

2.6.9: Dimensional Analysis

Learning Objectives

- To be introduced to the dimensional analysis and how it can be used to aid basic chemistry problem solving.
- To use dimensional analysis to identify whether an equation is set up correctly in a numerical calculation
- To use dimensional analysis to facilitate the conversion of units.

Dimensional analysis is amongst the most valuable tools physical scientists use. Simply put, it is the conversion between an amount in one unit to the corresponding amount in a desired unit using various conversion factors. This is valuable because certain measurements are more accurate or easier to find than others.

A Macroscopic Example: Party Planning

If you have every planned a party, you have used dimensional analysis. The amount of beer and munchies you will need depends on the number of people you expect. For example, if you are planning a Friday night party and expect 30 people you might estimate you need to go out and buy 120 bottles of sodas and 10 large pizza's. How did you arrive at these numbers? The following indicates the type of dimensional analysis solution to a party problem:

$$(30 \text{ humans}) \times \left(\frac{4 \text{ sodas}}{1 \text{ human}} \right) = 120 \text{ sodas} \quad (2.6.9.1)$$

$$(30 \text{ humans}) \times \left(\frac{0.333 \text{ pizzas}}{1 \text{ human}} \right) = 10 \text{ pizzas} \quad (2.6.9.2)$$

Notice that the units that canceled out are lined out and only the desired units are left (discussed more below). Finally, in going to buy the soda, you perform another dimensional analysis: should you buy the sodas in six-packs or in cases?

$$(120 \text{ sodas}) \times \left(\frac{1 \text{ six pack}}{6 \text{ sodas}} \right) = 20 \text{ six packs} \quad (2.6.9.3)$$

$$(120 \text{ sodas}) \times \left(\frac{1 \text{ case}}{24 \text{ sodas}} \right) = 5 \text{ cases} \quad (2.6.9.4)$$

Realizing that carrying around 20 six packs is a real headache, you get 5 cases of soda instead.

In this party problem, we have used dimensional analysis in two different ways:

- In the first application (Equations 2.6.9.1 and Equation 2.6.9.2), dimensional analysis was used to calculate how much soda is needed need. This is based on knowing: (1) how much soda we need for one person and (2) how many people we expect; likewise for the pizza.
- In the second application (Equations 2.6.9.3 and 2.6.9.4), dimensional analysis was used to **convert units** (i.e. from individual sodas to the equivalent amount of six packs or cases)

Using Dimensional Analysis to Convert Units

Consider the conversion in Equation 2.6.9.3

$$(120 \text{ sodas}) \times \left(\frac{1 \text{ six pack}}{6 \text{ sodas}} \right) = 20 \text{ six packs} \quad (2.6.9.5)$$

If we ignore the numbers for a moment, and just look at the units (i.e. **dimensions**), we have:

$$\text{soda} \times \left(\frac{\text{six pack}}{\text{sodas}} \right)$$

We can treat the dimensions in a similar fashion as other numerical analyses (i.e. any number divided by itself is 1). Therefore:

$$\text{soda} \times \left(\frac{\text{six pack}}{\text{sodas}} \right) = \cancel{\text{soda}} \times \left(\frac{\text{six pack}}{\cancel{\text{sodas}}} \right)$$

So, the dimensions of the numerical answer will be "six packs".

How can we use dimensional analysis to be sure we have set up our equation correctly? Consider the following alternative way to set up the above unit conversion analysis:

$$120 \cancel{\text{soda}} \times \left(\frac{6 \text{ sodas}}{\cancel{\text{six pack}}} \right) = 720 \frac{\text{sodas}^2}{1 \text{ six pack}}$$

- While it is correct that there are 6 sodas in one six pack, the above equation yields a value of 720 with units of **sodas²/six pack**.
- These rather bizarre units indicate that the equation has been setup **incorrectly** (and as a consequence you will have a ton of extra soda at the party).

Using Dimensional Analysis in Calculations

In the above case it was relatively straightforward keeping track of units during the calculation. What if the calculation involves powers, etc? For example, the equation relating kinetic energy to mass and velocity is:

$$E_{kinetics} = \frac{1}{2} \text{mass} \times \text{velocity}^2 \quad (2.6.9.6)$$

An example of units of mass is kilograms (kg) and velocity might be in meters/second (m/s). What are the dimensions of $E_{kinetic}$?

$$(kg) \times \left(\frac{m}{s} \right)^2 = \frac{kg \text{ m}^2}{s^2}$$

The $\frac{1}{2}$ factor in Equation 2.6.9.6 is neglected since pure numbers have no units. Since the velocity is squared in Equation 2.6.9.6 the **dimensions** associated with the numerical value of the velocity are also squared. We can double check this by knowing the the Joule (J) is a measure of energy, and as a composite unit can be decomposed thusly:

$$1 \text{ J} = kg \frac{m^2}{s^2}$$

Units of Pressure

Pressure (P) is a measure of the Force (F) per unit area (A):

$$P = \frac{F}{A}$$

$$120 \cancel{\text{kcal}} \left(\frac{1000 \cancel{\text{cal}}}{1 \cancel{\text{kcal}}} \right) \left(\frac{4.184 \cancel{\text{J}}}{1 \cancel{\text{cal}}} \right) \left(\frac{1 \text{ kJ}}{1000 \cancel{\text{J}}} \right) = 502 \text{ kJ}$$

We next convert the 7.00 oz of chocolate to grams:

$$7.00 \cancel{\text{oz}} \left(\frac{28.35 \text{ g}}{1 \cancel{\text{oz}}} \right) = 199 \text{ g}$$

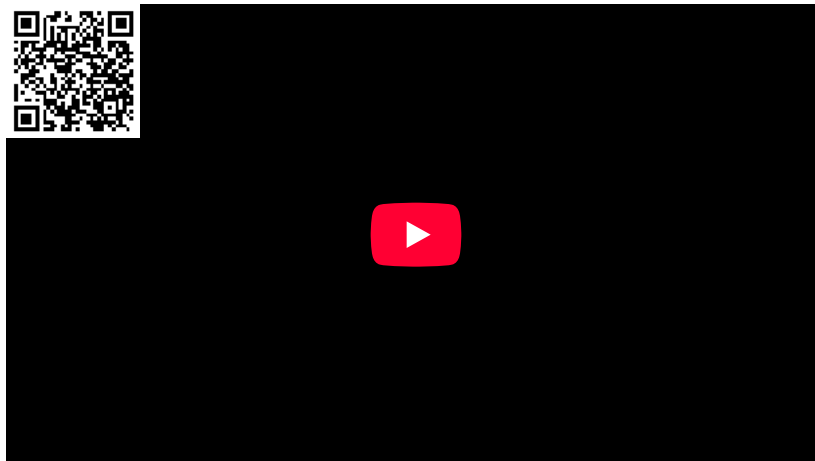
The number of kilojoules per gram is therefore

$$\frac{502 \text{ kJ}}{199 \text{ g}} = 2.52 \text{ kJ/g}$$

Alternatively, we could solve the problem in one step with all the conversions included:

$$\left(\frac{120 \cancel{\text{kcal}}}{7.00 \cancel{\text{oz}}} \right) \left(\frac{1000 \cancel{\text{cal}}}{1 \cancel{\text{kcal}}} \right) \left(\frac{4.184 \cancel{\text{J}}}{1 \cancel{\text{cal}}} \right) \left(\frac{1 \text{ kJ}}{1000 \cancel{\text{J}}} \right) \left(\frac{1 \cancel{\text{oz}}}{28.35 \text{ g}} \right) = 2.53 \text{ kJ/g}$$

The discrepancy between the two answers is attributable to rounding to the correct number of significant figures for each step when carrying out the calculation in a stepwise manner. Recall that all digits in the calculator should be carried forward when carrying out a calculation using multiple steps. In this problem, we first converted kilocalories to kilojoules and then converted ounces to grams.



Converting Between Units: [Converting Between Units, YouTube\(opens in new window\)](#) [youtu.be]

Summary

Dimensional analysis is used in numerical calculations, and in converting units. It can help us identify whether an equation is set up correctly (i.e. the resulting units should be as expected). Units are treated similarly to the associated numerical values, i.e., if a variable in an equation is supposed to be squared, then the associated dimensions are squared, etc.

Contributors and Attributions

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