7.1: Introduction to Symmetries, Invariance, and the Hamiltonian

The chapter 7 discussion of Lagrangian dynamics illustrates the power of Lagrangian mechanics for deriving the equations of motion. In contrast to Newtonian mechanics, which is expressed in terms of force vectors acting on a system, the Lagrangian method, based on d'Alembert's Principle or Hamilton's Principle, is expressed in terms of the scalar kinetic and potential energies of the system. The Lagrangian approach is a sophisticated alternative to Newton's laws of motion, that provides a simpler derivation of the equations of motion that allows constraint forces to be ignored. In addition, the use of Lagrange multipliers or generalized forces allows the Lagrangian approach to determine the constraint forces when these forces are of interest. The equations of motion, derived either from Newton's Laws or Lagrangian dynamics, can be non-trivial to solve mathematically. It is necessary to integrate second-order differential equations, which for n degrees of freedom, imply $2n$ constants of integration.

Chapter 7 will explore the remarkable connection between symmetry and invariance of a system under transformation, and the related conservation laws that imply the existence of constants of motion. Even when the equations of motion cannot be solved easily, it is possible to derive important physical principles regarding the first-order integrals of motion of the system directly from the Lagrange equation, as well as for elucidating the underlying symmetries plus invariance. This property is contained in **Noether's theorem** which states that conservation laws are associated with differentiable symmetries of a physical system.