13.2.2: Characteristics of Oscillations

Characteristics of Oscillations with Simple Harmonic Motion

The following characteristics are true for any oscillating system that can be described as simple harmonic.

- Acceleration is maximum when amplitude is maximum.
- Velocity is zero when acceleration and amplitude are maximum,
- Velocity is maximum when amplitude and displacement are zero.

For an object that starts with maximum amplitude, the relationship between position and period is as follows:

- t = 0, object is at maximum amplitude (x =A).
- t = $\frac{T}{4}$, object reaches equilibrium position (x = 0), headed towards x = -A.
- t = $\frac{T}{2}$, objects reaches maximum amplitude on other side of equilibrium position (x = -A).
- t = $\frac{3T}{4}$, object reaches equilibrium position (x = 0), headed towards x = A.
- t = T, object is back at maximum amplitude (x =A).

Mathematics of Simple Harmonic Motion

To describe an object oscillating back and forth between two endpoints, we need a function that also oscillates between two extremes. A sin or a cosine function has limits of +1 and -1, making it an ideal choice to describe these systems. Deriving the results means using differential equations, but the following relationships emerge for an object that starts at rest at maximum amplitude.

- Amplitude changes over time: $x = A \cos(2\pi \frac{t}{T}) = A \cos(2\pi f t)$.
- Velocity changes over time: $v = -(2\pi \frac{t}{T}) A \sin(2\pi \frac{t}{T}) = -(2\pi f) A \sin(2\pi f t)$.
- Acceleration changes over time: $x = [(2\pi \frac{t}{T})]^2 A \cos(2\pi \frac{t}{T}) = -[(2\pi f)]^2 A \cos(2\pi f t)$.

We saw in our study of energy that kinetic energy depends on the velocity of the object ($K = \frac{1}{2} mv^2$), but now the velocity of the object depends on the frequency of the object, so $K = \frac{1}{2} mv^2$ becomes $K = \frac{1}{2} m [-(2\pi f) A \sin(2\pi f t)]^2$.

The maximum kinetic energy of an oscillating mass happens when sin = 1 or -1, so $K_{max} = \frac{1}{2} m(4\pi^2 f^2)A^2 = 2m\pi^2 f^2 A^2$. The fact that the kinetic energy depends on the oscillation frequency will prove to be important in our study of electromagnetic energy.

Example 13.2.2.1

An oscillating mass on a spring has an amplitude of 2 meters and a period of 10 seconds. It is released from rest at t = 0 seconds. Calculate the following:

1. The position of the object when t = 6 seconds.
2. The velocity of the object when t = 9 seconds.
3. The acceleration of the object when t = 20 seconds.

Solution

Since period and frequency are inversely related, we can calculate f. f = 1 cycle / 10 seconds = 0.1 $\frac{\text{cycle}}{\text{second}}$

1. $x = 2$ meters $\cos(2\pi(0.1)(6)) = -1.62$ meters. This negative value means it is to the left of the equilibrium position.
2. $v = -(2\pi(0.1)(9))$ 2 meters $\sin(2\pi(0.1)(9)) = 3.32$ meters per second. The positive value means it is moving to the right.
3. $a = -[(2\pi(0.1)(20))]^2 A \cos(2\pi(0.1)(20)) = -157.9$ meters per second^2. The negative value means it is accelerating to the left.

From the solution to the differential equation that describes the system we can also find how the spring and the mass interact to produce the particular period of motion:

$$T = 2\pi\sqrt{M/k}$$

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From this we can see that some of our intuition was correct. Larger masses lead to longer periods, as expected. Stiffer springs lead to shorter periods, again as expected. Since there is no amplitude in the equation for the period, the period of a simple harmonic system doesn't depend on the amplitude.

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