8.4.1.1: Spring Potential Energy

Learning Objectives

- Explain the work done in deforming a spring.
- Describe the potential energy stored in a deformed spring.

Hooke's Law, $F=-kx$, describes force exerted by a spring being deformed. Here, F is the restoring force, x is the displacement from equilibrium or **deformation**, and k is a constant related to the difficulty in deforming the system. The minus sign indicates the restoring force is in the direction opposite to the displacement.

In order to produce a deformation, work must be done. That is, a force must be exerted through a distance, whether you pluck a guitar string or compress a car spring. If the only result is deformation, and no work goes into thermal, sound, or kinetic energy, then all the work is initially stored in the deformed object as some form of potential energy. The potential energy stored in a spring is $\mathrm{PE_{el}}=\frac{1}{2}kx^2$. Here, we generalize the idea to elastic potential energy for a deformation of any system that can be described by Hooke's law. Hence,

$$\mathrm{PE_{el}}=\frac{1}{2}kx^2,$$

where $\mathrm{PE_{el}}$ is the **elastic potential energy** stored in any deformed system that obeys Hooke's law and has a displacement x from equilibrium and a force constant k.

Connecting Ideas

Hooke's Law is the same equation that was used to develop the motion of waves. If we think about the oscillating motion of a compressed spring after the depressing force is released, this illustrates the reason for this connection.

It is possible to find the work done in deforming a system in order to find the energy stored. This work is performed by an applied force F_{app}. The applied force is exactly opposite to the restoring force (action-reaction), and so $F_{\mathrm{app}}=kx$. Figure 8.4.1.1.1 shows a graph of the applied force versus deformation x for a system that can be described by Hooke's law. Work done on the system is force multiplied by distance, which equals the area under the curve or $(1/2)kx^2$ (Method A in the figure). Another way to determine the work is to note that the force increases linearly from 0 to kx, so that the average force is $(1/2)kx$, the distance moved is x, and thus $W=F_{\mathrm{app}}d=[(1/2)kx](x)=(1/2)kx^2$ (Method B in the figure).

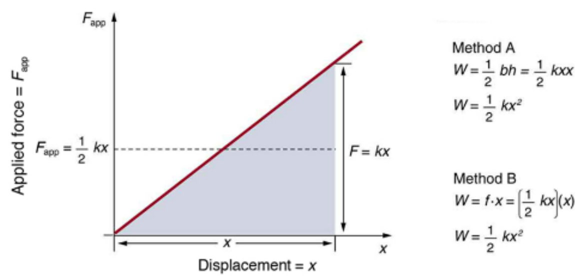

Figure 8.4.1.1.1: A graph of applied force versus distance for the deformation of a system that can be described by Hooke's law is displayed. The work done on the system equals the area under the graph or the area of the triangle, which is half its base multiplied by its height, or $W=(1/2)kx^2$.

8.4.1.1.1 Example : Calculating Stored Energy: A Tranquilizer Gun Spring

We can use a toy gun's spring mechanism to ask and answer two simple questions: (a) How much energy is stored in the spring of a tranquilizer gun that has a force constant of 50.0 N/m and is compressed 0.150 m? (b) If you neglect friction and the mass of the spring, at what speed will a 2.00-g projectile be ejected from the gun?

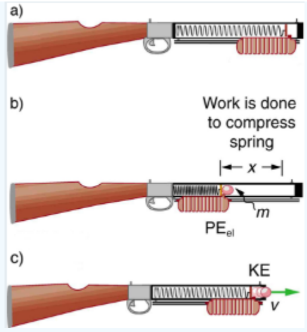

Figure 8.4.1.1.2 (a) In this image of the gun, the spring is uncompressed before being cocked. (b) The spring has been compressed a distance x, and the projectile is in place. (c) When released, the spring converts elastic potential energy $\mathrm{PE}_{\mathrm{el}}$ into kinetic energy.

Strategy for a

(a): The energy stored in the spring can be found directly from elastic potential energy equation, because k and x are given.

Solution for a

Entering the given values for k and x yields

$$\mathrm{PE}_{\mathrm{el}} = \frac{1}{2}kx^2 = \frac{1}{2}(50.0\ \mathrm{N/m})(0.150\ \mathrm{m})^2 = 0.563\ \mathrm{N \cdot m} = 0.563\ \mathrm{J}$$

Strategy for b

Because there is no friction, the potential energy is converted entirely into kinetic energy. The expression for kinetic energy can be solved for the projectile's speed.

Solution for b

1. Identify known quantities:

$$\mathrm{KE}_{\mathrm{f}} = \mathrm{PE}_{\mathrm{el}} \text{ or } 1/2mv^2 = (1/2)kx^2 = \mathrm{PE}_{\mathrm{el}} = 0.563\ \mathrm{J}$$

2. Solve for v:

$$v = \left[\frac{2\mathrm{PE}_{\mathrm{el}}}{m}\right]^{1/2} = \left[\frac{2(0.563\ \mathrm{J})}{0.002\ \mathrm{kg}}\right]^{1/2} = 23.7(\ \mathrm{J/kg})^{1/2}$$

3. Convert units: 23.7m/s

Discussion

(a) and (b): This projectile speed is impressive for a tranquilizer gun (more than 80 km/h). The numbers in this problem seem reasonable. The force needed to compress the spring is small enough for an adult to manage, and the energy imparted to the dart is small enough to limit the damage it might do. Yet, the speed of the dart is great enough for it to travel an acceptable distance.

8.4.1.1.1 Exercise

Envision holding the end of a ruler with one hand and deforming it with the other. When you let go, you can see the oscillations of the ruler. In what way could you modify this simple experiment to increase the rigidity of the system?

Answer

You could hold the ruler at its midpoint so that the part of the ruler that oscillates is half as long as in the original experiment.

8.4.1.1.2 Exercise

If you apply a deforming force on an object and let it come to equilibrium, what happened to the work you did on the system?

Answer

It was stored in the object as potential energy.

Section Summary

- Hooke's law describes force exerted by a spring being deformed,

$$F = -kx,$$

where F is the restoring force, x is the displacement from equilibrium or deformation, and k is the force constant of the system.
- Elastic potential energy $\mathrm{PE_{el}}$ stored in the deformation of a system that can be described by Hooke's law is given by

$$\mathrm{PE_{el}} = (1/2)kx^2.$$

Glossary

deformation
displacement from equilibrium

elastic potential energy
potential energy stored as a result of deformation of an elastic object, such as the stretching of a spring