6.9: The Schrödinger Equation Activities

A 1.0 kg ball is thrown directly upward at 10 m/s and the zero-point of gravitational potential energy is located at the position at which the ball leaves the thrower's hand. All heights are measured relative to the zero-point. Below is a graph of gravitational potential energy (U = mgh) vs. height. Answer the following questions.

a. Draw a line on the graph representing the total energy of the ball. Using the graph, determine the maximum height reached by the ball.

b. Using the graph, determine the kinetic energy of the ball when it is at one-half of its maximum height.

c. What would happen to the kinetic energy of the ball if it was at 1 m above its maximum height. What would this imply about the velocity of the ball at this location?

d. Does the ball spend more time, less time or the same amount time in the position interval between 3 m and 4 m or in the position interval between 4 m and 5 m? Why?

e. If a determination of the ball's position is made at a random time, is it more likely, less likely or equally likely to find the ball in the position interval between 3 m and 4 m or in the position interval between 4 m and 5 m? Why?
A 2.0 kg cart oscillates on a horizontal, frictionless surface attached to a k = 50 N/m spring. The cart passes through the spring equilibrium length (s = 0 m) at a speed of 10 m/s. Below is a graph of elastic potential energy (U = ½ ks2) vs. spring deformation (s). Answer the following questions.

a. Draw a line on the graph representing the total energy of the cart. Using the graph, determine the maximum elongation of the spring.

b. Using the graph, determine the kinetic energy of the cart when the spring is at one-half of its maximum elongation.

c. What would happen to the kinetic energy of the cart if it were 1m beyond the maximum elongation of the spring? What would this imply about the velocity of the cart at this location?

d. Does the cart spend more time, less time or the same amount time in the position interval between 0 m and 1 m or in the position interval between 1 m and 2 m? Why?

e. If a determination of the cart's position is made at a random time, is it more likely, less likely or equally likely to find the cart in the position interval between 0 m and 1 m or in the position interval between 1 m and 2 m? Why?

Below is a graph of potential energy (U) vs. position for a region of space occupied by a single macroscopic particle. There are no forces acting on the particle in this region of space other than the force that gives rise to the potential energy. The total energy of the particle is also indicated on the graph. The letters A through F mark six positions within this region of space.

a. Rank the potential energy of the particle at these positions.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

b. Rank the kinetic energy of the particle at these positions.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

c. If a determination of the particle's position is made at a random time, rank the probability of finding the particle in the immediate vicinity of these positions.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

Below is a graph of potential energy (U) vs. position for a region of space occupied by a single macroscopic particle. There are no forces acting on the particle in this region of space other than the force that gives rise to the potential energy. The total energy of the particle is also indicated on the graph. The letters A through F mark six positions within this region of space.

a. Rank the potential energy of the particle at these positions. If the particle is never at a position leave it out of the ranking.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

b. Rank the kinetic energy of the particle at these positions. If the particle is never at a position leave it out of the ranking.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

c. If a determination of the particle's position is made at a random time, rank the probability of finding the particle in the immediate vicinity of these positions.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

For each of the potential energy functions below, carefully sketch the wavefunction corresponding to the energy level indicated.

a.

b.

c.

d.

For each of the potential energy functions below, carefully sketch the wavefunction corresponding to the energy level indicated.

a.

b.

c.

d.

For each of the potential energy functions below, carefully sketch the continuation of the wavefunction as it passes the barrier or well.

a.

b.

c.

d.

A hypothetical atom has the four electron energy levels shown below.

a. How many spectral lines can be emitted by transitions between these four energy levels?

b. The transition from level ____ to level ____ emits the longest wavelength photon.

c. The transition from level ____ to level ____ involves the absorption of the largest energy photon.

d. If the energy of the n = 3 level was somehow reduced, how many of the six spectral lines would change energy?

e. If the energy of the n = 3 level was somehow reduced, which electron transition(s) would change to larger energy?

f. In which level(s) is the electron most likely to be detected in the immediate vicinity of x = 0?

g. In which level(s) does the electron spend the most time outside of the atom?

g. In which level(s) is the electron most likely to be detected within the right half of the atom?

A proton is trapped in an infinite potential well of length 1.0 x 10-15 m. Find the three longest wavelength photons emitted by the proton as it changes energy levels in the well.

Mathematical Analysis

An electron is trapped in an infinite potential well of length 1.0 x 10-10 m. Find the three longest wavelength photons emitted by the electron as it changes energy levels in the well.

Mathematical Analysis

Imagine eight neutrons trapped in an atomic nucleus. Model the nucleus as an infinite potential well of length 5.0 x 10-15 m.
a. Find the total kinetic energy of the eight neutrons.
b. If four of the neutrons changed into protons, calculate the new total kinetic energy. Ignore the mass difference between protons and neutrons.

Mathematical Analysis

Imagine eight neutrons and five protons trapped in an atomic nucleus. Model the nucleus as an infinite potential well of length 5.0 x 10-15 m.
a. Find the total kinetic energy of the nucleons.
b. If neutrons and protons can freely change into each other, what will happen?
c. Calculate the minimum total kinetic energy of the nucleons. Ignore the mass difference between protons and neutrons.

Mathematical Analysis

A photon is trapped in an infinite potential well of length L. Find the allowed energies for the photon. (Hint: You cannot use the Schrödinger equation to solve this problem.)

Mathematical Analysis

You may occasionally feel trapped in a classroom. If so, you may find yourself unable to be completely stationary. Assuming a 10 m wide classroom, and a 65 kg student, estimate your minimum energy and velocity.

Mathematical Analysis

An electron is trapped in an infinite potential well of length 1.0 x 10-10 m.
a. According to classical physics (i.e., common sense), what is the probability that the electron will be found in the middle fifth

(from 0.4 x 10-10 m to 0.6 x 10-10 m) of the well?
b. In its ground state, what is the probability that the electron will be found in the middle fifth of the well?
c. In its n=2 state, what is the probability that the electron will be found in the middle fifth of the well?
d. Based on the shape of the wavefunction, explain why (b) is greater than (a), and (a) is greater than (c)? (If they aren't, you did the problem incorrectly!)

Mathematical Analysis

A neutron is trapped in an infinite potential well of length 4.0 x 10-15 m.
a. According to classical physics (i.e., common sense), what is the probability that the neutron will be found in the left quarter (from 0.0 x 10-15 m to 1.0 x 10-15 m) of the well?
b. In its ground state, what is the probability that the neutron will be found in the left quarter of the well?
c. In its n=2 state, what is the probability that the neutron will be found in the left quarter of the well?
d. Based on the shape of the wavefunctions, do your answers for a, b, and c have the correct relative size?

Mathematical Analysis

A particle is trapped in a one-dimensional infinite potential well of length L.
a. Find the expectation values of the particle's position and momentum in the first excited state of this well. Compare these results to the results for the ground state.
b. Find the uncertainty in the particle's position and momentum in the first excited state of this well. Compare these results to the results for the ground state.
c. Show that the uncertainties in these values do not violate the uncertainty principle.

Mathematical Analysis

A particle is trapped in a one-dimensional infinite potential well of length L.
a. Find the expectation value of the particle's position as a function of energy level, n.
b. Find the uncertainty in the particle's position as a function of energy level, n.
c. In the limit of very large n, what is the uncertainty in the particle's position? (In classical physics, this value would be 0.289L.)

Mathematical Analysis

A particle is trapped in a one-dimensional infinite potential well of length L.
a. Find the expectation value of the particle's kinetic energy in the ground state of this well. (Hint: Kinetic energy can be expressed as p2/2m. This should allow you to construct an operator for kinetic energy.)
b. Find the uncertainty in the particle's kinetic energy in the ground state of this well.
c. Carefully explain what your answer for (b) implies about the time the particle can remain in the ground state.

Mathematical Analysis

Classically, a particle trapped in a one-dimensional infinite potential well of length L would have an equal probability of being detected anywhere in the well. Thus, its "classical" wavefunction would be:

where C is a constant.
a. Find the value of C by setting the total probability of finding the particle in the well equal to 1.
b. Find the expectation value of the particle's position.
c. Find the uncertainty in the particle's position.

Mathematical Analysis

A particle trapped in a one-dimensional parabolic potential well centered on x = 0 has a ground-state wavefunction given by:

Note that this type of well extends from x = -∞ to x = +∞.
a. Find A.
b. Find the expectation values of the particle's position and momentum in the ground state of this well.
c. Find the uncertainty in the particle's position and momentum in the ground state of this well.

d. Show that the uncertainties in these values do not violate the uncertainty principle.

Mathematical Analysis

A particle of mass m is trapped in an infinite potential well of x-length L and y-width 3L. For each of the pairs of quantum numbers below, state the locations (other than the boundaries) where the probability of detecting the particle is zero.
a. (nx, ny) = (1, 1)
b. (nx, ny) = (3, 1)
c. (nx, ny) = (2, 3)

Mathematical Analysis

A particle of mass m is trapped in an infinite potential well of x-length L, y-width L, and z-height 2L. For each of the triplets of quantum numbers below, state the locations (other than the boundaries) where the probability of detecting the particle is zero.
a. (nx, ny, nz) = (1, 1, 1)
b. (nx, ny, nz) = (2, 2, 1)
c. (nx, ny, nz) = (1, 2, 2)

Mathematical Analysis

A particle of mass m is trapped in an infinite potential well of x-length L and y-width L. Determine the 5 lowest energy levels and list them below.

Mathematical Analysis

Level nx ny E ( )
1 1 1

A particle of mass m is trapped in an infinite potential well of x-length 3L and y-width 2L. Determine the 5 lowest energy levels and list them below.

Mathematical Analysis

Level nx ny E ( )
1 1 1

Eight neutrons and five protons are trapped in an infinite potential well of x-length 4.0 x 10-15 m and y-width 4.0 x 10-15 m. Ignore the mass difference between protons and neutrons.
a. Determine the 5 lowest energy levels and list them below.
b. Find the total kinetic energy of the 13 particles.
c. Would the total energy decrease if a neutron turned into a proton? If so, by how much?

Mathematical Analysis

Level nx ny E ( ) # neutrons # protons
1 1 1 2 2

Eight protons and eleven neutrons are trapped in an infinite potential well of x-length 4.0 x 10-15 m and y-width 2.0 x 10-15 m. Ignore the mass difference between protons and neutrons.
a. Determine the 5 lowest energy levels and list them below.
b. Find the total kinetic energy of the 19 particles.
c. Would the total energy decrease if a neutron turned into a proton? If so, by how much?

Mathematical Analysis

Level nx ny E ( ) # neutrons # protons
1 1 1 2 2

A particle of mass m is trapped in an infinite potential well of x-length L, y-width L, and z-height L. Determine the 5 lowest energy levels and list them below.

Mathematical Analysis

Level nx ny nz E ( )
1 1 1 1

A particle of mass m is trapped in an infinite potential well of x-length L, y-width L, and z-height 2L. Determine the 5 lowest energy levels and list them below.

Mathematical Analysis

Level nx ny nz E ( )
1 1 1 1

A particle of mass m is trapped in an infinite potential well of x-length L, y-width 2L, and z-height 2L. Determine the 5 lowest energy levels and list them below.

Mathematical Analysis

Level nx ny nz E ( )
1 1 1 1

Find the allowed energy levels for a proton trapped in a semi-infinite potential well of width 3.0 fm and depth 40 MeV. Compare these values to those obtained assuming the well is infinitely deep.

Mathematical Analysis

Find the allowed energy levels for an electron trapped in a semi-infinite potential well of width 1.0 nm and depth 5.0 eV. Compare these values to those obtained assuming the well is infinitely deep.

Mathematical Analysis

Find the allowed energy levels for a neutron trapped in a semi-infinite potential well of width 7.0 fm and depth 50 MeV. Compare these values to those obtained assuming the well is infinitely deep.

Mathematical Analysis

A proton is incident on a rectangular potential barrier of height 50 MeV and width 5 fm. What is the approximate probability that the proton will tunnel through the barrier for each of the incident kinetic energies below?
a. 20 MeV
b. 30 MeV
c. 40 MeV
d. 45 MeV

Mathematical Analysis

a. Estimate the tunneling probability for an 18 MeV proton incident on a symmetric potential well with barrier height 20 MeV, barrier width 3 fm, well depth -50 MeV, and well width 15 fm.
b. If the proton successfully tunnels into the well, estimate the lifetime of the resulting state.

Mathematical Analysis

a. Estimate the tunneling probability for a 5 MeV alpha particle incident on a symmetric potential well with barrier height 40 MeV, barrier width 8 fm, well depth -50 MeV, and well width 15 fm.
b. If the proton successfully tunnels into the well, estimate the lifetime of the resulting state.

Mathematical Analysis

a. Estimate the tunneling probability for a 1.0 MeV proton incident on a symmetric potential well with barrier height 1.5 MeV, barrier width 1 fm, well depth -10 MeV, and well width 3.0 fm.
b. If the proton successfully tunnels into the well, estimate the lifetime of the resulting state.

Mathematical Analysis

In a sparkplug, a potential difference of about 20,000 V is needed for a spark to jump the 1.5 mm gap. Modeling this as a rectangular potential barrier of height 20 keV and width 1.5 mm:
a. Estimate the probability of an electron tunneling across the sparkplug gap when the potential difference across the gap is only 10,000 V. Based on this answer, should auto mechanics worry about quantum mechanics?
b. At what potential difference would tunneling provide a one-in-a-billion chance of a premature spark?

Mathematical Analysis

In a transistor a rule of thumb is "60 mV per decade", meaning that a voltage change of 60 mV should cause a tenfold increase (or decrease) in current. In a tunneling transistor we can image a rectangular potential barrier of width 10 nm with a height that can be adjusted by the applied voltage. What height barrier is needed so that a change of 60 mV (60meV of energy) causes a tenfold change in current?

Mathematical Analysis

In a scanning tunneling microscope (STM), a slender metal tip is positioned very close to a sample under study. Although no electric contact is made between tip and sample, electrons from the tip can tunnel across the empty space to the sample, resulting in an electric current. Since this current is exponentially dependent on the separation between the tip and the sample, incredibly precise measurements of surface features are possible.

Approximating the potential barrier between tip and sample to be a rectangular barrier with height equal to a typical metallic work function (4.0 eV), find the change in separation between tip and sample that will result in a 10% change in tunneling current. This change in separation is approximately the resolution of the STM.

Mathematical Analysis

In an ammonia molecule (NH3), the nitrogen atom is equally likely to be "above" or "below" the plane formed by the three hydrogen atoms. In fact, the nitrogen atom tunnels back and forth between these two equivalent orientations at an incredibly stable frequency. The stable frequency of ammonia inversion was used as the standard for the first generation of atomic clocks.

Approximating the potential barrier between the above and below orientations as a rectangular barrier with height U = 0.26 eV, and width L = 0.038 nm, find the frequency with which the nitrogen oscillates between these two states. The energy of the nitrogen atom in either well is E = 0.25 eV.

Mathematical Analysis