2.2.9: Percents Part 2 and Error Analysis

You may use a calculator throughout this module.

Recall: The amount is the answer we get after finding the percent of the original number. The base is the original number, the number we find the percent of. We can call the percent the rate.

When we looked at percents in a previous module, we focused on finding the amount. In this module, we will learn how to find the percentage rate and the base.

$$\text{Amount} = \text{Rate} \cdot \text{Base}$$

$$A = R \cdot B$$

We can translate from words into algebra.

- "is" means equals
- "of" means multiply
- "what" means a variable

Solving Percent Problems: Finding the Rate

Suppose you earned 56 points on a 60-point quiz. To figure out your grade as a percent, you need to answer the question "56 is what percent of 60?" We can translate this sentence into the equation $56 = R \cdot 60$.

? Exercises 2.2.9.1

1. 56 is what percent of 60?

2. What percent of 120 is 45?

Answer

1. 93% or 93.3%
2. 37.5%

Be aware that this method gives us the answer in decimal form and we must move the decimal point to convert the answer to a percent.

Also, if the instructions don't explicitly tell you how to round your answer, use your best judgment: to the nearest whole percent or nearest tenth of a percent, to two or three significant figures, etc.

Solving Percent Problems: Finding the Base

Suppose you earn 2% cash rewards for the amount you charge on your credit card. If you want to earn $\$\ 50$ in cash rewards, how much do you need to charge on your card? To figure this out, you need to answer the question "50 is 2% of what number?" We can translate this into the equation $50 = 0.02 \cdot B$.

? Exercises 2.2.9.1

3. $ 50 is 2% of what number?

4. 5% of what number is 36?

Answer

3. $ 2,500

4. 720

Solving Percent Problems: Using Proportions

Recall that a percent is a ratio, a fraction out of 100. Instead of translating word for word as we have just been doing, we can set up a proportion with the percentage rate over 100. Because the base is the original amount, it corresponds to 100%.

$$\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100}$$

Let's try Exercises 1 through 4 again, using proportions.

? Exercises 2.2.9.1

5. 56 is what percent of 60?

6. What percent of 120 is 45?

7. $ 50 is 2% of what number?

8. 5% of what number is 36?

Answer

5. 93% or 93.3%

6. 37.5%

7. $ 2,500

8. 720

Now that we have looked at both methods, you are free to use whichever method you prefer: percent equations or proportions.

? Exercises 2.2.9.1

9. An 18% tip will be added to a dinner that cost $ 107.50. What is the amount of the tip?

10. The University of Oregon women's basketball team made 13 of the 29 three-points shots they attempted during a game against UNC. What percent of their three-point shots did the team make?

11. 45% of the people surveyed answered "yes" to a poll question. If 180 people answered "yes", how many people were surveyed altogether?

Answer

9. $ 19.35

10. 44.8% or 45%

11. 400 people were surveyed

Solving Percent Problems: Percent Increase

When a quantity changes, it is often useful to know by what percent it changed. If the price of a candy bar is increased by 50 cents, you might be annoyed because it's it's a relatively large percentage of the original price. If the price of a car is increased by 50 cents, though, you wouldn't care because it's such a small percentage of the original price.

To find the percent of increase:

1. Subtract the two numbers to find the amount of increase.
2. Using this result as the amount and the **original** number as the base, find the unknown percent.

Notice that we always use the ***original*** number for the base, the number that occurred earlier in time. In the case of a percent increase, this is the smaller of the two numbers.

? Exercises 2.2.9.1

12. The price of a candy bar increased from $ 0.89 to $ 1.39. By what percent did the price increase?

13. The population of Portland in 2010 was 583,793. The estimated population in 2019 was 654,741. Find the percent of increase in the population.[1]

Answer

12. 56.2% increase
13. 12.2% increase

Solving Percent Problems: Percent Decrease

Finding the percent decrease in a number is very similar.

To find the percent of decrease:

1. Subtract the two numbers to find the amount of decrease.
2. Using this result as the amount and the **original** number as the base, find the unknown percent.

Again, we always use the ***original*** number for the base, the number that occurred earlier in time. For a percent decrease, this is the larger of the two numbers.

? Exercises 2.2.9.1

14. During a sale, the price of a candy bar was reduced from $ 1.39 to $ 0.89. By what percent did the price decrease?

15. The number of students enrolled at Clackamas Community College decreased from 7,439 in Summer 2019 to 4,781 in Summer 2020. Find the percent of decrease in enrollment.

Answer

14. 36.0% decrease
15. 35.7% decrease

Relative Error

In an earlier module, we said that a measurement will always include some error, no matter how carefully we measure. It can be helpful to consider the size of the error relative to the size of what is being measured. As we saw in the examples above, a difference of 50 cents is important when we're pricing candy bars but insignificant when we're pricing cars. In the same way, an error of an eighth of an inch could be a deal-breaker when you're trying to fit a screen into a window frame, but an eighth of an inch is insignificant when you're measuring the length of your garage.

The ***expected outcome*** is what the number would be in a perfect world. If a window screen is supposed to be exactly 25 inches wide, we call this the expected outcome, and we treat it as though it has infinitely many significant digits. In theory, the expected

outcome is $25.000000\ldots$

To find the ***absolute error***, we subtract the measurement and the expected outcome. Because we always treat the expected outcome as though it has unlimited significant figures, the absolute error should have the same precision (place value) as the *measurement*, not the expected outcome.

To find the ***relative error***, we divide the absolute error by the expected outcome. We usually express the relative error as a percent. In fact, the procedure for finding the relative error is identical to the procedures for finding a percent increase or percent decrease!

To find the relative error:

1. Subtract the two numbers to find the absolute error.
2. Using the **absolute error** as the amount and the **expected outcome** as the base, find the unknown percent.

? Exercisew 2.2.9.1

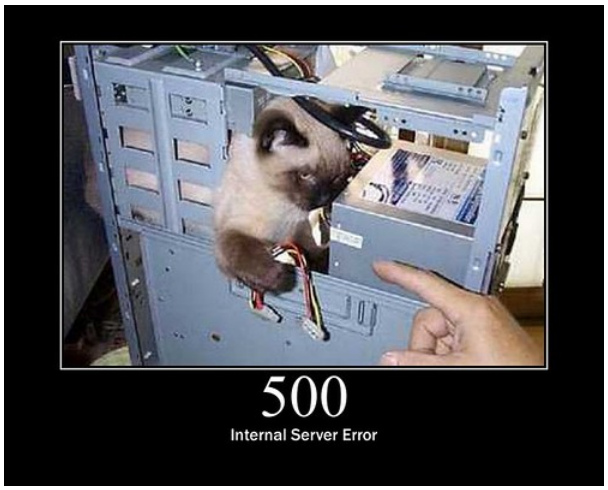
16. A window screen is measured to be $25\frac{3}{16}$ inches wide instead of the advertised 25 inches. Determine the relative error, rounded to the nearest tenth of a percent.

17. The contents of a box of cereal are supposed to weigh 10.8 ounces, but they are measured at 10.67 ounces. Determine the relative error, rounded to the nearest tenth of a percent.

Answer

16. $0.1875 \div 25 \approx 0.8\%$

17. $0.13 \div 10.8 \approx 1.2\%$



Tolerance

The ***tolerance*** is the maximum amount that a measurement is allowed to differ from the expected outcome. For example, the U.S. Mint needs its coins to have a consistent size and weight so that they will work in vending machines. A dime (10 cents) weighs 2.268 grams, with a tolerance of ± 0.091 grams.[2] This tells us that the minimum acceptable weight is $2.268 - 0.091 = 2.177$ grams, and the maximum acceptable weight is $2.268 + 0.091 = 2.359$ grams. A dime with a weight outside of the range $2.177 \leq \text{weight} \leq 2.359$ would be unacceptable.

Exercises 2.2.9.1

A U.S. nickel (5 cents) weighs 5.000 grams with a tolerance of ± 0.194 grams.

18. Determine the lowest acceptable weight and highest acceptable weight of a nickel.

19. Determine the relative error of a nickel that weighs 5.21 grams.

A U.S. quarter (25 cents) weighs 5.670 grams with a tolerance of ± 0.227 grams.

20. Determine the lowest acceptable weight and highest acceptable weight of a quarter.

21. Determine the relative error of a quarter that weighs 5.43 grams.

Answer

18. 4.806g; 5.194g

19. $0.21 \div 5.000 = 4.2\%$

20. 5.443g; 5.897g

21. $0.24 \div 5.670 \approx 4.2\%$

1. www.census.gov/quickfacts/fact/table/portlandcityoregon,OR,US/PST045219 ↵
2. https://www.usmint.gov/learn/coin-and-medal-programs/coin-specifications and https://www.thesprucecrafts.com/how-much-do-coins-weigh-4171330↵