

21.10: Wave Propagation on a Transmission Line

In Section 3.6, it is shown that expressions for the phasor representations of the potential and current along a transmission line are

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \quad (21.10.1)$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z} \quad (21.10.2)$$

where γ is the propagation constant and it is assumed that the transmission line is aligned along the z axis. In this section, we demonstrate that these expressions represent sinusoidal waves, and point out some important features. Before attempting this section, the reader should be familiar with the contents of Sections 3.4, 3.6 and 3.7. A refresher on fundamental wave concepts (Section 1.3) may also be helpful.

We first define real-valued quantities α and β to be the real and imaginary components of γ ; i.e.,

$$\alpha \triangleq \text{Re}\{\gamma\}$$

$$\beta \triangleq \text{Im}\{\gamma\}$$

and subsequently

$$\gamma = \alpha + j\beta$$

Then we observe

$$e^{\pm\gamma z} = e^{\pm(\alpha+j\beta)z} = e^{\pm\alpha z} e^{\pm j\beta z}$$

It may be easier to interpret this expression by reverting to the time domain:

$$\text{Re}\{e^{\pm\gamma z} e^{j\omega t}\} = e^{\pm\alpha z} \cos(\omega t \pm \beta z)$$

Thus, $e^{-\gamma z}$ represents a damped sinusoidal wave traveling in the $+z$ direction, and $e^{+\gamma z}$ represents a damped sinusoidal wave traveling in the $-z$ direction.

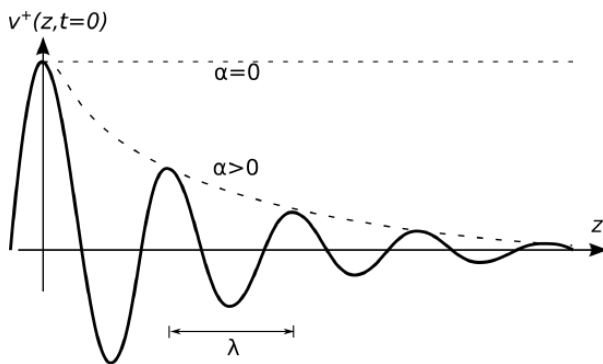


Figure 21.10.1 The potential $v^+(z, t)$ of the wave traveling in the $+z$

direction at $t = 0$ for $\psi = 0$.

Let's define $\tilde{V}^+(z)$ and $\tilde{I}^+(z)$ to be the potential and current associated with a wave propagating in the $+z$ direction. Then:

$$\tilde{V}^+(z) \triangleq V_0^+ e^{-\gamma z}$$

or equivalently in the time domain:

$$\begin{aligned} v^+(z, t) &= \text{Re}\{\tilde{V}^+(z) e^{j\omega t}\} \\ &= \text{Re}\{V_0^+ e^{-\gamma z} e^{j\omega t}\} \\ &= |V_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \psi) \end{aligned}$$

where ψ is the phase of V_0^+ . Figure 21.10.1 shows $v^+(z, t)$. From fundamental wave theory we recognize

$\beta \triangleq \text{Im}\{\gamma\}$ (rad/m) is the *phase propagation constant*, which is the rate at which phase changes as a function of distance.

Subsequently the wavelength in the line is

$$\lambda = \frac{2\pi}{\beta}$$

Also we recognize:

$\alpha \triangleq \text{Re}\{\gamma\}$ (1/m) is the *attenuation constant*, which is the rate at which magnitude diminishes as a function of distance.

Sometimes the units of α are indicated as “Np/m” (“nepers” per meter), where the term “neper” is used to indicate the units of the otherwise unitless real-valued exponent of the constant e .

Note that $\alpha = 0$ for a wave that does not diminish in magnitude with increasing distance, in which case the transmission line is said to be *lossless*. If $\alpha > 0$ then the line is said to be *lossy* (or possibly “low loss” if the loss can be neglected), and in this case the rate at which the magnitude decreases with distance increases with α .

Next let us consider the speed of the wave. To answer this question, we need to be a bit more specific about what we mean by “speed.” At the moment, we mean phase velocity; that is, the speed at which a point of constant phase seems to move through space. In other words, what distance Δz does a point of constant phase traverse in time Δt ? To answer this question, we first note that the phase of $v^+(z, t)$ can be written generally as

$$\omega t - \beta z + \phi$$

where ϕ is some constant. Similarly, the phase at some time Δt later and some point Δz further along can be written as

$$\omega(t + \Delta t) - \beta(z + \Delta z) + \phi$$

The phase velocity v_p is $\Delta z / \Delta t$ when these two phases are equal; i.e., when

$$\omega t - \beta z + \phi = \omega(t + \Delta t) - \beta(z + \Delta z) + \phi$$

Solving for $v_p = \Delta z / \Delta t$, we obtain:

$$v_p = \frac{\omega}{\beta}$$

Having previously noted that $\beta = 2\pi / \lambda$, the above expression also yields the expected result

$$v_p = \lambda f$$

The phase velocity $v_p = \omega / \beta = \lambda f$ is the speed at which a point of constant phase travels along the line.

Returning now to consider the current associated with the wave traveling in the $+z$ direction:

$$\tilde{I}^+(z) = I_0^+ e^{-\gamma z}$$

We can rewrite this expression in terms of the characteristic impedance Z_0 , as follows:

$$\tilde{I}^+(z) = \frac{V_0^+}{Z_0} e^{-\gamma z}$$

Similarly, we find that the current $\tilde{I}^-(z)$ associated with $\tilde{V}^-(z)$ for the wave traveling in the $-z$ direction is

$$\tilde{I}^-(z) = \frac{-V_0^-}{Z_0} e^{+\gamma z}$$

The negative sign appearing in the above expression emerges as a result of the sign conventions used for potential and current in the derivation of the telegrapher’s equations (Section 3.5). The physical significance of this change of sign is that wherever the potential of the wave traveling in the $-z$ direction is positive, then the current at the same point is flowing in the $-z$ direction.

It is frequently necessary to consider the possibility that waves travel in both directions simultaneously. A very important case where this arises is when there is reflection from a discontinuity of some kind; e.g., from a termination which is not perfectly impedance-matched. In this case, the total potential $\tilde{V}(z)$ and total current $\tilde{I}(z)$ can be expressed as the general solution to the wave equation; i.e., as the sum of the “incident” (+ z -traveling) wave and the reflected ($-z$ -traveling) waves:

$$\tilde{V}(z) = \tilde{V}^+(z) + \tilde{V}^-(z)$$

$$\tilde{I}(z) = \tilde{I}^+(z) + \tilde{I}^-(z)$$

The existence of waves propagating simultaneously in both directions gives rise to a phenomenon known as a *standing wave*. Standing waves and the calculation of the coefficients V_0^- and I_0^- due to reflection are addressed in Sections 3.13 and 3.12 respectively.

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