4.3: Equipotential Curves and Surfaces

Learning Objectives

By the end of this section, you will be able to:

- Define equipotential lines and and equipotential surfaces.
- Explain the relationships between equipotential lines and electric field lines.
- Map equipotential lines for one or two point charges.

Equipotentials

We can represent electric potentials (voltages) pictorially, just as we drew pictures to illustrate electric fields. This is not surprising, since the two concepts are related. Consider Figure $4.3.1$, which shows an isolated positive point charge and its electric field lines, which radiate out from a positive charge and terminate on negative charges. We use red arrows to represent the magnitude and direction of the electric field, and we use black lines to represent places where the electric potential is constant. These are called **equipotential surfaces** in three dimensions, or **equipotential lines** in two dimensions. The term **equipotential** is also used as a noun, referring to an equipotential line or surface. The potential for a point charge is the same anywhere on an imaginary sphere of radius r surrounding the charge. This is true because the potential for a point charge is given by $V = kq/r$ and thus has the same value at any point that is a given distance r from the charge. An equipotential sphere is a circle in the two-dimensional view of Figure $4.3.1$. Because the electric field lines point radially away from the charge, they are perpendicular to the equipotential lines.

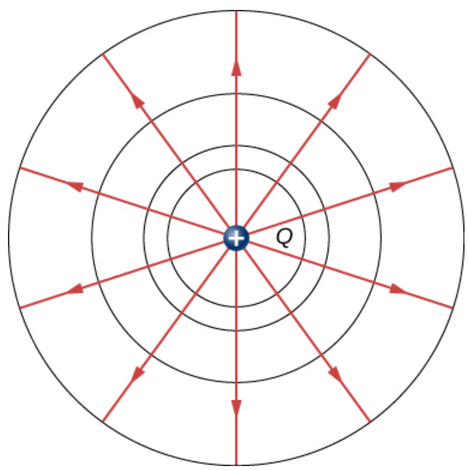

Figure $4.3.1$: An isolated point charge Q with its electric field lines with red arrows and equipotential lines in black. The potential is the same along each equipotential line, meaning that no work is required to move a charge anywhere along one of those lines. Work is needed to move a charge from one equipotential line to another. Equipotential lines are perpendicular to electric field lines in every case. For a three-dimensional visualization, see http://falstad.com/vector3de/ (select Field selection: point charge, Display: Equipotentials or Display: Field Lines, and "No Slicing"; adjust potential or field line density, as desired).

It is important to note that *equipotential lines are always perpendicular to electric field lines*. No work is required to move a charge along an equipotential, since $\Delta V = 0$. Thus, the work is

$$W = -\Delta U = -q\Delta V = 0. \tag{4.3.1}$$

Work is zero if the direction of the force is perpendicular to the displacement. Force is in the same direction as E, so motion along an equipotential must be perpendicular to E. More precisely, work is related to the electric field by

$$\begin{aligned} W &= \vec{F} \cdot \vec{d} \\ &= q\vec{E} \cdot \vec{d} \\ &= qEd \cos\theta \\ &= 0. \end{aligned} \tag{4.3.2}$$

Note that in Equation 4.3.2, E and F symbolize the magnitudes of the electric field and force, respectively. Neither q nor E is zero and d is also not zero. So cos θ must be 0, meaning θ must be $90°$. In other words, motion along an equipotential is perpendicular to $\vec{E}$.

In addition, the strength of the field is related to the density of the equipotentials, assuming that they have equal changes in potential. To see this, recall from Electric Field from Electric Potential that the component of the electric field in the s-direction is

$$E_s = -\frac{dV}{ds} \tag{4.3.3}$$

For small spatial intervals, the derivative can be approximated by a ratio of finite differences

$$E_s \approx -\frac{\Delta V}{\Delta s} \tag{4.3.4}$$

If ΔV is the same for adjacent equipotentials, then Equation 4.3.4 implies that the electric field strength will be higher for more closely-spaced equipotentials where Δs is smaller and lower where the spacing is higher, as illustrated in Figure 4.3.2. The negative sign means that the electric field will point in the direction of decreasing potential.

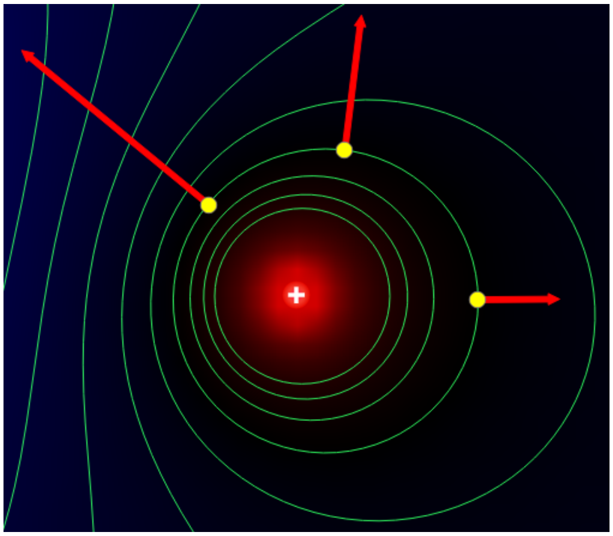

Figure 4.3.2: Relationship between equipotentials (green) and electric field (red arrows). The electric field direction is always perpendicular to the equipotentials and points toward decreasing potential. The electric field magnitude is inversely proportional to the spacing of equipotentials of equal intervals (Ronald Kumon, CC-BY-SA 4.0)

Equipotentials of Point-Charge Systems

Figure 4.3.3 shows the electric field and equipotential lines for two equal and opposite charges. Given the electric field lines, the equipotential lines can be drawn simply by making them perpendicular to the electric field lines.

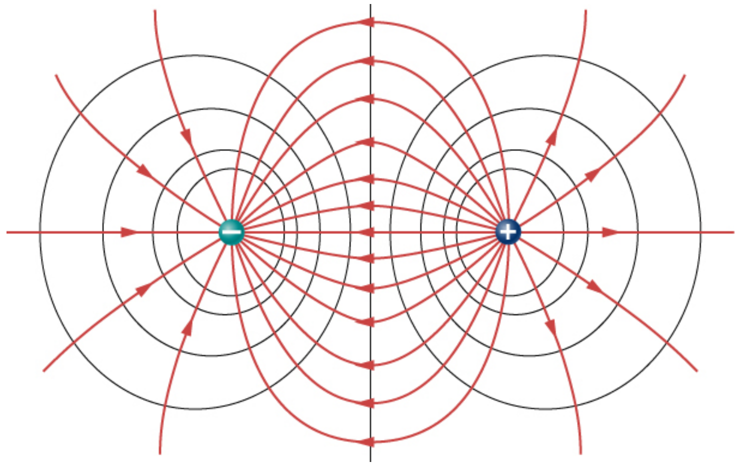

Figure $4.3.3$: The electric field lines (red arrows) and equipotential lines (black) for two equal but opposite charges. The equipotential lines can be drawn by making them perpendicular to the electric field lines, if those are known. Note that the potential is greatest (most positive) near the positive charge and least (most negative) near the negative charge. For a three-dimensional version, see http://falstad.com/vector3de/ (select Field selection: dipole, Display: Equipotentials or Display: Field Lines, and "No Slicing"; adjust settings for charge separation, and potential or field line density, as desired).

Conversely, given the equipotential lines, as in Figure $4.3.4a$, the electric field lines can be drawn by making them perpendicular to the equipotentials, as in Figure $4.3.4b$.

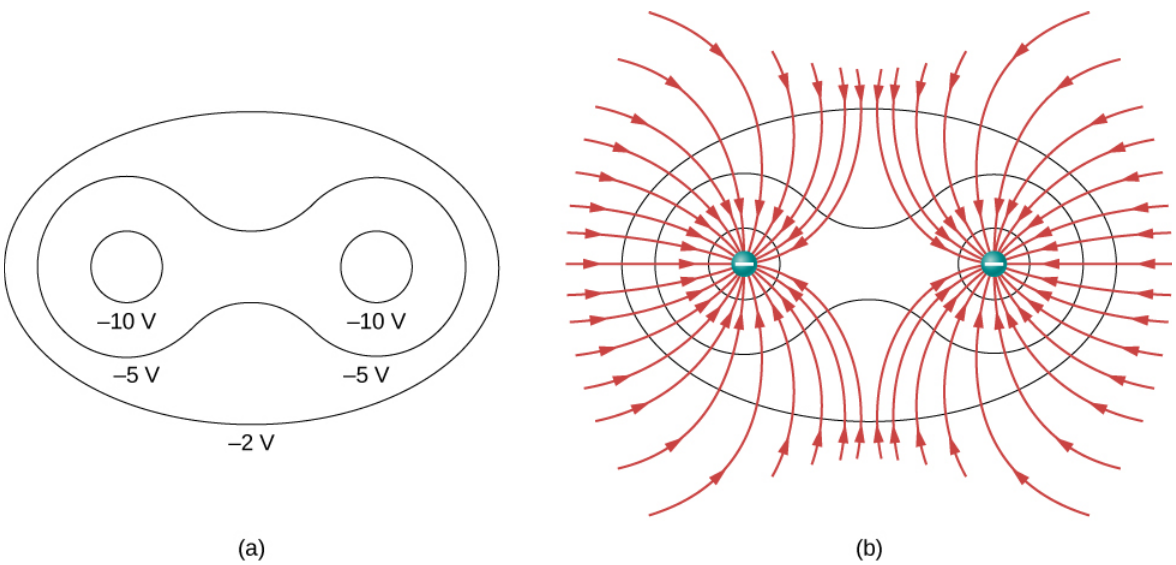

Figure $4.3.4$: (a) These equipotential lines might be measured with a voltmeter in a laboratory experiment. (b) The corresponding electric field lines are found by drawing them perpendicular to the equipotentials. Note that these fields are consistent with two equal negative charges. For a three-dimensional version, see http://falstad.com/vector3de/ (select Field selection: point charge double, Display: Equipotentials or Display: Field Lines, and "No Slicing"; adjust settings for charge separation, and potential or field line density, as desired).

Electric equipotential lines are sometimes compared to equipotential isolines (lines of constant elevation) for gravity on hills . If the hill has any extent at the same slope, the isolines along that extent would be parallel to each other. Furthermore, in regions of constant slope, the isolines would be evenly spaced. An example of real topographic lines is shown in Figure $4.3.5$.

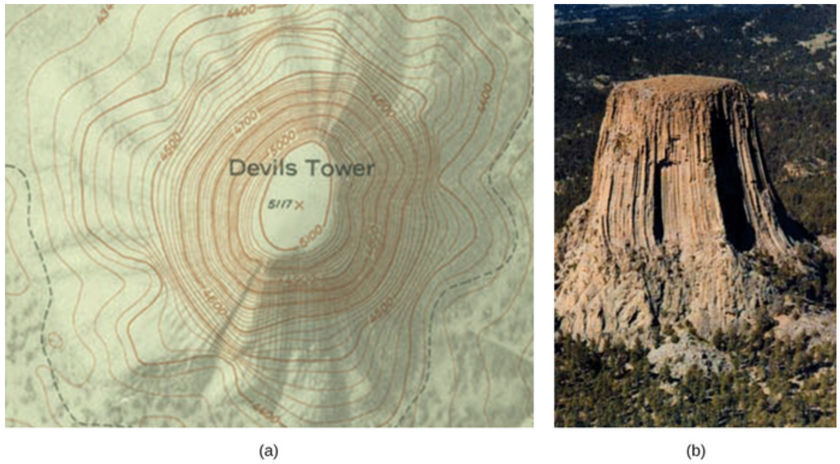

Figure 4.3.5. (a) A topographical map of Devil's Tower, Wyoming. Lines that are close together indicate very steep terrain. (b) A perspective photo of Devil's Tower shows just how steep its sides are. Notice the top of the tower has the same shape as the center of the topographical map

To improve your intuition, we show a three-dimensional variant of the potential in a system with two opposing charges. Figure 4.3.6 displays a three-dimensional map of electric potential, where lines on the map are for equipotential surfaces. The hill is at the positive charge, and the trough is at the negative charge. The potential is zero far away from the charges. Note that the cut off at a particular potential implies that the charges are on conducting spheres with a finite radius.

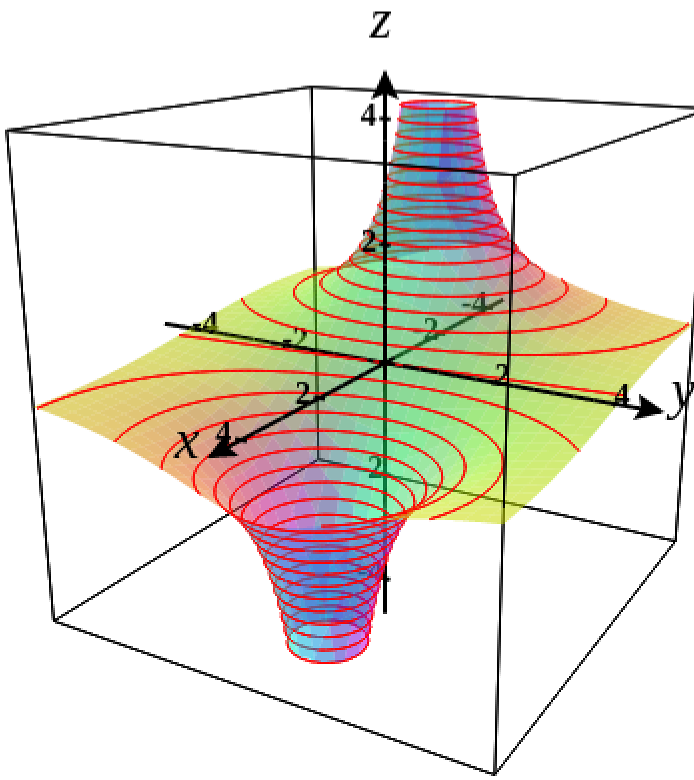

Figure 4.3.6: Electric potential map of two opposite charges of equal magnitude on conducting spheres. The potential is negative near the negative charge and positive near the positive charge. This dynamic image is powered by CalcPlot3D.

A two-dimensional map of the cross-sectional plane that contains both charges is shown in Figure 4.3.7. The line that is equidistant from the two opposite charges corresponds to zero potential, since at the points on the line, the positive potential from the positive charge cancels the negative potential from the negative charge. Equipotential lines in the cross-sectional plane are closed loops, which are not necessarily circles, since at each point, the net potential is the sum of the potentials from each charge.

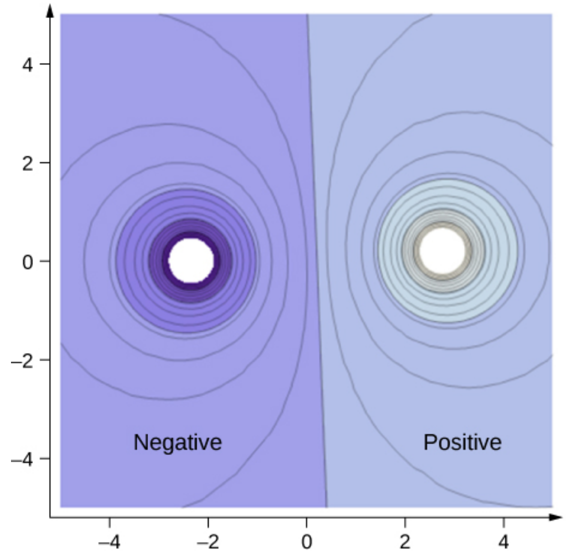

Figure 4.3.7: A cross-section of the electric potential map of two opposite charges of equal magnitude. The potential is negative near the negative charge and positive near the positive charge.

You can draw equipotential lines using computer simulations. Try to reproduce the equipotential curves for the dipole and other charge configurations using the following simulation.

PhET Simulation: Charges and Fields

In this use of the "Charges and Fields" simulation, you will investigate equipotentials and the relationship between equipotentials and the electric field, starting with the example of an electric dipole.

Instructions:

1. Uncheck "Electric Field" and check "Voltage" and "Grid" in the checkboxes in the upper right corner. (When "Voltage" is selected, the strength of the voltage is indicated by red where the voltage is positive, blue where the voltage is negative. The brightness of the color indicates the magnitude of the voltage.)
2. Drag a positive charge and a negative charge into the box. Place them on the same horizontal line 4 major grid units apart (20 minor grid units apart)
3. Drag the purple voltage meter into the box. Measure the voltage at various points around the charges.
4. Put the voltage meter at the midpoint between the two charges. Click on the "pencil" button on the meter to draw the equipotential line corresponding to the voltage on the meter.
5. Move the voltage meter along the line connecting the two charges until the voltage changes by 2 V from its starting value. Click on the "pencil" button to draw another equipotential. Continue this process at 2 V intervals on either side of the midpoint until you have at least 11 equipotentials drawn. Check the "Values" box to show the potential values on each equipotential curve.
6. Drag an electric field sensor into the box and move it around an equipotential curve. Does the electric field vector have the same magnitude (length) at every point on the equipotential curve? If not, where is the electric field the strongest (longest vector)? What is the density of equipotential curves near this location? Where is the electric field the weakest? What is the density of equipotential curves near this location?
7. Finally, check the "Electric Field" box. How do the directions of the arrows relate to the equipotential curves?
8. Remove the positive charge by dragging it back to the box at the bottom of the simulation windows, and then replace it with a negative charge b. How do the equipotentials and electric field vectors change?

Source: Charges and Fields

Credit: Simulation by PhET Interactive Simulations, University of Colorado Boulder, licensed under CC-BY-4.0.