6.13: The Lagrangian versus the Newtonian approach to classical mechanics

It is useful to contrast the differences, and relative advantages, of the Newtonian and Lagrangian formulations of classical mechanics. The Newtonian force-momentum formulation is vectorial in nature, it has cause and effect embedded in it. The Lagrangian approach is cast in terms of kinetic and potential energies which involve only scalar functions and the equations of motion come from a single scalar function, i.e. Lagrangian. The directional properties of the equations of motion come from the requirement that the trajectory is specified by the principle of least action. The directional properties of the vectors in the Newtonian approach assist in our intuition when setting up a problem, but the Lagrangian method is simpler mathematically when the mechanical system is more complex.

The major advantage of the variational approaches to mechanics is that solution of the dynamical equations of motion can be simplified by expressing the motion in terms of independent **generalized coordinates .** For Lagrangian mechanics these generalized coordinates can be any set of **independent variables**, q_i, where $1 \leq i \leq n$, plus the corresponding velocities $\dot{q}_i$. These independent generalized coordinates completely specify the scalar potential and kinetic energies used in the Lagrangian or Hamiltonian. The variational approach allows for a much larger arsenal of possible generalized coordinates than the typical vector coordinates used in Newtonian mechanics. For example, the generalized coordinates can be dimensionless amplitudes for the N normal modes of coupled oscillator systems, or action-angle variables. Moreover, very different generalized coordinates can be used for each of the n variables. The tremendous freedom plus flexibility of the choice of generalized coordinates is important when constraint forces are acting on the system. Generalized coordinates allow the constraint forces to be ignored by including auxiliary conditions to account for the kinematic constraints that lead to correlated motion. The Lagrange method provides an incredibly consistent and mechanistic problem-solving strategy for many-body systems subject to constraints. Expressed in terms of generalized coordinates, the Lagrange's equations can be applied to a wide variety of physical problems including those involving fields. The manipulation of scalar quantities in a configuration space of generalized coordinates can greatly simplify problems compared with being confined to a rigid orthogonal coordinate system characterized by the Newtonian vector approach.

The use of generalized coordinates in Lagrange's equations of motion can be applied to a wide range of physical phenomena including field theory, such as for electromagnetic fields, which are beyond the applicability of Newton's equations of motion. The superiority of the Lagrangian approach compared to the Newtonian approach for solving problems in mechanics is apparent when dealing with holonomic constraint forces. Constraint forces must be known and included explicitly in the Newtonian equations of motion. Unfortunately, knowledge of the equations of motion is required to derive these constraint forces. For holonomic constrained systems, the equations of motion can be solved directly without calculating the constraint forces using the minimal set of generalized coordinate approach to Lagrangian mechanics. Moreover, the Lagrange approach has significant philosophical advantages compared to the Newtonian approach.