13.25: Dynamic balancing of wheels

For rotating machinery It is crucial that rotors be both statically and dynamically balanced. *Static balance means that the center of mass is on the axis of rotation. Dynamic balance means that the axis of rotation is a principal axis.*

For example, consider the symmetric rotor that has its symmetry axis at an angle ϕ to the axis of rotation. In this case the system is statically balanced since the center of gravity is on the axis of rotation. However, the rotation axis is at an angle ϕ to the symmetry axis. This implies that the axle has to provide a torque to maintain rotation that is not along a principal axis. If you distort the front wheel of your car by hitting it sideways against the sidewalk curb, or if the wheel is not dynamically balanced, then you will find that the steering wheel can vibrate wildly at certain speeds due to the torques caused by dynamic imbalance shaking the steering mechanism. This can be especially bad when the rotation frequency is close to a resonant frequency of the suspension system. Insist that your automobile wheels are dynamically balanced when you change tires, static balancing will not eliminate the dynamic imbalance forces. Another example is that the ailerons, rudder, and elevator on aircraft usually are dynamically balanced to stop the build up of oscillations that can couple to flexing and flutter of the airframe which can lead to airframe failure.

Example $13.25.1$: Forces on the bearings of a rotating circular disk

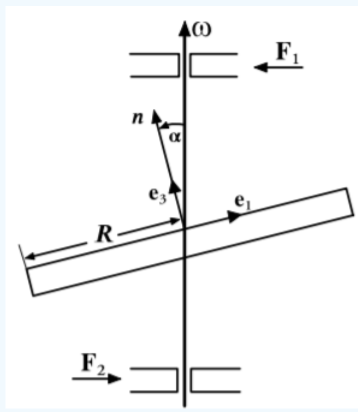

Figure 13.25.1: Rotation of circular disk about an axis that is at an angle α to the symmetry axis of the circular disk.

A homogeneous circular disk of mass M, and radius R, rotates with constant angular velocity ω about a body-fixed axis passing through the center of the circular disk as shown in the adjacent figure. The rotation axis is inclined at an angle α to the symmetry axis of the circular disk by bearings on both sides of the disk spaced a distance d apart. Determine the forces on the bearings.

Choose the body-fixed axes such that $\hat{e}_3$ is along the symmetry axis of the circular disk, and $\hat{e}_1$ points in the plane of the disk symmetry axis and the rotation axis. These axes are the principal axes for which the inertia tensor can be calculated to be

$$\mathbf{I} = \frac{MR^2}{4}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Note that for this thin plane laminae disk $I_{11} + I_{22} = I_{33}$.

The components of the angular velocity vector ω along the three body-fixed axes are given by

$$\boldsymbol{\omega} = (\omega \sin\alpha, 0, \omega\, cos\alpha)$$

Since it is assumed that $\dot{\omega} = 0$ then substituting into Euler's equations $(13.17.6)$ gives the torques acting to be

$$N_1 = N_3 = 0$$
$$N_2 = -\omega^2 \sin\alpha \cos\alpha \frac{1}{4} MR^2$$

That is, the torque is in the $\hat{e}_2$ direction. Thus the forces F on the bearings can be calculated since $\mathbf{N} = \mathbf{r} \times \mathbf{F}$, thus

$$|F| = \frac{|N_2|}{2d} = MR^2\omega^2 \frac{\sin 2\alpha}{16d}$$

Estimate the size of these forces for the front wheel of your car travelling at $70\,m.p.h.$ if the rotation axis is displaced by 2° from the symmetry axis of the wheel.

Figure 13.25.2: Forward two-and-a-half somersaults with two twists demonstrates unequivocally that a diver can initiate continuous twisting in midair. In the illustrated maneuver the diver does more than one full somersault before he starts to twist. To maintain the twisting the diver does not have to move his legs.[Fro80]