12.E: Non-inertial reference frames (Exercises)

1. Consider a fixed reference frame S and a rotating frame S'. The origins of the two coordinate systems always coincide. By carefully drawing a diagram, derive an expression relating the coordinates of a point P in the two systems. (This was covered in Chapter 2, but it is worth reviewing now.
2. The effective force observed in a rotating coordinate system is given by equation $(12.5.7)$.
   1. What is the significance of each term in this expression?
   2. Suppose you wanted to measure the gravitational force, both magnitude and direction, on a body of mass m at rest on the surface of the Earth. What terms in the effective force can be neglected?
   3. Suppose you wanted to calculate the deflection of a projectile fired horizontally along the Earth's surface. What terms in the effective force can be neglected?
   4. Suppose you wanted to calculate the effective force on a small block of mass m placed on a frictionless turntable rotating with a time-dependent angular velocity $\omega(t)$. What terms in the effective force can be neglected?
3. A plumb line is carried along in a moving train, with m the mass of the plumb bob. Neglect any effects due to the rotation of the Earth and work in the noninertial frame of reference of the train.
   1. Find the tension in the cord and the deflection from the local vertical if the train is moving with constant acceleration a_0.
   2. Find the tension in the cord and the deflection from the local vertical if the train is rounding a curve of radius ρ with constant speed v_0.
4. A bead on a rotating rod is free to slide without friction. The rod has a length L and rotates about its end with angular velocity ω. The bead is initially released from rest (relative to the rod) at the midpoint of the rod.
   1. Find the displacement of the bead along the wire as a function of time.
   2. Find the time when the bead leaves the end of the rod.
   3. Find the velocity (relative to the rod) of the bead when it leaves the end of the rod.
5. Here is a "thought experiment" for you to consider. Suppose you are in a small sailboat of mass M at the Earth's equator. At the equator there is very little wind (this is known as the "equatorial doldrums"), so your sailboat is, more or less, sitting still. You have a small anchor of mass m on deck and a single mast of height h in the middle of the boat. How can you use the anchor to put the boat into motion? In which direction will the boat move?
6. Does water really flow in the other direction when you flush a toilet in the southern hemisphere? What (if anything) does the Coriolis force have to do with this?
7. We are presently at a latitude λ (with respect to the equator) and Earth is rotating with constant angular velocity ω. Consider the following two scenarios: Scenario A: A particle is thrown upward with initial speed v_0. Scenario B: An identical particle is dropped (at rest) from the maximum height of the particle in Scenario A. Circle all the true statements regarding the Coriolis deflection assuming that the particles have landed for a) and b), .
   1. (a) The magnitude is greater in A than in B.
   2. (b) The direction in A and B are the same.
   3. (c) The direction in A does not change throughout flight.
8. If a projectile is fired due east from a point on the surface of the Earth at a northern latitude λ with a velocity of magnitude V_0 and at an inclination to the horizontal of α, show that the lateral deflection when the projectile strikes the Earth is
$$d=\frac{4V_0^3}{g^2}\omega\sin\lambda\sin^2\alpha\cos\alpha \tag{12.E.1}$$
   where ω is the rotation frequency of the Earth.
9. Obtain an expression for the angular deviation of a particle projected from the North Pole in a path that lies close to the surface of the earth. Is the deviation significant for a missile that makes a 4800-km flight in 10 minutes? What is the "miss distance" if the missile is aimed directly at the target? Is the miss difference greater for a 19300-km flight at the same velocity?
10. An automobile drag racer drives a car with acceleration a and instantaneous velocity v. The tires of radius r_0 are not slipping. Derive which point on the tire has the greatest acceleration relative to the ground. What is this acceleration?
11. Shot towers were popular in the eighteenth and nineteenth centuries for dropping melted lead down tall towers to form spheres for bullets. The lead solidified while falling and often landed in water to cool the lead bullets. Many such shot towers were built

in New York State. Assume a shot tower was constructed at latitude $42^{\circ}\ N$, and that the lead fell a distance of $27\ m$. In what direction and by how far did the lead bullets land from the direct vertical?