13.6: Principal Axis System

The *inertia tensor is a real symmetric matrix* because of the symmetry given by equation $(13.4.5)$. A property of real symmetric matrices is that there exists an orientation of the coordinate frame, with its origin at the chosen body-fixed point O, such that the inertia tensor is diagonal. The coordinate system for which the inertia tensor is diagonal is called the **Principal axis system** which has three perpendicular **principal axes**. Thus, in the principal axis system, the inertia tensor has the form

$$\{\mathbf{I}\} = \begin{pmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{pmatrix} \tag{13.6.1}$$

where I_{ij} are real numbers, which are called the **principal moments of inertia** of the body, and are usually written as I_j. When the angular velocity vector $\boldsymbol{\omega}$ points along any principal axis unit vector $\hat{j}$, then the angular momentum $\mathbf{L}$ is parallel to $\boldsymbol{\omega}$ and the magnitude of the principal moment of inertia about this principal axis is given by the relation

$$L_j\hat{j} = I_j\omega_j\hat{j} \tag{13.6.2}$$

The principal axes are fixed relative to the shape of the rigid body and they are invariant to the orientation of the body-fixed coordinate system used to evaluate the inertia tensor. The advantage of having the bodyfixed coordinate frame aligned with the principal axis coordinate frame is that then the inertia tensor is diagonal, which greatly simplifies the matrix algebra. Even when the body-fixed coordinate system is not aligned with the principal axis frame, if the angular velocity is specified to point along a principal axis then the corresponding moment of inertia will be given by 13.6.2.

In principle it is possible to locate the principal axes by varying the orientation of the angular velocity vector $\boldsymbol{\omega}$ to find those orientations for which the angular momentum $\mathbf{L}$ and angular velocity $\boldsymbol{\omega}$ are parallel which characterizes the principal axes. However, the best approach is to diagonalize the inertia tensor.

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