13.15: Kinetic energy in terms of Euler angular velocities

The kinetic energy is a scalar quantity and thus is the same in both stationary and rotating frames of reference. It is much easier to evaluate the kinetic energy in the rotating Principal-axis frame since the inertia tensor is diagonal in the Principal-axis frame as given in equation $(13.12.14)$

$$T_{rot} = \frac{1}{2}\sum_{i}^{3} I_{ii}\omega_i^2 \tag{13.15.1}$$

Using equation $(13.14.1 - 13.14.3)$for the body-fixed angular velocities gives the rotational kinetic energy in terms of the Euler angular velocities and principal-frame moments of inertia to be

$$T_{rot} = \frac{1}{2}\left[I_1(\dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi)^2 + I_2(\dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi)^2 + I_3(\dot{\phi}\cos\theta + \dot{\psi})^2\right] \tag{13.15.2}$$