10.6: Mathematical Formulas - Vector Identities

Algebraic Identities

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B}) \tag{10.6.1}$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B}) \tag{10.6.2}$$

Identities Involving Differential Operators

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0 \tag{10.6.3}$$

$$\nabla\times(\nabla f) = 0 \tag{10.6.4}$$

$$\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) + (\nabla f)\times\mathbf{A} \tag{10.6.5}$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B}) \tag{10.6.6}$$

$$\nabla\cdot(\nabla f) = \nabla^2 f \tag{10.6.7}$$

$$\nabla\times\nabla\times\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A} \tag{10.6.8}$$

$$\nabla^2\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla\times(\nabla\times\mathbf{A}) \tag{10.6.9}$$

Divergence Theorem

Given a closed surface $\mathcal{S}$ enclosing a contiguous volume $\mathcal{V}$,

$$\int_{\mathcal{V}} (\nabla\cdot\mathbf{A})\, dv = \oint_{\mathcal{S}} \mathbf{A}\cdot d\mathbf{s}$$

where the surface normal $d\mathbf{s}$ is pointing out of the volume.

Stokes' Theorem

Given a closed curve $\mathcal{C}$ bounding a contiguous surface $\mathcal{S}$,

$$\int_{\mathcal{S}} (\nabla\times\mathbf{A})\cdot d\mathbf{s} = \oint_{\mathcal{C}} \mathbf{A}\cdot d\mathbf{l}$$

where the direction of the surface normal $d\mathbf{s}$ is related to the direction of integration along $\mathcal{C}$ by the "right hand rule."