6.1: Circular Motion

Weather satellite orbiting Earth is an example of circular motion Figure 5.1.1

Weather satellites, like the one shown above, are found miles above the earth's surface. Satellites can be polar orbiting, meaning they cover the entire Earth asynchronously, or geostationary, in which they hover over the same spot.

Arthur C. Clarke was the first to propose that satellites be placed in "geosynchronous orbit" around the Earth. These satellites orbit with the Earth over a time period of 24 hours, causing them to appear to stand still from Earth. Many of the fixed satellite dishes we see today, such as the ones used at home for satellite TV, are aimed at satellites in geosynchronous orbit. Learn more about the circular motion of a satellite by exploring the Clarke's Dream simulation below:

Circular Motion

The earth is a sphere. If you draw a horizontal straight line from a point on the surface of the earth, the surface of the earth drops away from the line. The distance that the earth drops away from the horizontal line is very small – so small, in fact, that we cannot represent it well in a drawing. In the sketch below, if the blue line is 1600 m, the amount of drop (the red line) would be 0.20 m. If the sketch were drawn to scale, the red line would be too short to see.

Drawing a horizontal line from the surface of the Earth shows a very small vertical drop Figure 5.1.2

When an object is launched exactly horizontally in projectile motion, it travels some distance horizontally before it strikes the ground. In the present discussion, we wish to imagine a projectile fired horizontally on the surface of the earth such that while traveling 1600 m horizontally, the object would fall exactly 0.20 m. If this could occur, then the object would fall exactly the amount necessary during its horizontal motion to remain at the surface of the earth, but not touching it. In such a case, the object would travel all the way around the earth continuously and circle the earth, assuming there were no obstacles, such as mountains.

What initial horizontal velocity would be necessary for this to occur? We first calculate the time to fall the 0.20 m:

$t=(2d/a)^{(1/2)}=((2)(0.20\text{ m})/9.80\text{ m/s}^2)^{(1/2)}=0.20\text{ s}$

The horizontal velocity necessary to travel 1600 m in 0.20 s is 8000 m/s. Thus, the necessary initial horizontal velocity is 8000 m/s.

In order to keep an object traveling in a circular path, there must be an acceleration toward the center of the circle. This acceleration is called **centripetal acceleration.** In the case of satellites orbiting the earth, the centripetal acceleration is caused by gravity. If you were swinging an object around your head on a string, the centripetal acceleration would be caused by your hand pulling on the string toward the center of the circle.

It is important to note that the object traveling in a circle has a constant speed but does not have a constant velocity. This is because direction is part of velocity; when an object changes its direction, it is changing its velocity. Hence the object's acceleration. The acceleration in the case of uniform **circular motion** is the change in the direction of the velocity, but not its magnitude.

For an object traveling in a circular path, the centripetal acceleration is directly related to the square of the velocity of the object and inversely related to the radius of the circle.

$a_c=v^2/r$

Taking a moment to consider the validity of this equation can help to clarify what it means. Imagine a yo-yo. Instead of using it normally, let it fall to the end of the string, and then spin it around above your head. If we were to increase the speed at which we rotate our hand, we increase the velocity of the yo-yo - it is spinning faster. As it spins faster, it also changes direction faster. The acceleration increases. Now let's think about the bottom of the equation: the radius. If we halve the length of the yo-yo string (bring the yo-yo closer to us), we make the yo-yo's velocity greater. Again, it moves faster, which increases the acceleration. If we make the string longer again, this decreases the acceleration. We now understand why the relationship between the radius and the acceleration is an inverse relationship - as we decrease the radius, the acceleration increases, and visa versa.

Vectors of acceleration and velocity are perpendicular in uniform circular motion Figure 5.1.3

Examples

Example 5.1.1

A ball at the end of a string is swinging in a horizontal circle of radius 1.15 m. The ball makes exactly 2.00 revolutions per second. What is its centripetal acceleration?

Solution

We first determine the velocity of the ball using the facts that the circumference of the circle is $2\pi r$ and the ball goes around exactly twice per second.

$v=(2)(2\pi r)/t=(2)(2)(3.14)(1.15\text{ m})/1.00\text{ s}=14.4\text{ m/s}$

We then use the velocity and radius in the centripetal acceleration equation.

$a_c=v^2/r=(14.4\text{ m/s})^2/1.15\text{ m}=180.\text{ m/s}^2$

Example 5.1.2

The moon's nearly circular orbit around the earth has a radius of about 385,000 km and a period of 27.3 days. Calculate the acceleration of the moon toward the earth.

Solution

$v=2\pi r/T=(2)(3.14)(3.85\times10^8\text{ m})/(27.3\text{ d})(24.0\text{ h/d})(3600\text{ s/h})=1020\text{ m/s}$

$a_c=v^2/r=(1020\text{ m/s})^2/3.85\times10^8\text{ m}=0.00273\text{ m/s}^2$

As shown in the previous example, the velocity of an object traveling in a circle can be calculated by

$v=2\pi r/T$

Where r is the radius of the circle and T is the period (time required for one revolution).

This equation can be incorporated into the equation for centripetal acceleration as shown below.

$a_c=v^2/r=(2\pi r/T)^2/r=4\pi^2r/T^2$

Summary

- In order to keep an object traveling in a circular path, there must be an acceleration toward the center of the circle. This acceleration is called centripetal acceleration**.**
- The acceleration in the case of uniform circular motion changes the direction of the velocity but not its magnitude.
- Formulas for centripetal acceleration are ac=v2r and ac=4π2rT2.

Review

1. An automobile rounds a curve of radius 50.0 m on a flat road at a speed of 14 m/s. What centripetal acceleration is necessary to keep the car on the curve?
2. An object is swung in a horizontal circle on a length of string that is 0.93 m long. If the object goes around once in 1.18 s, what is the centripetal acceleration?

Explore More

Use this resource to answer the questions that follow.

1. What does centripetal mean?
2. What is uniform circular motion?
3. Why is centripetal acceleration always towards the center?

Additional Resources

Study Guide: Circular Motion Study Guide

Real World Application: Spinning on Ice

PLIX: Play, Learn, Interact, eXplore: Spinning on Ice

Interactive: Angular SpeedPhases Of The Moon

Video: