3.2.7: Displacement During Uniform Acceleration

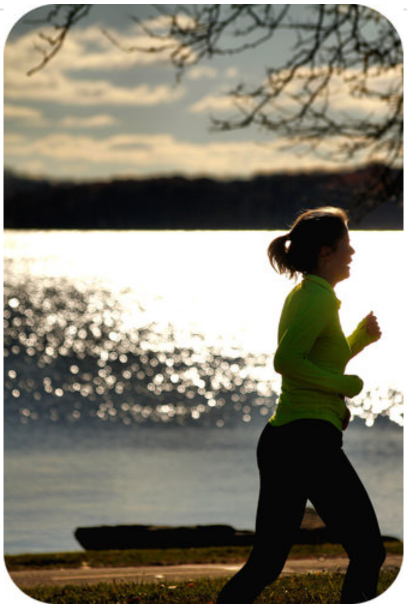

Figure 2.6.1

Long distance runners try to maintain constant velocity with very little acceleration or deceleration to conserve energy.

Displacement During Constant Acceleration

When acceleration is constant, there are three equations that relate displacement to two of the other three quantities we use to describe motion – time, velocity, and acceleration. These equations only work when acceleration is constant, but there are, fortunately, quite a few cases of motion where the acceleration is constant. One of the most common, if we ignore air resistance, are objects falling due to gravity.

When an object is moving with constant velocity, the displacement can be found by multiplying the velocity by the time interval, as shown in the equation below

$$d = vt$$

If the object is moving with constant acceleration, but not a constant velocity, we can use a derivation of this equation. Instead of using *v*, as velocity, we must calculate and use the average velocity using this equation:

$$v_{avg} = \frac{1}{2}(v_f + v_i)$$

The distance, then, for uniformly accelerating motion can be found by multiplying the average velocity by the time.

$$d = \frac{1}{2}(v_f + v_i)t \quad \textbf{(Equation 1)}$$

We know that the final velocity for constantly accelerated motion can be found by multiplying the acceleration times time and adding the result to the initial velocity, $v_f = v_i + at$.

The second equation that relates displacement, time, initial velocity, and final velocity is generated by substituting this equation into equation 1.

Start by distributing the 1/2 in equation 1 through:

$$d = \frac{1}{2}(v_f + v_i)t = \frac{1}{2}v_f t + \frac{1}{2}v_i t$$

We know that $v_f = v_i + at$. Therefore:

$$d = \frac{1}{2}v_i t + \frac{1}{2}(v_i + at)$$

$$d = \frac{1}{2}v_i t + \frac{1}{2}v_i t + \frac{1}{2}at^2$$

$$d = v_i t + \frac{1}{2}at^2 \quad \textbf{(Equation 2)}$$

The third equation is formed by combining $v_f = v_i + at$ and $d = \frac{1}{2}(v_f + v_i)t$. If we solve the first equation for t and then substitute into the second equation, we get

$$d = \frac{1}{2}(v_f + v_i)\frac{v_f - v_i}{a} = \frac{1}{2}\frac{(v_f^2 - v_i^2)}{a}$$

And solving for v_f^2 yields

$$v_f^2 = v_i^2 + 2ad \quad \textbf{(Equation 3)}$$

Keep in mind that these three equations are only valid when acceleration is constant. In many cases, the initial velocity can be set to zero and that simplifies the three equations considerably. When acceleration is constant and the initial velocity is zero, the equations can be simplified to:

$$d = \frac{1}{2}v_f t$$

$$d = \frac{1}{2}at^2 \text{ and}$$

$$v_f^2 = 2ad$$

Example 2.6.1

Suppose a planner is designing an airport for small airplanes. Such planes must reach a speed of 56 m/s before takeoff and can accelerate at 12.0 m/s^2. What is the minimum length for the runway of this airport?

Solution

The acceleration in this problem is constant and the initial velocity of the airplane is zero. Therefore, we can use the equation vf2=2ad and solve for d.

$d=(v_f^2)/2a=(56\text{ m/s})^2/((2)(12.0\text{ m/s}^2))=130\text{ m}$

Example 2.6.2

How long does it take a car to travel 30.0 m if it accelerates from rest at a rate of 2.00 m/s^2?

Solution

The acceleration in this problem is constant and the initial velocity is zero, therefore, we can use d=(1/2)at^2 solved for t.

t=(2d/a)^(1/2)=((2)(30.0 m)/(2.00 m/s^2))^(1/2)=5.48 s

Example 2.6.3

A baseball pitcher throws a fastball with a speed of 30.0 m/s. Assume the acceleration is uniform and the distance through which the ball is accelerated is 3.50 m. What is the acceleration?

Solution

Since the acceleration is uniform and the initial velocity is zero, we can use v_f^2=2ad solve for a.

$a=v_f^2/2d=(30.0\text{ m/s})^2/(2)(3.50\text{ m})=900\ (\text{m}^2/\text{s}^2)/7.00\text{ m}=129\text{ m/s}^2$

Suppose we plot the velocity versus time graph for an object undergoing uniform acceleration. In this first case, we will assume the object started from rest.

If the object has a uniform acceleration of 6.0 m/s^2 and started from rest, then each succeeding second, the velocity will increase by 6.0 m/s. Here is the table of values and the graph.

Velocity versus time graph

Figure 2.6.2

In displacement versus time graphs, the slope of the line is the velocity of the object. In this case of a velocity versus time graph, the slope of the line is the acceleration. If you take any segment of this line and determine the Δy to Δx ratio, you will get 6.0 m/s^2 which we know to be the constant acceleration of this object.

We know from geometry that the area of a triangle is calculated by multiplying one-half the base times the height. The area under the curve in the image above is the area of the triangle shown below. The area of this triangle would be calculated by area=(1/2)(6.0 s)(36 m/s)=108 m.

Velocity versus time graph with the area under the line being equal to the area of a triangle

Figure 2.6.3

By going back to equation 2, we know that displacement=(1/2)at^2 .

Using this equation, we can determine that the displacement of this object in the first 6 seconds of travel is =(1/2)(6.0 m/s2)(6.0 s)^2=108 m.

It is not coincidental that this number is the same as the area of the triangle. In fact, the area underneath the curve in a velocity versus time graph is always equal to the displacement that occurs during that time interval.

Use the PLIX Interactive below to analyze the motion of Jane's dog, Sparky, and visualize how acceleration and displacement can be derived from a velocity-time graph:

MAKING CONNECTIONS: TAKE-HOME EXPERIMENT—BREAKING NEWS

We have been using SI units of meters per second squared to describe some examples of acceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking acceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration, a=v/t. While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the acceleration in miles per hour per second. Convert this to meters per second squared and compare with other accelerations mentioned in this chapter. Calculate the distance traveled in braking.

Summary

- There are three equations we can use when acceleration is constant to relate displacement to two of the other three quantities we use to describe motion – time, velocity, and acceleration:
  - $d=\frac{1}{2}(v_f+v_i)t$ (Equation 1)
  - $d=v_it+\frac{1}{2}at^2$ (Equation 2)
  - $v_f^2=v_i^2+2ad$ (Equation 3)
- When the initial velocity of the object is zero, these three equations become:
  - $d=\frac{1}{2}v_ft$ (Equation 1')
  - $d=\frac{1}{2}at^2$ (Equation 2')
  - $v_f^2=2ad$ (Equation 3')
- The slope of a velocity versus time graph is the acceleration of the object.
- The area under the curve of a velocity versus time graph is the displacement that occurs during the given time interval.

Review

1. An airplane accelerates with a constant rate of 3.0 m/s^2 starting at a velocity of 21 m/s. If the distance traveled during this acceleration was 535 m, what is the final velocity?
2. An car is brought to rest in a distance of 484 m using a constant acceleration of -8.0 m/s^2. What was the velocity of the car when the acceleration first began?
3. An airplane starts from rest and accelerates at a constant 3.00 m/s^2 for 20.0 s. What is its displacement in this time?
4. A driver brings a car to a full stop in 2.0 s.
   1. If the car was initially traveling at 22 m/s, what was the acceleration?
   2. How far did the car travel during braking?

Explore More

Use this resource to answer the question that follows.

1. What does the area bounded by a velocity versus time graph represent?

Additional Resources

Study Guide: Motion Study Guide

Real World Application: Dangerous Pennies

Interactive Element: Model Rocket, Irwin 2D

PLIX: Play, Learn, Interact, eXplore: Constant Acceleration and Inertia

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