

22.6: Equivalent Circuit Model for Transmission; Radiation Efficiency

A radio transmitter consists of a source which generates the electrical signal intended for transmission, and an antenna which converts this signal into a propagating electromagnetic wave. Since the transmitter is an electrical system, it is useful to be able to model the antenna as an equivalent circuit. From a circuit analysis point of view, it should be possible to describe the antenna as a passive one-port circuit that presents an impedance to the source. Thus, we have the following question: What is the equivalent circuit for an antenna which is transmitting?

We begin by emphasizing that the antenna is passive. That is, the antenna does not add power. Invoking the principle of conservation of power, there are only three possible things that *can* happen to power that is delivered to the antenna by the transmitter:¹

- Power can be converted to a propagating electromagnetic wave. (The desired outcome.)
- Power can be dissipated within the antenna.
- Energy can be stored by the antenna, analogous to the storage of energy in a capacitor or inductor.

We also note that these outcomes can occur in any combination. Taking this into account, we model the antenna using the equivalent circuit shown in Figure 22.6.1.

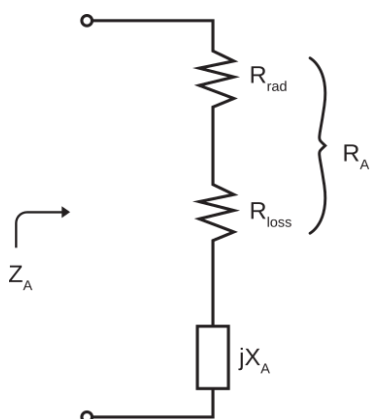


Figure 22.6.1 Equivalent circuit for an antenna which is transmitting. (CC BY-SA 4.0; S.

Lally)

Since the antenna is passive, it is reasonable to describe it as an impedance Z_A which is (by definition) the ratio of voltage \tilde{V}_A to current \tilde{I}_A at the terminals; i.e.,

$$Z_A \triangleq \frac{\tilde{V}_A}{\tilde{I}_A}$$

In the phasor domain, Z_A is a complex-valued quantity and therefore has, in general, a real-valued component and an imaginary component. We may identify those components using the power conservation argument made previously: Since the real-valued component must represent power transfer and the imaginary component must represent energy storage, we infer:

$$Z_A \triangleq R_A + jX_A$$

where R_A represents power transferred to the antenna, and X_A represents energy stored by the antenna. Note that the energy stored by the antenna is being addressed in precisely the same manner that we address energy storage in a capacitor or an inductor; in all cases, as reactance. Further, we note that R_A consists of components R_{rad} and R_{loss} as follows:

$$Z_A = R_{rad} + R_{loss} + jX_A$$

where R_{rad} represents power transferred to the antenna and subsequently radiated, and R_{loss} represents power transferred to the antenna and subsequently dissipated.

To confirm that this model works as expected, consider what happens when a voltage is applied across the antenna terminals. The current \tilde{I}_A flows and the time-average power P_A transferred to the antenna is

$$P_A = \frac{1}{2} \text{Re} \left\{ \tilde{V}_A \tilde{I}_A^* \right\}$$

where we have assumed peak (as opposed to root mean squared) units for voltage and current. Since $\tilde{V}_A = Z_A \tilde{I}_A$, we have:

$$P_A = \frac{1}{2} \text{Re} \left\{ (R_{rad} + R_{loss} + jX_A) \tilde{I}_A \tilde{I}_A^* \right\}$$

which reduces to:

$$P_A = \frac{1}{2} |\tilde{I}_A|^2 R_{rad} + \frac{1}{2} |\tilde{I}_A|^2 R_{loss} \quad (22.6.1)$$

As expected, the power transferred to the antenna is the sum of

$$P_{rad} \triangleq \frac{1}{2} |\tilde{I}_A|^2 R_{rad} \quad (22.6.2)$$

representing power transferred to the radiating electromagnetic field, and

$$P_{loss} \triangleq \frac{1}{2} |\tilde{I}_A|^2 R_{loss} \quad (22.6.3)$$

representing power dissipated within the antenna.

The reactance X_A will play a role in determining \tilde{I}_A given \tilde{V}_A (and vice versa), but does not by itself account for disposition of power. Again, this is exactly analogous to the role played by inductors and capacitors in a circuit.

The utility of this equivalent circuit formalism is that it allows us to treat the antenna in the same manner as any other component, and thereby facilitates analysis using conventional electric circuit theory and transmission line theory. For example: Given Z_A , we know how to specify the output impedance Z_S of the transmitter so as to minimize reflection from the antenna: We would choose $Z_S = Z_A$, since in this case the voltage reflection coefficient would be

$$\Gamma = \frac{Z_A - Z_S}{Z_A + Z_S} = 0$$

Alternatively, we might specify Z_S so as to maximize power transfer to the antenna: We would choose $Z_S = Z_A^*$; i.e., conjugate matching.

In order to take full advantage of this formalism, we require values for R_{rad} , R_{loss} , and X_A . These quantities are considered below.

Radiation resistance

(R_{rad}) is referred to as *radiation resistance*. Equation 22.6.2 tells us that

$$R_{rad} = 2P_{rad} |\tilde{I}_A|^{-2} \quad (22.6.4)$$

This equation suggests the following procedure: We apply current \tilde{I}_A to the antenna terminals, and then determine the total power P_{rad} radiated from the antenna in response. For an example of this procedure, see Section 10.2 ("Total Power Radiated by an Electrically-Short Dipole"). Given \tilde{I}_A and P_{rad} , one may then use Equation 22.6.4 to determine R_{rad} .

Loss resistance

Loss resistance represents the dissipation of power within the antenna, which is usually attributable to loss intrinsic to materials comprising or surrounding the antenna. In many cases, antennas are made from good conductors – metals, in particular – so that R_{loss} is very low compared to R_{rad} . For such antennas, loss is often so low compared to R_{rad} that R_{loss} may be neglected. In the case of the electrically-short dipole, R_{loss} is typically very small but R_{rad} is also very small, so both must be considered. In many other cases, antennas contain materials with substantially greater loss than metal. For example, a microstrip patch antenna implemented on a printed circuit board typically has non-negligible R_{loss} because the dielectric material comprising the antenna exhibits significant loss.

Antenna reactance

The reactance term jX_A accounts for energy stored by the antenna. This may be due to reflections internal to the antenna, or due to energy associated with non-propagating electric and magnetic fields surrounding the antenna. The presence of significant reactance

(i.e., $|X_A|$ comparable to or greater than $|R_A|$) complicates efforts to establish the desired impedance match to the source. For an example, see Section 10.4 (“Reactance of the Electrically-Short Dipole”).

Radiation efficiency

When R_{loss} is non-negligible, it is useful to characterize antennas in terms of their *radiation efficiency* e_{rad} , defined as the fraction of power which is radiated compared to the total power delivered to the antenna; i.e.,

$$e_{rad} \triangleq \frac{P_{rad}}{P_A}$$

Using Equations 22.6.1-22.6.3, we see that this efficiency can be expressed as follows:

$$e_{rad} = \frac{R_{rad}}{R_{rad} + R_{loss}} \quad (22.6.5)$$

Once again, the equivalent circuit formalism proves useful.

✓ Example 22.6.1: Impedance of an antenna

The total power radiated by an antenna is 60 mW when 20 mA (rms) is applied to the antenna terminals. The radiation efficiency of the antenna is known to be 70%. It is observed that voltage and current are in-phase at the antenna terminals. Determine (a) the radiation resistance, (b) the loss resistance, and (c) the impedance of the antenna.

Solution

From the problem statement, $P_{rad} = 60$ mW, $|\tilde{I}_A| = 20$ mA (rms), and $e_{rad} = 0.7$. Also, the fact that voltage and current are in-phase at the antenna terminals indicates that $X_A = 0$. From Equation 22.6.4, the radiation resistance is

$$R_{rad} \approx \frac{2 \cdot (60 \text{ mW})}{|\sqrt{2} \cdot 20 \text{ mA}|^2} = \underline{150 \Omega}$$

Solving Equation 22.6.5 for the loss resistance, we find:

$$R_{loss} = \frac{1 - e_{rad}}{e_{rad}} R_{rad} \cong \underline{64.3 \Omega}$$

Since $Z_A = R_{rad} + R_{loss} + jX_A$, we find $Z_A \cong \underline{214.3 + j0 \Omega}$. This will be the ratio of voltage to current at the antenna terminals regardless of the source current.

1. Note that “delivered” power means power accepted by the antenna. We are not yet considering power reflected from the antenna due to impedance mismatch.↩

This page titled 22.6: Equivalent Circuit Model for Transmission; Radiation Efficiency is shared under a CC BY-SA license and was authored, remixed, and/or curated by Steven W. Ellingson (Virginia Tech Libraries' Open Education Initiative).

- 10.5: Equivalent Circuit Model for Transmission; Radiation Efficiency by Steven W. Ellingson is licensed CC BY-SA 4.0. Original source: <https://doi.org/10.21061/electromagnetics-vol-2>.