16.5: Direct Calculation of Electrical Quantities from Charge Distributions (Summary)

Key Terms

continuous charge distribution	total source charge composed of so large a number of elementary charges that it must be treated as continuous, rather than discrete
infinite straight wire	straight wire whose length is much, much greater than either of its other dimensions, and also much, much greater than the distance at which the field is to be calculated
linear charge density	amount of charge in an element of a charge distribution that is essentially one-dimensional (the width and height are much, much smaller than its length); its units are C/m
surface charge density	amount of charge in an element of a two-dimensional charge distribution (the thickness is small); its units are C/m^2
volume charge density	amount of charge in an element of a three-dimensional charge distribution; its units are C/m^3

Key Equations

Coulomb’s law	$\overrightarrow{F_{12}}(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}^2}\hat{r_{12}}$
Superposition of electric forces	$\vec{F}(r) = \frac{1}{4\pi\varepsilon_0}Q\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{r}_i$
Electric force due to an electric field	$\vec{F} = Q\vec{E}$
Electric field at point P	$\vec{E}(P) \equiv \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{r}_i$
Field of an infinite wire	$\vec{E}(z) = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}\hat{k}$
Field of an infinite plane	$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{k}$
Dipole moment	$\vec{p} \equiv q\vec{d}$
Torque on dipole in external E-field	$\vec{\tau} = \vec{p}\times\vec{E}$
Electric field of a continuous charge distribution	$\vec{E} = k\int\frac{dq\hat{r}}{r}$
Electric potential of a continuous charge distribution	$V_P = k\int\frac{dq}{r}$

Summary

Calculating Electric Fields of Charge Distributions

- A very large number of charges can be treated as a continuous charge distribution, where the calculation of the field requires integration. Common cases are:
  - one-dimensional (like a wire); uses a line charge density λ
  - two-dimensional (metal plate); uses surface charge density σ
  - three-dimensional (metal sphere); uses volume charge density ρ
- The “source charge” is a differential amount of charge dq. Calculating dq depends on the type of source charge distribution:

$$dq = \lambda dl; dq = \sigma dA; dq = \rho dV\ .$$

- The field of continuous charge distributions may be calculated with $\vec{E} = k\int \frac{dq\hat{r}}{r}$.
- Symmetry of the charge distribution is usually key.
- Important special cases are the field of an “infinite” wire and the field of an “infinite” plane.

Electric Dipoles

- If a permanent dipole is placed in an external electric field, it results in a torque that aligns it with the external field.
- If a nonpolar atom (or molecule) is placed in an external field, it gains an induced dipole that is aligned with the external field.
- The net field is the vector sum of the external field plus the field of the dipole (physical or induced).
- The strength of the polarization is described by the dipole moment of the dipole, $\vec{p} = q\vec{d}$.

Calculating Electric Potential of Charge Distributions

- The potential of continuous charge distributions may be calculated with $V_P = k\int \frac{dq}{r}$.

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