# 17.E: Relativistic Mechanics (Exercises)

1. A relativistic snake of proper length $100\ cm$ is travelling to the right across a butcher's table at $v = 0.6c$. You hold two meat cleavers, one in each hand which are $100\ cm$ apart. You strike the table simultaneously with both cleavers at the moment when the left cleaver lands just behind the tail of the snake. You rationalize that since the snake is moving with $\beta = 0.6$, then the length of the snake is Lorentz contracted by the factor $\gamma = \frac{5}{4}$ and thus the Lorentz-contracted length of the snake is $80\ cm$ and thus will not be harmed. However, the snake reasons that relative to it the cleavers are moving at $\beta = 0.6$ and thus are only $80\ cm$ apart when they strike the $100\ cm$ long snake and thus it will be severed. Use the Lorentz transformation to resolve this paradox.

2. Explain what is meant by the following statement: "Lorentz transformations are orthogonal transformations in Minkowski space."

3. Which of the following are invariant quantities in space-time?

a. Energy
b. Momentum
c. Mass
d. Force
e. Charge
f. The length of a vector
g. The length of a four-vector

4. What does it mean for two events to have a spacelike interval? What does it mean for them to have a timelike interval? Draw a picture to support your answer. In which case can events be causally connected?

5. A supply rocket flies past two markers on the Space Station that are $50\ m$ apart in a time of $0.2\ \mu s$ as measured by an observer on the Space station.

a. What is the separation of the two markers as seen by the pilot riding in the supply rocket?
b. What is the elapsed time as measured by the pilot in the supply rocket?
c. What are the speeds calculated by the observer in the Space Station and the pilot of the supply rocket?

6. The Compton effect involves a photon of incident energy $E_i$ being scattered by an electron of mass $m_e$ which initially is stationary. The photon scattered at an angle $\theta$ with respect to the incident photon has a final energy $E_f$. Using the special theory of relativity derive a formula that related $E_f$ and $E_i$ to $\theta$.

7. Pair creation involves production of an electron-positron pair by a photon. Show that such a process is impossible unless some other body, such as a nucleus, is involved. Suppose that the nucleus has a mass $M$ and the electron mass $m_e$. What is the minimum energy that the photon must have in order to produce an electron-positron pair?

8. A $K$ meson of rest energy $494\ MeV$ decays into a $\mu$ meson of rest energy $106\ MeV$ and a neutrino of zero rest energy. Find the kinetic energies of the $\mu$ meson and the neutrino into which the $K$ meson decays while at rest.