2.2.11: Angles

You will need a calculator near the end of this module.

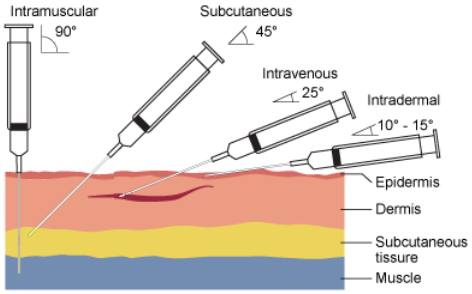
an angle grinder

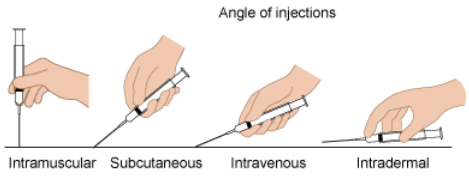
Angle measurement is important in construction, surveying, physical therapy, and many other fields. We can visualize an angle as the figure formed when two line segments share a common endpoint. We can also think about an angle as a measure of rotation. A full rotation or a full circle is 360°, so a half rotation or U-turn is 180°, and a quarter turn is 90°.

We often classify angles by their size.

- **Acute Angle:** between 0° and 90°
- **Right Angle:** exactly 90°
- **Obtuse Angle:** between 90° and 180°
- **Straight Angle:** exactly 180°
- **Reflexive Angle:** between 180° and 360°

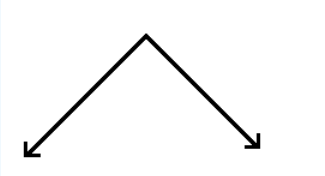
Lines that form a 90° angle are called ***perpendicular***. As shown below, the needle should be perpendicular to the body surface for an intramuscular injection.

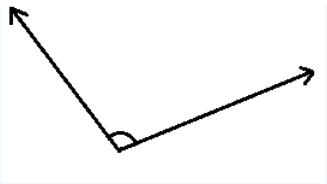


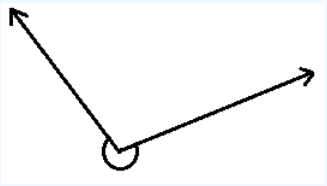


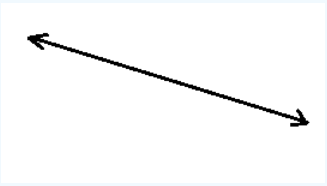
? Exercises 2.2.11.1

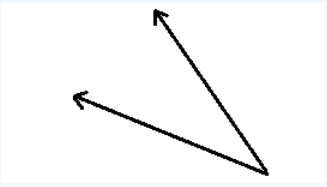
Identify each angle shown below as acute, right, obtuse, straight, or reflexive.

1. 

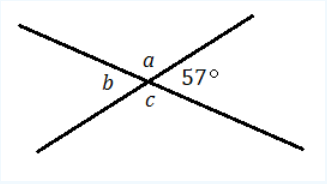
2. 

3. 

4. 

5. 

Find the measure of each unknown angle.

6. 

Answer

1. right angle
2. obtuse angle
3. reflexive angle
4. straight angle
5. acute angle
6. $a = 123^\circ$; $b = 57^\circ$; $c = 123^\circ$

Angles in Triangles

If you need to find the measures of the angles in a triangle, there are a few rules that can help.

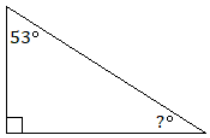
The sum of the angles of every triangle is 180°.

If any sides of a triangle have equal lengths, then the angles opposite those sides will have equal measures.

? Exercises 2.2.11.1

Find the measures of the unknown angles in each triangle.

7.



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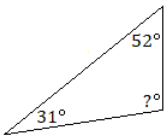

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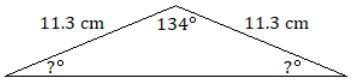

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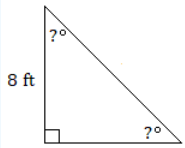

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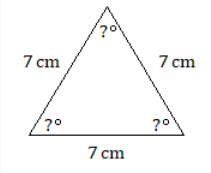
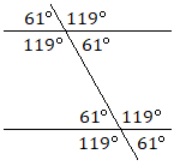


Answer

7. 37°
8. 97°
9. 23° each
10. 45° each
11. 60° each

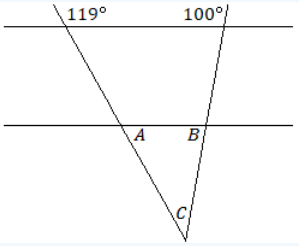
Angles and Parallel Lines

Two lines that point in the exact same direction and will never cross are called parallel lines. If two parallel lines are crossed by a third line, sets of equally-sized angles will be formed, as shown in the following diagram. All four acute angles will be equal in measure, all four obtuse angles will be equal in measure, and any acute angle and obtuse angle will have a combined measure of 180°.



Example 2.2.11.1

Find the measures of angles A, B, and C.



Solution

A. 61°

B. 80°

C. 39°

Degrees, Minutes, Seconds

It is possible to have angle measures that are not a whole number of degrees. It is common to use decimals in these situations, but the older method—called the ***degrees-minutes-seconds*** or DMS system—divides a degree using fractions out of 60: a minute is $\frac{1}{60}$ of a degree, and a second is $\frac{1}{60}$ of a minute, which means a second is $\frac{1}{3,600}$ of a degree. (Fortunately, these conversions work exactly like time; think of 1 degree as 1 hour.) For example, $2.5^\circ = 2^\circ 30'$.

We will look at the procedure for converting between systems, but there are online calculators such as the one at https://www.fcc.gov/media/radio/dms-decimal which will do the conversions for you.

If you have latitude and longitude in DMS, like N $18^\circ 54' 40''$ W $155^\circ 40' 51''$, and need to convert it to decimal degrees, the process is fairly simple with a calculator.

Converting from DMS to Decimal Degrees

Enter $\text{degrees} + \text{minutes} \div 60 + \text{seconds} \div 3600$ in your calculator. Round the result to the fourth decimal place, if necessary.[1]

Exercises 2.2.11.1

Convert each angle measurement from degrees-minutes-seconds into decimal form. Round to the nearest ten-thousandth, if necessary.

13. $67^\circ 48' 54''$

14. $19^\circ 37' 25''$

15. $34^\circ 14' 12''$

Answer

13. 67.815°
14. 19.6236°
15. 34.2367°

Going from decimal degrees to DMS is a more complicated process.

Converting from Decimal Degrees to DMS

1. The whole-number part of the angle measurement gives the number of degrees.
2. Multiply the decimal part by 60. The whole number part of this result is the number of minutes.
3. Multiply the decimal part *of the minutes* by 60. This gives the number of seconds (including any decimal part of seconds).

For example, let's convert 15.374°.

1. The *degrees* part of our answer will be 15.
2. The decimal part times 60 is $0.374 \cdot 60 = 22.44$ minutes. The *minutes* part of our answer will be 22.
3. The decimal part times 60 is $0.44 \cdot 60 = 26.4$ seconds. The *seconds* part of our answer will be 26.4.

So $15.374^\circ = 15^\circ 22' 26.4''$.

? Exercises 2.2.11.1

Convert each angle measurement from decimal into degrees-minutes-seconds form.

16. 26.785°

17. 58.216°

18. 41.13°

Answer

16. $26^\circ 47' 6''$
17. $58^\circ 12' 57.6''$
18. $41^\circ 7' 48''$

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1. We round to four decimal places because 1 second of angle is $\frac{1}{3,600}$ of a degree. This is a smaller fraction than $\frac{1}{1,000}$ so our precision is slightly better than the thousandths place. ↵
