9.2.4: Vector Addition

Adding Vectors in Two Dimensions

In the following image, vectors A and B represent the two displacements of a person who walked 90. m east and then 50. m north. We want to add these two vectors to get the vector sum of the two movements.

Graph of two vectors, representing two displacements of a person who walked 90 mi east and then 50 mi north

Figure 1.6.1

The graphical process for adding vectors in two dimensions is to place the tail of the second vector on the arrow head of the first vector as shown above.

The sum of the two vectors is the vector that begins at the origin of the first vector and goes to the ending of the second vector, as shown below.

Two vectors representing two displacements of a person who walked 90 mi east and then 50 mi north, with the third vector connecting the two, creating a triangle indicating the sum

Figure 1.6.2

If we are using totally graphic means of adding these vectors, the magnitude of the sum can be determined by measuring the length of the sum vector and comparing it to the original standard. We then use a compass to measure the angle of the summation vector.

If we are using calculation, we first determine the inverse tangent of 50 units divided by 90 units and get the angle of 29° north of east. The length of the sum vector can then be determined mathematically by the Pythagorean theorem, a^2+b^2=c^2. In this case, the length of the hypotenuse would be the square root of (8100 + 2500), or 103 units.

If three or four vectors are to be added by graphical means, we would continue to place each new vector head to toe with the vectors to be added until all the vectors were in the coordinate system. The resultant, or sum, vector would be the vector from the origin of the first vector to the arrowhead of the last vector; the magnitude and direction of this sum vector would then be measured.

Mathematical Methods of Vector Addition

We can add vectors mathematically using trig functions, the law of cosines, or the Pythagorean theorem.

If the vectors to be added are at right angles to each other, such as the example above, we would assign them to the sides of a right triangle and calculate the sum as the hypotenuse of the right triangle. We would also calculate the direction of the sum vector by using an inverse sin or some other trig function.

Suppose, however, that we wish to add two vectors that are not at right angles to each other. Let's consider the vectors in the following images.

Two vectors on a north-south, east-west grid, one originating between north and east, one originating between north and west

Figure 1.6.3

The two vectors we are to add are a force of 65 N at 30° north of east and a force of 35 N at 60° north of west.

We know that vectors in the same dimension can be added by regular arithmetic. Therefore, we can resolve each of these vectors into components that lay on the axes as pictured below. The **resolution of vectors** reduces each vector to a component on the north-south axis and a component on the east-west axis.

Sum of two vectors on a north-south, east-west grid, creating a triangle for each vector

Figure 1.6.4

After resolving each vector into two components, we can now mathematically determine the magnitude of the components. Once we have done that, we can add the components in the same direction arithmetically. This will give us two vectors that are perpendicular to each other and can be the legs of a right triangle.

The east-west component of the first vector is (65 N)(cos 30°) = (65 N)(0.866) = 56.3 N north

The north-south component of the first vector is (65 N)(sin 30°) = (65 N)(0.500) = 32.5 N north

The east-west component of the second vector is (35 N)(cos 60°) = (35 N)(0.500) = 17.5 N west

The north-south component of the second vector is (35 N)(sin 60°) = (35 N)(0.866) = 30.3 N north

The sum of the two east-west components is 56.3 N - 17.5 N = 38.8 N east

The sum of the two north-south components is 32.5 N + 30.3 N = 62.8 N north

We can now consider those two vectors to be the sides of a right triangle and find the length and direction of the hypotenuse using the Pythagorean Theorem and trig functions.

$c=38.82+62.82=74$ N

$\sin x=62.874$ so $x=\sin-10.84$ so $x=58\circ$

The direction of the sum vector is 74 N at 58° north of east.

Perpendicular vectors have no components in the other direction. For example, if a boat is floating down a river due south, and you are paddling the boat due east, the eastward vector has no component in the north-south direction and therefore, has no effect on the north-south motion. If the boat is floating down the river at 5 mph south and you paddle the boat eastward at 5 mph, the boat continues to float southward at 5 mph. The eastward motion has absolutely no effect on the southward motion. Perpendicular vectors have NO effect on each other.

Examples

A motorboat heads due east at 16 m/s across a river that flows due north at 9.0 m/s.

Example 1.6.1

What is the resultant velocity of the boat?

Sketch:

alt

Figure 1.6.5

Solution

Since the two motions are perpendicular to each other, they can be assigned to the legs of a right triangle and the hypotenuse (resultant) calculated.

$c=a2+b2=(16\text{ m/s})2+(9.0\text{ m/s})2=18\text{ m/s}$

$\sin\theta=9.018=0.500$ and therefore $\theta=30\circ$

The resultant is 18 m/s at 30° north of east.

Example 1.6.2

If the river is 135 m wide, how long does it take the boat to reach the other side?

Solution

The boat is traveling across the river at 16 m/s due to the motor. The current is perpendicular and therefore has no effect on the speed across the river. The time required for the trip can be determined by dividing the distance by the velocity.

$t=dv=135\text{ m}16\text{ m/s}=8.4\text{ s}$

Example 1.6.3

The boat is traveling across the river for 8.4 seconds and therefore, it is also traveling downstream for 8.4 seconds. We can determine the distance downstream the boat will travel by multiplying the speed downstream by the time of the trip.

Solution

$d_{\text{downstream}}=(v_{\text{downstream}})(t)=(9.0\text{ m/s})(8.4\text{ s})=76\text{ m}$

Use this PLIX Interactive to visualize how any vector can be broken down into separate x and y components:

Summary

- Vectors can be added mathematically using geometry and trigonometry.
- Vectors that are perpendicular to each other have no effect on each other.
- Vector addition can be accomplished by resolving the vectors to be added into components those vectors, and then completing the addition with the perpendicular components.

Review

1. A hiker walks 11 km due north from camp and then turns and walks 11 km due east.
    1. What is the total distance walked by the hiker?
    2. What is the displacement (on a straight line) of the hiker from the camp?
2. While flying due east at 33 m/s, an airplane is also being carried due north at 12 m/s by the wind. What is the plane's resultant velocity?
3. Two students push a heavy crate across the floor. John pushes with a force of 185 N due east and Joan pushes with a force of 165 N at 30° north of east. What is the resultant force on the crate?
4. An airplane flying due north at 90. km/h is being blown due west at 50. km/h. What is the resultant velocity of the plane?

Explore More

Use this resource to answer the questions that follow.

1. What are the steps the teacher undertakes in order to calculate the resultant vector in this problem?
2. How does she find the components of the individual vectors?
3. How does she use the individual vector's components to find the components of the resultant vector?
4. Once the components are determined, how does she find the overall resultant vector?

Additional Resources

Real World Application: Banked With No Friction

Video:

Video:

Video: