7.A: Radioactive Chains (Project)

After a radioactive nucleus decays, it often leaves behind a new radioactive nucleus, termed the daughter nucleus. This daughter then decays, often leaving behind another radioactive daughter. This chain of daughter nuclei can get very complicated, as each nucleus typically has a different decay constant, may decay by a different process, and emits a radioactive particle of different energy. For example, starting from a sample of 238U, you will end up with this lovely collection of nuclei:

In this activity, you will construct a spreadsheet that calculates the relative abundance of daughter nuclei over time for a radioactive chain of three members, as well as the energy released by the sample.

I. Constructing the Spreadsheet

Construct a spreadsheet that has the following general form. (The template RadioactiveChains is available in the PHY 262 course folder.)

Radioactive Chains
name:
name:
λ (unit) Q (unit)
Nuclei #1
Nuclei #2
Nuclei #3

time step = unit?

Time (unit?) N1 N2 N3 Energy (unit?)
0 100.00 0.00 0.00 0.00

The spreadsheet should allow you to enter the decay constants for three sequential members in a radioactive chain, the Q for each decay, and a reasonable time step. The abundance of element #1 is initially set to 100, while the initial abundances of the daughters are zero. Whatever unit (s-1, day-1, etc.) you use to express the decay constant should be the same unit (s, day, etc.) used to express the time step.

The Time column should include 100 time steps of the inputted duration, and your spreadsheet should also include graphs of the relative abundance of all three nuclei vs. time (all on one graph), and the total energy released by the sample vs. time.

A. Relative Abundances

The number of radioactive particles present at one instant is related to the number present a short time earlier, adjusted by the decay constant and time step. The decay constant (λ) is, crudely speaking, the probability that a nucleus will decay in some designated time interval. For example, a decay constant of λ = $0.5 textyear^{-1}$ means that there is a 50% chance of the nucleus decaying per year. Therefore, the number of nuclei actually decaying is the product of this decay constant (λ), the number of nuclei currently present (N), and the time interval (dt).

Mathematically,

$$\text{number of decays} = \lambda N dt \tag{7.A.1}$$

This means that the number of nuclei present at one instant is equal to the number present at a previous instant, minus the number decayed.

$$N(t+dt) = N(t) - \lambda N(t) dt \tag{7.A.2}$$

There is one complication, however. In addition to decaying, nuclei are created by the decay of higher mass nuclei. Therefore, the above equation is only correct for the mother, or highest mass, nucleus. For each of the daughters, you would have to add the number that are created by the decay of their mother. If the nuclei are labeled 1, 2, 3, … in order of creation, then

$$N_1(t+dt) = N_1(t) - \lambda_1 N_1(t) dt \tag{7.A.3}$$

$$N_1(t+dt) = N_2(t) - \lambda_2 N_2(t) dt + \lambda_1 N_1(t) dt \tag{7.A.4}$$

$$N_1(t+dt) = N_3(t) - \lambda_3 N_3(t)dt + \lambda_2 N_2(t)dt \tag{7.A.5}$$

Create a spreadsheet that incorporates these ideas for a chain of three hypothetical nuclei with $\lambda_1 = 0.006\text{year}^{-1}$, $\lambda_2 = 0.003\text{year}^{-1}$, and $\lambda_3 = 0.012\text{year}^{-1}$. Determine and graph the abundance of each type of nuclei as a function of time with dt = 10 years.

1. For each nuclei, record below the approximate time at which it is in greatest abundance, and the maximum number present at this time.

If nuclei #2 is not in greatest abundance after approximately 225 years, you've done something wrong.

B. Energy Released

If you were designing an enclosure for the radioactive sample described above, you would need to know how the relative abundances of the different nuclei change over time. Your spreadsheet can now perform that calculation. In addition, however, you would need to know how the amount of energy released by the sample also changes over time. In fact, contrary to what you may think, the amount of energy released by a radioactive sample often increases rather than decreases over time. This means the sample gets "hotter" before it ultimately decays toward stability. To study this process, you will need to incorporate information regarding the energy released by each of the decay processes.

For each radioactive decay reaction, a specific amount of energy is released. This energy is referred to as the Q for the reaction. Thus Q1 is released for every decay of nuclei #1, Q2 for every decay of nuclei #2, and Q3 for every decay of nuclei #3. Therefore,

Energy released = Q1 (decays of N1) + Q2 (decays of N2) + Q3 (decays of N3)

With Q1 = 0.4 MeV, Q2 = 2.6 MeV, and Q3 = 0.8 MeV, adjust your spreadsheet so that it calculates the total amount of energy released for each time step. Create a graph of energy released vs. time.

2. From your spreadsheet, determine the time at which the most energy is being released by the sample. How much energy is being released (per 100 initial nuclei of nuclei #1) at this time? How are you going to engineer a containment system that will still be operational after this amount of time? (I don't expect you to be able to answer that last one.)

II. Using the Spreadsheet

A.

$$^{210}\text{Rn} \xrightarrow{\lambda_1} {}^{210}\text{At} \xrightarrow{\lambda_2} {}^{210}\text{Po} \xrightarrow{\lambda_3} {}^{206}\text{Pb} \tag{7.A.6}$$

Analyze the above chain where $\lambda_1 = 6.93 day^{-1}$, $\lambda_2 = 2.05 day^{-1}$, and $\lambda_3 = 0.0051 day^{-1}$.

1. Determine the Q for $^{210}\text{Rn} \rightarrow {}^{210}\text{At}$.
2. Determine the Q for $^{210}\text{At} \rightarrow {}^{210}\text{Po}$.
3. Determine the Q for $^{210}\text{Po} \rightarrow {}^{206}\text{Pb}$.

Adjust your time step so that your graph shows all of the N1 and most of the N2 decayed. Print-out the first page of your spreadsheet calculations and both graphs and attach them to the end of this activity.

4. When (in days) will the amount of 210At be the greatest? What is the maximum amount present?

5. When (in days) will the amount of 210Po be the greatest? What is the maximum amount present? (You will have to use a different time step than above to answer this question.)

6. When (in days) is the maximum amount of energy released by this chain?

B. Thorium Series

Wikipedia (http://en.Wikipedia.org/wiki/Decay_chain) has a very thorough discussion of the major radioactive decay chains, including half-lives and Q values for each decay process. Create a spreadsheet model for the 3-steps leading from 228Ra to 224Ra.

Adjust your time step until you can clearly see the maximum of the 228Th abundance curve.

7. Why is it so difficult to choose the appropriate time step?

The large difference in half-life between the various daughters makes it impossible to pick a time step where, over only 100 cycles, you can get significant 228Ra decay but yet not have your calculation "blow-up" for 226Ac. There are two relatively simple solutions to this problem.

Typically, if you are interested in behavior over the span of years, half-lives on the order of days or less can be considered "instantaneous". Thus, you can approximate 228Ra as decaying directly to 228Th with a half-life of 5.75 yrs and a Q equal to the total energy released by the two processes, 2.170 MeV.

The other option is just to increase the number of cycles of your calculation. Instead of 100 time steps, use 10,000!

Using 10,000 cycles, adjust your time step until you can clearly see the maximum of the 228Th abundance curve. Print-out the first page of your spreadsheet calculations and both graphs and attach them to the end of this activity.

8. Clearly explain why 228Ac is never present in substantial amounts.

9. When (in years) will the amount of 228Th be the greatest? What is the maximum amount present?

10. When (in years) is the maximum amount of energy released by this chain?
