7.5: Cyclic Coordinates

Translational and rotational invariance occurs when a system has a cyclic coordinate q_k. *A cyclic coordinate is one that does not explicitly appear in the Lagrangian*. The term cyclic is a natural name when one has cylindrical or spherical symmetry. In Hamiltonian mechanics a cyclic coordinate often is called an *ignorable coordinate* . By virtue of Lagrange's equations

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k} = 0 \tag{7.5.1}$$

then a cyclic coordinate q_k, is one for which $\frac{\partial L}{\partial q_k} = 0$. Thus

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} = \dot{p}_k = 0 \tag{7.5.2}$$

that is, p_k *is a constant of motion if the conjugate coordinate* q_k *is cyclic*. This is just Noether's Theorem.