6.4: Orbital Motion

The International Space Station is a man made satellite orbiting the earth Figure 5.4.1

We commonly talk about satellites orbiting Earth. But what does that really mean? When a satellite, space shuttle, or some other object is orbiting a planet, it maintains a circular orbit around the planet a constant distance off the surface. Manmade satellites typically orbit between 200 and 400 miles. For example, the International Space Station (ISS) orbits at 370 km, or 230 miles.

The ISS has an average velocity of 7.66 km/sec tangential to its orbit. An orbiting satellite is close enough to be acted upon by Earth's gravity. This force is constantly pulling the satellite in toward the center of the earth – it is a centripetal force and causes a centripetal acceleration. At this height, however, Earth's gravity is only about 8.7 m/s^2. As was discussed in Motion in Two Dimensions: Circular Motion, the velocity and centripetal acceleration are perpendicular.

NASA scientists, in designing this and all other satellites, must carefully calculate the velocity necessary to keep the satellite orbiting. To keep the satellite from falling back to Earth, the horizontal velocity must be large enough. The satellite must travel far enough horizontally that it follows the curve of the planet, as shown below.

When the satellite is orbiting in this way, it is falling straight down towards Earth. Imagine standing in an elevator when the bottom drops out from under you. The elevator, you, and anything you might have had with you all fall straight down. If you had a backpack on your back, the weight of the books can no longer be felt because the books are in free fall as well. A similar thing is happening in the space shuttle or orbiting satellite, where objects are weightless.

For a full description of weight, look at Forces: Mass versus Weight. In sum, weight is the result of the force of gravity being opposed by the normal force. As we just learned, objects in orbit are in free fall; they have nothing exerting a normal force against them and thus no weight.

Examples

Example 5.4.1

Consider a satellite of the earth orbiting at 225 km above the surface of the earth. Keep in mind that this is NOT the radius of the satellite's orbit. The satellite's orbit is measured from the center of the earth, so its radius will be the radius of the earth, 6.37×10^6 m, plus the 225,000 m. The mass of the earth is 5.98×10^{24} kg. What is the velocity of the satellite?

Solution

Since the centripetal force keeping this satellite traveling in orbit is provided by the gravitational force of the earth, we can set the formula for centripetal force equal to the formula for gravitational force.

$F_c = mv^2/r$ and $F_g = Gm_1m_2/d^2$

$Gm_E m_s/r^2 = m_s v^2/r$ so $v = (Gm_E/r)$^(1/2)

$v = (6.67 \times 10^{-11}\ N \cdot m^2/kg^2)(5.98 \times 10^{24}\ kg)6.60 \times 10^6\ m = 7770\ m/s$

Example 5.4.2

Set the equation for Newton's law of gravity equal to the equation for centripetal force shown below and derive an equation for the period of a planet around the earth.

Solution

$F_g = Gm_E ms/r^2$ and $Fc = 4\pi^2 m_{sat} r/T^2$

$Gm_E ms/r^2 = 4\pi^2 msr/T^2$ and $T = 2\pi(r^3/GmE)$^(1/2)

Use the Space Station simulation below to observe how adjusting the initial radius of a satellite in orbit affects its orbital period, as well as the overall shape of the orbit:

Summary

- A satellite in a circular orbit accelerates toward the object it is orbiting at a rate equal to the acceleration of gravity at its orbital radius.
- As in a falling elevator, objects in orbit are in constant freefall.
- Objects in free fall have no normal force acting upon them, and thus no weight.

Review

1. Sally's mass on the earth is 50. kg.
    1. What is her weight on the earth?
    2. What is her weight on the moon?
    3. What is her mass on the moon?
2. The radius of the planet Mercury is 2.43×10^6 m and its mass is 3.2×10^{23} kg.
    1. Find the speed of a satellite in orbit 265,000 m above the surface.
    2. Find the period of the satellite.
3. A geosynchronous orbit is an orbit in which the satellite remains over the same spot on the earth as the earth turns. This is accomplished by matching the velocity of the satellite to the velocity of the turning earth. The orbital radius of a geosynchronous satellite is 4.23×10^7m (measured from the center of the earth).
    1. What is the speed of the satellite in orbit?
    2. What is its period?

Resources

What makes the moon go around the Earth? Watch this video to find out.

Additional Resources

Study Guide: Gravitation Study Guide

Real World Application: Mercury's Orbit

Interactive: Clarke's Dream, Newton's Cannon

PLIX: Play, Learn, Interact, eXplore: Orbital Motion: Satellites

Video: