7.A: Alpha Decay (Project)

The radioactive process known as alpha decay involves the tunneling of an alpha particle (a bound state of two protons and two neutrons) through the Coulomb barrier to escape the nuclear potential.

In this activity, you will construct a spreadsheet that calculates the tunneling probability for the alpha particle and, using this probability, calculates the half-life of the radioactive nucleus.

I. Constructing the Spreadsheet

Construct a spreadsheet that has the following general form. (The template AlphaDecay is available in the PHY 262 course folder.)

Alpha Decay
your name:
your name:

alpha energy = MeV T =
A = f =
Z = ▯ =
half-life = s
nuclear depth = MeV
nuclear radius = fm
exit radius = fm

Position Potential integrand
(fm) (MeV)

The spreadsheet should allow you to enter the mass number (A) and atomic number (Z) of a radioactive nucleus, the kinetic energy of the emitted alpha particle, and then calculate the half-life for the transition.

One of the several approximations you will make is to imagine the alpha particle as a point particle moving in the potential well created by the rest of the nucleus. Since the alpha particle consists of 4 nucleons (two of which are protons), the effective mass number of the rest of the nucleus is (A – 4) and the effective charge of the rest of the nucleus is (Z – 2).

For heavy nuclei, the depth of the nuclear potential (UNuclear) is approximately

$$U_{Nuclear} = -50 \text{ MeV} \tag{7.A.1}$$

In addition, the radius of the nuclear potential (R) is

$$R = (1.2 \text{ fm})(A-4)^{1/3} \tag{7.A.2}$$

The electrical potential energy between the alpha particle and the rest of the nucleus (UCoulomb) is given by

$$U_{Coulomb} = qV \tag{7.A.3}$$

$$U_{Coulomb} = (2e)\left(\frac{k((Z-2)e)}{r}\right) \tag{7.A.4}$$

$$U_{Coulomb} = \frac{2(Z-2)ke^2}{r} \tag{7.A.5}$$

(Note that in “nuclear” units, ke2 = 1.44 MeV fm.)

Therefore, the exit radius (RExit) can be determined by equating the alpha energy (EAlpha) to the electrical energy

$$E_{Alpha} = U_{Coulomb} \tag{7.A.6}$$

$$E_{Alpha} = \frac{2(Z-2)ke^2}{R_{Exit}} \tag{7.A.7}$$

$$R_{Exit} = \frac{2(Z-2)ke^2}{E_{Alpha}} \tag{7.A.8}$$

Using these definitions, complete the Position and Potential columns of the spreadsheet for 500 equally spaced points between the nuclear radius and the exit radius.

A. Tunneling Approximation Scheme

The probability of tunneling through a square potential barrier of height U and width x is given by:

$$T \cong e^{-x/\delta} \tag{7.A.9}$$

where

$$\delta = \frac{\hbar c}{\sqrt{8mc^2(U-E)}} \tag{7.A.10}$$

If the barrier is not square, we can approximate the barrier by a series of extremely thin barriers which are approximately square. In the limit of infinitesimally thin barriers, this becomes

$$T \cong e^{-\int dx/\delta} \tag{7.A.11}$$

$$T \cong exp\left(-\frac{\sqrt{8mc^2}}{\hbar c}\int\sqrt{U-E}\,dx\right) \tag{7.A.12}$$

Thus, to find the probability of the alpha particle tunneling through the barrier, you should calculate, at each of the 500 equally spaced points between the nuclear radius and the exit radius, the value of the integrand,

$$\sqrt{U-E}(\Delta x) \tag{7.A.13}$$

The sum of these contributions can then be used to find T. (Note that the rest energy of the alpha particle is 3728 MeV.)

The number of collisions per second the alpha particle makes with the nuclear "wall", f, can be determined from the speed of the alpha particle inside the nucleus and the size of the nucleus. The decay rate, λ, is the probability of decay per second, and can be determined from T and f. Finally, the half-life is directed related to λ. The specific relationships are left for you to determine.

II. Using the Spreadsheet

1. Calculate the half-life for $^{222}_{92}U$ emitting an alpha particle of energy 9.50 MeV and record it below. (Don't worry if you don't get exactly the answer shown in the data table. Due to the approximations we've made your answer should be a couple hundred times larger than the experimentally measured value.) Print the first page of your spreadsheet and attach it to the end of this activity.

2. Although your value for the half-life is larger than the measured value, your spreadsheet can still be used to explore how half-life depends on alpha energy. For each of the even-A uranium isotopes listed in the data table, determine the theoretical half-life using your spreadsheet. Record these results in a suitably labeled column in the data table.

3. To compare the theoretical dependence between half-life and alpha energy to the experimentally measured dependence, create a graph of the log of the half-life vs. alpha energy. Show both the theoretical and experimental data on this graph. Print this graph and attach it to the end of the activity.

4. Does your spreadsheet accurately reflect the dependence of half-life on alpha energy? Clearly explain why or why not.

5. Clearly explain why a reduction in alpha energy by a factor of approximately two (9.50 MeV to 4.27 MeV) can result in a change in half-life by a factor of approximately 1023 (!).