# 5.3: The Spatial Form of the Uncertainty Principle

The electrons in atoms are confined to a region of space approximately $10^{-10}$ m across. What is the minimum uncertainty in the velocity of atomic electrons?

The wave-like nature of matter forces certain restrictions on the precision with which a particle can be located in space and time. These restrictions are known as the Heisenberg Uncertainty Principle. (The word “uncertainty” is a poor choice. It is not that we are uncertain of the speed and location of the particle at a specific time; rather it is that the “particle” does not *have* a definite speed and location! The wave-like nature of the “particle” forces it to be spread out in space and time, analogous to the spreading of classical waves in space.)

The spatial form of Heisenberg’s Uncertainty Principle is

$$(\sigma_X)(\sigma_{p_x}) \geq \frac{\hbar}{2} \tag{5.3.1}$$

where

- $\sigma_x$ is the uncertainty, or variation, in the particle’s position,
- \sigma\)px is the uncertainty, or variation, in the particle’s momentum in the same direction,
- and ħ is Planck’s constant divided by 2p.

With the center of the atom designated as the origin, the position of the electron can be represented as

\[x = (0\pm0.5)\times10^{-10} \text{ m}\]

thus

$$\sigma_x = 0.05 \text{ nm} \tag{5.3.2}$$

Using the uncertainty principle results in

$$(\sigma_X)(\sigma_{p_x}) \approx \frac{\hbar}{2} \tag{5.3.3}$$

\sigma_{p_x} \approx \frac{\hbar}{2(\sigma_x}}

m\sigma_{v_x} \approx \frac{\hbar}{2(\sigma_x}}

\sigma_{v_x} \approx \frac{\hbar}{2m(\sigma_x}}

\sigma_{v_x} \approx \frac{\hbar c}{2mc^2(\sigma_x}}c

Just as “hc” will pop up in numerous equations throughout this course, the constant “ħc” is also quite common and has an equally friendly value, 197.4 eV nm. Thus,

$$\sigma_{v_x} \approx \frac{197.4 \text{ eVnm}}{2(511000 \text{ eV})(0.05 \text{ nm}}c \tag{5.3.4}$$

$$\sigma_{v_x} \approx 3.86 \times 10^{-3} c \tag{5.3.5}$$

$$\sigma_{v_x} \approx 1.2 \times 10^{6} \text{ m/s} \tag{5.3.6}$$

Thus the velocity of an atomic electron has an inherent, irreducible uncertainty of about a million meters per second! If anyone tells you they know how fast an atomic electron is moving to a greater precision than a million meters per second, you know what to tell them…