17.8: The General Theory of Relativity

The Special Theory of Relativity is restricted to inertial frames that are in uniform non-accelerated motion, and are assumed to exist over all of space-time. In 1916 Einstein published the General Theory of Relativity which expands the scope of relativistic mechanics to include non-inertial accelerating frames plus a unified theory of gravitation. The General Theory of Relativity incorporates both the Special Theory of Relativity as well as Newton's Law of Universal Gravitation. It provides a unified theory of gravitation that is a geometric property of space and time. In particular, the curvature of space-time is directly related to the four-momentum of matter and radiation. Unfortunately, Einstein's equations of general relativity are nonlinear partial differential equations that are difficult to solve exactly, and the theory requires knowledge of Riemannian geometry that goes beyond the scope of this book. However, it is useful to summarize the fundamental concepts upon which the theory is based, and some of the observable implications since the General Theory of Relativity is an important branch of classical mechanics.

The Fundamental Concepts

The development of general relativity by Einstein was strongly influenced by the following five principles.

Mach's Principle

The 1883 work "The Science of Mechanics" by the philosopher/physicist, Ernst Mach, criticized Newton's concept of an absolute frame of reference, and suggested that local physical laws are determined by the large-scale structure of the universe. The concept is that local motion of a rotating frame is determined by the large-scale distribution of matter, that is, relative to the fixed stars. Einstein's interpretation of Mach's statement was that the inertial properties of a body is determined by the presence of other bodies in the universe, and he named this concept Mach's Principle. Mach's Principle has never been developed into a quantitative physical theory that would explain a mechanism by which the large-scale distribution of matter can produce such an effect.

Equivalence Principle

The equivalence principle comprises closely-related concepts dealing with the equivalence of gravitational and inertial mass. The **weak equivalence principle** states that the inertial mass and gravitational mass of a body are identical, leading to acceleration that is independent of the nature of the body. This experimental fact usually is attributed to Galileo. Recent measurements have shown that this weak equivalence principle is obeyed to a sensitivity of 5×10^{-13}. **Einstein's equivalence principle** states that the outcome of any local non-gravitational experiment, in a freely falling laboratory, is independent of the velocity of the laboratory and its location in space-time. This principle implies that the result of local experiments must be independent of the velocity of the apparatus. Einstein's equivalence principle has been tested by searching for variations of dimensionless fundamental constants such as the fine structure constant. The **strong equivalence principle** combines the weak equivalence and Einstein equivalence principles, and implies that the gravitational constant is constant everywhere in the universe. The strong equivalence principle suggests that gravity is geometrical in nature and does not involve any fifth force in nature. Einstein's General Theory of Relativity satisfies the strong equivalence principle. Tests of the strong equivalence principle have involved searches for variations in the gravitational constant G and masses of fundamental particles throughout the life of the universe.

Principle of Covariance

A physical law expressed in a covariant formulation has the same mathematical form in all coordinate systems, and is usually expressed in terms of tensor fields. Maxwell's equations of electromagnetism are an example of such a covariant formulation. In the Special Theory of Relativity, the Lorentz, rotational, translational and reflection transformations between inertial coordinate frames all are covariant. The covariant quantities are the 4-scalars, and 4-vectors in Minkowski space-time. Einstein recognized that the principle of covariance, that is built into the Special Theory of Relativity, should apply equally to accelerated relative motion in the General Theory of Relativity. He exploited tensor calculus to extend the Lorentz covariance to the more general local covariance in the General Theory of Relativity. The reduction locally of the general metric tensor to the Minkowski metric corresponds to free-falling motion, that is geodesic motion, and thus encompasses gravitation. Unified field theory involves attempts to extend the General Theory of Relativity to incorporate other physical phenomena within a covariant framework in a purely geometric representation in space-time.

Correspondence principle

The Correspondence Principle states that the predictions of any new scientific theory must reduce to the pre dictions of well established earlier theories under circumstances for which the preceding theory was known to be valid. This also is referred to as the "correspondence limit". The Correspondence Principle is an important concept used both in quantum mechanics and relativistic mechanics. Einstein’s Special Theory of Relativity satisfies the Correspondence Principle because it reduces to classical mechanics in the limit of velocities small compared to the speed of light. The Correspondence Principle requires that the Gen eral Theory of Relativity must reduce to the Special Theory of Relativity for inertial frames, and should approximate Newton’s Theory of Gravitation in weak fields and at low velocities.

Principle of Minimal Gravitational Coupling

The principle of minimal gravitational coupling requires that the total Lagrangian for the field equations of general relativity consist of two additive parts, one part corresponding to the free gravitational Lagrangian, and the other part to external source fields in curved space-time. That is, no terms explicitly containing the curvature of space-time should be added in the extension from the special to general theories of relativity.

Einstein’s postulates for the General Theory of Relativity

Einstein realized that the Equivalence Principle relating the gravitational and inertial masses implies that the constancy of the velocity of light in vacuum cannot hold in the presence of a gravitational field. That is, the Minkowskian line element must be replaced by a more general line element that takes gravity into account. Einstein proposed that the Minkowskian line element in four-dimensional space-time, be replaced by introducing a four-dimensional Riemannian geometrical structure where space, time, and matter are combined. As described by Lancos[La49], [Har03], [Mu08] this astonishingly bold proposal implies that planetary motion is described as purely a geodesic phenomenon in a certain four-space of Riemannian structure, where the geodesic is the equation of a curve on a manifold for any possible set of coordinates. This implies that the concept of "gravitational force" is discarded, and planetary motion is a manifestation of a pure geodesic phenomenon for forceless motion in a four-dimensional Riemannian structure.

Chapter 5.10 showed that the Lagrangian and Hamiltonian representations of variational mechanics are powerful approaches for determining the equation governing geodesic constrained motion. In addition, these representations are independent of the chosen frame of reference as required by the General Theory of Relativity. Thus variational mechanics is the preeminent theoretical representation of the General Theory of Relativity and the predictions are consistent with the fundamental concepts described in chapter 16.8.

To summarize, the Special Theory of Relativity implies that the Newtonian concepts of absolute frame of reference and separation of space and time are invalid. The General Theory of Relativity goes beyond the Special Theory by implying that the gravitational force, and the resultant planetary motion, can be described as pure geodesic phenomena for forceless motion in a four-dimensional Riemannian structure.

Experimental evidence

The evidence in support of Einstein’s Theory of General Relativity is compelling. The following are typical experimental manifestations of the General Theory of Relativity.

Kepler problem

In 1915 Einstein showed that relativistic mechanics explained the anomalous advance of the perihelion of the planet mercury, that is, the axes of the elliptical Kepler orbit precess. Example $16.7.1$ discusses the analogue of this effect for the Bohr-Sommerfeld hydrogen atom.

Deflection of light

Eddington travelled to the island of Príncipe, near Africa, to watch the solar eclipse of 29 May 1919. During the eclipse, he took pictures of the stars in the region around the Sun. According to the theory of general relativity, stars with light rays that passed near the Sun would appear to have been slightly shifted because their light had been curved by the sun’s gravitational field. This effect is noticeable only during eclipses, since otherwise the Sun’s brightness obscures the affected stars. The results confirmed Einstein’s prediction of the deflection of light in a gravitational field which made Einstein famous.

Gravitational lensing

The deflection of light by the gravitational attraction of a massive object situated between a distant star and the observer results in the observation of multiple images of the distant quasar.

Gravitational time dilation and frequency shift

Processes occurring in a high gravitation field are slower than in a weak gravitational field; this is called gravitational time dilation. In addition, light climbing out of a gravitational well is red shifted. The gravitational time dilation has been measured many times and the continued operation of the Global Position System provides an ongoing validation. The gravitational red shift has been confirmed in the laboratory using the precise Mössbauer effect in nuclear physics. Tests in stronger gravitational fields are provided by studies of binary pulsars. All of these measurements confirm the general theory of relativity.

Gravitational waves detection

In 1916 Einstein predicted the existence of gravitational waves on the basis of the theory of general relativity. The first implied detection of gravitational waves were made in 1976 by Hulse and Taylor who detected a decrease in the orbital period due to significant energy loss which presumably was associated with emission of gravity waves by the compact neutron star in the binary pulsar $PSR1913+16$. The most compelling direct evidence for observation of a gravitational wave was made on 15 September 2015 by the LIGO Laser Interferometer Gravitational-Wave Observatories. The waveform detected by the two LIGO observatories matched the predictions of General Relativity for gravitational waves emanating from the inward spiral plus merger of a pair of black holes of around 36 and 29 solar masses, followed by the resultant binary black hole. The gravitational wave emitted by this cataclysmic merger reached Earth as a ripple in space-time that changed the length of the $4\ km$ LIGO arm by a thousandth of the width of the proton. The gravitational energy emitted was $3.0^{+0.5}_{-0.5}c^2$ solar masses. A second observation of gravitational waves was made on 26 December 2015, and four similar observations were made during 2017. The detection of such miniscule changes in space-time is a truly remarkable achievement. This direct detection of gravitational waves resulted in the award of the 2017 Nobel Prize to Rainer Weiss, Barry Barish, and Kip Thorn.

Black holes

If the mass to radius ratio of a massive object becomes sufficiently large, general relativity predicts formation of a black hole, which is a region of space from which neither light nor matter can escape. A supermassive black hole, with a mass that is $10^6 - 10^9$ solar masses, is thought to have played an important role in formation of the M87 galaxy. This black hole at the core of the massive elliptical M87 galaxy was observed April 2017 by the Event Horizon Telescope (EHT). Figure $17.8.1$ shows a polarized light image of this black hole, revealing a ring-like structure consistent with synchrotron emission from relativistic electrons that are gyrating around the inner edge of a vortex of magnetic field lines in the vicinity of the event horizon. (The Astrophysical Journal Letters, 910:L12, 20 March 2021).

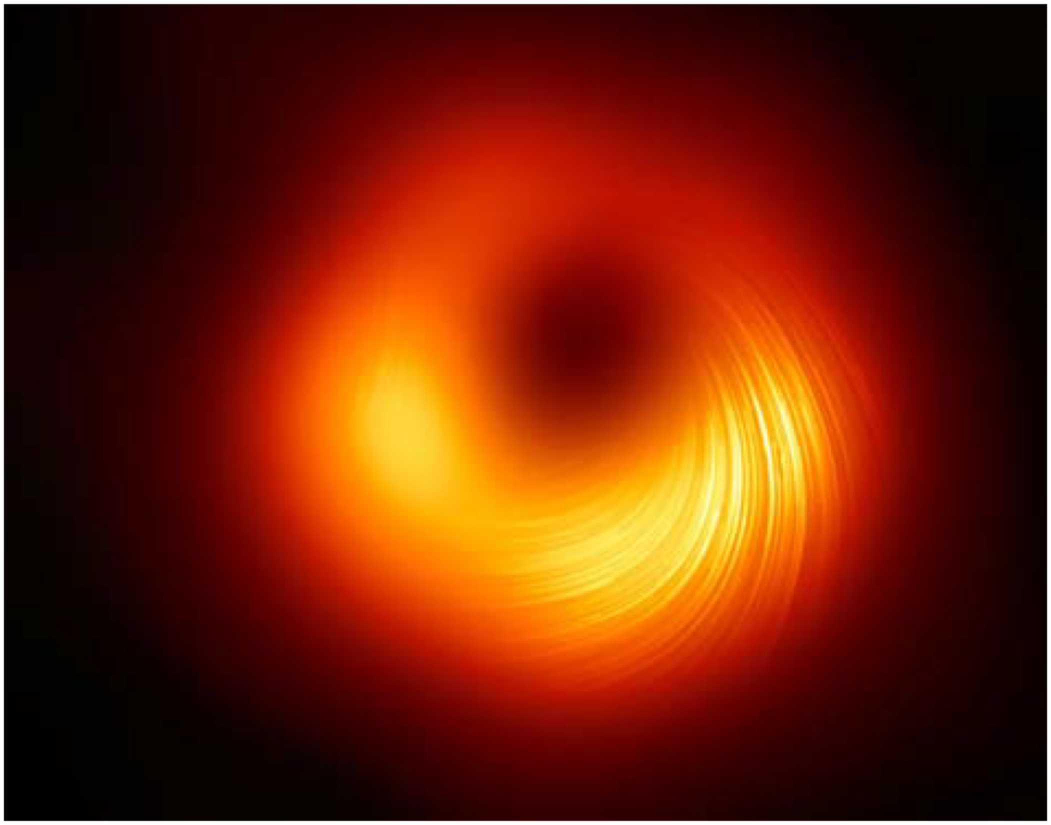

Figure 17.8.1: Polarized-light image of the M87 black hole