2.2.2: Negative Numbers

Negative numbers are a fact of life, from winter temperatures to our bank accounts. Let's practice evaluating expressions involving negative numbers.

Absolute Value

The ***absolute value*** of a number is its distance from 0. You can think of it as the size of a number without identifying it as positive or negative. Numbers with the same absolute value but different signs, such as 3 and -3, are called ***opposites***. The absolute value of -3 is 3, and the absolute value of 3 is also 3.

We use a pair of straight vertical bars to indicate absolute value; for example, $|-3|=3$ and $|3|=3$.

Exercises 2.2.2.1

Evaluate each expression

1. $|-5|$

2. $|5|$

Answer

1. 5
2. 5

Adding Negative Numbers

To add two negative numbers, add their absolute values (i.e., ignore the negative signs) and make the final answer negative.

Exercises 2.2.2.1

Perform each addition.

3. $-8+(-7)$

4. $-13+(-9)$

Answer

3. -15
4. -22

To add a positive number and a negative number, we *subtract* the smaller absolute value from the larger. If the positive number has the larger absolute value, the final answer is positive. If the negative number has the larger absolute value, the final answer is negative.

Exercises 2.2.2.1

Perform each addition.

5. $7+(-3)$

6. $-7+3$

7. $14+(-23)$

8. $-14+23$

9. The temperature at noon on a chilly Monday was $-7°F$. By the next day at noon, the temperature had risen $25°F$. What was the temperature at noon on Tuesday?

Answer

5. 4
6. -4
7. -9
8. 9
9. $18°F$

If an expression consists of only additions, we can break the rules for order of operations and add the numbers in whatever order we choose.

Exercises 2.2.2.1

Evaluate each expression using any shortcuts that you notice.

10. $-10+4+(-4)+3+10$

11. $-291+73+(-9)+27$

Answer

10. 3
11. -200

Subtracting Negative Numbers

The image below shows part of a paystub in which an $\$ 18$ payment needed to be made, but the payroll folks wanted to track the payment in the deductions category. Of course, a positive number in the deductions will subtract money away from the paycheck. Here, though, a deduction of negative 18 dollars has the effect of *adding*18 dollars to the paycheck. Subtracting a negative amount is equivalent to adding a positive amount.

Deductions after Federal Tax	
Faculty Union Dues	$27.00
Stipend for Part Time Faculty	-$18.00
Workers Compensation Hourly Assessment	$0.13

To subtract two signed numbers, we **add** the first number to the **opposite** of the second number.

Exercises 2.2.2.1

Perform each subtraction.

12. $5-2$

13. $2-5$

14. $-2-5$

15. $-5-2$

16. $2-(-5)$

17. $5-(-2)$

18. $-2-(-5)$

19. $-5-(-2)$

20. One day in February, the temperature in Portland, Oregon is $43°$F, and the temperature in Portland, Maine is $-12°$F. What is the difference in temperature?

Answer

12. 3

13. -3

14. -7

15. -7

16. 7

17. 7

18. 3

19. -3

20. 55°F difference

Multiplying Negative Numbers

Suppose you spend 3 dollars on a coffee every day. We could represent spending 3 dollars as a negative number, -3 dollars. Over the course of a 5-day work week, you would spend 15 dollars, which we could represent as -15 dollars. This shows that $-3\cdot 5=-15$, or $5\cdot -3=-15$.

If two numbers with **opposite** signs are multiplied, the product is negative.

Exercise 2.2.2.1

Find each product.

21. $-4\cdot 3$

22. $5(-8)$

Answer

21. -12

22. -40

Going back to our coffee example, we saw that $5(-3) = -15$. Therefore, the *opposite* of $5(-3)$ must be positive 15. Because -5 is the opposite of 5, this implies that $-5(-3) = 15$.

If two numbers with the **same** sign are multiplied, the product is positive.

WARNING! These rules are different from the rules for addition; be careful not to mix them up.

? Exercise 2.2.2.1

Find each product.

23. $-2(-9)$

24. $-3(-7)$

Answer

23. 18

24. 21

Recall that an exponent represents a repeated multiplication. Let's see what happens when we raise a negative number to an exponent.

? Exercises 2.2.2.1

Evaluate each expression.

25. $(-2)^2$

26. $(-2)^3$

27. $(-2)^4$

28. $(-2)^5$

Answer

25. 4

26. -8

27. 16

28. -32

If a negative number is raised to an **odd** power, the result is negative.
If a negative number is raised to an **even** power, the result is positive.

Dividing Negative Numbers

Let's go back to the coffee example we saw earlier: $-3 \cdot 5 = -15$. We can rewrite this fact using division and see that $-15 \div 5 = -3$; a negative divided by a positive gives a negative result. Also, $-15 \div -3 = 5$; a negative divided by a negative gives a positive result. This means that the rules for division work exactly like the rules for multiplication.

If two numbers with **opposite** signs are divided, the quotient is negative.
If two numbers with the **same** sign are divided, the quotient is positive.

Exercises 2.2.2.1

Find each quotient.

29. $-42 \div 6$

30. $32 \div (-8)$

31. $-27 \div (-3)$

32. $0 \div 4$

33. $0 \div (-4)$

34. $4 \div 0$

Answer

29. -7

30. -4

31. 9

32. 0

33. 0

34. undefined

Go ahead and check those last three exercises with a calculator. Any surprises?

- 0 divided by another number is 0.
- A number divided by 0 is undefined, or not a real number.

Here's a quick explanation of why $4 \div 0$ can't be a real number. Suppose that there is a mystery number, which we'll call n, such that $4 \div 0 = n$. Then we can rewrite this division as a related multiplication, $n \cdot 0 = 4$. But because 0 times any number is 0, the left side of this equation is 0, and we get the result that $0 = 4$, which doesn't make sense. Therefore, there is no such number n, and $4 \div 0$ cannot be a real number.

Order of Operations with Negative Numbers

P: Work inside of **parentheses** or grouping symbols, following the order PEMDAS as necessary.

E: Evaluate **exponents**.

MD: Perform **multiplications** and **divisions** from left to right.

AS: Perform **additions** and **subtractions** from left to right.

Let's finish up this module with some order of operations practice.

Exercises 2.2.2.1

Evaluate each expression using the order of operations

35. \((2-5)^2\cdot2+1\

36. $2 - 5^2 \cdot (2+1)$

37. $[7(-2)+16] \div 2$

38. $7(-2)+16 \div 2$

39. $\dfrac{1-3^4}{2(5)}$

40. $\dfrac{(1-3)^4}{2} \cdot 5$

Answer

35. 19

36. -73

37. 1

38. -6

39. -8

40. 40
