2.2: Newton's Laws of motion

Newton defined a vector quantity called linear momentum $\mathbf{p}$ which is the product of mass and velocity.

$$\mathbf{p} = m\dot{\mathbf{r}} \tag{2.2.1}$$

Since the mass m is a scalar quantity, then the velocity vector $\dot{r}$ and the linear momentum vector $\mathbf{p}$ are colinear.

Newton's laws, expressed in terms of linear momentum, are:

1. *Law of inertia*: A body remains at rest or in uniform motion unless acted upon by a force.
2. *Equation of motion*: A body acted upon by a force moves in such a manner that the time rate of change of momentum equals the force.

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \tag{2.2.2}$$

3. *Action and reaction*: If two bodies exert forces on each other these forces are equal in magnitude and opposite in direction.

Newton's second law contains the essential physics relating the force $\mathbf{F}$ and the rate of change of linear momentum $\mathbf{p}$.

Newton's first law, the law of inertia, is a special case of Newton's second law in that if

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = 0 \tag{2.2.3}$$

then $\mathbf{p}$ is a *constant of motion*.

Newton's third law also can be interpreted as a statement of the conservation of momentum, that is, for a two particle system with no external forces acting,

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \tag{2.2.4}$$

If the forces acting on two bodies are their mutual action and reaction, then Equation 2.2.4 simplifies to

$$\mathbf{F}_{12} = -\mathbf{F}_{21} = \frac{d\mathbf{p_1}}{dt} + \frac{d\mathbf{p_2}}{dt} = \frac{d}{dt}(\mathbf{p_1} + \mathbf{p_2}) = 0 \tag{2.2.5}$$

This implies that the total linear momentum $\mathbf{P} = \mathbf{p_1} + \mathbf{p_2}$ is a constant of motion. Combining Equations 2.2.1 and 2.2.2 leads to a second-order differential equation

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = m\frac{d^2\mathbf{r}}{dt^2} = m\ddot{\mathbf{r}} \tag{2.2.6}$$

Note that the force on a body $\mathbf{F}$, and the resultant acceleration $\mathbf{a} = \ddot{\mathbf{r}}$ are colinear. Appendix $19.3.2$ gives explicit expressions for the acceleration $\mathbf{a}$ in cartesian and curvilinear coordinate systems. The definition of force depends on the definition of the mass m. Newton's laws of motion are obeyed to a high precision for velocities much less than the velocity of light. For example, recent experiments have shown they are obeyed with an error in the acceleration of $\Delta a \leq 5 \times 10^{-14} m/s^2$.