda = \frac{v_{wave}}{f}$$

For water: $\lambda = \frac{2.25 \times 10^8 \text{meters per second}}{10^{10} \text{Hertz}}$ = 2.25 millimeters

For air: $\lambda = \frac{3 \times 10^8 \text{meters per second}}{10^{10} \text{Hertz}}$ = 3 millimeters

An overview of electromagnetic waves, their means of production and some applications are summarized in the table below:



Type of wave	Production	Applications	Issues
Radio	Accelerating charges	Communications Remote controls MRI	Requires control for band use
Microwaves	Accelerating charges and thermal agitation	Communications Ovens Radar Cell phone use	
Infrared	Thermal agitation and electronic transitions	Thermal imaging Heating	Absorbed by atmosphere Greenhouse effect
Visible light	Thermal agitation and electronic transitions	Photosynthesis Human vision	
Ultraviolet	Thermal agitation and electronic transitions	Sterilization Vitamin D production	Ozone depletion Cancer causing
X-rays	Inner electronic transitions and fast collisions	Security Medical diagnosis Cancer therapy	Cancer causing
Gamma rays	Nuclear decay	Nuclear medicine Security Medical diagnosis Cancer therapy	Cancer causing Radiation damage

Table of EM Radiation taken from OpenStax *University Physics vol. 2* and is licensed under CC-BY

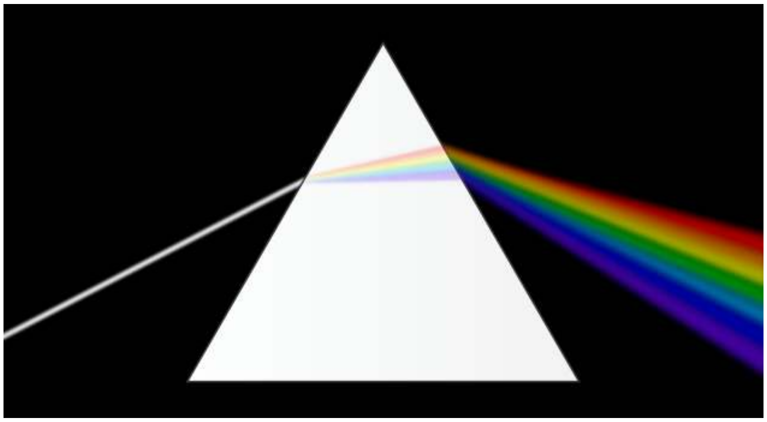
15.4: Electromagnetic Waves and Materials

When electromagnetic waves reach objects, the waves can either reflect from the object, pass through the object, or be absorbed by the object. These behaviors are part of the language we use when describing objects. 'Shiny' means that the surface is reflecting much of the energy that reaches it. 'Dull' means that it reflects less. When objects absorb most of the energy that reaches them, we call it 'opaque', but if most of the energy passes through it, it is 'transparent'. When we speak of a 'bright light', we mean the intensity is large. When the light is 'dim', the intensity is small and very little wave energy reaches out eyes..

The same object does not produce the same results for all wave frequencies. An object can be opaque to visible light but transparent to x-rays, for example. It might refect a wave of one frequency and absorb a wave with a different frequency. What we call the color of an object depends on which wavelengths are reflected from the object's surface and which ones are absorbed. Part of what influences the way a wave interacts with an object is the frequency of the source and the molecular properties of the object.

The behavior of the waves generally depends on the wave speed in the different materials. Imagine materials 1 and 2. Where they meet is called a boundary. Any changes in wave behavior happens at the boundary. A wave starts in material 1 and moves towards material 2. If the wave speed in material 2 is less than the wave speed in material 1, then part of the wave will be reflected from the boundary. The greater the difference in wave speeds, the greater the amount that is reflected. Waves will also change the direction they are traveling when they cross a boundary. If the wave slows down as it crosses the boundary, it will bend in one direction. If the wave speeds up, it will bend in the opposite direction as it crosses the boundary. These topics will be explored more completely in the chapter on optics.

Earlier, it was mentioned that the speed of an electromagnetic wave varies according to the frequency of the oscillation. That means that light that contains a mixture of different frequencies will show a difference in the way that the light interacts with it. A prism is a prime example of the effect of materials on the electromagnetic waves passing through them.



Prism taken from *Wikimedia Commons* by Astroskiandhike and is licensed under CC-BY-SA 4.0

A mix of all colors enters the prism at the left edge. Due to the difference in wave speeds for each different color (source frequency), the path traveled through the prism is also different. Red light travels the fastest through the prism and is deflected the least, Blue light travels the slowest and is affected more by passing through the prism.

In general, waves with low frequencies tend to be affected least by passing through materials, and waves with high frequencies tend to be affected the most. This difference in behavior helps explain why the sky is blue during the day, but becomes reddish at sunset. These topics will be explored further in the chapter on reflection and refraction.

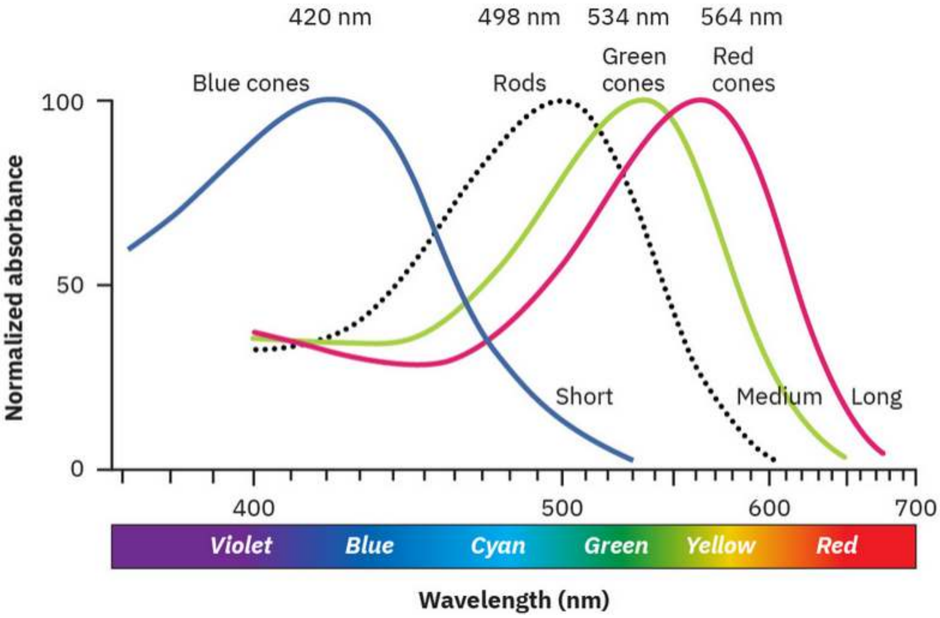
15.5: Light, Color and Perception

We've already discussed the wavelengths and frequencies of electromagnetic radiation and we've spoken of the narrow strip of frequencies that we can perceive in terms of colors. We can see the effect of a prism on beam of light as it splits into bands of color. But what does color mean? How is it that we can perceive wavelength differences as small as fifty-billionths of a meter? We've already explored sound and how it is perceived by us in different circumstances, now it is time to turn our attention to the visual.

For this portion of the textbook, we turn to human biology. When gathering data for experimental analysis it is important to understand the equipment being used, so you know how to interpret the data you've collected. Given that most of the information that we gather about the world comes to us through our eyes, understanding vision helps us better understand the data we collect about our world. As with most of this textbook, the treatment we give this topic is necessarily brief, as it encompasses biology, chemistry and psychology. What follows is a rudimentary discussion of the subject matter.

Light entering our eyes is focused on the retina. The retina is a collection of cells lining the eyeball that gather information from the light and send it to the occipital lobe of our brain via the optic nerve. There are two main types of *photoreceptors* (cells that sense light) in the retina:

- Cones: A cell concentrated near the center of the retina that work best in bright light and control high resolution vision. There are three types of cones and each type can detect a certain range of frequencies, peaking at a particular wavelength. The retina contains about six million cones.
- Rods: A cell that is far more sensitive to small energy differences than cones. They are essential for motion detection and for vision in dark environments. They do not record information about color, but provide vision in grey scale. They outnumber cones by a factor of twenty to one.



Plot of cone wavelengths taken from OpenStax *College Physics 2e* and is licensed under CC-BY

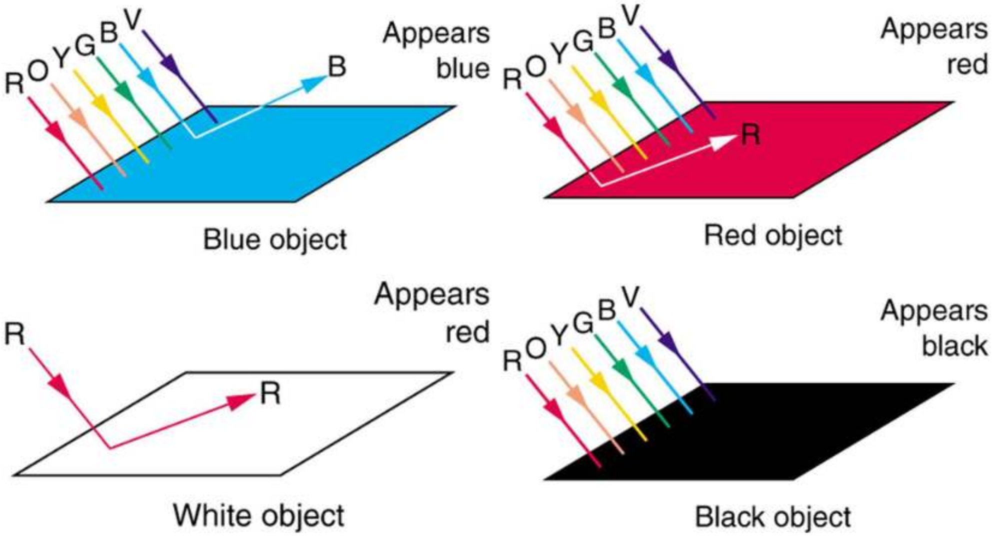
When light falls on the retina, certain cones respond to the wavelength of the incoming electromagnetic wave. The strength of the response has to do with the brightness (intensity) of the wave. In the simplest model of color vision, there are three primary colors that correspond to the red, blue and green cones. The colors we perceive depends on which cones are activated and the intensity of the cone activation.

Red and green cones can produce the perception of yellow if the intensities are properly balanced. If all three cone types are illuminated, we perceive white light. If blue and red cones are illuminated in one way, we perceive a bluish purple, another set of illuminations gives us a reddish purple. There is some evidence that our color vision is not quite as simple as this model would suggest, but the fundamental relationship between perceived color, cones and illumination levels is well established.

True Colors

In order for our eyes to see anything, light waves have to reach the object, be reflected from it and then reach our eyes where the rods and cones can then be activated. Opaque objects appear to be the colors we perceive them to be because of the light they reflect. White light is a mixture of all the visible colors of light, so if an object reflects all visible colors then it appears white to us. If an object is illuminated by white light and only reflects blue light (absorbing the rest of the colors) then the object appears blue. Similarly, if an object is illuminated by white light but only reflects red light, then it appears to be red. The *true color* of an object is defined to be the color perceived when illuminated by white light. A truly black object is one that absorbs all colors and reflects none.

However, if a white object is illuminated by red light, it will reflect that red light and appear red. If it is illuminated by red and green light, it could appear yellow. A red object illuminated by blue light will not reflect the blue light, and will appear black. Perceived colors can differ from true colors based on the color of the light that is illuminating the object.



Apparent colors taken from OpenStax(opens in new window) *College Physics 2e(opens in new window)* and is licensed under CC-BY

Light sources are assigned a color based on the frequencies (wavelengths) they emit. A neon sign appears red because it primarily emits red frequencies of light, while the Sun appears yellowish because the intensities of the yellow frequencies are greater than any of the other colors emitted.

Modified Model of Color Vision

Experiments have shown that the simple model presented above doesn't account for the full range of human color vision. It is known that there are nerve connections between rods and cones in the retina, and the optic nerve that carries information to the brain has fewer connections to the brain than there are rods and cones. This would imply that there is some sort of pre-processing that happens on the retina, before the information is sent to the occipital lobe. It is thought that the eye may make comparisons to information collected by adjacent receptors. If both receptors detect the same information, the redundant signals may be ignored. If the receptors report differences, additional processing power is allocated. This might help explain our sensitivity to detecting edges.

The founder of the Polaroid corporation, Edwin Land, proposed a model in which the three types of cones are organized into systems he called *retinexes*. Each retinex forms an image which is compared to the images formed by other retinexes. This may explain a visual ability called *color constancy*. Color constancy describes the ability of the eye to determine the color of an object under different ambient lighting conditions. It is possible to determine that a piece of paper is red whether it is illuminated by sunlight, candlelight or fluorescent light, even though the emitted color of each of these light sources is quite different. By comparing the paper to its surrounding in each case, the true color of the paper can be determined.

Our color vision is not completely understood, and it would appear that vision is far more complex than our initial model might suggest.

15.5.1: Color and Astronomy

Beyond its ability to provides us with a richer visual experience, the color of objects can be used in astronomy to help gather information about them. In particular, there is a link between the frequency of the light emitted by an object and the temperature of that object. The phrases 'red hot' and 'white hot' spring to mind as an example of the linguistic relationship between color and temperature.

Thermodynamics is a branch of physics that concerns itself with heat energy and how it is transferred between objects. Simple experiments show that the color emitted at the highest intensity is well correlated to the temperature of the object. Simply stated, as the temperature of the object increases, the wavelength it emits decreases. German physicist Wilhelm Wien discovered an experimental relationship between temperature and wavelength known as Wien's Law:

$$\lambda_{peak} = \frac{2.897\times10^{-3}\ \text{meter-Kelvin}}{\text{T}}$$

In this expression T is the temperature of the object in Kelvin.

The image below was taken by the Hubble Space Telescope and shows a region of the Milky Way Galaxy known as the Sagittarius Star Cloud.

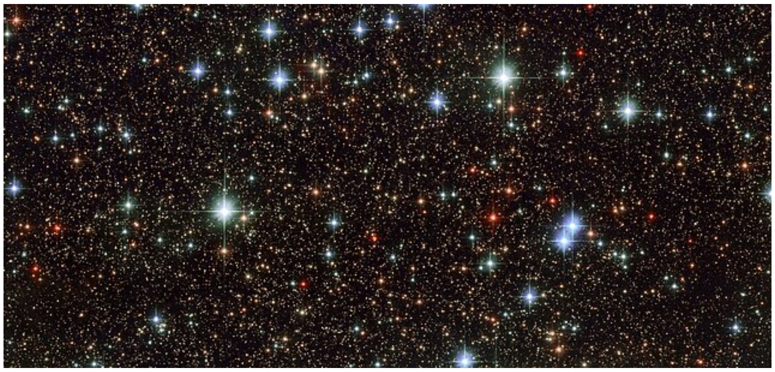


Image taken from ESA/Hubble and is licensed under CC-BY 4.0

We can use our understanding of electromagnetic waves, intensity and color to help make sense out of the image. Knowing that the stars are all located at a similar distance from the earth, we can conclude that the brighter (more intense) stars are emitting more energy per second than their dimmer companions. Furthermore, we can use Wien's Law to help categorize the approximate temperature of these stars. Since blue light has a shorter wavelength than red light, stars that appear blue-white are at a higher temperature than stars that appear reddish. The temperature of the star has an effect on which elements can be fused, and this information can be used to establish an approximate mass of the star. In the chapter on nuclear physics, the relationship between temperature and nucleosynthesis will be explored further.

To get a more accurate reading of the peak intensity color emitted by the star, filters are used that block all but a narrow range of wavelengths. The intensities of various wavelengths are compared to a reference star and a value is then determined.

Star Color	Approximate Temperature
Blue	25,000 Kelvin
White	10,000 Kelvin
Yellow	6000 Kelvin
Orange	4000 Kelvin
Red	3000 Kelvin

Observations show that the hottest visible stars have surface temperatures over 40,000 Kelvin and the lowest detectable temperatures are about 2000 Kelvin. Our Sun emits yellowish-green light with a surface temperature around 6000 Kelvin. It looks yellow from the surface of the planet because the shorter (bluer) wavelengths interact more with the molecules in the atmosphere, causing them to be randomly scattered, leaving the longer (yellow) wavelengths of light to travel straight to our eyes. This also explains why the sky appears blue.

Understanding the behavior of charges and the electromagnetic waves they produce has led to vast technological advances and the ability to examine the universe and make sense out of what we see. In the chapter on optics, differences in wave speeds will allow us to understand how the lenses in our eyes work, as well how to build microscopes and telescopes to expand the scales at which we can gather information.

15.6: An Alternate View of Electromagnetic Energy

Physicists soon realized that there were problems with the electromagnetic wave model. While it explained many phenomena such as radio communication, RADAR and large scale energy transfers, there were problems. The relationship between temperature and the color of the light that was emitted by the object didn't match what the models predicted. Different physicists worked on this problem, but couldn't resolve the issue. Some were able to create a model that matched the long wavelength data, but failed to match the short wavelength data. Others were able to match the short wavelengths, but not the long. This problem became known as the *Ultraviolet Catastrophe*. Since the models predicted that all waves should get shorter and shorter then all red light should becomes ultraviolet. Since this clearly did not happen, it pointed out that there was something fundamental that physicists did not understand about how electromagnetic energy behaved.

Planck's Hypothesis

In 1900, Max Planck decided to tackle the problem. He used almost the same approach as the others before him, treating waves as the result of charges performing simple harmonic motion. If you recall the section on transverse waves, we showed that the kinetic energy of a mass-spring system depended on the oscillation frequency. Planck added his own innovation and proposed that the charges could only oscillate at certain fixed frequencies. This meant that each oscillating charge could only have certain energy values. That meant that those oscillating charges could only change their energy by fixed amounts. This goes completely against the Newtonian model which says that masses can oscillate at any frequency, and can change their energy by any amount.

Planck's hypothesis of discrete energies (which he called *quanta*) meant that the oscillating charges had *quantized energies*. A particular oscillator could only oscillate in certain ways, though there might be a large number of allowed ways to oscillate. We choose 'n' to list the allowed energies. n = 1 is the first allowed oscillation energy and n = 5 is the fifth allowed energy. We say the oscillator is in a different *energy state* for each value of n. n = 1 is the first energy state, and n = 5 is the fifth energy state. n is called the *quantum number*. Planck's hypothesis is written as:

$$E_n = nhf \ (n = 1,2,3,4...)$$

If this model were true, then the oscillator could only change its energy in multiples of hf, where f is the oscillator frequency and h is a proportionality constant called Planck's Constant. $h = 6.626 \times 10^{-34}$ Joule-seconds = 4.136×10^{-15} electron-Volt-seconds. An oscillator in the n = 2 state would have energy of 2hf.

The model predicts that an oscillating charge (representing an atom in some material) can absorb energy from an incoming electromagnetic wave, but the amount of energy absorbed must be a multiple of hf. By absorbing energy of 3hf, the oscillating charge would move from the n = 2 state to the n = 5 state. If the absorbed energy were to be 3.25hf, the oscillator would 'ignore' the wave and not absorb **any** energy. The oscillator could also emit energy, as long as it was a multiple of hf. The n = 5 oscillator could emit 2hf worth of energy and become an n = 3 oscillator with energy of 3hf.

Planck was able to use this hypothesis to construct a model that exactly followed the experimental relationship between the temperature and the color (electromagnetic wavelength) of the hot object. Planck was also able to show that the Wien Law followed from his hypothesis, as well as another experimental relationship called Stefan's Law, which related the energy emitted by stars and their temperature.

Because this idea went against everything that physicists believed to be true about the way that energy is transferred between objects, even Planck himself did not fully believe that energy behaved this way. He considered quantization a mathematical trick that happened to produce a model that fit well with the experimental evidence. It would be the work of Albert Einstein that lifted this idea to prominence.

15.6.1: Photoelectric Effect

In 1887, a German physicist named Heinrich Hertz was working with radio waves, when he discovered that light could be used to eject electrons from metal surfaces. The ejected electrons were called *photoelectrons* to link them to this specific behavior, which was dubbed the *photoelectric effect*.

In order to ionize an atom and cause the electron to be ejected a certain amount of energy has to be provided, called the electron binding energy. The classical model suggested that the rate at which energy was provided was unimportant. A little bit of energy over a long time period should work as well as a lot of energy over a short time period. The expectation was that if small amounts of energy were provided there should be a lag, or delay, before the photoelectron appeared. This was not supported by the evidence. A second prediction of the classical model was that more intense waves should give the electrons more kinetic energy, and that the color (wavelength) of the light used should not matter.

However, the results were strongly dependent on the wavelength and not at all dependent on the intensity. In fact, there were no photoelectrons at all above a certain wavelength. This made no sense in the classical model, and it was the work of Albert Einstein in 1905 that led to a consistent explanation. Einstein reasoned that Planck model of quantized energy could explain this seemingly bizarre behavior. He extended Planck's hypothesis to suggest that electromagnetic waves were actually a 'swarm' of quanta, meaning that the wave itself is quantized. An analogy might be to look at a school of fish, or a flock of birds. Each one is a separate entity, but the entire collective moves as a single unit.

Einstein called each unit of the wave a *photon.* An individual photon moves at the speed of light (c). A light 'wave' is a collection of photons, each of which has a particular frequency. The energy of a photon is quantized:

$$\mathrm{E_f = hf} = \frac{hc}{\lambda}$$

Because Planck's oscillators can only receive or emit energy that is a multiple of their oscillation frequency, a photon with the wrong wavelength (frequency) will not eject an electron. Photons of the correct frequency (or integer multiples of the correct frequency) will transfer their energy to the electron, causing it to be ejected. This "all or nothing" approach to energy transfers is a hallmark of quantum phenomena. In this model, an intense electromagnetic wave is composed of a large number of photons, so 'bright' light means a large photon number and 'dim' light is the result of fewer photons. The photon model also explained why there were no photoelectrons above a certain wavelength. If the wavelength was too long, the frequency would be too low and the photons would not have enough energy to eject an electron. Since an electromagnetic wave could transfer momentum, each individual photon also had to have momentum. The momentum of a photon could be written as follows:

$$\mathrm{p} = \frac{h}{\lambda}$$

Einstein's photon model accounted for all the effects that the classical model could not. It suggested that the Planck hypothesis was not simply a mathematical trick and energy quantization was apparently the correct way of looking at the world. In some ways, this idea should be somewhat familiar. After all, a stream of water is composed of huge numbers of water molecules. A sand dune is a pile of trillions of sand grains. A squirrel is an aggregate of different cells. At a large scale, these seem like continuous objects because we cannot see the details but as we 'zoom in' a new picture emerges.

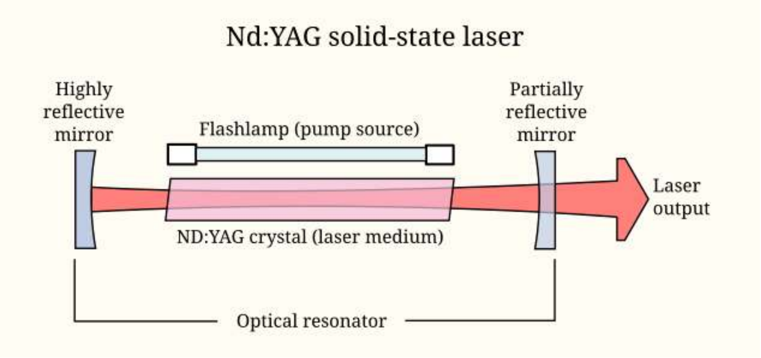
These and other experiments that could be best explained by the photon model helped cement the idea of energy quantization. This would form the basis for the Bohr model of the atom and explain the colors emitted by different types of atoms, eventually allowing us to determine the atmospheric composition of distant planets. The ideas surrounding quantization would eventually give rise to Quantum Mechanics, which is a branch of physics focusing on the behaviors of very small numbers of very small objects. This branch of physics gave rise to semiconductors and are used on a daily basis in many devices, including the memory of your smart phone.

15.6.2: Lasers

One of the more transformative technologies to come from the photon model of light and the Bohr model of the quantized atom is the laser. Lasers are used in an enormous number of ways upon which our technological society has come to depend, such as:

- Communications: Fiber optic cables use lasers to transfer data at extreme speeds.
- Medicine: Lasers are used in many medical procedures including eye surgery, dermatology and precision surgery.
- Manufacturing: To cut, weld, drill and mark materials.
- Research: For spectroscopy, microscopy and in particle accelerators.
- Military: For range finding, laser guidance of munitions, remote sensing and surveillance.
- Entertainment: Light shows, stage lighting and projection systems.
- Electronics: Barcode scanners. laser printers, optical mice, CD, DVD and Blu-Ray drives.
- Instrumentation: Atomic clocks, interferometers, holographic displays.
- Environment: Lidar weather systems, pollutant and gas detection.
- Art Conservation: Cleaning artifacts, sculptures and buildings without damage.

The word is an acronym of the process that creates laser light: Light Amplification (by) Stimulated Emission (of) Radiation. The process creates an 'avalanche' of photons, each with the same wavelength and therefore the same energy. Thanks to our study of the atomic model, we can understand the principles that make it work.



Laser Schematic taken from Wikipedia and is licensed under CC-BY-SA 3.0

A laser has three fundamental components. A *gain medium*, a power supply, and a mechanism to create optical feedback. The gain medium is a material which can absorb electromagnetic energy. You've probably heard of Helium-Neon lasers, or Carbon Dioxide lasers, both of which describe the gain medium. The gain medium can also be crystals, glasses, semiconductors and liquids for more exotic types of lasers.

Recall that under normal conditions, atomic electrons like to be in the lowest possible energy state. When the gain medium absorbs energy, electrons are promoted from low energy states to higher energy states. The characteristics of a gain medium are such that these excited electrons do not return to the lower energy states immediately, as they normally would. Instead they remain in a *metastable state* called a *population inversion*, where the lower energy states remain empty and the high energy states remain filled.

Obtaining the population inversion requires that energy be transferred to the gain medium by the power supply. This process is known as *laser pumping*. Pumping can be achieved by electrical currents, discharge lamps (essentially a turbo-charged camera flash), chemical reactions, nuclear fission or beams of high energy electrons. Once complete, the gain medium has stored a significant amount of power supply energy in the metastable states of the gain medium's orbital electrons.

When one of the excited electrons returns to the low energy state it came from, a photon is emitted. The photon has a frequency which describes its energy. When this photon disturbs one of the other excited electrons, it causes that electron to drop to the lower energy state and emit a second photon identical to the first. Now there are two identical photons, each of which triggers a drop in energy of other electrons. One becomes two, two becomes four, four become eight and so on. There is a similar process involved in the production of nuclear power, which will be described in a later chapter.

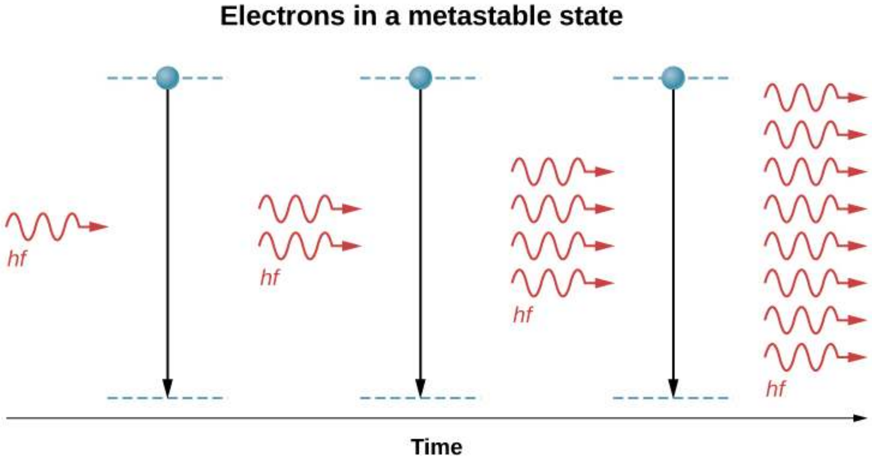


Diagram taken from OpenStax *University Physics vol. 3* and is licensed under CC-BY

To maximize the chances for these photons to stimulate the emission of additional photons, it is useful to 'trap' them in the gain medium. This is often accomplished by a pair of mirrors, called the *optical resonator*. One is perfectly reflective and the other one is partially reflective, meaning that some photons escape but many are returned to the gain medium to promote the production of more photons. If enough time passes, all the electrons will have returned to the lower energy states and the process would stop. A poor analogy would be pushing someone on a swing. If you only push once, the person will stop swinging after some time has passed. To maintain their motion, you must periodically push on them and add energy. To maintain laser operation, the power supply must pump the gain medium repeatedly.

15.7: A New Paradigm

At this point it would seem fair to ask the question "Is light a wave, or is light a bunch of photons"? How could Maxwell's equations describing light as a continuous disturbance be compatible with the Planck-Einstein approach of treating light as a particle? They aren't. Is there compelling evidence for both models? There is. The wavelike nature of light has been experimentally confirmed many times, and much of the telecommunications industry is based on the wavelike nature of light. Likewise, the particulate nature of light has been verified in countless experiments, despite its controversial nature. How can we reconcile these different approaches to the same phenomenon?

To further complicate matters, we return to the description of an electron. It is well documented that electrons are discrete objects. They have a set charge and a measurable mass. Atoms are typically ionized by removing electrons from the orbital shells, further emphasizing their discrete nature. However, in 1924 Louis de Broglie rearranged the photon momentum equation to show that any object having momentum should also be described by a wavelength. In its original form, the equation for the momentum of a photon is:

$$\mathrm{p} = \frac{h}{\lambda}$$

The de Broglie relationship is:

$$\lambda = \frac{h}{p}$$

This was used to suggest that electrons in atomic orbitals could be thought of as standing waves, and furthermore that all matter of a particulate nature could have corresponding wavelike properties. In essence, de Broglie thought that matter could be thought of as a collection of waves that traveled with a common speed and had an effective mass. Both of these properties depended on the energy of the 'particle' and this was connected to the work of Einstein in the preceding years.

In 1925 and 1926 Edwin Schroedinger used the particle-wave duality of electrons to develop a wave equation that described the motion of electrons. This became known as the Schroedinger equation and directly led to the later development of quantum mechanics as a robust area of physics exploration. There were discussions about how to 'prove' the wavelike behavior of electrons using diffraction effects. Interference and diffraction are characteristically wavelike properties, so having electrons display diffraction would validate their treatment as continuous waves instead of discrete 'lumps' of matter.

In 1927, the wave nature of electrons was confirmed by two different sets of diffraction experiments. As was discussed in the chapter about reflection and refraction, diffraction is a distinctly wavelike behavior. Having electrons display diffraction effects verified that electrons 'are' waves. These results were expanded upon in the 1930s with beams of helium atoms and hydrogen molecules. In both cases, the wavelike nature of atoms and molecules was confirmed. This indicated that wavelike behavior is not limited to electrons and is apparently a general property of matter at the microscopic scale.

Waves and particles are two very different models for physical systems. Waves obey wave equations, are continuous and vary over time. Waves display diffraction and wave interference effects. Water waves, seismic waves, radio waves, sound waves and more are all modeled using wave equations and all show exceptionally good agreement between the model and the experimental results.

Particles obey classical mechanics, have trajectories where their position and velocities vary over time and have a center of mass. They do not display interference or diffraction effects. Stars, planets, spacecraft, tennis balls, bullets and grains of sand can all be modeled as particles and again the experimental results validate this approach.

So, Particle or Wave?

While this is a perfectly natural question to ask, it turns out that it is not a terribly useful question to ask. The answer depends on other factors. Sometimes small objects behave like waves and sometimes they behave like particles. Some quantum experiments show particle-like collisions when two objects interact, other experiments show wave-like diffraction and interference effects. As a sort of catch-all statement, physics is able to suggest that energy propagates from one point to another like a wave (with wavelengths, wave speeds and amplitudes), but interactions happen in a particle-like fashion (with momentum, kinetic energies and forces).

It may seem disconcerting to try and hold two contradictory models in our thoughts, but the question is less about truth and more about utility. The only question that is reasonable to ask is "will my model match the experimental evidence better if I treat this as a wave or as a particle?". We will need to practice the same sort of mental flexibility when we try and understand the workings of the nucleus. It turns out that a lifetime of physics experience on earth has little to offer when we try and understand the workings of things beyond our direct experiences.

As dismissive as it might appear upon first reading the statement, Neil deGrasse Tyson was correct in saying "The Universe is under no obligation to make sense to you." Whether or not it 'makes sense', the goal of science is to understand how things behave. If they behave in ways that seem intuitive and are easy to grasp, so much the better. If the behaviors are strange and unsettling to our preconceived notions, it is our duty to adjust our thinking.

"It is far better to grasp the universe as it really is than to persist in delusion, however satisfying and reassuring." - Carl Sagan.

15.8: End of Chapter Activity

End of Chapter Activity: Creating a Lesson Plan on Electromagnetic Radiation, Wavelength, Frequency, and Color with AI and Bloom's Taxonomy

Teaching complex scientific concepts like electromagnetic radiation, wavelength, frequency, and color to 1st graders can be challenging. It requires simplifying the material to make it accessible and engaging for young minds. Your task is to create a succinct lesson plan for 1st graders that introduces them to the basics of these concepts. To help you with this, you will use AI tools and incorporate Bloom's Taxonomy to ensure a comprehensive learning experience. This lesson plan will go towards your digital notebook, a portfolio filled with lesson plans, activities, and labs for future use.

Activity Prompt:

Objective: Use AI and Bloom's Taxonomy to develop a lesson plan that effectively teaches 1st graders about the fundamentals of electromagnetic radiation, wavelength, frequency, and color, including simple experiments and creative activities.

Understanding the Concepts:

Knowledge (Remembering): Define key terms related to light and color, such as light, wavelength, frequency, and color.

Comprehension (Understanding): Explain these concepts in simple, age-appropriate language, using relatable examples and visuals.

Planning the Lesson:

Application: Design an engaging activity that allows students to observe and understand the basics of light and color. For example, use a prism to show how white light splits into different colors or use colored filters to explore how light changes.

Analysis: Use AI tools to create visual aids or interactive simulations that illustrate how light waves work. For instance, create a simple animation showing how light waves with different wavelengths correspond to different colors.

Deepening Understanding:

Synthesis (Creating): Ask students to create their own simple art projects that demonstrate their understanding of color mixing and light. For example, they could use colored cellophane to create a rainbow or draw pictures using primary and secondary colors.

Evaluation: Have students discuss and reflect on their art projects and the properties of light and color they observed. Encourage them to think about how light and color are part of their everyday lives.

Using AI in the Classroom:

Explore AI tools like educational apps or platforms that provide interactive content for teaching about light and color. Use these tools to create quizzes, flashcards, or interactive stories that reinforce the lesson's concepts.

Use AI to assess student understanding through formative assessments and provide instant feedback.

Deliverable:

Submit a detailed lesson plan that includes:

1. **A brief overview of the key concepts covered:** Outline the foundational concepts of light, wavelength, frequency, and color that will be taught.
2. **A description of the activities and experiments designed:** Detail the hands-on activities and experiments you will use to help students understand these concepts.
3. **Examples of AI tools used and how they enhance the learning experience:** Describe the AI tools you plan to incorporate, such as simulations or interactive quizzes, and explain how they will help students grasp complex concepts.
4. **An explanation of how Bloom's Taxonomy was applied in the lesson plan to ensure a well-rounded educational experience:** Illustrate how each level of Bloom's Taxonomy (Remembering, Understanding, Applying, Analyzing, Creating, and Evaluating) is addressed in your lesson plan.

Additionally, include a creative project component where students create a digital story or a simple animation that explains a concept related to light and color, using AI tools to enhance their projects.

Example Lesson Plan:

Grade: 1st Grade
Topic: Electromagnetic Radiation, Wavelength, Frequency, and Color
Duration: 1 Week

Overview:
Students will learn about the basics of light, wavelength, frequency, and color through engaging activities and creative projects.

Day 1: Introduction to Light and Color

Objective: Define basic concepts related to light and color and provide examples.

- **Remembering:** Define key terms (light, wavelength, frequency, color).
- **Understanding:** Explain the concepts using examples from everyday life (e.g., sunlight, rainbows).

Activity:
Watch a short, animated video (created using AI tools) explaining what light is, how it travels in waves, and how different colors correspond to different wavelengths.

Day 2: Exploring Colors with a Prism

Objective: Understand how light splits into different colors.

- **Applying:** Conduct an activity using a prism to show how white light splits into a rainbow of colors.

Activity:
Students use prisms to observe how white light splits into different colors. They draw pictures of the rainbows they see and discuss why this happens.

Day 3: Color Mixing with Filters

Objective: Explore how light changes with colored filters.

- **Applying:** Conduct an activity using colored cellophane or filters to mix different colors of light.

Activity:
Students use colored cellophane to mix different colors and observe the results. They create art projects by overlaying different colors to see how new colors are formed.

Day 4: Creative Project – Light and Color Art

Objective: Create art projects that demonstrate understanding of light and color.

- **Creating:** Students create their own art projects using colored materials.

Activity:
In groups, students use colored paper, cellophane, and other materials to create art projects that demonstrate their understanding of light and color. They explain their projects and the colors they used.

Day 5: Reflection and Digital Story

Objective: Reflect on what they have learned and create a digital story about light and color.

- **Evaluating:** Discuss and reflect on the activities and projects.
- **Creating:** Use AI tools to create a digital story or simple animation about light and color.

Activity:
Students create a short digital story or animation using AI tools that explains what they have learned about light and color. They can include drawings, photos of their projects, and voice recordings. They present their projects to the class, using their digital stories to enhance their explanations.

By incorporating these strategies and activities, educators can effectively teach 1st graders about electromagnetic radiation, wavelength, frequency, and color, helping them understand and appreciate these fundamental concepts in a fun and engaging way.

15.9: End of Chapter Key Terms

Definition: Key Terms

1. Source Charge: A charge that produces an electromagnetic wave when it is oscillated.
2. Amplitude: The strength of the electric field.
3. Wavelength: The distance between maximum electric field locations.
4. Frequency: The number of oscillations per second completed by the source charge.
5. Wave Speed: The rate at which electromagnetic energy propagates. In vacuum this is the speed of light: 3×10^8 meters per second.
6. Wave Interference: When two or more waves combine to create a larger or smaller wave.
7. Intensity: The power delivered by a wave per unit of area.
8. Radiation Pressure: The force per unit area delivered by an electromagnetic wave.
9. Electromagnetic Spectrum: The range of frequencies and wavelengths that encompass electromagnetic waves.
10. Infrared Light: Wavelengths of light longer than red light.
11. Ultraviolet Light: Wavelengths of light shorter than violet light.
12. Photoreceptors: Light sensing cells in the retina of human eyes.
13. True Color: The color of an object when illuminated by white light.
14. Retinex: A collection of photoreceptors in the retina.
15. Color Constancy: The ability of the eye to perceive true colors under a variety of lighting.
16. Wien's Law: An experimental relationship between temperature of an object and the wavelength of light it emits.
17. Ultraviolet Catastrophe: The failure of the electromagnetic wave model to recreate Wien's Law.
18. Quanta: The smallest bit of energy possible in Planck's Hypothesis.
19. Quantized Energy: The idea that all energy must be a multiple of a fundamental value.
20. Energy State: A way to describe the value of the energy associated with a particular object.
21. Quantum Number: A number that is used to multiply the quanta to obtain the energy state.
22. Photoelectrons: Electrons ejected from surfaces when light shines upon them.
23. Photoelectric Effect: The relationship between the wavelength of light and the appearance of photoelectrons.
24. **Electromagnetic Radiation**: Waves of electric and magnetic fields that propagate through space.
25. Photon: The smallest unit of an electromagnetic wave in the particle model of light.
26. Gain Medium: The part of a laser that produces the laser light.
27. Metastable State: A condition where excited electrons remain in an excited state for some period of time.
28. Population Inversion: When there are more electrons in an excited state than in lower energy states.
29. Laser Pumping: The process of transferring energy from the power supply to the gain medium.
30. Optical Resonator: A component of a laser that traps and amplifies electromagnetic radiation before it is released as a beam.
31. Particle-WAve Duality: The concept that objects display both wave-like and particle-like properties.
32. **Quantum Mechanics**: The branch of physics that studies the behavior of particles on the atomic and subatomic level.

CHAPTER OVERVIEW

16: Reflections and Refraction of Waves

16.1: Introduction and Learning Objectives

Chapter Learning Objectives

1. **Understanding Wave Behavior at Boundaries:**
    - Explain reflection and refraction principles.
    - Differentiate behaviors of mechanical and electromagnetic waves in new mediums.
    - Analyze reflection and refraction laws, including Snell's Law.
2. **Exploring the Refraction of Light:**
    - Understand the refractive index and light speed variations in different mediums.
    - Examine factors influencing light bending between materials.
    - Apply refraction concepts to lenses, prisms, and optical fibers.
3. **Investigating Reflection in Various Contexts:**
    - Explore specular and diffuse reflection principles.
    - Understand surface effects on wave reflection.
    - Analyze reflection in technology and natural phenomena like mirrors and echoes.
4. **Studying Applications of Refraction and Reflection:**
    - Investigate practical applications in technology, medicine, and communication.
    - Understand principles in designing optical instruments and industry applications.
5. **Educational Strategies for Teaching Reflection and Refraction:**
    - Develop engaging methods for teaching reflection and refraction.
    - Create interactive experiments and demonstrations.
    - Use multimedia and simulation tools to enhance learning.
    - Address student misconceptions and enhance understanding with real-world examples.

Introduction to Reflections and Refractions of Waves

In the fascinating realm of physical science, the behaviors of waves at boundaries—specifically reflection and refraction—play crucial roles in our understanding of natural phenomena and technological advancements. When waves encounter a boundary between two different mediums, they exhibit behaviors such as reflection, where waves bounce back into the original medium, and refraction, where waves change direction and speed as they enter a new medium. These principles govern a wide array of wave types, from mechanical waves like sound and water waves to electromagnetic waves, including light. Fundamental laws such as the law of reflection, which states that the angle of incidence equals the angle of reflection, and Snell’s Law, which describes the relationship between angles of incidence and refraction in different mediums, form the backbone of these phenomena.

Refraction of light, a key focus in this chapter, involves understanding how the refractive index affects light speed in various materials, causing light to bend at boundaries. This bending of light underpins essential technologies like lenses, prisms, and optical fibers, which are integral to fields such as optics, communication, and medical imaging. Similarly, the study of reflection, whether specular (mirror-like) or diffuse (scattered), provides insights into everyday occurrences and advanced technologies, from the simple use of mirrors to complex seismic wave analysis.

For educators, conveying these scientific principles effectively in a K-12 setting involves not just theoretical explanations but also practical applications and engaging teaching methods. Demonstrating wave behavior through interactive experiments, utilizing multimedia resources, and creating real-world connections can significantly enhance students' comprehension and interest. By integrating these concepts into classroom activities, educators can help students appreciate the relevance of wave reflections and refractions in both everyday life and advanced technological applications, fostering a deeper understanding and curiosity about the physical world.

SECTION OVERVIEW

16.2: Optics

16.2.1: Reflection

Reflection of a sign in water

Figure 14.1.1

The designers of this sign used the fact that light can reflect off of many surfaces, including water, to make the sign legible. It takes a bit of thought, but words can be written in such a way that their reflection is legible, even though the words themselves are not.

Reflection of Light

Wave reflecting off a barrier

Figure 14.1.2

The Law of Reflection

When a light ray strikes a reflecting surface, the angle of incidence (measured from the normal line) is equal to the angle of reflection (also measured from the normal line). This is called the **Law of Reflection.**

If the reflecting surface is a very smooth surface, the reflection will be a **regular reflection,** in which the light rays maintain their position relative to each other and objects will be visible and identifiable in the reflected image. If the reflecting surface is rough, the reflection will be a **diffuse reflection,** and objects will not be visible or identifiable in the reflection. When you are considering the size of things on the scale of wavelengths of light, even surfaces that appear smooth may be very rough in terms of light waves, and most surfaces produce diffuse reflection.

Comparison of the reflection off a rough and off a smooth surface

Figure 14.1.3

Left and Right Reversal in a Plane Mirror

The images that appear in a plane (flat) mirror are reversed in some ways and not reversed in other ways. In the image below, the man's right hand is labeled. The same hand in the mirror image, however, looks like a left hand. While the left and right of the image are reversed, the top and bottom of the image are not.

Right and left reversal in a mirror

Figure 14.1.4

In the simulation below, the young man is getting ready for the prom. He needs to stand at a particular distance and have the mirror be a particular length to be able to view his entire body. Rays of light are emerging from various light sources in the room and bouncing off him into the mirror. Play around with rays from different locations, try different mirror lengths, and see what you can discover about the reflection of light in a mirror.

Summary

- The law of reflection states that, when a light ray strikes a reflecting surface, the angle of incidence (measured from the normal line) is equal to the angle of reflection (also measured from the normal line).
- If the reflecting surface is a very smooth surface, the reflection will be regular, in which the light rays maintain their position relative to each other.
- If the reflecting surface is rough, the reflection will be diffuse and objects will be distorted in the reflection.

- Images in a plane mirror are reversed left and right but not reversed top and bottom.

Review

1. How does regular reflection differ from diffuse reflection?
2. If a light ray strikes a mirrored surface at an angle of 25° to the surface, what is the angle of incidence?
3. For problem #2, what will be the angle of reflection?
4. A dry cement road is a diffuse reflector. When it rains, the water fills in all the little holes and cracks in the cement road and it becomes a smooth regular reflector. At night, when you are depending on the light from your headlights to show you the lines on the road, a wet road becomes much darker and it is more difficult to see the lines. Explain why this occurs.

Explore More

Use this resource to answer the questions that follow.

1. Both the angle of incidence and the angle of reflection are measured from the __________.
2. In reflection, the angle of incidence __________ the angle of reflection.

Additional Resources

Study Guide: Geometric Optics Study Guide

Videos: Geometric Optics 1: Reflection - Overview

Real World Application: Oil and Water

16.2.2: Refraction

A pencil looks bent in a glass of water due to refraction

Figure 14.2.1

When a light ray passes at an angle through the boundary between optically different media, the light does not travel in a straight line. The pencil in the glass of liquid shown above is a normal straight pencil. The light that travels from the pencil through the liquid, through the glass, and into the air is bent differently than light from the portion of the pencil that is not in the liquid. Your eye assumes the light from both portions of the pencil moved in a straight line, but the two portions of the pencil do not appear to be lined up. Your eye thinks the pencil is broken.

Refraction of Light

The speed of light is different in different media. If the speed of light is slower in a particular medium, that medium is said to be more **optically dense**. When a wave front enters a new medium at an angle, it will change directions. If the light is entering a more optically dense medium, the light bends toward the normal line. If the light is entering a less optically dense medium, the light will bend away from the normal line. Remember that the normal line is the line perpendicular to the medium interface.

In the sketch below, light wave fronts are moving upward from the bottom of the page and encounter a boundary into a more optically dense medium. The light waves bend toward the normal line. Because the right end of the wave fronts enter the new medium first, they slow down first. When the right side of the wave front is moving more slowly that the left side, the wave front will change directions.

Refraction of a wave front

Figure 14.2.2

When light is traveling from air into another medium, **Snell's Law** states the relationship between the angle of incidence and angle of refraction is

$n=\sin\theta_i/\sin\theta_r$

where θ_i is the angle of incidence, θ_r is the angle of refraction, and n is the ratio of the two sines and is called the **index of refraction**. Snell's Law may be stated that a ray of light bends in such a way that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant.

The index of refraction is also related to the relative speeds of light in a vacuum and in the medium.

n=speed of light in a vacuum/speed of light in the medium

When a ray of light is traveling from medium into another medium, Snell's Law can be written as $n_i\sin\theta_i=n_r\sin\theta_r$.

Indices of Refraction

Medium	n_
Vacuum	1.00
Air	1.0003
Water	1.36
Ethanol	1.36
Crown Glass	1.52
Quartz	1.54
Flint Glass	1.61
Diamond	2.42

Example 14.2.1

A ray of light traveling through air is incident upon a slab of Flint glass at an angle of 40.0°. What is the angle of refraction?

Solution

$n=\sin\theta_i/\sin\theta_r$ so $\sin\theta_r=\sin\theta_i/n=0.643/1.61=0.399$

The angle of refraction$=\sin^{-1}.399=23.5^{\circ}$

Example 14.2.2

What is the speed of light in a diamond?

Solution

speed of light in diamond=speed of light in a vacuum/n$=3.00\times10^{8}$m/s /2.42

speed of light in diamond$=1.24\times10^{8}$ m/s

In the Least Time simulation below, you can adjust the sliders so that light travels from air to a denser material like glass or diamond. Then, drag the incident ray and refracted rays to the correct angles according to Snell’s Law. When you have found the path of least time - a red light ray will shoot out across the both materials! Very exciting!

Effects of Refraction

Bending the Sun’s Rays

Because air is slightly more optically dense than a vacuum, when sunlight passes from the vacuum of space into our atmosphere, it bends slightly towards the normal. When the sun is below the horizon and thus not visible on a direct line, the light path will bend slightly and thus make the sun visible by refraction. Observers can see the sun before it actually comes up over the horizon, or after it sets.

Refraction of the Sun's rays

Figure 14.2.3

Mirages

In the Figure below, the sun shines on the road, heating the air just above the road. The difference in density between the hot air over the road and the surrounding air causes the hot air to refract light that passes through it. When you look at the road, you see a **mirage**. What appears to be water on the road is actually light coming from the sky that has been refracted as it passes through the hot air above the road. This phenomenon is common on hot roads and in the desert.

A mirage on a road

Figure 14.2.4

Summary

- The speed of light is different in different media.
- When a wave front enters a new medium at an angle, it will change directions. If the light is entering a more optically dense medium, the light bends toward the normal line. If the light is entering a less optically dense medium, the light will bend away from the normal line.
- When light is traveling from air into another medium, Snell’s Law states that $n=\sin\theta i\sin\theta r$.
- The index of refraction is also related to the relative speeds of light in a vacuum and in the medium. n=speed of light in a vacuum/speed of light in the medium
- When a ray of light is traveling from one medium into another medium, Snell’s Law can be written as $ni\sin\theta i=nr\sin\theta r$.

Review

1. Light moving through air is incident on a piece of crown glass at an angle of 45°. What is the angle of refraction?
2. A ray of light passes from air into water at an incident angle of 60°. Find the angle of refraction.
3. Light passes from water into a block of transparent plastic. The angle of incidence from the water is 31∘ and the angle of refraction in the block is 27°. What is the index of refraction for the plastic?
4. The index of refraction of water is 1.36. What is the speed of light in water?
5. If the speed of light in a piece of plastic is 2.00×10^8 m/s, what is the index of refraction for the plastic?

Additional Resources

Study Guide: Geometric Optics Study Guide

Real World Application: Is That A Mirage?

PLIX: Play, Learn, Interact, eXplore: Refraction: Light Entering Water, Refraction Slider

Videos:

16.2.3: Total Internal Reflection

Total internal reflection in a fiber optic cable

Figure 14.3.1

Total internal reflection allows the light to travel down the optical fiber and not pass through the sides of the tube. The light continuously reflects from the inside of the tube and eventually comes out the end. Optical fibers make interesting lamps but they are also used to transport telephone and television signals.

Total Internal Reflection

We already know that when light passes from one medium into a second medium where the index of refraction is smaller, the light refracts away from the normal.

In the image below, the light rays are passing into an optically less dense medium; therefore, the rays bend away from the normal. As the angle of incidence increases, the light ray bends even further away from the normal. Eventually, the angle of incidence will become large enough that the angle of refraction equals $90\circ$, meaning the light ray will not enter the new medium at all.

Diagram illustrating the conditions for total internal reflection

Figure 14.3.1

Consider a ray of light passing from water into air. The index of refraction for air is 1.00 and for water is 1.36. Using Snell’s Law, $n_i \sin\theta_i = n_r \sin\theta_r$, and allowing the angle of refraction to be 90°, we can solve for the angle of incidence which would cause the light ray to stay in the old medium.

$n_i \sin\theta_i = n_r \sin\theta_r$

$(1.36)(\sin\theta_i) = (1.00)(\sin 90^\circ)$

$\sin\theta_i = 0.735$ *and* $\theta_i = 47^\circ$

This result tells us that when light is passing from water into air, if the angle of incidence exceeds 47°, the light ray will not enter the new medium. The light ray will be completely reflected back into the original medium. This is called **total internal reflection**. The minimum angle of incidence for total internal reflection to occur is called the **critical angle**.

Total internal reflection is the principle behind **fiber optics**. A bundle of fibers made out of glass or plastic only a few micrometers in diameter is called a light pipe since light can be transmitted along it with almost no loss. Light passing down the fibers makes glancing collisions with the walls so that total internal reflection occurs.

Use the Diamond Cut simulation below to adjust the shape of a diamond with the three sliders related to depth, crown, and table size. Try to find a combination of sliders for which the light is trapped inside - achieving total internal reflection - and a very sparkly diamond!

Summary

- When light passes from one medium into a second medium with a smaller index of refraction, the light refracts away from the normal.
- If the angle of incidence becomes large enough that the angle of refraction equals $90\circ$, the light ray will not enter the new medium with the smaller angle of refraction.
- Total internal reflection means the light ray will not enter the new medium but will be completely reflected back into the original medium.

Review

1. Find the critical angle for light passing from diamond into air, given $n_{\text{diamond}}=2.42$.
2. When two swimmers are under water in a swimming pool, it is possible for the interface between the water and the air to act as a mirror, allowing the swimmers to see images of each other if they look up at the underside of the surface. Explain this phenomenon.
3. Robert shines a laser beam through a slab of plastic and onto the interface between the slab of plastic and the air on the other side. The index of refraction for the plastic is 1.62. If the angle of incidence in the plastic is 54 , will the laser beam pass out of the plastic into the air?

Explore More

Use this resource to answer the questions that follow.

1. What phenomenon occurs when the light does not enter the new medium and remains in the old medium?
2. When does this phenomenon occur?

Additional Resources

Study Guide: Geometric Optics Study Guide

Videos: Geometric Optics 6: Total Internal Reflection - Overview

Real World Application: Light Pipes

PLIX: Play, Learn, Interact, eXplore: Total Internal Reflection

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16.2.4: Plane Mirrors

Identical twins that look like they are in a mirror

Figure 14.4.1

Is this a picture of one girl in front of a mirror or is this a picture of twins designed to look like a mirror image? The writing on the shirt is properly mirror reversed, and, as in plane mirror images, the right hand of the object becomes the left hand of the image. However, if you look closely, you will see that in the picture one girl is wearing a ring and the other isn't.

Image in a Plane Mirror

The sketch below shows how we see an image in a plane mirror. Plane mirrors work because the light rays create a **virtual image** behind the mirror. Light rays from the object strike the mirror and reflect according to the law of reflection. When some of the light rays enter our eye, our eye and brain interpret these rays as having traveled in a straight line path. Therefore, our eye and brain track the light rays backward to a position from which they appear to have come. At this position, we see an image.

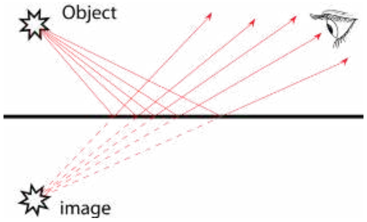


Figure 14.4.2

In a plane mirror, the image will be the same size as the object and will be the same distance behind the mirror as the object is in front of the mirror. This image is called a virtual image because the light does not actually pass through the image.

Picture of a woman and her reflection in a mirror

Figure 14.4.3

When you look at an image of yourself in a plane mirror, there are some differences that are apparent. In the image above, you see a woman cleaning a mirror. The object woman has a cleaning cloth in her left hand. The image woman, however, appears to be holding the cleaning cloth in her right hand. Although the images in a plane mirror appear to flip horizontally, it is actually a result of them reversing in depth along the z-axis (in and out).

To learn more about why images in a mirror appear to flip horizontally watch the video below:

Launch the PLIX Interactive below to further explore virtual images and try to determine why you see a "mirror image" of yourself when you look at your reflection:

Example 14.4.1

A person 1.80 m tall stands in front of a plane mirror. What is the minimum height of the mirror, and how high must its lower edge be above the floor for the person to be able to see his/her whole body? Assume the person's eyes are 6.0 cm below the top of the head.

Solution

The law of reflection tells us that the angle of incidence equals the angle of reflection. From this, we know that the light ray leaving the person's toes will strike the mirror halfway between his toes and his eyes. The distance from the person's toes to eyes is 1.74 m, so the bottom of the mirror must be 0.87 m above the floor. The light ray that leaves the top of the person's head and reflects from the mirror into his eyes must strike the mirror 3.0 cm below the top of his head.

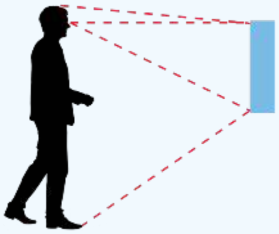


Figure 14.4.4

Therefore, the top of the mirror is 1.77 m above the floor and the bottom of the mirror is 0.87 m above the floor. The height of the mirror is 1.77 m−0.87 m=0.90 m.

It doesn't take much analysis to recognize that the distance the person stands from the mirror does NOT affect the results. If you can't see your feet in a mirror, getting closer or farther away from the mirror won't make any difference. Try it out in this simulation below by adjusting the distance to the mirror and the length of the mirror:

Summary

- Our eye and brain interpret any light rays that enter an eye as having traveled in a straight line path.
- For a plane mirror, the image will be the same size as the object and will be the same distance behind the mirror as the object is in front of the mirror.
- Since the light forming the image does not pass through the image, it is called a virtual image.
- Images in a plane mirror are reversed left and right but are not reversed top and bottom.

Review

1. A virtual image is one:
    1. toward which light rays converge but do not pass through.
    2. from which light rays diverge but do not pass through.
    3. from which light rays diverge as they pass through.
    4. toward which light rays converge and pass through.
    5. with a ray normal to a mirror passing through.
2. An object is 2.0 m in front of a plane mirror. Its image is:
    1. virtual, inverted, and 2.0 m behind the mirror.
    2. virtual, inverted, and 2.0 m in front of the mirror.
    3. virtual, erect, and 2.0 m in front of the mirror.
    4. real, erect, and 2.0 m behind the mirror.
    5. virtual, erect, and 2.0 m behind the mirror.
3. If the angle of incidence for an object and a plane mirror is 30°, what is the angle between the object and its image?
    1. 30°
    2. 60°
    3. 90°
    4. 120°
    5. 180°
4. If the angle of a car's windshield is 45° to vertical, what position of the sun is most likely to reflect into oncoming driver's eyes?
    1. Low in the sky behind the oncoming driver's car
    2. Low in the sky opposite the oncoming driver's car

   3. Directly overhead
5. A 50. cm tall object is 3.0 m from a plane mirror.
   1. How tall will the image be?
   2. How far from the mirror will the image be?
   3. Will the image be real or virtual?
   4. Will the image be upright or inverted?
6. Which statement is true about the image produced by a plane mirror?
   1. It appears to be located on the same side of the mirror as the object.
   2. It appears to be larger than the object.
   3. It appears to be inverted relative to the object.
   4. It appears to be reversed left and right.
7. A light ray strikes a plane mirror at an angle of 80° to the normal. What is the angle that the reflected ray makes with the surface of the mirror?
8. A laser beam strikes a plane mirror with an angle of incidence of 38°. What is the angle between the incident beam and the reflected beam?

Explore More

Use this resource to answer the questions that follow.

1. Are the images in the plane mirror upside down?
2. Are the images in the plane mirror reversed left and right?

Additional Resources

Videos:

Real World Application: The Person In The Mirror

Study Guide: Geometric Optics Study Guide

16.2.5: Concave Mirrors

Concave mirror of the Hubble Space Telescope

Figure 14.5.1

Concave mirrors are used in a number of applications. They form upright, enlarged images, and are therefore useful in makeup application or shaving. They are also used in flashlights and headlights because they project parallel beams of light, and in telescopes because they focus light to produce greatly enlarged images.

The photograph above shows the grinding of the primary mirror in the Hubble space telescope. The Hubble Space Telescope is a reflecting telescope with a mirror approximately eight feet in diameter, and was deployed from the Space Shuttle Discovery on April 25, 1990.

Image in a Concave Mirror

Reflecting surfaces do not have to be flat. The most common curved mirrors are spherical. A **spherical mirror** is called a **concave mirror** if the center of the mirror is further from the viewer than the edges are. A spherical mirror is called **a convex mirror** if the center of the mirror is closer to the viewer than the edges are.

To see how a concave mirror forms an image, consider an object that is very far from the mirror so that the incoming rays are essentially parallel. For an object that is infinitely far away, the incoming rays would be exactly parallel. Each ray would strike the mirror and reflect according to the law of reflection (angle of reflection is equal to the angle of incidence). As long as the section of mirror is small compared to its radius of curvature, all the reflected rays will pass through a common point, called the **focal point (*f*)**.

Illustrates how a concave mirror forms an image

Figure 14.5.2

If too large a piece of the mirror is used, the rays reflected from the top and bottom edges of the mirror will not pass through the focal point and the image will be blurry. This flaw is called **spherical aberration** and can be avoided either by using very small pieces of the spherical mirror or by using parabolic mirrors.

A line drawn to the exact center of the mirror and perpendicular to the mirror at that point is called the **principal axis**. The distance along the principle axis from the mirror to the focal point is called the **focal length**. The focal length is also exactly one-half of the radius of curvature of the spherical mirror. That is, if the spherical mirror has a radius of 8 cm, then the focal length will be 4 cm.

Objects Outside the Center Point

Object located outside the center point

Figure 14.5.3

Above is a spherical mirror with the principle axis, the focal point, and the center of curvature (*C)* identified on the image. An object has been placed well beyond *C*, and we will treat this object as if it were infinitely far away. There are two rays of light leaving any point on the object that can be traced without any drawing tools or measuring devices. The first is a ray that leaves the object and strikes the mirror parallel to the principle axis that will reflect through the focal point. The second is a ray that leaves the object and strikes the mirror by passing through the focal point; this ray will reflect parallel to the principle axis.

These two rays can be seen in the image below. The two reflected rays intersect after reflection at a point between *C* and *F*. Since these two rays come from the tip of the object, they will form the tip of the image.

Reflected rays off a concave mirror

Figure 14.5.4

If the image is actually drawn in at the intersection of the two rays, it will be smaller and inverted, as shown below. Rays from every point on the object could be drawn so that every point could be located to draw the image. The result would be the same as shown here. This is true for all concave mirrors with the object outside *C*: the image will be between *C* and *F*, and the image will be inverted and diminished (smaller than the object).

Image formed by a concave mirror

Figure 14.5.5

The heights of the object and the image are related to their distances from the mirror. In fact, the ratio of their heights is the same ratio as their distances from the mirror. If d_o is object distance, d_i the image distance, h_o the object height and h_i the image height, then

$h_o/h_i=d_o/d_i$.

It can also be shown that $d_o/d_i=(d_o-f)/f$ and from this, we can derive the **mirror equation**,

$(1/d_o)+(1/d_i)=(1/f)$.

In this equation, f is the focal length d_o is the object distance, and d_i is the image distance.

The **magnification equation** for a mirror is the image size divided by the object size, where m gives the magnification of the image.

$m=-d_i/d_o$

Example 14.5.1

A 1.50 cm tall object is placed 20.0 cm from a concave mirror whose radius of curvature is 30.0 cm. What is the image distance and what is the image height?

Solution

$(1/d_o)+(1/d_i)=(1/f)$ The focal length is one-half the center of curvature so it is 15.0 cm. $(1/20.0)+(1/x)=(1/15)$ multiply both sides by $60x$ to get $3x+60=4x$ and $x=60.0$ cm.

The image distance is 60.0 cm. The image is 3 times as far from the mirror as the object so it will be 3 times as large, or 4.5 cm tall.

Objects Between the Center Point and the Focal Point

Regardless of where the object is, its image's size and location can be determined using the equations given earlier in this section. Nonetheless, patterns emerge in these characteristics. We already know that the image of an object outside the center point is closer and smaller than the object. When an object is between the center point and the focal point, the image is larger and closer. These characteristics can also be determined by drawing the rays coming off of the object; this is called a **ray diagram.**

Image formed when the object is between the center point and the focal point

Figure 14.5.6

Look again at the image above that was shown earlier in the lesson. If you consider the smaller arrow to be the object and follow the rays backward, the ray diagram makes it clear that if an object is located between the center point and focal point, the image is inverted, larger, and at a greater distance.

Example 14.5.2

If an 3.00 cm tall object is held 15.00 cm away from a concave mirror with a radius of 20.00 cm, describe its image.

Solution

To solve this problem, we must determine the height of the image and the distance from the mirror to the image. To find the distance, use the mirror equation:

$(1/d_i)=(1/f)-(1/d_o)=(1/10)-(1/15)=(1/30)=30\text{cm}$

Next determine the height of the image:

$(h_o/h_i)=(d_o/d_i)=(3/h_i)=(15/30)$

Using this equation, we can determine that the height of the image is 6 cm.

This image is a **real image**, which means that the rays of light are real rays. These are represented in the ray diagram as solid lines, while virtual rays are dotted lines.

Objects Inside the Focal Point

Consider what happens when the object for a concave mirror is placed between the focal point and the mirror. This situation is sketched at below.

Object inside the focal point

Figure 14.5.7

Once again, we can trace two rays to locate the image. A ray that originates at the focal point and passes through the tip of the object will reflect parallel to the principal axis. The second ray we trace is the ray that leaves the tip of the object and strikes the mirror parallel to the principal axis.

Below is the ray diagram for this situation. The rays are reflected from the mirror and as they leave the mirror, they diverge. These two rays will never come back together and so a real image is not possible. When an observer looks into the mirror, however, the eye will trace the rays backward as if they had followed a straight line. The dotted lines in the sketch show the lines the rays would have followed behind the mirror. The eye will see an image behind the mirror just as if the rays of light had originated behind the mirror. The image seen will be enlarged and upright. Since the light does not actually pass through this image position, the image is virtual.

Virtual image formed by a concave mirror

Figure 14.5.8

Example 14.5.3

A 1.00 cm tall object is placed 10.0 cm from a concave mirror whose radius of curvature is 30.0 cm. Determine the image distance and image size.

Solution

Since the radius of curvature is 30.0 cm, the focal length is 15.0 cm. The object distance of 10.0 cm tells us that the object is between the focal point and the mirror.

$(1/d_o)+(1/d_i)=(1/f)$ and plugging known values yields $(1/10.0)+(1/x)=(1/15)$

Multiplying both sides of the equation by 30x yields $3x+30=2x$ and $x=-30.0$ cm.

The negative image distance indicates that the image is behind the mirror. We know the image is virtual because it is behind the mirror. Since the image is 3 times as far from the mirror as the object, it will be 3 times as tall. Therefore, the image height is 3.00 cm.

A Cassegrain telescope uses a concave primary mirror to magnify light from very distant objects to form an image we can see. Play around with a Cassegrain telescope in the simulation below and see if you can until you get a clear view of Jupiter in the eyepiece:

Summary

- A spherical mirror is concave if the center of the mirror is further from the viewer than the edges.
- For an object that is infinitely far away, incoming rays would be exactly parallel.
- As long as the section of mirror is small compared to its radius of curvature, all the reflected rays will pass through a common point, called the focal point.
- The distance along the principle axis from the mirror to the focal point is called the focal length, *f*. This is also exactly one-half of the radius of curvature.
- The mirror equation is $(1/d_o)+(1/d_i)=1/f$.
- The magnification equation is $m=-d_i/d_o$.
- For concave mirrors, when the object is outside *C*, the image will be between *C* and *F* and the image will be inverted and diminished (smaller than the object).

- For concave mirrors, when the object is between C and F, the image will be beyond C and will be enlarged and inverted.
- For concave mirrors, when the object is between F and the mirror, the image will be behind the mirror and will be enlarged and upright.

Review

1. A concave mirror is designed so that a person 20.0 cm in front of the mirror sees an upright image magnified by a factor of two. What is the radius of curvature of this mirror?
2. If you have a concave mirror whose focal length is 100.0 cm, and you want an image that is upright and 10.0 times as tall as the object, where should you place the object?
3. A concave mirror has a radius of curvature of 20.0 cm. Locate the image for an object distance of 40.0 cm. Indicate whether the image is real or virtual, enlarged or diminished, and upright or inverted.
4. A dentist uses a concave mirror to examine a tooth that is 1.00 cm in front of the mirror. The image of the tooth forms 10.0 cm behind the mirror.
    1. What is the mirror’s radius of curvature?
    2. What is the magnification of the image?
5. When a man stands 1.52 m in front of a shaving mirror, the image produced is inverted and has an image distance of 18.0 cm. How close to the mirror must the man place his face if he wants an upright image with a magnification of 2.0?

Explore More

Use this resource to answer the questions that follow.

1. What are the two main rays you need to draw a real image?
2. What is the third ray? What can you use it for?
3. How do light rays through the focal point and vertex behave the same? How are they different?

Additional Resources

Study Guide: Geometric Optics Study Guide

Real World Application: Distorted Images

Video:

- **14.5: Concave Mirrors** by CK-12 Foundation is licensed CK-12. Original source: https://flexbooks.ck12.org/cbook/ck-12-physics-flexbook-2.0.

16.2.6: Convex Mirrors

A car rearview mirror is a convex mirror

Figure 14.6.1

Convex mirrors provide a wide angle view and they provide an upright image. Wide angle images have benefits in rearview mirrors, in-store customer monitors, and for viewing large areas like intersections and parking lots.

Images in a Convex Mirror

In **convex mirrors**, the reflecting surface is on the outside of the sphere, making the center of curvature and the focal point on the opposite side of the mirror from the object. Since the focal point is on the opposite side of the mirror from the object, the focal length is assigned a negative number.

Light rays that come to the mirror aimed at the focal point on the back side will be reflected parallel to the principal axis. In the sketch, blue rays approach the mirror as if they would continue through the mirror toward the focal point (dotted blue lines). They reflect along the purple lines parallel to the principal axis.

Light rays reflecting off a convex mirror

Figure 14.6.2

Similarly, light rays that approach the mirror parallel to the principal axis reflect as if they came from the focal point.

Example 14.6.1

A convex mirror has a radius of curvature of 40.0 cm. If the object distance is 1000.0 cm, find the image distance and the magnification.

Solution

$(1/d_o)+(1/d_i)=(1/f)$ *so* $(1/1000)+(1/x)=(1/-20.0)$

Multiplying both sides by 1000x yields $x+1000=-50x$ and $51x=-1000$ and $x=-19.6$ cm.

Since the image distance is negative, it means the image is behind the mirror and is virtual. The image will be upright.

$m=-d_i/d_o=-(-19.6)/1000=0.0196$ or 1/51

The image is reduced by a factor of 51.

For a convex mirror, if the object is at infinity, the image will be a dot on the focal point. As the object moves from infinity toward the mirror, the image moves along the principal axis toward the mirror. When the object right next to the mirror, the image will be right next to the mirror on the other side.

All ray diagrams for convex mirrors look essentially like the image below, with the placement of the image somewhere between the mirror and the focal point.

Image formed by a convex mirror

Figure 14.6.3

The ray tracing for convex mirrors follow this general sketch. Two rays leave the tip of the object, one approaches the mirror parallel and reflects as if it came from the focal point. The top blue dotted line shows this imaginary route. The second ray leaves the object tip and approaches the mirror toward the focal point but reflects parallel at the mirror. The green solid line shows the reflected ray and the dotted green line shows the imaginary route behind the mirror. Where the two dotted lines intersect is the tip of the image. All images in convex mirrors are upright, virtual, and diminished. As the object moves toward the mirror, the image also moves toward the mirror and increases in size. This can be determined using the mirror and magnification equations.

When light enters a Cassegrain telescope from a distant planet, first it bounces off a concave primary mirror and then off a secondary convex mirror. Play around with a Cassegrain telescope in the simulation below and see if you can until you get a clear view of Jupiter in the eyepiece:

Summary

- For convex mirrors, the reflecting surface is on the outside of the sphere, placing the center of curvature and the focal point on the opposite side of the mirror from the object.
- Light rays that approach the mirror aimed at the focal point will be reflected parallel to the principal axis.
- Light rays that come to the mirror parallel to the principal axis reflect as if they came from the focal point.
- Since the focal point is on the opposite side of the mirror from the object, the focal length is assigned a negative number.
- Both the mirror equation and the magnification equation are true with convex mirrors.

Review

1. Find the image distance of an object placed 5.00 cm in front of a convex mirror whose focal length is 8.00 cm.
2. Find the image distance of an object placed 3.00 cm in front of a convex mirror whose focal length is 8.00 cm.
3. A 4.0 cm tall light bulb is placed a distance 35.5 cm from a convex mirror with a focal length of -12.2 cm.
    1. Find the image distance.
    2. Find the image size.

Explore More

Use this resource to answer the questions that follow.

1. Images in a convex mirror are always ____________ and always ____________.
2. The image in a convex mirror will always be on the ____________ (same or opposite) side of the mirror from the object.

Additional Resources

Study Guide: Geometric Optics Study Guide

Real World Application: Closer than They Appear

Videos:

- **14.6: Convex Mirrors** by CK-12 Foundation is licensed CK-12. Original source: https://flexbooks.ck12.org/cbook/ck-12-physics-flexbook-2.0.

16.2.7: Double Convex Lenses

Refracting telescope at the Lick Observatory

Figure 14.7.1

Refracting telescopes, such as the one shown here, use lenses to focus the image. The telescope in this picture is one of the largest refracting telescopes in the world, and can be found in the Lick Observatory in San Jose, California.

Images in Double Convex Lenses

Lenses are made of transparent material such as glass or plastic with an index of refraction greater than that of air. At least one of the faces is a part of a sphere; a **convex lens** is thicker at the center than the edges, and a **concave lens** is thicker at the edges than the center. Convex lenses are called converging lenses, because they refract parallel light rays so that they meet. They are one of the most useful and important parts of all optical devices, and are found in eyeglasses, telescopes, microscopes, magnifying glasses, cameras and many other objects.

Both the mirror equation and the magnification equation also apply to lenses. However, when dealing with lenses, the mirror equation is renamed the Lens Equation.

Image formed by a convex lens

Figure 14.7.2

Double convex lenses have focal points on both sides of the lens, but it is also necessary to use points at twice the focal length to locate objects and images. Therefore, along the principal axis, there are points identified as *F* and as *2F* on both sides of the lens.

As with mirrors, we only need to trace two rays in order to locate the image for lenses. Both rays change direction while inside the lens, and their convergence point on the opposite side of the lens is the image location. As can be seen in the figure above, Ray 1 approaches the lens parallel to the principal axis and is refracted through the focal point on the other side. Ray 2 travels through the focal point and is then refracted parallel to the principal axis. The yellow arrow on the right of the lens is the inverted image.

The diagram above shows the situation when the object is outside *2F*. In this situation, the image will be between *F* and *2F* on the other side and will be inverted, diminished, and real. A real image can be projected on a screen. That is, if you placed a sheet of paper at the image position, the image would actually appear on the paper.

If the object is placed between *2F* and *F*, the image will appear beyond *2F* on the other side. The image will be real, inverted, and enlarged. You can do a ray tracing like the one shown to demonstrate this is true.

If the object is placed inside *F* (between *F* and the lens), the image will be on the same side of the lens as the object and it will be virtual, upright, and enlarged.

In the sketch below, the object is red and has been placed inside *F*. The ray that approaches the mirror parallel to the principal axis is dotted yellow. It refracts through the focal point, also shown in dotted yellow. The ray that approaches the mirror through the focal point is dotted blue and refracts parallel to the principal axis, also shown in dotted blue. As you can see, the refracted rays diverge, so there will be no real image. If the eye is placed beyond the object around the *2F* shown in the sketch, the eye will see the rays as if they have traveled in a straight line. These imaginary rays will converge at the tip of the green arrow which is the image position.

Virtual image formed by a convex lens

Figure 14.7.3

Examples

Example 14.7.1

An object is 40.0 cm to the left of a convex lens of +8.00 cm focal length. Determine the image distance.

Solution

$(1/d_o)+(1/d_i)=(1/f)$ *plugging in values* $(1/40.0)+(1/x)=(1/8.00)$

Multiplying both sides by 40x yields $x+40=5x$ so $4x=40$ and $x=10.0$ cm.

Example 14.7.2

An object 1.00 cm high is 8.00 cm to the left of a convex lens of 6.00 cm focal length. Find the image location and image height.

Solution

Add text here.

$(1/d_o)+(1/d_i)=(1/f)$ *plugging in values* $(1/8.00)+(1/x)=(1/6.00)$

Multiplying both sides by 24x yields $3x+24=4x$ so $x=24.0$ cm.

Since the image distance is three times the object distance, the image height will be three times the object height or 3.00 cm.

Launch the Magnifying Glass simulation below to investigate how this double convex lens causes the image of a candle to appear larger, smaller, and upside down:

Summary

- Lenses are made of transparent materials, such as glass or plastic, with an index of refraction greater than that of air.
- One or both of the lens faces is part of a sphere and can be concave or convex.
- A lens is called a convex lens if it is thicker at the center than at the edges.
- Both the mirror equation and the magnification equation apply to lenses. Generally, when dealing with lenses, the mirror equation is renamed the Lens Equation.
- Double convex lenses have focal points on both sides of the lens; these and the points at twice the focal length are used to locate objects and images.
- When the object is outside *2F*, the image will be between *F* and *2F* on the other side and will be inverted, diminished, and real.
- If the object is placed between *2F* and *F*, the image will appear beyond *2F* on the other side. The image will be real, inverted, and enlarged.
- For convex lenses, when the object is placed inside *F*, the image will be on the same side of the lens as the object and it will be virtual, upright, and enlarged.

Review

1. An object is placed to the left of a 25 cm focal length convex lens so that its image is the same size as the object. Determine the object and image locations.
2. A lens is needed to create an inverted image twice as large as the object when the object is 5.00 cm in front of the lens. What focal length lens is needed? Hint: 1di+1do=1f
3. If you have a convex lens whose focal length is 10.0 cm, where would you place an object in order to produce an image that is virtual?
4. Describe how a convex lens could be used to make a magnifying lens.
5. Determine the image distance and image height for a 5.00 cm tall object placed 45.0 cm from a double convex lens with a focal length of 15.0 cm.
6. Determine the image distance and image height for a 5.00 cm tall object placed 30.0 cm from a double convex lens with a focal length of 15.0 cm.
7. Determine the image distance and image height for a 5.00 cm tall object placed 20.0 cm from a double convex lens with a focal length of 15.0 cm.
8. Determine the image distance and image height for a 5.00 cm tall object placed 10.0 cm from a double convex lens with a focal length of 15.0 cm.

Explore More

Use this resource to answer the questions that follow.

1. What four points are marked on the principal axis of a lens?
2. A ray of light from the object that passes through the focal point on the same side of the lens as the object will refract ___________.
3. Under what circumstances will a convex lens form a virtual image?

Additional Resources

Study Guide: Geometric Optics Study Guide

Interactive: Contact Lens

PLIX: Play, Learn, Interact, eXplore: Double Convex Lens

Video:

16.2.8: Double Concave Lenses

Light rays diverging from a concave lens

Figure 14.8.1

The three light rays traveling into the concave lens shown above travel away from each other. For this reason, concave lens are also called diverging lenses. As a result of this light divergence, concave lenses create only virtual images.

Images in Double Concave Lenses

Every **concave lens** causes all rays to diverge. Rays that approach the lens parallel to the principal axis refract as if they came from the focal point.

Light rays passing through a concave lens

Figure 14.8.2

As you can see in the figure above, the light rays hit the lens and refract away from each other. Since none of these rays will intersect, a real image cannot exist. Instead, all images created by a double concave lens are virtual images. Like in all ray diagrams, images can be found using two rays.

Image formed by a concave lens

Figure 14.8.3

The first ray, shown above, begins from the tip of the image and travels to the lens parallel to the principal axis. Within the lens, this ray is refracted away from the principal axis such that the virtual ray (shown as a dotted line) travels back to the focal point. The second ray also leaves from the tip of the object, and travels straight through the center of the lens. The image will be where these two rays intersect - one real and one virtual. Since one ray is a virtual ray, the image will always be virtual, as well as upright and diminished.

Like for convex lenses, the lens equation and magnification equations can be used to calculate image size and distance for double concave lenses. When using the lens equation with a concave lens, however, the focal length must be assigned a negative value.

In the Contact Lens simulation below, the Lens Slider allows you to choose between a converging (convex) contact lens and a diverging (concave) contact lens. Try to view different objects at different distances using these two lenses and determine what is the best shape for the contact lens:

Summary

- A concave lens causes all rays to diverge.
- Concave lenses create only virtual images. After the rays are refracted, they never converge and so there will be no real images.
- All concave lens images will be upright, virtual, and diminished, and can be found between the *F* and the lens.
- When using the lens equation with a concave lens, the focal length must be assigned a negative value ($(1/d_i)+(1/d_o)=(1/-f)$).

Review

1. An object is placed 15.0 cm in front of a concave lens with a focal length of 8.00 cm. Find the image distance. Hint: 1di+1do=1−f

2. An object is placed 3.00 cm in front of a concave lens with a focal length of 5.00 cm. Find the image distance. Hint: 1di+1do=1−f
3. What physical characteristic of a lens distinguishes a converging lens from a diverging lens?
4. An 2.00 cm tall object is placed 20.0 cm in front of a concave lens with a focal length of 5.00 cm. Find the image distance and the height of the image. Hint: 1di+1do=1−f and hiho=−dido

Explore More

Use this resource to answer the questions that follow.

1. Concave lenses are __________________ (thicker or thinner) in the middle.
2. Concave lenses are also called __________________ lenses.
3. All images from concave lenses are __________________ (real or virtual).

Additional Resources

Study Guide: Geometric Optics Study Guide

Real World Application: A View Through a Lens

Video:

16.3: End of Chapter Activity

End of Chapter Activity: Creating a Lesson Plan on Reflection and Refraction of Waves with AI and Bloom's Taxonomy

Now that you have explored the principles of wave behavior at boundaries, including reflection and refraction, it's time to put your knowledge into practice. Your task is to create a succinct lesson plan for 10th graders that introduces them to these complex concepts. To help you with this, you will use AI tools and incorporate Bloom's Taxonomy to ensure a comprehensive learning experience. This lesson plan will go towards your digital notebook, a portfolio filled with lesson plans, activities, and labs for future use.

Activity Prompt:

Objective: Use AI and Bloom's Taxonomy to develop a lesson plan that effectively teaches 10th graders about the fundamentals of reflection and refraction of waves, including the physics behind these phenomena and their practical applications.

Understanding the Concepts:

Knowledge (Remembering): Define key terms related to wave behavior, such as reflection, refraction, angle of incidence, angle of reflection, refractive index, specular reflection, and diffuse reflection.

Comprehension (Understanding): Explain these concepts in detail, using diagrams and real-world examples to illustrate them.

Planning the Lesson:

Application: Design engaging activities that allow students to observe and understand reflection and refraction. For example, use mirrors to demonstrate the law of reflection and lenses to explore refraction.

Analysis: Use AI tools to create visual aids or interactive simulations that illustrate the behavior of waves at boundaries. For instance, create a simulation showing how light waves bend when transitioning between different mediums.

Deepening Understanding:

Synthesis (Creating): Ask students to design their own experiments or projects that demonstrate principles of reflection and refraction. For example, they could create an experiment to measure the angle of refraction using a water tank and a laser pointer.

Evaluation: Have students discuss and reflect on their experiments and the principles they demonstrate. Encourage them to think critically about the applications of reflection and refraction in technology and natural phenomena.

Using AI in the Classroom:

Explore AI tools like educational apps or platforms that provide interactive content for teaching about wave behavior. Use these tools to create quizzes, flashcards, or interactive stories that reinforce the lesson's concepts.

Use AI to assess student understanding through formative assessments and provide instant feedback.

Deliverable:

Submit a detailed lesson plan that includes:

1. **A brief overview of the key concepts covered:** Outline the foundational concepts of reflection and refraction that will be taught.
2. **A description of the activities and experiments designed:** Detail the hands-on activities and experiments you will use to help students understand these concepts.
3. **Examples of AI tools used and how they enhance the learning experience:** Describe the AI tools you plan to incorporate, such as simulations or interactive quizzes, and explain how they will help students grasp complex concepts.
4. **An explanation of how Bloom's Taxonomy was applied in the lesson plan to ensure a well-rounded educational experience:** Illustrate how each level of Bloom's Taxonomy (Remembering, Understanding, Applying, Analyzing, Creating, and Evaluating) is addressed in your lesson plan.

Additionally, include a creative project component where students create a digital presentation or a video tutorial that explains a concept related to reflection or refraction, using AI tools to enhance their projects.

Example Lesson Plan:

Grade: 10th Grade
Topic: Reflection and Refraction of Waves
Duration: 1 Week

Overview:
Students will learn about the basics of wave behavior at boundaries, including reflection and refraction, through engaging activities and creative projects.

Day 1: Introduction to Reflection and Refraction

Objective: Define basic and advanced concepts related to reflection and refraction and provide examples.

- **Remembering:** Define key terms (reflection, refraction, angle of incidence, angle of reflection, refractive index).
- **Understanding:** Explain the concepts using examples from everyday life (e.g., mirrors, lenses).

Activity:
Watch a detailed video (created using AI tools) explaining what reflection and refraction are, how they occur, and the principles behind them.

Day 2: Exploring Reflection

Objective: Understand the law of reflection and its applications.

- **Applying:** Conduct an activity using mirrors to demonstrate the law of reflection.

Activity:
Students use mirrors and laser pointers to observe how light reflects off different surfaces. They measure the angles of incidence and reflection and confirm that they are equal. They also explore the differences between specular and diffuse reflection using various surfaces.

Day 3: Exploring Refraction

Objective: Understand the law of refraction (Snell's Law) and its applications.

- **Applying:** Conduct an activity using lenses or a water tank to demonstrate refraction.

Activity:
Students use a water tank and laser pointers to observe how light bends when it passes from air into water. They measure the angles of incidence and refraction and use Snell's Law to calculate the refractive index of water. They also explore how lenses bend light to form images.

Day 4: Creative Project – Designing Reflection and Refraction Experiments

Objective: Design and conduct experiments to demonstrate principles of reflection and refraction.

- **Creating:** Students design their own experiments to visualize reflection and refraction.

Activity:
In groups, students design experiments such as using a prism to split light into its component colors or creating a simple telescope using lenses. They document each step, explain the principles involved, and present their projects to the class.

Day 5: Reflection and Digital Presentation

Objective: Reflect on what they have learned and create a digital presentation about reflection and refraction.

- **Evaluating:** Discuss and reflect on the experiments and activities.
- **Creating:** Use AI tools to create a digital presentation or video tutorial explaining a concept related to reflection or refraction.

Activity:
Students create a digital presentation or a video tutorial using AI tools that explains what they have learned about reflection and refraction. They can include diagrams, photos of their experiments, and voice recordings. They present their projects to the class, using their digital presentations to enhance their explanations.

By incorporating these strategies and activities, educators can effectively teach 10th graders about reflection and refraction of waves, helping them understand and appreciate these fundamental concepts and their applications in the real world.

16.4: End of Chapter Key Terms

Definition: Reflection and Refraction Waves

- **Wave**: A disturbance that transfers energy through matter or space, characterized by its wavelength, frequency, and amplitude.
- **Reflection**: The bouncing back of a wave when it hits a surface that it cannot pass through.
- **Law of Reflection**: States that the angle of incidence (the angle at which a wave hits a surface) is equal to the angle of reflection (the angle at which the wave bounces off).
- **Angle of Incidence**: The angle between the incoming wave and the normal (a line perpendicular to the surface at the point of incidence).
- **Angle of Reflection**: The angle between the reflected wave and the normal.
- **Refraction**: The bending of a wave as it passes from one medium to another, caused by a change in its speed.
- **Index of Refraction**: A measure of how much a wave slows down in a given medium, defined as the ratio of the speed of light in a vacuum to its speed in the medium.
- **Snell's Law**: Describes the relationship between the angles of incidence and refraction, stating that the ratio of the sines of these angles is equal to the ratio of the indices of refraction of the two media.
- **Normal Line**: An imaginary line perpendicular to the surface where a wave is incident.
- **Critical Angle**: The angle of incidence above which total internal reflection occurs when a wave travels from a medium with a higher index of refraction to one with a lower index of refraction.
- **Total Internal Reflection**: The complete reflection of a wave back into its original medium when the angle of incidence exceeds the critical angle.
- **Dispersion**: The separation of a wave into its component wavelengths (colors), typically seen with light passing through a prism.
- **Wavefront**: A surface over which a wave has a constant phase, often used to visualize how waves propagate.
- **Plane Mirror**: A flat mirror that reflects light to form an upright, virtual image the same size as the object.
- **Concave Mirror**: A mirror with a surface that curves inward, capable of converging light rays to a focal point.
- **Convex Mirror**: A mirror with a surface that curves outward, causing light rays to diverge and form a smaller, virtual image.
- **Lens**: A transparent object that refracts light waves to converge or diverge them to form an image.
- **Converging Lens**: A lens that brings parallel light rays to a focus (also known as a convex lens).
- **Diverging Lens**: A lens that spreads out parallel light rays (also known as a concave lens).
- **Focal Point**: The point where converging light rays meet or appear to meet after passing through a lens or reflecting off a mirror.
- **Focal Length**: The distance between the lens or mirror and its focal point.
- **Real Image**: An image formed by converging light rays that can be projected onto a screen.
- **Virtual Image**: An image formed by diverging light rays that cannot be projected onto a screen and can only be seen by looking into the optical device (e.g., a mirror or lens).

CHAPTER OVERVIEW

17: Nuclear Physics

17.1: Introduction and Learning Objectives

Introduction to Nuclear Physics

- *This Chapter is authored and remixed by Claude Mona from San Diego Community College District*

In this chapter, we delve into the fascinating and complex world of nuclear physics and its applications to chemistry, exploring fundamental concepts that underpin much of modern science and technology. We begin with an overview of the timeline for the development of nuclear physics and note its relatively recent discoveries. Not all aspects of nuclear physics are settled, and the Standard Model of Physics leaves some gaps.

We then discuss some core ideas in nuclear physics and examine standard notation and units used in the discipline. This prepares us for discussing the way the nucleus is modeled, which provides a framework for understanding subsequent topics. Atomic theory describing the nature of atoms and their components—protons, neutrons, and electrons—has evolved significantly since its inception. Understanding atomic structure and the forces at play within the nucleus allows us to grasp how elements are formed and why certain isotopes are stable while others are radioactive.

Atomic decay, another central topic in nuclear physics, involves the transformation of an unstable nucleus into a more stable one through the emission of particles or radiation. This process, which includes alpha, beta, and gamma decay, has profound implications for both natural phenomena and technological applications. Fission, the splitting of a heavy nucleus into smaller nuclei, and fusion, the merging of light nuclei to form a heavier nucleus, are both central to our understanding of how atomic nuclei interact. These reactions are not only critical for the production of energy in nuclear power plants and stars but also offer insights into the forces that hold nuclei together.

The uses of nuclear energy span a wide range of applications, from generating electricity to powering space missions and advancing medical technology. By harnessing the energy released from nuclear reactions, humanity has developed powerful tools for progress, albeit with significant considerations regarding safety and environmental impact. Understanding the principles behind these applications helps us appreciate both the potential and the challenges of nuclear energy.

Finally, integrating these advanced scientific concepts into a K-12 educational environment requires creativity and thoughtful pedagogy. Simplifying the complexities of nuclear physics and chemistry for younger students can be achieved through engaging activities, hands-on experiments, and visual aids that illustrate abstract ideas in tangible ways. For example, demonstrating the basics of atomic structure with models, simulating fission and fusion with safe classroom experiments, and discussing the everyday uses of nuclear energy can make these topics accessible and fascinating. By incorporating real-world examples and interactive learning strategies, educators can inspire a new generation to appreciate and pursue the wonders of nuclear science.

Learning Objectives

1. Gain understanding of the development of nuclear physics.

- Recent discoveries
- Indirect evidence

2. Become familiar with standard notation, nomenclature and units used in nuclear physics.

- Nucleons
- Atomic Mass Units
- Mass-Energy conversion

3. Understand the current model of the nucleus.

- Structure
- Stability
- Droplet
- Rigid Sphere

4. Examine radioactive decay.

- Statistical Basis
- Decay Types
- Activity

5. Explore nuclear fission.

- Binding Energy
- BEN curve
- Water Drop Model

6. Understand nuclear fusion.

- BEN curve
- Nucleosynthesis
- Fusion as power supply

7. Investigate the beneficial and harmful biological aspects to nuclear physics.

- Nuclear Medicine
- Ionizing Radiation
- Radiation Dose

17.2: A Brief History of Nuclear Physics

Fundamental Particles

The idea that everything is made of tiny pieces of ‘stuff’ dates back to ancient Greece and India. In fact, the word atom comes from the ancient Greek word *atomos*, which means “uncuttable” or indivisible. The idea that there was some smallest piece of matter was supported by the work of John Dalton (1766-1844). He collected data on the weights of chemical compounds and found that they could be explained if the smallest piece of each element had a different weight. He decided to call these smallest pieces ‘atoms’. For about 100 years afterwards, everyone thought these were the smallest, most fundamental pieces of matter.

In 1897, a physicist named J.J. Thomson was able to show that atoms were *not* indivisible by discovering electrons and showing that the electrons weigh much less than atoms. He found that they have a negative charge, and the movement of these electrons was an electric current. Since atoms are usually neutral (uncharged), that meant that the electrons’ negative charge had to be balanced by some positive charge somewhere in the atom. The atom would be held together by the attractive Coulomb force between oppositely charged objects. Thomson thought that the positive and negative charges were evenly blended throughout the atom, but that model was not supported by experimental evidence.

The Atomic Model

A trio of physicists (Ernest Rutherford, Hans Geiger and Ernest Marsden) performed experiments between 1908 and 1913 that showed that the positive charges in the atom were packed into a tiny space at the center of the atom. This space was called the *nucleus* and the experimental evidence suggested that most of the atomic mass was located in the nucleus. Furthermore, the experiments showed that the nucleus was thousands of times smaller than the atom.

This meant that most of the atom was empty space and another physicist, Niels Bohr, developed a model of the atom that had the negative electrons orbiting around the positive center at certain fixed distances, much like the planets orbit the sun in our solar system. That line of thought eventually led to the development of quantum mechanics, a subject beyond the scope of this text.

Rutherford deduced that the hydrogen atom is the simplest atom, made of a single electron and a positive nucleus. He named the positive charge of the hydrogen nucleus the *proton*. The proton has the same amount of charge as the electron, but it has the opposite sign. Since protons and electrons have the same amount of charge, in order for an atom to be electrically neutral, the number of protons have to equal the number of electrons.

The Nuclear Model

Rutherford was also able to show that the mass of many elements was larger than the mass of the protons and the electrons combined. To explain the missing mass, he hypothesized a neutral particle of about the same mass as the proton, a *neutron*, that was also part of the nucleus. In 1932, James Chadwick’s experiments confirmed the existence of the neutron. For the lighter elements, the number of protons and neutrons are roughly the same, but for the heavier elements there were typically more neutrons than protons.

The nucleus was now thought to be a collection of protons and neutrons, giving it a positive charge and accounting for most of the mass of the atom. There was a problem. The same Coulomb force that caused the electrons and protons to be attracted also would make the protons repel each other. How could a collection of positively charged protons and uncharged neutrons be packed into the nucleus? The Coulomb force trying to push the protons apart would be immense at those tiny distances!

Physicists hypothesized that there must be an attractive force between the protons and neutrons, and that it had to be stronger than the Coulomb force to keep the nucleus from flying apart. This new force was called the *strong force* and it could only act at very small distances, to keep the nucleus intact. Now the nucleus is modeled by two different forces. The strong force that keeps the nucleus together and the Coulomb force that exists between the protons that tries to tear the nucleus apart.

Radioactivity, Quarks and the Standard Model

Some elements spontaneously transform into different elements through a process that is called *radioactive decay*. It seems that some nuclei are unstable. The strong force is just barely large enough to keep the nucleus together, so sometimes it falls apart. Trying to understand that process led to the discovery that neutrons and protons are not truly fundamental particles.

Although we once thought atoms were fundamental, we learned that an atom is composed of smaller objects: protons, neutrons and electrons. In a similar way, we thought that protons and neutrons are fundamental particles but we've learned that the nucleons are made up of even smaller objects, called *quarks.*

It currently appears at though quarks are truly fundamental particles. Trying to understand how quarks behave has led to the development of the *Standard Model of Particle Physics* as a way to understand the universe. This framework is well-supported by many experimental results but is known to be incomplete.

17.3: Fundamental Concepts

It has been said that a picture is worth a thousand words. Similarly, understanding the symbols and units used to discuss nuclear physics allows us to communicate a lot of information in a compact notation. We are continuing the discussion of atoms but we are focusing on the nucleus, so our terminology is very specific.

Terminology

As you learned earlier, a particular atom on the periodic table of the elements is typically specified by one or more letters and several numbers:

$${}^{A}_{Z}\mathrm{X}$$

Z is the *proton number,* A is the *atomic mass* and X is replaced by the letter(s) related to the name of the element. A neutral atom will also have Z electrons.

The value of Z determines the element and its placement on the periodic table: Z = 1 is hydrogen (H), while Z = 6 is carbon (C) and Z = 47 is silver (Ag).

Nuclear physics uses a very similar notation, but there are certain small but important differences. Nuclear physics generally doesn't involve the orbital electrons. That means the mass numbers used will be slightly different than the values on the periodic table. There is also a slightly different notation:

$${}^{A}_{Z}\mathrm{X}_{N}$$

Z is the *proton number*, A is the *nucleon number*, N is the *neutron number*. Although the neutron number is important, it is often not explicitly stated. Since the nucleon number refers to the total number of protons and neutrons that make up the nucleus: A = Z + N, only two of the three values are needed. It is possible for atoms to have the same proton number, but different nucleon numbers. These are called *isotopes* of the element.

✓ Example 17.3.1

Here are the symbolic representations of two silver isotopes.

$$\boxed{{}^{105}_{47}Ag}$$

$$\boxed{{}^{111}_{47}Ag}$$

Determine the neutron number for each.

Solution

Since A = Z + N, then N = A - Z: 105 - 47 = 58 and 111 - 47 = 64

Units and common physical constants

Because we will be dealing with very small objects, there is a system of units designed for this scale. After all, you wouldn't want to measure your height in fractions of a mile, or your weight in fractions of a ton. Similarly, the units of kilograms, meters and Joules are far too large for describing the nuclear world.

Mass

Nuclear masses are reported in terms of the *atomic mass unit*. This is abbreviated as *amu* or u. The amu is defined as 1/12 of the mass of a neutral Carbon 12 atom in its ground state. Since neutrons and protons have almost the same mass, we will say that a nucleon has a mass of approximately 1 u. This is incorrect, but it will give us a rough estimate of the true value.

- Proton mass: 1.00728 u
- Neutron mass: 1.00867 u
- Electron mass: 0.00054 u

In kilograms, $1\mathrm{u} = 1.6606 \times 10^{-27}\ \mathrm{kg}$.

Example 17.3.2

Convert the mass of the proton from atomic mass units to kilograms.

Solution

The proton mass is 1.00728 u.

1.00728 u $(1.6606 \times 10^{-27}\ \mathrm{kg/u})$ = 1.6727×10^{-27} kg

Energy

Nuclear energies are described in terms of the electron-Volt, abbreviated as eV. The electron-Volt is equal to the kinetic energy gained by an electron as it is accelerated through a potential difference of one Volt. 1 eV is equal to $1\mathrm{eV} = 1.6 \times 10^{-19}\ \mathrm{J}$.

Atomic energies are usually in the range of thousands of electron-Volts (10^3 eV, or kilo electron-Volts, keV), while nuclear energies are typically in the range of millions of electron-Volts (10^6 eV, or mega electron-Volts, MeV), about a thousand times larger.

Because Einstein was able to relate mass and energy by the equation $E = mc^2$, we will often express masses in terms of energy units. This may be confusing at first, but it helps make some of the calculations simpler. To convert between energy (in MeV) and mass (in u) the conversion factor is the square of the speed of light. In these units we have: $c^2 = 931.502\ \mathrm{MeV/u}$.

- Proton mass: 1.00728 u = 938.280 MeV/c^2
- Neutron mass: 1.00867 u = 938.573 MeV/c^2
- Electron mass: 0.00054 u = 0.511 MeV/c^2

Length

Typical atomic scales are measured in Angstroms. One Angstrom is 10^{-10} meters, but this is still too large for the nuclear scale. Typical lengths in nuclear physics are thousands of times smaller than the atom. Nuclear physics uses the femtometer (fm), also called the Fermi (F). 1 fm = 1 F = 10^{-15} meters.

Time

Events involving the nucleus can be almost instantaneous or they can span trillions of years (or more). Because of this wide range, many different time units are used.

Relevant constants

Physical constants that are regularly used in nuclear physics include:

- The speed of light in vacuum: c = 299, 792, 458 meters per second, often rounded to 3 x 10^8 m/s.
- The charge of the electron, and the proton: e = 1.602176487 x 10^{-19} Coulomb. Usually rounded to 1.602 x 10^{-19} C.
- The Planck constant: h = 6.62606896 × 10^{-34} Joule-seconds, or 6.626 × 10^{-34} J-s.
- Avogadro's Number: N_a = 6.022 x 10^{23} objects/mol.

Since nuclear physics overlaps with many different areas of science, there are a huge number of related variables. A good online reference is NIST, and this site also includes a tool to convert energy to different units.

17.4: Modeling the Nucleus

The Size of the Nucleus

When Rutherford was performing his experiments, he launched helium nuclei at a thin sheet of gold. The data he collected suggested that the nucleus is roughly spherical. Some of the helium nuclei he launched at the gold atoms bounced off and reversed direction, indicating that the nucleus acted as if it were rigid. It was also thousands of times smaller than the atom. Given the size of the nucleus, it would seem that the nucleons are very tightly packed inside of it. The view of the nucleus was of a small, very dense sphere positioned at the center of the atom, surrounded by orbiting electrons. We have since learned that this model is also not exactly correct, but for the moment it does a reasonably good job of letting us imagine what is happening.

By assuming that the total volume of the nucleus was the sum of the volumes of the nucleons, physicists were able to develop a relationship between the number of nucleons and the effective radius of the nucleus. The volume of spherical nucleus of radius R, is (4/3) π R^3. The volume of a single spherical nucleon with radius r_0 is (4/3) π r_0^3. Because there are A nucleons in a particular nucleus, then the volume of the nucleus should be A(4/3) π r_0^3 = (4/3) π R^3 and the *empirical radius relationship* may be found:

$$R = r_0 A^{1/3}$$

In this expression, r_0 is the approximate radius of a single nucleon, if we think of them them as rigid spheres. It has a value of about 1.2 fm.

This expression agrees well with experimental evidence, but it raises a question. If you pour a bunch of marbles into a jar, the volume they occupy is larger than the sum of the volumes of each marble. This is because there is some empty space in between each of the rigid marbles, where the edges meet. But if you add drops of water to a jar then the total volume of water is the sum of the volumes of each of the drops. Since the water drops are not rigid, they can combine. So, are nucleons more like marbles or are they more like water drops? The answer seems to be that it depends on how the nucleons are interacting. Sometimes they act like marbles and sometimes they act like water drops. We'll touch on this again later in the chapter.

✓ Example 17.4.1

Determine the approximate radius of a gold nucleus. The gold atom has a radius of 144 x 10^{-12} meters. How many times larger than the nucleus is the atom?

Solution

Gold has atomic number 197. Using the equation provided: $R = r_0 179^{1/3}$. The cube root of 179 is about 5.636.

This makes the radius of the gold nucleus R = 1.2 fm (5.636) = 6.763 fm = 6.763 x 10^{-15} meters.

The ratio of the radii: $\frac{144\times10^{-12}\text{ meters}}{6.736\times10^{-15}\text{ meters}}$ is about 21,292. The gold atom is more than twenty-one thousand times the size of the gold nucleus.

The Mass Defect

Imagine that you want to assemble a particular nucleus that will have nucleon number A. You will need Z protons and (A-Z) neutrons. Label the mass of a proton as m_p and the mass of a neutron as m_n. The amount of mass that you start with is (Z)m_p + (A-Z)m_n. The amount of mass that you end with is the mass of the nucleus, m_{nuc}.

You would think that these numbers would be the same. You might believe they should be the same. However, the data from numerous experiments shows that the mass of a nucleus (m_{nuc}) is *less than* the sum of the masses of the nucleons. The difference in masses is called the *mass defect* and can be found by:

$$\Delta m = [\, m_{nuc}] - [Zm_p + (A-Z)m_n]$$

This shows that a collection of loose nucleons has more mass than those same nucleons when they are in the nucleus. Using Einstein's mass-energy relationship ($E = mc^2$) then a difference in masses means that there is a difference in system energy when we compare a collection of loose neutrons and protons to an atomic nucleus. Specifically, combining all the nucleons into a nucleus causes the system to lose energy. That energy is emitted (radiated) away from the nucleus.

$$\Delta E = (\Delta m)c^2$$

A collection of nucleons that combine to form a nucleus has less energy that if those nucleons were free (unbound). The lower energy results in a more stable system.

Stable systems are more resistant to changes, and it is more difficult to change the behavior of the nucleus than to change the behavior of any single nucleon. There is an analagous process that occurs when water cools and forms into ice. The heat is radiated away, which lowers the energy of the system. The random motion of the water molecules changes and as the water changes to ice, they becomes arranged in a regular pattern. It is difficult to move a particular H_2O atom when it is part of block of ice, but it is very simple when it is in liquid form. While you should *not* think about a nucleus forming in the same way that water freezes, the overlapping idea is that systems can become more stable by releasing energy to the environment.

✓ Example 17.4.2

Tritium is an isotope of hydrogen made of one proton and two neutrons. Calculate the mass deficit of tritium. Use 3.016049 u as the nuclear mass of tritium. Answer in units of u.

Solution

$\Delta m = [\ m_{nuc}] - [(Z)m_p + (A\text{-}Z)m_n]$, Z = 1 for hydrogen, and N = 2. $\Delta m = [\ 3.016049\ \text{u}] - [(1\text{x}\ 1.00728\ \text{u}) + (2\ \text{x}\ 1.00867\ \text{u}\ )] = -8.571\ \text{x}\ 10^{-3}$ u.

We interpret the result to mean that the mass of the tritium nucleus is 8.571 x 10^{-3} u less than the masses of the neutrons and proton. We report the mass deficit as a positive number for this reason.

Binding Energy and Stability

Since the strong force causes the nucleons to stick together, this means that we have to do work to remove one of the nucleons from the nucleus. This is similar to the effect of the gravitational force on a rock. In order to lift an object away from the earth, we had to do work on the object and its energy increased. In order to remove an electron from an atom, energy had to be added to the system. For electrons, this was called the ionization energy. For the rock, it was the gravitational potential energy. In the case of nuclear physics, we refer to the nuclear *binding energy*.

$$E_{binding} = \Delta mc^2$$

This value represents how much work it would take to disassemble the nucleus. You would have to add energy to the system to pull the nucleons apart. Taking the total binding energy and dividing it by the total number of nucleons gives the *Binding Energy per Nucleon* (BEN). The larger the BEN, the more work is required to pull the nucleons apart. Smaller BEN values mean that it is easier to disassemble that nucleus, and that nucleus is less stable.

$$BEN = E_{binding}/A = \Delta mc^2/A$$

Because the mass deficit is negative, then the binding energy and BEN is also negative. The negative value represents the system losing energy as the nucleus formed. Even though it is a negative value, it is most often referenced as a positive value. This may seem confusing, but thinking about it as a debt may help. If you owe someone fifty dollars, you are in debt to them. The fifty dollars represents how much money you need to give them in order to get out of debt. In a somewhat similar way, the BEN reflects how much energy needs to be added to the system to 'pay off' the 'energy debt' of the nucleons and set them free.

✓ Example 17.4.3

Using the results of example two, calculate the BEN for tritium. Answer in units of MeV.

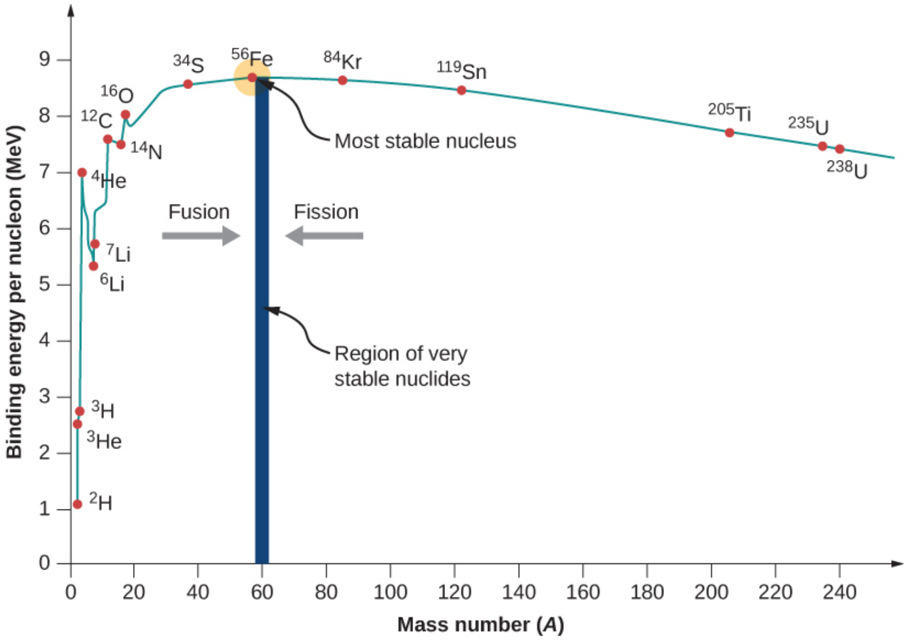
Solution

-8.571 x 10^{-3} u was the mass deficit. Divided by three, this gives a BEN of -2.857 x 10^{-3} u. Converting this to units of MeV involves a conversion factor to replace c^2. BEN = -(2.857 x 10^{-3} u) x (931.502 MeV/u) = -2.6613 MeV. This means 2.6613 mega electron-volts would have to be added to the nucleus in order to disassemble it.

Experimental results have produced a graph of the BEN, which is shown below. You can see that the BEN is small for Helium, which means it is relatively easy to break up a Helium nucleus. As the atomic mass increases towards Iron (Fe) it becomes more and more difficult to remove a nucleon from the nucleus which makes iron very stable. For nuclei with masses larger than iron, it becomes easier to cause them to break apart. You can see that the BEN for Carbon (C) is very similar to the BEN for Uranium (U). We can understand this in terms of the strong force between nucleons and the Coulomb force between protons.

For small proton numbers the strong force is dominant but as the proton numbers increase, the repulsive Coulomb force becomes more significant and the atoms tend to become less stable as a result since the Coulomb force is trying break the nucleus apart. For the less massive atoms, increasing the number of nucleons by nuclear fusion raises the BEN. For more massive atoms, decreasing the number of nucleons by nuclear fission will raise the BEN.

The BEN curve has applications for nuclear power generation, whether by fusion (smashing atoms together) or fission (splitting atoms apart).



BEN curve taken from OpenStax *University Physics vol. 3* and is licensed under CC-BY

17.5: Radioactivity

Nuclear decay or *radioactive decay* is the process by which atoms of a certain element spontaneously transform into atoms of a different element, while radiating (emitting) energy. This spontaneous decay process occurs for atoms with more than 82 protons. The concept of the BEN and the BEN curve help us to understand that when massive atoms split apart, the result is more stable atoms. Einstein's mass-energy relationship lets us calculate how much energy is released during this process. We can do all this without knowing why any of it happens; we're just collecting and analyzing data.

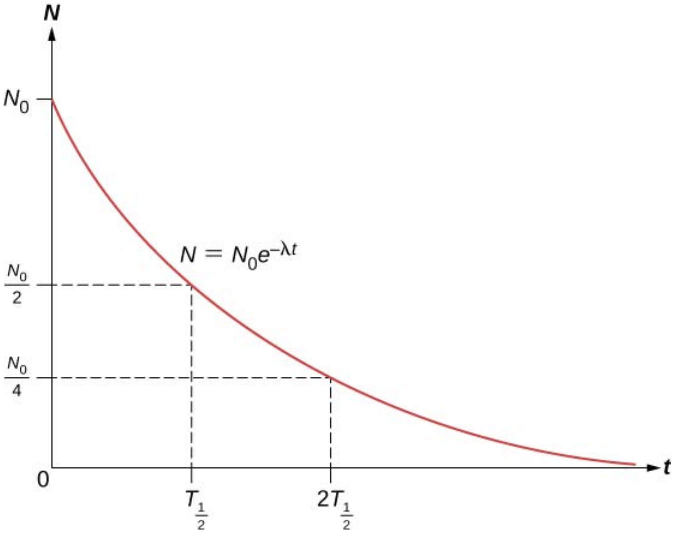
Experimental data shows that different elements decay at different rates and so we say that each radioactive element has a different *decay constant*. We use the Greek letter lambda (λ) to represent this value. By statistical analysis of the data it is possible to determine the *Radioactive Decay Law*:

$$N = N_0e^{-\lambda t}$$

This equation tells us that if you start with some number of atoms (N_0), then after some time (t), you will have N atoms left. The rest will have transformed into a different element. After some amount of time, you will half one-half of the original number of atoms that you started with. We call that amount of time the *half-life*. By setting $N = 1/2(N_0)$ and solving the decay equation you obtain a relationship between the decay constant and the half-life.

$$T_{1/2} = (0.693)/\lambda$$

Below is a graph of the number of atoms remaining in the sample as a function of time. You can see that the number of atoms decreases quickly, and that after each period of time equal to one half-life, the number of atoms remaining has decreased by a factor of two.



Radioactive Decay Curve taken from OpenStax *University Physics vol. 3* and is licensed under CC-BY

✓ Example 17.5.1

There are 10,000 atoms of a particular type. How many remain after five half-lives?

Solution

After one half-life, there are 5000 atoms remaining. After two, there are 2500. After three, 1250. Four half-lives leaves 625 and five gives 312.5 atoms remaining. Of course, you can't have a half-atom. An atom has either transformed, or it hasn't. So, after five half-lives, you would have either 312 or 313 atoms remaining.

How many atoms transform each second is called the *activity* of the element. High activities are paired with short half-lives and large decay constants. Atoms that are very stable have low activities, long half-lives and small decay constants. The activity may be determined from the decay law:

$$\text{Activity} = \lambda N = \lambda N_0 e^{-\lambda t}$$

Although the decay law is valid for a sample with a large number of atoms in it, there is no way to tell which atom will decay at any given time. At best, statistics can be used to identify the average amount of time an atom exists before decaying. We call this the *mean lifetime* of an atom, often just called the lifetime.

$$T_{\text{lifetime}} = 1/\lambda$$

✓ Example 17.5.2

The half-life of strontium-90 is 28.8 years. Determine the decay constant and the activity of a 1.50 gram sample of the material.

Solution

The decay constant is found by the half-life, which should be converted into seconds:

$$\lambda = \frac{0.693}{9.101 \times 10^8 \text{seconds}} = 7.61 \times 10^{-10}\ \text{s}^{-1}$$

To determine the activity, the number of atoms in the sample must be determined. Strontium-90 has a molar mass of 87.62 grams per mol. Therefore $1.50 \text{ grams} \times \frac{1 \text{mol}}{87.62 \text{ grams}} \times \frac{6.02 \times 10^{23} \text{atoms}}{1 \text{ mol}} = 1.0309 \times 10^{22} \text{ atoms}$

$$A_0 = 7.61 \times 10^{-10}\ \text{s}^{-1} \times 1.0309 \times 10^{22} \text{ atoms} = 7.845 \times 10^{12}\ \frac{\text{decays}}{\text{second}}$$

Radioactive Dating

Understanding the decay rates of different atoms is the basis for *radioactive dating*. Probably the most familiar example of this is carbon-14 dating. Carbon-14 is a naturally occurring isotope of carbon. That means it behaves like carbon-12, so it is part of every living organism. As long as the organism is living, it takes in carbon and a fraction of this is carbon-14. However, carbon-14 decays over time with a half-life of 5730 years. By comparing the current amount of carbon-14 in an organism to the expected amount, it is possible to determine how much time has passed since the organism died. Because carbon-14 has a fairly short half-life, this technique has a limit of about 50,000 years. Items older than this will have practically no carbon-14 left in them and so different elements must be used. Uranium-238 has a half-life of more than four billion years, allowing for determining age on a much longer time scale.

Types of Radioactive Decay

Regardless of the lifetime, the data has shown that there are only a small number of ways that atoms can decay. Initially, there is a 'parent nucleus' that decays into multiple 'daughter nuclei'. The process can be written as a type of equation that describes how the decay takes place. The parent is on the left, and an arrow points towards the daughters.

The first type of decay that was measured is called *alpha decay*. In this decay mode, the parent nucleus splits into a daughter and a helium nucleus. The Helium nucleus consists of two neutrons and two protons and is often called an *alpha particle*.

For alpha decay, the process can be written symbolically as:

$${}^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\alpha$$

In *beta decay* processes the parent transforms into a daughter and two other particles. There are actually two types of beta decays. In the first type, the parent becomes a daughter, an electron and a third particle called an electron antineutrino. In the second process, the parent becomes a daughter, a positron and an electron neutrino.

One of the stranger things that comes from the study of nuclear physics is the concept of the *antiparticle*. Antiparticles have the same mass as the 'normal' version, but if the particle has a net charge, the antiparticle will have the same amount of charge but the opposite sign of charge. An antielectron (also called a positron) is a particle that has the same amount of mass and charge as an electron, but the charge is positive. The electron antineutrino is the antiparticle to the electron neutrino. Neutrinos and antineutrinos are particles that have no charge and incredibly small masses.

Beta decay is the result of a nucleon in the parent nucleus transforming from one type to another. For example, a neutron can decay into a proton, electron and an electron antineutrino.

$$^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z+1}\mathrm{Y} + {}^{0}_{-1}\mathrm{e} + {}^{0}_{0}\bar{\nu}_e$$

You can see that the nucleon number is the same, and the increase in the positive charge is offset by by the appearance of the electron. It is also possible for a proton to transform into a neutron, a positron and an electron neutrino:

$$^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z-1}\mathrm{Y} + {}^{0}_{+1}\mathrm{e} + {}^{0}_{0}\nu_e$$

It is important to understand that the daughter particles are created during the decay process by mass-energy conversion. They do not exist before the decay. This is analogous to the behavior of atomic electrons. When an electron moves from an outer orbital to one closer to the nucleus, the electron moves to a lower energy state and so the atom has to lose energy. The way the atom loses energy is to emit a photon. The photon didn't exist before the electron transition, it was produced by the electron transition.

Similarly, when the neutron decays into a proton, an electron, and an antineutrino, only the neutron existed before the decay. The other three particles are created by the decay. Although perhaps counterintuitive, this is consistent with Einstein's mass-energy relationship.

There is one final way that a nucleus can lose energy. This process is completely analagous to the way an excited atom loses energy. The excited nucleus sheds energy by emitting a very high energy photon. This process is called *gamma decay*. In this process the nucleus does not transform into another element. The asterisk (*) is used to denote the excited nucleus.

$$^{A}_{Z}\mathrm{X}^{*} \rightarrow {}^{A}_{Z}\mathrm{X} + \gamma$$

Why exactly would a neutron suddenly transform into a proton, or vice versa? The question is obvious, and an obvious answer is that decay tends to increase the stability of the system. How that is understood requires that we enter the world of subatomic particle physics and quarks. Those topics are generally beyond the scope of this text, but the general idea is that neither protons nor neutrons are fundamental particles.

In much the same way that a nucleus can be treated as a single object or as a collection of nucleons, a nucleon can be thought of in the same way. On one level, a nucleon is a single object. On another scale, a nucleon is a collection of objects called quarks. Quarks have properties like mass and charge, but they also have properties that are less familiar. Quarks are also described in terms of charm and strangeness. Quark-quark interactions follow certain rules that physicists have been trying to decipher since the first evidence of quarks was detected in 1968.

Radioactive Decay Series

Radioactive parent elements will spontaneously split to raise the daughter's BEN and make the daughters more stable than the parent. It is quite likely that the daughters can also become more stable by decaying. The daughters from the first decay become the parent for the second decay, and so on. The decay series will end when the BEN of the final stage is near the maximum possible. Whether the decay happens by alpha, beta or gamma decay depends on many different factors that are beyond the scope of this text. An example involving Thorium decaying into Lead is shown below.

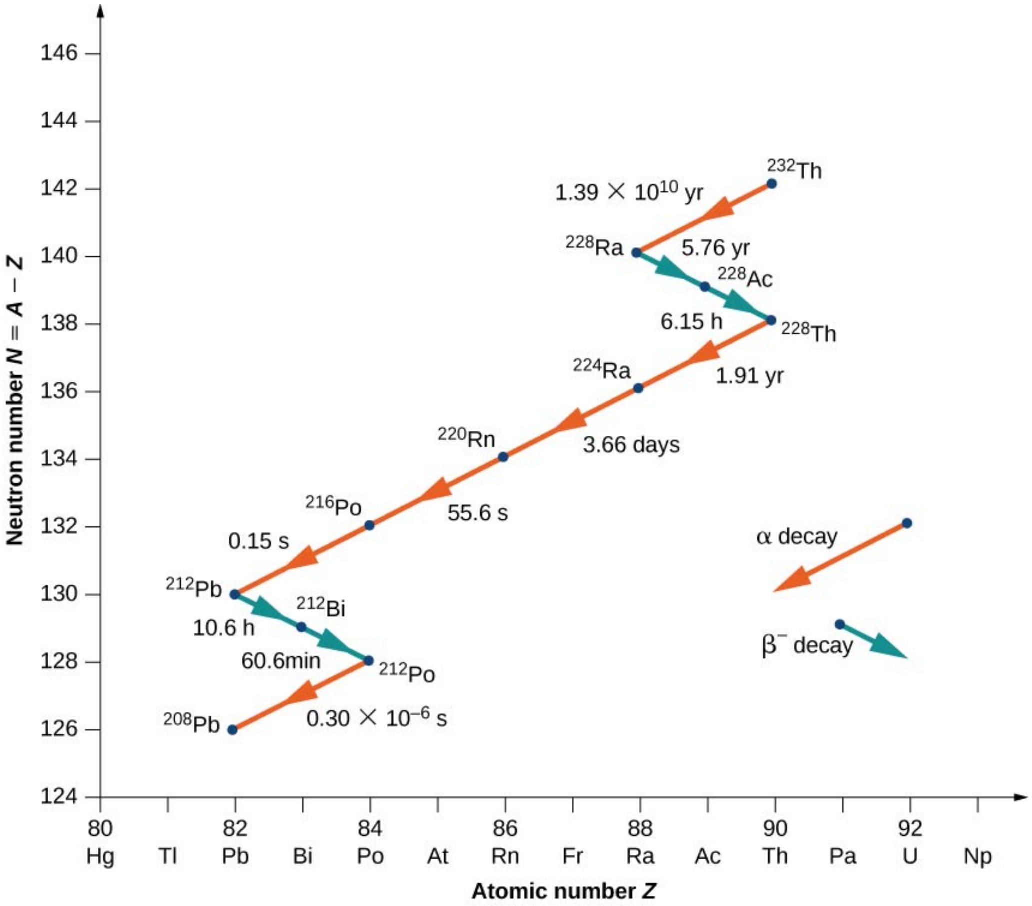
$$^{232}_{90}\mathrm{Th} \rightarrow {}^{228}_{88}\mathrm{Ra} + {}^{4}_{2}\alpha$$

$$^{228}_{88}\mathrm{Ra} \rightarrow {}^{228}_{89}\mathrm{Ac} + {}^{0}_{-1}\mathrm{e} + {}^{0}_{0}\bar{\nu}_e$$

$$^{228}_{89}\mathrm{Ac} \rightarrow {}^{228}_{90}\mathrm{Th} + {}^{0}_{-1}\mathrm{e} + {}^{0}_{0}\bar{\nu}_e$$

$$^{228}_{90}\mathrm{Th} \rightarrow {}^{224}_{86}\mathrm{Ra} + {}^{4}_{2}\alpha$$

After a series of alpha and beta decays the Thorium atoms become stable isotopes of Lead. The half-lives for each of the decays is shown on the graph. Some portions of the decay series occur over very long times and others take only fractions of a second, according to the decay constant for each element.



Thorium Decay Chain taken from OpenStax *University Physics vol. 3* and is licensed under CC-BY

The energy released to the environment by the radioactive decay of atoms includes the kinetic energy of the daughters and other particles created by the decay, and the energy of any photons emitted. The release of energy by radioactive decay helps geologists understand why the earth's core is hotter than expected. This energy transfer forms the basis for the nuclear power plant and also appears in medicine as both the cause of, and cure for, certain cancers.

17.6: Nuclear Fission

The process of splitting of the nucleus into less massive daughters is called *fission*. We know that some elements will fission spontaneously. We've learned that isotopes are created when the same number of protons are paired with different numbers of neutrons. In general, isotopes become less stable as the number of neutrons increase. Enrico Fermi started bombarding elements with neutrons in the early 1930s. He assumed this process would lead to less stable nuclei and increased activity, but had difficulty in identifying the daughter products. The idea that scientists could increase the chances of a fission event revolutionized physics and ushered in new science and applications.

Fermi's experiments were reproduced by several other physicists from that era, including Lise Meitner, Otto Hahn and Fritz Strassman. They used uranium as the target and they were able to verify that the neutron bombardment caused the uranium nucleus to split into less massive daughters. Each of the daughters had a larger BEN than the parent and were more stable as a result. When a neutron is added to a uranium nucleus there are multiple possible outcomes, some of which are shown here:

$$ {}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\ {}^{1}_{0}\mathrm{n} + Q $$

$$ {}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + 2\ {}^{1}_{0}\mathrm{n} + Q $$

$$ {}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{132}_{50}\mathrm{Sn} + {}^{101}_{42}\mathrm{Mo} + 3\ {}^{1}_{0}\mathrm{n} + Q $$

We label the energy associated with a fission event as Q. If Q is positive, the fission will release energy into the environment. The energy is in the form of the kinetic energy of the daughters and we know that collisions can transfer kinetic energy between objects. We can calculate the value of Q by examining the mass deficit.

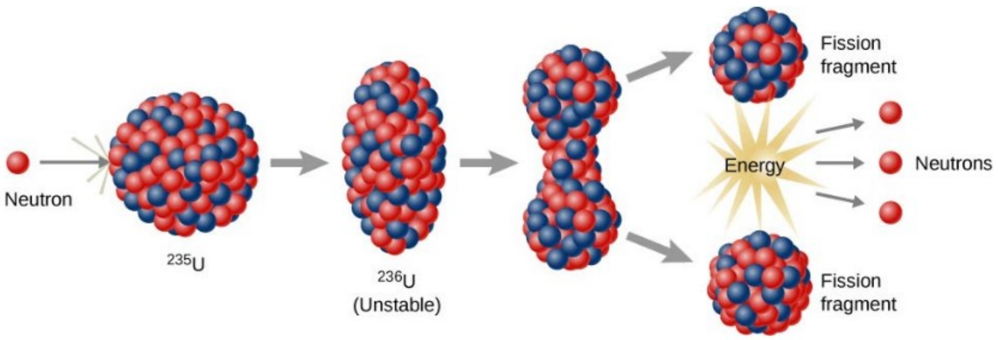
✓ Example 17.6.1

Calculate Q, the total energy released in the following reaction: ${}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + 2\ {}^{1}_{0}\mathrm{n} + Q$. Answer in units of MeV. You will need to look up the masses of products and reactants to three decimal places.

Solution

The Q value is calculated by $Q = \Delta mc^2 = ((1.008\mathrm{u} + 235.043\mathrm{u}) - (140.883\mathrm{u} + 91.926\mathrm{u} + 3 \times 1.008\mathrm{u}))\ c^2 = 0.218\mathrm{u}\ (931.502 \frac{\mathrm{MeV}}{\mathrm{u}}) = 203.07\ \mathrm{MeV}$

Neils Bohr and John Wheeler used a *liquid drop model* to explain this process. In the water drop model, a nucleus can absorb a nucleon in much the same way that two drops of water can combine to form a larger drop. We saw hints of this behavior when we examined the model for the nuclear radius. Just as smaller drops can combine to form a larger one, a large drop can break into smaller droplets if it is disturbed. A neutron fired into the nucleus can cause it to vibrate. If the vibration is violent enough the nucleus will divide into smaller nuclei and emits two or three neutrons.



Liquid drop model of fission taken from OpenStax *University Physics vol. 3* and is licensed under CC-BY

This process is the basis for how a nuclear power plant and a nuclear weapon works. When the first nucleus fissions and emits multiple neutrons, each of those neutrons can cause a fission event in another nearby uranium atom. One fission becomes three, and three become nine and then twenty seven and so on. This cascade of fission events release large amounts of energy into the environment. If that energy release is controlled, then the heat can be used to boil water and turn a steam turbine to produce electricity. If the energy release is uncontrolled then this becomes a nuclear weapon.

As with most energy production processes, whether they involve burning fossil fuels or splitting atoms, the combustion byproducts are hazardous. Coal, oil and gas-burning power plants contribute to ground, air, and water pollution and all the ills associated with that pollution. Fission reactors produce radioactive waste, composed of elements that continue to decay and radiate energy into the environment. Radioactive waste has the potential to damage the environment and pose a risk to human health. One of the great challenges facing humanity is the disposal of the waste produced by combustion processes of all sorts.

17.7: Nuclear Fusion

In the discussion of nuclear binding energy, we noted that heavy elements can become more stable by splitting into smaller nuclei and releasing energy as they do so. This formed the basis for the discussion of nuclear fission. At the less massive end of the BEN curve, we see that nuclei become more stable as they increase in nucleon number. The process of combining lighter nuclei is called *nuclear fusion*. Since the BEN increases as smaller nuclei are fused together, energy is released in the process. The energy that is released can come in the form of kinetic energy, or the energy can be radiated away as a photon. The fusion process can only occur at high temperatures because the positively charged nuclei repel each other via the Coulomb force. Only atoms moving at very large speeds can come close enough for the strong force to overcome the Coulomb repulsion and cause the nuclei to merge into a single nucleus. We imagine this process to be similar to the behavior of two droplets of liquid that combine into a single drop. The edges of the droplets have to touch before surface tension can merge them.

The Solar Spectrum

Investigation of the spectrum of light emitted by the sun shows an abundance of hydrogen and helium. How this relates to the sun's energy output was not proposed until 1938, when Hans Bethe showed that solar energy is the result of hydrogen fusion in the sun's interior. This fusion process is known as the proton-proton chain and is shown by the following reactions:

$$^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{2}_{1}\mathrm{H} + {}^{0}_{+1}\mathrm{e} + {}^{0}_{0}\nu_e + Q$$

$$^{1}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + \gamma + Q$$

$$^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} + Q$$

✓ Example 17.7.1

Calculate the Q value of the final part of the proton-proton chain. Use 3.016 u for the mass of ${}^{3}_{2}\mathrm{He}$, 4.003 u for ${}^{4}_{2}\mathrm{He}$ and 1.008 u for the mass of hydrogen. Answer in units of MeV.

Solution

Again, the relevant equation is the mass-deficit equation. 2(3.016 u) - (4.003 u + 2(1.008 u)) = 1.3 x 10^{-2} u. Using the conversion factor between u and MeV gives a final value of 12.109 MeV released per fusion.

The fusion of hydrogen forms an important part of the life cycle of a star. The energy that is released in the sun's interior leads to a pressure expansion that counteracts the gravitational forces acting to compress the sun. As long as the sun has hydrogen to fuse, it can remain in a stable state. The current model has stars fusing hydrogen into helium, and once the hydrogen is most fused the star begins to fuse helium nuclei to produce heavier elements. This process is called *nucleosynthesis*. Possible reactions include the following fusion chain:

$$^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{8}_{4}\mathrm{Be} + \gamma$$

$$^{8}_{4}\mathrm{Be} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{12}_{6}\mathrm{C} + \gamma$$

$$^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{16}_{8}\mathrm{O} + \gamma$$

It is believed that the carbon and oxygen nuclei that are produced in these processes migrate to the surface of the star by convection. As stars reach the end of their life cycle, they shed their outer layers and eject these heavier elements into the cosmos. Spectral analysis of nebulae show the presence of oxygen atoms, thus supporting the hypothesis. It is now believed that many of the massive nuclei found on Earth were fused in the cores of stars.

However, in order to fuse the heavier elements, larger and hotter stars are required. As the proton numbers increase it becomes increasingly difficult to overcome the Coulomb repulsion, so these atoms need to be moving at much higher speeds. If the stars are massive enough their core temperatures will be sufficiently high to produce more complex nuclei:

$$^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{23}_{11}\mathrm{Na} + {}^{1}_{1}\mathrm{H}$$

$$^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{24}_{12}\mathrm{Mg} + \gamma$$

$$^{12}_{6}\mathrm{C} + {}^{16}_{8}\mathrm{O} \rightarrow {}^{28}_{14}\mathrm{Si} + \gamma$$

Nucleosynthesis will continue until the core of the star is primarily iron and nickle. Iron is one of the most stable nuclei, so it will not be exothermic, but it will require energy in order to fuse into heavier atoms. This means that an iron core will not produce energy, so there is nothing to offset the gravitational pressure trying to collapse the star. Therefore, the star collapses into itself which causes the core temperature to spike, reaching temperatures of billions of degrees. The resulting pressure waves cause the star to explode and during this process elements heavier than iron can be formed. The resulting heavy elements are flung into space from the explosion.

Eventually, these atoms will be recycled into new stars and their accompanying planets, pulled together by the gravitational force. Images from the Hubble Space Telescope show evidence for this process in the Serpens cloud core located about 750 light years from earth, as new stars form from cooling dust and gases. These new stars will begin the nucleosynthesis process all over again. As Carl Sagan said: “The nitrogen in our DNA, the calcium in our teeth, the iron in our blood, the carbon in our apple pies were made in the interiors of collapsing stars. We are made of starstuff.”

Fusion on Earth

It is the process of nucleosynthesis that scientists are trying to utilize when they design and build fusion reactors. In particular, this two-step process is thought to be the most promising for practical use:

$${}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{1}\mathrm{H} + {}^{1}_{+1}\mathrm{H}$$

$${}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}$$

There are still a number of technological challenges left to solve. As mentioned earlier, trying to force two positively charged objects close enough together that the strong force can cause them to attract is extremely difficult. Extremely high temperatures are needed for the nuclei to fuse, on the order of ten million Kelvin. Designing materials that can withstand these temperatures and then finding a way to harness the resulting energy are significant challenges.

The reason fusion is so appealing is that it would be the first power supply that wouldn't produce harmful combustion by-products. The result of the fusion cycle is to produce helium and an extra neutron. The neutron can be used to create more of the hydrogen isotope needed for the fusion process, and helium is not biologically damaging. Access to power without pollution will be transformative.

17.8: Nuclear Physics and Biology

The discovery of nuclear physics revolutionized the way we interact with the world. Radioactivity helps us to determine the ages of ancient artifacts, discover and treat cancers, and provides power to our electrical grid. At the same time, radioactive decay can cause damage to our cells and cause cancers. The same process that generates power also produces hazardous waste that threatens the environment if not properly contained. As with all discoveries made by humanity, with the potential for great rewards comes the potential for immense harm.

Medical Benefits

Radioimmunoassay

The difference in behaviors of radioactive and nonradioactive substances underpins all applications of nuclear medicine. An example of this is the technique of *radioimmunoassay* (RIA) which was developed to detect and measure the precise amount of some substance. Antibodies are special proteins that can bind to other proteins, and they can be designed to bind to a specific protein of interest. In order to test for the presence of some substance in a blood or urine sample, an antibody that binds to that substance is manufactured. Some of the antibody is made radioactive by including an isotope of a particular atom. Initially only the radioactive antibody is present, so the sample emits the largest amount of radiation possible.

Next, the antibodies drawn from the person to be tested are added to the sample. If the same antibody is present in the patient's sample, it will preferentially bind to the receptor sites and dislodge the radioactive antibodies. The sample is 'washed' and the unbound radioactive antibodies are flushed from the sample. This means the sample becomes less radioactive. The amount by which the radioactivity decreases is directly proportional to the amount of antibodies in the patient's sample. Using a standard reference curve, the precise amount of substance in the patient sample can be determined.

RIA is used to detect the presence of many hormones and drugs. It is extremely precise and highly sensitive, allowing the detection of nanograms of antibodies. RIA is also used in the detection of hepatitis and early stage cancers.

Radiopharmaceuticals

Another application of radioactive substances in medicine is the use of *radiopharmaceuticals*. These are special drugs that contain an unstable isotope of one of the constituent atoms. The drug is administered and tends to concentrate in areas of the body showing signs of Inflammation. As the drug accumulates, detectors can pinpoint the location of radioactive activity. The act of adding an unstable isotope to a substance is called *radioactive tagging* and this allows doctors to track how drugs move around in a patient's body. This same technique allows for the detection of Alzheimer's disease, brain tumors and some bone cancers. Radioactive tagging can also monitor blood flow, the activity of the heart muscle and can identify a malfunctioning thyroid gland. It can be used to image the skeletal structure, the lungs and kidneys.

An idea radioactive tag will have a short half-life to minimize the radiation effect on the patient, and also have a unique 'fingerprint'. This fingerprint is most often a gamma ray with a well-defined energy, so it is clear where the isotope is concentrated. It is common to use an isotope of technetium $({}^{90}_{43}\mathrm{Tc})$ because it has a six-hour half life and produces an easily identifiable gamma ray when it decays.

Radiation Imaging

You 've probably had an x-ray at some point in your life. There, a high energy photon is used to image the interior of the body. X-ray images are two dimensional and face some technical challenges to achieving good image quality for diagnostic purposes. Positron Emission Tomography (PET) provides superior, three-dimensional imaging by taking advantage of matter-antimatter annihilation.

A radiopharmaceutical that decays by positron emission is administered to the patient, and the patient is placed inside a radiation detector. When the isotope decays and produces a positron, it will immediately encounter an electron and the two will annihilate each other. This process completely converts the masses of the electron and positron into identical gamma rays that travel in opposite directions. Detecting the gamma rays allows software to locate the point in the patient's body where the decay occurred.

This technique allows for resolution at a scale of one-half centimeter and is especially useful in identifying and studying areas of specific brain activity, such as speech, motor function and sight. It is also used to monitor how the brain utilizes oxygen and water. Regions of decreased brain metabolism are often linked to Alzheimer's disease, so this type of monitoring can identify areas of concern before symptoms manifest in the patient.

Radioactive Seeds

One of the by-products of radioactive decay is energy. If energy is added to a region, the temperature of the region increases. The greater the rate of energy transfer, the faster the temperature rises. High temperatures tend to kill cells, so radioactive seeds are used to treat certain types of cancers, such as prostate, breat and lung cancer. A radioactive seed is a small, radioactive cylinder that resembles a tiny nail. The seeds is inserted into the cancerous region of the body and decays in place, releasing energy the entire time. Seeds are also inserted into tumors, effectively 'cooking' the tumor from the inside. The use of radioactive seeds in the treatment of tumors and cancers is called *brachytherapy*.

Biological Risks

The same processes that aids in the treatment and cure of cancers can also be the source of biological harm, including cancer. The transfer of energy to the environment from radioactive decay processes is at the heart of all these effects. In particular, how the released energy is transferred to the environment is important. In a nuclear reactor, the decay energy is transferred to large volume of water, which transforms into steam and the steam is used to produce electrical energy. In brachytherapy the energy is released into the small volume of a tumor, thereby destroying it.

On the large scale, exposure to radioactive decay can lead to skin burns and the death of hair follicles with subsequent hair loss. Nausea and vomiting can result from radiation exposure as the cells lining the digestive system die. Blindness may also result if the energy release happens quickly, as in a nuclear explosion. On the smaller scale, if energy is transferred to a molecule there are different possible outcomes. If the energy transfer is small, the one or more of the atoms making up that molecule may become excited, but can then dissipate that energy without ill effect. If the energy transfer is larger, one or more of the atoms may become ionized. This can have dramatic effects on the molecule. It may change its shape or break apart, both of which change the way it functions. One benchmark for whether or not a particular radiation is dangerous to biological organisms at the cellular level is its ability to ionize atoms and change the way molecules function.

The helical DNA molecules that regulate the behavior of cells can become damaged by *ionizing radiation*. DNA normally can repair itself, and does so on a regular and ongoing basis. Of particular concern is when the ionizing radiation inhibits the DNA repair ability. In these cases, the damaged cell can become senescent - a sort of cellular coma where it ceases to do anything. Or, the cell can undergo programmed cell death - analagous to cellular suicide. Finally, the cell may begin dividing in an uncontrolled fashion. This third outcome can result in cancers and tumors.

Radiation Dose

The heart of determining *radiation dose* is measuring the amount of energy delivered to (absorbed by) each kilogram of body tissue. The *radiation dose unit*, also known as a *rad*, is defined to be:

$$1 \text{ rad} = \frac{1 \text{ Joule}}{100 \text{ kilogram}} = 0.01 \frac{\text{Joule}}{\text{kilogram}}$$

This is also referred to as the "whole body radiation dose".

As seen earlier, there are different types of radioactive decay products and each poses a different risk to cells even if the decay energy is the same. In general, alpha particles are extremely damaging, but only at very small distances. Beta decays are less damaging, but can deliver energy over larger distances. Gamma rays have the greatest range, but are also generally less damaging than alpha particles. This leads to the concept of *relative biological effectiveness* (RBE). RBE takes into account both the energy and the type of radiation to estimate the 'harm multiplier'. Typical RBE values are shown in the table:

A table of RBE values to determine relative health hazard.

Type of Radiation and Decay Energy	Body RBE (approximate values)
γ rays, β decay > 32 keV	1
β decay < 32 keV, neutrons < 20 keV	2
neutrons and protons from 1 to 10 MeV	10
α, heavy ions from particle accelerators	10 to 20

Table of RBE taken from OpenStax *University Physics vol. 3* and is licensed under CC-BY

Note that the mass and charge of the decay product both play a role in determining biological damage. Highly charged and massive particles, such as alphas are very damaging. Uncharged, yet massive particles are also damaging because they produce secondary ionization. The determination of RBE values are approximations determined from an understanding of radiation and its effects on living tissue. Data was collected

Rads, Rems and Sieverts

A more meaningful unit of radiation exposure takes into account the type of radiation, so the *roentgen equivalent man* (rem) is used:

$$\text{rem} = \text{rad}(\text{RBE})$$

Since the rad - and therefore the rem - are defined to be a tiny amount of energy per unit of mass, the *sievert* (Sv) is then defined:

$$1 \text{ Sv} = 100 \text{ rem}$$

A low dose is anything less than 10 rem, while a dose of 10 to 100 rem is called a moderate dose. Doses above 100 rem are considered high. A dose of about 4.5 Sv will kill 50% of the people thus exposed within about a month. A dose of over 20 Sv will prove fatal in a matter of hours as the entire central nervous system collapses.

Exposure to low levels of radiation is common and ongoing, no matter where you are on the planet. The earth emits radiation due to the presence of uranium, thorium and potassium isotopes in the soil. As a matter of fact, bananas are radioactive though in negligible amounts, due to the presence of naturally occurring potassium isotopes. Fertilizers also contain radioactive isotopes and these are taken up by plants, making them radioactive as well. The wood used for buildings and the granite used for countertops also contain unstable isotopes. We are also constantly being bombarded by gamma rays originating high in the atmosphere. Additionally, medical visits can increase radiation doses through nuclear medicine or even routine dental x-rays. There are no places on earth that are devoid of radiation, so it is a good thing our DNA has a robust repair mechanism.

The average person will absorb between 250 to 400 millirem per year, an incredibly small dose overall. Still, any ionizing radiation exposure has the possibility of damaging DNA. As a result, there is a roughly 20% chance that a person will develop some form of cancer during their lifetime, typically in their later years. There are regional differences in both frequency of cancers and health outcomes. For example, Australians are statistically more likely to develop cancer, but Mongolians are more likely to die from the disease. Diet and lifestyles can contribute significantly to the chances of developing cancer, and The World Cancer Research Fund International estimates that 40% of global cancer cases could be prevented by addressing diet, nutrition and physical activity.

17.9: End of Chapter Key Terms

Definition: Nuclear Physics

- Atom: Once thought to be the smallest piece of matter. Composed of protons, neutrons and electrons.
- Electron: A negatively charged fundamental particle that orbits the nucleus of the atom.
- Atomic Nucleus: The center of the atom, containing most of the atom's mass; composed of protons and neutrons.
- Proton: A positively charged particle located in the atom's nucleus.
- Neutron: An uncharged (neutral) particle with about the same mass as the proton, located in the nucleus.
- Nucleon(s): A (collective) term for protons and neutrons in general.
- Isotope: Atoms with the same number of protons, but different numbers of neutrons.
- Empirical Radius Relationship: A way to find the nuclear radius as a function of nucleon number.
- Coulomb Force: A force of attraction or repulsion between charged particles.
- Strong Force, AKA the Strong Nuclear Force: An attractive force between nucleons that counteracts the Coulomb force.
- Radioactive Decay: The spontaneous transformation of one element into another with the emission of radiation.
- Half-Life: The time required for half the atoms in a radioactive sample to decay.
- Decay Constant: A measure of how likely the atom is to undergo radioactive decay. Related to half-life.
- Mean (average) Lifetime: How long (on average) that a particular atom will exist before decaying.
- Mass Defect: The difference in mass between the initial state of the system and the final state of the system.
- Binding Energy: The energy required to disassemble a nucleus.
- Binding Energy per Nucleon: The energy required to remove a single nucleon from the nucleus. A measure of nuclear stability.
- Antiparticle: A particle that has the same mass as another but opposite charge and magnetic moment.
- Alpha Decay/Alpha Particle: A decay mode that emits an alpha particle ($^{2}_{2}\alpha$).
- Beta Decay/Beta Particle: A decay mode that emits a beta particle, which is an electron ($^{0}_{-1}\text{e}$) or a positron ($^{0}_{+1}\text{e}$).
- Gamma Decay: A decay mode that emits a high energy photon.
- Fission: The splitting of a heavy atomic nucleus into two lighter nuclei, releasing energy.
- Liquid Drop Model: Treating the nucleus like a drop of liquid to explain fission by neutron bombardment.
- Fusion: The process of combining two light atomic nuclei to form a heavier nucleus, releasing energy.
- Quarks: Fundamental particles that combine to form everything except electrons.
- Radioimmunoassay: Nuclear medicine technique used to screen for diseases and drugs.
- Radiopharmaceutical: Drug containing radioactive isotope. Used to track blood flow, drug absorption and perform imaging.
- Brachytherapy: The practice of inserting radioactive 'seeds' into the body in order to kill cancer cells.
- Ionizing Radiation: Radioactive decay products that change the structure of molecules in cells by ionizing atoms.
- Radiation Dose/rad: Definition of radiation exposure in terms of energy per kilogram of body mass.
- Relative Biological Effectiveness: A multiplier that affects radiation dose based on the type of radiation involved.
- Roentgen Equivalent Man: A way to estimate effective biological dose; the product of rad and RBE.

17.10: End of Chapter Activity

End of Chapter Activity: Creating a Lesson Plan on Nuclear Physics and Chemistry of Astronomy with AI and Bloom's Taxonomy

Teaching the complex concepts of nuclear physics and the chemistry of astronomy to 6th graders can be challenging. Your task is to create a succinct lesson plan that introduces these topics in an accessible and engaging way. To help you with this, you will use AI tools and incorporate Bloom's Taxonomy to ensure a comprehensive learning experience. This lesson plan will go towards your digital notebook, a portfolio filled with lesson plans, activities, and labs for future use.

Activity Prompt:

Objective: Use AI and Bloom's Taxonomy to develop a lesson plan that effectively teaches 6th graders about the basics of nuclear physics and the chemistry of astronomy, including concepts such as nuclear fusion, the life cycle of stars, and the elements formed in stars.

Understanding the Concepts:

Knowledge (Remembering): Define key terms related to nuclear physics and astronomy, such as nuclear fusion, stars, elements, and the periodic table.

Comprehension (Understanding): Explain these concepts in simple, age-appropriate language, using relatable examples and visuals.

Planning the Lesson:

Application: Design an engaging activity that allows students to observe and understand the basic concepts of nuclear fusion and the life cycle of stars. For example, use a model to demonstrate how stars fuse hydrogen to form helium and other elements.

Analysis: Use AI tools to create visual aids or interactive simulations that illustrate the process of nuclear fusion in stars and the formation of elements. For instance, create a simulation showing how stars evolve and produce different elements over their lifetimes.

Deepening Understanding:

Synthesis (Creating): Ask students to create their own simple projects that demonstrate their understanding of nuclear fusion and star formation. For example, they could design a poster that explains the life cycle of a star or create a model of a star using craft materials.

Evaluation: Have students discuss and reflect on their projects and the principles they observed. Encourage them to think about how nuclear fusion and the elements formed in stars are important for life on Earth.

Using AI in the Classroom:

Explore AI tools like educational apps or platforms that provide interactive content for teaching about nuclear physics and astronomy. Use these tools to create quizzes, flashcards, or interactive stories that reinforce the lesson's concepts.

Use AI to assess student understanding through formative assessments and provide instant feedback.

Deliverable:

Submit a detailed lesson plan that includes:

1. **A brief overview of the key concepts covered:** Outline the foundational concepts of nuclear physics and astronomy that will be taught.
2. **A description of the activities and experiments designed:** Detail the hands-on activities and experiments you will use to help students understand these concepts.
3. **Examples of AI tools used and how they enhance the learning experience:** Describe the AI tools you plan to incorporate, such as simulations or interactive quizzes, and explain how they will help students grasp complex concepts.
4. **An explanation of how Bloom's Taxonomy was applied in the lesson plan to ensure a well-rounded educational experience:** Illustrate how each level of Bloom's Taxonomy (Remembering, Understanding, Applying, Analyzing, Creating, and Evaluating) is addressed in your lesson plan.

Additionally, include a creative project component where students create a digital presentation or a simple stop-motion video that explains a concept related to nuclear physics and astronomy, using AI tools to enhance their projects.

Example Lesson Plan:

Grade: 6th Grade
Topic: Nuclear Physics and Chemistry of Astronomy
Duration: 1 Week

Overview:
Students will learn about the basics of nuclear physics and the chemistry of astronomy, including nuclear fusion, the life cycle of stars, and the elements formed in stars, through engaging activities and creative projects.

Day 1: Introduction to Nuclear Physics and Astronomy

Objective: Define basic concepts related to nuclear physics and astronomy and provide examples.

- **Remembering:** Define key terms (nuclear fusion, stars, elements, periodic table).
- **Understanding:** Explain the concepts using examples from everyday life (e.g., the Sun as a star undergoing nuclear fusion).

Activity:
Watch a short, animated video (created using AI tools) explaining what nuclear fusion is, how it occurs in stars, and how elements are formed in stars.

Day 2: Exploring Nuclear Fusion

Objective: Understand the process of nuclear fusion in stars.

- **Applying:** Conduct an activity using a model to demonstrate nuclear fusion.

Activity:
Students use a simple model (e.g., using colored balls to represent protons) to visualize how hydrogen atoms fuse to form helium in stars. They learn about the energy released during this process.

Day 3: The Life Cycle of Stars

Objective: Understand the life cycle of stars and the formation of elements.

- **Applying:** Conduct an activity using a timeline or flowchart to explore the stages of a star's life cycle.

Activity:
Students create a timeline or flowchart that shows the life cycle of a star, from its formation in a nebula to its final stages as a white dwarf, neutron star, or black hole. They include information about the elements formed at each stage.

Day 4: Creative Project – Designing Star Models

Objective: Design and create models to demonstrate the life cycle of stars and nuclear fusion.

- **Creating:** Students design their own models to visualize star formation and nuclear fusion.

Activity:
In groups, students use craft materials to create models of different stages of a star's life cycle. They explain their models and how nuclear fusion occurs at each stage, highlighting the elements formed.

Day 5: Reflection and Digital Presentation

Objective: Reflect on what they have learned and create a digital presentation about nuclear physics and astronomy.

- **Evaluating:** Discuss and reflect on the projects and activities.
- **Creating:** Use AI tools to create a digital presentation or simple stop-motion video that explains a concept related to nuclear physics and astronomy.

Activity:
Students create a digital presentation or a simple stop-motion video using AI tools that explains what they have learned about nuclear physics and the life cycle of stars. They can include drawings, photos of their models, and voice recordings. They present their projects to the class, using their digital presentations to enhance their explanations.

By incorporating these strategies and activities, educators can effectively teach 6th graders about nuclear physics and the chemistry of astronomy, helping them understand and appreciate these fundamental concepts in a fun and engaging way.

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