

## 7.11: Hamiltonian for Cyclic Coordinates

It is interesting to discuss the properties of the Hamiltonian for cyclic coordinates  $q_k$  for which  $\frac{\partial L}{\partial q_k} = 0$ . Ignoring the external and Lagrange multiplier terms,

$$\dot{p}_k = \frac{\partial L}{\partial q_k} = -\frac{\partial H}{\partial q_k} = 0 \quad (7.11.1)$$

That is, a cyclic coordinate has a constant corresponding momentum  $p_k$  for the Hamiltonian as well as for the Lagrangian. Conversely, if a generalized coordinate does not occur in the Hamiltonian, then the corresponding generalized momentum is conserved. Cyclic coordinates were discussed earlier when discussing symmetries and conservation-law aspects of the Lagrangian. For example, if the Lagrangian, or Hamiltonian do not depend on a linear coordinate  $x$ , then  $p_x$  is conserved. Similarly for  $\theta$  and  $p_\theta$ . An extension of this principle has been derived for the relationship between time independence and total energy of a system, that is, the Hamiltonian equals the total energy if the transformation to generalized coordinates is time independent and the potential is velocity independent.

A valuable feature of the Hamiltonian formulation is that it allows elimination of cyclic variables which reduces the number of degrees of freedom to be handled. As a consequence, cyclic variables are called **ignorable variables** in Hamiltonian mechanics. For example, consider that the Lagrangian has one cyclic variable  $q_n$ . As a consequence, the Lagrangian does not depend on  $q_n$ , and thus it can be written as

$$L = L(q_1, \dots, q_{n-1}; \dot{q}_1, \dots, \dot{q}_n; t).$$

The Lagrangian still contains  $n$  generalized velocities, thus one still has to treat  $n$  degrees of freedom even though one degree of freedom  $q_n$  is cyclic. However, in the Hamiltonian formulation, only  $n - 1$  degrees of freedom are required since the momentum for the cyclic degree of freedom is a constant  $p_n = \alpha$ . Thus the Hamiltonian can be written as

$$H = H(q_1, \dots, q_{n-1}; p_1, \dots, p_{n-1}; \alpha; t).$$

that is, the Hamiltonian includes only  $n - 1$  degrees of freedom. Thus the dimension of the problem has been reduced by one since the conjugate cyclic (ignorable) variables  $(q_n, p_n)$  are eliminated. Hamiltonian mechanics can significantly reduce the dimension of the problem when the system involves several cyclic variables. This is in contrast to the situation for the Lagrangian approach as discussed in chapters 8 and 15.

---

This page titled [7.11: Hamiltonian for Cyclic Coordinates](#) is shared under a [CC BY-NC-SA 4.0](#) license and was authored, remixed, and/or curated by [Douglas Cline](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.