2.4: Interstellar Travel – Energy Issues (Project)

In a previous investigation, you discovered some of the fundamental difficulties in traveling to even a relatively nearby star. In this investigation, you will be introduced to a much more important factor, the incredible amounts of energy needed to accelerate a macroscopic object to relativistic speeds.

I. Energy Considerations

Consider a spaceship fully fueled of mass M. Some portion of the mass of the spaceship is in the form of its fuel, such that after "burning" its fuel the ship will have a smaller mass. This mass is the "useful" mass of the spacecraft, the payload, and we will designate it fM (a fraction, f, of the total loaded mass). For example, if the ship is 5% fuel, $f = 0.95$.

1. What is an expression for the amount of fuel on the fully loaded spacecraft?

A. Fusion powered spaceship

The most efficient energy source available using today's technology is fusion energy, and given that it is the energy source used by stars, it's probably a good bet that it may be the most efficient large scale energy source available in the universe.

Imagine a spacecraft powered by fusion energy. The most efficient fusion reaction effectively combines four hydrogen nuclei into a single helium nucleus. Since a helium nucleus is less massive than four hydrogen nuclei, the energy released by this reaction can be calculated using Einstein's relation:

$$E = mc^2. \tag{2.4.1}$$

1. Determine the energy released by this reaction by converting the mass difference between the helium nucleus and the four hydrogen nuclei into a measure of energy.

2. The efficiency of the reaction, ϵ, is defined to be the ratio of energy released to energy present in the initial hydrogen nuclei. Calculate the efficiency of this fusion reaction.

3. Using your expression for the total amount of fuel on the spacecraft, and the efficiency of the fusion reaction, write a symbolic expression (use ϵ, not its numerical value) for the total energy released by fusing all of the fuel.

The energy released via fusion will go toward providing kinetic energy to the spacecraft as well as kinetic energy to the exhaust helium. Calculating the portion of this energy that goes into moving the spacecraft is a complicated problem, so we'll simply approximate that **all** of the energy goes into kinetic energy of the ship! (This is a huge over-estimation, but we will find that even with this approximation, a fusion powered spaceship is not very feasible.)

4. Equate the kinetic energy of the payload (fM) with the total energy released by fusing all of the fuel. From this equation, solve for f, the fraction of the initial mass of the ship that is payload.

5. In the last investigation, you found that at a cruising speed of $v = 0.9992c$, a spacecraft could reach Vega in a little less than 3 years. Using the result above, what fraction of the mass of that spaceship can be payload, and how much of the ship must be made up of fuel?

You should have found that 99.97% of the mass of your spaceship must be fuel! A more useful way to express the amount of fuel needed for acceleration is to calculate the fuel to payload ratio. This is the number of kg of fuel needed to accelerate a single kg of payload.

6. Calculate the fuel to payload ratio for your spaceship.

For every 1 kg of payload you want to take to Vega, you would have to supply the ship with over 3000 kg of fuel. Actually, the situation is far worse. This is simply the fuel required to accelerate the payload up to the cruising speed. For a complete round trip, one would have to carry enough fuel to boost the payload to the cruising speed, decelerate it to rest at the destination, boost it to cruising speed again for the return trip home, and decelerate it upon reaching earth.

7. How much fuel is required for a complete round trip? Express your answer as a multiple of the payload mass, i.e., the number of kg of fuel per kg of payload. Explain why the answer is not simply four times the fuel required to boost the payload alone to the cruising speed.

This number borders on the ridiculous. You would need over $10^{14}\ kg$ of fuel to take a single one kg mass on a round-trip to Vega.

8. If you aren't as anxious to get to Vega quickly, you could travel at a cruising speed of $0.5\,c$. Calculate the amount of fuel, as a multiple of payload mass, needed for this roundtrip.

9. This should be substantially less fuel, but how long, ignoring acceleration and deceleration, would a roundtrip journey to Vega (25 c yr away) take at the relatively slow speed of 0.5 c?

B. Collecting fuel along the way

One possible way around the problem discovered above is to collect the hydrogen needed for fuel along the way to Vega. Interstellar space contains approximately one hydrogen atom per cubic meter. With a large enough collecting "mouth" perhaps this would solve the fuel problem.

In addition to the problem of the size of the "mouth" needed to collect enough hydrogen, a more fundamental problem remains. The hydrogen is approximately at rest with respect to Vega. However, in the frame of the spaceship, it is moving toward the ship at the same speed that the ship is moving toward Vega. Thus, the hydrogen atoms have a large kinetic energy as they are scooped up by the ship. The collision between the atoms and the ship will effectively slow the ship. When the kinetic energy of the atoms (which effectively slows the ship) is approximately equal to the energy that can be extracted from the atoms to accelerate the ship, this technique of collecting fuel along the way becomes relatively useless.

1. Setting the kinetic energy of the collected atoms equal to the energy that can be extracted from the atoms, calculate the speed above which collecting fuel as you go becomes a losing proposition.

2. Cruising at the speed calculated above, ignoring acceleration and deceleration, how long would it take you to reach Vega?

C. Matter-Antimatter Powered Spaceship

There is one final possibility. There exists a reaction that has an efficiency of 1.0, where all of the initial mass energy present is converted into kinetic energy. This is the annihilation of matter with antimatter. If a proton is combined with an antiproton, the total mass of both protons is converted into the kinetic energy of the resulting photons. If this reaction does not provide a reasonable fuel source for our trip to Vega, then a trip to Vega is simply not feasible.

There are many very serious problems with using matter-antimatter annihilation as a fuel source. The first is that there is effectively no naturally occurring antimatter in the universe! All of the antimatter has to be "built". We currently are technologically able to manufacture small amounts of antimatter, so we'll assume that in the future this may not be an insurmountable problem. Of course, the energy that is released when antimatter is annihilated has to be "put into" the antimatter in the first place, but this manufacturing can be done on a home planet and need not weigh down our spaceship. Storing large amounts of antimatter and combining it in a controlled manner on the spaceship is a different story, but let's pretend we've mastered that as well.

1. Imagine a rocket engine that combines matter and antimatter in a controlled way, and focuses the resulting photons into a tight beam traveling away from the stern of the spaceship. For this engine, determine f, the payload fraction of the initial mass of the ship, to achieve a cruising speed v. (Hint: The ship can essentially be considered to be a particle of mass M at rest that decays into a big flash of light and a smaller particle (the payload) of known mass fM traveling at a known speed v. Conserve both energy and momentum.)

2. Using the result above, what fraction of the mass of the spaceship can be payload, and how much of the ship must be made up of fuel?

3. Calculate the fuel to payload ratio for a matter-antimatter spaceship. How much fuel, per kilogram of payload, is needed for a roundtrip journey?

This is less ridiculous, but still requires over a million kg of fuel per kg of payload.

4. If you aren't as anxious to get to Vega quickly, you could travel at a cruising speed of 0.8 c. Calculate the amount of fuel, as a multiple of payload mass, needed for this roundtrip.

5. How long would such a roundtrip journey take at the relatively slow speed of 0.8 c.

D. One last concern

1. If relativistic travel were ever possible, we would be wise to avoid bumping into any interstellar dust. Calculate the kinetic energy of a speck of dust (10-6 g) traveling at 0.8 c. This energy is comparable to how many 1000 kg cars traveling at 65 mph?