22.9: Decibel Scale for Power Ratio

In many disciplines within electrical engineering, it is common to evaluate the ratios of powers and power densities that differ by many orders of magnitude. These ratios could be expressed in scientific notation, but it is more common to use the logarithmic *decibel* (dB) scale in such applications.

In the conventional (linear) scale, the ratio of power P_1 to power P_0 is simply

$$G = \frac{P_1}{P_0} \quad \text{(linear units)}$$

Here, "G" might be interpreted as "power gain." Note that $G < 1$ if $P_1 < p_0$ and $G > 1$ if $P_1 > P_0$. In the decibel scale, the ratio of power P_1 to power P_0 is

$$\boxed{G \triangleq 10 \log_{10} \frac{P_1}{P_0} \quad \text{(dB)}} \tag{22.9.1}$$

where "dB" denotes a unitless quantity which is expressed in the decibel scale. Note that $G < 0$ dB (i.e., is "negative in dB") if $P_1 < P_0$ >0 and $G > 0$ dB if $P_1 > P_0$.

> The power gain P_1/P_0 in dB is given by Equation 22.9.1.

Alternatively, one might choose to interpret a power ratio as a loss L with $L \triangleq 1/G$ in linear units, which is $L = -G$ when expressed in dB. Most often, but not always, engineers interpret a power ratio as "gain" if the output power is expected to be greater than input power (e.g., as expected for an amplifier) and as "loss" if output power is expected to be less than input power (e.g., as expected for a lossy transmission line).

> Power loss L is the reciprocal of power gain G. Therefore, $L = -G$ when these quantities are expressed in dB.

Example 22.9.1: Power loss from a long cable

A 2 W signal is injected into a long cable. The power arriving at the other end of the cable is 10 μW. What is the power loss in dB?

Solution

In linear units:

$$G = \frac{10\ \mu\text{W}}{2\ \text{W}} = 5 \times 10^{-6} \quad \text{(linear units)}$$

In dB:

G &= 10\log_{10}\left(5 \times 10^{-6}\right) \cong -53.0~\mbox{dB} \nonumber \\ L &= -G \cong \underline{+53.0~\mbox{dB}} \nonumber

The decibel scale is used in precisely the same way to relate ratios of spatial power densities for waves. For example, the loss incurred when the spatial power density is reduced from S_0 (SI base units of W/m^2) to S_1 is

$$L = 10 \log_{10} \frac{S_0}{S_1} \quad \text{(dB)}$$

This works because the common units of m^{-2} in the numerator and denominator cancel, leaving a power ratio.

A common point of confusion is the proper use of the decibel scale to represent voltage or current ratios. To avoid confusion, simply refer to the definition expressed in Equation 22.9.1. For example, let's say $P_1 = V_1^2/R_1$ where V_1 is potential and R_1 is the impedance across which V_1 is defined. Similarly, let us define $P_0 = V_0^2/R_0$ where V_0 is potential and R_0 is the impedance across which V_0 is defined. Applying Equation 22.9.1:

$$
\begin{aligned}
G &\triangleq 10 \log_{10} \frac{P_1}{P_0} \quad (\mathrm{dB}) \\
&= 10 \log_{10} \frac{V_1^2/R_1}{V_0^2/R_0} \quad (\mathrm{dB})
\end{aligned} \tag{22.9.2}
$$

Now, if $R_1 = R_0$, then

$$
\begin{aligned}
G &= 10 \log_{10} \frac{V_1^2}{V_0^2} \quad (\mathrm{dB}) \\
&= 10 \log_{10} \left(\frac{V_1}{V_0}\right)^2 \quad (\mathrm{dB}) \\
&= 20 \log_{10} \frac{V_1}{V_0} \quad (\mathrm{dB})
\end{aligned} \tag{22.9.3}
$$

However, note that this is *not* true if $R_1 \neq R_0$.

> A power ratio in dB is equal to $20 \log_{10}$ of the voltage ratio only if the associated impedances are equal.

Adding to the potential for confusion on this point is the concept of *voltage gain* G_v:

$$
G_v \triangleq 20 \log_{10} \frac{V_1}{V_0} \quad (\mathrm{dB})
$$

which applies regardless of the associated impedances. Note that $G_v = G$ only if the associated impedances are equal, and that these ratios are different otherwise. Be careful!

The decibel scale simplifies common calculations. Here's an example. Let's say a signal having power P_0 is injected into a transmission line having loss L. Then the output power $P_1 = P_0/L$ in linear units. However, in dB, we find:

$$
\begin{aligned}
10 \log_{10} P_1 &= 10 \log_{10} \frac{P_0}{L} \\
&= 10 \log_{10} P_0 - 10 \log_{10} L
\end{aligned} \tag{22.9.4}
$$

Division has been transformed into subtraction; i.e.,

$$
P_1 = P_0 - L \quad (\mathrm{dB}) \tag{22.9.5}
$$

This form facilitates easier calculation and visualization, and so is typically preferred.

Finally, note that the units of P_1 and P_0 in Equation 22.9.5 are not dB *per se*, but rather dB with respect to the original power units. For example, if P_1 is in mW, then taking $10 \log_{10}$ of this quantity results in a quantity having units of dB relative to 1 mW. A power expressed in dB relative to 1 mW is said to have units of "dBm." For example, "0 dBm" means 0 dB relative to 1 mW, which is simply 1 mW. Similarly $+10$ dBm is 10 mW, -10 dBm is 0.1 mW, and so on.
