18.4: Lagrangian Representation in Quantum Theory

The classical notion of canonical coordinates and momenta, has a simple quantum analog which has allowed the Hamiltonian theory of classical mechanics, that is based on canonical coordinates, to serve as the foundation for the development of quantum mechanics. The alternative Lagrangian formulation for classical dynamics is described in terms of coordinates and velocities, instead of coordinates and momenta. The Lagrangian and Hamiltonian formulations are closely related, and it may appear that the Lagrangian approach is more fundamental. The Lagrangian method allows collecting together all the equations of motion and expressing them as stationary properties of the action integral, and thus it may appear desirable to base quantum mechanics on the Lagrangian theory of classical mechanics. Unfortunately, the Lagrangian equations of motion involve partial derivatives with respect to coordinates, and their velocities, and the meaning ascribed to such derivatives is difficult in quantum mechanics. The close correspondence between Poisson brackets and the commutation rules leads naturally to Hamiltonian mechanics. However, Dirac showed that Lagrangian mechanics can be carried over to quantum mechanics using canonical transformations such that the classical Lagrangian is considered to be a function of coordinates at time t and $t+dt$ rather than of coordinates and velocities.

The motivation for Feynman's 1942 Ph.D thesis, entitled "*The Principle of Least Action in Quantum Mechanics*", was to quantize the classical action at a distance in electrodynamics. This theory adopted an overall space-time viewpoint for which the classical Hamiltonian approach, as used in conventional formulations of quantum mechanics, is inapplicable. Feynman used the Lagrangian, plus the principle of least action, to underlie his development of quantum field theory. To paraphrase Feynman's Nobel Lecture, he used a physical approach that is quite different from the customary Hamiltonian point of view for which the system is discussed in great detail as a function of time. That is, you have the field at this moment, then a differential equation gives you the field at a later moment and so on; that is, the Hamiltonian approach is a time differential method. In Feynman's least-action approach the action describes the character of the path throughout all of space and time. The behavior of nature is determined by saying that the whole space-time path has a certain character. The use of action involves both advanced and retarded terms that make it difficult to transform back to the Hamiltonian form. The Feynman space-time approach is far beyond the scope of this course. This topic will be developed in advanced graduate courses on quantum field theory