14.4: Center of Mass Oscillations

Transforming the coordinates into the center of mass of the two oscillating masses elucidates an interesting feature of the normal modes for the two-coupled linear oscillator. As illustrated in Figure $(14.2.1)$, the center-of-mass coordinate for the two mass system is

$$\begin{aligned} 2R_{cm} &= l + x_1 + l + l' + x_2 \\ &= 2l + l' + \eta_2 \end{aligned}$$

while the relative separation distance is

$$r = (l + l' + x_2) - (l + x_1) = l' - \eta_1$$

That is, the two normal modes are

$$\begin{aligned} \eta_1 &= l' - r \\ \eta_2 &= 2R_{cm} - 2l - l' \end{aligned} \tag{14.4.1}$$

The η_1 mode, which has angular frequency $\omega_1 = \sqrt{\frac{\kappa+2\kappa'}{M}}$ corresponds to an oscillations of the relative separation r, while the center-of-mass location R_{cm} is stationary. By contrast, the η_2 mode, with angular frequency $\omega_2 = \sqrt{\frac{\kappa}{M}}$ corresponds to an oscillation of the center of mass R_{cm} with the relative separation r being a constant.

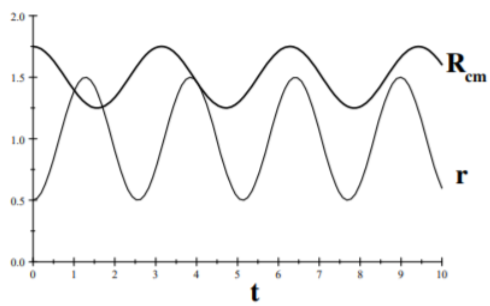

Figure 14.4.1: Time dependence of the center-of-mass R_{cm} and relative separation r for two coupled linear oscillators assuming spring constants of $\kappa = 4M$ and $\kappa' = M$.

Figure $14.4.1$ illustrates the decoupled center-of-mass R_{cm}, and relative motions r for both normal modes of the coupled double-oscillator system. The difference in angular frequencies and amplitudes is readily apparent. It is of interest to consider the special case where the spring constant $\kappa = 0$ for the two outside springs. Then the angular frequencies are $\omega_1 = \sqrt{\frac{2\kappa'}{M}}$ and $\omega_2 = 0$ for the two normal modes. When $\kappa = 0$ the η_2 mode is a spurious center-of-mass mode since it corresponds to an oscillation with $\omega_2 = 0$ in spite of the fact that there are no forces acting on the center of mass. That is, the center-of-mass momentum must be a constant of motion. This spurious center-of-mass oscillation is a consequence of measuring the displacements (x_1, x_2) with respect to an arbitrary external reference that is not related to the center of mass of the coupled system. Spurious center-of-mass modes are encountered frequently in many-body coupled oscillator systems such as molecules and nuclei. In such cases it is necessary to project out the center-of-mass motion to eliminate such spurious solutions as will be discussed later.