14.1: Introduction to Coupled Linear Oscillators

Chapter 3 discussed the behavior of a single linearly-damped linear oscillator subject to a harmonic force. No account was taken for the influence of the single oscillator on the driver for the case of forced oscillations. Many systems in nature comprise complicated free or forced oscillations of coupled-oscillator systems. Examples of coupled oscillators are; automobile suspension systems, electronic circuits, electromagnetic fields, musical instruments, atoms bound in a crystal, neural circuits in the brain, networks of pacemaker cells in the heart, etc. Energy can be transferred back and forth between coupled oscillators as the motion evolves. However, it is possible to describe the motion of coupled linear oscillators in terms of a sum over independent normal coordinates, i.e. normal modes, even though the motion may be very complicated. These normal modes are constructed from the original coordinates in such a way that the normal modes are uncoupled. The topic of finding the normal modes of coupled oscillator systems is a ubiquitous problem encountered in all branches of science and engineering. As discussed in chapter 3, oscillatory motion of non-linear systems can be complicated. Fortunately most oscillatory systems are approximately linear when the amplitude of oscillation is small. This discussion assumes that the oscillation amplitudes are sufficiently small to ensure linearity.