2.8: Motion of a Charged Particle in an Electric Field

As we have seen, when a particle with charge q is placed in an electric field $\vec{E}$, the field causes an electric force $\vec{F}_E = q\vec{E}$ on the charge, as illustrated in Figure 2.8.1. A positive charge has a force in the same direction of the electric field, while a negative charge has an electric force in the opposite direction of the field.

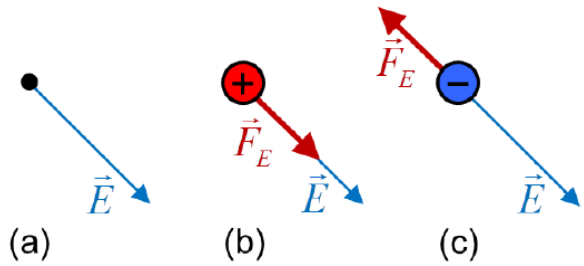

Figure 2.8.1: (a) Electric field at a point in space. (b) A positive charge placed at the same point in space will have a force in the same direction of the electric field. (c) A positive charge placed at the same point in space will have a force in the opposite direction of the electric field. (Ronald Kumon, CC-BY-SA 4.0)

According to Newton's Second Law, the net force $\vec{F}_{\text{net}} = m\vec{a}$, where m is the mass of the object upon which the force is exerted, and $\vec{a}$ is the acceleration of the object. If the electric force is the only force acting on the charged particle, then the acceleration of the particle will be

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q}{m}\vec{E} \tag{2.8.1}$$

We see that the acceleration is proportional to the electric field by a factor of the charge-to-mass ratio of the particle. In general, the electric field will vary from point to point in space and so the acceleration will also vary from point to point, and computational methods may be needed to determine the motion of the particle.

Charged Particle Motion in a Uniform Electric Field

in the special case that the electric field is uniform, then the acceleration is constant, and kinematic equations can be used to calculate the position and velocity of the particle.

Example 2.8.1

A thin square conducting plate 0.500 m on each side is given a charge of +0.200 nC. An electron at rest is near the center of the place and 1.00 cm above the plate.

(a) How long does it take for the electron to hit the plate?

(b) What is the velocity of the electron when it hits the plate?

Solution

PLAN

We will model the plate as an infinite plane. This assumption is plausible because the electron is away from the edges of the plate and its distance away from the plate is small compared to the width of the plate. According to Common Models of Electric Field, the electric field for an infinite plane is uniform, and therefore we can use kinematic equations (Motion with Constant Acceleration) to solve for the electron's travel time and final velocity. We will also ignore the effects of gravity on the electron given its very small mass and the effects of drag due its very small size.

SKETCH

We sketch out a side view of the plate, showing the electron at a distance $d = 1.00\text{ cm} = 0.0100\text{ m}$ above the plate.

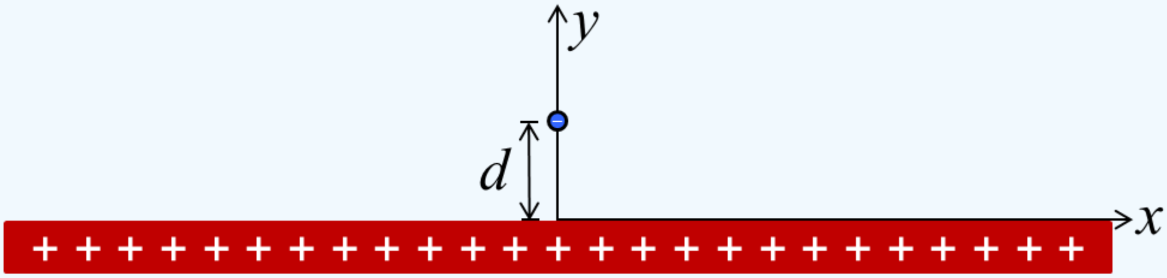

Figure 2.8.2: Electron at a distance d above a positively-charged plate. (Ronald Kumon, CC-BY-SA 4.0)

CALCULATE

We will first calculate the electric field at the position of the electron (see section on the "Infinite Plane" in Common Models of Electric Field). Observe that the surface charge density σ on the square plate is the charge on the plate divided by its surface area

$$\sigma = \frac{Q}{L^2} = \frac{+0.200\times10^{-9}\ \mathrm{C}}{(0.500\ \mathrm{m})^2} = 8.00\times10^{-10}\mathrm{C/m^2}\ . \tag{2.8.2}$$

The electric field is related to the surface charge density according to

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\ \hat{j} = \frac{2.00\times10^{-7}\ \mathrm{C/m^2}}{2\cdot 8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}}\ \hat{j} = 45.2\ \mathrm{N/C}\ \hat{j}\ . \tag{2.8.3}$$

The field is directed upwards above the plate because the plate is positively charged. By Eq. 2.8.1, the acceleration of the particle is then

$$\vec{a} = \frac{q}{m}\vec{E} = \frac{-1.6\times10^{-19}\ \mathrm{C}}{9.11\times10^{-31}\ \mathrm{kg}}(45.2\ \mathrm{N/C}\ \hat{j}) = (-7.94\times10^{12}\ \mathrm{m/s^2})\ \hat{j}\ . \tag{2.8.4}$$

This result indicates the acceleration is downward toward the plate, as expected.

(a) According to the kinematic equations, the vertical position of the particle $y = y_0 + v_{y0}\Delta t + \frac{1}{2}a_y(\Delta t)^2$, where y_0 is the initial position and v_{y0} is the initial y-component of the velocity (zero in this case because the particle is starting at rest). Solving for the time of flight yields

$$\Delta t = \left(\frac{2y_0}{a_y}\right)^{1/2} = \left(\frac{2\cdot 0.0100\ \mathrm{m}}{1.98\times10^{15}\ \mathrm{m/s^2}}\right)^{1/2} = 5.02\times10^{-8}\ \mathrm{s}\ . \tag{2.8.5}$$

(b) The vertical component of the velocity of the particle can be computed from the kinematic equation

$$v_y = v_{y0} + a_y\Delta t = 0\ \mathrm{m/s} + (-1.98\times10^{15}\ \mathrm{m/s^2})(3.17\times10^{-9}\ \mathrm{s}) = -3.98\times10^{5}\ \mathrm{m/s}\ . \tag{2.8.6}$$

CHECK

(a) The mass of the electron is very small and the acceleration is very high, so the travel time is very small, as expected.

(b) The final speed of the particle is large but plausible given the large value of the acceleration.

? Exercise 2.8.1

For the scenario as in Example 2.8.1, what is the minimum speed that the particle can be traveling in the x-direction to miss the landing on the plate?

Answer

The motion in the x- and y-directions are independent. In the x-direction, the kinematic equation is

$$x = x_0 + v_{x0}\Delta t + \frac{1}{2}a_x(\Delta t)^2 = v_{x0}\Delta t\,. \tag{2.8.7}$$

because the particle starts with $x = 0$ in the coordinate system in the Example 2.8.1 and there is no force or acceleration acting in the x-direction. Solving for the x-component of the velocity yields

$$v_{x0} = \frac{x}{\Delta t} = \frac{0.5\text{ m}}{5.02\times 10^{-8}\text{ s}} = 9.36\times 10^{6}\text{ m/s} \tag{2.8.8}$$

This is a high velocity but plausible given the short time of travel before the electron hits the plate.

The principle illustrated by Example 2.8.1 and Exercise 2.8.1 is applied in *electrostatic precipitators* [1], which use the principle to pull unwanted particulate matter from the air. By charging the unwanted particles (e.g., fly ash from a coal-fired power plant) and passing the particles over charged plates, the particles can be filtered out of the air, as illustrated in Figure 2.8.3.

Large electrostatic precipitators are used industrially to remove over 99% of the particles from stack gas emissions associated with the burning of coal and oil. Home precipitators, often in conjunction with the home heating and air conditioning system, are very effective in removing polluting particles, irritants, and allergens.

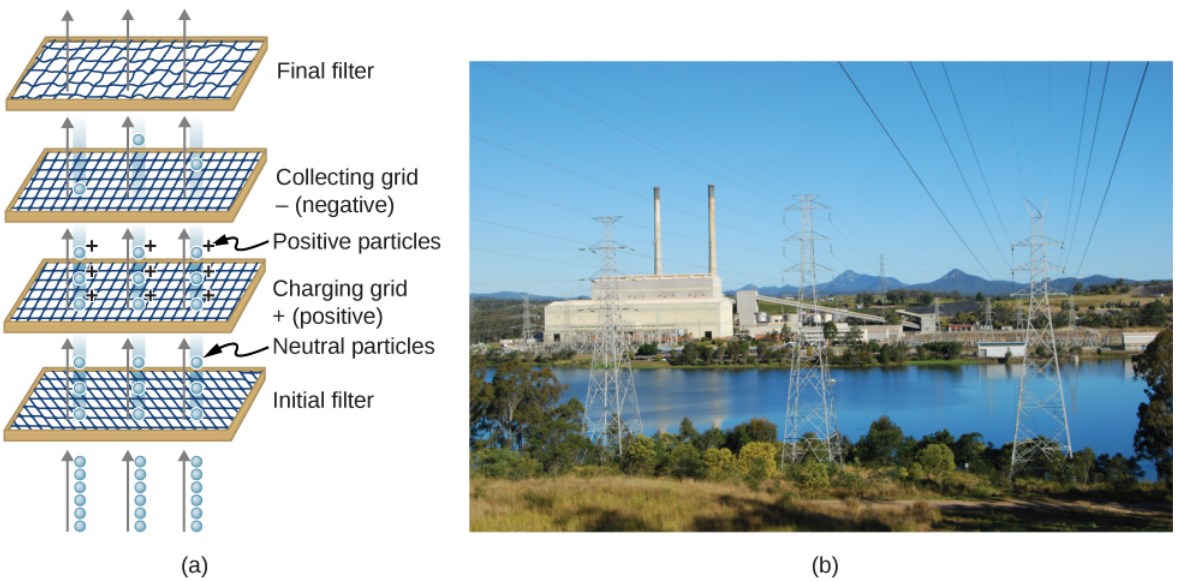

Figure 2.8.3: (a) Schematic of an electrostatic precipitator. Air is passed through grids of opposite charge. The first grid charges airborne particles, while the second attracts and collects them. (b) The dramatic effect of electrostatic precipitators is seen by the absence of smoke from this power plant. (credit b: modification of work by “Cmdalgleish”/Wikimedia Commons) (Figure from Applications of Electrostatics).

Electric Field in the Ionosphere

Electric fields exist naturally in the atmosphere, and these fields can affect charged particles in the air. Atoms in the upper atmosphere are ionized by energy from the sun in a layer of the atmosphere called the *ionosphere*. (We will see later that the ionosphere can play an important role in the propagation of radio waves.) After ionization, the freed electrons are still attracted to the positive ions and so are not completely lost out into space. The resulting separation charge creates a *polarization electric field* in the ionosphere, sometimes also called an *ambipolar electric field*. We will explore some possible consequences of this field on the atmosphere in Example 2.8.2.

Example 2.8.2

The polarization electric field in a part of the ionosphere in altitude has been measured to be 1.09×10^{-6} N/C in the upward direction (directed radially away from the surface of the earth) [2].

(a) What is the acceleration of a hydrogen ion in this region? What is the direction of the net acceleration?

(b) How long will it take for the ion to reach the speed of sound on the earth's surface (340 m/s) with this acceleration?

Solution

PLAN

To find the acceleration of the ion, we need to first calculate the net force. We will consider the effect of both the electric field and gravitational field on the ion as these fields are acting in opposite directions. If we assume that the acceleration is approximately constant, we can estimate the time to increase the ion's speed to a supersonic value.

SKETCH

We draw a free-body diagram for the ion, as seen in Figure 2.8.4 to see the relative effects of the forces. (The vector lengths come from the calculations below.)

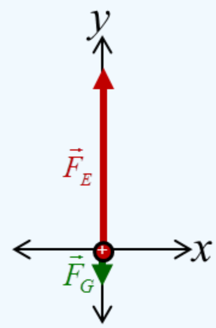

Figure 2.8.1: Free-body diagram for the hydrogen ion. (Ronald Kumon, CC-BY-SA 4.0)

CALCULATE

(a) A hydrogen ion is just a proton, which has mass of $m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$ and a charge of $q_p = e = +1.60 \times 10^{-19}\,\mathrm{C}$. The electric force on the ion is then

$$\overrightarrow{F_E} = q_p \vec{E} = (+1.6 \times 10^{-19}\,\mathrm{C})(+1.09 \times 10^{-6}\,\mathrm{N/C}\,\hat{j}) = 1.74 \times 10^{-25}\,\mathrm{N}\,\hat{j} \tag{2.8.9}$$

The gravitational force on the ion can be approximated by

$$\overrightarrow{F_G} = m_p \vec{g} = (1.67 \times 10^{-27}\,\mathrm{kg})(-9.8\,\mathrm{m/s^2}\,\hat{j}) = -1.64 \times 10^{-26}\,\mathrm{N}\,\hat{j}\,. \tag{2.8.10}$$

(This is an approximation because the acceleration of gravity decreases with altitude according to Newton's Law of Universal Gravitation; see Figure 1 in [3].)

The net force on the ion is then

$$\vec{F}_{\mathrm{net}} = \vec{F}_E + \vec{F}_G = 1.74 \times 10^{-25}\,\mathrm{N}\,\hat{j} + -0.164 \times 10^{-25}\,\mathrm{N}\,\hat{j} = 1.58 \times 10^{-25}\,\mathrm{N}\,\hat{j}\,. \tag{2.8.11}$$

The acceleration of the ion is

$$\vec{a} = \frac{\vec{F}_{\mathrm{net}}}{m_p} = \frac{1.58 \times 10^{-25}\,\mathrm{N}\,\hat{j}}{1.67 \times 10^{-27}\,\mathrm{kg}} = 94.6\,\mathrm{m/s^2}\,\hat{j}\,, \tag{2.8.12}$$

which is around 9.7 times the acceleration of gravity on the surface of the earth

(b) If we assume that the acceleration is approximately constant and the ion starts at rest, then the time to reach 340 m/s can be estimated from the kinematic equation $v_y = v_{y0} + a_y \Delta t$ such that

$$\Delta t = \frac{340\,\mathrm{m/s}}{94.6\,\mathrm{m/s^2}} = 3.59\,\mathrm{s}\,. \tag{2.8.13}$$

(The value is an estimate because speed of sound increases with altitude [4], so it would actually take longer to reach the local sound speed in the ionosphere.)

CHECK

The calculations indicate that the ambipolar electric field should create supersonic jets in the ionosphere. These *polar winds* have been experimentally observed (see news article [5] and visualizations [6]). The value of the acceleration is close to value of $10.6g$ given in [2], which is the acceleration that would be estimated by including the effect of the electric field only. Of course, we have simplified the problem considerably here (e.g., by neglecting the effect of the earth's magnetic field), and the actual physics are considerably more complicated; see [2] for further discussion.

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