# 8.1: Introduction

The three major formulations of classical mechanics are

1. **Newtonian mechanics** which is the most intuitive vector formulation used in classical mechanics.
2. **Lagrangian mechanics** is a powerful algebraic formulation of classical mechanics derived using either d'Alembert's Principle, or Hamilton's Principle. The latter states *"A dynamical system follows a path that minimizes the time integral of the difference between the kinetic and potential energies"*.
3. **Hamiltonian mechanics** has a beautiful superstructure that, like Lagrangian mechanics, is built upon variational calculus, Hamilton's principle, and Lagrangian mechanics.

Hamiltonian mechanics is introduced at this juncture since it is closely interwoven with Lagrange mechanics. Hamiltonian mechanics plays a fundamental role in modern physics, but the discussion of the important role it plays in modern physics will be deferred until chapters 15 and 18 where applications to modern physics are addressed.

The following important concepts were introduced in chapter 7:

The **generalized momentum** was defined to be given by

$$p_i \equiv \frac{\partial L(\mathbf{q},\dot{\mathbf{q}},t)}{\partial \dot{q}_i} \tag{8.1.1}$$

Note that, as discussed in chapter 7.2, if the potential is velocity dependent, such as the Lorentz force, then the generalized momentum includes terms in addition to the usual mechanical momentum.

Jacobi's **generalized energy function** $h(\mathbf{q},\dot{\mathbf{q}},t)$ was introduced where

$$h(\mathbf{q},\dot{\mathbf{q}},t) = \sum_i^n \left(\dot{q}_i \frac{\partial L}{\partial \dot{q}_i}\right) - L(\mathbf{q},\dot{\mathbf{q}},t) \tag{8.1.2}$$

The **Hamiltonian function** was defined to be given by expressing the generalized energy function, Equation 8.1.2, in terms of the generalized momentum. That is, the Hamiltonian $H(\mathbf{q},\mathbf{p},t)$ is expressed as

$$H(\mathbf{q},\mathbf{p},t) = \sum_i^n p_i \dot{q}_i - L(\mathbf{q},\dot{\mathbf{q}},t) \tag{8.1.3}$$

The symbols $\mathbf{q}$, $\mathbf{p}$, designate vectors of $n$ generalized coordinates, $\mathbf{q} \equiv (q_1, q_2, ..q_n)$, $\mathbf{p} \equiv (p_1, p_2, ..p_n)$. Equation 8.1.3 can be written compactly in a symmetric form using the scalar product $\mathbf{p}\cdot\dot{\mathbf{q}} = \sum_i p_i \dot{q}_i$.

$$H(\mathbf{q},\mathbf{p},t) + L(\mathbf{q},\dot{\mathbf{q}},t) = \mathbf{p}\cdot\dot{\mathbf{q}} \tag{8.1.4}$$

A crucial feature of Hamiltonian mechanics is that the Hamiltonian is expressed as $H(\mathbf{q},\mathbf{p},t)$, that is, *it is a function of the $n$ generalized coordinates and their conjugate momenta, which are taken to be independent*, plus the independent variable, time. This contrasts with the Lagrangian $L(\mathbf{q},\dot{\mathbf{q}},t)$ which is a function of the $n$ generalized coordinates $q_j$, and the corresponding velocities $\dot{q}_j$, that is the time derivatives of the coordinates $q_i$, plus the independent variable, time.