

10.2: Origins of Nonconservative Motion

Nonconservative degrees of freedom involve irreversible processes, such as dissipation, damping, and also can result from course-graining, or ignoring coupling to active degrees of freedom. The nonconservative role of ignored active degrees of freedom is illustrated by the weakly-coupled double harmonic oscillator system discussed below. Let the two harmonic oscillators have masses (m_1, m_2) , uncoupled angular frequencies (ω_1, ω_2) , and oscillation amplitudes (q_1, q_2) . Assume that the coupling potential energy is $U = \lambda q_1 q_2$. The Lagrangian for this weakly-coupled double oscillator is

$$L(q_1, q_2, \dot{q}_1, \dot{q}_2, t) = \frac{m_1}{2} (\dot{q}_1^2 - \omega_1^2 q_1^2) + \lambda q_1 q_2 + \frac{m_2}{2} (\dot{q}_2^2 - \omega_2^2 q_2^2) \quad (10.2.1)$$

Note that the total Lagrangian is conservative since the Lagrangian is explicitly time independent. As shown in chapter 14.2, the solution for the amplitudes of the oscillation for the coupled system are given by

$$\begin{aligned} q_1(t) &= D \sin\left[\left(\frac{\omega_1 + \omega_2}{2}\right)t\right] \sin\left[\left(\frac{\omega_1 - \omega_2}{2}\right)t\right] \\ q_2(t) &= D \cos\left[\left(\frac{\omega_1 + \omega_2}{2}\right)t\right] \cos\left[\left(\frac{\omega_1 - \omega_2}{2}\right)t\right] \end{aligned}$$

The system exhibits the common "beats" behavior where the coupled harmonic oscillators have an angular frequency that is the average oscillator frequency $\omega_{average} = \left(\frac{\omega_1 + \omega_2}{2}\right)$, and the oscillation intensities are modulated at the difference frequency, $\omega_{difference} = \left(\frac{\omega_1 - \omega_2}{2}\right)$. Although the total energy is conserved for this conservative system, this shared energy flows back and forth between the two coupled harmonic oscillators at the difference frequency. If the equations of motion for oscillator 1 ignore the coupling to the motion of oscillator 2, that is, assume a constant average value $q_2 = \langle q_2 \rangle$ is used, then the intensity $|q_1|^2$ and energy of the first oscillator still is modulated by the $\left|\sin\left(\frac{\omega_1 - \omega_2}{2}t\right)\right|^2$ term. Thus the total energy for this truncated coupled-oscillator system is no longer conserved due to neglect of the energy flowing into and out of oscillator 1 due to its coupling to oscillator 2. That is, the solution for the truncated system of oscillator 1 is not conservative since it is exchanging energy with the coupled, but ignored, second oscillator. This elementary example illustrates that ignoring active degrees of freedom can transform a conservative system into a nonconservative system, for which the equations of motion derived using the truncated Lagrangian is incorrect.

The above example illustrates the importance of including all active degrees of freedom when deriving the equations of motion, in order to ensure that the total system is conservative. Unfortunately, nonconservative systems due to viscous or frictional dissipation typically result from weak thermal interactions with an enormous number of nearby atoms, which makes inclusion of all of these degrees of freedom impractical. Even though the detailed behavior of such dissipative degrees of freedom may not be of direct interest, all the active degrees of freedom must be included when applying Lagrangian or Hamiltonian mechanics.

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