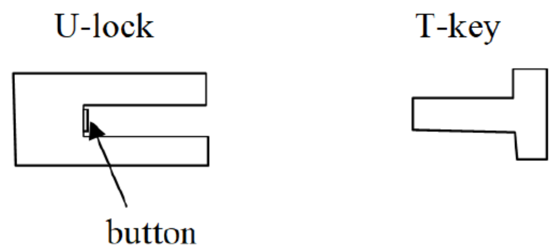
1.7: The Lock and Key Paradox (Project)

Imagine a U-shaped lock that opens only when a T-shaped key strikes a button in the deepest part of the lock, as illustrated below:



To prevent the lock from being easily opened, the depth of the lock (L_o, when at rest) is greater than the length of the key (K0, when at rest). Naïve locksmiths unfamiliar with special relativity would therefore believe the lock is unopenable. However, educated locksmiths argue that if the lock is in motion, relative to the key, the depth of the lock will shrink allowing the key to open the lock. Unfortunately, other educated locksmiths argue that in the same situation, in the frame of reference of the lock, the key would be in motion, causing it to shrink, thereby not opening the lock.

This situation is more complicated than the rather mundane pole-vaulter-and-the-barn paradox (or equivalently the farmer-and-the-tractor paradox). In those situations, the “solution” to the paradox involves understanding that the order of two events is not the same in both frames. In this situation, we don’t really care when the key strikes the button (if it does), but whether it objectively happens or not. To resolve this paradox, we will need to directly address the limits of information travel time.

I. The Lock Frame

Let the relative speed between the lock and key be v, corresponding to a Lorentz factor . Answer the following questions in the frame of reference of the lock.

1. Write an expression for the length of the key, K.

Let’s assume that when the rear of the key strikes the front edge of the lock, the rear of the key instantaneously stops. Although any real material would deform and come to rest over a finite time, no fundamental law of physics limits the lower value of this time. However, a fundamental law of physics does imply that the front edge of the key will continue to move forward until the information that the rear edge has stopped is transmitted to it. The front edge cannot stop simultaneously with the rear edge. The information that the rear edge has stopped can be transmitted no faster than the speed of light!

2. Write an expression for T, the time it takes for a message to be sent from the rear edge of the key to its front edge. Assume the message is sent at maximum speed, c. (Hint: Note that the tip of the key is moving away from this propagating signal.)

3. Write an expression for the distance the tip travels (K) after the rear has been stopped.

4. Write an expression for the length of the key when the front tip finally stops, K*. If K* is greater than or equal to the rest length of the lock, L0, the button will be pressed.

5. Show that K* is greater than K0. This means that the length of the key is larger than its proper length when it finally stops moving. Thus, during the process of stopping, the key will overshoot its proper length before settling back to its proper length!

II. The Key Frame

Answer the following questions in the frame of reference of the key.

1. Write an expression for the depth of the lock, L.

Again assume that when the front of the lock strikes the rear of the key, the front of the lock instantaneously stops. However, the rear of the lock (and hence the button) will continue to move forward until the information that the front edge has stopped is transmitted to it.

2. Write an expression for T, the time it takes for a message to be sent from the front edge of the lock to the location of the button. Assume the message is sent at maximum speed, c. (Hint: Note that the button is moving toward this propagating signal.)

3. Write an expression for the distance the button travels (L) after the rear has been stopped.

4. Write an expression for the depth of the lock when the button finally stops, L*. If L* is less than or equal to the rest length of the key, K0, the button will be pressed.

5. Show that L* is smaller than L. This means that the depth of the lock is smaller than its Lorentz contracted length when it finally stops moving. Thus, during the process of stopping, the lock will overcontract before re-expanding back to its proper length!

III. Consistent Results?

1. Show that the necessary condition to trigger the button in the lock frame (K* = L0, from I. 4.) is exactly the same as the necessary condition to trigger the button in the key frame (L* = K0, from II. 4.)

IV. A Numerical Example

Let L0 = 0.10 m, K0 = 0.09 m, and v = 0.5 c.

1. Ignoring the overcontraction of the lock, show that in the key frame the button is hit.

2. Ignoring the overexpansion of the key, show that in the lock frame the button would not be hit.

3. Including the overexpansion of the key, how long is the key when it finally stops? Is this sufficient to contact the button?
