13.26: Rotation of Deformable Bodies

The discussion in this chapter has assumed that the rotating body is a rigid body. However, there is a broad and important class of problems in classical mechanics where the rotating body is deformable that leads to intriguing new phenomena. The classic example is the cat, which, if dropped upside down with zero angular momentum, is able to distort its body plus tail in order to rotate such that it lands on its feet in spite of the fact that there are no external torques acting and thus the angular momentum is conserved. Another example is the high diver doing a forward two—and-a-half somersault with two twists.[Fro80] Once the diver leaves the board then the total angular momentum must be conserved since there are no external torques acting on the system. The diver begins a somersault by rotating about a horizontal axis which is a principal axis that is perpendicular to the axis of his body passing through his hips. Initially the angular momentum, and angular velocity, are parallel and point perpendicular to the symmetry axis. Initially the diver goes into a tuck which greatly reduces his moment of inertia along the axis of his somersault which concomitantly increases his angular velocity about this axis and he performs one full somersault prior to initiating twisting. Then the diver twists its body and moves its arms to destroy the axial symmetry of his body which changes the direction of the principal axes of the inertia tensor. This causes the angular velocity to change in both direction and magnitude such that the angular momentum remains conserved. The angular velocity now is no longer parallel to the angular momentum resulting in a component along the length of the body causing it to twist while somersaulting. This twisting motion will continue until the symmetry of the diver's body is restored which is done just before entering the water. By skilled timing, and body movement, the diver restores the symmetry of his body to the optimum orientation for entering the water. Such phenomena involving deformable bodies are important to motion of ballet dancers, jugglers, astronauts in space, and satellite motion. The above rotational phenomena would be impossible if the cat or diver were rigid bodies having a fixed inertia tensor. Calculation of the dynamics of the motion of deformable bodies is complicated and beyond the scope of this book, but the concept of a time dependent transformation of the inertia tensor underlies the subsequent motion. The theory is complicated since it is difficult even to quantify what corresponds to rotation as the body morphs from one shape to another. Further information on this topic can be found in the literature. [Fro80]