

Preface

This Libretexts book introduces the reader to the intellectual beauty, and philosophical implications, of the fact that nature obeys variational principles that underlie the Lagrangian and Hamiltonian analytical formulations of classical mechanics. These variational methods, developed for classical mechanics during the 18th – 19th century, have become the preeminent formalisms for classical dynamics, as well as for many other branches of science and engineering. This book leads the reader from the intuitive Newtonian vectorial formulation, to introduction of the more abstract variational principles that underlie Hamilton’s Principle, and the related Lagrangian and Hamiltonian analytical formulations. This culminates in discussion of the contributions of variational principles to classical mechanics, relativistic mechanics, and quantum physics. This book attempts to unify the undergraduate physics curriculum by bridging the chasm that divides the Newtonian vector-differential, from the integral variational formulations of classical mechanics, and the corresponding philosophical approaches adopted in classical and quantum mechanics.

Development of this textbook was influenced by two graduate textbooks: The Variational Principles of Mechanics by Cornelius Lanczos (1949) [La49], and Classical Mechanics (1950) by Herbert Goldstein[Go50]. The present textbook presents the techniques and philosophical implications of the variational approaches to classical mechanics, with a breadth and depth close to that provided by Goldstein and Lanczos, but in a format that better matches the needs of undergraduate students. This book is based on lecture notes written in support of the physics junior/senior undergraduate course P235W entitled “Variational Principles in Classical Mechanics” that the author taught at the University of Rochester between 1993 – 2015. This course typically comprised $\approx 70\%$ junior/senior undergraduates, $\approx 25\%$ sophomores, $\leq 5\%$ graduate students, and the occasional well-prepared freshman. The target audience was physics and astrophysics majors, but the course attracted a significant fraction of majors from other disciplines such as mathematics, chemistry, optics, engineering, music, and the humanities. As a consequence, the book includes introductory level physics, plus mathematical review material, to accommodate the wide range of prior preparation of the students. To conform with modern literature in this field, this book follows the widely-adopted nomenclature used in Goldstein [Go50], with recent additions by Johns [Jo05] and the present textbook.

The scientific content of the Libretexts book is identical to the Third Edition of Variational Principles in Classical Mechanics, version 3.1, by Douglas Cline, published in 2021 by Amazon. This Libretexts version of the book introduces the convenience of modern interactive on-line computer access. All versions of this book review the role of variational principles in bridging the gap between classical mechanics and quantum mechanics. They illustrate the pivotal role that variational principles have played in the development of classical, relativistic, quantal, and statistical mechanics. Skill at solving problems is essential to fully exploit variational principles in classical mechanics. Compilations of worked problems, with corresponding solutions, [La10, Li94, Th04] can be used to develop this skill.

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Douglas Cline,
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