11.1: Introduction to Conservative two-body Central Forces

Conservative two-body central forces are important in physics because of the pivotal role that the Coulomb and the gravitational forces play in nature. The Coulomb force plays a role in electrodynamics, molecular, atomic, and nuclear physics, while the gravitational force plays an analogous role in celestial mechanics. Therefore this chapter focusses on the physics of systems involving conservative two-body central forces because of the importance and ubiquity of these conservative two-body central forces in nature.

A conservative two-body central force has the following three important attributes.

1. **Conservative:** A conservative force depends only on the particle position, that is, the force is not time dependent. Moreover the work done by the force moving a body between any two points 1 and 2 is path independent. Conservative fields are discussed in chapter 2.10.
2. **Two-body:** A two-body force between two bodies depends only on the relative locations of the two interacting bodies and is not influenced by the proximity of additional bodies. For two-body forces acting between n bodies, the force on body 1 is the vector superposition of the two-body forces due to the interactions with each of the other $n-1$ bodies. This differs from three-body forces where the force between any two bodies is influenced by the proximity of a third body.
3. **Central:** A central force field depends on the distance r_{12} from the origin of the force at point 1, to the body location at point 2, and the force is directed along the line joining them, that is, $\hat{\mathbf{r}}_{12}$.

A conservative, two-body, central force combines the above three attributes and can be expressed as,

$$\mathbf{F}_{21}=f(r_{12})\hat{\mathbf{r}}_{12} \tag{11.1.1}$$

The force field $\mathbf{F}_{21}$ has a magnitude $f(r_{12})$ that depends only on the magnitude of the relative separation vector $\mathbf{r}_{12}=\mathbf{r}_2-\mathbf{r}_1$ between the origin of the force at point 1 and point 2 where the force acts, and the force is directed along the line joining them, that is, $\hat{\mathbf{r}}_{12}$.

Chapter 2.10 showed that if a two-body central force is conservative, then it can be written as the gradient of a scalar potential energy $U(r)$ which is a function of the distance from the center of the force field.

$$\mathbf{F}_{21}=-\nabla U(r_{12}) \tag{11.1.2}$$

As discussed in chapter 2, the ability to represent the conservative central force by a scalar function $U(r)$ greatly simplifies the treatment of central forces.

The Coulomb and gravitational forces both are true conservative, two-body, central forces whereas the nuclear force between nucleons in the nucleus has three-body components. Two bodies interacting via a two-body central force is the simplest possible system to consider, but Equation $11.1.1$ is applicable equally for n bodies interacting via two-body central forces because the superposition principle applies for two-body central forces. This chapter will focus first on the motion of two bodies interacting via conservative two-body central forces followed by a brief discussion of the motion for $n>2$ interacting bodies.