7.3: Alpha and Beta Decay

$^{226}\mathrm{Ra}$ undergoes alpha decay with a half-life of 1600 years. Consider a 100 g sample of pure $^{226}\mathrm{Ra}$. What is the initial power output of this sample?

An alpha particle is a bound state of two neutrons and two protons. In many nuclei with very large numbers of nucleons (typically more than 200), alpha particle near the “top” of the potential well “see” a well with the barrier height (~20 MeV) and well depth (~ -50 MeV) shown below:

If this is the case, the alpha particle can escape the nucleus by tunneling through the barrier. This tunneling process is alpha decay. The half-life for alpha decay can be calculated from the tunneling probability.

Generically, alpha decay can be written as:

$$ {}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X} \Rightarrow {}^{\mathrm{A}-4}_{\mathrm{Z}-2}\mathrm{X} + {}^{4}_{2}\mathrm{He} \tag{7.3.1}$$

For this example, this reduces to,

$$ {}^{226}_{88}\mathrm{Ra} \Rightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He} \tag{7.3.2}$$

The energy released by this decay can be calculated by simply finding the mass difference between the initial and final nuclear states, and converting this mass difference into an energy difference. The energy released in a nuclear transformation is typically referred to as the Q-value of the reaction.

One possible complication with calculating Q-values is that only atomic masses are tabulated, while the difference in nuclear mass determines Q. To convert atomic masses to nuclear masses, multiples of the electron mass must be subtracted from each term. Generically,

$$Q = (m_{X,atomic}c^2 - Zm_ec^2) - (m_{X,atomic}c^2 - (Z-2)m_ec^2) - (m_{He,atomic}c^2 - 2m_ec^2) \tag{7.3.3}$$

$$Q = m_{X,atomic}c^2 - Zm_ec^2 - m_{X,atomic}c^2 - (Z-2)m_ec^2 - m_{He,atomic}c^2 - 2m_ec^2 \tag{7.3.4}$$

$$Q = (m_{X,atomic} - m_{X,atomic} - m_{He,atomic}c^2 \tag{7.3.5}$$

Since the number of electrons on each side of the reaction is equal, you can use atomic masses to determine Q.

$$Q = (m_{Ra} - m_{Rn} - m_{He})c^2 \tag{7.3.6}$$

$$Q = (226.025402u - 222.017570u - 4.002603)c^2 \tag{7.3.7}$$

$$Q = 5.229 \times 10^{-3} uc^2 \tag{7.3.8}$$

$$Q = 4.87 \text{ MeV} \tag{7.3.9}$$

This is the energy released per alpha decay. If we can determine the activity of the sample (the number of decays per second), the product of activity and Q will be the power of the sample.

The number of atoms in the sample is

$$100g\left(\frac{6.02 \times 10^{23} atoms Ra}{226g}\right) = 2.66 \times 10^{23} atoms Ra \tag{7.3.10}$$

Since the decay constant is

$$\lambda = \frac{\log(2)}{t_{1/2}} \tag{7.3.11}$$

$$\lambda = \frac{\log(2)}{1600yr} \tag{7.3.12}$$

$$\lambda = 4.33 \times 10^{-4} yr^{-1} \tag{7.3.13}$$

$$\lambda = 1.37 \times 10^{-11} s^{-1} \tag{7.3.14}$$

the activity is

$$A = (2.66 \times 10^{23} atoms Ra)(1.37 \times 10^{-11} s^{-1} \tag{7.3.15}$$

$$A = 3.65 \times 10^{12} decays/s \tag{7.3.16}$$

and the power is

$$P = QA \tag{7.3.17}$$

$$P = (4.87 \text{ MeV}/decay)(3.65 \times 10^{12} decays/s \tag{7.3.18}$$

$$P = 1.78 \times 10^{13} \text{ MeV/s} \tag{7.3.19}$$

$$P = 2.85 J/s \tag{7.3.20}$$

$$P = 2.85 W \tag{7.3.21}$$

Beta Decay

^{81}Kr is unstable. How will it decay? Calculate the Q value for this decay.

In addition to alpha decay, which typically occurs only for very large nuclei, another possible nuclear transformation involves the spontaneous transformation of a proton into a neutron, or vice-versa. There are actually three different decay processes that involve this type of transformation, which is governed by the weak force. These decays are generically referred to as beta decay.

\Beta^{-} Decay

Beta-minus decay involves the transformation of a neutron into a proton, electron, and anti-neutrino:

$$n \Rightarrow p^{+} + e^{-} + \bar{v} \tag{7.3.22}$$

The neutron can decay by this reaction both inside the nucleus and as a free particle.

Generically, beta-minus decay can be written as

$${}^{A}_{Z}X \Rightarrow {}^{A}_{Z+1}X' + e^{-} + \bar{v} \tag{7.3.23}$$

resulting in a Q-value of:

$$Q = (m_{X,atomic}c^2 - Zm_ec^2) - (m_{X',atomic}c^2 - (Z+1)m_ec^2) - (m_ec^2) \tag{7.3.24}$$

$$Q = m_{X,atomic}c^2 - Zm_ec^2 - m_{X',atomic}c^2 + (Z+1)m_ec^2 - m_ec^2 \tag{7.3.25}$$

$$Q = (m_{X,atomic} - m_{X',atomic})c^2 \tag{7.3.26}$$

\Beta^{+} Decay

Beta-plus decay involves the transformation of a proton into a neutron, positron, and neutrino:

$$p^{+} \Rightarrow n + e^{+} + v \tag{7.3.27}$$

This process can only occur inside the nucleus.

Generically, beta-plus decay can be written as

$${}^{A}_{Z}X \Rightarrow {}^{A}_{Z-1}X' + e^{+} + v \tag{7.3.28}$$

resulting in a Q-value of:

$$Q = (m_{X,atomic}c^2 - Zm_ec^2) - (m_{X',atomic}c^2 - (Z-1)m_ec^2) - (m_ec^2) \tag{7.3.29}$$

$$Q = m_{X,atomic}c^2 - Zm_ec^2 - m_{X',atomic}c^2 + (Z-1)m_ec^2 - m_ec^2 \tag{7.3.30}$$

$$Q = (m_{X,atomic} - m_{X',atomic} - 2m_e)c^2 \tag{7.3.31}$$

Electron Capture

Electron capture involves a proton in the nucleus absorbing an inner shell electron:

$$p^+ + e^- \Rightarrow n + v \tag{7.3.32}$$

Generically, electron capture can be written as

$$^A_ZX + e^- \Rightarrow ^A_{Z-1} X' + v \tag{7.3.33}$$

resulting in a Q-value of:

$$Q = (m_{X,atomic}c^2 - Zm_ec^2) + (m_ec^2) - (m_{X',atomic}c^2 - (Z-1)m_ec^2) \tag{7.3.34}$$

$$Q = m_{X,atomic}c^2 - Zm_ec^2 + m_ec^2 - m_{X',atomic}c^2 + (Z-1)m_ec^2 \tag{7.3.35}$$

$$Q = (m_{X,atomic} - m_{X',atomic})c^2 \tag{7.3.36}$$

Applied to this example, the three processes yield the following reactions:

\[

$$\begin{array}{lll} \text{beta}^-\text{decay} & ^{81}_{36}Kr \Rightarrow ^{81}_{37}Rb + e^- + \bar{v} & Q = (m_{Kr} - m_{Rb})c^2 \\ \text{beta}^+\text{decay} & ^{81}_{36}Kr \Rightarrow ^{81}_{35}Br + e^+ + \bar{v} & Q = (m_{Kr} - m_{Br} - 2m_{e^-})c^2 \\ \text{electron capture} & ^{81}_{36}Kr + e^- \Rightarrow ^{81}_{35}Br + v & Q = (m_{Kr} - m_{Br})c^2 \end{array} \tag{7.3.37}$$

]

To determine how 81Kr will decay, calculate the Q-value for each hypothetical reaction. Only Q-values greater than zero (reactions that release energy) occur spontaneously.

\[

$$\begin{array}{lll} \text{beta}^-\text{decay} & ^{81}_{36}Kr \Rightarrow ^{81}_{37}Rb + e^- + \bar{v} & Q = (80.916593u - 80.916291u)c^2 \\ & & Q < 0 \\ \text{beta}^+\text{decay} & ^{81}_{36}Kr \Rightarrow ^{81}_{35}Br + e^+ + \bar{v} & Q = (80.916593u - 80.916291u - 2(5.4858 \times 10^{-4}u))c^2 \\ & & Q < 0 \\ \text{electron capture} & ^{81}_{36}Kr + e^- \Rightarrow ^{81}_{35}Br + v & Q = (80.916593u - 80.916291u)c^2 \\ & & Q = 0.281 MeV \end{array} \tag{7.3.38}$$

]

Therefore, 81Kr will decay via electron capture, and release 0.281 MeV of energy per decay.

If more than one decay involves a positive Q, the one that releases the most energy will typically dominate. The exception to this rule involves electron capture. Both beta-plus and beta-minus, if allowed, always dominate electron capture since electron capture involves the relatively rare occurrence of a sizable overlap between electron and proton wavefunctions.

Beta decay can be understood conceptually by looking carefully at the differences in the potential wells for protons and neutrons, and the order in which the available energy levels are filled.
For example, consider 24Na, which consists of 13 neutrons and 11 protons. The filled energy levels would look like the well on the left.

The overall energy of the nucleus would be reduced (and its stability increased) if the "stray" neutron at the top of the neutron well could somehow transform itself into a proton and jump down to the lower energy state in the proton well. Well, nature allows this transformation and we call it - decay!

As another example, consider 18F, which consists of 9 neutrons and 9 protons. Its filled energy levels would look like the well on the left.

The overall energy of this nucleus would be reduced if a proton could somehow transform itself into a neutron. Well, nature allows this transformation and we call it + decay!
