1.6: Interstellar Travel – Kinematic Issues (Project)

Imagine you would like to visit Vega, a nearby star approximately 25 light-years from earth, and then return to earth to tell your friends. At first glance, it would appear that the Special Theory of Relativity requires this to be a very long trip. Since no material object can travel faster than light, it would seem to require more than 50 years to make the roundtrip. Then you remember time dilation! If you traveled fast enough, you could make the roundtrip in much less than 50 years (of course, your poor friends would still have aged 50+ years). However, as you will see in this and a following investigation, other, potentially overwhelming, problems may make your trip to Vega incredibly unlikely.

I. Kinematic Considerations

A. Vega in one year, in spaceship frame
Assume you would like to be able to reach Vega in one year, as measured on your clock.

1. Ignoring accelerations at the beginning and end of your journey, how fast must you travel (in units of c) to get to Vega (25 c yr from earth) in 1.0 yr?

B. Time needed to accelerate, in spaceship frame
In part A, we neglected the time it would take to reach cruising speed. This could be a major concern. The human body cannot withstand extended periods of acceleration greater than 9.8 m/s2, or 1 g. Imagine the sensation of riding a rollercoaster around a sharp bend. This is approximately an acceleration of 2g. Could you withstand that feeling for the days, weeks or months required to reach very high speed? An acceleration of 2g is like a 150 lb person walking around all day and night carrying another 150 lb person on their back. This cannot be tolerated for long, so we will restrict our acceleration to 1 g.

1. With an acceleration of 1 g, as measured on the spaceship, how long will it take (in days measured on the ship) to reach the cruising speed calculated in A?

You should have found that it would take approximately a year just to reach cruising speed. So much for the idea of reaching Vega in a year! You have only just reached cruising speed! And don't forget you would need an additional year just to slow down and land or even turn around.

C. Distance needed to accelerate, in spaceship frame
Even though it takes nearly a year to accelerate to cruising speed, you would be covering some of the 25 c yr to Vega.

1. Calculate the distance traveled, in light-years in the frame of the spaceship, while the spaceship is accelerating.

The answer above would be the odometer reading on the spaceship when it has reached cruising speed. However, it's not clear what the total distance to Vega is in this frame, because the distance is continually being contracted by different amounts as the spaceship accelerates.

Therefore, a more useful number would be the distance traveled by the spaceship in the frame of Earth. This number could then be compared to the 25 c yr to Vega.

D. Distance needed to accelerate, in Earth's frame
This calculation is much trickier than the preceding ones. If the ship traveled at constant speed, the distance traveled in the earth's frame is proportional to the distance traveled in the ship's frame:
dearth = dship

However, changes as the ship accelerates. To handle a changing , we will have to perform an integral over the ship's journey.

First, the distance traveled by the ship, in the ship's frame, can be written:

Note that if you evaluated this integral, you would get the familiar result for distance traveled from rest. The key observation is that the distance traveled measured on earth is still simply the product of and the distance traveled by the ship, just remembering that since is not constant, it is inside the integral.

Insert the term "a (1/a)" into the integral.

Note that "a" can be written "dv/dt".

Since the acceleration of the ship is assumed constant, "(1/a)" can come out of the integral, and note that the dt's "cancel".

1. Evaluate the integral, from rest to final speed vc. Simplify the result but do not yet plug in numerical values.

2. Using the cruising speed determined in A, calculate the distance in light-years the spaceship travels while accelerating, in the earth's frame.

You should have found that the ship has traveled less than one light-year during acceleration, and would, of course, need to begin to slow down one light-year, as measured on earth, before reaching Vega.

E. Total travel time, in spaceship frame
The total time to travel to Vega includes the time needed to accelerate, the time needed to decelerate, and the time needed to cruise at constant speed between these two portions of the journey.

1. Calculate the total travel time, in years in the spaceship frame, to Vega.

2. To summarize this process in the spaceship frame, complete the following motion table:

Ship's Frame
Blast-off!

t1 = 0 yr
r1* = 0 c-yr
v1* = 0 c
a12 = Reaches cruising speed

t2 =
r2 =
v2 =
a23 = Begins to slow

t3 =
r3 =
v3 =
a34 = Reaches Vega

t4 =
r4 =
v4 =

* Of course, the position (r) and velocity (v) of the ship in the ship's frame are always zero. Use these rows to tabulate the odometer reading of the ship and the speed with which Vega approaches the ship.

F. Total travel time, in Earth's frame
Just as we needed to do an integral to find the acceleration distance in the earth's frame (since the distance traveled by the ship is contracted by a varying amount), we need to do an integral to determine the acceleration time in the earth's frame since time is dilated by a varying amount.

If the ship traveled at constant speed, the time measured in the earth's frame is proportional to the time measured in the ship's frame:

tearth = tship

However, changes as the ship accelerates. To handle a changing , we have to perform an integral over the ship's journey. Thus,

Since the ship starts at rest, v = at, and

1. Evaluate the integral from rest to the time at which cruising speed is reached in the ship frame, tc.

2. To summarize this process in the Earth frame, complete the following motion table:

Earth's Frame
Blast-off!

t1 = 0 yr
r1 = 0 c-yr

v1 = 0 c
a12 = XXX Reaches cruising speed

t2 =
r2 =
v2 =
a23 = Begins to slow

t3 =
r3 =
v3 =
a34 = XXX Reaches Vega

t4 =
r4 =
v4 =
