5.5: The Meaning of the Uncertainty Principle

There is much confusion regarding the meaning of the Uncertainty Principle. In fact, the uncertainty principle is really just a statement about waves and how simple waves can be combined to form *wave packets*. A wave packet is a localized wave disturbance.

Unlike simple sine and cosine representations of waves, such as:

$$\Psi(x, t) = A \sin(kx - \omega t) \tag{5.5.1}$$

which extend at equal amplitude to $\pm\infty$, a wave packet has amplitude that is larger in one region of space than another. Mathematically, wave packets are formed by adding together appropriately chosen simple sine and cosine waves.

For example,

note that the wave packet (C) has larger amplitude in some regions of space than in other regions. We can say that the wave packet C is *localized* in space. If we want to more narrowly localize C in space we will need to add together a larger range of different wavelength waves. Thus, the spatial localization of C is inversely related to the range of wavelengths used to construct C. Since a larger range of wavelengths corresponds to a larger range of momenta (by DeBroglie's hypothesis), spatial localization comes at the price of an increased range of momenta. This is all the uncertainty principle says! The width of spatial localization is inversely proportional to the range in momenta. This is true of ***all*** waves and is not special, in any way, to matter waves.

Additionally, since the wave packet is mathematically comprised of many waves with different momenta, these constituent waves all move through space at different rates. The speed of each constituent wave is referred to as the *phase velocity*. The speed of the wave packet is the *group velocity*. Since the different constituent waves have different phase velocities, this leads, over time, to a change in the overall shape and extent of the wave packet. This change in shape of the wave packet over time is termed *dispersion* and is illustrated at left (time increases as you scan from top to bottom).

This spreading of the wave packet is natural and is completely analogous to, for example, the spreading of water waves on a pond. In the case of a matter wave, however, this spreading is interpreted as the increasing uncertainty as to the location of the "particle" that the wave represents. The wave (actually the square of the wave) represents the probability of finding the particle at a certain location in space when a measurement is performed.

Before the measurement is made, however, the "particle" must be thought of as existing at all of the locations where the probability is non-zero.