3.3.2: Projectile Motion for an Object Launched at an Angle

Understanding the physics before a human cannonball launch is very important

Figure 4.3.1

In the case of the human cannonball shown, all the vector and gravitational calculations must be worked out perfectly before the first practice session. With this activity, you cannot afford trial and error – the first miss might be the last trial.

Projectile Motion for an Object Launched at an Angle

When an object is projected from rest at an upward angle, its initial velocity can be resolved into two components. These two components operate independently of each other. The upward velocity undergoes constant downward acceleration which will result in it rising to a highest point and then falling backward to the ground. The horizontal motion is constant velocity motion and undergoes no changes due to gravity.The analysis of the motion involves dealing with the two motions independently.

Parabolic path of an object launched at an angle

Figure 4.3.2

Example 4.3.1

A cannon ball is fired with an initial velocity of 100.0 m/s at an angle of 45° above the horizontal. What maximum height will it reach and how far will it fly horizontally?

Solution

The first step in the analysis of this motion is to resolve the initial velocity into its vertical and horizontal components.

$$v_{i-up}=(100.0\ m/s)(\sin 45^{\circ})=(100.0\ m/s)(0.707)=70.7\ m/s$$

$$v_{i-horizontal}=(100.0\ m/s)(\cos 45^{\circ})=(100.0\ m/s)(0.707)=70.7\ m/s$$

We will deal with the vertical motion first. The vertical motion is symmetrical. As the object rises to its highest point and then falls back down, it will travel the same distance in each direction, and take the same amount of time. This is often hard to accept, but the amount of time the object takes to come to a stop at its highest point is the same amount of time it takes to return to where it was launched from. Similarly, the initial velocity upward will be the same magnitude (opposite in direction) as the final velocity when it returns to its original height. There are several ways we could approach the upward motion. We could calculate the time it would take gravity to bring the initial velocity to rest. Or, we could calculate the time it would take gravity to change the initial velocity from +70.7 m/s to -70.0 m/s. Yet another way would be to calculate the time it takes for the object to return to its original height.

$$v_f=v_i+at \text{ so } t=(v_f-v_i)/a$$

If we calculate the time required for the ball to rise up to its highest point and come to rest, the initial velocity is 70.7 m/s and the final velocity is 0 m/s. Since we have called the upward velocity positive, then the acceleration must be negative or -9.80 m/s^2.

$$t=(v_f-v_i)/a=(0\ m/s-70.7\ m/s)/-9.80\ m/s^2=7.21\ s$$

Since this is the time required for the cannon ball to rise up to its highest point and come to rest, then the time required for the entire trip up and down would be double this value, or 14.42 s. The maximum height reached can be calculated by multiplying the time for the upward trip by the average vertical velocity. Since the object's velocity at the top is 0 m/s, the average upward velocity during the trip up is one-half the initial velocity.

$$v_{up-ave}=(12)(70.7\ m/s)=35.3\ m/s$$

$$height=(v_{up-ave})(t_{up})=(35.3\ m/s)(7.21\ s)=255\ m$$

The horizontal distance traveled during the flight is calculated by multiplying the total time by the constant horizontal velocity.

$$d_{horizontal}=(14.42\ s)(70.7\ m/s)=1020\ m$$

Example 4.3.2

A golf ball was hit into the air with an initial velocity of 4.47 m/s at an angle of 66° above the horizontal. How high did the ball go and how far did it fly horizontally?

Solution

A golf ball was hit into the air with an initial velocity of 4.47 m/s at an angle of 66° above the horizontal. How high did the ball go and how far did it fly horizontally?

$$v_{i-up}=(4.47\text{ m/s})(\sin 66^\circ)=(4.47\text{ m/s})(0.913)=4.08\text{ m/s}$$

$$v_{i-hor}=(4.47\text{ m/s})(\cos 66^\circ)=(4.47\text{ m/s})(0.407)=1.82\text{ m/s}$$

$$t_{up}=(v_f-v_i)/a=(0\text{ m/s}-4.08\text{ m/s})/-9.80\text{ m/s}^2=0.416\text{ s}$$

$$v_{up-ave}=(12)(4.08\text{ m/s})=2.04\text{ m/s}$$

$$\text{height}=(v_{up-ave})(t_{up})=(2.04\text{ m/s})(0.416\text{ s})=0.849\text{ m}$$

$$t_{total\ trip}=(2)(0.416\text{ s})=0.832\text{ s}$$

$$d_{horizontal}=(0.832\text{ s})(1.82\text{ m/s})=1.51\text{ m}$$

Example 4.3.3

Suppose a cannon ball is fired downward from a 50.0 m high cliff at an angle of 45° with an initial velocity of 80.0 m/s. How far horizontally will it land from the base of the cliff?

Solution

Suppose a cannon ball is fired downward from a 50.0 m high cliff at an angle of 45° with an initial velocity of 80.0 m/s. How far horizontally will it land from the base of the cliff?

In this case, the initial vertical velocity is downward and the acceleration due to gravity will increase this downward velocity.

$$v_{i-down}=(80.0\text{ m/s})(\sin 45^\circ)=(80.0\text{ m/s})(0.707)=56.6\text{ m/s}$$

$$v_{i-hor}=(80.0\text{ m/s})(\cos 45^\circ)=(80.0\text{ m/s})(0.707)=56.6\text{ m/s}$$

$$d=v_{i-down}t+(1/2)at^2$$

$$50.0=56.6t+4.9t^2$$

Changing to standard quadratic form yields $4.9t^2+56.6t-50.0=0$

This equation can be solved with the quadratic formula. The quadratic formula will produce two possible solutions for t:

$$t=(-b+(b2-4ac)\wedge(1/2))/2a \text{ and } t=(-b-(b2-4ac)\wedge(1/2))/2a$$

$$t=(-56.6+((56.6)^2-(4)(4.9)(-50))\wedge(1/2))/(2)(4.9)=0.816\text{ s}$$

The other solution to the quadratic formula is -12.375 *s*. Clearly, the cannon ball doesn't take -12 seconds to fly. Therefore, we take the positive answer. Using the quadratic formula will give you two answers; be careful to think about the answer you get - does it make sense?

To solve the problem, we plug the speed and time into the equation for distance:

$$d_{horizontal}=(0.816\text{ s})(56.6\text{ m/s})=46.2\text{ m}$$

In the following simulation, we can see that water from a fountain falls in a parabolic motion and lands at different positions based on its initial height, velocity and launch angle. Practice calculating where the water should land and then move the catcher to see if you are correct:

Summary

- To calculate projectile motion at an angle, first resolve the initial velocity into its horizontal and vertical components.
- Analysis of projectile motion involves dealing with two motions independently.

- Vertical components will always have the acceleration of gravity acting on them.
- Vertical motion is symmetrical - the distance and time are the same in the rise as in the fall; the final velocity will have the same magnitude as the initial velocity but in the opposite direction.
- Horizontal components will never be effected by gravity; it is constant velocity motion.

Review

1. A player kicks a football from ground level with a velocity of magnitude 27.0 m/s at an angle of 30.0° above the horizontal.
    1. Find the time the ball is in the air.
    2. Find the maximum height of the ball.
    3. Find the horizontal distance the ball travels.
2. A person standing on top of a 30.0 m high building throws a ball with an initial velocity of 20. m/s at an angle of 20.0° below horizontal. How far from the base of the building will the ball land?
3. An arrow is fired downward at an angle of 45 degrees from the top of a 200 m cliff with a velocity of 60.0 m/s.
    1. How long will it take the arrow to hit the ground?
    2. How far from the base of the cliff will the arrow land?

Explore More

Use this video to answer the questions that follow.

1. What is the cannon ball in this video?
2. What is used as the monkey in this video?
3. Does the velocity of the cannon ball matter, or will it hit the monkey at any velocity?

Additional Resources

Interactive: Archery

Real World Application: To Hit An Ace

Videos: Parabolic Projectile Motion - Overview, The Physics Behind a Curveball

Study Guide: Motion Study Guide