10.3: Algebraic Mechanics for Nonconservative Systems

Since Lagrangian and Hamiltonian formulations are invalid for the nonconservative degrees of freedom, the following three approaches are used to include nonconservative degrees of freedom directly in the Lagrangian and Hamiltonian formulations of mechanics.

1. Expand the number of degrees of freedom used to include all active degrees of freedom for the system, so that the expanded system is conservative. This is the preferred approach when it is viable. Hamilton's action principle based on initial conditions, introduced in chapter 9.2.4, doubles the number of degrees of freedom, which can be used to account for the dissipative forces providing one approach to solve nonconservative systems. However, this approach typically is impractical for handling dissipated processes because of the large number of degrees of freedom that are involved in thermal dissipation.
2. Nonconservative forces can be introduced directly at the equations of motion stage as generalized forces Q_j^{EXC}. This approach is used extensively. For the case of linear velocity dependence, the Rayleigh's dissipation function provides an elegant and powerful way to express the generalized forces in terms of scalar potential energies.
3. New degrees of freedom or effective forces can be postulated that are then incorporated into the Lagrangian or the Hamiltonian in order to mimic the effects of the nonconservative forces.

Examples that exploit the above three ways to introduce nonconservative dissipative forces in algebraic formulations are given below.