16.3: Modeling the Nucleus

The Size of the Nucleus

When Rutherford was performing his experiments, he launched helium nuclei at a thin sheet of gold. The data he collected suggested that the nucleus is roughly spherical. Some of the helium nuclei he launched at the gold atoms bounced off and reversed direction, indicating that the nucleus acted as if it were rigid. It was also thousands of times smaller than the atom. Given the size of the nucleus, it would seem that the nucleons are very tightly packed inside of it. The view of the nucleus was of a small, very dense sphere positioned at the center of the atom, surrounded by orbiting electrons. We have since learned that this model is also not exactly correct, but for the moment it does a reasonably good job of letting us imagine what is happening.

By assuming that the total volume of the nucleus was the sum of the volumes of the nucleons, physicists were able to develop a relationship between the number of nucleons and the effective radius of the nucleus. The volume of spherical nucleus of radius R, is (4/3) π R^3. The volume of a single spherical nucleon with radius r_0 is (4/3) π r_0^3. Because there are A nucleons in a particular nucleus, then the volume of the nucleus should be A(4/3) π r_0^3 = (4/3) π R^3 and the *empirical radius relationship* may be found:

$$R = r_0 A^{1/3}$$

In this expression, r_0 is the approximate radius of a single nucleon, if we think of them them as rigid spheres. It has a value of about 1.2 fm.

This expression agrees well with experimental evidence, but it raises a question. If you pour a bunch of marbles into a jar, the volume they occupy is larger than the sum of the volumes of each marble. This is because there is some empty space in between each of the rigid marbles, where the edges meet. But if you add drops of water to a jar then the total volume of water is the sum of the volumes of each of the drops. Since the water drops are not rigid, they can combine. So, are nucleons more like marbles or are they more like water drops? The answer seems to be that it depends on how the nucleons are interacting. Sometimes they act like marbles and sometimes they act like water drops. We'll touch on this again later in the chapter.

Example 16.3.1

Determine the approximate radius of a gold nucleus. The gold atom has a radius of 144 x 10^{-12} meters. How many times larger than the nucleus is the atom?

Solution

Gold has atomic number 197. Using the equation provided: $R = r_0 179^{1/3}$. The cube root of 179 is about 5.636.

This makes the radius of the gold nucleus R = 1.2 fm (5.636) = 6.763 fm = 6.763 x 10^{-15} meters.

The ratio of the radii: $\frac{144 \times 10^{-12} \text{ meters}}{6.736 \times 10^{-15} \text{ meters}}$ is about 21,292. The gold atom is more than twenty-one thousand times the size of the gold nucleus.

The Mass Defect

Imagine that you want to assemble a particular nucleus that will have nucleon number A. You will need Z protons and (A-Z) neutrons. Label the mass of a proton as m_p and the mass of a neutron as m_n. The amount of mass that you start with is $(Z)m_p + (A-Z)m_n$. The amount of mass that you end with is the mass of the nucleus, m_{nuc}.

You would think that these numbers would be the same. You might believe they should be the same. However, the data from numerous experiments shows that the mass of a nucleus (m_{nuc}) is *less than* the sum of the masses of the nucleons. The difference in masses is called the *mass defect* and can be found by:

$$\Delta m = [\ m_{nuc}] - [Zm_p + (A-Z)m_n]$$

This shows that a collection of loose nucleons has more mass than those same nucleons when they are in the nucleus. Using Einstein's mass-energy relationship ($E = mc^2$) then a difference in masses means that there is a difference in system energy when we compare a collection of loose neutrons and protons to an atomic nucleus. Specifically, combining all the nucleons into a nucleus causes the system to lose energy. That energy is emitted (radiated) away from the nucleus.

$$\Delta E = (\Delta m)c^2$$

A collection of nucleons that combine to form a nucleus has less energy that if those nucleons were free (unbound). The lower energy results in a more stable system.

Stable systems are more resistant to changes, and it is more difficult to change the behavior of the nucleus than to change the behavior of any single nucleon. There is an analagous process that occurs when water cools and forms into ice. The heat is radiated away, which lowers the energy of the system. The random motion of the water molecules changes and as the water changes to ice, they becomes arranged in a regular pattern. It is difficult to move a particular H_2O atom when it is part of block of ice, but it is very simple when it is in liquid form. While you should *not* think about a nucleus forming in the same way that water freezes, the overlapping idea is that systems can become more stable by releasing energy to the environment.

Example 16.3.2

Tritium is an isotope of hydrogen made of one proton and two neutrons. Calculate the mass deficit of tritium. Use 3.016049 u as the nuclear mass of tritium. Answer in units of u.

Solution

$\Delta m = [\ m_{nuc}] - [(Z)m_p + (A\text{-}Z)m_n]$, Z = 1 for hydrogen, and N = 2. $\Delta m = [\ 3.016049\ u] - [(1\text{x}\ 1.00728\ u) + (2\ \text{x}\ 1.00867\ u\ )] = -8.571\ \text{x}\ 10^{-3}$ u.

We interpret the result to mean that the mass of the tritium nucleus is $8.571\ \text{x}\ 10^{-3}$ u less than the masses of the neutrons and proton. We report the mass deficit as a positive number for this reason.

Binding Energy and Stability

Since the strong force causes the nucleons to stick together, this means that we have to do work to remove one of the nucleons from the nucleus. This is similar to the effect of the gravitational force on a rock. In order to lift an object away from the earth, we had to do work on the object and its energy increased. In order to remove an electron from an atom, energy had to be added to the system. For electrons, this was called the ionization energy. For the rock, it was the gravitational potential energy. In the case of nuclear physics, we refer to the nuclear *binding energy*.

$$E_{binding} = \Delta mc^2$$

This value represents how much work it would take to disassemble the nucleus. You would have to add energy to the system to pull the nucleons apart. Taking the total binding energy and dividing it by the total number of nucleons gives the *Binding Energy per Nucleon* (BEN). The larger the BEN, the more work is required to pull the nucleons apart. Smaller BEN values mean that it is easier to disassemble that nucleus, and that nucleus is less stable.

$$BEN = E_{binding}/A = \Delta mc^2/A$$

Because the mass deficit is negative, then the binding energy and BEN is also negative. The negative value represents the system losing energy as the nucleus formed. Even though it is a negative value, it is most often referenced as a positive value. This may seem confusing, but thinking about it as a debt may help. If you owe someone fifty dollars, you are in debt to them. The fifty dollars represents how much money you need to give them in order to get out of debt. In a somewhat similar way, the BEN reflects how much energy needs to be added to the system to 'pay off' the 'energy debt' of the nucleons and set them free.

Example 16.3.3

Using the results of example two, calculate the BEN for tritium. Answer in units of MeV.

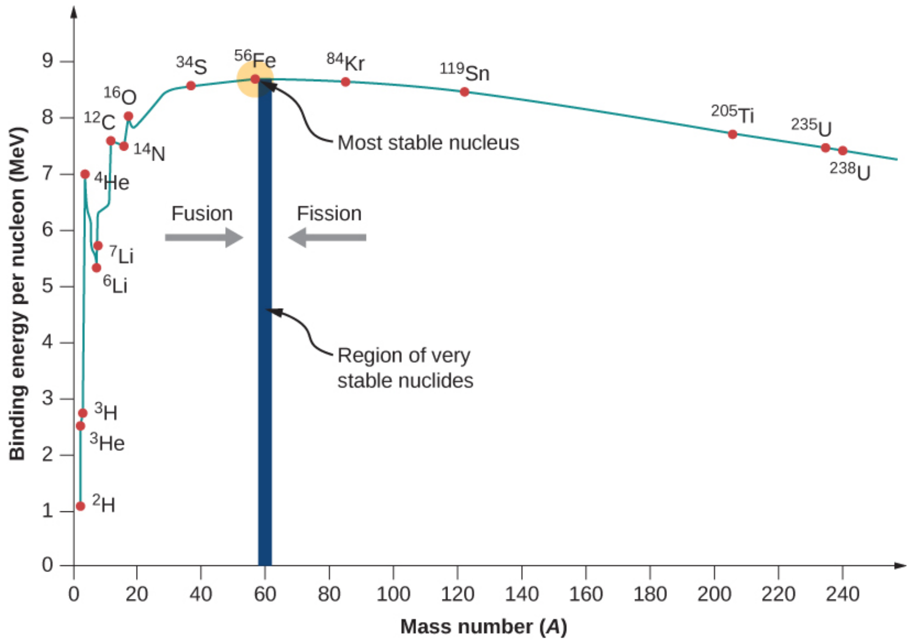
Solution

$-8.571\ \text{x}\ 10^{-3}$ u was the mass deficit. Divided by three, this gives a BEN of $-2.857\ \text{x}\ 10^{-3}$ u. Converting this to units of MeV involves a conversion factor to replace c^2. BEN = $-(2.857\ \text{x}\ 10^{-3}\ u)\ \text{x}\ (931.502\ \text{MeV/u}) = -2.6613$ MeV. This means 2.6613 mega electron-volts would have to be added to the nucleus in order to disassemble it.

Experimental results have produced a graph of the BEN, which is shown below. You can see that the BEN is small for Helium, which means it is relatively easy to break up a Helium nucleus. As the atomic mass increases towards Iron (Fe) it becomes more and more difficult to remove a nucleon from the nucleus which makes iron very stable. For nuclei with masses larger than iron, it becomes easier to cause them to break apart. You can see that the BEN for Carbon (C) is very similar to the BEN for Uranium (U). We can understand this in terms of the strong force between nucleons and the Coulomb force between protons.

For small proton numbers the strong force is dominant but as the proton numbers increase, the repulsive Coulomb force becomes more significant and the atoms tend to become less stable as a result since the Coulomb force is trying break the nucleus apart. For the less massive atoms, increasing the number of nucleons by nuclear fusion raises the BEN. For more massive atoms, decreasing the number of nucleons by nuclear fission will raise the BEN.

The BEN curve has applications for nuclear power generation, whether by fusion (smashing atoms together) or fission (splitting atoms apart).



BEN curve taken from OpenStax *University Physics vol. 3* and is licensed under CC-BY