8.5: Common Magnetic Field Models

Learning Objectives

- Explain the concept of continuous distribution of current elements and explain why it is used.
- List and apply magnetic field models for some common geometric distributions of current.

Magnetic Fields due to Current Distributions

By using the Biot-Savart Law, it is possible to calculate the magnetic field at a point in space for any distribution of current elements. In general, the mathematics of calculating the magnetic field can be challenging. In some special cases, we can construct the integrals, perform the integration analytically, and then write down closed-form formulas for the magnetic field. In other special cases of high symmetry, it is possible to use a technique involving Ampère's Law to make the calculations much easier. However, in general, it is often necessary to use computer simulations to calculate the magnetic field throughout the space surrounding an arbitrary current distribution.

In this chapter, we will summarize several models corresponding to the most common geometric distributions of current. In each model, the magnetic field can be written in a closed-formed expression for a specified region of space around the current distribution. Students who are interested in the details of the calculations should refer to the chapter Calculation of Magnetic Quantities from Currents. We will make use of these results in later chapters.

Magnetic Fields of Common Current Current Distributions

In the following results, it is assumed that the current is uniform throughout the current-carrying wire and that the current is steady in time. The width of the current-carrying wire is assumed to be small enough that the wire can be effectively treated as one-dimensional.

Magnetic Field due to a Thin Straight Wire

Figure $8.5.1$ shows a section of an infinitely long, straight wire that carries a current I. What is the magnetic field at a point P, located a distance R from the wire?

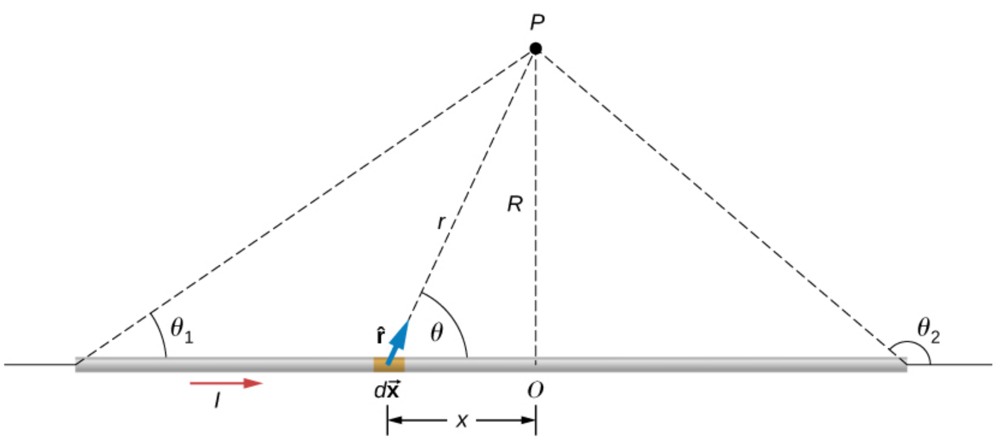

Figure $8.5.1$: A section of a thin, straight current-carrying wire. The independent variable θ has the limits θ_1 and θ_2.

Using the Biot-Savart to integrate along the length of the wire, the magnetic field strength at a distance R is given by

$$B = \frac{\mu_0 I}{2\pi R}. \quad \text{(infinite, thin, straight wire)} \tag{8.5.1}$$

The magnetic field lines of the infinite wire are circular and centered at the wire (Figure $8.5.2$), and they are identical in every plane perpendicular to the wire. Since the field decreases with distance from the wire, the spacing of the field lines must increase correspondingly with distance. The direction of this magnetic field may be found with a form of the **right-hand rule** (Figure $8.5.2$). If you hold the wire with your right hand so that your thumb points along the current, then your fingers wrap around the wire in the same sense as $\vec{B}$.

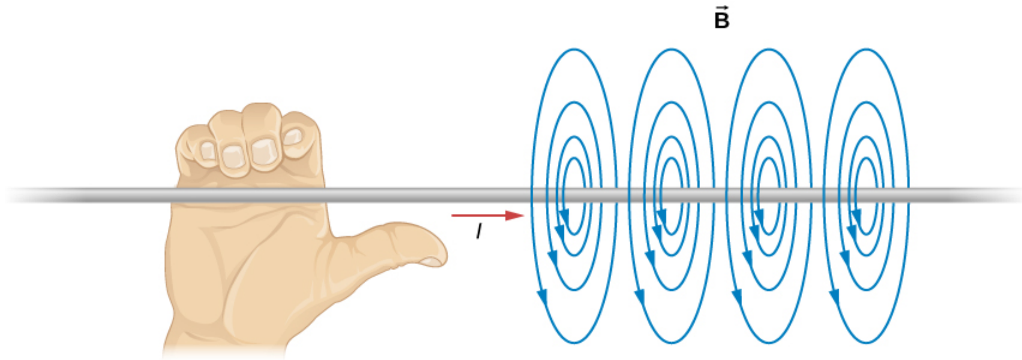

Figure 8.5.2:. Some magnetic field lines of an infinite wire. The direction of B can be found with a form of the right-hand rule.

The direction of the field lines can be observed experimentally by placing several small compass needles on a circle near the wire, as illustrated in Figure $8.5.3a$. When there is no current in the wire, the needles align with Earth's magnetic field. However, when a large current is sent through the wire, the compass needles all point tangent to the circle. Iron filings sprinkled on a horizontal surface also delineate the field lines, as shown in Figure $8.5.3b$.

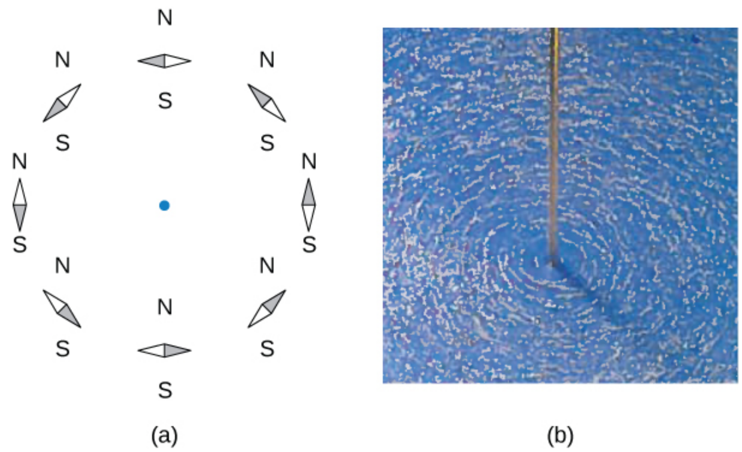

Figure 8.5.3: The shape of the magnetic field lines of a long wire can be seen using (a) small compass needles and (b) iron filings.

Magnetic Field due to a Current Loop

The circular loop of Figure 8.5.4 has a radius R, carries a current I, and lies in the xz-plane. We will assume that the loop in an ideal circular loop. This will be a reasonable approximation for a real loop if the gap between the input and output wires is small compared to the radius of the loop. What is the magnetic field due to the current at an arbitrary point P along the axis of the loop?

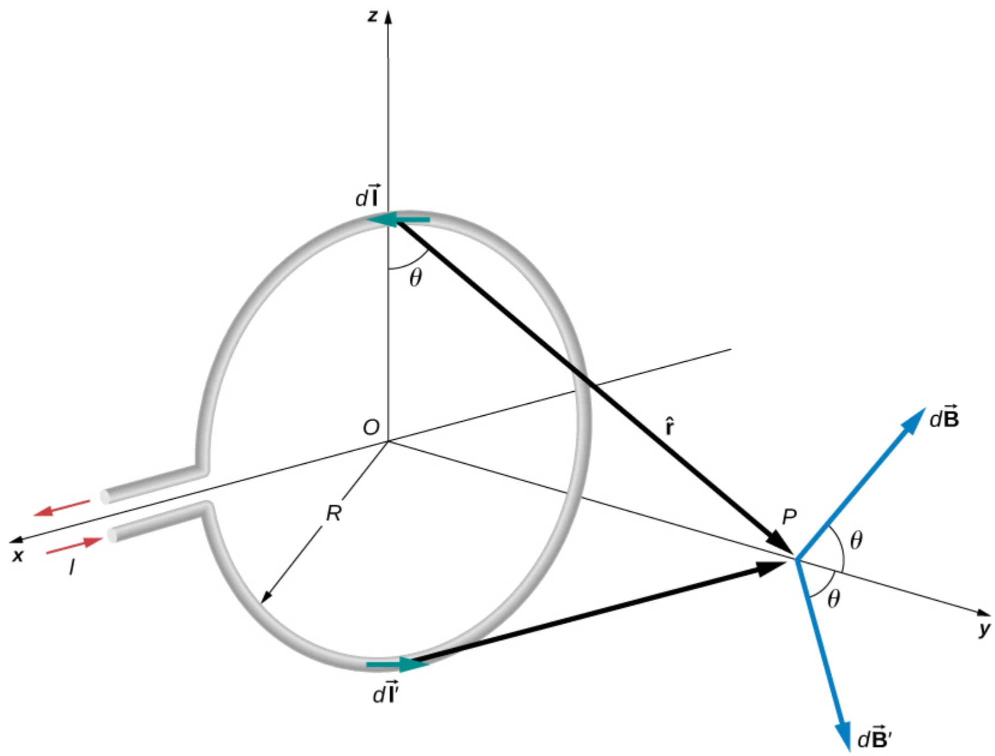

Figure 8.5.4: Determining the magnetic field at point P along the axis of a current-carrying loop of wire.

Using the Biot-Savart Law and integrating around the loop, the magnetic field along the y-axis at point P can shown to be

$$\vec{B} = \frac{\mu_0 R^2 I \hat{j}}{2(y^2 + R^2)^{3/2}}. \tag{8.5.2}$$

By setting $y = 0$ in Equation 8.5.2, we obtain the magnetic field at the center of the single loop:

$$\vec{B} = \frac{\mu_0 I}{2R} \hat{j} \text{ (single loop)}. \tag{8.5.3}$$

For a flat coil of N loops, this equation becomes

$$B = \frac{\mu_0 N I}{(2R)} \text{ (short coil of multiple loops)} \tag{8.5.4}$$

This latter equation is also approximately valid for short coils, where the length of the coil is small compared to the diameter of the coil.

If we consider $y >> R$ in Equation 8.5.2, the expression reduces to an expression known as the magnetic field from a dipole:

$$\vec{B} = \frac{\mu_0 R^2 I}{2y^3}. \tag{8.5.5}$$

The calculation of the magnetic field due to the circular current loop at points off-axis requires rather complex mathematics, so we will just look at the results. The magnetic field lines are shaped as shown in Figure 8.5.5. Notice that one field line follows the axis of the loop. This is the field line we just found. Also, very close to the wire, the field lines are almost circular, like the lines of a long straight wire.

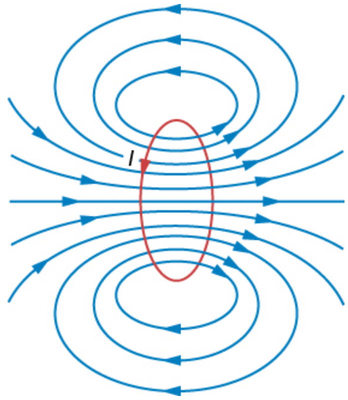

Figure 8.5.5: Sketch of the magnetic field lines of a circular current loop.

Magnetic Field due to a Solenoid

A long wire wound in the form of a helical coil is known as a **solenoid**. Solenoids are commonly used in experimental research requiring magnetic fields. A solenoid is generally easy to wind, and near its center, its magnetic field is quite uniform and directly proportional to the current in the wire.

Figure $8.5.6$ shows a solenoid consisting of N turns of wire tightly wound over a length L. A current I is flowing along the wire of the solenoid.

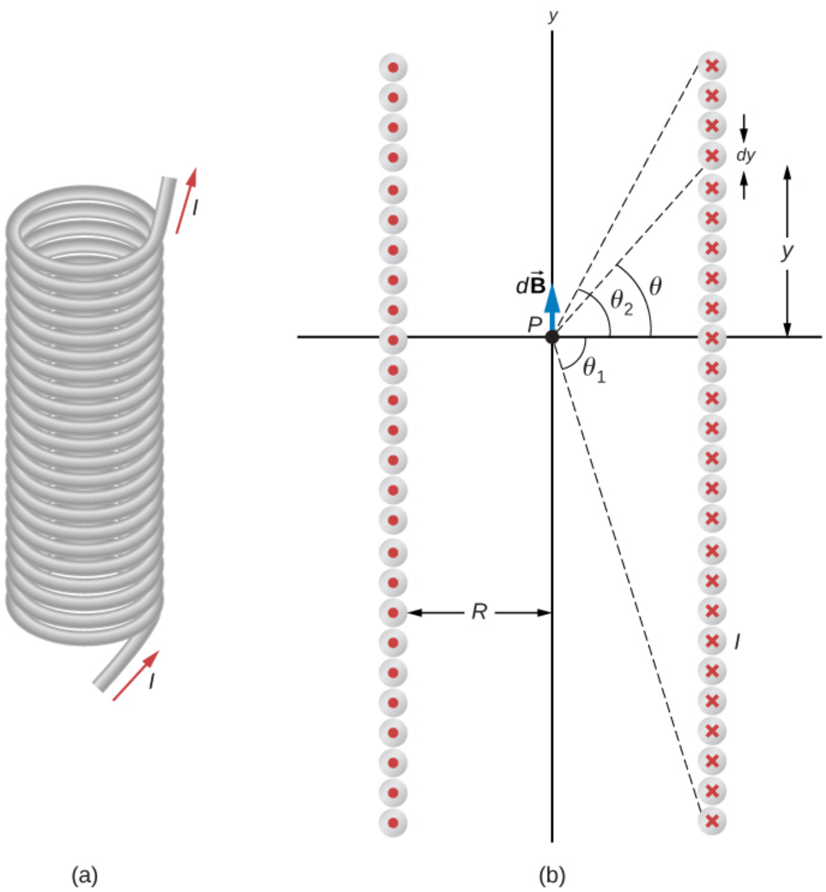

Figure 8.5.6: (a) A solenoid is a long wire wound in the shape of a helix. (b) The magnetic field at the point **P** on the axis of the solenoid is the net field due to all of the current loops.

Using the Biot-Savart Law and adding up the contributions of all the loops by integrating with respect to θ, we find that the magnetic field along the central axis of the finite solenoid is

$$\vec{B} = \frac{\mu_0 I N}{2L}(\sin\theta_2 - \sin\theta_1)\hat{j}. \quad \text{(finite solenoid)} \tag{8.5.6}$$

The infinitely long solenoid, for which $L \to \infty$ is of special interest and yields a particularly simple result. From a practical point of view, the infinite solenoid is one whose length is much larger than its radius $(L >> R)$. In this case, $\theta_1 = -\pi/2$ and $\theta_2 = \pi/2$. Then from Equation 8.5.6, the magnetic field along the central axis of an infinite solenoid is

$$\vec{B} = \frac{\mu_0 IN}{L}\hat{j} = \mu_0 nI\hat{j}, \quad \text{(infinite solenoid)} \tag{8.5.7}$$

where $n = N/L$ is the number of turns per unit length. You can find the direction of $\vec{B}$ with a right-hand rule: Curl your fingers in the direction of the current, and your thumb points along the magnetic field in the interior of the solenoid.

Example 8.5.1: Magnetic Field Inside a Solenoid

A solenoid has 300 turns wound around a cylinder of diameter 1.20 cm and length 14.0 cm. If the current through the coils is 0.410 A, what is the magnitude of the magnetic field inside and near the middle of the solenoid?

Strategy

We are given the number of turns and the length of the solenoid so we can find the number of turns per unit length. Therefore, the magnetic field inside and near the middle of the solenoid is given by Equation 8.5.7. Outside the solenoid, the magnetic field is zero.

Solution

The number of turns per unit length is

$$n = \frac{300 \text{ turns}}{0.140 \text{ m}} = 2.14 \times 10^3 \text{ turns/m}. \tag{8.5.8}$$

The magnetic field produced inside the solenoid is

$$B = \mu_0 nI = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2.14 \times 10^3 \text{ turns/m})(0.410 \text{ A}) \tag{8.5.9}$$

$$B = 1.10 \times 10^{-3} \text{ T}. \tag{8.5.10}$$

Significance

This solution is valid only if the length of the solenoid is reasonably large compared with its diameter. This example is a case where this is valid.

Exercise 8.5.1

What is the ratio of the magnetic field produced from using a finite formula over the infinite approximation for an angle θ of (a) 85°? (b) 89°? The solenoid has 1000 turns in 50 cm with a current of 1.0 A flowing through the coils.

Solution

a. 1.00382; b. 1.00015

Magnetic Field due to Toroid

A **toroid** is a donut-shaped coil closely wound with one continuous wire, as illustrated in part (a) of Figure 8.5.7. If the toroid has N windings and the current in the wire is I, what is the magnetic field both inside and outside the toroid?

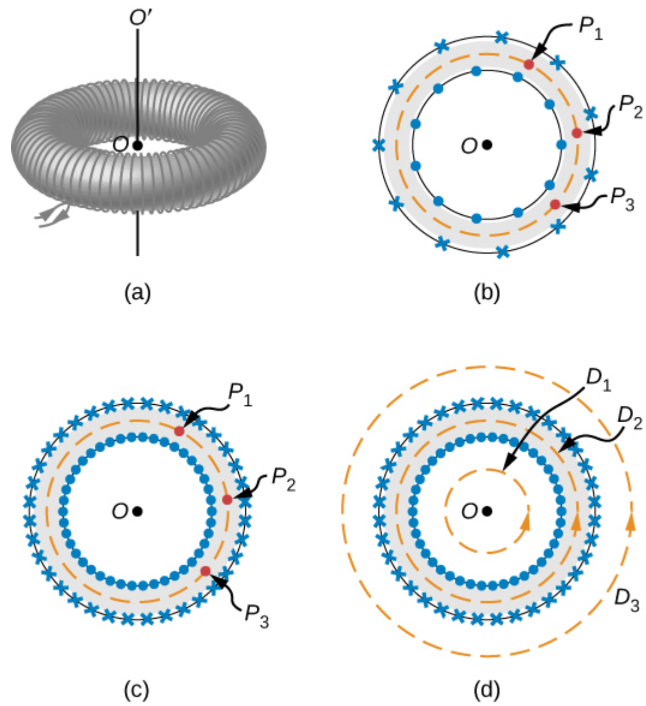

Figure 8.5.7: (a) A toroid is a coil wound into a donut-shaped object. (b) A loosely wound toroid does not have cylindrical symmetry. (c) In a tightly wound toroid, cylindrical symmetry is a very good approximation. (d) Several paths of integration for Ampère's law.

Assuming cylindrical symmetry around the axis OO', Ampère's law can be used to show that

$$B = \frac{\mu_0 NI}{2\pi r} \text{ (within the toroid).} \tag{8.5.11}$$

Actually, this assumption of circular symmetry is not precisely correct, for as part (b) of Figure 8.5.4 shows, the view of the toroidal coil varies from point to point (for example, P_1, P_2 and P_3) on a circular path centered around OO'. However, if the toroid is tightly wound, all points on the circle become essentially equivalent [part (c) of Figure 8.5.4], and cylindrical symmetry is an accurate approximation.

The magnetic field is directed in the counterclockwise direction for the windings shown. When the current in the coils is reversed, the direction of the magnetic field also reverses.

The magnetic field inside a toroid is not uniform, as it varies inversely with the distance r from the axis OO' However, if the central radius R (the radius midway between the inner and outer radii of the toroid) is much larger than the cross-sectional diameter of the coils r, the variation is fairly small, and the magnitude of the magnetic field may be calculated by Equation 8.5.11 where $r = R$.