19.6: Resonance in an AC Circuit

Learning Objectives

By the end of the section, you will be able to:

- Determine the peak ac resonant angular frequency for a RLC circuit
- Explain the width of the average power versus angular frequency curve and its significance using terms like bandwidth and quality factor

In the **RLC** series circuit of Figure 15.4.1, the current amplitude is, from Equation 15.4.7,

[Math Processing Error]

If we can vary the frequency of the ac generator while keeping the amplitude of its output voltage constant, then the current changes accordingly. A plot of *[Math Processing Error]* versus *[Math Processing Error]* is shown in Figure *[Math Processing Error]*.

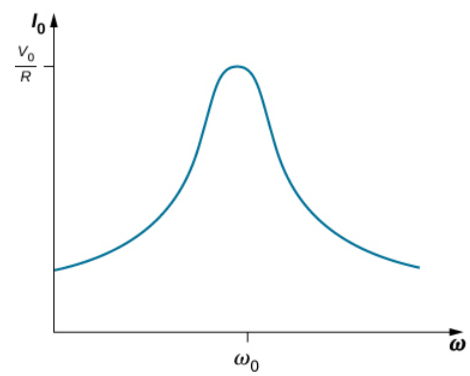

Figure *[Math Processing Error]*: At an RLC circuit's resonant frequency, *[Math Processing Error]*, the current amplitude is at its maximum value.

In Oscillations, we encountered a similar graph where the amplitude of a damped harmonic oscillator was plotted against the angular frequency of a sinusoidal driving force (see Forced Oscillations). This similarity is more than just a coincidence, as shown earlier by the application of Kirchhoff's loop rule to the circuit of Figure 15.4.1. This yields

[Math Processing Error]

or

[Math Processing Error]

where we substituted **dq**(t)/**dt** for **i**(t). A comparison of Equation *[Math Processing Error]* and, from Oscillations, Damped Oscillations for damped harmonic motion clearly demonstrates that the driven **RLC** series circuit is the electrical analog of the driven damped harmonic oscillator.

The **resonant frequency** *[Math Processing Error]* of the **RLC** circuit is the frequency at which the amplitude of the current is a maximum and the circuit would oscillate if not driven by a voltage source. By inspection, this corresponds to the angular frequency *[Math Processing Error]* at which the impedance **Z** in Equation *[Math Processing Error]* is a minimum, or when

[Math Processing Error]

and

[Math Processing Error]

This is the resonant angular frequency of the circuit. Substituting *[Math Processing Error]* into Equation 15.4.5, Equation 15.4.7, and Equation 15.4.8, we find that at resonance,

[Math Processing Error]

Therefore, at resonance, an **RLC** circuit is purely resistive, with the applied emf and current in phase.

What happens to the power at resonance? Equation 15.5.18 tells us how the average power transferred from an ac generator to the **RLC** combination varies with frequency. In addition, *[Math Processing Error]* reaches a maximum when *[Math Processing Error]*, which depends on the frequency, is a minimum, that is, when *[Math Processing Error]* and *[Math Processing Error]*. Thus, at resonance, the average power output of the source in an **RLC** series circuit is a maximum. From Equation 15.5.18, this maximum is *[Math Processing Error]*.

Figure *[Math Processing Error]* is a typical plot of *[Math Processing Error]* versus *[Math Processing Error]* in the region of maximum power output. The **bandwidth** *[Math Processing Error]* of the resonance peak is defined as the range of angular frequencies *[Math Processing Error]* over which the average power *[Math Processing Error]* is greater than one-half the maximum value of *[Math Processing Error]*. The sharpness of the peak is described by a dimensionless quantity known as the **quality factor Q** of the circuit. By definition,

[Math Processing Error]

where *[Math Processing Error]* is the resonant angular frequency. A high **Q** indicates a sharp resonance peak. We can give **Q** in terms of the circuit parameters as

[Math Processing Error]

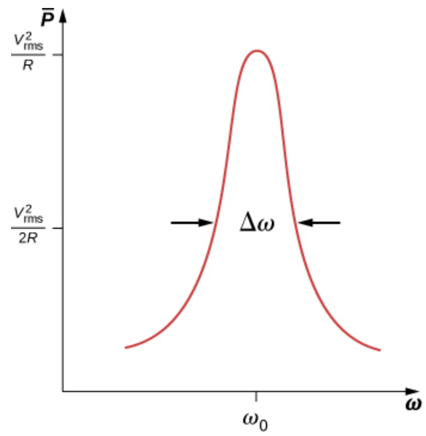

Figure *[Math Processing Error]*: Like the current, the average power transferred from an ac generator to an **RLC** circuit peaks at the resonant frequency.

Resonant circuits are commonly used to pass or reject selected frequency ranges. This is done by adjusting the value of one of the elements and hence “tuning” the circuit to a particular resonant frequency. For example, in radios, the receiver is tuned to the desired station by adjusting the resonant frequency of its circuitry to match the frequency of the station. If the tuning circuit has a high **Q**, it will have a small bandwidth, so signals from other stations at frequencies even slightly different from the resonant frequency encounter a high impedance and are not passed by the circuit. Cell phones work in a similar fashion, communicating with signals of around 1 GHz that are tuned by an inductor-capacitor circuit. One of the most common applications of capacitors is their use in ac-timing circuits, based on attaining a resonant frequency. A metal detector also uses a shift in resonance frequency in detecting metals (Figure *[Math Processing Error]*).

Figure *[Math Processing Error]*: When a metal detector comes near a piece of metal, the self-inductance of one of its coils changes. This causes a shift in the resonant frequency of a circuit containing the coil. That shift is detected by the circuitry and transmitted to the diver by means of the headphones.

Example *[Math Processing Error]*: Resonance in an RLC Series Circuit

a. What is the resonant frequency of the circuit of Example 15.3.1?
b. If the ac generator is set to this frequency without changing the amplitude of the output voltage, what is the amplitude of the current?

Strategy

The resonant frequency for a **RLC** circuit is calculated from Equation *[Math Processing Error]*, which comes from a balance between the reactances of the capacitor and the inductor. Since the circuit is at resonance, the impedance is equal to the resistor. Then, the peak current is calculated by the voltage divided by the resistance.

Solution

1. The resonant frequency is found from Equation *[Math Processing Error]*: *[Math Processing Error]*
2. At resonance, the impedance of the circuit is purely resistive, and the current amplitude is *[Math Processing Error]*

Significance

If the circuit were not set to the resonant frequency, we would need the impedance of the entire circuit to calculate the current.

Example *[Math Processing Error]*: Power Transfer in an RLC Series Circuit at Resonance

a. What is the resonant angular frequency of an **RLC** circuit with *[Math Processing Error]*, and *[Math Processing Error]*?
b. If an ac source of constant amplitude 4.00 V is set to this frequency, what is the average power transferred to the circuit?
c. Determine **Q** and the bandwidth of this circuit.

Strategy

The resonant angular frequency is calculated from Equation *[Math Processing Error]*. The average power is calculated from the rms voltage and the resistance in the circuit. The quality factor is calculated from Equation *[Math Processing Error]* and by knowing the resonant frequency. The bandwidth is calculated from Equation *[Math Processing Error]* and by knowing the quality factor.

Solution

1. The resonant angular frequency is *[Math Processing Error]*
2. At this frequency, the average power transferred to the circuit is a maximum. It is *[Math Processing Error]*
3. The quality factor of the circuit is *[Math Processing Error]* We then find for the bandwidth *[Math Processing Error]*

Significance

If a narrower bandwidth is desired, a lower resistance or higher inductance would help. However, a lower resistance increases the power transferred to the circuit, which may not be desirable, depending on the maximum power that could possibly be transferred.

? Exercise *[Math Processing Error]*

In the circuit of Figure 15.4.1, *[Math Processing Error]*, and *[Math Processing Error]*.

a. What is the resonant frequency?
b. What is the impedance of the circuit at resonance?
c. If the voltage amplitude is 10 V, what is **i(t)** at resonance?
d. The frequency of the AC generator is now changed to 200 Hz. Calculate the phase difference between the current and the emf of the generator.

Answer

a. 160 Hz; b. *[Math Processing Error]*; c. *[Math Processing Error]*; d. 0.023 rad

? Exercise *[Math Processing Error]*

What happens to the resonant frequency of an **RLC** series circuit when the following quantities are increased by a factor of 4: (a) the capacitance, (b) the self-inductance, and (c) the resistance?

Answer

a. halved; b. halved; c. same

? Exercise *[Math Processing Error]*

The resonant angular frequency of an **RLC** series circuit is *[Math Processing Error]*. An ac source operating at this frequency transfers an average power of *[Math Processing Error]* to the circuit. The resistance of the circuit is *[Math Processing Error]*. Write an expression for the emf of the source.

Answer

[Math Processing Error]