17.S: Relativistic Mechanics (Summary)

Special Theory of Relativity

The Special Theory of Relativity is based on Einstein's postulates;

1. *The laws of nature are the same in all inertial frames of reference.*
2. *The velocity of light in vacuum is the same in all inertial frames of reference.*

For a primed frame moving along the x_1 axis with velocity v Einstein's postulates imply the following Lorentz transformations between the moving (primed) and stationary (unprimed) frames

Table $17.S.1$

$x' = \gamma(x - vt)$	$x = \gamma(x' + vt')$
$y' = y$	$y = y'$
$z' = z$	$z = z'$
$t' = \gamma\left(t + \frac{vx}{c^2}\right)$	$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$

where the Lorentz γ factor $\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

Lorentz transformations were used to illustrate Lorentz contraction, time dilation, and simultaneity. An elementary review was given of relativistic kinematics including discussion of velocity transformation, linear momentum, center-of-momentum frame, forces and energy.

Geometry of space-time

The concepts of four-dimensional space-time were introduced. A discussion of four-vector scalar products introduced the use of contravariant and covariant tensors plus the Minkowski metric g where the scalar product was defined. The Minkowski representation of space time and the momentum-energy four vector also were introduced.

Lorentz-invariant formulation of Lagrangian mechanics

The Lorentz-invariant extended Lagrangian formalism, developed by Struckmeier[Str08], based on the parametric approach pioneered by Lanczos[La49], provides a viable Lorentz-invariant extension of conventional Lagrangian mechanics that is applicable for one-body motion in the realm of the Special Theory of Relativity.

Lorentz-invariant formulation of Hamiltonian mechanics

The Lorentz-invariant extended Hamiltonian formalism, developed by Struckmeier based on the parametric approach pioneered by Lanczos, was introduced. It provides a viable Lorentz-invariant extension of conventional Hamiltonian mechanics that is applicable for one-body motion in the realm of the Special Theory of Relativity. In particular, it was shown that the Lorentz-invariant extended Hamiltonian is conserved making it ideally suited for solving complicated systems using Hamiltonian mechanics via use of the Poisson-bracket representation of Hamiltonian mechanics, canonical transformations, and the Hamilton-Jacobi techniques.

The General Theory of Relativity

An elementary summary was given of the fundamental concepts of the General Theory of Relativity and the resultant unified description of the gravitational force plus planetary motion as geodesic motion in a four-dimensional Riemannian structure. Variational mechanics were shown to be ideally suited to applications of the General Theory of Relativity.

Philosophical implications

Newton's equations of motion, and his Law of Gravitation, that reigned supreme from 1687 to 1905, have been toppled from the throne by Einstein's theories of relativistic mechanics. By contrast, the complete independence to coordinate frames in Lagrangian, and Hamiltonian formulations of classical mechanics, plus the underlying Principle of Least Action, are equally valid in both the relativistic and non-relativistic regimes. As a consequence, relativistic Lagrangian and Hamiltonian formulations underlie much of modern physics, especially quantum physics, which explains why relativistic mechanics plays such an important role in classical dynamics.
