13.2: Rigid-body Coordinates

Motion of a rigid body is a special case for motion of the N-body system when the relative positions of the N bodies are related. It was shown in chapter 2 that the motion of a rigid body can be broken into a combination of a linear translation of some point in the body, plus rotation of the body about an axis through that point. This is called **Chasles' Theorem**. Thus the position of every particle in the rigid body is fixed with respect to one point in the body. If the fixed point of the body is chosen to be the center of mass, then, as discussed in chapter 2, it is possible to separate the kinetic energy, linear momentum, and angular momentum into the center-of-mass motion, plus the motion about the center of mass. Thus the behavior of the body can be described completely using only six independent coordinates governed by six equations of motion, three for translation and three for rotation.

Referred to an inertial frame, the translational motion of the center of mass is governed by

$$\mathbf{F}^E = \frac{d\mathbf{P}}{dt} \tag{13.2.1}$$

while the rotational motion about the center of mass is determined by

$$\mathbf{N}^E = \frac{d\mathbf{L}}{dt} \tag{13.2.2}$$

where the external force $\mathbf{F}^E$ and external torque $\mathbf{N}^E$ are identified separately from the internal forces acting between the particles in the rigid body. It will be assumed that the internal forces are central and thus do not contribute to the angular momentum.

The location of any fixed point in the body, such as the center of mass, can be specified by three generalized cartesian coordinates with respect to a fixed frame. The rotation of the body-fixed axis system about this fixed point in the body can be described in terms of three independent angles with respect to the fixed frame. There are several possible sets of orthogonal angles that can be used to describe the rotation. This book uses the Euler angles ϕ, θ, ψ which correspond first to a rotation ϕ about the z-axis, then a rotation θ about the x axis subsequent to the first rotation, and finally a rotation ψ about the new z axis following the first two rotations. The Euler angles will be discussed in detail following introduction of the inertia tensor and angular momentum.