

2.6: Accuracy and Significant Figures



In the first few modules, we rarely concerned ourselves with rounding; we assumed that every number we were told was exact and we didn't have to worry about any measurement error. However, every measurement contains some error. A standard sheet of paper is 8.5 inches wide and 11 inches high, but it's possible that the actual measurements could be closer to 8.4999 and 11.0001 inches. Even if we measure something very carefully, with very sensitive instruments, we should assume that there could be some small measurement error.

Exact Values and Approximations

A number is an exact value if it is the result of counting or a definition.

A number is an approximation if it is the result of a measurement or of rounding.

? Exercises 2.6.1

Identify each number as an exact value or an approximation.

1. An inch is $\frac{1}{12}$ of a foot.
2. This board is 78 inches long.
3. There are 14 students in class.
4. A car's tachometer reads 3,000 rpm.
5. A right angle measures 90° .
6. The angle of elevation of a ramp is 4° .

Answer

1. exact value
2. approximation
3. exact value
4. approximation
5. exact value
6. approximation

Suppose a co-worker texts you that they will arrive in 20 minutes. It's hard to tell how precise this number is, because we often round off to the nearest 5 or 10 minutes. You might reasonably expect them to arrive anytime within the next 15 to 25 minutes. If your co-worker texts that they will arrive in 17 minutes, though, it is likely that their GPS told them that more precise number, and you could reasonably expect them to arrive within 16 to 18 minutes.

Accuracy and Significant Figures

Because measurements are inexact, we need to consider how accurate they are. This requires us to think about significant figures—often abbreviated “sig figs” in conversation—which are the digits in the measurement that we trust to be correct. The accuracy of a

number is equal to the number of significant figures.^[1] The following rules aren't particularly difficult to understand but they can take time to absorb and internalize, so we'll include lots of examples and exercises.



Figure 2.6.1: African-American women were vital to NASA's success in the 1960s, as shown in the movie Hidden Figures.

Significant Figures

1. All nonzero digits are significant.
Ex: 12,345 has five sig figs, and 123.45 has five sig figs.
2. All zeros between other nonzero digits are significant.
Ex: 10,045 has five sig figs, and 100.45 has five sig figs.
3. Any zeros to the right of a decimal number are significant.
Ex: 123 has three sig figs, but 123.00 has five sig figs.
4. Zeros on the left of a decimal number are NOT significant.
Ex: 0.123 has three sig figs, and 0.00123 has three sig figs.
5. Zeros on the right of a whole number are NOT significant unless they are marked with an overbar.
Ex: 12,300 has three sig figs, but 12,300̄ has five sig figs.

Another way to think about #4 and #5 above is that zeros that are merely showing the place value—where the decimal point belongs—are NOT significant.

? Exercises 2.6.1

Determine the accuracy (i.e., the number of significant figures) of each number.

7. 63,400
8. 63,040
9. 63,004
10. 0.085
11. 0.0805
12. 0.08050

Answer

7. three significant figures
8. four significant figures
9. five significant figures
10. two significant figures
11. three significant figures
12. four significant figures

As mentioned above, we use an **overbar** to indicate when a zero that looks insignificant is actually significant. For example, the precision^[2] of 7,400 is the hundreds place; if we rounded anything from 7,350 to 7,449 to the nearest hundred, we would write the result as 7,400. An overbar shows that the number is more precise than it appears. If we rounded anything from 7,395 to 7,404 to the nearest ten, the result would be 7,400, but it isn't clear anymore that the number was rounded to the tens place. Therefore, to show the level of precision, we write the result as 7,40 $\bar{0}$. If we rounded anything from 7,399.5 to 7,400.4 to the nearest one, the result would be again 7,400, and we again can't see how precise the rounded number really is. Therefore, to show that the number is precise to the ones place, we write the result as 7,40 $\bar{0}$.

? Exercises 2.6.1

Determine the accuracy (i.e., the number of significant figures) of each number.

13. 8,000

14. 8, $\bar{0}$ 00

15. 8,0 $\bar{0}$ 0

16. 8,00 $\bar{0}$

Answer

- 13. one significant figure
- 14. two significant figures
- 15. three significant figures
- 16. four significant figures

Two things to remember: we don't put an overbar over a nonzero digit, and we don't need an overbar for any zeros on the right of a decimal number because those are already understood to be significant.

Accuracy-Based Rounding

As we saw in a previous module about decimals, it is often necessary to round a number. We often round to a certain place value, such as the nearest hundredth, but there is another way to round. **Accuracy-based rounding** considers the number of significant figures rather than the place value.

Accuracy-based rounding:

1. Locate the **rounding digit** to which you are rounding by counting from the left until you have the correct number of significant figures.
2. Look at the **test digit** directly to the right of the rounding digit.
3. If the test digit is 5 or greater, increase the rounding digit by 1 and drop all digits to its right. If the test digit is less than 5, keep the rounding digit the same and drop all digits to its right.

? Exercises 2.6.1

Round each number so that it has the indicated number of significant figures.

17. 21,837 (two sig figs)

18. 21,837 (three sig figs)

19. 21,837 (four sig figs)

20. 4.2782 (two sig figs)

21. 4.2782 (three sig figs)

22. 4.2782 (four sig figs)

Answer

- 17. 22, 000
- 18. 21, 800
- 19. 21, 840
- 20. 4.3
- 21. 4.28
- 22. 4.278

When the rounding digit of a whole number is a 9 that gets rounded up to a 0, we must write an overbar above that 0.

Similarly, when the rounding digit of a decimal number is a 9 that gets rounded up to a 0, we must include the 0 in that decimal place.

? Exercises 2.6.1

Round each number so that it has the indicated number of significant figures. Be sure to include trailing zeros or an overbar if necessary.

- 23. 13, 997(two sig figs)
- 24. 13, 997(three sig figs)
- 25. 13, 997(four sig figs)
- 26. 2.5996(two sig figs)
- 27. 2.5996(three sig figs)
- 28. 2.5996(four sig figs)



Mt. Everest, Lohtse, and Nupse in the early morning

The mountain we know as Mt. Everest is called *Sagarmatha* in Nepal and *Chomolungma* in Tibet. On December 8, 2020, it was jointly announced by Nepal and China that the summit has an elevation of 29, 031.69ft, replacing the previously accepted elevation of 29, 029ft.^[3]

- 29. Round 29, 031.69ft to two sig figs.
- 30. Round 29, 031.69ft to three sig figs.
- 31. Round 29, 031.69ft to four sig figs.
- 32. Round 29, 031.69ft to five sig figs.
- 33. Round 29, 031.69ft to six sig figs.

Answer

- 23. 14, 000
- 24. 14, $\overline{000}$
- 25. 14, $\overline{000}$
- 26. 2.6

27. 2.60
28. 2.600
29. 29,000 ft
30. 29,000 ft
31. 29,030 ft
32. 29,032 ft
33. 29,031.7 ft

Accuracy when Multiplying and Dividing

Suppose you needed to square the number $3\frac{1}{3}$. You could rewrite $3\frac{1}{3}$ as the improper fraction $\frac{10}{3}$ and then figure out that $(\frac{10}{3})^2 = \frac{100}{9}$, which equals the repeating decimal 11.111...

Because most people prefer decimals to fractions, we might instead round $3\frac{1}{3}$ to 3.3 and find that $3.3^2 = 10.89$. However, this is not accurate because 11.111...rounded to the nearest hundredth should be 11.11. The answer 10.89 looks very accurate, but it is a false accuracy because there is round-off error involved. If we round our answer 10.89 to the nearest tenth, we would get 10.9, which is still not accurate because 11.111...rounded to the nearest tenth should be 11.1. If we round our answer 10.89 to the nearest whole number, we would get 11, which is accurate because 11.111...rounded to the nearest whole number is indeed 11. It turns out that **we should be focusing on the number of significant figures rather than the place value**; because 3.3 has only two sig figs, our answer must be rounded to two sig figs.

Suppose instead that we round $3\frac{1}{3}$ to 3.33 and find that $3.33^2 = 11.0889$. Again, this is not accurate because 11.111...rounded to the nearest ten-thousandth should be 11.1111. If we round 11.0889 to the nearest thousandth, we would get 11.089, which is still not accurate because 11.111...rounded to the nearest thousandth should be 11.111. If we round 11.0889 to the nearest hundredth, we would get 11.09, which is still not accurate because 11.111...rounded to the nearest hundredth should be 11.11. Only when we round to the nearest tenth do we get an accurate result: 11.0889 rounded to the nearest tenth is 11.1, which is accurate because 11.111...rounded to the nearest tenth is indeed 11.1. As above, we need to focus on the number of significant figures rather than the place value; because 3.33 has only three sig figs, our answer must be rounded to three sig figs.

When **multiplying or dividing** numbers, the answer must be rounded to the same number of significant figures as the **least** accurate of the original numbers.

Don't round off the original numbers; do the necessary calculations first, then round the answer as your last step.

? Exercises 2.6.1

Use a calculator to multiply or divide as indicated. Then round to the appropriate level of accuracy.

34. $8.75 \cdot 12.25$

35. $355.12 \cdot 1.8$

36. $77.3 \div 5.375$

37. $53.2 \div 4.5$

38. Suppose you are filling a 5-gallon can of gasoline. The gasoline costs \$2.579 per gallon, and you estimate that you will buy 5.0 gallons. How much should you expect to spend?

Answer

34. 107

35. 640

36. 14.4

37. 12

38. \$ 12.90

1. The terms "significant digits" and "significant figures" are used interchangeably. ↩
2. Precision is different from accuracy, as we'll learn in the next module, but it is mentioned here because it can be difficult to explain one without the other. ↩
3. https://www.washingtonpost.com/world/asia_pacific/mount-everest-height-nepal-china/2020/12/08/a7b3ad1e-389a-11eb-aad9-8959227280c4_story.html ↩

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