3.5: Falling into a Black Hole - Easy Version (Project)

Consider falling feet first into a black hole of mass M. Although you may think that this would be a rather unpleasant experience, the truth may surprise you. Sure, you will be increasingly stretched until you are torn apart near the end of your journey, but for most of your journey you feel quite pleasant. You are freely floating through space, enjoying the ride. It's only when you are so close to the hole that your feet feel a greater "force" than your head that you begin to be stretched into spaghetti and the pain increases. We can approximate the duration of this painful period (t_{ouch}) to see if it's over quickly or is a slow, agonizing demise. We will use Newtonian physics to calculate t_{ouch}. (You get a very similar answer when doing this calculation correctly with General Relativity.)

In Newton's Theory of Gravitation,

$$\frac{GM_{blackhole}m_{you}}{r^2} = m_{you}g \tag{3.5.1}$$

$$\frac{GM_{blackhole}}{r^2} = g \tag{3.5.2}$$

We will assume that pain begins when the *difference* between the acceleration of your head and your feet is approximately the value of g at the earth's surface (i.e., your head is "pulled" upward at $4.9\,m/s^2$ and your feet are "pulled" downward at $4.9\,m/s^2$). Thus, pain begins when $\Delta g = -9.8\,m/s^2$ and $\Delta r \approx 2m$, the distance between your head and feet.

1. Take the derivative, dg/dr, of the expression for gravitational acceleration in Newtonian physics.

2. Substitute the values of Δg and Δr in for dg and dr and solve for r_{pain}, the radius at which you begin to feel "uncomfortable", as a function of M, the mass of the black hole.

Black holes come in various sizes. A typical stellar black hole (a star that has collapsed to form a black hole) may have a mass of 5 solar masses, while a galactic black hole (formed from millions of stars collapsing together) may have a mass a million solar masses.

3. Find the radius of pain for a 5 solar mass black hole.

4. Is r_{pain} inside or outside the event horizon of the hole? (If outside the horizon, others can witness you screaming in agony.)

5. Find the radius of pain for a 10^6 solar mass black hole.

6. Is r_{pain} inside or outside the event horizon of the hole? (If inside the horizon, when you begin to scream in agony no one can ever hear you!)

How long you feel uncomfortable (and whether you even have time to scream) depends on how long it takes you to reach $r = 0$ (where you will surely be dead). To find this time, we need your speed when you reach r_{pain}.

7. Assume you fell into the black hole from rest from a great distance. Using energy conservation in Newtonian physics, find an expression for your speed when you reach r_{pain}.

8. Calculate your speed at r_{pain} for the 5 solar mass hole. If this speed is slow compared to c, the Newtonian approximation is valid. Is the approximation valid?

9. Calculate your speed at r_{pain} for the 10^6 solar mass hole. Is the Newtonian approximation valid for a galactic black hole?

You should find that the approximation is not valid for a very large black hole. Therefore, we will not be able to calculate touch for this hole. However, trust me, it won't hurt for long.

10. Even though you will continue to accelerate as you fall further into the 5 solar mass hole, find the time it would take to reach the center traveling at the speed calculated above. This is the maximum amount of time for which you would suffer. Can you take the pain for this long?