19.5: Power in an AC Circuit

Learning Objectives

By the end of the section, you will be able to:

- Describe how average power from an ac circuit can be written in terms of peak current and voltage and of rms current and voltage
- Determine the relationship between the phase angle of the current and voltage and the average power, known as the power factor

A circuit element dissipates or produces power according to *[Math Processing Error]*, where **I** is the current through the element and *[Math Processing Error]* is the voltage across it. Since the current and the voltage both depend on time in an ac circuit, the instantaneous power *[Math Processing Error]* is also time dependent. A plot of *[Math Processing Error]* for various circuit elements is shown in Figure *[Math Processing Error]*. For a resistor, *[Math Processing Error]* and *[Math Processing Error]* are in phase and therefore always have the same sign. For a capacitor or inductor, the relative signs of *[Math Processing Error]* and *[Math Processing Error]* vary over a cycle due to their phase differences. Consequently, *[Math Processing Error]* is positive at some times and negative at others, indicating that capacitive and inductive elements produce power at some instants and absorb it at others.

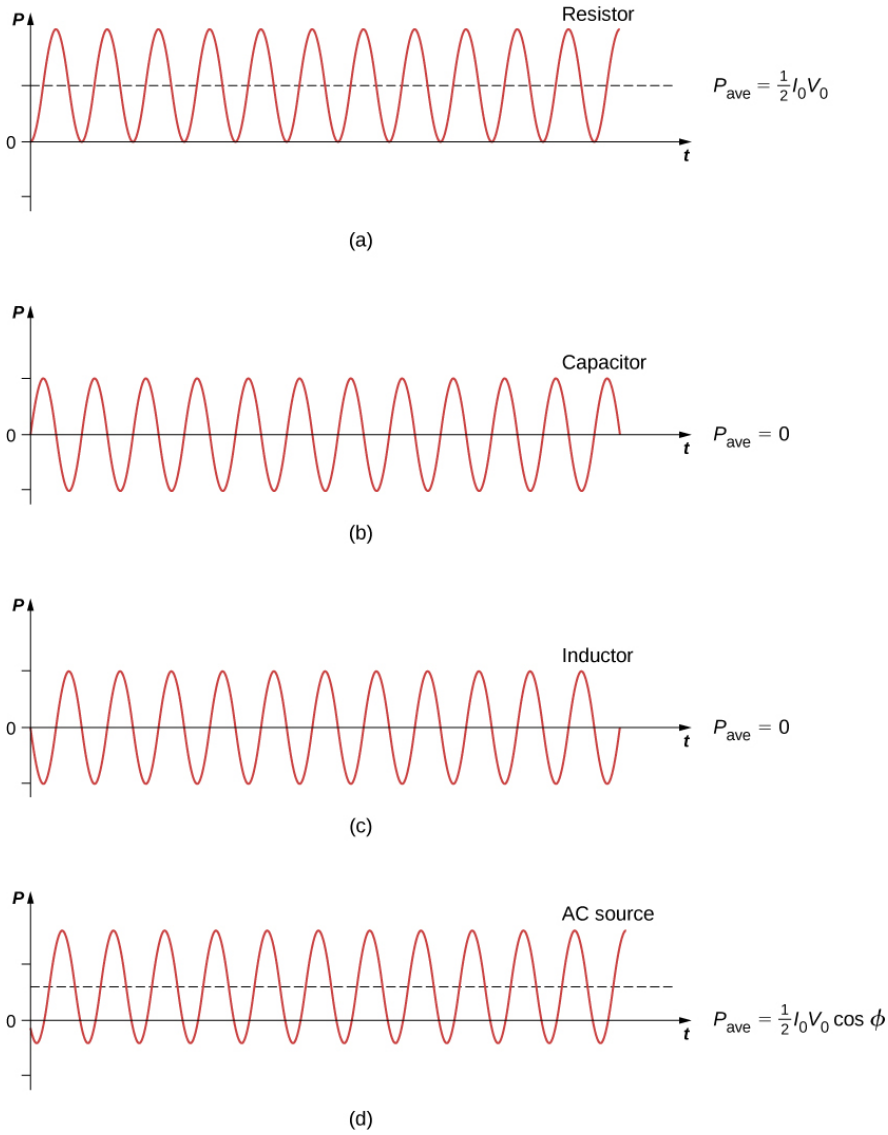

Figure *[Math Processing Error]*: Graph of instantaneous power for various circuit elements. (a) For the resistor, *[Math Processing Error]*, whereas for (b) the capacitor and (c) the inductor, *[Math Processing Error]*. (d) For the source, *[Math Processing Error]*, which may be positive, negative, or zero, depending on *[Math Processing Error]*.

Because instantaneous power varies in both magnitude and sign over a cycle, it seldom has any practical importance. What we're almost always concerned with is the power averaged over time, which we refer to as the **average power**. It is defined by the time average of the instantaneous power over one cycle:

[Math Processing Error]

where *[Math Processing Error]* is the period of the oscillations. With the substitutions *[Math Processing Error]* and *[Math Processing Error]*, Equation *[Math Processing Error]* becomes

[Math Processing Error]

Using the trigonometric difference identity

[Math Processing Error]

we obtain

[Math Processing Error]

Evaluation of these two integrals yields

[Math Processing Error]

and

[Math Processing Error]

Hence, the average power associated with a circuit element is given by

[Math Processing Error]

In engineering applications, *[Math Processing Error]* is known as the **power factor**, which is the amount by which the power delivered in the circuit is less than the theoretical maximum of the circuit due to voltage and current being out of phase. For a resistor, *[Math Processing Error]*, so the average power dissipated is

[Math Processing Error]

A comparison of *[Math Processing Error]* and *[Math Processing Error]* is shown in Figure *[Math Processing Error]*. To make Equation *[Math Processing Error]* look like its dc counterpart, we use the rms values *[Math Processing Error]* and *[Math Processing Error]* of the current and the voltage. By definition, these are

[Math Processing Error]

and

[Math Processing Error]

where

[Math Processing Error]

and

[Math Processing Error]

With *[Math Processing Error]* and *[Math Processing Error]*, we obtain

[Math Processing Error]

and

[Math Processing Error]

We may then write for the average power dissipated by a resistor,

[Math Processing Error]

This equation further emphasizes why the rms value is chosen in discussion rather than peak values. Both Equations *[Math Processing Error]* and *[Math Processing Error]* are correct for average power, but the rms values in the formula give a cleaner representation, so the extra factor of 1/2 is not necessary.

Alternating voltages and currents are usually described in terms of their rms values. For example, the 110 V from a household outlet is an rms value. The amplitude of this source is *[Math Processing Error]*. Because most ac meters are calibrated in terms of rms values, a typical ac voltmeter placed across a household outlet will read 110 V.

For a capacitor and an inductor, *[Math Processing Error]* and *[Math Processing Error]*, respectively. Since *[Math Processing Error]*, we find from Equation *[Math Processing Error]* that the average power dissipated by either of these elements is *[Math Processing Error]*. Capacitors and inductors absorb energy from the circuit during one half-cycle and then discharge it back to the circuit during the other half-cycle. This behavior is illustrated in the plots of Figures *[Math Processing Error]* and *[Math Processing Error]* which show *[Math Processing Error]* oscillating sinusoidally about zero.

The phase angle for an ac generator may have any value. If *[Math Processing Error]*, the generator produces power; if *[Math Processing Error]*, it absorbs power. In terms of rms values, the average power of an ac generator is written as

[Math Processing Error]

For the generator in an **RLC** circuit,

[Math Processing Error] and

[Math Processing Error]

Hence the average power of the generator is

[Math Processing Error]

This can also be written as

[Math Processing Error]

which designates that the power produced by the generator is dissipated in the resistor. As we can see, Ohm's law for the rms ac is found by dividing the rms voltage by the impedance.

Example *[Math Processing Error]*: Power Output of a Generator

An ac generator whose emf is given by

[Math Processing Error]

is connected to an **RLC** circuit for which *[Math Processing Error]*, and *[Math Processing Error]*.

a. What is the rms voltage across the generator?
b. What is the impedance of the circuit?
c. What is the average power output of the generator?

Strategy

The rms voltage is the amplitude of the voltage times *[Math Processing Error]*. The impedance of the circuit involves the resistance and the reactances of the capacitor and the inductor. The average power is calculated by Equation *[Math Processing Error]* because we have the impedance of the circuit *[Math Processing Error]*, the rms voltage *[Math Processing Error]*, and the resistance *[Math Processing Error]*.

Solution

1. Since *[Math Processing Error]*, the rms voltage across the generator is *[Math Processing Error]*
2. The impedance of the circuit is *[Math Processing Error]*
3. From Equation *[Math Processing Error]*, the average power transferred to the circuit is *[Math Processing Error]*

Significance

If the resistance is much larger than the reactance of the capacitor or inductor, the average power is a dc circuit equation of *[Math Processing Error]*, where **V** replaces the rms voltage.

Exercise *[Math Processing Error]*

An ac voltmeter attached across the terminals of a 45-Hz ac generator reads 7.07 V. Write an expression for the emf of the generator.

Answer

[Math Processing Error]

Exercise *[Math Processing Error]*

Show that the rms voltages across a resistor, a capacitor, and an inductor in an ac circuit where the rms current is *[Math Processing Error]* are given by *[Math Processing Error]*, and *[Math Processing Error]*, respectively. Determine these values for the components of the **RLC** circuit of Equation *[Math Processing Error]*.

Answer

2.00 V; 10.01 V; 8.01 V