4.4: Photons and Matter

Consider a beam of photons incident on a chunk of matter. The photons can be absorbed by the electrons, scatter from the electrons, and create pairs of particles by interacting with the nucleus. Two questions remains, however. Which of these processes will occur for any particular photon and how far will the photon penetrate the matter before one of these processes occurs. These questions require an understanding of the *probability* of each of these processes occurring. The probability of a particular process occurring is represented by the *cross section* for that process, denoted σ.

Basically, the idea is to imagine the atom as a dart board. Each process (photoelectric effect, scattering, pair production) is represented by an area of the dart board. If the photon strikes that area, that process occurs.

Thus, the probability of a process occurring per atom is given by the ratio of the cross section for that process per atom (the area of that portion of the dart board) divided by the total area of the target:

$$\frac{\text{Probability}}{\text{atom}}=\frac{\sigma}{A} \tag{4.4.1}$$

$$\text{Probability}=\frac{\sigma}{A}N_{atoms} \tag{4.4.2}$$

where N_atoms is the total number of atoms in the target.

To convert this idea into a more useable form, imagine a beam of parallel photons incident on a thick slab of material (containing many microscopic targets).

The decrease in the number of photons in the beam due to interactions in a thin slab of material is given by the product of the probability of interaction and the number of photons present:

$$\Delta N_{photons}=-\left(\frac{\sigma}{A}N_{atoms}\right)N_{photons} \tag{4.4.3}$$

The number of atoms in the slim slab of material is given by the product of the number of atoms per unit volume (n) and the volume of the slab ($A\Delta x$). Thus,

$$\Delta N_{photons}=-\left(\frac{\sigma}{A}nA\Delta x\right)N_{photons} \tag{4.4.4}$$

$$\Delta N_{photons}=-\sigma n\left(\Delta x\right)N_photons \tag{4.4.5}$$

Since the mass per unit volume (ρ) is a more commonly known value than the number per unit volume (n), let's replace n with ρ and redefine the cross section to have units of area per unit mass (typically cm^2/g).

$$\Delta N_{photons}=-\sigma\rho(\Delta x)N_{photons} \tag{4.4.6}$$

$$\frac{\Delta N_{photons}}{\Delta x}=-\sigma\rho N_{photons} \tag{4.4.7}$$

In the limit of very thin slabs, this becomes a differential equation with a known solution:

$$\frac{dN_{photons}}{dx}=-\sigma\rho N_{photons} \tag{4.4.8}$$

$$N_{photons}(x)-N_0e^{-\sigma\rho X} \tag{4.4.9}$$

Thus, the number of photons in the beam decreases exponentially, with dependence on both the cross section for the interaction of interest and the density of the target material.

Photons and Lead

The wonderful website:
physics.nist.gov/PhysRefData/...Text/XCOM.html
tabulates the cross sections for all of the photon interactions we have discussed for almost any element or compound you can imagine. These cross sections are crucial information for a wide variety of important activities, from calculating dosage for radiation therapy for cancer to determining the necessary shielding for nuclear reactors.
Below is a graphical representation of the cross sections for photons with energy between 1.0 keV and 1000 MeV interacting with lead. Note that at different energies different processes dominate. At lower energy, the dominant process is photoelectric absorption. Around 1.0 MeV, however, incoherent scattering begins to dominate. (Incoherent scattering is Compton scattering, in which the wavelength of the photon changes during the scattering event. Coherent scattering is when the photon scatters without changing its wavelength. This occurs, for example, when the photon scatters off of the nucleus. Note that if you substitute the nuclear mass into the Compton formula, you would find that the wavelength of the photon does not change.) Finally, above about 10 MeV pair production involving the nucleus dominates. Pair production can also occur with the electron "absorbing" the necessary momentum but this is much less likely.

Consider the problem below:

1.0 MeV photons are incident on a thick lead (ρ = 11.34 g/cm^3) slab.

a. If the slab is 1.0 cm thick, what percentage of the photons will undergo the photoelectric effect?

b. What thickness of lead is needed to stop 95% of the photons?
c. What thickness of lead is needed to stop or scatter 95% of the photons?

Since the first question concerns just the photoelectric effect, we need the photoelectric effect cross section at 1.0 MeV. From the website, σ photoelectric = $1.81x10^{-2}$cm2/g. Thus,

$$N_{photons}(x) = N_0 e^{\sigma\rho x} \tag{4.4.10}$$

$$N_{photons} = N_0 e^{-(.0181)(11.34)(1)} \tag{4.4.11}$$

$$N_{photons} = N_0(0.814) \tag{4.4.12}$$

If 81.4 percent of the photons have not undergone the photoelectric effect, then 18.6% of them have.

Normally, photons can be "stopped" by either absorption (photoelectric effect) or pair production. Since 1.0 MeV is too low an energy for pair production, the only stopping mechanism is the photoelectric effect. Thus,

$$N_{photons}(x) = N_0 e^{\sigma\rho x} \tag{4.4.13}$$

$$\frac{N_{photons}}{N_0} = e^{-(.0181)(11.34)x} \tag{4.4.14}$$

$$0.05 = e^{-0.205x} \tag{4.4.15}$$

$$\ln(0.05) = -0.205x \tag{4.4.16}$$

$$x = 14.6 \text{ cm} \tag{4.4.17}$$

The cross section for stopping and scattering is the total cross section at 1.0 MeV, (\sigma\)total = 7.1 x 10-2 cm2/g. Thus,

$$N_{photons}(x) = N_0 e^{\sigma\rho x} \tag{4.4.18}$$

$$\frac{N_{photons}}{N_0} = e^{-(.071)(11.34)x} \tag{4.4.19}$$

$$0.05 = e^{-0.805x} \tag{4.4.20}$$

$$\ln(0.05) = -0.805x \tag{4.4.21}$$

$$x = 3.72 \text{ cm} \tag{4.4.22}$$

The decrease in beam intensity due to both absorption and scattering is referred to as the attenuation of the beam.
