7.9: Capacitance (Summary)

Key Terms

capacitance	amount of charge stored per unit volt
capacitor	device that stores electrical charge and electrical energy
dielectric	insulating material used to fill the space between two plates
dielectric breakdown	phenomenon that occurs when an insulator becomes a conductor in a strong electrical field
dielectric constant	factor by which capacitance increases when a dielectric is inserted between the plates of a capacitor
dielectric strength	critical electrical field strength above which molecules in insulator begin to break down and the insulator starts to conduct
energy density	energy stored in a capacitor divided by the volume between the plates
induced electric-dipole moment	dipole moment that a nonpolar molecule may acquire when it is placed in an electrical field
induced electrical field	electrical field in the dielectric due to the presence of induced charges
induced surface charges	charges that occur on a dielectric surface due to its polarization
parallel combination	components in a circuit arranged with one side of each component connected to one side of the circuit and the other sides of the components connected to the other side of the circuit
parallel-plate capacitor	system of two identical parallel conducting plates separated by a distance
RC circuit	circuit that contains both a resistor and a capacitor
series combination	components in a circuit arranged in a row one after the other in a circuit

Key Equations

Capacitance	$C=\frac{Q}{V}$
Capacitance of a parallel-plate capacitor	$C=\varepsilon_0\frac{A}{d}$
Capacitance of a vacuum spherical capacitor	$C=4\pi\varepsilon_0\frac{R_1R_2}{R_2-R_1}$
Capacitance of a vacuum cylindrical capacitor	$C=\frac{2\pi\varepsilon_0 l}{ln(R_2/R_1)}$
Capacitance of a series combination	$\frac{1}{C_S}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\cdots$
Capacitance of a parallel combination	$C_P=C_1+C_2+C_3+\cdots$
Energy density	$u_E=\frac{1}{2}\varepsilon_0E^2$
Energy stored in a capacitor	$U_C=\frac{1}{2}V^2C=\frac{1}{2}\frac{Q^2}{C}=\frac{1}{2}QV$
Capacitance of a capacitor with dielectric	$C=\kappa C_0$

Energy stored in an isolated capacitor with dielectric	$U = \frac{1}{\kappa} U_0$
Dielectric constant	$\kappa = \frac{E_0}{E}$
Induced electrical field in a dielectric	$\vec{E}_i = (\frac{1}{\kappa} - 1)\vec{E}_0$
Charge on a charging capacitor	$q(t) = C\varepsilon(1 - e^{-\frac{t}{RC}}) = Q(1 - e^{-\frac{t}{\tau}})$
Time constant	$\tau = RC$
Current during charging of a capacitor	$I = \frac{\varepsilon}{R} e^{-\frac{t}{RC}} = I_o e^{-\frac{t}{RC}}$
Charge on a discharging capacitor	$q(t) = Qe^{-\frac{t}{\tau}}$
Current during discharging of a capacitor	$I(t) = -\frac{Q}{RC} e^{-\frac{t}{\tau}}$

Summary

Capacitors and Capacitance

- A capacitor is a device that stores an electrical charge and electrical energy. The amount of charge a vacuum capacitor can store depends on two major factors: the voltage applied and the capacitor's physical characteristics, such as its size and geometry.
- The capacitance of a capacitor is a parameter that tells us how much charge can be stored in the capacitor per unit potential difference between its plates. Capacitance of a system of conductors depends only on the geometry of their arrangement and physical properties of the insulating material that fills the space between the conductors. The unit of capacitance is the farad, where $1F = 1C/1V$.

Capacitors in Series and in Parallel

- When several capacitors are connected in a series combination, the reciprocal of the equivalent capacitance is the sum of the reciprocals of the individual capacitances.
- When several capacitors are connected in a parallel combination, the equivalent capacitance is the sum of the individual capacitances.
- When a network of capacitors contains a combination of series and parallel connections, we identify the series and parallel networks, and compute their equivalent capacitances step by step until the entire network becomes reduced to one equivalent capacitance.

Energy Stored in a Capacitor

- Capacitors are used to supply energy to a variety of devices, including defibrillators, microelectronics such as calculators, and flash lamps.
- The energy stored in a capacitor is the work required to charge the capacitor, beginning with no charge on its plates. The energy is stored in the electrical field in the space between the capacitor plates. It depends on the amount of electrical charge on the plates and on the potential difference between the plates.
- The energy stored in a capacitor network is the sum of the energies stored on individual capacitors in the network. It can be computed as the energy stored in the equivalent capacitor of the network.

Capacitor with a Dielectric

- The capacitance of an empty capacitor is increased by a factor of κ when the space between its plates is completely filled by a dielectric with dielectric constant κ.
- Each dielectric material has its specific dielectric constant.
- The energy stored in an empty isolated capacitor is decreased by a factor of κκ when the space between its plates is completely filled with a dielectric with dielectric constant κ.

Molecular Model of a Dielectric

- When a dielectric is inserted between the plates of a capacitor, equal and opposite surface charge is induced on the two faces of the dielectric. The induced surface charge produces an induced electrical field that opposes the field of the free charge on the capacitor plates.

- The dielectric constant of a material is the ratio of the electrical field in vacuum to the net electrical field in the material. A capacitor filled with dielectric has a larger capacitance than an empty capacitor.
- The dielectric strength of an insulator represents a critical value of electrical field at which the molecules in an insulating material start to become ionized. When this happens, the material can conduct and dielectric breakdown is observed.

Application - RC Circuits

- An **RC** circuit is one that has both a resistor and a capacitor.
- The time constant τ for an **RC** circuit is $\tau = RC$.
- When an initially uncharged ($q = 0$ at $t = 0$) capacitor in series with a resistor is charged by a dc voltage source, the capacitor asymptotically approaches the maximum charge.
- As the charge on the capacitor increases, the current exponentially decreases from the initial current: $I_0 = \varepsilon / R$.
- If a capacitor with an initial charge **Q** is discharged through a resistor starting at $t = 0$, then its charge decreases exponentially. The current flows in the opposite direction, compared to when it charges, and the magnitude of the charge decreases with time.

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