2.13: Solution of many-body equations of motion

The following are general methods used to solve Newton's many-body equations of motion for practical problems.

Analytic solution

In practical problems one has to solve a set of equations of motion since the forces depend on the location of every body involved. For example one may be dealing with a set of coupled oscillators such as the many components that comprise the suspension system of an automobile. Often the coupled equations of motion comprise a set of coupled second-order differential equations.

The first approach to solve such a system is to try an analytic solution comprising a general solution of the inhomogeneous equation plus one particular solution of the inhomogeneous equation. Another approach is to employ numeric integration using a computer.

Successive approximation

When the system of coupled differential equations of motion is too complicated to solve analytically one can use the method of successive approximation. The differential equations are transformed to integral equations. Then one starts with some initial conditions to make a first order estimate of the functions. The functions determined by this first order estimate then are used in a second iteration and this is repeated until the solution converges.

An example of this approach is when making Hartree-Foch calculations of the electron distributions in an atom. The first order calculation uses the electron distributions predicted by the one-electron model of the atom. This result then is used to compute the influence of the electron charge distribution around the nucleus on the charge distribution of the atom for a second iteration etc.

Perturbation method

The perturbation technique can be applied if the force separates into two parts $F = F_1 + F_2$ where $F_1 >> F_2$ and the solution is known for the dominant F_1 part of the force. Then the correction to this solution due to addition of the perturbation F_2 usually is easier to evaluate. As an example, consider that one of the Space Shuttle thrusters fires. In principle one has all the gravitational forces acting plus the thrust force of the thruster. The perturbation approach is to assume that the trajectory of the Space Shuttle in the earth's gravitational field is known. Then the perturbation to this motion due to the very small thrust, produced by the thruster, is evaluated as a small correction to the motion in the Earth's gravitational field. This perturbation technique is used extensively in physics, especially in quantum physics. An example from my own research is scattering of a $1GeV$ ^{208}Pb ion in the Coulomb field of a ^{197}Au nucleus. The trajectory for elastic scattering is simple to calculate since neither nucleus is excited and thus the total energy and momenta are conserved. However, usually one of these nuclei will be internally excited by the electromagnetic interaction. This is called Coulomb excitation. The effect of the Coulomb excitation usually can be treated as a perturbation by assuming that the trajectory is given by the elastic scattering solution and then calculate the excitation probability assuming the Coulomb excitation of the nucleus is a small perturbation to the trajectory.