12.S: Non-inertial reference frames (Summary)

This chapter has focussed on describing motion in non-inertial frames of reference. It has been shown that the force and acceleration in non-inertial frames can be related using either Newtonian or Lagrangian mechanics by introducing additional inertial forces in the non-inertial reference frame.

Translational acceleration of a reference frame

In a primed frame, that is undergoing translational acceleration $\mathbf{A}$, the motion in this non-inertial frame can be calculated by addition of an inertial force $-m\mathbf{A}$, that leads to an equation of motion

$$m\mathbf{a}' = \mathbf{F} - m\mathbf{A} \tag{12.S.1}$$

Note that the primed frame is an inertial frame if $\mathbf{A} = 0$.

Rotating reference frame

It was shown that the time derivatives of a general vector $\mathbf{G}$ in both an inertial frame and a rotating reference frame are related by

$$\left(\frac{d\mathbf{G}}{dt}\right)_{fixed} = \left(\frac{d\mathbf{G}}{dt}\right)_{rotating} + \boldsymbol{\omega} \times \mathbf{G} \tag{12.S.2}$$

where the $\boldsymbol{\omega} \times \mathbf{G}$ term originates from the fact that the unit vectors in the rotating reference frame are time dependent with respect to the inertial frame.

Reference frame undergoing both rotation and translation

Both Newtonian and Lagrangian mechanics were used to show that for the case of translational acceleration plus rotation, the effective force in the non-inertial (double-primed) frame can be written as

$$\mathbf{F}_{eff} = m\mathbf{a}'' = \mathbf{F} - m(\mathbf{A} + \boldsymbol{\omega} \times \mathbf{V} + 2\boldsymbol{\omega} \times \mathbf{v}'' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') + \dot{\boldsymbol{\omega}} \times \mathbf{r}') \tag{12.S.3}$$

These inertial correction forces result from describing the system using a non-inertial frame. These inertial forces are felt when in the rotating-translating frame of reference. Thus the notion of these inertial forces can be very useful for solving problems in non-inertial frames. For the case of rotating frames, two important inertial forces are the centrifugal force, $-\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}')$, and the Coriolis force $-2\boldsymbol{\omega} \times \mathbf{v}''$.

Routhian reduction for rotating systems

It was shown that for non-inertial systems, identical equations of motion are derived using Newtonian, Lagrangian, Hamiltonian, and Routhian mechanics.

Terrestrial manifestations of rotation

Examples of motion in rotating frames presented in the chapter included projectile motion with respect to the surface of the Earth, rotation alignment of nucleons in rotating nuclei, and weather phenomena.