

5.11: Kirchoff's Voltage Law for Electrostatics - Differential Form

The integral form of Kirchoff's Voltage Law for electrostatics (KVL; Section 5.10) states that an integral of the electric field along a closed path is equal to zero:

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0$$

where \mathbf{E} is electric field intensity and C is the closed curve. In this section, we derive the differential form of this equation. In some applications, this differential equation, combined with boundary conditions imposed by structure and materials (Sections 5.17 and 5.18), can be used to solve for the electric field in arbitrarily complicated scenarios. A more immediate reason for considering this differential equation is that we gain a little more insight into the behavior of the electric field, disclosed at the end of this section.

The equation we seek may be obtained using Stokes' Theorem (Section 4.9), which in the present case may be written:

$$\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{s} = \oint_C \mathbf{E} \cdot d\mathbf{l} \quad (5.11.1)$$

where S is any surface bounded by C , and $d\mathbf{s}$ is the normal to that surface with direction determined by right-hand rule. The integral form of KVL tells us that the right hand side of the above equation is zero, so:

$$\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{s} = 0$$

The above relationship must hold regardless of the specific location or shape of S . The only way this is possible for all possible surfaces is if the integrand is zero at every point in space. Thus, we obtain the desired expression:

$$\boxed{\nabla \times \mathbf{E} = 0} \quad (5.11.2)$$

Summarizing:

The differential form of Kirchoff's Voltage Law for electrostatics (Equation 5.11.2) states that the curl of the electrostatic field is zero.

Equation 5.11.2 is a partial differential equation. As noted above, this equation, combined with the appropriate boundary conditions, can be solved for the electric field in arbitrarily-complicated scenarios. Interestingly, it is not the only such equation available for this purpose – Gauss' Law (Section 5.7) also does this. Thus, we see a *system* of partial differential equations emerging, and one may correctly infer that the electric field is not necessarily fully constrained by either equation alone.

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