

10.6: Mathematical Formulas - Vector Identities

Algebraic Identities

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) \quad (10.6.1)$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B}) \quad (10.6.2)$$

Identities Involving Differential Operators

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0 \quad (10.6.3)$$

$$\nabla \times (\nabla f) = 0 \quad (10.6.4)$$

$$\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) + (\nabla f) \times \mathbf{A} \quad (10.6.5)$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B}) \quad (10.6.6)$$

$$\nabla \cdot (\nabla f) = \nabla^2 f \quad (10.6.7)$$

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \quad (10.6.8)$$

$$\nabla^2 \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla \times (\nabla \times \mathbf{A}) \quad (10.6.9)$$

Divergence Theorem

Given a closed surface \mathcal{S} enclosing a contiguous volume \mathcal{V} ,

$$\int_{\mathcal{V}} (\nabla \cdot \mathbf{A}) dv = \oint_{\mathcal{S}} \mathbf{A} \cdot d\mathbf{s}$$

where the surface normal $d\mathbf{s}$ is pointing out of the volume.

Stokes' Theorem

Given a closed curve \mathcal{C} bounding a contiguous surface \mathcal{S} ,

$$\int_{\mathcal{S}} (\nabla \times \mathbf{A}) \cdot d\mathbf{s} = \oint_{\mathcal{C}} \mathbf{A} \cdot d\mathbf{l}$$

where the direction of the surface normal $d\mathbf{s}$ is related to the direction of integration along \mathcal{C} by the “right hand rule.”

This page titled [10.6: Mathematical Formulas - Vector Identities](#) is shared under a [CC BY-SA 4.0](#) license and was authored, remixed, and/or curated by [Steven W. Ellingson](#) (Virginia Tech Libraries' Open Education Initiative).

- **10.6: Mathematical Formulas - Vector Identities** by [Steven W. Ellingson](#) is licensed [CC BY-SA 4.0](#). Original source: <https://doi.org/10.21061/electromagnetics-vol-1>.