1.2: Length Contraction

If the time interval between two events depends on the relative motion of the observer, Einstein realized that the spatial separation between the events must also be observer-dependent. Consider a hypothetical spaceship journey from earth to a distant star. Assume the star is a distance L_o from the earth, as measured by stationary earth-bound observers. Therefore, the elapsed time for the spaceship to reach the star, as measured on earth, is

$$\Delta t = \frac{L_o}{v} \tag{1.2.1}$$

where v is the speed of the spaceship measured on earth.

By time dilation, however, the elapsed time for the spaceship to reach the star, as measured on the spaceship (a proper time), is

$$\Delta t_o = \frac{\Delta t}{\gamma} \tag{1.2.2}$$

$$= \frac{L_o}{v\gamma} \tag{1.2.3}$$

The distance the spaceship travels (L), as measured on the spaceship, is simply the product of its speed and the elapsed time measured on the ship

$$L = v\Delta t_o \tag{1.2.4}$$

$$= v\left(\frac{L_o}{v\gamma}\right) \tag{1.2.5}$$

$$= \frac{L_o}{\gamma} \tag{1.2.6}$$

Since gamma is greater than one, the distance between the earth and the star as measured on the ship is less than the distance as measured on the earth. To moving observers, the distance to the star shrinks.

Using Length Contraction

The star Vega is approximately 25 light-years from Earth (as measured by observers on Earth).

a. *How fast must a spaceship travel in order to reach Vega in 30 years, as measured on Earth?*
b. *How fast must a spaceship travel in order to reach Vega in 30 years, as measured on the spaceship?*

The distance between two events, for example leaving Earth and arriving at Vega, depends on who makes the measurements. Within any particular reference system, the familiar results of classical physics are valid. However, in comparing results between observers in different reference systems, a method of relating one observer's measurements to another is needed.

The formula for length contraction is

where
L0 the proper length, the distance between two events in the frame of reference in which both events are at rest,
and L is the distance between the same two events in a different frame, moving at relative speed v.

For part a, relativity theory is unnecessary. Both the distance and time measurements are made from the same frame of reference. Therefore, results from classical physics are valid.

(Note the use of the speed of light as a unit. Rather than substituting 3.0 x 108 m/s for c, simply leave c as a unit of velocity.)
For part b, the distance is measured in the Earth's frame while the time is in the frame of the spaceship. To solve part b, you must either convert the distance into the spaceship frame or the time into the Earth frame. You can convert the distance into the spaceship frame by realizing that the distance to Vega as measured on Earth is a proper length. Therefore,