# 2.2.18: Area of Regular Polygons

**You may use a calculator throughout this module.**

The Pentagon building spans $28.7$ acres ($116,000 \text{ m}^2$), and includes an additional $5.1$ acres ($21,000 \text{ m}^2$) as a central courtyard.[1] A pentagon is an example of a regular polygon.

A regular polygon has all sides of equal length and all angles of equal measure. Because of this symmetry, a circle can be inscribed—drawn inside the polygon touching each side at one point—or circumscribed—drawn outside the polygon intersecting each vertex. We'll focus on the inscribed circle first.

Let's call the radius of the inscribed circle lowercase $r$; this is the distance from the center of the polygon perpendicular to one of the sides.[2]

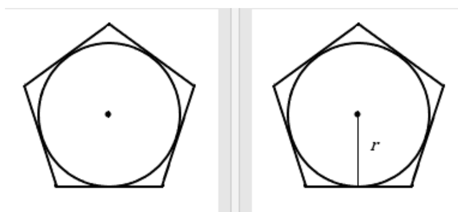



Figure $2.2.18.1$: (left) inscribed circle (right) inscribed circle with radius.

### Area of a Regular Polygon (with a radius drawn to the center of one side) [3]

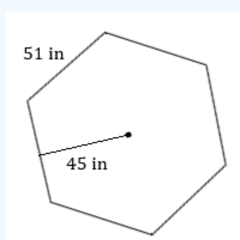
For a regular polygon with $n$ sides of length $s$, and inscribed (inner) radius $r$,

$$A = nsr \div 2$$

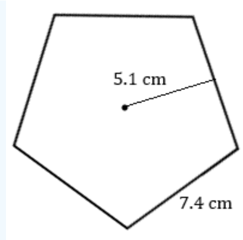
Note: This formula is derived from dividing the polygon into $n$ equally-sized triangles and combining the areas of those triangles.

### Exercises 2.2.18.1

**1.** Calculate the area of this regular hexagon.




**2.** Calculate the area of this regular pentagon.




**3.** A stop sign has a a height of 30 inches, and each edge measures 12.5 inches. Find the area of the sign.


**Answer**

1. $6,900 \text{ in}^2$
2. $94 \text{ cm}^2$
3. $750 \text{ in}^2$

Okay, but what if we know the distance from the center to one of the corners instead of the distance from the center to an edge? We'll need to imagine a circumscribed circle.

Let's call the radius of the circumscribed circle capital $R$; this is the distance from the center of the polygon to one of the vertices (corners).

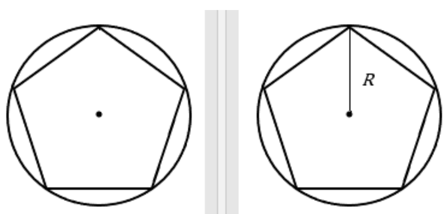



Figure $2.2.18.2$ (left) circumscribed circle and (right) circumscried circles with circle capital.

### Area of a Regular Polygon (with a radius drawn to a vertex) [4]

For a regular polygon with $n$ sides of length $s$, and circumscribed (outer) radius $R$,

$$A = 0.25ns\sqrt{4R^2 - s^2}$$

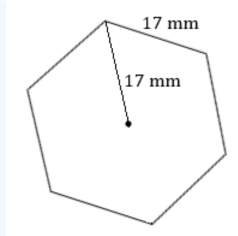
or

$$A = ns\sqrt{4R^2 - s^2} \div 4$$

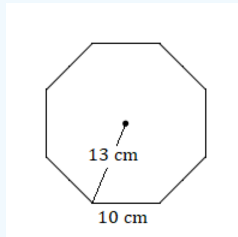
Note: This formula is also derived from dividing the polygon into $n$ equally-sized triangles and combining the areas of those triangles. This formula includes a square root because it involves the Pythagorean theorem.

**Exercises 2.2.18.1**

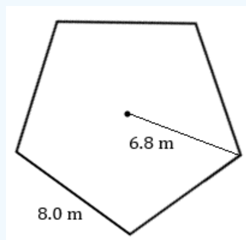
**4.** Calculate the area of this regular hexagon.




**5.** Calculate the area of this regular octagon.




**6.** Calculate the area of this regular pentagon.



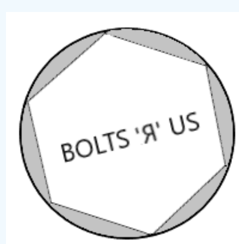

**Answer**

4. $750 \text{ mm}^2$
5. $480 \text{ cm}^2$
6. $110 \text{ m}^2$

As you know, a composite figure is a geometric figure which is formed by joining two or more basic geometric figures. Let's look at a composite figure formed by a circle and a regular polygon.

## ? Exercise 2.2.18.1

**7.** The hexagonal head of a bolt fits snugly into a circular cap with a circular hole with inside diameter 46 mm as shown in this diagram. Opposite sides of the bolt head are 40 mm apart. Find the total empty area in the hole around the edges of the bolt head.




**Answer**

$280 \text{ mm}^2$ (the area of the circle $\approx 1,660 \text{ mm}^2$ and the area of the hexagon is $1,380 \text{ mm}^2$)

1. https://en.Wikipedia.org/wiki/The_Pentagon ↩

2. The inner radius is more commonly called the *apothem* and labeled $a$, but we are trying to keep the jargon to a minimum in this textbook. ↵
3. This formula is more commonly written as one-half the apothem times the perimeter: $A = \frac{1}{2}ap$ ↵
4. Your author created this formula because every other version of it uses trigonometry, which we aren't covering in this textbook. ↵

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