

5.11: Variational Approach to Classical Mechanics

This chapter has introduced the general principles of variational calculus needed for understanding the Lagrangian and Hamiltonian approaches to classical mechanics. Although variational calculus was developed originally for classical mechanics, now it has grown to be an important branch of mathematics with applications to many other fields outside of physics. The prologue of this book emphasized the dramatic differences between the differential vectorial approach of Newtonian mechanics, and the integral variational approaches of Lagrange and Hamiltonian mechanics. The Newtonian vectorial approach involves solving Newton's differential equations of motion that relate the force and momenta vectors. This requires knowledge of the time dependence of all the force vectors, including constraint forces, acting on the system which can be very complicated. Chapter 2 showed that the first-order time integrals, equations (2.4.1), (2.4.7), relate the initial and final total momenta without requiring knowledge of the complicated instantaneous forces acting during the collision of two bodies. Similarly, for conservative systems, the first-order spatial integral, equation (2.4.12), relates the initial and final total energies to the net work done on the system without requiring knowledge of the instantaneous force vectors. The first-order spatial integral has the advantage that it is a scalar quantity, in contrast to time integrals which are vector quantities. These first-order integral relations are used frequently in Newtonian mechanics to derive solutions of the equations of motion that avoid having to solve complicated differential equations of motion.

This chapter has illustrated that variational principles provide a means of deriving more detailed information, such as the trajectories for the motion between given initial and final conditions, by requiring that scalar functionals have extrema values. For example, the solution of the brachistochrone problem determined the trajectory having the minimum transit time, based on only the magnitudes of the kinetic and gravitational potential energies. Similarly, the catenary shape of a suspended chain was derived by minimizing the gravitational potential energy. The calculus of variations uses Euler's equations to determine directly the differential equations of motion of the system that lead to the functional of interest being stationary at an extremum. The Lagrangian and Hamiltonian variational approaches to classical mechanics are discussed in chapters 6 – 16. The broad range of applicability, the flexibility, and the power provided by variational approaches to classical mechanics and modern physics will be illustrated.

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