SPIRAL PHYSICS - CALCULUS BASED

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CHAPTER OVERVIEW

1: Ideal Gases

In this section, we will investigate a new, mechanical model that can be viewed as an application of the models previously studied. Although we will take a step back in complexity and examine a particle, rather than a rigid-body, model, to compensate for this simplification we will not look at one particle, or two particles, but rather *many* particles. "Many" in this case means approximately 10^{23}!

Since there are *so* many particles we will no longer be interested in the motion of any one individual particle, but rather in the average motions of the entire group of particles. We will try to use our understanding of mechanics, applied to individual *microscopic* particles, to learn about the *macroscopic* properties of the entire group.

To simplify matters we will assume that the particles do not interact except via elastic collisions with each other and collisions with the walls of the container in which they are contained. Thus, there are no non-contact interactions, such as the force of gravity, nor contact interactions, such as connecting ropes, springs,etc., between the particles.

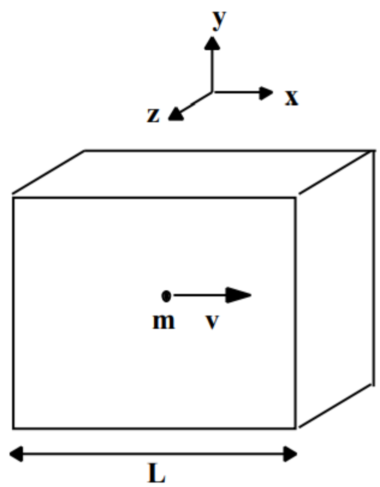
If we imagine our collection of particles to represent individual atoms in a gas this model is referred to as the *ideal gas model*. The study of the ideal gas forms the entrance into the field of physics known as thermodynamics.

Thumbnail: In an ordinary gas, so many molecules move so fast that they collide billions of times every second. (Public Domain; Greg L via Wikipedia)

1.0: Concepts and Principles

Applying Newton's Second Law

Although the ultimate goal is to study a collection of a large number of particles, we'll start out by looking at just a single particle. Imagine a particle of mass m and speed v confined inside an elastic cube of edge length L.



Since we're imagining, imagine the particle to be moving in the +x-direction, perpendicular to a face of the cube. The particle will strike the wall and bounce off. Let's apply Newton's second law to the particle during the time interval in which it is in contact with the wall, $\Delta t_{collision}$:

$$\Sigma F = ma$$

$$F_{\text{average}} = m\left(\frac{\Delta v}{\Delta t_{\text{collision}}}\right)$$

Notice that we have replaced the instantaneous force acting on the particle with the average force over the time interval of interest. Since the wall is perfectly elastic, the collision should just reverse the direction of the particle's speed.

$$F_{\text{average}} = m\left(\frac{v_f - v_i}{\Delta t_{\text{collision}}}\right)$$

$$F_{\text{average}} = m\left(\frac{(-v) - (v)}{\Delta t_{\text{collision}}}\right)$$

$$F_{\text{average}} = \frac{-2mv}{\Delta t_{\text{collision}}}$$

This is the average force on the particle. The average force on the wall is equal in magnitude but opposite in direction.

$$F_{\text{averageonull}} = \frac{2mv}{\Delta t_{\text{collision}}}$$

This is the average force on the wall during the relatively small time interval of the collision. There are long lulls (when the particle is moving toward the other wall and then returning) that the average force on the wall is zero. If we want the average force on the wall during the entire motion of the particle (and we do) we have to divide not by $\Delta t_{\text{collision}}$ but by $\Delta t_{\text{round-trip}}$, the time to make a complete round-trip journey. Thus, the average force on the wall during the entire journey of the particle is:

$$F_{\text{averageonuall}} = \frac{2mv}{\Delta t_{\text{round - trip}}}$$

Since we know the speed of the particle, we know $F_{\text{averageonuall}} = \frac{2mv}{\Delta t_{\text{round - trip}}}$. A total distance traveled of 2L, at a speed of v, takes a time of:

$$\Delta t_{\text{round-trip}} = \frac{2L}{v}$$

so

$$F_{\text{averageonvall}} = \frac{2mv}{\left(\frac{2L}{v}\right)}$$

$$F_{\text{averageonwall}} = \frac{mv^2}{L}$$

Thus, we have determined the average force a *single* particle of mass m and speed v would exert on the wall of a cube of edge length L. The question remains, however, what would this average force be if we had N particles?

To generalize our relation we must make two adjustments. First, if we had N identical particles, all moving in an identical manner, it would seem obvious that the average force on the wall would be N times greater,

$$F_{\text{averageonwall}} = \frac{Nmv^2}{L}$$

This would be true, however, only if all N particles were moving with the same speed in the same direction, in this case the x direction.

$$F_{\text{averageonwall}} = \frac{Nmv_x^2}{L}$$

What if the particles were moving in all directions at many possible speeds? *On average*, there is no reason why the particles should be going faster in the x than the y or the z direction. Thus,

$$\left(v_x^2\right)_{\text{average}} = \left(v_y^2\right)_{\text{average}} = \left(v_z^2\right)_{\text{average}}$$

Since

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

we will assume that

$$\left(v^2\right)_{\text{average}} = \left(v_x^2\right)_{\text{average}} + \left(v_y^2\right)_{\text{average}} + \left(v_z^2\right)_{\text{average}}$$
$$\left(v^2\right)_{\text{average}} = \left(v_x^2\right)_{\text{average}} + \left(v_x^2\right)_{\text{average}} + \left(v_x^2\right)_{\text{average}}$$
$$\left(v^2\right)_{\text{average}} = 3\left(v_x^2\right)_{\text{average}}$$
$$\left(v_x^2\right)_{\text{average}} = \frac{1}{3}\left(v^2\right)_{\text{average}}$$

So, averaging over all the various velocities leads to:

$$F_{\text{averageonuall}} = \frac{Nm\left(v^2\right)_{\text{average}}}{3L}$$

Let’s try to remember that the force and the squared speed are averages, yet state the relation as:

$$F = \frac{Nmv^2}{3L}$$

It would be nice to have a relationship that does not explicitly depend on the length of the container. We can accomplish this by defining the *pressure* (p) of the gas as the average force exerted by the gas on one square meter of its container. Thus, pressure is measured in Newtons per square meter, which is given the formal SI name of *pascal*, abbreviated Pa.

If we take the above expression and divide it by the surface area of the wall, L^2, the resulting ratio will be the pressure of the gas on the wall of the container.

$$F = \frac{Nmv^2}{3L}$$

$$\frac{F}{L^2} = \frac{Nmv^2}{3L^3}$$

$$p = \frac{Nmv^2}{3L^3}$$

$$p = \frac{Nmv^2}{3V}$$

with L^3 replaced by V, the volume of the container.

We now have a relationship that combines macroscopic quantities, such as the pressure and volume of the gas, and microscopic quantities, such as the mass and average squared velocity of individual particles.

To complete the macroscopic description, I will make a radical definition: I hereby define the average kinetic energy of an individual particle to be proportional to a quantity I will call *temperature* (T).[1]

$$\frac{1}{2}mv^2 \propto T$$

Thus, temperature is a macroscopic measurement of a microscopic quantity; an individual particle's average kinetic energy. Selecting a proportionality constant so that the resulting relationship is as simple as possible leads me to select "$\frac{3}{2}$ k" as the constant. The "$\frac{3}{2}$" makes the resulting relation simple and the "k" is in honor of Ludwig Boltzmann and is called the Boltzmann constant.[2]

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$mv^2 = 3kT$$

Note

[1] We will measure temperature in kelvin, abbreviated K.

[2] The Boltzmann constant is experimentally determined to be 1.38 x 10^{-23} joules/kelvin.

Therefore our relationship between macroscopic variables becomes

$$p = \frac{Nmv^2}{3V}$$

$$p = \frac{N(3kT)}{3V}$$

$$p = \frac{NkT}{V}$$

$$pV = NkT$$

This relation is termed the *equation of state* for an ideal gas because it forms a relationship between the parameters that define the state the gas is in; its pressure, volume and temperature.

An alternative form of the same relation, written in terms of the number of moles of particles[3] (n) rather than number of particles (N) is

$$pV = nRT$$

By comparing the two relations you should notice that R, the *universal gas constant*, is equal to the product of the Boltzmann constant and the number of particles in a mole. Thus, R = 8.31 joules/kelvin.

Note

[3] 6.02 x 10^{23} particles comprise one mole of particles.

Applying the Work-Energy Relation

The equation of state relates the *state variables* of a gas: pressure, volume and temperature. If you know the value of two of these variables, you can always determine the third. However, the relation says nothing about the *process* by which the gas arrived at that state. The easiest way to investigate processes is through the application of the work-energy relation.

In general terms, the work-energy relation states that the energy of a particle is changed through the application of work. Schematically,

$$\textit{initial energy} + \textit{work} = \textit{final energy}$$

Thus, in general terms, work is the transfer of energy to a particle. We could state the workenergy relation in these more general terms.

$$\textit{initial energy} + \textit{energy transfer} = \textit{final energy}$$

Since our collection of particles is non-interacting, there is no gravitational or elastic potential energy present. The only form of energy present is kinetic.

$$\textit{initial kinetic energy} + \textit{energy transfer} = \textit{final kinetic energy}$$

This form of the work-energy relation must hold for each and every particle. If we apply this result to all of the particles in the gas, and sum the results, we arrive at

$$\textit{total initial KE of all particles} + \textit{total energy transfer to gas} = \textit{total final}\,\mathrm{KE}\,\textit{of all particles}$$

The total kinetic energy of all of the particles in a gas is termed the *internal energy* (U_{internal}) of the gas. Thus,

$$U_{\text{internal},\, i} + \textit{total energy transfer to gas} = U_{\text{internal},\, f}$$

with

$$U_{\text{internal}} = N\left(\frac{1}{2}mv^2\right)$$

$$U_{\text{internal}} = N\left(\frac{3}{2}kT\right)$$

$$U_{\text{internal}} = \frac{3}{2}NkT$$

or

$$U_{\text{internal}} = \frac{3}{2}nRT$$

Now comes the tricky part. What is the total energy transfer to the gas? Remember that the energy transfer to each particle is due to the application of forces external to that particle. Since the particles do not interact with each other, except through elastic collisions, all the energy transfer must be done through collisions of the particles with the walls of the container. This transfer of energy to the gas is via two, distinct methods.

Macroscopic Energy Transfer

First imagine a collision of a particle with a stationary wall. An analogy to this situation would be a tennis ball bouncing off of a stationary tennis racket. The tennis ball rebounds off of the racket with approximately the same speed, and hence the same energy, as it had when it struck the racket. Thus, there is no energy transfer to the ball during this type of collision.

Now imagine a tennis ball striking a moving racket. If the racket is moving toward the ball, the ball will rebound off the racket with a greatly increased speed. Hence, there is energy transfer *to* the ball during this collision. If the racket is moving away *from* the ball, the ball will rebound with a decreased speed. Hence, there is energy transfer from the ball during this collision.

From this analogy, we can see that energy can be transferred to or from the gas of particles if the walls of the container are in motion. If the walls of the container are in motion, then the volume of the container must be changing. Thus, if the volume of the container is changing, an energy transfer is taking place. This energy transfer is termed *macroscopic* because the mechanism causing it is the motion of a macroscopic object, a container wall.

Since the container walls are moved by the application of forces acting over distances, this energy transfer is the familiar energy transfer termed *work*. In fact, from the definition of work,

$$W = \int (F\cos\phi)dr$$

with F the average force acting on the container wall and dr the distance the container wall moves.

The force acting on the container wall can be written in terms of the pressure on the wall and the area of the wall, and let dr be the distance the container wall moves in the direction of the force:

$$W = \int (pA)dr$$

A change in wall position, dr, leads to a change in container volume, dV, given by

$$dV = A(dr)$$

Thus,

$$W = \int p(dV)$$

This is the work done *on* the container by the gas. Since we are interested in the energy transfer to the gas, we need the work done *on* the gas *by* the container, which is

$$W = -\int p(dV)$$

Work is the macroscopic transfer of energy to the gas by means of changes in gas volume.

Microscopic Energy Transfer

Now let's look more closely at the collision of a gas particle with the container wall. Realistically, the container wall is composed of a huge number of particles. The container wall also has a definite temperature, implying that the particles comprising the wall have average kinetic energies proportional to this temperature. Thus, *even if the wall is macroscopically stationary, the individual particles comprising the wall are in motion*. Therefore, the collision of a gas particle is not really with a stationary wall but rather with a microscopically moving wall particle! In addition, since we already know that energy can be transferred during an elastic collision between two moving objects, we should realize that energy can be transferred even when the walls are macroscopically stationary. This *microscopic* transfer of energy is termed *heat*.

However, let's not get carried away. Granted, the wall particles are in motion, but aren't the collisions between gas particles and wall particles equally likely to occur when the wall particles are moving toward the gas particles (transfer of energy to the gas) as when the wall particles are moving away from the gas particles (transfer of energy from the gas)? If this is true, the net energy flow should be zero!

The above argument *is* true, assuming the colliding particles have the same kinetic energy as the wall particles. The argument is simply a complicated way of saying that if the wall and the gas particles have the same kinetic energy, and are therefore at the same temperature, no net energy will be transferred from the wall to the gas. There will be no transfer of energy via heat if both objects are at the same temperature.

But what if the temperature of the wall is higher than that of the gas? This means that the wall particles have more kinetic energy than the gas particles. Although you will still have some collisions in which energy is transferred from the gas (when the wall particles are moving away from the gas particles), the amount of energy transferred to the gas (when the wall particles are moving toward the gas particles) will be greater. Overall, the particles with more kinetic energy will transfer energy to the particles with less kinetic energy. Hence, energy will flow from high kinetic energy particles to low kinetic energy particles, or, put more simply, from high temperature to low temperature. Again, this microscopic flow of energy from high temperature to low temperature particles is called heat. Physicists use the symbol Q to represent the amount of energy transferred via heat.

Thus, our application of the work-energy relation to an ideal gas leaves us with

$$\mathrm{U}_{\text{internal, i}}+\text{ total energy transfer to gas }=\mathrm{U}_{\text{internal, f}}$$

$$\mathrm{U}_{\text{internal, i}}+\mathrm{W}+\mathrm{Q}=\mathrm{U}_{\text{internal, f}}$$

This relationship, often referred to as the First Law of Thermodynamics, states that the internal energy of a gas can be changed through energy transfer to the gas via work or heat.

1.1: Analysis Tools

A Constant Pressure ProcessNote

An approximately spherical, 9.4 cm radius helium balloon is brought from a chilly room (15 ^{0}C) into a warm room (20 ^{0}C).

To analyze this situation, we should first carefully determine and define the sequence of events that take place. At each of these instants, let's tabulate what we know about the state of the helium gas. Then, we can analyze the process that leads from one state to another.

Event 1: The instant before the balloon is brought into the warmer room	Event 2: The instant the balloon stops expanding
p_1 = $V_1 = \frac{4}{3}\pi(0.094\text{ m})^3$ T_1 = 15 0C = 288 K	p_2 = V_2 = T_2 = 20 0C = 293 K

Process:

W_{12} =

Q_{12} =

The situation describes a gas that undergoes a process taking it from one state to another. Therefore, at each state I have tabulated the value (if know) of each of the state variables. Between the states I have indicated a location for the recording of the process undertaken as well as the energy transfer during the process, via work or heat.

Do I know anything else about the initial state of the gas in the balloon? The force that the helium gas exerts on the inner surface of the balloon must be equal in magnitude to the force that the outside air exerts on the outer surface of the balloon plus the additional force inward due to the elastic nature of the balloon. If I ignore this additional force due to the elastic balloon, the pressure of the helium must be equal in magnitude to the outside air pressure. Average air pressure on the surface of the earth, referred to as an atmosphere (atm), is

$$1\text{ atm} = 1.01325 \times 10^5\text{ Pa}$$

In addition, since the outside air pressure doesn't change, neither does the pressure of the helium gas. Therefore, the process is at constant pressure.

Event 1: The instant before the balloon is brought into the warmer room	Event 2: The instant the balloon stops expanding
p_1 = 1.01 x 10^5 Pa V_1 = 3.48 x 10^{-3} m^3 T_1 = 15 0C = 288 K	p_2 = 1.01 x 10^5 Pa V_2 = T_2 = 20 0C = 293 K

Process: Constant Pressure

W_{12} =

Q_{12} =

The next step is to apply the equation of state to the two states of the gas:

$p_1V_1 = nRT_1$ (1.01 x 10^5 Pa)(3.48 x 10^{-3} m^3) = n(8.31 J/K)(288 K) n = 0.147 moles	$p_2V_2 = nRT_2$ (1.01 x 10^5 Pa) V_2 = (0.147)(8.31 J/K)(293 K) V_2 = 3.54 x 10^{-3} m^3 Thus, the final radius of the balloon is: r_2 = 9.46 cm

We have completely defined the initial and final states of the helium gas in the balloon, but what about the process that took the balloon from the first state to the second state? From the First Law of Thermodynamics,

$$U_{\text{internal, i}} + W + Q = U_{\text{internal, f}}$$

$$\frac{3}{2}nRT_1 + \left(-\int pdV\right) + Q = \frac{3}{2}nRT_2$$

Since the process is at constant pressure,

$$W = -p\int dV$$

$$W = -p\left(V_2 - V_1\right)$$

$$W = -\left(1.01 \times 10^5\ \text{Pa}\right)(3.54 - 3.48) \times 10^{-3}\ \text{m}^3$$

$$W = -6.06\ \text{J}$$

Thus,

$$\frac{3}{2}nRT_1 + \left(-\int pdV\right) + Q = \frac{3}{2}nRT_2$$

$$\frac{3}{2}(0.147)(8.31)(288) - 6.06 + Q = \frac{3}{2}(0.147)(8.31)(293)$$

$$Q = 15.2\ \text{J}$$

During the constant pressure process, 15.2 J of energy were transferred to the gas via heat (from the 20 ^{0}C room) and 6.06 J of energy were transferred out of the gas via the work done by the gas in expanding the balloon.

A Constant Volume Process

> *A miniature (1.1 m^3), robotic submarine is loaded with scientific apparatus at the ocean's surface, sealed, and programmed to dive from the surface (15 ^{0}C) to a depth of 3000 m (3 ^{0}C).*

When the submarine is open at the ocean's surface, the air inside the sub is at a pressure of 1 atm. The sub is then sealed so that no air can leak in or out. Assume the sub is made of strong material so that its volume does not change during its descent.

Event 1: The instant the submarine is sealed at the surface	Event 2: The instant the submarine achieves temperature equilibrium at a depth of 3000 m
p_1 = 1.01 x 10^5 Pa V_1 = 1.1 m^3 T_1 = 288 K	p_2 = V_2 = 1.1 m^3 T_2 = 276 K

Process: Constant Volume

W_{12} =

Q_{12} =

The next step is to apply the equation of state to the two states of the gas:

$p_1V_1 = nRT_1$ (1.01 x 10^5 Pa)(1.1 m^3) = n(8.31 J/K)(288 K) n = 46.4 moles	$p_2V_2 = nRT_2$ p_2 (1.1 m^3) = (46.4)(8.31 J/K)(276 K) p_2 = 0.968 x 10^5 Pa

We have completely defined the initial and final states of the gas in the sub, but what about the process that took the gas from the first state to the second state? From the First Law of Thermodynamics,

$$U_{\text{internal },i} + W + Q = U_{\text{internal, f}}$$

$$\frac{3}{2}nRT_1 + \left(-\int p\,dV\right) + Q = \frac{3}{2}nRT_2$$

Since the process is at constant volume, no energy transfer can take place through work. Thus,

$$\frac{3}{2}nRT_1 + 0 + Q = \frac{3}{2}nRT_2$$

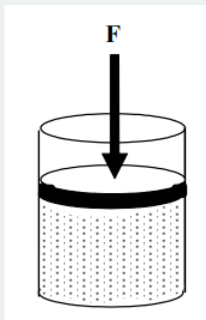
$$\frac{3}{2}(46.4)(8.31)(288) + Q = \frac{3}{2}(46.4)(8.31)(276)$$

$$Q = -6940\text{ J}$$

During the sub's descent, 6940 J of energy were transferred out of the gas into the surrounding ocean water via heat.

A Constant Temperature Process

The piston at right is immersed in an ice-water bath. With a 100 N force applied to the circular, 2 cm radius piston head, the volume of gas in the piston is 200 cm^3. The external force is slowly reduced to zero, leaving only atmospheric pressure pressing on the piston head.



Since the piston is immersed in ice water, the initial temperature of the gas is 0 ^{0}C. If the external force is removed slowly, the temperature of the gas should always remain in equilibrium with the ice water bath. Hence, the final temperature of the gas is also 0 ^{0}C. Note that the initial pressure of the gas is equal to the sum of atmospheric pressure plus the pressure due to the applied force.

Event 1: The instant before the force is decreased	Event 2: The instant the applied force reaches 0 N
p_1 = 1.01 x 10^5 Pa + 100 N/(π(0.02 m)^2) p_1 = 1.81 x 10^5 Pa V_1 = 200 cm^3 = 2 x 10^{-4} m^3 T_1 = 273 K	p_2 = 1.01 x 10^5 Pa V_2 = T_2 = 273 K

Process: Constant Temperature

W_{12} =

Q_{12} =

The next step is to apply the equation of state to the two states of the gas:

$p_1V_1 = nRT_1$ (1.81 x 10^5 Pa)(2 x 10^{-4} m^3) = n(8.31 J/K)(273 K) n = 0.0159 moles	$p_2V_2 = nRT_2$ (1.01 x 10^5) V_2 = (0.0159)(8.31 J/K)(273 K) V_2 = 3.57 x 10^{-4} m^3

We have completely defined the initial and final states of the gas in the piston, but what about the process that took the gas from the first state to the second state? From the First Law of Thermodynamics,

$$U_{\text{internal, i}} + W + Q = U_{\text{internal, f}}$$

$$\frac{3}{2}\mathrm{nRT_1} + \left(-\int \mathrm{p\,dV}\right) + Q = \frac{3}{2}\mathrm{nRT_2}$$

Let's examine the work done on the gas.

$$W = -\int \mathrm{pdV}$$

Since the volume is the variable of integration, we must express the pressure in terms of the volume. From the equation of state,

$$pV = nRT$$

$$p = \frac{nRT}{V}$$

Therefore,

$$W = -\int \frac{nRT}{V} dV$$

Since the temperature is constant,

$$W = -nRT \int \frac{dV}{V}$$

$$W = -nRT\left(\ln(V_2) - \ln(V_1)\right)$$

$$W = -nRT \ln\left(\frac{V_2}{V_1}\right)$$

$$W = -(0.0159)(8.31)(273)\ln\left(\frac{3.57\times10^{-4}}{2.0\times10^{-4}}\right)$$

$$W = -20.9J$$

Since the temperature is constant, the internal energy is the same in both states. Thus,

$$\frac{3}{2}\mathrm{nRT_1} - 20.9 + Q = \frac{3}{2}\mathrm{nRT_2}$$

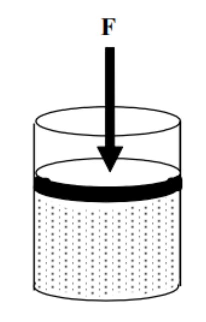
$$\frac{3}{2}\mathrm{nRT_1} - 20.9 + Q = \frac{3}{2}\mathrm{nRT_1}$$

$$Q = 20.9\ \mathrm{J}$$

During the removal of the applied force, 20.9 J of energy were transferred to the gas via heat and 20.9 J of energy were transferred out of the gas via the work done by the expanding gas.

An Adiabatic Process

The piston at right is immersed in an ice-water bath. With a 100 N force applied to the circular, 2 cm radius piston head, the volume of gas in the piston is 200 cm^3. The external force is quickly removed, leaving only atmospheric pressure pressing on the piston head.



Since the piston is immersed in ice water, the initial temperature of the gas is 0 ^{0}C. If the external force is quickly removed, the gas will quickly expand, transferring energy out of the gas via work. In the previous problem, since the expansion was slow, energy was able to flow into the gas, via heat, as this process took place, thus maintaining a constant temperature. If the expansion is fast, there is not sufficient time for energy transfer to take place via heat. A process in which there is no transfer of energy via heat is termed adiabatic. Adiabatic processes occur either very quickly or in systems with thermal insulation.

Event 1: The instant before the force is removed	Event 2: The instant the gas reaches its new equilibrium state
p_1 = 1.81 x 10^5 Pa V_1 = 200 cm^3 = 2 x 10^{-4} m^3 T_1 = 273 K	p_2 = 1.01 x 10^5 Pa V_2 = T_2 =

Process: Adiabatic

W_{12} =

Q_{12} =

The next step is to apply the equation of state to the two states of the gas:

$p_1V_1 = nRT_1$ (1.81 x 10^5 Pa)(2 x 10^{-4} m^3) = n(8.31 J/K)(273 K) n = 0.0159 moles	$p_2V_2 = nRT_2$ (1.01 x 10^5)V_2 = (0.0159)(8.31 J/K) T2 Uh-oh, two variables in one equation.

Let’s see if the First Law of Thermodynamics can help us out,

$$\mathrm{U_{internal,\,i}} + \mathrm{W} + \mathrm{Q} = \mathrm{U_{internal\,,f}}$$

$$\frac{3}{2}\mathrm{nRT_1} + \mathrm{W} + 0 = \frac{3}{2}\mathrm{nRT_2}$$

$$\mathrm{W} = \frac{3}{2}\mathrm{nR}\left(\mathrm{T_2} - \mathrm{T_1}\right)$$

If we knew the final temperature, we could determine the work done by the gas. If we knew the work done by the gas, we could determine the final temperature. Unfortunately, we don’t know either. In previous examples, since one variable was held constant (p, V or T), the equation of state was sufficient to determine the remaining state variables, and the First Law was sufficient to quantify the process. Somehow, from somewhere, we need more information.

Let’s look again at the equation of state (pay attention, this gets tricky):

$$\mathrm{pV} = \mathrm{nRT}$$

If this relation is valid, so is its differential form,

$$\mathrm{d}\left(\mathrm{pV}\right) = \mathrm{d}\left(\mathrm{nRT}\right)$$

Using the product rule and noting that n and R are constants,

$$\mathrm{p\ dV + V\ dp = nR\ dT}$$

Since

$$\mathrm{W} = -\int \mathrm{p\ dV}$$

$$\mathrm{dW = -p\ dV}$$

and for an adiabatic process (illustrated above using the First Law),

$$\mathrm{dW} = \frac{3}{2}\mathrm{nR\ dT}$$

$$-\mathrm{p\ dV} = \frac{3}{2}\mathrm{nR\ dT}$$

Therefore,

$$\mathrm{p\ dV + V\ dp = nR\ dT}$$

$$-\frac{3}{2}\mathrm{nR\ dT + V\ dp = nR\ dT}$$

$$\mathrm{V\ dp} = \frac{5}{2}\mathrm{nR\ dT}$$

$$\frac{3}{5}\ \mathrm{V\ dp} = \frac{3}{2}\mathrm{nR\ dT}$$

It is reasonable for you to ask at this point, “What’s the point?”. The point is that we can now relate p dV and V dp:

$$-\mathrm{p\ dV} = \frac{3}{2}\mathrm{nR\ dT} \qquad \frac{3}{5}\ \mathrm{V\ dp} = \frac{3}{2}\mathrm{nR\ dT}$$

$$-\mathrm{p\ dV} = \frac{3}{5}\ \mathrm{V\ dp}$$

rearranging

$$-\frac{5}{3}(1/\mathrm{V})\ \mathrm{dV} = (1/\mathrm{p})\ \mathrm{dp}$$

and integrating

$$-\frac{5}{3}\int(1/\mathrm{V})\ \mathrm{dV} = \int(1/\mathrm{p})\ \mathrm{dp}$$

$$-\frac{5}{3}\ln(\mathrm{V_2/V_1}) = \ln(\mathrm{p_2/p_1})$$

Let’s define the factor $\frac{5}{3}$ to be γ, termed the *adiabatic constant*.

$$-\gamma\ln(\mathrm{V_2/V_1}) = \ln(\mathrm{p_2/p_1})$$

$$\ln\left(\mathrm{V_2/V_1}\right)^{-\gamma} = \ln(\mathrm{p_2/p_1})$$

“Applying” the exponential function to both sides

$$\exp\left[\ln\left(\mathrm{V_2/V_1}\right)^{-\gamma}\right] = \exp[\ln(\mathrm{p_2/p_1})]$$

yields

$$\left(\mathrm{V_2/V_1}\right)^{-\gamma} = \mathrm{p_2/p_1}$$

$$\left(\mathrm{V_1/V_2}\right)^{\gamma} = \mathrm{p_2/p_1}$$

$$\mathrm{p_1\ V_1^{\gamma} = p_2\ V_2^{\gamma}}$$

We now have a second relationship between the state variables of a gas, valid only for an adiabatic process. Using this relationship (with $\gamma = \frac{5}{3}$),

$$\mathrm{p}_1 \mathrm{~V}_1^\gamma = \mathrm{p}_2 \mathrm{~V}_2^\gamma$$

$$\left(1.81 \times 10^5 \mathrm{~Pa}\right)\left(2 \times 10^{-4} \mathrm{~m}^3\right)^\gamma = \left(1.01 \times 10^5\right) \mathrm{V}_2^\gamma$$

$$0.124 = \left(1.01 \times 10^5\right) \mathrm{V}_2^\gamma$$

$$1.23 \times 10^{-6} = \mathrm{V}_2^\gamma$$

$$\mathrm{V}_2 = \left(1.23 \times 10^{-6}\right)^{1/\gamma}$$

$$\mathrm{V}_2 = 2.84 \times 10^{-4} \mathrm{~m}^3$$

Now, by re-examining the second state:

$$\mathrm{p}_2 \mathrm{~V}_2 = \mathrm{nRT}_2$$

$$\left(1.01 \times 10^5\right)\left(2.84 \times 10^{-4} \mathrm{~m}^3\right) = (0.0159)(8.31 \mathrm{~J}/\mathrm{K})\mathrm{T}_2$$

$$\mathrm{T}_2 = 217 \mathrm{~K}$$

and from the First Law,

$$\mathrm{W} = \frac{3}{2}(0.0159)(8.31 \mathrm{~J}/\mathrm{K})(217 \mathrm{~K} - 273 \mathrm{~K})$$

$$\mathrm{W} = -11.1 \mathrm{~J}$$

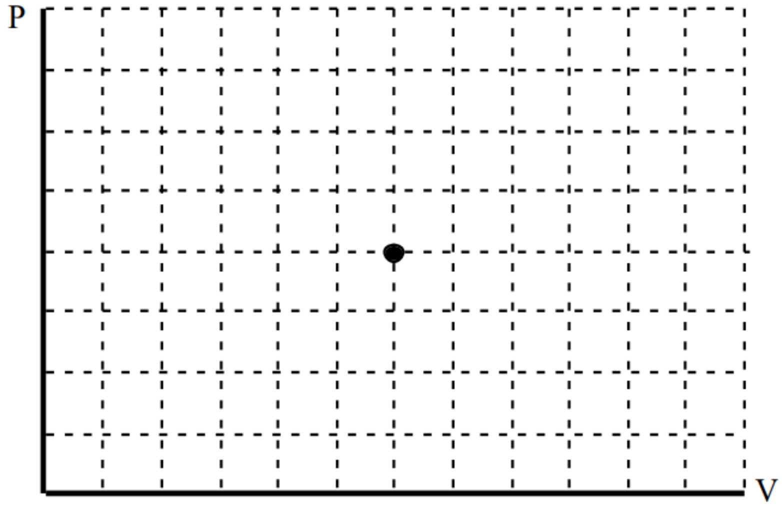
During the abrupt removal of the applied force, 11.1 J of energy was transferred out of the gas via the work done by the expanding gas.

Other Processes

The preceding examples are not meant to imply that all (or even most) thermodynamic processes that occur in nature fall into one of the 4 categories examined above. However, at this level of inquiry, only processes in which one of the thermodynamic state variables is held constant, or the process occurs adiabatically, will be investigated in numerical detail.

1.2: Activities

a. Imagine a piston containing a sample of ideal gas. Assume the piston head is perfectly free to move, unless locked in place, and the walls of the piston readily allow the transfer of energy via heat, unless wrapped in insulation. The gas is at the pressure and volume indicated on the graph below and is in equilibrium with a large thermal reservoir at room temperature. Draw a curve, labeled with the appropriate number, to represent each of the following actions. After each action the piston is reset to its initial equilibrium state.

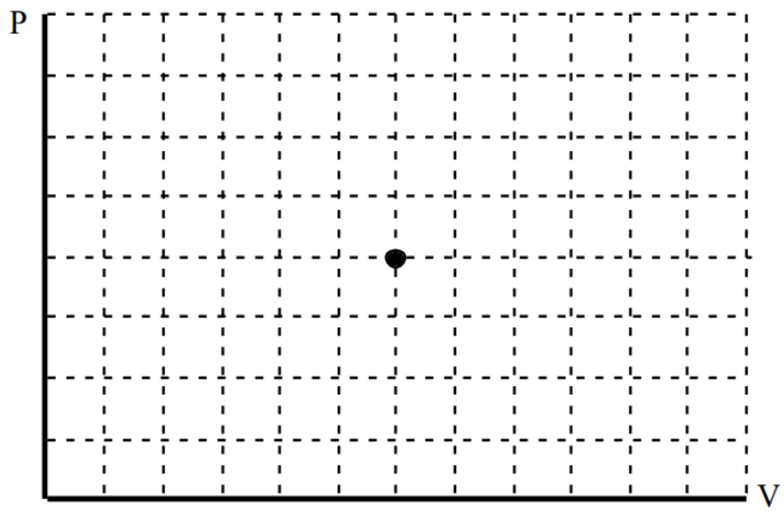


1. Lock the piston head in place. Hold the piston above a very hot flame.
2. Slowly push the piston down.
3. Pull the piston up abruptly.
4. Plunge the piston into very cold water.
5. Wrap the piston in insulation. Slowly pull the piston up.

b. For each of the actions, state whether the energy transferred to the gas via work, the energy transferred to the gas via heat, and the change in the internal energy of the gas are positive, negative, or zero.

Action	W	Q	ΔU
1			
2			
3			
4			
5			

a. Imagine a piston containing a sample of ideal gas. Assume the piston head is perfectly free to move, unless locked in place, and the walls of the piston readily allow the transfer of energy via heat, unless wrapped in insulation. The gas is at the temperature and volume indicated on the graph below and is in equilibrium with a large thermal reservoir at room temperature. Draw a curve, labeled with the appropriate number, to represent each of the following actions. After each action the piston is reset to its initial equilibrium state.

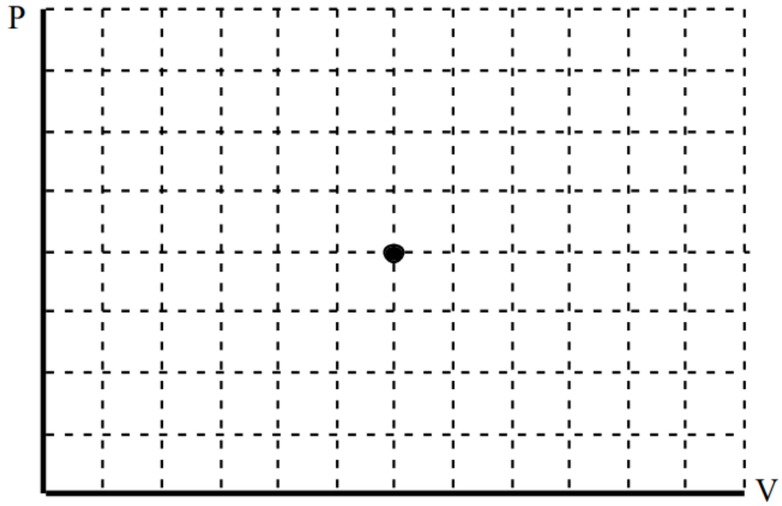


1. Hold the piston above a very hot flame.
2. Push the piston down abruptly.
3. Wrap the piston in insulation. Slowly push the piston down.
4. Lock the piston head in place. Plunge the piston into very cold water.
5. Slowly pull the piston up.

b. For each of the actions, state whether the energy transferred to the gas via work, the energy transferred to the gas via heat, and the change in the internal energy of the gas are positive, negative, or zero.

Action	W	Q	ΔU
1			
2			
3			
4			
5			

a. Imagine a piston containing a sample of ideal gas. Assume the piston head is perfectly free to move, unless locked in place, and the walls of the piston readily allow the transfer of energy via heat, unless wrapped in insulation. The gas is at the temperature and pressure indicated on the graph below and is in equilibrium with a large thermal reservoir at room temperature. Draw a curve, labeled with the appropriate number, to represent each of the following actions. After each action the piston is reset to its initial equilibrium state.

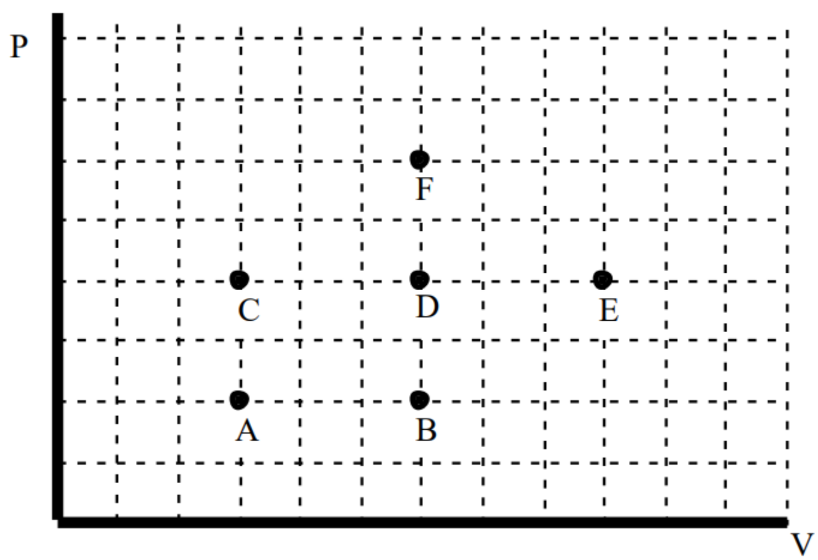


1. Hold the piston above a very hot flame.
2. Push the piston down abruptly.
3. Wrap the piston in insulation. Slowly push the piston down.
4. Lock the piston head in place. Plunge the piston into very cold water.
5. Slowly pull the piston up.

b. For each of the actions, state whether the energy transferred to the gas via work, the energy transferred to the gas via heat, and the change in the internal energy of the gas are positive, negative, or zero.

Action	W	Q	ΔU
1			
2			
3			
4			
5			

Below are representations of six thermodynamic states of the same ideal gas sample.



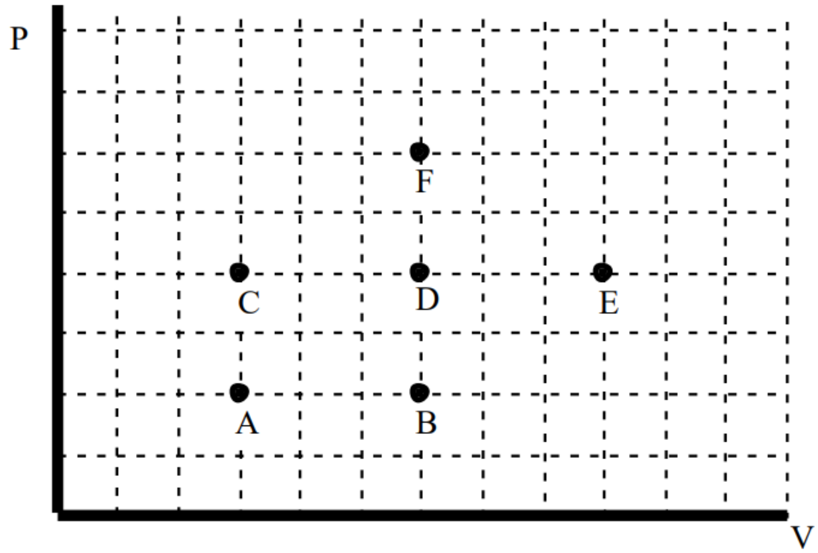
Rank the states on the basis of the temperature of the gas sample at each state.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are representations of six thermodynamic states of the same ideal gas sample.



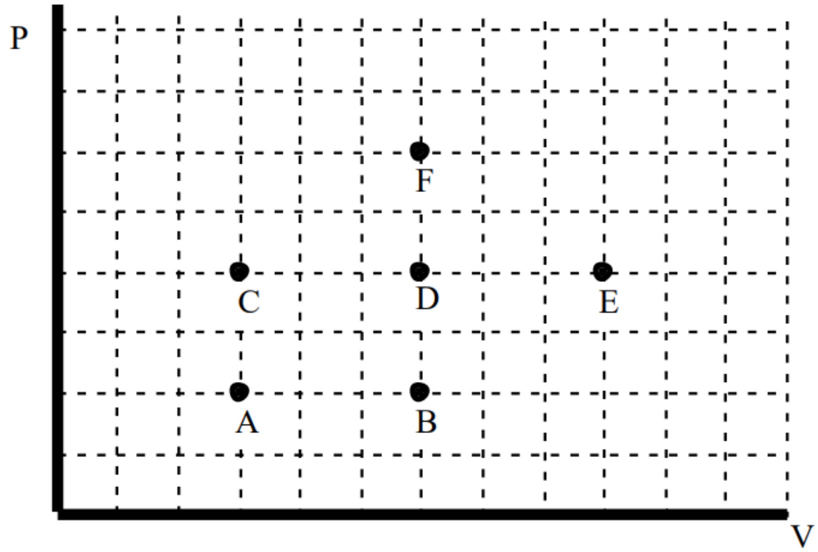
Rank the states on the basis of the volume of the gas sample at each state.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are representations of six thermodynamic states of the same ideal gas sample.



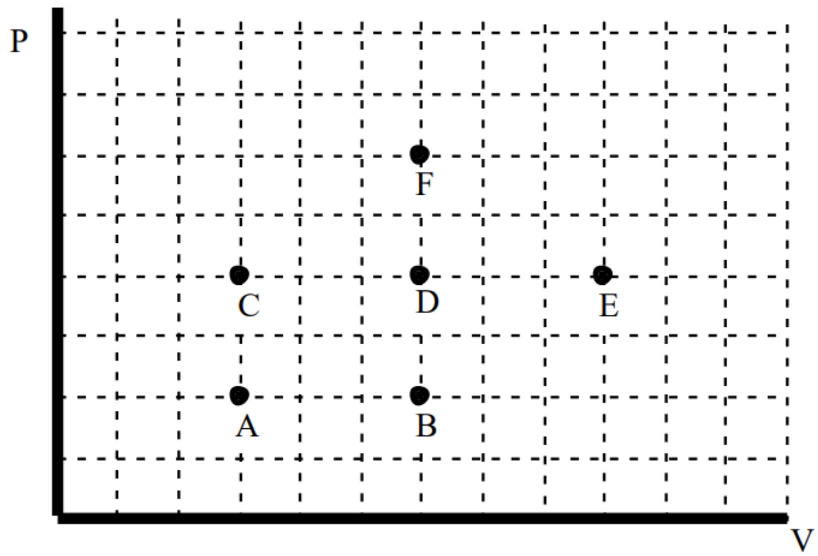
Rank the states on the basis of the average kinetic energy of the atoms in the gas sample at each state.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are representations of six thermodynamic processes undertaken with the same ideal gas sample starting from the same initial state. Letters near each end-state label each process.



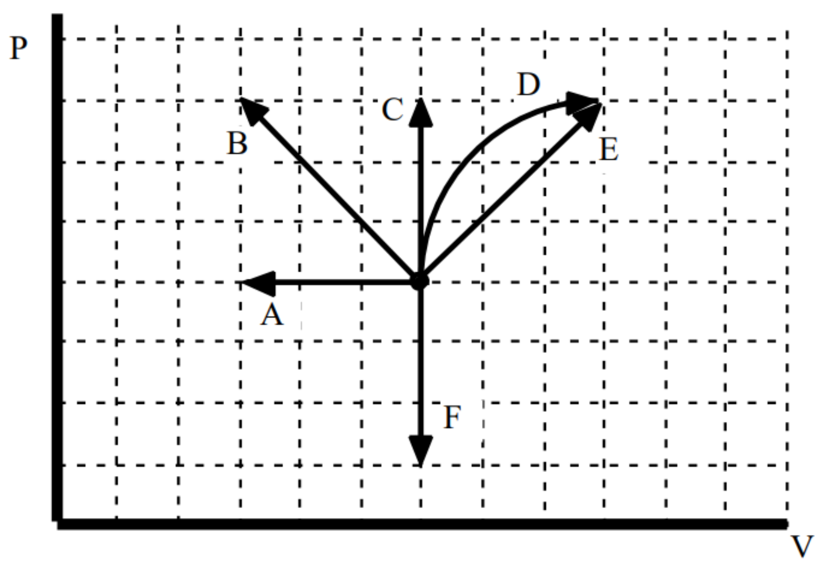
Rank the states on the basis of the average kinetic energy of the atoms in the gas sample at each state.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are representations of six thermodynamic processes undertaken with the same ideal gas sample starting from the same initial state. Letters near each end-state label each process.



a. Rank the processes on the basis of the amount of energy transferred to the gas via work during each process.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

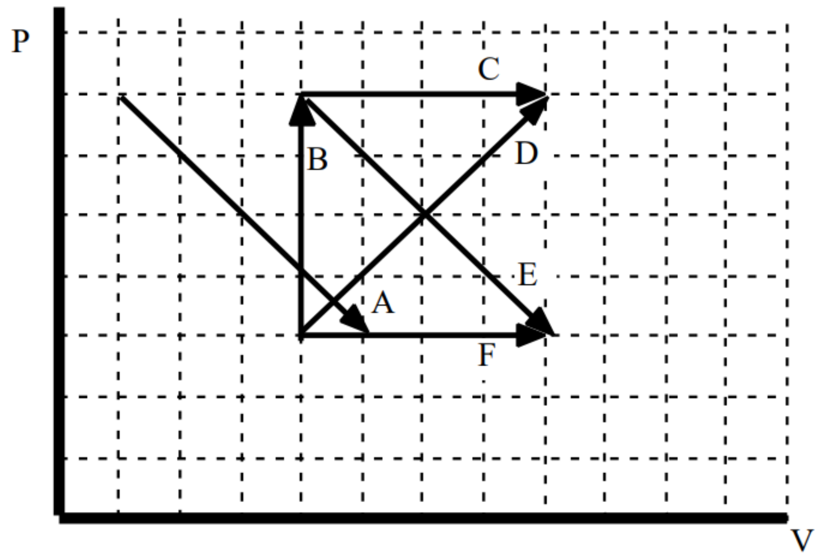
b. Rank the processes on the basis of the change in internal energy of the gas sample.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are representations of six thermodynamic processes undertaken with the same ideal gas sample. Letters near each end-state label each process.



a. Rank the processes on the basis of the amount of energy transferred to the gas via work during each process.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

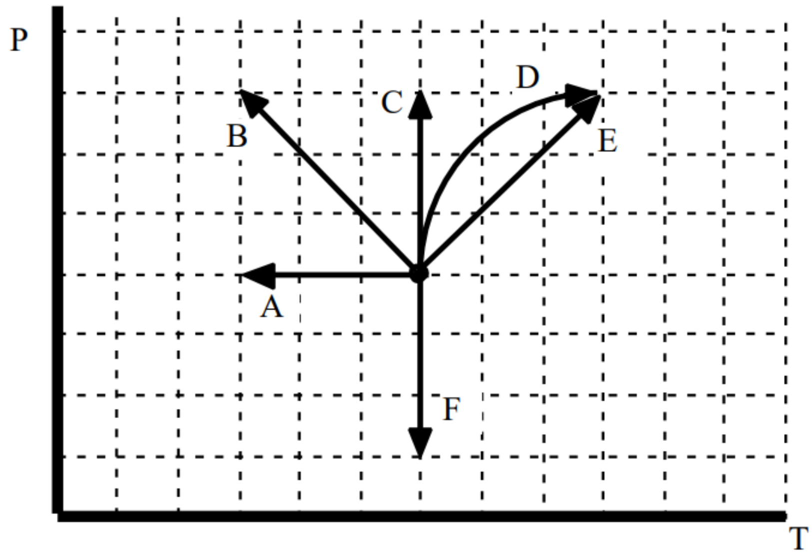
b. Rank the processes on the basis of the change in internal energy of the gas sample.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are representations of six thermodynamic processes undertaken with the same ideal gas sample starting from the same initial state. Letters near each end-state label each process.

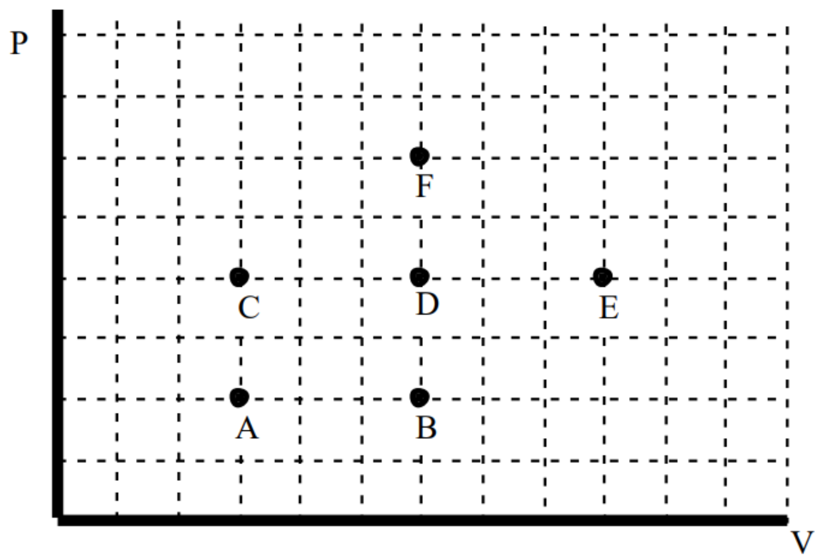


Rank the processes on the basis of the change in internal energy of the gas sample.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:



An ideal gas sample is processed between equilibrium states represented by the letters A through F on the pressure vs. volume graph above. All processes connect states via straight line segments.

Compare the energy transferred to the gas via work. Circle the correct comparison symbol.

a. **(A to B)** > = < ? **(B to A)**

Explanation:

b. **(C to B)** > = < ? **(B to E)**

Explanation:

c. **(A to B to D to C to A)** > = < ? **(C to B to D to F to C)**

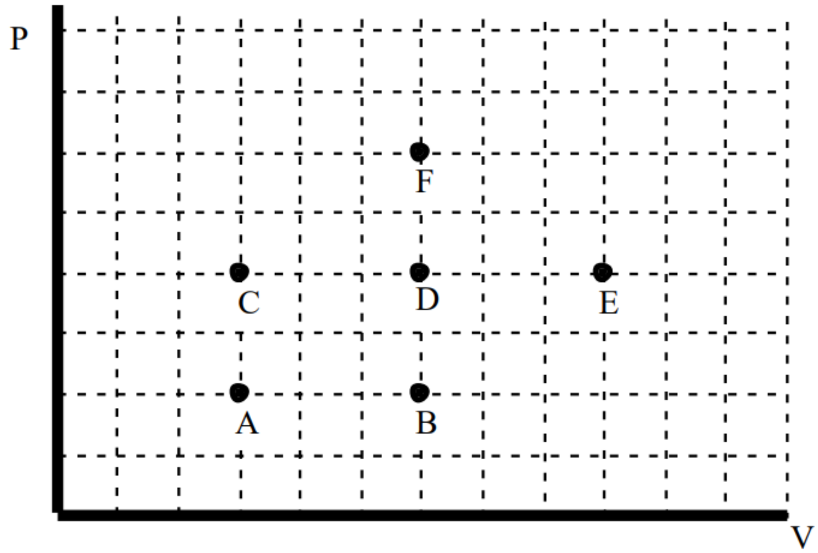
Explanation:

d. **(B to A to C to B)** > = < ? **(B to C to A to B)**

Explanation:

e. **(E to F to D to E)** > = < ? **(B to C to D to B)**

Explanation:



An ideal gas sample is processed between equilibrium states represented by the letters A through F on the pressure vs. volume graph above. All processes connect states via straight line segments.

Compare the energy transferred to the gas via heat. Circle the correct comparison symbol.

a. **(A to B)** > = < ? **(B to A)**

Explanation:

b. **(A to B)** > = < ? **(A to C)**

Explanation:

c. **(A to E)** > = < ? **(A to F)**

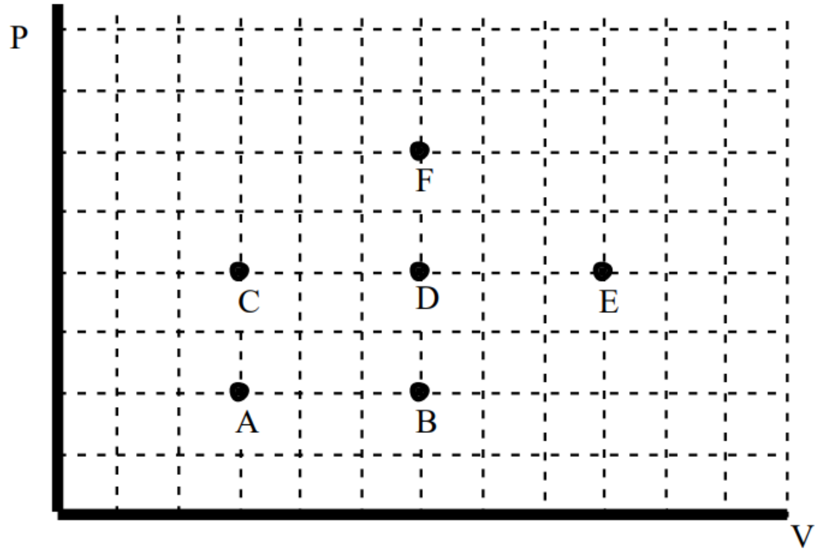
Explanation:

d. **(A to B to D to C to A)** > = < ? **(C to B to D to F to C)**

Explanation:

e. **(B to A to C to B)** > = < ? **(B to C to A to B)**

Explanation:



An ideal gas sample is processed between equilibrium states represented by the letters A through F on the pressure vs. volume graph above. All processes connect states via straight line segments.

Compare the change in internal energy of the gas. Circle the correct comparison symbol.

a. **(A to B)** > = < ? **(B to A)**

Explanation:

b. **(A to D)** > = < ? **(B to E)**

Explanation:

c. **(A to B to D to C to A)** > = < ? **(A to B to C to A)**

Explanation:

d. **(B to A to C to B)** > = < ? **(B to C to A to B)**

Explanation:

e. **(E to F to D to E)** > = < ? **(B to C to A to B)**

Explanation:

What, if anything, is wrong with the following answers? Correct all the mistakes that you find.

1. Can heat be transferred to an ideal gas sample and the sample's temperature remain constant? If so, describe how this could be done.

No. Temperature is a measure of the amount of heat that has been added to a gas. Therefore, if you transfer additional heat to a gas, the temperature of the gas must increase.

2. Must the pressure always increase when the volume of an ideal gas sample is decreased? Why?

Yes. When a gas sample is compressed, the molecules of the sample now must occupy a smaller volume. Thus, they will collide with the walls of the container more often. More collisions mean a higher pressure.

3. Can work be done on an ideal gas sample and the sample's volume remain constant? If so, describe how this could be done.

Normally, when work is done on a gas sample, the gas volume decreases. However, when heat is normally transferred to a gas sample, the gas volume increases. Therefore, it is possible to both do work on the gas, and transfer heat to the gas, in the proper ratio, such that the volume remains constant.

4. Can the volume of an ideal gas sample decrease even though the pressure and temperature remain constant? If so, describe how this could be done.

No. The equation of state of an ideal gas (pV = nRT) clearly indicates that if p and T remain constant, so must V, assuming no gas has leaked out of the sample.

> A 5 cubic meter weather balloon is filled with helium at the surface of the earth on a summer (28° C) day. When the balloon reaches an altitude of 6 km, the air pressure has dropped to about half of its original value and the temperature is now - 50° C. Assume the balloon has minimal resistance to expansion.

Thermodynamic Information

Event 1:	Event 2:
p_1 = V_1 = T_1 =	p_2 = V_2 = T_2 =
Process: W_{12} = Q_{12} =	

Mathematical Analysis

> A robotic airplane takes off from the surface of the earth on a summer (26° C) day. When the airplane reaches an altitude of 6 km, the temperature outside has dropped to - 50° C. Assume the airplane was sealed shut at the earth's surface and is made of strong enough material such that its volume does not change.

Thermodynamic Information

Event 1:	Event 2:
p_1 = V_1 = T_1 =	p_2 = V_2 = T_2 =
Process: W_{12} = Q_{12} =	

Mathematical Analysis

> Upon leaving your house on a cold (0° C) morning, you check your tire pressure. The gauge reads 32 lb/in2 (2.2 x 10^5 Pa). Upon arriving at school, the gauge reads 35 lb/in2 . Assume the increase in the volume of the tire is small enough to be ignored. Note that the gauge displays the amount by which the tire pressure exceeds atmospheric pressure.

Thermodynamic Information

Event 1:	Event 2:
p_1 = V_1 = T_1 =	p_2 = V_2 = T_2 =
Process: W_{12} = Q_{12} =	

Mathematical Analysis

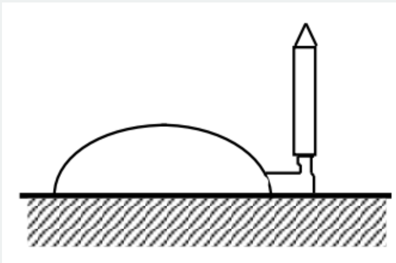
Upon leaving your house on a cold (0° C) morning, you check your tire pressure. The gauge reads 32 lb/in^2 (2.2 x 10^5 Pa). Upon arriving at school, the gauge reads 35 lb/in^2. Assume the volume of the tire increased by approximately 5% during the drive. Note that the gauge displays the amount by which the tire pressure exceeds atmospheric pressure.

Thermodynamic Information

Event 1:	Event 2:
p_1 = V_1 = T_1 =	p_2 = V_2 = T_2 =
Process: W_{12} = Q_{12} =	

Mathematical Analysis

A stomp rocket is a device in which a sample of air trapped in a heavy-duty, hemispherical balloon is very quickly compressed (by stomping on it) causing an abrupt increase in pressure. This increased pressure launches a toy rocket attached to the gas sample by a nozzle. Assume the gas sample is initially at 20° C and 1.0 atm and the volume of the gas sample is decreased by 50% during the stomp.



Thermodynamic Information

Event 1:	Event 2:
p_1 = V_1 = T_1 =	p_2 = V_2 = T_2 =
Process: W_{12} = Q_{12} =	

Mathematical Analysis

A bubble of volume V is formed at the bottom of the ocean and rises to the top. The pressure at the bottom of the ocean is approximately 6 x 10^7 Pa and the temperature is 3° C. The temperature of the ocean's surface is approximately 12° C.

Thermodynamic Information

Event 1:	Event 2:
p_1 = V_1 = T_1 =	p_2 = V_2 = T_2 =

Event 1:	Event 2:
Process: W_{12} = Q_{12} =	

Mathematical Analysis

> After diving, a diver has a small bubble of oxygen of volume V trapped in his bloodstream. As his heart beats, the bubble's size varies. His blood pressure is approximately 120 / 80, where '120' is measured in mm of Hg, a unit used to measure pressure. 120 mm of Hg corresponds to a pressure of 1.6 x 10^4 Pa above atmospheric pressure.

Thermodynamic Information

Event 1:	Event 2:
p_1 = V_1 = T_1 =	p_2 = V_2 = T_2 =
Process: W_{12} = Q_{12} =	

Mathematical Analysis

> The pressure in a fluid as a function of depth beneath the surface (for fluids at a uniform temperature) is $p = p_o + \rho g d$, where p_o is the pressure at the surface, ρ is the density of the fluid, and d is the distance below the surface. A bubble of volume V is formed at the bottom of a 0.9 m high 'yard of beer' and rises to the top. The density of beer is approximately 1.0 x 10^3 kg/m^3.

Thermodynamic Information

Event 1:	Event 2:
p_1 = V_1 = T_1 =	p_2 = V_2 = T_2 =
Process: W_{12} = Q_{12} =	

Mathematical Analysis

> In an automobile engine, an air-fuel mixture initially at atmospheric pressure and 85° C is quickly compressed by a factor of four before ignition. Assume the mixture behaves like an ideal gas.

Thermodynamic Information

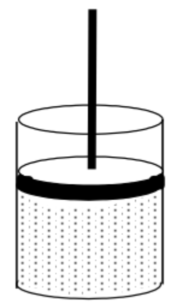
Event 1:	Event 2:
p_1 = V_1 = T_1 =	p_2 = V_2 = T_2 =

Event 1:	Event 2:
Process: W_{12} = Q_{12} =	

Mathematical Analysis

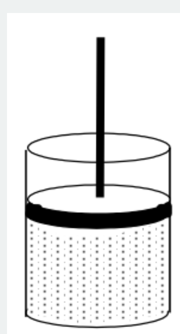
> An amateur inventor claims that the following process will “remove heat” from the surroundings of the piston, a water bath at 80° C. Initially, the volume of the gas in the piston is 200 cm^3 and the pressure exerted on the piston head is 1.5 atm above atmospheric pressure. The external pressure is quickly removed, leaving only atmospheric pressure on the piston head. Once the gas stops its rapid expansion, the piston head is locked in place and the sample is allowed to return to thermal equilibrium with the water bath.

Thermodynamic Information



Event 1:	Event 2:	Event 3:	Event 4:	Event 5:
p_1 = V_1 = T_1 =	p_2 = V_2 = T_2 =	p_3 = V_3 = T_3 =	p_4 = V_4 = T_4 =	p_5 = V_5 = T_5 =
Process: W_{12} = Q_{12} =	Process: W_{23} = Q_{23} =	Process: W34 = Q34 =	Process: W45 = Q45 =	

Mathematical Analysis

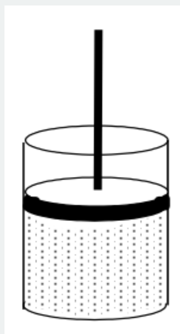
> An amateur inventor claims that the following process will “remove heat” from the surroundings of the piston, a water bath at 80° C. Initially, the volume of the gas in the piston is 200 cm^3 and the pressure exerted on the piston head is 1.5 atm above atmospheric pressure. The external pressure is quickly removed, leaving only atmospheric pressure on the piston head. Once the gas stops its rapid expansion, the piston head is locked in place and the sample is allowed to return to thermal equilibrium with the water bath. Then, the external pressure is slowly increased until the sample returns to its original volume.
>
> 

Thermodynamic Information

Event 1:	Event 2:	Event 3:	Event 4:	Event 5:
p_1 = V_1 = T_1 =	p_2 = V_2 = T_2 =	p_3 = V_3 = T_3 =	p_4 = V_4 = T_4 =	p_5 = V_5 = T_5 =
Process: W_{12} = Q_{12} =	Process: W_{23} = Q_{23} =	Process: W34 = Q34 =	Process: W45 = Q45 =	

Mathematical Analysis

An amateur inventor is unsure what effect the following process will have on the surroundings of the piston, a water bath at 80° C. Initially, the volume of the gas in the piston is 200 cm^3 and the pressure exerted on the piston head is 1.5 atm above atmospheric pressure. The external pressure is very slowly removed, until only atmospheric pressure remains on the piston head. Once the gas stops its expansion, an additional external pressure is quickly applied to the piston head until the sample returns to its original volume. Then, the piston head is locked in place and the sample is allowed to return to thermal equilibrium with the water bath.

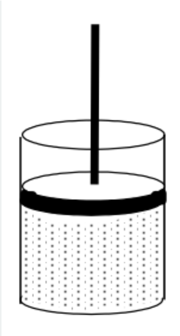


Thermodynamic Information

Event 1:	Event 2:	Event 3:	Event 4:	Event 5:
p_1 = V_1 = T_1 =	p_2 = V_2 = T_2 =	p_3 = V_3 = T_3 =	p_4 = V_4 = T_4 =	p_5 = V_5 = T_5 =
Process: W_{12} = Q_{12} =	Process: W_{23} = Q_{23} =	Process: W34 = Q34 =	Process: W45 = Q45 =	

Mathematical Analysis

An amateur inventor has no idea what he's doing. He starts with 400 cm^3 of gas in the piston at atmospheric pressure, with the piston in an ice water bath. After locking the piston head in place, he then plunges the piston into a hot water bath (90° C). Once thermal equilibrium is reached, he unlocks the piston head and allows the gas sample to slowly expand. Once the expansion is complete, he returns the sample to the ice water bath (without locking the piston head).

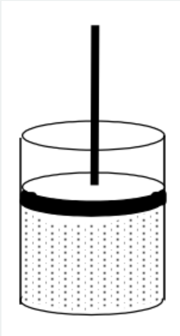


Thermodynamic Information

Event 1:	Event 2:	Event 3:	Event 4:	Event 5:
p_1 = V_1 = T_1 =	p_2 = V_2 = T_2 =	p_3 = V_3 = T_3 =	p_4 = V_4 = T_4 =	p_5 = V_5 = T_5 =
Process: W_{12} = Q_{12} =	Process: W_{23} = Q_{23} =	Process: W34 = Q34 =	Process: W45 = Q45 =	

Mathematical Analysis

An amateur inventor has no idea what she's doing. She starts with 400 cm^3 of gas in the piston at atmospheric pressure, with the piston in an ice water bath. Attaching a vacuum pump, she slowly reduces the pressure on the piston head to 0.3 atm. After locking the piston head in place, she removes the vacuum pump. She then places the piston in a flame until the pressure of the gas is once again 1.0 atm. Once this is complete, she unlocks the piston head and she returns the sample to the ice water bath.

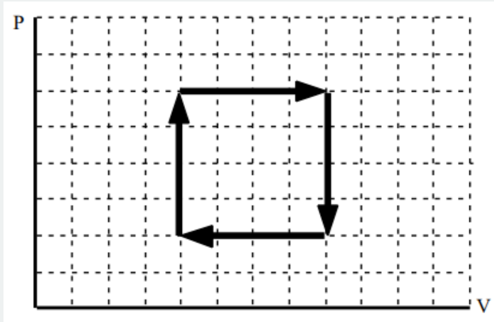


Thermodynamic Information

Event 1:	Event 2:	Event 3:	Event 4:	Event 5:
p_1 = V_1 = T_1 =	p_2 = V_2 = T_2 =	p_3 = V_3 = T_3 =	p_4 = V_4 = T_4 =	p_5 = V_5 = T_5 =
Process: W_{12} = Q_{12} =	Process: W_{23} = Q_{23} =	Process: W34 = Q34 =	Process: W45 = Q45 =	

Mathematical Analysis

In a certain industrial process, thirty moles of gas are processed through the cycle at right. The gas begins the cycle at a pressure of 1.5 atm and temperature 300 K. The first step of the cycle is an expansion to twice the initial volume. The minimum temperature of the gas during the cycle is 180 K. The graph is not to scale.

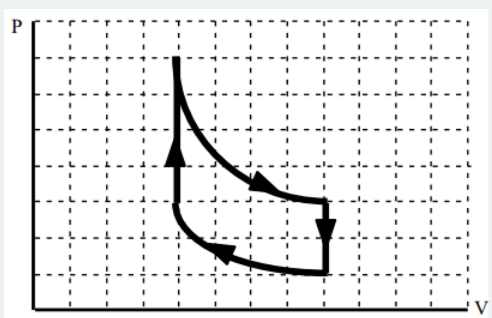


Thermodynamic Information

Event 1:	Event 2:	Event 3:	Event 4:	Event 5:
p_1 = V_1 = T_1 =	p_2 = V_2 = T_2 =	p_3 = V_3 = T_3 =	p_4 = V_4 = T_4 =	p_5 = V_5 = T_5 =
Process: W_{12} = Q_{12} =	Process: W_{23} = Q_{23} =	Process: W34 = Q34 =	Process: W45 = Q45 =	

Mathematical Analysis

In a certain industrial process, a gas sample is processed through the cycle at right. The gas starts at a pressure of 2 atm, volume of 0.6 m^3 and a temperature of 450 K. The gas expands isothermally (at constant temperature) to twice its original volume. The pressure is then decreased at constant volume. The gas is then isothermally compressed to its original volume and a pressure of 1.7 atm. The gas is then returned to its initial state. The graph is not to scale.



Thermodynamic Information

Event 1:	Event 2:	Event 3:	Event 4:	Event 5:
p_1 = V_1 = T_1 =	p_2 = V_2 = T_2 =	p_3 = V_3 = T_3 =	p_4 = V_4 = T_4 =	p_5 = V_5 = T_5 =
Process: W_{12} = Q_{12} =	Process: W_{23} = Q_{23} =	Process: W34 = Q34 =	Process: W45 = Q45 =	

CHAPTER OVERVIEW

2: Model 1 - The One-Dimensional, Constant-Force, Particle Model

Thumbnail: A JR Central L0 series five-car maglev (magnetic levitation) train undergoing a test run on the Yamanashi Test Track. The maglev train’s motion can be described using kinematics, the subject of this chapter. (credit: modification of work by “Maryland GovPics”/Flickr)

2.0: Introduction

Model building in physics

Physicists build models.

Physicists then explore these models in the hopes of gaining insight into how the actual physical world operates.

Therefore, what *you* will do throughout this course is build models, of increasing complexity, of the real world and by closely examining these models *you* may gain insight into how the world operates. Some of the early models you will examine will be obviously limited, but keep in mind that even the most advanced physicists are merely model-builders, and the models they typically study are as superficial to them as the models we will study are to us.

Model building is necessary because of the overwhelming complexity of the real world. To attempt to study a real phenomenon, with all of its many details intact, is extremely difficult. Moreover, models often allow you to focus on the important aspects of a phenomenon without the distracting details.

For example, a model of reality that everyone is familiar with is a map. Imagine if Google Maps showed every driveway and alleyway regardless of your level of zoom! Although these details exist, a model that tried to encompass all of these details at all times would be *less* useful than one in which everyone's driveway was omitted. In fact it would be an unreadable mess. *Thus, it is possible to omit detail, to be a poorer reflection of reality, yet to be a better, more effective, and more useful model*. The correct "zoom" level for driving across town ignores driveways but includes most, if not all, streets. However if your task was to drive across the state, not only are the driveways be omitted but so are the vast majority of side streets; probably only state and federal highways are included on the map. Thus, a good model is closely tied to the task at hand. What can be a very useful model for one task can be useless for another.

Thus, when we build models where the effects of friction are neglected, or the shape of an object is ignored, it is not the case that this is a deficient model of the situation. It may well be the case that if these details were included some important features of the scenario would be masked by the complexity. Simplifications made in constructing models of reality are not always limitations to the usefulness of the model, often they are the key to building a useful and productive model.

Units

In this course we will exclusively use the *International System of Units*. In this system, all times are measured in seconds (s), all positions in meters (m), and all masses in kilograms (kg). For the sake of clarity, I will not include units during every step of a calculation, but it can safely be assumed that these standard units are in use throughout all calculations.

Active reading

If you do not currently have a pen or pencil in your hand, pick one up.

As you read this text, you should be writing down explanatory notes to yourself, questions to be asked in class, and any flashes of insight you may have. Don't be afraid to write in the text. I promise that actively wrestling with the ideas on the page rather than passively reading the words on the page will make a huge difference in your understanding. If you find yourself reading page after page of the text without spontaneously thinking of questions, either you are not really digesting the material or you should be in a more advanced class.

In addition, the concepts and principles of physics are complex, even the ones that appear to be simple. (If the principles really were simple, it would not have taken humankind thousands of years to understand the motion of a simple falling object!) They will become clear to you only after careful study. With this in mind, the text is not meant to be read (and written in) once. It should be re-read (and re-written in!) as you work through the various activities included. Hopefully, as you complete the activities the concepts will come into better focus.

2.1: Model Specifics

For our first pass through the study of mechanics, we will make a set of simplifying assumptions leading to a model of reality termed the one-dimensional, constant-force, particle model. We will assume that:

The object moves along a perfectly straight line

Thus, if we are studying the motion of a car we will assume that the road is perfectly smooth with no bumps, the road is perfectly straight with no curves, and the driver is perfectly steady at the wheel with no swaying to the right or left.

The object is acted on by constant forces

For example, when a basketball bounces off the ground the force of the ground on the basketball is:

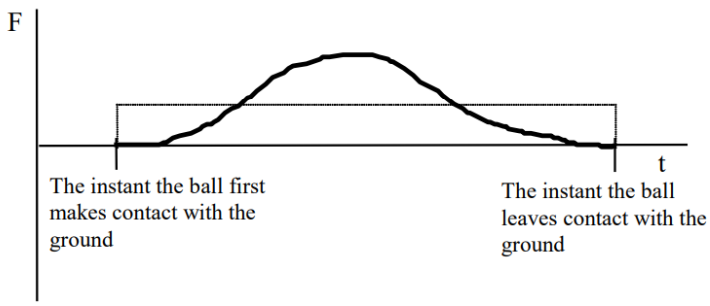
- zero before the basketball strikes the ground,
- non-zero and increasing in magnitude as the basketball deforms in its contact with the ground,
- non-zero and decreasing in magnitude as the basketball springs back into shape while still in contact with the ground,
- and once again zero as the ball leaves contact with the ground.

In reality, the force between the basketball and the ground is quite complicated.

However, in our model we will approximate the force on the basketball due to the ground as:

- zero before the basketball strikes the ground,
- non-zero and constant in magnitude the entire time the ball is in contact with the ground,
- and once again zero as the ball leaves contact with the ground.

The difference between reality and our model is illustrated by the graph below. The dark line represents the actual force between the ground and the ball, the dashed line represents how we will approximate the force in our current model.



In addition, you will soon learn that the total force acting on an object is proportional to its *acceleration*. Thus, in this model, we will approximate all accelerations as constant.

The object's size and shape are unimportant

In physics, the word particle refers to a hypothetical entity having no size and therefore no shape. The statement that objects will be thought of as particles simply means that we will ignore any effects their actual size and shape could have on the scenarios we investigate.

The object is classical

For this entire course, we will assume the objects under investigation are much larger than an atom and move much slower than the speed of light.

2.2: Kinematics

Concepts and Principles

Kinematics is the formal language physicists use to describe motion. The need for a formal language is evidenced by a simple experiment: drop an object from about shoulder height and ask two people to independently describe the motion of the object. Chances are that the descriptions will not be in perfect agreement, even though both observers described the same motion. Obviously, a more formal way of describing motion is necessary to eliminate this type of descriptive ambiguity. Kinematics is the formal method of describing motion.

Three parameters are carefully defined and used by physicists to describe motion. Specifying these three parameters at all times forms a complete description of the motion of an object.

Position

The position of an object is its location relative to a well-defined coordinate system at a particular instant of time. Without a specified coordinate system, position is a meaningless concept. A coordinate system is comprised of a *zero*, a specified *positive direction*, and a *scale*.

For example, in the hypothetical experiment in which the object was dropped from shoulder height, a coordinate system could have been defined in which the zero position was at ground level, the positive direction was up, and the scale used was meters. Using this coordinate system, the position of the object could have been specified at any particular instant of time. Of course, choosing the zero at the location at which the object was dropped, the positive direction as down, and the scale in feet is also perfectly acceptable. It doesn't matter what you choose as a coordinate system, only that you explicitly choose one. Depending on the coordinate system chosen, the position of an object can be positive, negative, or zero.

We will use the symbol r to designate position, and measure it in meters (m).

Velocity

Although the word velocity is often used loosely in everyday conversation, its meaning in physics is specific and well-defined. To physicists, the velocity is the rate at which the position is changing. The velocity can be specified at any particular instant of time.

For example, if the position is changing quickly the velocity is large and if the position is not changing at all the velocity is zero. A mathematical way to represent this definition is

$$v=\frac{\Delta r}{\Delta t}=\frac{r_{\text{final }}-r_{\text{initial }}}{t_{\text{final }}-t_{\text{initial }}}$$

Thus, velocity is measured in meters per second (m/s).

Actually, this is the definition of the *average* velocity of the object over the time interval Δt, but as the time interval becomes smaller and smaller the value of this expression becomes closer and closer to the actual rate at which the position is changing at one particular instant of time.

Since the final position of the object (r_{final}) may be either positive, negative, or zero, and either larger, smaller, or the same as the initial position ($r_{initial}$), the velocity may be positive, negative, or zero. The sign of the velocity depends on the coordinate system chosen to define the position. A positive velocity simply means that the object is moving in the positive direction, as defined by the coordinate system, while a negative velocity means the object is traveling in the other direction.

Acceleration

Again, although the word acceleration is often used loosely in everyday conversation, its meaning in physics is specific and well-defined. To physicists, the acceleration is the rate at which the velocity is changing. Again, the acceleration can be specified at any particular instant of time.

For example, if the velocity is changing quickly the acceleration is large in magnitude, and if the velocity is not changing the acceleration is zero. If an object has non-zero acceleration, it does *not* mean that the object is speeding up. It simply means that the velocity is changing. Moreover, even if an object has a *positive* acceleration, it does not mean that the object is speeding up! A positive acceleration means that the change in the velocity points in the positive direction. (I can almost guarantee you will experience confusion about this. Take some time to think about the preceding statement right now.)

A mathematical way to represent acceleration is

$$a=\frac{\Delta v}{\Delta t}=\frac{v_{\text{final}}-v_{\text{initial}}}{t_{\text{final}}-t_{\text{initial}}}$$

Thus, acceleration is measured in meters per second per second (m/s^2).

Again, this is actually the average acceleration of the object over the time interval Δt, but as the time interval becomes smaller and smaller, the value of this expression becomes closer and closer to the actual rate at which the velocity is changing at one particular instant of time.

Since v_{final} may be either positive, negative, or zero, and either larger, smaller, or the same as $v_{initial}$, the acceleration may be positive, negative, or zero. The algebraic sign of the acceleration depends on the coordinate system chosen to define the position. A negative acceleration means that the change in the velocity points in the negative direction. For example, the velocity could be in the positive direction and the object slowing down *or* the velocity could be in the negative direction and the object speeding up. Both of these scenarios would result in a negative acceleration. Conversely, a positive acceleration means that the change in the velocity points in the positive direction.

Kinematics is the correct use of the parameters position, velocity, and acceleration to describe motion. Learning to use these three terms correctly can be made much easier by learning a few tricks of the trade. These tricks, or analysis tools, are detailed in the following section.

Analysis Tools

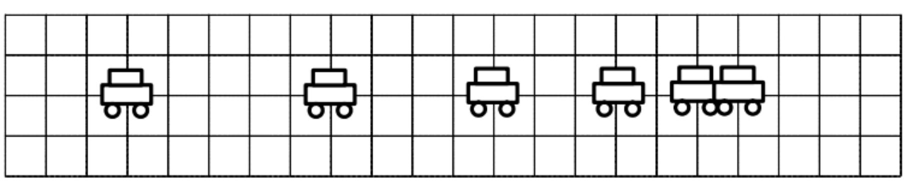
The words used by physicists to describe the motion of objects are defined above. However learning to use these terms correctly is more difficult than simply memorizing definitions. An extremely useful tool for bridging the gap between a normal, conversational description of a situation and a physicists' description is the motion diagram. A motion diagram is the first step in translating a verbal description of a phenomenon into a physicists' description.

Start with the following verbal description of a physical situation:

> *The driver of an automobile traveling at 15 m/s, noticing a red-light 30 m ahead, applies the brakes of her car until she stops just short of the intersection.*

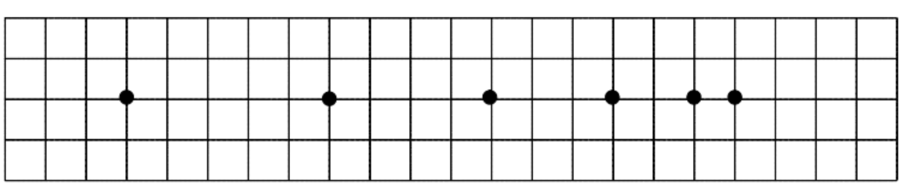
Determining the position from a motion diagram

A motion diagram can be thought of as a multiple-exposure photograph of the physical situation, with the image of the object displayed at equal time intervals. For example, a multiple-exposure photograph of the situation described above would look something like this:



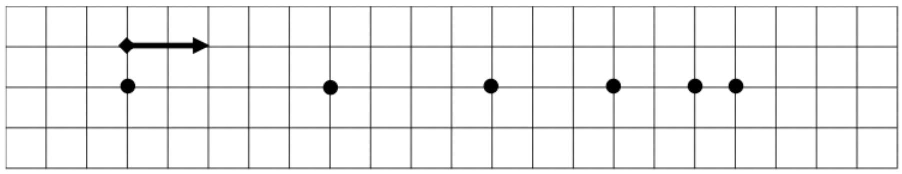
Note that the images of the automobile are getting closer together near the end of its motion because the car is traveling a smaller distance between the equally-timed exposures.

In general, in drawing motion diagrams it is better to represent the object as simply a dot, unless the actual shape of the object conveys some interesting information. Thus, a better motion diagram would be:



Since the purpose of the motion diagram is to help us describe the car's motion, a coordinate system is necessary. Remember, to define a coordinate system you must choose a zero, define a positive direction, and select a scale. We will always use meters as our position scale in this course, so you must only select a zero and a positive direction. Remember, there is no correct answer. Any coordinate system is as correct as any other.

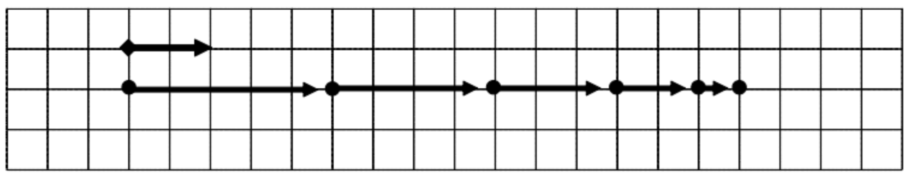
The choice below indicates that the initial position of the car is the origin, and positions to the right of that are positive. (In the text I'll always use a little diamond to indicate the zero with an arrow pointing in the positive direction.)



We can now describe the position of the car. The car starts at position zero and then has positive, increasing positions throughout the remainder of its motion.

Determining the velocity from a motion diagram

Since velocity is the change in position of the car during a corresponding time interval, and we are free to select the time interval as the time interval between exposures on our multiple exposure photograph, the velocity is simply the change in the position of the car "between dots." Thus, the arrows (vectors) on the motion diagram below represent the velocity of the car.



We can now describe the velocity of the car. Since the velocity vectors always point in the positive direction, the velocity is always positive. The car starts with a large, positive velocity which gradually declines until the velocity of the car is zero at the end of its motion.

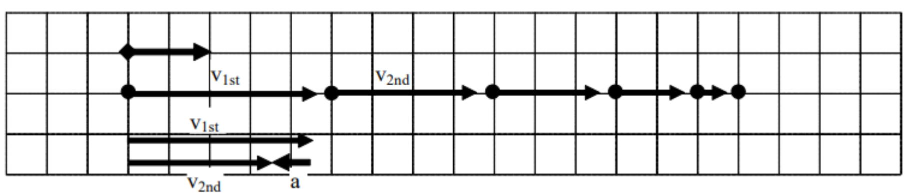
Determining the acceleration from a motion diagram

Since acceleration is the change in velocity of the car during a corresponding time interval, and we are free to select the time interval as the time interval between exposures on our multiple-exposure photograph, we can determine the acceleration by comparing two successive velocities. The change in these velocity vectors will represent the acceleration.

To determine the acceleration,

- select two successive velocity vectors,
- draw them starting from the same point,
- construct the vector (arrow) that connects the tip of the *first* velocity vector to the tip of the *second* velocity vector.
- The vector you have constructed represents the acceleration.

Comparing the first and second velocity vectors leads to the acceleration vector shown below:



Thus, the acceleration points to the left and is therefore negative. You could construct the acceleration vector at every point in time, but hopefully you can see that as long as the velocity vectors continue to point toward the right and decrease in magnitude, the acceleration will remain negative.

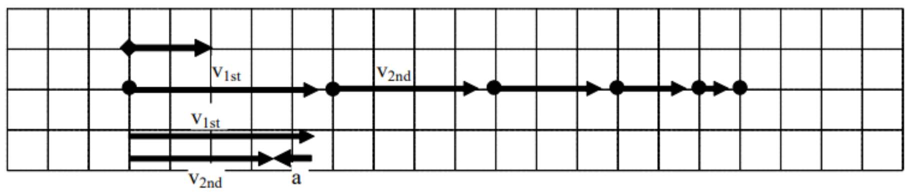
Thus, with the help of a motion diagram, you can extract lots of information about the position, velocity, and acceleration of an object. You are well on your way to a complete kinematic description.

Drawing Motion Graphs

Another useful way to describe the motion of an object is by constructing graphs of the object's position, velocity, and acceleration vs. time. A graphical representation is a very effective means of presenting information concerning an object's motion and, moreover, it is relatively easy to construct motion graphs if you have a correct motion diagram.

Examine the same situation as before:

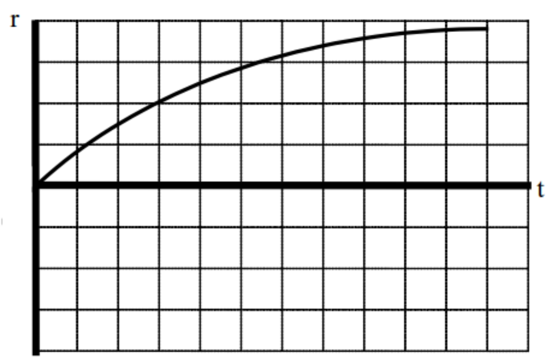
> *The driver of an automobile traveling at 15 m/s, noticing a red-light 30 m ahead, applies the brakes of her car until she stops just short of the intersection.*



The verbal representation of the situation has already been translated into a motion diagram. A careful reading of the motion diagram allows the construction of the motion graphs.

Drawing the position vs. time graph

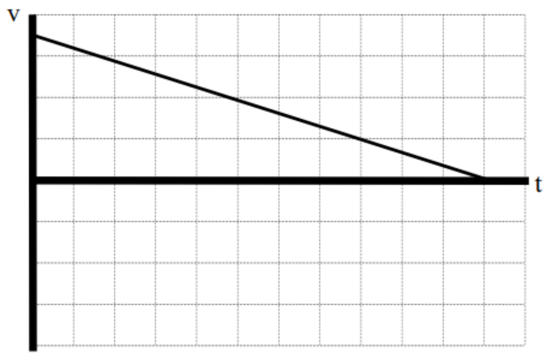
We already know, from the motion diagram, that the car starts at position zero, then has positive, increasing positions throughout the remainder of its motion. This information can be transferred onto a position vs. time graph.



Notice that the position is zero when the time is equal to zero, the position is always positive, and the position increases as time increases. Also note that in each subsequent second, the car changes its position by a smaller amount. This leads to the graph of position vs. time gradually decreasing in slope until it achieves a slope of zero. Once the car stops, the position of the car should not change.

Drawing the velocity vs. time graph

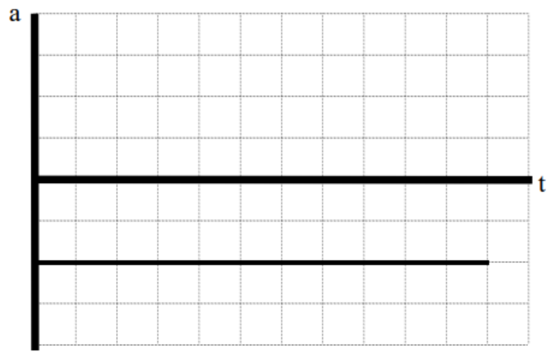
From the motion diagram, we know that the velocity of the car is always positive, starts large in magnitude, and decreases until it is zero. This information can be transferred onto a velocity vs. time graph.



How do we know that the slope of the line is constant? The slope of the line represents the rate at which the velocity is changing, and the rate at which the velocity is changing is termed the acceleration. Since in this model of mechanics we will only consider particles undergoing constant acceleration, the slope of a line on a velocity vs. time graph must be constant.

Drawing the acceleration vs. time graph

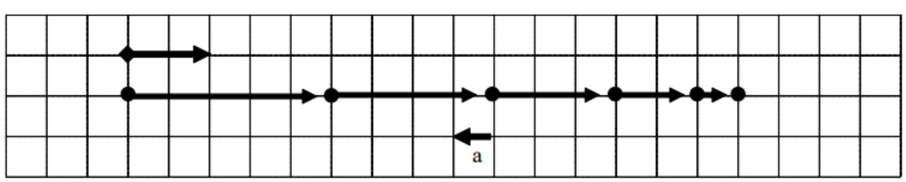
From the motion diagram, the acceleration of the car can be determined to be negative at every point. Again, in this pass through mechanics we will only be investigating scenarios in which the acceleration is constant. Thus, a correct acceleration vs. time graph is shown below.



Tabulating Motion Information

After constructing the two qualitative representations of the motion (the motion diagram and the motion graphs), we are ready to tackle the quantitative aspects of the motion.

Utilizing the motion diagram,



you can now assign numerical values to several of the kinematic variables. A glance at the situation description should indicate that information is presented about the car at two distinct events. Information is available about the car at the instant the driver applies the brakes (the velocity is given), and the instant the driver stops (the position is given). Other information can also be determined by referencing the motion diagram. To tabulate this information, you should construct a *motion table*.

Event 1: The instant the driver first applies the brakes.	Event 2: The instant the car finally stops.
$t_1 = 0$ s	$t_2 =$
$r_1 = 0$ m	$r_2 = +30$ m

Event 1: The instant the driver first applies the brakes.	Event 2: The instant the car finally stops.
v_1 = +15 m/s	v_2 = 0 m/s
a_{12} =	

In addition to the information explicitly given, the velocity at the first event and the position at the second event, other information can be extracted from the problem statement and the motion diagram.

For example,

- the position of the car at the first event is zero because the origin of the coordinate system is at this point,
- the time at the first instant can be "set to zero" by imagining a hypothetical stopwatch that is clicked on as the car begins to brake,
- and the velocity at the second event is zero because the car is stopped.

Since you are working under the assumption in this model that the acceleration is constant, the acceleration *between* the two instants in time is some unknown, constant value. To remind you that this assumption is in place, the acceleration is not labeled at the first instant, a_1, or the second instant, a_2, but rather as the acceleration *between* the two instants in time, a_{12}.

You now have a complete tabulation of all the information presented, both explicitly and implicitly, in the situation description. Moreover, you now can easily see that the only kinematic information not known about the situation is the assumed constant acceleration of the auto and the time at which it finally stops. Thus, to complete a kinematic description of the situation these two quantities must be determined. What you may not know is that you have already been presented with the information needed to determine these two unknowns.

Doing the Math

In the concepts and principles portion of this unit, you were presented with two formal, mathematical relationships, the definitions of velocity and acceleration. In the example that you are working on, there are two unknown kinematic quantities. You should remember from algebra that two equations are sufficient to calculate two unknowns. Thus, by applying the two definitions you should be able to determine the acceleration of the car and the time at which it comes to rest.

Although you can simply apply the two definitions directly, normally the two definitions are rewritten, after some algebraic re-arranging, into two different relationships. This rearrangement is simply to make the algebra involved in solving for the unknowns easier. It is by no means necessary to solve the problem. In fact, the two definitions can be written in a large number of different ways, although this does not mean that there are a large number of different formulas you must memorize in order to analyze kinematic situations. There are only two independent kinematic relationships. The two kinematic relationships[1] we will use when the acceleration is constant are:

$$v_2 = v_1 + a_{12}(t_2 - t_1)$$

and

$$r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$$

To finish the analysis of this situation,

Event 1: The instant the driver first applies the brakes.	Event 2: The instant the car finally stops.
t_1 = 0 s	t_2 =
r_1 = 0 m	r_2 = +30 m
v_1 = +15 m/s	v_2 = 0 m/s
a_{12} =	

Note

1 These two relationships are simply the definitions of velocity and acceleration rearranged. I show all the steps in the addendum to this chapter.

simply write the two kinematic relationships, input the known kinematic variables from the motion table, and solve the two relations for the two unknowns. (This process is not physics, it's algebra.)

$$v_2 = v_1 + a_{12}(t_2 - t_1)$$
$$0 = 15 + a_{12}(t_2 - 0)$$
$$a_{12} = \frac{-15}{t_2}$$

$$r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$$
$$30 = 0 + 15(t_2 - 0) + \frac{1}{2}a_{12}(t_2 - 0)^2$$
$$30 = 15t_2 + \frac{1}{2}a_{12}t_2^2$$
$$30 = 15t_2 + \frac{1}{2}\left(\frac{-15}{t_2}\right)t_2^2 \qquad \text{substitue in } a_{12} \text{ from above}$$
$$30 = 15t_2 - 7.5t_2$$
$$30 = 7.5t_2$$
$$t_2 = 4.0 \text{ s}$$

Substitute this result back into the original equation:

$$a_{12} = \frac{-15}{4}$$
$$a_{12} = -3.8 \text{ m/s}^2$$

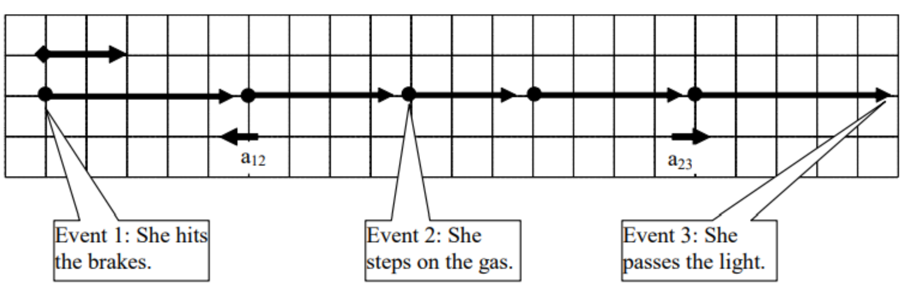
Thus, the car must have accelerated at 3.8 m/s^2 in the negative direction, and stopped after 4.0 seconds. The kinematic description of the situation is complete.

Analyzing a More Complex Motion

Let's re-visit our scenario, although this time the light turns green while the car is slowing down:

> *The driver of an automobile traveling at 15 m/s, noticing a red-light 30 m ahead, applies the brakes of her car. When she is 10 m from the light, and traveling at 8.0 m/s, the light turns green. She instantly steps on the gas and is back at her original speed as she passes under the light.*

Our first step in analyzing this motion should be to draw a motion diagram.



I've noted on the motion diagram the important events that take place during the motion. Notice that between the instant she hits the brakes and the instant she steps on the gas the acceleration is negative, while between the instant she steps on the gas and the instant she passes the light the acceleration is positive. Thus, in tabulating the motion information and applying the kinematic relations we will have to be careful not to confuse kinematic variables between these two intervals. Below is a tabulation of motion information using the coordinate system established in the motion diagram.

Event 1: She hits the brakes.	Event 2: She steps on the gas.	Event 3: She passes the light
t_1 = 0 s	t_2 =	t_3 =
r_1 = 0 m	r_2 = +20 m	r_3 = +30 m
v_1 = +15 m/s	v_2 = +8.0 m/s	v_3 = +15 m/s
a_{12} =	a_{23} =	

First, notice that during the time interval between "hitting the brakes" and "stepping on the gas" there are two kinematic variables that are unknown. Recall that by using your two kinematic relations you should be able to determine these values. Second, notice that during the second time interval again two variables are unknown. Once again, the two kinematic relations will allow you to determine these values. Thus, before I actually begin to do the algebra I *know* the unknown variables can be determined!

First let's examine the motion between hitting the brakes and stepping on the gas:

$$\begin{aligned} v_2 &= v_1 + a_{12}(t_2 - t_1) \\ 8 &= 15 + a_{12}(t_2 - 0) \\ a_{12} &= \frac{-7}{t_2} \end{aligned}$$

$$\begin{aligned} r_2 &= r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2 \\ 20 &= 0 + 15(t_2 - 0) + \frac{1}{2}a_{12}(t_2 - 0)^2 \\ 20 &= 15t_2 + \frac{1}{2}a_{12}t_2^2 \\ 20 &= 15t_2 + \frac{1}{2}\left(\frac{-7}{t_2}\right)t_2^2 && \text{substitue in } a_{12} \text{ from above} \\ 20 &= 15t_2 - 3.5t_2 \\ 20 &= 11.5t_2 \\ t_2 &= 1.74 s \end{aligned}$$

Substitute this result back into the original equation:

$$\begin{aligned} a_{12} &= \frac{-7}{1.74} \\ a_{12} &= -4.03 \text{ m/s}^2 \end{aligned}$$

Now, using these results, examine the kinematics between stepping on the gas and passing the light. Note that the initial values of the kinematic variables are denoted by '2' and the final values by '3', since we are examining the interval between event 2 and event 3.

$$\begin{aligned} v_3 &= v_2 + a_{23}(t_3 - t_2) \\ 15 &= 8 + a_{23}(t_3 - 1.74) \\ a_{23} &= \frac{7}{(t_3 - 1.74)} \end{aligned}$$

$$\begin{aligned} r_3 &= r_2 + v_2(t_3 - t_2) + \frac{1}{2}a_{23}(t_3 - t_2)^2 \\ 30 &= 20 + 8(t_3 - 1.74) + \frac{1}{2}a_{23}(t_3 - 1.74)^2 \\ 10 &= 8(t_3 - 1.74) + \frac{1}{2}a_{23}(t_3 - 1.74)^2 \\ 10 &= 8(t_3 - 1.74) + \frac{1}{2}\left(\frac{7}{(t_3 - 1.74)}\right)(t_3 - 1.74)^2 && \text{substitue in } a_{23} \text{ from above} \\ 10 &= 8(t_3 - 1.74) + 3.5(t_3 - 1.74) \\ 10 &= 11.5(t_3 - 1.74) \\ 0.87 &= t_3 - 1.74 \\ t_3 &= 2.61 \text{ s} \end{aligned}$$

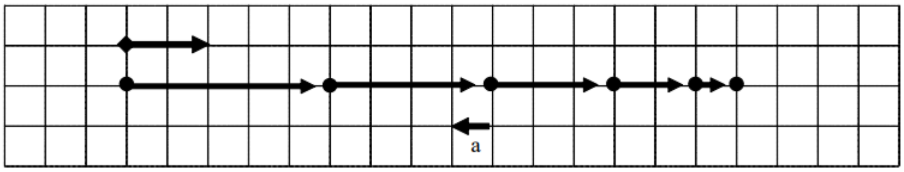
We now have a complete kinematic description of the motion.

Symbolic Analysis

Consider the following situation:

> *The driver of an automobile suddenly sees an obstacle blocking her lane. Determine the total distance the auto travels between seeing the obstacle and stopping (d) as a function of the initial velocity of the car (v_i) and the magnitude of its acceleration while stopping (a_s).*

As always, the first step in analyzing motion is to draw a motion diagram.



Rather than calculate the stopping distance for particular values of initial velocity and acceleration, the goal of this activity is to determine, in general, how the stopping distance depends on these two parameters. If we can construct this function we can then use the result to calculate the stopping distance for *any* car if we know its initial velocity and stopping acceleration.

Although this sounds like a different task from what we've done in the previous two examples we will approach this task exactly the same way, by tabulating what we know about the situation,

Event 1: The instant the driver first applies the brakes.	Event 2: The instant the car finally stops.
$t_1 = 0$ s	$t_2 =$
$r_1 = 0$ m	$r_2 = d$
$v_1 = v_i$	$v_2 = 0$ m/s
$a_{12} = -a_s$	

and then applying our two kinematic relationships:

$$v_2 = v_1 + a_{12}(t_2 - t_1) \qquad r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$$

$$0 = v_i - a_s t_2 \qquad d = 0 + v_i t_2 - \frac{1}{2}a_s t_2^2$$

Since our goal is to determine d as a function of v_i and as, we must eliminate t_2. To do this, solve for t_2 in the left equation and substitute this expression into the right equation.

$$t_2 = \frac{v_i}{a_s}$$

$$d = v_i\left(\frac{v_i}{a_s}\right) - \frac{1}{2}a_s\left(\frac{v_i}{a_s}\right)^2$$

$$d = \frac{v_i^2}{a_s} - \frac{1}{2}\frac{v_i^2}{a_s}$$

$$d = \frac{v_i^2}{2a_s}$$

Thus, the stopping distance appears to be proportional to the *square* of the initial velocity and inversely proportional to the stopping acceleration. Does this make sense?

To determine if a symbolic expression is sensible it is often useful to check *limiting* cases. A limiting case is when one of the variables in the expression takes on an extreme value, typically zero or infinity. For example, if the initial velocity of the car was zero the stopping distance would have to also be zero, since the car was never moving! Allowing vi to equal zero in the above expression results in a stopping distance of zero, so our expression "passes" this logical test.

Another limiting case would be setting the acceleration of the car equal to zero. With no acceleration, the car should never stop. In our expression, setting the acceleration equal to zero results in an infinite stopping distance, which again agrees with commonsense. If our expression didn't give the correct results in these limiting cases we would know we made an error somewhere in the derivation (and, of course, we would then go back and find our mistake and fix it because we are good students …).

Hints and Suggestions

Algebraic Signs

Confusion about the meaning of algebraic signs is common among beginning physics students. The best way to clarify this confusion is to remember that algebraic signs are simply a mathematical way to describe direction. Instead of saying up and down, or east and west, physicists construct coordinate systems and translate the words east and west into the symbols '+' and '-', or even '-' and '+' if we choose a different coordinate system. The key to the translation is the coordinate system. A coordinate system is very similar to the English-French dictionary you might take with you on your first trip to France. When you see a '-', use your coordinate system to translate it into a verbal description of direction.

Do not fall into the common habit of translating a '-' into the word "decreasing". A negative acceleration, for example, does NOT imply that the object is slowing down. It implies an acceleration that points in the negative direction. It is *impossible* to determine whether an object is speeding up or slowing down by looking at the sign of the acceleration! Conversely, the word "deceleration", which does mean that an object is slowing down, does *not* give any information regarding the sign of the acceleration. I can decelerate in the positive direction as easily as I can decelerate in the negative direction.

Addendum

Deriving the kinematic relationships

Let's construct the two independent kinematic relationships that you will use whenever the acceleration is constant. In a later chapter, we will return to the case in which the acceleration is not constant.

From the definition of acceleration:

$$a=\frac{v_f-v_i}{t_f-t_i}$$
$$v_f-v_i=a\left(t_f-t_i\right)$$
$$v_f=v_i+a\left(t_f-t_i\right)$$

The above relationship is our first kinematic relationship. The acceleration in this relationship is really the average acceleration. However, since the acceleration is constant in this model the average acceleration is the same as the acceleration at any instant between the initial and final state.

From the definition of velocity:

$$v=\frac{r_f-r_i}{t_f-t_i}$$
$$r_f-r_i=v\left(t_f-t_i\right)$$
$$r_f=r_i+v\left(t_f-t_i\right)$$

You must remember, however, that the velocity in this formula is really the average velocity of the object over the time interval selected. To keep you from having to remember this fact, we can rewrite the average velocity as the sum of the initial velocity and the final velocity divided by two:

$$r_f=r_i+\left(\frac{v_i+v_f}{2}\right)\left(t_f-t_i\right)$$
$$r_f=r_i+\frac{1}{2}\left(v_i+v_f\right)\left(t_f-t_i\right)$$

Substituting in the first kinematic relationship for the final velocity yields:

$$r_f=r_i+\frac{1}{2}\left(v_i+v_i+a\left(t_f-t_i\right)\right)\left(t_f-t_i\right)$$
$$r_f=r_i+\frac{1}{2}\left(2v_i\left(t_f-t_i\right)+a\left(t_f-t_i\right)^2\right)$$
$$r_f=r_i+v_i\left(t_f-t_i\right)+\frac{1}{2}a\left(t_f-t_i\right)^2$$

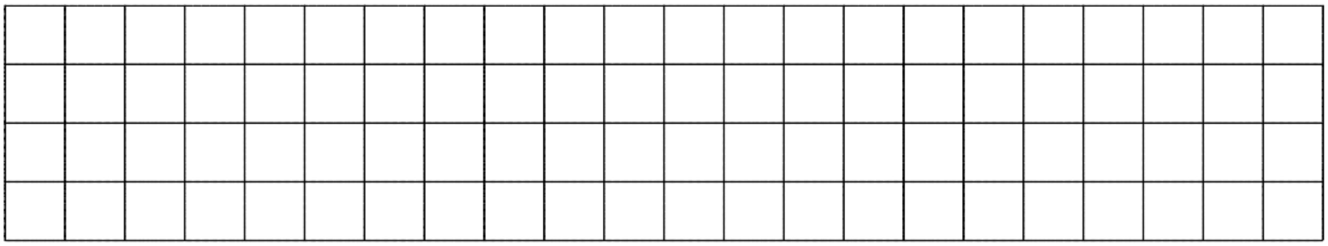
The above relationship is our second kinematic relationship.

Although we could keep deriving new kinematic equations forever, it is *impossible* that any other derived equation could allow us to calculate some quantity that these equations do not allow us to calculate.

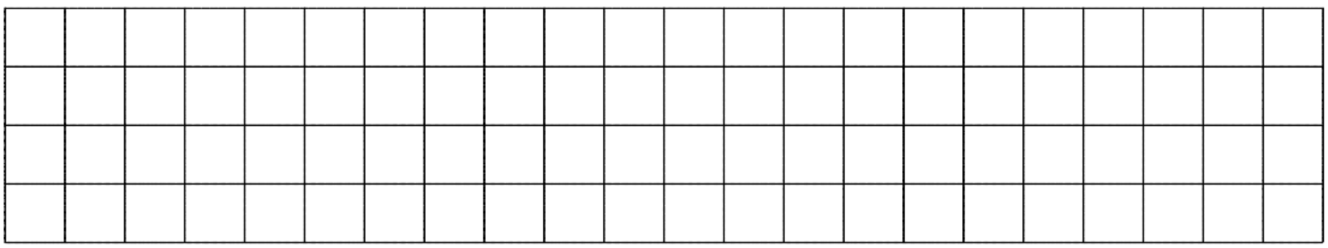
Activities

Construct motion diagrams for the motions described below.

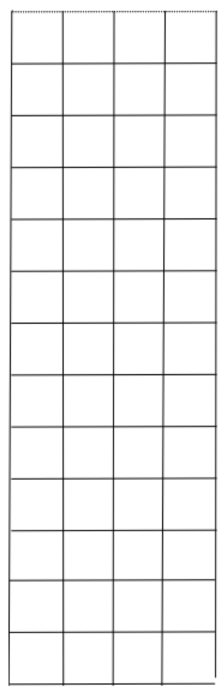
a. A subway train in Washington, D.C., starts from rest and accelerates at 2.0 m/s^2 for 12 seconds.



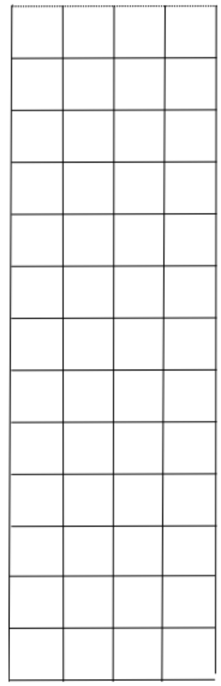
b. The driver of a car traveling at 35m/s suddenly sees a police car. The driver attempts to reach the speed limit of 25 m/s by accelerating at 2.5 m/s^2. The driver has a reaction time of 0.55 s. (The reaction time is the time between first seeing the police car and pressing the brake.)



c. A pole-vaulter, just before touching the cushion on which she lands after a jump, is falling downward at a speed of 10 m/s. The pole-vaulter sinks about 2.0 m into the cushion before stopping.

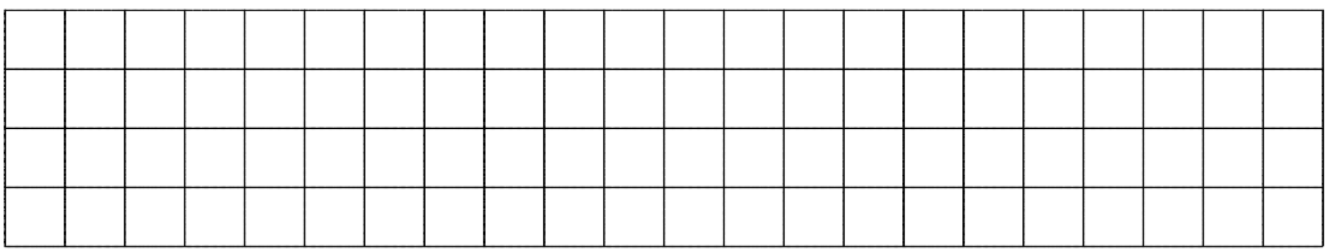


d. An elevator is moving downward at 4.0 m/s for 3.5 s before someone presses the emergency stop button. The elevator comes to rest after traveling 2.9 m.

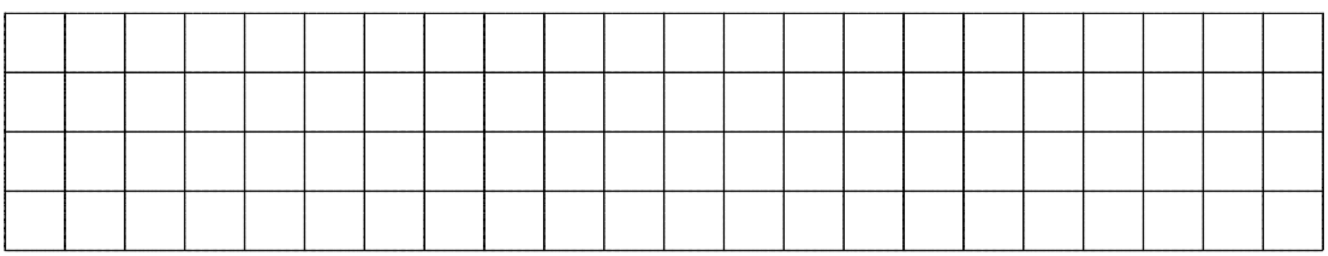


Construct motion diagrams for the motions described below.

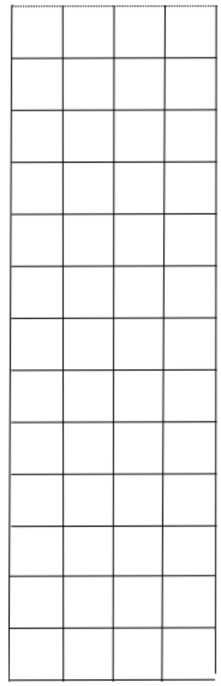
a. An automobile comes to rest after skidding 35 m. The car's acceleration while skidding is known to be 6.0 m/s^2.



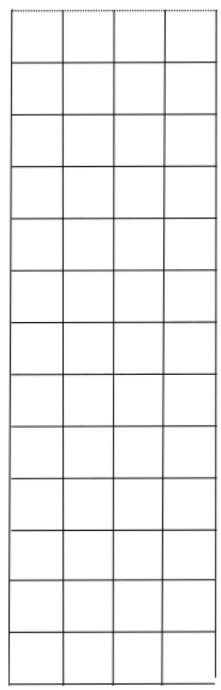
b. A car, initially traveling at 20 m/s to the east, accelerates toward the west at 2.0 m/s^2. At the same time that the car starts moving, a truck, 60 m west of the car and moving at 16 m/s toward the east, starts to move faster, accelerating at 1.0 m/s^2. It's a one-lane road and both drivers are too busy texting to notice each other.



c. A child is hanging from a rope by her hands. She exerts a burst of strength and 2.0 s later is traveling at 1.4 m/s up the rope.

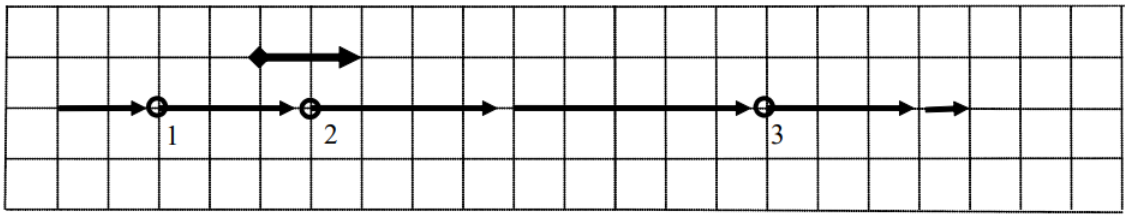


d. A two-stage rocket initially accelerates upward from rest at 13 m/s^2 for 5.0s before the second stage initiates a 14 s long upward acceleration of 25 m/s^2 .



For each of the motion diagrams below, determine the algebraic sign (+, - or zero) of the position, velocity, and acceleration of the object at the location of the three open circles. Describe an actual motion that could be represented by each motion diagram.

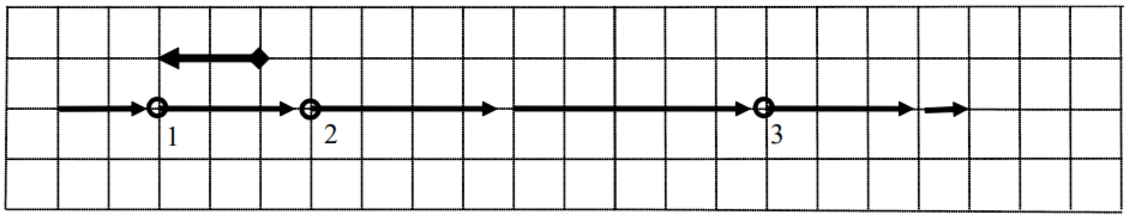
a.



Description:

	1	2	3
r			
v			
a			

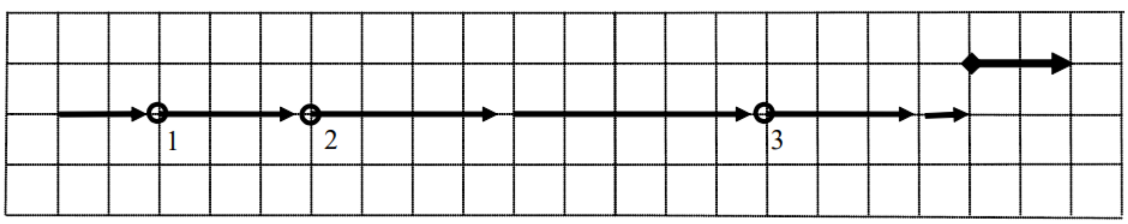
b.



Description:

	1	2	3
r			
v			
a			

c.



Description:

	1	2	3
r			
v			
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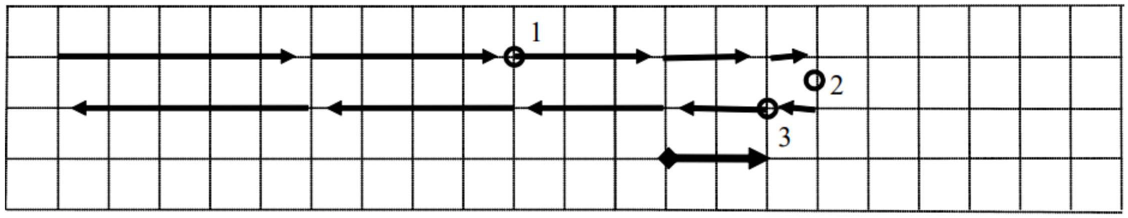
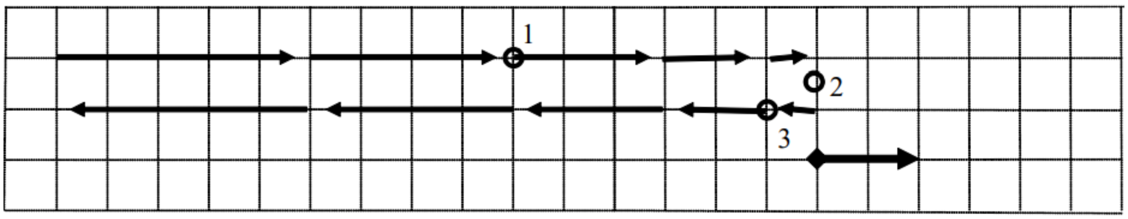


Description:

	1	2	3
r			
v			
a			

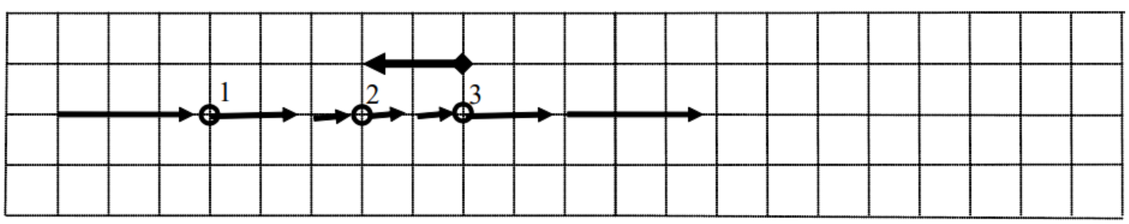
b.



Description:

	1	2	3
r			
v			
a			

c.

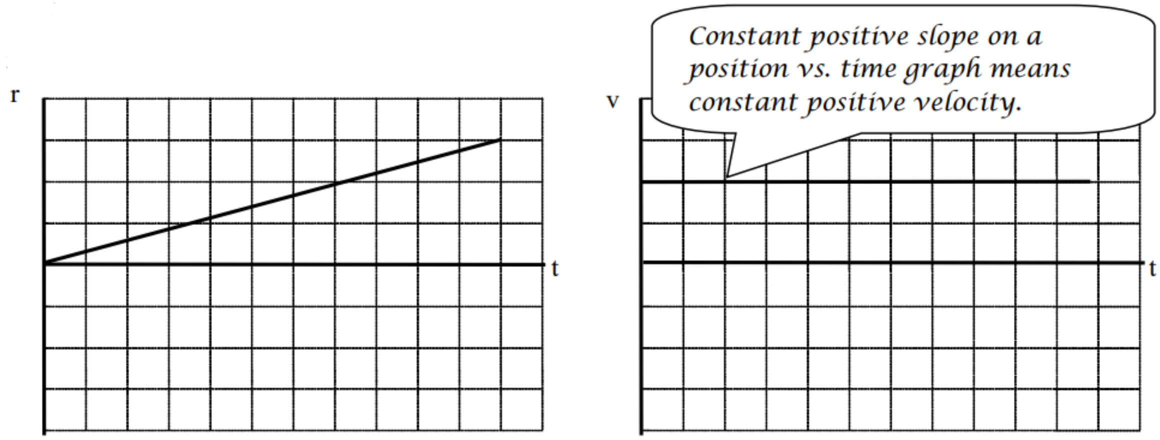


Description:

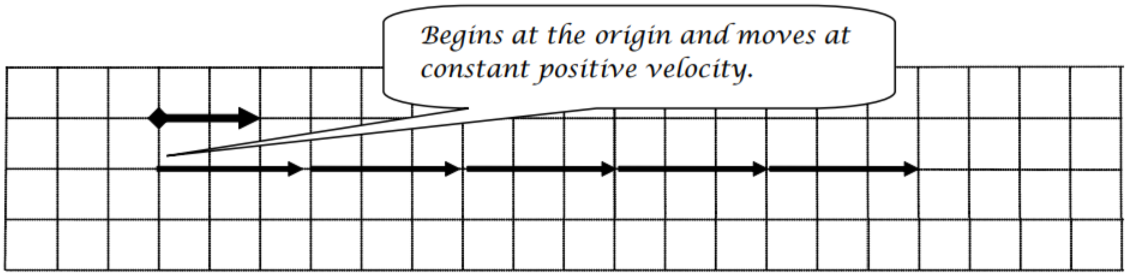
	1	2	3
r			
v			
a			

For each of the position vs. time graphs below, construct a corresponding motion diagram and velocity vs. time graph.

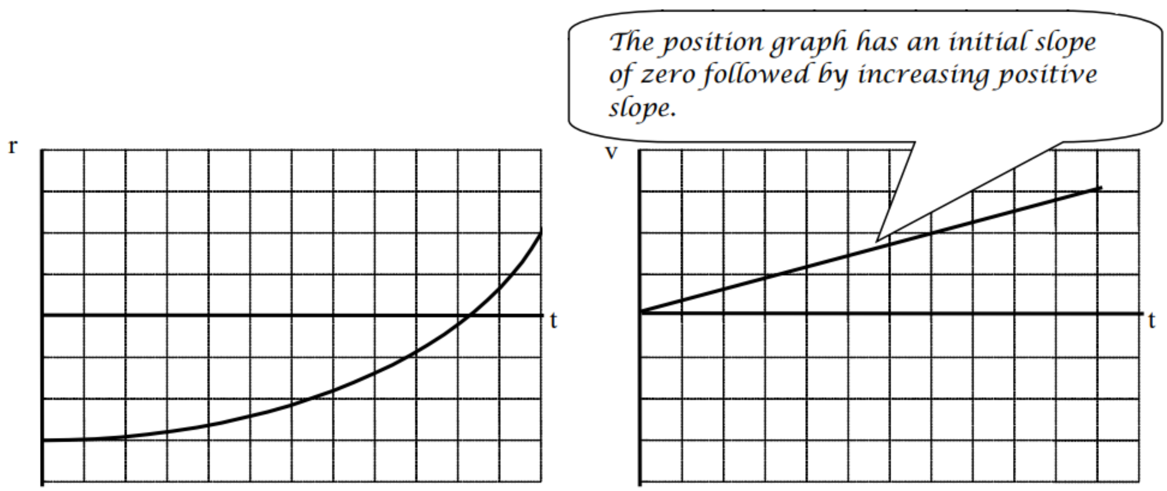
a. **Motion Graphs**



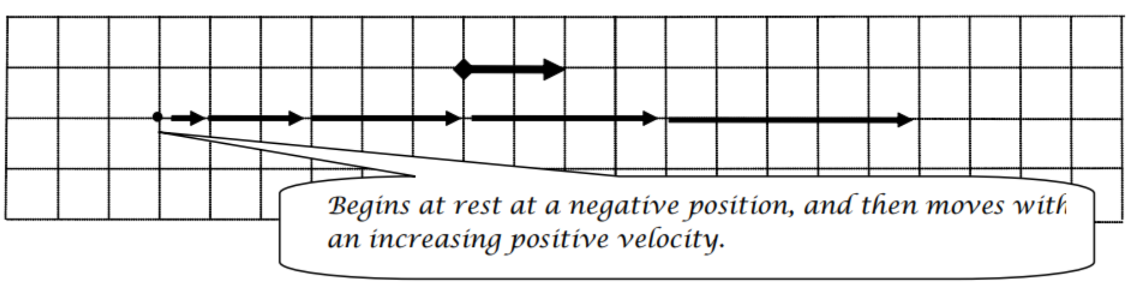
Motion Diagram



b. **Motion Graphs**



Motion Diagram



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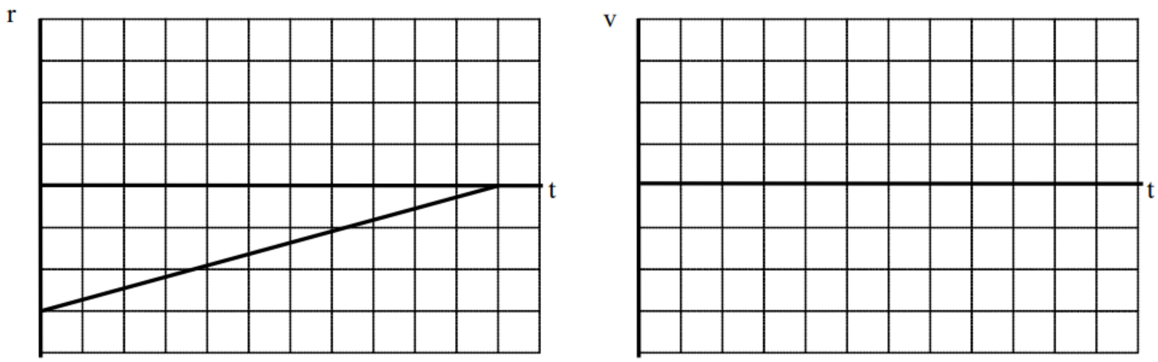
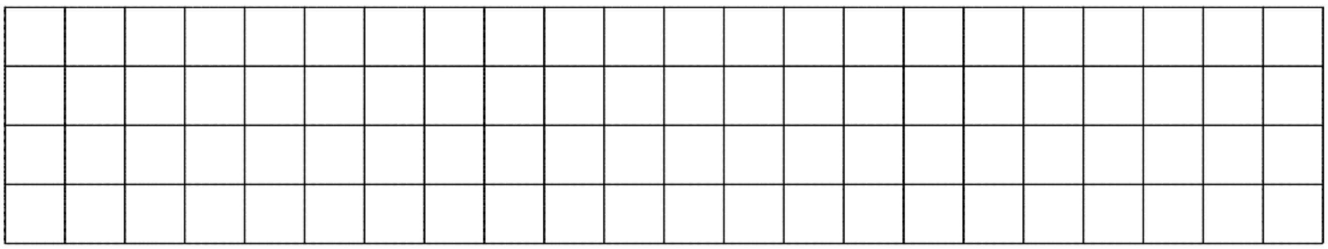
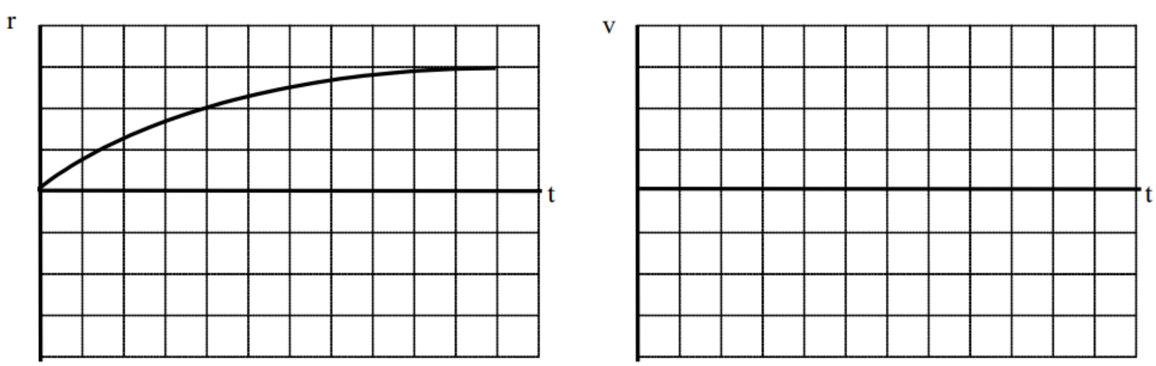


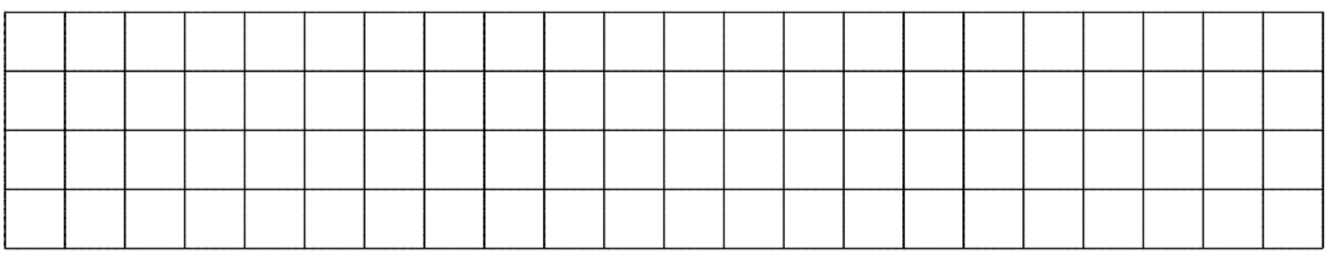
Motion Diagram



b. **Motion Graphs**

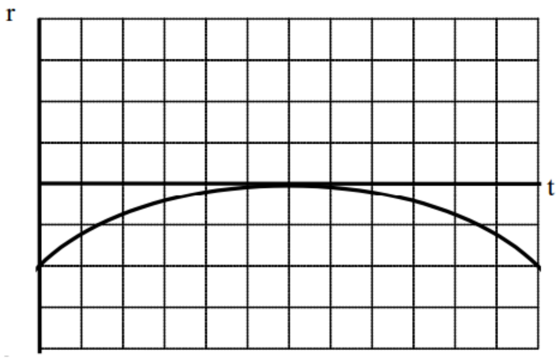


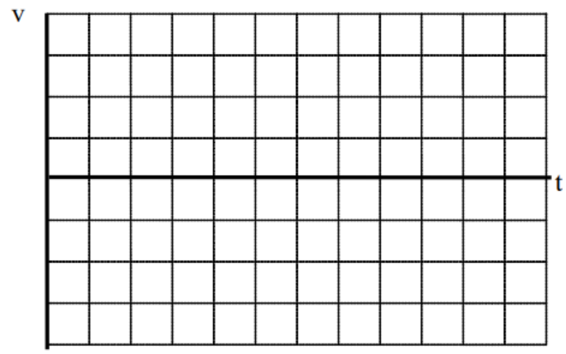
Motion Diagram



For each of the graphs below, construct a corresponding graph and motion diagram.

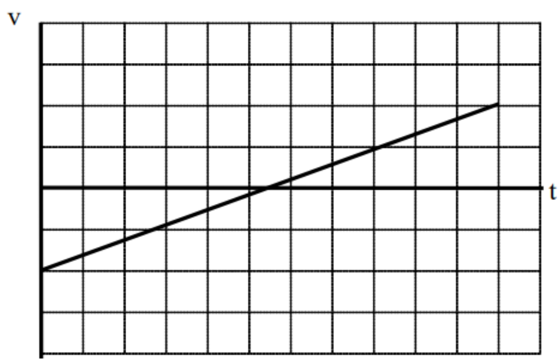
a. **Motion Graphs**

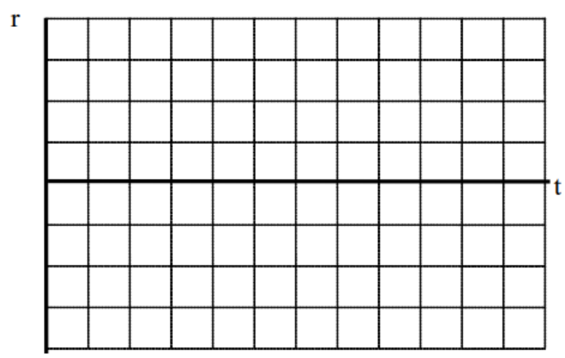




Motion Diagram

b. **Motion Graphs**





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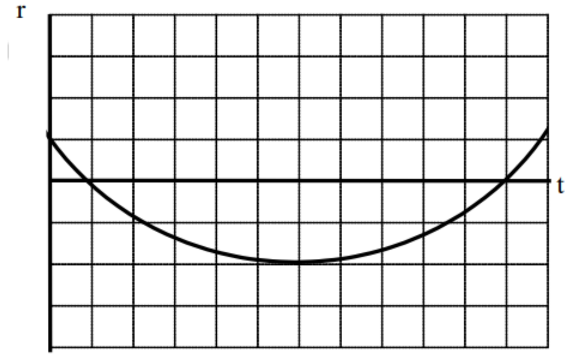
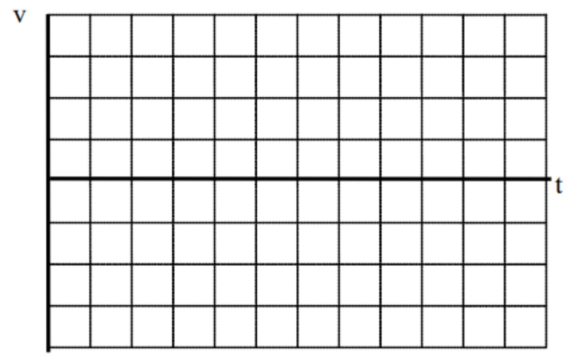
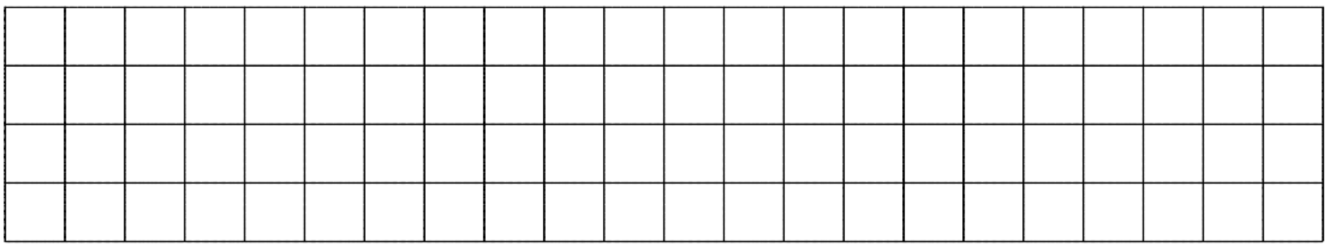
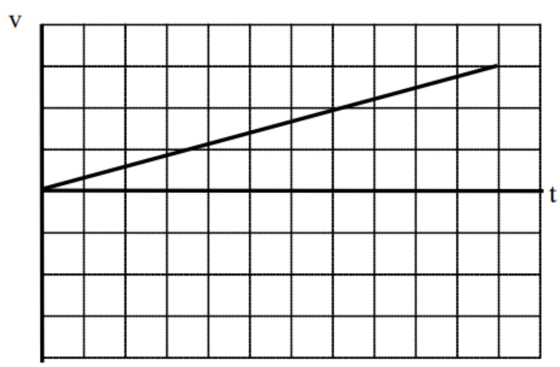


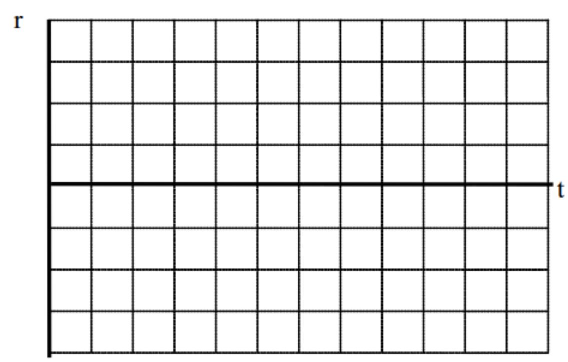


Motion Diagram

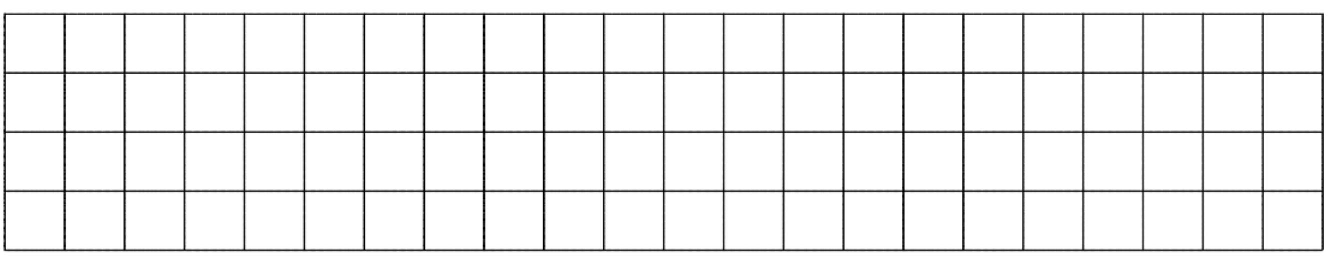


b. **Motion Graphs**



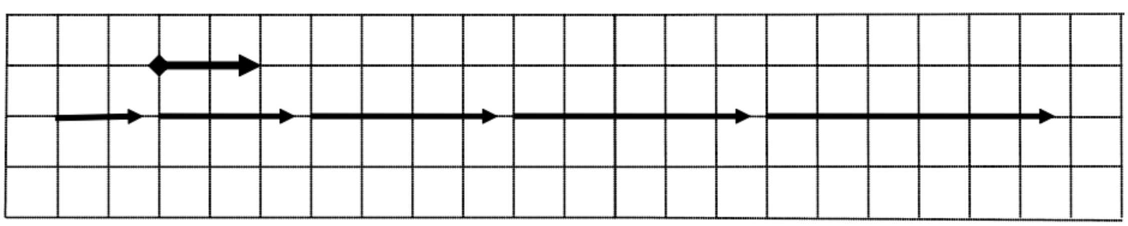


Motion Diagram



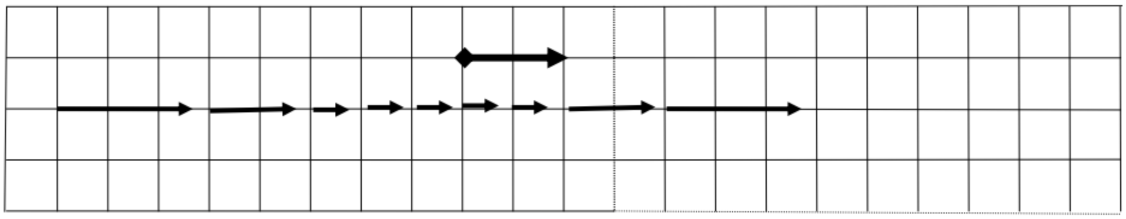
For each of the graphs below, construct a corresponding graph and motion diagram.

a. **Motion Graphs**



Motion Diagram

b. **Motion Graphs**



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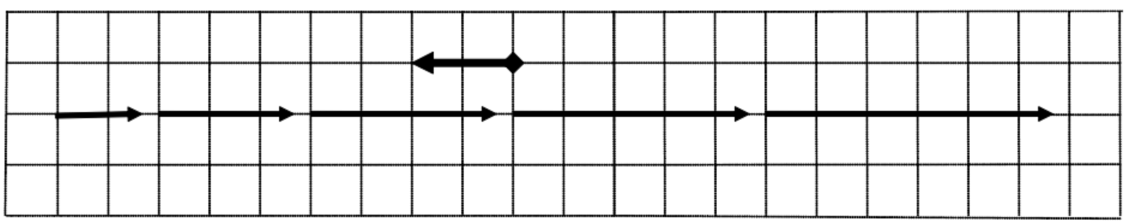
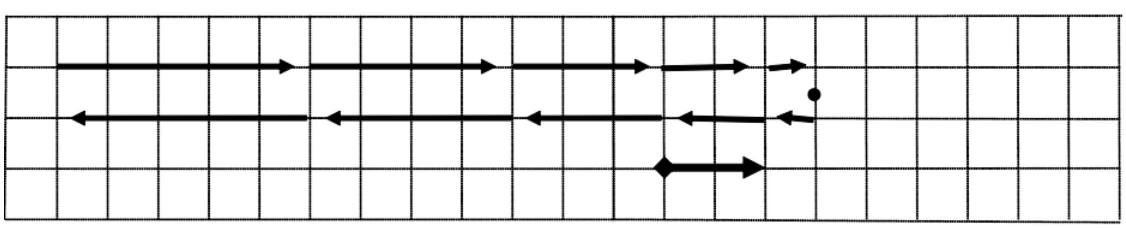
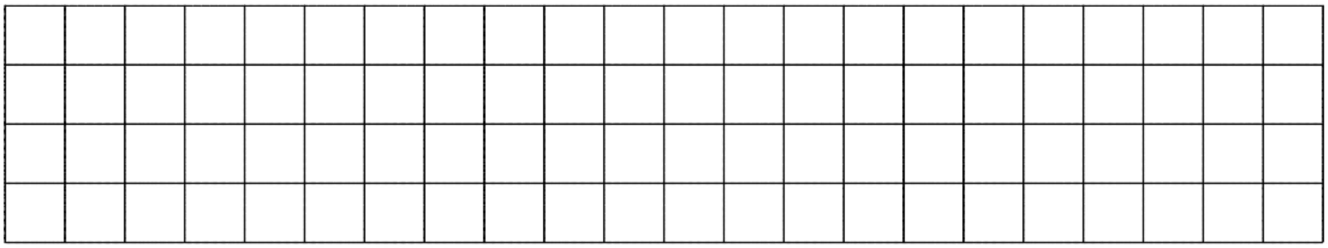


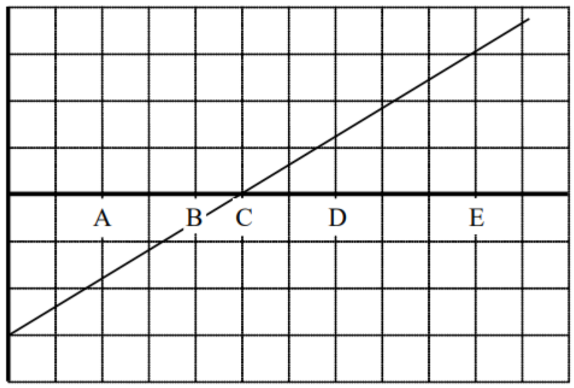
Motion Diagram

b. **Motion Graphs**



Motion Diagram





An object's motion is represented by the position vs. time graph at the top of the page.

a. Rank the object's position at the lettered times.

Largest Positive 1. E 2. D 3. C 4. B 5. A Largest Negative

b. Rank the object's velocity at the lettered times.

Largest Positive 1. ABCDE 2. 3. 4. 5. Largest Negative

Constant slope means constant velocity (and zero acceleration).

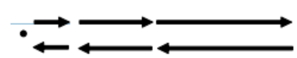
c. Rank the object's acceleration at the lettered times.

Largest Positive 1. 2. 3. ABCDE 4. 5. Largest Negative

An object's motion is represented by the velocity vs. time graph at the top of the page.

d. Rank the object's position at the lettered times.

Largest Positive 1. E 2. A 3. D 4. B 5. C Largest Negative



The motion diagram for the object is sketched above. Notice that regardless of where the origin is located, the turnaround point (C) is the smallest position.

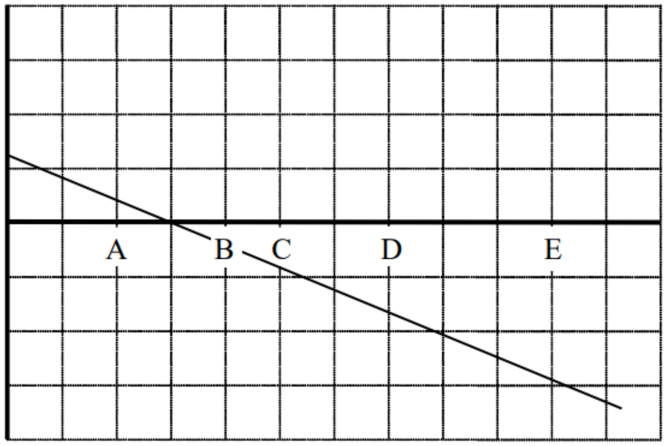
e. Rank the object's velocity at the lettered times.

Largest Positive 1. E 2. D 3. C 4. B 5. A Largest Negative

f. Rank the object's acceleration at the lettered times.

Largest Positive 1. ABCDE 2. 3. 4. 5. Largest Negative

Constant slope means constant acceleration.



An object's motion is represented by the position vs. time graph above.

a. Rank the object's position at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

b. Rank the object's velocity at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

c. Rank the object's acceleration at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

An object's motion is represented by the velocity vs. time graph above.

d. Rank the object's position at the lettered times.

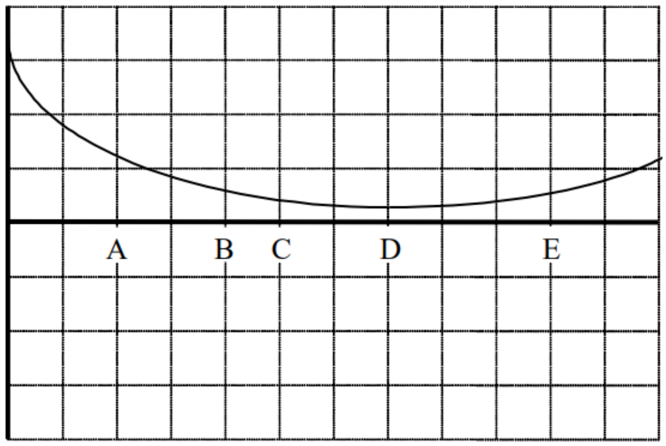
Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

e. Rank the object's velocity at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

f. Rank the object's acceleration at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative



An object's motion is represented by the position vs. time graph above.

a. Rank the object's position at the lettered times.

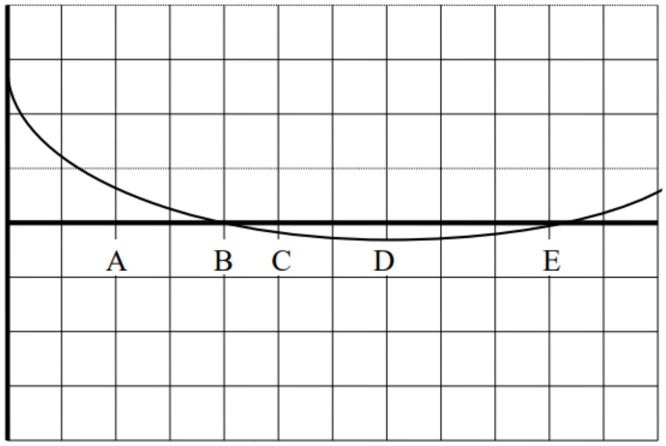
Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

b. Rank the object's velocity at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

c. Rank the object’s acceleration at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative



An object’s motion is represented by the position vs. time graph above.

d. Rank the object’s position at the lettered times.

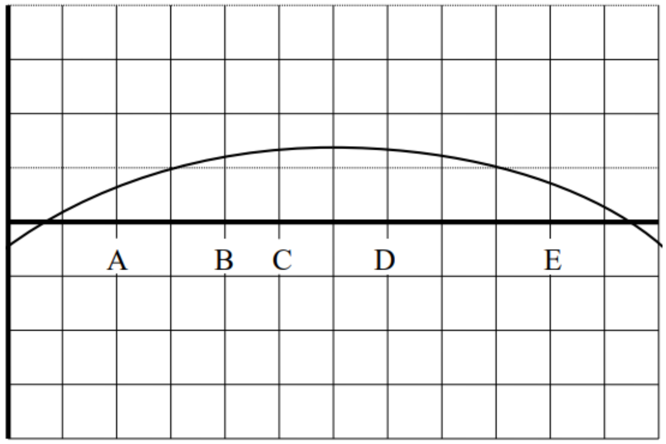
Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

e. Rank the object’s velocity at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

f. Rank the object’s acceleration at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative



An object’s motion is represented by the position vs. time graph above.

a. Rank the object’s position at the lettered times.

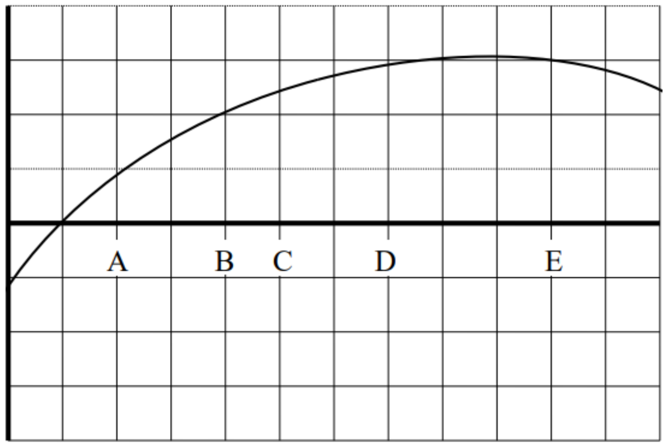
Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

b. Rank the object’s velocity at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

c. Rank the object’s acceleration at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative



An object’s motion is represented by the position vs. time graph above.

d. Rank the object’s position at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

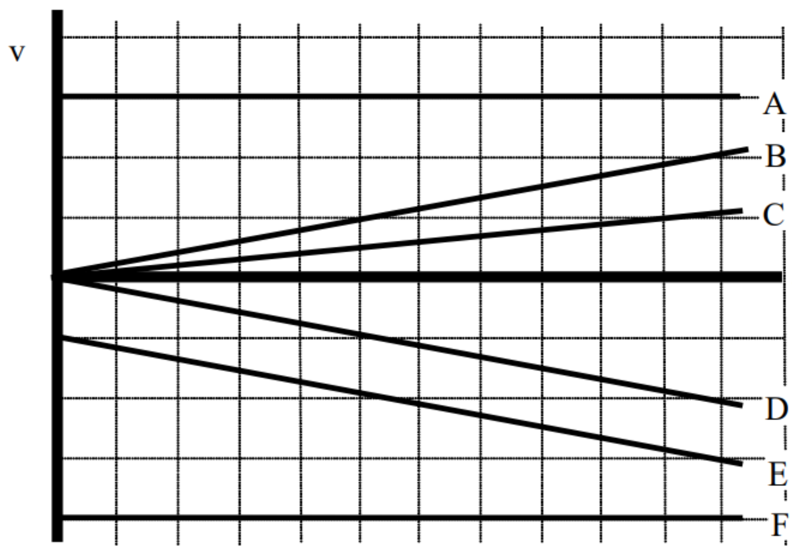
e. Rank the object’s velocity at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

f. Rank the object’s acceleration at the lettered times.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ Largest Negative

Below are velocity vs. time graphs for six different objects.



Rank these graphs on the basis of the distance traveled by each object.

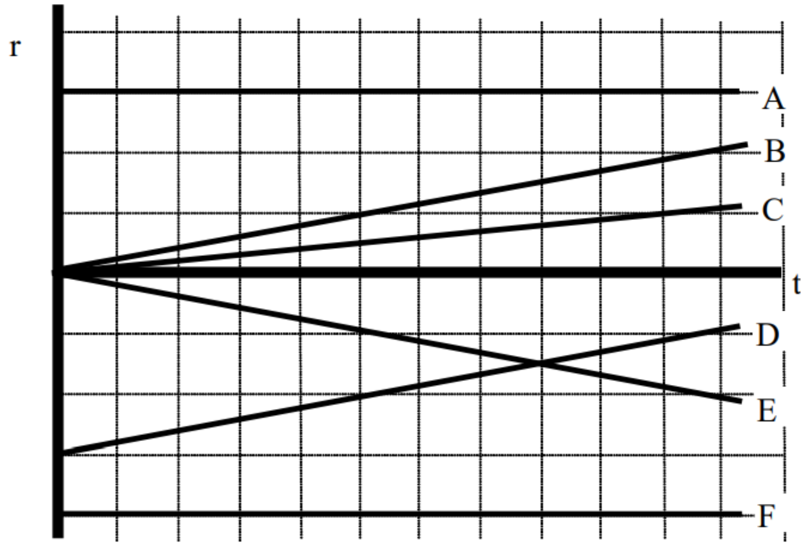
Largest 1. __F__ 2. __A__ 3. __E__ 4. _B D_ 5. __C__ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The faster you travel, the more distance you travel. The direction you are headed is not important. Therefore, F travels the largest distance because it is always moving the fastest, followed by A. E is consistently traveling faster than D, so it covers a larger distance than D. B and D always travel at the same speed (although in opposite directions) so they cover the same distance, therefore they are ranked as equal. C travels the slowest so it covers the least distance.

Below are position vs. time graphs for six different objects.



a. Rank these graphs on the basis of the velocity of the object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

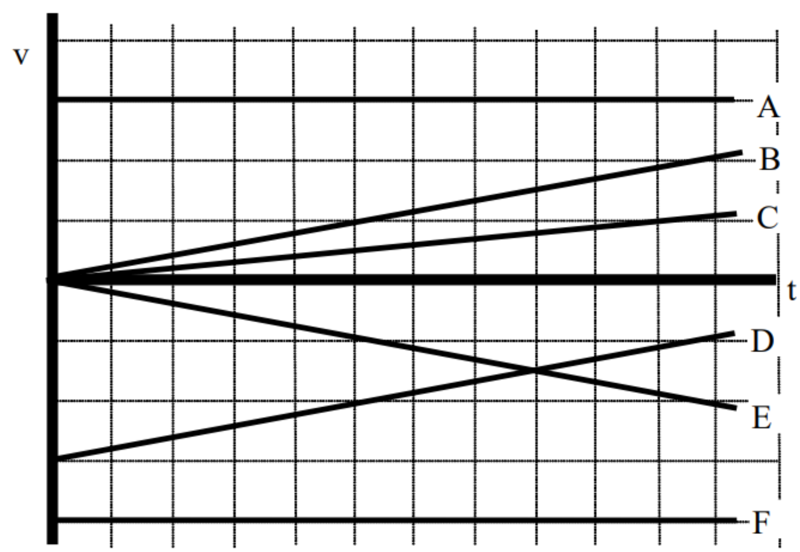
b. Rank these graphs on the basis of the acceleration of the object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are velocity vs. time graphs for six different objects.



a. Rank these graphs on the basis of the final position of each object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank these graphs on the basis of the acceleration of the object.

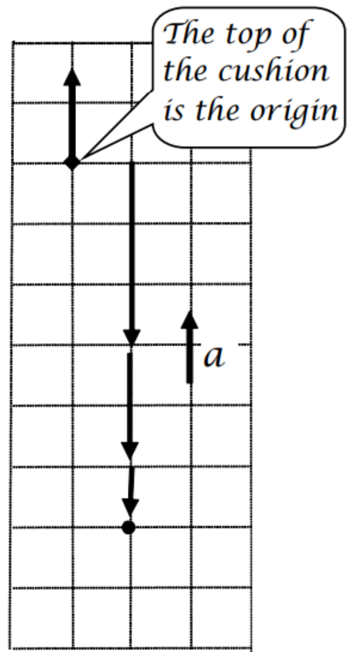
Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

> *A pole-vaulter, just before touching the cushion on which she lands after a jump, is falling downward at a speed of 10 m/s. The pole-vaulter sinks about 2.0 m into the cushion before stopping.*

Motion Diagram



Motion Information

Event 1: The instant she strikes the cushion. *(The problem begins when she hits the cushion, not while she's falling.)*	Event 2: The instant she stops.
$t_1 = 0$ s	$t_2 =$
$r_1 = 0$ m	$r_2 = -2.0$ m *(She stops below the origin.)*
$v_1 = -10$ m/s *(She's initially moving downward, which was chosen as the negative direction.)*	$v_2 = 0$ m/s
$a_{12} =$	

Mathematical Analysis

$$v_2 = v_1 + a_{12}(t_2 - t_1)$$
$$0 = -10 + a_{12}(t_2 - 0)$$
$$a_{12} = \frac{10}{t_2}$$

$$
\begin{aligned}
r_2 &= r_1 + v_1\left(t_2 - t_1\right) + \tfrac{1}{2} a_{12}\left(t_2 - t_1\right)^2 \\
-2 &= 0 - 10\left(t_2 - 0\right) + \tfrac{1}{2} a_{12}\left(t_2 - 0\right)^2 \\
-2 &= -10 t_2 + \tfrac{1}{2} a_{12} t_2^2 \\
-2 &= -10 t_2 + \tfrac{1}{2}\left(\tfrac{10}{t_2}\right) t_2^2 && \text{substitute in } \mathrm{a}_{12} \\
-2 &= -10 t_2 + 5 t_2 \\
-2 &= -5 t_2 \\
t_2 &= 0.4 \text{ s}
\end{aligned}
$$

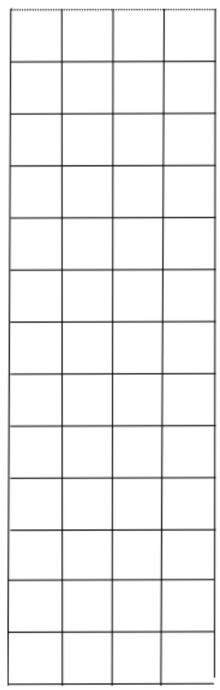
Substitute this result back into the original equation:

$$
\begin{aligned}
a_{12} &= \tfrac{10}{0.4} \\
a_{12} &= 25 \text{ m/s}^2
\end{aligned}
$$

The acceleration is positive, as it should be since the jumper is moving downward and slowing down.

A child is hanging from a rope by her hands. She exerts a burst of strength and 2.0 s later is traveling at 1.4 m/s up the rope.

Motion Diagram



Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
v_1 =	v_2 =
a_{12} =	

Mathematical Analysis[1]

An elevator is moving downward at 4.0 m/s when someone presses the emergency stop button. The elevator comes to rest after traveling 2.9 m.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
v_1 =	v_2 =
a_{12} =	

Mathematical Analysis[2]

The driver of a car traveling at 16 m/s suddenly sees a truck that has entered from a side street and blocks the car's path. The car's maximum magnitude acceleration while braking is 6.0 m/s^2 and the driver has a reaction time of 0.75 s. (The reaction time is the time between first seeing the truck and pressing the brake.) The driver stops just in time to avoid an accident.

Motion Diagram

Motion Information

Event 1:	Event 2:	Event 3:

Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_{12} =	a_{23} =	

Mathematical Analysis[3]

> *The driver of a car traveling at 35 m/s suddenly sees a police car. The driver attempts to reach the speed limit of 25 m/s by accelerating at 2.5 m/s^2. The driver has a reaction time of 0.55 s. (The reaction time is the time between first seeing the police car and pressing the brake.)*

Motion Diagram

Motion Information

Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_{12} =	a_{23} =	

Mathematical Analysis[4]

> *An automobile, initially traveling at 15 m/s, begins to slow down as it approaches a red light. After traveling 15 m, and slowing to 3.0 m/s, the light turns green and the driver steps on the gas and accelerates for 2.4 seconds until she reaches her original speed.*

Motion Diagram

Motion Information

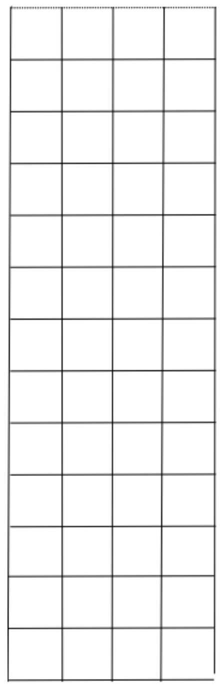
Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =

Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_{12} =	a_{23} =	

Mathematical Analysis[5]

> *A two-stage rocket initially accelerates upward from rest at 13 m/s^2 for 5.0s before the second stage ignites. The second stage "burns" for 14 s and results in the rocket achieving a speed of 415 m/s at the end of this stage.*

Motion Diagram



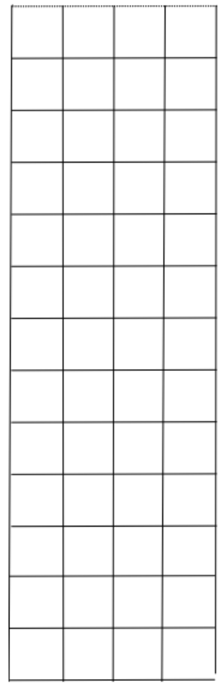
Motion Information

Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_{12} =	a_{23} =	

Mathematical Analysis[6]

> Rather than pushing the button for the correct floor a man prefers to hit the emergency stop button when an elevator approaches his floor, and then pry the doors apart. An elevator is moving at a constant speed of 2.8 m/s when it is 16 m from his floor. With uncanny timing, and an elevator that can slow at 3.5 m/s^2 , he makes the elevator stop precisely at his floor.

Motion Diagram



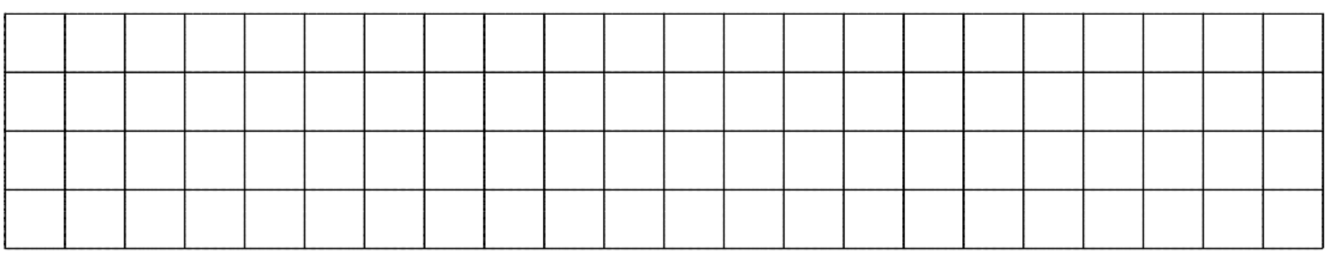
Motion Information

Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_{12} =	a_{23} =	

Mathematical Analysis[7]

> A subway train in Washington, D.C., starts from rest and accelerates at 2.0 m/s^2 for 12 s. The train travels at a constant speed for 65 s. The speed of the train then decreases for 25 s until it reaches the next station.

Motion Diagram



Motion Information

Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_{12} =	a_{23} =	

Mathematical Analysis[8]

> A rocket ship is launched from rest from a space station. Its destination is 1.0 x 10^{11} m away. The ship is programmed to accelerate at 7.4 m/s^2 for 12 hours. After 12 hours, the ship will travel at constant velocity until it comes within 1.0 x 10^{10} m of its destination. Then, it will fire its retrorockets to land safely.

Motion Diagram

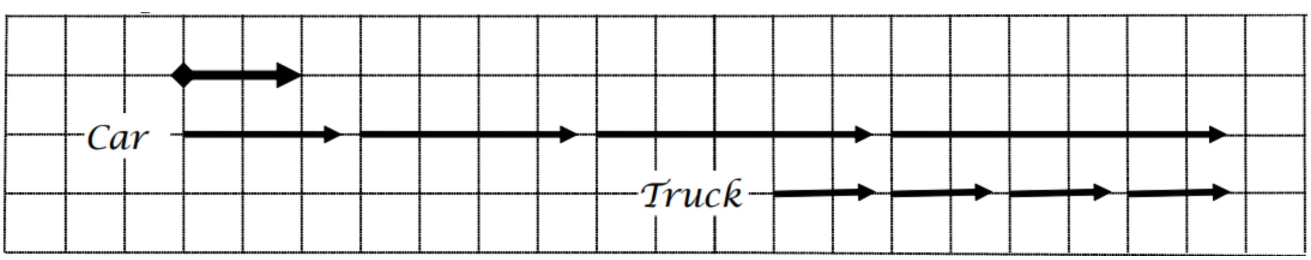
Motion Information

Event 1:	Event 2:	Event 3:	Event 4:
t_1 =	t_2 =	t_3 =	t_4 =
r_1 =	r_2 =	r_3 =	r_4 =
v_1 =	v_2 =	v_3 =	v_4 =
a_{12} =	a_{23} =		a_{34} =

Mathematical Analysis[9]

> The driver of a car traveling at 16 m/s sees a truck 20 m ahead traveling at a constant speed of 12 m/s. The car starts without delay to accelerate at 4.0 m/s^2 in an attempt to rear-end the truck. The truck driver is too busy talking on his cell phone to notice the car.

Motion Diagram



Motion Information

Object: Car Event 1: Car begins to accelerate.	Event 2: Collision!	Object: Truck Event 1: Car begins to accelerate.	Event 2: Collision!
t_1 = 0 s	t_2 =	t_1 = 0 s	t_2 =
r_1 = 0 m	r_2 =	r_1 = 20 m	r_2 =
v_1 = 16 m/s	v_2 =	v_1 = 12 m/s	v_2 =
a_{12} = +4 m/s^2		a_{12} = 0 m/s^2	

Mathematical Analysis

At first glance, there appear to be six unknowns in the motion table. This should concern you since you only have four equations (the two kinematic equations applied to the car and the same two applied to the truck). However, since the car and truck collide at event 2, t_2 and r_2 for the car and truck must be equal at this event. Thus, the only four variables are t_2, r_2, v_{2Car}, and v_{2Truck}. These can be determined by the four kinematic equations. Specifically, set the position equation for the car equal to the position equation for the truck and solve for t_2:

$$\begin{gathered} r_{2\text{Car}} = r_{2\text{ Truck}} \\ 0+16\left(t_2-0\right)+\frac{1}{2}(4)\left(t_2-0\right)^2=20+12\left(t_2-0\right)+\frac{1}{2}(0)\left(t_2-0\right)^2 \\ 16 t_2+2 t_2^2=20+12 t_2 \\ 0=20-4 t_2-2 t_2^2 \end{gathered}$$

Using the quadratic formula, t_2 = 2.32 s. Plugging this back into either position equation yields,

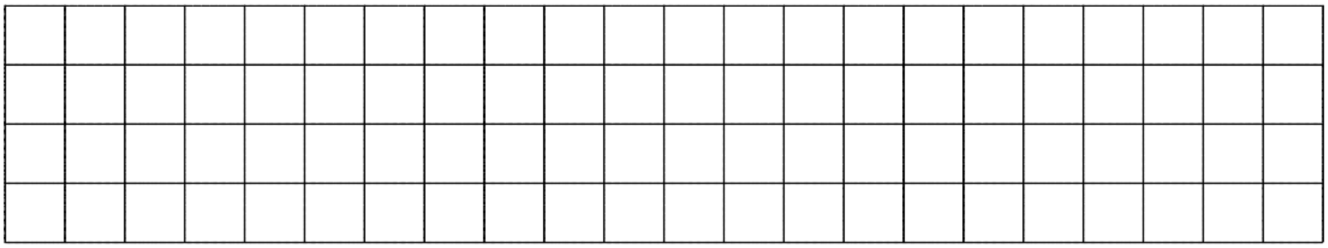
$$\begin{gathered} r_{2\text{Car}}=0+16(2.32-0)+\frac{1}{2}(4)(2.32-0)^2 \\ r_{2\text{Car}}=47.9 \text{ m} \end{gathered}$$

Solving the two velocity equations gives:

$$\begin{array}{ll} v_{2Car}=16+4(2.32) & v_{2Truck}=12+0(2.32) \\ v_{2Car}=25.3 m/s & v_{2Truck}=12 m/s \end{array}$$

> *A car, initially at rest, accelerates toward the west at 2.0 m/s^2. At the same time that the car starts, a truck, 350 m west of the car and moving at 16 m/s toward the east, starts to move slower, accelerating at 1.0 m/s^2. The car and truck pass safely.*

Motion Diagram



Motion Information

Object: Event 1:	Event 2:	Object: Event 1:	Event 2:
t_1 =	t_2 =	t_1 =	t_2 =
r_1 =	r_2 =	r_1 =	r_2 =
v_1 =	v_2 =	v_1 =	v_2 =
a_{12} =		a_{12} =	

Mathematical Analysis[10]

> *A car, initially traveling at 20 m/s to the east, accelerates toward the west at 2.0 m/s^2. At the same time that the car starts, a truck, 60 m west of the car and moving at 16 m/s toward the east, starts to move faster, accelerating at 1.0 m/s^2. It's a one-lane road and both drivers are too busy texting to notice each other.*

Motion Diagram

Motion Information

Object: Event 1:	Event 2:	Object: Event 1:	Event 2:
$t_1 =$	$t_2 =$	$t_1 =$	$t_2 =$
$r_1 =$	$r_2 =$	$r_1 =$	$r_2 =$
$v_1 =$	$v_2 =$	$v_1 =$	$v_2 =$
$a_{12} =$		$a_{12} =$	

Mathematical Analysis[11]

> *A helium balloon begins to rise from rest with an acceleration of 2.0 m/s^2 until it reaches a height of 50 m, and then continues upward at constant speed. However, a passenger in the balloon forgot their cell phone on the ground and threatens to jump from the balloon because of separation anxiety. Therefore a second balloon is launched 5.0 s after the first and accelerates upwards at 3.0 m/s^2 to reunite the passenger and his phone.*

Motion Diagram

Motion Information

Object: Event 1:	Event 2:	Event 3:	Object: Event 1:	Event 2:	Event 3:
$t_1 =$	$t_2 =$	$t_3 =$	$t_1 =$	$t_2 =$	$t_3 =$

Object: Event 1:	Event 2:	Event 3:	Object: Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =	t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =	r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =	v_1 =	v_2 =	v_3 =
a_{12} =	a_{23} =			a_{12} =	a_{23} =

Mathematical Analysis[12]

A car traveling at 16 m/s sees a truck 20 m ahead traveling at a constant speed of 12 m/s. After thinking it over for 0.80 s, the car starts to accelerate at 4.0 m/s^2 in an attempt to rear-end the truck. The truck driver has paid-up insurance so doesn't care. The car driver's motive is unknown.

Motion Diagram

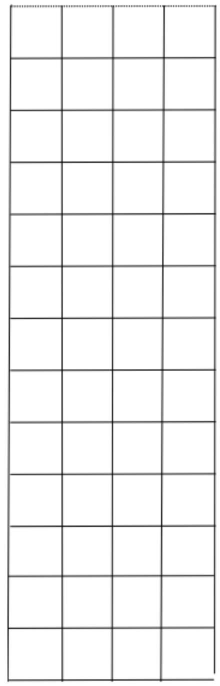
Motion Information

Object: Event 1:	Event 2:	Event 3:	Object: Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =	t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =	r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =	v_1 =	v_2 =	v_3 =
a_{12} =	a_{23} =			a_{12} =	a_{23} =

Mathematical Analysis[13]

A pole-vaulter lands on a cushion after a vault. Determine her acceleration as she sinks into the cushion ($a_{cushion}$) as a function of her velocity when she hits the cushion (v_i) and the distance she sinks into the cushion (d).

Motion Diagram



Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
v_1 =	v_2 =
a_{12} =	

Mathematical Analysis

Questions

If v_i= 0 m/s, what should $a_{cushion}$ equal? Does your function agree with this observation?

If d = 0 m, what should $a_{cushion}$ equal? Does your function agree with this observation?

What would result in a larger magnitude acceleration, hitting the cushion twice as fast or sinking one-half as far into the cushion?

> *The driver of an automobile suddenly sees an obstacle blocking her lane. Determine the total distance the car travels between seeing the obstacle and stopping (d) as a function of the initial velocity of the car (v_i) and the magnitude of the car's acceleration while stopping (a_s). Ignore the driver's reaction time.*

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =

Event 1:	v_1 =	Event 2:	v_2 =
	t_1 =	a_{12} =	t_2 =
	r_1 =		r_2 =
	v_1 =		v_2 =
		a_{12} =	

Mathematical Analysis

Questions

If vi= 0 m/s, what should d equal? Does your function agree with this observation?

If as = 0 m/s^2, what should d equal? Does your function agree with this observation?

If the car was traveling twice as fast how much further would the car travel before stopping?

> *The driver of an automobile suddenly sees an obstacle blocking his lane. Determine the total distance the car travels between seeing the obstacle and stopping (d) as a function of his reaction time (t_R), the initial velocity of the car (v_i), and the magnitude of the car's acceleration while stopping (a_s).*

Motion Diagram

Motion Information

Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_2 =
v_1 =	v_2 =	v_2 =
a_{12} =	a_{23} =	

Mathematical Analysis

Questions

If vi= 0 m/s, what should d equal? Does your function agree with this observation?

If as = 0 m/s^2, what should d equal? Does your function agree with this observation?

Does the car travel a greater distance during the reaction phase or the braking phase? Estimate necessary quantities assuming a panic stop from highway speeds.

> *Two automobiles are involved in a race over level ground. Both cars begin at rest. The Audi accelerates at A for T seconds and then travels at constant velocity for the rest of the race. The Buick accelerates at A/2 for 2T seconds and then travels at constant velocity for the rest of the race. Determine the distance (d) between the two cars, at the instant t = 2T, as a function of A and T.*

Motion Diagram

Motion Information

Object: Event 1:	Event 2:	Event 3:	Object: Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =	t_1 =	t_2 =	t_3 =

Object: Event 1: r_1 = v_1 =	Event 2: r_2 = v_2 =	Event 3: r_3 = v_3 =	Object: Event 1: r_1 = v_1 =	Event 2: r_2 = v_2 =	Event 3: r_3 = v_3 =
t_1 = a_{12} =	t_2 = a_{23} =	t_3 =	t_1 =	t_2 = a_{12} =	t_3 = a_{23} =
r_1 =	r_2 =	r_3 =	r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =	v_1 =	v_2 =	v_3 =
a_{12} =	a_{23} =			a_{12} =	a_{23} =

Mathematical Analysis

Questions

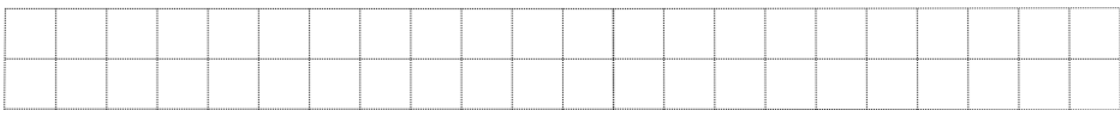
If A = 0 m/s^2, what should d equal? Does your function agree with this observation?

Assume the race takes longer than 2T to finish. What is the difference in speed between the two cars as they cross the finish line?

Assume the race takes longer than 2T to finish. What is the distance between the two cars as they cross the finish line?

> *Two automobiles are involved in a game of chicken. The two cars start from rest at opposite ends of a long, one-lane road. The Audi accelerates at A for T seconds and then travels at constant velocity. The Buick accelerates at A/2 for 2T seconds and then travels at constant velocity. Determine the elapsed time when the car's collide ($t_{collide}$) as a function of the length of the road (d), A and T. The collision takes place after both cars have reached constant velocity.*

Motion Diagram



Motion Information

Object: Event 1:	Event 2:	Event 3:	Object: Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =	t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =	r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =	v_1 =	v_2 =	v_3 =
a_{12} =	a_{23} =			a_{12} =	a_{23} =

Mathematical Analysis

Questions

If A = 0 m/s^2, what should $t_{collide}$ equal? Does your function agree with this observation?

If d = ∞, what should $t_{collide}$ equal? Does your function agree with this observation?

What is the minimum length of road needed to guarantee the two cars collide at top speed?

2.3: Dynamics

Concepts and Principles

Newton's First Law

Dynamics is the study of the cause of motion, or more precisely the cause of *changes* in motion. In the late 1600's Isaac Newton hypothesized that motion does not require a cause, rather *changes* in motion require causes. An object experiences a change in motion only when it interacts with some aspect of its surroundings. This bold hypothesis, referred to as Newton's First Law of Motion, is summarized by the idea that an object will maintain its state of motion, whether at rest or traveling at high speed, unless acted upon by some aspect of its surroundings.

Using the kinematic terminology developed in the last unit, this means that an object's velocity (state of motion) is constant unless it interacts with some outside agent. An external interaction is not necessary for an object to move, it is only necessary if the object's velocity changes. Thus, what is *caused* is not velocity, but acceleration. This concept is one of the most subtle, and complex, in all of physics.

Newton's Second Law

Newton also hypothesized that the sum total of all interactions with the external environment, which he termed *forces*, is directly proportional to the acceleration of the object. Moreover, the proportionality constant between the sum of all forces acting on an object and the acceleration of the object measures the "resistance" of an object to changes in its motion. This resistance to changes in motion is termed the *inertia*.

For example, an object with great inertia (quantified by a large proportionality constant) responds to the application of forces with a relatively small acceleration. An object with little inertia (a small proportionality constant) responds to the application of the same forces with a relatively large acceleration. The amount of inertia an object has is measured by the *inertial mass* of the object.

In summary, this relationship, known as Newton's Second Law of Motion, and can be written mathematically as:

$$\Sigma F = ma$$

where

- F is a force acting on the object from its surroundings, measured in Newtons (N),
- Σ (sigma) is a shorthand reminder to sum <u>all</u> of the forces acting on the object,
- and m is the mass of the object, measured in kilograms (kg).

The sum of all of the forces acting on an object will be referred to as the *total force* acting on the object.

Newton's Third Law

Newton's third great contribution to the study of dynamics was his vision of force, defined to be the interaction between an object and some aspect of its surroundings. Newton theorized that since objects *interact* with other objects in their surroundings, always in pairs, a certain symmetry exists in nature. The distinction between the actor and the acted-upon is arbitrary. It would be just as easy to switch focus and consider the object in the surroundings as the actedupon and the original object of interest the actor.

If nature exhibits this symmetry, then the force that one object exerts on another must *always* be equal in magnitude to the force that the second object exerts on the first. To speak of one object as exerting a force on another is to speak of only one-half of the picture. This idea, known as Newton's Third Law of Motion, is of central importance in the study of forces. In summary, objects *interact* with each other, and equal magnitude forces are exerted on each of the two objects interacting. A simplistic way of picturing this is the idea that you cannot touch something without being touched, and moreover that the harder you touch the harder you will be touched in return.

Investigating the dynamics of a situation involves the identification of all interactions an object experiences with other objects in its surroundings. To help in the identification of these interactions, and to use this information to better describe the ultimate motion of the object, a number of useful analysis tools are detailed below.

Analysis Tools

Drawing Free-Body Diagrams

The free-body diagram is by far the most important analysis tool for determining the interactions between an object and its surroundings. There are three distinct steps to creating a free-body diagram. Let's walk through the steps for the situation described below:

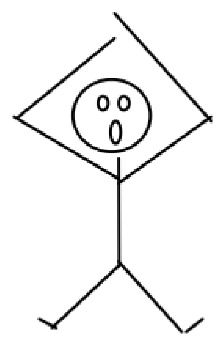
> *A child pulls herself up a rope using only her hands.*

1. Select the object you would like to study.

In this example, it is probably safe to assume that the object we would like to study is the child. However, depending on what we are investigating it may be the rope or even the ceiling we are interested in. Selecting the correct object to represent by a free-body diagram is a crucial step, especially in more complicated situations. With practice you will develop a knack for selecting the correct object to represent.

2. Draw a picture of the object of interest free from all other objects.

Notice that the rope does not appear in the diagram. As the name *free-bod*y implies, the object is drawn free of all external constraints.



3. Indicate on the diagram all interactions of the object with its environment.

Now comes the most difficult part of constructing a free-body diagram. It is crucial not to miss an interaction. If an interaction is overlooked, than the total of the forces will be incorrect, and the acceleration will be incorrect, and your entire analysis will be incorrect.

Also, only the portion of the interaction that acts *on the girl* should be indicated on a freebody diagram of the girl. For example, she is interacting with the rope. The rope's action *on the girl* will be indicated, not the action of the girl *on the rope*.

To aid in the search for interactions, we will divide the types of interactions that the girl can be part of into two types, non-contact and contact.

- Non-Contact Interactions

  Non-contact interactions include all interactions that can occur between the girl and objects in her surroundings that do not require direct physical contact between the two objects. Non-contact interactions include the interaction of the girl with the gravitational and electromagnetic fields in her vicinity. (How these fields are created and how they can affect the girl will slowly be incorporated into our physics model.)

  At the current level of complexity, however, the only non-contact interaction you need worry about is the interaction of the girl with the gravitational field created by the earth, which we will simply term the force of gravity. The direction of this force is down, toward the center of the earth.
- Contact Interactions

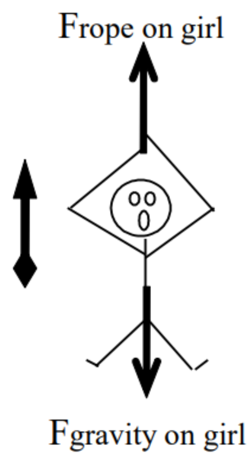
  Contact interactions occur at every point on the girls' body in which she is in direct physical contact with an external object. The most obvious of these is the rope. The girl is in contact with the rope, so the rope and girl exert forces on each other. These forces are equal in magnitude. Remember, however, that it is only the force exerted *on* the girl that is indicated on a free-body diagram of the girl. The location of this force is at the girls' hands, and the direction of this force is up. (The direction of this force cannot be down, because that would imply that the rope is *pushing* the girl, as opposed to *pulling* her. It is impossible for a rope to push someone, unless it is a very stiff rope. Very stiff ropes will be called rods.)

The only other objects to actually make contact with the girl are air molecules. The air molecules interact with the girl on all sides, each exerting a small force directly inward, perpendicular to the girl's body. Although each of these forces is very small, their sum is not always small. For example, if the girl were falling freely from an airplane the vast numbers of air molecules colliding with the girl from underneath, versus the rather small number colliding from above, and the strength of these collisions, would add to a very large force acting upward on the girl. This force could easily be equal in magnitude to the force of gravity on the girl. The force of air molecules on an object, referred to as air resistance, is often ignored in analyzing scenarios simply because of the difficulty of dealing with the complexity.

However, in many cases the effects of the air molecules are negligible compared to the other forces acting on the object. This is the case with the girl climbing the rope. The forces exerted by the air molecules are probably very close to being uniformly distributed around the girl's surface. Thus, for every air molecule pushing her to the right, there is probably an air molecule pushing her to the left. These forces will add to a total force very close to zero.

A correct free-body diagram for the girl is shown below:

Since a coordinate system is crucial for translating motion diagrams and free-body diagrams into mathematical relationships, a coordinate system has been added to the free-body diagram. It is always a good idea to use the same coordinate system for both the free-body diagram and the motion diagram.



Calculating the Force of Gravity near the Surface of the Earth

In addition to creating the three laws of motion mentioned earlier, Newton also postulated the Law of Universal Gravitation. This law states that every object of mass in the universe creates a gravitational field, and every object of mass in the universe senses and interacts with every other objects' field. That's an awful lot of forces! To try to identify and estimate the magnitude of all of these forces on an object near the surface of the earth would be a lifelong task.

Luckily, the strength of the gravitational field depends on the mass of the object producing the field, and inversely as the square of the distance from the object. The more massive the object, the stronger the field. The closer the object, the stronger the field. Thus, since the earth is much more massive than any other nearby object, when creating free-body diagrams for objects near the surface of the earth we can safely include just the gravitational field due to the earth, ignoring all the other, relatively small, gravitational fields.

The magnitude of the gravitational field of a massive object (g) depends on the mass of the object (M), the distance from the center of the object (d), and a constant called, appropriately, the gravitational constant (G). The relationship is:

$$g=\frac{GM}{d^2}$$

Near the surface of the earth, the gravitational field has a magnitude of approximately 9.8 N/kg. Although the gravitational field strength varies with the distance from the surface of the earth, we will ignore this slight variation unless explicitly told to include its effects.

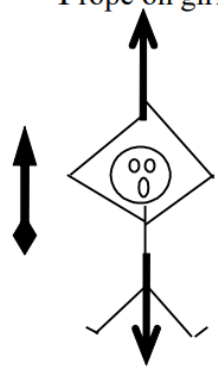
The gravitational force felt by a massive object in the presence of a gravitational field is given by the product of the object's mass and the magnitude of the gravitational field at the location of the object:

$$F_{\text{gravity}} = mg$$

Applying Newton's Second Law

Let's return to the scenario under investigation and make some quantitative information more explicit. Then, we can attempt to further investigate the situation using Newton's Second Law.

> *A 30 kg child pulls herself up a rope at approximately constant speed using only her hands.*



Newton's Second Law states:

$$\Sigma F = ma$$

ΣF refers to the sum of all of the forces acting on the girl, the force of the rope (which is positive in our coordinate system) and the force of gravity (which is negative in our coordinate system). Thus,

$$F_{\text{rope}} - F_{\text{gravity}} = ma$$

Since m = 30 kg, and a = 0 m/s^2 (since she climbs at constant speed), the equation becomes:

$$F_{\text{rope}} - F_{\text{gravity}} = 0$$

By Newton's relationship for the force of gravity:

$$\begin{aligned} F_{\text{gravity}} &= mg \\ F_{\text{gravity}} &= (30kg)\left(9.8\frac{N}{kg}\right) \\ F_{\text{gravity}} &= 294N \end{aligned}$$

Therefore:

$$\begin{aligned} F_{\text{rope}} - 294N &= 0 \\ F_{\text{rope}} &= 294N \end{aligned}$$

Thus, Newton's Second Law allows us to determine the force with which the rope pulls on the girl. Of course, by Newton's Third Law, the force with which the girl pulls on the rope is equal in magnitude, so the girl exerts a 294 N force on the rope.

If the girl had not climbed the rope at approximately constant speed her acceleration would have to be determined, either from an explicit mention in the description or through using the kinematic relations developed in the last unit, and then inserted into

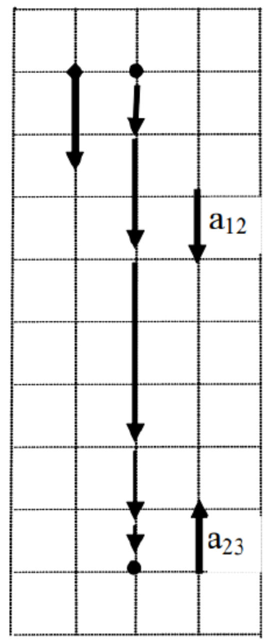
Newton’s Second Law. If her acceleration had been directed upwards (positive) the force of the rope on the girl would have had to be larger. If her acceleration had been directed downwards (negative) the force of the rope on the girl would have had to be smaller.

Analyzing a More Complex Scenario

Before you start analyzing dynamics scenarios on your own, let's work our way through a more complex scenario.

> *To practice falling, a 55 kg pole-vaulter falls from rest off of a wall 5.0 m above a foam cushion. The pole-vaulter sinks about 1.8 m into the cushion before stopping.*

Before we begin analyzing the forces acting on this pole-vaulter, I think we should try to get a handle on the kinematics of the situation. Therefore, our first step in analyzing this situation is to draw a motion diagram and tabulate motion information.

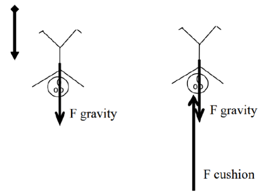


Event 1: The instant she leaves the wall.	Event 2: The instant she hits the cushion.	Event 3: The instant she comes to rest.
$t_1 = 0$ s	$t_2 =$	$t_3 =$
$r_1 = 0$ m	$r_2 = 5.0$ m	$r_3 = 6.8$ m
$v_1 = 0$ m/s	$v_2 =$	$v_3 = 0$ m/s
$a_{12} =$		$a_{23} =$

Notice that between the instant she leaves the wall and the instant she hits the cushion the acceleration is positive (down), while between the instant she hits the cushion and the instant she comes to rest the acceleration is negative (up). Thus, when applying the kinematic relationships and Newton's Second Law we will have to be careful not to confuse variables between these two intervals.

What should jump out at you is the fact that this kinematic scenario cannot be solved! There are *five* unknown kinematic quantities and only *four* kinematic equations. Something else is needed in order to complete the kinematic description. Let's look at the forces acting on the pole-vaulter to see if we can figure out another piece of kinematic information.

Between the first two instants, the only force acting on the pole-vaulter is the force of gravity. Once she hits the cushion, however, there are two forces acting on the pole-vaulter, the force of gravity and the force of the cushion. Correct free-body diagrams for these *two distinct phases of her motion* are given below.



The diagram on the left corresponds to the first time interval and the diagram on the right to the second time interval. For *each* of these free-body diagrams, I will apply Newton's Second Law:

$$\begin{aligned} \Sigma F &= m a_{12} \\ +F_{\text{gravity}} &= (55 \text{ kg}) a_{12} \end{aligned} \qquad \begin{aligned} \Sigma F &= m a_{23} \\ +F_{\text{gravity}} - F_{\text{cushion}} &= (55 \text{ kg}) a_{23} \end{aligned}$$

Since

$$\begin{aligned} F_{\text{gravity}} &= m g \\ F_{\text{gravity}} &= (55 \text{ kg}) \left(9.8 \frac{\text{N}}{\text{kg}}\right) \\ F_{\text{gravity}} &= 539 \text{ N} \end{aligned}$$

$$\begin{aligned} 539 N &= (55 \text{ kg}) a_{12} \\ a_{12} &= 9.8 \text{ m/s}^2 \end{aligned} \qquad 539 N - F_{\text{cushion}} = (55 \text{ kg}) a_{23}$$

Thus, from Newton's Second Law, we know that the acceleration *during the fall* is 9.8 m/s^2. (We still don't know what the acceleration was during the impact portion of the motion.) Substituting this value back into the motion table yields:

Event 1: The instant she leaves the wall.	Event 2: The instant she hits the cushion.	Event 3: The instant she comes to rest.
t_1 = 0 s	t_2 =	t_3 =
r_1 = 0 m	r_2 = +5.0 m	r_3 = +6.8 m
v_1 = 0 m/s	v_2 =	v_3 = 0 m/s
a_{12} = 9.8 m/s^2		a_{23} =

This is now solvable, using strictly kinematics, for the four remaining unknowns. Try to do the math on your own, and compare your result to:

Event 1: The instant she leaves the wall.	Event 2: The instant she hits the cushion.	Event 3: The instant she comes to rest.
t_1 = 0 s	t_2 = 1.0 s	t_3 = 1.36 s
r_1 = 0 m	r_2 = +5.0 m	r_3 = +6.8 m
v_1 = 0 m/s	v_2 = +9.9 m/s	v_3 = 0 m/s
a_{12} = 9.8 m/s^2		a_{23} = -27 m/s^2

We now have a complete kinematic description of the motion.

Returning to Newton's Second Law for the impact portion of the motion,

$$\begin{aligned} 539 N - F_{\text{cushion}} &= (55 \text{ kg}) a_{23} \\ 539 N - F_{\text{cushion}} &= (55 \text{ kg}) \left(-27 \text{ m/s}^2\right) \\ 539 N - F_{\text{cushion}} &= -1485 \text{ N} \\ F_{\text{cushion}} &= 2024 \text{ N} \end{aligned}$$

The cushion exerts a force of about 2000 N on the pole-vaulter to stop her fall.

Hints and Suggestions

The Magnitude of the Gravitational Field

Quite often, students make a pair of mistakes when dealing with the magnitude of the gravitational field, g.

1. 'g' is never negative.

Since g is the *magnitude* of the gravitational field, it cannot be a negative number. As a magnitude, *it does not have a direction associated with i*t! Resist all temptation to replace 'g' with the value "-9.8 N/kg"!

Part of the confusion lies with the fact that the gravitational *field* does have an associated direction. The gravitational field of the earth is directed downward toward the center of the earth. Even so, the gravitational field is *not* negative. Negative only makes sense relative to a coordinate system, and since you are always free to choose any system you want, the gravitational field is just as likely to be oriented in the positive as the negative direction

2. 'g' is not an acceleration.

Often, students have learned that 'g' is the "acceleration due to gravity." However, as I sit here in a chair writing this book, the force of gravity *is* acting on me and I am most definitely *not* accelerating at 9.8 m/s^2! In fact, the force of gravity has acted on me for every second of my life and only very rarely have I accelerated at 9.8 m/s^2. 'g' measures the strength of the gravitational field. As such, it is related to the gravitational force, which, like all forces, can give raise to accelerations. However, it is the *total* force acting on an object that determines its acceleration, not simply the force of gravity.

It is true that the units of 'g', N/kg, are also the units of acceleration, since a Newton is defined to be a kg m/s^2. It is also true that in a *very specific scenario*[2], when the only force acting on an object is the force of gravity, the magnitude of the object's acceleration is numerically equal to 'g'. However, there are also very specific scenarios in which the acceleration of an object is numerically equal to 4.576 m/s^2, or 62.31452 m/s^2. The strength of physics is its ability to analyze diverse scenarios with the same small set of tools, not to develop specialized tools tailored to every different specific scenario. Newton's Second Law will always allow you to determine an object's acceleration, whether the force of gravity acts alone or not.

> **Note**
>
> 2 When the only force acting on an object is the force of gravity, the situation is termed *freefall*.

Newton's Third Law

Many physics students have heard the saying, "For every action there is an equal and opposite re-action." I was forced to memorize this statement in a middle-school science class, and was told it was called Newton's Third Law. I'm sure I had no idea what it really meant. It states that there is a *reaction* to every action, which seems to imply the "action" happens first. This isn't what the law means. There really is no separation or possible distinction between action and reaction. A better way to look at it is that there is an *interaction* between two objects, and the two "sides" of this interaction experience exactly the same force. Of course, the *effect* of this mutually symmetrical force acting on the two objects need not be identical.

As a test of your understanding of Newton's Third Law, try to answer the following question:

> **Note**
>
> *As you drive along the highway, a mosquito splats against your car windshield. During the collision between the mosquito and the car,*
>
> *a. the force on the mosquito was greater in magnitude than the force on the car.*
>
> *b. the force on the car was greater in magnitude than the force on the mosquito.*
>
> *c. the force on the mosquito was equal in magnitude to the force on the car.*
>
> *d. it is impossible to determine the relative sizes of the forces without more information.*

As strange as it may seem, the correct answer is 'c'. The forces exerted on the mosquito and the car are equal in magnitude. In the terminology used in this chapter, the mosquito and car interact (probably an unpleasant interaction for the mosquito), and in an interaction the two agents involved exert equal forces on each other.

However, obviously *something* is different about the interaction from the mosquito's perspective. What is different is not the force acting on the mosquito but rather its acceleration. Although the forces acting on the mosquito and car are the same, the mosquito's acceleration is *much greater* than the car's acceleration because the mosquito's mass is *much smaller* than the car's mass. The acceleration of the car is so small that it is not even noticed by the driver, while the acceleration of the mosquito is certainly noticed by the mosquito!

Activities

Construct free-body diagrams for the objects described below.

a. When throwing a ball vertically upward, my hand moves through a distance of about 1.0 m before the ball leaves my hand. The 0.80 kg ball reaches a maximum height of about 20 m above my hand.

while the ball is in my handafter the ball leaves my hand

b. To practice falling, a 55 kg pole-vaulter falls off of a wall 6.0 m above a 2.0 m thick foam cushion resting on the ground. However, he misses the cushion.

while falling through the airwhile being stopped by the ground

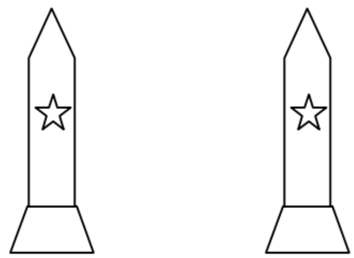
c. A decorative light fixture in an elevator consists of a 2.0 kg light suspended by a cable from the ceiling of the elevator. From this light, a separate cable suspends a second 0.80 kg light.

the top light *the bottom light*

Construct free-body diagrams for the objects described below.

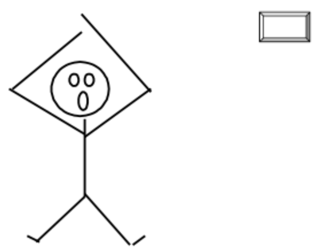
a. A 4000 kg rocket's engine produces a thrust of 70,000 N for 15 s. The rocket is fired vertically upward.

while the engine is firing *after the engine turns off*



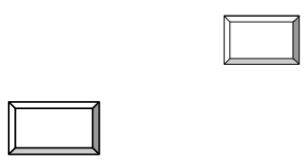
b. Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. An 84 kg block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the 84 kg block is released, the student is pulled up to his dorm room.

the student *the block*



c. A 1.0 kg block is stacked on top of a 2.0 kg block on the floor of an elevator moving downward at constant speed.

the 1.0 kg block *the 2.0 kg block*

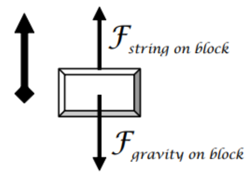


A block hangs from the ceiling of an elevator by a string. For each of the following situations, circle the correct relationship symbol between the magnitude of the force of the string on the block and the magnitude of the force of gravity on the block and explain your reasoning.

a. The elevator is at rest.

$$F_{\text{string on block}} \; > \; = \; < \; ? \; F_{\text{gravity on block}}$$

Explanation:



Since the block is not accelerating, the two forces acting on it must be equal in magnitude.

b. The elevator is moving upward at an constant speed.

$$F_{\text{string on block}} \; > \; \boxed{=} \; < \; ? \quad F_{\text{gravity on block}}$$

Explanation:

Since the block is still not accelerating, the two forces acting on it must be equal in magnitude.

c. The elevator is moving downward at a decreasing speed.

$$F_{\text{string on block}} \; \boxed{>} \; = \; < \; ? \quad F_{\text{gravity on block}}$$

Explanation:

Since the block is accelerating upward, the force directed upward (the force of the string) must be larger than the force directed downward (the force of gravity).

d. The elevator is moving upward at an increasing speed.

$$F_{\text{string on block}} \; \boxed{>} \; = \; < \; ? \quad F_{\text{gravity on block}}$$

Explanation:

The block is accelerating upward, so the force directed upward must be larger than the force directed downward

A man stands on a bathroom scale inside of an elevator. For each of the following situations, circle the correct relationship symbol between the magnitude of the force of the scale on the man and the magnitude of the force of gravity on the man and explain your reasoning.

a. The elevator is at rest.

$$F_{\text{scale on man}} \quad > \; = \; < \; ? \quad F_{\text{gravity on man}}$$

Explanation:

b. The elevator is moving downward at a constant speed.

$$F_{\text{scale on man}} \quad > \; = \; < \; ? \quad F_{\text{gravity on man}}$$

Explanation:

c. The elevator is moving downward at a increasing speed.

$$F_{\text{scale on man}} \quad > \; = \; < \; ? \quad F_{\text{gravity on man}}$$

Explanation:

d. The elevator is moving upward at a decreasing speed.

$$F_{\text{scale on man}} \quad > \; = \; < \; ? \quad F_{\text{gravity on man}}$$

Explanation:

Two blocks are stacked on top of each other on the floor of an elevator. For each of the following situations, circle the correct relationship symbol between the two force magnitudes and explain your reasoning.

a. The elevator is moving downward at a constant speed.

$$F_{\text{bottom block on top block}} \quad > \; = \; < \; ? \quad F_{\text{top block on bottom block}}$$

Explanation:

$$F_{\text{bottom block on top block}} \quad > \; = \; < \; ? \quad F_{\text{gravity on top block}}$$

Explanation:

b. The elevator is moving downward at an increasing speed.

$$F_{\text{bottom block on top block}} \quad > \; = \; < \; ? \quad F_{\text{top block on bottom block}}$$

Explanation:

$$F_{\text{bottom block on top block}} \quad > \; = \; < \; ? \quad F_{\text{gravity on top block}}$$

Explanation:

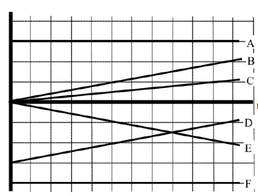
c. The elevator is moving upward at a decreasing speed.

$$F_{\text{bottom block on top block}} \quad > \; = \; < \; ? \quad F_{\text{top block on bottom block}}$$

Explanation:

$$F_{\text{bottom block on top block}} \quad > \; = \; < \; ? \quad F_{\text{gravity on top block}}$$

Explanation:



a. If the graph is of position vs. time, rank these graphs on the basis of the total force acting on the object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

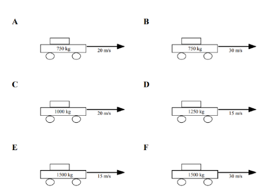
b. If the graph is of velocity vs. time, rank these graphs on the basis of the total force acting on the object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six automobiles traveling at constant velocity. The automobiles have different masses and velocities. Rank these automobiles on the basis of the magnitude of the total force acting on them.

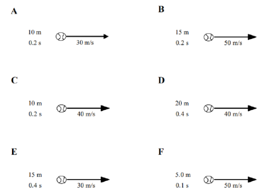


Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six identical baseballs shortly after being thrown. At the instant shown, the baseball's velocity is indicated, along with the distance the ball has traveled and the elapsed time since leaving the thrower's hand. Rank these baseballs on the basis of the magnitude of the force of the thrower's hand currently acting on them.

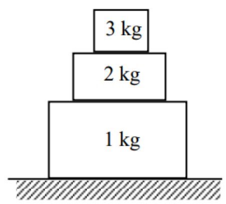


Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are three blocks stacked on top of each other at rest. Rank the magnitude of the forces referred to below from largest to smallest.



A The force of the 3 kg block on the 2 kg block

B The force of the 2 kg block on the 3 kg block

C The force of the 3 kg block on the 1 kg block

D The force of the 1 kg block on the 3 kg block

E The force of the 2 kg block on the 1 kg block

F The force of the 1 kg block on the 2 kg block

G The force of the 1 kg block on the floor

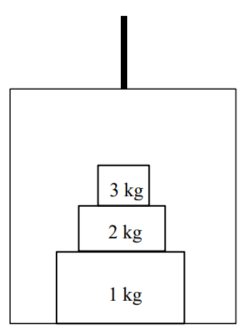
H The force of the floor on the 1 kg block

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are three blocks stacked on top of each other inside of an elevator moving upward at increasing speed. Rank the magnitude of the forces referred to below from largest to smallest.



A The force of the 3 kg block on the 2 kg block

B The force of the 2 kg block on the 3 kg block

C The force of the 3 kg block on the 1 kg block

D The force of the 1 kg block on the 3 kg block

E The force of the 2 kg block on the 1 kg block

F The force of the 1 kg block on the 2 kg block

G The force of the 1 kg block on the floor of the elevator

H The force of the floor of the elevator on the 1 kg block

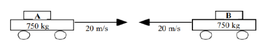
Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

For each of the collisions illustrated below, circle the correct relationship symbol between the magnitude of the force of car A on car B and the magnitude of the force of car B on car A and explain your reasoning.

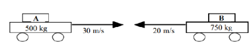
a.



$$F_{\text{car } A \text{ on car B}} \quad > \quad = \quad < \quad ? \quad F_{\text{car B on car A}}$$

Explanation:

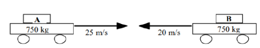
b.



$$F_{\text{car } A \text{ on car B}} \quad > \quad = \quad < \quad ? \quad F_{\text{car B on car A}}$$

Explanation:

c.

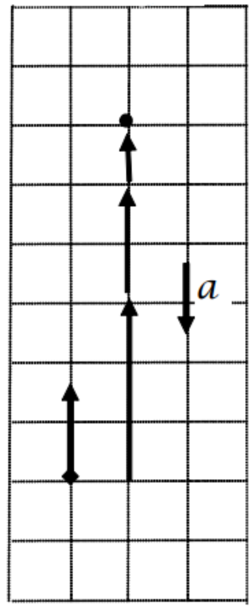


$$F_{\text{car } A \text{ on car B}} \quad > \quad = \quad < \quad ? \quad F_{\text{car B on car A}}$$

Explanation:

A 100 kg man concerned about his weight decides to weigh himself in an elevator. He stands on a bathroom scale in an elevator that is moving upward at 3.0 m/s. As the elevator reaches his floor, it slows to a stop over a distance of 2.0 m.

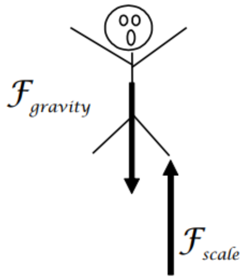
Motion Diagram



Motion Information

Event 1: The instant the elevator begins to slow.	Event 2: The instant it stops.
$t_1 = 0$ s	$t_2 =$
$r_1 = 0$ m	$r_2 = +2.0$ m
$v_1 = 3.0$ m/s	$v_2 = 0$ m/s
$a_{12} =$	

Free-Body Diagram



Mathematical Analysis

Since there are only two unknown kinematic quantities, we can determine them by our two kinematic equations.

$$v_2 = v_1 + a_{12}(t_2 - t_1)$$
$$0 = 3 + a_{12}t_2 - 0)$$
$$a_{12} = \frac{-3}{t_2}$$

Now substitute this expression into the other equation:

$$\begin{aligned}
&r_2 = r_1 + v_1\left(t_2 - t_1\right) + \tfrac{1}{2}a_{12}\left(t_2 - t_1\right)^2 \\
&2 = 3t_2 + \tfrac{1}{2}a_{12}t_2^2 \\
&2 = 3t_2 + \tfrac{1}{2}\left(\tfrac{-3}{t_2}\right)t_2^2 \\
&2 = 3t_2 - 1.5t_2 \\
&2 = 1.5t_2 \\
&t_2 = 1.33s
\end{aligned}$$

Substitute this result back into the original equation:

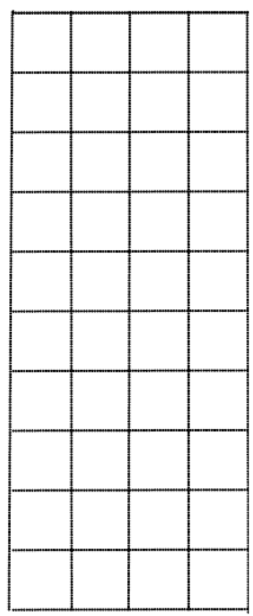
$$\begin{aligned}
&a_{12} = \tfrac{-3}{1.33} \\
&a_{12} = -2.25\ \mathrm{m/s^2}
\end{aligned}$$

Now apply Newton’s Second Law to the man:

$$\begin{aligned}
&\Sigma F = ma \\
&F_{\text{scale}} - F_{\text{gravity}} = (100)(-2.25) \\
&F_{\text{scale}} - (100)(9.8) = -225 \\
&F_{\text{scale}} = 755N
\end{aligned}$$

A 40 kg child is hanging from a rope by her hands. She exerts a burst of strength and 2.0 s later is traveling at 1.4 m/s up the rope.

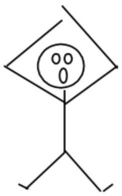
Motion Diagram



Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
v_1 =	v_2 =
a_{12} =	

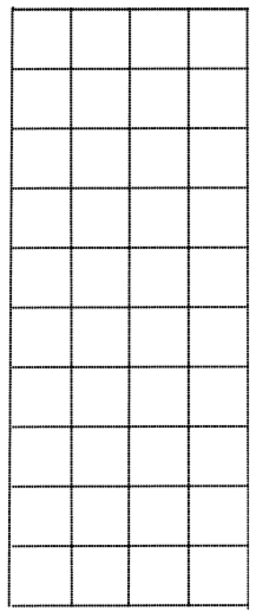
Free-Body Diagram



Mathematical Analysis[14]

A 55 kg pole-vaulter, just before touching the cushion on which she lands after a jump, is falling downward at a speed of 10 m/s. The pole-vaulter sinks about 2.0 m into the cushion before stopping.

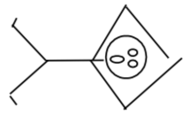
Motion Diagram



Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
v_1 =	v_2 =
a_{12} =	

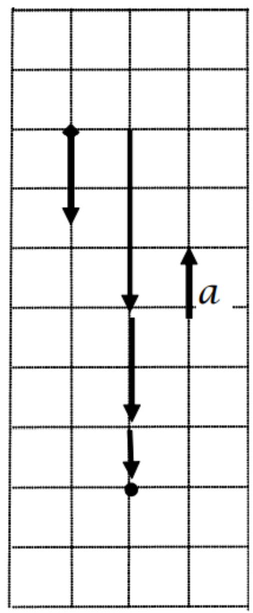
Free-Body Diagram



Mathematical Analysis[15]

A decorative light fixture in an elevator consists of a 2.0 kg light suspended by a cable from the ceiling of the elevator. From this light, a separate cable suspends a second 0.80 kg light. The elevator is moving downward at 4.0 m/s when someone presses the emergency stop button. The elevator comes to rest in 1.2 seconds.

Motion Diagram

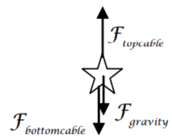


Motion Information

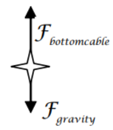
Event 1: The stop button is pressed	Event 2: The elevator stops.
$t_1 = 0$ s	$t_2 = 1.2$ s
$r_1 = 0$ m	$r_2 =$
$v_1 = 4.0$ m/s	$v_2 = 0$ m/s
$a_{12} =$	

Free-Body Diagram

the top light



the bottom light



Mathematical Analysis

Since there are only two unknown kinematic quantities, we can determine them by our two kinematic equations. Note that both lights have the same kinematic description.

$$0 = 4 + a_{12}(1.2 - 0)$$
$$a_{12} = -3.33 \text{ m/s}^2$$

$$r_2 = 0 + 4(1.2 - 0) + \frac{1}{2}(-3.33)(1.2 - 0)^2$$
$$r_2 = 2.4m$$

Now apply Newton's Second Law to the two lights:

bottom light

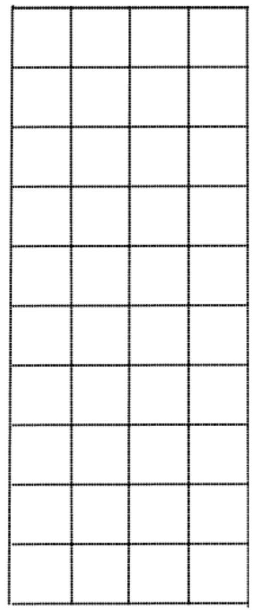
$$-F_{\text{bottomcable}} + F_{\text{gravity}} = ma$$
$$-F_{\text{bottomcable}} + (0.8)(9.8) = (0.8)(-3.33)$$
$$F_{\text{bottomcable}} = 10.5N$$

top light

$$-F_{\text{topcable}} + F_{\text{bottomcable}} + F_{\text{gravity}} = ma$$
$$-F_{\text{topcable}} + 10.5 + (2.0)(9.8) = (2.0)(-3.33)$$
$$F_{\text{topcable}} = 36.8N$$

A decorative light fixture in an elevator consists of a 2.0 kg light suspended by a cable from the ceiling of the elevator. From this light, a separate cable suspends a second 0.80 kg light. The elevator is moving downward at 4.0 m/s when someone presses the emergency stop button. During the stop, the upper cable snaps. The elevator engineer says that the cable could withstand a force of 40 N without breaking.

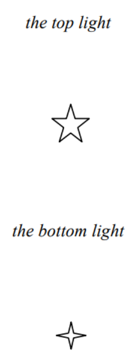
Motion Diagram



Motion Information

Event 1:	Event 2:
$t_1 =$	$t_2 =$
$r_1 =$	$r_2 =$
$v_1 =$	$v_2 =$
$a_{12} =$	

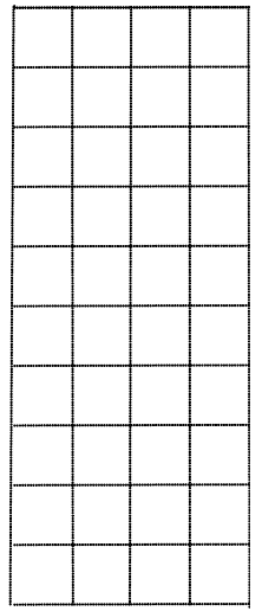
Free-Body Diagram



Mathematical Analysis[16]

A 70 kg student is 120 m above the ground, moving upward at 3.5 m/s, while hanging from a rope hanging from a 280 kg helium balloon. The lift on the balloon due to the buoyant force is 3000 N.

Motion Diagram



Motion Information

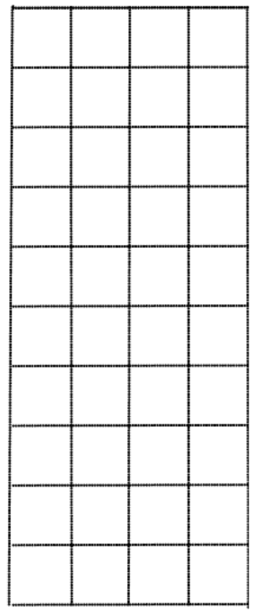
Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
v_1 =	v_2 =
a_{12} =	

Free-Body Diagram

Mathematical Analysis[17]

To practice falling, a 55 kg pole-vaulter falls off of a wall 6.0 m above a foam cushion. The pole-vaulter sinks about 1.4 m into the cushion before stopping.

Motion Diagram

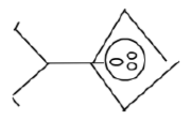


Motion Information

Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_{12} =		a_{23} =

Free-Body Diagram

before hitting cushion

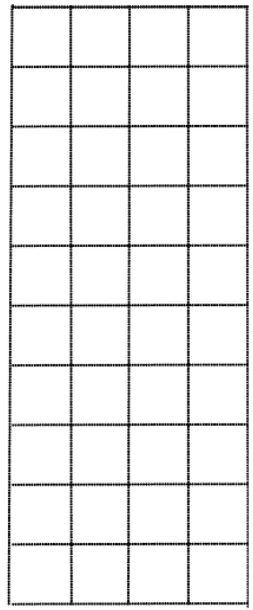


after hitting cushion

Mathematical Analysis[18]

To practice falling, a 55 kg pole-vaulter falls off of a wall 6.0 m above a 2.0 m thick foam cushion resting on the ground. However, he misses the cushion. The pole-vaulter sinks about 0.10 m into the ground before stopping.

Motion Diagram

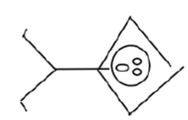


Motion Information

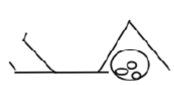
Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_{12} =		a_{23} =

Free-Body Diagram

before hitting cushion



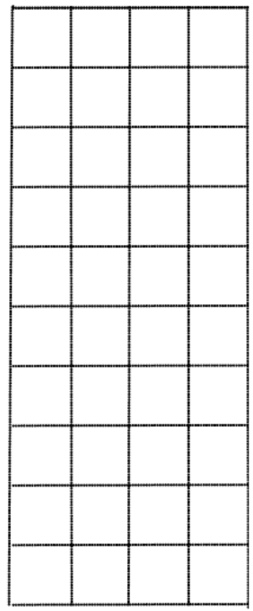
after hitting ground



Mathematical Analysis[19]

When throwing a 0.80 kg ball vertically upward, my hand moves through a distance of about 1.0 m before the ball leaves my hand. The ball leaves my hand at 35 m/s.

Motion Diagram



Motion Information

Event 1:	Event 2:	Event 3: Ball reaches its max height
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_{12} =		a_{23} =

Free-Body Diagram

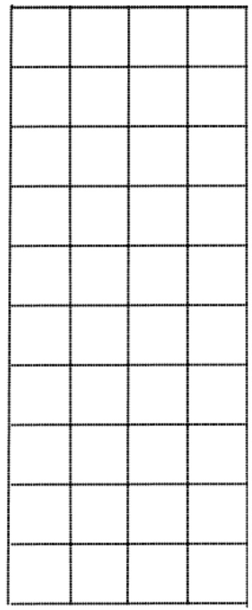
before ball leaves hand

after ball leaves hand

Mathematical Analysis[20]

A 4000 kg rocket's engine produces a thrust of 70,000 N for 15 s. The rocket is fired vertically upward.

Motion Diagram



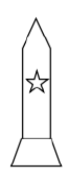
Motion Information

Event 1:	Event 2:	Event 3: Rocket reaches its max height
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_{12} =		a_{23} =

Free-Body Diagram

before engine turns off

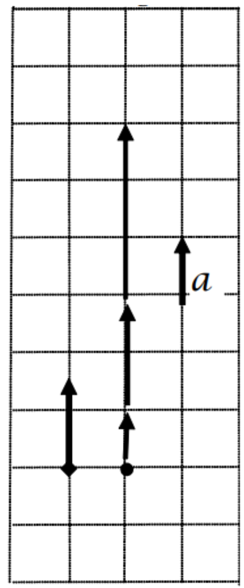
after engine turns off



Mathematical Analysis[21]

Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. An 84 kg block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the 84 kg block is released, the student is pulled up to his dorm room, 8.0 m off the ground.

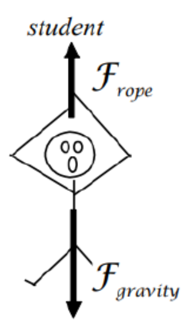
Motion Diagram

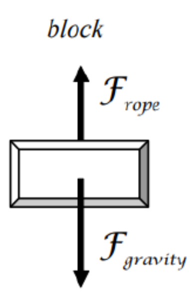


Motion Information

Object: Student	
Event 1: Block is released	Event 2: Student reaches room.
t_1 = = 0 s	t_2 =
r_1 = 0 m	r_2 = +8.0 m
v_1 = 0 m/s	v_2 =
a_{12} =	

Free-Body Diagram





Mathematical Analysis

Since there are three kinematic variables, we will have to analyze the forces first:

student

$$F_{\text{rope}} - F_{\text{gravity}} = ma$$
$$F_{\text{rope}} - (80)(9.8) = 80a_{\text{student}}$$

block

$$F_{\text{rope}} - F_{\text{gravity}} = ma$$
$$F_{\text{rope}} - (84)(9.8) = 84a_{\text{block}}$$

Because they are tied together, the acceleration of the student and the acceleration of the block are equal in magnitude, but opposite in direction. Therefore, $a_{\text{block}} = -a_{\text{student}}$.

$$F_{\text{rope}} - 784 = 80a_{\text{student}}$$
$$F_{\text{rope}} = 80a_{\text{student}} + 784$$

$$F_{\text{rope}} - 823 = 84\left(-a_{\text{student}}\right)$$
$$\left(80a_{\text{student}} + 784\right) - 823 = -84a_{\text{student}}$$
$$164a_{\text{student}} = 39$$
$$a_{\text{student}} = 0.24 \text{ m/s}^2$$

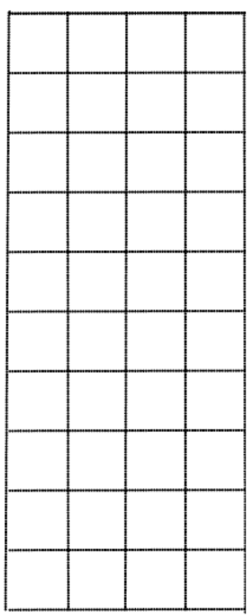
We can now complete the kinematic description of the student's motion:

$$8 = 0 + 0\left(t_2 - 0\right) + \frac{1}{2}(0.24)\left(t_2 - 0\right)^2$$
$$t_2 = 8.18s$$

$$v_2 = 0 + 0.24(8.18 - 0)$$
$$v_2 = 1.96 \text{ m/s}$$

Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. A block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room, 8 m off the ground, in a time of 1.8 s.

Motion Diagram

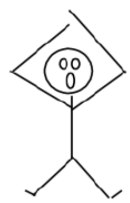


Motion Information

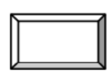
Object:	
Event 1:	Event 2:
t_1 = =	t_2 =
r_1 =	r_2 =
v_1 =	v_2 =
a_{12} =	

Free-Body Diagram

student



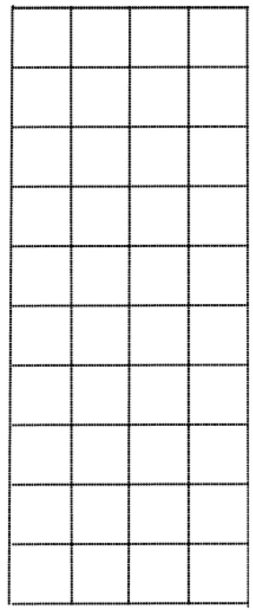
block



Mathematical Analysis[22]

Tired of walking down the stairs, an 80 kg engineering student designs an ingenious device for reaching the ground from his third floor dorm room. A block, at rest on the ground, is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the student steps out of the window, she falls the 8 m to the ground in a time of 1.8 s.

Motion Diagram

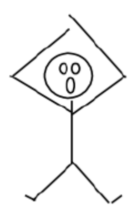


Motion Information

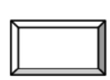
Object:	
Event 1:	Event 2:
t_1 = =	t_2 =
r_1 =	r_2 =
v_1 =	v_2 =
a_{12} =	

Free-Body Diagram

student

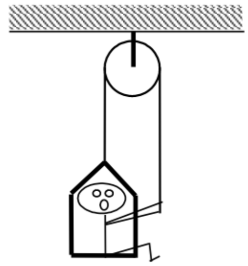


block

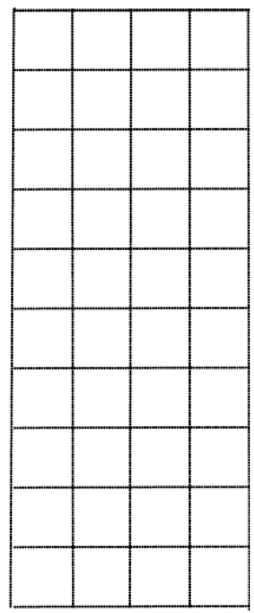


Mathematical Analysis[23]

A 60 kg student lifts herself from rest to a speed of 1.5 m/s in 2.1 s. The chair has a mass of 35 kg.



Motion Diagram



Motion Information

Object:	
Event 1:	Event 2:
t_1 = =	t_2 =
r_1 =	r_2 =
v_1 =	v_2 =
a_{12} =	

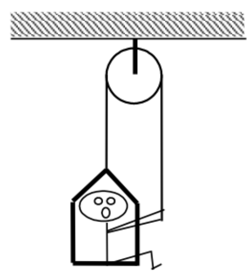
Free-Body Diagram

student

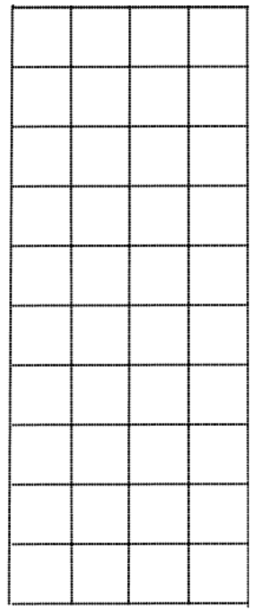
chair

Mathematical Analysis[24]

A 60 kg student lowers herself down 40 m at a constant speed of 1.0 m/s. The chair has a mass of 35 kg.



Motion Diagram



Motion Information

Object:	
Event 1:	Event 2:
t_1 = =	t_2 =
r_1 =	r_2 =
v_1 =	v_2 =
a_{12} =	

Free-Body Diagram

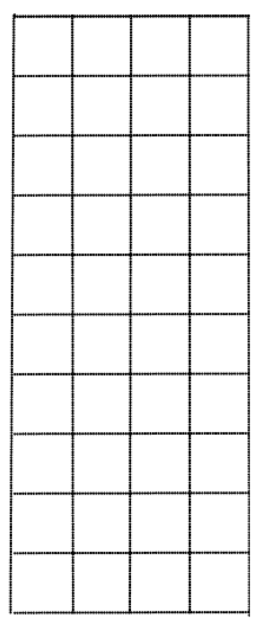
student

chair

Mathematical Analysis[25]

A man of mass m, concerned about his weight, decides to weigh himself in an elevator. He stands on a bathroom scale in an elevator moving upward at speed v. As the elevator reaches his floor, it slows to a stop over a distance, d. Determine the reading on the bathroom scale (F_{scale}) as a function of m, v, d, and g.

Motion Diagram

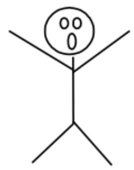


Motion Information

Object:	
Event 1:	Event 2:
t_1 = =	t_2 =

Object:	
r_1 =	r_2 =
v_1 =	v_2 =
a_{12} =	

Free-Body Diagram



Mathematical Analysis

Questions

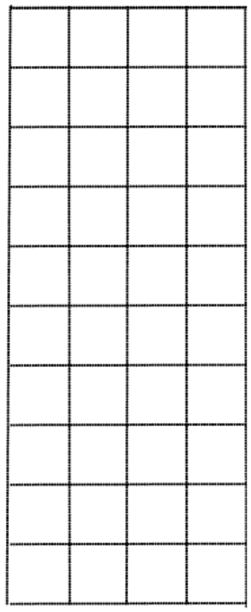
If $v = 0$ m/s, what should F_{scale} equal? Does your function agree with this observation?

If $d = \infty$, what should F_{scale} equal? Does your function agree with this observation?

For what stopping distance, d, would the bathroom scale read 0 N? Would the scale also read 0 N for this stopping distance if the elevator was initially moving downward?

A falling pole-vaulter of mass m lands on a cushion at speed v. The pole-vaulter sinks a distance d into the cushion before stopping. Determine the force exerted on the pole-vaulter due to the cushion ($F_{cushion}$) as a function of m, v, d, and g.

Motion Diagram



Motion Information

Object:	
Event 1:	Event 2:

Object:	
t_1 = =	t_2 =
r_1 =	r_2 =
v_1 =	v_2 =
a_{12} =	

Free-Body Diagram

Mathematical Analysis

Questions

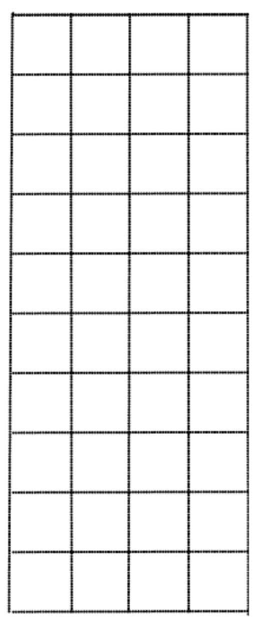
If v = 0 m/s, what should $F_{cushion}$ equal? Does your function agree with this observation?

If d = 0 m, what should $F_{cushion}$ equal? Does your function agree with this observation?

What would be worse for the pole-vaulter, hitting the cushion at twice her original speed or sinking half of the original distance into the cushion?

A pole-vaulter of mass m falls off a wall a distance D above a cushion. The pole-vaulter sinks a distance d into the cushion before stopping. Determine the force exerted on the pole-vaulter due to the cushion ($F_{cushion}$) as a function of m, D, d, and g.

Motion Diagram



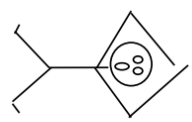
Motion Information

Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =

Event 1:	Event 2:	Event 3:
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_{12} =		a_{23} =

Free-Body Diagram

before hitting cushion



after hitting cushion

Mathematical Analysis

Questions

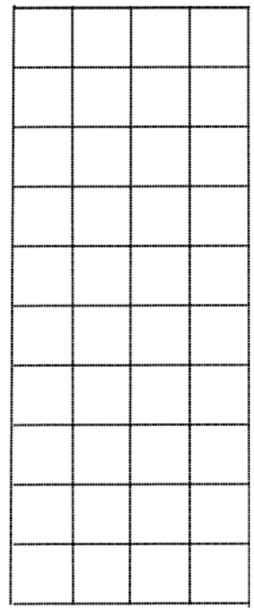
If D = ∞, what should $F_{cushion}$ equal? Does your function agree with this observation?

If d = 0 m, what should $F_{cushion}$ equal? Does your function agree with this observation?

What would be worse for the pole-vaulter, starting at twice the initial distance above the cushion or sinking half of the original distance into the cushion?

Tired of walking up the stairs, an engineering student of mass m designs an ingenious device for reaching his third floor dorm room. A block of mass M is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room in a time T. Determine the velocity of the student (v) when he reaches his room as a function of m, M, T and g.

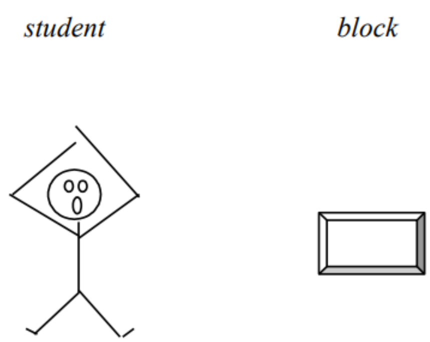
Motion Diagram



Motion Information

Object:	
Event 1:	Event 2:
t_1 = =	t_2 =
r_1 =	r_2 =
v_1 =	v_2 =
a_{12} =	

Free-Body Diagram



Mathematical Analysis

Questions

If $g = 0\ m/s^2$, what should v equal? Does your function agree with this observation?

If m = M, what should v equal? Does your function agree with this observation?

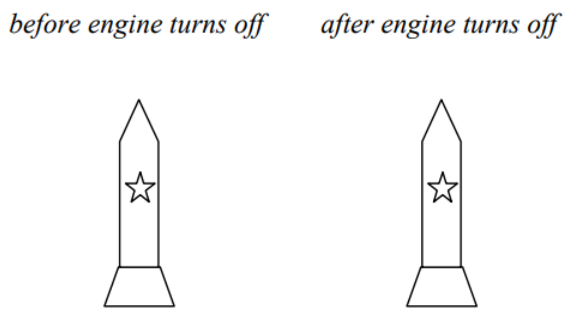
If M = ∞, what should v equal? Does your function agree with this observation?

A rocket of mass m is fired vertically upward from rest. The rocket's engine produces a thrust of constant magnitude F_{thrust} for t_{thrust} seconds. Determine the maximum height reached by the rocket (H) as a function of F_{thrust}, t_{thrust}, m, and g.

Motion Information

Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_{12} =		a_{23} =

Free-Body Diagram



Mathematical Analysis

Questions

If g = 0 m/s^2, what should H equal? Does your function agree with this observation?

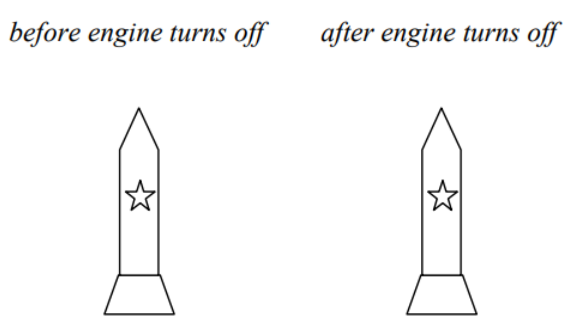
If F_{thrust} = mg, what should H equal? Does your function agree with this observation?

A rocket of mass m is fired vertically upward from rest. The rocket's engine produces a thrust of constant magnitude F_{thrust} for t_{thrust} seconds. Determine the time it takes the rocket to reach its apex (t_{apex}) as a function of F_{thrust}, t_{thrust}, m, and g.

Motion Information

Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_{12} =		a_{23} =

Free-Body Diagram



Mathematical Analysis

Questions

If $g = 0$ m/s^2, what should t_{apex} equal? Does your function agree with this observation?

If $F_{thrust} = mg$, what should t_{apex} equal? Does your function agree with this observation?

Conservation Laws

Concepts and Principles

What is a Conservation Law?

In general, a conservation law is a statement that a certain quantity does not change over time. If you know how much of this quantity you have today, you can be assured that the exact same amount of the quantity will be available tomorrow. A famous (at least to physicists) explanation of the nature of a conservation law was given by Richard Feynman.

> *Imagine your child has a set of 20 wooden blocks. Every day before bedtime you gather up your child's blocks to put them away. As you gather up the blocks, you keep count in your head. Once you reach 20, you know you have found all of the blocks and it is unnecessary for you to search any longer. This is because the number of blocks is conserved. It is the same today as it was yesterday.*
>
> *If one day you only find 18 blocks, you know to keep looking until you find the missing 2 blocks. Also, with experience, you discover the typical hiding places for the blocks. You know to check under the sofa, or behind the curtains.*
>
> *If your child is rambunctious, you may even have to look outside of the room. Perhaps he threw a block or two out of the window. Even though blocks can disappear from inside of the room, and appear out in the yard, if you search everywhere you will always find the 20 blocks.*

Physicists have discovered a number of quantities that behave exactly like the number of wooden blocks. We will examine two of these quantities, energy and momentum, below.

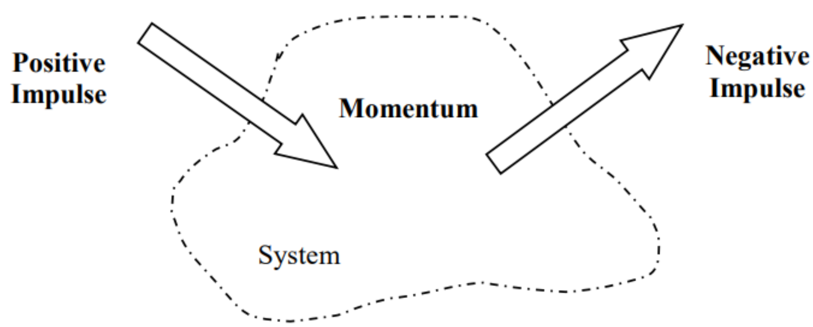
The Impulse-Momentum Relation

While Newton's Second Law directly relates the total force that acts on an object at a specific time to the object's acceleration at that exact same time, conservation laws relate the amount of a certain quantity present at one time to the amount present at a later time.

The first conserved quantity we will investigate is *momentum*. Of course, just because momentum is conserved doesn't mean that the momentum of any particular object or system of objects is always constant. The momentum of a single object, like the number of blocks in the playroom, can change. Just as blocks can be thrown out of the window of the playroom, the momentum of a single object can be changed by applying *impulse* to it. The relationship between impulse and momentum is, conceptually,

$$\text{initial momentum} + \text{impulse} = \text{final momentum}$$

Pictorially, we can visualize this as



In practice, we will identify an object or collection of objects (a *system*) and determine the amount of momentum the system contains at some initial time. This quantity cannot change unless impulse is done to the system. We call processes that bring momentum into the system as positive impulses, and processes that remove momentum from the system as negative impulses.

Mathematically this is written as

$$\text{initial momentum } + \text{ impulse } = \text{ final momentum}$$
$$P_i + J_{if} = P_f$$
$$\Sigma m v_i + \Sigma F(\Delta t) = \Sigma m v_f$$

where

- momentum (P) is the product of an object's mass and velocity,
- impulse (J) is the product of a force *external to the system* and the time interval over which it acts,
- and Σ indicates that you must sum the momentum of all of the objects in the system and all of the impulses acting on the system.

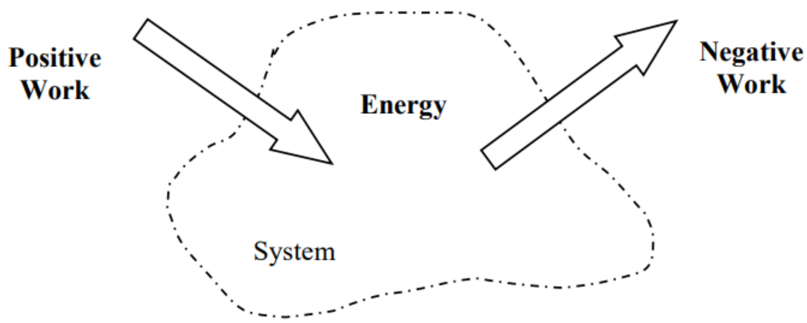
In short, if no impulse is applied to a system, its momentum will remain constant. However, if an impulse is applied to the system, its momentum will change by an amount exactly equal to the impulse applied. This momentum does not appear or disappear without a trace. It is simply transferred to the object *supplying* the impulse. In this sense, impulse is the transfer of momentum into or out of a system, analogous to tossing blocks into or out of a playroom.

The Work-Energy Relation

The second conserved quantity we will investigate is *energy*. Just like momentum, or wooden blocks, the conservation of energy doesn't mean that the energy of any particular object is always constant. The energy of a single object or system of objects can be changed by doing *work* to it. The relationship between work and energy is, conceptually,

$$\text{initial energy } + \text{ work } = \text{ final energy}$$

Pictorially, we can visualize this as



The similarity between momentum and energy is not complete, however. While there is only one form of momentum (i.e., one hiding place for momentum "blocks") there are several forms of energy. These different forms of energy will be introduced as you progress through more and more complicated models of the physical world. For now, the only "hiding place" I want to discuss is *kinetic energy*. In terms of kinetic energy, the above conceptual relationship between work and energy becomes, expressed mathematically,

$$\text{initial energy } + \text{ work } = \text{ final energy}$$
$$KE_i + W_{if} = KE_f$$
$$\Sigma \frac{1}{2} m v_i^2 + \Sigma |F||\Delta r| \cos\phi = \Sigma \frac{1}{2} m v_f^2$$

where

- kinetic energy (KE) is the product of one-half an object's mass and squared velocity,
- work (W) is the product of a force *(even an internal force)* and the displacement over which it acts (with more subtle details discussed below),
- Σ indicates that you must sum the kinetic energy of all of the objects in the system and all of the work done to the system,
- and we define a new unit, Joule (J), as J = kg $(m/s)^2$ = N m

Unlike anything we've studied up to this point, the work-energy relation is a *scalar* equation. This will become especially important when we study objects moving in more than one dimension. For now, all this means is that in the expression for work, $|F||\Delta r| \cos\phi$, we should use the *magnitude* of the force and the *magnitude* of the change in position. This product is then multiplied by $\cos\phi$, where ϕ is defined to be the angle between the applied force and the displacement of the object. If the force and displacement are in the same direction $\phi = 0^{\circ}$, and the work is positive (the object gains energy). If the force and displacement are in the opposite direction $\phi = 180^{\circ}$, and the work is negative (the object loses energy). Note that the actual directions of the force and the displacement are unimportant, only their directions *relative to each other* affect the work.

In general, if no work is done to a system, its kinetic energy will remain constant. However, if work is done to system, its total energy will change by an amount exactly equal to the work done. Work is the transfer of energy from one system to another, again analogous to tossing blocks from the playroom into the yard.

Analysis Tools

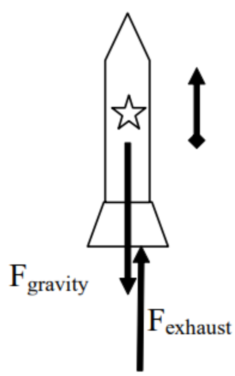
Applying the Impulse-Momentum Relation to a Single Object

Let's investigate the following scenario:

> *A 0.35 kg model rocket is fitted with an engine that produces a thrust of 11.8 N for 1.8 s. The rocket is launched vertically upward.*

To apply the impulse-momentum relation, you must clearly specify the initial and final events at which you will tabulate the momentum. For example:

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches maximum height.
$P_1 = 0$	$P_2 = (0.35)\ v_2$
$J_{12} = +F_{\text{exhaust}}\ (1.8) - F_{\text{gravity}}\ (1.8)$	



Note that each external force acting on the rocket is multiplied by the time interval over which it acts. (Also note that the rocket's engine does not produce a force on the rocket! The engine produces a downward force on the hot exhaust gases emitted from the engine and these hot gases exert an equal magnitude force back up on the rocket. That is why the force on the rocket is labeled as F_{exhaust} rather than F_{engine}.)

Applying impulse-momentum to the rocket during this time interval yields:

$$\begin{aligned} &P_1 + J_{12} = P_2 \\ &0 + F_{\text{exhaust}}\ (1.8) - F_{\text{gravity}}\ (1.8) = 0.35 v_2 \\ &0 + (11.8)(1.8) - (0.35)(9.8)(1.8) = 0.35 v_2 \\ &v_2 = 43.0 \text{ m/s} \end{aligned}$$

Thus, the rocket is traveling at 43.0 m/s at the instant the engine shuts off.

Of course, there is no reason why we had to analyze the rocket's motion between the two instants of time we selected above. We could have selected the events:

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches maximum height.
$P_1 =$	$P_2 = 0$

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches maximum height.
$\mathrm{J}_{12}=+F_{\text{exhaust}}\,(1.8)-F_{\text{gravity}}\,(\Delta t)$	

During this time interval, the force of the exhaust gases only act on the rocket for a *portion* of the entire time interval. Noting that the rocket's velocity when it reaches its maximum height is zero, impulse-momentum would look like this:

$$\begin{aligned}&P_1+J_{12}=P_2\\&0+F_{\text{exhaust}}\,(1.8)-F_{\text{gravity}}\,(\Delta t)=0\\&0+(11.8)(1.8)-(0.35)(9.8)(\Delta t)=0\\&\Delta t=6.19s\end{aligned}$$

Thus, the rocket is in the air for 6.19 s before reaching its maximum height.

Applying the Work-Energy Relation to a Single Object

The work-energy relation also has many uses for investigating physical scenarios. For example, let's look again at our model rocket:

> *A 0.35 kg model rocket is fitted with an engine that produces a thrust of 11.8 N for 1.8 s. The rocket is launched vertically upward.*

Assuming we've already analyzed this scenario using impulse-momentum, what additional information can we extract using work-energy?

Event 1: The instant the engine is ignited.	Event 2: The instant the engine shuts off.
$KE_1 = 0$	$\mathrm{KE}_2=1/2\,(0.35)(43)^2$
$\mathrm{W}_{12}=F_{\text{exhaust}}\,(\Delta r)\cos 0+F_{\text{gravity}}\,(\Delta r)\cos 180$	

Therefore,

$$\begin{aligned}&KE_1+W_{12}=KE_2\\&0+|F_{\text{exhaust}}\,||\Delta r|\cos 0+|F_{\text{gravity}}\,||\Delta r|\cos 180=\tfrac{1}{2}(0.35)(43.2)^2\\&0+11.8(\Delta r)(1)+(0.35)(9.8)(\Delta r)(-1)=327\\&11.8\Delta r-3.43\Delta r=327\\&\Delta r=39.1\text{ m}\end{aligned}$$

Thus, the rocket rises to a height of 39.1 m before the engines shuts off.

What if we apply work-energy between the following two events:

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches its maximum height
$KE_1 = 0$	$KE_2 = 0$
$\mathrm{W}_{12}=F_{\text{exhaust}}\,(39)\cos 0+F_{\text{gravity}}\,(\Delta r)\cos 180$	

During this time interval, the force of the exhaust gases only act on the rocket for a *portion* of the entire displacement, namely 39 m, while the force of gravity acts over the entire displacement.

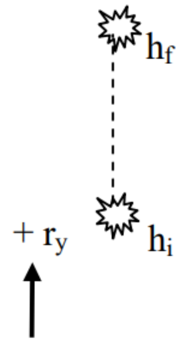
$$\begin{aligned}&KE_1+W_{12}=KE_2\\&0+(11.8)(39)\cos 0+(0.35)(9.8)(\Delta r)\cos 180=0\\&0+460-3.43\Delta r=0\\&\Delta r=134m\end{aligned}$$

Thus, the maximum height reached by the rocket is 134 m.

Gravitational Potential Energy

In any situation in which an object changes its height above the surface of the earth, the force of gravity does work on the object. It is possible to calculate this work in general, and to rewrite the work-energy relation in such a way as to incorporate the effects of this work. This is referred to as constructing a *potential energy function* for the work done by gravity.

Let's imagine an object of mass, m, located an initial height, h_i, above the zero of a vertical coordinate system, with the upward direction designated positive. It moves to a final height of h_f.



To calculate the work done by gravity on this object:

$$\begin{aligned} W_{\text{gravity}} &= |F||\Delta r| \cos \phi \\ W_{\text{gravity}} &= (mg)\left(h_f - h_i\right) \cos 180 \\ W_{\text{gravity}} &= -mgh_f + mgh_i \end{aligned}$$

The "mgh" terms are referred to as *gravitational potential energy*. Thus, the work done by gravity can be thought of as changing the gravitational potential energy of the object. Let's plug the above result into the work-energy relation:

$$\begin{aligned} &\tfrac{1}{2} m v_i^2 + \Sigma|F||\Delta r| \cos \phi = \tfrac{1}{2} m v_f^2 \\ &\tfrac{1}{2} m v_i^2 + \Sigma|F||\Delta r| \cos \phi + W_{\text{gravity}} = \tfrac{1}{2} m v_f^2 \\ &\tfrac{1}{2} m v_i^2 + \Sigma|F||\Delta r| \cos \phi - mgh_f + mgh_i = \tfrac{1}{2} m v_f{}^2 \\ &\tfrac{1}{2} m v_i^2 + mgh_i + \Sigma|F||\Delta r| \cos \phi = \tfrac{1}{2} m v_f^2 + mgh_f \end{aligned}$$

Therefore, this final relation:

$$\begin{aligned} &KE_i + GE_i + W_{if} = KE_f + GE_f \\ &\tfrac{1}{2} m v_i^2 + mgh_i + \Sigma|F||\Delta r| \cos \phi = \tfrac{1}{2} m v_f^2 + mgh_f \end{aligned}$$

can (and will) be used in place of the standard work-energy relation provided:

1. You do not include the force of gravity a second time by calculating the work done by gravity. Basically, in this relationship gravity is no longer thought of as a force that does work on objects but rather as a source of potential energy.
2. You calculate the initial and final heights, h_i and h_f, using a coordinate system in which the upward direction is positive.

Applying Work-Energy with Gravitational Potential Energy

Let's use the work-energy relation, with gravitational potential energy terms, to re-analyze the previous scenario:

> *A 0.35 kg model rocket is fitted with an engine that produces a thrust of 11.8 N for 1.8 s. The rocket is launched vertically upward.*

Let's apply work-energy between the following two events, setting the initial elevation of the rocket equal to zero:

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches its maximum height
$KE_1 = 0$	$KE_2 = 0$

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches its maximum height
$GE_1 = 0$	$GE_2 = (0.35)(9.8)\ h_2$
$W_{12} = F_{\text{exhaust}}\,(39)\cos 0$	

During this time interval, the force of the exhaust gases only act on the rocket for a *portion* of the entire displacement, namely 39 m. Remember, the force of gravity *does not do work* in this way of modeling nature, rather the gravitational energy of the rocket changes as it changes its elevation.

$$
\begin{aligned}
KE_i + GE_i + W_{if} &= KE_f + GE_f \\
0 + 0 + (11.8)(39)\cos 0 &= 0 + (0.35)(9.8)h_f \\
0 + 0 + 460 &= 0 + 3.43 h_f \\
h_f &= 134 m
\end{aligned}
$$

results in, of course, the same maximum height reached by the rocket.

Applying the Impulse-Momentum Relation to a Collision

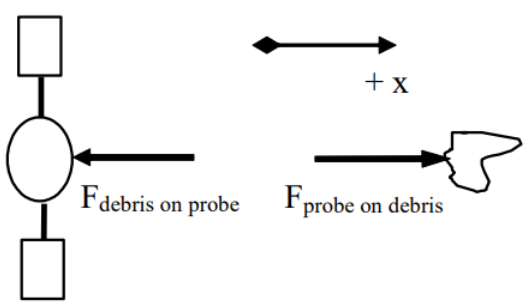
Probably the most useful application of the impulse-momentum relation is in the study of collisions. For example:

> *Far from the earth, a 250 kg space probe, moving at 5 km/s, collides head-on with a 60 kg piece of space debris initially at rest. The debris becomes entangled in the probe's solar collectors.*

Let's choose:

Event 1: The instant before the collision.	Event 2: The instant the debris and probe reach a common velocity.
Object: Space Probe	
$P_1 = (250)(5000)$	$P_2 = 250\ v_2$
$J_{12} = -F_{\text{debrisonpobe}}\,(\Delta t)$	
Object: Debris	
$P_1 = 0$	$P_2 = 60\ v_2$
$J_{12} = +F_{\text{probeondebris}}\,(\Delta t)$	

The free-body diagrams for the two objects during this time interval are shown below.



Applying the impulse-momentum relation to each object separately yields:

Probe	*Debris*
$P_1 + J_{12} = P_2$	$P_1 + J_{12} = P_2$
$250(5000) - F_{\text{debrisonprobe}}\,(\Delta t) = 250 v_2$	$0 + F_{\text{probeondebris}}\,(\Delta t) = 60 v_2$

$1250000-F_{\text{debrisonprobe}}\,(\Delta t)=250v_2$	$F_{\text{probeondebris}}\,(\Delta t)=60v_2$

Notice that the final velocities of the two objects are the same, because they remain joined together following the collision. Also, the Δt's are the same because the time interval over which the force of the debris acts on the probe must be the same as the time interval over which the force of the probe acts on the debris. In fact, these two forces must be equal to each other in magnitude by Newton's Third Law.

Thus, the impulses must cancel if the two equations are added together:

$$\begin{aligned}&1250000-F_{\text{debrisonprobe}}\,(\Delta t)=250v_2\\&\underline{F_{\text{probeondebris}}\,(\Delta t)=60v_2}\\&1250000=310v_2\\&v_2=4032\text{ m/s}\end{aligned}$$

The probe slows to a speed of 4032 m/s (and the debris changes direction and accelerates to a speed of 4032 m/s) via the collision. Thus, even though we do not know the magnitude of the force involved, or the duration of the collision, we can calculate the final velocities of the two objects colliding. This is because the forces involved comprise an interaction, and by Newton's Third Law forces that comprise an interaction are always equal in magnitude and opposite in direction.

In fact, in problems involving collisions (or explosions, which to physicists are merely collisions played backward in time!), you should almost always apply the impulse-momentum relation to the interacting objects because the forces involved comprise an interaction. Thus, by adding your equations together, these terms will always add to zero. This will often allow you to determine the final velocities of the colliding objects.

In conclusion, I should point out that the probe loses momentum during the collision and that the debris gains the exact same amount of momentum. (Check the numbers to verify this statement.) The momentum is transferred from the probe to the debris through the action of the impulse the probe and debris exert on each other. The momentum transfer from the probe to the debris is analogous to throwing a wooden block from the playroom into the yard: The playroom now has one less block and the yard has one more!

Applying the Work-Energy Relation to the Same Collision

Let's return to the collision scenario discussed above and attempt to investigate it using workenergy.

> *Far from the earth, a 250 kg space probe, moving at 5 km/s, collides head-on with a 60 kg piece of space debris initially at rest. The debris becomes entangled in the probe's solar collectors.*

Event 1: The instant before the collision.	Event 2: The instant the debris and probe reach a common velocity.
Object: Space Probe	
$\text{KE}_1=1/2\,(250)5000^2$	$\text{KE}_2=1/2\,(250)\text{v}_2{}^2$
$GE_1 = 0$	$GE_2 = 0$
$\text{W}_{12}=F_{\text{on P}}\,(\Delta r_P)\cos 180$	
Object: Debris	
$KE_1 = 0$	$\text{KE}_2=1/2\,(60)\text{v}_2{}^2$
$GE_1 = 0$	$GE_2 = 0$
$\text{W}_{12}=F_{\text{on D}}\,(\Delta r_D)\cos 0$	

Applying the work-energy relation to each object separately yields:

Probe	*Debris*

$KE_i + GE_i + W_{if} = KE_f + GE_f$	$KE_i + GE_i + W_{if} = KE_f + GE_f$
$\frac{1}{2}(250)(5000)^2 + (F_{onP})(\Delta r_{\text{probe}})\cos 180 = \frac{1}{2}(250)(v_{2\,\text{probe}})^2$	$0 + (F_{onD})(\Delta r_{\text{debris}})\cos 0 = \frac{1}{2}(60)(v_{2\,\text{debris}})^2$
$3.13 \times 10^9 - F_{\text{onP}}\ (\Delta r_{\text{probe}}) = 125 v_{2\,\text{probe}}{}^2$	$F_{\text{onD}}\ (\Delta r_{\text{debris}}) = 30 v_{2\,\text{debris}}{}^2$

The final velocities of the two objects are the same, because they remain joined together following the collision, and the two forces are the same by Newton's Third Law. **However, these two equations cannot be added together and solved because the two distances over which the forces act, Δr_{probe} and Δr_{debris}, are not necessarily equal.** During the collision, the center of the probe will move a different distance that the center of the debris[3]. Since these two distances are different, the works will *not* cancel as the impulses did, and the equations are *not* solvable!

> 3 If the two objects were *actually* particles, rather than being *approximated* as particles, then the two distances would have to be the same and the two works would cancel when the equation were added together.

In fact, since we know v_2 = 4032 m/s from our momentum analysis,

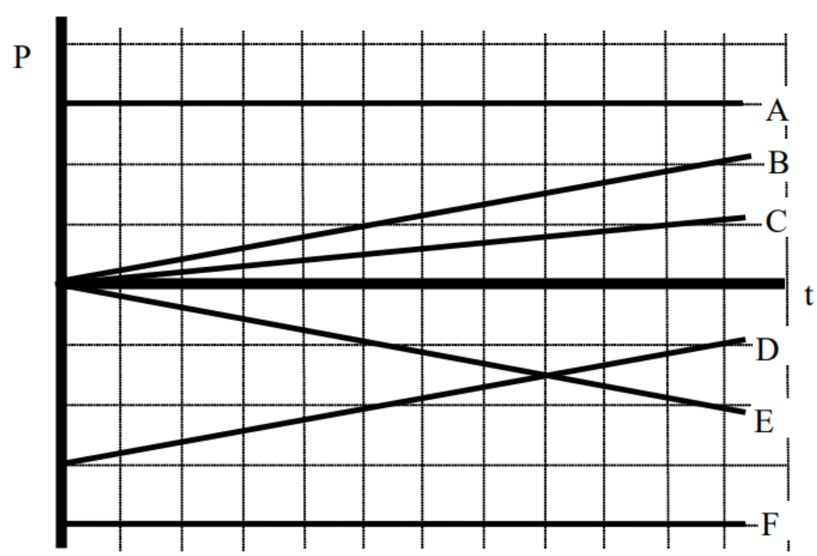
Probe	*Debris*
$3.13 \times 10^9 - F_{\text{onP}}\ (\Delta r_{\text{probe}}) = 125(4032)^2$	$F_{\text{onD}}\ (\Delta r_{\text{debris}}) = 30(4032)^2$
$3.13 \times 10^9 - F_{\text{onP}}\ (\Delta r_{\text{probe}}) = 2.03 \times 10^9$	$W_{\text{ondebris}} = F_{\text{onD}}\ (\Delta r_{\text{debris}}) = 0.49 \times 10^9 J$
$W_{\text{onprobe}} = -F_{\text{onP}}\ (\Delta r_{\text{probe}}) = -1.1 \times 10^9$ J	

Obviously, the two works do not cancel. In fact, the *internal work*, or work done by the objects on each other, totals -0.61 x 10^9 J. This means that there is 0.61 x 10^9 J *less* kinetic energy in the system of the probe and the debris after the collision than before the collision. This is sometimes referred to as the energy lost in the collision, although the energy is not lost but rather converted into other forms of energy (i.e., other hiding places for the wooden blocks that have yet to be discussed), such as thermal energy.

In short, the work-energy relation (as it now stands) cannot be used to effectively analyze collisions unless additional information regarding the internal energy is available. Occasionally, an approximation is made in which the total internal work is zero. When this approximation is made, the collision is referred to as an *elastic* collision. Realistic collisions, in which the total internal energy is not zero and kinetic energy is "lost", are referred to as *inelastic* collisions.

Activities

Below are momentum vs. time graphs for six different objects.



a. Rank these graphs on the basis of the change in momentum of the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

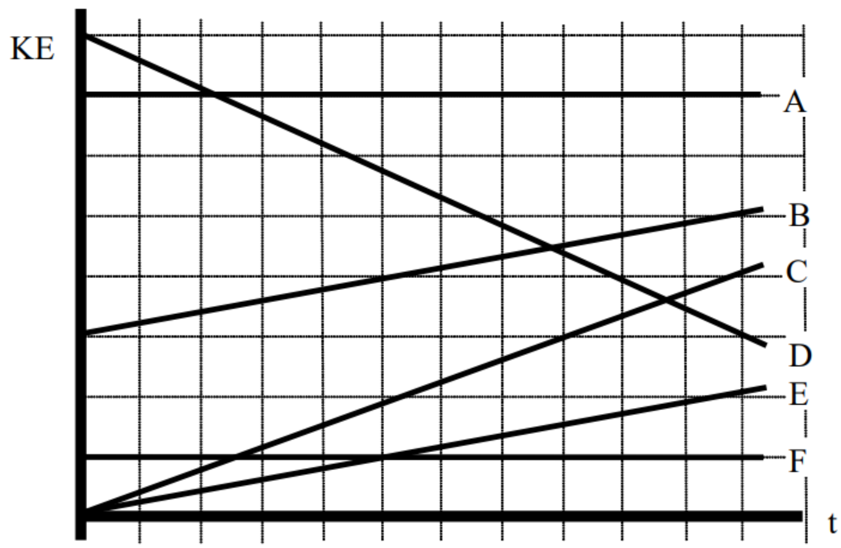
b. Rank these graphs on the basis of the total impulse on the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are kinetic energy vs. time graphs for six different objects. All of the objects move horizontally.



a. Rank these graphs on the basis of the change in kinetic energy of the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

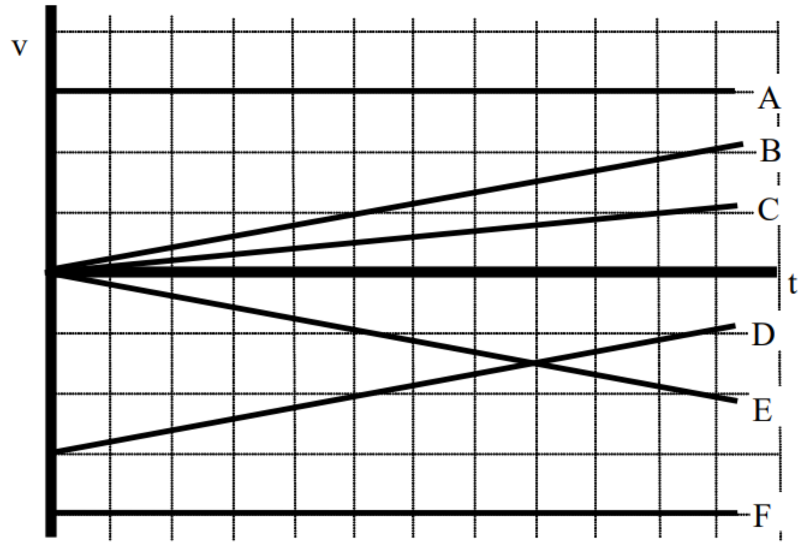
b. Rank these graphs on the basis of the total work on the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are velocity vs. time graphs for six equal-mass objects.



a. Rank these graphs on the basis of the change in momentum of the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

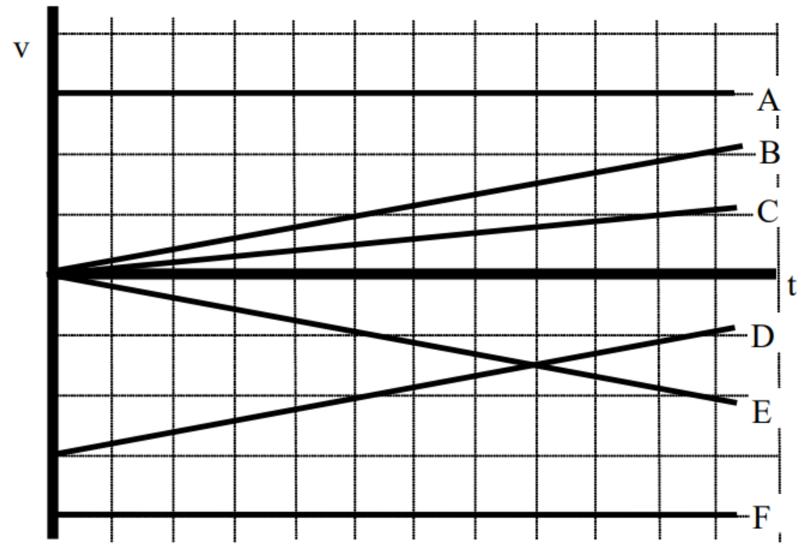
b. Rank these graphs on the basis of the change in kinetic energy of the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are velocity vs. time graphs for six equal-mass objects. All of the objects move horizontally.



a. Rank these graphs on the basis of the total impulse on the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

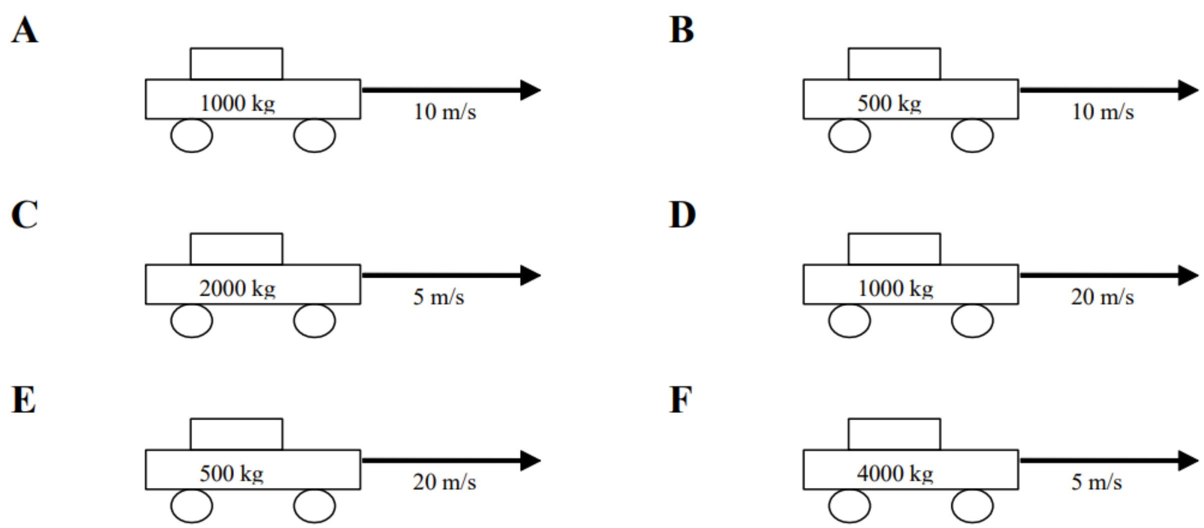
b. Rank these graphs on the basis of the total work on the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six automobiles initially traveling at the indicated velocity. The automobiles have different masses and velocities.



a. All automobiles will be stopped in the same amount of time. Rank these automobiles on the basis of the magnitude of the force needed to stop them.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

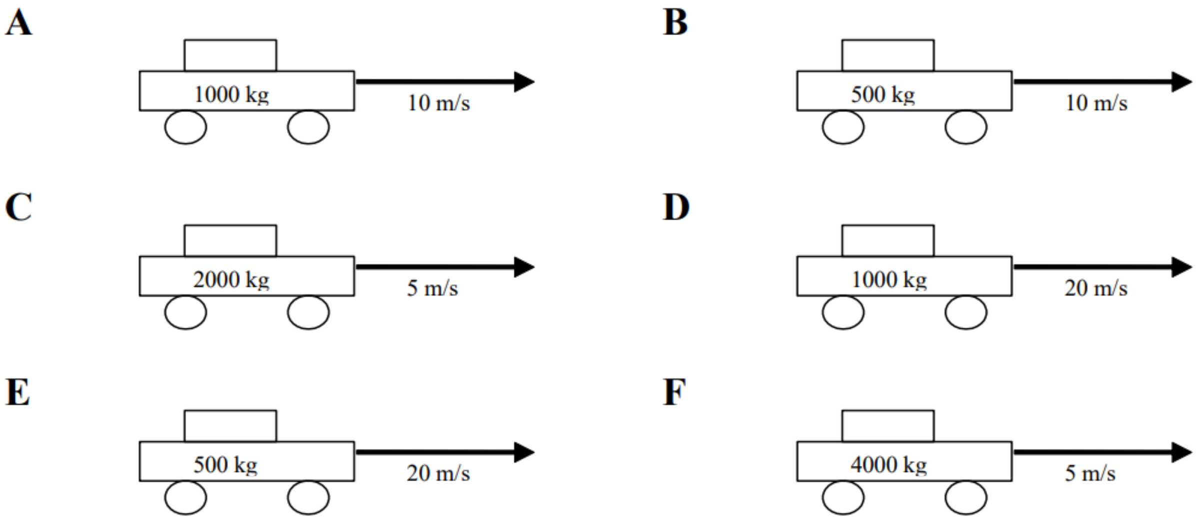
b. All automobiles will be stopped in the same amount of distance. Rank these automobiles on the basis of the magnitude of the force needed to stop them.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six automobiles initially traveling at the indicated velocity. The automobiles have different masses and velocities.



a. Rank these automobiles on the basis of the magnitude of the force needed to stop them.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank these automobiles on the basis of the magnitude of the work needed to stop them.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

c. Rank these automobiles on the basis of the magnitude of the impulse needed to stop them.

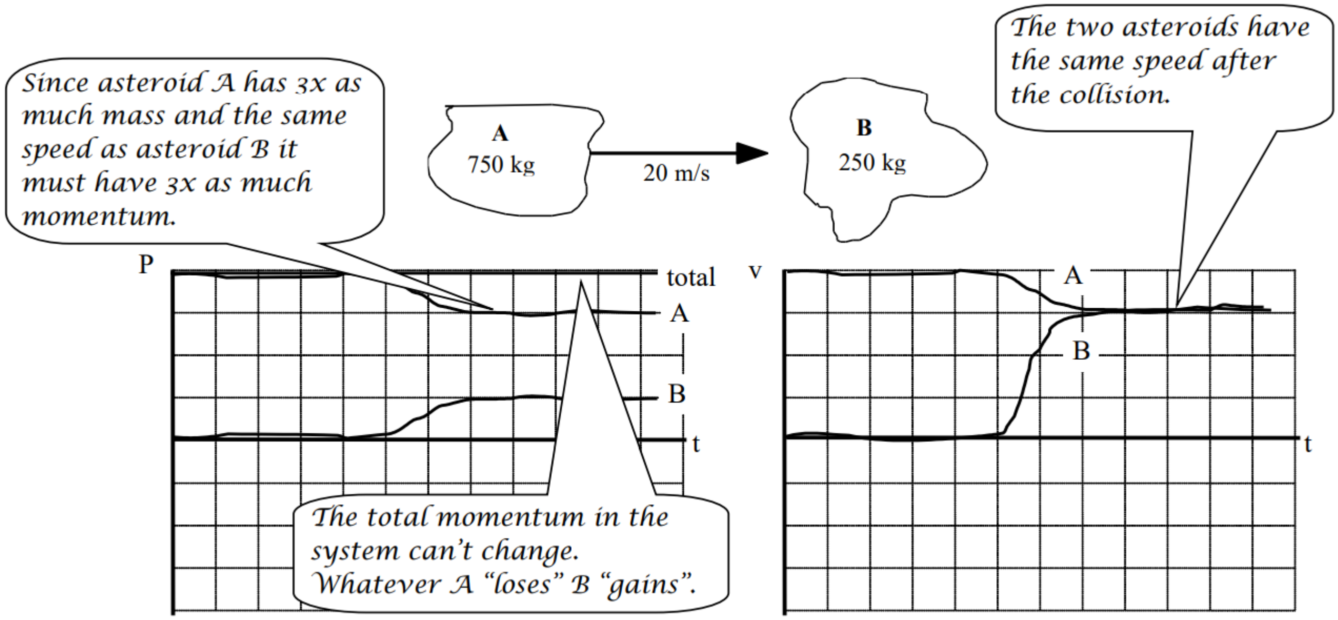
Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

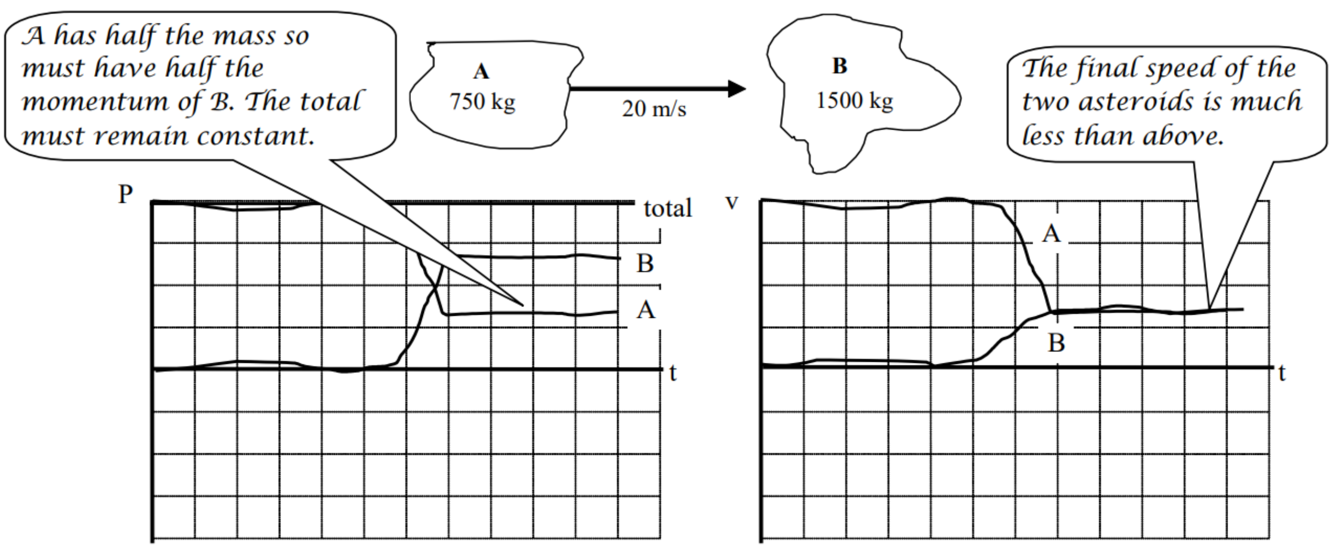
Explain the reason for your ranking:

For each of the collisions illustrated below, sketch a graph of the momentum and velocity of asteroid A, the momentum and velocity of asteroid B, and the total momentum in the system of the two asteroids. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

a. The two asteroids remain joined together after the collision.

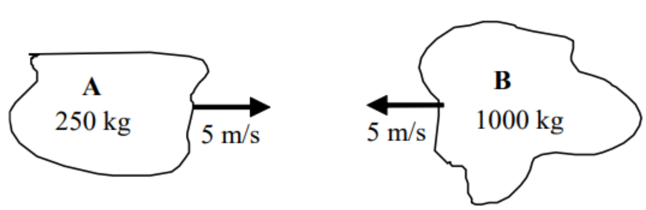


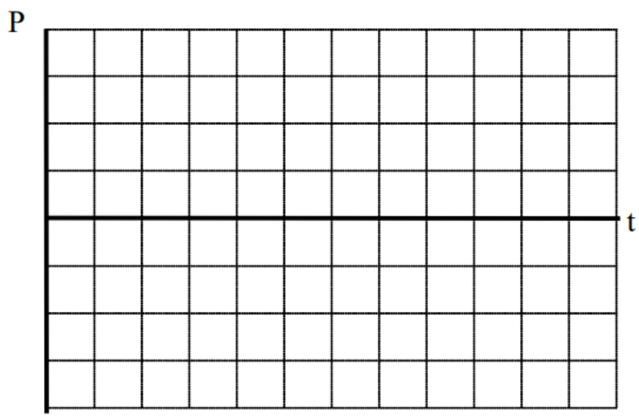
b. The two asteroids remain joined together after the collision.

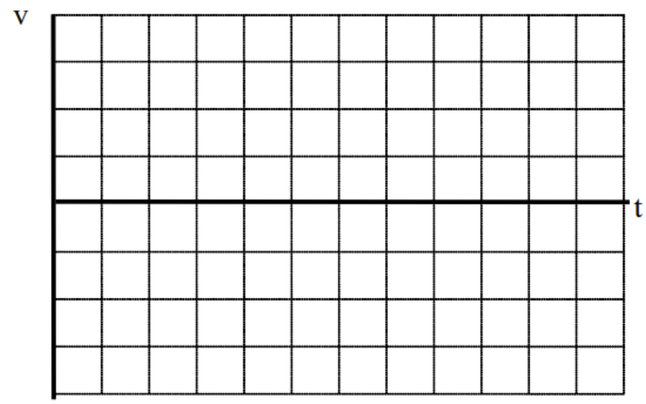


For each of the collisions illustrated below, sketch a graph of the momentum and velocity of asteroid A, the momentum and velocity of asteroid B, and the total momentum in the system of the two asteroids. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

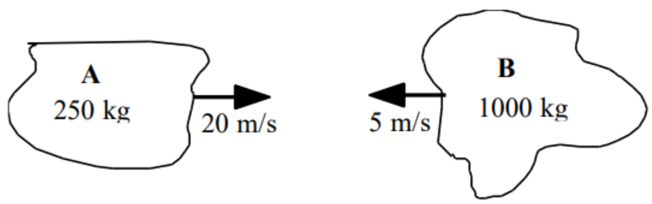
a. The two asteroids remain joined together after the collision.

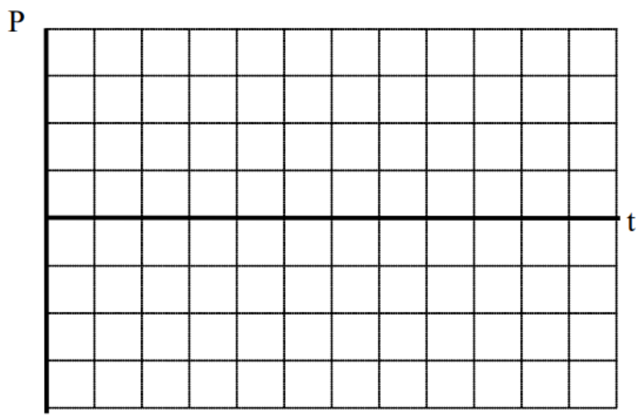


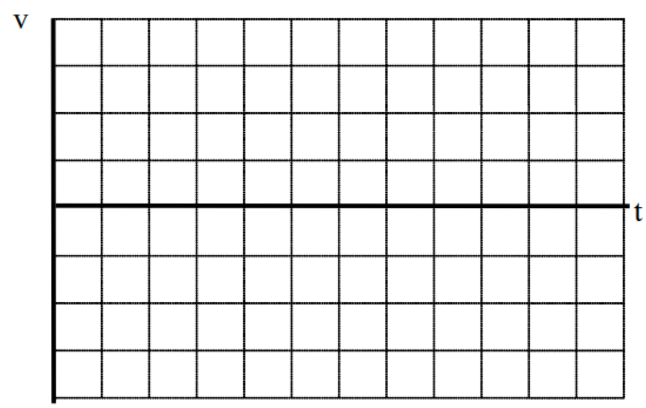




b. The two asteroids remain joined together after the collision.

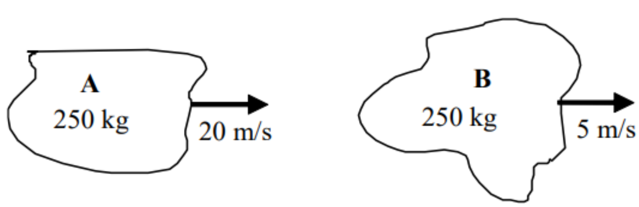


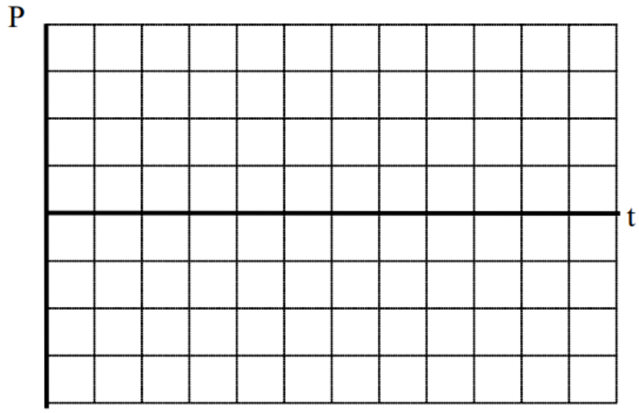


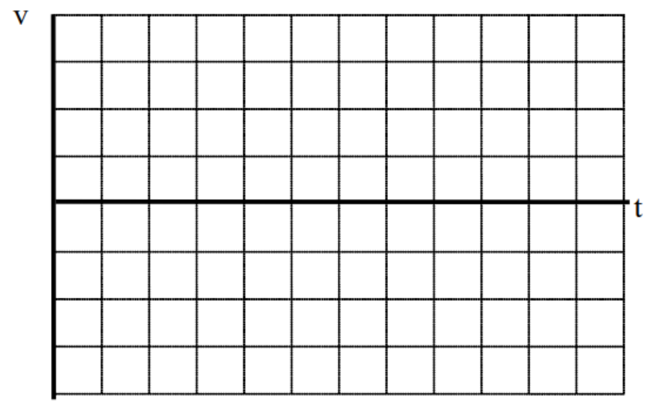


For each of the collisions illustrated below, sketch a graph of the momentum and velocity of asteroid A, the momentum and velocity of asteroid B, and the total momentum in the system of the two asteroids. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

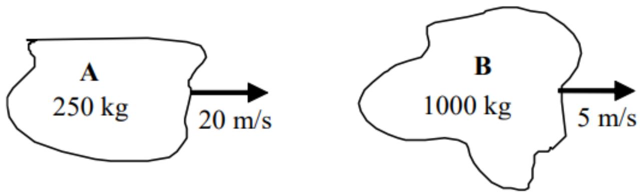
a. The two asteroids remain joined together after the collision.

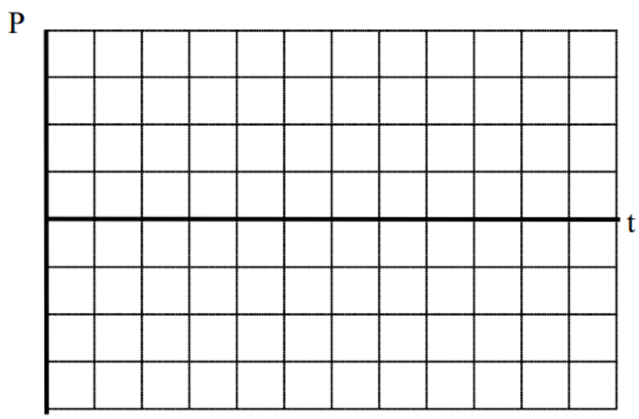


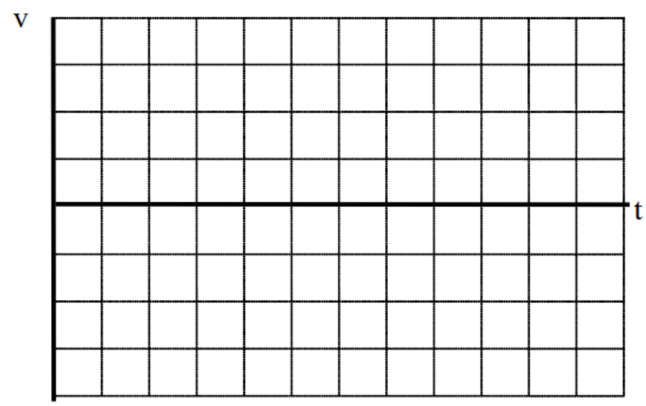




b. The two asteroids remain joined together after the collision.

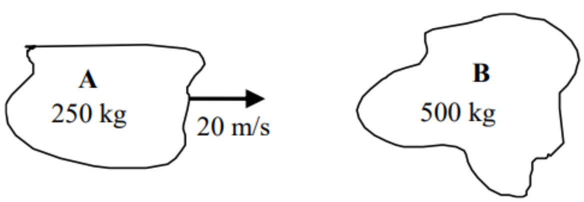


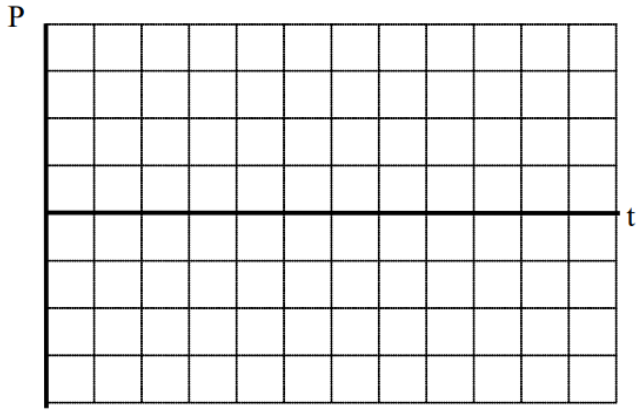


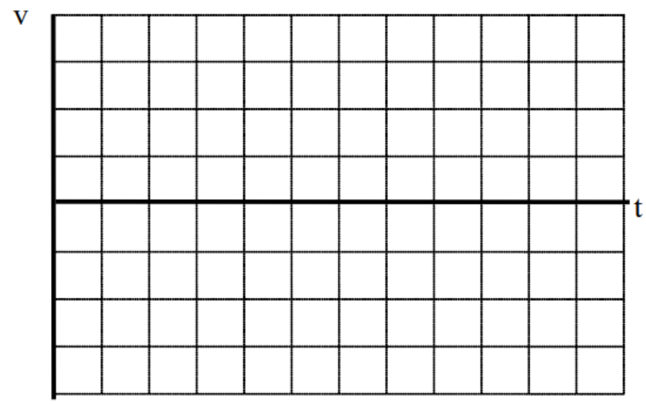


For each of the collisions illustrated below, sketch a graph of the momentum and velocity of asteroid A, the momentum and velocity of asteroid B, and the total momentum in the system of the two asteroids. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

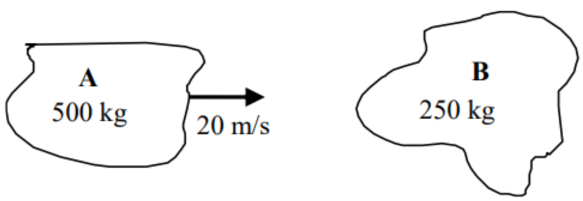
a. Asteroid A rebounds at 5 m/s after the collision.

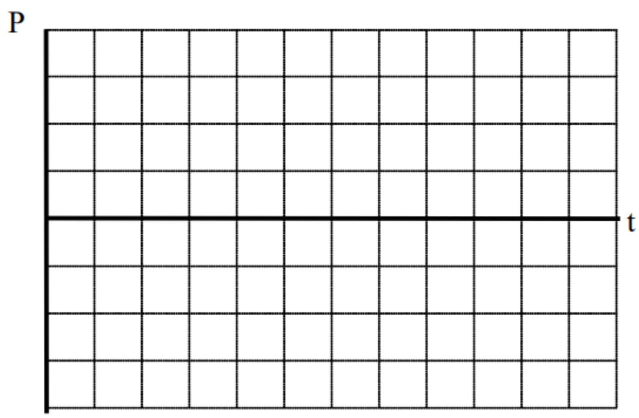


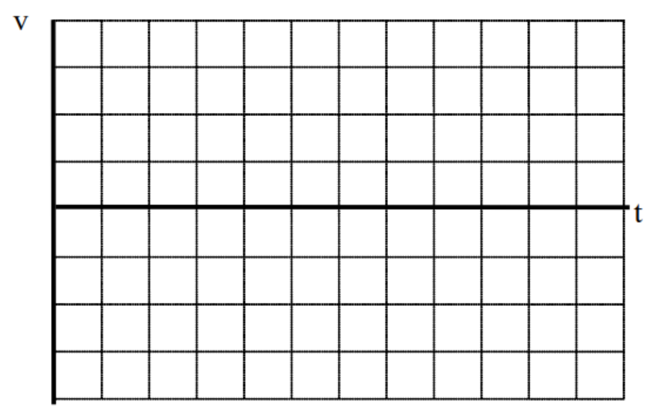




b. Asteroid B moves at 20 m/s after the collision.

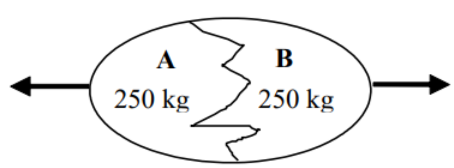


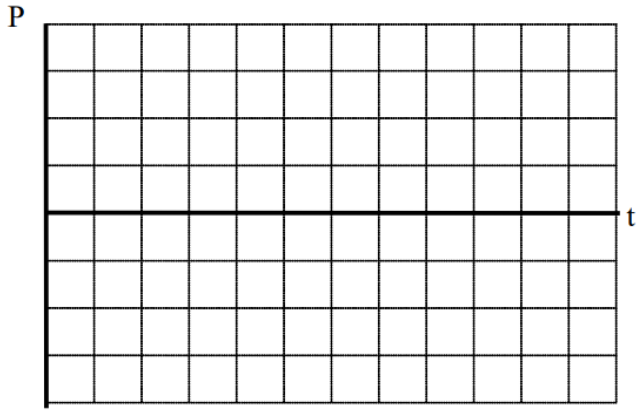


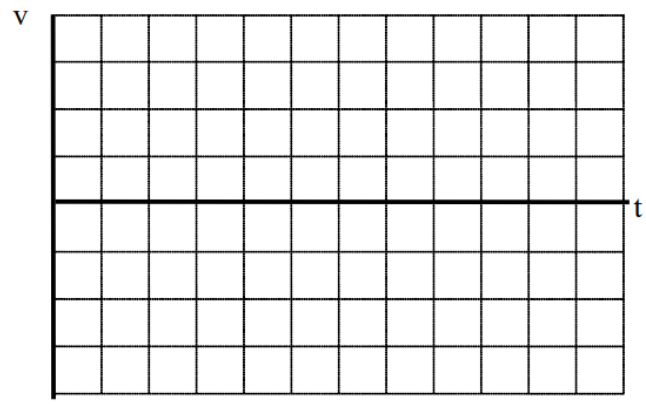


For each of the explosions illustrated below, sketch a graph of the momentum and velocity of fragment A, the momentum and velocity of fragment B, and the total momentum in the system of the two fragments. Begin your graph before the explosion takes place and continue it while the fragments travel away from the sight of the explosion. Use a consistent coordinate system and scale on all graphs.

a. The exploding egg is initially at rest.

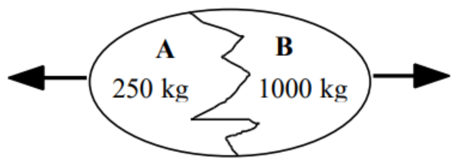


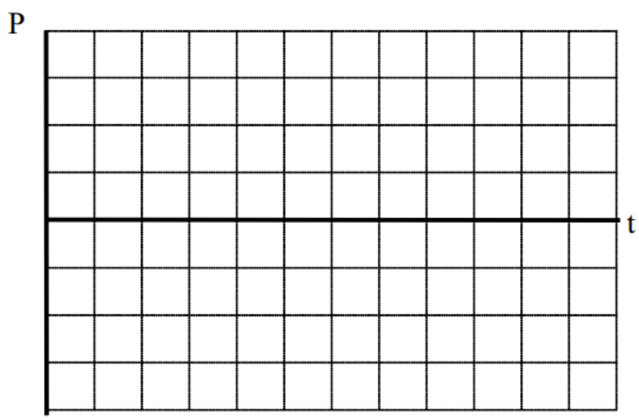


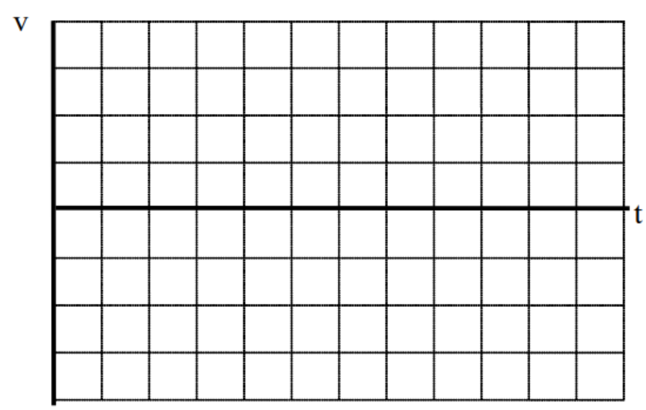


b. The exploding egg is initially at rest.

The exploding egg is initially at rest.

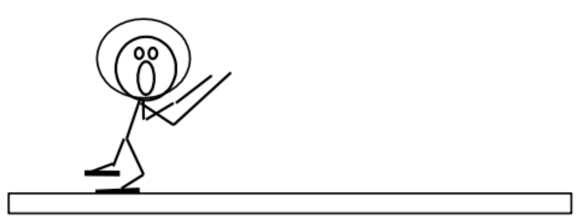


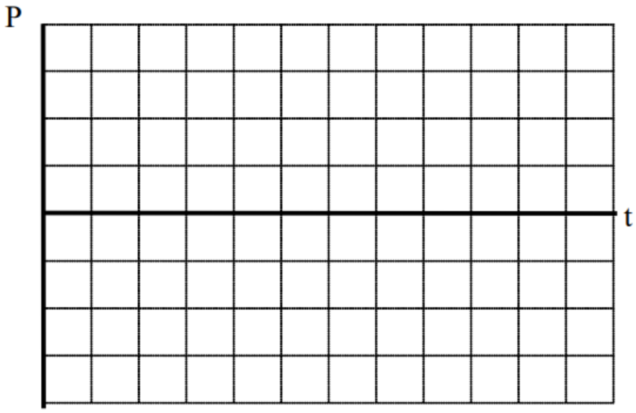


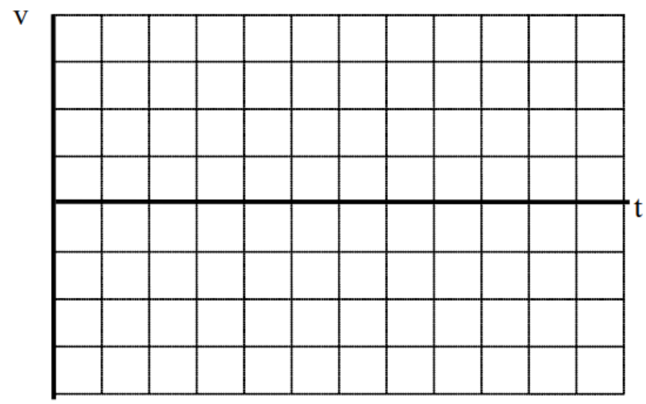


A 200 kg astronaut is initially at rest on the extreme edge of a 1000 kg space platform. She wears special magnetic shoes that allow her to walk along the metal platform. For each of the situations illustrated below, sketch a graph of the momentum and velocity of the astronaut, the momentum and velocity of the platform, and the total momentum in the system of the two objects. Begin your graph before the astronaut begins to walk and continue it while she walks along the platform. Use a consistent coordinate system and scale on all graphs.

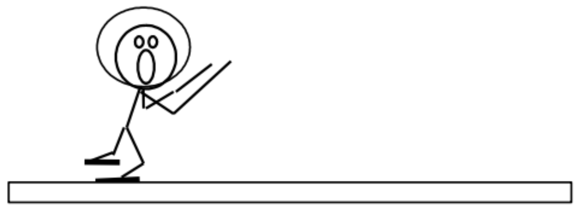
a. The astronaut and platform are initially at rest.

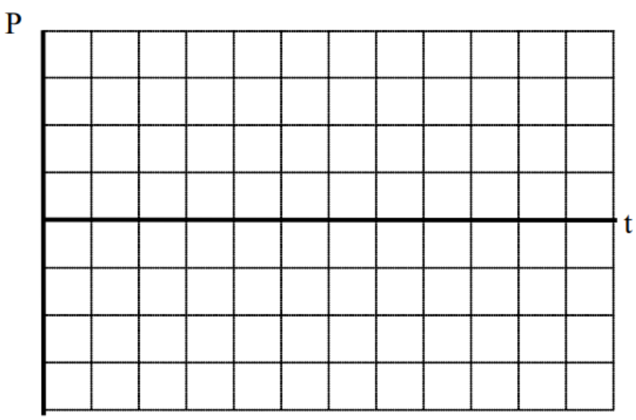


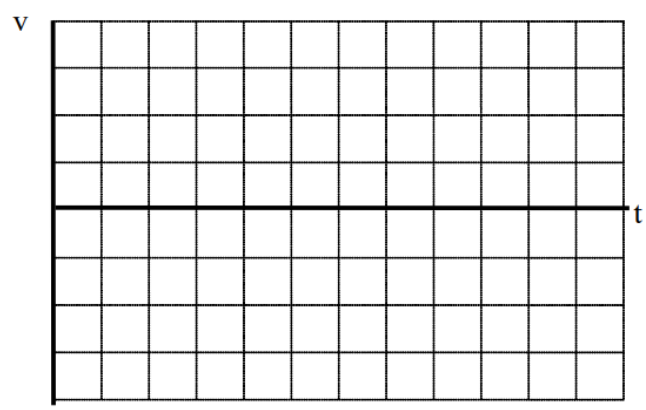




b. The astronaut and platform are initially drifting to the right.

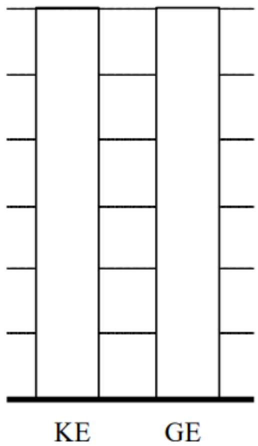




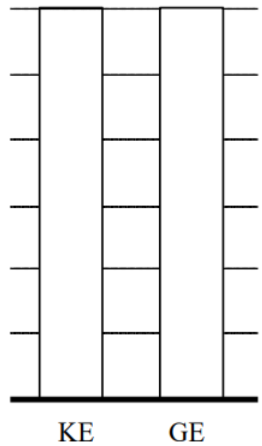


For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of the object at each of the events listed. Use a consistent scale throughout each motion. Set the lowest point of the motion as the zero-point of gravitational potential energy.

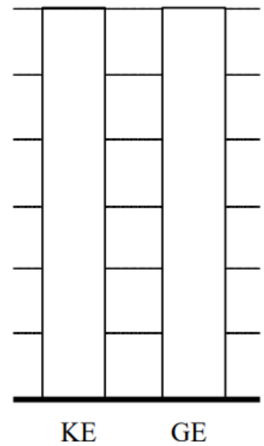
a. *A 4000 kg rocket’s engine produces a thrust of 70,000 N for 15 s. The rocket is fired vertically upward.*



When the engine is first turned one.

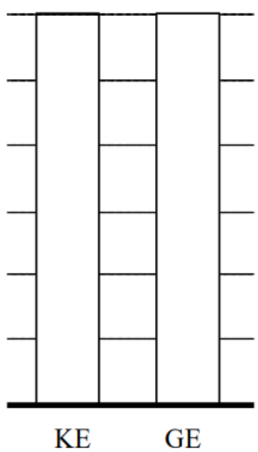


When the engine turns off.

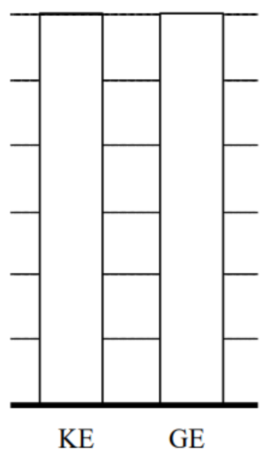


When the rocket reaches its maximum height.

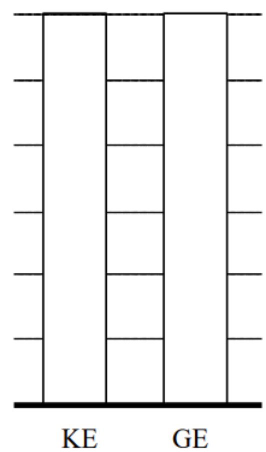
b. *To practice falling, a 55 kg pole-vaulter falls off of a wall 6.0 m above a 2.0 m thick foam cushion resting on the ground. However, he misses the cushion. The pole-vaulter sinks about 0.10 m into the ground before stopping.*



When the person first begins to fall.



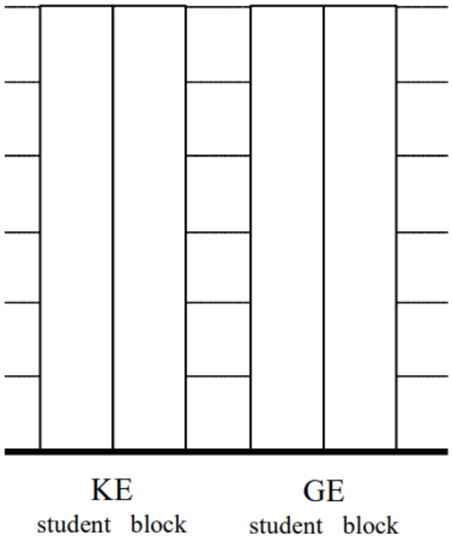
When the person hits the ground.



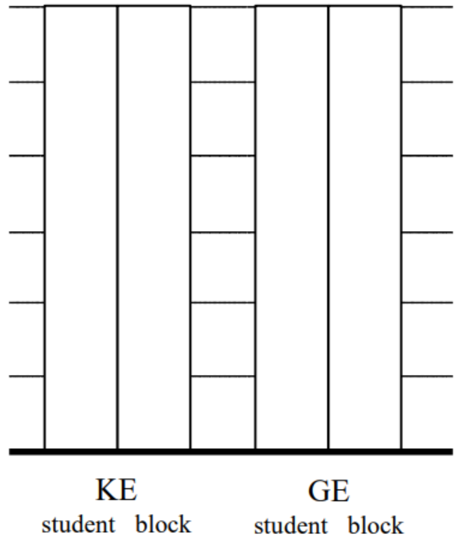
When the person finally stops.

For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of each object at each of the events listed. Use a consistent scale throughout each motion. Set the lowest point of the motion as the zero-point of gravitational potential energy

a. *Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. An 84 kg block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the 84 kg block is released, the student is pulled up to his dorm room, 8.0 m off the ground.*

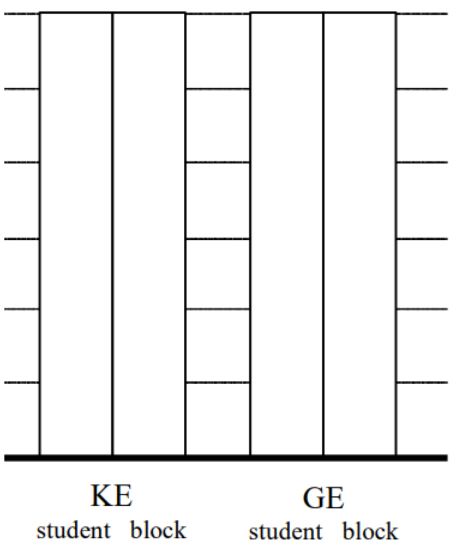


When the block is released.

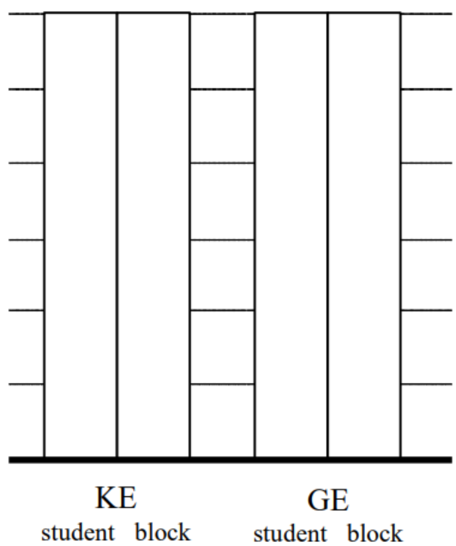


When the student reaches his room.

b. Tired of walking down the stairs, a 75 kg engineering student designs an ingenious device for reaching the ground from his third floor dorm room. A 60 kg block at rest on the ground is attached to a rope that passes over a pulley. The student grabs the other end of the rope and steps out of his window.



When the student steps out of the window.



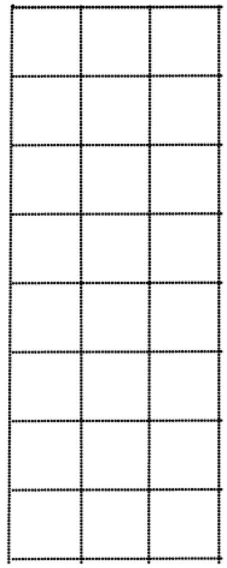
When the student reaches the ground.

A 100 kg man concerned about his weight decides to weigh himself in an elevator. He stands on a bathroom scale in an elevator that is moving upward at 3.0 m/s. As the elevator reaches his floor, it slows to a stop.

a. If the elevator slows to a stop over a distance of 2.0 m, what is the reading on the bathroom scale?

b. If the elevator slows to a stop in 1.5 s, what is the reading on the bathroom scale?

Motion Diagram



a. Motion Information

Event 1:	Event 2:
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Mathematical Analysis[26]

Free-Body Diagram

b. Motion Information

Event 1:	Event 2:
P1 =	P2 =
J12 =	

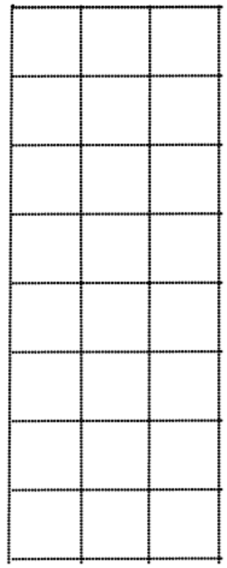
Mathematical Analysis

A 70 kg student is 120 m above the ground, moving upward at 3.5 m/s, while hanging from a rope hanging from a 280 kg helium balloon. The lift on the balloon due to the buoyant force is 3000 N.

a. With what speed does the student hit the ground?

b. How long does it take the student to reach the ground?

Motion Diagram



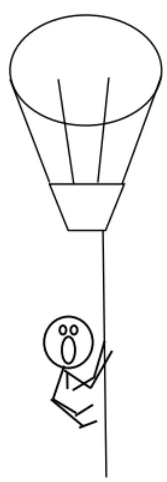
a. Motion Information

Event 1:	Event 2:
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Mathematical Analysis[27]

Free-Body Diagram

student & balloon



b. Motion Information

Event 1:	Event 2:
P_1 =	P_2 =

Event 1:	Event 2:
J_{12} =	

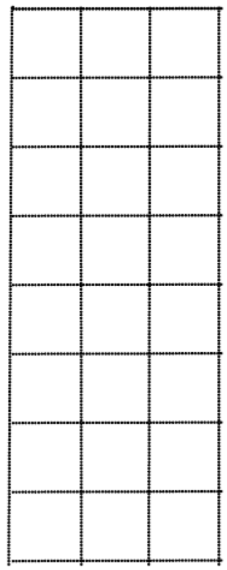
Mathematical Analysis

A 4000 kg rocket’s engine produces a thrust of 70,000 N for 15 s. The rocket is fired vertically upward.

a. What is the speed of the rocket when its engine turns off?

b. How long does it take the rocket to reach its maximum height?

Motion Diagram



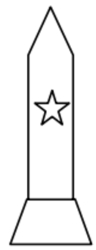
a. Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	

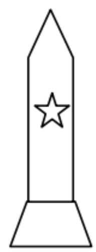
Mathematical Analysis[28]

Free-Body Diagram

while engine fires



after engine turns off



b. Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	

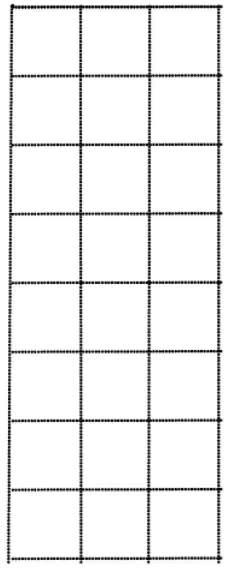
Mathematical Analysis

To practice falling, a 55 kg pole-vaulter falls off of a wall 6.0 m above a 2.0 m thick foam cushion resting on the ground. However, he misses the cushion. The pole-vaulter sinks about 0.10 m into the ground before stopping.

a. What is the speed of the pole-vaulter when he hits the ground?

b. What is the force exerted on the pole-vaulter by the ground as he comes to rest?

Motion Diagram

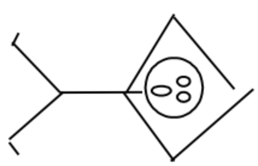


a. Motion Information

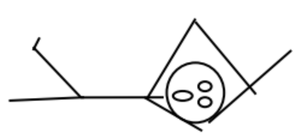
Event 1:	Event 2:
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Mathematical Analysis[29]

Free-Body Diagram



while dying



b. Motion Information

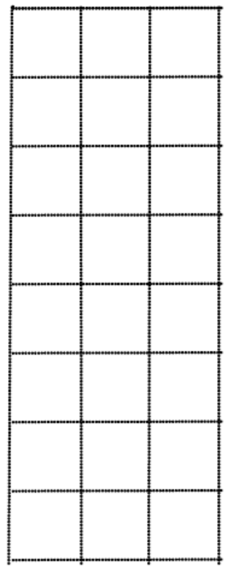
Event 1:	Event 2:
KE_1 =	KE_2 =
GE_1 =	GE_2 =

Event 1:	Event 2:
W_{12} =	

Mathematical Analysis

A decorative light fixture in an elevator consists of a 2.0 kg light suspended by a cable from the ceiling of the elevator. From this light, a separate cable suspends a second 0.80 kg light. The elevator is moving downward at 4.0 m/s when someone presses the emergency stop button. During the stop, the upper cable snaps. The elevator engineer says that the cable could withstand a force of 40 N without breaking. Find the maximum time and distance over which the elevator stopped.

Motion Diagram



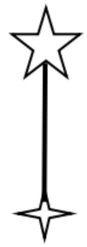
Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Mathematical Analysis[30]

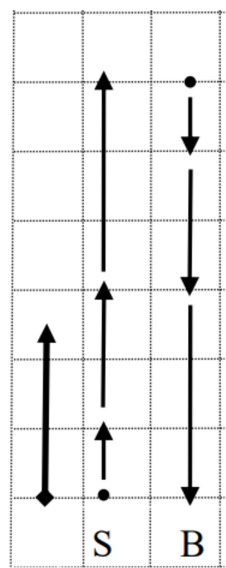
Free-Body Diagram

the two lights



Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. An 84 kg block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the 84 kg block is released, the student is pulled up to his dorm room, 8.0 m off the ground.

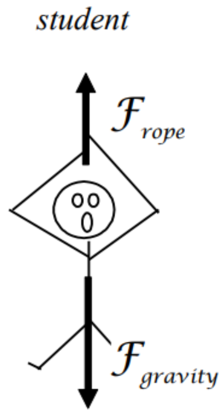
Motion Diagram

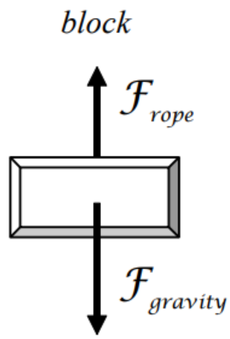


Motion Information

Event 1: The block is released	Event 2: The student reaches the room.
Object: Student	
$KE_1 = 0$	$KE_2 = 1/2(80)\mathcal{V}_f^2$
$GE_1 = 0$	$GE_2 = 80(9.8)(8)$
$W_{12} = \mathcal{F}_R(8)\cos 0^\circ$	
Object: Block	
$KE_1 = 0$	$KE_2 = 1/2(84)\mathcal{V}_f^2$
$GE_1 = 84(9.8)(8)$	$GE_2 = 0$
$W_{12} = \mathcal{F}_R(8)\cos 180^\circ$	

Free-Body Diagram





Since the distance the student and block travel is known, applying work-energy should allow us to solve the problem. I'll apply it separately to each object.

student	block
$0+F_{\text{rope}}(8)=\frac{1}{2}80v_f^2+80(9.8)(8)$	$84(9.8)(8)-F_{\text{rope}}(8)=\frac{1}{2}84v_f^2$
$8F_{\text{rope}}=40v_f^2+6272$	$6586-8F_{\text{rope}}=42v_f^2$

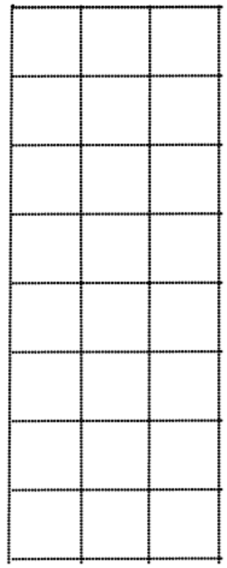
$\mathcal{F}_{\text{rope}}$ *is the same in both equations, as is the final speed. Thus the two equations can be added together to yield:*

$$6586=40v_f^2+42v_f^2+6272$$
$$314=82v_f^2$$
$$v_f=1.96\text{ m/s}$$

Notice that if you applied work-energy to the entire system you would have generated this same equation. Initially, the only form of energy present is the gravitational energy of the block (mgh = 6586 J). At the second event, both objects have kinetic energy plus the student has gravitational potential energy (mgh = 6272 J).

Tired of walking up the stairs, an engineering student designs an ingenious device for reaching his third floor dorm room. A 100 kg block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room, 8.0 m off the ground. He is traveling at 2.2 m/s when he reaches his room.

Motion Diagram

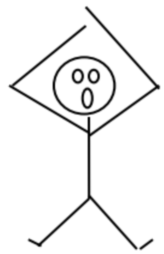


Motion Information

Event 1:	Event 2:
Object:	
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	
Object:	
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Free-Body Diagram

student

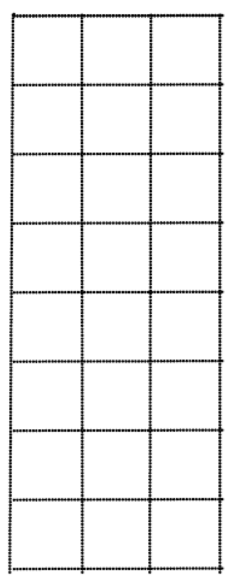


block

Mathematical Analysis[31]

Tired of walking down the stairs, a 75 kg engineering student designs an ingenious device for reaching the ground from his third floor dorm room. A 60 kg block at rest on the ground is attached to a rope that passes over a pulley. The student grabs the other end of the rope and steps out of his window. He falls for 5.5 s before reaching the ground.

Motion Diagram



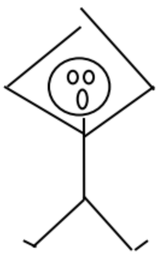
Motion Information

Event 1:	Event 2:
Object:	

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Free-Body Diagram

student

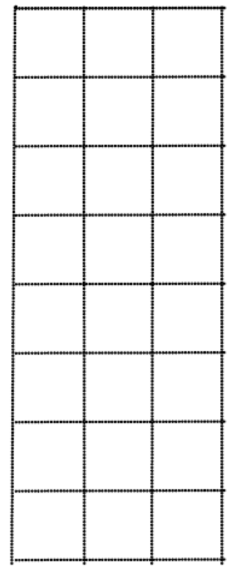


block

Mathematical Analysis[32]

Tired of walking down the stairs, a 75 kg engineering student designs an ingenious device for reaching the ground from his dorm room. A 60 kg block at rest on the ground is attached to a rope that passes over a pulley. The student grabs the other end of the rope and steps out of his window. He hits the ground at 3.3 m/s.

Motion Diagram

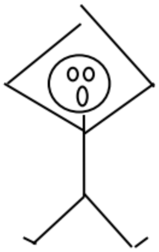


Motion Information

Event 1:		Event 2:	
Object:		Object:	
P_1 =	P_2 =	P_1 =	P_2 =
J_{12} =		J_{12} =	
P_1 =	P_2 =	P_1 =	P_2 =
KE_1 =	KE2 =	KE_1 =	KE2 =
GE_1 =	GE2 =	GE_1 =	GE2 =
W_{12} =		W_{12} =	

Free-Body Diagram

student

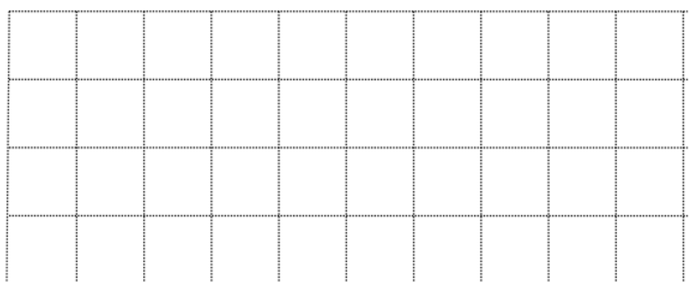


block

Mathematical Analysis[33]

Far from any other masses, a 2000 kg asteroid traveling at 12 m/s collides with a 1200 kg asteroid traveling in the other direction at 16 m/s. After the collision they remain joined together and move with a common velocity.

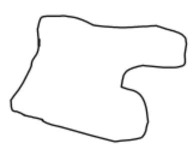
Motion Diagram



Free-Body Diagrams

2000 kg asteroid

1200 kg asteroid



Motion Information

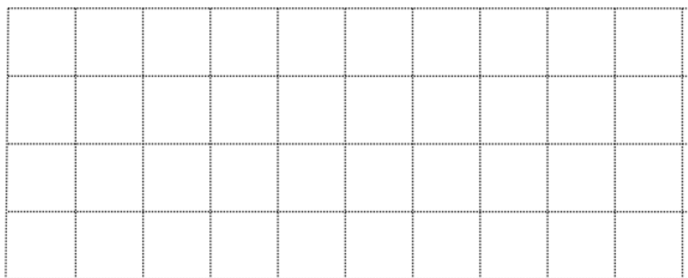
Event 1:	Event 2:
Object:	
P_1 =	P_2 =

Event 1:	Event 2:
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

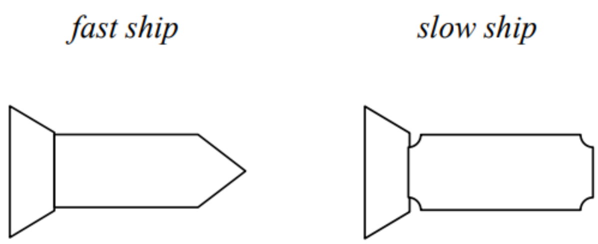
Mathematical Analysis[34]

On a remote stretch of intergalactic highway, a 7.5×10^6 kg spaceship traveling at 10 percent the speed of light ($0.10c = 3.0 \times 10^7$ m/s) doesn't notice the slower spaceship ahead clogging the lane. The fast-moving ship rear-ends the slower ship, an older 5.5×10^6 kg model, and the two ships get entangled and drift forward at 0.07c.

Motion Diagram



Free-Body Diagrams



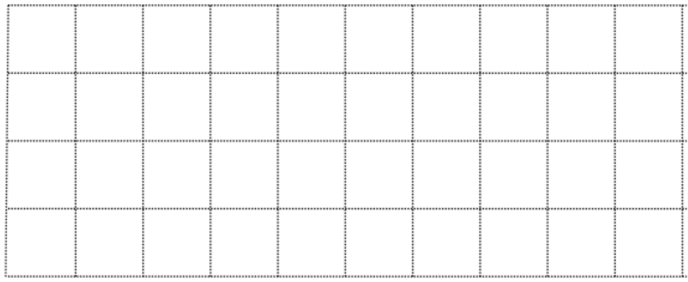
Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

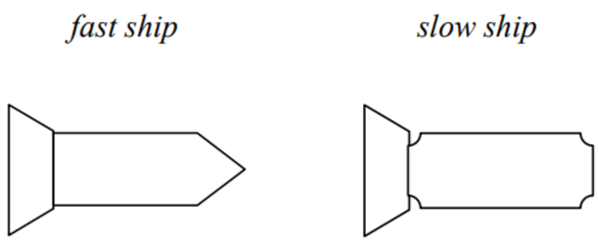
Mathematical Analysis[35]

On a remote stretch of intergalactic highway, a 7.5×10^6 kg spaceship traveling at 10 percent the speed of light ($0.10c = 3.0 \times 10^7$ m/s) doesn't notice the slower spaceship ahead, moving at 0.05c, clogging the lane. The fast-moving ship rear-ends the slower ship, an older 4.5×10^6 kg model, and the slower ship gets propelled forward at 0.13c.

Motion Diagram



Free-Body Diagrams



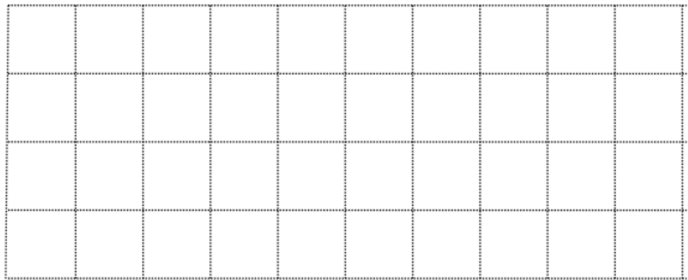
Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

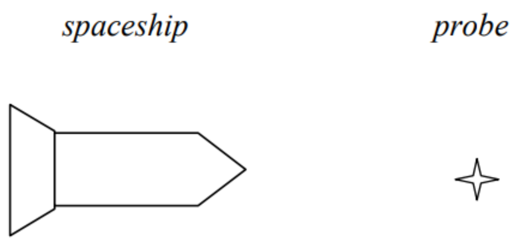
Mathematical Analysis[36]

In the farthest reaches of deep space, an 8000 kg spaceship, including contents, is at rest relative to a space station. The spaceship recoils after it launches a 600 kg scientific probe with a speed of 300 m/s relative to the space station.

Motion Diagram



Free-Body Diagrams



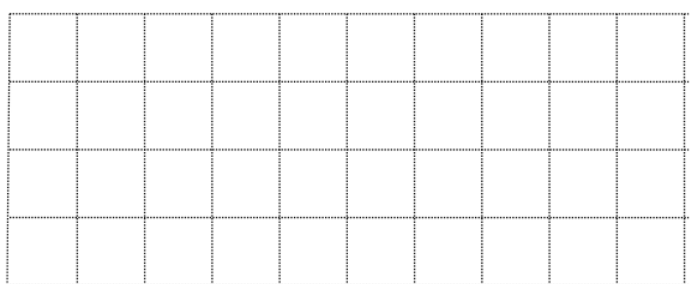
Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

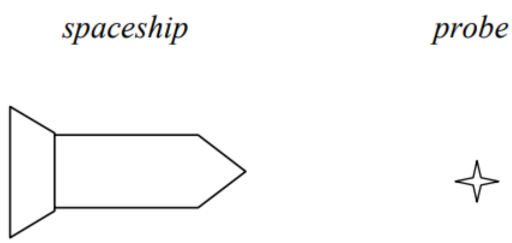
Mathematical Analysis[37]

In the farthest reaches of deep space, an 8000 kg spaceship, including contents, is drifting at 50 m/s relative to a space station. The spaceship is brought to rest, relative to the space station, by the recoil from launching a 600 kg scientific probe.

Motion Diagram



Free-Body Diagrams



Motion Information

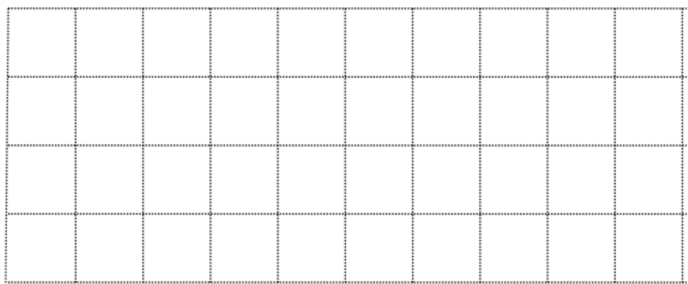
Event 1:	Event 2:
Object:	
P_1 =	P_2 =

Event 1:	Event 2:
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

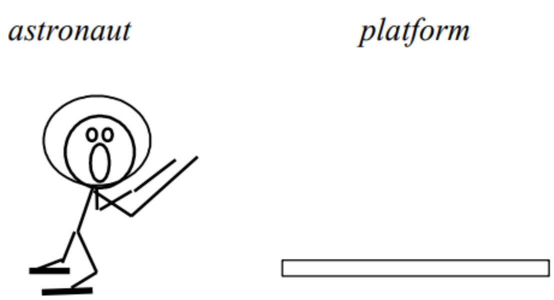
Mathematical Analysis[38]

A 140 kg astronaut is standing on the extreme edge of a 1000 kg space platform, at rest relative to the mother ship. She begins to walk toward the other edge of the platform, reaching a speed of 2.0 m/s relative to the mother ship. (She wears special magnetic shoes that allow her to walk along the metal platform.)

Motion Diagram



Free-Body Diagrams



Motion Information

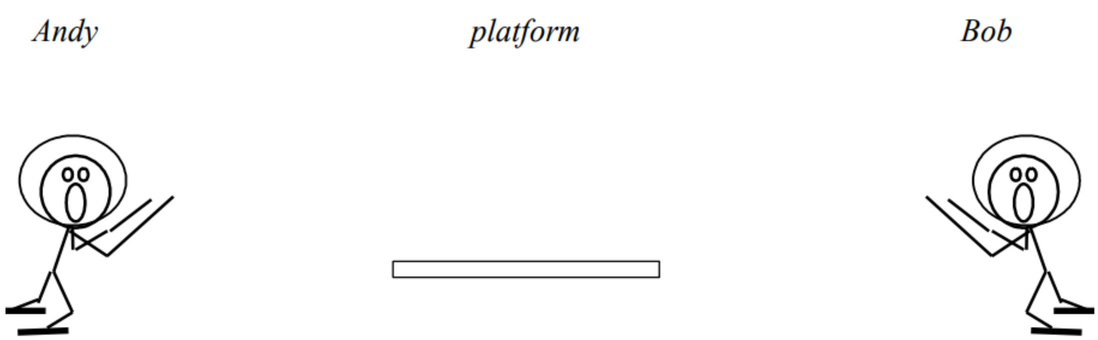
Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Mathematical Analysis[39]

Two astronauts, 140 kg Andy and 170 kg Bob, are standing on opposite edges of a 1000 kg space platform, at rest relative to the mother ship. They each begin to walk toward the opposite ends of the platform, Andy reaching a speed of 2.0 m/s and Bob 1.5 m/s, both relative to the mother ship. (They wear special magnetic shoes that allow them to walk along the metal platform.)

Motion Diagram

Free-Body Diagrams



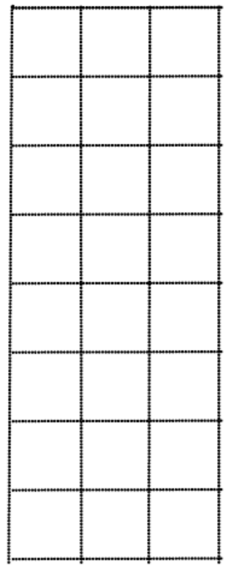
Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Mathematical Analysis[40]

A 70 kg student is hanging from a 280 kg helium balloon. The balloon is rising at a constant speed of 8.0 m/s relative to the ground. The lift on the balloon due to the buoyant force is constant. The student begins to climb up the rope at a speed of 15 m/s relative to the ground. The balloon's upward speed is decreased as the student climbs.

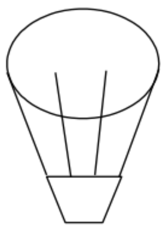
Motion Diagram



Free-Body Diagrams

student

balloon



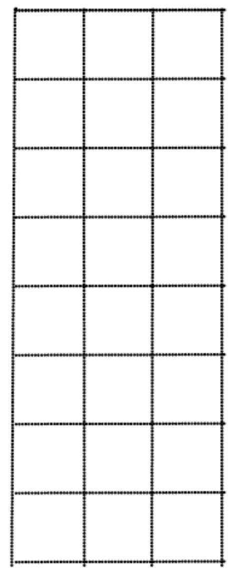
Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Mathematical Analysis[41]

A man of mass m, concerned about his weight, decides to weigh himself in an elevator. He stands on a bathroom scale in an elevator which is moving upward at v. As the elevator reaches his floor, it slows to a stop over a time interval, T. Determine the reading on the bathroom scale (F_{scale}) as a function of m, v, T, and g.

Motion Diagram



Free-Body Diagrams

Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	

Questions

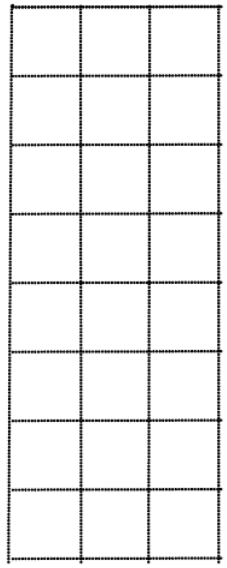
If T = ∞, what should F_{scale} equal? Does your function agree with this observation?

For what combination of v and T would the bathroom scale read 0 N?

If the elevator were initially going down, would the above combination of v and T also lead to a scale reading of 0 N?

A rocket of mass m is fired vertically upward from rest. The rocket's engine produces a thrust of constant magnitude F for t_{thrust} seconds. Determine the time it takes the rocket to reach its apex (t_{apex}) as a function of F, t_{thrust}, m, and g.

Motion Diagram



Free-Body Diagrams

Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	

Questions

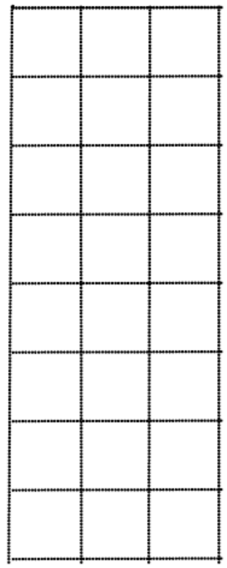
If $g = 0\ m/s^2$, what should t_{apex} equal? Does your function agree with this observation?

If $F = mg$, what should t_{apex} equal? Does your function agree with this observation?

For what value of F would $t_{apex} = 2t_{thrust}$?

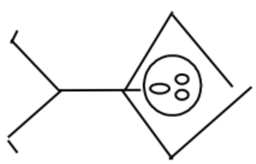
To practice falling, a pole-vaulter of mass m falls off of a wall a distance D above a thick foam cushion. The pole-vaulter sinks a distance d into the cushion before stopping. Determine the force exerted on the pole-vaulter due to the cushion ($F_{cushion}$) as a function of m, D, d, and g.

Motion Diagram

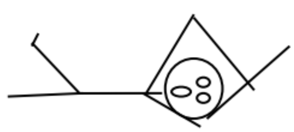


Free-Body Diagrams

while falling



while dying



Motion Information

Event 1:	Event 2:
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Questions

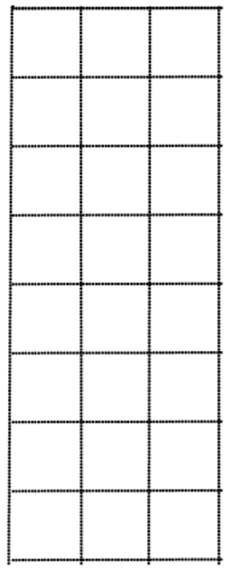
If D = ∞, what should $F_{cushion}$ equal? Does your function agree with this observation?

If d = 0 m, what should $F_{cushion}$ equal? Does your function agree with this observation?

What would be worse for the pole-vaulter, starting at twice the initial distance above the cushion or sinking half of the original distance into the cushion?

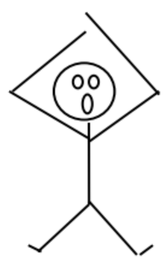
Tired of walking up the stairs, an engineering student of mass m designs an ingenious device for reaching his third floor dorm room. A block of mass M is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room in a time T. Determine the velocity of the student (v) when he reaches his room as a function of m, M, T and g.

Motion Diagram



Free-Body Diagrams

student



block

Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Questions

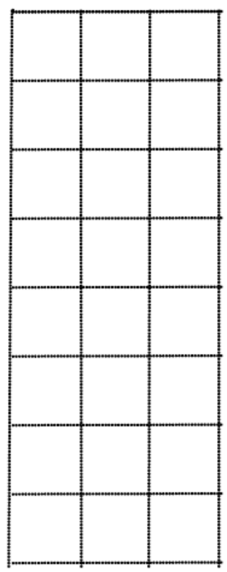
If g = 0 m/s^2, what should v equal? Does your function agree with this observation?

If m = M, what should v equal? Does your function agree with this observation?

If M = ∞, what should v equal? Does your function agree with this observation?

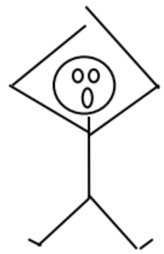
Tired of walking down the stairs, an engineering student of mass m designs an ingenious device for reaching the ground from her dorm room. A block of mass M at rest on the ground is attached to a rope that passes over a pulley. The student grabs the other end of the rope and steps out of her window a distance D above the ground. Determine the velocity of the student (v) when she reaches the ground as a function of m, M, D and g.

Motion Diagram

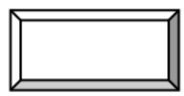


Free-Body Diagrams

student



block



Motion Information

Event 1:	Event 2:
Object:	
KE_1 =	KE2 =
GE_1 =	GE_2 =
W_{12} =	
Object:	
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Questions

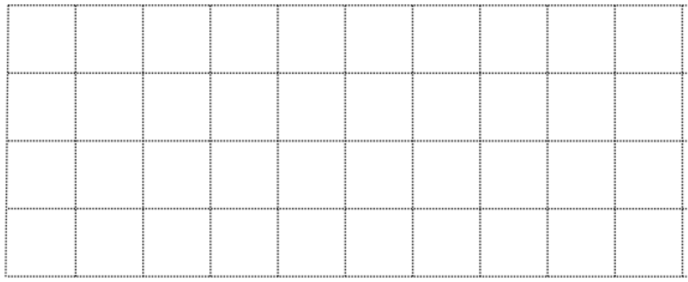
If g = 0 m/s^2, what should v equal? Does your function agree with this observation?

If m = M, what should v equal? Does your function agree with this observation?

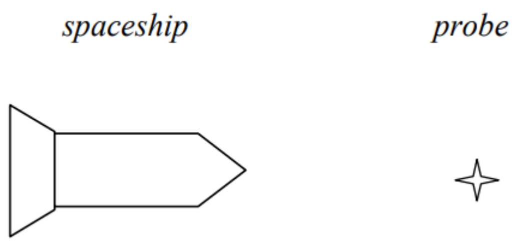
If M = ∞, what should v equal? Does your function agree with this observation?

In the farthest reaches of deep space, a spaceship of mass M, including contents, is at rest relative to a space station. The spaceship recoils after it launches a scientific probe of mass m at a speed v relative to the space station. Determine the recoil speed of the spaceship (V) as a function of M, m, and v.

Motion Diagram



Free-Body Diagrams



Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Questions

If M = 2m, what should V equal? Does your function agree with this observation?

If M = ∞, what should V equal? Does your function agree with this observation?

Model One

Summary Problems

1. A sprinter in a 100 meter dash starts from rest and accelerates at 2.5 m/s^2 for 3.9 s. She then runs at a constant speed, until she either tires or finishes the race. She can run at a constant velocity for at most 7.0 s without tiring. Once tired, she begins to slow, accelerating at 1.0 m/s^2. How long does it take her to finish the race?
2. The *Schindler Mobile* is a self-propelled elevator, powered by a small motor attached to the bottom of the elevator cab that drives *the cab up and* down the tracks of two high-strength aluminum columns. Assume the Schindler Mobile can travel from the 1^{st} floor to the 10^{th} floor (approximately 40 m) in 15 s. Assume the elevator both speeds up and slows
3. *The Thrust SSC*, a 7 ton jet mobile powered by two Royal Air Force Phantom jets providing 110,000 hp, was designed to break the speed of sound. The "car" was tested at a 15 mile long track in the Black Rock desert of Nevada. The car accelerated for nearly five miles, then moved through a measured mile at top speed. The car slowed by cutting power and releasing parachutes for five additional miles before applying brakes at speeds below 300 mph. Complete a kinematic description for the car's motion, assuming it reaches a maximum speed of Mach 1 (750 mph at the temperatures encountered at the Black Rock desert raceway).

4. Often the reason for the formation of congested traffic is obvious; accidents, lane closings, or other bottlenecks. However, you have probably also experienced "phantom" traffic jams, which emerge without any obvious reason, seemingly *out of nothing*. This phenomenon can be understood by the collective behavior of many drivers. If one vehicle drives more slowly than others, the vehicle behind has to brake to maintain the desired *safety time*. (The safety time is the elapsed time between the two objects passing the same point. Thus, the distance associated with this "time cushion" varies with the speed of the traffic.) Consequently, the next vehicle behind has to brake, and so on. If traffic flow is unstable, each following vehicle has to brake stronger than its predecessor. Thus, a small initial perturbation triggers a backward propagating "wave" of slower vehicles with increasing amplitude. Finally, the vehicles come nearly to a stop; a full-fledged traffic jam has evolved. The driver having caused the small perturbation by driving unusually slowly escapes without even realizing what he has triggered!

   To get a better feel for the kinematics involved in instigating phantom jams, imagine a car approaching a slow-moving truck on a one-lane road. The car is initially traveling at 120 km/hr, while the truck moves at 70 km/hr. The car is 100 m behind the truck when the driver first notices the truck. Find the minimum acceleration necessary for the car to come to equilibrium behind the truck and achieve a safety time of 2 s. Assume the truck does not accelerate.
5. When modeling traffic flow, various psychological factors must be incorporated. One is the politeness factor. The *politeness factor* quantifies how much one weights disadvantages imposed on other drivers against one's own advantage when considering a lane change. Lane changes are more common when the politeness factor is low. Different regions of the country have, on average, different politeness factors. In addition, urban vs. rural drivers differ in politeness factor. At high values of the politeness factor, drivers run the risk of getting stuck permanently behind slow-moving vehicles or other obstacles.

   To get a better feel for how "politeness" affects traffic flow, imagine a car stuck behind a 5 m long, stationary obstacle blocking its lane. 30 m ahead of the car is a turn in the road. Therefore, the driver of the car cannot see an approaching car or truck until it is 30 m from the car.
    a. If the car pulls out to go around the obstacle (with acceleration 4 m/s^2) just as a truck moving at 55 mph rounds the bend, is this an “extremely polite" maneuver? (A maneuver is extremely polite when the truck does not need to slow down in order to avoid an accident.)
    b. What acceleration is necessary for an extremely polite driver to pull out from behind the barrier? (If the car cannot generate this acceleration, an extremely polite driver must spend the rest of their life stuck behind the barrier!)
6. Traffic engineers are concerned with selecting the proper "yellow time" to ensure safe passage through stoplights. To understand this scenario, imagine yourself driving down a relatively empty road. Up ahead, the traffic light turns yellow. If you are close enough to the traffic light you can pass through the intersection before the light turns red. If you are far from the traffic light you can safely slow down and stop before the intersection. But what if you are in-between, in what is termed the "no-win" zone, and are too far to make it and too close to stop? Traffic engineers design the duration of the yellow signal to eliminate this no-win zone.
    a. You are driving at the speed limit (45 mph) on a straight, empty road with perfect visibility. Your maximum acceleration while braking is 7.0 m/s^2. The yellow time is 1.0 s. Determine the location of the no-win zone (i.e., the range of positions from which you cannot safely traverse the intersection). Assume you don't speed up to "run" the yellow, since this is an illegal activity.
    b. If you do want to "run" the yellow from anywhere in the no-win zone, what minimum acceleration is needed? Is this feasible? How fast would you be traveling as you go through the intersection?
7. Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. A heavy block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room, 8.0 m off the ground. However, the student has a weak stomach and will get nauseous if he accelerates at greater than 4.0 m/s^2. Also, the rope he used can transmit a force of only 1100 N before breaking. If possible, what mass ballast block should he use to avoid breaking the rope and avoid getting nauseous?
8. Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching her third floor dorm room. A heavy block is attached to a rope that passes over a pulley. The student holds the other end of the rope. However, the dorm room window is 12 m off the ground and the block is initially only 10 m off the ground. The student wants to choose a mass for the block such that when the block hits the ground, the student is “launched” upward, and reaches her window at the apex of her motion. (That will make it easiest to climb in the window.) What mass block should she use?

Selected Answers

[1] r2 = 1.4 m

[2] t2 = 1.45 m

[3] t3 = 3.4 s

[4] t3 = 4.55 s

[5] r3 = 36.6 m

[6] r3 = 3520 m

[7] r2 = 14.9 m

[8] r4 = 2000 m

[9] t4 = 3.7 x 105 s

[10] t2 = 14.9 s

[11] t2 = 7.8 s

[12] t3 = 15.7 s

[13] t3 = 2.87 s

[14] F rope = 420 N

[15] F cushion = 1910 N

[16] a ≥ 4.49 m/s2

[17] t2 = 17.1 s to reach ground

[18] F cushion = 2830 N

[19] F ground = 43700 N

[20] r3 = 63.5 m

[21] r3 = 1550 m

[22] m block = 240 kg

[23] m block = 26 kg

[24] F rope = 500 N

[25] F rope = 466 N

[26] a. F scale = 755 N b. F scale = 780 N

[27] a. v = 17.5 m/s b. t = 17.1 s

[28] a. v = 116 m/s b. t = 26.8 s

[29] a. v = 12.5 m/s b. F ground = 43700 N

[30] t2 = 0.89 s r2 = 1.78 m

[31] m student = 94 kg

[32] v2 = 6.0 m/s

[33] Student falls 5.0 m in 3.03 s

[34] v2 = 1.5 m/s

[35] v1 slowship = 0.029c = 8.73×10^6 m/s

[36] v2 fastship = 0.052c = 1.56×10^7 m/s

[37] v2 ship = 24.3 m/s

[38] v probe = 667 m/s

[39] v2 platform = 0.28 m/s

[40] v2 platform = 0.025 m/s

[41] v2 balloon = 6.3 m/s

2.4: Conservation Laws

Concepts and Principles

What is a Conservation Law?

In general, a conservation law is a statement that a certain quantity does not change over time. If you know how much of this quantity you have today, you can be assured that the exact same amount of the quantity will be available tomorrow. A famous (at least to physicists) explanation of the nature of a conservation law was given by Richard Feynman.

> *Imagine your child has a set of 20 wooden blocks. Every day before bedtime you gather up your child's blocks to put them away. As you gather up the blocks, you keep count in your head. Once you reach 20, you know you have found all of the blocks and it is unnecessary for you to search any longer. This is because the number of blocks is conserved. It is the same today as it was yesterday.*
>
> *If one day you only find 18 blocks, you know to keep looking until you find the missing 2 blocks. Also, with experience, you discover the typical hiding places for the blocks. You know to check under the sofa, or behind the curtains.*
>
> *If your child is rambunctious, you may even have to look outside of the room. Perhaps he threw a block or two out of the window. Even though blocks can disappear from inside of the room, and appear out in the yard, if you search everywhere you will always find the 20 blocks.*

Physicists have discovered a number of quantities that behave exactly like the number of wooden blocks. We will examine two of these quantities, energy and momentum, below.

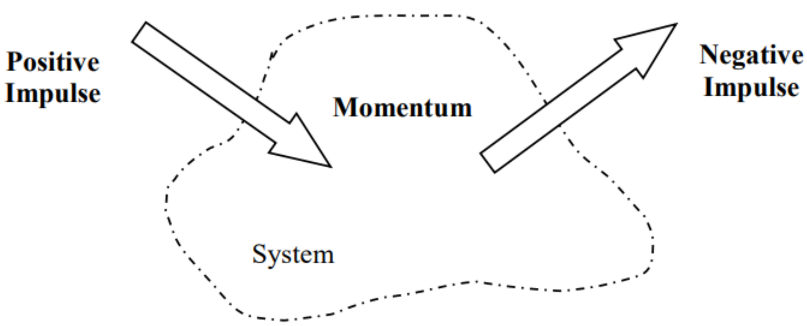
The Impulse-Momentum Relation

While Newton's Second Law directly relates the total force that acts on an object at a specific time to the object's acceleration at that exact same time, conservation laws relate the amount of a certain quantity present at one time to the amount present at a later time.

The first conserved quantity we will investigate is *momentum*. Of course, just because momentum is conserved doesn't mean that the momentum of any particular object or system of objects is always constant. The momentum of a single object, like the number of blocks in the playroom, can change. Just as blocks can be thrown out of the window of the playroom, the momentum of a single object can be changed by applying *impulse* to it. The relationship between impulse and momentum is, conceptually,

$$\text{initial momentum} + \text{impulse} = \text{final momentum}$$

Pictorially, we can visualize this as



In practice, we will identify an object or collection of objects (a *system*) and determine the amount of momentum the system contains at some initial time. This quantity cannot change unless impulse is done to the system. We call processes that bring momentum into the system as positive impulses, and processes that remove momentum from the system as negative impulses.

Mathematically this is written as

$$\text{initial momentum} + \text{impulse} = \text{final momentum}$$
$$P_i + J_{if} = P_f$$
$$\Sigma m v_i + \Sigma F(\Delta t) = \Sigma m v_f$$

where

- momentum (P) is the product of an object's mass and velocity,
- impulse (J) is the product of a force *external to the system* and the time interval over which it acts,
- and Σ indicates that you must sum the momentum of all of the objects in the system and all of the impulses acting on the system.

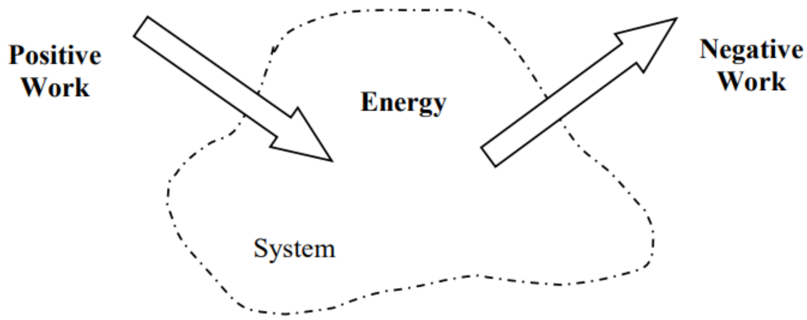
In short, if no impulse is applied to a system, its momentum will remain constant. However, if an impulse is applied to the system, its momentum will change by an amount exactly equal to the impulse applied. This momentum does not appear or disappear without a trace. It is simply transferred to the object *supplying* the impulse. In this sense, impulse is the transfer of momentum into or out of a system, analogous to tossing blocks into or out of a playroom.

The Work-Energy Relation

The second conserved quantity we will investigate is *energy*. Just like momentum, or wooden blocks, the conservation of energy doesn't mean that the energy of any particular object is always constant. The energy of a single object or system of objects can be changed by doing *work* to it. The relationship between work and energy is, conceptually,

$$\text{initial energy } + \text{ work } = \text{ final energy}$$

Pictorially, we can visualize this as



The similarity between momentum and energy is not complete, however. While there is only one form of momentum (i.e., one hiding place for momentum "blocks") there are several forms of energy. These different forms of energy will be introduced as you progress through more and more complicated models of the physical world. For now, the only "hiding place" I want to discuss is *kinetic energy*. In terms of kinetic energy, the above conceptual relationship between work and energy becomes, expressed mathematically,

$$\begin{gathered}\text{initial energy } + \text{ work } = \text{ final energy} \\ KE_i + W_{if} = KE_f \\ \Sigma \frac{1}{2} m v_i^2 + \Sigma |F||\Delta r| \cos\phi = \Sigma \frac{1}{2} m v_f^2\end{gathered}$$

where

- kinetic energy (KE) is the product of one-half an object's mass and squared velocity,
- work (W) is the product of a force *(even an internal force)* and the displacement over which it acts (with more subtle details discussed below),
- Σ indicates that you must sum the kinetic energy of all of the objects in the system and all of the work done to the system,
- and we define a new unit, Joule (J), as J = kg (m/s)^2 = N m

Unlike anything we've studied up to this point, the work-energy relation is a *scalar* equation. This will become especially important when we study objects moving in more than one dimension. For now, all this means is that in the expression for work, $|F||\Delta r| \cos\phi$, we should use the *magnitude* of the force and the *magnitude* of the change in position. This product is then multiplied by $\cos\phi$, where ϕ is defined to be the angle between the applied force and the displacement of the object. If the force and displacement are in the same direction $\phi = 0^\circ$, and the work is positive (the object gains energy). If the force and displacement are in the opposite direction $\phi = 180^\circ$, and the work is negative (the object loses energy). Note that the actual directions of the force and the displacement are unimportant, only their directions *relative to each other* affect the work.

In general, if no work is done to a system, its kinetic energy will remain constant. However, if work is done to system, its total energy will change by an amount exactly equal to the work done. Work is the transfer of energy from one system to another, again analogous to tossing blocks from the playroom into the yard.

Analysis Tools

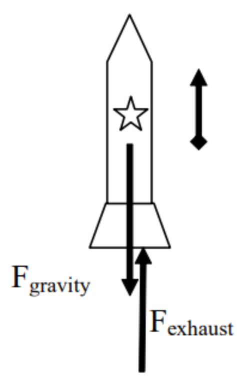
Applying the Impulse-Momentum Relation to a Single Object

Let's investigate the following scenario:

> *A 0.35 kg model rocket is fitted with an engine that produces a thrust of 11.8 N for 1.8 s. The rocket is launched vertically upward.*

To apply the impulse-momentum relation, you must clearly specify the initial and final events at which you will tabulate the momentum. For example:

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches maximum height.
$P_1 = 0$	$P_2 = (0.35)\ v_2$
$J_{12} = +F_{\text{exhaust}}\ (1.8) - F_{\text{gravity}}\ (1.8)$	



Note that each external force acting on the rocket is multiplied by the time interval over which it acts. (Also note that the rocket's engine does not produce a force on the rocket! The engine produces a downward force on the hot exhaust gases emitted from the engine and these hot gases exert an equal magnitude force back up on the rocket. That is why the force on the rocket is labeled as $F_{exhaust}$ rather than F_{engine}.)

Applying impulse-momentum to the rocket during this time interval yields:

$$
\begin{aligned}
&P_1 + J_{12} = P_2 \\
&0 + F_{\text{exhaust}}\ (1.8) - F_{\text{gravity}}\,(1.8) = 0.35 v_2 \\
&0 + (11.8)(1.8) - (0.35)(9.8)(1.8) = 0.35 v_2 \\
&v_2 = 43.0\ \text{m/s}
\end{aligned}
$$

Thus, the rocket is traveling at 43.0 m/s at the instant the engine shuts off.

Of course, there is no reason why we had to analyze the rocket's motion between the two instants of time we selected above. We could have selected the events:

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches maximum height.
$P_1 =$	$P_2 = 0$
$J_{12} = +F_{\text{exhaust}}\ (1.8) - F_{\text{gravity}}\ (\Delta t)$	

During this time interval, the force of the exhaust gases only act on the rocket for a *portion* of the entire time interval. Noting that the rocket's velocity when it reaches its maximum height is zero, impulse-momentum would look like this:

$$P_1 + J_{12} = P_2$$
$$0 + F_{\text{exhaust}}\,(1.8) - F_{\text{gravity}}\,(\Delta t) = 0$$
$$0 + (11.8)(1.8) - (0.35)(9.8)(\Delta t) = 0$$
$$\Delta t = 6.19s$$

Thus, the rocket is in the air for 6.19 s before reaching its maximum height.

Applying the Work-Energy Relation to a Single Object

The work-energy relation also has many uses for investigating physical scenarios. For example, let's look again at our model rocket:

> *A 0.35 kg model rocket is fitted with an engine that produces a thrust of 11.8 N for 1.8 s. The rocket is launched vertically upward.*

Assuming we've already analyzed this scenario using impulse-momentum, what additional information can we extract using work-energy?

Event 1: The instant the engine is ignited.	Event 2: The instant the engine shuts off.
$KE_1 = 0$	$KE_2 = 1/2\ (0.35)(43)^2$
$W_{12} = F_{\text{exhaust}}\ (\Delta r)\cos 0 + F_{\text{gravity}}\ (\Delta r)\cos 180$	

Therefore,

$$KE_1 + W_{12} = KE_2$$
$$0 + |F_{\text{exhaust}}\,||\Delta r|\cos 0 + |F_{\text{gravity}}|\,|\Delta r|\cos 180 = \frac{1}{2}(0.35)(43.2)^2$$
$$0 + 11.8(\Delta r)(1) + (0.35)(9.8)(\Delta r)(-1) = 327$$
$$11.8\Delta r - 3.43\Delta r = 327$$
$$\Delta r = 39.1\text{ m}$$

Thus, the rocket rises to a height of 39.1 m before the engines shuts off.

What if we apply work-energy between the following two events:

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches its maximum height
$KE_1 = 0$	$KE_2 = 0$
$W_{12} = F_{\text{exhaust}}\ (39)\cos 0 + F_{\text{gravity}}\ (\Delta r)\cos 180$	

During this time interval, the force of the exhaust gases only act on the rocket for a *portion* of the entire displacement, namely 39 m, while the force of gravity acts over the entire displacement.

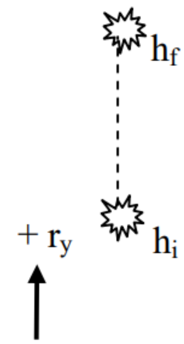
$$KE_1 + W_{12} = KE_2$$
$$0 + (11.8)(39)\cos 0 + (0.35)(9.8)(\Delta r)\cos 180 = 0$$
$$0 + 460 - 3.43\Delta r = 0$$
$$\Delta r = 134m$$

Thus, the maximum height reached by the rocket is 134 m.

Gravitational Potential Energy

In any situation in which an object changes its height above the surface of the earth, the force of gravity does work on the object. It is possible to calculate this work in general, and to rewrite the work-energy relation in such a way as to incorporate the effects of this work. This is referred to as constructing a *potential energy function* for the work done by gravity.

Let's imagine an object of mass, m, located an initial height, h_i, above the zero of a vertical coordinate system, with the upward direction designated positive. It moves to a final height of h_f.



To calculate the work done by gravity on this object:

$$\begin{aligned} W_{\text{gravity}} &= |F||\Delta r|\cos\phi \\ W_{\text{gravity}} &= (mg)\left(h_f - h_i\right)\cos 180 \\ W_{\text{gravity}} &= -mgh_f + mgh_i \end{aligned}$$

The “mgh” terms are referred to as *gravitational potential energy*. Thus, the work done by gravity can be thought of as changing the gravitational potential energy of the object. Let’s plug the above result into the work-energy relation:

$$\begin{aligned} &\tfrac{1}{2}mv_i^2 + \Sigma|F||\Delta r|\cos\phi = \tfrac{1}{2}mv_f^2 \\ &\tfrac{1}{2}mv_i^2 + \Sigma|F||\Delta r|\cos\phi + W_{\text{gravity}} = \tfrac{1}{2}mv_f^2 \\ &\tfrac{1}{2}mv_i^2 + \Sigma|F||\Delta r|\cos\phi - mgh_f + mgh_i = \tfrac{1}{2}m{v_f}^2 \\ &\tfrac{1}{2}mv_i^2 + mgh_i + \Sigma|F||\Delta r|\cos\phi = \tfrac{1}{2}mv_f^2 + mgh_f \end{aligned}$$

Therefore, this final relation:

$$\begin{aligned} &KE_i + GE_i + W_{if} = KE_f + GE_f \\ &\tfrac{1}{2}mv_i^2 + mgh_i + \Sigma|F||\Delta r|\cos\phi = \tfrac{1}{2}mv_f^2 + mgh_f \end{aligned}$$

can (and will) be used in place of the standard work-energy relation provided:

1. You do not include the force of gravity a second time by calculating the work done by gravity. Basically, in this relationship gravity is no longer thought of as a force that does work on objects but rather as a source of potential energy.
2. You calculate the initial and final heights, h_i and h_f, using a coordinate system in which the upward direction is positive.

Applying Work-Energy with Gravitational Potential Energy

Let’s use the work-energy relation, with gravitational potential energy terms, to re-analyze the previous scenario:

> *A 0.35 kg model rocket is fitted with an engine that produces a thrust of 11.8 N for 1.8 s. The rocket is launched vertically upward.*

Let’s apply work-energy between the following two events, setting the initial elevation of the rocket equal to zero:

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches its maximum height
$KE_1 = 0$	$KE_2 = 0$
$GE_1 = 0$	$GE_2 = (0.35)(9.8)\ h_2$
$W_{12} = F_{\text{exhaust}}\ (39)\cos 0$	

During this time interval, the force of the exhaust gases only act on the rocket for a *portion* of the entire displacement, namely 39 m. Remember, the force of gravity *does not do work* in this way of modeling nature, rather the gravitational energy of the rocket changes as it changes its elevation.

$$\begin{aligned} &KE_i + GE_i + W_{if} = KE_f + GE_f \\ &0 + 0 + (11.8)(39)\cos 0 = 0 + (0.35)(9.8)h_f \\ &0 + 0 + 460 = 0 + 3.43h_f \\ &h_f = 134m \end{aligned}$$

results in, of course, the same maximum height reached by the rocket.

Applying the Impulse-Momentum Relation to a Collision

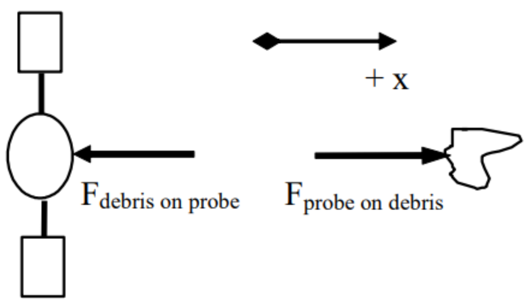
Probably the most useful application of the impulse-momentum relation is in the study of collisions. For example:

> *Far from the earth, a 250 kg space probe, moving at 5 km/s, collides head-on with a 60 kg piece of space debris initially at rest. The debris becomes entangled in the probe's solar collectors.*

Let's choose:

Event 1: The instant before the collision.	Event 2: The instant the debris and probe reach a common velocity.
Object: Space Probe	
P_1 = (250)(5000)	P_2 = 250 v_2
$J_{12} = -F_{\text{debrisonpobe}}\ (\Delta t)$	
Object: Debris	
P_1 = 0	P_2 = 60 v_2
$J_{12} = +F_{\text{probeondebris}}\ (\Delta t)$	

The free-body diagrams for the two objects during this time interval are shown below.



Applying the impulse-momentum relation to each object separately yields:

Probe	*Debris*
$P_1 + J_{12} = P_2$	$P_1 + J_{12} = P_2$
$250(5000) - F_{\text{debrisonprobe}}\ (\Delta t) = 250v_2$	$0 + F_{\text{probeondebris}}\ (\Delta t) = 60v_2$
$1250000 - F_{\text{debrisonprobe}}\ (\Delta t) = 250v_2$	$F_{\text{probeondebris}}\ (\Delta t) = 60v_2$

Notice that the final velocities of the two objects are the same, because they remain joined together following the collision. Also, the Δt's are the same because the time interval over which the force of the debris acts on the probe must be the same as the time interval over which the force of the probe acts on the debris. In fact, these two forces must be equal to each other in magnitude by Newton's Third Law.

Thus, the impulses must cancel if the two equations are added together:

$$1250000 - F_{\text{debrisonprobe}}(\Delta t) = 250 v_2$$
$$F_{\text{probeondebris}}(\Delta t) = 60 v_2$$
$$1250000 = 310 v_2$$
$$v_2 = 4032 \text{ m/s}$$

The probe slows to a speed of 4032 m/s (and the debris changes direction and accelerates to a speed of 4032 m/s) via the collision. Thus, even though we do not know the magnitude of the force involved, or the duration of the collision, we can calculate the final velocities of the two objects colliding. This is because the forces involved comprise an interaction, and by Newton's Third Law forces that comprise an interaction are always equal in magnitude and opposite in direction.

In fact, in problems involving collisions (or explosions, which to physicists are merely collisions played backward in time!), you should almost always apply the impulse-momentum relation to the interacting objects because the forces involved comprise an interaction. Thus, by adding your equations together, these terms will always add to zero. This will often allow you to determine the final velocities of the colliding objects.

In conclusion, I should point out that the probe loses momentum during the collision and that the debris gains the exact same amount of momentum. (Check the numbers to verify this statement.) The momentum is transferred from the probe to the debris through the action of the impulse the probe and debris exert on each other. The momentum transfer from the probe to the debris is analogous to throwing a wooden block from the playroom into the yard: The playroom now has one less block and the yard has one more!

Applying the Work-Energy Relation to the Same Collision

Let's return to the collision scenario discussed above and attempt to investigate it using workenergy.

> *Far from the earth, a 250 kg space probe, moving at 5 km/s, collides head-on with a 60 kg piece of space debris initially at rest. The debris becomes entangled in the probe's solar collectors.*

Event 1: The instant before the collision.	Event 2: The instant the debris and probe reach a common velocity.
Object: Space Probe	
$KE_1 = 1/2\ (250) 5000^2$	$KE_2 = 1/2\ (250) v_2{}^2$
$GE_1 = 0$	$GE_2 = 0$
$W_{12} = F_{\text{on P}}\ (\Delta r_P) \cos 180$	
Object: Debris	
$KE_1 = 0$	$KE_2 = 1/2\ (60) v_2{}^2$
$GE_1 = 0$	$GE_2 = 0$
$W_{12} = F_{\text{on D}}\ (\Delta r_D) \cos 0$	

Applying the work-energy relation to each object separately yields:

Probe	*Debris*
$KE_i + GE_i + W_{if} = KE_f + GE_f$	$KE_i + GE_i + W_{if} = KE_f + GE_f$
$\frac{1}{2}(250)(5000)^2 + (F_{onP})(\Delta r_{\text{probe}})\cos 180 = \frac{1}{2}(250)(v_{2\text{ probe}})^2$	$0 + (F_{onD})(\Delta r_{\text{debris}})\cos 0 = \frac{1}{2}(60)(v_{2\text{ debris}})^2$
$3.13 \times 10^9 - F_{\text{onP}}\ (\Delta r_{\text{probe}}) = 125 v_{2\text{ probe}}{}^2$	$F_{\text{onD}}\ (\Delta r_{\text{debris}}) = 30 v_{2\text{ debris}}{}^2$

The final velocities of the two objects are the same, because they remain joined together following the collision, and the two forces are the same by Newton's Third Law. **However, these two equations cannot be added together and solved because the two distances over which the forces act, Δr_{probe} and Δr_{debris}, are not necessarily equal.** During the collision, the center of the probe will move a different distance that the center of the debris[3]. Since these two distances are different, the works will *not* cancel as the impulses did, and the equations are *not* solvable!

> 3 If the two objects were *actually* particles, rather than being *approximated* as particles, then the two distances would have to be the same and the two works would cancel when the equation were added together.

In fact, since we know v_2 = 4032 m/s from our momentum analysis,

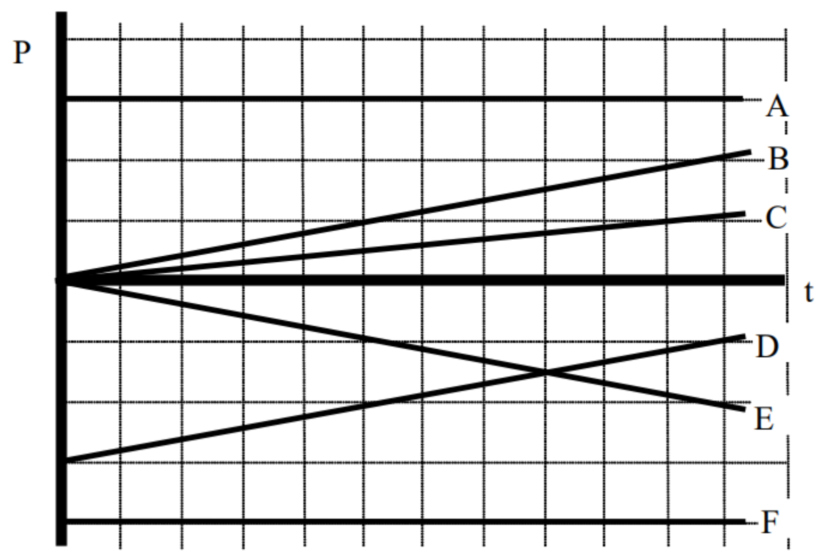
Probe	*Debris*
$3.13 \times 10^9 - F_{\mathrm{onP}}\left(\Delta r_{\text{probe}}\right)=125(4032)^2$	$F_{\mathrm{onD}}\left(\Delta r_{\text{debris}}\right)=30(4032)^2$
$3.13 \times 10^9 - F_{\mathrm{onP}}\left(\Delta r_{\text{probe}}\right)=2.03 \times 10^9$	$W_{\text{ondebris}}=F_{\mathrm{onD}}\left(\Delta r_{\text{debris}}\right)=0.49 \times 10^9 J$
$W_{\text{onprobe}}=-F_{\mathrm{onP}}\left(\Delta r_{\text{probe}}\right)=-1.1 \times 10^9$ J	

Obviously, the two works do not cancel. In fact, the *internal work*, or work done by the objects on each other, totals -0.61 x 10^9 J. This means that there is 0.61 x 10^9 J *less* kinetic energy in the system of the probe and the debris after the collision than before the collision. This is sometimes referred to as the energy lost in the collision, although the energy is not lost but rather converted into other forms of energy (i.e., other hiding places for the wooden blocks that have yet to be discussed), such as thermal energy.

In short, the work-energy relation (as it now stands) cannot be used to effectively analyze collisions unless additional information regarding the internal energy is available. Occasionally, an approximation is made in which the total internal work is zero. When this approximation is made, the collision is referred to as an *elastic* collision. Realistic collisions, in which the total internal energy is not zero and kinetic energy is "lost", are referred to as *inelastic* collisions.

Activities

Below are momentum vs. time graphs for six different objects.



a. Rank these graphs on the basis of the change in momentum of the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

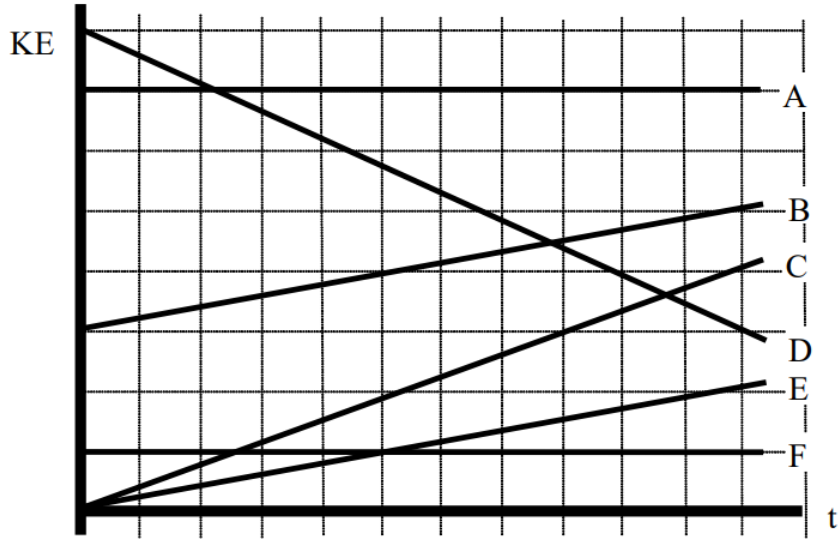
b. Rank these graphs on the basis of the total impulse on the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are kinetic energy vs. time graphs for six different objects. All of the objects move horizontally.



a. Rank these graphs on the basis of the change in kinetic energy of the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

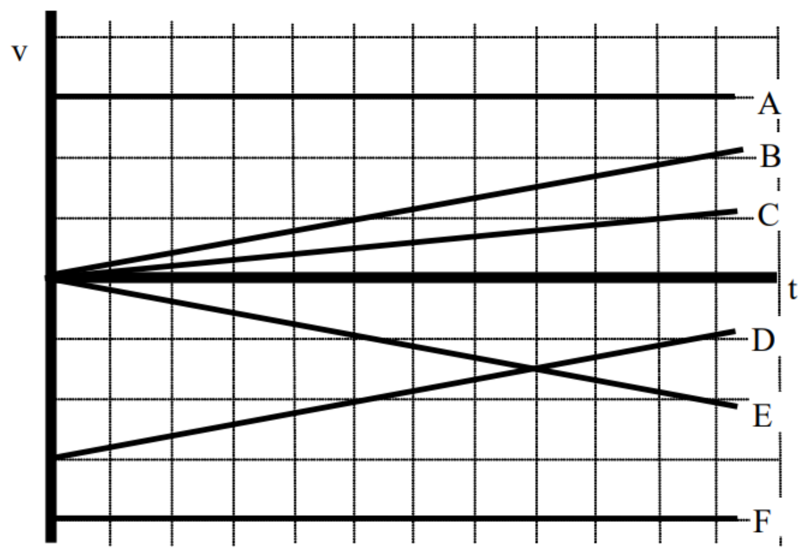
b. Rank these graphs on the basis of the total work on the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are velocity vs. time graphs for six equal-mass objects.



a. Rank these graphs on the basis of the change in momentum of the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

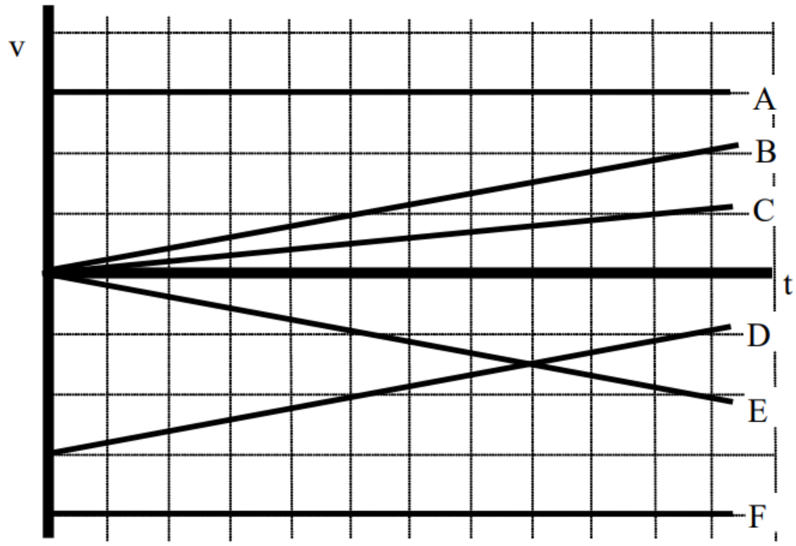
b. Rank these graphs on the basis of the change in kinetic energy of the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are velocity vs. time graphs for six equal-mass objects. All of the objects move horizontally.



a. Rank these graphs on the basis of the total impulse on the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

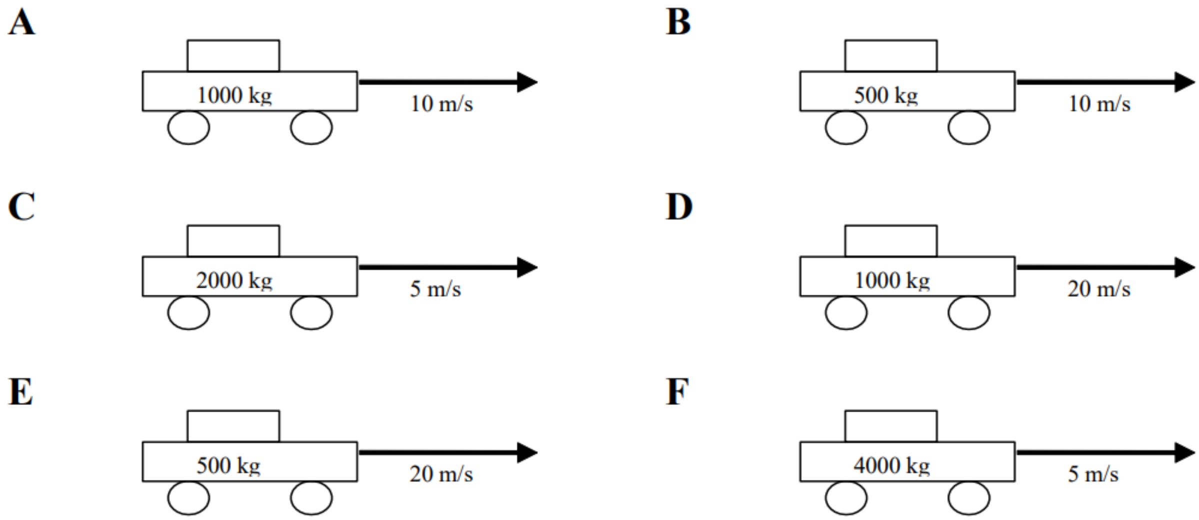
b. Rank these graphs on the basis of the total work on the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six automobiles initially traveling at the indicated velocity. The automobiles have different masses and velocities.



a. All automobiles will be stopped in the same amount of time. Rank these automobiles on the basis of the magnitude of the force needed to stop them.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

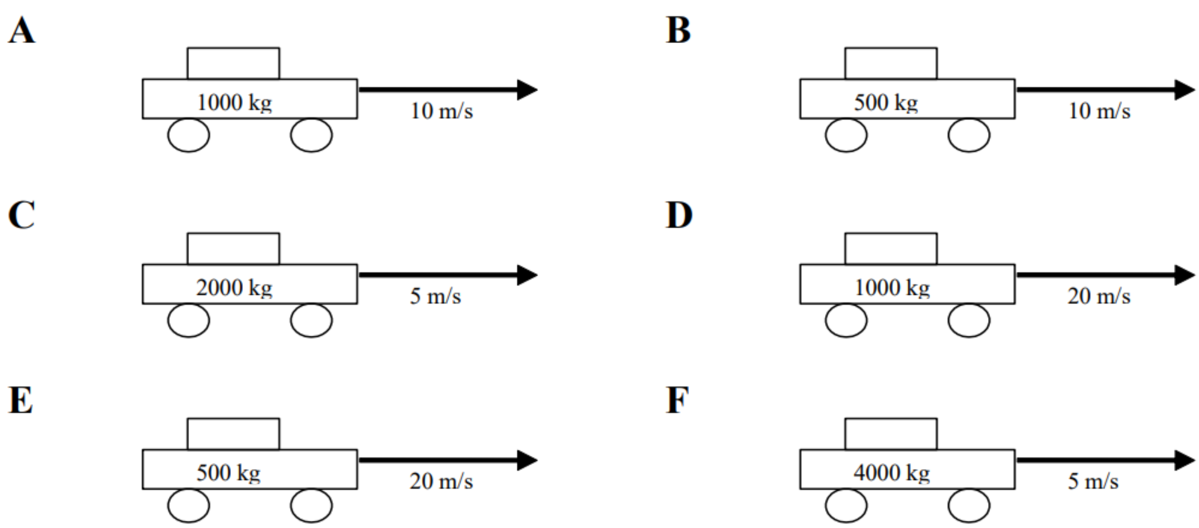
b. All automobiles will be stopped in the same amount of distance. Rank these automobiles on the basis of the magnitude of the force needed to stop them.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six automobiles initially traveling at the indicated velocity. The automobiles have different masses and velocities.



a. Rank these automobiles on the basis of the magnitude of the force needed to stop them.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank these automobiles on the basis of the magnitude of the work needed to stop them.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

c. Rank these automobiles on the basis of the magnitude of the impulse needed to stop them.

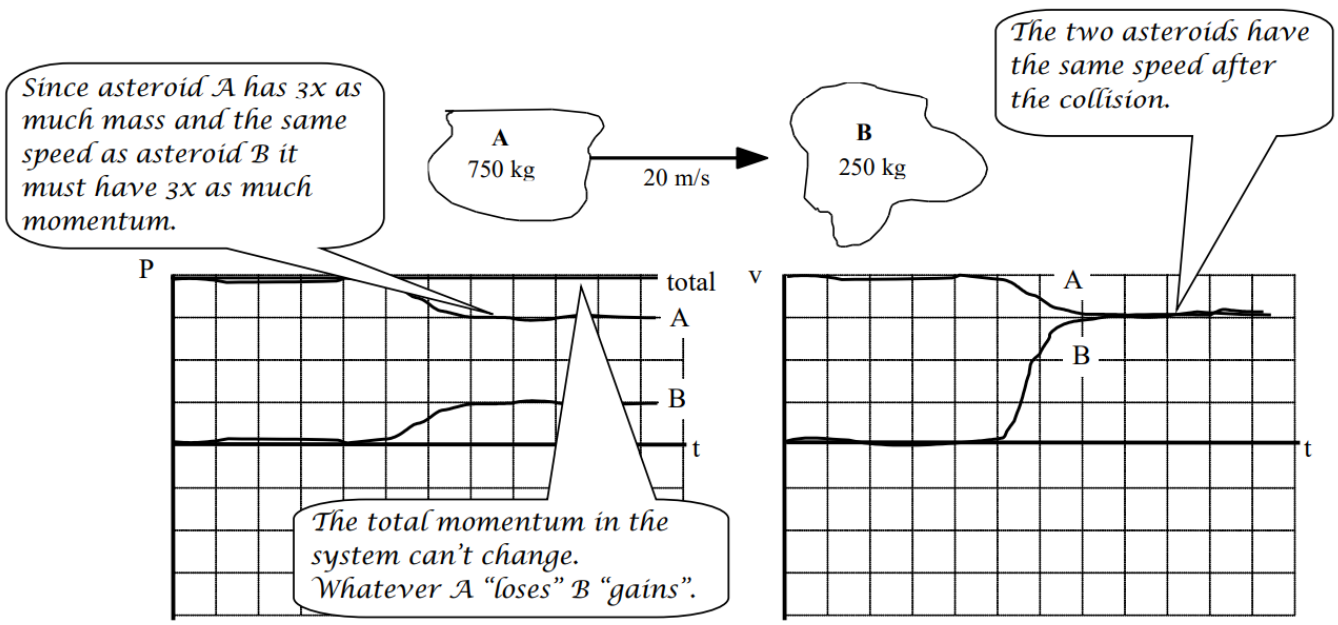
Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

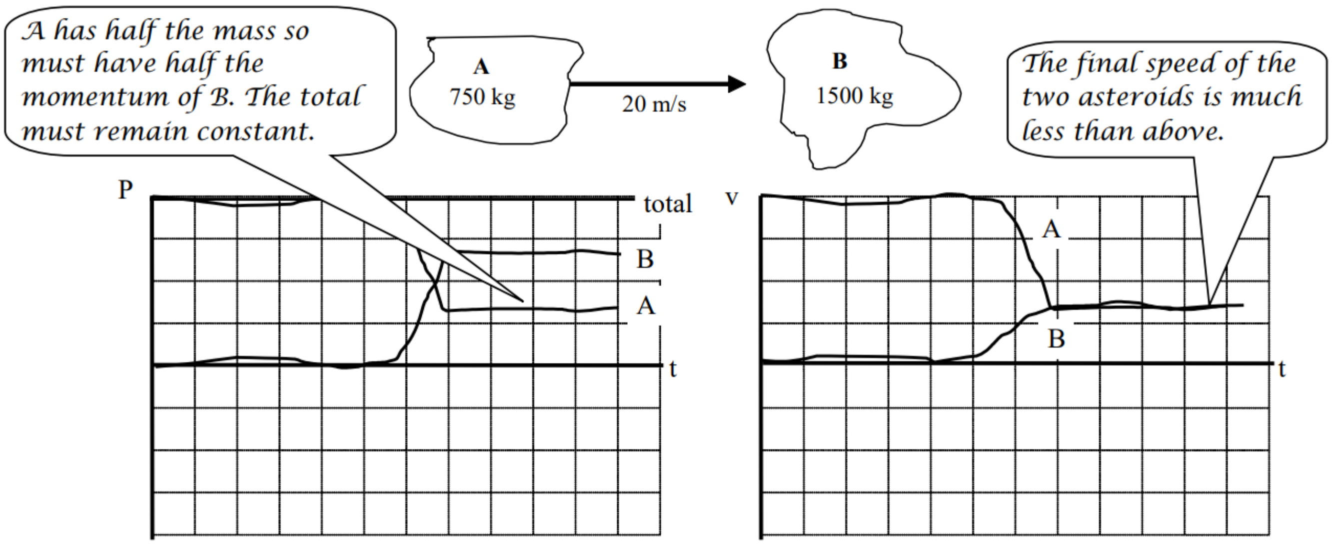
Explain the reason for your ranking:

For each of the collisions illustrated below, sketch a graph of the momentum and velocity of asteroid A, the momentum and velocity of asteroid B, and the total momentum in the system of the two asteroids. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

a. The two asteroids remain joined together after the collision.



b. The two asteroids remain joined together after the collision.



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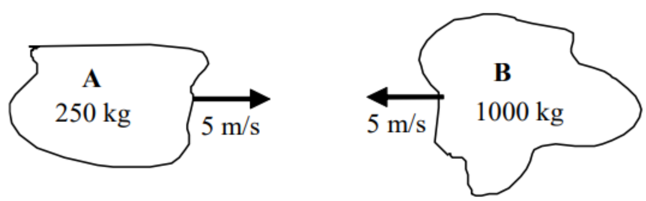
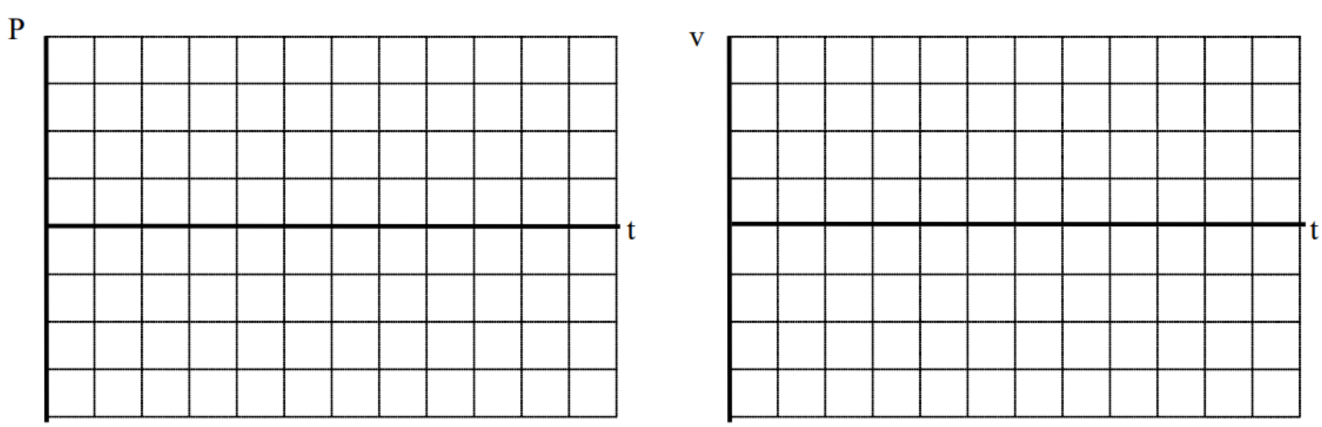
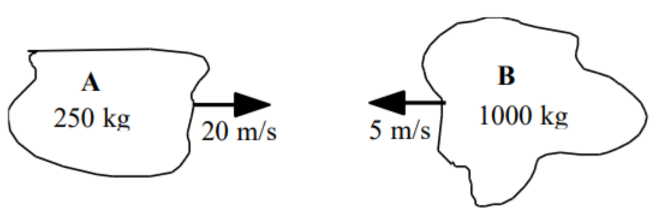
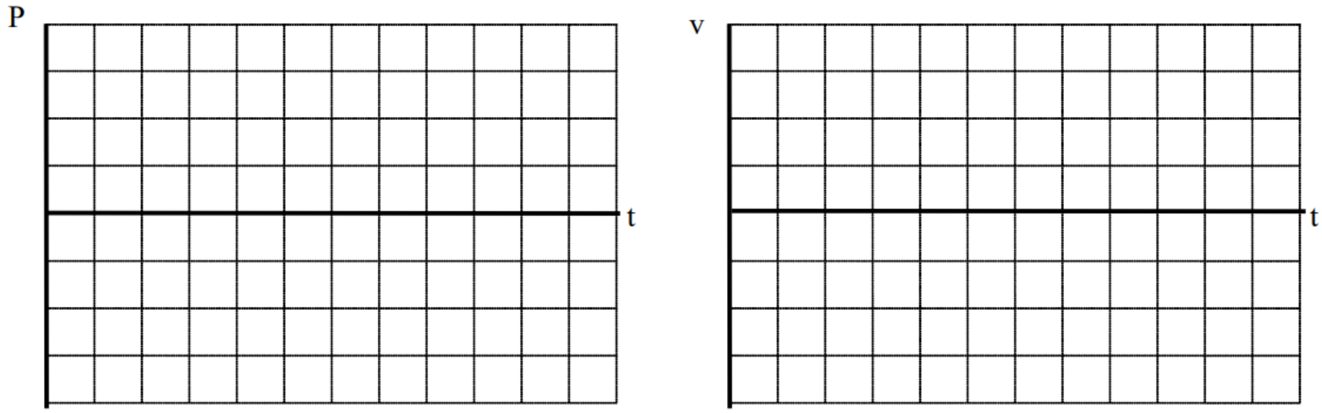




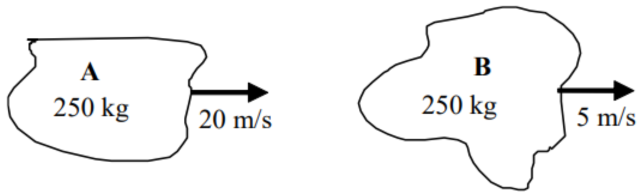
b. The two asteroids remain joined together after the collision.

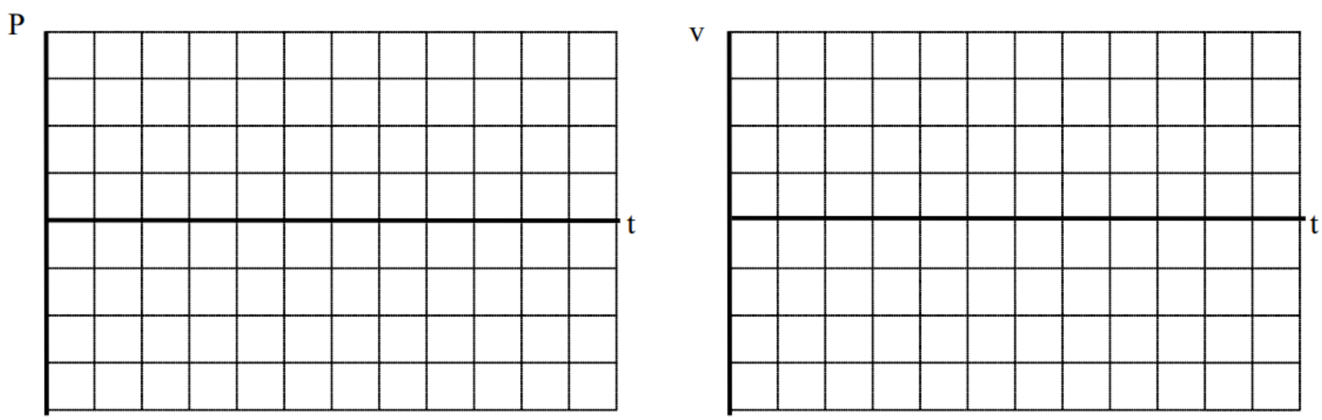




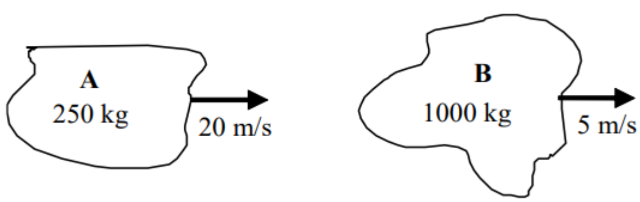
For each of the collisions illustrated below, sketch a graph of the momentum and velocity of asteroid A, the momentum and velocity of asteroid B, and the total momentum in the system of the two asteroids. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

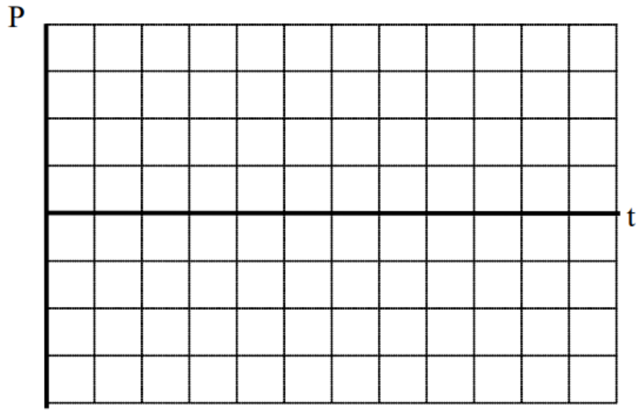
a. The two asteroids remain joined together after the collision.

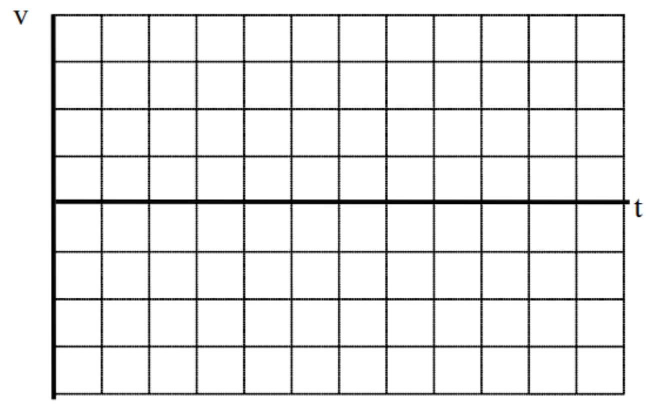




b. The two asteroids remain joined together after the collision.

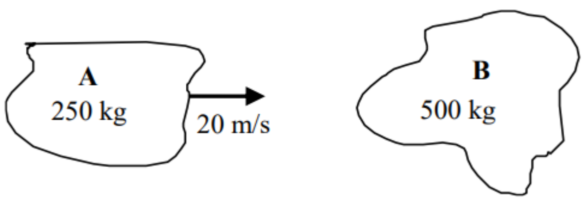


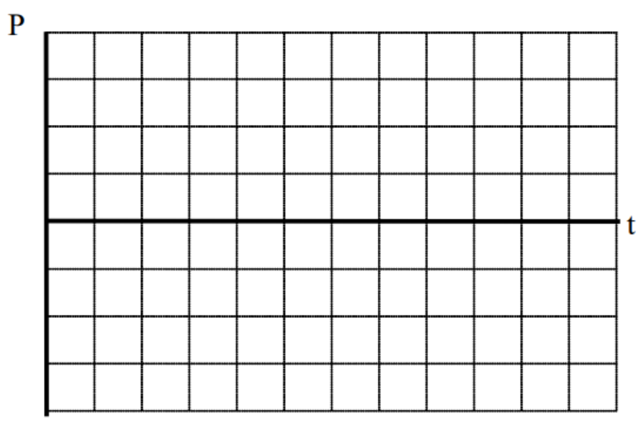


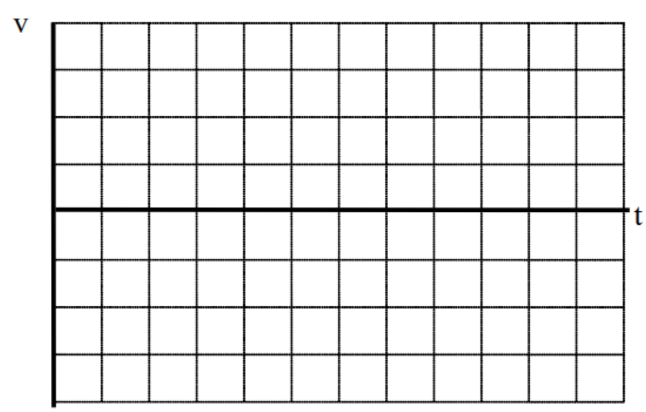


For each of the collisions illustrated below, sketch a graph of the momentum and velocity of asteroid A, the momentum and velocity of asteroid B, and the total momentum in the system of the two asteroids. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

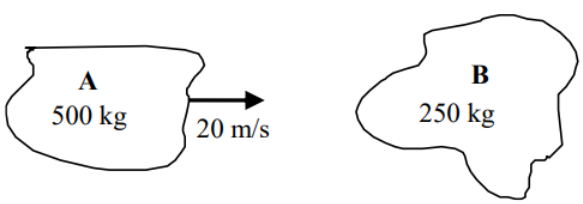
a. Asteroid A rebounds at 5 m/s after the collision.

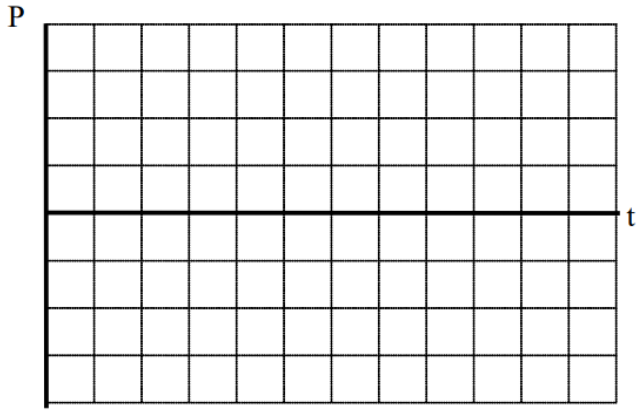


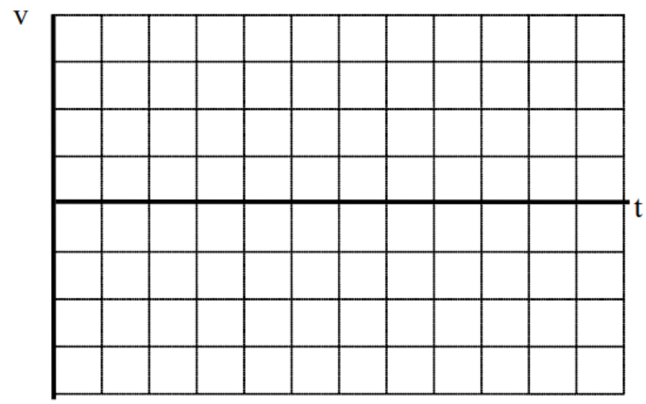




b. Asteroid B moves at 20 m/s after the collision.

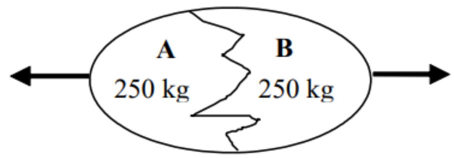


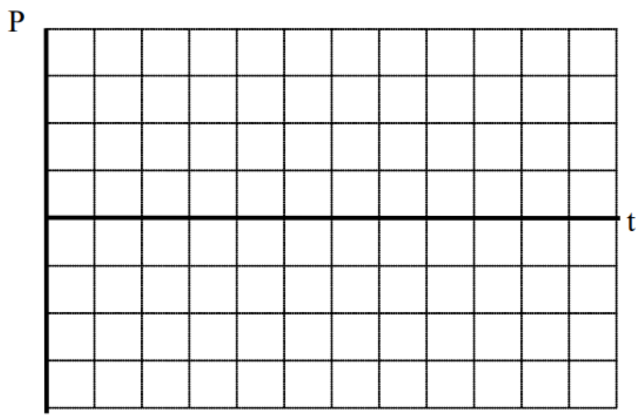


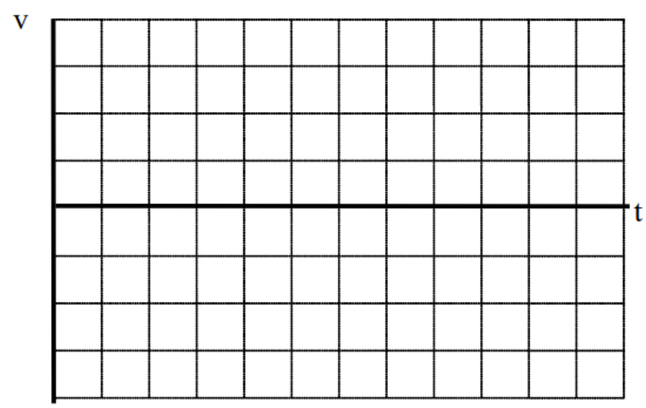


For each of the explosions illustrated below, sketch a graph of the momentum and velocity of fragment A, the momentum and velocity of fragment B, and the total momentum in the system of the two fragments. Begin your graph before the explosion takes place and continue it while the fragments travel away from the sight of the explosion. Use a consistent coordinate system and scale on all graphs.

a. The exploding egg is initially at rest.

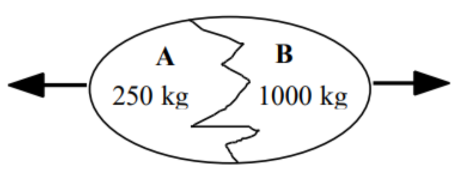


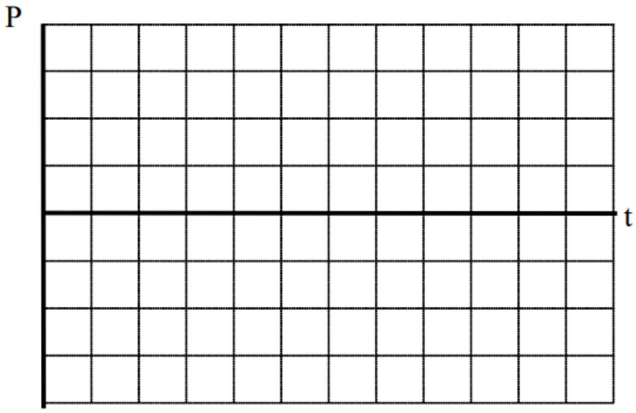


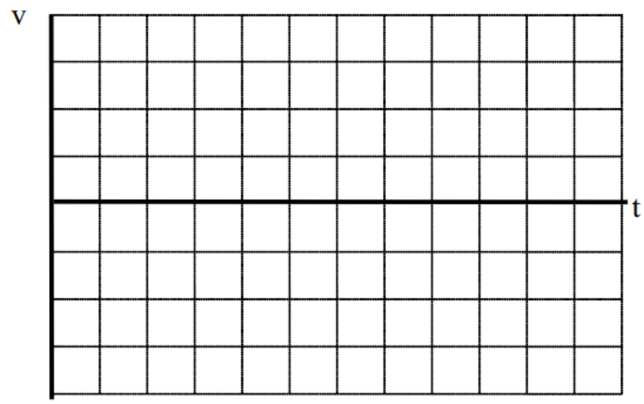


b. The exploding egg is initially at rest.

The exploding egg is initially at rest.

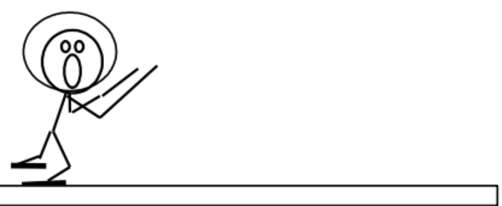


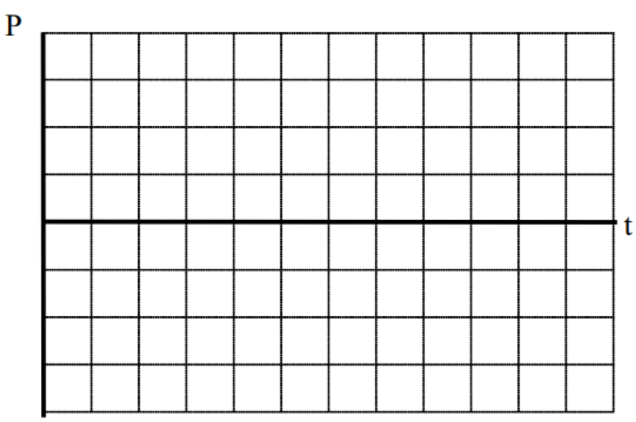


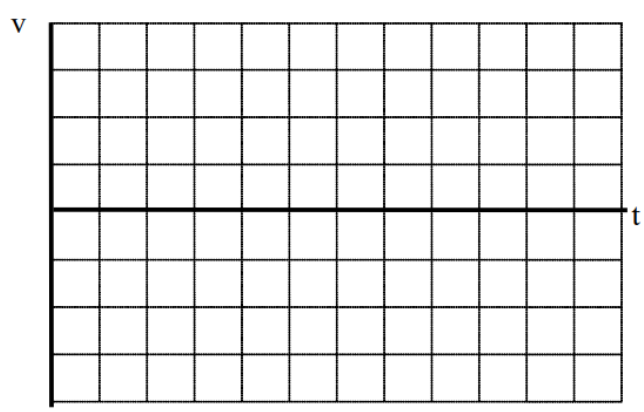


A 200 kg astronaut is initially at rest on the extreme edge of a 1000 kg space platform. She wears special magnetic shoes that allow her to walk along the metal platform. For each of the situations illustrated below, sketch a graph of the momentum and velocity of the astronaut, the momentum and velocity of the platform, and the total momentum in the system of the two objects. Begin your graph before the astronaut begins to walk and continue it while she walks along the platform. Use a consistent coordinate system and scale on all graphs.

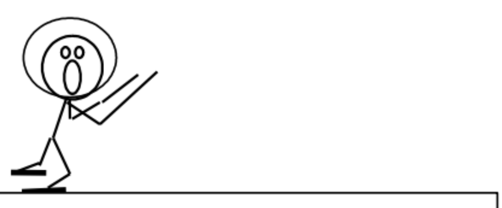
a. The astronaut and platform are initially at rest.

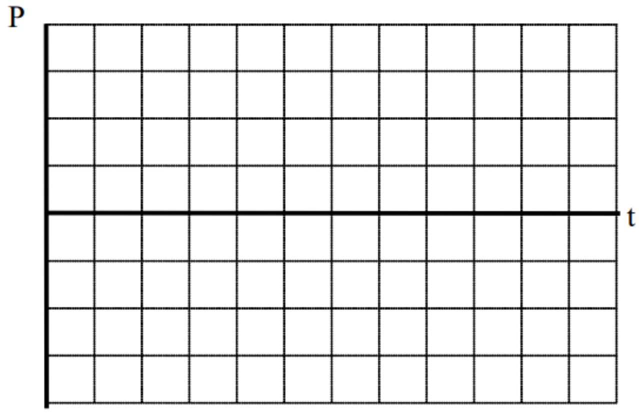


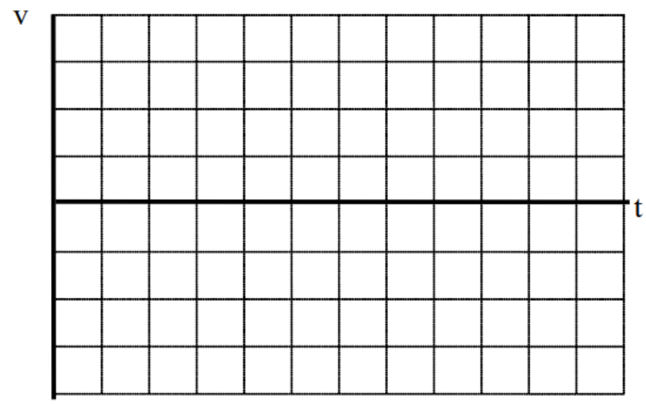




b. The astronaut and platform are initially drifting to the right.

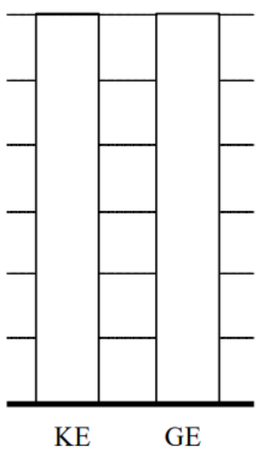




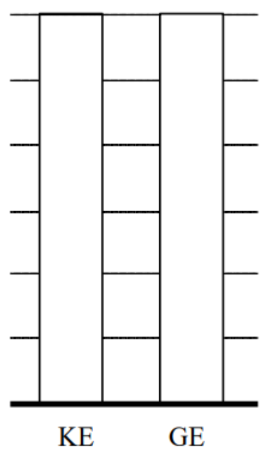


For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of the object at each of the events listed. Use a consistent scale throughout each motion. Set the lowest point of the motion as the zero-point of gravitational potential energy.

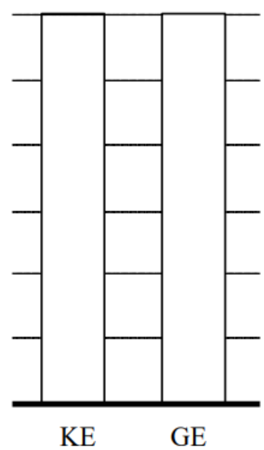
a. *A 4000 kg rocket's engine produces a thrust of 70,000 N for 15 s. The rocket is fired vertically upward.*



When the engine is first turned one.

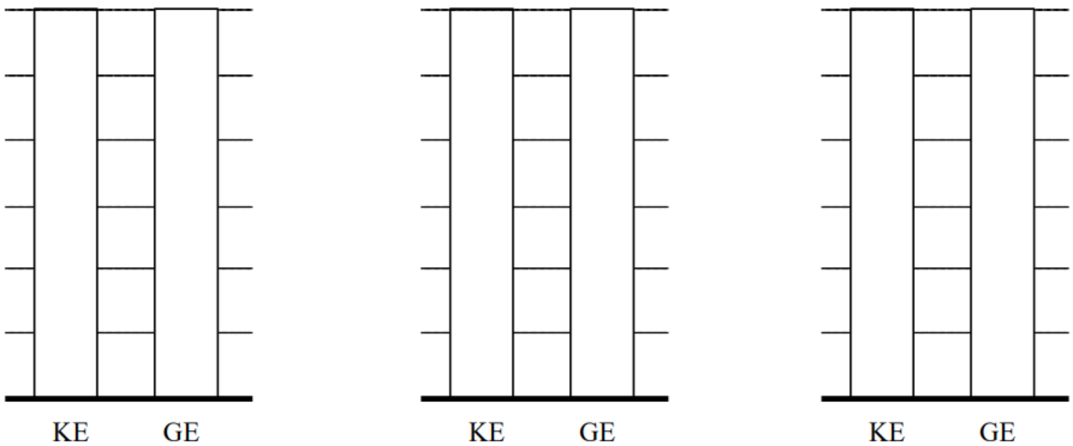


When the engine turns off.



When the rocket reaches its maximum height.

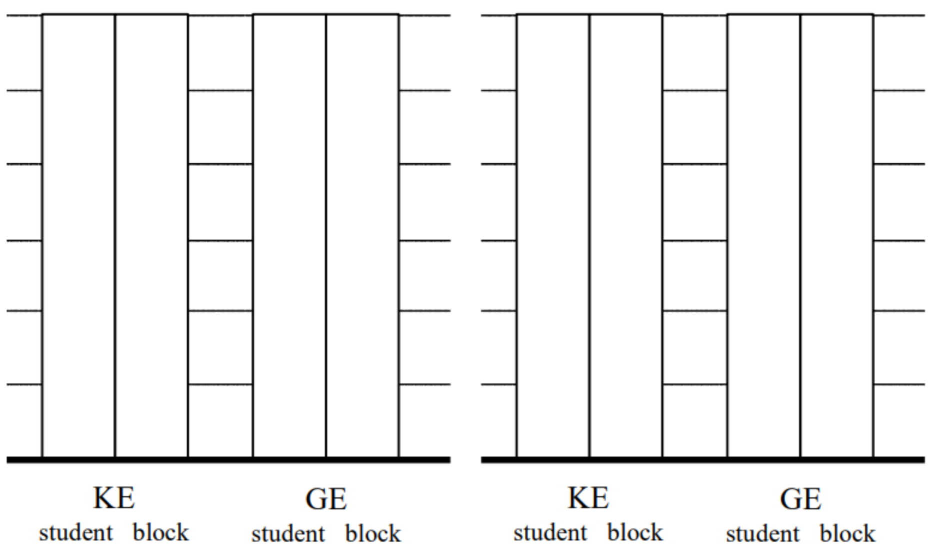
b. *To practice falling, a 55 kg pole-vaulter falls off of a wall 6.0 m above a 2.0 m thick foam cushion resting on the ground. However, he misses the cushion. The pole-vaulter sinks about 0.10 m into the ground before stopping.*



When the person first begins to fall. *When the person hits the ground.* *When the person finally stops.*

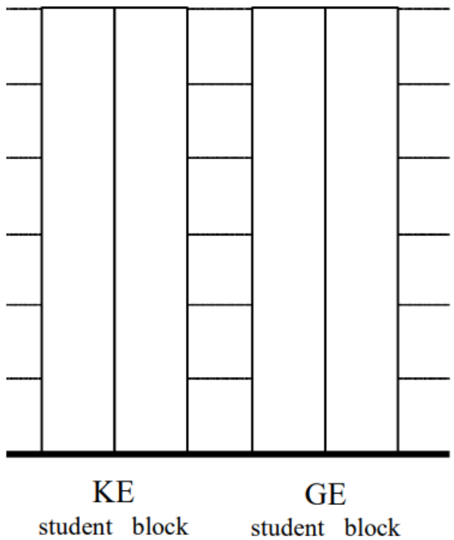
For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of each object at each of the events listed. Use a consistent scale throughout each motion. Set the lowest point of the motion as the zero-point of gravitational potential energy

a. *Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. An 84 kg block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the 84 kg block is released, the student is pulled up to his dorm room, 8.0 m off the ground.*

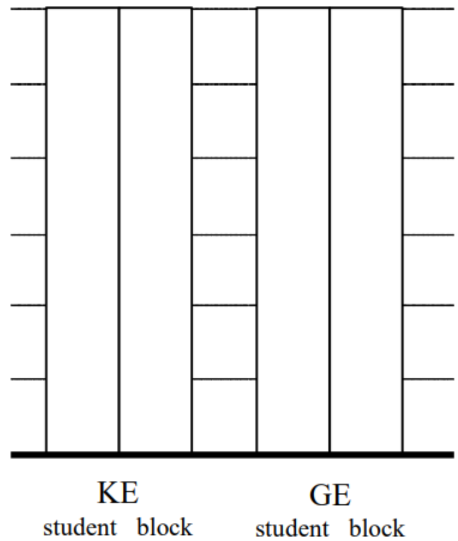


When the block is released. *When the student reaches his room.*

b. *Tired of walking down the stairs, a 75 kg engineering student designs an ingenious device for reaching the ground from his third floor dorm room. A 60 kg block at rest on the ground is attached to a rope that passes over a pulley. The student grabs the other end of the rope and steps out of his window.*



When the student steps out of the window.



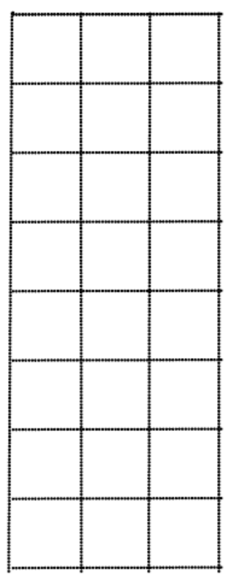
When the student reaches the ground.

A 100 kg man concerned about his weight decides to weigh himself in an elevator. He stands on a bathroom scale in an elevator that is moving upward at 3.0 m/s. As the elevator reaches his floor, it slows to a stop.

a. If the elevator slows to a stop over a distance of 2.0 m, what is the reading on the bathroom scale?

b. If the elevator slows to a stop in 1.5 s, what is the reading on the bathroom scale?

Motion Diagram



a. Motion Information

Event 1:	Event 2:
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Mathematical Analysis[26]

Free-Body Diagram

b. Motion Information

Event 1:	Event 2:
P1 =	P2 =
J12 =	

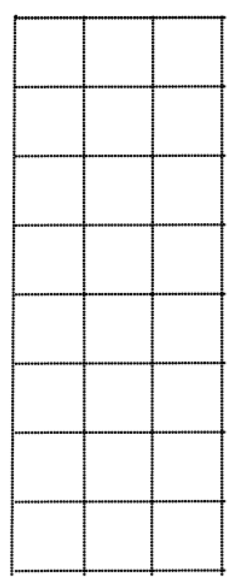
Mathematical Analysis

A 70 kg student is 120 m above the ground, moving upward at 3.5 m/s, while hanging from a rope hanging from a 280 kg helium balloon. The lift on the balloon due to the buoyant force is 3000 N.

a. With what speed does the student hit the ground?

b. How long does it take the student to reach the ground?

Motion Diagram



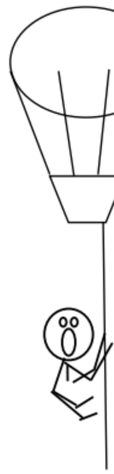
a. Motion Information

Event 1:	Event 2:
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Mathematical Analysis[27]

Free-Body Diagram

student & balloon



b. Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	

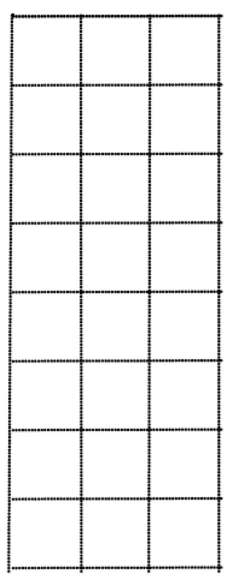
Mathematical Analysis

A 4000 kg rocket's engine produces a thrust of 70,000 N for 15 s. The rocket is fired vertically upward.

a. What is the speed of the rocket when its engine turns off?

b. How long does it take the rocket to reach its maximum height?

Motion Diagram



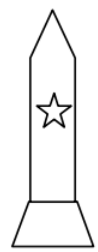
a. Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	

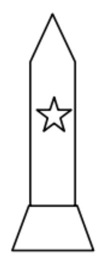
Mathematical Analysis[28]

Free-Body Diagram

while engine fires



after engine turns off



b. Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	

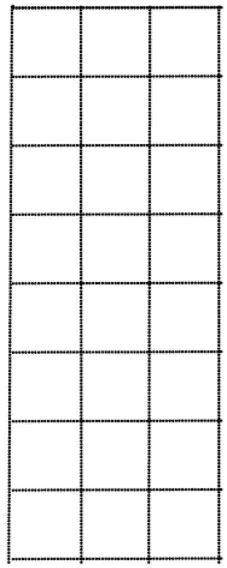
Mathematical Analysis

To practice falling, a 55 kg pole-vaulter falls off of a wall 6.0 m above a 2.0 m thick foam cushion resting on the ground. However, he misses the cushion. The pole-vaulter sinks about 0.10 m into the ground before stopping.

a. What is the speed of the pole-vaulter when he hits the ground?

b. What is the force exerted on the pole-vaulter by the ground as he comes to rest?

Motion Diagram

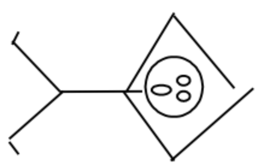


a. Motion Information

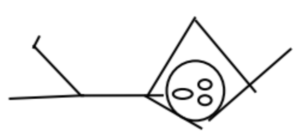
Event 1:	Event 2:
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Mathematical Analysis[29]

Free-Body Diagram



while dying



b. Motion Information

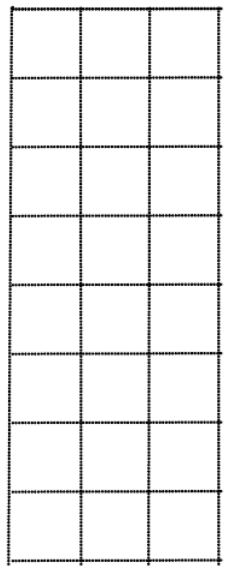
Event 1:	Event 2:
KE_1 =	KE_2 =
GE_1 =	GE_2 =

Event 1:	Event 2:
W_{12} =	

Mathematical Analysis

A decorative light fixture in an elevator consists of a 2.0 kg light suspended by a cable from the ceiling of the elevator. From this light, a separate cable suspends a second 0.80 kg light. The elevator is moving downward at 4.0 m/s when someone presses the emergency stop button. During the stop, the upper cable snaps. The elevator engineer says that the cable could withstand a force of 40 N without breaking. Find the maximum time and distance over which the elevator stopped.

Motion Diagram



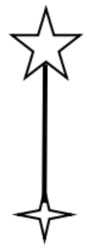
Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Mathematical Analysis[30]

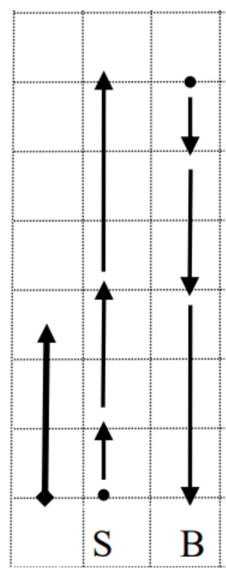
Free-Body Diagram

the two lights



Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. An 84 kg block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the 84 kg block is released, the student is pulled up to his dorm room, 8.0 m off the ground.

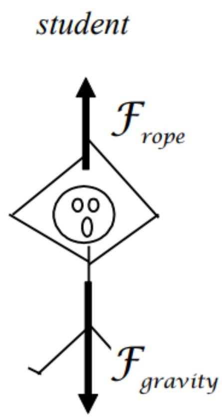
Motion Diagram

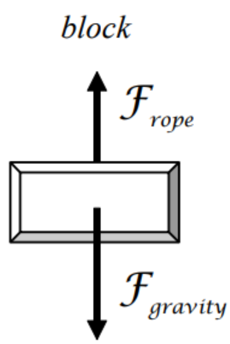


Motion Information

Event 1: The block is released	Event 2: The student reaches the room.
Object: Student	
$KE_1 = 0$	$KE_2 = 1/2(80)\mathcal{V}_f^2$
$GE_1 = 0$	$GE_2 = 80(9.8)(8)$
$W_{12} = \mathcal{F}_R(8)\cos 0^\circ$	
Object: Block	
$KE_1 = 0$	$KE_2 = 1/2(84)\mathcal{V}_f^2$
$GE_1 = 84(9.8)(8)$	$GE_2 = 0$
$W_{12} = \mathcal{F}_R(8)\cos 180^\circ$	

Free-Body Diagram





Since the distance the student and block travel is known, applying work-energy should allow us to solve the problem. I'll apply it separately to each object.

student	block
$0 + F_{\text{rope}}(8) = \frac{1}{2}80v_f^2 + 80(9.8)(8)$	$84(9.8)(8) - F_{\text{rope}}(8) = \frac{1}{2}84v_f^2$
$8F_{\text{rope}} = 40v_f^2 + 6272$	$6586 - 8F_{\text{rope}} = 42v_f^2$

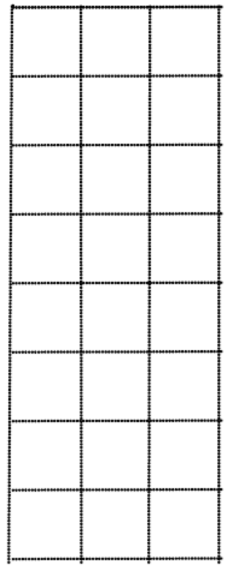
$\mathcal{F}_{\text{rope}}$ *is the same in both equations, as is the final speed. Thus the two equations can be added together to yield:*

$$6586 = 40v_f^2 + 42v_f^2 + 6272$$
$$314 = 82v_f^2$$
$$v_f = 1.96 \text{ m/s}$$

Notice that if you applied work-energy to the entire system you would have generated this same equation. Initially, the only form of energy present is the gravitational energy of the block (mgh = 6586 J). At the second event, both objects have kinetic energy plus the student has gravitational potential energy (mgh = 6272 J).

Tired of walking up the stairs, an engineering student designs an ingenious device for reaching his third floor dorm room. A 100 kg block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room, 8.0 m off the ground. He is traveling at 2.2 m/s when he reaches his room.

Motion Diagram

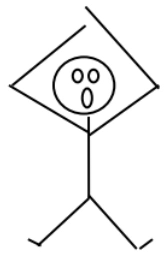


Motion Information

Event 1:	Event 2:
Object:	
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	
Object:	
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Free-Body Diagram

student

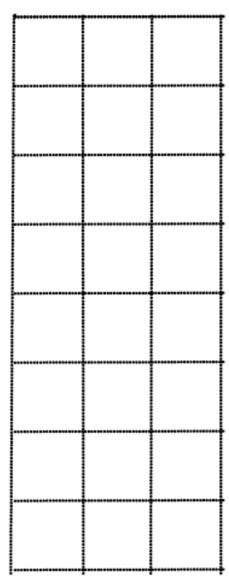


block

Mathematical Analysis[31]

Tired of walking down the stairs, a 75 kg engineering student designs an ingenious device for reaching the ground from his third floor dorm room. A 60 kg block at rest on the ground is attached to a rope that passes over a pulley. The student grabs the other end of the rope and steps out of his window. He falls for 5.5 s before reaching the ground.

Motion Diagram



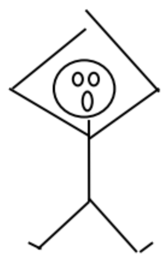
Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =

Event 1:	Event 2:
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Free-Body Diagram

student

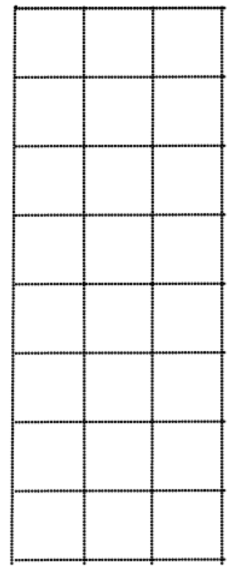


block

Mathematical Analysis[32]

Tired of walking down the stairs, a 75 kg engineering student designs an ingenious device for reaching the ground from his dorm room. A 60 kg block at rest on the ground is attached to a rope that passes over a pulley. The student grabs the other end of the rope and steps out of his window. He hits the ground at 3.3 m/s.

Motion Diagram

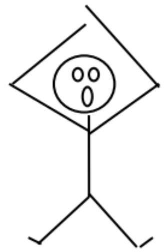


Motion Information

Event 1:		Event 2:	
Object:		Object:	
P_1 =	P_2 =	P_1 =	P_2 =
J_{12} =		J_{12} =	
P_1 =	P_2 =	P_1 =	P_2 =
KE_1 =	KE2 =	KE_1 =	KE2 =
GE_1 =	GE2 =	GE_1 =	GE2 =
W_{12} =		W_{12} =	

Free-Body Diagram

student

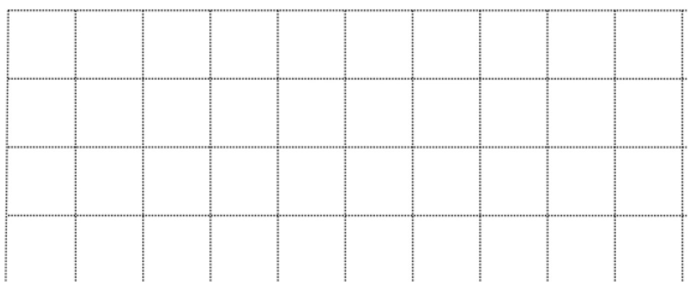


block

Mathematical Analysis[33]

Far from any other masses, a 2000 kg asteroid traveling at 12 m/s collides with a 1200 kg asteroid traveling in the other direction at 16 m/s. After the collision they remain joined together and move with a common velocity.

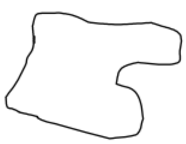
Motion Diagram



Free-Body Diagrams

2000 kg asteroid

1200 kg asteroid



Motion Information

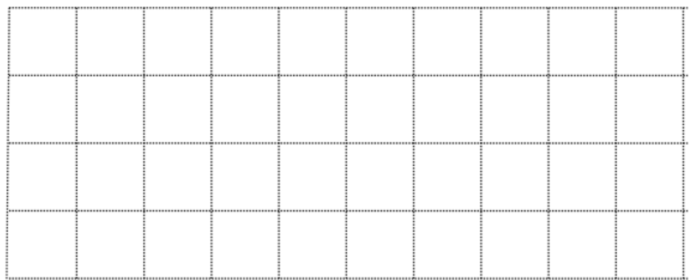
Event 1:	Event 2:
Object:	
P_1 =	P_2 =

Event 1:	Event 2:
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

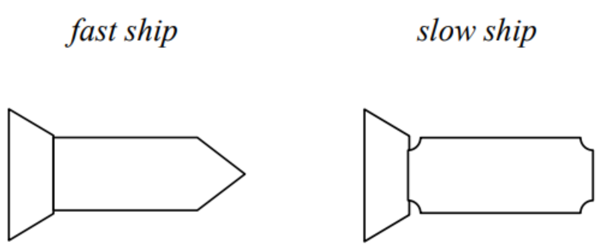
Mathematical Analysis[34]

On a remote stretch of intergalactic highway, a 7.5 x 10^6 kg spaceship traveling at 10 percent the speed of light (0.10c = 3.0 x 10^7 m/s) doesn't notice the slower spaceship ahead clogging the lane. The fast-moving ship rear-ends the slower ship, an older 5.5 x 10^6 kg model, and the two ships get entangled and drift forward at 0.07c.

Motion Diagram



Free-Body Diagrams



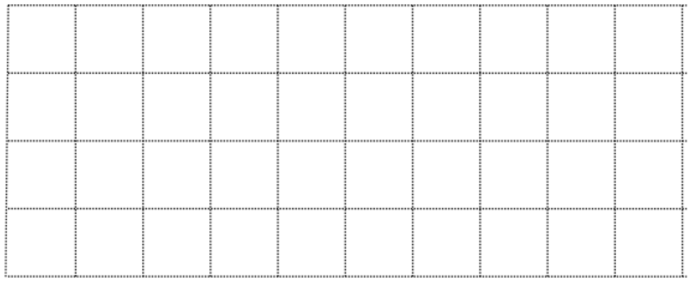
Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

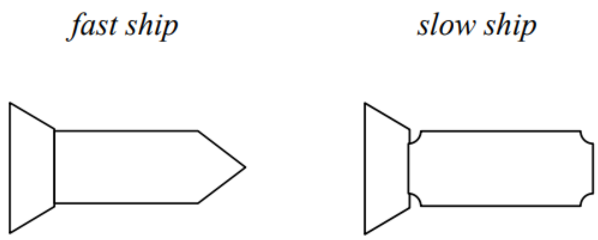
Mathematical Analysis[35]

On a remote stretch of intergalactic highway, a 7.5 x 10^6 kg spaceship traveling at 10 percent the speed of light (0.10c = 3.0 x 10^7 m/s) doesn't notice the slower spaceship ahead, moving at 0.05c, clogging the lane. The fast-moving ship rear-ends the slower ship, an older 4.5 x 10^6 kg model, and the slower ship gets propelled forward at 0.13c.

Motion Diagram



Free-Body Diagrams



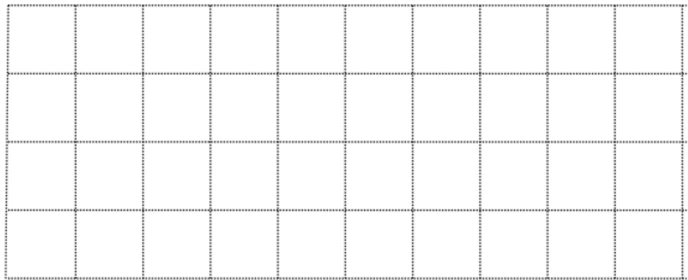
Motion Information

Event 1:	Event 2:
Object:	
$P_1 =$	$P_2 =$
$J_{12} =$	
Object:	
$P_1 =$	$P_2 =$
$J_{12} =$	

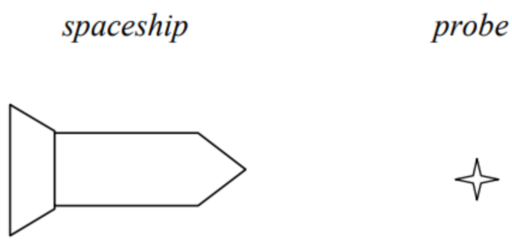
Mathematical Analysis[36]

In the farthest reaches of deep space, an 8000 kg spaceship, including contents, is at rest relative to a space station. The spaceship recoils after it launches a 600 kg scientific probe with a speed of 300 m/s relative to the space station.

Motion Diagram



Free-Body Diagrams



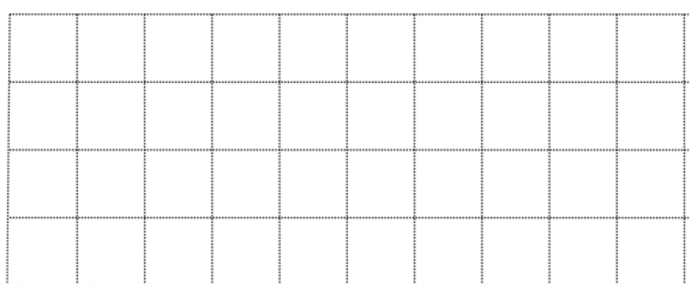
Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

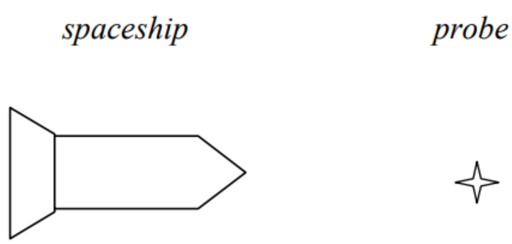
Mathematical Analysis[37]

In the farthest reaches of deep space, an 8000 kg spaceship, including contents, is drifting at 50 m/s relative to a space station. The spaceship is brought to rest, relative to the space station, by the recoil from launching a 600 kg scientific probe.

Motion Diagram



Free-Body Diagrams



Motion Information

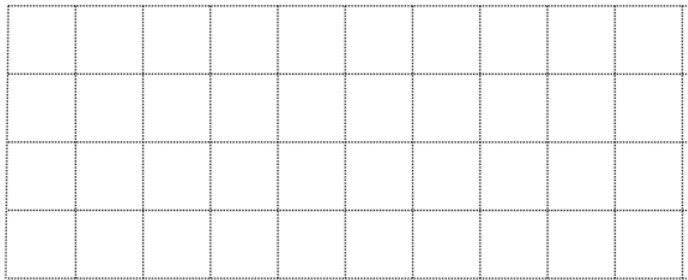
Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	

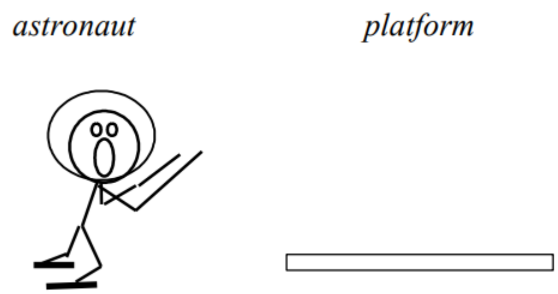
Mathematical Analysis[38]

A 140 kg astronaut is standing on the extreme edge of a 1000 kg space platform, at rest relative to the mother ship. She begins to walk toward the other edge of the platform, reaching a speed of 2.0 m/s relative to the mother ship. (She wears special magnetic shoes that allow her to walk along the metal platform.)

Motion Diagram



Free-Body Diagrams



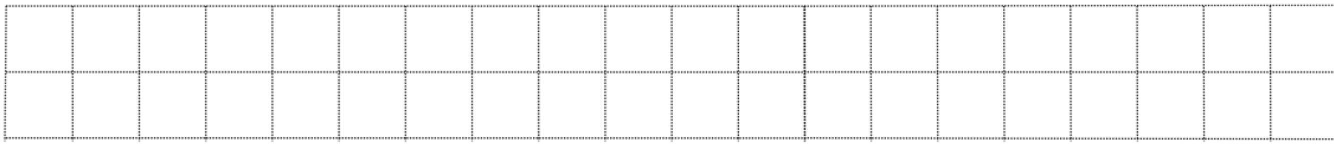
Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

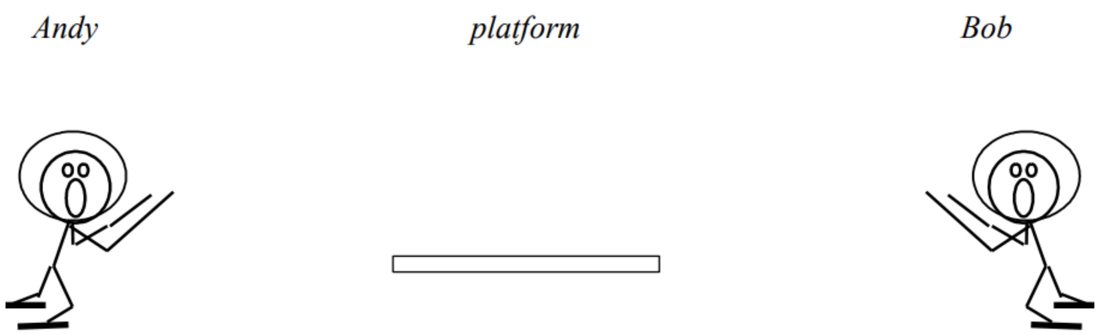
Mathematical Analysis[39]

Two astronauts, 140 kg Andy and 170 kg Bob, are standing on opposite edges of a 1000 kg space platform, at rest relative to the mother ship. They each begin to walk toward the opposite ends of the platform, Andy reaching a speed of 2.0 m/s and Bob 1.5 m/s, both relative to the mother ship. (They wear special magnetic shoes that allow them to walk along the metal platform.)

Motion Diagram



Free-Body Diagrams



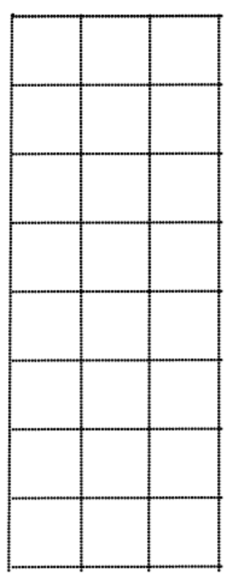
Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Mathematical Analysis[40]

A 70 kg student is hanging from a 280 kg helium balloon. The balloon is rising at a constant speed of 8.0 m/s relative to the ground. The lift on the balloon due to the buoyant force is constant. The student begins to climb up the rope at a speed of 15 m/s relative to the ground. The balloon's upward speed is decreased as the student climbs.

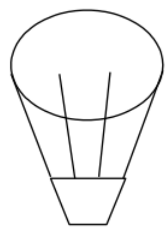
Motion Diagram



Free-Body Diagrams

student

balloon



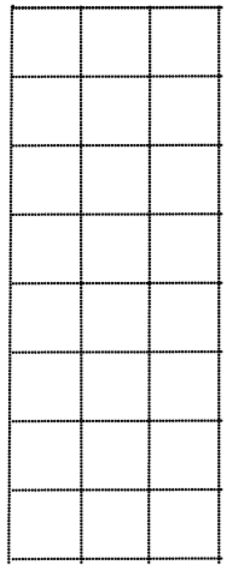
Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Mathematical Analysis[41]

A man of mass m, concerned about his weight, decides to weigh himself in an elevator. He stands on a bathroom scale in an elevator which is moving upward at v. As the elevator reaches his floor, it slows to a stop over a time interval, T. Determine the reading on the bathroom scale (F_{scale}) as a function of m, v, T, and g.

Motion Diagram



Free-Body Diagrams

Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	

Questions

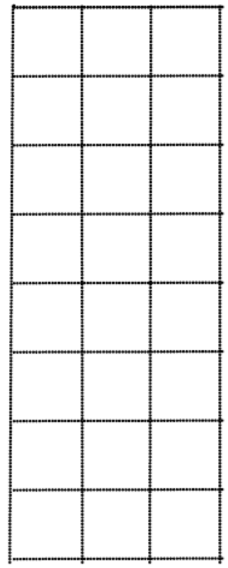
If $T = \infty$, what should F_{scale} equal? Does your function agree with this observation?

For what combination of v and T would the bathroom scale read 0 N?

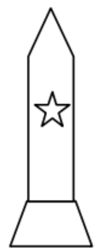
If the elevator were initially going down, would the above combination of v and T also lead to a scale reading of 0 N?

A rocket of mass m is fired vertically upward from rest. The rocket's engine produces a thrust of constant magnitude F for t_{thrust} seconds. Determine the time it takes the rocket to reach its apex (t_{apex}) as a function of F, t_{thrust}, m, and g.

Motion Diagram



Free-Body Diagrams



Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	

Questions

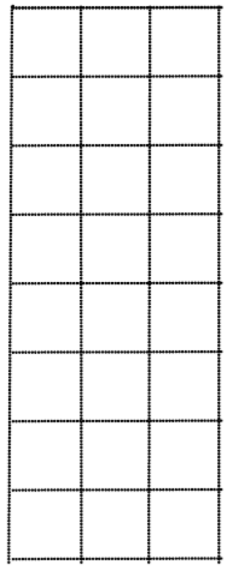
If $g = 0\ m/s^2$, what should t_{apex} equal? Does your function agree with this observation?

If $F = mg$, what should t_{apex} equal? Does your function agree with this observation?

For what value of F would $t_{apex} = 2t_{thrust}$?

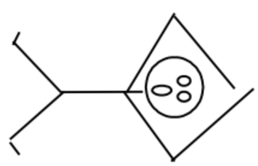
To practice falling, a pole-vaulter of mass m falls off of a wall a distance D above a thick foam cushion. The pole-vaulter sinks a distance d into the cushion before stopping. Determine the force exerted on the pole-vaulter due to the cushion ($F_{cushion}$) as a function of m, D, d, and g.

Motion Diagram

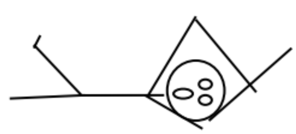


Free-Body Diagrams

while falling



while dying



Motion Information

Event 1:	Event 2:
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Questions

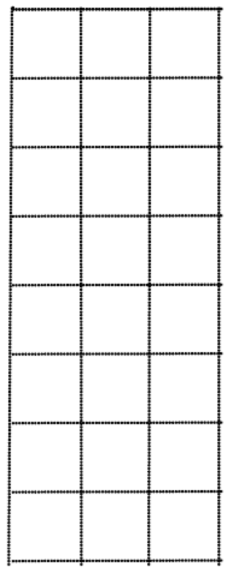
If $D = \infty$, what should $F_{cushion}$ equal? Does your function agree with this observation?

If d = 0 m, what should $F_{cushion}$ equal? Does your function agree with this observation?

What would be worse for the pole-vaulter, starting at twice the initial distance above the cushion or sinking half of the original distance into the cushion?

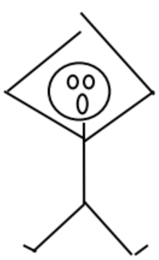
Tired of walking up the stairs, an engineering student of mass m designs an ingenious device for reaching his third floor dorm room. A block of mass M is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room in a time T. Determine the velocity of the student (v) when he reaches his room as a function of m, M, T and g.

Motion Diagram



Free-Body Diagrams

student



block

Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Questions

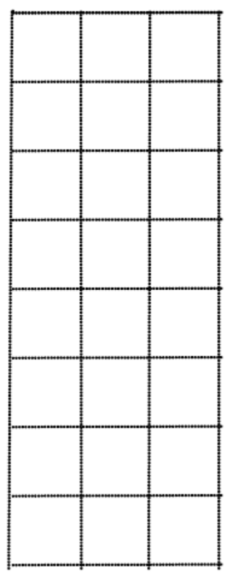
If g = 0 m/s^2, what should v equal? Does your function agree with this observation?

If m = M, what should v equal? Does your function agree with this observation?

If M = ∞, what should v equal? Does your function agree with this observation?

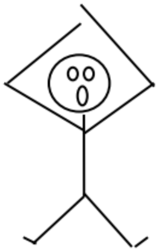
Tired of walking down the stairs, an engineering student of mass m designs an ingenious device for reaching the ground from her dorm room. A block of mass M at rest on the ground is attached to a rope that passes over a pulley. The student grabs the other end of the rope and steps out of her window a distance D above the ground. Determine the velocity of the student (v) when she reaches the ground as a function of m, M, D and g.

Motion Diagram

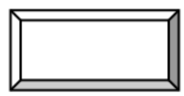


Free-Body Diagrams

student



block



Motion Information

Event 1:	Event 2:
Object:	
KE_1 =	KE2 =
GE_1 =	GE_2 =
W_{12} =	
Object:	
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Questions

If g = 0 m/s^2, what should v equal? Does your function agree with this observation?

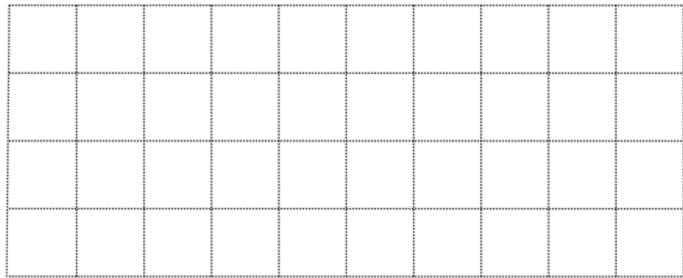
If m = M, what should v equal? Does your function agree with this observation?

If M = ∞, what should v equal? Does your function agree with this observation?

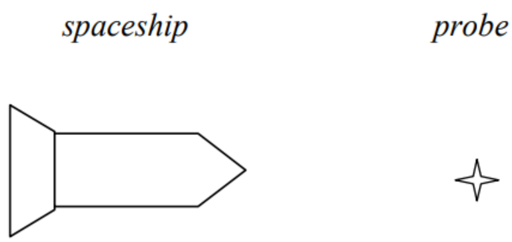
In the farthest reaches of deep space, a spaceship of mass M, including contents, is at rest relative to a space station. The spaceship recoils after it launches a scientific probe of mass m at a speed v relative to the space station. Determine the recoil speed of the spaceship (V) as a function of M, m, and v.

Motion Diagram

![LibreTexts logo]



Free-Body Diagrams



Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Questions

If M = 2m, what should V equal? Does your function agree with this observation?

If M = ∞, what should V equal? Does your function agree with this observation?

Model One

Summary Problems

1. A sprinter in a 100 meter dash starts from rest and accelerates at 2.5 m/s^2 for 3.9 s. She then runs at a constant speed, until she either tires or finishes the race. She can run at a constant velocity for at most 7.0 s without tiring. Once tired, she begins to slow, accelerating at 1.0 m/s^2. How long does it take her to finish the race?
2. The *Schindler Mobile* is a self-propelled elevator, powered by a small motor attached to the bottom of the elevator cab that drives *the cab up and* down the tracks of two high-strength aluminum columns. Assume the Schindler Mobile can travel from the 1st floor to the 10th floor (approximately 40 m) in 15 s. Assume the elevator both speeds up and slows
3. *The Thrust SSC*, a 7 ton jet mobile powered by two Royal Air Force Phantom jets providing 110,000 hp, was designed to break the speed of sound. The "car" was tested at a 15 mile long track in the Black Rock desert of Nevada. The car accelerated for nearly five miles, then moved through a measured mile at top speed. The car slowed by cutting power and releasing parachutes for five additional miles before applying brakes at speeds below 300 mph. Complete a kinematic description for the car's motion, assuming it reaches a maximum speed of Mach 1 (750 mph at the temperatures encountered at the Black Rock desert raceway).
4. Often the reason for the formation of congested traffic is obvious; accidents, lane closings, or other bottlenecks. However, you have probably also experienced "phantom" traffic jams, which emerge without any obvious reason, seemingly *out of nothing*.

This phenomenon can be understood by the collective behavior of many drivers. If one vehicle drives more slowly than others, the vehicle behind has to brake to maintain the desired *safety time*. (The safety time is the elapsed time between the two objects passing the same point. Thus, the distance associated with this "time cushion" varies with the speed of the traffic.) Consequently, the next vehicle behind has to brake, and so on. If traffic flow is unstable, each following vehicle has to brake stronger than its predecessor. Thus, a small initial perturbation triggers a backward propagating "wave" of slower vehicles with increasing amplitude. Finally, the vehicles come nearly to a stop; a full-fledged traffic jam has evolved. The driver having caused the small perturbation by driving unusually slowly escapes without even realizing what he has triggered!

To get a better feel for the kinematics involved in instigating phantom jams, imagine a car approaching a slow-moving truck on a one-lane road. The car is initially traveling at 120 km/hr, while the truck moves at 70 km/hr. The car is 100 m behind the truck when the driver first notices the truck. Find the minimum acceleration necessary for the car to come to equilibrium behind the truck and achieve a safety time of 2 s. Assume the truck does not accelerate.

5. When modeling traffic flow, various psychological factors must be incorporated. One is the politeness factor. The *politeness factor* quantifies how much one weights disadvantages imposed on other drivers against one's own advantage when considering a lane change. Lane changes are more common when the politeness factor is low. Different regions of the country have, on average, different politeness factors. In addition, urban vs. rural drivers differ in politeness factor. At high values of the politeness factor, drivers run the risk of getting stuck permanently behind slow-moving vehicles or other obstacles.

   To get a better feel for how "politeness" affects traffic flow, imagine a car stuck behind a 5 m long, stationary obstacle blocking its lane. 30 m ahead of the car is a turn in the road. Therefore, the driver of the car cannot see an approaching car or truck until it is 30 m from the car.
   a. If the car pulls out to go around the obstacle (with acceleration 4 m/s^2) just as a truck moving at 55 mph rounds the bend, is this an “extremely polite" maneuver? (A maneuver is extremely polite when the truck does not need to slow down in order to avoid an accident.)
   b. What acceleration is necessary for an extremely polite driver to pull out from behind the barrier? (If the car cannot generate this acceleration, an extremely polite driver must spend the rest of their life stuck behind the barrier!)
6. Traffic engineers are concerned with selecting the proper "yellow time" to ensure safe passage through stoplights. To understand this scenario, imagine yourself driving down a relatively empty road. Up ahead, the traffic light turns yellow. If you are close enough to the traffic light you can pass through the intersection before the light turns red. If you are far from the traffic light you can safely slow down and stop before the intersection. But what if you are in-between, in what is termed the "no-win" zone, and are too far to make it and too close to stop? Traffic engineers design the duration of the yellow signal to eliminate this no-win zone.
   a. You are driving at the speed limit (45 mph) on a straight, empty road with perfect visibility. Your maximum acceleration while braking is 7.0 m/s^2. The yellow time is 1.0 s. Determine the location of the no-win zone (i.e., the range of positions from which you cannot safely traverse the intersection). Assume you don't speed up to "run" the yellow, since this is an illegal activity.
   b. If you do want to "run" the yellow from anywhere in the no-win zone, what minimum acceleration is needed? Is this feasible? How fast would you be traveling as you go through the intersection?
7. Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. A heavy block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room, 8.0 m off the ground. However, the student has a weak stomach and will get nauseous if he accelerates at greater than 4.0 m/s^2. Also, the rope he used can transmit a force of only 1100 N before breaking. If possible, what mass ballast block should he use to avoid breaking the rope and avoid getting nauseous?
8. Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching her third floor dorm room. A heavy block is attached to a rope that passes over a pulley. The student holds the other end of the rope. However, the dorm room window is 12 m off the ground and the block is initially only 10 m off the ground. The student wants to choose a mass for the block such that when the block hits the ground, the student is “launched” upward, and reaches her window at the apex of her motion. (That will make it easiest to climb in the window.) What mass block should she use?

Selected Answers

[1] r2 = 1.4 m

[2] t2 = 1.45 m

[3] t3 = 3.4 s

[4] t3 = 4.55 s

[5] r3 = 36.6 m

[6] r3 = 3520 m

[7] r2 = 14.9 m

[8] r4 = 2000 m

[9] t4 = 3.7 x 105 s

[10] t2 = 14.9 s

[11] t2 = 7.8 s

[12] t3 = 15.7 s

[13] t3 = 2.87 s

[14] F rope = 420 N

[15] F cushion = 1910 N

[16] a ≥ 4.49 m/s2

[17] t2 = 17.1 s to reach ground

[18] F cushion = 2830 N

[19] F ground = 43700 N

[20] r3 = 63.5 m

[21] r3 = 1550 m

[22] m block = 240 kg

[23] m block = 26 kg

[24] F rope = 500 N

[25] F rope = 466 N

[26] a. F scale = 755 N b. F scale = 780 N

[27] a. v = 17.5 m/s b. t = 17.1 s

[28] a. v = 116 m/s b. t = 26.8 s

[29] a. v = 12.5 m/s b. F ground = 43700 N

[30] t2 = 0.89 s r2 = 1.78 m

[31] m student = 94 kg

[32] v2 = 6.0 m/s

[33] Student falls 5.0 m in 3.03 s

[34] v2 = 1.5 m/s

[35] v1 slowship = 0.029c = 8.73 x 10^6 m/s

[36] v2 fastship = 0.052c = 1.56 x 10^7 m/s

[37] v2 ship = 24.3 m/s

[38] v probe = 667 m/s

[39] v2 platform = 0.28 m/s

[40] v2 platform = 0.025 m/s

[41] v2 balloon = 6.3 m/s

2.5: Selected Answers

[1] r2 = 1.4 m

[2] t2 = 1.45 m

[3] t3 = 3.4 s

[4] t3 = 4.55 s

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[41] v2 balloon = 6.3 m/s

CHAPTER OVERVIEW

3: Model 2 - The Constant Force Particle Model

3.0: Model Specifics

For our second pass through the study of mechanics, we will eliminate one of the major restrictions in our original model. We will now allow objects to move through three dimensional space. We will still, however, restrict our model with the following approximations.

The object is acted on by constant forces.

The object's size and shape are unimportant.

The object is classical.

3.1: Kinematics

Concepts and Principles

An empirical fact about nature is that motion in one direction (for example, the horizontal) does not appear to influence aspects of the motion in a perpendicular direction (the vertical). Imagine a coin dropped from shoulder height. The elapsed time for the coin to hit the ground, the rate at which its vertical position is changing, and its vertical acceleration are the same whether you do this experiment in a stationary bus or one traveling down a smooth, level highway at 65 mph. The horizontal motion of the coin does not affect these aspects of its vertical motion.[1]

Thus, to completely describe the motion of an object moving both horizontally and vertically you can first ignore the horizontal motion, and describe only the vertical *component* of the motion, and then ignore the vertical motion, and describe the horizontal component. Putting these kinematic components together gives you a complete description of the motion. This experimental fact about nature will make analyzing multi-dimensional motion no more conceptually difficult than analyzing one-dimensional motion.

Given this independence between motions in perpendicular directions, the same kinematic concepts and relationships utilized in one-dimensional motion will be utilized for multidimensional motion.

Note

[1] Actually, at extremely high speeds the horizontal and vertical motions are not independent. At speeds comparable to the speed of light, the interdependence between horizontal and vertical motion (because of time dilation) becomes noticeable.

Position

The position of an object is its location relative to a well-defined coordinate system. In multidimensional situations, however, you must designate coordinate systems for all perpendicular directions of interest. The zero and positive direction for one dimension is completely independent of the zero and positive direction for another direction. The location at which all coordinate system zeros intersect is referred to as the origin of the coordinate system.

Velocity

The velocity is the rate at which the position is changing. Thus, we will define the velocity component in the vertical direction, for example, as the rate at which the vertical position is changing. The velocity component in the vertical direction is *completely* independent of the horizontal position or the rate at which the horizontal position changes.

As long as the coordinate directions are perpendicular, the speed, or magnitude of the object's velocity, can be determined by:

$$v=\sqrt{v_x^2+v_y^2+v_z^2}$$

The direction of the object's velocity can be determined via right-angle trigonometry.

Acceleration

The acceleration is the rate at which the velocity is changing. Thus, we will define the acceleration component in the vertical direction, for example, as the rate at which the velocity component in the vertical direction is changing. The acceleration component in the vertical direction is *completely* independent of the velocity component in the horizontal direction or the rate at which the velocity component in the horizontal direction changes.

As long as the coordinate directions are perpendicular, the magnitude of the object's acceleration can always be determined by:

$$a=\sqrt{a_x^2+a_y^2+a_z^2}$$

The direction of the object's acceleration can be determined via right-angle trigonometry.

Doing kinematics in multiple dimensions involves a concerted effort on your part to disregard motion in one direction when considering motion in a perpendicular direction. The ability to mentally break down a complicated motion into its component motions requires considerable practice.

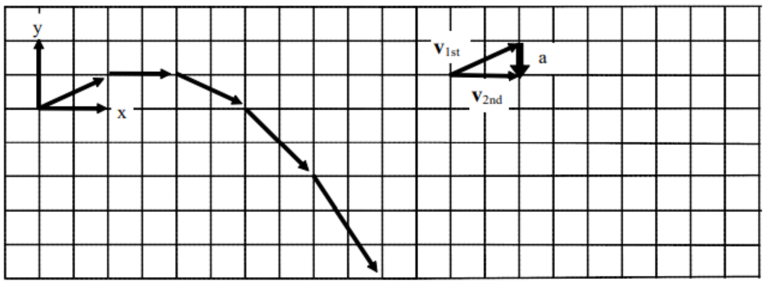
Analysis Tools

Drawing Motion Diagrams

Beginning your analysis by drawing a motion diagram is always the correct first step:

> In the shot put, a large mass is thrown at an angle of 22^o above horizontal, from a position of 2 m above the ground, a horizontal distance of 25 m.

A motion diagram for this scenario is sketched below.



- Horizontal (x) and vertical (y) coordinate systems are clearly indicated.
- In constructing the motion diagram, only a portion of the entire motion of the shot put is illustrated. For this motion diagram, analysis begins **the instant after the shot put leaves the putter's hand**, and analysis ends **the instant before the shot put hits the ground**. It is of extreme importance to clearly understand the beginning and the end of the motion that you will describe. The acceleration of the shot put while in the putter's hand, and the acceleration upon contact with the ground, has been conveniently left out of this analysis. Unless explicit information is either provided or desired about these accelerations, it is best to focus analysis on the simplest portion of the motion, i.e., when it is flying freely through the air.
- The acceleration is determined by the same method as in one-dimensional motion. In this case, the acceleration was determined near the beginning of the motion. Determining the acceleration at any other time will also indicate that its direction is straight downward, since we have focused our analysis on the time interval when the shot put is being acted on by only the force of gravity.

Drawing Motion Graphs

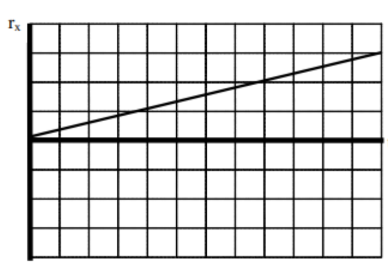
The verbal representation of the situation has already been translated into a pictorial representation, the motion diagram. A careful reading of the motion diagram allows the construction of the motion graphs.

Drawing the position vs. time graph

First, examine the position of the shot put as it moves through the air. Remember, the analysis of the horizontal position must be independent of the analysis of the vertical position.

Horizontal Position

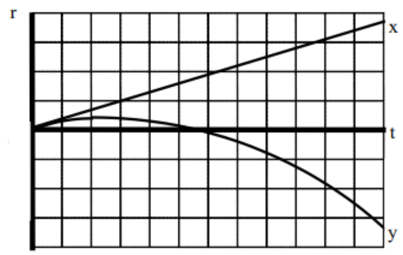
From the motion diagram, the shot put starts at position zero, and then has positive, increasing positions throughout the remainder of its motion. The horizontal position increases by even amounts in even time intervals.



Vertical Position

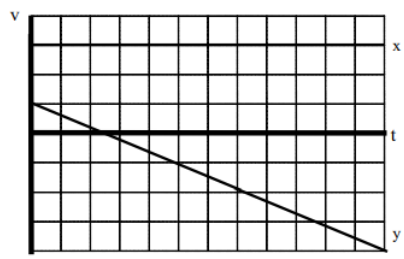
The shot put starts at position zero, increases its vertical position at a rate that is decreasing, then begins to decrease its vertical position at a rate that is increasing, even as it drops to negative positions.

Typically, both the horizontal and vertical positions are displayed on the same axis.



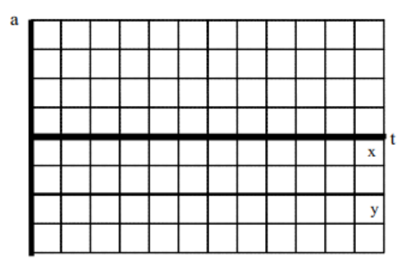
Drawing the velocity vs. time graph

In the horizontal direction, the rate at which the position changes is constant. Hence, the horizontal component of velocity is constant, and positive. In the vertical direction, the velocity component begins positive, decreases to zero, and then increases in the negative direction.



Drawing the acceleration vs. time graph

From the motion diagram, the acceleration of the shot-put can be determined to be directed downward at every point. Thus, the horizontal component of acceleration is zero and the vertical component is negative, and approximately constant due to our model's approximations.



Tabulating Motion Information

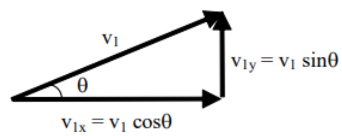
> In the shot put, a large mass is thrown at an angle of 22^o above horizontal, from a position of 2 m above the ground, a horizontal distance of 25 m.

Now that you have constructed a motion diagram and motion graphs, you should be able to assign numerical values to several of the kinematic variables. A glance at the situation description should indicate that information is presented about the shot put at two distinct events: when the shot put leaves the putter's hand and when the shot put strikes the ground. Other information can also be determined about these events by referencing the motion diagram. To organize this information, you should construct a motion table.

Event 1: The instant after the shot put leaves the hand.	Event 2: The instant before the shot put hits the ground.
$t_1 = 0$ s	$t_2 =$
$r_{1x} = 0$ m	$r_{2x} = +25$ m
$r_{1y} = 0$ m	$r_{2y} = -2$ m
$v_{1x} = v_1 \cos 22°$	$v_{2x} =$
$v_{1y} = v_1 \sin 22°$	$v_{2y} =$
$a_{12x} = 0$ m/s^2	
$a_{12y} = -9.8$ m/s^2	

In addition to the information explicitly given (the initial and final positions), information is available about both the initial velocity and the acceleration.

- **Initial velocity:** Although the magnitude of the initial velocity (v_1) is unknown, its orientation in space is known. Thus, via the right-angle trigonometry shown below, the components of this unknown magnitude velocity in the horizontal and vertical directions can be determined. Since we will analyze the x- and y-motion separately, we *must* break the initial velocity into its x- and y-components.



- **Acceleration:** The only force acting on the shot-put *during the time interval of interest* is the force of gravity, which acts directly downward. This is because the analysis of the motion is restricted to the time interval *after* leaving the thrower's hand and *before* striking the ground. Thus, there is no horizontal acceleration of the shot-put and the vertical acceleration has a magnitude of 9.8 m/s^2 .

Doing the Math

In Model 1, you were presented with two kinematic relationships. These relationships are valid in both the horizontal and vertical directions. Thus, you have a total of four relationships with which to analyze the scenario given. In the example above, there are four unknown kinematic variables. You should remember from algebra that four equations are sufficient to calculate four unknowns. Thus, by applying the kinematic relations in both the horizontal and vertical directions, you should be able to determine the initial velocity of the shot-put, the time in the air, and the final horizontal and vertical velocity components.

First, let's examine the horizontal component of the motion. Note that the positions, velocities, and accelerations in the following equations are all horizontal components.

x-direction

$$v_2 = v_1 + a_{12}(t_2 - t_1) \qquad r_2 = r_1 + v_1(t_2 - t_1) + \tfrac{1}{2}a_{12}(t_2 - t_1)^2$$

$$v_{2x} = (v_1 \cos 22) + 0(t_2 - 0) \qquad 25 = 0 + (v_1 \cos 22)(t_2 - 0) + \tfrac{1}{2}(0)(t_2 - 0)^2$$

$$v_{2x} = 0.927 v_1 \qquad 25 = 0.927 v_1 t_2$$

$$v_1 = \frac{27.0}{t_2}$$

Now let's examine the vertical component of the motion. All the positions, velocities, and accelerations in the following equations are now vertical components.

y-direction

$$v_2 = v_1 + a_{12}(t_2 - t_1) \qquad r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$$
$$v_{2y} = (v_1 \sin 22) + (-9.8)(t_2 - 0) \qquad -2 = 0 + (v_1 \sin 22)(t_2 - 0) + \frac{1}{2}(-9.8)(t_2 - 0)^2$$
$$v_{2y} = 0.375v_1 - 9.8t_2 \qquad -2 = 0.375v_1t_2 - 4.9{t_2}^2$$

Substituting the value of v_1 from above yields:

$$-2 = 0.375\left(\frac{27.0}{t_2}\right)t_2 - 4.9{t_2}^2$$
$$-2 = 10.1 - 4.9t_2^2$$
$$-12.1 = -4.9{t_2}^2$$
$$2.47 = {t_2}^2$$
$$t_2 = 1.57s$$

Plugging t_2 = 1.57 s into all of the remaining equations gives:

$$v_1 = 17.2 \text{ m/s} \qquad v_{2x} = 15.9 \text{ m/s} \qquad v_{2y} = -8.94 \text{ m/s}$$

Hints and Suggestions

Selecting Events

Let's look again at the shot-putter.

> In the shot put, a large mass is thrown at an angle of 22° above horizontal, from a position of 2 m above the ground, a horizontal distance of 25 m.

Imagine a video of the shot put event. Fast-forward over the frames showing the shot putter picking up the shot and stepping into the ring. Begin to watch the imaginary video frame-byframe as the shot putter begins to push the shot off of her shoulder and forward. Stop the video on the frame when the shot first leaves the putter's hand

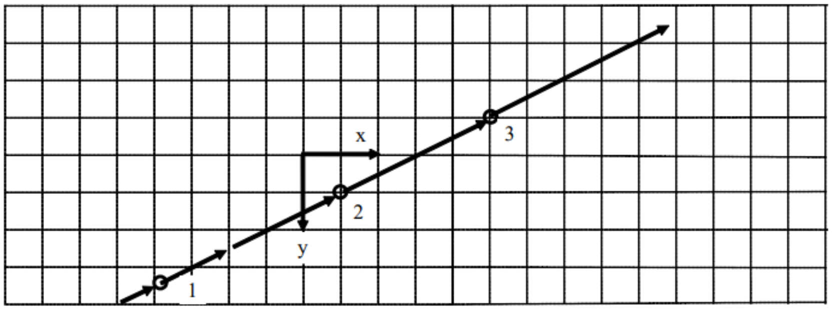
Why is it so important that we begin the analysis at this frame and **explicitly disregard** all the motion that has taken place before this frame? The reason is that in every frame preceding this frame, the shot put was in contact with the putter. Thus, the putter was exerting a force on the shot. Since no information is presented concerning this force, we have no way to determine the acceleration during these frames and hence no way to determine any other kinematic variables. Thus, we disregard all motion preceding the instant the shot leaves the putter's hand because that portion of the motion is simply impossible to analyze with the information provided. Once the shot leaves her hand, the only force acting on the shot is the force of gravity, which greatly simplifies the analysis.

Continue playing the imaginary video forward. Begin playing the tape frame-by-frame as the shot approaches the ground. Stop the video the frame before the shot hits the ground. We will stop our analysis at this frame. Why? Because starting with the next frame, the shot is in contact with the ground. Once in contact with the ground, an additional, unknown magnitude force begins to act on the shot. Once an unknown magnitude force begins to act, the acceleration of the shot becomes unknown and we are stuck. Thus, we conveniently stop our analysis before things get too complicated!

Since our analysis stops the instant before contact, note that the shot is still moving at this instant. (If it wasn't, how could it ever reach the ground?) Thus, resist the temptation to think that the velocity of the shot is zero at the end of analysis. The velocity of the shot is ultimately equal to zero (after it makes a big divot into the ground) but that happens long after it strikes the ground and hence long after our analysis is finished.

Activities

For each of the motion diagrams below, determine the algebraic sign (+, - or zero) of the x- and y-position, velocity, and acceleration of the object at location of the three open circles.
a.

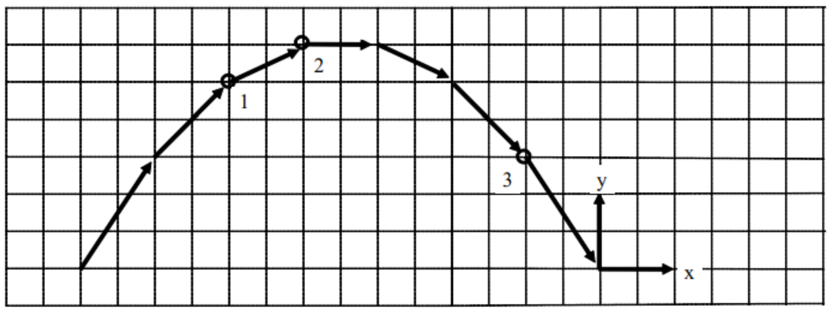


1	x	y
r		
v		
a		

2	x	y
r		
v		
a		

3	x	y
r		
v		
a		

b.



1	x	y
r		
v		
a		

2	x	y
r		
v		
a		

3	x	y
r		
v		
a		

For each of the motion diagrams below, determine the algebraic sign (+, - or zero) of the x- and y-position, velocity, and acceleration of the object at location of the three open circles.

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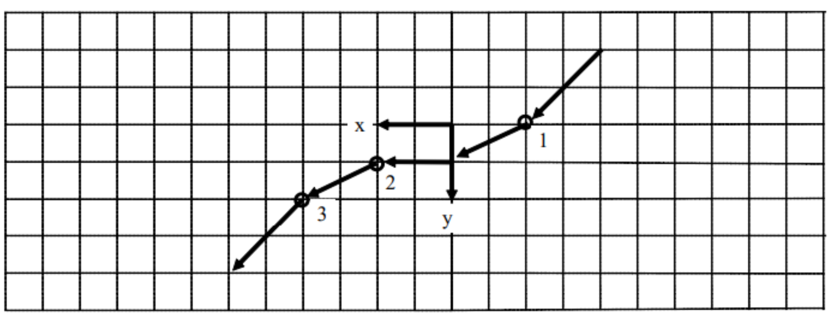
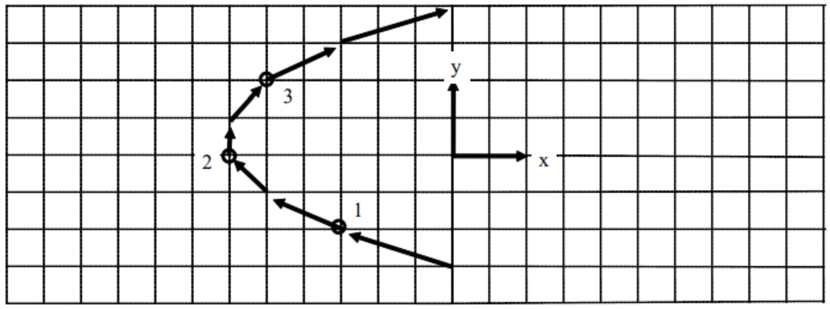


1	x	y
r		
v		
a		

2	x	y
r		
v		
a		

3	x	y
r		
v		
a		

b.



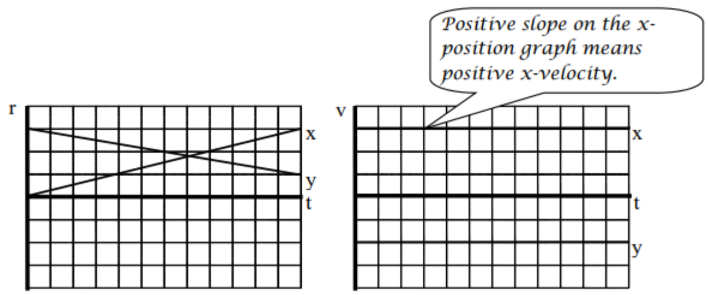
1	x	y
r		
v		
a		

2	x	y
r		
v		
a		

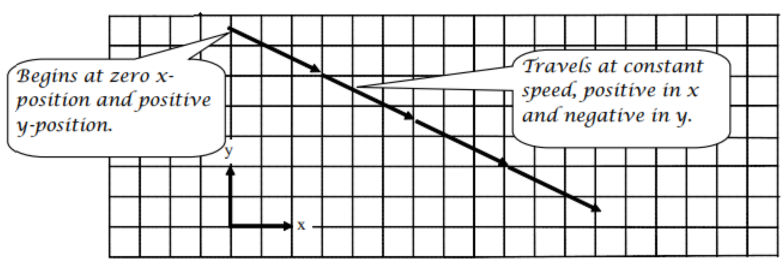
3	x	y
r		
v		
a		

Construct the missing motion graphs and/or motion diagram.

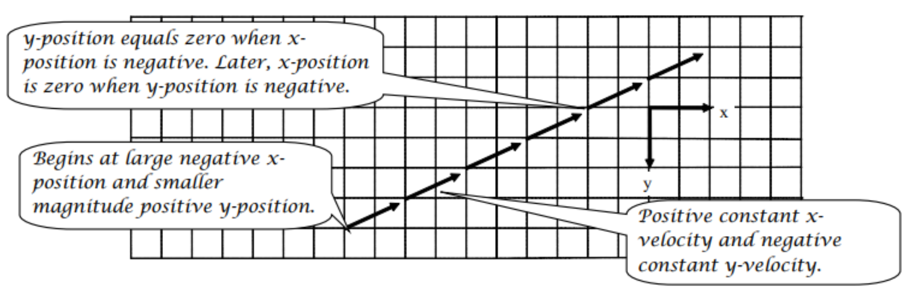
a. **Motion Graphs**



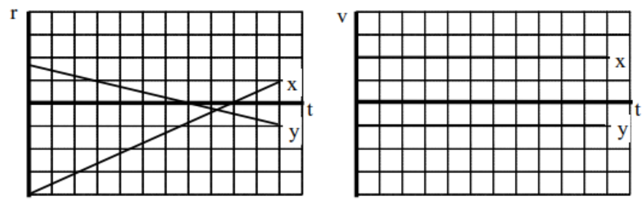
Motion Diagram



b. **Motion Diagram**

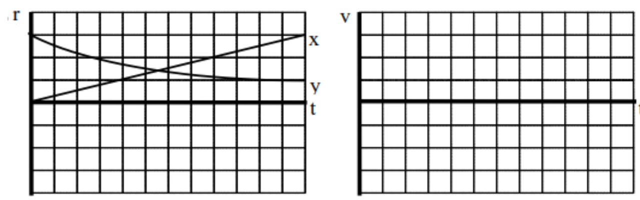


Motion Graphs

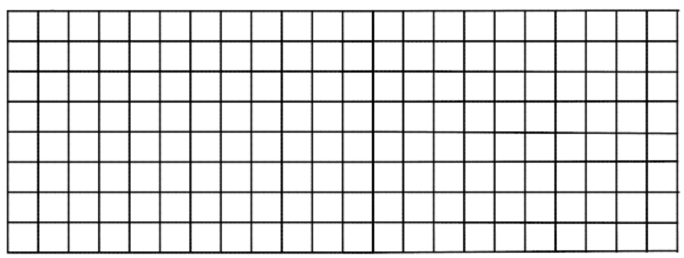


Construct the missing motion graphs and/or motion diagram.

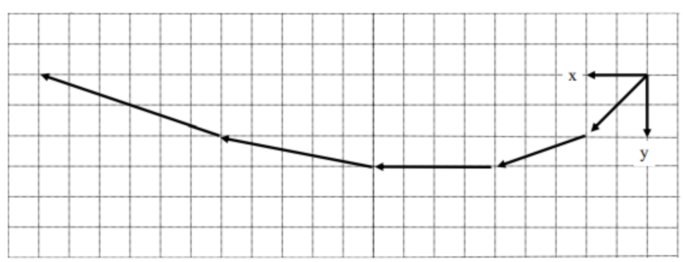
a. **Motion Graphs**



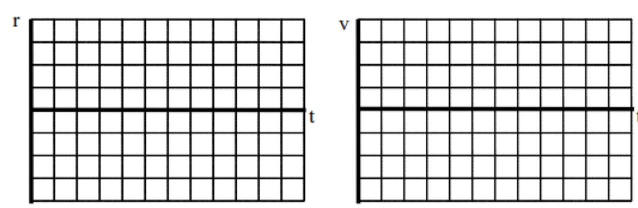
Motion Diagram



b. **Motion Diagram**



Motion Graphs



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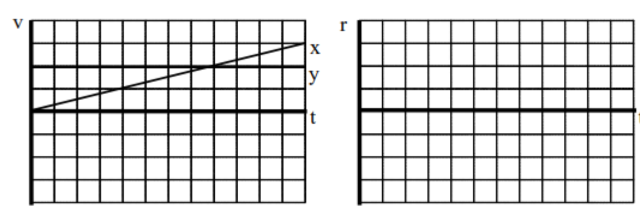
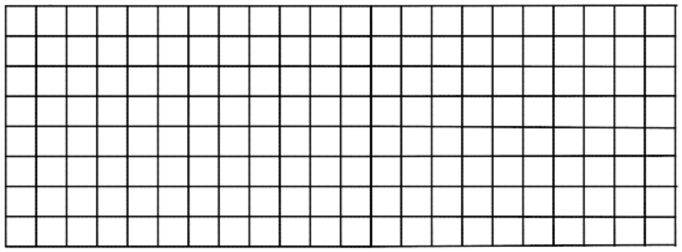
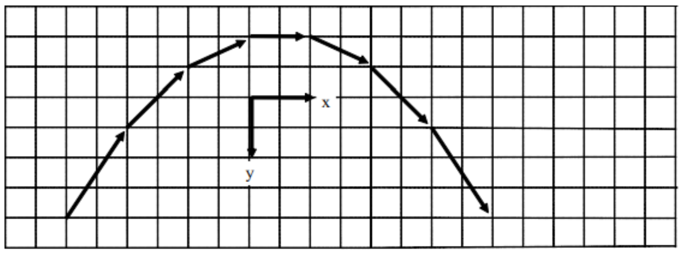


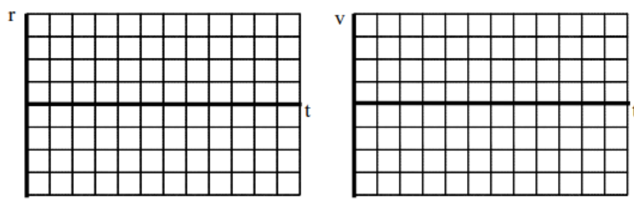
Motion Diagram



b. **Motion Diagram**

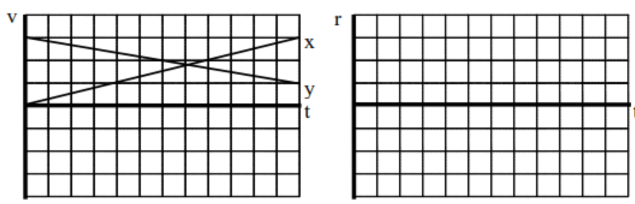


Motion Graphs

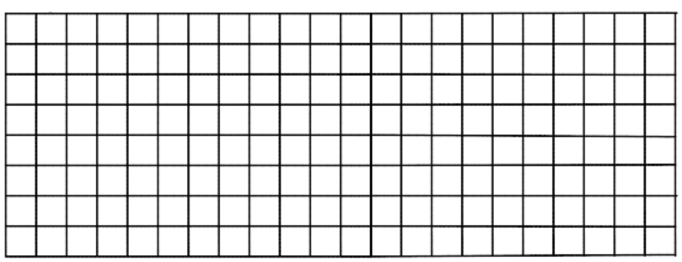


Construct the missing motion graphs and/or motion diagram.

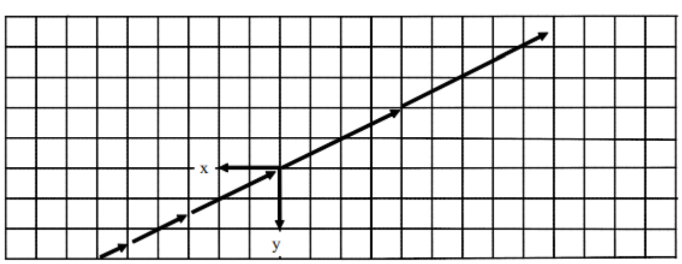
a. **Motion Graphs**



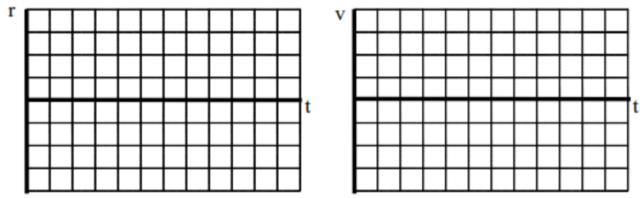
Motion Diagram



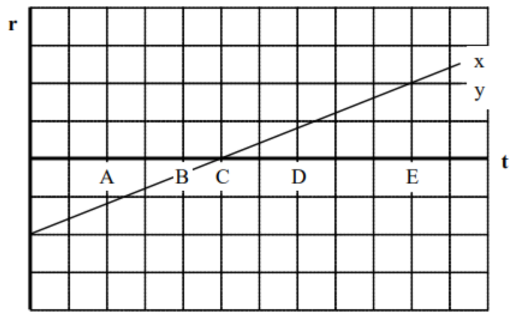
b. **Motion Diagram**



Motion Graphs



An object's motion is represented by the position vs. time graph below. Both the x- and y-position components are indicated on the graph.



a. Rank the object's distance from the origin at the lettered times.

Largest 1. __E__ 2. __A__ 3. __D__ 4. __B__ 5. __C__ Smallest

_____ The ranking cannot be determined based on the information provided.

Since distance is given by Pythagoras' Theorem, $D=\sqrt{r_x^2+r_y^2}$ *, and* r_y *is constant, the distance from the origin is proportional to the magnitude of the x-position.*

b. Rank the object's speed at the lettered times

Largest 1. ABCDE 2. ____ 3. ____ 4. ____ 5. ____ Smallest

_____ The ranking cannot be determined based on the information provided.

The object moves with constant speed in the positive x-direction.

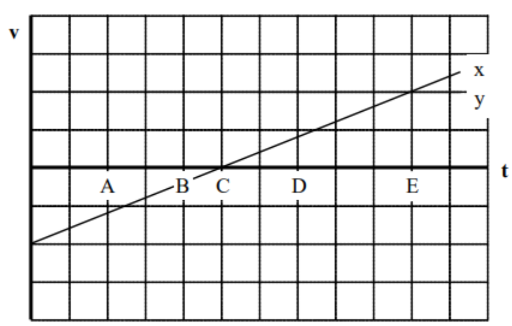
c. Rank the angle between the object's velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 90º.)

Largest 1. ABCDE 2. ____ 3. ____ 4. ____ 5. ____ Smallest

_____ The ranking cannot be determined based on the information provided.

Since the object moves with constant speed in the positive x-direction, the angle of its velocity vector is 0°.

An object's motion is represented by the velocity vs. time graph below. Both the x- and y-velocity components are indicated on the graph.



a. Rank the object's distance from the origin at the lettered times.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

___X__ The ranking cannot be determined based on the information provided.

Since a velocity graph doesn't specify the location of the coordinate system, you can't determine the distance from the origin of the coordinate system.

b. Rank the object's speed at the lettered times.

Largest 1. __E__ 2. __A__ 3. __D__ 4. __B__ 5. __C__ Smallest

______ The ranking cannot be determined based on the information provided.

Since speed is given by $v = \sqrt{v_x^2 + v_y^2}$ *, and* v_y *is constant, the speed is proportional to the magnitude of the x-position velocity.*

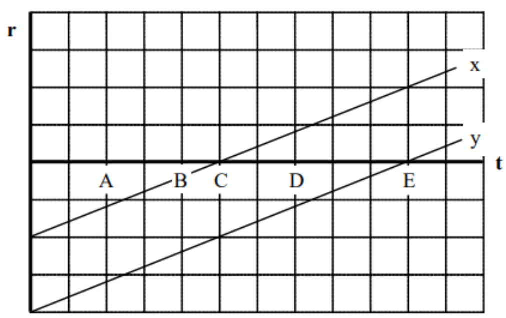
c. Rank the angle between the object's velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 90°.)

Largest 1. __A__ 2. __B__ 3. __C__ 4. __D__ 5. __E__ Smallest

______ The ranking cannot be determined based on the information provided.

Draw a motion diagram! The y-velocity is constant and positive, so all of the vectors are in the first and second quadrant. A and B are at > 90°, C is at 90°, and D and E are at < 90°.

An object's motion is represented by the position vs. time graph below. Both the x- and y-position components are indicated on the graph.



a. Rank the object's distance from the origin at the lettered times

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

b. Rank the object's speed at the lettered times.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

c. Rank the angle between the object's velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 90°.)

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

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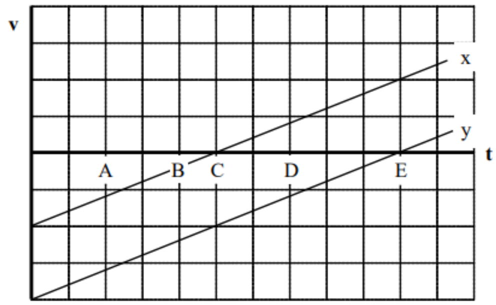
An object's motion is represented by the velocity vs. time graph below. Both the x- and y-velocity components are indicated on the graph.



a. Rank the object's distance from the origin at the lettered times.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

b. Rank the object's speed at the lettered times.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

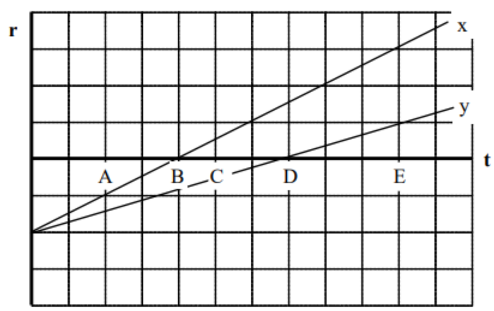
______ The ranking cannot be determined based on the information provided.

c. Rank the angle between the object's velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 90^o.)

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

An object's motion is represented by the position vs. time graph below. Both the x- and y-position components are indicated on the graph.



a. Rank the object's distance from the origin at the lettered times.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

b. Rank the object's speed at the lettered times.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

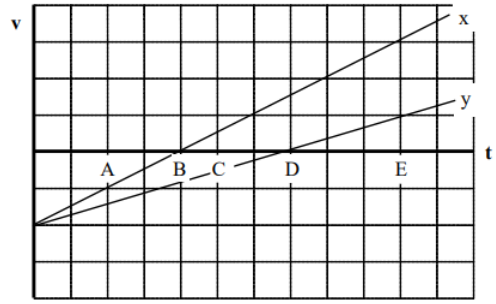
______ The ranking cannot be determined based on the information provided.

c. Rank the angle between the object's velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 90^o.)

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

An object's motion is represented by the velocity vs. time graph below. Both the x- and y-velocity components are indicated on the graph.



a. Rank the object’s distance from the origin at the lettered times.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

b. Rank the object’s speed at the lettered times.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

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c. Rank the angle between the object’s velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 90^o.)

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

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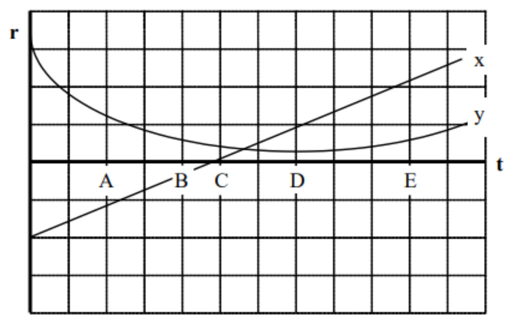

a. Rank the object’s distance from the origin at the lettered times.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

b. Rank the object’s speed at the lettered times.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

c. Rank the angle between the object’s velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 90^o.)

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

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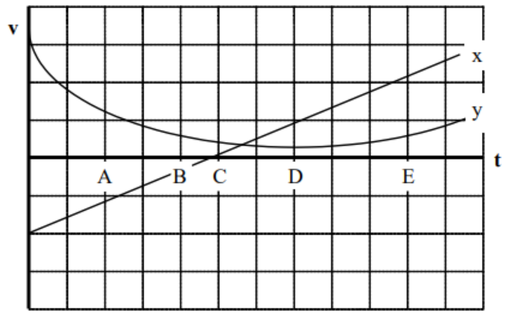
An object’s motion is represented by the velocity vs. time graph below. Both the x- and y-velocity components are indicated on the graph.



a. Rank the object's distance from the origin at the lettered times.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

b. Rank the object's speed at the lettered times.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

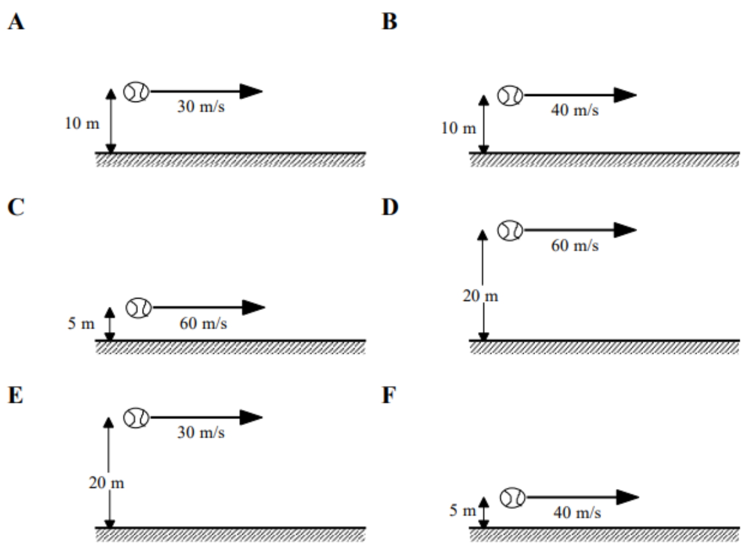
______ The ranking cannot be determined based on the information provided.

c. Rank the angle between the object's velocity and the x-axis at the lettered times, measuring all angles counterclockwise from +x. (Thus, the +y axis is at 90^o.)

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

Below are six identical baseballs thrown horizontally at different speeds from different heights above the ground. Assume the effects of air resistance are negligible.



a. Rank these baseballs on the basis of the elapsed time before they hit the ground.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

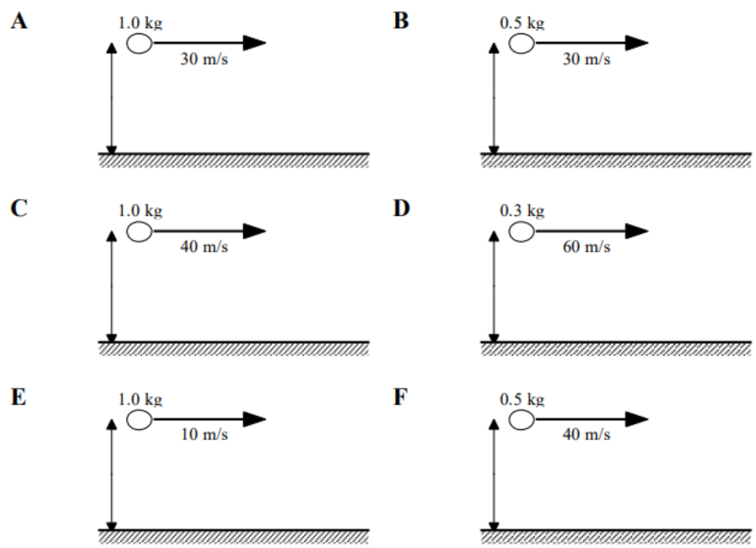
b. Rank these baseballs on the basis of the magnitude of their vertical velocity when they hit the ground.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six balls of different mass thrown horizontally at different speeds from the same height above the ground. Assume the effects of air resistance are negligible.



a. Rank these baseballs on the basis of the elapsed time before they hit the ground.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

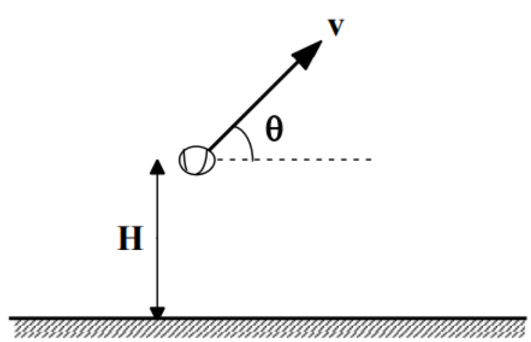
b. Rank these baseballs on the basis of the horizontal distance traveled before they hit the ground.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six different directions and speeds at which a baseball can be thrown. In all cases the baseball is thrown at the same height, H, above the ground. Assume the effects of air resistance are negligible.



	v	θ
A	30 m/s	30^0
B	45 m/s	0^0
C	30 m/s	60^0
D	15 m/s	60^0
E	20 m/s	45^0
F	15 m/s	90^0

a. Rank these baseballs on the basis of the maximum height the baseball reaches above the ground.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank these baseballs on the basis of the elapsed time before they hit the ground.

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ Smallest

______ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

At a circus, a human cannonball is shot from a cannon at 15 m/s at an angle of 40^o above horizontal. She leaves the cannon 1.0 m off the ground and lands in a net 2.0 m off the ground.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[1]

x-direction y-direction

At the buzzer, a basketball player shoots a desperation shot. She is 10 m from the basket and the ball leaves her hands exactly 1.2 m below the rim. She shoots at 35^o above the horizontal and the ball goes in!

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[2]

x-direction y-direction

With 1.0 s left on the clock, a basketball player shoots a desperation shot. The ball leaves her hands exactly 0.9 m below the rim at an angle of 35^o above the horizontal and the ball goes in just as the buzzer sounds!

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[3]

x-direction y-direction

A mountaineer must leap across a 3.0 m wide crevasse. The other side of the crevasse is 4.0 m below the point from which the mountaineer leaps. The mountaineer leaps at 35^o above horizontal and successfully makes the jump.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[4]

x-direction y-direction

The right fielder flawlessly fields the baseball and throws a perfect strike to the catcher who tags out the base runner trying to score. The right fielder is approximately 300 feet (90 m) from home plate and throws the ball at an initial angle of 30° above horizontal. The catcher catches the ball on the fly exactly 1.7 m below the height from which it was thrown.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =

Event 1:	Event 2:
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[5]

x-direction y-direction

The right fielder flawlessly fields the baseball and throws the ball at 94 mph (42 m/s) at an initial angle of 20° above horizontal toward home plate. The fielder is 80 m from the catcher and the ball leaves his hand exactly 1.6 m above the ground.

Motion Diagram

Motion Information

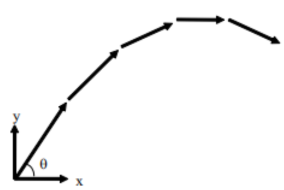
Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[6]

x-direction y-direction

A fire hose, with muzzle velocity of 24 m/s, is used to put out an apartment building fire. The fire is raging inside an apartment 5.0 m above the level of the hose and 10 m, measured horizontally, from the end of the hose. Ignore the effects of air resistance on the water.

Motion Diagram



Motion Information

Event 1: Water leaves hose	Event 2: Water hits flames
$t_1 = 0$ s	t_2 =
$r_{1x} = 0$ m	$r_{2x} = 10$ m
$r_{1y} = 0$ m	$r_{2y} = 5$ m
$v_{1x} = (24\text{ m/s})\cos\theta$	v_{2x} =

Event 1: Water leaves hose	Event 2: Water hits flames
v_{1y} = (24 m/s) sinθ	v_{2y} =
a_{12x} = 0 m/s^2	
a_{12y} = -9.8 m/s^2	

Mathematical Analysis

x-direction

$$r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$$
$$10 = 0 + (24\cos\theta)t_2 + 0$$
$$t_2 = \frac{10}{24\cos\theta}$$

substitute into y-equation:

y-direction

$$r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$$
$$5 = 0 + (24\sin\theta)t_2 + \frac{1}{2}(-9.8)t_2^2$$
$$5 = (24\sin\theta)\left(\frac{10}{24\cos\theta}\right) - 4.9\left(\frac{10}{24\cos\theta}\right)^2$$
$$5 = 10\tan\theta - \frac{0.851}{\cos^2\theta}$$
$$0 = 10\tan\theta - \frac{0.851}{\cos^2\theta} - 5$$

This equation can be solved by using a “solver” program, by knowing a few trig identities, or, most conveniently, by graphing the righthand-side of the equation and finding where it crosses zero. The solution is θ = 32°.

Therefore, t_2 = 0.49 s, and

$$v_2 = v_1 + a_{12}(t_2 - t_1) \qquad v_2 = v_1 + a_{12}(t_2 - t_1)$$
$$v_{2x} = 24\cos 32 + 0 \qquad v_{2y} = 24\sin 32 - 9.8(0.49)$$
$$v_{2x} = 20.4\text{ m/s} \qquad v_{2y} = 7.92\text{ m/s}$$

A mountaineer must leap across a 3.0 m wide crevasse. The other side of the crevasse is 4.0 m below the point from which the mountaineer leaps. The mountaineer leaps at a speed of 3.5 m/s and barely makes the jump.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[7]

x-direction y-direction

The right fielder flawlessly fields the baseball and must throw a perfect strike to the catcher, 90 m away, to tag out the base runner trying to score. The right fielder knows she can throw a baseball at 80 mph (36 m/s) and calculates the proper angle at which to throw so that the catcher will catch the ball on the fly exactly 1.8 m below the height from which it was thrown. However, her calculation is so time-consuming that the ball arrives too late and the runner scores

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[8]

x-direction y-direction

At the buzzer, a basketball player shoots a desperation shot. She is 14 m from the basket and the ball leaves her hands exactly 1.4 m below the rim. She throws the ball at 18 m/s. Can she make the shot?

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[9]

x-direction y-direction

At a circus, a human cannonball will be shot from a cannon at 15 m/s. She will leave the cannon 1.0 m off the ground and hopefully land in a net 3.0 m off the ground, after flying a horizontal distance of 22 m. Do you want this job?

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[10]

x-direction y-direction

At a circus, a human cannonball will be shot from a cannon at 24 m/s. She will leave the cannon 1.0 m off the ground and hopefully land in a net 3.0 m off the ground, after flying a horizontal distance of 22 m. Do you want this job?

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[11]

x-direction y-direction

A ball is rolled off a level 0.80 m high table at 15 m/s. The floor beyond the table slopes down at a constant 5^0 below the horizontal.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[12]

x-direction y-direction

A golf ball leaves the club at 18 m/s at an angle of 65^0 above the horizontal. The ground ahead slopes upward at 4°.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =

Event 1:	Event 2:
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[13]

x-direction y-direction

A ski-jumper leaves the ramp at an angle of 11^0 above the horizontal, 3.0 m above the ground. The ground slopes downward at 33° from this point. The jumper lands 140 m down the slope.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[14]

x-direction y-direction

A kayaker 120 m east and 350 m north of his campsite is moving with the current at 2 m/s to the south. He begins to paddle west, giving the kayak an acceleration of 0.2 m/s^2 for 15 s.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[15]

x-direction y-direction

A kayaker 120 m east and 80 m north of her campsite is moving with the current at 2 m/s to the south. She begins to paddle west, giving the kayak a constant acceleration. She lands right at her campsite.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[16]

x-direction y-direction

An astronaut on a spacewalk is 30 m from her spaceship and moving at 0.8 m/s away from her ship, at an angle of 17° from a line between her and the ship. She engages her Manned Maneuvering Unit (MMU) for 15 s. The MMU imparts an acceleration of 0.1 m/s^2 to her in the direction she was originally moving.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[17]

x-direction y-direction

An astronaut on a spacewalk is 30 m from her spaceship and moving at 0.8 m/s away from her ship, at an angle of 17° from a line between her and the ship. She engages her Manned Maneuvering Unit (MMU) for 20 s. The MMU imparts an acceleration of 0.1 m/s^2 to her in the direction initially toward her ship. (She does not change this direction during the maneuver.)

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =

Event 1:	Event 2:
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[18]

x-direction y-direction

An astronaut on a spacewalk is 30 m from his spaceship and moving at 0.8 m/s away from his ship, at an angle of 17° from a line between him and the ship. He engages his Manned Maneuvering Unit (MMU), which supplies an acceleration of 0.1 m/s^2 in a constant direction. He returns to his ship safely.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis[19]

x-direction y-direction

Determine the time-of-flight (T) of a rock thrown horizontally off of a cliff as a function of the initial velocity (vi), the height of the cliff (H), and g. Assume the ground at the base of the cliff is level.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis

x-direction y-direction

Questions

If H = ∞, what should T equal? Does your function agree with this observation?

If g = 0 m/s^2, what should T equal? Does your function agree with this observation?

If v_i is doubled, what happens to T?

Determine the horizontal range (R) of a rock thrown horizontally off of a cliff as a function of the initial velocity (v_i), the height of the cliff (H), and g. Assume the ground at the base of the cliff is level.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis

x-direction y-direction

Questions

f H = ∞, what should R equal? Does your function agree with this observation?

If g = ∞, what should R equal? Does your function agree with this observation?

If v_i is doubled, what happens to R?

Determine the maximum height (H) of a projectile launched over level ground as a function of the initial velocity (v_i), the launch angle (θ), and g.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis

x-direction y-direction

Questions

If g = ∞, what should H equal? Does your function agree with this observation?

If θ = 0°, what should H equal? Does your function agree with this observation?

If v_i is doubled, what happens to H?

Determine the range (R) of a projectile launched over level ground as a function of the initial velocity (v_i), the launch angle (θ), and g.

Determine the range (R) of a projectile launched over level ground as a function of the initial velocity (v_i), the launch angle (θ), and g.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Mathematical Analysis

x-direction y-direction

Questions

If g = 0 m/s^2, what should R equal? Does your function agree with this observation?

If θ = 90°, what should R equal? Does your function agree with this observation?

If vi is doubled, what happens to R?

A projectile is launched from the top of an decline of constant angle ϕ. Determine the distance the projectile travels along the decline (D) as a function of the initial velocity (v_i), the launch angle above horizontal (θ), the decline angle (ϕ), and g.

Motion Diagram

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	

Event 1:	Event 2:
a_{12y} =	

Mathematical Analysis

x-direction y-direction

Questions

If ϕ = 90°, what should D equal? Does your function agree with this observation?

If θ = 90°, what should D equal? Does your function agree with this observation?

3.2: Dynamics

Concepts and Principles

Just like in kinematics, it's an empirical fact about nature that when a force acts on an object in one direction (for example, the horizontal) this action does not appear to cause changes in the motion in a perpendicular direction (the vertical). Therefore, to investigate the effects of forces on the motion of an object in the vertical direction, you can ignore all forces acting in the horizontal direction. Of course, many forces will simultaneously act in both the horizontal and vertical directions. As in kinematics, the effect of these forces can be examined by concentrating on the *components* of the forces in the various directions. Again, as long as the directions of interest are perpendicular, the force components can be determined through right-angle trigonometry, and the magnitude of the force can always be determined by:

$$F=\sqrt{F_x^2+F_y^2+F_z^2}\,.$$

Thus, Newton's second law,

$$\Sigma F=ma$$

is independently valid in any member of a set of perpendicular directions. The total force in the horizontal direction, for example, is equal to the mass times the acceleration in that direction. Note that the mass has been verified to be independent of direction, meaning that objects possess the same inertia in all directions.

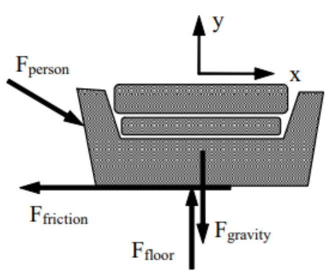
Analysis Tools

Drawing Free-Body Diagrams

The free-body diagram is still the most important analysis tool for determining the forces that act on a particular object. As an example, start with a verbal description of a situation:

> While rearranging furniture, a 600 N force is applied at an angle of 25^0 below horizontal to a 100 kg sofa at rest.

A free-body diagram for the sofa is sketched below:



This is a complete free-body diagram for the couch.

The only non-contact interaction is the force of gravity, directed vertically downward.

The couch is in contact with two external objects, the person, pushing the couch across the floor, and the floor, exerting a force directed upward to prevent the couch from sinking into the floor. In addition, experience tells us that the interaction between the couch and the floor also hinders the motion of the couch in the direction of the person's push. This portion of the couch-floor interaction is commonly referred to as *friction*.

The Force of Friction

The interaction between objects in direct contact typically consists of two parts. One part of the interaction is directed perpendicular to the surface of contact.[2] The other part of the interaction is the portion commonly called friction. The frictional portion of the interaction depends on many variables.

For most situations, a *model* of friction limiting the number of variables effecting the interaction to two is adequate. These two variables are the magnitude of the perpendicular portion of the interaction, generically called the *contact force*, and a unit-less constant that reflects the relative roughness of the surface-to-surface contact, termed the *coefficient of friction*. This linear model of sliding friction further differentiates between the frictional interaction when the two surfaces are moving with respect to each other, termed *kinetic friction*, and when they are not, termed *static friction*.

Note

[2] The portion of the interaction directed perpendicular to the surface of contact is sometimes referred to as the *normal* force, where normal has its mathematical definition of perpendicular.

Kinetic friction

The kinetic friction model states that the frictional interaction between the surfaces is approximately equal to the product of the contact force, $F_{contact}$ and the coefficient of friction for kinetic situations, μ_k:

$$F_{\text{friction}} = \mu_k F_{\text{contact}}$$

The direction of this force on a particular object is in opposition to the relative motion of the two surfaces in contact.

Static friction

The static friction model states that the frictional interaction between the surfaces must be less than, or at most equal to, the product of the contact force, $F_{contact}$, and the coefficient of friction for static situations, μ_s:

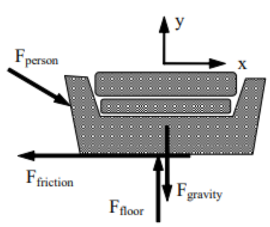
$$F_{\text{friction}} \leq \mu_s F_{\text{contact}}$$

The direction of this force on a particular object is in opposition to the motion that *would* result if the frictional interaction were not present.

Applying Newton's Second Law

Using this model for friction, we can now quantitatively analyze the original situation. Note that the two coefficients of friction will typically be given as an ordered pair, (μ_s, μ_k).

While rearranging furniture, a 600 N force is applied at an angle of 25^0 below horizontal to a 100 kg sofa at rest. The coefficient of friction between the sofa and the floor is (0.5, 0.4).



Hopefully you realize that two quite different outcomes can result from this push. Either the person pushes too weakly to move the couch or the push is sufficient to make the couch move. Since the frictional forces acting in these two cases are quite different, we can't really numerically analyze the situation until we make an *assumption* as to the outcome of the push. Of course, we will then have to check the validity of our assumption once we have completed our analysis. If our assumption turns out to be incorrect, we will then have to re-analyze the situation using the other possible outcome.

I will assume the couch doesn't move for the analysis below, and then later check the assumption. Assuming the couch doesn't move is equivalent to assuming $a_x = 0$ m/s^2 and that the relevant type of friction to use is static friction.

Applying Newton's Second Law independently in the horizontal (x) and vertical (y) directions yields:

$$\begin{array}{ll}
\underline{\mathbf{x-direction}} & \underline{\mathbf{y-direction}} \\
\Sigma F=ma & \Sigma F=ma \\
F_{\text{person}}\ \cos 25-F_{\text{staticfriction}}=ma_x & -F_{\text{person}}\ \sin 25-F_{\text{gravity}}+F_{\text{floor}}\ =ma_y \\
600\cos 25-F_{\text{staticfriction}}=100(0) & -600\cos\sin 25-(100)(9.8)+F_{\text{floor}}\ =100(0) \\
F_{\text{staticfriction}}=544N & F_{\text{floor}}\ =1234N
\end{array}$$

Notice that the acceleration of the couch in the vertical direction must be zero regardless of my assumption, unless the couch begins to levitate or crash through the floor.

Assuming the couch doesn't move leads to a calculated value of static friction equal to 544 N. Can static friction create a force of this magnitude to prevent the couch's motion? I can check this calculated value against the allowed values for static friction:

$$\begin{array}{l}
F_{\text{staticfriction}}\leq\mu_s F_{\text{contact}} \\
F_{\text{staticfriction}}\leq(0.5)(1234) \\
F_{\text{staticfriction}}\leq 617\text{ N}
\end{array}$$

Since the calculated value of the static frictional force is below the maximum possible value of the static frictional force, my analysis and assumption are valid, the couch does not budge. The person is not pushing hard enough to overcome the static frictional force that acts to prevent the couch's motion relative to the floor.

Therefore, in this scenario the actual value of the static frictional force is 544 N (remember, it can be any value less than or equal to 617 N) and the acceleration of the couch is equal to zero.

How would the analysis change if the couch *was* initially in motion? Assume you enlisted a friend to help get the couch moving, but as soon as it began to move your friend stopped pushing. Would the couch stop immediately, gradually slow down to a stop, or could you keep the couch in motion across the room?

If the couch was initially moving, two things must change in our analysis. First, the horizontal acceleration of the couch is no longer necessarily zero. Second, the frictional force acting on the couch is kinetic

Applying Newton's Second Law independently in the horizontal (x) and vertical (y) directions now yields:

$$\begin{array}{ll}
\underline{\mathbf{x-direction}} & \underline{\mathbf{y-direction}} \\
\Sigma F=ma & \Sigma F=ma \\
F_{person}\cos 25-F_{kineticfriction}\ =100a_x & -F_{person}\sin 25-F_{gravity}+F_{floor}=ma_y \\
600\cos 25-F_{kineticfriction}\ =100a_x & -600\cos\sin 25-(100)(9.8)+F_{floor}=100(0) \\
544-F_{kineticfriction}\ =100a_x & F_{floor}=1234N
\end{array}$$

To finish the analysis, we need to calculate the kinetic frictional force.

$$\begin{array}{l}
F_{\text{kineticfriction}}=\mu_k F_{\text{contact}} \\
F_{\text{kineticfriction}}=(0.4)(1234) \\
F_{\text{kineticfriction}}=494\text{ N}
\end{array}$$

Substituting this into the x-equation above yields:

$$\begin{array}{l}
544-494=100a_x \\
50=100a_x 50=100a_x \\
a_x=0.50\text{ m/s}^2
\end{array}$$

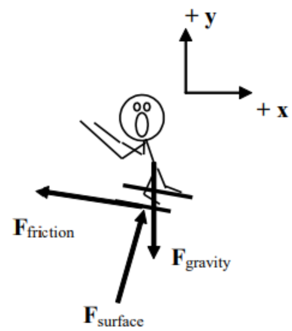
Thus, if the couch is already moving the kinetic frictional force is 494 N and the couch accelerates toward the right at 0.50 m/s^2. In summary, if the couch is initially moving it will continue to move and accelerate at 0.50 m/s^2 to the right. If it is initially at rest, the person pushing on it will not be able to get it to move.

Choosing a Coordinate System

In analyzing a scenario, you are always free to choose whatever coordinate system you like. If you make up negative, or left positive, this will not make you get the wrong answer. However, certain coordinate systems may make the mathematical analysis simpler than other coordinate systems. For example;

> *A 75 kg skier starts from rest at the top of a 20^0 slope. He's a show-off, so he skies down the hill backward. The frictional coefficient between his skies and the snow is (0.10,0.05).*

In attempting to analyze this situation, first draw a free-body diagram

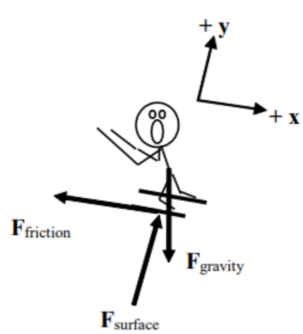


Notice that I have chosen the traditional horizontal and vertical coordinate system. I could analyze the situation using this coordinate system, but there are two difficulties with this choice.

1. Neither the force of the surface nor the force of friction is oriented in the x- or y-direction. (The force of gravity is oriented in the negative y-direction.) Therefore, I will have to use trigonometry to determine the x- and y-components of both of these forces.
2. The skier is accelerating down the inclined slope. Thus, I will also need trigonometry to determine the x- and y-components of the acceleration.

Although these difficulties are by no means insurmountable, why make the task more difficult than it has to be?

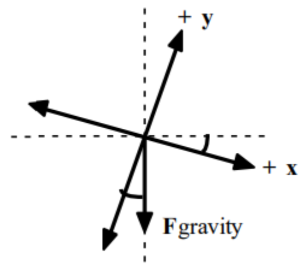
Contrast the above choice of coordinate system with a coordinate system in which the xdirection is tilted parallel to the surface on which the skier slides and the y-direction, remaining perpendicular to the x, is perpendicular to the surface.



1. Using the tilted coordinate system, the only force not oriented in the x- or y-direction is the force of gravity. Therefore, I will only need to use trigonometry to determine the x- and ycomponents of one force rather than two.
2. The skier is accelerating down the inclined slope. Since the xdirection is oriented parallel to the slope, the skier has an acceleration in the x-direction and zero acceleration in the ydirection.

This simple rotation of the coordinate system has made the mathematical analysis of this situation much easier. Applying Newton's second law in the x- and y-direction leads to:

$\underline{\mathbf{x-direction}}$	$\underline{\mathbf{y-direction}}$
$\Sigma F = m\,a$	$\Sigma F = ma$
$F_{gravity}(\sin 20) - F_{friction} = ma_x$	$-F_{gravity}(\cos 20) + F_{surface} = m(0)$
$(75)(9.8)(\sin 20) - F_{friction} = 100a_x$	$-(75)(9.8)(\cos 20) + F_{surface} = m(0)$
$251 - F_{friction} = 100a_x$	$F_{surface} = 691\ N$



Notice that if the x-axis is rotated by 20^0 from horizontal to become parallel to the slope, the y-axis is rotated by 20^0 from vertical. Since the force of gravity is always oriented vertically downward, it's now 20^0 from the y-axis.

Thus, the force of gravity has a component in the positive x-direction of $F_{gravity}$ (sin 20^0) and a component in the negative y-direction of $F_{gravity}$ (cos 20^0).

Now that the contact force between the skier and the slope is known, the static friction force can be determined.

$$\begin{gathered}F_{\text{friction}} \leq \mu_s F_{\text{contact}} \\ F_{\text{friction}} \leq (0.10)(691) \\ F_{\text{friction}} \leq 69 \text{ N}\end{gathered}$$

Since the x-component of the force of gravity on the skier (251 N) is larger than the force of static friction (69 N), the skier will accelerate down the hill. Once he begins to move, the frictional force must be calculated using the kinetic friction model.

$$\begin{gathered}F_{\text{friction}} = \mu_k F_{\text{contact}} \\ F_{\text{friction}} = (0.05)(691) \\ F_{\text{friction}} = 35 \text{ N}\end{gathered}$$

Examining the x-component of Newton's second law:

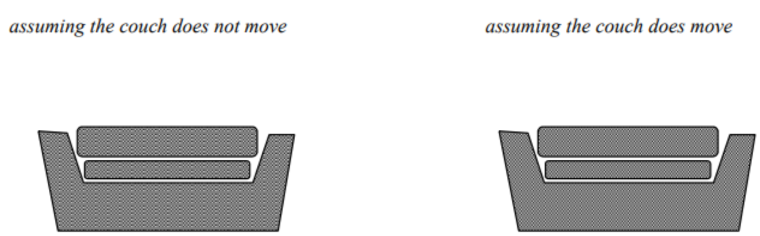
$$\begin{gathered}251 - F_{\text{friction}} = 100 a_x \\ 251 - 35 = 100 a_x \\ a_x = 2.2 \text{ m/s}^2\end{gathered}$$

The skier accelerates down the slope with an acceleration of 2.2 m/s^2.

Activities

Construct free-body diagrams for the objects described below

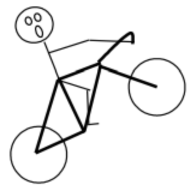
a. Someone mistakenly put a lovely couch out at the curb on garbage day and you decide to take it back to your apartment. You push horizontally on the 80 kg couch with a force of 320 N. The frictional coefficient is (0.40,0.35).



b. A 100 kg bicycle and rider initially move at 16 m/s up a 15^0 hill. The rider slams on the brakes and skids to rest. The coefficient of friction is (0.8,0.7).

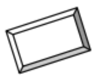
while skidding

when the bike is at rest on the incline

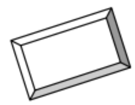


c. A 10 kg box is stacked on top of a 25 kg box. The boxes are at rest on an 8° incline.

the top box



the bottom box



Construct free-body diagrams for the objects described below.

a. Someone mistakenly put a lovely couch out at the curb on garbage day and you decide to take it back to your apartment. You pull on the 110 kg couch with a force of 410 N directed at 35^0 above horizontal. The frictional coefficient is (0.40,0.35).

assuming the couch does not move

assuming the couch does move

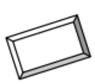
b. A 60 kg skier starts from rest at the top of a 100 m, 25^0 slope. He doesn't push with his poles because he's afraid of going too fast. The frictional coefficient is (0.10,0.05).

while skiing downhill

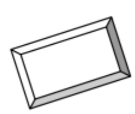
while being pulled back uphill by the towrope

c. A 10 kg box is stacked on top of a 25 kg box. The boxes are sliding down an 18° incline at increasing speed. The top box is not moving relative to the bottom box.

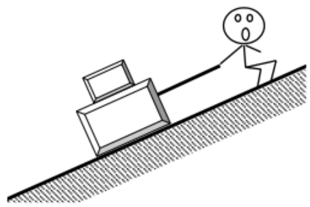
the top box



the bottom box

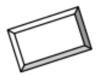
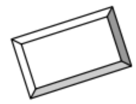


The strange man below is trying to pull the pair of boxes up the incline. Construct the requested free-body diagrams.



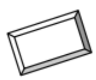
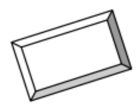
a. The boxes *almost* move up the incline.

the top box | *the bottom box*

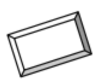
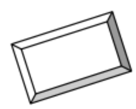
b. The boxes move up the incline.

the top box | *the bottom box*

c. The bottom box moves up the incline but the top box slides off the bottom box.

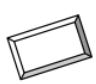
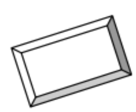
the top box | *the bottom box*

The strange man below is trying to prevent himself from getting crushed by the boxes. Construct the requested free-body diagrams.

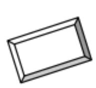
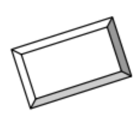
a. The boxes *almost* move down the incline.

the top box | *the bottom box*

b. The boxes *almost* move up the incline.

the top box | *the bottom box*

c. The bottom box *almost* moves down the incline but the top box slides off the bottom box.

A constant magnitude force is applied to a rope attached to a crate. The crate is on a level surface. For each of the following situations, circle the correct relationship symbol between the two force magnitudes and explain your reasoning.

a. The crate moves at constant speed and the rope is horizontal.

$$\begin{array}{lcl} F_{gravity} & > \ = \ < \ ? & F_{surface} \\ F_{rope} & > \ = \ < \ ? & F_{friction} \end{array}$$

Explanation:

b. The crate does not move and the rope is horizontal.

$$\begin{array}{lcl} F_{gravity} & > \ = \ < \ ? & F_{surface} \\ F_{rope} & > \ = \ < \ ? & F_{friction} \end{array}$$

Explanation:

c. The crate moves at constant speed and the rope is inclined above the horizontal

$$\begin{array}{lcl} F_{gravity} & > \ = \ < \ ? & F_{surface} \\ F_{rope} & > \ = \ < \ ? & F_{friction} \end{array}$$

Explanation:

A constant magnitude force is applied to a rope attached to a crate. The crate is on an inclined surface. For each of the following situations, circle the correct relationship symbol between the two force magnitudes and explain your reasoning.

a. The crate does not move and the rope is parallel to the incline and directed up the incline.

$$\begin{array}{lcl} F_{gravity} & > \ = \ < \ ? & F_{surface} \\ F_{rope} & > \ = \ < \ ? & F_{friction} \end{array}$$

Explanation:

b. The crate does not move and the rope is parallel to the incline and directed down the incline.

$$\begin{array}{lcl} F_{gravity} & > \ = \ < \ ? & F_{surface} \\ F_{rope} & > \ = \ < \ ? & F_{friction} \end{array}$$

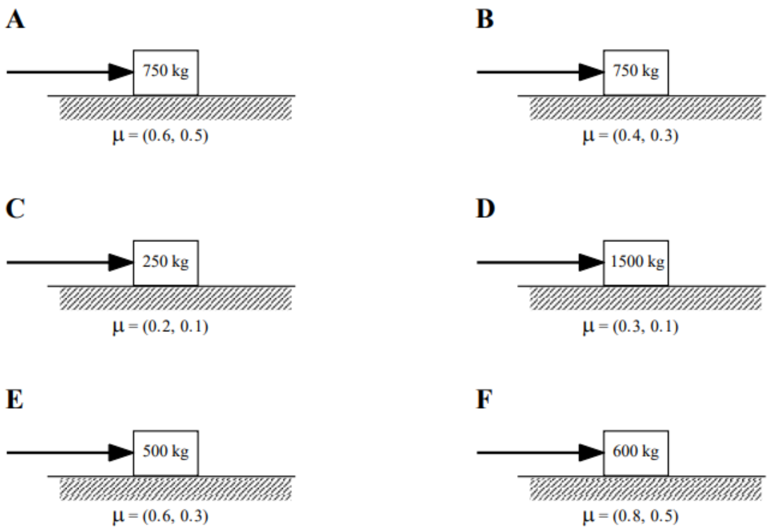
Explanation:

c. The crate moves at constant speed up the incline and the rope is parallel to the incline and directed up the incline.

$$\begin{array}{lcl} F_{gravity} & > \ = \ < \ ? & F_{surface} \\ F_{rope} & > \ = \ < \ ? & F_{friction} \end{array}$$

Explanation:

Below are six crates at rest on level surfaces. The crates have different masses and the frictional coefficients between the crates and the surfaces differ. The same external force is applied to each crate, but none of the crates move. Rank the crates on the basis of the magnitude of the frictional force acting on them.

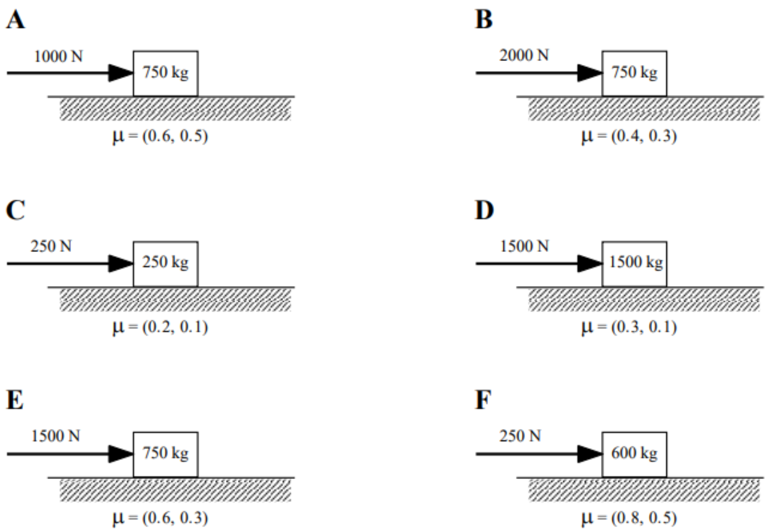


Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six crates at rest on level surfaces. The masses, frictional coefficients between the crates and the surfaces, and the external applied force all differ.



a. If none of the crates move, rank the crates on the basis of the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

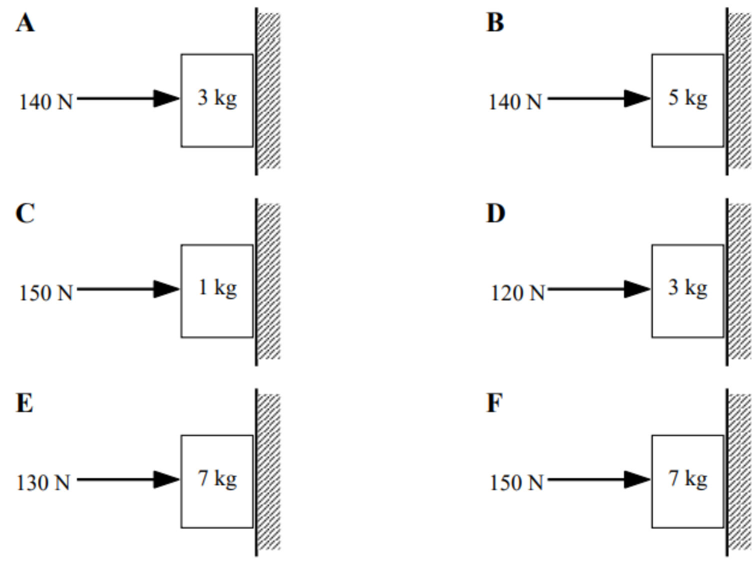
b. If the crates are moving, rank the crates on the basis of the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six boxes held at rest against a wall. The coefficients of friction between each box and the wall are identical.



a. Rank the boxes on the basis of the magnitude of the force of the wall acting on them

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

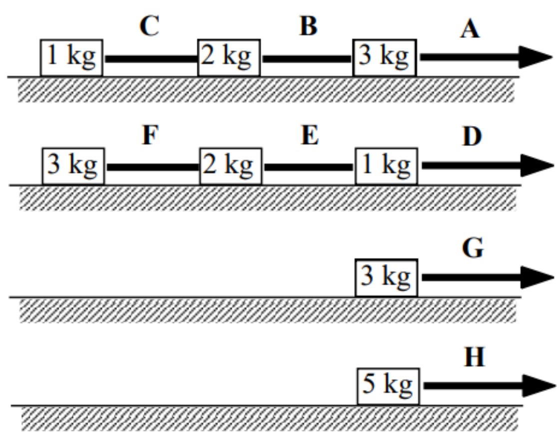
b. Rank the boxes on the basis of the magnitude of the frictional force acting on them

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are eight crates of differing mass. Each crate is being pulled to the right at the same constant speed.



a. Rank the magnitude of the force exerted by each rope on the crate immediately to its left if the frictional coefficient between each crate and the surface is the same non-zero value

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

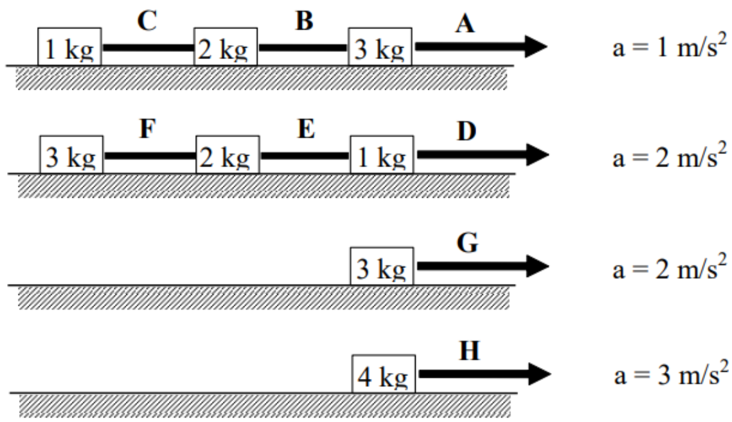
b. Rank the magnitude of the force exerted by each rope on the crate immediately to its left if the frictional coefficient between each crate and the surface is zero.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are eight crates of differing mass. The frictional coefficients between each crate and the surface on which they slide are so small that the force of friction is negligible on all crates. Each crate is being pulled to the right and accelerating. The acceleration of each crate or chain of crates is given. Rank the magnitude of the force exerted by each rope on the crate immediately to its left.



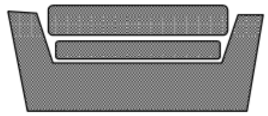
Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Someone mistakenly put a lovely couch out at the curb on garbage day and you decide to take it back to your apartment. You push on the 110 kg couch with a force of 410 N directed at 35^0 below horizontal. The couch doesn't move.

Free-Body Diagram

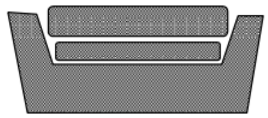


Mathematical Analysis[20]

x-direction y-direction

Someone mistakenly put a lovely couch out at the curb on garbage day and you decide to take it back to your apartment. You push on the 80 kg couch with a force of 320 N directed at 15^0 below horizontal. The frictional coefficient is (0.40,0.35).

Free-Body Diagram



Mathematical Analysis[21]

x-direction y-direction

Someone mistakenly put a lovely couch out at the curb on garbage day and you decide to take it back to your apartment. You pull on the 110 kg couch with a force of 510 N directed at 35^0 above horizontal. The frictional coefficient is (0.40,0.35).

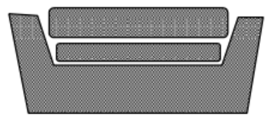
Free-Body Diagram

Mathematical Analysis[22]

x-direction y-direction

You get into a fight with another person over a garbage-day couch. You push on the 80 kg couch with a force of 660 N directed at 15^0 below horizontal. She claims ownership by sitting on the couch while you try to push it. You still manage to just barely get the couch moving. The frictional coefficient is (0.40,0.35).

Free-Body Diagram



Mathematical Analysis[23]

x-direction y-direction

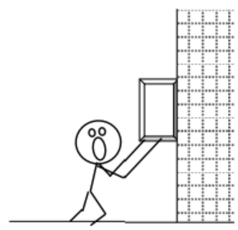
You get into a fight with another person over a garbage-day couch. You push on the 80 kg couch with a force of 420 N directed at 15^0 below horizontal. She pushes on the other side of the couch with a force of 510 N directed at 25^0 below horizontal. The frictional coefficient is (0.40,0.35).

Free-Body Diagram

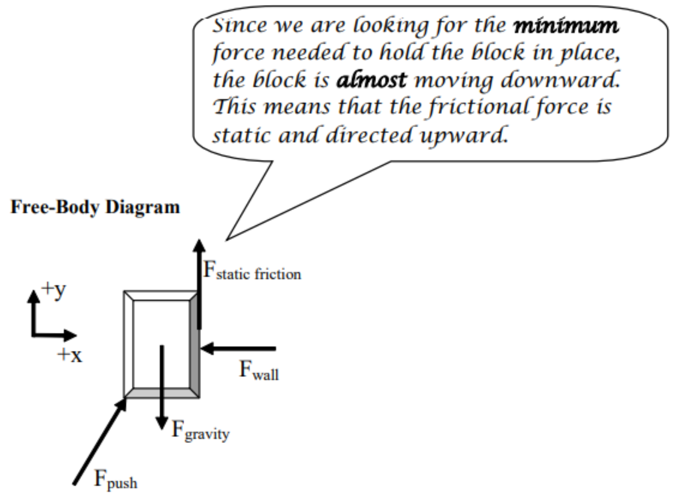
Mathematical Analysis[24]

x-direction y-direction

The person at right exerts the minimum force necessary to support the 100 kg block. He pushes at an angle of 50^0 above the horizontal. The coefficient of friction is (0.6,0.5).



Free-Body Diagram



Mathematical Analysis

x-direction

$$\begin{aligned} F_{\text{push}} \cos 50 - F_{\text{wall}} &= 100(0) \\ F_{\text{wall}} &= 0.643 F_{\text{push}} \end{aligned}$$

y-direction

$$F_{\text{push}} \sin 50 - (100)(9.8) + F_{\text{staticfriction}} = 100(0)$$

> **Note**
>
> *Since the block is not moving, both accelerations equal zero.*

friction

$$\begin{aligned} F_{\text{staticfriction}} &\leq 0.6 F_{\text{wall}} \\ F_{\text{staticfriction}} &\leq 0.6\left(0.643 F_{\text{push}}\right) \\ F_{\text{staticfriction}} &\leq 0.386 F_{\text{push}} \end{aligned}$$

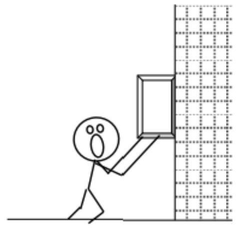
Since we are looking for the minimum force needed to support the block, the block is almost moving. This means that static friction is at its maximum value. Therefore we can substitute $0.386F_{push}$ into the y-equation and solve.

$$\begin{aligned} 0.766 F_{\text{push}} - 980 + 0.386 F_{\text{push}} &= 0 \\ 1.152 F_{\text{push}} &= 980 \\ F_{\text{push,min}} &= 851 \text{ N} \end{aligned}$$

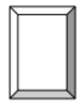
If we were looking for the maximum force, everything would be the same except for the direction of the frictional force (it would be downward). Therefore, the maximum force would be

$$\begin{aligned} 0.766 F_{\text{push}} - 980 - 0.386 F_{\text{push}} &= 0 \\ 0.380 F_{\text{push}} &= 980 \\ F_{\text{push, max}} &= 2579 \text{ N} \end{aligned}$$

The person at right exerts an 850 N force on the 90 kg block at an angle of 55^0 above the horizontal. The coefficient of friction is (0.6,0.5).



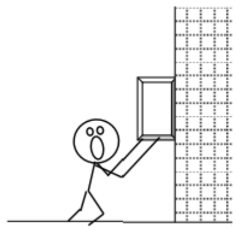
Free-Body Diagram



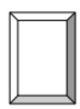
Mathematical Analysis[25]

x-direction y-direction

The person at right exerts a 620 N force on the 70 kg block at an angle of 40^0 above the horizontal. The coefficient of friction is (0.5,0.4).



Free-Body Diagram



Mathematical Analysis[26]

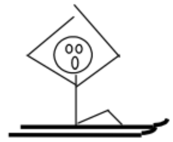
x-direction y-direction

A boy pulls a 30 kg sled, including the mass of his kid brother, along ice. The boy pulls on the tow rope, oriented at 60^0 above horizontal, with a force of 110 N until his kid brother begins to cry. Like clockwork, his brother always cries upon reaching a speed of 2.0 m/s. The frictional coefficient is (0.20,0.15).

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Free-Body Diagram



Mathematical Analysis[27]

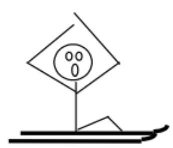
x-direction y-direction

Starting from rest, a girl can pull a sled, carrying her kid brother, 20 m in 8 s. The girl pulls on the tow rope, oriented at 30^0 above horizontal, with a force of 90 N. The frictional coefficient is (0.15,0.10).

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Free-Body Diagram



Mathematical Analysis[28]

x-direction y-direction

A 60 kg skier starts from rest at the top of a 100 m, 25^0 slope. He doesn't push with his poles because he's afraid of going too fast. The frictional coefficient is (0.10,0.05).

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =

Event 1:	Event 2:
a_{12x} =	
a_{12y} =	

Free-Body Diagram

Mathematical Analysis[29]

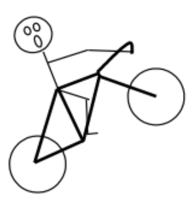
x-direction y-direction

A 100 kg bicycle and rider initially move at 16 m/s up a 15^0 hill. The rider slams on the brakes and skids to rest. The coefficient of friction is (0.8,0.7).

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Free-Body Diagram



Mathematical Analysis[30]

x-direction y-direction

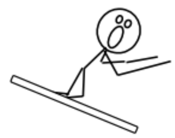
A 70 kg snow-boarder starts from rest at the top of a 270 m, 20^0 slope. She reaches the bottom of the slope in 14.5 seconds.

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =

Event 1:	Event 2:
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Free-Body Diagram

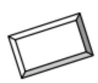


Mathematical Analysis[31]

x-direction y-direction

At a UPS distribution center, a 60 kg crate is at rest on an 8° ramp. A worker applies the minimum horizontal force needed to push the crate up the ramp. The coefficient of friction between the crate and the ramp is (0.3, 0.2).

Free-Body Diagram



Mathematical Analysis[32]

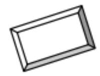
x-direction y-direction

At a UPS distribution center, a 40 kg crate is sliding down an 8° ramp at 3 m/s. A worker applies a horizontal force to the crate and brings the crate to rest in 1.5 s. The coefficient of friction between the crate and the ramp is (0.3, 0.2).

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

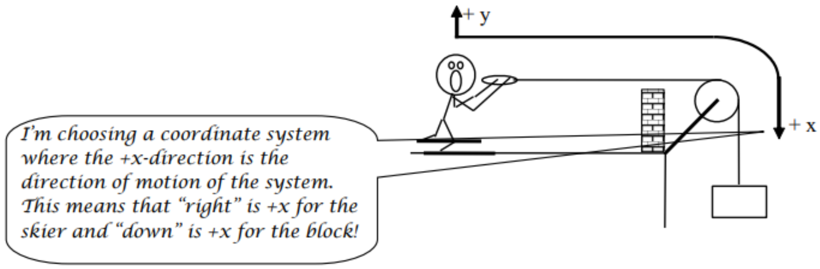
Free-Body Diagram



Mathematical Analysis[33]

x-direction y-direction

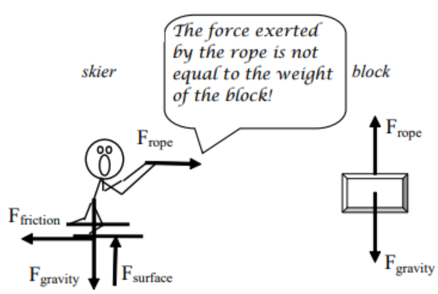
The device at right guarantees all the excitement of skiing without the need for hills. The skier begins from rest 35 m from the brick wall. The block has a mass of 50 kg and the skier has a mass of 75 kg. The coefficient of friction is (0.15,0.13).



Motion Information

Object: Skier	
Event 1: Block is released	Event 2: Smashes into wall
$t_1 = 0$ s	$t_2 =$
$r_{1x} = 0$ m	$r_{2x} = 35$ m
$r_{1y} = 0$ m	$r_{2y} = 0$ m
$v_{1x} = 0$ m/s	$v_{2x} =$
$v_{1y} = 0$ m/s	$v_{2y} = 0$ m
$a_{12x} =$	
$a_{12y} = 0$ m/s^2	

Free-Body Diagram



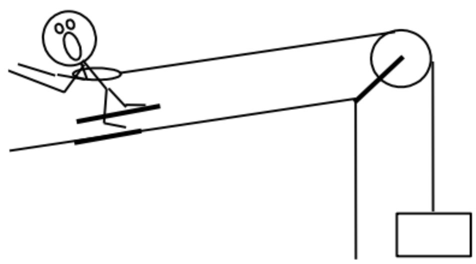
Mathematical Analysis

skier	block

skier	block
y-direction $F_{\text{surface}} - 75(9.8) = 75(0)$ $F_{\text{surface}} = 735 \text{ N}$ *friction* $F_{\text{kineticfriction}} = 0.13F_{\text{surface}}$ $F_{\text{kinetifriction}} = 0.13(735)$ $F_{\text{kineticfriction}} = 95.6 \text{ N}$ *x-direction* $F_{\text{rope}} - F_{\text{friction}} = 75a_{\text{skier}}$ $F_{\text{rope}} - 95.6 = 75a_{\text{skier}}$	*x-direction* $-F_{\text{rope}} + 50(9.8) = 50a_{\text{block}}$ $-F_{\text{rope}} + 490 = 50a_{\text{block}}$ In the coordinate system above, $a_{\text{skier}} = a_{\text{block}} = a$, so the two x-equations can be added to yield: $F_{\text{rope}} - 95.6 = 75a$ $-F_{\text{rope}} + 490 = 50a$ $394.4 = 125a$ $a = 3.16 \text{ m/s}^2$

Kinematics can be used to find v_2 = 14.9 m/s and t_2 = 4.71 s.

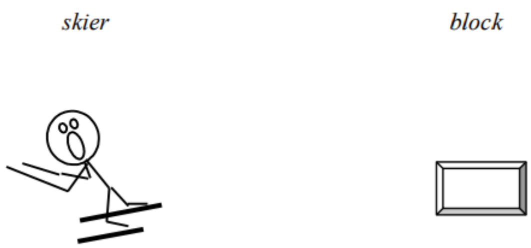
The device at right allows novices to ski downhill at reduced speeds. The block has a mass of 15 kg and the skier has a mass of 70 kg. The coefficient of friction is (0.05,0.04). The skier starts from rest at the top of a 30 m, 20^0 slope.



Motion Information

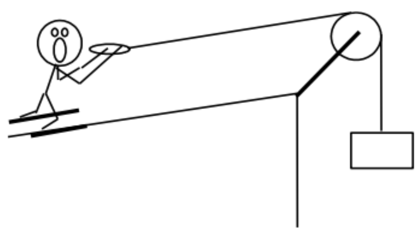
Object:	
Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Free-Body Diagram



Mathematical Analysis[34]

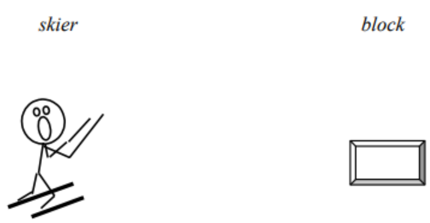
The device at right allows you to ski uphill. The ballast block has a mass of 30 kg and the skier has a mass of 60 kg. The coefficient of friction is (0.07,0.06). The skier starts from rest at the bottom of a 30 m, 20^0 slope.



Motion Information

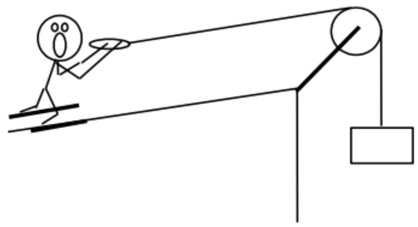
Object:	
Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
v_{1x} =	v_{2x} =
v_{1y} =	v_{2y} =
a_{12x} =	
a_{12y} =	

Free-Body Diagram



Mathematical Analysis[35]

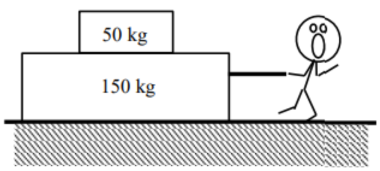
The device at right ***may*** *allow you to ski uphill (or it* ***may*** *allow you to ski downhill backward). The ballast block has a mass of 20 kg and the skier has a mass of 70 kg. The coefficient of friction is (0.1,0.09). The ramp is inclined at 20^0 above horizontal.*



Free-Body Diagram

Mathematical Analysis[36]

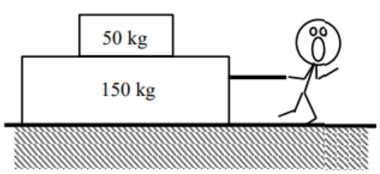
The strange man at right wants to pull the two blocks to the other side of the room in as short a time as possible. However, he doesn't want the top block to slide relative to the bottom block. The coefficient of friction between the bottom block and the floor is (0.25,0.20) and the coefficient of friction between the top block and the bottom block is (0.30,0.25). The blocks start from rest.



Free-Body Diagram

Mathematical Analysis[37]

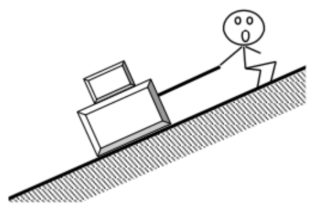
The strange man at right wants to pull the two blocks to the other side of the room in as short a time as possible by pulling on the top block. However, he doesn't want the top block to slide relative to the bottom block. The coefficient of friction between the bottom block and the floor is (0.10,0.05) and the coefficient of friction between the top block and the bottom block is (0.60,0.50). The blocks start from rest.



Free-Body Diagram

Mathematical Analysis[38]

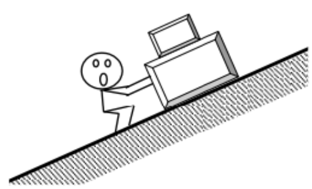
The strange man at right wants to pull the two blocks to the top of the hill in as short a time as possible. However, he doesn't want the top block to slide relative to the bottom block. The coefficient of friction between the 150 kg bottom block and the floor is (0.25,0.20) and the coefficient of friction between the 50 kg top block and the bottom block is (0.30,0.25). The hill is inclined at 150 above horizontal. The blocks start from rest.



Free-Body Diagram

Mathematical Analysis[39]

The strange man at right applies the minimum force necessary to not get crushed by the bottom block. (The top block may or may not crush him.) The coefficient of friction between the 150 kg bottom block and the floor is (0.25,0.20) and the coefficient of friction between the 50 kg top block and the bottom block is (0.40,0.35). The hill is inclined at 20^0 above horizontal. The blocks are initially at rest.



Free-Body Diagram

Mathematical Analysis[40]

You should know the story by now. You push on a garbage-day couch at an angle θ below horizontal. Determine the minimum force (F_{min}) needed to move the couch as a function of the couch's mass (m), θ, the appropriate coefficient of friction, and g.

Free-Body Diagram

Mathematical Analysis

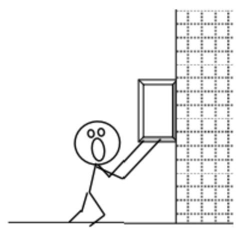
Questions

If $\theta = 0°$, what should F_{min} equal? Does your function agree with this observation?

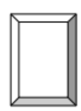
If $\theta = 90°$, what should F_{min} equal? Does your function agree with this observation?

If $m = \infty$, what should F_{min} equal? Does your function agree with this observation?

The man at right exerts a force on the block at an angle θ above horizontal. Determine the minimum force (F_{min}) needed to begin to slide the block up the wall as a function of the block's mass (m), θ, the appropriate coefficient of friction, and g.



Free-Body Diagram



Mathematical Analysis

Questions

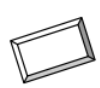
If $\theta = 90°$, what should F_{min} equal? Does your function agree with this observation?

If $\theta = 0°$, what should F_{min} equal? Does your function agree with this observation?

Below what angle θ is it impossible to slide the block up the wall?

A crate is held at rest on a ramp inclined at $theta$ from horizontal. Determine the minimum force (F_{min}), applied parallel to the incline, needed to prevent the crate from sliding down the ramp as a function of the crate's mass (m), $theta$, the appropriate coefficient of friction, and g.

Free-Body Diagram



Mathematical Analysis

Questions

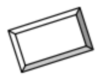
If $\theta = 0°$, what should F_{min} equal? Does your function agree with this observation?

If $\theta = 90°$, what should F_{min} equal? Does your function agree with this observation?

If $m = \infty$, what should F_{min} equal? Does your function agree with this observation?

A crate is held at rest on a ramp inclined at $theta$ from horizontal. Determine the maximum force (F_{max}), applied horizontally, before the crate begins to move as a function of the crate's mass (m), $theta$, the appropriate coefficient of friction, and g.

Free-Body Diagram



Mathematical Analysis

Questions

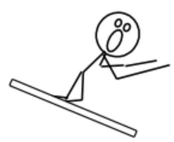
If $m = \infty$, what should F_{max} equal? Does your function agree with this observation?

If $g = \infty$, what should F_{min} equal? Does your function agree with this observation?

If $\theta = 0°$, what should F_{max} equal? Does your function agree with this observation?

A skier of mass m starts from rest at the top of a ski run of incline θ. Determine the minimum angle (θ_{min}) such that the skier will begin to slide down the slope without pushing off as a function of m, the appropriate coefficient of friction, and g.

Free-Body Diagram



Mathematical Analysis

Questions

If $\mu = 0$, what should θ_{min} equal? Does your function agree with this observation?

If $g = 0\ m/s^2$, what should θ_{min} equal? Does your function agree with this observation?

If m was twice as large, what should θ_{min} equal? Does your function agree with this observation?

3.3: Conservation Laws

Concepts and Principles

The Impulse-Momentum Relation

Just like the kinematic relations and Newton's second law, the impulse-momentum relation is independently valid in any member of a set of perpendicular directions. Thus, we will typically apply the impulse-momentum relation in its component forms:

$$mv_{xi} + \Sigma\left(F_x(\Delta t)\right) = mv_{xf}$$
$$mv_{y_i} + \Sigma\left(F_y(\Delta t)\right) = mv_{y_f}$$
$$mv_{zi} + \Sigma\left(F_z(\Delta t)\right) = mv_{zf}$$

The Work-Energy Relation

From Model 1, our expression for the Work-Energy Relation, with gravitational potential energy terms, is:

$$\frac{1}{2}mv_i^2 + mgh_i + \Sigma(|F||\Delta r|\cos\phi) = \frac{1}{2}m{v_f}^2 + mgh_f$$

It's very important to remember that the work-energy relation is a *scalar* equation, meaning it cannot be broken into components and "solved" separately in the x-, y-, and z-directions. This is even more important to remember now that we are working in multiple dimensions. This observation results in two important points:

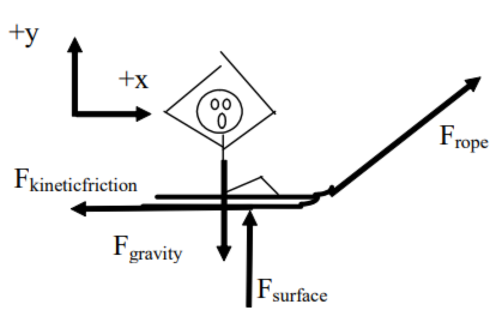
- The work-energy relation involves the actual initial and final velocities, *not* their components. The kinetic energy of an object does not depend on the direction of travel of the object.
- In the expression for work, $|F||\Delta r|\cos\phi$, the product of the *magnitude* of the force and the *magnitude* of the displacement is multiplied by $\cos\phi$, where ϕ is defined to be the angle between the applied force and the displacement of the object. If the force and displacement are in the same direction $\phi = 0^\circ$, and the work is positive (the object gains energy). If the force and displacement are in the opposite direction ϕ = 180°, and the work is negative (the object loses energy). If the force and displacement are perpendicular, no work is done. Note that the actual directions of the force and the displacement are unimportant, only their directions *relative to each other* affect the work.

Analysis Tools

Applying the Impulse-Momentum Relation to a Single Object

Let's investigate the following scenario:

> *A boy pulls a 30 kg sled, including the mass of his kid brother, along ice. The boy pulls on the tow rope, oriented at 60^0 above horizontal, with a force of 110 N until his kid brother begins to cry. Like clockwork, his brother always cries upon reaching a speed of 2.0 m/s. The frictional coefficient is (0.20,0.15).*



To apply the impulse-momentum relation, you must clearly specify the initial and final events at which you will tabulate the momentum. For example:

Event 1: The instant before the sled begins to move.	Event 2: The instant the sled reaches 2.0 m/s.

Event 1: The instant before the sled begins to move.	Event 2: The instant the sled reaches 2.0 m/s.
$P_{1x} = 0$	$P_{2x} = 30\ (2.0) = 60$
$P_{1y} = 0$	$P_{2y} = 0$
$J_{12x} = 110\cos 60(\Delta t) - F_{kf}(\Delta t)$	
$J_{12y} = 110\sin 60(\Delta t) - (30)(9.8)(\Delta t) + F_{\text{surface}}(\Delta t)$	

Applying impulse-momentum separately in the x- and y-directions yields:

$$\begin{array}{ll} \underline{\mathbf{x-direction}} & \underline{\mathbf{y-direction}} \\ P_1 + J_{12} = P_2 & P_1 + J_{12} = P_2 \\ 0 + 110\cos 60(\Delta t) - F_{kf}(\Delta t) = 60 & 0 + 110\sin 60(\Delta t) - (30)(9.8)(\Delta t) + F_{\text{surface}}(\Delta t) = 0 \\ 55(\Delta t) - 0.15F_{\text{surface}}(\Delta t) = 60 & 95(\Delta t) - 294(\Delta t) + F_{\text{surface}}(\Delta t) = 0 \\ & F_{\text{surface}} = 199N \end{array}$$

Substituting the value for the force of the surface into the x-equation,

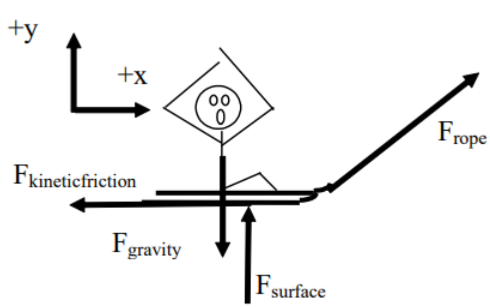
$$\begin{aligned} &55(\Delta t) - 0.15(199)(\Delta t) = 60 \\ &55(\Delta t) - 30(\Delta t) = 60 \\ &25(\Delta t) = 60 \\ &\Delta t = 2.4s \end{aligned}$$

The kid brother begins to cry after only 2.4 s.

Applying the Work-Energy Relation to a Single Object

What will the work-energy relation tell us about the same scenario?

> A boy pulls a 30 kg sled, including the mass of his kid brother, along ice. The boy pulls on the tow rope, oriented at 60^0 above horizontal, with a force of 110 N until his kid brother begins to cry. Like clockwork, his brother always cries upon reaching a speed of 2.0 m/s. The frictional coefficient is (0.20,0.15).



To apply the work-energy relation, you must clearly specify the initial and final events at which you will tabulate the energy. For example:

Event 1: The instant before the sled begins to move.	Event 2: The instant the sled reaches 2.0 m/s
$KE_1 = 0$	$KE_2 = \frac{1}{2}\,30(2.0)^2 = 60$
$GE_1 = 0$	$GE_2 = 0$
$W_{12} = 110(\Delta r)\cos 60 + F_{\text{surface}}(\Delta r)\cos 90 + F_{\text{kineticfriction}}(\Delta r)\cos 180$	

Applying the work-energy relation yields:

$$KE_1 + GE_1 + W_{12} = KE_2 + GE_2$$
$$0 + 0 + 110(\Delta r)\cos 60 + F_{\text{surface}}(\Delta r)\cos 90 + F_{\text{kineticfriction}}(\Delta r)\cos 180 = 60 + 0$$
$$55(\Delta r) - 0.15 F_{\text{surface}}(\Delta r) = 60$$

Notice that the force of the surface does no work, the force of the rope does positive work, and the force of friction does negative work. Each of these terms should make sense if you remember that work is the transfer of energy into (positive) or out of (negative) the system of interest. Also recall that in this form of the work-energy relation we conceptualize gravity as a source of potential energy, not as a force that does work.

Using the result for the force of the surface determined in the first example, $F_{surface}$ = 199 N, gives:

$$55(\Delta r) - 0.15 F_{\text{surface}}(\Delta r) = 60$$
$$55(\Delta r) - 0.15(199)(\Delta r) = 60$$
$$55(\Delta r) - 30(\Delta r) = 60$$
$$25(\Delta r) = 60$$
$$\Delta r = 2.4m$$

The kid brother begins to cry after traveling 2.4 m.

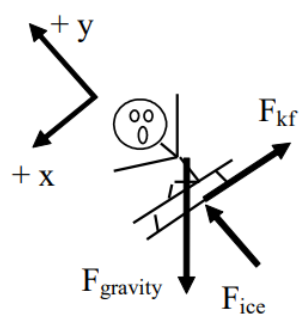
Applying Work-Energy with Gravitational Potential Energy

Let’s use the work-energy relation, with gravitational potential energy terms, to analyze the following scenario:

> *A 30 kg child on his 15 kg sled slides down his parents’ 10 m long, 15^0 above horizontal driveway after an ice storm. The coefficient of friction between the sled and the driveway is (0.10, 0.08).*

To calculate the gravitational energy terms let the bottom of the driveway be zero and up positive. The coordinate system used to calculate gravitational energy does not in general have to be the same as the system you use for the rest of the problem. In fact, since the workenergy relation is a scalar equation, the other portions of the equation should not depend on your choice of coordinate system at all!

Event 1: The instant before the sled begins to move.	Event 2: The instant the sled reaches the bottom of the driveway
Event 2: The instant the sled reaches the bottom of the driveway	$KE_2 = \frac{1}{2} 45v^2$
GE_1 = 45 (9.8) (10 sin 15^0) = 1140	GE_2 = 0
$W_{12} = F_{ice}(10)\cos 90 + F_{\text{kineticfriction}}(10)\cos 180$	



$$KE_1 + GE_1 + W_{12} = KE_2 + GE_2$$
$$0 + 1140 + F_{ice}(10)\cos 90 + F_{kf}(10)\cos 180 = \frac{1}{2}45v_2^2 + 0$$
$$1140 - 10F_{kf} = 22.5v_2^2$$

Note:

- The only forces that *could* do work are the force of the ice and the force of friction, since the action of the force of gravity is already incorporated into the gravitational potential energy terms.
- The heights in the gravitational potential energy function were measured from the bottom of the driveway, with the positive direction as upward, as required. Notice that the initial height is not the same as the length of the driveway. Since the driveway is 10 m long, at an angle of 15°, the height of the top of the driveway relative to the bottom is (10 m) sin 15°. The height at the bottom of the driveway is defined to be 0 m.

To finish the analysis we need to determine the kinetic frictional force. Since this depends on the force of the ice, apply Newton's Second Law in the y-direction and find:

$$\Sigma F = ma$$
$$+F_{ice} - F_{\text{gravity}} \cos 15 = 45(0)$$
$$F_{ice} - (45)(9.8) \cos 15 = 0$$
$$F_{ice} = 426N$$

$$F_{kf} = \mu_s F_{ice}$$
$$F_{kf} = (0.08)(426)$$
$$F_{kf} = 34N$$

Plugging this value into the work-energy relation yields:

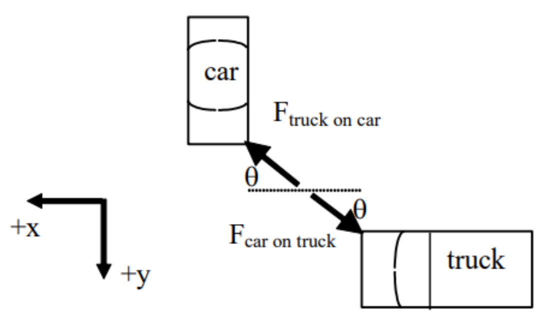
$$1140 - 10(34) = 22.5v_2^2$$
$$1140 - 340 = 22.5v_2^2$$
$$800 = 22.5v_2^2$$
$$v_2 = 5.96m/s$$

A Two-Dimensional Collision

Let's try a two-dimensional collision.

> At a busy intersection, an impatient driver heading south runs a red-light and collides with a delivery truck originally moving at 15 m/s west. The vehicles become entangled and the skid marks from the wreckage are at 22° south of west. The auto mass is 755 kg and the truck mass is 1250 kg.

Partial free-body diagrams (top view) for both the car and the truck during the time interval during the collision are shown below.



These are only partial free-body diagrams because:

- Forces perpendicular to the earth's surface (the force of gravity and the force of the road) are not shown.
- During a collision, the force *between* the colliding objects is normally much greater in magnitude than any other forces acting on the objects. Therefore we will often ignore the other forces acting on colliding objects for the duration of a collision. This approximation is termed the impulse approximation. Under the *impulse approximation*, the frictional forces between the car and truck and the road are ignored.

Also note that the direction of the force acting between the car and truck is unknown. The angle θ is not determined from the situation description.

Event 1: The instant before the car and truck collide	Event 2: The instant they reach a common velocity
Object: Car	
$P_{1x} = 0$	$P_{2x} = 755\ (v_2 \cos 22^o) = 700\ v_2$
$P_{1y} = 755\ v_{car}$	$P_{2y} = 755\ (v_2 \sin 22^o) = 283\ v_2$
$J_{12x} = F_{truckoncar} \cos\theta(\Delta t)$	
$J_{12y} = -F_{truckoncar} \sin\theta(\Delta t)$	
Object: Truck	
$P_{1x} = 1250\ (15) = 18750$	$P_{2x} = 1250\ (v_2 \cos 22^o) = 1160\ v_2$
$P_{1y} = 0$	$P_{2y} = 1250\ (v_2 \sin 22^o) = 468\ v_2$
$J_{12x} = -F_{carontruck} \cos\theta(\Delta t)$	
$J_{12y} = F_{carontruck} \sin\theta(\Delta t)$	

Applying the impulse-momentum relation to the car and truck yields:

Car

$$\begin{array}{ll} \underline{\mathbf{x-direction}} & \underline{\mathbf{y-direction}} \\ P_1 + J_{12} = P_2 & P_1 + J_{12} = P_2 \\ 0 + F_{\text{truckoncar}} \cos\theta(\Delta t) = 700 v_2 & 755 v_{\text{car}} - F_{\text{truckoncar}} \sin\theta(\Delta t) = 283 v_2 \end{array}$$

Truck

$$\begin{array}{ll} \underline{\mathbf{x-direction}} & \underline{\mathbf{y-direction}} \\ P_1 + J_{12} = P_2 & P_1 + J_{12} = P_2 \\ 18750 - F_{\text{carontruck}} \cos\theta(\Delta t) = 1160 v_2 & 0 + F_{\text{carontruck}} \sin\theta(\Delta t) = 468 v_2 \end{array}$$

Since the magnitude of the force on the car due to the truck and the force on the truck due to the car are equal, when the x-equations for the car and truck are added, the impulses cancel!

$$\begin{array}{l} \underline{\mathbf{x-direction}} \\ 0 + 18750 = 700 v_2 + 1160 v_2 \\ 18750 = 1860 v_2 \\ v_2 = 10.1 \text{ m/s} \end{array}$$

This is the speed of the wreckage immediately after the collision. Note that this is exactly the same equation we would have written if we had considered the *system* of the car and truck right from the start. Try it!

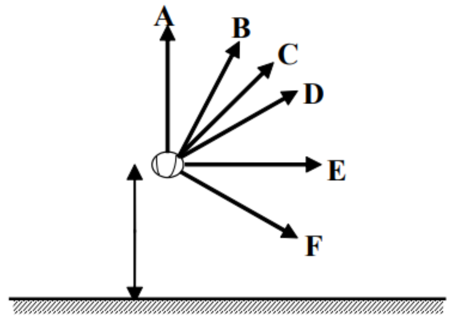
Adding the two y-equations yields:

$$\begin{array}{l} \underline{\mathbf{x-direction}} \\ 755 v_{\text{car}} + 0 = 283 v_2 + 468 v_2 \\ 755 v_{\text{car}} = 751(10.1) \\ v_{\text{car}} = 10.0 \text{ m/s} \end{array}$$

This is the speed of the car immediately before colliding with the truck.

Activities

Below are six different directions in which a baseball can be thrown. In all cases the baseball is thrown at the same initial speed from the same height above the ground. Assume the effects of air resistance are negligible.



a. Rank these baseballs on the basis of their horizontal speed the instant before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

b. Rank these baseballs on the basis of their vertical speed the instant before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

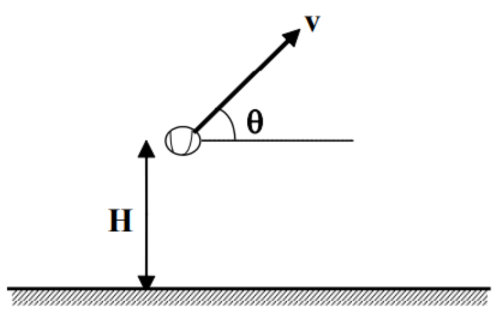
c. Rank these baseballs on the basis of their speed the instant before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

Below are six different directions and heights from which a baseball can be thrown. In all cases the baseball is thrown at the same speed, v. Assume the effects of air resistance are negligible.



	H	θ
A	10 m	30^{0}
B	10 m	0^{0}
C	10 m	60^{0}
D	20 m	0^{0}
E	15 m	45^{0}
F	5 m	90^{0}

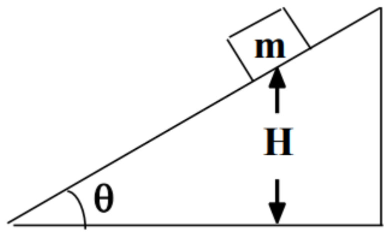
Rank these baseballs on the basis of their speed the instant before they hit the ground.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

A crate is released from rest along an inclined surface. The mass of the crate and the angle of the incline vary. The frictional coefficients between the crates and the surfaces are identical and so small that the effect of friction is negligible. All crates are released from the same vertical height, H, above the bottom of the incline.



	m	θ
A	10 kg	30^0
B	20 kg	15^0
C	10 kg	60^0
D	20 kg	60^0
E	15 kg	45^0
F	5 kg	85^0

a. Rank these scenarios on the basis of the kinetic energy of the crate the instant it reaches the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

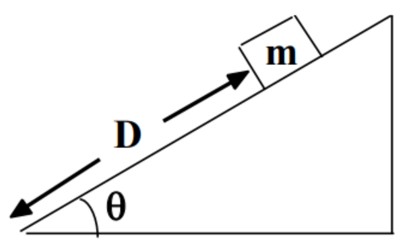
b. Rank these scenarios on the basis of the speed of the crate the instant it reaches the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

A crate is released from rest along an inclined surface. The mass of the crate and the angle of the incline vary. The frictional coefficients between the crates and the surfaces are identical and so small that the effect of friction is negligible. All crates are released from the same distance, D, along the incline.



	m	θ
A	10 kg	30^0
B	20 kg	15^0
C	10 kg	60^0
D	20 kg	60^0
E	15 kg	45^0
F	5 kg	85^0

a. Rank these scenarios on the basis of the kinetic energy of the crate the instant it reaches the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

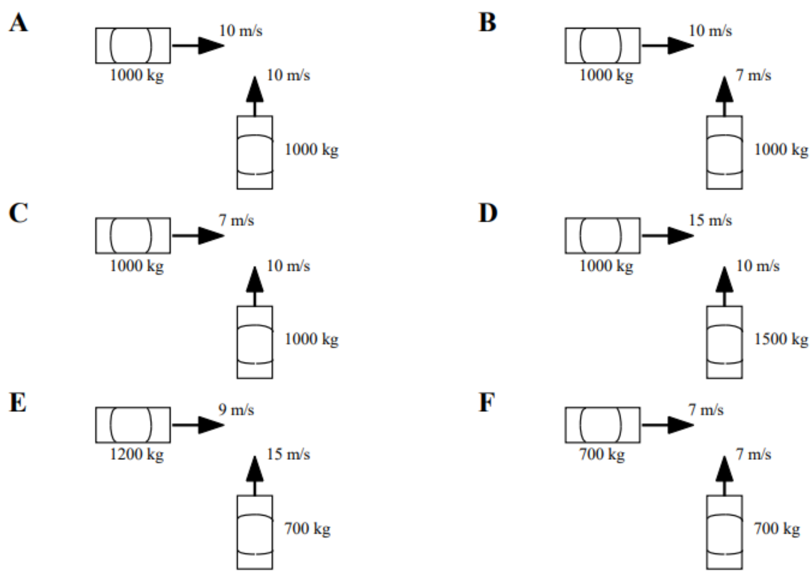
b. Rank these scenarios on the basis of the speed of the crate the instant it reaches the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

Below are bird's-eye views of six automobile crashes an instant before they occur. The automobiles have different masses and velocities. All automobiles will remain joined together after the impact and skid to rest. Rank these automobile crashes on the basis of the angle at which the wreckage skids. Let 0^0 be the angle oriented directly toward the right and measure angles counterclockwise from 0^0.

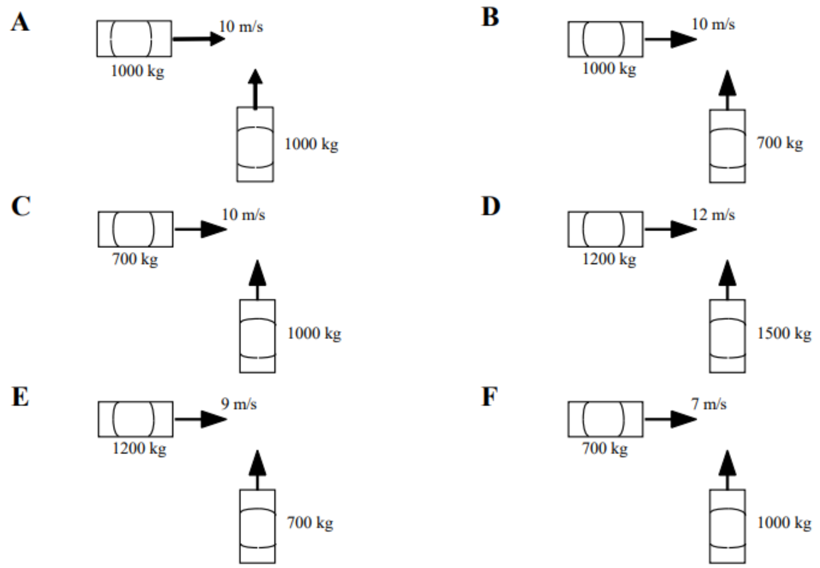


Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

Below are bird's-eye views of six automobile crashes an instant before they occur. The automobiles have different masses and velocities. All automobiles will remain joined together after the impact and skid to rest at the same angle, as measured from a line oriented directly toward the right. Rank these scenarios on the basis of the initial speed of the auto traveling toward the top of the page.



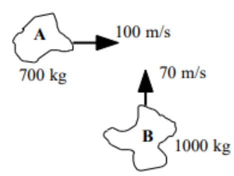
Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

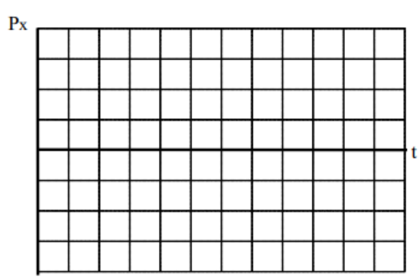
_____ The ranking cannot be determined based on the information provided.

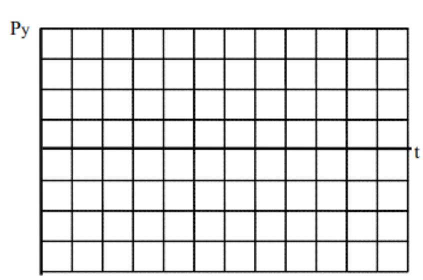
Explain the reason for your rankings:

For each of the collisions illustrated below, sketch a graph of the momentum of asteroid A, the momentum of asteroid B, and the total momentum in the system of the two asteroids. Sketch the horizontal and vertical momentum separately. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

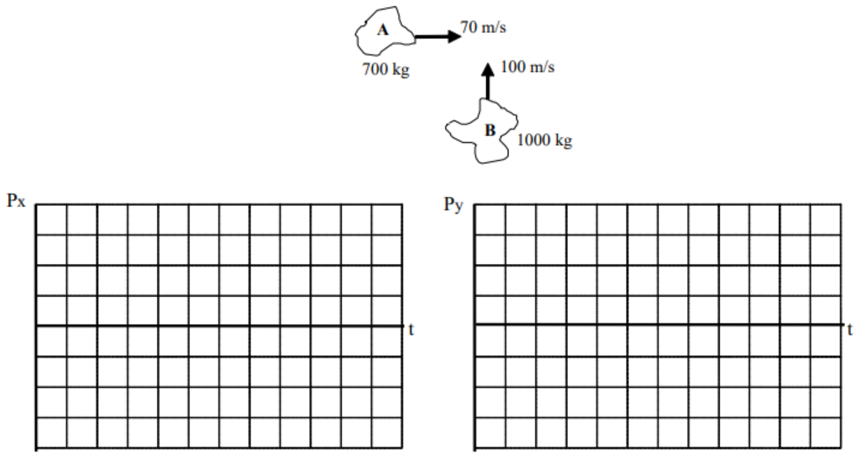
a. The two asteroids remain joined together after the collision.





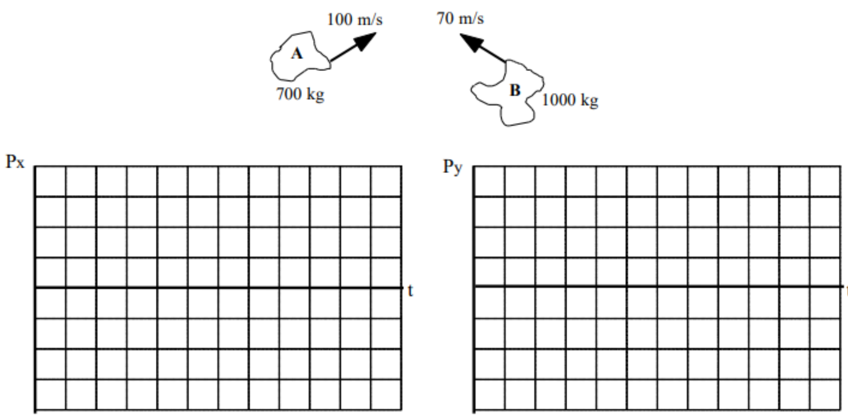


b. The two asteroids remain joined together after the collision.

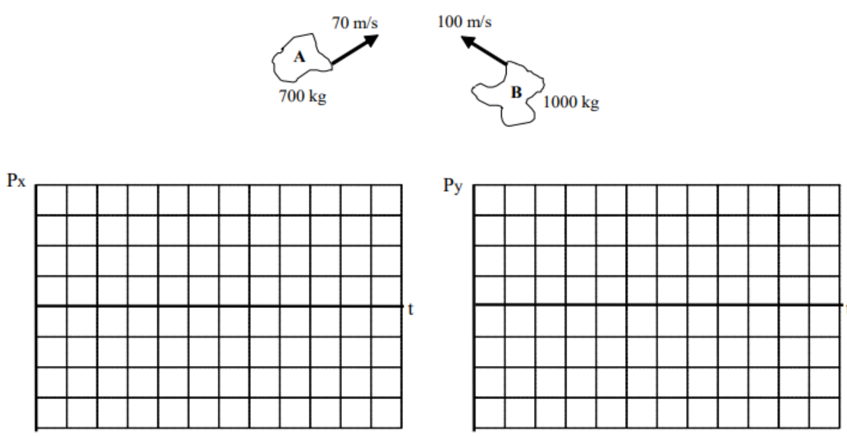


For each of the collisions illustrated below, sketch a graph of the momentum of asteroid A, the momentum of asteroid B, and the total momentum in the system of the two asteroids. Sketch the horizontal and vertical momentum separately. Begin your graph before the collision takes place and continue it after the collision is over. The asteroids' initial velocities are both oriented at the same angle from horizontal. Use a consistent scale on all graphs.

a. The two asteroids remain joined together after the collision.

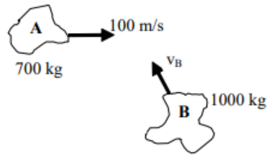


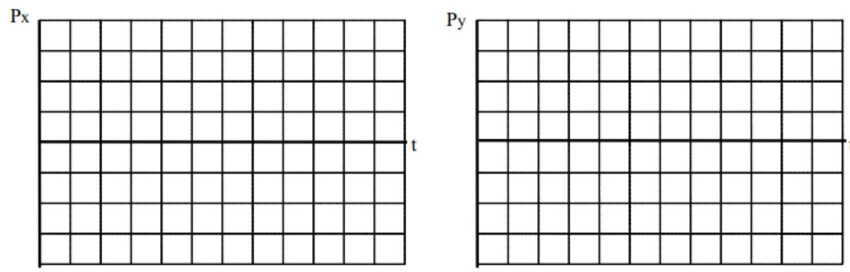
b. The two asteroids remain joined together after the collision.



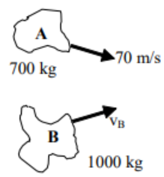
For each of the collisions illustrated below, sketch a graph of the momentum of asteroid A, the momentum of asteroid B, and the total momentum in the system of the two asteroids. Sketch the horizontal and vertical momentum separately. Begin your graph before the collision takes place and continue it after the collision is over.

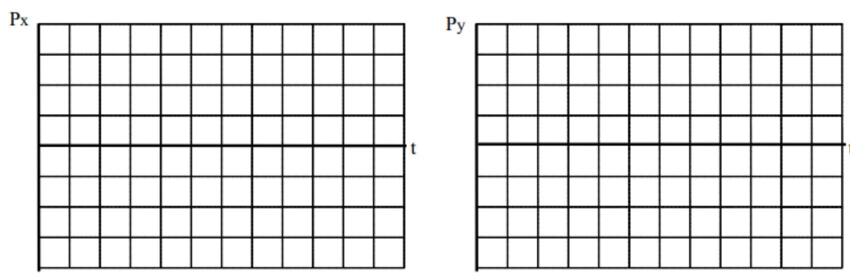
a. The two asteroids remain joined together after the collision and move directly toward the top of the page.





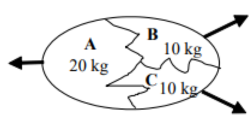
b. The two asteroids remain joined together after the collision and move directly toward the right. The asteroids' initial velocities are both oriented at the same angle from horizontal.

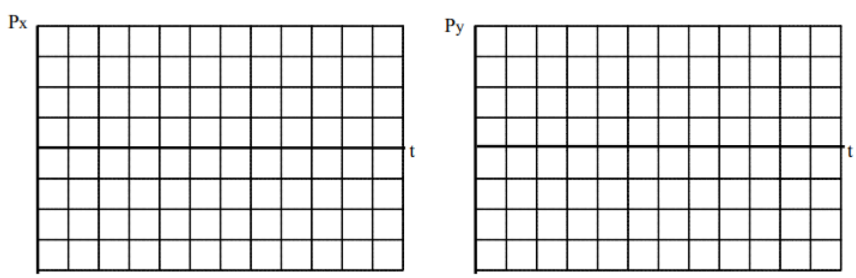




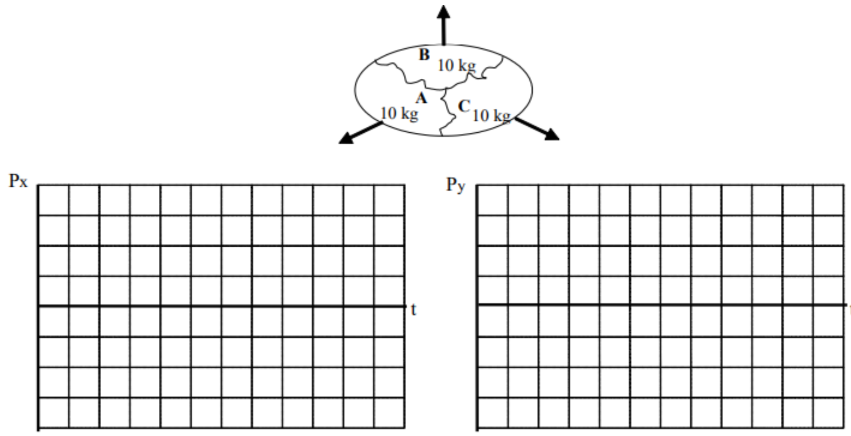
For each of the explosions illustrated below, sketch a graph of the momentum of fragment A, B, and C, and the total momentum in the system of the three asteroids. Sketch the horizontal and vertical momentum separately. Begin your graph before the explosion takes place and continue it as the fragments move apart. The exploding egg is initially at rest.

a. Fragment A moves horizontally and fragments B and C move at the same angle from horizontal after the explosion.



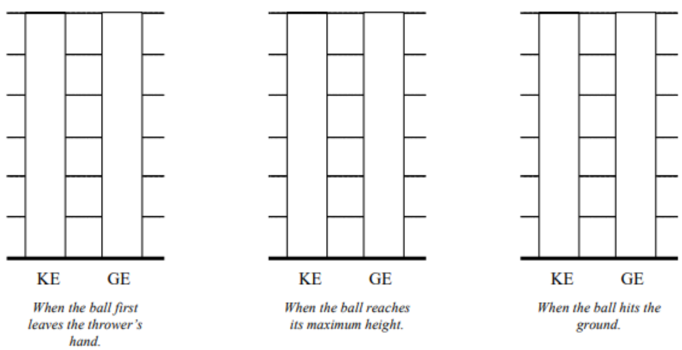


b. Fragment B moves vertically and fragments A and C move at the same angle from vertical after the explosion.

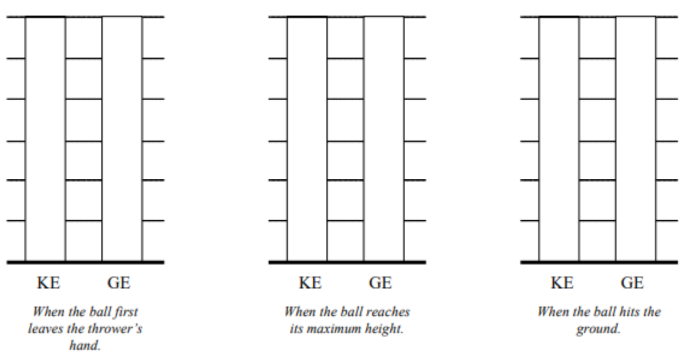


For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of the object at each of the events listed. Use a consistent scale throughout both motions. Set ground level as the zero-point of gravitational potential energy

a. A baseball is thrown at 30 m/s at an angle of 30° above horizontal over level ground.

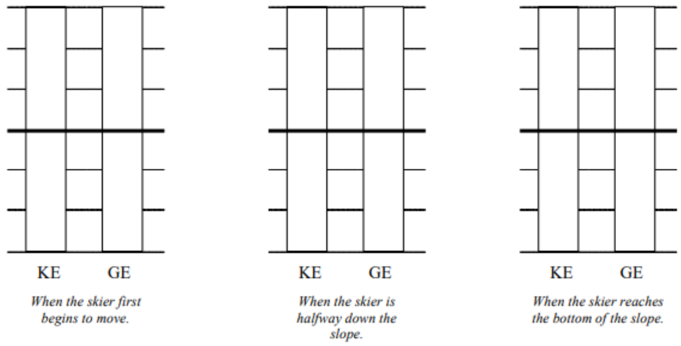


b. A baseball is thrown at 30 m/s at an angle of 60° above horizontal over level ground.

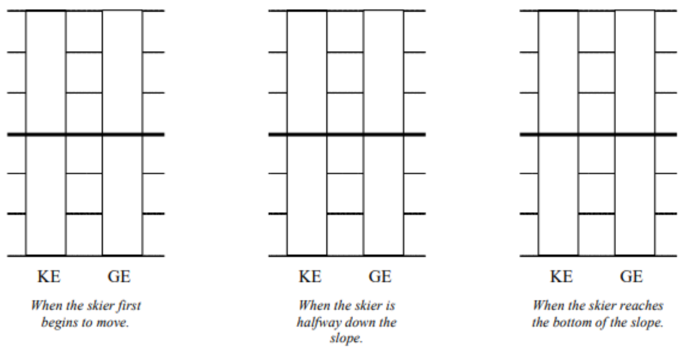


For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of the object at each of the events listed. Use a consistent scale throughout both motions

a. A 60 kg skier starts from rest at the top of a 100 m, 25^0 slope. He doesn't push with his poles because he's afraid of going too fast. Set the bottom of the slope as the zero-point of gravitational potential energy

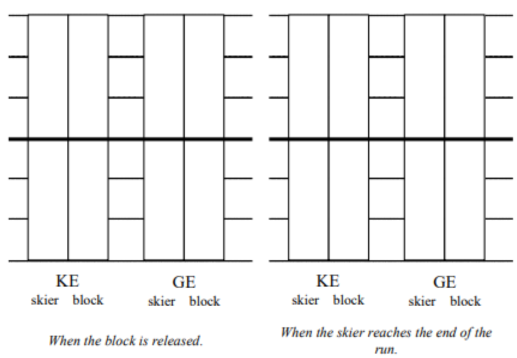


b. A 60 kg skier starts from rest at the top of a 100 m, 25^0 slope. He doesn't push with his poles because he's afraid of going too fast. Set the top of the slope as the zero-point of gravitational potential energy

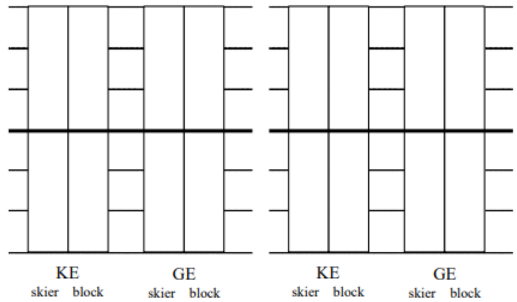


For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of each object at each of the events listed. Use a consistent scale throughout both motions. Set the initial positions of the skier and block as the zero-points of gravitational potential energy.

a. In a horizontal skiing device, the skier begins from rest 35 m from the end of the skiing run. The ballast block has a mass of 50 kg and the skier has a mass of 75 kg. The coefficient of friction is extremely small.



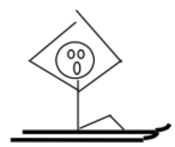
b. In an inclined skiing device, the skier begins from rest 35 m from the end of the 20^0 above horizontal inclined skiing run. The ballast block has a mass of 50 kg and the skier has a mass of 75 kg. The coefficient of friction is extremely small.



When the block is released. *When the skier reaches the end of the run.*

A girl pulls a 35 kg sled, including the mass of her kid sister, along ice. The girl pulls on the tow rope, oriented at 40^0 above horizontal, with a force of 120 N until her sister begins to cry. Like clockwork, her sister always cries upon reaching a speed of 3.0 m/s. The frictional coefficient is (0.10,0.08).

Free-Body Diagram



Mathematical Analysis[41]

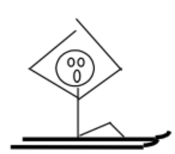
Event 1:	Event 2:
P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	
J_{12y} =	
K_{E1} =	K_{E2} =
G_{E1} =	G_{E2} =
W_{12} =	

a. How far has the sled moved before the little sister begins to cry?

b. What is the elapsed time before the little sister begins to cry?

A boy pulls a 30 kg sled, including the mass of his kid brother, along ice. The boy pulls on the tow rope, oriented at 60° above horizontal, with a force of 110 N for 3.0 s. At the end of the 3.0 s pull, his kid brother begins to cry. The frictional coefficient is (0.20,0.15).

Free-Body Diagram



Mathematical Analysis[42]

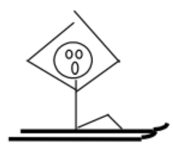
Event 1:	Event 2:
P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	
J_{12y} =	
K_{E1} =	K_{E2} =
G_{E1} =	G_{E2} =
W_{12} =	

a. How fast is the sled moving before the little brother begins to cry?

b. How far has the sled moved before the little brother begins to cry?

Starting from rest, a girl can pull a sled, carrying her kid brother, 20 m in 8 s. The girl pulls on the tow rope, oriented at 30^0 above horizontal, with a force of 90 N. The frictional coefficient is (0.15,0.10).

Free-Body Diagram

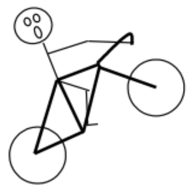


Mathematical Analysis[43]

Event 1:	Event 2:
P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	
J_{12y} =	
K_{E1} =	K_{E2} =
G_{E1} =	G_{E2} =
W_{12} =	

A 100 kg bicycle and rider initially move at 16 m/s up a 15^0 hill. The rider slams on the brakes and skids to rest. The coefficient of friction is (0.8,0.7).

Free-Body Diagram



Mathematical Analysis[44]

Event 1:	Event 2:
P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	
J_{12y} =	
K_{E1} =	K_{E2} =
G_{E1} =	G_{E2} =
W_{12} =	

a. How far does the bike skid?

b. What is the elapsed time before the bike stops skidding?

A 60 kg skier starts from rest at the top of a 100 m, 25^0 slope. He doesn't push with his poles because he's afraid of going too fast. The frictional coefficient is (0.10,0.05).

Free-Body Diagram

Mathematical Analysis[45]

Event 1:	Event 2:
P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	
J_{12y} =	
K_{E1} =	K_{E2} =
G_{E1} =	G_{E2} =
W_{12} =	

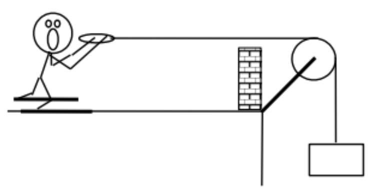
A 70 kg snowboarder starts from rest at the top of a 270 m, 20^0 slope. At the bottom of the hill she's moving at 33 m/s.

Free-Body Diagram

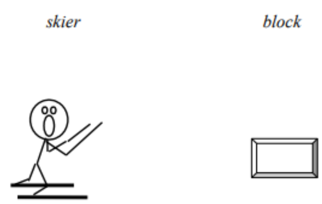
Mathematical Analysis[46]

Event 1:	Event 2:
P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	
J_{12y} =	
K_{E1} =	K_{E2} =
G_{E1} =	G_{E2} =
W_{12} =	

The device at right guarantees all the excitement of skiing without the need for hills. The skier begins from rest 35 m from the brick wall. The block has a mass of 50 kg and the skier has a mass of 75 kg. The coefficient of friction is (0.15,0.13).



Free-Body Diagram

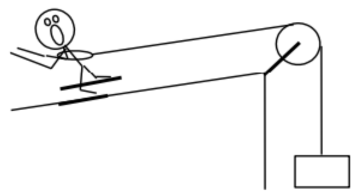


Mathematical Analysis[47]

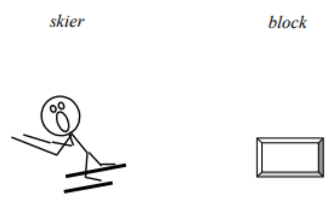
Event 1:	Event 2:
Object:	
K_{E1} =	K_{E2} =
G_{E1} =	G_{E2} =
W12 =	
Object:	
K_{E1} =	K_{E2} =

Event 1:	Event 2:
G_{E1} =	G_{E2} =
W12 =	

The device at right allows novices to ski downhill at reduced speeds. The block has a mass of 10 kg and the skier has a mass of 80 kg. The coefficient of friction is (0.08,0.07). The skier starts from rest at the top of a 30 m, 20^0 slope.



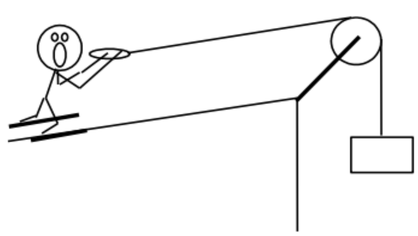
Free-Body Diagram



Mathematical Analysis[48]

Event 1:	Event 2:
Object:	
K_{E1} =	K_{E2} =
G_{E1} =	G_{E2} =
W12 =	
Object:	
K_{E1} =	K_{E2} =
G_{E1} =	G_{E2} =
W12 =	

The device at right allows you to ski uphill (until you smash into the pulley). The ballast block has a mass of 60 kg and the skier has a mass of 70 kg. The coefficient of friction is (0.1,0.09). The ramp is inclined at 200 above horizontal and the pulley is 45 m away.



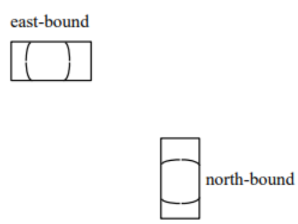
Free-Body Diagram

Mathematical Analysis[49]

Event 1:	Event 2:
Object:	
K_{E1} =	K_{E2} =
G_{E1} =	G_{E2} =
W12 =	
Object:	
K_{E1} =	K_{E2} =
G_{E1} =	G_{E2} =
W12 =	

Two identical 800 kg automobiles, one moving east at 10 m/s and the other moving north at 15 m/s, collide. After the collision they remain joined together and move with a common velocity.

Free-Body Diagrams



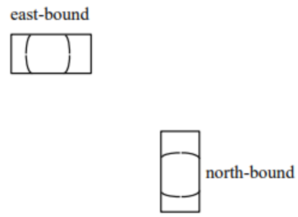
Mathematical Analysis[50]

Event 1:	Event 2:
Object:	
P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	
J_{12y} =	
Object:	
P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	

J_{12y} =	

Two identical 750 kg automobiles, one moving east at 10 m/s and the other moving north, collide. After the collision they remain joined together and move with a common velocity. The wreckage skids at 30^0 north of east.

Free-Body Diagrams

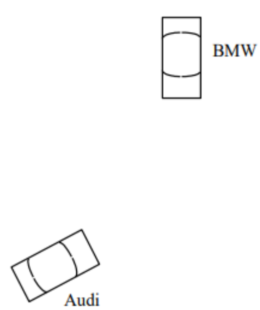


Mathematical Analysis[51]

Event 1:	Event 2:
Object:	
P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	
J_{12y} =	
Object:	
P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	
J_{12y} =	

In a demolition derby, a 700 kg Audi is traveling at 15 m/s 30^0 north of east. An 800 kg BMW is traveling at 5.0 m/s south. They collide. After the collision, the Audi is redirected to 100 north of east and the BMW is redirected to 400 east of south.

Free-Body Diagrams



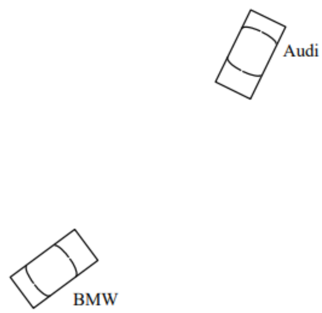
Mathematical Analysis[52]

Event 1:	Event 2:
Object:	

P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	
J_{12y} =	
Object:	
P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	
J_{12y} =	

In a demolition derby, a 600 kg Audi is traveling at 15 m/s 30^0 west of south. A 700 kg BMW is traveling at 10 m/s 400 north of east. They collide. After the collision, the Audi is redirected to 200 north of west and the BMW is redirected to 50^0 south of east.

Free-Body Diagrams

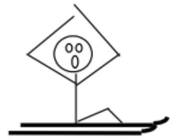


Mathematical Analysis[52]

Event 1:	Event 2:
Object:	
P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	
J_{12y} =	
Object:	
P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	
J_{12y} =	

A boy pulls an initially stationary sled of mass m (including the mass of the strange neighborhood kid riding the sled) along a level surface. He exerts a force of magnitude F at an angle of θ above the horizontal. Determine the velocity (v) of the sled as a function of the distance pulled (d), the appropriate coefficient of friction between the sled and the surface, m, F, θ, and g.

Free-Body Diagram



Mathematical Analysis

Event 1:	Event 2:
K_{E1} =	K_{E2} =
G_{E1} =	G_{E2} =
W12 =	

Questions

If d = 0 m, what should v equal? Does your function agree with this observation?

If m = 0 kg, what should v equal? Does your function agree with this observation?

If F = 0 N, what should v equal? Does your function agree with this observation?

A skier of mass m starts from rest at the top of a slope of length D inclined at θ above horizontal. She does not push with her poles. Determine the speed of the skier at the bottom of the slope (v) as a function of the appropriate coefficient of friction between the skies and the snow, D, m, θ, and g.

Free-Body Diagram

Mathematical Analysis

Event 1:	Event 2:
K_{E1} =	K_{E2} =
G_{E1} =	G_{E2} =
W12 =	

Questions

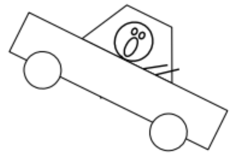
If g = 0 m/s^2, what should v equal? Does your function agree with this observation?

If θ = 0°, what should v equal? Does your function agree with this observation?

If the D is doubled, what will happen to v?

The driver of an automobile of mass m, traveling down an incline of angle θ, suddenly sees an obstacle blocking her lane. Ignoring her reaction time, determine the time elapsed (T) before the car skids to a stop as a function of the initial velocity (v), the appropriate coefficient of friction between the tires and the road, m, θ and g.

Free-Body Diagram



Mathematical Analysis

Event 1:	Event 2:
P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	
J_{12y} =	

Questions

If μ = 0, what should T equal? Does your function agree with this observation?

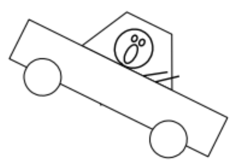
If m is doubled, what will happen to T?

If v is doubled, what will happen to T?

Is there a maximum angle above which the car will not stop? If so, determine an expression for this angle.

The driver of an automobile of mass m, traveling down an incline of angle $theta$, suddenly sees an obstacle blocking her lane. Ignoring her reaction time, determine the distance (D) the car skids before stopping as a function of the initial velocity (v), the appropriate coefficient of friction between the tires and the road, m, θ and g

Free-Body Diagram



Mathematical Analysis

Event 1:	Event 2:
K_{E1} =	K_{E2} =
G_{E1} =	G_{E2} =
W_{12} =	

Questions

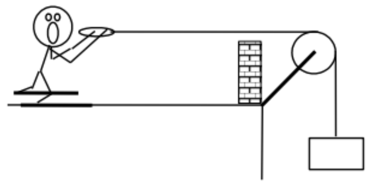
If μ = 0, what should T equal? Does your function agree with this observation?

If m is doubled, what will happen to T?

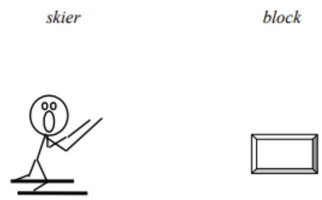
If v is doubled, what will happen to T?

Is there a maximum angle above which the car will not stop? If so, determine an expression for this angle.

The device at right guarantees all the excitement of skiing without the need for hills. A skier of mass m begins from rest a distance D from the brick wall. Determine the speed of the skier as he crashes into the wall (v) as a function of the appropriate coefficient of friction between the skies and the snow, the mass of the block (M), D, m, and g.



Free-Body Diagram



Mathematical Analysis[47]

Event 1:	Event 2:
Object:	
K_{E1} =	K_{E2} =
G_{E1} =	G_{E2} =
W12 =	
Object:	
K_{E1} =	K_{E2} =
G_{E1} =	G_{E2} =
W12 =	

Questions

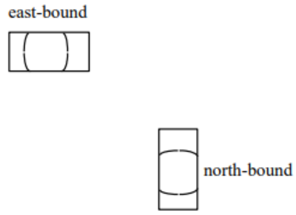
If M = 0, what should v equal? Does your function agree with this observation?

If D is doubled, what will happen to v?

What is the minimum block mass needed for the skier to move?

Two identical automobiles, one moving east at v_E and the other moving north at v_N, collide. After the collision they remain joined together and move with a common velocity. Determine the angle at which the wreckage skids (θ), measured counterclockwise from east, as a function of v_E and v_N.

Free-Body Diagrams



Mathematical Analysis[51]

Event 1:	Event 2:
Object:	
P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	
J_{12y} =	
Object:	
P_{1x} =	P_{2x} =
P_{1y} =	P_{2y} =
J_{12x} =	
J_{12y} =	

Questions

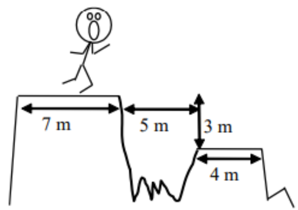
If $v_E = \infty$, what should θ equal? Does your function agree with this observation?

If $v_E = v_N$, what should θ equal? Does your function agree with this observation?

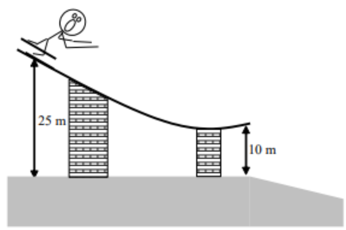
3.4: Summary Problems and Projects

Summary Problems

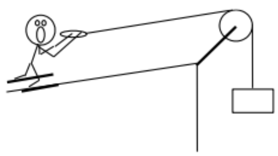
1. A physicist/baseball player hits a ball to very deep centerfield. It's going, going, ... Video analysis shows that the ball leaves his bat at a speed of 35 m/s, an angle of 40° above horizontal, and exactly 1.2 m above the ground. The centerfield fence is 400 feet (120 m) away and 4.0 m high. Starting at the top of the fence is a grassy hillside sloping away from the field at 10° above horizontal. The game is being played in the Vacu-Dome (ignore air resistance). Where on the field or on the hill or on the wall does the ball strike?
2. The strange man at right wants to leap safely down to the plateau below, without landing on the jagged rocks. He can accelerate at 2 m/s^2. At what range of angles, if any, can he jump to land safely? Assume he doesn't slow down as he redirects his velocity on lift-off and he doesn't slide off the plateau on landing.



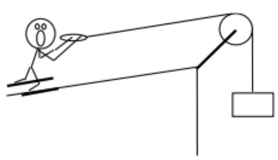
3. In the ski jump, a 55 kg skier (with *really* slippery skis) starts from rest at the top of a 25 m starting ramp declined at 40°. At the end of the ramp is a sharp bend that allows the skier to leave the ramp oriented at 10° above horizontal. At this point, the skier is 10 m directly above the mountainside. The mountainside slopes downward at 10° below horizontal. Where (along the mountainside) does the skier land?



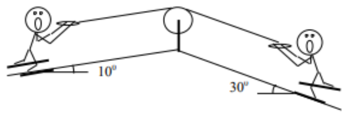
4. If possible, design an uphill skiing device (select the necessary incline angle, ballast mass, and coefficient of friction) such that people with mass less than 50 kg move up the incline while people with mass more than 75 kg move down the incline. Do not use an incline steeper than 15°, or absurd values for the coefficient of friction.



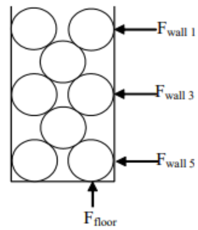
5. In the uphill skiing device at right, the block is released from rest 15 m above the ground. However, the 45 kg skier starts 22 m from the tip of the incline. If possible, select the necessary incline angle, ballast mass, and coefficient of friction such that skier travels all of the way to the tip of the incline before sliding back down. Do not use an incline steeper than 15°, or absurd values for the coefficient of friction.



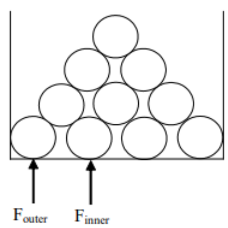
6. The tandem skiing device at right is very rare, and nearly useless. In fact, for the situation pictured neither skier moves. The skier on the left slope has a mass of 65 kg, and the frictional coefficient between her skis and the slope is (0.15, 0.10). The frictional coefficient between the skier on the right slope's skis and the slope is (0.25, 0.20). Find the range of possible values for the mass of the skier on the right slope.



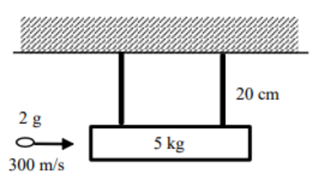
7. In a tall container of water, the largest forces are exerted on the bottom surface of the container. In addition, the force exerted on the wall of the container increases with depth below the surface. These intuitive facts are not necessarily true with a tall container filled with a granular material, such as sand. To see this, examine the hypothetical container at right. Each of the spheres represent a grain of sand, with mass M and radius R. The width of the container is 5 R.



   a. The forces exerted by the sand grains on the walls of the container and on the floor of the container are labeled at right. (The forces in the diagram are the Newton's Third Law partners to these forces. The numbering system refers to the sand level, with the top level labeled 1, the next layer underneath 2, etc. Thus, the sand does not touch the wall at level 2 or 4.) To verify the statement above, show that the largest of these four forces is not F_{floor}, and that the largest wall force is not $F_{wall\ 5}$. Ignore any frictional effects between the sand grains.
   b. Find the minimum width of the container such that F_{floor} is less than or equal to $F_{wall\ 3}$. For narrower containers, the force exerted on the floor is indeed larger than any of the forces exerted on the walls.
8. If you were buried underneath a sandpile (I hope this fate never befalls you), the force pressing down on you would not necessarily be a maximum under the center of the pile. To see this, examine the hypothetical sandpile at right. Each of the spheres represent a grain of sand, with mass M and radius R. The width of the container is 9.5 R.



   a. The forces exerted by the sand grains on the floor of the container are labeled F_{outer} (for the outer grains) and F_{inner} for the inner grains. (The forces in the diagram are the Newton's third law partners to these forces.) To verify the statement above, show that F_{inner} is no larger than F_{outer}.
   b. The configuration shown requires a frictional force to act between the bottom surface of the container and the sand grains. Determine this frictional force on each grain of the bottom level.
9. Two identical 750 kg automobiles, one moving east and the other moving north, collide. After the collision they remain joined together and move with a common velocity. The wreckage skids 43 m at 300 north of east. The coefficient of friction is (0.6, 0.5). How fast were the automobiles traveling before the collision?
10. A bullet is fired at a suspended, stationary, slab of wood. The bullet remains embedded in the wood after the collision, and the slab swings out to the right. To what maximum angle from vertical does the slab swing?



Projects

Hawking Goes Zero-g

Note

Based on: “Hawking goes zero-G,” www.msnbc.msn.com/id/18334489/

World-famous physicist Stephen Hawking experienced eight rounds of weightlessness on Thursday during a better-than-expected airplane flight that he saw as the first step toward a trip in space.

"It was amazing," Hawking told reporters afterward, using his well-known computerized voice. "The zeroG part was wonderful, and the high-G part was no problem. I could have gone on and on.

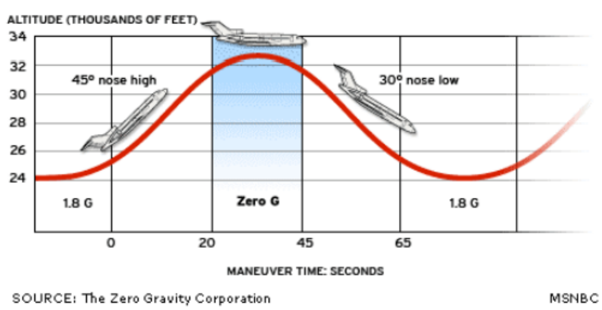
Hawking's host, Zero Gravity Corp. co-founder and chief executive officer Peter Diamandis, said before the flight that he'd claim success if Hawking had just a single half-minute float in weightlessness aboard the company's specially modified Boeing 727 jet. It turned out that Hawking took eight turns with ease

"He would have flown more if we let him," said Noah McMahon, one of Hawking's coaches as well as Zero Gravity's chief marketing officer. "He was all smiles all the time."

Zero Gravity had originally planned to bring Hawking back to NASA's Shuttle Landing Facility here after six ups-and-downs. "We negotiated and agreed to do two more," Diamandis told reporters jokingly. After the landing, Hawking's fellow fliers gave him a round of applause.

Hawking is one of the globe's best-known scientists — not only because of his best-selling works on the mysteries of black holes and the origins of the universe, but also because of his increasing disability due to a degenerative nerve disease known as amyotrophic lateral sclerosis. He is almost completely paralyzed and can communicate only via facial gestures and a gesture-controlled computer system.

The zero-G airplane flights conducted by Zero Gravity, as well as government space programs, duplicate the sense of weightlessness that astronauts feel in orbit for about a half-minute at a time. The plane follows a parabolic, roller-coaster course through the sky. During the top half of each parabola, airplane passengers feel as if they're in free-fall — but when the plane pulls out of its descent, they feel more than the normal pull of gravity.



The graphic above illustrates the idealized flight plan for a zero-g flight. Assuming the jet enters the shaded “zero-g” portion of the flight at the illustrated 45° nose high position, exits at 30° nose low, and the passengers experience “zero-g” for the entire shaded region, complete a kinematic description of the plane’s motion. Based on your analysis, is the illustrated change in altitude consistent with the above assumptions?

Steep Streets[3]

Note

Based on: “Here in Beechview,” postgazette.com, January 30, 2005

Despite the twin "Do Not Enter" signs at its midpoint, Canton Avenue isn't a one-way street. It's a no-way street. No way you're going to drive up it. Not this time of year, when it's covered with ice and snow. This Beechview byway is way too steep–even to plow.

It's the steepest street in this hilly town and, probably, the region, with a grade of 37 percent -- that is, rising 37 feet per 100 feet of run. So confirm records from the city Department of Engineering and Construction. Canton could be the steepest street anywhere. Figures can be fuzzy, but the best San Francisco can do are grades of 31.5 percent. The world's steepest claim is made by Baldwin Street in Dunedin, New Zealand, but its steepest part, according to the town's own Web site, is only 35 percent. Could Pittsburgh have a world record hidden in the trees high above Banksville Road?

Whatever the case, over a distance that would be about two blocks in a normal city, Canton goes from almost flat to free-fall. However, that doesn't stop people from trying to drive up it. Not many make it. So says Dolores Love, who lives at the very base of Canton, on the other side of Coast Avenue (also in the Top 20, with a grade of nearly 18 percent). The view from her living room is straight up Canton, and she and her husband, Ed, are in the process of improving that vista while improving their house.

"Part of the reason we put these big windows in is so we can watch the goofballs try to drive up the hill," she says. "I'm serious. ... I live for it."

Sure enough, the first big snow this month brought the first fools attempting to conquer Canton. She watched the car climb about a third of the way up, then slide back and over a curb, where it teetered like something out of a cartoon. Last Sunday, the Loves were heading out to watch the Steelers game when they witnessed the spectacle of three guys in a four-wheel-drive pickup trying to muscle up the street -- because, you know, it's there. The truck slid all the way back and into a tiny guardrail next door.

The handful of households up top are supposed to drive out on Hampshire Avenue (a mere 23 percent grade). The two families who live on the sheer stretch know to park down on Coast when it snows and walk up the steps that are Canton's sidewalk. This adds a new dimension to unloading groceries and other tasks.

The five steepest streets in Pittsburgh

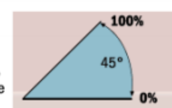
Street	Neighborhood	Grade
Canton Avenue	Beechview	37%
Dornbush Street	Homewood	32%
Boustead Street	Beechview	29%
East Woodford Avenue	Carrick	27%
Rialto Street	Troy Hill	25%

Some of the other steep streets

- Cutler Street on North Side
- Capital Avenue in Brookline
- Tesla Street in Hazelwood
- Potomac Avenue in Banksville
- Hampshire Avenue in Beechview

Making the grade

The percentage expresses the steepness of the hill as the rise over run expressed as a percentage. A 0% grade is perfectly flat and a 100% grade is 45 degrees from the horizontal.



Note

[3] The street was recently featured in an Audi commercial available at: https://www.youtube.com/watch?v=DH7h...layer_embedded

a. What is the minimum coefficient of friction necessary to safely park your car on Canton Avenue? Is this a static or kinetic coefficient?

b. What is the minimum coefficient of friction necessary to drive your car up Canton Avenue? Is this a static or kinetic coefficient?

c. The steep part of Canton Avenue is 64 m long. Assume you begin from rest at the top of Canton Avenue and allow your car to roll halfway down (without applying the brakes). What minimum coefficient of friction would be necessary to safely stop your car at the base of the hill?

Selected Answers

[1] t2 = 1.86 s

[2] v1 = 11.2 m/s

[3] δrx = 8.3 m

[4] v1 = 3.28 m/s

[5] v1 = 31.4 m/s

[6] The ball hits people in the stands behind home plate. (It sails 10.7 m above home plate.)

[7] θ = 44°

[8] θ = 20.1°, 68.8°

[9] θ = 18.6°, 77.2°

[10] No.

[11] Yes, as long as the cannon is set to 16.4°.

[12] t_2 = 0.56 s

[13] t_2 = 3.22 s

[14] t_2 = 3.22 s

[15] 335 m from home

[16] t_2 = 40 s a = 0.15 m/s^2

[17] 52.6 m from ship

[18] 25.7 m from ship

[19] t_2 = 33.5 s θ = 188° from line initially between ship and man

[20] $\mu_s \geq 0.256$

[21] a = 0 m/s^2

[22] a = 0.94 m/s^2

[23] m = 65.2 kg

[24] F_{sf} = 56 N

[25] F_{sf} = 186 N up

[26] a = 1.39 m/s^2 down

[27] a = 0.84 m/s^2

[28] m = 51.4 kg

[29] t_2 = 7.4 s

[30] r_{2x} = 14 m

[31] μ_k = 0.085

[32] F = 270 N

[33] F = 55.9 N

[34] a = 0.73 m/s^2

[35] a = 0.66 m/s^2

[36] a = 0 m/s^2

[37] F_{max} = 980 N

[38] F_{max} = 359 N

[39] F_{max} = 947 N

[40] F_{min} = 210 N

[41] a. r_2 = 2.23 m b. t_2 = 1.49 s

[42] a. v_2 = 2.52 m/s b. r_2 = 3.78 m

[43] m = 51.4 kg

[44] a. r2 = 14 m b. t2 = 1.75 s

[45] v2 = 27 m/s

[46] μ = 0.14

[47] v2 = 14.9 m/s

[48] v2 = 8.89 m/s

[49] v2 = 14.3 m/s

[50] θ = 56°

[51] v2 = 5.8 m/s

[52] v2audi = 12.8 m/s

[53] v2audi = 4.86 m/s

3.5: Selected Answers

[1] t2 = 1.86 s

[2] v1 = 11.2 m/s

[3] δrx = 8.3 m

[4] v1 = 3.28 m/s

[5] v1 = 31.4 m/s

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[9] θ = 18.6°, 77.2°

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[12] t2 = 0.56 s

[13] t2 = 3.22 s

[14] t2 = 3.22 s

[15] 335 m from home

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[18] 25.7 m from ship

[19] t2 = 33.5 s θ = 188° from line initially between ship and man

[20] μs ≥ 0.256

[21] a = 0 m/s^2

[22] a = 0.94 m/s^2

[23] m = 65.2 kg

[24] Fsf = 56 N

[25] Fsf = 186 N up

[26] a = 1.39 m/s^2 down

[27] a = 0.84 m/s^2

[28] m = 51.4 kg

[29] t2 = 7.4 s

[30] r2x = 14 m

[31] μk = 0.085

[32] F = 270 N

[33] F = 55.9 N

[34] a = 0.73 m/s^2

[35] a = 0.66 m/s^2

[36] a = 0 m/s^2

[37] Fmax = 980 N

[38] Fmax = 359 N

[39] Fmax = 947 N

[40] Fmin = 210 N

[41] a. r2 = 2.23 m b. t2 = 1.49 s

[42] a. v2 = 2.52 m/s b. r2 = 3.78 m

[43] m = 51.4 kg

[44] a. r2 = 14 m b. t2 = 1.75 s

[45] v2 = 27 m/s

[46] μ = 0.14

[47] v2 = 14.9 m/s

[48] v2 = 8.89 m/s

[49] v2 = 14.3 m/s

[50] θ = 56°

[51] v2 = 5.8 m/s

[52] v2audi = 12.8 m/s

[53] v2audi = 4.86 m/s

CHAPTER OVERVIEW

4: Model 3 - The Particle Model

4.0: Model Specifics

For our third pass through the study of mechanics, we will finally allow forces to vary in both magnitude and direction. Peeling away this approximation requires us to utilize a new mathematical tool, calculus. We will still, however, require the following two approximations.

The object's size and shape are unimportant.

The object is classical.

4.1: Kinematics

Concepts and Principles

If the forces acting on an object are not constant, then the acceleration of the object is not constant. To analyze the kinematics of an object undergoing non-constant acceleration requires the use of calculus. By re-examining our original definitions, valid in the limit of very small time intervals during which the acceleration is *approximately* constant, the relationships between position, velocity, and acceleration can be constructed in terms of the derivative.

Position

Let r(t) be the location of the object at every time, t, in the time interval of interest.

Velocity

Our original definition of velocity,

$$v=\frac{r_{\text{final}}-r_{\text{initial}}}{t_{\text{final}}-t_{\text{initial}}}$$

remains valid if the acceleration is constant. The acceleration will always be constant in the limit of infinitesimally small time intervals. By the fundamental definition of calculus, the above expression, in the limit of infinitesimally small time intervals, becomes the derivative of the position function. Thus, v(t), the velocity of the object at every time, t, is defined to be the derivative of the position function, r(t).

$$v(t)=\frac{dr(t)}{dt}$$

Acceleration

Our original definition of acceleration,

$$a=\frac{v_{\text{final}}-v_{\text{initial}}}{t_{\text{final}}-t_{\text{initial}}}$$

also remains valid only in the limit of infinitesimally small time intervals. By the fundamental definition of calculus, the above expression, in the limit of infinitesimally small time intervals, becomes the derivative of the velocity function. Thus, a(t), the acceleration of the object at every time, t, is defined to be the derivative of the velocity function, v(t).

$$a(t)=\frac{dv(t)}{dt}$$

Thus, if the position of the object is known as a function of time, the velocity and acceleration functions can be constructed through differentiation of r(t). On the other hand, if the acceleration of the object is known as a function of time, the velocity and position functions can be constructed through anti-differentiation, or integration, of a(t).

An important distinction, however, is that when integrating a(t) to form v(t), an arbitrary constant will be introduced into the expression for v(t). This constant can often be determined from knowledge of the object's velocity at some specific instant in time. Another integration, to form r(t), will introduce an additional arbitrary constant that can often be determined from knowledge of the object's position at some specific instant in time.

In closing, please remember that the kinematic relations that have been used throughout this course were derived assuming a constant acceleration. If the acceleration is not constant, those relations are *incorrect*, and the correct kinematic relationships *must* be determined through direct integration and differentiation.

Analysis Tools

Using the Calculus

Investigate the scenario described below.

> *In a 100 m dash, detailed video analysis indicates that a particular sprinter's speed can be modeled as a quadratic function of time at the beginning of the race, reaching 10.6 m/s in 2.70 s, and as a decreasing linear function of time for the remainder of*

the race. She finished the race in 12.6 seconds.

To analyze this situation, we should first carefully determine and define the sequence of events that take place. At each of these instants, let's tabulate what we know about the motion.

Event 1: The race begins	Event 2: She reaches 10.6 m/s	Event 3: She crosses the finish line
t_1 = 0 s	t_2 = 2.7 s	t_3 = 12.6 s
r_1 = 0 m	r_2 =	r_3 = 100 m
v_1 = 0 m/s	v_2 = 10.6 m/s	v_3 =
a_1 =	a_2 =	a_3 =

Since the acceleration is no longer necessarily constant between instants of interest, it is no longer useful to speak of a_{12} or a_{23}. The acceleration, like the position and the velocity, is a *function*. What the table represents is the value of that *function* at specific instants of time.

Between event 1 and 2, the sprinter's velocity can be modeled by a generic quadratic function of time[1], or

$$v(t) = At^2 + B$$

Note

[1] When I say a "quadratic function of time", I mean a function that only contains a term in which the time variable is squared (along with a time-independent constant). I don't mean a polynomial of degree two, i.e., $At^2 + Bt + C$. This may or may not agree with the terminology you learned in math class.

Our job is to first determine (if possible) the arbitrary constants A and B, and then use this velocity function to find the position and acceleration functions.

Since the sprinter starts from rest, we can evaluate the function at t = 0 s and set the result equal to 0 m/s:

$$v(0) = A(0)^2 + B = 0$$
$$B = 0$$

Since we also know the sprinter reaches a speed of 10.6 m/s in 2.7 s, we can evaluate the function at t = 2.7 s and set the result equal to 10.6 m/s:

$$v(2.7) = A(2.7)^2 + 0 = 10.6$$
$$A = 1.45$$

Now that we know the two constants in the velocity function, we have a complete description of the sprinter's speed during this time interval:

$$v(t) = 1.45t^2$$

Once the velocity function is determined, we can differentiate to determine her acceleration function.

$$a(t) = \frac{dv(t)}{dt}$$
$$a(t) = \frac{d}{dt}1.45t^2$$
$$a(t) = 2.90t$$

Evaluating this function at t = 0 s and t = 2.7 s yields a_1 = 0 m/s^2 and a_2 = 7.83 m/s^2.

We can also integrate to determine her position function.

$$r(t) = \int v(t) dt$$
$$r(t) = \int 1.45t^2 dt$$
$$r(t) = \frac{1.45}{3}t^3 + C$$

Since we know r = 0 m when t = 0 s, we can determine the integration constant:

$$r(0) = \frac{1.45}{3}(0)^3 + C = 0$$
$$C = 0$$

Therefore, the position of the sprinter is given by the function:

$$r(t) = \frac{1.45}{3}t^3$$

Evaluating this function at t = 0 s and t = 2.7 s yields r_1 = 0 m and r_2 = 9.51 m.

During the second portion of the race, when her speed is decreasing linearly, her acceleration is constant. Therefore, we can use the kinematic relations developed for constant acceleration, and her acceleration is simply a constant value, denoted a_{23}.

$$r_3 = r_2 + v_2(t_3 - t_2) + \frac{1}{2}a_{23}(t_3 - t_2)^2$$
$$100 = 9.51 + 10.6(12.6 - 2.7) + \frac{1}{2}a_{23}(12.6 - 2.7)^2$$
$$a_{23} = -0.295 \text{ m/s}^2$$

$$v_3 = v_2 + a_{23}(t_3 - t_2)$$
$$v_3 = 10.6 - 0.295(12.6 - 2.7)$$
$$v_3 = 7.68 \text{ m/s}$$

She crosses the finish line running at 7.68 m/s.

Another Example

Investigate the scenario described below.

> *A sports car can accelerate from rest to a speed of 40 m/s while traveling a distance of 200 m. Assume the acceleration of the car can be modeled as a decreasing linear function of time, with a maximum acceleration of 10.4 m/s^2.*

Event 1: The car begins from rest	Event 2: The car reaches 40 m/s
t_1 = 0 s	t_2 =
r_1 = 0 m	r_2 = 200 m
v_1 = 0 m/s	v_2 = 40 m/s
a_1 = 10.4 m/s^2	a_2 =

Between event 1 and 2, the car's acceleration can be modeled by a generic linear function of time, or

$$a(t) = At + B$$

Since the acceleration is decreasing, the maximum value occurs at t = 0 s,

$$a(0) = A(0) + B = 10.4$$
$$B = 10.4$$

Since we don't know the value of the acceleration at t_2, or even the value of t_2, we can't determine A, and all we can currently say about the acceleration function is that it is given by:

$$a(t) = At + 10.4$$

Nonetheless, we can still integrate the acceleration to determine the velocity,

$$v(t) = \int a(t) dt$$
$$v(t) = \int (At + 10.4) dt$$
$$v(t) = \frac{1}{2} At^2 + 10.4t + C$$

Since we know v = 0 m/s when t = 0 s, we can determine the integration constant:

$$v(0) = \frac{1}{2} A(0)^2 + 10.4(0) + C = 0$$
$$C = 0$$

We also know that v = 40 m/s at t_2, so:

$$v(t_2) = \tfrac{1}{2} A{t_2}^2 + 10.4t_2 = 40$$

This equation can't be solved, since it involves two unknowns. However, if we can generate a second equation involving the same two unknowns, we can solve the two equations simultaneously. This second equation must involve the position function of the car:

$$r(t) = \int v(t) dt$$
$$r(t) = \int \left(\frac{1}{2} At^2 + 10.4t\right) dt$$
$$r(t) = \frac{1}{6} At^3 + 5.2t^2 + D$$

Since we know r = 0 when t = 0 s, we can determine the integration constant:

$$r(0) = \frac{1}{6} A(0)^3 + 5.2(0)^2 + D = 0$$
$$D = 0$$

We also know that r = 200 m at t_2, so:

$$r(t_2) = \tfrac{1}{6} A{t_2}^3 + 5.2{t_2}^2 = 200$$

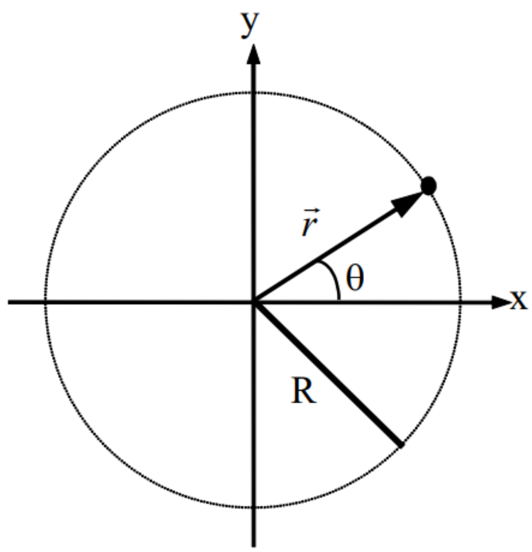
These two equations,

$$\frac{1}{2} A{t_2}^2 + 10.4t_2 = 40$$
$$\frac{1}{6} A{t_2}^3 + 5.2{t_2}^2 = 200$$

can be solved by substitution (or by using a solver). Solve the first equation for A, and substitute this expression into the second equation. This will result in a quadratic equation for t_2. The solution is t_2 = 7.57 s, the time for the car to reach 40 m/s. Plugging this value back into the original equations allows you to complete the description of the car's motion.

Circular Motion

If the acceleration of an object is not constant, in either magnitude or direction, the development of a kinematic description necessitates the use of calculus. A very common class of motion, in which the acceleration is guaranteed to change in at least direction, is the motion of an object on a circular path. Let's examine general circular motion in more detail before we attempt to describe a specific situation.



The x- and y-position of the object moving along a circular path of radius R can always be described by the functions:

$$r_x(t) = R\cos\theta(t)$$
$$r_y(t) = R\sin\theta(t)$$

assuming the origin of the coordinate system is placed at the center of the circle.

Defining angular position, angular velocity and angular acceleration

Angular Position

The function θ(t) specifies the *angular position* of the object, and is typically measured in radians. It specifies where, *along the circle*, the object is at every instant of time. For example, if θ(t) is a constant, the object doesn't move. If θ(t) is a linear function of time, the object moves with constant velocity around the circle. If θ(t) is a more complex function, the object speeds up or slows down as it moves around the circle.

Angular Velocity

The rate at which θ(t) is changing, $\frac{d\theta(t)}{dt}$, is termed the *angular velocity* of the object, and denoted ω(t). (ω is the lower-case Greek letter "omega".[2]) Since ω(t) is the rate at which the angular position is changing, it has units of rad/s.

> **Note**
>
> [2] Call it "omega" don't call it "double-you". It will make you sound smarter.

Angular Acceleration

The rate at which α(t) is changing, $\frac{d\omega(t)}{dt}$, is termed the angular acceleration of the object, and denoted α(t). (α is the lower-case Greek letter "alpha".) Since α(t) is the rate at which the angular velocity is changing, it has units of rad/s^2.

Deriving relationships for velocity and acceleration

Now that we have the definitions of the angular quantities out of the way, let's determine the velocity and acceleration of an object undergoing circular motion. I'll begin by writing the position function in $\hat{i}\hat{j}\hat{k}$ notation, a common "short-hand" method of writing the x-, y-, and zcomponents of a vector all together. In this notation, the $\hat{i}$ simply stands for the x-component, the $\hat{j}$ for the y-component, and the $\hat{k}$ for any z-component. Hold onto your hat and try not to get lost in the calculus.

Position

$$\vec{r}(t) = R(\cos\theta(t)\hat{i} + \sin\theta(t)\hat{j})$$

Velocity

$$\vec{v}(t)=\frac{d}{dt}\vec{r}(t)$$
$$\vec{v}(t)=\frac{d}{dt}[R(\cos\theta(t)\hat{i}+\sin\theta(t)\hat{j})]$$
$$\vec{v}(t)=R\left(\frac{d}{dt}\cos\theta(t)\hat{i}+\frac{d}{dt}\sin\theta(t)\hat{j}\right)$$
$$\vec{v}(t)=R\left(-\sin\theta(t)\frac{d}{dt}\theta(t)\hat{i}+\cos\theta(t)\frac{d}{dt}\theta(t)\hat{j}\right)$$
$$\vec{v}(t)=R(-\sin\theta(t)\omega(t)\hat{i}+\cos\theta(t)\omega(t)\hat{j})$$
$$\vec{v}(t)=R\omega(t)(-\sin\theta(t)\hat{i}+\cos\theta(t)\hat{j})$$

Acceleration

$$\vec{a}(t)=\frac{d}{dt}\vec{v}(t)$$
$$\vec{a}(t)=\frac{d}{dt}[R\omega(t)(-\sin\theta(t)\hat{i}+\cos\theta(t)\hat{j})]$$

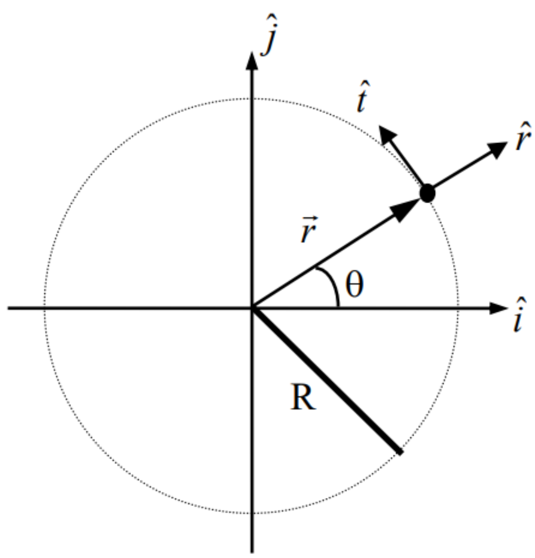
Remembering to use the chain rule for differentiation,

$$\vec{a}(t)=R\left[\frac{d}{dt}\omega(t)\right](-\sin\theta(t)\hat{i}+\cos\theta(t)\hat{j})+R\omega(t)\left[\frac{d}{dt}(-\sin\theta(t)\hat{i}+\cos\theta(t)\hat{j})\right]$$
$$\vec{a}(t)=R\alpha(t)(-\sin\theta(t)\hat{i}+\cos\theta(t)\hat{j})+R\omega(t)\left(-\cos\theta(t)\frac{d}{dt}\theta(t)\hat{i}+-\sin\theta(t)\frac{d}{dt}\theta(t)\hat{j}\right)$$
$$\vec{a}(t)=R\alpha(t)(-\sin\theta(t)\hat{i}+\cos\theta(t)\hat{j})+R\omega(t)(-\cos\theta(t)\omega(t)\hat{i}+-\sin\theta(t)\omega(t)\hat{j})$$
$$\vec{a}(t)=R\alpha(t)(-\sin\theta(t)\hat{i}+\cos\theta(t)\hat{j})-R\omega^2(t)(\cos\theta(t)\hat{i}+\sin\theta(t)\hat{j})$$

These relationships for velocity and acceleration *look* intimidating, but are actually rather simple. (You don't have to believe me just yet ...) The problem is that they are written using an awkward choice of coordinate system. In a previous Model, we used inclined coordinates for situations involving objects moving on an inclined surface. For an object moving in a circle, it's almost as if the surface upon which the object moves is continually changing its angle of incline! Perhaps we should use a coordinate system in which the orientation of the system continually changes, always keeping one axis parallel and one axis perpendicular to the motion. This is exactly what we will do. This coordinate system is referred to as the *polar coordinate system*.

Polar Coordinates

In the polar coordinate system, one axis (the radial axis, or $\hat{r}$) is perpendicular to the surface of the circular path pointing radially away from the center, and the other axis (the tangential, or $\hat{t}$) is parallel to the surface of the circular path pointing in the counterclockwise direction.



Notice that $\hat{r}$ is inclined by an angle θ from the positive x-axis. Therefore, in terms of $\hat{i}$ and $\hat{j}$;

$\hat{r} = \cos\theta(t)\hat{i} + \sin\theta(t)\hat{j}$

$\hat{t}$, on the other hand, is inclined by an angle θ to the left of the positive y-axis. Therefore, in terms of $\hat{i}$ and $\hat{j}$;

$\hat{t} = -\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j}$

Now re-examine the relationships for position, velocity and acceleration.

Position

$\vec{r}(t) = R(\cos\theta(t)\hat{i} + \sin\theta(t)\hat{j})$

becomes[3]

$\vec{r}(t) = R\hat{r}$

In component form this is:

$$r_r = R$$
$$r_t = 0$$

This means that the position of an object undergoing circular motion is *only* in the radial direction, and has a constant magnitude equal to the radius of the circle. Basically, the coordinate system is constructed so that the location of the object *defines* the radial direction.

Note

[3] There's always a bit of complaining regarding this formula. It looks like it says "r is equal to r times r" but these three "r's" have completely different meanings. $\vec{r}$ is the position of the object in space and has components in 2 (or more generally 3) directions. It can be expressed in any coordinate system. R is the magnitude of the position in polar coordinates. $\hat{r}$ is simply notation telling you that the position magnitude is measured radially away from the origin.

Velocity

$\vec{v}(t) = R\omega(t)(-\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j})$

becomes

$\vec{v}(t) = R\omega(t)\hat{t}$

In component form this is:

$$v_r = 0$$
$$v_t = R\omega$$

This means that the velocity of an object undergoing circular motion is *only* in the tangential direction, and has a magnitude equal to the product of the radius and angular velocity. The only way an object can have a radial velocity is if the radius of its path changes, but that can't happen for an object moving along a circular path. If the object moved along an elliptical path, for example, then it would have both tangential and radial velocities.

Acceleration

$\vec{a}(t) = R\alpha(t)(-\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j}) - R\omega^2(t)(\cos\theta(t)\hat{i} + \sin\theta(t)\hat{j})$

becomes

$\vec{a}(t) = R\alpha(t)\hat{t} - R\omega^2(t)\hat{r}$

In component form this is:

$$a_r = -R\omega^2$$
$$a_t = R\alpha$$

The acceleration of an object undergoing circular motion has two components. If the object is speeding up or slowing down, the angular acceleration does not equal zero and there is an acceleration component in the tangential direction. The magnitude of the tangential acceleration is equal to the product of the radius and angular acceleration.

However, *even if the object is moving at constant speed*, there is an acceleration component in the negative radial direction, i.e., pointing toward the center of the circle. By virtue of traveling in a circle, the velocity vector of an object continually changes its orientation. This change in orientation is directed toward the center of the circle. Draw a motion diagram and convince yourself of this fact!

The magnitude of the radial acceleration is equal to the product of the radius and the *square* of the angular *velocity*.

The Kinematics of Circular Motion

Let's try out our new tools by examining the following scenario.

> *An automobile enters a U-turn of constant radius of curvature 95 m. The car enters the U-turn traveling at 33 m/s north and exits at 22 m/s south. Assume the speed of the car can be modeled as a quadratic function of time.*

To analyze this situation, we should first carefully determine and define the sequence of events that take place. At each of these events we will tabulate the position, velocity and acceleration in polar coordinates as well as the angular position, angular velocity, and angular acceleration.

Event 1: The auto enters the turn.	Event 2: The auto exits the turn.
t_1 = 0 s	t_2 =
r_{1r} = 95 m	r_{2r} = 95 m
r_{1t} = 0 m	r_{2t} = 0 m
θ_1 = 0 rad	$\theta_2 = \pi$ rad
v_{1r} = 0 m/s	v_{2r} = 0 m/s
v_{1t} = 33 m/s	v_{2t} = 22 m/s
ω_1 =	ω_2 =
a_{1r} =	a_{2r} =
a_{1t} =	a_{2t} =
α_1 =	α_1 =

Notice that the position of the car is strictly in the radial direction and the velocity of the car is strictly in the tangential direction. It is *impossible*, for an object moving along a circular path, to have a non-zero tangential position or radial velocity.

Also notice that since the car completely reverses its direction of travel, it must have traveled halfway around a circular path. Thus, $\theta_2 = \pi$ rad.

We can quickly determine the angular velocity of the car at the two events by using the relationship:

$$v_t = R\omega$$

With R = 95 m, the angular velocity of the car as it enters the turn is 0.35 rad/s, and as it exits the turn, 0.23 rad/s.

Between event 1 and 2, the car's speed can be modeled by a generic quadratic function of time, or

$$v_t(t) = At^2 + B$$

Since the car enters the turn at 33 m/s, we can evaluate the function at t = 0 s and set the result equal to 33 m/s:

$$v_t(0) = A(0)^2 + B = 33$$
$$B = 33$$

The car exits the turn at 22 m/s, but since we don't know the value of t_2 we can't determine A. We do know, however, that:

$$v_t\left(t_2\right) = A{t_2}^2 + 33 = 22$$

If we can determine another equation involving A and t_2 we can solve these two equations simultaneously.

The only other important piece of information regarding the motion is that $\theta_2 = \pi$. To make use of that information, we must "convert" our velocity equation into an angular velocity (ω) equation and then integrate the resulting equation into an equation for θ. To determine the angular velocity function,

$$v_t = R\omega$$
$$\omega = \frac{v_t}{R}$$
$$\omega(t) = \frac{At^2 + 33}{95}$$
$$\omega(t) = \frac{A}{95}t^2 + 0.35$$

The angular position (θ) is the integral of the angular velocity,

$$\theta(t) = \int \omega(t) dt$$
$$\theta(t) = \int \left(\frac{A}{95}t^2 + 0.35\right) dt$$
$$\theta(t) = \frac{A}{285}t^3 + 0.35t + D$$

Since we know θ = 0 when t = 0 s, we can determine the integration constant:

$$\theta(0) = \frac{A}{285}(0)^3 + 0.35(0) + D = 0$$
$$D = 0$$

We also know that $\theta = \pi$ at t_2, so:

$$\theta(t_2) = \tfrac{A}{285}{t_2}^3 + 0.35t_2 = \pi$$

These two equations,

$$\frac{A}{285}{t_2}^3 + 0.35t_2 = \pi$$
$$A{t_2}^2 + 33 = 22$$

can be solved by substitution (or by using a solver). Solve the second equation for A, and substitute this expression into the first equation. You can solve the resulting equation for t_2. The solution is t_2 = 10.0 s, the time for the car to complete the turn. Plugging this value back into the original equation allows you to determine A = -0.11.

Once you know A, you can complete the rest of the motion table. For example, since

$$\omega(t) = \frac{A}{95}t^2 + 0.35$$
$$\omega(t) = -0.00116t^2 + 0.35$$

We can now determine angular acceleration,

$$\alpha(t) = \frac{d\omega(t)}{dt}$$
$$\alpha(t) = \frac{d}{dt}\left(-0.00116t^2 + 0.35\right)$$
$$\alpha(t) = -0.00232t$$

radial acceleration,

$$a_r(t) = -R\omega(t)^2$$
$$a_r(t) = -95\left(-0.00116t^2 + 0.35\right)^2$$

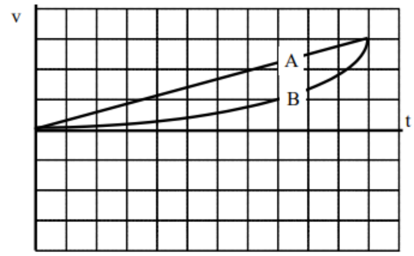
and tangential acceleration.

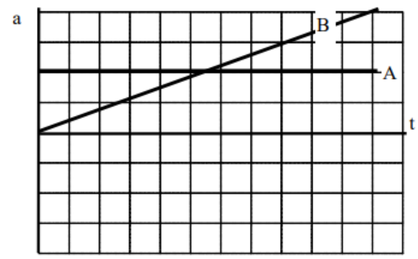
$$a_t(t) = R\alpha(t)$$
$$a_t(t) = 95(-0.00232t)$$
$$a_t(t) = -0.22t$$

Each of these functions could be evaluated at $t_1 = 0$ s and $t_2 = 10.0$ s to complete the table.

Activities

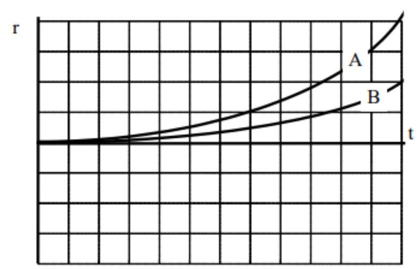
In a drag race, two cars begin from rest at the starting line and move according to the velocity vs. time graph below. Construct the position and acceleration vs. time graphs for each car.





Acceleration is the derivative of velocity, or the slope of a velocity vs. time graph. Car A has a constant, positive slope, and car B has a positive increasing slope.

Since the two cars have the same final velocity, the areas under the acceleration vs. time graphs must be equal. (Since the integral of acceleration is change in velocity, and both cars have the same change in velocity, the integrals, or areas, must be equal.)

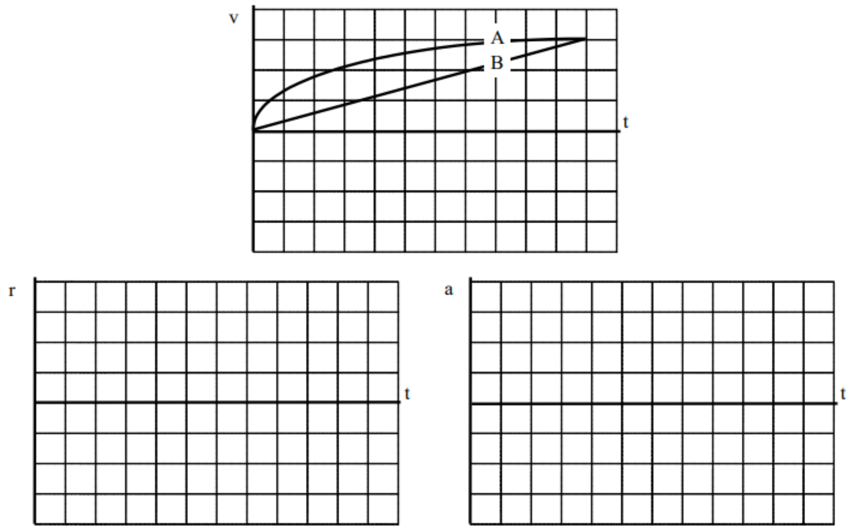


Velocity is the derivative of position, so since car A has a linearly increasing velocity its position must be a quadratic function of time. Car B has a smaller velocity at every instant, so it must be behind car A (have a smaller position) throughout the race.

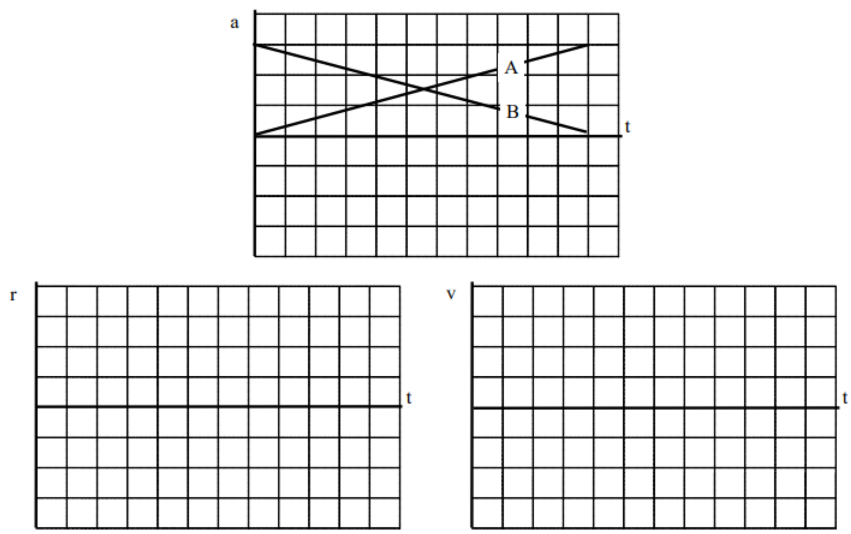
Since the area under car A's velocity graph is larger than the area under car B's, car A must have a larger change in position than car B (i.e., the faster car is always leading the race).

In a drag race, two cars begin from rest at the starting line and move according to the given kinematic graph. Construct the other two kinematic graphs for each car.

a.

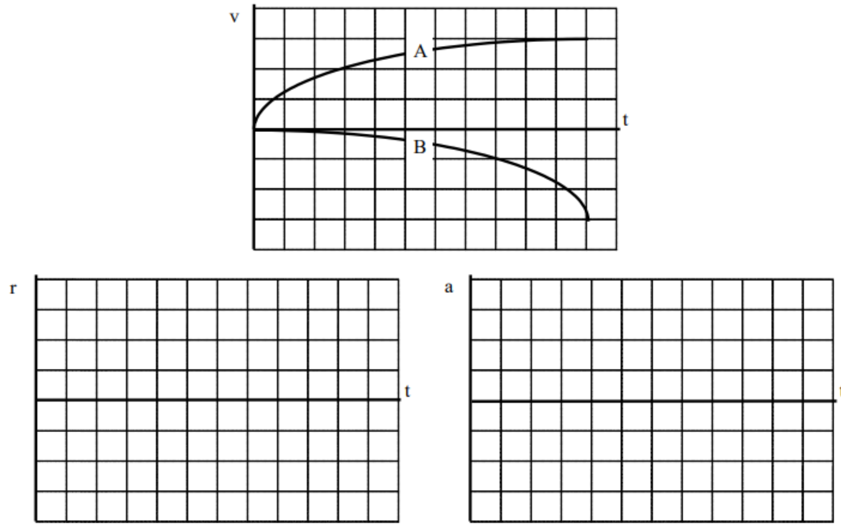


b.

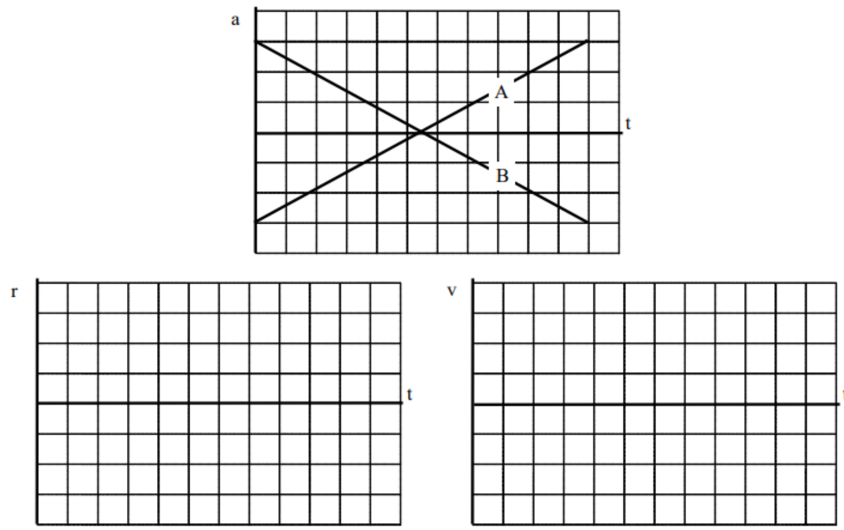


In a strange type of race, two cars begin from rest at the starting line and move according to the given kinematic graph. Construct the other two kinematic graphs for each car.

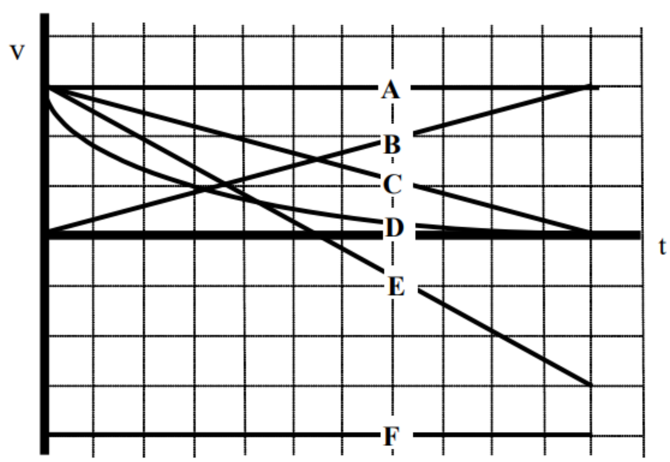
a.



b.



Below are velocity vs. time graphs for six different objects



a. Rank these objects on the basis of their change in position during the time interval shown.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

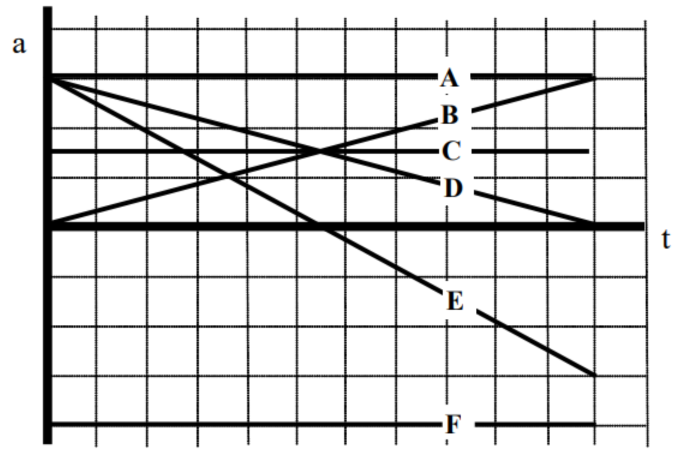
b. Rank these objects on the basis of their change in acceleration during the time interval shown.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are acceleration vs. time graphs for six different cars. All six cars begin a race at rest at the starting line.



a. Rank these cars on the basis of their final velocity at the end of the time interval shown.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank these cars on the basis of their final position at the end of the time interval shown.

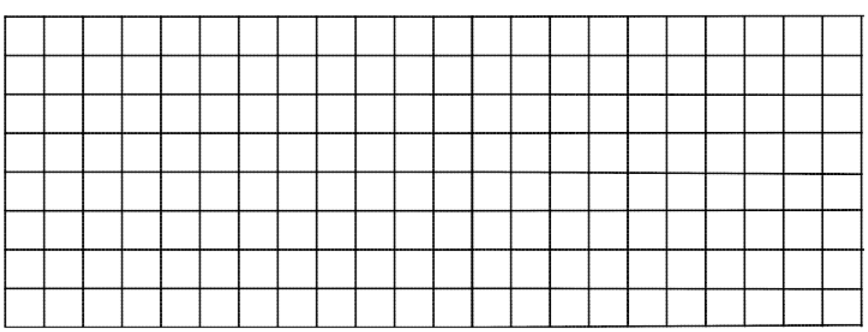
Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

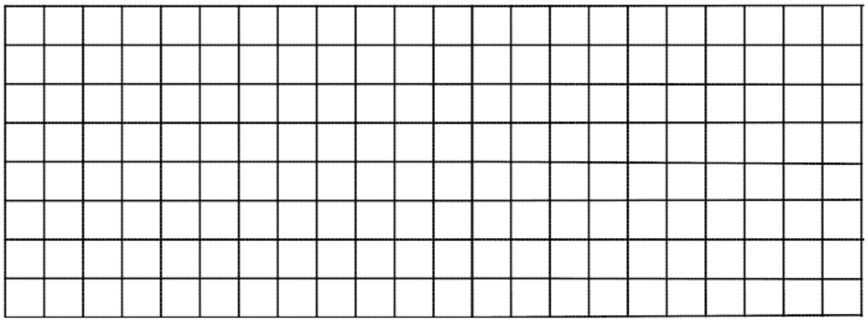
Explain the reason for your ranking:

Construct motion diagrams for the motions described below.

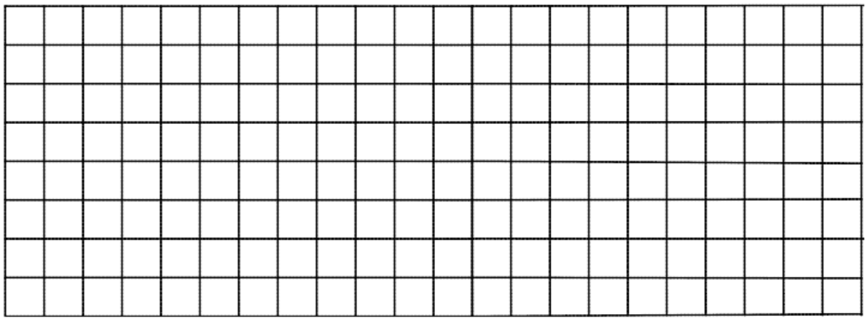
a. A satellite has been programmed to circle a stationary space station at a radius of 10 km and a constant angular speed of 0.02 rad/s.



b. A rider on a merry-go-round, 3 m from the axis, is traveling at 4 m/s. The merry-go-round slows, and the rider reaches a speed of 0.5 m/s in 11 seconds.

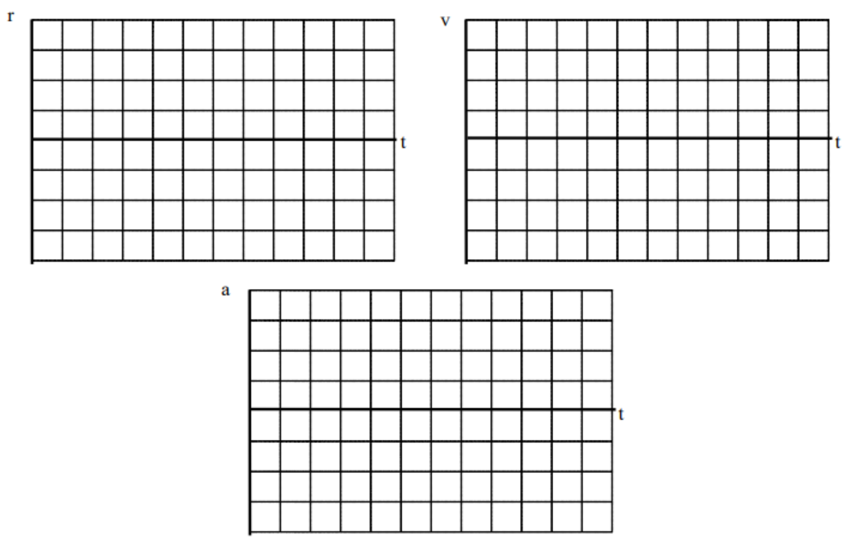


c. In a device built to acclimate astronauts to large accelerations, astronauts are strapped into a pod that is swung in a 6 m radius circle at high speed. The linear speed of the pod is increased from rest to a speed of 17 m/s in a time interval of 25 seconds.

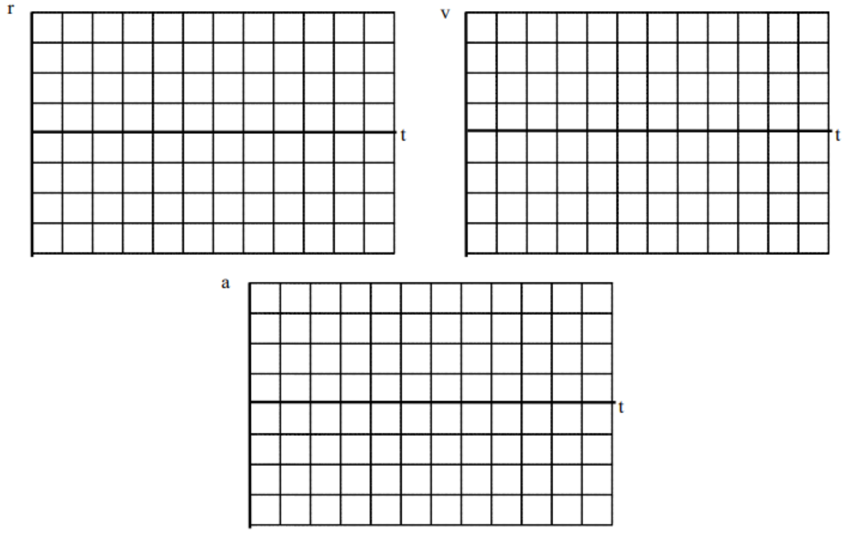


Sketch position, velocity, and acceleration vs. time graphs for one complete cycle of a rider on a merry-goround turning at constant angular speed. Set the origin of the coordinate system at the center of the merrygo-round.

a. Sketch the x- and y-components of this motion separately.

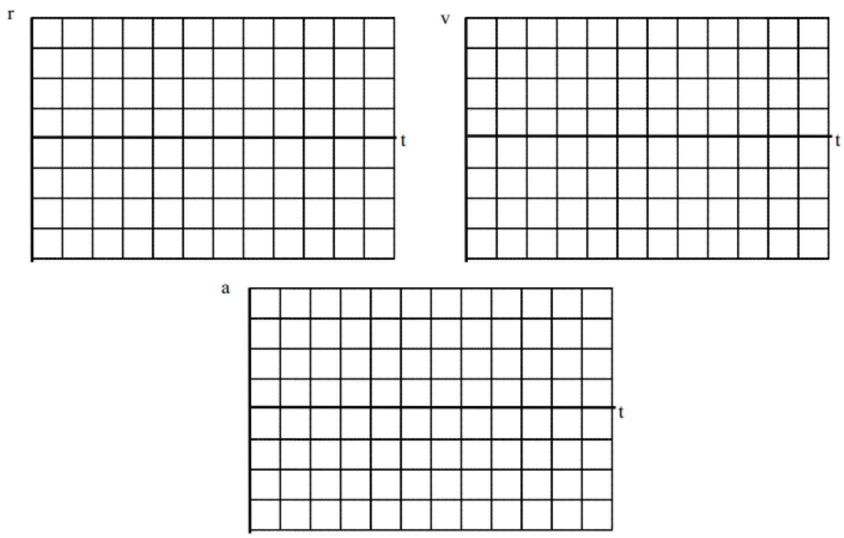


b. Sketch the r- and t-components of this motion separately.

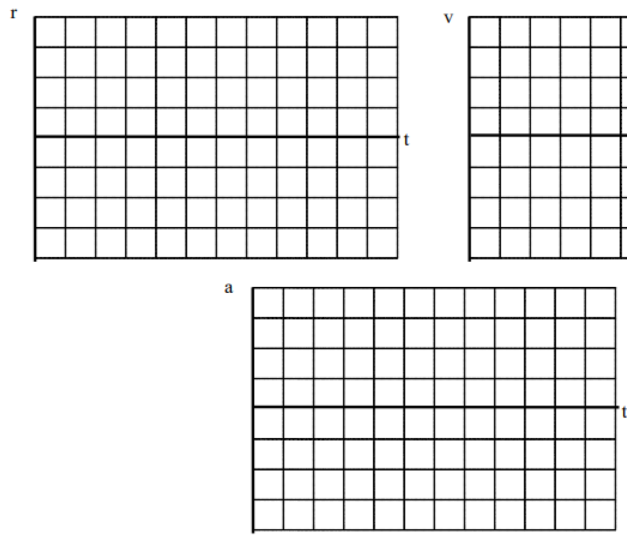


Sketch position, velocity, and acceleration vs. time graphs for one complete cycle of a rider on a Ferris wheel as it increases its angular speed from rest. Set the origin of the coordinate system at the center of the Ferris wheel.

a. Sketch the x- and y-components of this motion separately.

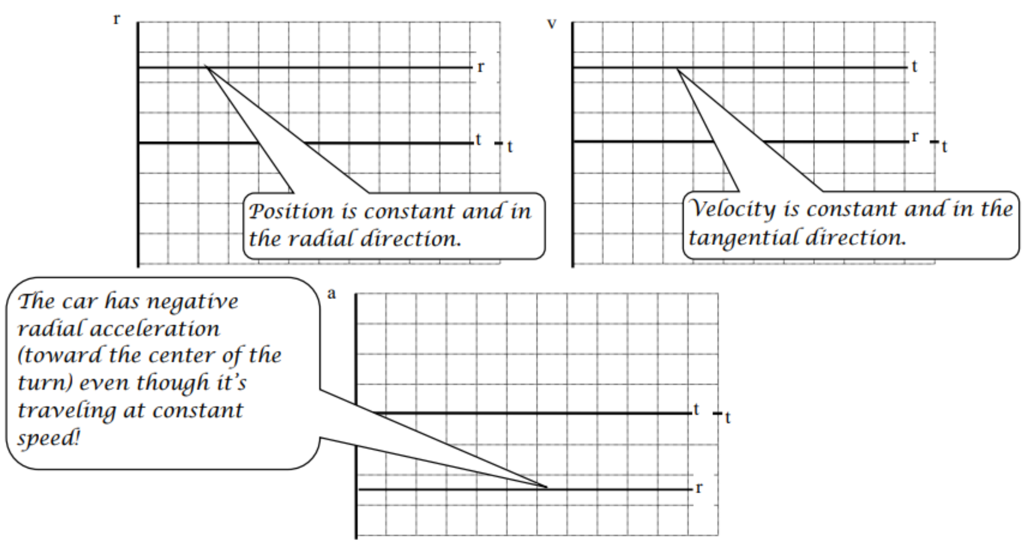


b. Sketch the r- and t-components of this motion separately.

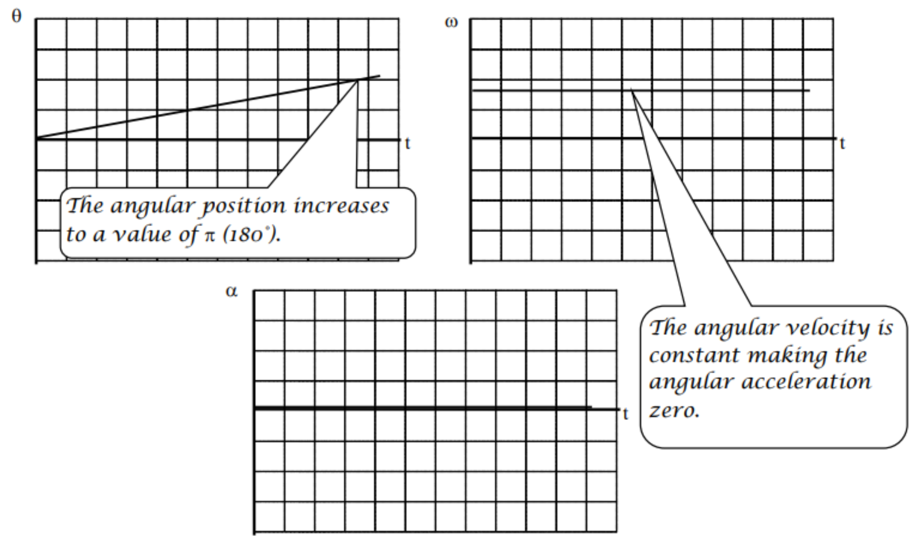


A car drives around a semi-circular U-turn at constant speed. Set the origin of the coordinate system at the center-of-curvature of the U-turn.

a. Using polar coordinates, sketch position, velocity, and acceleration vs. time graphs for this motion.

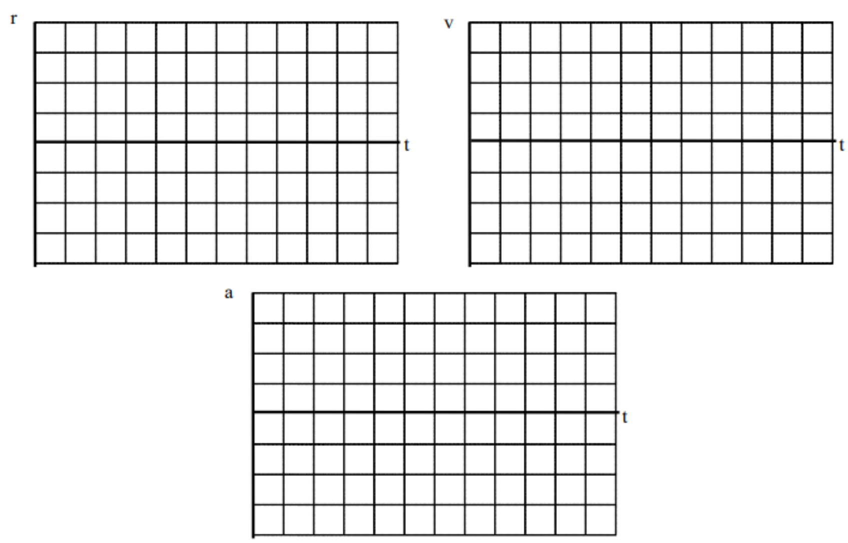


b. Sketch angular position, angular velocity, and angular acceleration vs. time graphs for this motion.

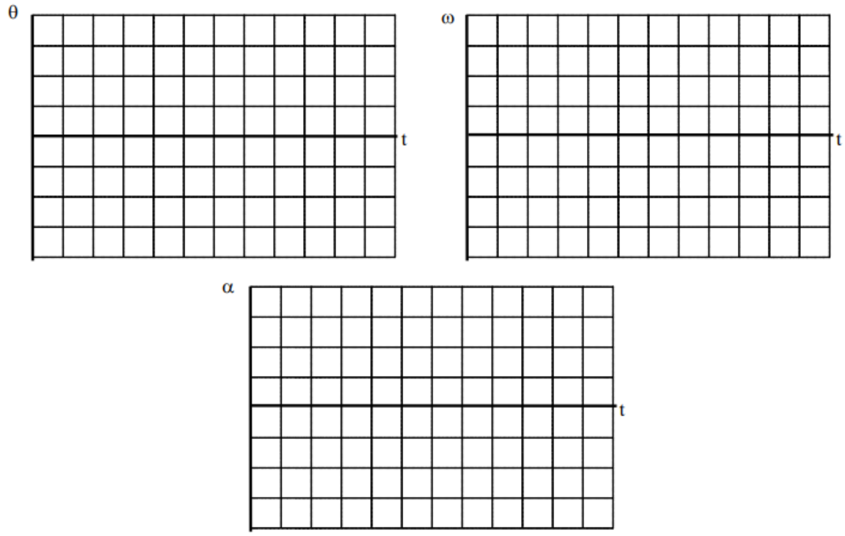


A car drives around a semi-circular U-turn at decreasing speed. Set the origin of the coordinate system at the center-of-curvature of the U-turn.

a. Using polar coordinates, sketch position, velocity, and acceleration vs. time graphs for this motion.

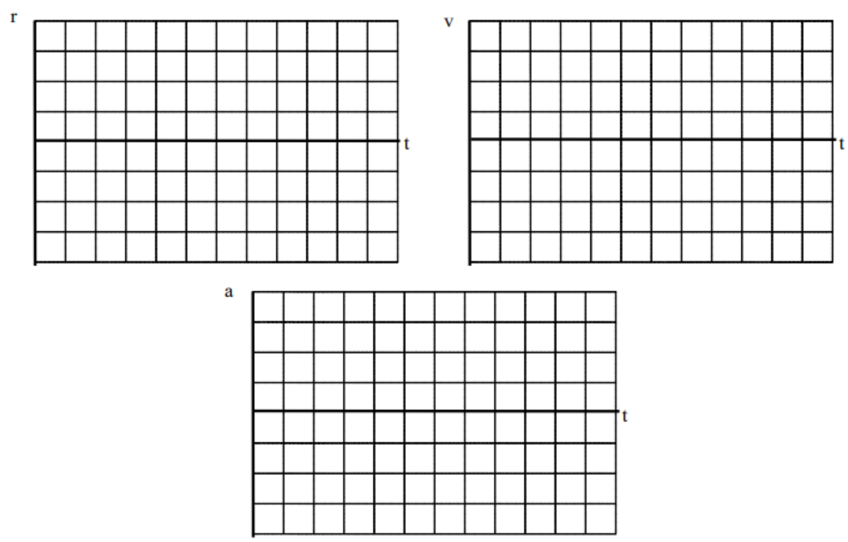


b. Sketch angular position, angular velocity, and angular acceleration vs. time graphs for this motion.

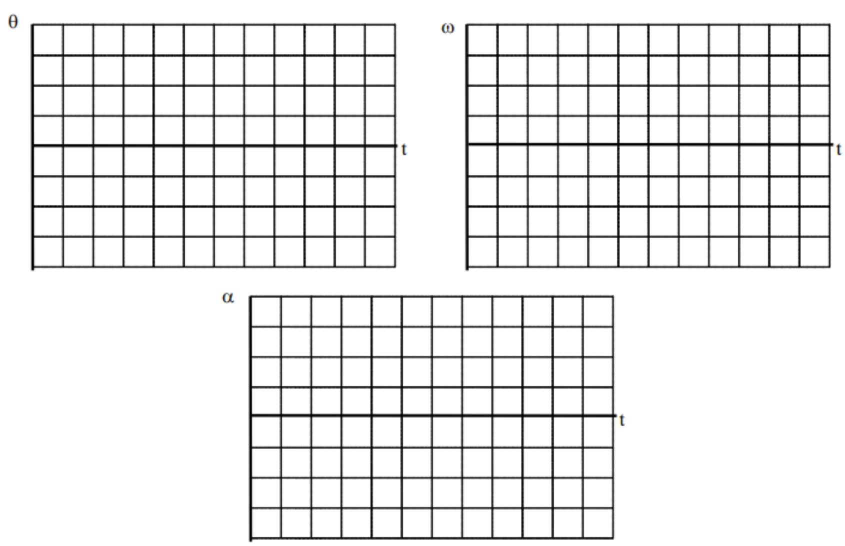


A pendulum completes one cycle of its motion. Set the origin of the coordinate system at the point of attachment of the pendulum.

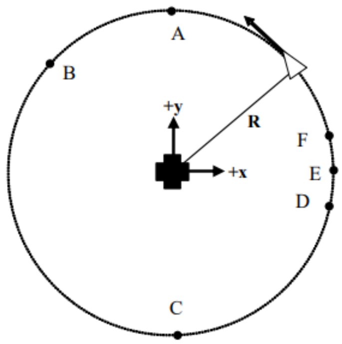
a. Using polar coordinates, sketch position, velocity, and acceleration vs. time graphs for this motion.



b. Sketch angular position, angular velocity, and angular acceleration vs. time graphs for this motion.



An artificial satellite circles a space station at constant speed. The satellite passes through the six labeled points. For all questions below, use the indicated coordinate system.



a. Rank the x-velocity of the satellite at each of the labeled points.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

b. Rank the y-velocity of the satellite at each of the labeled points.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

c. Rank the radial velocity of the satellite at each of the labeled points.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

d. Rank the tangential velocity of the satellite at each of the labeled points.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

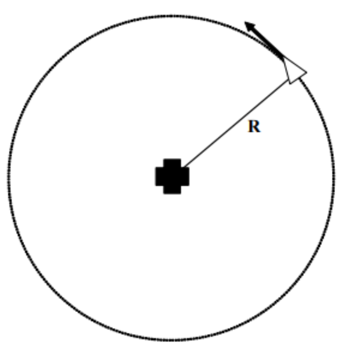
_____ The ranking cannot be determined based on the information provided.

e. Rank the angular velocity of the satellite at each of the labeled points.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Six artificial satellites of identical mass circle a space station with constant period T. The satellites are located a distance R from the space station.



	R	T
A	5000 m	160 hrs
B	2500 m	40 hrs
C	2500 m	80 hrs
D	10000 m	160 hrs
E	5000 m	120 hrs
F	10000 m	80 hrs

a. Rank these satellites on the basis of the magnitude of their angular velocity.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

b. Rank these satellites on the basis of the magnitude of their radial velocity.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

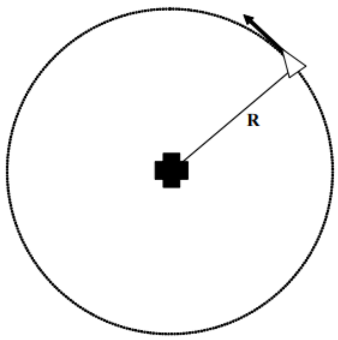
c. Rank these satellites on the basis of the magnitude of their tangential velocity

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

Six artificial satellites of identical mass circle a space station at constant speed v. The satellites are located a distance R from the space station.



	R	v
A	5000 m	160 m/s
B	2500 m	40 m/s
C	2500 m	80 m/s
D	10000 m	160 m/s
E	5000 m	120 m/s
F	10000 m	80 m/s

a. Rank these satellites on the basis of the magnitude of their angular acceleration.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

b. Rank these satellites on the basis of the magnitude of their radial acceleration.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

c. Rank these satellites on the basis of the magnitude of their tangential acceleration.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

In a 100 m dash, detailed video analysis indicated that a particular sprinter's speed can be modeled as a quadratic function of time at the beginning of a race, reaching a speed of 12.1 m/s in 1.7 s, and then as a linear function of time for the remainder of the race. She finished the race in 10.6 seconds.

Motion Information

Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_1 =	a_2 =	a_3 =

Mathematical Analysis[1]

In a 400 m race, detailed video analysis indicated that a particular sprinter's speed can be modeled as a cubic function of time at the beginning of a race, reaching a speed of 8.5 m/s in 7.1 s, and as a linear function of time for the remainder of the race. She *crossed the finish line traveling at 7.4 m/s.*

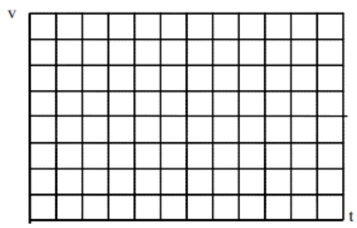
Motion Information

Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_1 =	a_2 =	a_3 =

Mathematical Analysis[2]

Two cars, an Audi and a BMW, can accelerate from rest to a speed of 25 m/s in a time of 6.2 s. The velocity of the Audi increases as a linear function of time and the velocity of the BMW increases as a quadratic function of time.

Motion Graph



Motion Information

Audi		BMW	
Event 1:	Event 2:	Event 1:	Event 2:
t_1 =	t_2 =	t_1 =	t_2 =
r_1 =	r_2 =	r_1 =	r_2 =
v_1 =	v_2 =	v_1 =	v_2 =
a_1 =	a_2 =	a_1 =	a_2 =

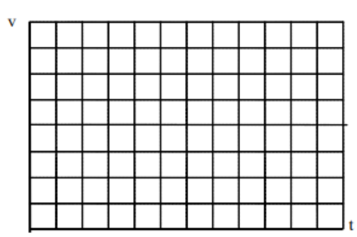
Question

Based only on the graph, which car travels a larger distance in 6.2 s? Explain.

Mathematical Analysis[3]

Two cars, an Audi and a BMW, can accelerate from rest to a speed of 40 m/s after traveling a distance of 400 m. The velocity of the Audi increases as a linear function of time and the velocity of the BMW increases as a quadratic function of time.

Motion Graph



Motion Information

Audi		BMW	
Event 1:	Event 2:	Event 1:	Event 2:
t_1 =	t_2 =	t_1 =	t_2 =
r_1 =	r_2 =	r_1 =	r_2 =
v_1 =	v_2 =	v_1 =	v_2 =
a_1 =	a_2 =	a_1 =	a_2 =

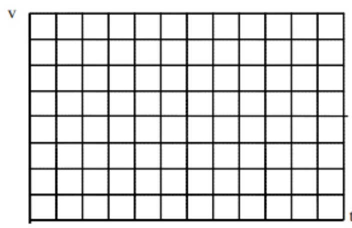
Question

Based only on the graph, which car will take longer to reach 40 m/s? Explain.

Mathematical Analysis[4]

Two cars, an Audi and a BMW, accelerating from rest, can travel a distance of 400 m in 16.2 s. The velocity of the Audi increases as a linear function of time and the velocity of the BMW increases as a quadratic function of time.

Motion Graph



Motion Information

Audi		BMW	
Event 1:	Event 2:	Event 1:	Event 2:
t_1 =	t_2 =	t_1 =	t_2 =
r_1 =	r_2 =	r_1 =	r_2 =
v_1 =	v_2 =	v_1 =	v_2 =
a_1 =	a_2 =	a_1 =	a_2 =

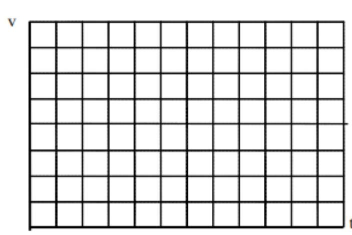
Question

Based only on the graph, which car is traveling faster at 16.2 s? Explain.

Mathematical Analysis[5]

Two cars, an Audi and a BMW, can accelerate from 15 m/s to 25 m/s in a time of 3.9 s. The velocity of the Audi increases as a linear function of time and the velocity of the BMW increases as a quadratic function of time.

Motion Graph



Motion Information

Audi		BMW	
Event 1:	Event 2:	Event 1:	Event 2:
t_1 =	t_2 =	t_1 =	t_2 =
r_1 =	r_2 =	r_1 =	r_2 =
v_1 =	v_2 =	v_1 =	v_2 =
a_1 =	a_2 =	a_1 =	a_2 =

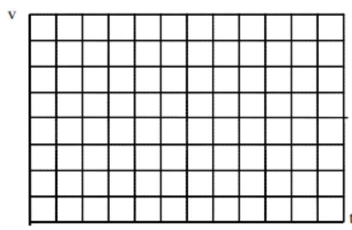
Question

Based only on the graph, which car travels a larger distance while accelerating? Explain.

Mathematical Analysis[6]

Two cars, an Audi and a BMW, can slow from 35 m/s to 5 m/s over a distance of 70 m. The velocity of the Audi decreases as a linear function of time and the velocity of the BMW decreases as a quadratic function of time.

Motion Graph



Motion Information

Audi		BMW	
Event 1:	Event 2:	Event 1:	Event 2:
t_1 =	t_2 =	t_1 =	t_2 =
r_1 =	r_2 =	r_1 =	r_2 =
v_1 =	v_2 =	v_1 =	v_2 =
a_1 =	a_2 =	a_1 =	a_2 =

Question

Based only on the graph, which car takes a longer time to slow to 5 m/s? Explain.

Mathematical Analysis[7]

In a hypothetical universe, the acceleration of an object subject to the gravitational force field of an Earthlike planet decreases with the amount of time in seconds, t, spent in the field as

$$a=\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right) e^{-0.1 t}$$

A ball is released from rest 100 m above the ground.

Motion Information

Event 1: The ball is released.	Event 2: The ball hits the ground.
t_1 =	t_2 =
r_1 =	r_2 =

Event 1: The ball is released.	Event 2: The ball hits the ground.
t_1 =	t_2 =
r_1 =	r_2 =
v_1 =	v_2 =
a_1 =	a_2 =

Mathematical Analysis[8]

In a hypothetical universe, the acceleration of an object subject to the gravitational force field of an Earthlike planet decreases with the amount of time in seconds, t, spent in the field as

$$a = \left(9.8 \text{ m/s}^2\right) e^{-0.1t}$$

A ball is thrown vertically upward at 40 m/s.

Motion Information

Event 1: The ball is released.	Event 2: The ball reaches its apex.	Event 3: The ball returns to your hand.
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_1 =	a_2 =	a_3 =

Mathematical Analysis[9]

In a hypothetical universe, the acceleration of an object subject to the gravitational force field of an Earthlike planet increases with the amount of time in seconds, t, spent in the field as

$$a = \left(9.8\, m/s^2\right)\left(1 - e^{-0.1t}\right)$$

A ball is released from rest 100 m above the ground.

Motion Information

Event 1: The ball is released.	Event 2: The ball hits the ground.
t_1 =	t_2 =
r_1 =	r_2 =
v_1 =	v_2 =
a_1 =	a_2 =

Mathematical Analysis[10]

In a hypothetical universe, the acceleration of an object subject to the gravitational force field of an Earthlike planet increases with the amount of time in seconds, t, spent in the field as

$$a = \left(9.8\, m/s^2\right)\left(1 - e^{-0.1t}\right)$$

A ball is thrown vertically upward at 40 m/s.

Motion Information

Event 1: The ball is released.	Event 2: The ball reaches its apex.	Event 3: The ball returns to your hand.
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =

Event 1: The ball is released.	Event 2: The ball reaches its apex.	Event 3: The ball returns to your hand.
a_1 =	a_2 =	a_3 =
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_1 =	a_2 =	a_3 =

Mathematical Analysis[11]

A rider on a merry-go-round is traveling at a constant speed of 4.0 m/s, and completes three revolutions in 14 s.

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1r} =	r_{2r} =
r_{1t} =	r_{2t} =
θ_1 =	θ_2 =
v_{1r} =	v_{2r} =
v_{1t} =	v_{2t} =
ω_1 =	ω_2 =
a_{1r} =	a_{2r} =
a_{1t} =	a_{2t} =
α_1 =	α_1 =

Mathematical Analysis[12]

A rider on a merry-go-round, 3.0 m from the axis, is traveling at 4.0 m/s. The merry-go-round slows to rest over three complete revolutions. The rider's speed decreases as a linear function of time.

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1r} =	r_{2r} =
r_{1t} =	r_{2t} =
θ_1 =	θ_2 =
v_{1r} =	v_{2r} =
v_{1t} =	v_{2t} =
ω_1 =	ω_2 =
a_{1r} =	a_{2r} =
a_{1t} =	a_{2t} =
α_1 =	α_1 =

Mathematical Analysis[13]

A rider on a 15 m diameter Ferris wheel is initially at rest. The angular speed of the Ferris wheel is increased to 1.5 rad/s over a time interval of 3.5 s. The angular acceleration of the Ferris wheel increases from 0 rad/s 2 as a square-root function of time.

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1r} =	r_{2r} =
r_{1t} =	r_{2t} =
θ_1 =	θ_2 =
v_{1r} =	v_{2r} =
v_{1t} =	v_{2t} =
ω_1 =	ω_2 =
a_{1r} =	a_{2r} =
a_{1t} =	a_{2t} =
α_1 =	α_1 =

Mathematical Analysis[14]

A rider on a 16 m diameter Ferris wheel is initially at traveling at 10 m/s. The Ferris wheel slows to rest over two complete revolutions. During the slow-down, the magnitude of the angular acceleration of the Ferris wheel decreases linearly from its maximum value to 0 rad/s^2.

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1r} =	r_{2r} =
r_{1t} =	r_{2t} =
θ_1 =	θ_2 =
v_{1r} =	v_{2r} =
v_{1t} =	v_{2t} =
ω_1 =	ω_2 =
a_{1r} =	a_{2r} =
a_{1t} =	a_{2t} =
α_1 =	α_1 =

Mathematical Analysis[15]

An automobile enters a constant 90 m radius of curvature turn traveling at 25 m/s north and exits the curve traveling at 35 m/s east. Assume the speed of the car can be modeled as a square-root function of time.

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1r} =	r_{2r} =
r_{1t} =	r_{2t} =

Event 1:	Event 2:
θ_1 =	θ_2 =
v_{1r} =	v_{2r} =
v_{1t} =	v_{2t} =
ω_1 =	ω_2 =
a_{1r} =	a_{2r} =
a_{1t} =	a_{2t} =
α_1 =	α_1 =

Mathematical Analysis[16]

An automobile enters a constant 90 m radius of curvature turn traveling at 25 m/s north and exits the curve traveling east. The car completes the turn in 5.4 s. Assume the speed of the car can be modeled as a quadratic function of time.

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1r} =	r_{2r} =
r_{1t} =	r_{2t} =
θ_1 =	θ_2 =
v_{1r} =	v_{2r} =
v_{1t} =	v_{2t} =
ω_1 =	ω_2 =
a_{1r} =	a_{2r} =
a_{1t} =	a_{2t} =
α_1 =	α_1 =

Mathematical Analysis[17]

In a device built to acclimate astronauts to large accelerations, astronauts are strapped into a pod that is swung in a 6.0 m radius circle at high speed. The angular speed of the pod is increased quadratically from rest to an angular speed of 2.8 rad/s in a time interval of 20 seconds. The device is then linearly slowed to rest over a time interval of 40 seconds.

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1r} =	r_{2r} =
r_{1t} =	r_{2t} =
θ_1 =	θ_2 =
v_{1r} =	v_{2r} =
v_{1t} =	v_{2t} =

Event 1:	Event 2:
ω_1 =	ω_2 =
a_{1r} =	a_{2r} =
a_{1t} =	a_{2t} =
α_1 =	α_1 =

Mathematical Analysis[18]

In a device built to acclimate astronauts to large accelerations, astronauts are strapped into a pod that is swung in a 6.0 m radius circle at high speed. The linear speed of the pod is increased quadratically from rest to a speed of 17 m/s after three complete revolutions of the pod. The device is then linearly slowed to rest over a time interval of 35 seconds.

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1r} =	r_{2r} =
r_{1t} =	r_{2t} =
θ_1 =	θ_2 =
v_{1r} =	v_{2r} =
v_{1t} =	v_{2t} =
ω_1 =	ω_2 =
a_{1r} =	a_{2r} =
a_{1t} =	a_{2t} =
α_1 =	α_1 =

Mathematical Analysis[19]

4.2: Dynamics

Concepts and Principles

We are now prepared to investigate forces that are free to vary in magnitude and direction. However, you may recall that at no time in the explanation of Newton's laws was the conversation restricted to either constant acceleration or constant force. In fact, Newton's laws are completely valid regardless of the nature of the forces acting on an object, or its acceleration.[4] Thus, the relation

$$\Sigma F = ma$$

remains valid, and is still the central relationship in the study of mechanics.

Note

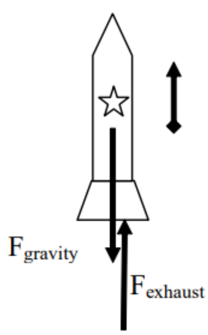
[4] Actually, Newton's Second Law is known to be inadequate to explain phenomenon occurring at extremely high speeds or at extremely small distances. The speeds must be comparable to the speed of light or the distances must be comparable to the size of the atom for these effects to be noticeable.

Analysis Tools

Time-Dependent Forces

A 2.0 kg toy rocket is fitted with an engine that provides a thrust roughly modeled by the function F(t) = (60 N/s) t - (15 N/s²) t², for 0 < t < 4.0 s, and zero thereafter. The rocket is launched directly upward.

Of course, the first step to analyzing any situation involving forces is to construct a free-body diagram. Below is a free-body diagram for the rocket during the time interval 0 < t < 4.0 s.



Given that you are not presented with information pertaining to the frictional force acting on the rocket, you must analyze the rocket's trajectory in a hypothetical, friction-free environment.

From Newton's second law:

$$\begin{gathered}
\Sigma F = ma \\
F_{\text{exhaust }} - F_{\text{gravity }} = ma \\
60t - 15t^2 - 2(9.8) = 2a \\
-7.5t^2 + 30t - 9.8 = a
\end{gathered}$$

Thus, the acceleration of the rocket is not constant, and the velocity and position of the rocket must be determined by integrating the acceleration.

Before we do this, however, note that the acceleration of the rocket is initially directed downward. (At t = 0s, a = -9.8 m/s^2.) This is because the supporting force exerted on the rocket by the launch platform has been ignored. This force would hold the rocket in place until the force of the exhaust gases on the rocket is equal to, and then exceeds, the force of gravity on the rocket. In reality, the acceleration of the rocket is zero until $F_{exhaus}t = F_{gravity}$. This occurs when:

$$F_{\text{exhaust}} = F_{\text{gravity}}$$
$$60t - 15t^2 = 2(9.8)$$
$$-15t^2 + 60t - 19.6 = 0$$
$$t = 0.36\text{ s}$$

The correct motion information is tabulated below.

Event 1: The engine is ignited	Event 2: Rocket leaves the pad	Event 3: The thrust ends	Event 4: The rocket reaches its apex
t_1 = 0 s	t_2 = 0.36 s	t_3 = 4.0 s	t_4 =
r_1 = 0 m	r_2 = 0 m	r_3 =	r_4 =
v_1 = 0 m/s	v_2 = 0 m/s	v_3 =	v_4 = 0 m/s
a_1 = 0 m/s^2	a_2 = 0 m/s^2	a3 =	a_4 = -9.8 m/s^2

Between event 2 and 3, the rocket's acceleration is given by the function above:

$$a(t) = -7.5t^2 + 30t - 9.8$$

We can integrate the acceleration to determine the velocity,

$$v(t) = \int a(t)dt$$
$$v(t) = \int \left(-7.5t^2 + 30t - 9.8\right) dt$$
$$v(t) = -2.5t^3 + 15t^2 - 9.8t + C$$

Since we know v = 0 m/s when t = 0.36 s, we can determine the integration constant:

$$v(0.36) = -2.5(0.36)^3 + 15(0.36)^2 - 9.8(0.36) + C = 0$$
$$C = 1.70\text{ m/s}$$

Therefore,

$$v(t) = -2.5t^3 + 15t^2 - 9.8t + 1.7$$

and when the thrust ends, at t_3 = 4.0 s, v_3 = 42.5 m/s.

To find the position of the rocket when the thrust ends, integrate the velocity function:

$$r(t) = \int v(t)dt$$
$$r(t) = \int \left(-2.5t^3 + 15t^2 - 9.8t + 1.7\right) dt$$
$$r(t) = -0.625t^4 + 5t^3 - 4.9t^2 + 1.7t + D$$

Since we know r = 0 m when t = 0.36 s, we can determine the integration constant:

$$r(0.36) = -0.625(0.36)^4 + 5(0.36)^3 - 4.9(0.36)^2 + 1.7(0.36) + D = 0$$
$$D = -0.178$$

Therefore,

$$r(t) = -0.625t^4 + 5t^3 - 4.9t^2 + 1.7t - 0.178$$

and when the thrust ends, at t_3 = 4.0 s, r_3 = 88.2 m.

Between event 3 and 4 the acceleration is constant, so we can use our constant-acceleration kinematic equations:

$$\begin{array}{ll} v_4 = v_3 + a_{34}\left(t_4 - t_3\right) & r_4 = r_3 + v_3\left(t_4 - t_3\right) + 1/2\, a_{34}\left(t_4 - t_3\right)^2 \\ 0 = 42.5 + (-9.8)\left(t_4 - 4\right) & r_4 = 88.2 + 42.5(8.34 - 4) + 1/2\,(-9.8)(8.34 - 4)^2 \\ t_4 = 8.34\text{ s.} & r_4 = 180\text{ m} \end{array}$$

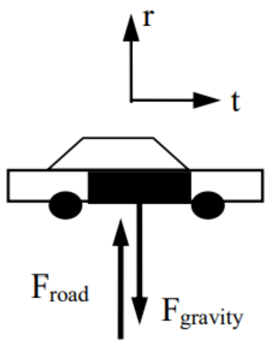
Thus, the rocket would achieve a maximum height of 180 m in a friction-free environment.

Applying Newton's Second Law to Circular Motion

Investigate the situation below, in which an object travels on a curved path.Note

> *During a high speed auto chase in San Francisco, a 1745 kg police cruiser traveling at constant speed loses contact with the road as it goes over the crest of a hill with radius of curvature 80 m.*

A free-body diagram for the car at the instant it's at the crest of the hill is sketched below.



It's doubtful that the shape of the hill that the car travels on is circular. However, at every point on the hill we can approximate the path of the car with a circle of appropriate radius. This "instantaneous circular path" has a radius equal to the given radius of curvature. *Any* curved path can be approximated as a series of circular paths. Therefore, since the car is traveling along a circular path, it's best to analyze the situation using polar coordinates. Remember, in polar coordinates the radial direction points away from the center of the circular path.

In general you should apply Newton's second law independently in the radial (r) and tangential (t) directions, although since the cruiser is moving at constant speed there is no acceleration in the tangential direction and therefore no net force in the tangential direction.

$$\underline{\mathbf{r-direction}}$$
$$\Sigma F = ma$$
$$F_{\text{road}} - F_{\text{gravity}} = ma_r$$
$$F_{\text{road}} - F_{\text{gravity}} = m\left(-R\omega^2\right)$$

Since the cruiser loses contact with the road, $F_{road} = 0$ N.

$$0 - 1745(9.8) = -1745(80)\omega^2$$
$$\omega = 0.35\text{rad/s}$$

Since v = R ω,

$$v = (80\text{ m})(0.35\text{rad/s})$$
$$v = 28\text{ m/s}$$

For the car to lose contact with the road, it must be traveling at a speed of 28 m/s or greater when it reaches the crest of the hill. At lower speeds, the car would remain in contact with the road.

Since it is often useful to directly relate the radial acceleration of an object to its velocity, let's construct a direct relationship between these two variables:

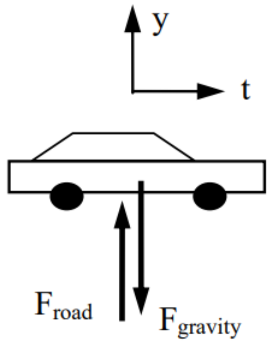
$$a_r = -R\omega^2$$
$$a_r = -R\left(\frac{v}{R}\right)^2$$
$$a_r = -\frac{v^2}{R}$$

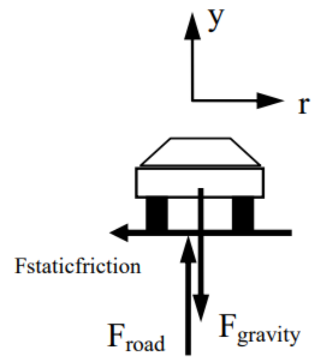
This relationship, although mathematically equivalent to $a_r = -R\omega^2$, is in a more "useful" form.

Another Circular Motion Scenario

> *A 950 kg car, traveling at a constant 30 m/s, safely makes a lefthand-turn with radius of curvature 75 m.*

First, let's draw a pair of free-body diagrams for the car, a side-view (on the left) and a rearview (on the right)





The free-body diagram on the left is a side-view of the car. Notice that the upward direction is the y-direction, and the forward direction, tangent to the turn, is the t-direction.

The free-body diagram on the right is a rear-view of the car. This is what you would see if you stood directly behind the car. Notice that the upward direction is still the y-direction, and the horizontal direction, *perpendicular* to the direction of travel and hence directed radially outward, is the r-direction. The frictional force indicted is *perpendicular* to the tread on the tire. This force causes the car to accelerate *toward* the center of the turn. Remember, if the car is going to travel along a circular path, it must have an acceleration directed toward the center of the circle. *Something* has to be supplying the force that creates this acceleration. This something is the *static* friction between the tire and the road that acts to prevent the car from sliding out of the turn. Since the car has no velocity in the radial direction, the frictional force that points in this direction must be static!

Now that we have all that straightened out (maybe), let's apply Newton's Second Law.

$\underline{\mathbf{y-direction}}$	$\underline{\mathbf{r-direction}}$
$\Sigma F = ma$	$\Sigma F = ma$
$F_{\text{road}} - F_{\text{gravity}} = ma_y$	$-F_{\text{staticfriction}} = ma_r$
$F_{\text{road}} - 950(9.8) = 950(0)$	$-F_{\text{staticfriction}} = m\left(-\frac{v^2}{R}\right)$
$F_{\text{road}} = 9310N$	$F_{\text{staticfriction}} = 950\left(\frac{30^2}{75}\right)$
	$F_{\text{staticfriction}} = 11400 \text{ N}$

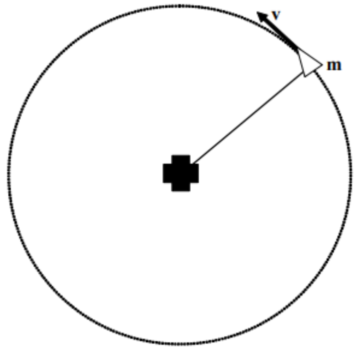
Thus, to safely make this turn requires at least 11400 N of static friction. Using this value, I should be able to compute the minimum coefficient of friction necessary for the car to safely round this turn at this speed.

$$F_{\text{staticfriction}} \leq \mu_s F_{\text{road}}$$
$$11400 \leq \mu_s(9310)$$
$$\mu_s \geq 1.22$$

Although this is a large value for the coefficient of static friction, it is an attainable value for a sports car with performance tires.

Activities

Six artificial satellites of mass, m, circle a space station at constant speed, v.



	m	v
A	200 kg	160 m/s
B	100 kg	160 m/s
C	400 kg	80 m/s
D	800 kg	40 m/s
E	200 kg	120 m/s
F	300 kg	80 m/s

a. If the distance between the space station and the satellites is the same for all satellites, rank the magnitude of the net force acting on each satellite.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

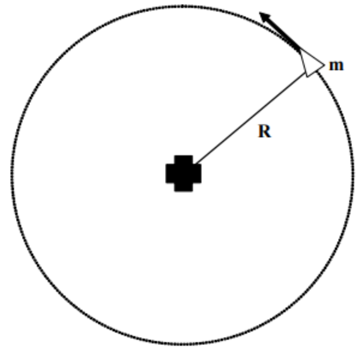
b. If the period is the same for all satellites, rank the magnitude of the net force acting on each satellite.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Six artificial satellites of mass, m, circle a space station at distance, R.



	m	R
A	200 kg	5000 m
B	100 kg	10000 m
C	400 kg	2500 m
D	800 kg	5000 m
E	100 kg	2500 m
F	300 kg	7500 m

a. If the tangential speed is the same for all satellites, rank the magnitude of the net force acting on each satellite.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

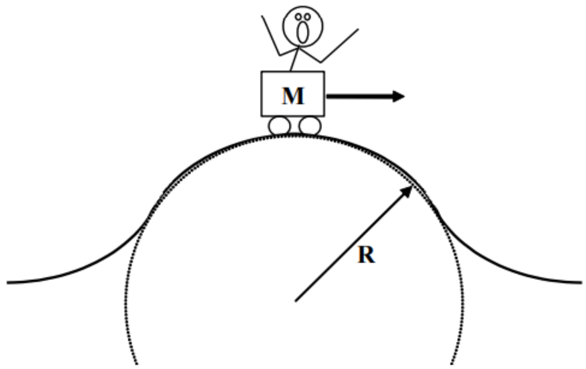
b. If the period is the same for all satellites, rank the magnitude of the net force acting on each satellite.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Six rollercoaster carts pass over semi-circular “bumps”. The mass of each cart (including passenger), M, and the force of the track on the cart at the apex of each bump, F, are given below.



	M	F
A	200 kg	400 N
B	100 kg	400 N
C	400 kg	200 N
D	800 kg	800 N
E	100 kg	800 N
F	300 kg	300 N

a. If the radius of each bump is the same, rank the speed of the cart as it passes over the apex of the bump.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

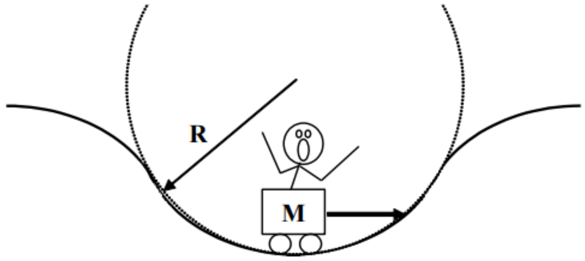
b. If the speed of each cart is the same at the apex, rank the radius of each bump.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A rollercoaster track has six semi-circular “dips” with different radii of curvature, R. The rollercoaster cart rides through each dip at a different speed, v.



	R	v
A	30 m	16 m/s
B	60 m	16 m/s
C	15 m	8 m/s
D	30 m	4 m/s
E	15 m	12 m/s
F	45 m	4m/s

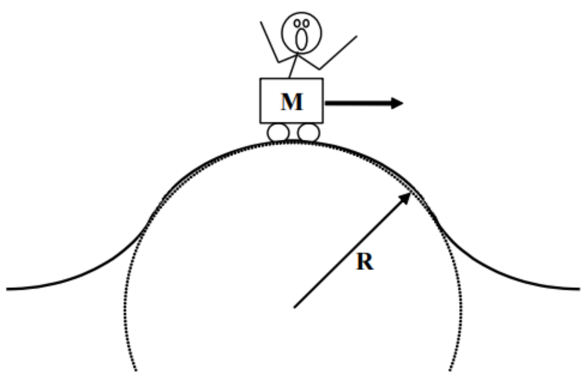
Rank the magnitude of the force of the rollercoaster track on the cart at the bottom of each dip.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A rollercoaster track has a semi-circular "bump" of radius of curvature R. A rollercoaster cart (including passenger) of total mass M rides over the bump.



	M	R
A	200 kg	30 m
B	100 kg	60 m
C	400 kg	15 m
D	800 kg	30 m
E	100 kg	15 m
F	300 kg	45 m

Rank the minimum speed necessary for the cart to momentarily lose contact with the track at the top of the bump

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

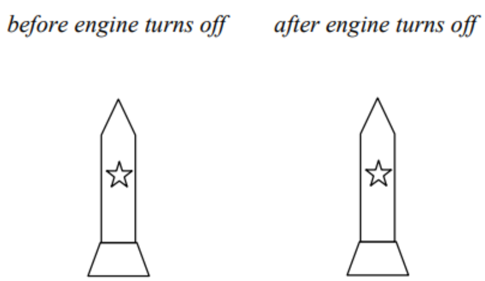
Explain the reason for your ranking:

A 100 kg rocket is fired vertically upward. Its engine supplies an upward force of magnitude $F = (5000 - 5.0t^2)$ N (where t is in seconds) until F = 0 N, then the engine shuts off.

Motion Information

Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_1 =	a_2 =	a3 =

Free-Body Diagrams



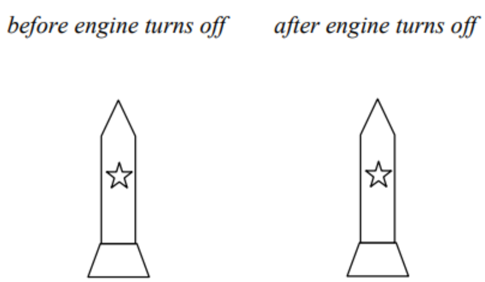
Mathematical Analysis[20]

A 75 kg rocket is launched directly upward. The force on the rocket due to its engine increases from 0 N to 5000 N over 8.0 s as a linear function of time. The thrust then drops to zero almost instantaneously.

Motion Information

Event 1:	Event 2:	Event 3:	Event 4:
t_1 =	t_2 =	t_3 =	t_4 =
r_1 =	r_2 =	r_3 =	r_4 =
v_1 =	v_2 =	v_3 =	v_4 =
a_1 =	a_2 =	a_3 =	a_4 =

Free-Body Diagrams



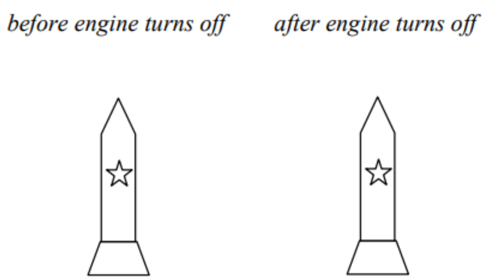
Mathematical Analysis[21]

A 75 kg rocket is launched directly upward. The force on the rocket due to its engine decreases from 5000 N to 0 N as a quadratic function of time. The rocket reaches a speed of 200 m/s during the time interval that its engine fires.

Motion Information

Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_1 =	a_2 =	a3 =

Free-Body Diagrams



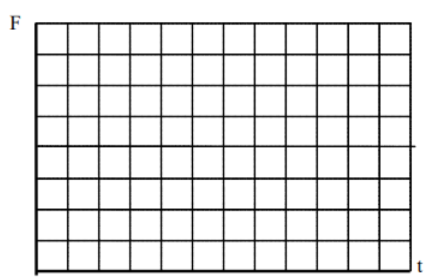
Mathematical Analysis[22]

Two 75 kg rockets, the A-57X and the B-44ZC, are launched directly upward. The force on each rocket due to its engine decreases from 5000 N to 0 N over 8.0 s. The force on the A-57X decreases as a linear function of time while the force on the B-44ZC decreases as a quadratic function of time.

Motion Information

A-57X		B-44ZC	
Event 1:	Event 2:	Event 1:	Event 2:
t_1 =	t_2 =	t_1 =	t_2 =
r_1 =	r_2 =	r_1 =	r_2 =
v_1 =	v_2 =	v_1 =	v_2 =
a_1 =	a_2 =	a_1 =	a_2 =

Force Graph



Question

Based only on the graph, which rocket is traveling faster after 8.0 s? Explain.

Mathematical Analysis[23]

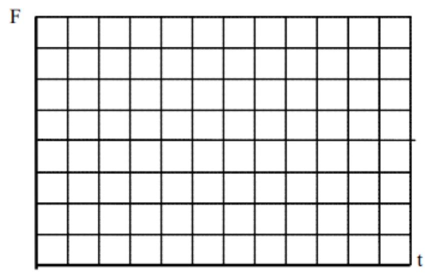
Two 75 kg rockets, the A-57X and the B-44ZC, are launched directly upward. Both rockets reach a speed of 200 m/s after traveling for 7.1 s. The force on the A-57X due to its engine decreases to 0 N in 7.1 s as a linear function of time while the force on the B-

44ZC due to its engine decreases to 0 N in 7.1 s as a quadratic function of time.

Motion Information

A-57X		B-44ZC	
Event 1:	Event 2:	Event 1:	Event 2:
$t_1 =$	$t_2 =$	$t_1 =$	$t_2 =$
$r_1 =$	$r_2 =$	$r_1 =$	$r_2 =$
$v_1 =$	$v_2 =$	$v_1 =$	$v_2 =$
$a_1 =$	$a_2 =$	$a_1 =$	$a_2 =$

Force Graph



Question

Based only on the graph, which rocket's engine exerts the larger maximum force? Explain.

Mathematical Analysis[24]

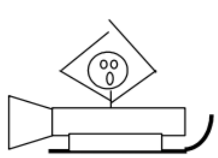
The horizontal thrust acting on a 270 kg rocket sled increases as a cubic function of time from 0 N to 5400 N in 4.3 s. The thrust then drops to zero almost instantaneously. The effective frictional coefficient acting between the rocket sled and the ground is (0.4, 0.3). The rocket sled starts from rest on a level surface.

Motion Information

Event 1:	Event 2:	Event 3:	Event 4:
$t_1 =$	$t_2 =$	$t_3 =$	$t_4 =$
$r_1 =$	$r_2 =$	$r_3 =$	$r_4 =$
$v_1 =$	$v_2 =$	$v_3 =$	$v_4 =$
$a_1 =$	$a_2 =$	$a_3 =$	$a_4 =$

Free-Body Diagram

before engine turns off



Mathematical Analysis[25]

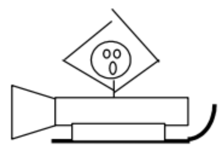
The horizontal thrust acting on a 270 kg rocket sled decreases from 5000 N to 0 N as a quadratic function of time. The rocket sled is traveling at 195 m/s when the engine shuts down. The effective frictional coefficient acting between the rocket sled and the *ground is (0.4, 0.3). The rocket sled starts from rest on a level surface.*

Motion Information

Event 1:	Event 2:	Event 3:
t_1 =	t_2 =	t_3 =
r_1 =	r_2 =	r_3 =
v_1 =	v_2 =	v_3 =
a_1 =	a_2 =	a_3 =

Free-Body Diagram

before engine turns off



Mathematical Analysis[26]

A rollercoaster cart travels through a semicircular "bump" of radius 20 m followed by a "dip" of radius 25 m. At the apex of the bump, the 50 kg passenger momentarily loses contact with her seat, and at the bottom of the dip the bathroom scale she's sitting on reads 1020 N.

a. How fast is the cart moving at the apex of the bump?

Free-Body Diagram

Mathematical Analysis[27]

b. How fast is the cart moving at the bottom of the dip?

Free-Body Diagram

Mathematical Analysis

A 60 kg woman rides in a Ferris wheel of radius 16 m. In order to better understand physics, she takes along a bathroom scale and sits on it. When at the top, the scale reads 540 N. The Ferris wheel maintains the same speed throughout its motion. She checks the scale again at the bottom of the motion.

a. What is the angular speed of the Ferris wheel?

Free-Body Diagram

Mathematical Analysis[28]

b. What does the bathroom scale read at the bottom of the motion?

Free-Body Diagram

Mathematical Analysis

A 1.5 kg pendulum bob swings from the end of a 2 m long string. The maximum angle from vertical reached by the pendulum is 15°, and the bob reaches a speed of 1.16 m/s as it passes through its lowest point.

a. What is the force exerted on the bob by the string at the maximum angle?

Free-Body Diagram

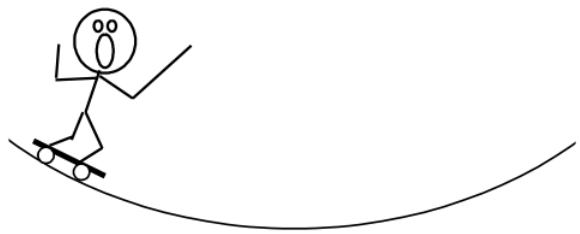
Mathematical Analysis[29]

b. What is the force exerted on the bob by the string at the lowest point?

Free-Body Diagram

Mathematical Analysis

The 65 kg strange man at right is skateboarding in a halfpipe. The half-pipe is a half-circle with a radius of 5 m. At the instant shown he is 2 m from the bottom of the halfpipe, measured vertically, and is moving at 7.5 m/s. He reaches a speed of 9.5 m/s at the bottom.



a. What is the force exerted by the skateboard on the man at the instant shown?

Free-Body Diagram

Mathematical Analysis[30]

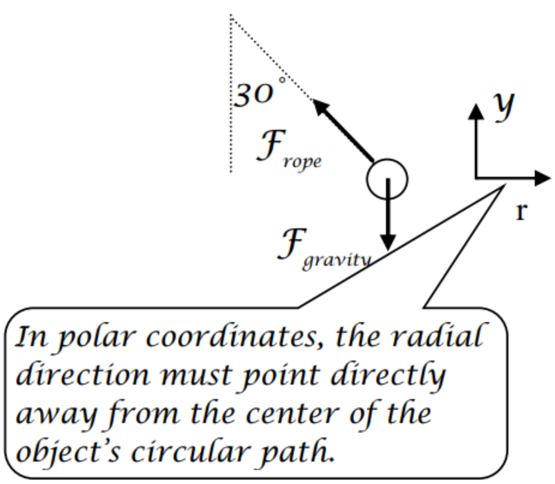
b. What is the force exerted by the skateboard on the man at the bottom of the half-pipe?

Free-Body Diagram

Mathematical Analysis

A 2.0 kg tether ball is attached to a vertical pole by a 1.5 m long rope. The ball is swung around the pole at an angle of 30^0 from vertical.

Free-Body Diagram



Mathematical Analysis

y-direction

$$\Sigma F = ma$$
$$F_{\text{rope}} \cos 30 - F_{\text{gravity}} = ma_y$$
$$0.866 F_{\text{rope}} - 2(9.8) = 2(0)$$
$$F_{\text{rope}} = 22.6N$$

> *There's no acceleration in the vertical direction if the angle of the rope is constant.*

x-direction

$$\begin{aligned}
&\Sigma F = ma \\
&-F_{rope}\sin 30 = ma_r \\
&-22.6(0.5) = 2\left(-\frac{v^2}{R}\right) \\
&-11.3 = 2\left(-\frac{v^2}{1.5\sin 30}\right) \\
&-11.3 = -2.67v^2 \\
&v = 2.06\ \mathrm{m/s}
\end{aligned}$$

> *The radius of the ball's circular path is not equal to the length of the rope.*

A 2.0 kg tether ball swings around a vertical pole attached to two 1.5 m long ropes, each at an angle of 30^0 from vertical. One rope is attached to the top of the ball and the top of the pole, the other rope is attached to the bottom of the ball and the bottom of the pole. The ball is traveling at 3.5 m/s.

Free-Body Diagram

Mathematical Analysis[31]

In the hammer throw, a 7.3 kg steel ball at the end of a 1.22 m wire is swung in an approximately circular path around a thrower's head. Assume the ball is traveling at a constant speed of 7 m/s.

Free-Body Diagram

Mathematical Analysis[32]

A 1000 kg car rounds a 50 m radius curve at constant speed without slipping. The coefficient of friction between the car's tires and the road is (0.6,0.5).

Free-Body Diagram

rear view

Mathematical Analysis[33]

An 400 m radius, banked, highway curve is designed to allow cars to drive through the curve at a speed of 20 m/s and not slip, even when the road is extremely icy.

Free-Body Diagram

rear view

Mathematical Analysis[34]

A 100 m radius, 8^0 banked, highway curve has just been built. You decide to find maximum velocity with which you can drive your 1200 kg pick-up truck through this curve without sliding up or down the incline. The coefficient of friction between your enormous tires and the road is (0.7, 0.6).

Free-Body Diagram

rear view

Mathematical Analysis[35]

You enter an 80 m radius, 6^0 banked, highway curve at 30 m/s. The coefficient of friction between your tires and the road is (0.9, 0.8) .

Free-Body Diagram

rear view

Mathematical Analysis[36]

In an amusement park ride called the Rotor, a circular, 4.0 m radius room is spun around at high speed and then the floor is removed. The people riding the Rotor feel that they are being pressed against the wall with such a large force that they do not slide down the wall to the floor. (Obviously they do not understand physics.) The coefficient of friction between the riders and the wall is (0.6,0.5).

Free-Body Diagram

Mathematical Analysis[37]

A rollercoaster track has a semicircular "bump" of radius R. A passenger of mass m sits on a bathroom scale in a rollercoaster cart. Determine the reading of the bathroom scale (F_{scale}) as the cart goes over the top of the bump as a function of the speed of the cart (v), m, R, and g.

Free-Body Diagram

Mathematical Analysis

Questions

If $R = \infty$, what should F_{scale} equal? Does your function agree with this observation?

If $v = 0$ m/s, what should F_{scale} equal? Does your function agree with this observation?

At what speed will the rider lose contact with the scale?

*A woman of mass m rides in a Ferris wheel of radius R. In order to better understand physics, she takes along a bathroom scale and sits on it. Determine the **difference** in scale readings between the bottom and top of the Ferris wheel (ΔF_{scale}) as a function of the constant angular speed of the Ferris wheel (ω), m, R, and g.*

Free-Body Diagram

top of Ferris wheel

bottom of Ferris wheel

Mathematical Analysis

Questions

If ω = 0 rad/s, what should ΔF_{scale} equal? Does your function agree with this observation?

If ω was twice as large, what would happen to ΔF_{scale}?

A banked highway curve with radius of curvature R has just been built. With bald tires on an icy morning, determine the maximum speed (v_{max}) with which you can make the turn without skidding as a function of the banking angle from horizontal (θ), the car's mass, R, and g.

Free-Body Diagram

rear view

Mathematical Analysis

Questions

If the mass of the car was twice as large, what would happen to v_{max}?

If θ = 0°, what should v_{max} equal? Does your function agree with this observation?

If θ = 90°, what should v_{max} equal? Does your function agree with this observation?

A tether ball of mass m is attached to a vertical pole by a rope of length L. Determine the angle the rope makes with the pole (θ) as an implicit function of the speed of the ball (v), m, L, and g.

Free-Body Diagram

Mathematical Analysis

Questions

If the mass of the ball was twice as large, what would happen to θ*?*

If v = 0 m/s, what should θ *equal? Does your function agree with this observation?*

If v = ∞, what should θ *equal? Does your function agree with this observation?*

4.3: Conservation Laws

Concepts and Principles

The Impulse-Momentum Relation

We've already used the impulse-momentum relation to analyze situations involving constant forces. The relation is typically applied in its component form:

$$mv_{xi} + \Sigma\left(F_x(\Delta t)\right) = mv_{xf}$$
$$mv_{y_i} + \Sigma\left(F_y(\Delta t)\right) = mv_{y_f}$$
$$mv_{zi} + \Sigma\left(F_z(\Delta t)\right) = mv_{zf}$$

Hopefully it's not too much of a stretch to argue that for forces that vary in magnitude or direction the simple summation over a time interval (δt) must be replaced by an integral over an infinitesimal time (dt):

$$mv_{x_i} + \int_{t_i}^{t_f} F_x dt = mv_{x_f}$$
$$mv_{y_i} + \int_{t_i}^{t_f} F_y dt = mv_{y_f}$$
$$mv_{zi} + \int_{t_i}^{t_f} F_z dt = mv_{zf}$$

Regardless of whether the forces acting on an object are constant or not, the impulse they exert on the object is precisely equal to the change in the object's momentum.

The Work-Energy Relation

Our previous encounter with the work-energy relation resulted in:

$$\frac{1}{2}m{v_i}^2 + mgh_i + \Sigma(|F||\Delta r|\cos\phi) = \frac{1}{2}m{v_f}^2 + mgh_f$$

where the forces acting on the object of interest were constant in both magnitude and direction. Again, for forces that vary, I will generalize this result to:

$$\frac{1}{2}m{v_i}^2 + mgh_i + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}m{v_f}^2 + mgh_f$$

where the angle ϕ is the angle between the instantaneous force acting on the object and the instantaneous displacement of the object (dr). Thus, this angle can change as the object moves along its path. Technically, this integral is termed a *line integral* and its evaluation can be rather complicated.

Also recall from our previous discussion of work-energy that this is *not* a vector equation, meaning it is not applied independently in each of the coordinate directions.

Analysis Tools

Applying the Impulse-Momentum Relation

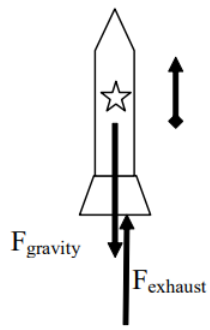
Let's re-examine the same situation we examined at the beginning of the previous chapter, a rocket launched directly upward with a time-dependent thrust.

> *A 2.0 kg toy rocket is fitted with an engine that provides a thrust roughly modeled by the function F(t) = (60 N/s) t - (15 N/s^2) t^2, for 0 < t < 4.0 s, and zero thereafter. The rocket is launched directly upward.*

For analysis, we'll apply the impulse-momentum relation between:

Event 1: The instant the rocket leaves the launch-pad.

Event 2: The instant the thrust drops to zero.



$$mv_{y_i} + \int_{t_i}^{t_f} F_y dt = mv_{y_f}$$

$$2(0) + \int_{t_i}^{t_f} (F_{\text{thrust}} - F_{\text{gravity}})\, dt = 2v_{y_f}$$

$$\int_{0.36}^{4} \left(60t - 15t^2 - 2(9.8)\right) dt = 2v_{y_f}$$

Remember from last chapter that the rocket does not leave the launch pad until 0.36 s after the engine is ignited.

$$\int_{0.36}^{4} \left(60t - 15t^2 - 19.6\right) dt = 2v_{y_f}$$

$$\left[30t^2 - 5t^3 - 19.6t\right]_{0.36}^{4} = 2v_{y_f}$$

$$\left(30(4)^2 - 5(4)^3 - 19.6(4)\right) - \left(30(0.36)^2 - 5(0.36)^3 - 19.6(0.36)\right) = 2v_{y_f}$$

$$v_{y_f} = 42.5 \text{ m/s}$$

When the engine shuts off, the rocket is traveling at 42.5 m/s upward.

We could also apply the impulse-momentum relation between:

Event 1: The instant the thrust drops to zero.

Event 2: The instant the rocket reaches its maximum height.

During this interval, the only force acting on the rocket is the force of gravity, and the impulse-momentum relation is:

$$mv_{y_i} + \int_{t_i}^{t_f} F_y dt = mv_{y_f}$$

$$2(42.5) + \int_{4}^{t_f} (-F_{\text{gravity}})\, dt = 2(0)$$

$$85 + \int_{4}^{t_f} (-19.6) dt = 0$$

$$85 - 19.6\,(t_f) + 19.6(4) = 0$$

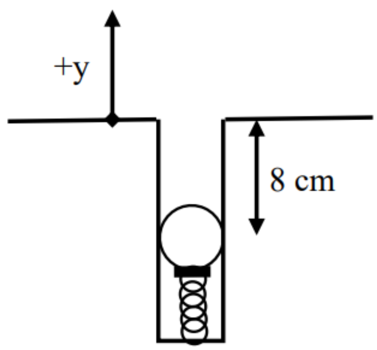
$$t_f = 8.34s$$

Thus, the rocket reaches its highest altitude 8.34 s after launch.

It’s important to note that when a force is a function of time, it’s relatively easy to integrate the function and determine the impulse. However, it should be clear that it would *not* be easy to determine the work done by a force of this type. Since work is expressed as an integral of a force with respect to a displacement (dr), the force function has to be expressed in terms of position, r. In general, it’s not an easy (or sometimes possible) task to “convert” a function of time into a function of position, so work-energy is not a particularly useful way to analyze systems when the forces acting are time dependent. However, if the forces depend upon the *position* of the object, work-energy is a powerful analysis tool.

> *A 0.15 kg ball is launched vertically upward by means of a spring-loaded plunger, pulled back 8.0 cm and released. It requires a force of about 10 N to push the plunger back 8.0 cm.*

The Force Exerted by a Spring



The force that the spring exerts on the ball depends on the amount by which the spring is compressed. The more the spring is compressed, the larger the force it exerts on the ball. A common *model* is that the force exerted by a spring is directly proportional to the amount of deformation of the spring, in this case compression. Deformation (s) is defined to be the difference between the current length of the spring (L) and the equilibrium length (L_0):

$$s = L - L_0$$

If we define the positive coordinate direction to point in the same direction as positive deformation (stretch), then

$$F_{\text{spring}} = -ks$$

with the proportionality constant, k, referred to as the *spring constant*.

For the plunger, since it takes 10 N to compress the spring by 8.0 cm,

$$\begin{aligned} &F_{\text{spring}} = -ks \\ &10 = -k(-0.08) \\ &k = 125\frac{N}{m} \end{aligned}$$

For analysis, we'll apply the work-energy relation between:

Event 1: The instant the plunger is released.

Event 2: The instant the ball reaches its maximum height.

For these two events, work-energy looks like this:

$$\frac{1}{2}mv_i{}^2 + mgh_i + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}mv_f{}^2 + mgh_f$$

$$0 + 0.15(9.8)(-0.08) + \int_{y_i}^{y_f}(F_{\text{spring}}\cos\phi)\,dy = 0 + 0.15(9.8)h_f$$

To do the integral, we must express F_{spring} in terms of the variable of integration, y. For the coordinate system chosen, s and y are identical. Also note that the force of the spring and the direction of motion of the ball point in the same direction. Thus, $\phi = 0$.

$$-0.118+\int_{-0.08}^{0}(-125s)(\cos 0)ds = 1.47h_f$$
$$-0.118+\left[-62.5s^2\right]_{-0.08}^{0} = 1.47h_f$$
$$-0.118+\left[0+62.5(-0.08)^2\right] = 1.47h_f$$
$$-0.118+0.40 = 1.47h_f$$
$$h_f = 0.19m$$

The ball reaches a maximum height of 19 cm above the top of the plunger.

Elastic Potential Energy

When an object interacts with a spring, or other elastic material, a common model is that the material reacts linearly, i.e., with a force directly proportional to the deformation of the material. It is possible to calculate the work done by the linear material in general, and to rewrite the work-energy relation in such a way as to incorporate the effects of this work from the start. This is referred to as constructing a *potential energy function* for the work done by the elastic material. We did exactly the same thing earlier for the force of gravity.

Imagine a spring of spring constant k, initially deformed by a distance s_i. It changes its deformation, ultimately resulting in deformation s_f. To calculate the work done by the spring on the object causing the deformation:

$$\text{Work } = \int_{r_i}^{r_f}(F\cos\phi)dr$$
$$\text{Work } = \int_{r_i}^{r_f}(-ks)(\cos\phi)dr$$

Choosing a coordinate system in which s and r are interchangeable (the origin is located at the point where the spring is at its natural length and the direction of elongation is positive) and allowing the force and the displacement to be in the same direction ($\phi = 0$) results in,

$$\text{Work } = \int_{s_i}^{s_f}(-ks)(\cos 0)ds$$
$$\text{Work } = -\frac{1}{2}k{s_f}^2+\frac{1}{2}k{s_i}^2$$

The terms, $\frac{1}{2}\mathrm{ks}^2$, are referred to as *elastic potential energy*.

Inserting this result into the work-energy relation results in

$$\frac{1}{2}m{v_i}^2+\frac{1}{2}k{s_i}^2+mgh_i+\int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}m{v_f}^2+\frac{1}{2}k{s_f}^2+mgh_f$$

with the understanding that the forces remaining in the equation, which may do work on the system, do not include the force of gravity or the force of the spring. The work done by the force of gravity and the force of the spring are already included in the relation via the inclusion of the potential energy terms.

Applying the Work-Energy Relation with Elastic Potential Energy

> *A 0.15 kg ball is launched vertically upward by means of a spring-loaded plunger, pulled back 8.0 cm and released. It requires a force of about 10 N to push the plunger back 8.0 cm.*

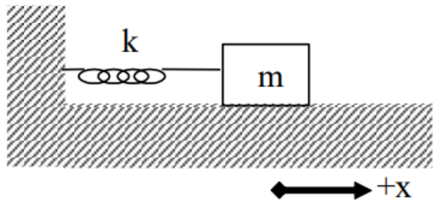
The spring constant of the plunger is known to be 125 N/m from above. Applying the work-energy relation with potential energy terms between the instant the plunger is released and the instant the ball reaches its maximum height results in,

$$\frac{1}{2}m{v_i}^2+\frac{1}{2}k{s_i}^2+mgh_i+\int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}m{v_f}^2+\frac{1}{2}k{s_f}^2+mgh_f$$
$$0+\frac{1}{2}(125)(-0.08)^2+0.15(9.8)(-0.08)+0 = 0+0+0.15(9.8)h_f$$
$$h_f = 0.19m$$

Of course, this results in the same answer as before.

Applying Newton's Second Law to a Spring-Mass System

Since we didn't analyze systems involving springs in the previous dynamics section, we should make up for that omission now.



The system above consists of a spring with spring constant k attached to a block of mass m resting on a frictionless surface. The origin of the coordinate system is located at the position in which the spring is unstretched.

Now imagine the block is pulled to the right and let go. Hopefully you can convince yourself that the block will oscillate back and forth. Let's apply Newton's Second Law at the instant the mass is at an arbitrary position, x. The only force acting on the mass in the x-direction is the force of the spring.

$$\begin{aligned} \Sigma F &= ma \\ F_{\text{spring}} &= ma \\ -ks &= ma \end{aligned}$$

Because of our choice of coordinate system, the stretch of the spring (s) is exactly equal to the location of the block (x). Therefore,

$$-kx = ma$$

Note that when the block is at a positive position, the force of the spring is in the negative direction and when the block is at a negative position, the force of the spring is in the positive direction. Thus, the force of the spring always acts to return the block to equilibrium.

Rearranging gives

$$\begin{aligned} -\frac{k}{m}x &= a \\ -\frac{k}{m}x &= \frac{d^2x}{dt^2} \end{aligned}$$

and defining a constant, ω^2, as

(Granted, it seems pretty silly to define $\frac{k}{m}$ as the square of a constant, but just play along. You may also find it frustrating to learn that this "omega" is *not* an angular velocity. The block doesn't even have an angular velocity!)

yields,

$$-\omega^2 x = \frac{d^2x}{dt^2}$$

Therefore, the position function for the block must have a second time derivative equal to the product of $(-\omega^2)$ and itself. The only functions whose second time derivative is equal to the product of a negative constant and itself are the sine and the cosine functions. Therefore, a solution to this differential equation[5]

$$-\omega^2 x = \frac{d^2x}{dt^2}$$

can be written:

$$x(t) = A\cos(\omega t + \phi)$$

or equivalently with the sine function, where A and ϕ are arbitrary constants.[6]

- **A** is the *amplitude* of the oscillation. The amplitude is the maximum displacement of the object from equilibrium.

- ϕ is the *phase angle*. The phase angle is used to adjust the function forward or backward in time. For example, if the particle is at the origin at t = 0 s, ϕ must equal $+\pi/2$ or $-\pi/2$ to ensure that the cosine function evaluates to zero at t = 0 s. If the particle is at its maximum position at t = 0 s, then the phase angle must be zero or π to ensure that the cosine function evaluates to +1 or -1 at t = 0 s.
- ω is the *angular frequency* of the oscillation.[7]

Note

[5] A differential equation is an equation involving a function and its derivative(s).

[6] To prove to yourself that this is indeed the solution to the equation, you should substitute the function, x(t), into the left side of the equation and the second derivative of x(t) into the right side. This will verify that the two sides of the equation are equal. In addition to mathematically verifying this solution, you should verify the solution physically by sketching a graph of the motion that you *know* would result if the block were displaced to the right and comparing that sketch to a sketch of the function.

[7] Again, note that ω is *not* the angular velocity. The block is not rotating; it does *not* have an angular velocity.

Note that the cosine function repeats itself when its argument increases by 2π. Thus, when

$$\Delta(\omega t+\phi)=2\pi$$

the function repeats. Since ω and ϕ are constant,

$$\Delta(\omega t+\phi)=\omega\Delta t$$

Therefore, the time interval when

$$\omega\Delta t=2\pi$$

is the time interval for one complete cycle of the oscillatory motion. The time for one complete cycle of the motion is termed the *period*, T. Thus,

$$T=\frac{2\pi}{\omega}$$

Therefore, the physical significance of the angular frequency is that it is inversely proportional to the period.

Substituting in the definition of ω:

$$\omega=\sqrt{\frac{k}{m}}$$

yields

$$T=2\pi\sqrt{\frac{m}{k}}$$

In summary, a mass attached to a spring will oscillate about its equilibrium position with a position function given by:

$$x(t)=A\cos(\omega t+\phi)$$

This function repeats with a period of

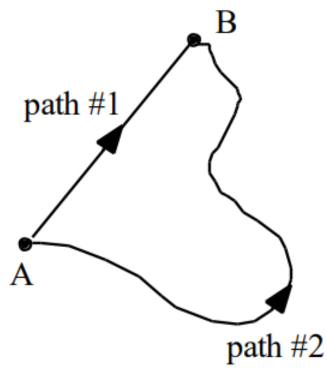
$$T=2\pi\sqrt{\frac{m}{k}}$$

Potential Energy Functions

The potential energy functions for the work done by gravity and by springs make analyzing many situations much easier. In light of this, why don't we construct potential energy functions for the work done by every different type of force that could possibly act on an object? Of course, one reason is that there are too many different types of forces. Having a potential energy function for the work done by every one of them would lead to so many potential energy functions that it would be hard to keep them all straight.

Another, more subtle reason is that it is impossible to construct potential energy functions for certain forces. A potential energy function must, by definition, be a *function*. Mathematically speaking, a potential energy *function* of position must assign a single, specific value of potential energy to every position. Functions must be single-valued. Notice that the gravitational and elastic potential energies are single valued. If you specify a height off of the ground, or the deformation of the spring, the potential energy function tells you exactly how much energy the system possesses at that position, regardless of the path the object took to reach that position.

In light of this observation, let's try to create a potential energy function to replace the work done by friction.



Imagine sliding an object along a rough surface from point A to point B. If you slide the object along path #1, the force of friction will do a certain amount of work on the object. (This work will be negative because the direction of the force of friction is always in opposition to the change in position of the object.)

If you slide the object along path #2, you should see that the magnitude of the work done by friction will be greater. (Although the frictional force will be the same in magnitude, the distance over which the frictional force acts will be larger. Thus, a larger amount of negative work will be done by friction.)

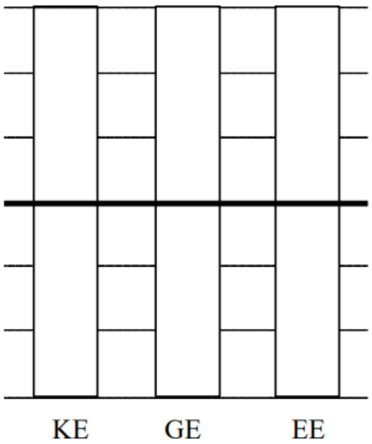
Now let's try to create a potential energy function for the work done by friction. Since we are always free to choose a coordinate system, we can choose a system in which the potential energy at A is zero. What is the value for the potential energy at B? Since the work done by friction depends on the path taken from A to B, so must the potential energy. However, this leaves us with a potential energy at B that can be either one of two values, depending on the path taken! Since a function must be single-valued, the work done by friction cannot be represented by a function. You cannot create a potential energy function for the work done by friction!

There are also other forces whose work cannot be represented by a potential energy function. In general, forces whose work can be represented by a potential energy function are termed *conservative* forces, while those for which potential energy functions cannot be constructed are termed *non-conservative*.

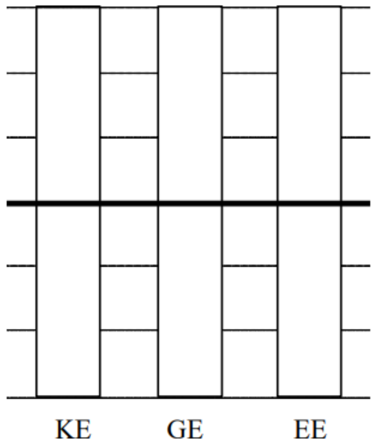
Activities

For each of the scenarios described below, indicate the amount of kinetic energy, gravitational potential energy, and elastic energy in the system at each of the events listed. Use a consistent scale throughout both motions.

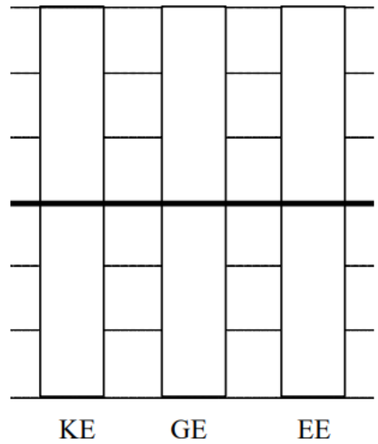
a. *A 75 kg bungee jumper steps off a platform high above a raging river and plummets downward. The elastic bungee cord has an effective spring constant of 50 N/m and is initially slack, although it begins to stretch the moment the jumper steps off of the platform. Set the lowest point of the jumper as the zero-point of gravitational potential energy*



When the jumper first steps off the platform.

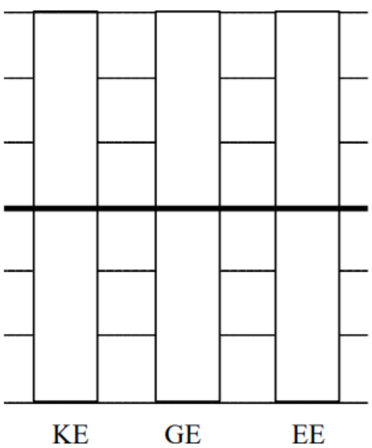


When the jumper has fallen exactly halfway.

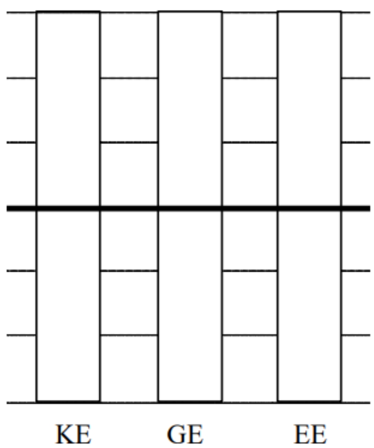


When the jumper reaches his lowest point.

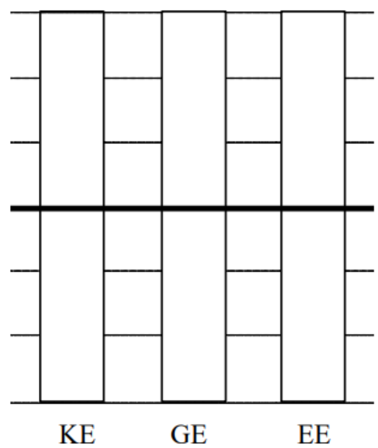
b. *A 75 kg bungee jumper steps off a platform high above a raging river and plummets downward. The elastic bungee cord has an effective spring constant of 50 N/m and is initially slack, although it begins to stretch the moment the jumper steps off of the platform. Set the platform as the zero-point of gravitational potential energy*



When the jumper first steps off the platform.

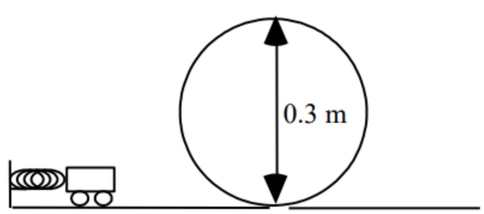


When the jumper has fallen exactly halfway.

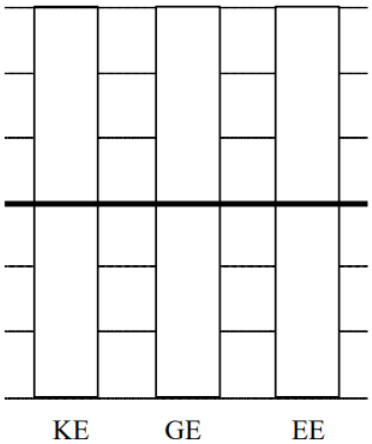


When the jumper reaches his lowest point.

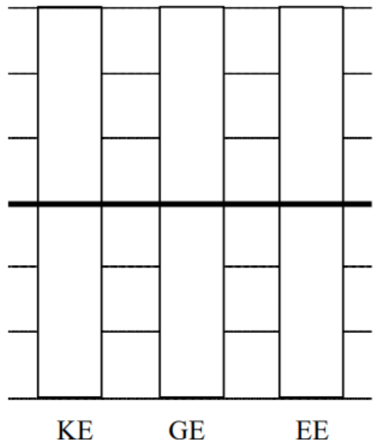
For each of the scenarios described below, indicate the amount of kinetic energy, gravitational potential energy, and elastic energy in the system at each of the events listed. Use a consistent scale throughout each motion. Set the initial position of the object as the zero-point of gravitational potential energy



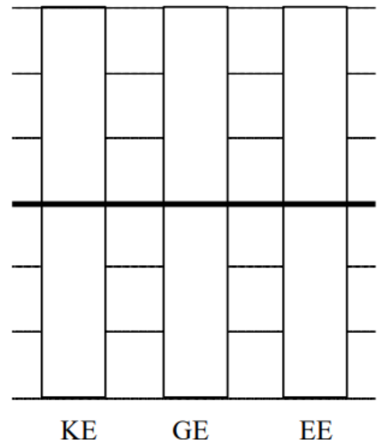
a. *A 0.27 kg toy car is held at rest against a 84 N/m compressed spring. When released, the car travels around a 0.30 m high loop. The car's speed at the top of the loop is 2.2 m/s. Assume friction is so small that it can be ignored.*



When the car is first released.

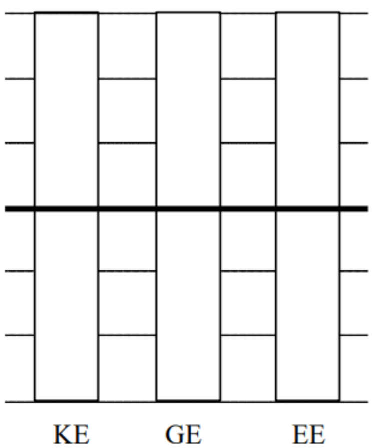


When the car is at the bottom of the loop.

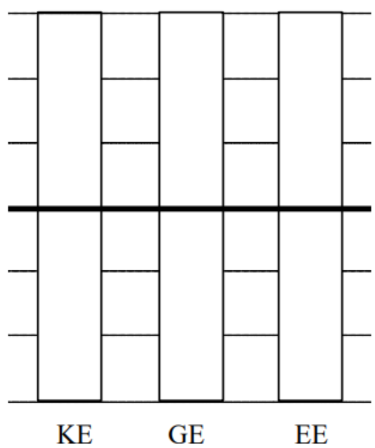


When the car is at the top of the loop.

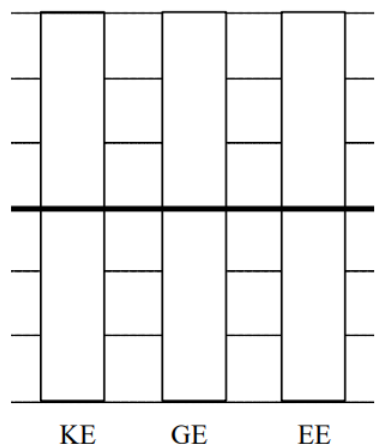
b. *A 0.10 kg pinball is launched into a pinball machine by means of a plunger, pulled back 8.0 cm and released. The surface of the pinball machine is inclined at* 13^0 *from horizontal. Assume friction between the ball and the surface of the pinball machine during the launch of the ball is so small it can be ignored.*



When the plunger is first released.

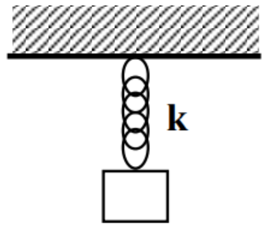


When the ball leaves the plunger.



When the ball reaches its maximum height.

Six crates of different mass are hanging at rest from six springs. Each spring has a stiffness (k), a natural length (L_0), and a current length (L). The natural length of the spring is its length before the crate is hung from it. The current length of the spring is its length when the crate is hung from it.



	k	L_0	L
A	20 N/m	0.4 m	0.8 m
B	20 N/m	0.3 m	0.6 m
C	10 N/m	0.4 m	0.8 m
D	10 N/m	0.3 m	1.2 m
E	30 N/m	0.4 m	0.6 m
F	30 N/m	0.2 m	0.4 m

a. Rank the magnitude of the force of each spring acting on each crate.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

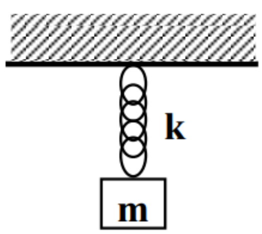
b. Rank the mass of each crate.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Six crates of different mass (m) are attached to springs of different stiffness (k). The masses are held in place such that none of the springs are initially stretched. All springs are initially the same length. The masses are released and the springs stretch.



	m	k
A	5 kg	20 N/m
B	20 kg	5 N/m
C	10 kg	10 N/m
D	15 kg	20 N/m
E	5 kg	5 N/m
F	15 kg	10 N/m

a. Rank the maximum elongation of the spring in each system.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

b. Rank the maximum speed of the crate in each system.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

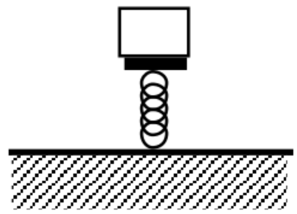
c. Rank the maximum acceleration magnitude of the crate in each system.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

Six identical mass crates are at rest on six springs. Each spring has a natural length (L_0) and a current length (L). The natural length of the spring is its length before the crate is placed on top of it. The current length of the spring is its length when the crate is on top of it.



	L_0	L
A	0.8 m	0.4 m
B	0.6 m	0.3 m
C	0.8 m	0.2 m
D	1.2 m	0.3 m
E	0.6 m	0.4 m
F	0.4 m	0.2 m

a. Rank the magnitude of the force of the spring acting on each crate.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

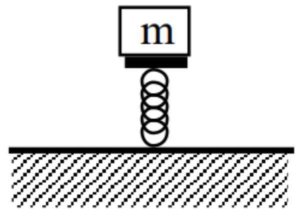
b. Rank the stiffness of each spring.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

Six crates of different mass (m) are at rest on six springs. Each spring has a natural length (L_0) and a current length (L). The natural length of the spring is its length before the crate is placed on top of it. The current length of the spring is its length when the crate is on top of it.



	m	L_0	L
A	10 kg	0.8 m	0.4 m
B	20 kg	0.6 m	0.3 m
C	10 kg	0.8 m	0.2 m
D	5 kg	1.2 m	0.3 m
E	5 kg	0.6 m	0.4 m
F	20 kg	0.4 m	0.2 m

a. Rank the stiffness of each spring.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

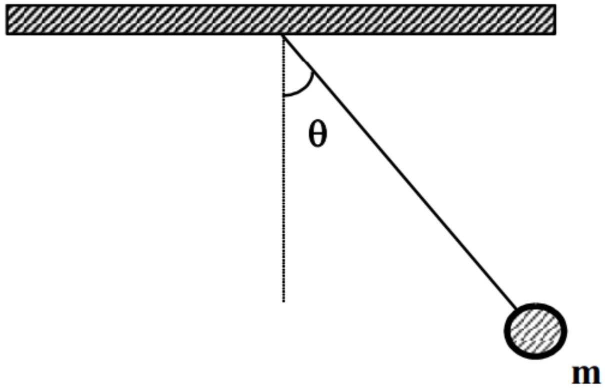
b. Rank the elastic potential energy stored in each spring

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

A pendulum of mass m is released from rest from an angle θ from vertical. All pendulums are the same length.



	m	θ
A	2 kg	60°
B	1 kg	60°
C	4 kg	30°
D	8 kg	15°
E	2 kg	30°
F	3 kg	45°

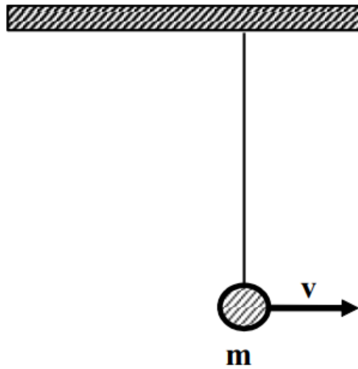
Rank the maximum speed of each pendulum.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

A pendulum of mass m is moving at velocity v as it passes through the vertical. All pendulums are the same length.



	m	v
A	2 kg	4 m/s
B	1 kg	4 m/s
C	4 kg	2 m/s
D	8 kg	1 m/s
E	2 kg	2 m/s
F	3 kg	3 m/s

Rank the maximum angle from vertical reached by the pendulum.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your rankings:

A 100 kg rocket is fired vertically upward. Its engine supplies an upward force of magnitude $F = (5000 - 5.0t^2)$ N (where t is in seconds) until $F = 0$ N, then the engine shuts off.

a. How fast is the rocket moving when the engine shuts off?

Free-Body Diagram

Mathematical Analysis[38]

Event 1:

Event 2:

b. How long does it take the rocket to reach its maximum height?

Free-Body Diagram

Mathematical Analysis

Event 1:

Event 2:

A 75 kg rocket is launched directly upward. The force on the rocket due to its engine increases from 0 N to 5000 N over 8.0 s as a linear function of time. The thrust then drops to zero almost instantaneously.

a. How fast is the rocket moving when the engine shuts off?

Free-Body Diagram

Mathematical Analysis[39]

Event 1:

Event 2:

b. How long does it take the rocket to reach its maximum height?

Free-Body Diagram

Mathematical Analysis

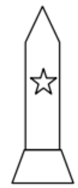
Event 1:

Event 2:

A 75 kg rocket is launched directly upward. The rocket is traveling at a speed of 200 m/s when its engine turns off. The force on the rocket due to its engine decreases to 0 N in 22 s as a quadratic function of time.

What is the maximum thrust acting on the rocket?

Free-Body Diagram



Mathematical Analysis[40]

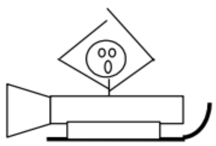
Event 1:

Event 2:

The horizontal thrust acting on a 270 kg rocket sled increases as a cubic function of time from 0 N to 5400 N in 4.3 s. The thrust then drops to zero almost instantaneously. The effective frictional coefficient acting between the rocket sled and the ground is (0.4, 0.3). The rocket sled starts from rest on a level surface.

How fast is the rocket sled moving when the engine shuts off?

Free-Body Diagram



Mathematical Analysis[41]

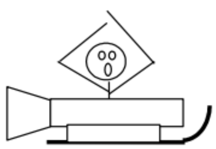
Event 1:

Event 2:

The horizontal thrust acting on a 270 kg rocket sled decreases from 8000 N to 0 N as a quadratic function of time. The rocket sled is traveling at 195 m/s when the engine shuts down. The effective frictional coefficient acting between the rocket sled and the ground is (0.4, 0.3). The rocket sled starts from rest on a level surface.

What is the total elapsed time between the engine turning on and the sled ultimately coming to rest?

Free-Body Diagram



Mathematical Analysis[42]

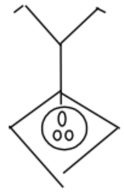
Event 1:

Event 2:

A 75 kg bungee jumper is about to step off of a platform high above a raging river and plummet downward. The elastic bungee cord has an effective spring constant of 50 N/m and is initially slack, although it begins to stretch the moment the jumper steps off of the platform.

a. How far does the bungee jumper fall?

Free-Body Diagram



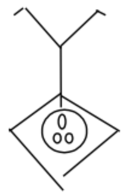
Mathematical Analysis[43]

Event 1:

Event 2:

b. What is the maximum speed of the bungee jumper?

Free-Body Diagram



Mathematical Analysis

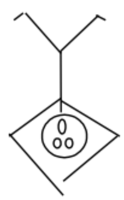
Event 1:

Event 2:

A 75 kg bungee jumper is about to step off of a platform 65 m above a raging river and plummet downward. He hopes to get just the top of his head wet. The elastic bungee cord acts as a linear spring and is initially slack, although it begins to stretch the moment the jumper steps off of the platform.

a. What is the spring constant of the bungee cord?

Free-Body Diagram



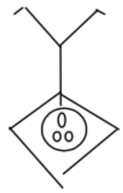
Mathematical Analysis[44]

Event 1:

Event 2:

b. What is the maximum speed of the bungee jumper?

Free-Body Diagram



Mathematical Analysis

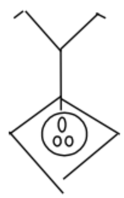
Event 1:

Event 2:

A 45 kg bungee jumper is about to step off of a platform 65 m above a raging river and plummet downward. She hopes to get just the top of her head wet. The elastic bungee cord acts as a linear spring and is initially slack. The cord does not begin to stretch until the jumper has fallen 10 m.

a. What is the spring constant of the bungee cord?

Free-Body Diagram



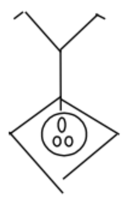
Mathematical Analysis[45]

Event 1:

Event 2:

b. What is the maximum speed of the bungee jumper?

Free-Body Diagram



Mathematical Analysis

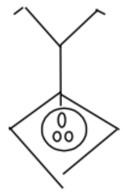
Event 1:

Event 2:

A 55 kg bungee jumper is about to step off of a platform 45 m above a raging river and plummet downward. She hopes to get just the top of her head wet. The elastic bungee cord acts as a linear spring with spring constant 30 N/m and is initially slack, but does not immediately begin to stretch when she steps off the platform.

a. How far can she safely fall before the bungee cord must begin to stretch?

Free-Body Diagram



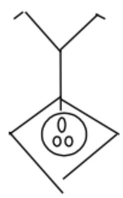
Mathematical Analysis[46]

Event 1:

Event 2:

b. How fast is she moving when she hits the river if she falls 15 m before the bungee cord begins to stretch?

Free-Body Diagram



Mathematical Analysis

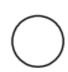
Event 1:

Event 2:

A 0.10 kg pinball is launched into a pinball machine by means of a plunger, pulled back 8.0 cm and released. The surface of the pinball machine is inclined at 13^0 *from horizontal. It requires a force of 12 N to pull the plunger back 5.0 cm. Assume friction between the ball and the surface of the pinball machine during the launch of the ball is so small it can be ignored.*

a. What is the speed of the ball as it leaves the plunger?

Free-Body Diagram



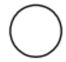
Mathematical Analysis[47]

Event 1:

Event 2:

b. How far does the ball travel after leaving the plunger, assuming friction is small enough to be ignored?

Free-Body Diagram



Mathematical Analysis

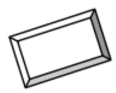
Event 1:

Event 2:

At a UPS distribution center, a 40 kg crate is sliding down an 8° ramp at 3 m/s. At the bottom of the ramp, 8 m away, is a 150 N/m spring designed to bring the crate to rest. The coefficient of friction between the crate and the ramp is (0.2, 0.1).

a. What is the speed of the crate when it hits the spring?

Free-Body Diagram



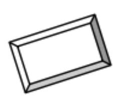
Mathematical Analysis[48]

Event 1:

Event 2:

b. How far does the spring compress before bringing the crate to rest?

Free-Body Diagram

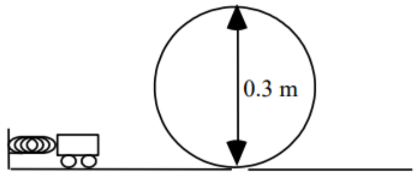


Mathematical Analysis

Event 1:

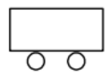
Event 2:

A 0.27 kg toy car is held at rest against a 84 N/m compressed spring. When released, the car travels around a 0.30 m high loop. The car's speed at the top of the loop is 2.2 m/s. Assume friction is so small that it can be ignored.



a. What is the initial compression of the launcher?

Free-Body Diagram



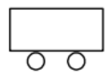
Mathematical Analysis[49]

Event 1:

Event 2:

b. With what speed does the cart enter the loop?

Free-Body Diagram



Mathematical Analysis

Event 1:

Event 2:

A rocket of mass m is fired vertically upward. The interaction between its engine and the surroundings produces an upward force of magnitude $F = F_{max} - Ct^2 N$ *(where t is in seconds) until* $F = 0$ *N, then the engine shuts off. Determine the speed of the rocket at the instant the engine shuts off,* v_{off}*, as a function of* F_{max}*, C, m, and g.*

Free-Body Diagram

Mathematical Analysis

Event 1:

Event 2:

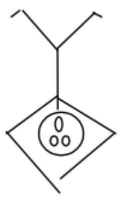
Questions

If $C = 0$ *N/s*^{2}*, what should* v_{off} *equal? Does your function agree with this observation?*

If $F_{max} < mg$, what should v_{off} equal? Does your function agree with this observation?

A bungee jumper of mass m is about to step off of a platform a distance D above a rocky ravine and plummet downward. He hopes to not get the top of his head bloody. The elastic bungee cord acts as a linear spring and is initially slack. The cord does not begin to stretch until the jumper has fallen a distance d. Determine the minimum spring constant (k) needed for a safe jump as a function of m, g, D, and d.

Free-Body Diagram



Mathematical Analysis

Event 1:

Event 2:

Questions

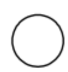
If g = 0 m, what should k equal? Does your function agree with this observation?

If d = D, what should k equal? Does your function agree with this observation?

If m is twice as large, what effect will this have on k?

A pinball of mass m is launched into a pinball machine by means of a spring-driven plunger pulled back a distance s and released. The surface of the pinball machine is inclined at $theta$ from horizontal. Determine the speed of the ball as it leaves the plunger (v_0) as a function of m, s, θ, g, and the effective spring constant of the plunger system. Assume friction between the ball and the surface of the pinball machine during the launch of the ball is so small it can be ignored.

Free-Body Diagram



Mathematical Analysis

Event 1:

Event 2:

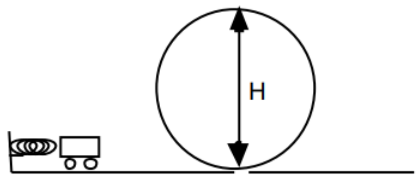
Questions

If s = 0 m, what should v_0 equal? Does your function agree with this observation?

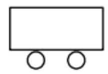
If k = 0 N/m, what should v_0 equal? Does your function agree with this observation?

If the plunger is pulled back twice as far, what effect will this have on v_0?

A toy car of mass m is held at rest against a compressed spring. When released, the car travels around a loop. Determine the speed of the cart (v) as a function of the angular position of the cart on the loop (let straight down be $\theta = 0°$), the spring constant (k), the initial compression of the spring (s), H, m, and g. Assume friction is so small that it can be ignored.



Free-Body Diagram



Mathematical Analysis

Event 1:

Event 2:

Questions

If s = 0 m, what should v equal? Does your function agree with this observation?

If k = 0 N/m, what should v equal? Does your function agree with this observation?

If θ = 0°, what should v equal? Does your function agree with this observation?

Selected Answers

[1] v3 = 8.83 m/s

[2] t3 = 55.5 s

[3] r2 bmw = 51.7 m

[4] t2 bmw = 30 s

[5] v2 bmw = 74.1 m/s

[6] r2 bmw = 71.5 m

[7] t2 bmw = 2.8 s

[8] t = 4.9 s

[9] t3 = 11.6 s

[10] t2 = 9.3 s

[11] t3 = 19.3 s

[12] rρ = 2.97 m

[13] t2 = 28.3 s

[14] θ2 = 2.1 rad

[15] t2 = 30.2 s

[16] t2 = 4.46 s

[17] v2 = 28.5 m/s

[18] θ3 = 74.7 rad

[19] θ3 = 68.4 rad

[20] t3 = 108 s

[21] t4 = 27.8 s

[22] t3 = 26.2 s

[23] v2B = 278 m/s

[24] r2B = 950 m

[25] t4 = 8.96 s

[26] t3 = 87.1 s

[27] a. v = 14.0 m/s b. v = 16.3 m/s

[28] a. ω = 0.224 rad/s b. F = 636 N

[29] a. F = 14.2 N b. F = 15.7 N

[30] a. F = 1114 N b. F = 1810 N

[31] F top rope = 44 N

[32] θ = 76.8°

[33] vmax = 17.1 m/s

[34] θ = 5.8°

[35] vmax = 30 m/s

[36] too fast!

[37] ωmin = 2 rad/s

[38] a. v2 = 746 m/s b. t3 = 107.7 s

[39] a. v2 = 194 m/s b. t3 = 27.8 s

[40] Fmax = 2125 N

[41] v = 13.7 m/s

[42] t = 11.6 s

[43] a. δr = 29.4 m b. v2 = 12 m/s

[44] a. k = 22.6 N/m b. v2 = 17.9 m/s

[45] a. k = 19 N/m b. v2 = 20.6 m/s

[46] a. d = 4.8 m b. v = 19.8 m/s

[47] a. v2 = 3.87 m/s b. δr = 3.4 m

[48] a. v2 = 3.91 m/s b. s = 2.13 m

[49] a. s = 0.19 m b. v2 = 3.27 m/s

4.4: Selected Answers

[1] v3 = 8.83 m/s

[2] t3 = 55.5 s

[3] r2 bmw = 51.7 m

[4] t2 bmw = 30 s

[5] v2 bmw = 74.1 m/s

[6] r2 bmw = 71.5 m

[7] t2 bmw = 2.8 s

[8] t = 4.9 s

[9] t3 = 11.6 s

[10] t2 = 9.3 s

[11] t3 = 19.3 s

[12] rρ = 2.97 m

[13] t2 = 28.3 s

[14] θ2 = 2.1 rad

[15] t2 = 30.2 s

[16] t2 = 4.46 s

[17] v2 = 28.5 m/s

[18] θ3 = 74.7 rad

[19] θ3 = 68.4 rad

[20] t3 = 108 s

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[22] t3 = 26.2 s

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[32] θ = 76.8°

[33] vmax = 17.1 m/s

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[36] too fast!

[37] ωmin = 2 rad/s

[38] a. v2 = 746 m/s b. t3 = 107.7 s

[39] a. v2 = 194 m/s b. t3 = 27.8 s

[40] Fmax = 2125 N

[41] v = 13.7 m/s

[42] t = 11.6 s

[43] a. δr = 29.4 m b. v2 = 12 m/s

[44] a. k = 22.6 N/m b. v2 = 17.9 m/s

[45] a. k = 19 N/m b. v2 = 20.6 m/s

[46] a. d = 4.8 m b. v = 19.8 m/s

[47] a. v2 = 3.87 m/s b. δr = 3.4 m

[48] a. v2 = 3.91 m/s b. s = 2.13 m

[49] a. s = 0.19 m b. v2 = 3.27 m/s

CHAPTER OVERVIEW

5: Model 4 - The Rigid-body Model

Thumbnail: The torque caused by the normal force – Fg and the weight of the top causes a change in the angular momentum L in the direction of that torque. This causes the top to precess. (CC-BY-SA-2.5; Xavier Snelgrove).

5.0: Model Specifics

We are now ready to consider how the actual size and shape of an object affects its motion. The most immediate ramification of this Model is that the objects we study can now **rotate**. However, there are still three important restrictions to the types of motions will we investigate.

The object is rigid.

The size and shape of the object under investigation do not change during the motion.

The rotation is planar.

This restriction may take some explaining. Imagine an object spinning in place. The axis (real or imaginary) about which the object spins is referred to as the rotation axis. Note that all points that lie on the rotation axis do not move. All other points on the object, however, exhibit circular motion around the rotation axis.

If the object, in addition to spinning, is also moving, this model will restrict us to motions where the center-of-mass moves exclusively in the plane perpendicular to the rotation axis. I will refer to this as *planar rotation*.

For example, a yo-yo is an object that typically moves in a plane perpendicular to its rotation axis. Thus, we can study the motion of most yo-yos. The motion of a wheel is typically in a plane perpendicular to its rotation axis. Thus, we can study the motion of most wheels. The motion of the earth around the sun, however, is beyond this model's capabilities since the rotation axis of the earth is tilted relative to the plane of the earth's motion. A wobbling top is also beyond this model's capabilities (although a steadily spinning top is not).

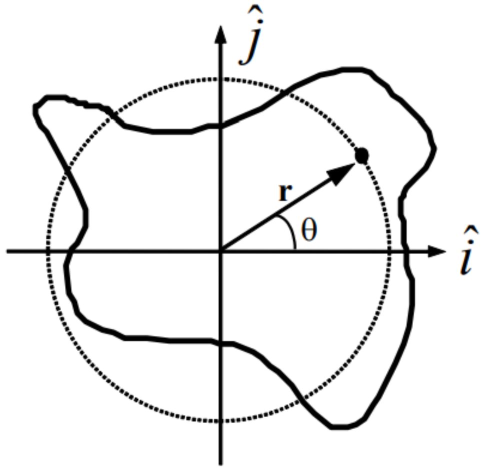
The object is classical.

5.1: Kinematics

Concepts and Principles

Rotation about a Fixed Axis (Spinning)

Imagine a rigid body constrained to rotate about a fixed axis. Non-physicists would say that the object is *spinning*.



Place the origin of a coordinate system at the location of the rotation axis. Examine an arbitrary point on the object, denoted by the position vector

$$\vec{r}(t) = r(\cos\theta(t)\hat{i} + \sin\theta(t)\hat{j})$$

or

$$\vec{r}(t) = r\hat{r}$$

Notice that as the object spins, this point undergoes circular motion (denoted by the dashed line). Although the actual object may be of irregular shape, as it spins *every point on the object undergoes circular motion*. Moreover, since each and every point on the object has to complete an entire cycle around the rotation axis in the same amount of time, every point must undergo circular motion with the same angular speed (ω(t)) and the same angular acceleration (α(t)).

Since every point on a rigid body must have the same angular speed and the same angular acceleration, we will speak of the angular speed and angular acceleration *of* the object, rather than the angular speed and angular acceleration of some point *on* the object.

Given that every point on a spinning object undergoes circular motion, the results from our study of circular motion will be very important in analyzing spinning objects. Recall that with θ(t) defined as the angular position of an arbitrary point on the rigid-body,

$$\omega(t) = \frac{d\theta(t)}{dt}$$

and

$$\alpha(t) = \frac{d\omega(t)}{dt}$$

Moreover, the velocity and acceleration of any point on a spinning, rigid body can be related to the angular quantities:

$$v_t(t) = r\omega(t)$$

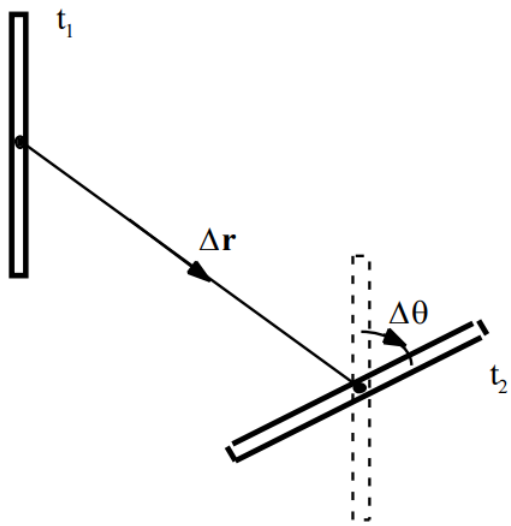
and

$$a_t(t) = r\alpha(t)$$
$$a_r(t) = -r\omega^2(t)$$

Rotation and Translation

To describe the motion of an object undergoing pure rotation (spinning), we have to describe the circular motion each point on the object undergoes. What must we do if the object is simultaneously rotating and translating (moving in a plane perpendicular to the rotation axis), like a wheel rolling down an incline?

The answer lies in the independence of these two types of motion. In much the same manner as we attacked kinematics in two dimensions by independently analyzing the horizontal and vertical motions, we will attack rigid-body kinematics by independently analyzing the rotational and translational motions. In short, *we will model any motion of a rigid-body as a superposition of a translation of the object's center-of-mass (CM) (which we will analyze by particle kinematics) and a rotation about an axis passing through the CM (which we will analyze by the kinematics of spinning, detailed above).*



For example, examine the motion of the thin rod between t_1 and t_2. Although the rod may have been spinning crazily through space between these two times, we can model its motion as a superposition of a simple translation of its CM without rotation (denoted by the vector $\delta \mathbf{r}$), which leaves the rod in the orientation denoted by the dashed lines, and a simple rotation about an axis through its CM without translation (denoted by $\delta\theta$), which leaves the rod in its proper, final orientation. If we imagine the time difference (t_2 - t_1) shrinking toward zero, hopefully it becomes plausible that we can model any motion through this method.

In summary, to describe the motion of an arbitrary rigid body we will break the motion down into a pure translation of the CM and a pure rotation about the CM. We will use particle kinematics to describe the translational portion of the motion and the kinematics of circular motion to describe the rotational portion. The velocity (or acceleration) of any point on the object is then determined by the sum of the velocity (or acceleration) due to the translation and the velocity (or acceleration) due to the rotation.

Analysis Tools

Pure Rotation

> *After the off button is pressed, a ceiling fan takes 22 s to come to rest. During this time, it completes 18 complete revolutions.*

To analyze this situation, we should first carefully determine and define the sequence of events that take place. At each of these instants, let's tabulate what we know about the motion. Since we are dealing with pure rotation, the relevant kinematic variables are the angular position, velocity, and acceleration of the fan. Also, let's take the direction that the fan is initially rotating to be the positive direction.

Event 1: The 'off' button is pressed	Event 2: The ceiling fan stops
t1 = 0 s	t2 = 22 s
θ_1 = 0 rad	θ_2 = 18 (2π) = 113 rad
ω_1 =	ω_2 = 0 rad/s

Event 1: The 'off' button is pressed	Event 2: The ceiling fan stops
α_1 =	α_2 =

Since no specific information concerning the angular acceleration of the fan is given, let's assume the angular acceleration is constant. (Thus, $\alpha_1=\alpha_2=\alpha_{12}$.) Since the relationships between angular position, velocity, and acceleration are the same as the relationships between linear position, velocity and acceleration, the kinematic equations for constant linear acceleration must have direct analogies for constant angular acceleration. Thus,

$$\begin{aligned}&\omega_2=\omega_1+\alpha_{12}\left(t_2-t_1\right)\\&0=\omega_1+\alpha_{12}(22)\\&\omega_1=-22\alpha_{12}\end{aligned}$$

Now substitute this expression into the other equation:

$$\begin{aligned}&\theta_2=\theta_1+\omega_1\left(t_2-t_1\right)+\frac{1}{2}\alpha_{12}\left(t_2-t_1\right)^2\\&113=0+\omega_1(22)+\frac{1}{2}\alpha_{12}(22)^2\\&113=22\left(-22\alpha_{12}\right)+242\alpha_{12}\\&113=-242\alpha_{12}\\&\alpha_{12}=-0.47\mathrm{rad}/\mathrm{s}^2\end{aligned}$$

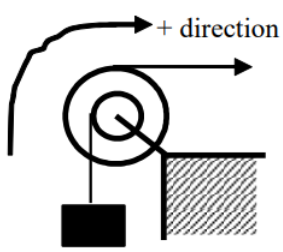
Substitute this result back into the original equation:

$$\begin{aligned}&\omega_1=-22(-0.47)\\&\omega_1=10.3\mathrm{rad}/\mathrm{s}\end{aligned}$$

Notice that the sign of the angular acceleration is negative. This indicates that the angular acceleration is in the opposite direction of the angular velocity, as it should be since the fan is slowing down.

Connecting Pure Rotation to Pure Translation

The device at right is used to lift a heavy load. The free rope is attached to a truck which accelerates from rest at a rate of 1.5 m/s^2. The inner radius of the pulley is 20 cm and the outer radius is 40 cm. The load must be raised 15 m.



The coordinate system chosen indicates that the block moving upward, the pulley rotating clockwise, and the truck moving to the right are all positive.

There are three different objects that we should be able to describe kinematically; the truck, the pulley, and the block. Let's tabulate everything we know about each object:

Truck		Pulley		Block	
Event 1: Truck begins to move	Event 2: Block raised 15 m	Event 1: Truck begins to move	Event 2: Block raised 15 m	Event 1: Truck begins to move	Event 2: Block raised 15 m
t_1 = 0 s	t_2 =	t_1 = 0 s	t_2 =	t_1 = 0 s	t_2 =
r_1 = 0 m	r_2 =	θ_1 = 0 rad	θ_2 =	r_1 = 0 m	r_2 = 15 m

Truck		Pulley		Block	
Event 1: Truck begins to move	Event 2: Block raised 15 m	Event 1: Truck begins to move	Event 2: Block raised 15 m	Event 1: Truck begins to move	Event 2: Block raised 15 m
v_1 = 0 m/s	v_2 =	ω_1 = 0 rad/s	ω_2 =	v_1 = 0 m/s	v_2 =
a_1 = 1.5 m/s^2	a_2 = 1.5 m/s^2	α_1 =	α_2 =	a_1 =	a_2 =

The truck and block exhibit translational motion, so we only have to tabulate linear variables. The pulley spins, so the relevant variables are the angular variables. Notice that just because the block moves 15 m *doesn't* mean the truck moves 15 m. Also, the acceleration of the block is *not* equal to the acceleration of the truck. However, these variables are related to each other since both objects are attached to the same pulley.

Also notice that we can't currently solve this problem. Each of the objects has three unknown quantities. However, since the kinematics of the three objects are related, we will be able to solve the problem once we've worked out the exact relationship between each object's kinematics.

Let's start with the acceleration. Assuming the rope from the *truck* does not slip on the pulley, the point on the pulley in contact with the rope must be accelerating at the same rate as the truck. Notice that this acceleration is tangent to the pulley, and this rope is located 0.4 m from the center of the pulley. Therefore, from

$$a_t = r\alpha$$
$$1.5 = 0.4\alpha$$
$$\alpha = 3.75 \mathrm{rad/s^2}$$

The pulley must have an angular acceleration of 3.75 rad/s^2 since it is attached to the truck.

In addition, assuming the rope from the *block* does not slip on the pulley, the point on the pulley in contact with this rope must be accelerating at:

$$a_t = r\alpha$$
$$a_t = 0.2(3.75)$$
$$a_t = 0.75 \mathrm{~m/s^2}$$

Since the point on the pulley attached to the block is accelerating at 0.75 m/s^2, the block itself must be accelerating at 0.75 m/s^2. In a nutshell, what we've done is used the acceleration of the truck to find the angular acceleration of the pulley, and then used the angular acceleration of the pulley to find the acceleration of the block. This chain of reasoning is very common.

Now that we know the block's acceleration, we can solve the problem. Applying the two kinematic equations to the block yields:

$$r_2 = r_1 + v_1(t_2 - t_1) + \frac{1}{2}a_{12}(t_2 - t_1)^2$$
$$15 = 0 + 0 + \frac{1}{2}(0.75)t_2{}^2$$
$$t_2 = 6.32 \mathrm{~s}$$

$$v_2 = v_1 + a_{12}(t_2 - t_1)$$
$$v_2 = 0 + 0.75(6.32)$$
$$v_2 = 4.74 \mathrm{~m/s}$$

Now that we know the final speed of the block, we can find the final angular speed of the pulley and the final speed of the truck:

Again assuming the rope from the *block* does not slip on the pulley, the point on the pulley in contact with the rope must be moving at the same rate as the block. Notice that this velocity is tangent to the pulley, and this rope is located 0.2 m from the center of the pulley. Therefore, from

$$v_t = r\omega$$
$$4.74 = 0.2\omega$$
$$\omega = 23.7 \mathrm{rad/s}$$

The pulley must have an angular velocity of 23.7 rad/s since it is attached to the block.

In addition, assuming the rope from the *truck* does not slip on the pulley, the point on the pulley in contact with this rope must be moving at:

$$v_t = r\omega$$
$$v_t = 0.4(23.7)$$
$$v_t = 9.48 \text{ m/s}$$

The truck is moving at 9.48 m/s when the block reaches 15 m.

We can relate the displacement of the block to the angular displacement of the pulley and the displacement of the truck using

$$s = r\theta$$

where s is (technically) the arc length over which a rope is wrapped around the pulley and θ is the angular displacement of the pulley. Of course, the amount of rope wrapped around a pulley is exactly equal to the displacement of the object attached to the rope. Therefore, relating the block to the pulley yields

$$s = r\theta$$
$$15 = 0.2\theta$$
$$\theta = 75\text{rad}$$

The pulley must have turned through 75 rad since it is attached to the block.

Relating the pulley to the truck results in:

$$s = r\theta$$
$$s = 0.4(75)$$
$$s = 30m$$

The truck moved 30 m while the block moved 15 m.

Notice that in all cases, the values of the kinematic variables for the truck are exactly twice the values for the block. This is not a coincidence. Since the truck is attached to the pulley at twice the distance from the axle that the block is attached, all of its kinematic variables will have twice the value. Once you understand why this is the case, you can use this insight to simplify your analysis.

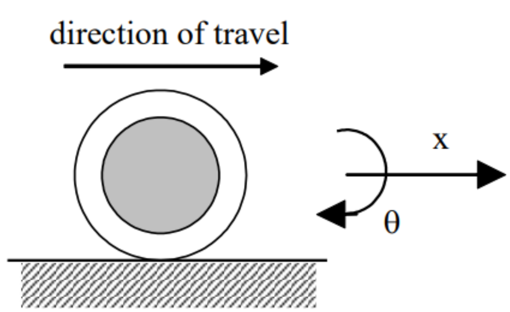
Rotating and Translating

> *Accelerating from rest, a Cadillac Sedan de Ville can reach a speed of 25 m/s in a time of 6.2 s. During this acceleration, the Cadillac's tires do not slip in their contact with the road. The diameter of the Cadillac's tires is 0.80 m.*

Let's examine the motion of one of the Cadillac's tires. Obviously, the tire both translates and rotates. We will imagine the motion to be a superposition of a pure translation of the CM of the tire and a pure rotation about the CM of the tire.

To analyze the situation, we should define the sequence of events that take place and tabulate what we know about the motion at each event. Since we are dealing with translation as well as rotation, we will need to keep track of both the linear kinematic variables and the angular kinematic variables.

Let's take the positive x direction to be the direction that the Cadillac translates and the positive θ direction to be the direction in which the tires rotate.



Event 1: The Cadillac begins to accelerate	Event 2: The Cadillac reaches 25 m/s
$t_1 = 0$ s	$t_2 = 6.2$ s

Event 1: The Cadillac begins to accelerate	Event 2: The Cadillac reaches 25 m/s
r_1 = 0 m	r_2 =
θ_1 = 0 rad	θ_2 =
v_1 = 0 m/s	v_2 = 25 m/s
ω_1 = 0 rad/s	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

First, let's examine the translational portion of the tire's motion.

$$\begin{aligned} v_2 &= v_1 + a_{12}(t_2 - t_1) \\ 25 &= 0 + a_{12}(6.2) \\ a_{12} &= 4.03 \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} r_2 &= r_1 + v_1(t_2 - t_1) + \frac{1}{2} a_{12}(t_2 - t_1)^2 \\ r_2 &= 0 + 0 + \frac{1}{2}(4.03)(6.2)^2 \\ r_2 &= 77.5 \text{ m} \end{aligned}$$

What about the rotational kinematics of the tire? The motion of the tire is the superposition of the pure translational motion of the CM and the pure rotational motion about the CM[1]. Thus, the velocity of any point on the tire is given by

$$\begin{aligned} \vec{v}_{\text{any pointon tire}} &= \vec{v}_{\text{due to translation of CM}} + \vec{v}_{\text{due to rotation about CM}} \\ \vec{v}_{\text{any pointon tire}} &= \vec{v}_{\text{CM}} + r\omega\hat{t} \end{aligned}$$

where r is the distance between the point of interest and the rotation axis, i.e., the distance from the CM.

The key insight into studying the rotation of the tire is to realize that *at any instant the velocity of the point on the tire in contact with the road is zero because the tire never slips in its contact with the road*. Thus, if the CM of the tire is moving forward at

$$\vec{v}_{\text{CM}} = (25 \text{ m/s})\hat{i}$$

the point on the bottom of the tire must be moving backward *relative to the CM* at the exact same speed in order for its velocity *relative to the ground* to be zero! Thus, only a very particular value for ω will allow the tire to roll without slipping.

From above,

$$\begin{aligned} \vec{v}_{\text{any pointon tire}} &= \vec{v}_{\text{CM}} + r\omega\hat{t} \\ 0 &= (25 \text{ m/s})\hat{i} + r\omega\hat{t} \end{aligned}$$

Note

[1] As stated in the text, to describe the general motion of the tire we will break the motion down into a pure translation of the CM and a pure rotation about the CM. Another way to envision the motion is that the tire is rotating about the point on the tire in contact with the ground. The rotation axis of the tire passes through this contact point. (Imagine the point on the tire in contact with the ground as being glued to the ground and the tire rotating about it.) However, an instant later this point is no longer in contact with the ground, and the rotation axis passes through the *next* point on the tire in contact with the ground. There are advantages and disadvantages to both conceptualizations of the tire's motion, however, we will restrict ourselves to the view that the tire undergoes pure rotation about the CM.

At the bottom of the tire, $\hat{t} = -\hat{i}$, since motion tangent to the tire at the bottom of the tire is motion to the left. Thus,

$$
\begin{aligned}
&0=(25m/s)\hat{i}-r\omega\hat{i}\\
&r\omega\hat{i}=(25m/s)\hat{i}\\
&(0.4m)\omega=(25m/s)\\
&\omega=62.5\mathrm{rad/s}
\end{aligned}
$$

Therefore, at the end of the acceleration, the tire has an angular speed of 62.5 rad/s.

I included all of the vector notation to ensure I did the calculation correctly, but hopefully we can understand the result without getting bogged down in notation. Basically, the bottom of the tire can't be moving relative to the ground if the wheel rolls without slipping. Since the CM of the tire is moving forward at 25 m/s, the bottom of the tire must be moving backward at 25 m/s relative to the CM. From the CM frame of reference, the tire is just spinning, and I can apply v = r ω to calculate the angular speed of the tire.

$$
\begin{aligned}
&v_t=r\omega\\
&25=0.4\omega\\
&\omega=62.5\mathrm{rad/s}
\end{aligned}
$$

We can find the angular acceleration and angular position of the tire by using the linear variables determined above and the method described in the previous example, or by using the equations for constant angular acceleration.

$$
\begin{aligned}
&\omega_2=\omega_1+\alpha_{12}\left(t_2-t_1\right)\\
&62.5=0+\alpha_{12}(6.2)\\
&\alpha_{12}=10.1\mathrm{rad/s}^2
\end{aligned}
$$

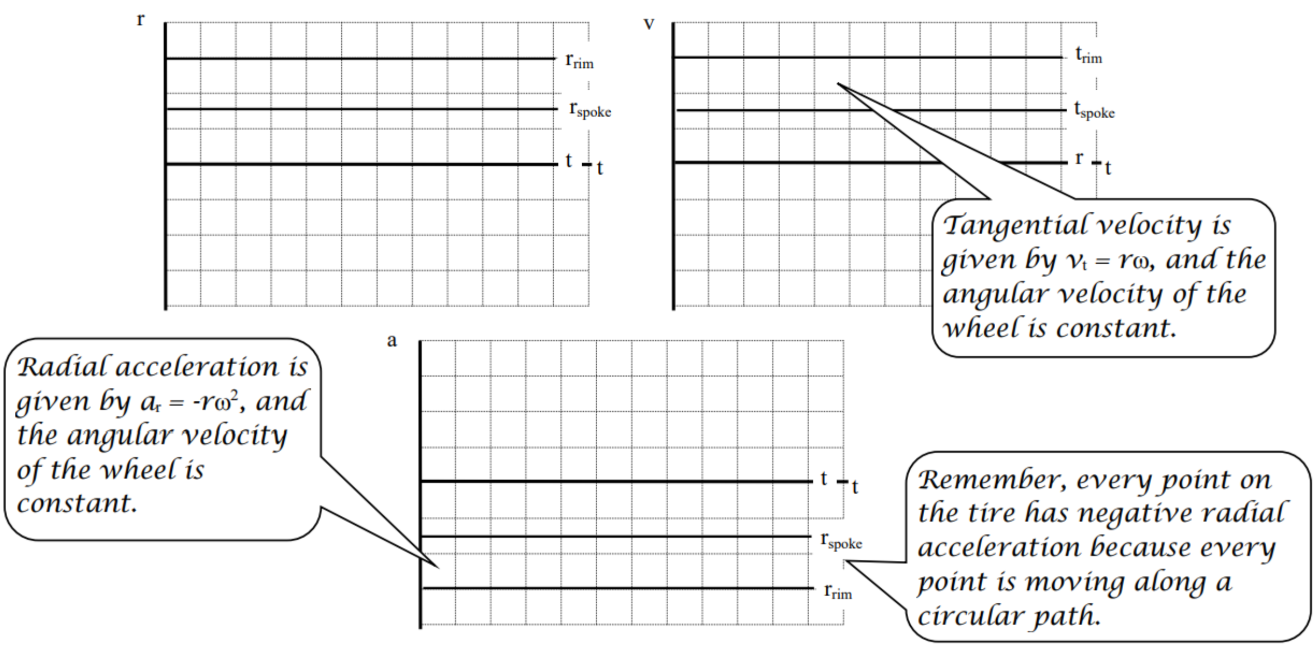
$$
\begin{aligned}
&\theta_2=\theta_1+\omega_1\left(t_2-t_1\right)+\frac{1}{2}\alpha_{12}\left(t_2-t_1\right)^2\\
&\theta_2=0+0+\frac{1}{2}(10.1)(6.2)^2\\
&\theta_2=194\mathrm{rad}
\end{aligned}
$$

Thus, the wheel rotates through 194 rad, or 30.8 revolutions, while accelerating.

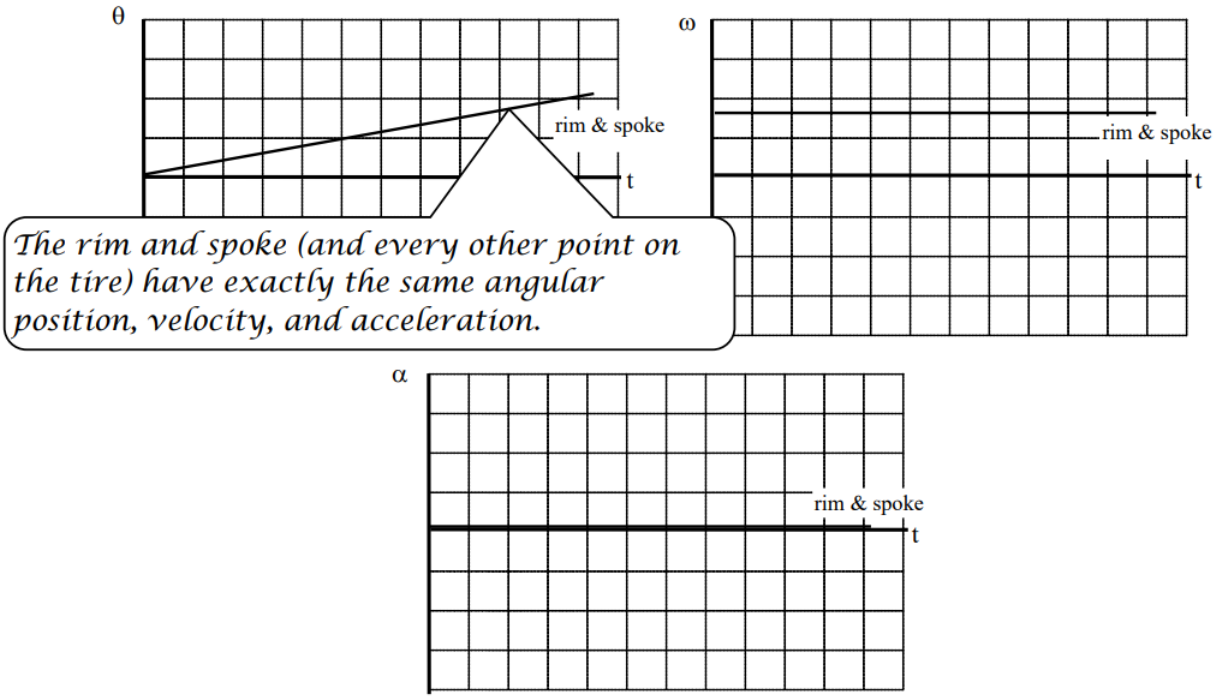
Activities

A girl pedals her bicycle at constant speed along a level path. Examine the motion of a point on the rim of her tire and a point midway along a spoke for one complete cycle. Set the origin of the coordinate system at the center of the tire.

a. Using polar coordinates, sketch position, velocity, and acceleration vs. time graphs for each point.

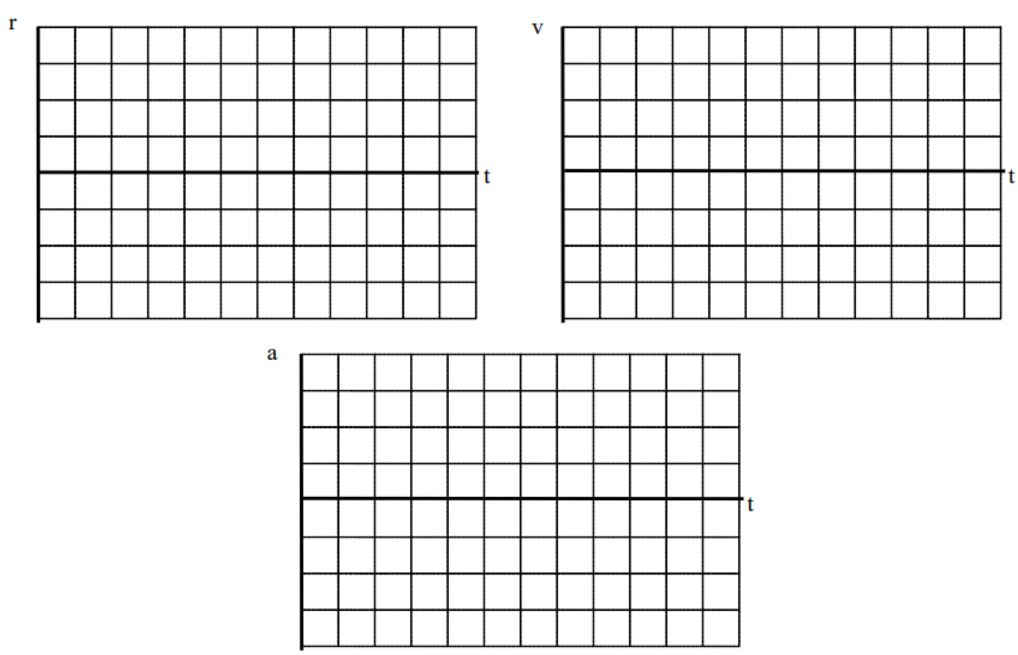


b. Sketch angular position, angular velocity, and angular acceleration vs. time graphs for each point.

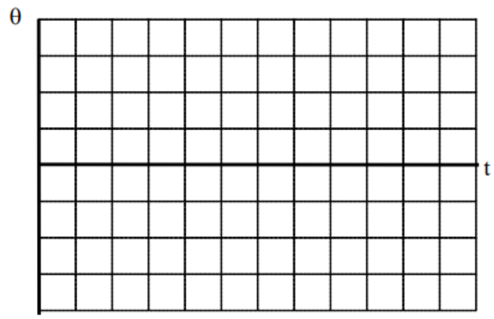


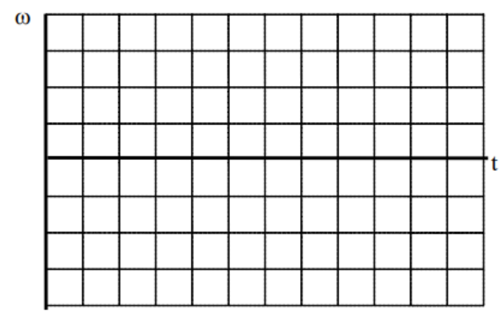
A girl pedals her bicycle, from rest, at a constantly increasing speed along a level path. Examine the motion of a point on the rim of her tire and a point midway along a spoke for one complete cycle, starting from rest. Set the origin of the coordinate system at the center of the tire.

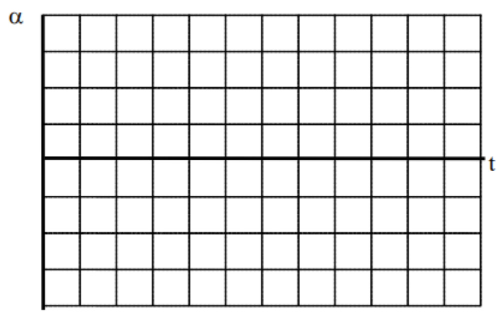
a. Using polar coordinates, sketch position, velocity, and acceleration vs. time graphs for each point.



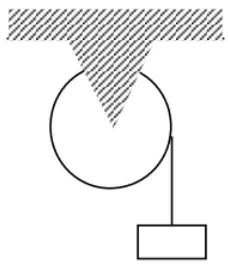
b. Sketch angular position, angular velocity, and angular acceleration vs. time graphs for each point.



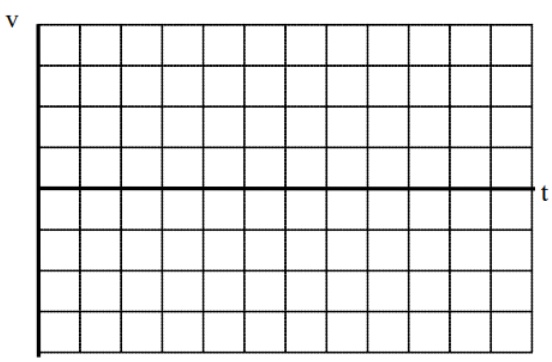


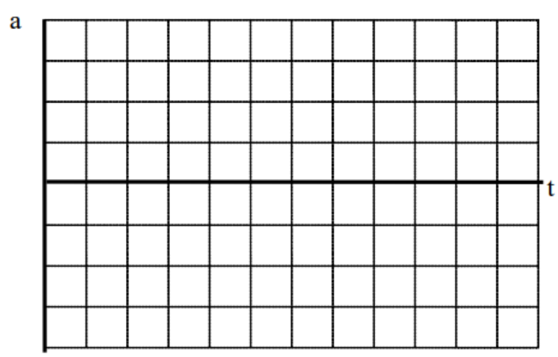


A block is attached to a rope that is wound around a pulley. Sketch velocity vs. time and acceleration vs. time graphs for a point on the rim of the pulley and for the block. Set the origin of the coordinate system at the center of the pulley.

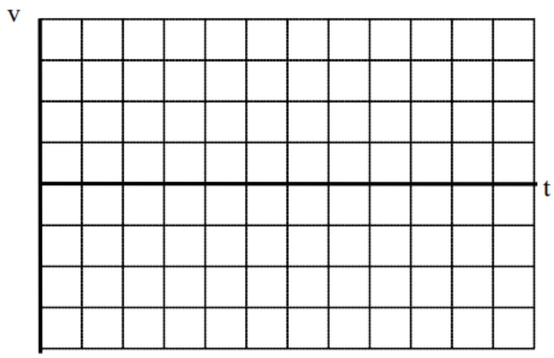


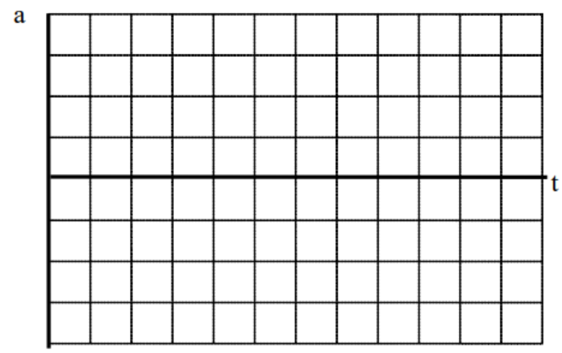
a. Sketch the polar components of the point on the rim's motion and the Cartesian components of the block's motion when the block is lowered at constant speed.



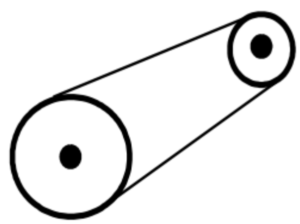


b. Sketch the polar components of the point on the rim's motion and the Cartesian components of the block's motion when the block is released from rest.

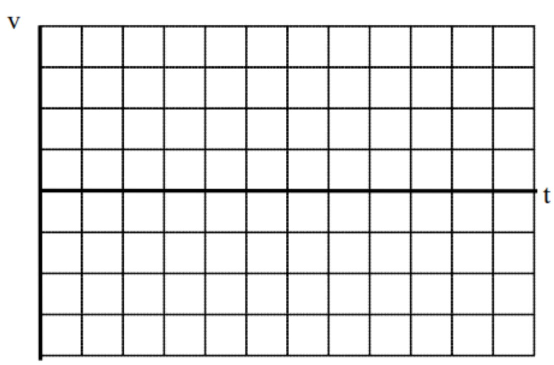


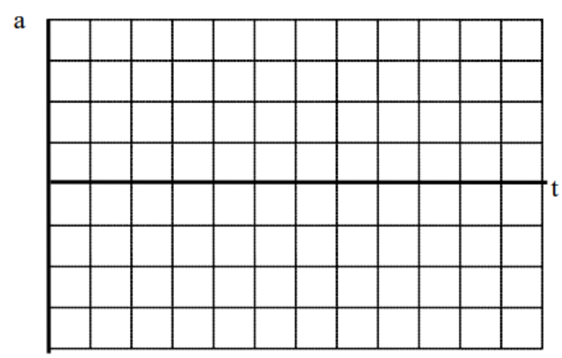


The two wheels at right are coupled together by an elastic conveyor belt that does not slip on either wheel. Examine the motion of a point on the rim of the large wheel and a point on the rim of the small wheel when the conveyor belt's speed increases from rest. Set the origin of a coordinate system at the center of each pulley.

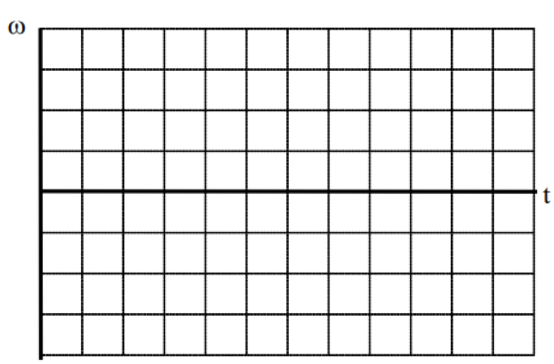


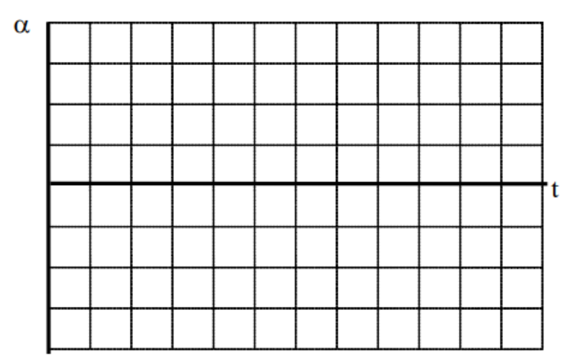
a. Using polar components, sketch velocity and acceleration vs. time graphs for each point.



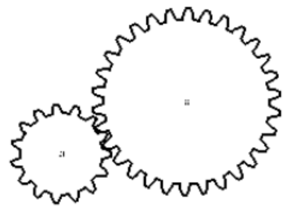


b. Sketch angular velocity and angular acceleration vs. time graphs for each point.

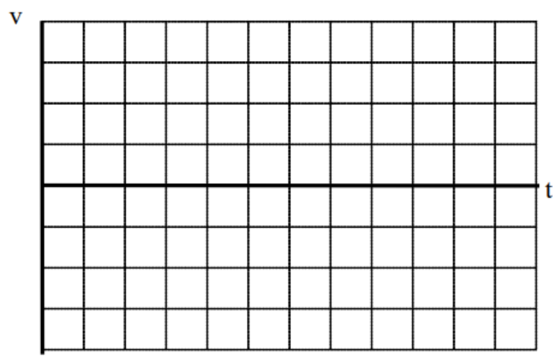


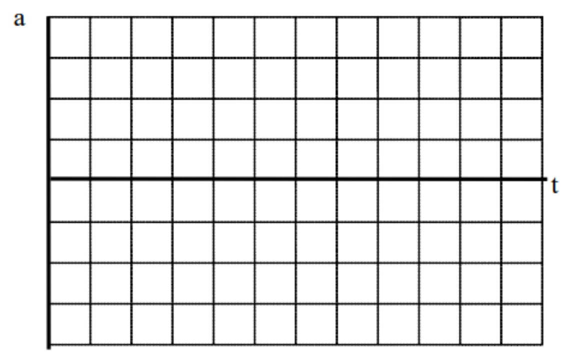


The two gears at right have equal teeth spacing and are well-meshed. Examine the motion of a point on the rim of the large gear and a point on the rim of the small gear when the small gear's speed increases from rest in the counterclockwise direction. Set the origin of a coordinate system at the center of each gear and let counterclockwise be the positive direction.

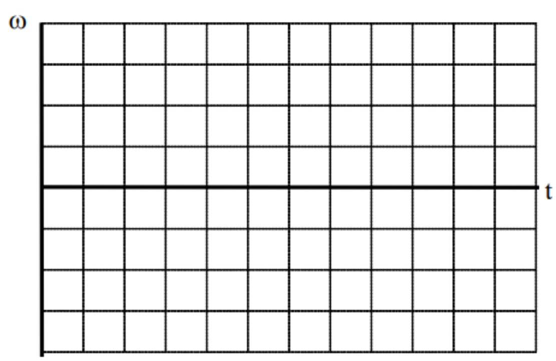


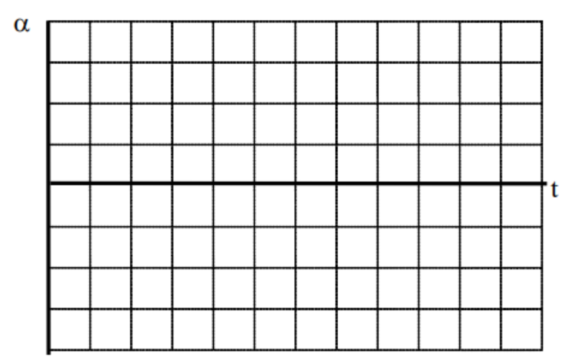
a. Using polar components, sketch velocity and acceleration vs. time graphs for each point.



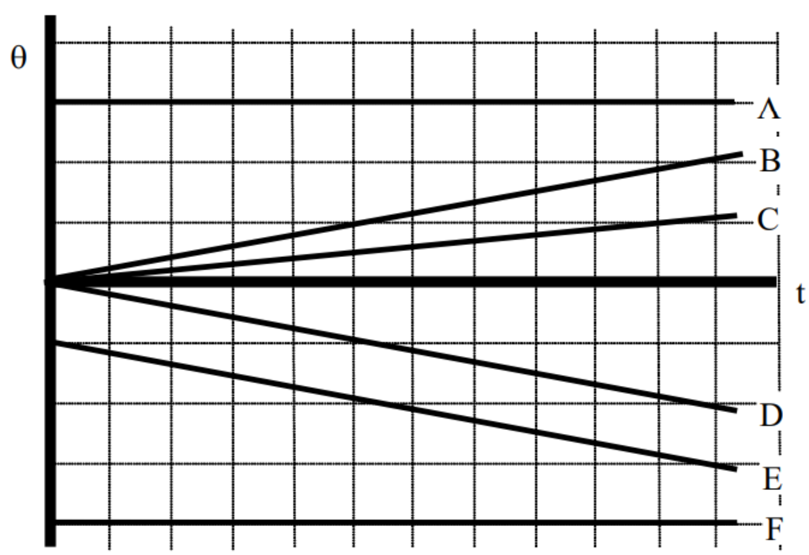


b. Sketch angular velocity and angular acceleration vs. time graphs for each point.





Below are angular position vs. time graphs for six different objects.



a. Rank these graphs on the basis of the angular velocity of the object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

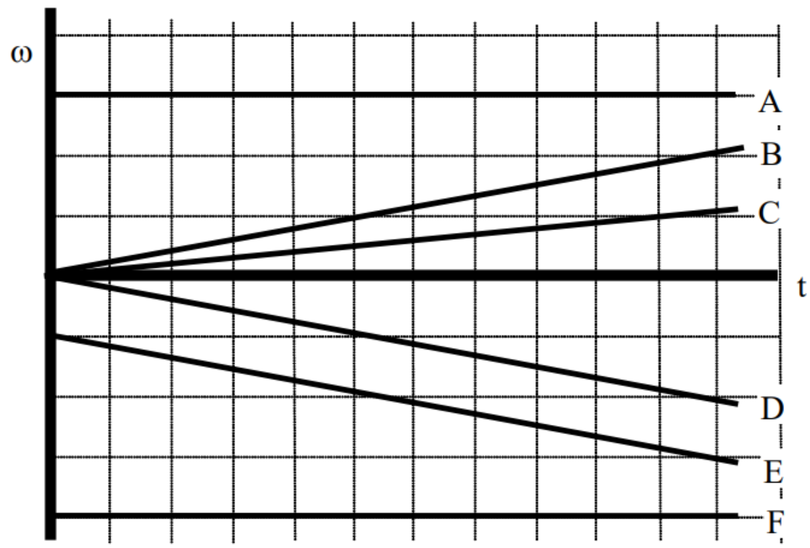
b. Rank these graphs on the basis of the angular acceleration of the object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are angular velocity vs. time graphs for six different objects.



a. Rank these graphs on the basis of the angular acceleration of the object.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

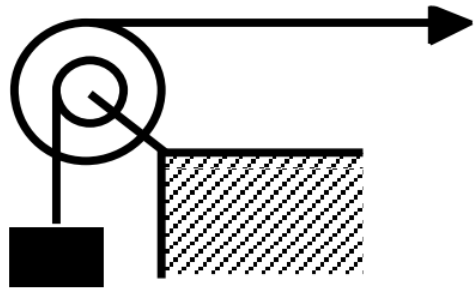
b. Rank these graphs on the basis of the angular displacement of the object over the time interval shown.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The pulley below has the outer radius and inner radius indicated. In all cases, the horizontal rope is pulled to the right at the same, constant speed.



	R_{outer}	R_{inner}
A	0.4 m	0.2 m
B	0.4 m	0.3 m
C	0.8 m	0.4 m
D	0.6 m	0.5 m
E	0.2 m	0.1 m
F	0.6 m	0.2 m

a. Rank these scenarios on the basis of the angular speed of the pulley.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

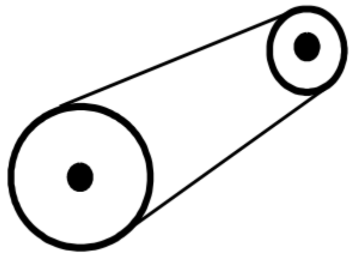
b. Rank these scenarios on the basis of the speed of the block.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The two wheels, with radii indicated below, are linked together by an elastic conveyor belt that does not slip on either wheel. In all cases, the small wheel is turning at the same, constant angular speed.



	R_{large}	R_{small}
A	0.4 m	0.2 m
B	0.4 m	0.3 m
C	0.8 m	0.4 m
D	0.6 m	0.5 m
E	0.2 m	0.1 m
F	0.6 m	0.2 m

a. Rank the scenarios on the basis of the speed of the belt.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

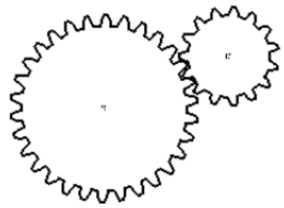
b. Rank the scenarios on the basis of the angular speed of the large wheel.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The two gears, with radii indicated below, have equal teeth spacing and are well-meshed. In all cases, the large gear is turning at the same, constant angular speed.



	R_{large}	R_{small}
A	0.4 m	0.2 m
B	0.4 m	0.3 m
C	0.8 m	0.4 m
D	0.6 m	0.5 m
E	0.2 m	0.1 m
F	0.6 m	0.2 m

a. Rank the scenarios on the basis of the speed of the teeth on the small gear.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

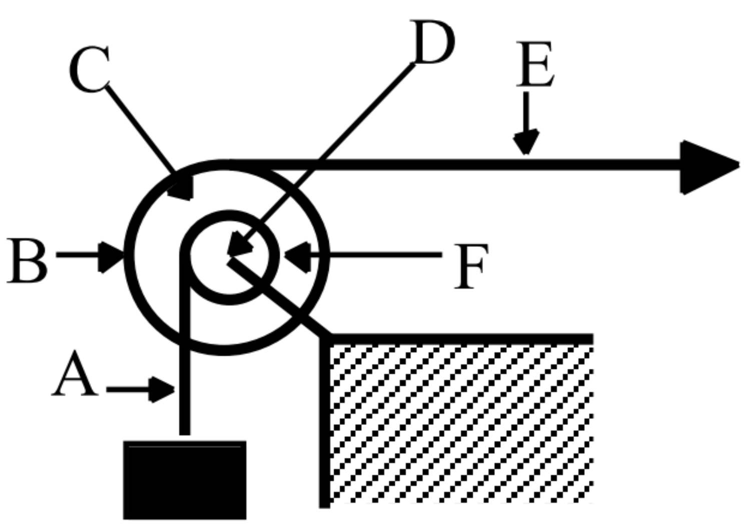
b. Rank the scenarios on the basis of the angular speed of the small gear.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A pulley system is illustrated below. The horizontal rope is pulled to the right at constant speed. Each letter designates a point on either the pulley or on one of the two ropes. Neither rope slips in its contact with the pulley.



a. Rank the designated points on the basis of their speed.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

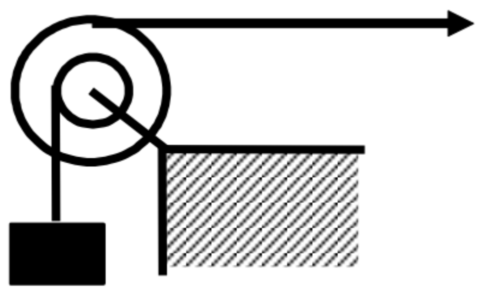
b. Rank the designated points on the basis of the magnitude of their acceleration.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

At the instant shown, the pulley below has the outer radius, inner radius, angular velocity, and angular acceleration indicated. Positive angular quantities are counterclockwise.



	R_{outer}	R_{inner}	ω	α
A	0.4 m	0.2 m	10 rad/s	0 rad/s^2
B	0.4 m	0.3 m	10 rad/s	1 rad/s^2
C	0.8 m	0.4 m	10 rad/s	0 rad/s^2
D	0.6 m	0.5 m	5 rad/s	-1 rad/s^2
E	0.2 m	0.1 m	20 rad/s	-4 rad/s^2
F	0.6 m	0.2 m	15 rad/s	2 rad/s^2

a. Rank these scenarios on the basis of the speed of the block.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

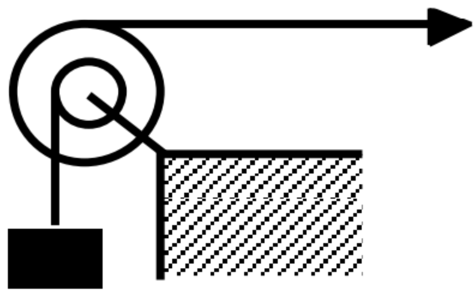
b. Rank these scenarios on the basis of the magnitude of the acceleration of the block.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

At the instant shown, the pulley below has the outer radius, inner radius, angular velocity, and angular acceleration indicated. Positive angular quantities are counterclockwise.



	R_{outer}	R_{inner}	ω	α
A	0.4 m	0.2 m	10 rad/s	0 rad/s^2
B	0.4 m	0.3 m	10 rad/s	0 rad/s^2
C	0.8 m	0.4 m	10 rad/s	0 rad/s^2
D	0.6 m	0.5 m	5 rad/s	0 rad/s^2
E	0.2 m	0.1 m	20 rad/s	0 rad/s^2
F	0.6 m	0.2 m	15 rad/s	0 rad/s^2

a. Rank these scenarios on the basis of the magnitude of the acceleration of a point on the inner rim of the pulley.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank these scenarios on the basis of the magnitude of the acceleration of the block.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

After being turned on, a record player (an antique device used by hipsters to listen to music) reaches its rated angular speed of 45 rpm (4.71 rad/s) in 1.5 s.

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
θ_1 =	θ_2 =
ω_1 =	ω_2 =
α_1 =	α_2 =

Mathematical Analysis[1]

A clothes dryer spins clothes at an angular speed of 6.8 rad/s. Starting from rest, the drier reaches its operating speed with an average angular acceleration of 7.0 rad/s^2.

Motion Information

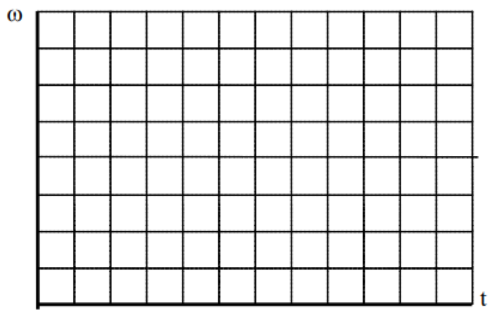
Event 1:	Event 2:

Event 1:	Event 2:
t_1 =	t_2 =
θ_1 =	θ_2 =
ω_1 =	ω_2 =
α_1 =	α_2 =

Mathematical Analysis[2]

In a secret Las Vegas research laboratory, two roulette wheels (Aladdin's and Bally's) are undergoing extensive testing. Both wheels are spun at 25 rad/s. After 6.4 s, both wheels have rotated through 100 rad. However, the angular speed of Aladdin's wheel decreases as a linear function of time while the angular speed of Bally's wheel decreases as an exponential function of time, $\omega = Ae^{-Bt}$.

Motion Graph



Motion Information

Aladdin's Wheel		Bally's Wheel	
Event 1:	Event 2:	Event 1:	Event 2:
t_1 =	t_2 =	t_1 =	t_2 =
θ_1 =	θ_2 =	θ_1 =	θ_2 =
ω_1 =	ω_2 =	ω_1 =	ω_2 =
α_1 =	α_2 =	α_1 =	α_2 =

Question

Based only on the graph, which roulette wheel is spinning faster after 6.4s? Explain.

Mathematical Analysis[3]

A diver rotating at approximately constant angular velocity completes four revolutions before hitting the water. She jumped vertically upward with an initial velocity of 5 m/s from a diving board 4 m above the water.

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =
v_1 =	v_2 =

Event 1:	Event 2:
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Mathematical Analysis[4]

A baton twirler throws a spinning baton directly upward. As it goes up and returns to the twirler's hand, it turns through four complete revolutions. The constant angular speed of the baton while in the air is 10 rad/s.

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =
v_1 =	v_2 =
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Mathematical Analysis[6]

A quarterback throws a pass that is a perfect spiral, spinning at 50 rad/s. The ball is thrown at 19 m/s at an angle of 35^0 above the horizontal. The ball leaves the quarterback's hand 2.0 m above the Astroturf and is caught just before it hits the turf. (The opposing coach thinks the ball was trapped. We are still waiting for the result of the challenge.)

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_{1x} =	r_{2x} =
r_{1y} =	r_{2y} =
θ_1 =	θ_2 =
v_{1x} =	v_{2x} =
v_{1y} =	v_{1y} =
ω_1 =	ω_2 =
a_{1x} =	a_{2x} =
a_{1y} =	a_{2y} =
α_1 =	α_2 =

Mathematical Analysis[6]

A car, with 0.75 m diameter tires, slows from 35 m/s to 15 m/s over a distance of 70 m. The car's tires do not slip in their contact with the road.

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =
v_1 =	v_2 =
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Mathematical Analysis[7]

A yo-yo of inner diameter 0.70 cm and outer diameter 8.0 cm is released from rest. The string is 0.80 m long and the yo-yo is moving downward at 0.5 m/s when it reaches the bottom of its motion. Assume the string does not slip on the inner diameter of the yo-yo during this portion of its motion.

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =
v_1 =	v_2 =
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Mathematical Analysis[8]

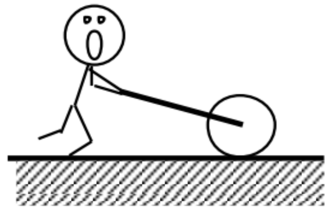
A bowling ball of diameter 21.6 cm is rolled down the alley at 4.7 m/s. The ball slows with an acceleration of 0.2 m/s^2 until it strikes the pins. The pins are located 18.3 m from the release point of the ball.

Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =
v_1 =	v_2 =
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Mathematical Analysis[9]

The strange man at right is pretending to be a steamroller by rolling a heavy cylindrical object around his backyard. The cylinder has a diameter of 0.70 m. Starting from rest, the man can make the cylinder rotate through three complete revolutions in 4.5 s. The cylinder does not slip in its contact with the ground.

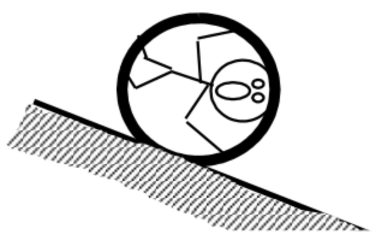


Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =
v_1 =	v_2 =
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Mathematical Analysis[10]

The man at right is trapped inside a section of large pipe. If that's not bad enough, the pipe begins to roll from rest down a 35 m long, 18^0 incline! The pipe has a diameter of 1.2 m. The pipe (and very dizzy man) reach the bottom of the incline after 6.32 s.

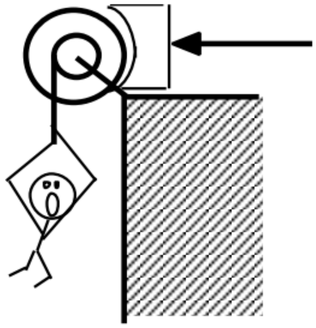


Motion Information

Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =
v_1 =	v_2 =
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Mathematical Analysis[11]

The unlucky man is falling at 20 m/s, 75 m above the crocodile-infested waters below! In an attempt to save him, the brake shoe is pressed against the spinning pulley. The action of the brake shoe gives the pulley an angular acceleration of 7.5 rad/s². The man is saved! (Barely.)

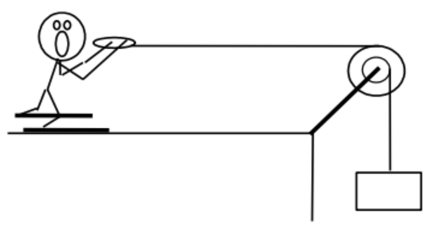


Motion Information

Lucky Man		Pulley	
Event 1:	Event 2:	Event 1:	Event 2:
t_1 =	t_2 =	t_1 =	t_2 =
r_1 =	r_2 =	θ_1 =	θ_2 =
v_1 =	v_2 =	ω_1 =	ω_2 =
a_1 =	a_2 =	α_1 =	α_2 =

Mathematical Analysis[12]

The device at right guarantees all the excitement of skiing without the need for hills. The man begins from rest and reaches a speed of 17 m/s after the block falls 10 m. The inner and outer pulley diameters are 0.40 m and 0.90 m, respectively.



Motion Information

Skier		Pulley		Block	
Event 1:	Event 2:	Event 1:	Event 2:	Event 1:	Event 2:
t_1 =	t_2 =	t_1 =	t_2 =	t_1 =	t_2 =
r_1 =	r_2 =	θ_1 =	θ_2 =	r_1 =	r_2 =
v_1 =	v_2 =	ω_1 =	ω_2 =	v_1 =	v_2 =
a_1 =	a_2 =	α_1 =	α_2 =	a_1 =	a_2 =

Mathematical Analysis[13]

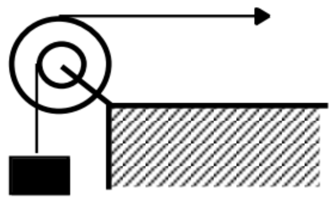
Tired of walking up the stairs, an engineering student designs an ingenious device for reaching her third floor dorm room. An block is attached to a rope that passes over the outer diameter of a 0.7 m outer diameter, disk-shaped compound pulley. The student holds a second rope, wrapped around the inner 0.35 m diameter of the pulley. When the block is released, the student is pulled up to her dorm room, 8 m off the ground, in 11.2 s.

Motion Information

Student		Pulley		Block	
Event 1:	Event 2:	Event 1:	Event 2:	Event 1:	Event 2:
t_1 =	t_2 =	t_1 =	t_2 =	t_1 =	t_2 =
r_1 =	r_2 =	θ_1 =	θ_2 =	r_1 =	r_2 =
v_1 =	v_2 =	ω_1 =	ω_2 =	v_1 =	v_2 =
a_1 =	a_2 =	α_1 =	α_2 =	a_1 =	a_2 =

Mathematical Analysis[14]

The device at right is used to lift a heavy load. The free rope is attached to a truck that, from rest, accelerates to the right with a constant acceleration. The block is lifted 25 m in 45 s. The inner and outer pulley diameters are 0.40 m and 0.90 m, respectively.

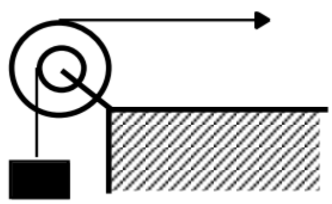


Motion Information

Truck		Pulley		Block	
Event 1:	Event 2:	Event 1:	Event 2:	Event 1:	Event 2:
t_1 =	t_2 =	t_1 =	t_2 =	t_1 =	t_2 =
r_1 =	r_2 =	θ_1 =	θ_2 =	r_1 =	r_2 =
v_1 =	v_2 =	ω_1 =	ω_2 =	v_1 =	v_2 =
a_1 =	a_2 =	α_1 =	α_2 =	a_1 =	a_2 =

Mathematical Analysis[15]

The device at right is used to lift a load quickly. The free rope is attached to a truck that, from rest, accelerates to the right with a constant acceleration. The block is lifted 25 m in 15 s. The inner and outer pulley diameters are 0.40 m and 0.90 m, respectively.



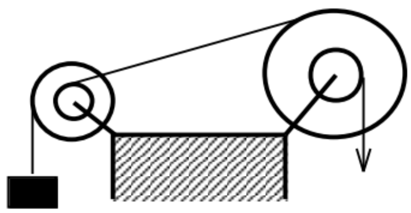
Motion Information

Truck	Pulley	Block

Truck Event 1:	Event 2:	Pulley Event 1:	Event 2:	Block Event 1:	Event 2:
Event 1:	Event 2:	Event 1:	Event 2:	Event 1:	Event 2:
t_1 =	t_2 =	t_1 =	t_2 =	t_1 =	t_2 =
r_1 =	r_2 =	θ_1 =	θ_2 =	r_1 =	r_2 =
v_1 =	v_2 =	ω_1 =	ω_2 =	v_1 =	v_2 =
a_1 =	a_2 =	α_1 =	α_2 =	a_1 =	a_2 =

Mathematical Analysis[16]

The device at right is used to quickly lift the block shown. The free rope is pulled downward at a constant speed of 2 m/s for 15 s. The inner and outer diameters of the big pulley are 0.40 m and 0.90 m, and the inner and outer diameters of the small pulley are 0.2 m and 0.6 m.



Motion Information

Big Pulley		Little Pulley		Block	
Event 1:	Event 2:	Event 1:	Event 2:	Event 1:	Event 2:
t_1 =	t_2 =	t_1 =	t_2 =	t_1 =	t_2 =
r_1 =	r_2 =	θ_1 =	θ_2 =	r_1 =	r_2 =
v_1 =	v_2 =	ω_1 =	ω_2 =	v_1 =	v_2 =
a_1 =	a_2 =	α_1 =	α_2 =	a_1 =	a_2 =

Mathematical Analysis[17]

5.2: Dynamics

Concepts and Principles

To study the dynamics of an arbitrary rigid body we will break the motion down into a pure translation of the CM and a pure rotation about the CM. We will use particle dynamics, i.e., Newton's second law applied to the CM of the object, to study the translational portion of the motion. The study of the rotational portion of the motion requires a pair of new concepts. We will "invent" these concepts through the use of an analogy with linear dynamics.

In linear dynamics, Newton's second law states that the linear acceleration of an object is proportional to the total force acting on the object and inversely proportional to the mass, or inertia, of the object. It would seem *plausible* that the angular acceleration of an object would depend on analogous concepts in the same manner.

We will replace the concept of force, often thought of as the push or pull applied to an object, with a quantity measuring the *twist* applied to an object. We will call this new quantity *torque*, symbolized τ.

We will replace the concept of mass, the measure of the resistance of the object to changes in its linear velocity, with a quantity measuring the resistance of the object to changes in its angular velocity. We will call this new quantity *rotational inertia*, symbolized I.

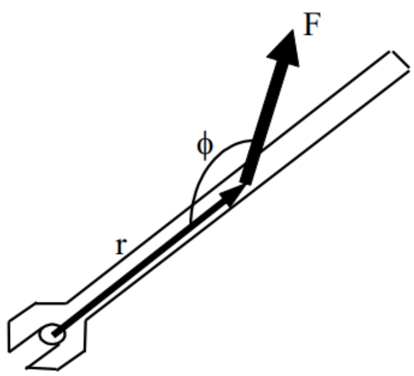
In summary,

$$\Sigma\tau = I\alpha$$

Before we go any further, however, let's define these new concepts more clearly.

Torque

In simple English, torque measures the twist applied to an object. The question remains, however, how do we *quantify* twist?



Let's examine a common device used to generate twist, a wrench. The magnitude, location, and orientation of the force applied to the wrench by the person's hand are indicated. Each of these three parameters effects the amount of twist the person delivers to the wrench (and therefore to the bolt).

If you've turned many bolts in your life, two things about this person's bolt-turning technique should grab you. First, why is this person applying the force at such a silly angle? She would generate much more twist if she applied the same magnitude force perpendicular to the wrench, rather than at an angle far from 90^0. Second, why is she not applying the force at the far edge of the wrench? She would generate far more twist if she applied the same magnitude force at the far edge of the wrench.

If the preceding paragraph makes sense to you, you understand how to quantify torque. To maximize torque, you should:

1. Apply the force far from the axis of rotation (the bolt).
2. Apply the force perpendicular to the position vector between the axis of rotation and the force.
3. Apply a large magnitude force.

Mathematically, this is summarized by:

$$\tau = rF\sin\phi$$

Note that this function has a maximum when r is large, F is large, and $\phi = 90^0$. Torque will also be assigned a direction, either clockwise or counterclockwise, depending upon the direction of the twist applied to the object.

Rotational Inertia

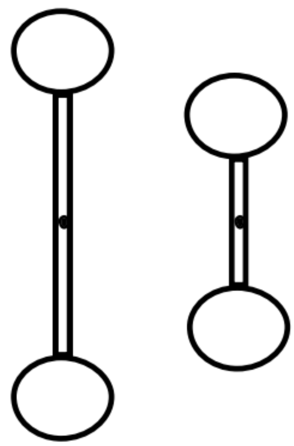
We have constructed a rotational analogy to Newton's second law,

$$\Sigma\tau = I\alpha$$

Our next task is to better define what we mean by I, the rotational inertia.

The rotational inertia is a measure of the resistance of the object to changes in its angular velocity. Imagine applying the same torque to two objects, initially at rest. After applying the torques for some set amount of time, measure the angular velocity of the objects. The object with the smaller angular velocity has the larger rotational inertia, because it has the larger resistance to angular acceleration.

Since more massive objects are harder to get moving linearly, it seems plausible that rotational inertia should depend on the mass of the object. It also seems plausible that rotational inertia should depend on the shape of the object.



For example, it would be easier to get the smaller dumbbell spinning than the larger dumbbell, even though they have the same total mass.

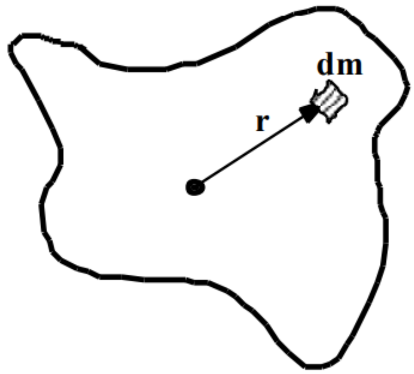
Let's try to get more quantitative. Examining

$$\Sigma\tau = I\alpha$$

we can see that the units of I must be the units of torque (N·m) divided by the units of angular acceleration (s^{-2}). Remembering that a Newton is equivalent to $kg{\cdot}m{\cdot}s^{-2}$ leads to the units of rotational inertia being $kg{\cdot}m^2$. Thus, rotational inertia must be the product of a mass and a distance squared.

It seems plausible (there's that word again!) that the distance that is squared in the relationship for rotational inertia is the distance from the rotation axis. The farther a piece of mass is from the rotation axis, the more difficult it is to give the object an angular acceleration.

Enough with the "plausibilities", let's finally just define the rotational inertia to be:



$$I=\int r^{2}dm$$

Imagine the object of interest divided into a large number of infinitesimally small chunks of mass, each with mass dm. Each chunk of mass is a distance r from the rotation axis. If you take the product of the mass of each chunk and the distance of the chunk from the rotation axis, squared, and sum this quantity over all the chunks of mass, you have the rotational inertia of the object of interest.

In summary, we now have quantitative relationships for measuring torque and rotational inertia,

$$\tau=rF\sin\phi$$
$$I=\int r^{2}dm$$

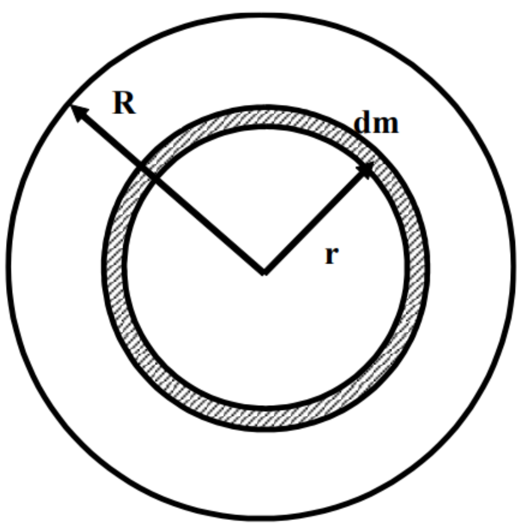
and the hypothesis that these two quantities are related in a manner analogous to Newton's second law,

$$\sum\tau=I\alpha$$

Analysis Tools

Calculating the Rotational Inertia

To fully utilize Newton's second law in rotational form, we must be able to set up and evaluate the integral that determines the rotational inertia. (To be honest, this is a lie. For the vast majority of common shapes, and many quite uncommon shapes, these integrals have already been evaluated. A table of selected results is at the end of this section.) To test our understanding of the relationship for rotational inertia, let's calculate the rotational inertia for a thin disk about an axis passing through its center of mass and perpendicular to its circular face.



1. Choose the chunks of mass, dm, to be ring-shaped. This is because you must multiply each dm by the distance of the chunk from the rotation axis squared. To facilitate doing this, it's crucial that every point in the chunk be the same distance from the axis, i.e., have the same r. If the ring-shaped chunk is thin enough, for example dr (infinitesimally) thick, then this is true.
2. Realize that the mass of the little chunk is directly proportional to its volume, assuming the disk has a constant density. If it does have a constant density, the ratio of the chunk's mass to its volume must be the same as the ratio of the total mass of the disk to its volume.

$$\frac{dm}{dV} = \frac{M}{V}$$
$$dm = \frac{M}{V} dV$$
$$dm = \frac{M}{\pi R^2 T} dV$$

   where R is the radius of the disk and T is its thickness. The volume of the ring-shaped chunk, dV, is equal to the product of the circumference of the ring (2πr), the thickness of the disk (T), and the thickness of the ring (dr). Thus,

$$dm = \frac{M}{\pi R^2 T} 2\pi r T(dr)$$

3. Plug the expression for dm into

$$I = \int r^2 dm$$
$$I = \int r^2 \frac{M}{\pi R^2 T} 2\pi r T(dr)$$
$$I = \frac{2M}{R^2} \int r^3 (dr)$$

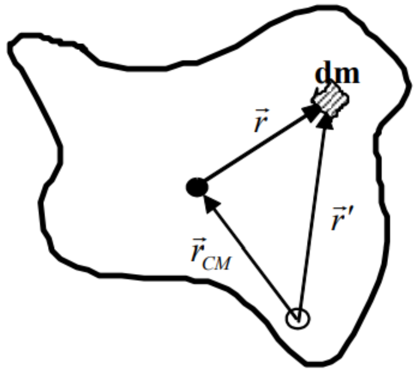
   To include all the chunks of mass, the integral must go from r = 0 m up to r = R.

$$I = \frac{2M}{R^2} \int_0^R r^3 (dr)$$
$$I = \frac{2M}{R^2} \frac{R^4}{4}$$
$$I = \frac{1}{2} M R^2$$

Thus, the rotational inertia of a thin disk about an axis through its CM is the product of onehalf the total mass of the disk and the square of its radius. Notice that the thickness of the disk does not effect its rotational inertia. A consequence of this fact is that a cylinder has the same rotational inertia as a disk, when rotated about an axis through its CM and perpendicular to its circular face.

The Parallel-Axis Theorem

Now let's imagine we need to calculate the rotational inertia of a thin disk about an axis perpendicular to its circular face and along the edge of the disk. It would be convenient if we could determine the rotational inertia about an axis along the edge using the rotational inertia about an axis through the CM (which we've already calculated). In fact, there is a *very* convenient method to determine the rotational inertia about any axis *parallel* to an axis through the CM if we know the rotational inertia about an axis through the CM.



Imagine you want to determine the rotational inertia of an arbitrarily shaped object about an arbitrary axis. The solid circle denotes an axis through the CM, the hollow circle the axis of interest. The two axes are parallel.

Notice that

$\vec{r}' = \vec{r}_{CM} + \vec{r}$

with $\vec{r}_{CM} = x_{CM}\hat{i} + y_{CM}\hat{j}$

and $\vec{r} = x\hat{i} + y\hat{j}$

Thus,

$$\vec{r}' = (x_{CM} + x)\,\hat{i} + (y_{CM} + y)\,\hat{j}$$
$$r'^2 = (x_{CM} + x)^2 + (y_{CM} + y)^2$$
$$r'^2 = x_{CM}{}^2 + x^2 + 2x_{CM}x + y_{CM}{}^2 + y^2 + 2y_{CM}y$$

The rotational inertia about the axis of interest is given by:

$$I = \int r'^2 dm$$
$$I = \int \left(x_{CM}{}^2 + x^2 + 2x_{CM}x + y_{CM}{}^2 + y^2 + 2y_{CM}y\right) dm$$
$$I = \int \left(\left(x_{CM}{}^2 + y_{CM}{}^2\right) + \left(x^2 + y^2\right) + 2x_{CM}x + 2y_{CM}y\right) dm$$
$$I = \int \left(r_{CM}{}^2 + r^2 + 2x_{CM}x + 2y_{CM}y\right) dm$$
$$I = \int r_{CM}{}^2 dm + \int r^2 dm + \int 2x_{CM}x\,dm + \int 2y_{CM}y\,dm$$

Note that x_{CM} , y_{CM} , and r_{CM} are constants that depend only on the distance between the two axes. Thus, x_{CM} , y_{CM} , and r_{CM} can by brought outside of the integral.

$$I = r_{CM}{}^2 \int dm + \int r^2 dm + 2x_{CM} \int x\,dm + 2y_{CM} \int y\,dm$$

Now comes the key observation in the derivation. Examine the term $\int \text{x dm}$. Remember that x is the horizontal distance from the CM. If this distance is integrated over all the chunks of mass, dm, throughout the entire object, this integral must equal zero because the CM is defined to be in exactly the spot where a mass-weighted average over distance is equal to zero. $\int \text{x dm}$ and $\int \text{y dm}$ are equal to zero by the definition of CM! (Pretty cool, huh?)

Thus,

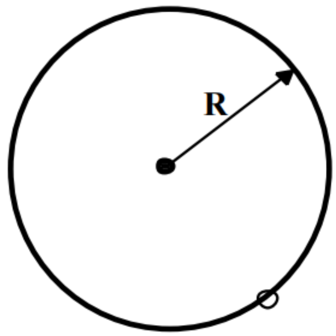
$$I = r_{CM}{}^2 \int dm + \int r^2 dm$$

Noting that $\int dm$ is the total mass of the object, M, and $\int r^2\ dm$ is the rotational inertia about the CM, I_{CM}, then

$$I = M{r_{CM}}^2 + I_{CM}$$

This result states that the rotational inertia about an axis parallel to an axis through the CM, I, is equal to the rotational inertia about an axis through the CM, I_{CM}, plus the product of the total mass of the object and the distance between the axes, r_{CM}, squared.

To answer the original question, let's determine the rotational inertia of a thin disk about an axis perpendicular to its circular face and along the edge of the disk using the parallel-axis theorem.



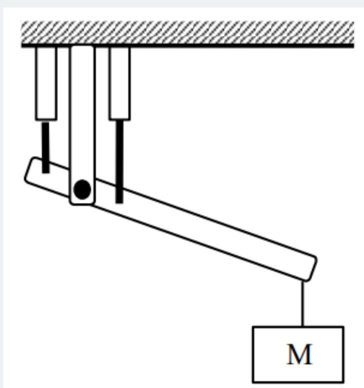
We know that the rotational inertia for a thin disk about an axis passing through its center of mass and perpendicular to its circular face is $\frac{1}{2}MR^2$ and that the distance between the CM axis and the axis of interest is R. Thus,

$$I = M{r_{CM}}^2 + I_{CM}$$
$$I = MR^2 + \frac{1}{2}MR^2$$
$$I = \frac{3}{2}MR^2$$

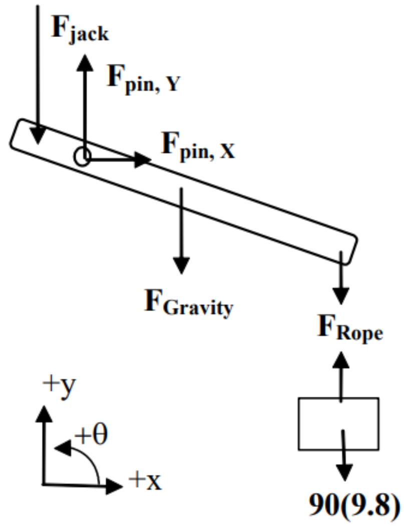
Applying Newton's Second Law in Translational and Rotational Form - I

Investigate the scenario described below.

> *The robotic arm at right consists of a pair of hydraulic jacks, each attached 10 cm from the pin "elbow". The elbow is 15 cm from the extreme edge of the 40 kg, 0.90 m long "forearm". The forearm and attached 90 kg load are held stationary at an angle of 20° below horizontal. Since the jacks primarily exert forces through extension, the front, or "biceps", jack is exerting negligible force.*



To study the dynamics of this situation, we will first need a free-body diagram of the forearm (and the attached load).



The forces due to the jack, the rope, and gravity should not need much explanation. However, the forces due to the pin elbow may need some explanation.

First, the free-body diagram drawn at left is of the forearm, not including the pin that serves as the elbow. Thus, the pin is external to the object of interest, and thus its interactions with the object are forces that must be indicated on the diagram. If you were to include the pin as part of the forearm, then the upper part of the arm would be external to the object of interest (pin plus forearm) and its interactions with the "pin plus forearm" would have to be indicated as forces on the diagram. Both of these approaches are completely valid.

Second, the direction of the force of the pin on the forearm may not be obvious. Does the pin push straight down on the forearm, pull up on the elbow, or push at some unspecified angle? The only thing that's clear is that this force is directed somewhere in the xy-plane. If the force is in the xy-plane, then it must have components along both the x- and y-axis (although one of these components may turn out to have a magnitude of zero). Thus, to handle the generality of the situation you should include both an x- and y-component for the force of the pin on the forearm. For convenience, I'll draw these forces as pointing in the positive x and y directions. (If my guess is wrong, the mathematics will tell me.)

Now that we have a free-body diagram, we can apply Newton's second law, both the linear form (in the x- and y-directions) and the rotational form (in the θ-direction).

$$\begin{array}{ll}\underline{\mathbf{x-direction}} & \underline{\mathbf{y-direction}} \\ \Sigma F = ma & \Sigma F = ma \\ F_{pin,x} = 40a_x & -F_{\text{jack}} + F_{pin,y} - F_{\text{gravity}} - F_{\text{rope}} = 40a_y\end{array}$$

Since the forearm is held stationary, both a_x and a_y are equal to zero. Also, F_{rope} is equal to the force of gravity on the load, 882 N.

$$F_{pin,x} = 0 \qquad -F_{jack} + F_{pin,y} - 40(9.8) - 882 = 40(0)$$
$$-F_{jack} + F_{pin,y} = 1274$$

Obviously, we need another equation in order to determine the two unknown forces. The obvious choice is Newton's second law in rotational form.

Before we begin, we should determine the rotational inertia for a thin rod (the closest thing to a forearm in our table) rotated about an axis not at its CM. A thin rod rotated about its CM has rotational inertia $\frac{1}{12}ML^2$. We are interested in its rotational inertia about an axis not at its CM, so we must use the parallel-axis theorem with r_{CM} = 0.30 m. (The CM is at the center of the forearm, 0.45 m from either end. Since the elbow is 0.15 m from one end, the distance between the elbow and the CM is 0.30 m.) Thus,

$$I = M{r_{CM}}^2 + I_{CM}$$
$$I = (40)(0.30)^2 + \frac{1}{12}(40)(0.90)^2$$
$$I = 6.3\text{kgm}^2$$

We must also determine the torque due to each force. (Remember, the angle, ϕ, in the relation for torque is the angle between r (oriented along the forearm) and F.)

Pin, Y	Jack	Gravity	Rope
$\tau = rF\sin\phi$ $\tau = (0)F_{pin,y}\sin(0)$ $\tau = 0$	$\tau = rF\sin\phi$ $\tau = (0.10)F_{\text{jack}}\ \sin(70)$ $\tau = 0.094F_{\text{jack}}$	$\tau = rF\sin\phi$ $\tau = (0.30)(392)\sin(110)$ $\tau = 111\text{Nm}$	$\tau = rF\sin\phi$ $\tau = (0.75)(882)\sin(110)$ $\tau = 622\text{Nm}$

Finally, let's apply Newton's second law in rotational form, paying careful attention to the algebraic sign of each torque. Note that our coordinate system indicates that all torques acting counterclockwise are positive. Therefore, all torques acting clockwise are negative.

$$\begin{aligned}\Sigma\tau &= I\alpha \\ +\tau_{\text{jack}} - \tau_{\text{gravity}} - \tau_{\text{rope}} &= 6.3\alpha \\ 0.094F_{\text{jack}} - 111 - 622 &= 6.3\alpha\end{aligned}$$

Since the forearm is held stationary, α is equal to zero.

$$\begin{aligned}0.094F_{\text{jack}} - 111 - 622 &= 0 \\ 0.094F_{\text{jack}} &= 733 \\ F_{\text{jack}} &= 7796\text{ N}\end{aligned}$$

The jack must exert a force of 7796 N to hold the forearm stationary.

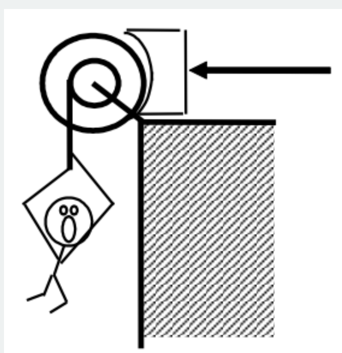
Plugging this value into our y-equation yields:

$$\begin{aligned}-F_{\text{jack}} + F_{\text{pin, }y} &= 1274 \\ -7796 + F_{\text{pin, }y} &= 1274 \\ F_{\text{pin ,}y} &= 9070\text{ N}\end{aligned}$$

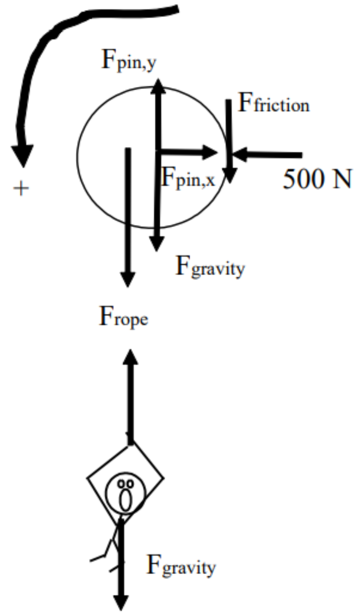
The pin must exert a force of 9070 N upwards, since the mathematics determined that the algebraic sign of $F_{Pin,y}$ was positive.

Applying Newton's Second Law in Translational and Rotational Form - II

A 75 kg man is attached to a rope wrapped around a 35 kg disk-shaped pulley, with inner and outer diameters 0.60 m and 0.90 m, respectively. The man is initially at rest. The brake shoe is pressed against the pulley with a force of 500 N. The coefficient of friction between the brake shoe and the pulley is (0.9, 0.8).



First, we have to determine whether the force applied to the brake shoe is sufficient to hold the man stationary. If it's not, we have to find the man's acceleration. To accomplish this, we need free-body diagrams for both the man and the pulley.



We’ll use counterclockwise as the positive θ-direction and down as the positive y-direction.

The pin, or axle, at the center of the pulley exerts forces in the x- and y-direction, although the exact directions may be unknown. We’ll just label these forces as $F_{pin,x}$ and $F_{pin,y}$.

The frictional force that acts on the right edge of the pulley acts in a way to prevent the man from falling (or, if he does fall, to slow down the man’s fall).

To begin my analysis, I'll assume that the man is stationary, solve for the value of the frictional force required to keep him stationary, and then determine whether this frictional force is possible given the applied force of 500 N and the coefficient of static friction.

Let’s apply Newton's second law to the man in the y-direction. (Remember, we are assuming he is not falling.)

$$\begin{aligned}\Sigma F &= ma \\ F_{\text{gravity}} - F_{\text{rope}} &= ma \\ 75(9.8) - F_{\text{rope}} &= 75(0) \\ F_{\text{rope}} &= 735 N\end{aligned}$$

Now look at Newton's second law applied to the pulley in the θ-direction.

$$\begin{aligned}\sum \tau &= I\alpha \\ +\tau_{\text{rope}} - \tau_{\text{friction}} &= I(0)\end{aligned}$$

Notice that all of the other forces acting on the pulley do *not* exert torques on the pulley. They are either located at r = 0 m or have angular orientations of $\phi = 180^\circ$.

$$\begin{aligned}(0.30)(735)\sin 90 - (0.45)F_{\text{friction}} \sin 90 = 0 \\ F_{\text{friction}} = 490 N\end{aligned}$$

Therefore, to hold the man stationary requires 490 N of friction. However,

$$\begin{aligned}F_{\text{friction}} &\leq \mu_s F_{\text{contact}} \\ F_{\text{friction}} &\leq (0.9)(500) \\ F_{\text{friction}} &\leq 450 \text{ N}\end{aligned}$$

Therefore, the man must accelerate downward.

Now that we know the man must fall, let's write the same two equations as before, although this time the accelerations (both angular for the pulley and linear for the man) are not zero.

For the pulley:

$$+\tau_{\text{rope}} - \tau_{\text{friction}} = \left(\frac{1}{2}MR^2\right)\alpha$$
$$(0.30)F_{\text{rope}}\sin 90 - (0.45)F_{\text{friction}}\sin 90 = \left(\frac{1}{2}35(0.45)^2\right)\alpha$$
$$0.30F_{\text{rope}} - 0.45F_{\text{friction}} = 3.54\alpha$$

This assumes that the rotational inertia of a "compound" pulley, one with more than one location where a rope can be wrapped, is the same as a regular pulley, and that the outermost radius of the pulley determines the inertia.

Since the man is falling, the frictional force is now kinetic, and

$$F_{\text{friction}} = \mu_k F_{\text{contact}}$$
$$F_{\text{friction}} = (0.8)(500)$$
$$F_{\text{friction}} = 400\text{ N}$$

Thus our “pulley” equation becomes:

$$0.30F_{\text{rope}} - 0.45(400) = 3.54\alpha$$
$$0.30F_{\text{rope}} - 180 = 3.54\alpha$$

Examining Newton's second law for the man,

$$F_{\text{gravity}} - F_{\text{rope}} = ma$$
$$75(9.8) - F_{\text{rope}} = 75(a)$$
$$735 - F_{\text{rope}} = 75a$$

We now have two equations with three variables. However, the angular acceleration of the pulley and the linear acceleration of the man are directly related. Since the rope that the man is attached to is wrapped 0.3 m from the center of the pulley, the man accelerates at the same rate as the tangential acceleration of a point on the pulley 0.3 m from the center. Thus,

$$a_{\text{man}} = r\alpha_{\text{pulley}}$$
$$a = 0.3\alpha$$

Using this relationship allows me to rewrite the two Newton's second law equations as:

$$0.30F_{\text{rope}} - 180 = 3.54\left(\frac{a}{0.3}\right)$$
$$735 - F_{\text{rope}} = 75a$$

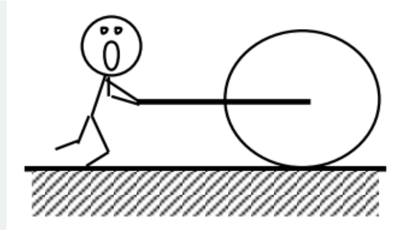
This pair of equations has the solution

$$a = 1.18\text{ m/s}^2$$
$$F = 647\text{ N}$$

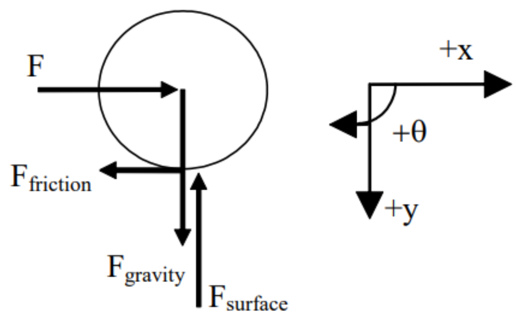
The man falls with this acceleration and the rope exerts this force (upward on the man and downward on the pulley).

Applying Newton's Second Law to Rolling Motion

> *The child at right is pretending to be a steamroller by pushing a 24 kg, 0.60 m radius cylindrical object around his backyard. The boy pushes horizontally with a force of 70 N. The coefficient of friction between the cylinder and the ground is (0.5, 0.4).*



Let’s draw a free-body diagram and write Newton’s second law in the x-, y-, and θ-directions.



x-direction	y-direction	θ-direction
$\Sigma F = ma$ $F - F_{\text{friction}} = ma$ $70 - F_{\text{friction}} = 24a$	$\Sigma F = ma$ $F_{\text{surface}} - F_{\text{gravity}} = ma$ $F_{\text{surface}} - 24(9.8) = 24(0)$ $F_{\text{surface}} = 235N$	$\Sigma\tau = I\alpha$ $+\tau_{\text{friction}} = I\alpha$ $(0.6)\ F_{\text{friction}}\ \sin 90 = \left(\frac{1}{2}MR^2\right)\alpha$ $0.6F_{\text{friction}} = \left(\frac{1}{2}(24)(0.6)^2\right)\alpha$ $F_{\text{friction}} = 7.2\alpha$

Note that the description does not specify whether the cylinder *rolls* or *skids* when the child pushes it. We will have to make an assumption, continue with the calculation, and then check our assumption for validity. Let’s assume that the bottom of the cylinder *does not slip* in its contact with the ground, which means the cylinder rolls without slipping around the backyard.

If the cylinder bottom does not slip in its contact with the ground, the horizontal acceleration (and velocity) of the cylinder bottom must equal zero. Since the acceleration of the cylinder bottom is the sum of the acceleration due to translation of the CM and the acceleration due to rotation about the CM, for the bottom to have zero horizontal acceleration means that the acceleration due to rotation,

$$a_t = r\alpha$$

must be equal in magnitude, and opposite in direction, to the translational acceleration of the CM. Thus,

$$a = (0.6)\alpha$$

Combining this with our y- and θ-equations yields:

$$70 - F_{\text{friction}} = 24a$$
$$F_{\text{friction}} = 7.2\left(\frac{a}{0.6}\right)$$

This pair of equations has the solution

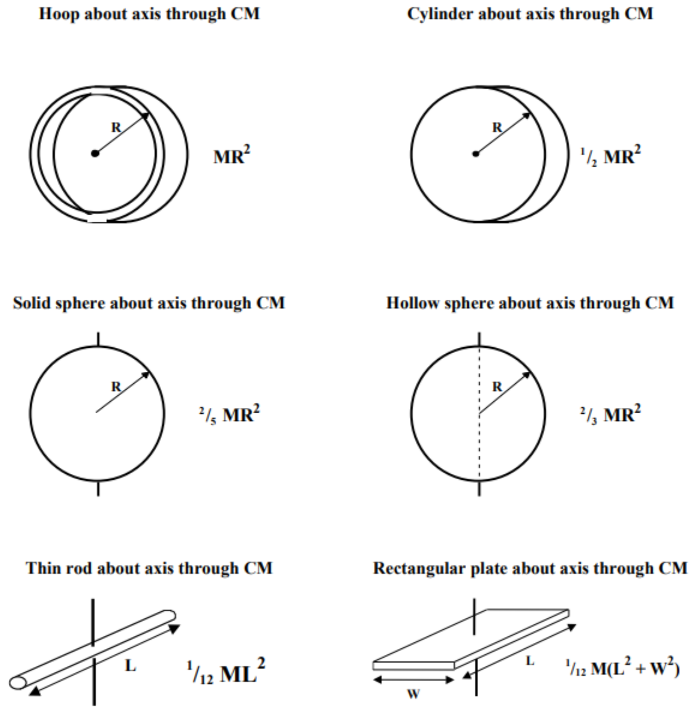
$$a = 1.94\ \text{m/s}^2$$
$$F_{\text{friction}} = 23.3\ \text{N}$$

Now we must check the validity of our assumption. If the cylinder rolls without slipping, the frictional force is static. Thus,

$$F_{\text{friction}} \leq \mu_s F_{\text{contact}}$$
$$F_{\text{friction}} \leq (0.5)(235)$$
$$F_{\text{friction}} \leq 118N$$

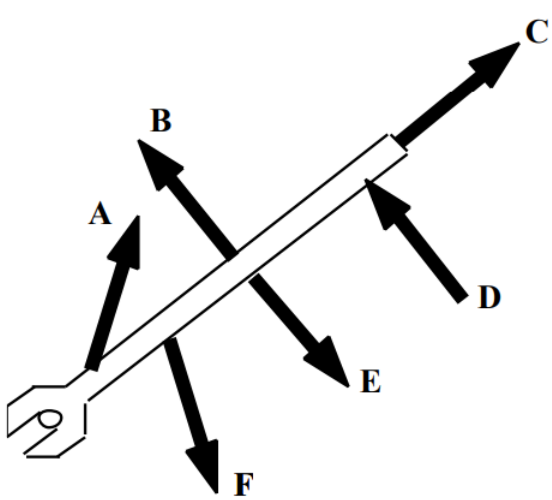
Since our calculated value for F_{friction} is less than 118 N, the cylinder does remain in static contact with the ground during its motion, our assumption is validated, and our numerical results are correct.

Rotational Inertia of Common, Uniform Solids



Activities

The wrench below has six equal magnitude forces acting on it.



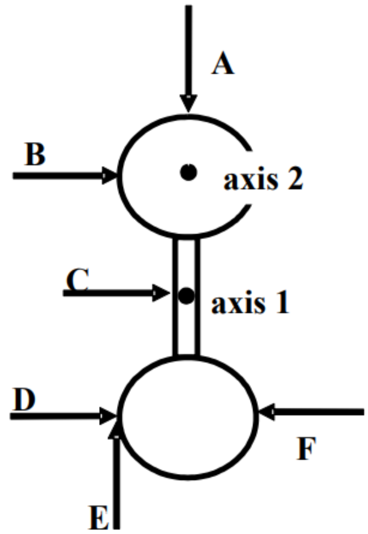
Rank these forces on the basis of the magnitude of the torque they apply to the wrench, measured about an axis centered on the bolt.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Two identical spheres are attached together by a thin rod. The rod lies on a line connecting the centers-ofmass of the two spheres. The length of the rod is equal to the diameter of each sphere. The object has six equal magnitude forces acting on it at the locations shown.



a. Rank these forces on the basis of the torque they apply to the object, measured about axis 1. Let the counterclockwise direction be positive.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

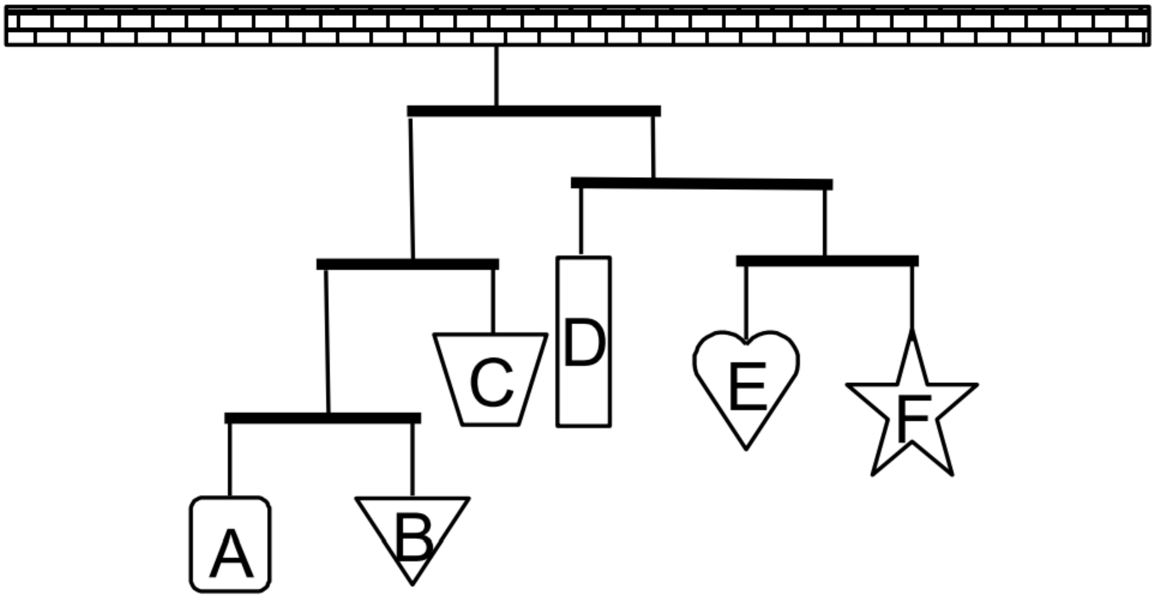
b. Rank these forces on the basis of the torque they apply to the object, measured about axis 2. Let the counterclockwise direction be positive.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

An artist constructs the mobile shown below. The highest two crossbars are 3 units long, and are hung from a point ⅓ along their length. The lower three crossbars are 2 units long, and are hung from their midpoint. In this configuration, the mobile is perfectly balanced.



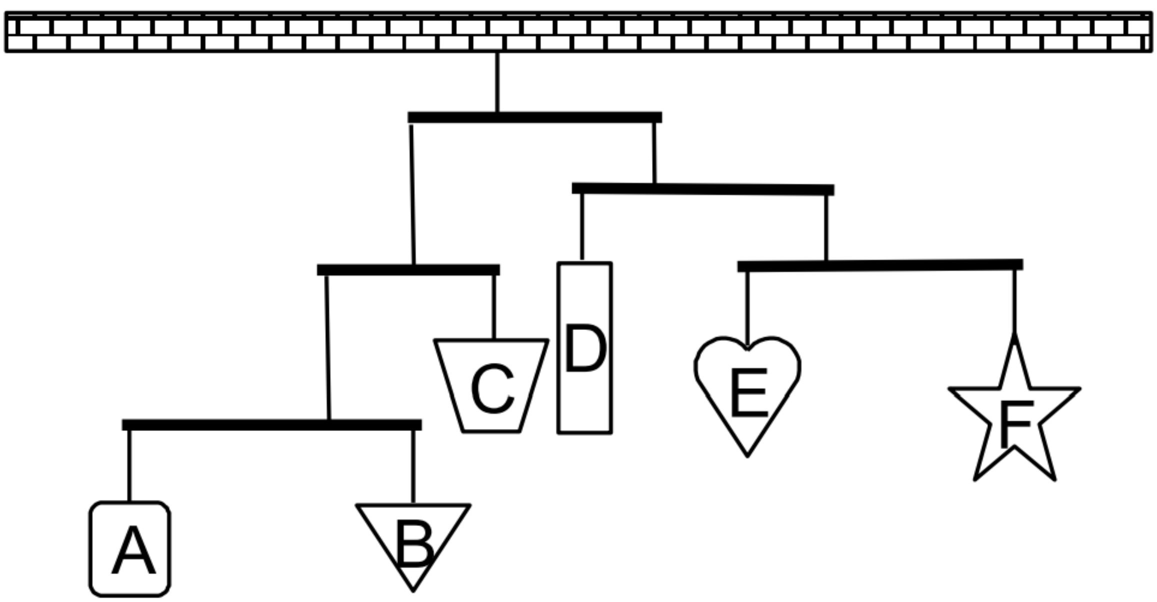
Rank the masses of the six hanging shapes.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

An artist constructs the mobile shown below. The crossbars are either 3 units long, and hung from a point ⅓ along their length, or are 2 units long, and hung from their midpoint. In this configuration, the mobile is perfectly balanced.



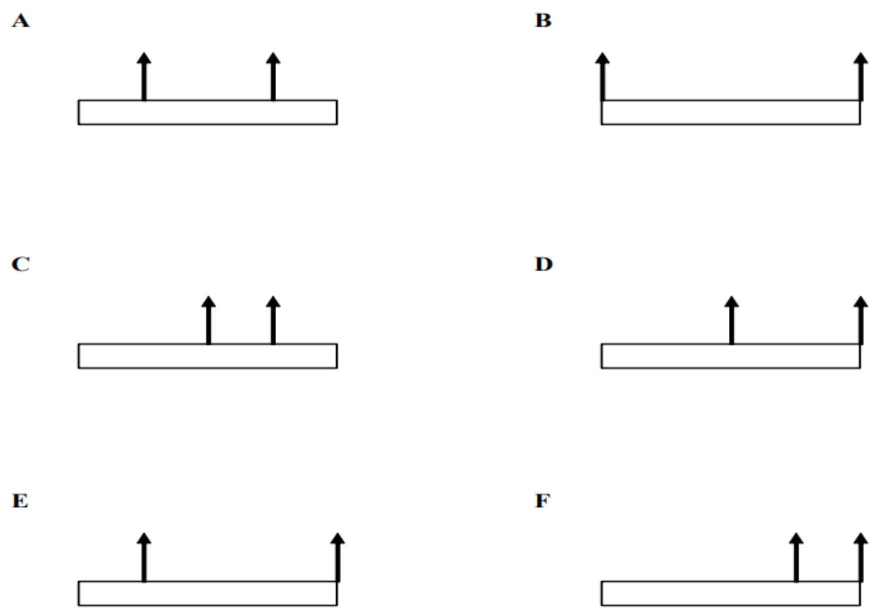
Rank the masses of the six hanging shapes

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The six platforms below are initially at rest in deep space. The indicated forces act on the platforms at their endpoints, quarter-points, or midpoints. All forces have the same magnitude.



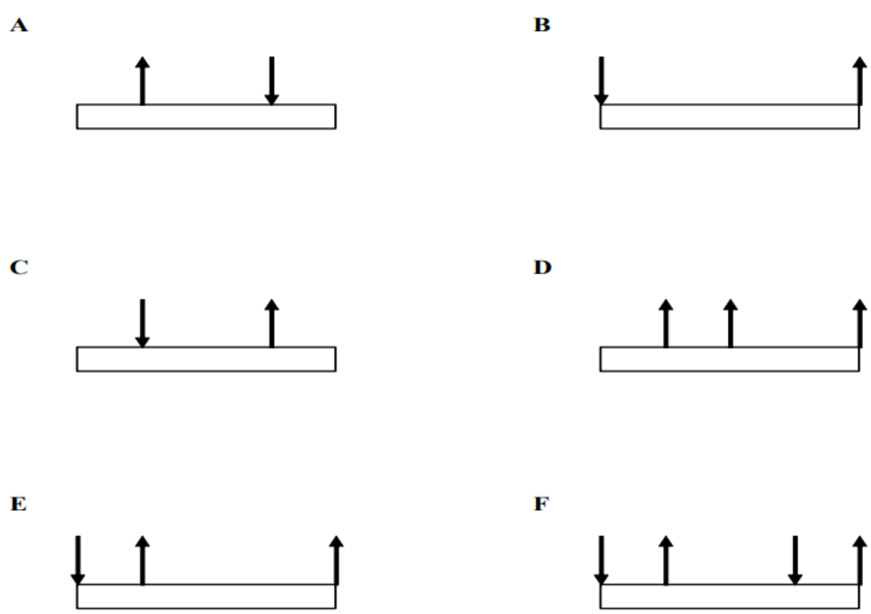
Rank these platforms on the basis of the torque that acts on them, measured about their CM. Let the counterclockwise direction be positive.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The six platforms below are initially at rest in deep space. The indicated forces act on the platforms at their endpoints, quarter-points, or midpoints. All forces have the same magnitude.



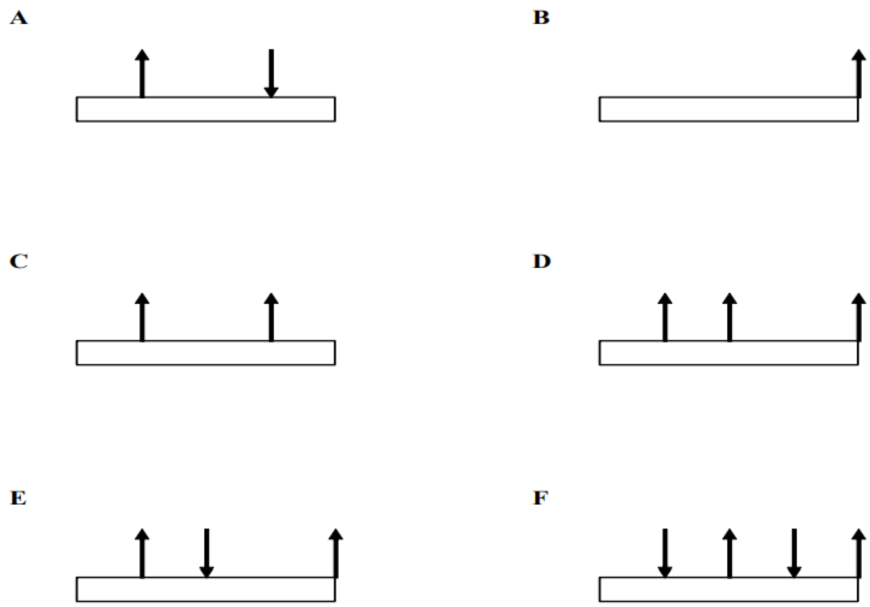
Rank these platforms on the basis of the torque that acts on them, measured about their CM. Let the counterclockwise direction be positive.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The six platforms below are initially at rest in deep space. The indicated forces act on the platforms at their endpoints, quarter-points, or midpoints. All forces have the same magnitude.



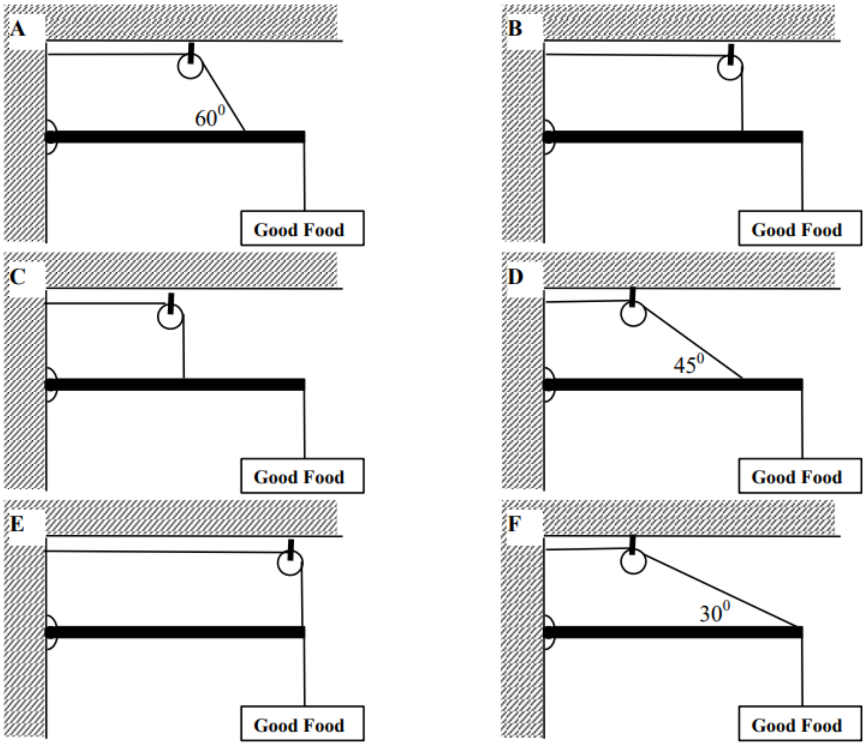
Rank these platforms on the basis of the force that must be applied to them, at their left-edge, to keep them from rotating. Let the positive direction be upward.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A roadside sign is to be hung from the end of a thin pole, and the pole supported by a single cable. Your design firm brainstorms the following six scenarios. In scenarios A, B, and D, the rope is attached to the pole ¾ of the distance between the hinge and the sign. In C, the rope is attached to the mid-point of the pole.



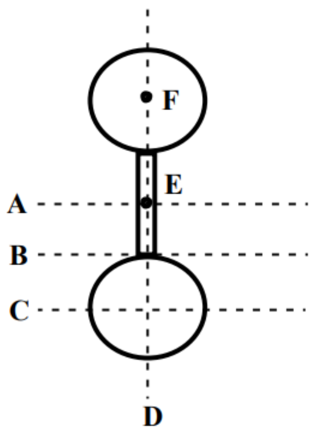
Rank the design scenarios on the basis of the tension in the supporting cable.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Two identical, uniform, solid spheres are attached together by a solid, uniform thin rod. The rod lies on a line connecting the centers-of-mass of the two spheres. The axes A, B, C, and D are in the plane of the page (which contains the centers-of-mass of the spheres and the rod), while axes E and F are perpendicular to the page.



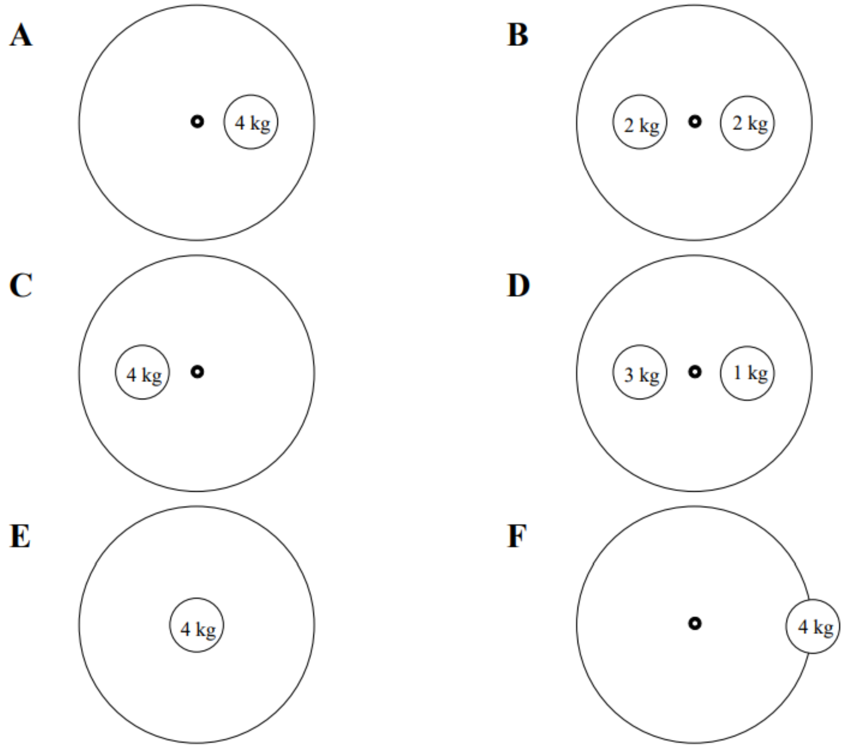
Rank the rotational inertia of this object about the axes indicated.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are top views of six identical turntables, upon which are fastened different masses. The distance from the center of the turntable to the center of the mass is either zero, one-half the radius of the turntable, or the radius of the turntable.



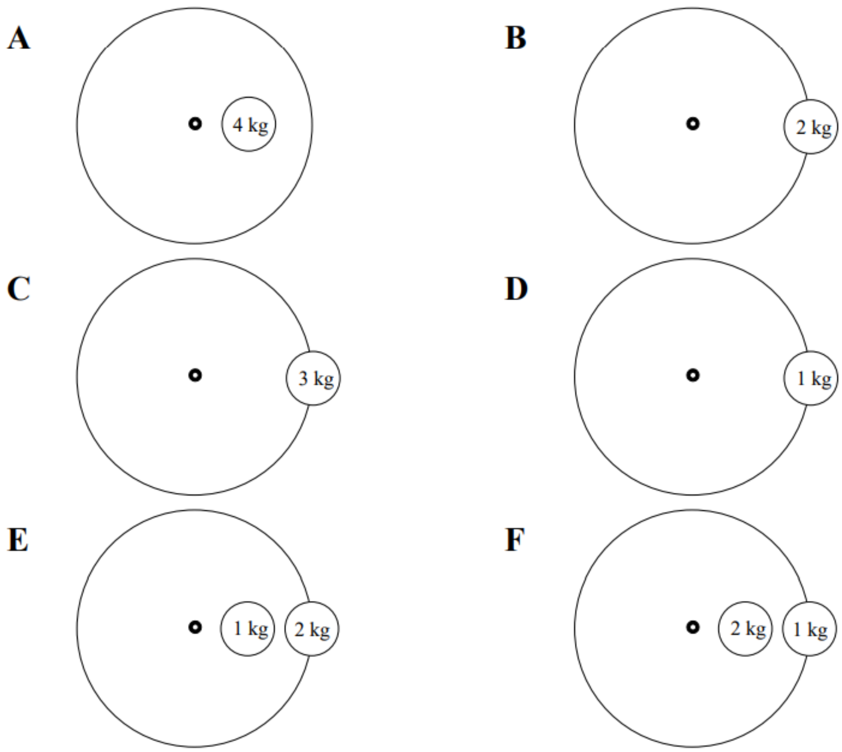
Rank the rotational inertia of these turntable-mass systems.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are top views of six identical turntables, upon which are fastened different masses. The distance from the center of the turntable to the center of the mass is either zero, one-half the radius of the turntable, or the radius of the turntable.



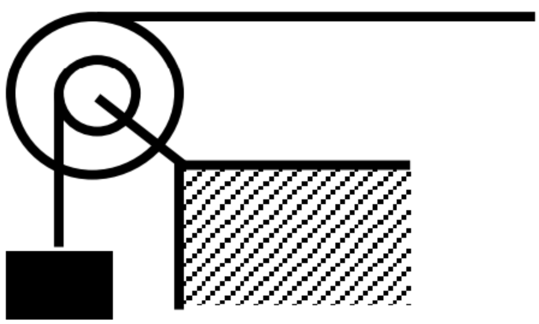
Rank the rotational inertia of these turntable-mass systems.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A block of mass M is attached to the inner radius (R_{inner}) of the pulley shown below. A second rope is attached to outer radius (R_{outer}) of the pulley. Assume friction in the pulley mount is very small.



	M	R_{inner}	R_{outer}
A	10 kg	10 cm	20 cm
B	5 kg	20 cm	40 cm
C	10 kg	5 cm	20 cm
D	10 kg	5 cm	10 cm
E	20 kg	5 cm	10 cm
F	20 kg	20 cm	30 cm

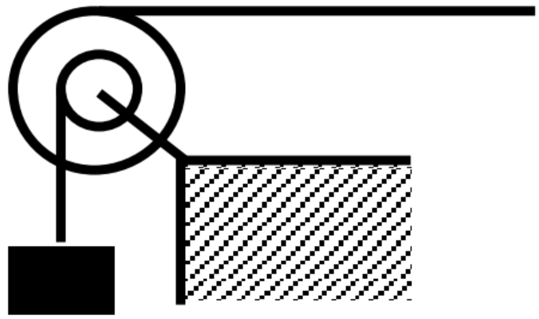
Rank these scenarios on the basis of the magnitude of the force that must be applied to the free rope to hold the block stationary.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A 10 kg block is attached to the inner radius (R_{inner}) of the pulley shown below. A second rope, attached to outer radius (R_{outer}) of the pulley, is used to raise the block at constant speed v. Assume friction in the pulley mount is very small.



	v	R_{inner}	R_{outer}
A	10 m/s	10 cm	20 cm
B	20 m/s	20 cm	40 cm
C	5 m/s	5 cm	20 cm
D	10 m/s	5 cm	10 cm
E	5 m/s	5 cm	10 cm
F	10 m/s	20 cm	30 cm

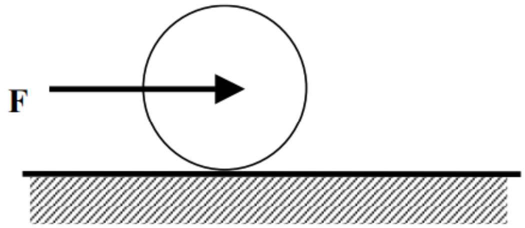
Rank these scenarios on the basis of the magnitude of the force that must be applied to the free rope to raise the block at the speed indicated.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A disk of mass m is pushed along a level surface by a force acting at its center-of-mass. The disk begins at rest and rolls without slipping along the surface. The coefficient of friction between the disk and the surface, μ, is listed below. All disks are pushed by the same magnitude force and have the same radius.



	m	μ
A	1 kg	(0.2, 0.1)
B	2 kg	(0.2, 0.1)
C	3 kg	(0.6, 0.4)
D	2 kg	(0.5, 0.3)
E	1 kg	(0.6, 0.5)
F	2 kg	(0.4, 0.3)

a. Rank the disks on the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

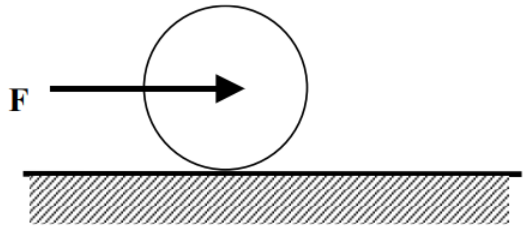
b. Rank the disks on the magnitude of their acceleration.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A sphere of radius R is pushed along a level surface by a force acting at its center-of-mass. The sphere begins at rest and rolls without slipping along the surface. The coefficient of friction between the sphere and the surface, μ, is listed below. All spheres are pushed by the same magnitude force and have the same mass.



	R	μ
A	10 cm	(0.2, 0.1)
B	20 cm	(0.2, 0.1)
C	30 cm	(0.6, 0.4)
D	20 cm	(0.5, 0.3)
E	10 cm	(0.6, 0.5)
F	20 cm	(0.4, 0.3)

a. Rank the spheres on the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

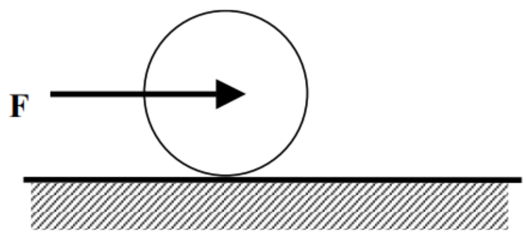
b. Rank the spheres on the magnitude of their acceleration.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A hollow sphere is pushed along a level surface by a force, F, acting at its center-of-mass. The hollow sphere begins at rest and rolls without slipping along the surface. The coefficient of friction between the hollow sphere and the surface, μ, is listed below. All hollow spheres have the same mass and radius.



	μ	F
A	(0.6, 0.4)	30 N
B	(0.6, 0.5)	30 N
C	(0.7, 0.5)	30 N
D	(0.3, 0.2)	60 N
E	(0.4, 0.2)	45 N
F	(0.9, 0.6)	60 N

a. Rank the hollow spheres on the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

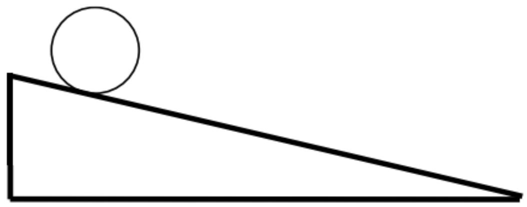
b. Rank the hollow spheres on the magnitude of their acceleration.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A disk of mass m is released from rest from the top of an incline. The disk rolls without slipping down the incline. The coefficient of friction between the disk and the incline, μ, is listed below. All disks are released from the same point on the incline and have the same radius.



	m	μ
A	1 kg	(0.2, 0.1)
B	2 kg	(0.2, 0.1)
C	3 kg	(0.6, 0.4)
D	2 kg	(0.5, 0.3)
E	1 kg	(0.6, 0.5)
F	2 kg	(0.4, 0.3)

a. Rank the disks on their speed at the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

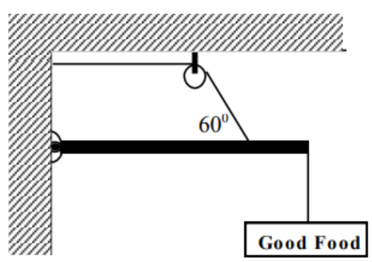
b. Rank the disks on the magnitude of the frictional force acting on them.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

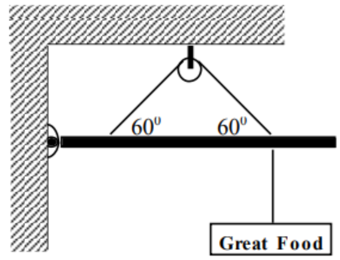
The roadside sign at right has a mass of 22 kg. It hangs from the end of a 1.6 m long, 14 kg support pole hinged to the wall. A support cable is attached to the pole 1.1 m from the wall.



Free-Body Diagram

Mathematical Analysis[18]

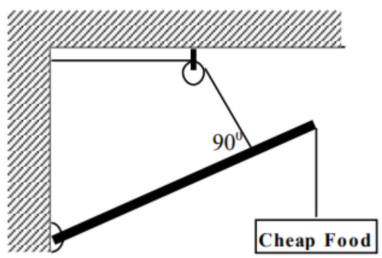
The roadside sign at right has a mass of 22 kg. It hangs from a 1.6 m long, 14 kg support pole hinged to the wall. A support cable is attached to the pole at both 0.4 m and 1.2 m from the wall. The sign is hung directly below the outer support cable.



Free-Body Diagram

Mathematical Analysis[19]

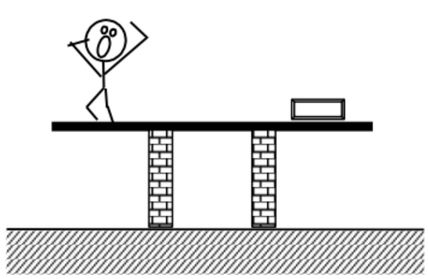
The roadside sign at right has a mass of 22 kg. It hangs from the end of a 1.6 m long, 14 kg support pole hinged to the wall at an angle of 25^0 above horizontal. A support cable is attached to the pole at a point 1.1 m from the wall along the pole.



Free-Body Diagram

Mathematical Analysis[20]

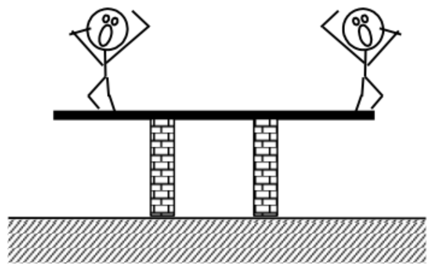
The 70 kg strange man has built a simple scaffold by placing a 6 m long, 35 kg board on top of two piles of bricks. The brick supports are 2 m apart and centered on the CM of the board. The man's 10 kg toolbox is 1 m from the right edge of the board. The man is 1.1 m from the left edge of the board when he stops and realizes he probably can't walk all the way to the left edge of the board.



Free-Body Diagram

Mathematical Analysis[21]

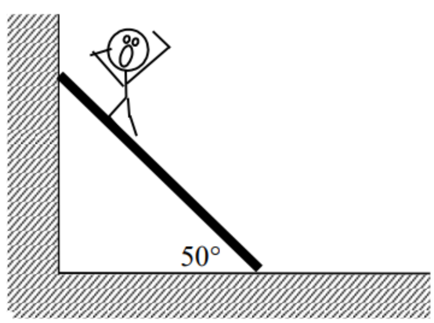
The two 70 kg strange twins have built a simple scaffold by placing a 6 m long, 35 kg board on top of two piles of bricks. The brick supports are 2 m apart and centered on the CM of the board. One twin is at the extreme right edge of the board while the other is 1.2 m from the left edge.



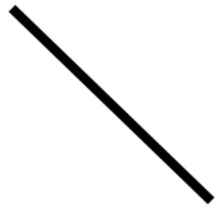
Free-Body Diagram

Mathematical Analysis[22]

The 70 kg strange man has climbed three-quarters of the way up the 20 kg, 10 m long ladder when he stops and realizes that 50° may not be a very safe angle for a ladder. The coefficient of friction between the base of the ladder and the ground is (0.6, 0.5). The friction between the ladder and the wall is negligible.

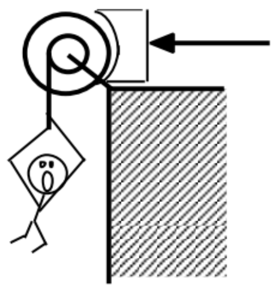


Free-Body Diagram



Mathematical Analysis[23]

The 75 kg man is falling at 20 m/s, 75 m above the crocodile-infested waters below! In an attempt to save him, the brake shoe is pressed against the spinning pulley. The coefficient of friction between the brake shoe and the pulley is (0.9, 0.8), and the 35 kg disk-shaped pulley has inner and outer diameters of 0.60 m and 0.90 m, respectively.



Motion Information

Object:	
Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =
v_1 =	v_2 =
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Free-Body Diagrams

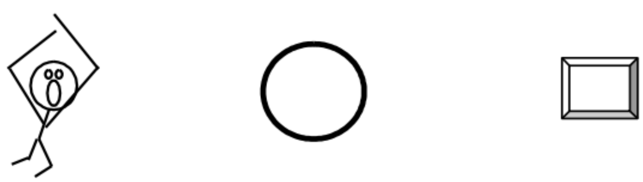
Mathematical Analysis[24]

Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. An 84 kg block is attached to a rope that passes over a 0.7 m diameter, 15 kg, disk-shaped pulley. The student holds the other end of the rope. When the 84 kg block is released, the student is pulled up to his dorm room, 8 m off the ground.

Motion Information

Object:	
Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =
v_1 =	v_2 =
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Free-Body Diagrams



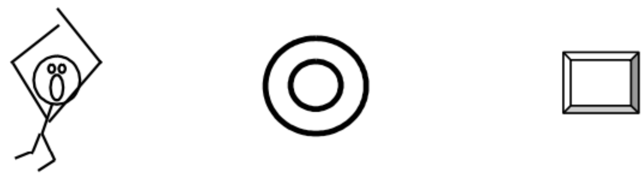
Mathematical Analysis[25]

Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching her third floor dorm room. An 42 kg block is attached to a rope that passes over the outer diameter of a 0.7 m outer diameter, 15 kg, disk-shaped compound pulley. The student holds a second rope, wrapped around the inner 0.35 m diameter of the pulley. When the 42 kg block is released, the student is pulled up to her dorm room, 8 m off the ground.

Motion Information

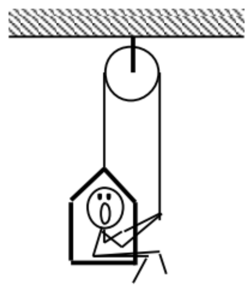
Object:	
Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =
v_1 =	v_2 =
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Free-Body Diagrams



Mathematical Analysis[26]

A 60 kg student lifts herself from rest to a speed of 1.5 m/s in 2.1 s. The boson's chair has a mass of 35 kg. The 20 kg, disk-shaped pulley has a diameter of 1.1 m.



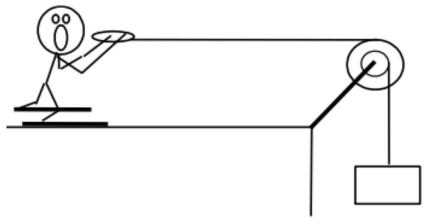
Motion Information

Object:	
Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =
v_1 =	v_2 =
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Free-Body Diagrams

Mathematical Analysis[27]

The device at right guarantees all the excitement of skiing without the need for hills. The 80 kg man begins from rest and reaches a speed of 34 m/s in 7.2 s. The 10 kg, disk-shaped pulley has inner and outer diameters of 0.40 m and 0.90 m, respectively. Assume friction is so small that it can be ignored.



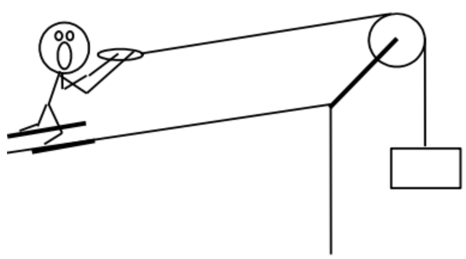
Motion Information

Object:	
Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =
v_1 =	v_2 =
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Free-Body Diagrams

Mathematical Analysis[28]

The device at right allows you to ski uphill. The ballast block has a mass of 30 kg and the skier has a mass of 70 kg. The 10 kg, disk-shaped pulley has diameter 0.90 m. Assume friction is so small that it can be ignored. The ramp is 45 m long and inclined at 20^0 above horizontal.



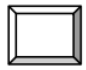
Motion Information

Object:

Object: Event 1:	Event 2:
Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =
v_1 =	v_2 =
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Free-Body Diagrams





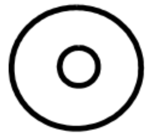
Mathematical Analysis[29]

A yo-yo of mass 0.60 kg, inner diameter 0.70 cm, and outer diameter 8.0 cm is released from rest. The string is 0.80 m long. Assume the rotational inertia of the yo-yo is similar to a simple disk and that the unwinding of the string does not effect the rotational inertia.

Motion Information

Object:	
Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =
v_1 =	v_2 =
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Free-Body Diagrams



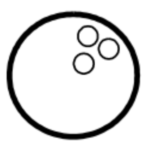
Mathematical Analysis[30]

A bowling ball of mass 7.1 kg and diameter 21.6 cm is thrown down the alley at a speed of 8.7 m/s. The bowling ball is initially skidding, with no angular velocity, down the alley. The coefficient of friction between the ball and the alley is (0.30, 0.25).

Motion Information

Object:	
Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =
v_1 =	v_2 =
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Free-Body Diagrams



Mathematical Analysis[31]

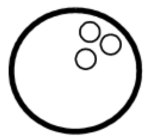
A bowling ball of mass 7.1 kg and diameter 21.6 cm is thrown down the alley at a speed of 9.5 m/s. The bowling ball is initially skidding, with no angular velocity, down the alley. The ball travels 9.6 m before it begins to roll without slipping.

Motion Information

Object:	
Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =

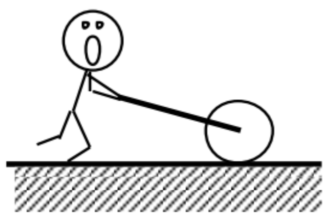
Object:	
Event 1:	Event 2:
v_1 =	v_2 =
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Free-Body Diagrams

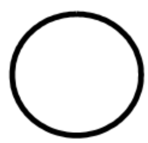


Mathematical Analysis[32]

The strange man at right is pretending to be a steamroller by pushing a heavy cylindrical object around his backyard. The cylinder has a mass of 50 kg and a diameter of 0.70 m. The man pushes with a force of 400 N at an angle of 18^0 below horizontal. The coefficient of friction between the cylinder and the ground is (0.6, 0.4).

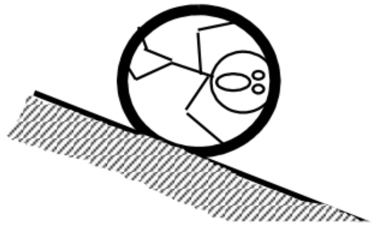


Free-Body Diagram



Mathematical Analysis[33]

The 65 kg man at right is trapped inside a section of large pipe. If that's not bad enough, the pipe begins to roll, from rest, down a 35 m long, 18^0 incline! The pipe has a mass of 180 kg and a diameter of 1.2 m. (Assume the man's presence inside the pipe has a negligible effect on the pipe's rotational inertia.) The coefficient of friction between the pipe and the ground is (0.5, 0.4).



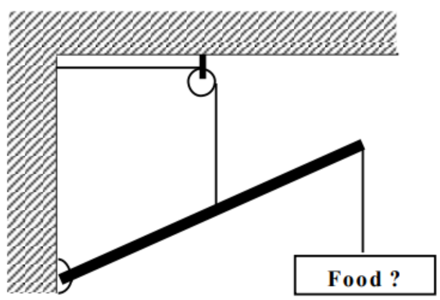
Motion Information

Object:	
Event 1:	Event 2:
t_1 =	t_2 =
r_1 =	r_2 =
θ_1 =	θ_2 =
v_1 =	v_2 =
ω_1 =	ω_2 =
a_1 =	a_2 =
α_1 =	α_2 =

Free-Body Diagrams

Mathematical Analysis[34]

The roadside sign at right has mass M. It hangs from the end of a support pole of length L and mass m, hinged to the wall at an angle of θ above horizontal. A support cable is attached vertically to the pole at its midpoint. Determine the force exerted by the cable on the pole (F_{cable}) as a function of M, m, g, L, and θ.



Free-Body Diagrams

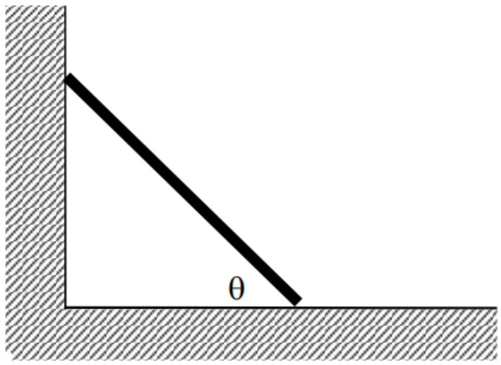
Mathematical Analysis

Questions

If M = 0 kg, what should F_{cable} equal? Does your function agree with this observation?

If the length of the pole was doubled, what would happen to F_{cable}?

Determine the minimum angle (θ_{min}) with which the ladder can be leaned against the wall and not slip as a function of the ladder's mass (m), length (L), g, and the appropriate coefficient of friction. Assume the friction between the ladder and the wall is negligible.



Free-Body Diagrams

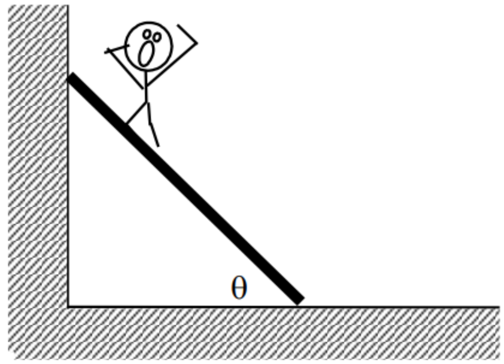
Mathematical Analysis

Questions

If μ = 0, what should θ_{min} equal? Does your function agree with this observation?

If the mass of the ladder was doubled, what would happen to θ_{min}?

Determine the minimum coefficient of static friction (μ_{min}) needed for the ladder not to slip as a function of the ladder's mass (m) and length (L), the man's mass (M) and position along the ladder (d), g, and the angle the ladder makes with the floor (θ). Assume the friction between the ladder and the wall is negligible.



Free-Body Diagrams

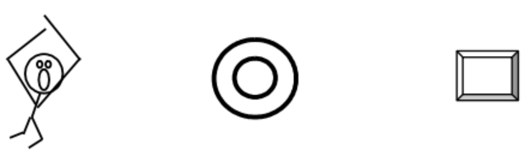
Mathematical Analysis

Questions

If θ = 90°, what should $\mu_{\min}$ equal? Does your function agree with this observation?

Tired of walking up the stairs, an engineering student of mass m designs an ingenious device for reaching her third floor dorm room. An block of mass M is attached to a rope that passes over the outer diameter, D, of a disk-shaped compound pulley of mass M_p. The student holds a second rope, wrapped around the inner diameter, d, of the pulley. When the block is released, the student is pulled up to her dorm room. Determine the acceleration of the student (a_s) as a function of m, M, M_p, d, D, and g.

Free-Body Diagrams



Mathematical Analysis

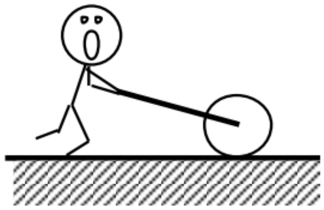
Questions

If $M_p = \infty$, what should a_s equal? Does your function agree with this observation?

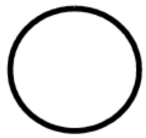
If d = D and m = M, what should a_s equal? Does your function agree with this observation?

If md > MD, what should a_s equal? Does your function agree with this observation?

The strange man at right is pretending to be a steamroller by pushing a heavy cylindrical object around his backyard. The cylinder has a mass, M, and a diameter, D. The man pushes at an angle of θ below horizontal. Determine the maximum force (F_{max}) with which the man can push and the "steamroller" roll without slipping as a function of M, D, g, θ, and the appropriate coefficient of friction between the cylinder and the ground.



Free-Body Diagram



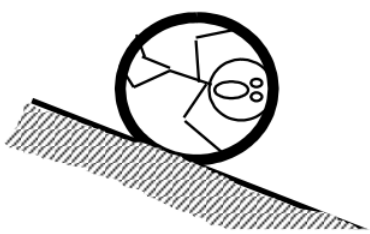
Mathematical Analysis

Questions

If μ = 0, what should F_{max} equal? Does your function agree with this observation?

Above what angle can the man push as hard as he can and still have the steamroller roll without slipping?

A man of mass m is trapped inside a pipe of mass M and diameter D initially at rest on an incline of angle θ. Determine the minimum angle (θ_{min}) above which the pipe will slide down the incline as a function of m, M, D, g, and the appropriate coefficient of friction between the pipe and the ground. Assume the man's presence inside the pipe has a negligible effect on the pipe's rotational inertia.



Free-Body Diagram

Mathematical Analysis

Questions

If μ = 0, what should θ_{min} equal? Does your function agree with this observation?

5.3: Conservation Laws

Concepts and Principles

The Angular Impulse-Angular Momentum Relation

We've already used the impulse-momentum relation to analyze situations involving translations through three-dimensional space. The relation is typically applied in its component form:

$$mv_{xi} + \int_{t_i}^{t_f} F_x dt = mv_{x_f}$$
$$mv_{y_i} + \int_{t_i}^{t_f} F_y dt = mv_{y_f}$$
$$mv_{zi} + \int_{t_i}^{t_f} F_z dt = mv_{zf}$$

Arguing by analogy, if the change in momentum in the x-, y-, and z-directions is equal to the impulse applied to the object in each direction doesn't it seem plausible that a similar relation would hold in the "θ-direction"? If so, this relation should be constructed between the torques acting on an object, the time interval over which the torques act, and the change in angular velocity of an object.[2] The relation should be:

The product of rotational inertia and angular velocity is termed the *angular momentum* of the object, typically denoted L, and the product of torque and the time interval over which it acts is termed the *angular impulse* applied to the object.

Thus, if no angular impulse is applied to an object, its angular momentum will remain constant. This special case is referred to as *angular momentum conservation*. However, if an angular impulse is applied to the object, the angular momentum will change by an amount exactly equal to the angular impulse applied. *Angular momentum is changed through angular impulse*.

Note

[2] Please remember that this model, and hence the relationships derived under it, are restricted to rigid bodies and motions in which the rotation axis is perpendicular to the plane in which the center-of-mass moves. Since objects must be rigid, their rotational inertia must remain constant. In addition, motions must either be about a stationary rotation axis or, if not, the rotation axis is taken to be through the CM.

Incorporating Rotation in the Work-Energy Relation

Our previous encounter with the work-energy relation resulted in:

$$\frac{1}{2}m{v_i}^2 + \frac{1}{2}k{s_i}^2 + mgh_i + \int_{r_i}^{r_f} (F\cos\phi)dr = \frac{1}{2}m{v_f}^2 + \frac{1}{2}k{s_f}^2 + mgh_f$$

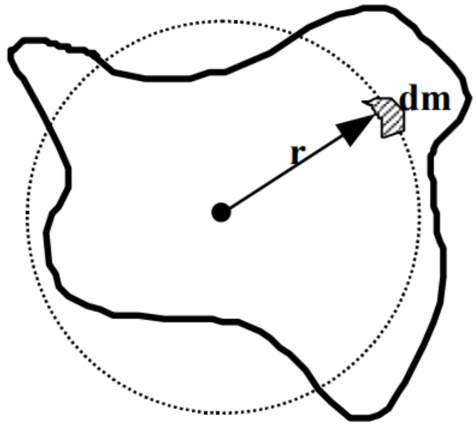
Recall from our previous discussion of work-energy that this is *not* a vector equation, meaning it is not applied independently in each of the coordinate directions. Generalizing this equation to include rotation will involving adding terms to this equation, not creating a separate "angular" energy equation.

Recall that we model the motion of an arbitrary rigid body as a superposition of a pure translation of the CM and a pure rotation about the CM. Let's investigate the effect of this model on our calculation of the kinetic energy of the object.

The translation portion of the motion is easy. We envision the object as a point particle, localized at the CM of the real object, traveling with the velocity of the CM of the object. Thus, this portion of the motion contributes a kinetic energy,

$$KE_{\text{translation}} = \frac{1}{2}m{v_{CM}}^2$$

What about the kinetic energy due to the pure rotation of the object about the axis through the CM?



Every small chunk of the object, dm, moves in a circle around the CM. Each of these pieces of mass has a velocity magnitude given by

$$v = r\omega$$

Thus, the kinetic energy of each piece is

$$KE_{dm} = \frac{1}{2}(dm)v^2$$
$$KE_{dm} = \frac{1}{2}(dm)(r\omega)^2$$

Therefore, the total kinetic energy due to rotation of all the little pieces is:

$$KE_{\text{rotation}} = \int \frac{1}{2}(dm)(r\omega)^2$$
$$KE_{\text{rotation}} = \frac{1}{2}\omega^2 \int r^2(dm)$$
$$KE_{\text{rotation}} = \frac{1}{2}I\omega^2$$

Combining the kinetic energy due to rotation with the kinetic energy due to translation leads to a total kinetic energy of:

$$KE = \frac{1}{2}m{v_{CM}}^2 + \frac{1}{2}I\omega^2$$

and a work-energy relation of:

$$\frac{1}{2}m{v_i}^2 + \frac{1}{2}I{\omega_i}^2 + \frac{1}{2}k{s_i}^2 + mgh_i + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}m{v_f}^2 + \frac{1}{2}I{\omega_f}^2 + \frac{1}{2}k{s_f}^2 + mgh_f$$

Unless a scenario involves springs or other elastic material, I'll typically write this relationship as:

$$\frac{1}{2}m{v_i}^2 + \frac{1}{2}I{\omega_i}^2 + mgh_i + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}m{v_f}^2 + \frac{1}{2}I{\omega_f}^2 + mgh_f$$

Analysis Tools

Applying the Work-Energy Relation including Rotation - I

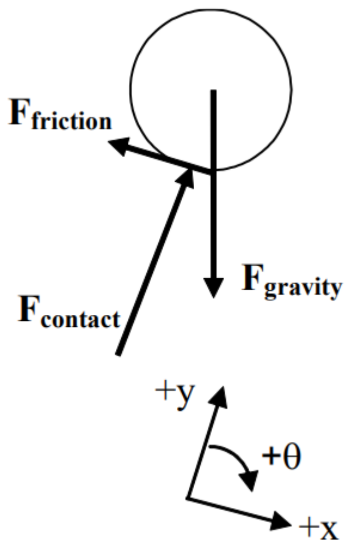
> *A mischievous child releases his mother's bowling ball from the top of the family's 25 m long, 15^0 above horizontal driveway. The ball rolls without slipping down the driveway and at the bottom plows into the mailbox. The 6.4 kg ball has a diameter of 24 cm.*

Let's determine the speed of the ball when it hits the mailbox. To determine this value, we can apply the work-energy relation to the ball between:

Event 1: The instant the ball is released.

Event 2: The instant the ball hits the mailbox.

For these two events, work-energy looks like this:



$$\frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 + mgh_i + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + mgh_f$$

$$0 + 0 + 6.4(9.8)(25\sin 15) + \int_{r_i}^{r_f}(F\cos\phi)dr = \frac{1}{2}(6.4)v_f^2 + \frac{1}{2}\left(\frac{2}{5}(6.4)(.12)^2\right)\omega_f^2 + 0$$

$$406 + \int_{r_i}^{r_f}(F\cos\phi)dr = 3.2v_f^2 + 0.0184\omega_f^2$$

where the bottom of the driveway is the zero for gravitational potential energy and the rotational inertia of the bowling ball is taken to be that of a sphere.

Now we must carefully determine which, if any, of the forces on the bowling ball do work on the bowling ball.

First, we don't have to worry about the force of gravity. The gravitational potential energy function was developed to automatically incorporate the work done by the force of gravity.

Second, the contact force can do no work on the ball because the contact force is always perpendicular to the motion of the ball.

Finally, what about the force of friction? It does appear, at first glance, that the force of friction is applied over the entire motion of the ball down the driveway. However, let's pay closer attention to the actual point at which the force acts.

The frictional force is static in nature, because since the ball rolls without slipping down the driveway the bottom of the ball is always in static contact with the ground (i.e., there is no relative velocity between the bottom of the ball and the ground). If the bottom of the ball has a velocity of zero, then the force that acts on the bottom of the ball (static friction) can act through no distance. During the instant at which the frictional force acts on a particular point on the bottom of the ball, that point is not moving. That point on the ball only moves when it is no longer in contact with the ground, but by that time the frictional force is acting on a *different* point. To summarize (and stop saying the same thing over and over), *the force of static friction can do no work because it acts on a point that does not move*.

Thus, our equation simplifies to:

$$406 = 3.2v_f^2 + 0.0184\omega_f^2$$

The linear and angular velocity of the ball must be related. Since the ball rolls without slipping, the bottom of the ball has no linear velocity. Since the velocity of the bottom of the ball is the sum of the velocity due to translation of the CM and the velocity due to rotation about the CM, the velocity due to rotation must be equal in magnitude to the velocity of the CM:

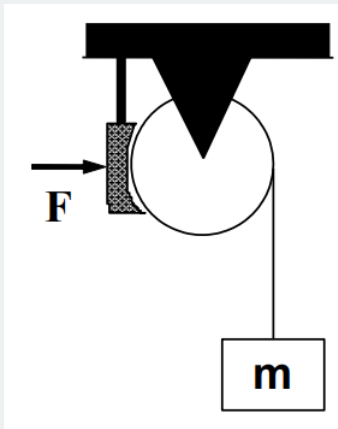
$$v = r\omega$$
$$v_{CM} = 0.12\omega$$

Substituting this into our equation yields:

$$406 = 3.2{v_f}^2 + 0.0184\left(\frac{v}{0.12}\right)^2$$
$$406 = 3.2{v_f}^2 + 1.28{v_f}^2$$
$$v_f = 9.52 \text{ m/s}$$

Applying the Work-Energy Relation including Rotation - II

The crate is descending with speed v_0 when a brake shoe is applied to the disk-shaped pulley of mass M and radius R. The coefficient of friction between the shoe and the pulley is (μ_s, μ_k). Determine the distance (D) the crate moves before stopping as a function of M, m, F, R, v_0, g, and the appropriate coefficient of friction.



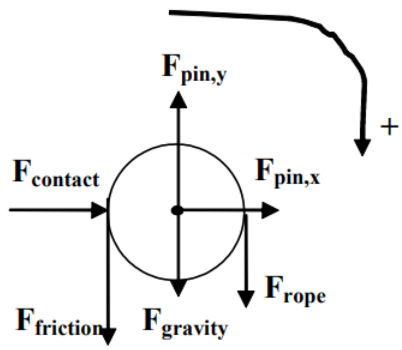
To determine this function, let's apply the work-energy relation to both the crate and the pulley between:

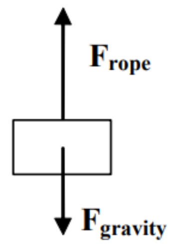
Event 1: The instant the brake shoe is applied.

Event 2: The instant the crate comes to rest.

In addition to defining the two instants of interest, we'll need free-body diagrams for both the crate and the pulley.

Pulley





$$\frac{1}{2}m{v_i}^2+\frac{1}{2}I{\omega_i}^2+mgh_i+\int_{r_i}^{r_f}(F\cos\phi)dr=\frac{1}{2}m{v_f}^2+\frac{1}{2}I{\omega_f}^2+mgh_f$$
$$0+\frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_0}{R}\right)^2+0+\int_{r_i}^{r_f}(F\cos\phi)dr=0+0+0$$

To find the work on the pulley, note that $F_{contact}$ and F_{pin} do no work since the distance over which these forces act is zero. However, both F_{rope} and $F_{friction}$ do work.

If the crate falls a distance D, both of these forces act over a distance D. However, note that this displacement is in the same direction as F_{rope} (ϕ = 0) on the right side of the pulley but in the opposite direction on the other side of the pulley (opposite to $F_{friction}$ and therefore ϕ = 180.) Since these forces are constant, there's no need to actually integrate:

$$\frac{1}{4}M{v_0}^2+(F_{\text{rope}}\cos 0)D+(F_{\text{friction}}\cos 180)D=0$$
$$\frac{1}{4}M{v_0}^2+DF_{\text{rope}}-DF_{\text{friction}}=0$$

Note that since the brake shoe does not accelerate, the external force applied to the shoe, F, is the same magnitude as the contact force between the shoe and the pulley.

$$\frac{1}{4}M{v_0}^2+DF_{\text{rope}}-D(\mu_kF)=0$$

Crate

$$\frac{1}{2}m{v_i}^2+\frac{1}{2}I\omega_i^2+mgh_i+\int_{r_i}^{r_f}(F\cos\phi)dr=\frac{1}{2}m{v_f}^2+\frac{1}{2}I{\omega_f}^2+mgh_f$$
$$\frac{1}{2}m{v_0}^2+0+mgD+(F_{rope}\cos 180)D=0+0+0$$
$$\frac{1}{2}m{v_0}^2+mgD-DF_{rope}=0$$

where the final position of the crate is the zero for gravitational potential energy.

The two equations can be added together to yield:

$$\frac{1}{4}M{v_0}^2 - \mu_k FD + \frac{1}{2}m{v_0}^2 + mgD = 0$$
$$\frac{1}{4}M{v_0}^2 + \frac{1}{2}m{v_0}^2 = \mu_k FD - mgD$$
$$D = \frac{\frac{1}{4}M{v_0}^2 + \frac{1}{2}m{v_0}^2}{\mu_k F - mg}$$

Thus, in order for D to have a physical (positive) value, the frictional force on the pulley must be greater in magnitude than the force of gravity on the crate, which agrees with common sense. Note that the numerator is simply the total kinetic energy of the pulley plus crate. The larger this sum, the larger the distance needed to stop the crate's fall, which again agrees with common sense.

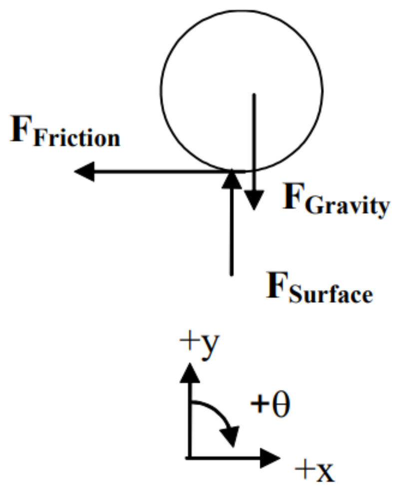
Applying the Impulse-Momentum Relations (Linear and Angular) - I

> A bowling ball of mass M and radius R leaves a bowler's hand with CM velocity v_0 and no rotation. The ball skids down the alley before it begins to roll without slipping. The coefficient of friction is (μ_s, μ_k). Determine the elapsed time (T) before the ball begins to roll without slipping as a function of M, R, v_0, g, and the appropriate coefficient of friction.

To determine this function, let's apply the impulse-momentum relations to the ball between:

Event 1: The instant the ball leaves the bowler's hand.

Event 2: The instant the ball begins to roll without slipping.



Since the ball both translates and rotates, we must write both the linear and rotational forms of the impulse-momentum relation. Remember, the rotation of the bowling ball is modeled to be about an axis through the CM. (Since all the forces are constant, we'll write the relations without the use of the integral.)

x-direction (linear momentum)	θ-direction (angular momentum)
$mv_{xi} + \int_{t_i}^{t_f} F_x dt = mv_{xf}$ $Mv_0 - F_{\text{friction}}(T) = Mv_f$ $Mv_0 - \mu_k MgT = Mv_f$ $v_0 - \mu_k gT = v_f$	$I\omega_i + \int_{t_i}^{t_f} \tau dt = I\omega_f$ $0 + (RF_{\text{friction}}(\sin 90))T = \left(\frac{2}{5}MR^2\right)\omega_f$ $R(\mu_k Mg)T = \frac{2}{5}MR^2\omega_f$ $\mu_k gT = \frac{2}{5}R\omega_f$

The final CM velocity and final angular velocity are related because at this instant the ball begins to roll without slipping. When the ball rolls without slipping, the bottom of the ball is in static contact with the ground, i.e., it has no linear velocity. Since the velocity of the bottom of the ball is the sum of the velocity due to translation of the CM and the velocity due to rotation about the CM, the velocity due to rotation must be equal in magnitude to the velocity of the CM:

$$v = r\omega$$
$$v_f = R\omega_f$$

Substituting this into our angular equation yields:

$$\mu_k g T = \frac{2}{5} R\left(\frac{v_f}{R}\right)$$
$$\mu_k g T = \frac{2}{5} v_f$$
$$v_f = \frac{5}{2}\mu_k g T$$

Plugging this into the linear equation:

$$v_0 - \mu_k g T = \frac{5}{2}\mu_k g T$$
$$v_0 = \frac{7}{2}\mu_k g T$$
$$T = \frac{2v_0}{7\mu_k g}$$

Thus, the faster you throw the ball, or the smaller the kinetic coefficient of friction, the longer it will take for the ball to begin to roll.

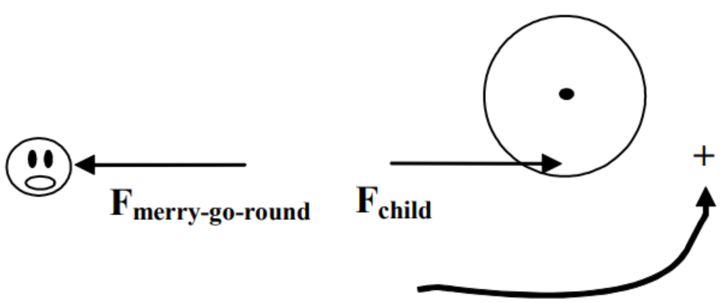
Applying the Impulse-Momentum Relations (Linear and Angular) - II

> *A 30 kg child running at 4.0 m/s leaps onto the outer edge of an initially stationary merry-go-round. The merry-go-round is a flat disk of radius 2.4 m and mass 70 kg. Ignore the frictional torque in the bearings of the merry-goround.*

Let's imagine we're interested in determining the final angular velocity of the merry-go-round after the child has safely come to rest on its surface. To determine this angular velocity, we can apply the impulse-momentum relations to both the child and the merry-go-round between:

Event 1: The instant before the child lands on the merry-go-round

Event 2: The instant after the child comes to rest on the merry-go-round.



The child and the merry-go-round interact via some unknown magnitude force. (Contrary to the free-body diagram, this force does not necessarily act on the child's disembodied head.) There are also other forces due to gravity and supporting structures that act on both the child and the merry-go-round, however, these forces are in the vertical direction and supply no torque.

Child	Merry-Go-Round
x-direction	***θ*-direction**

Child	Merry-Go-Round
$mv_{xi}+\int_{t_i}^{t_f} F_x dt=mv_{xf}$	$I\omega_i+\int_{t_i}^{t_f} \tau dt=I\omega_f$
$30(4)+\int_{t_i}^{t_f}\left(-F_{\text{merrygoround}}\right) dt=30v_f$	$0+\int_{t_i}^{t_f}\left(2.4F_{\text{child}}\ \sin 90\right) dt=\left(\frac{1}{2}70(2.4)^2\right)\omega_f$
$120-\int_{t_i}^{t_f} F_{\text{merrygoround}}\ dt=30v_f$	$\int_{t_i}^{t_f} F_{\text{child}}\ dt=84\omega_f$

The final velocity of the child and final angular velocity of the merry-go-round are related because at this instant the child is at rest (hanging on to) the merry-go-round. Therefore,

$$v=r\omega$$
$$v_f=2.4\omega_f$$

Substituting this into our angular equation yields:

$$\int_{t_i}^{t_f} F_{child} dt=84\left(\frac{v_f}{2.4}\right)$$
$$\int_{t_i}^{t_f} F_{\text{child}}\ dt=35v_f$$

By Newton's Third Law, the force of the child on the merry-go-round and the force of the merry-go-round on the child must be equal in magnitude. Therefore our two equations can be summed and the impulses cancel. This gives:

$$120=30v_f+35v_f$$
$$v_f=1.85\ \mathrm{m/s}$$

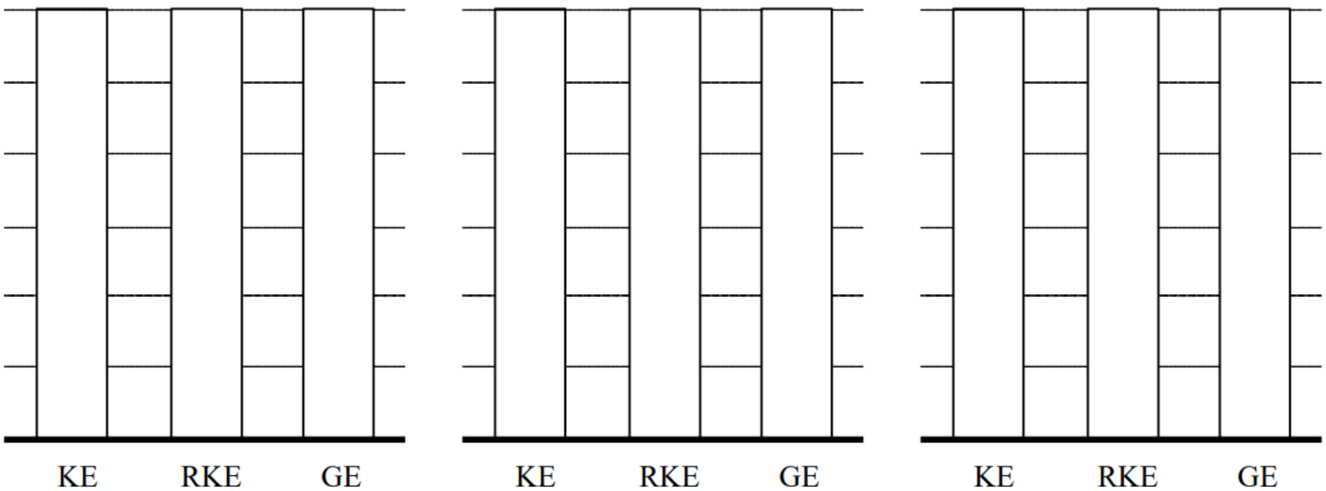
The child is slowed from 4 m/s to 1.85 m/s by jumping onto the merry-go-round. The merrygo-round, however, is accelerated from rest to:

$$1.85=2.4\omega_f$$
$$\omega_f=0.77\mathrm{rad/s}$$

Activities

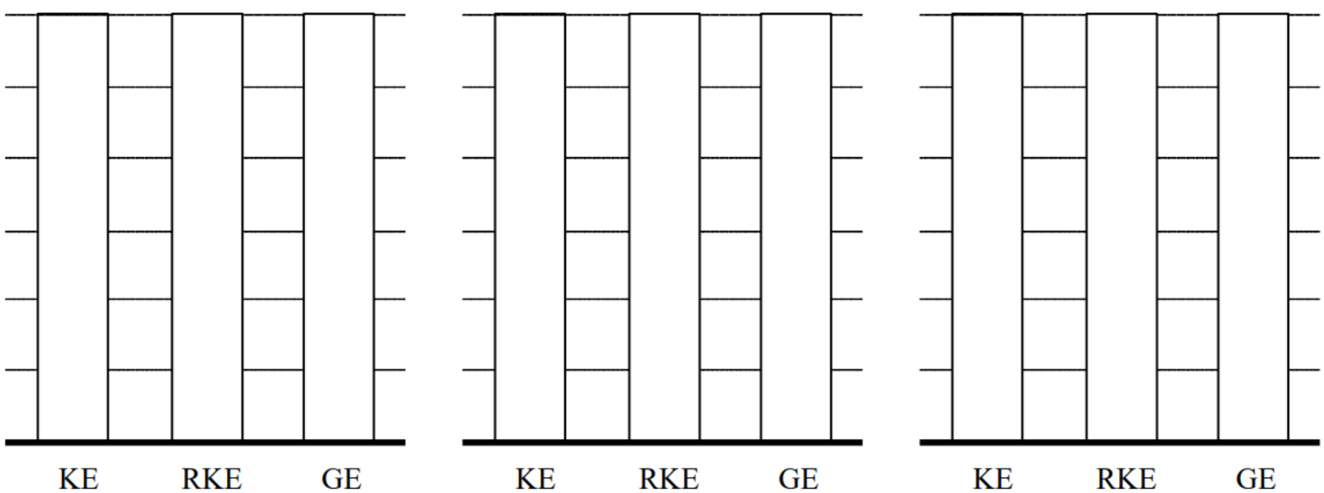
For each of the scenarios described below, indicate the amount of linear kinetic energy, rotational kinetic energy, and gravitational potential energy in the system at each of the events listed. Use a consistent scale for each motion. Set the lowest point of each motion as the zero-point of gravitational potential energy.

a. *A mischievous child releases his mother's bowling ball from the top of the family's 15 m long, 80 above horizontal driveway. The ball rolls without slipping down the driveway and at the bottom plows into the mailbox.*



When the ball is first released. *When the ball is halfway down the driveway.* *Just before the ball hits the mailbox.*

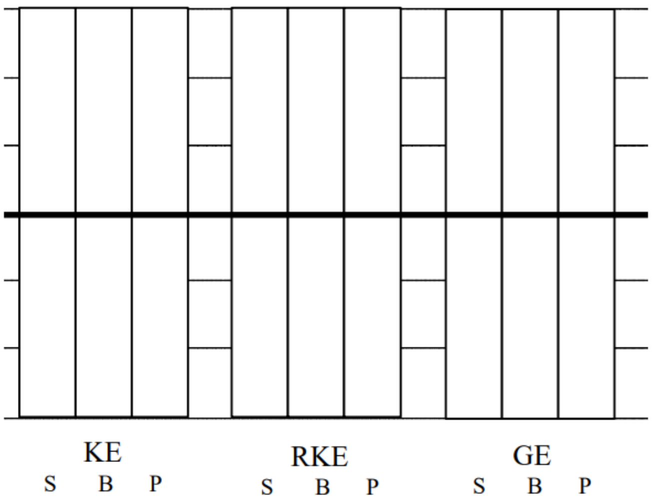
b. *A yo-yo of inner diameter 0.70 cm and outer diameter 8.0 cm is released from rest. The string is 0.80 m long. Assume the string does not slip on the inner diameter of the yo-yo as the yo-yo falls..*



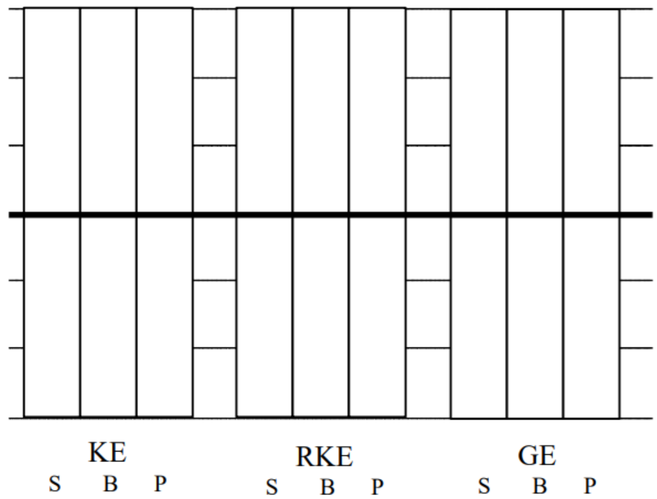
When the yo-yo is first released. *When the yo-yo is halfway down the string.* *Just before the yo-yo reaches the end of its string*

For each of the scenarios described below, indicate the amount of linear kinetic energy, rotational kinetic energy, and gravitational potential energy of each object at each of the events listed. Use a consistent scale for each motion. Set the initial positions of the objects as the zero-points of gravitational potential energy.

a. *Tired of walking up the stairs, an 80 kg engineering student (S) designs an ingenious device for reaching his third floor dorm room. An 84 kg block (B) is attached to a rope that passes over a 0.70 m diameter, 15 kg, disk-shaped pulley (P). The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room.*

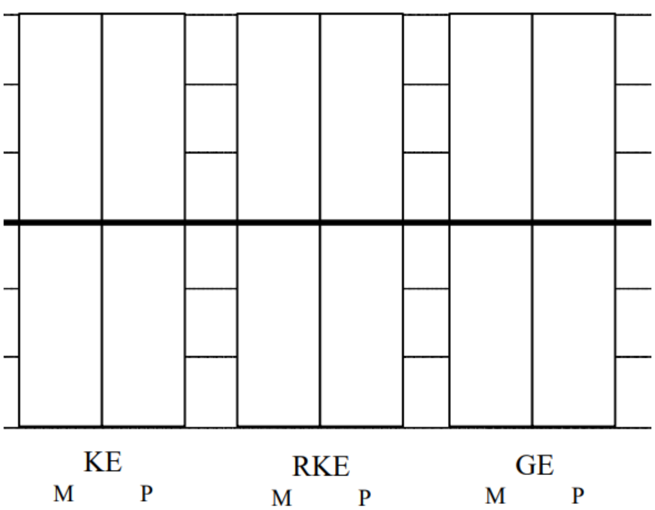


When the block is released.

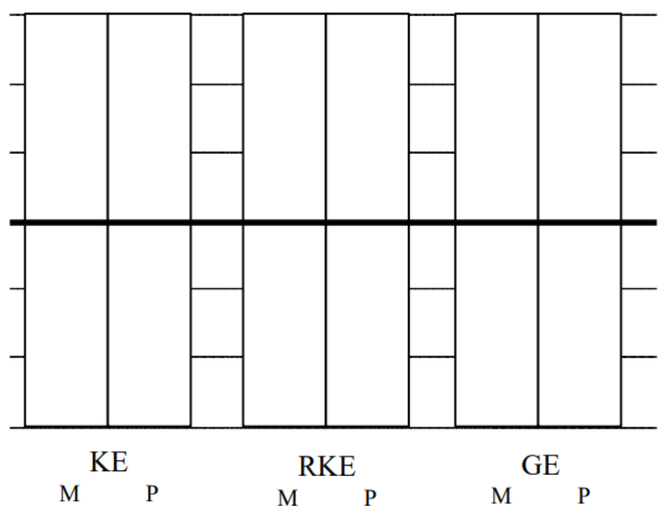


When the student reaches his room.

b. *A 75 kg man (M) is falling at 20 m/s, 75 m above crocodile infested waters! He holds a rope attached to a 15 kg simple pulley (P). In an attempt to save him, a brake shoe is pressed against the spinning pulley. The man is saved, but barely.*



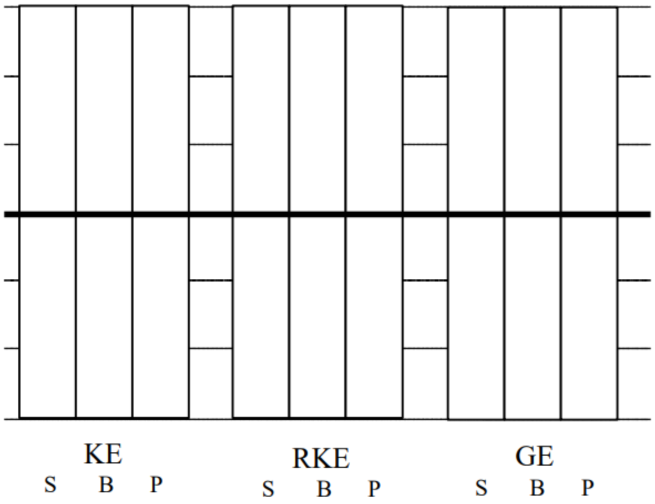
When the brake is first applied.



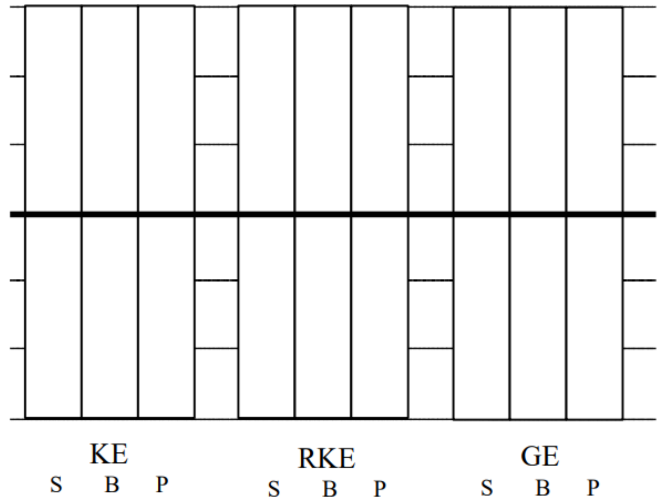
When the man stops falling.

For each of the scenarios described below, indicate the amount of linear kinetic energy, rotational kinetic energy, and gravitational potential energy of each object at each of the events listed. Use a consistent scale throughout both motions. Set the initial positions of the objects as the zero-points of gravitational potential energy

a. *In a horizontal skiing device, the skier begins from rest 35 m from the end of the skiing run. The skier (S) has a mass of 75 kg, the block (B) has a mass of 50 kg, and the simple pulley (P) has a mass of 15 kg. The coefficient of friction is extremely small.*

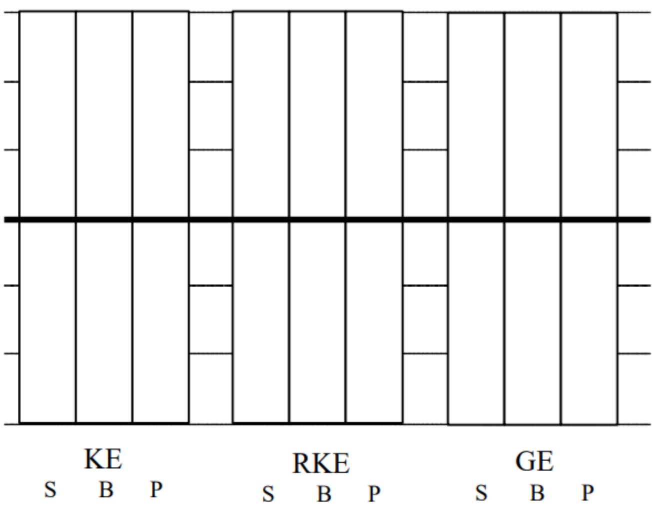


When the block is released.

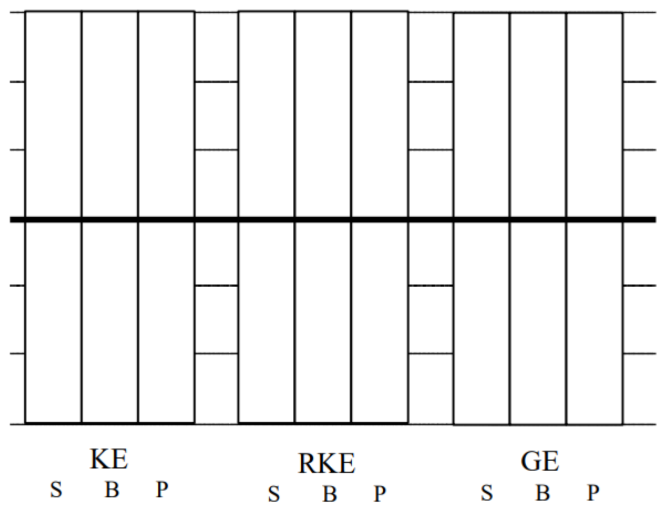


When the skier reaches the end of the run.

b. *In an inclined skiing device, the skier begins from rest 35 m from the end of the 20^0 above horizontal inclined skiing run. The skier (S) has a mass of 75 kg, the block (B) has a mass of 50 kg, and the simple pulley (P) has a mass of 15 kg. The coefficient of friction is extremely small.*

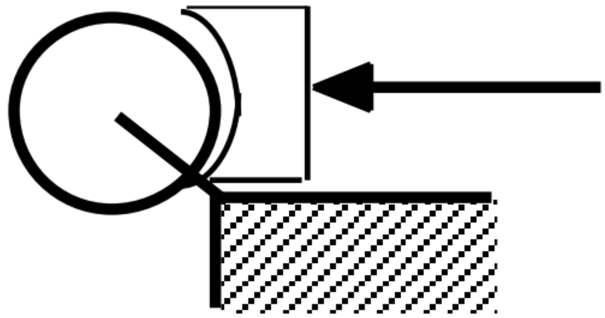


When the block is released.



When the skier reaches the end of the run.

A disk-shaped pulley of mass M and radius R is rotating at angular velocity ω. The friction in its bearings is so small that it can be ignored. A brake shoe is pressed against the pulley in order to stop it. In all cases, the brake shoe is pressed against the pulley with the same force and the coefficient of friction between the brake shoe and the pulley is the same.



	M	R	ω
A	10 kg	0.8 m	12 rad/s
B	10 kg	0.4 m	24 rad/s
C	20 kg	0.4 m	6 rad/s
D	40 kg	0.2 m	3 rad/s
E	20 kg	0.2 m	3 rad/s
F	30 kg	0.6 m	8 rad/s

a. Rank the scenarios below on the amount of time it takes to stop the pulley.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

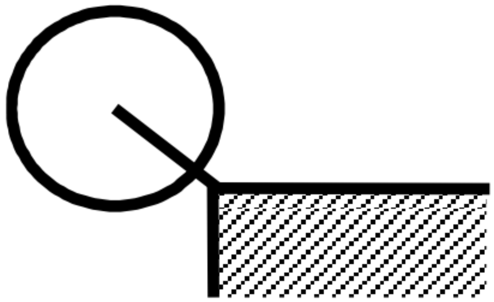
b. Rank the scenarios below on the angle through which the pulley rotates before stopping.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A disk-shaped pulley of mass M and radius R is rotating at angular velocity ω. The friction in its bearings is constant and ultimately causes the pulley to stop rotating.



	M	R	ω
A	10 kg	0.8 m	12 rad/s
B	10 kg	0.4 m	24 rad/s
C	20 kg	0.4 m	6 rad/s
D	40 kg	0.2 m	3 rad/s
E	20 kg	0.2 m	3 rad/s
F	30 kg	0.6 m	8 rad/s

a. Rank the scenarios below on the amount of time it takes to stop the pulley.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank the scenarios below on the angle through which the pulley rotates before stopping.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A sphere of mass m and radius R is released from rest at the top of an incline and rolls without slipping down the incline. All spheres are released from rest from the same location on the incline.

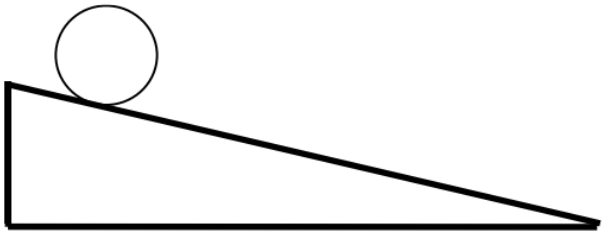
Rank the speed of the sphere at the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

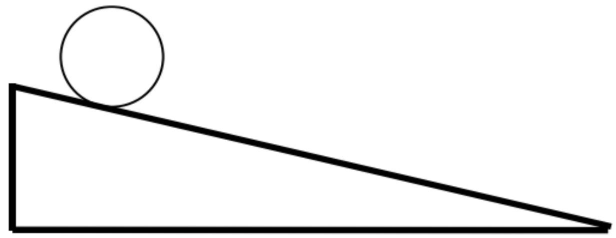
_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Six different equal-mass objects are released from rest from the same location at the top of an incline and roll without slipping down the incline.



	Object	R
A	solid sphere	30 cm
B	hollow sphere	60 cm
C	solid disk	10 cm
D	hoop	10 cm
E	solid cylinder	20 cm



	m	R
A	1 kg	30 cm
B	2 kg	60 cm
C	3 kg	10 cm
D	2 kg	15 cm
E	1 kg	15 cm
F	2 kg	30 cm

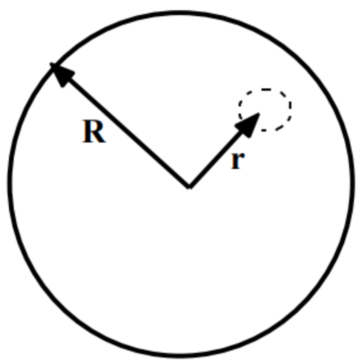
Rank the speed of the objects at the bottom of the incline.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A merry-go-round of radius R is rotating at constant angular speed. The friction in its bearings is so small that it can be ignored. A sandbag of mass m is dropped onto the merry-go-round, at a position designated by r. The sandbag does not slip or roll upon contact with the merry-go-round.



	m	r
A	10 kg	0.50 R
B	10 kg	0.25 R
C	20 kg	0.25 R
D	40 kg	0.25 R
E	10 kg	1.00 R
F	15 kg	0.75 R

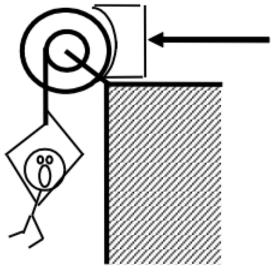
Rank the scenarios on the basis of the angular speed of the merry-go-round after the sandbag “sticks” to the merry-go-round.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

The 75 kg man is falling at 20 m/s, 75 m above the crocodile infested waters below! In an attempt to save him, the brake shoe is pressed against the spinning pulley. The coefficient of friction between the brake shoe and the pulley is (0.9, 0.8), and the 35 kg disk-shaped pulley has inner and outer diameters of 0.60 m and 0.90 m, respectively. The man is saved, but barely.



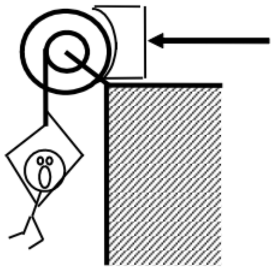
Free-Body Diagrams

Mathematical Analysis[35]

Event 1:

Event 2:

The 75 kg man is falling at 15 m/s! In an attempt to stop his fall, the brake shoe is pressed against the spinning pulley with a force of 1000 N. The coefficient of friction between the brake shoe and the pulley is (0.9, 0.8), and the 35 kg disk-shaped pulley has inner and outer diameters of 0.60 m and 0.90 m, respectively.



Free-Body Diagrams

Mathematical Analysis[36]

Event 1:

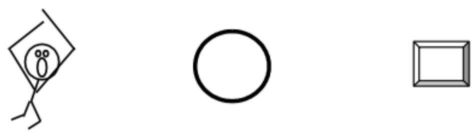
Event 2:

a. How long does it take to stop the man's fall?

b. How far does the man fall?

Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. An 84 kg block is attached to a rope that passes over a 0.70 m diameter, 15 kg, disk-shaped pulley. The student holds the other end of the rope. When the 84 kg block is released, the student is pulled up to his dorm room in 5.3 s.

Free-Body Diagrams



Mathematical Analysis[37]

Event 1:

Event 2:

Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching her third floor dorm room. An 42 kg block is attached to a rope that passes over the outer diameter of a 0.70 m outer diameter, 15 kg, disk-shaped compound pulley. The student holds a second rope, wrapped around the inner 0.35 m diameter of the pulley. When the 42 kg block is released, the student is pulled up to her dorm room, 8.0 m off the ground.

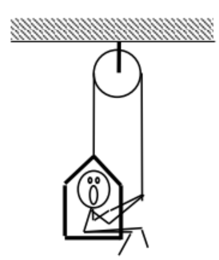
Free-Body Diagrams

Mathematical Analysis[38]

Event 1:

Event 2:

A 60 kg student lifts herself from rest to a speed of 1.5 m/s in 2.1 s. The boson's chair has a mass of 35 kg. The 20 kg, disk-shaped pulley has a diameter of 1.1 m.



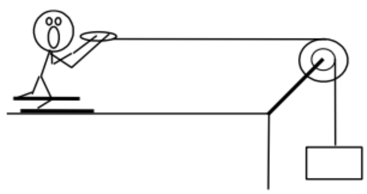
Free-Body Diagrams

Mathematical Analysis[39]

Event 1:

Event 2:

The device at right guarantees all the excitement of skiing without the need for hills. The 80 kg man begins from rest and reaches a speed of 34 m/s in 7.2 s. The 10 kg, disk-shaped pulley has inner and outer diameters of 0.40 m and 0.90 m, respectively. Assume friction is so small that it can be ignored.



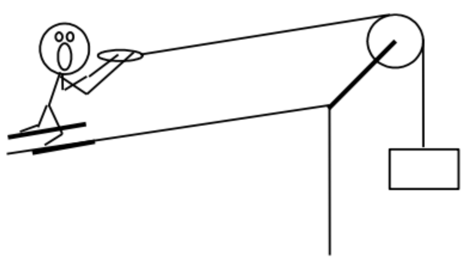
Free-Body Diagrams

Mathematical Analysis[40]

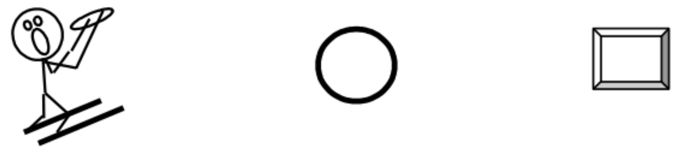
Event 1:

Event 2:

The device at right allows you to ski uphill. The 85 kg skier begins from rest and reaches a speed of 14 m/s after traveling 25 m up the incline. The 10 kg, disk-shaped pulley has diameter 0.90 m. Assume friction is so small that it can be ignored. The ramp is inclined at 20° above horizontal.



Free-Body Diagrams



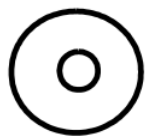
Mathematical Analysis[41]

Event 1:

Event 2:

A yo-yo of mass 0.60 kg, inner diameter 0.70 cm, and outer diameter 8.0 cm is released from rest. The string is 0.80 m long. Assume the rotational inertia of the yo-yo is similar to a simple disk and that the unwinding of the string does not affect the rotational inertia.

Free-Body Diagrams



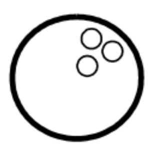
Mathematical Analysis[42]

Event 1:

Event 2:

A bowling ball of mass 7.1 kg and diameter 21.6 cm is thrown down the alley at a speed of 9.5 m/s. The bowling ball is initially skidding, with no angular velocity, down the alley. The ball skids for 1.8 s before it begins to roll without slipping.

Free-Body Diagrams



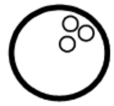
Mathematical Analysis[43]

Event 1:

Event 2:

A mischievous child releases his mother's bowling ball from the top of the family's 15 m long, 80 above horizontal driveway. The ball rolls without slipping down the driveway and at the bottom plows into the mailbox. The 6.4 kg ball has a diameter of 21.6 cm.

Free-Body Diagrams

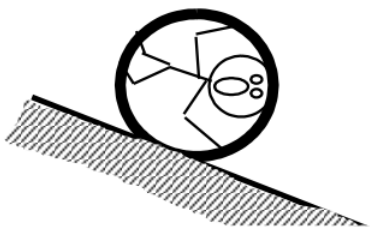


Mathematical Analysis[44]

Event 1:

Event 2:

The 65 kg strange man at right is trapped inside a section of large pipe. If that's not bad enough, the pipe begins to roll from rest down a 35 m long, 18^0 incline! The pipe rolls down the hill without slipping. The pipe has a mass of 180 kg and a diameter of 1.2 m. (Assume the man's presence inside the pipe has a negligible effect on the pipe's rotational inertia.)



Free-Body Diagrams

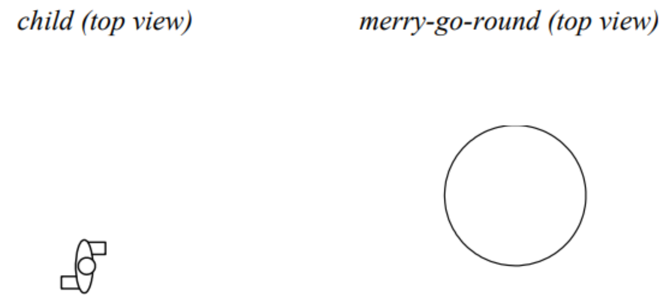
Mathematical Analysis[45]

Event 1:

Event 2:

A 30 kg child running at 3.0 m/s leaps onto the outer edge of an initially stationary merry-go-round. The merry-go-round is a flat disk of radius 2.4 m and mass 50 kg. Ignore the frictional torque in the bearings of the merry-go-round.

Free-Body Diagrams



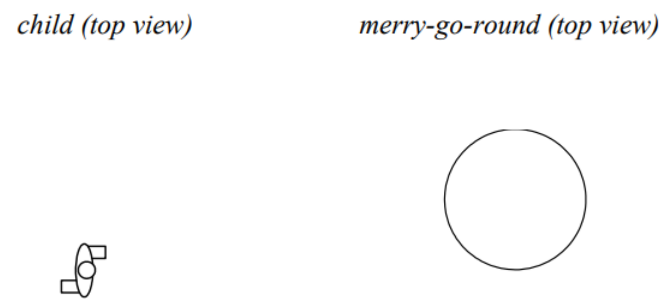
Mathematical Analysis[46]

Event 1:

Event 2:

A 40 kg child running at 4.0 m/s leaps onto the outer edge of a merry-go-round initially rotating in the opposite direction. The merry-go-round is brought to rest. The merry-go-round is a flat disk of radius 2.4 m and mass 50 kg. Ignore the frictional torque in the bearings of the merry-go-round.

Free-Body Diagrams



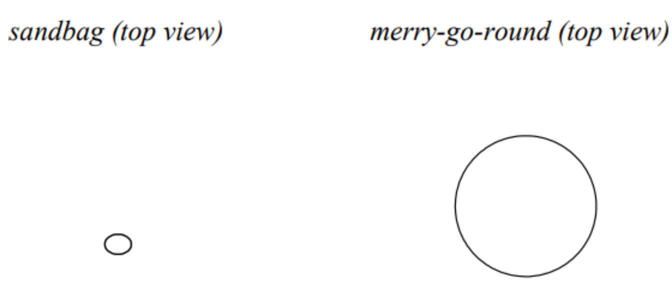
Mathematical Analysis[47]

Event 1:

Event 2:

A 30 kg bag of sand is dropped onto a merry-go-round 1.6 m from its center. The merry-go-round was rotating at 3.2 rad/s before the bag was dropped. The merry-go-round is a flat disk of radius 2.4 m and mass 50 kg. Ignore the frictional torque in the bearings of the merry-go-round.

Free-Body Diagrams



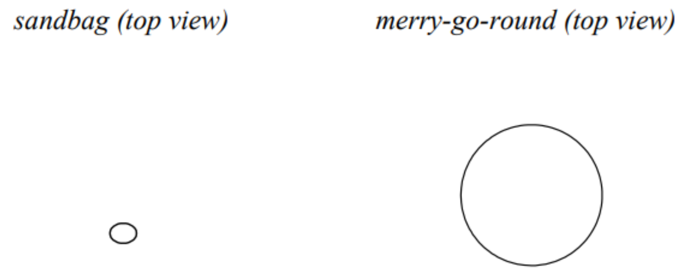
Mathematical Analysis[48]

Event 1:

Event 2:

A 30 kg bag of sand is dropped onto a merry-go-round. The merry-go-round was rotating at 2.2 rad/s before the bag was dropped, and 1.3 rad/s after the bag comes to rest. The merry-go-round is a flat disk of radius 2.4 m and mass 60 kg. Ignore the frictional torque in the bearings of the merry-go-round.

Free-Body Diagrams



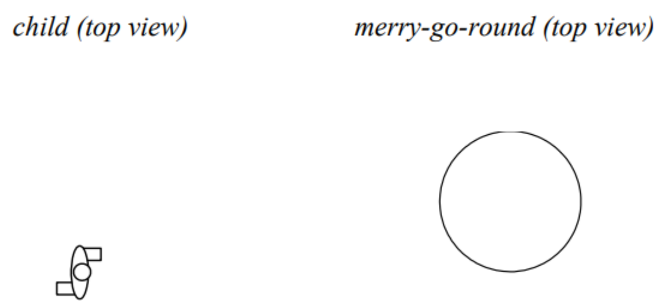
Mathematical Analysis[49]

Event 1:

Event 2:

A child leaps onto the outer edge of an initially stationary merry-go-round. The merry-go-round is a flat disk of radius R and mass M. Determine the angular speed of the merry-go-round (ω) after the child jumps on as a function of the child's mass (m), initial speed (v), M, and R. Ignore the frictional torque in the bearings of the merry-go-round.

Free-Body Diagrams



Mathematical Analysis

Event 1:

Event 2:

Questions

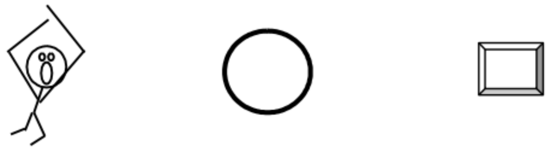
If $v = 0$ m/s, what should ω equal? Does your function agree with this observation?

If $M = \infty$, what should ω equal? Does your function agree with this observation?

If $m = \infty$, what should ω equal? Does your function agree with this observation?

Tired of walking up the stairs, an engineering student of mass m designs an ingenious device for reaching her third floor dorm room. An block of mass M is attached to a rope that passes over a disk-shaped pulley of mass M_p and radius R. The student holds the other end of the rope. When the block is released, the student is pulled up to her dorm room. Determine the velocity of the student (v_S) as she reaches her room as a function of m, M, M_p, R, g, and her distance off the ground (D).

Free-Body Diagrams



Mathematical Analysis

Event 1:

Event 2:

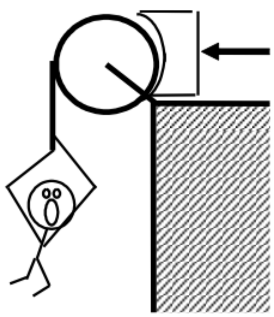
Questions

If D = 0 m, what should v_S equal? Does your function agree with this observation?

If m = M, what should v_S equal? Does your function agree with this observation?

If $M_p = \infty$, what should v_S equal? Does your function agree with this observation?

The man of mass m is falling at speed v! In an attempt to save him, the brake shoe is pressed against the spinning pulley with force F. The pulley has mass M and radius R. Determine the time needed to stop the man's fall (T) as a function of m, M, v, R, g, F, and the appropriate coefficient of friction between the brake shoe and the pulley.



Free-Body Diagrams

Mathematical Analysis

Event 1:

Event 2:

Questions

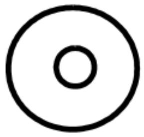
If μ = 0, what should T equal? Does your function agree with this observation?

If m = ∞, what should T equal? Does your function agree with this observation?

If F = ∞, what should T equal? Does your function agree with this observation?

A yo-yo of mass m, inner diameter d, and outer diameter D is released from rest. Assume the rotational inertia of the yo-yo is similar to a simple disk of diameter D and that the unwinding of the string does not affect the rotational inertia. Determine the angular velocity of the yo-yo (ω) as a function of the elapsed time since release (t), g, d, and D.

Free-Body Diagrams



Mathematical Analysis

Event 1:

Event 2:

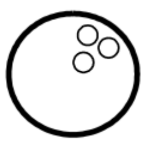
Questions

If D = ∞, what should ω equal? Does your function agree with this observation?

If g = 0 m/s^2, what should ω equal? Does your function agree with this observation?

A bowling ball of mass m and radius R is thrown down the alley at a speed v. The bowling ball is initially skidding, with no angular velocity, down the alley. Determine the time the ball skids (T) before beginning to roll without slipping as a function of m, R, g, v, and the appropriate coefficient of friction between the ball and the lane.

Free-Body Diagrams



Mathematical Analysis

Event 1:

Event 2:

Questions

If μ = 0, what should T equal? Does your function agree with this observation?

If v = ∞, what should T equal? Does your function agree with this observation?

5.4: Selected Answers

[1] $\theta 2$ = 3.53 rad

[2] t = 0.97 s

[3] $\theta B2$ = 8.98 rad/s

[4] t = 1.55 s

[5] v1 = 12.3 m/s

[6] t = 2.39 s

[7] t2 = 2.8 s

[8] α = 45.7 rad/s^2

[9] $\omega 2$ = 35.6 rad/s

[10] a = 0.65 m/s^2

[11] α = 2.92 rad/s^2

[12] t2 = 7.5 s

[13] t2 = 2.65 s

[14] ablock = 0.256 m/s^2

[15] atruck = 0.056 m/s^2

[16] atruck = 0.099 m/s^2

[17] Δrblock = 203 m

[18] F rope = 477 N

[19] F rope = 266 N

[20] F rope = 375 N

[21] Frightsupport = 9.8 N Fleftsupport = 1117 N

[22] Frightsupport = 1269 N Fleftsupport = 446 N

[23] Fsf = 514 N

[24] F minimum = 867 N

[25] a student = 0.23 m/s^2

[26] a student = 0.14 m/s^2

[27] F right rope = 503 N

[28] m block = 117 kg

[29] a skier = 0.57 m/s^2

[30] a = 0.15 m/s^2

[31] t2 = 1.01 s

[32] t2 = 1.18 s

[33] a = 5.07 m/s^2

[34] a = 1.75 m/s^2

[35] Fonbrake = 867 N

[36] a. t = 3.69 s b. r = 27.7 m

[37] v2 = 1.21 m/s

[38] v2student = 1.5 m/s

[39] Fright rope = 503 N

[40] mblock = 116 kg

[41] mblock = 108 kg

[42] v2 = 0.49 m/s

[43] v2 = 6.8 m/s

[44] v2 = 5.4 m/s

[45] v2 = 11 m/s

[46] v2 = 1.64 m/s

[47] ω1 = -2.67 rad/s

[48] ω2 = 2.1 rad/s

[49] R = 2.0 m

CHAPTER OVERVIEW

6: Linear Oscillations

Thumbnail: Mavericks Surf Contest 2010. (CC-SA-BY; Shalom Jacobovitz).

6.0: Model Specifics

Linear Oscillations

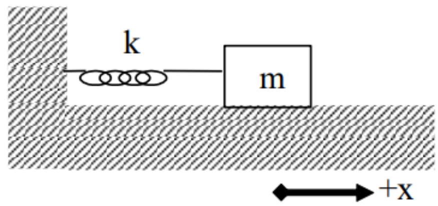
In this section, we will investigate a new model that can be viewed as an extension, or application, of the models previously studied. This application concerns the motion, of either a particle or a rigid body, after it is displaced from its *equilibrium position*. (The equilibrium position is the position at which the total force acting on the object is zero.) The model hypothesizes the existence of a force that acts to return the object to its equilibrium position. This force is termed the *restoring force*. Furthermore, the magnitude of this restoring force will be restricted to be *linearly dependent* on the magnitude of the initial displacement from equilibrium. This means that the farther the object is displaced from equilibrium, the larger the magnitude of the force acting to return it to equilibrium. When these two criteria are met, we will show that the object's position will oscillate about the location of its original equilibrium position.

6.1: Concepts and Principles

Imagine an object at rest at its equilibrium position, for example, a pendulum hanging straight downward, an object dangling from an elastic cord, or a diving board perfectly horizontal. When these objects are displaced from equilibrium, and released, they return toward their equilibrium position. However, their trip back toward equilibrium involves repeated oscillations about the location of equilibrium. If the initial disturbance is small enough, all of these systems exhibit mathematically similar behavior. This chapter explores the similarities in their oscillatory behavior.

We will analyze in detail the motion of a common oscillator; a mass attached to the end of a spring. We will investigate the resulting motion both without including the effects of friction (undamped) and including the effects of friction (damped). You should then be able to generalize these results for arbitrary oscillatory systems.

Undamped Spring-Mass Oscillator



One of the simplest systems exhibiting oscillatory behavior is the spring-mass system pictured at left. A linear spring of constant k is attached to a mass m. The equilibrium position of the spring-mass system is denoted as the origin of a coordinate system.

Now imagine the block is pulled to the right and let go. Hopefully you can convince yourself that the block will oscillate back and forth. Let's apply Newton's Second Law at the instant the mass is at an arbitrary position, x. The only force acting on the mass in the x-direction is the force of the spring.

$$\begin{aligned} \Sigma F &= ma \\ F_{\text{spring}} &= ma \\ -ks &= ma \end{aligned}$$

Because of our choice of coordinate system, the stretch of the spring (s) is exactly equal to the location of the block (x). Therefore,

$$-kx = ma$$

Note that when the block is at a positive position, the force of the spring is in the negative direction and when the block is at a negative position, the force of the spring is in the positive direction. Thus, the force of the spring always acts to return the block to equilibrium.

Rearranging gives

$$\begin{aligned} -\frac{k}{m}x &= a \\ -\frac{k}{m}x &= \frac{d^2x}{dt^2} \end{aligned}$$

and defining a constant, ω^2, as

$$\omega^2 = \frac{k}{m}$$

(Granted, it seems pretty silly to define $\frac{k}{m}$ as the square of a constant, but just play along. You may also find it frustrating to learn that this "omega" is *not* an angular velocity. The block doesn't even have an angular velocity!)

yields,

$$-\omega^2 x = \frac{d^2x}{dt^2}$$

Therefore, the position function for the block must have a second time derivative equal to the product of $(-\omega^2)$ and itself. The only functions whose second time derivative is equal to the product of a negative constant and itself are the sine and the cosine functions. Therefore, a solution to this differential equation[1]

$$-\omega^2 x = \frac{d^2 x}{dt^2}$$

can be written:

$$x(t) = A\cos(\omega t + \phi)$$

or equivalently with the sine function, where A and ϕ are arbitrary constants.[2]

- A is the *amplitude* of the oscillation. The amplitude is the maximum displacement of the object from equilibrium.
- ϕ is the *phase angle*. The phase angle is used to adjust the function forward or backward in time. For example, if the particle is at the origin at t = 0 s, ϕ must equal $+\pi/2$ or $-\pi/2$ to ensure that the cosine function evaluates to zero at t = 0 s. If the particle is at its maximum position at t = 0 s, then the phase angle must be zero or π to ensure that the cosine function evaluates to +1 or -1 at t = 0 s.
- ω is the *angular frequency* of the oscillation.[3]

Note

[1] A differential equation is an equation involving a function and its derivative(s).

[2] To prove to yourself that this is indeed the solution to the equation, you should substitute the function, x(t), into the left side of the equation and the second derivative of x(t) into the right side. This will verify that the two sides of the equation are equal. In addition to mathematically verifying this solution, you should verify the solution physically by sketching a graph of the motion that you *know* would result if the block were displaced to the right and comparing that sketch to a sketch of the function.

[3] Again, note that ω is *not* the angular velocity. The block is *not* rotating; it does not have an angular velocity

Note that the cosine function repeats itself when its argument increases by 2π. Thus, when

$$\Delta(\omega t + \phi) = 2\pi$$

the function repeats. Since ω and ϕ are constant,

$$\Delta(\omega t + \phi) = \omega \Delta t$$

Therefore, the time interval when

$$\omega \Delta t = 2\pi$$

is the time interval for one complete cycle of the oscillatory motion. The time for one complete cycle of the motion is termed the *period*, T. Thus,

$$T = \frac{2\pi}{\omega}$$

Therefore, the physical significance of the angular frequency is that it is inversely proportional to the period.

Substituting in the definition of ω:

$$\omega = \sqrt{\frac{k}{m}}$$

yields

$$T = 2\pi\sqrt{\frac{m}{k}}$$

In summary, a mass attached to a spring will oscillate about its equilibrium position with a position function given by:

$$x(t)=A\cos(\omega t+\phi)$$

This function repeats with a period of

$$T=2\pi\sqrt{\frac{m}{k}}$$

Since we have an understanding of the position of the oscillating object as a function of time, we have a complete kinematic description of the motion.

Damped Spring-Mass Oscillator

Imagine the same scenario investigated above, a block displaced to the right of equilibrium by a distance x. However, let's try to incorporate the frictional forces acting on the block. Let's imagine that the primary frictional force acting on the block is proportional to the block's velocity.[4]

$$\begin{aligned} F_{\text{friction}} &= -bv \\ F_{\text{friction}} &= -b\frac{dx}{dt} \end{aligned}$$

The negative sign implies that the direction of the frictional force is opposite to the direction of the block's velocity. The constant, b, is termed the *drag coefficient*.

Applying Newton's Second Law now yields:

$$\begin{aligned} \Sigma F &= ma \\ -kx-b\frac{dx}{dt} &= m\frac{d^2x}{dt^2} \end{aligned}$$

This is obviously a much more complicated differential equation to solve. Let's see if we can make any headway by thinking physically about the equation.

I would expect a solution that still oscillates like a cosine function, but with amplitude that gradually decreases due to friction. Therefore, a possible solution is

$$x(t)=Ae^{-\alpha t}\cos(\omega' t+\phi)$$

where A and ϕ are arbitrary constants. If you substitute this function and its derivatives into the above equation, you will see that it is a solution of the differential equation only if:

$$\alpha=\frac{b}{2m}$$

and

$$\omega'=\sqrt{\omega^2-\alpha^2}$$

Although this result was produced in a whirlwind of (unseen) mathematics, it does have a number of physical properties that make it plausible. For one, if there is no damping (b = 0), then $\alpha=0$, $\omega'=\omega$, and x(t) is exactly the same as it was in our original undamped case, as it must be. Also, notice that $\omega'<\omega$ if damping is present. With a smaller angular frequency, the period of the oscillation is longer, which it should be if friction is present.

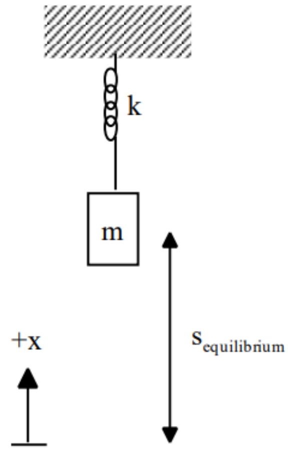
Note

[4] This is not the same type of frictional force investigated in previous models. The sliding friction model introduced in Model 2 is independent of the velocity of the object. The friction model introduced here, termed drag, is directly proportional to velocity.

6.2: Analysis Tools

A Vertical Spring-Mass System

> *An 80 kg bungee jumper is about to step off of a platform high above a raging river and plummet downward. The elastic bungee cord has an effective spring constant of 35 N/m and is initially slack, although it begins to stretch the moment the jumper steps off of the platform.*



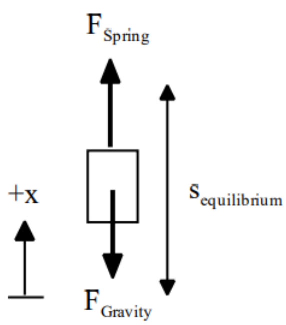
Our goal is to show that a vertically-oriented spring-mass system (the bungee jumper and bungee cord) obeys the same differential equation as the horizontally-oriented spring-mass system. Once we show this to be true, all of the results derived for the horizontal case will also be true for the vertical case.

First, let's place the origin of our coordinate system at the equilibrium location of the bungee jumper. Note, the initial position of the jumper is *not* the equilibrium position. The equilibrium position is where the net force on the bungee jumper is zero.

To find the equilibrium position:

$$\begin{aligned} \Sigma F &= ma \\ -F_{\text{gravity}} + F_{\text{spring}} &= ma \\ -mg + ks_{\text{equilibrium}} &= m(0) \\ S_{\text{equilibrium}} &= \frac{mg}{k} \end{aligned}$$

Now let's apply Newton's second law when the jumper is at an arbitrary position, x, with the origin of the coordinate system at the equilibrium position.



$$\begin{aligned} \Sigma F &= ma \\ -F_{\text{gravity}} + F_{\text{spring}} &= ma \\ -mg + ks &= ma \end{aligned}$$

Where "s" is the stretch of the spring. The spring is stretched by an amount

$$S = S_{\text{equilibrium}} - x$$

Therefore,

$$\begin{aligned}
&-mg + k\left(s_{\text{equilibrium}} - x\right) = ma \\
&-mg + k\left(\frac{mg}{k} - x\right) = ma \\
&-mg + mg - kx = ma \\
&-kx = ma \\
&-\frac{k}{m}x = \frac{d^2x}{dt^2}
\end{aligned}$$

This is exactly the same differential equation solved earlier. Thus, the motion of the bungee jumper must be described by

$$x(t) = A\cos(\omega t + \phi)$$

Now let’s determine the values of ω, A and ϕ:

From

$$\begin{aligned}
\omega &= \sqrt{\frac{k}{m}} \\
\omega &= \sqrt{\frac{35N/m}{80kg}} \\
\omega &= 0.661s^{-1}
\end{aligned}$$

Since A is the maximum displacement of the object from equilibrium,

$$\begin{aligned}
A &= S_{\text{equilibrium}} \\
A &= \frac{mg}{k} \\
A &= \frac{80(9.8)}{35} \\
A &= 22.4 \text{ m}
\end{aligned}$$

So now we have

$$x(t) = 22.4\cos(0.661t + \phi)$$

and all that’s left to determine is ϕ. At t = 0 s, the bungee jumper is at x = 22.4 m. Thus,

$$\begin{aligned}
&x(t) = 22.4\cos(0.661t + \phi) \\
&x(0) = 22.4\cos(0.661(0) + \phi) \\
&22.4 = 22.4\cos(\phi) \\
&1 = \cos(\phi) \\
&\phi = 0
\end{aligned}$$

This really shouldn’t surprise you since if you graphed the jumper’s motion it would be a perfect cosine function with no initial phase shift. Thus, the position of the bungee jumper as a function of time is:

$$x(t) = 22.4\cos(0.661t)$$

Since we have the position of the jumper at all times, we can determine *everything* about the jumper’s motion. For example, her velocity

$$\begin{aligned}
v(t) &= \frac{dx(t)}{dt} \\
v(t) &= \frac{d}{dt}(22.4\cos(0.661t)) \\
v(t) &= -14.8\sin(0.661t)
\end{aligned}$$

and her maximum speed

$$|v_{\max}| = 14.8 \text{ m/s}$$

which occurs as she passes through the equilibrium position.

Her acceleration

$$a(t) = \frac{dv(t)}{dt}$$
$$a(t) = \frac{d}{dt}(-14.8 \sin(0.661t))$$
$$a(t) = -9.8 \cos(0.661t)$$

and her maximum acceleration

$$|a_{\max}| = 9.8 \text{ m/s}^2$$

which occurs the instant she steps off of the platform (and at the very bottom of her motion).

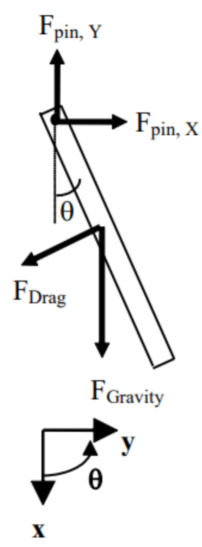
The time for her to travel through an entire cycle

$$T = \frac{2\pi}{\omega}$$
$$T = \frac{2\pi}{0.661}$$
$$T = 9.51s$$

And on and on and on...

The Physical Pendulum

> *A 0.4 kg meterstick is hung from a pin attached to one end. The meterstick is displaced by 0.1 radian (5.7^0) from equilibrium (straight down) and released. After ten complete cycles, the amplitude of the oscillation has decreased to 0.08 radian. Assume a drag force proportional to the speed of the CM of the stick acts at the location of the CM.*



At left is a free-body diagram of the meterstick when it is at an angle θ from equilibrium, and moving counterclockwise, i.e., swinging up to the right. Since the oscillatory behavior is occurring in the angular direction, let's write Newton's second law in angular form:

$$\begin{aligned}&\Sigma\tau = I\alpha\\&\tau_{\text{drag}} + \tau_{\text{gravity}} = I\alpha\\&-\frac{L}{2}F_{\text{drag}}\sin 90 - \frac{L}{2}(mg)\sin\theta = I\alpha\end{aligned}$$

A meterstick can be approximated as a thin rod, and a thin rod rotated about its CM has rotational inertia $\frac{1}{12}\ \mathrm{mL}^2$. We need the rotational inertia about an axis at one end, so we must use the parallel-axis theorem with $\mathrm{r_{CM}} = \frac{L}{2}$. Thus,

$$\begin{aligned}&I = m{r_{CM}}^2 + I_{CM}\\&I = m\left(\frac{L}{2}\right)^2 + \frac{1}{12}mL^2\\&I = \frac{1}{4}mL^2 + \frac{1}{12}mL^2\\&I = \frac{1}{3}mL^2\end{aligned}$$

So

$$\begin{aligned}&-\frac{L}{2}F_{\text{drag}}\sin 90 - \frac{L}{2}(mg)\sin\theta = \left(\frac{1}{3}mL^2\right)\alpha\\&-\frac{1}{2}F_{\text{drag}} - \frac{1}{2}mg\sin\theta = \frac{1}{3}mL\alpha\end{aligned}$$

Since the magnitude of the drag force is proportional to the speed of the CM,

$$F_{drag} = bv_{CM}$$

and[5]

$$\begin{aligned}&v = r\omega\\&v_{CM} = \frac{L}{2}\omega\end{aligned}$$

the drag can be written as

$$F_{\text{drag}} = b\left(\frac{L}{2}\omega\right)$$

resulting in:

$$\begin{aligned}&-\frac{1}{2}\left(b\left(\frac{L}{2}\omega\right)\right) - \frac{1}{2}mg\sin\theta = \frac{1}{3}mL\alpha\\&-\frac{1}{4}bL\omega - \frac{1}{2}mg\sin\theta = \frac{1}{3}mL\alpha\end{aligned}$$

Multiplying by 3/L yields:

$$\begin{aligned}&-\frac{3b}{4}\omega - \frac{3mg}{2L}\sin\theta = m\alpha\\&-\frac{3mg}{2L}\sin\theta - \frac{3b}{4}\frac{d\theta}{dt} = m\frac{d^2\theta}{dt^2}\end{aligned}$$

Since the angle of displacement is small, we will use a common approximation for small angles:

$$\sin\theta \approx \theta$$

(For the maximum angle attained by our meterstick, sin (0.1) = 0.998)

Note

[5] The ω in the following relationship is the angular velocity of the meterstick (which varies as the meterstick oscillates), **not** the angular frequency of the meterstick (which is a constant). I know they are the same symbol, but try to keep them straight by examining the *context* in which they appear.

Using this approximation, our equation becomes:

$$-\frac{3mg}{2L}\theta - \frac{3b}{4}\frac{d\theta}{dt} = m\frac{d^2\theta}{dt^2}$$

This is exactly the same differential equation as the damped linear oscillator,

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

if:

- k is replaced by $\frac{3mg}{2L}$
- b is replaced by $\frac{3b}{4}$
- and the position (x) is replaced by the angular position (θ).

Thus, the *effective* spring constant is $\frac{3mg}{2L}$ and the *effective* damping coefficient is $\frac{3b}{4}$.

Since it is the same differential equation, the motion of the meterstick must be described by the same function:

$$\theta(t) = Ae^{-\alpha t}\cos(\omega' t + \phi)$$

Now let's determine the values of A, ϕ, α, and ω':

- Since the meterstick is displaced by 0.1 radian from equilibrium and then released,

$$\mathrm{A} = 0.1 \text{ rad}$$

- Since the maximum displacement of the meterstick occurs at t = 0 s,

$$\phi = 0$$

- In the original differential equation

$$\alpha = \frac{b}{2m}$$

  Since the effective damping coefficient is $\frac{3b}{4}$,

$$\alpha = \frac{\left(\frac{3b}{4}\right)}{2m}$$
$$\alpha = \frac{3b}{8m}$$
$$\alpha = \frac{3b}{8(0.4)}$$
$$\alpha = 0.938b$$

- In the original differential equation

$$\omega = \sqrt{\frac{k}{m}}$$

  Since the effective spring constant is $\frac{3mg}{2L}$

$$\omega=\sqrt{\frac{\left(\frac{3mg}{2L}\right)}{m}}$$
$$\omega=\sqrt{\frac{3g}{2L}}$$

Notice that the angular frequency in the undamped case does not depend on the mass of the meterstick.

$$\omega=\sqrt{\frac{3\left(9.8m/s^2\right)}{2(1m)}}$$
$$\omega=3.83s^{-1}$$

Finally

$$\omega'=\sqrt{\omega^2-\alpha^2}$$
$$\omega'=\sqrt{3.83^2-(0.938b)^2}$$
$$\omega'=\sqrt{14.7-0.880b^2}$$

Putting it all together,

$$\theta(t)=0.1e^{-0.938bt}\cos(\omega' t)$$

with

$$\omega'=\sqrt{14.7-0.880b^2}$$

Now we can use the information about the effects of damping to determine b. We know that at a time equal to ten periods, the amplitude is 0.08 radians. Since the initial amplitude of the motion was 0.1 radian, the exponential factor, exp(0.938bt) , must be equal to 0.8 when t = 10 T:

$$\exp(-0.938b(10T))=0.8$$
$$\exp\left(-9.38b\left(\frac{2\pi}{\omega'}\right)\right)=0.8$$
$$\exp\left(-58.9\frac{b}{\omega'}\right)=0.8$$
$$-58.9\frac{b}{\omega'}=\ln(0.8)$$
$$-58.9\frac{b}{\omega'}=-0.223$$
$$\frac{b}{\omega'}=3.79\times10^{-3}$$
$$b=3.79\times10^{-3}\omega'$$
$$b=3.79x10^{-3}\sqrt{14.7-0.880b^2}$$
$$b^2=1.43x10^{-5}\left(14.7-0.880b^2\right)$$
$$b^2=2.11x10^{-4}-1.26x10^{-5}b^2$$
$$b^2=2.11x10^{-4}$$
$$b=0.0145\text{ kg/s}$$

Thus,

$$\omega'=\sqrt{14.7-0.880b^2}$$
$$\omega'=\sqrt{14.7-0.000185}$$
$$\omega'=3.83s^{-1}$$

Notice that the change in angular frequency due to damping is extremely small. (It is so small that it does not have an effect in the first three significant figures of the angular frequency.)

Finally,

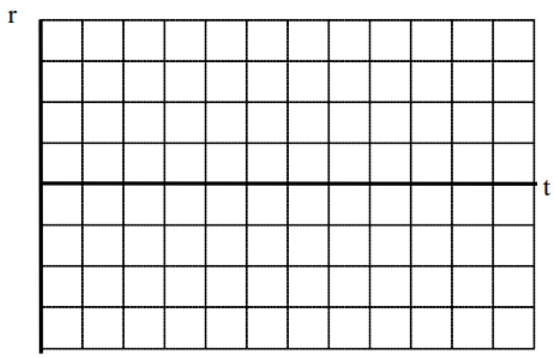
$$\theta(t) = 0.1e^{-0.938(0.0145)t}\cos(3.83t)$$
$$\theta(t) = 0.1e^{-0.0136t}\cos(3.83t)$$

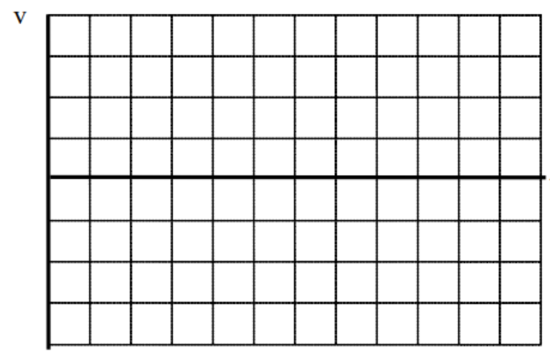
This is a complete description of the motion of the meterstick. As in the first example, we can determine anything we desire about the motion of this meterstick through a manipulation of this function.

6.3: Activities

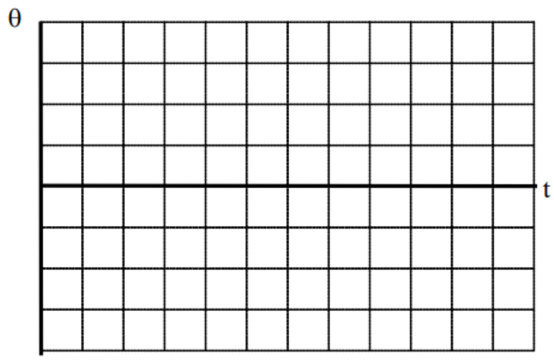
Sketch the indicated motion graphs for one complete cycle of the motions described below. Set the origin of the coordinate system at the equilibrium position of the mass.

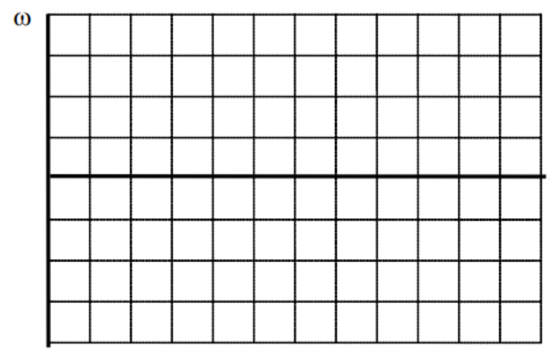
a. A mass is attached to a vertically-oriented spring. The mass is pulled a short distance below its equilibrium position and released. Begin the graph the instant the mass is released. Assume air resistance is so small that it can be ignored.





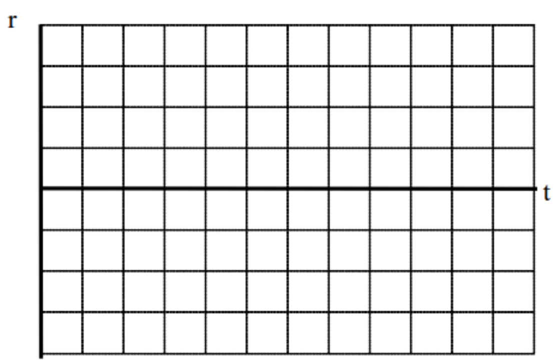
b. A pendulum is displaced from its equilibrium position and released. Begin the graph the instant the pendulum is released. Assume air resistance is so small that it can be ignored.

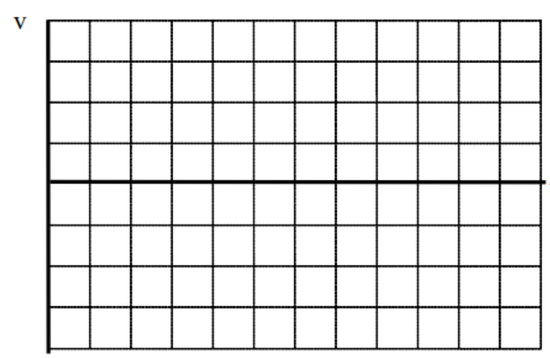




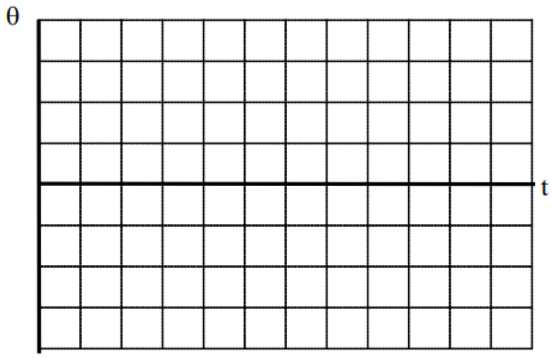
Sketch the indicated motion graphs for one complete cycle of the motions described below. Set the origin of the coordinate system at the equilibrium position of the mass.

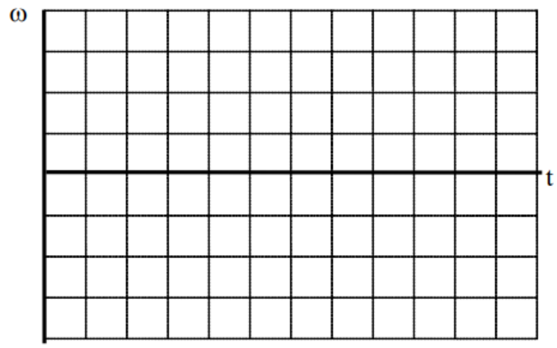
a. A mass is attached to a vertically-oriented spring. The mass is pulled a short distance below its equilibrium position and released. Begin the graph the instant the mass is released. Assume air resistance is so large that it cannot be ignored.



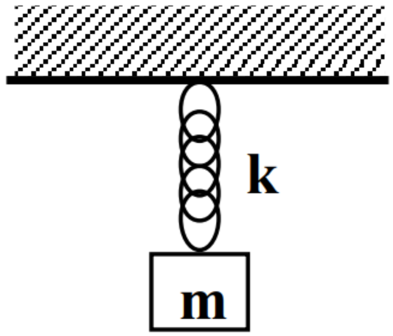


b. A pendulum is displaced from its equilibrium position and released. Begin the graph the instant the pendulum is released. Assume air resistance is so large that it cannot be ignored.





A crate of mass m is attached to a spring of stiffness k. The crates are held in place such that none of the springs are initially stretched. All springs are initially the same length. The crates are released and the springs stretch.



	m	k
A	5 kg	20 N/m
B	20 kg	5 N/m
C	10 kg	10 N/m
D	15 kg	20 N/m
E	5 kg	5 N/m
F	15 kg	10 N/m

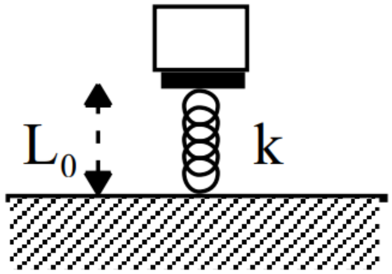
Rank the time required for the crates to return to their initial position.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Different mass crates are placed on top of springs of uncompressed length L0 and stiffness k. The crates are released and the springs compress to a length L before bringing the crates back up to their original positions.



	k	L_0	L
A	5 N/m	1.0 m	0.5 m
B	10 N/m	1.0 m	0.5 m
C	5 N/m	2.0 m	1.0 m
D	10 N/m	2.0 m	0.5 m
E	15 N/m	1.5 m	1.0 m
F	20 N/m	1.5 m	0.5 m

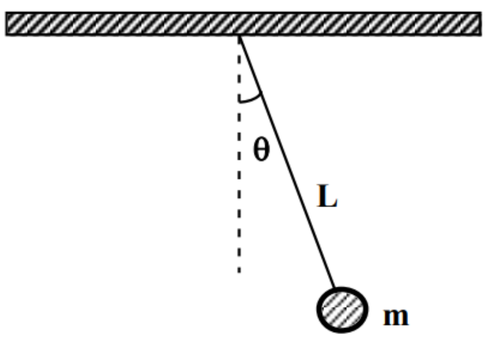
Rank the time required for the crates to return to their initial positions.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A pendulum of mass m and length L is released from rest from an angle θ from vertical.



	m	L
A	2 kg	1 m
B	1 kg	4 m
C	4 kg	1 m
D	4 kg	2 m
E	2 kg	2 m
F	4 kg	4 m

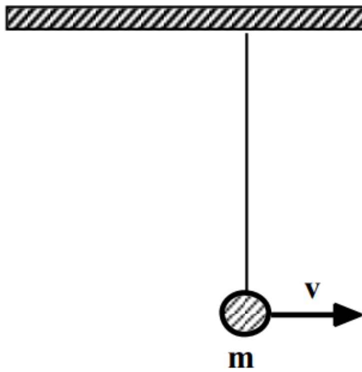
Rank the number of complete cycles of motion each pendulum goes through per minute.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A 2 m long pendulum of mass m is moving at velocity v as it passes through its equilibrium position.



	m	v
A	2 kg	4 m/s
B	1 kg	4 m/s
C	4 kg	2 m/s
D	8 kg	1 m/s
E	2 kg	2 m/s
F	3 kg	3 m/s

Rank the period of the pendulum.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

A 75 kg bungee jumper is about to step off of a platform high above a raging river and plummet downward. The elastic bungee cord has an effective spring constant of 50 N/m and is initially slack, although it begins to stretch the moment the jumper steps off of the platform.

Free-Body Diagram

Mathematical Analysis

Questions

What is the bungee jumper's maximum velocity?

How far does the bungee jumper fall?

If the river is only 25 m below the platform, how fast is the jumper going when she hits the river?

A 75 kg bungee jumper is about to step off of a platform 65 m above a raging river and plummet downward. She hopes to get just the top of her hair wet. The elastic bungee cord exerts a linear restoring force and is initially slack, although it begins to stretch the moment the jumper steps off of the platform.

Free-Body Diagram

Mathematical Analysis

Questions

What is the period of the jumper's motion?

What is the maximum force acting on the jumper?

How fast is the jumper moving 2.0 s after leaving the platform?

A 85 kg bungee jumper is about to step off of a platform high above a raging river and plummet downward. The elastic bungee cord has an effective spring constant of 40 N/m and is initially slack, although it begins to stretch the moment the jumper steps off of the platform. The amplitude of the jumper's motion decreases by approximately 20% after one complete cycle of his motion due to a drag force proportional to his speed.

Free-Body Diagram

Mathematical Analysis

Questions

What is the bungee jumper's maximum speed on his second cycle downward?

How far does the bungee jumper fall?

The suspension of an automobile sags by 8 cm when the 900 kg frame is placed on it. To check the shock absorbers, the autoworkers drop the car from a small height and note that the amplitude of oscillation decreases by about 80% during each oscillation. Assume the shock absorbers exert a damping force proportional to the car's speed.

Free-Body Diagram

Mathematical Analysis

Questions

What is the effective spring constant of the suspension?

What is the period of oscillation of the car?

What is the effective damping coefficient of the suspension?

A man steps onto a bathroom scale. The bathroom scale acts as an initially uncompressed, undamped spring of spring constant 40,000 N/m. The scale displays the force exerted by the spring on the man, and reaches a maximum display of 1574 N shortly after the man steps on the scale.

Free-Body Diagram

Mathematical Analysis

Question

What is the man's mass?

What is the period of oscillation of the scale's reading?

A man steps onto a bathroom scale. The bathroom scale acts as an initially uncompressed spring of spring constant 40,000 N/m. The scale displays the force exerted by the spring on the man, and reaches a maximum display of 1574 N shortly after the man steps on the scale. After the next oscillation, the maximum display is 1234 N. Assume the internal damping is proportional to the speed of the oscillating man.

Free-Body Diagram

Mathematical Analysis

Question

What is the man's mass?

What does the scale read 5 s after the man steps on?

A 70 kg skier is zooming down a mountain out of control when he decides to veer off the trail into the Safety-Spring. The Safety-Spring is a 900 N/m spring into which the skier can collide and be brought to rest much more gently than if he hit a tree. The skier *is traveling at 20 m/s when he pulls off onto a horizontal path and collides with and sticks to the spring. Assume friction is so small that it can be ignored.*

Free-Body Diagram

Mathematical Analysis

Question

What is the maximum force that acts on the man?

How long does it take for the Safety-Spring to stop the man's motion?

A 50 kg skier is zooming down a mountain out of control when she decides to veer off the trail into the new and improved Safety-Spring. The new and improved Safety-Spring is a 900 N/m spring into which the skier can collide and be brought to rest much more gently than if she hit a tree. The new and improved part is that a baffle has been built into the Safety-Spring which provides a damping coefficient of 2.2 Ns/m. The skier is traveling at 25 m/s when she pulls off onto a horizontal path and collides with and sticks to the spring. Assume sliding friction is so small that it can be ignored.

Free-Body Diagram

Mathematical Analysis

Question

What is the skier's speed at the end of one complete oscillation?

How long does it take for the Safety-Spring to reduce the skier's maximum speed during oscillation to less than 5 m/s?

A 65 kg physicist digs a hole through the center of the earth and out the other side and jumps in. She knows that the force of gravity acting on a particle ***below*** *the earth's surface is* $F_G = G M m r / R^3$*, where G is the universal gravitational constant (6.7 x* 10^{-11} Nm^2/kg^2*), M is the mass of the earth (6.0 x* 10^{24} *kg), m is the mass of the particle, R is the radius of the earth (6.4 x* 10^6*m), and r is the distance between the particle and the center of the earth.*

Free-Body Diagram

Mathematical Analysis

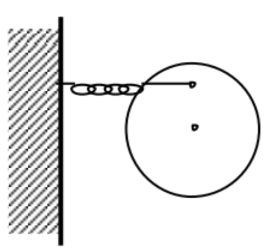
Question

How long does it take the physicist to reach the other side of the earth?

What is the maximum speed of the physicist during her journey?

If the physicist's 75 kg partner jumps into the hole the same time as the physicist, which one reaches the other side of the earth first?

The device at right is the top view of a vibrating dance floor. A 2200 kg, disk-shaped dance floor of radius 15 m is mounted horizontally. At a position 10 m from the rotation axis, a spring of constant 40,000 N/m is attached to the underside of the dance floor. The spring is unstretched in the orientation shown. The disk is rotated by 5° and released. Assume friction in the mount is so small it can be ignored.



Free-Body Diagram

Mathematical Analysis

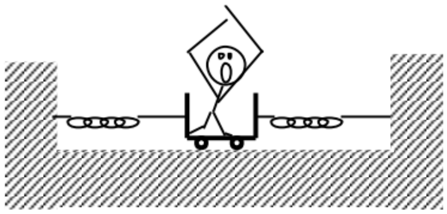
Question

What is the period of oscillation of the dance floor?

What is the maximum speed of someone standing on the outer edge of the dance floor?

What is the maximum acceleration of someone standing on the outer edge of the dance floor?

The SkakerCar at right is designed to test your balance. At present, a woman of mass m is standing in the mass M ShakerCar. Two springs, of constant k_1 and k_2, are attached to the car and are initially unstretched. The car is displaced a distance D to the left and released. Assume friction is so small it can be ignored.



Free-Body Diagram

Mathematical Analysis

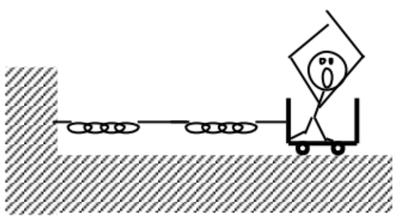
Question

What is the period of oscillation of the ShakerCar without the woman in it?

What is the maximum speed of the occupant during her journey?

What is the maximum acceleration of the occupant during her journey?

The SkakerCar at right is designed to test your balance. At present, a woman of mass m is standing in the mass M ShakerCar. Two springs, both of constant k, are attached to each other and are initially unstretched. The car is displaced a distance D to the left and released. Assume friction is so small it can be ignored.



Free-Body Diagram

Mathematical Analysis

Question

What is the period of oscillation of the ShakerCar without the woman in it?

What is the maximum speed of the occupant during her journey?

What is the maximum acceleration of the occupant during her journey?

Stranded on a desert island with only a meterstick and a nail, a physicist builds a pendulum clock by nailing one end of the meterstick to a tree branch. He then displaces the meterstick by 5° and lets it go. Assume friction in the mount is so small that it can be ignored.

Free-Body Diagram

Mathematical Analysis

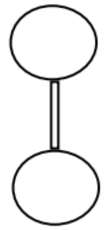
Question

What is the period of the pendulum clock?

What is the maximum speed of the bottom edge of the meterstick?

If the meterstick is cut in half, how will the period change?

Two identical spheres of mass M and radius R are connected as shown by a very thin rod of negligible inertia and length L, forming a dumbbell. In designing a pendulum clock, the designer considered mounting the dumbbell on an axis located a distance D from the CM, and perpendicular to the drawing at right, and displacing the dumbbell by a small angle.



Free-Body Diagram

Mathematical Analysis

Question

What is the period of oscillation of the dumbbell as a function of D?

What is the period of oscillation of the dumbbell if D = 0? Why?

For what value of D is the period a minimum?

SECTION OVERVIEW

Spiral Electricity and Magnetism (Calculus-Based)

Spiral Physics is a research based introductory physics curriculum developed at Monroe Community College. There are several important features of this curriculum. It integrates text and workbook activities in a modular fashion, and arranges topics so that students receive repeated exposure to concepts with increased complexity. It makes use of alternative problem types, including goal-less problem statements, ranking tasks, and critical analysis tasks. It restricts the equation set available for student use and is designed to facilitate active learning.

8: Electromagnetic Waves

Paul D'Alessandris (Monroe Community College)

CHAPTER OVERVIEW

1: Electric Fields

Topic hierarchy

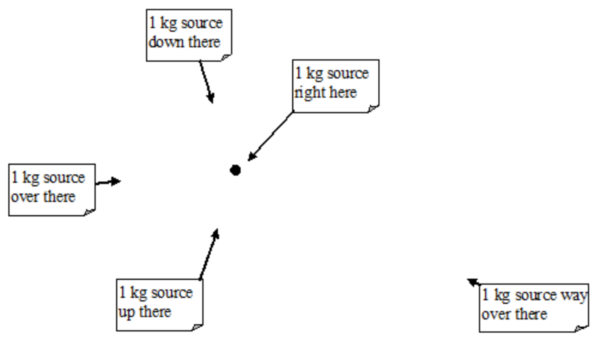
Paul D’Alessandris (Monroe Community College)

1.1: Concepts and Principles

Electric Charge, Electric Field and a Goofy Analogy

We all know that electrons and protons have electric charge. But what *is* electric charge and what does it mean for a particle, like a neutron, to not have electric charge[1]? On one level, the answer is that electric charge is the ablility to create and interact with an electric field. Of course, this begs the question, what is an electric field? To try to answer this question, let's look at mass and the gravitational field.

In Newton's theory of gravitational, every object that has mass creates a gravitational field. The object with mass is termed the *source* of the gravitational field. The source's gravitational field, which fills all of space, encodes information as to the location and mass of the source into space itself. It's almost as if an infinite number of little business cards have been printed and distributed throughtout space with detailed information concerning the source's characteristics. The gravitational field can be thought of as a huge number of business cards embedded into the fabric of space.(No, I am not joking, it looks like this.)



Typically when we draw the gravitational field we translate the message on the business cards into a mathematically equivalent message. Here's a picture of the gravitational field near the surface of the earth.

pic

The relationship that allows you to "translate the business cards" is Newton's expression for the gravitational field surrounding a mass,

pic

where

- G is the *gravitational constant*, equal to 6.67 x 10^{-11} N m^2/kg^2.
- m is the source mass, the mass that creates the gravitational field.
- r is the distance between the source mass and the location of the business card.
- (r hat) is the unit vector that points *from* the source mass *to* the business card. Notice that the direction of the gravitational field, g, is opposite to this direction.

Every mass in the universe has a gravitational field described by this formula surrounding it. Additionally, every mass in the universe has the ability to "read the business cards" of *every other mass*. This transfer of information from one mass to another is the gravitational force that attracts masses together. (Once one mass reads another masses' business card, the first mass feels a strange urge to go visit the second mass. The more enticing the business card (the larger the value of g) the stronger the urge.)

Let's drop the business card analogy and return to electric charges and electric fields. Basically, every object that has electric charge surrounds itself with an electric field given by a formula incredibly similar to the one for the gravitational field. How incredibly similar? Below is the formula for the electric field at a particular point in space, from a single source charge:

pic

where

- k is the *electrostatic constant*, equal to 9.00 x 10^9 N m^2/C^2.
- q is the source charge, the electric charge that creates the electric field. Its value can be positive or negative, and is measured in coulombs (C).
- r is the distance between the source charge and the point of interest.
- (r hat) is the unit vector that points *from* the source charge *to* the point of interest.

Every charge object is surrounded by a field given by this relationship. Every *other* charged object in the universe can "read" this field and will respond to its information by feeling an electric force. Object without electric charge neither create nor interact with electric fields (they can't read the business cards).

In this chapter you'll learn how to calculate the electric field produced by charge objects. In the next chapter, you'll learn how the electric field can be "sensed" by other electric charges resulting in the electric force.

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1.2: Charge and Charge Density

Charge and Charge Density

Macroscopic objects are normally neutral (or very close to neutral) because they contain equal numbers of protons and electrons. All charged objects are charged because of either an excess of lack of electrons. (It's much easier to add or remove electrons from an object than trying to add or remove the protons tightly bound inside the nuclei of its atoms.) Thus, the electric charge of any object is always an integer multiple of the electric charge on an electron.

Because of its fundamental importance, the magnitude of the charge on an electron is termed the *elementary charge* and denoted by the symbol e. In a purely logical world, the charge on any object would be reported as a multiple of e. However, since the charge on a macroscopic system can be *many* multiples of e, a more user-friendly unit, the coulomb (C), typically used to quantify electric charge. In this system,

pic

Thus, you can consider the charge on an electron as an incredibly small fraction of a coulomb, or a coulomb of charge as an incredibly large number of electrons.

In many applications, in addition to knowing the total charge on an object you will need to know how the charge is distributed. The distribution of charge on an object can be defined in several different ways. For objects such as wires or other thin cylinders, a *linear charge density*, l, will often be defined. This is the amound of charge per unit length of the object. if the charge is uniformly distributed, this is simply

pic

where Q is the total charge on the object[1] and L its total length. However, if the charge density varies over the length of the object, its value at any point must be defined as the ratio of the charge on a differential element at that location to the length of the element:

pic

For objects such as flat plates or the surfaces of cylinders and spheres, a *surface charge density*, s, can be defined. This is the amount of charge per unit area of the object. If the charge is uniformly distributed, this is

pic

or if the charge density varies over the surface:

pic

Lastly, for objects that have charge distributed throughout their volume, a *volume charge density*, r, can be defined. This is the amount of charge per unit volume of the object. If the charge is uniformly distributed, this is

pic

or if the charge density varies inside the object:

pic

To add to the confusion, you must realize that the same object can be described as having two different charge densities. For example, consider a plastic rod with charge distributed throughout its volume. Obviously, the charge per unit volume, r, can be defined for this object. However, you can also define the object as having linear charge density, l, reporting the amount of charge present per meter of length. These two parameters will have different values but refer to exactly the same object.

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[1]I will use lowercase q to designate the charge on a point particle and uppercase Q to designate the total charge distributed on macroscopic objects.

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1.3: Perfect Conductors and Perfect Insulators

Determining how electric charges in real materials respond to electric fields is incredibly important but also incredibly complicated. In light of this, we will initially restrict ourselves to two types of hypothetical materials.

In a *perfect conductor*, electric charges are free to move without any resistance to their motion. Metals provide a reasonable approximation to perfect conductors, although, of course, in a real metal a small amount of resistance to motion is present. When I refer to a material as a metal, we will approximate the metal as a perfect conductor.

In a *perfect insulator* electric charges cannot move, regardless of the amount of force applied to them. Many materials act as insulators, but all real materials experience electrical breakdown if the forces acting on charges become so great that the charges begin to move. When I refer to an insulating material, like plastic, for example, we will approximate the material as a perfect insulator.

Since electric fields create forces on electric charges, there cannot be static electric fields present inside perfect conductors. If a field was present inside a perfect conductor, the charges inside the conductor would feel an electric force and hence move in response to that force. They would continue to move until they redistributed themselves inside of the conductor in such a way as to cancel the electric field. The system could not reach equilibrium as long as an electric field was present. This re-arranging process would typically occur very quickly and we will always assume our analysis takes place after it is completed.

1.4: Analysis Tools - Continuous Charge Distribution

1. Continuous Charge Distribution

Continuous Charge Distribution

The plastic rod of length L below has uniform charge density l. Find the electric field at all points on the x-axis to the right of the rod.

pic

Electric charge is discrete, meaning it comes in integer multiples of electron and proton charge. Therefore, the electric field can always be calculated by summing the electric field from each of the electrons and protons that make up an object. However, macroscopic objects contain a *lot* of electrons and protons, so this summation has many, many terms:

pic

As described earlier, we will assume that the charge on macroscopic objects is continuous, and distributed throughout the object. Mathematically, this means we will replace a summation over a very large number of discrete charges with an integral over a hypothetically continuous distribution of charge. This leads to a relationship for the electric field at a particular point in space, from a continuous distribution of charge, of:

pic

where dq is the charge on a infinitesimally small portion of the object, and the integral is over the entire physical object.

Finding the electric field from a continuous distribution of charge involves several distinct steps. Until you become very comfortable setting up and evaluating electric field integrals, I would suggest you systematically walk through these steps.

1. *Carefully identify and label the location of the differential element on a diagram of the situation.*
2. *Carefully identify and label the location of the point of interest on a diagram of the situation.*
3. *Write an expression for dq, the charge on the differential element.*
4. *Write an expression for r, the distance between the differential element and the point of interest.*
5. *Write an expression for (r hat), the unit vector representing the direction from the element to the point of interest.*
6. *Insert your expressions into the integral for the electric field.*
7. *Carefully choose the limits of integration.*
8. *Evaluate the integral.*

I'll demonstrate below each of these steps for the scenario under investigation.

1. *Carefully identify and label the location of the differential element on a diagram of the situation.*

The differential element is a small (infinitesimal) piece of the object that we will treat like a point charge. The location of this differential element must be *arbitrary*, meaning it is not at a "special" location like the top, middle, or bottom of the rod. Its location must be represented by a variable, where this variable is the variable of integration and determines the limits of the integral.

For this example, select the differential element to be located a distance "y" above the center of the rod. The length of this element is "dy". (Later, you will select the limits of integration ot go from-L/2 to +L/2 to allow this arbitrary element to "cover" the entire rod.)

pic

2. *Carefully identify and label the location of the point of interest on a diagram of the situation.*

You are interested in the electric field at all points along the x-axis to the right of the rod. Therefore, select an *arbitrary* location along the x-axis and label it with its location.

pic

3. *Write an expression for dq, the charge on the differential element*.

The rod has a uniform charge density l, meaning the amount of charge per unit length along the rod is constant. Since the differential element has a length dy, the total charge on this elment (dq) is the product of the density and the length:

pic

4. *Write an expression for r, the distance between the differential element and the point of interest.*

By Pythagoras' theorem, the distance between the differential element and the point of interest is:

pic

5. *Write an expression for (r hat), the unit vector representing the direction from the element to the point of interest.*

The vector going *from* the element *to* the point of interest heads "down" a distance y and then to the right a distance x. This leads to a unit vector:

pic

6. *Insert your expressions into the integral for the electric field.*

pic

7. *Carefully choose the limits of integration.*

The limits of integration are determined by the range over which the differential element must be "moved" to cover the entire object. The location of the element must vary between the bottom of the rod (-L/2) and the top of the rod (+L/2) in order to include every part of the rod. The two ends of the rod form the two limits of integration.

pic

8. *Evaluate the integral.*

pic

It's typically easier to think of a vector integral as two, separate scalar integrals, one "in the x-direction" and one "in the y-direction", as above.

Examining the x-integral (and using an integral table):

pic

I'll leave it to you to evaluate the y-integral, but you should find that it equals zero. (Without doing calculation whatsoever, you should *know* that the y-component of the electric field along the x-axis *must* be zero because of the symmetry of the situation. If it's not clear why the y-component of the field must be zero, talk to you instructor!)

Thus, the electric field along the x-axis is given by:

pic

Does this function make sense? Two ways to check whether this field is reasonable are to determine how the funciton behaves for very small and very large values of x.

As x gets very small, you are getting closer and closer to the charged rod. This *should* lead to an electric field that grows larger and larger (without bound). Notice that the limit of the function as x approaches zero does "go to infinity", so the function does have the proper behavior for small x.

As x gets very large, you are getting farther and farther to the charged rod. Not only should the field decrease, but as you get very far from the rod should begin to look like a point charge. Notice that as x gets large, the term (L^2/4) becomes negligible compared to x^2. This leads to a field

pic

Since the total charge on the rod (Q) is simply the product of the charge density and the total length of the rod, this reduces to

pic

This is exactly the expression for the electric field from a point charge. Thus, as you move farther and farther from the rod, the rod does indeed begin to look like a point charge.

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1.4: Analysis Tools - Point Charges

Find the electric field at the indicated point. The charges are separated by a distance 4a.

pic

The electric field at this point will be the vector sum of the electric field from the left charge (special e1) and the electric field from the right charge (special e2).

Let's examine the left charge first. Since the problem is expressed symbolically, the charge is simply "2q", and the distance, r, between the charge and the indicated point of interest should be expressed in terms of "a". Since each square in the diagram has width and height a, this distance can be expressed as:

pic

All that's to determine is the unit vector. The unit vector is simply a mathematical description of how to get from the source charge to the point of interest. In English, to get from the source charge to the point of interest you should move 3a in the x-direction an a in the y-direction[1]. This can be written as:

pic

This is the vector that points from the source charge to point of interest. However, this is not a *unit* vector that points from the source charge to the point of interest. However, this is not a *unit* vector since its magnitude isn't 1. (A unit vector should convey a *direction* in space without altering the *magnitude* of the rest of the equation. It can accomplish this only if its magnitude is equal to 1.)

However, it's simple to convert a regular vector into a unit vector, just divide the vector by its magnitude. This leads to:

pic

Putting this all together yields:

pic

Repeating for the right charge gives:

pic

Adding these two contributions together yields

pic

Thus, the electric field at the location indicated points to the right and slightly downward.

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[1]For the sake of consistency, we will use a common coordinate system with the +x-direction pointing to the right and the +y-direction pointing to the top of the page. For three dimensional systems, the +z-direction will point directly out of the page. The coordinate axes will be indicated in the vast majority of diagrams.

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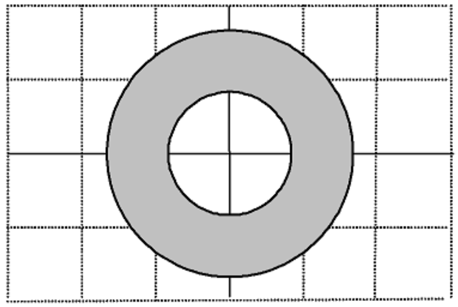
1.6: Analysis Tools - Gauss's Law

*The long, hollow plastic cyclinder at right has inner radius a, outer radius b, and uniform charge dens*ity ρ. Find *the electric field at all points in a plane perpendicular to the cyclinder near its midpoint.*



For certain situations, typically ones with a high degree of symmetry, Gauss' Lae allows you to calculate the electric field relatively easily. Gauss's Law, mathematically, states:

$$\oint E \cdot dA = \frac{q_{enclosed}}{\epsilon_o} \tag{1.6.1}$$

Let's describe what this means in English. The left side of the equation involves the vector dot product between the electric field and an infinitesimally small area that is a piece of a larger closed surface (termed the *Gaussian surface*). This dot product between electric field and area is termed *electric flux*, and is often visualized as the amount of field that "passes through" the little piece of area. The integral simply tells us to add up all of these infinitesimal electric fluxes to get the total flux through the entire closed surface.

The gist of Gauss' Law is that this total electric flux is exactly equal to the total amount of electric charge enclosed within the gaussian surface, divided by a constant, e0. (e0 is the *permittivity of free space,* a constant equal to 8.85 x 10^{-12} C^2/Nm^2.)

Somewhat counter intuitively, the key to applying Gauss' Law is to choose a gaussian surface such that you never really have to do the integral on the left side of the equation! To try to help you understand what I'm talking about, let's walk through the solution of the above problem. The following sequence of steps will help you understand the process of applying Gauss's law:

1. *Choose the appropriate gaussian surface.*
2. *Carefully draw the hypothetical gaussian surface at the location of interest.*
3. *Carefully draw the electric field at all points on the gaussian surface.*
4. *Write an expression for the surface area parallel to the electric field.*
5. *Write an expression for* $q_{enclosed}$*, the charge enclosed within the gaussian surface.*
6. *Apply Gauss' Law and determine the electric field at all points on this hypothetical surface.*

Since we want to find the electric field at all points in space, there are three distinct regions we will have to investigate:

- the region inside the "hole" in the cylinder ($r < a$),
- the region within the actual material of the cylinder ($a < r < b$),
- and the region outside of the cylinder ($r > b$).

Let's start outside of the cylinder.

Outside of the cylinder: r > b

1. *Choose the appropriate gaussian surface.*

Gauss' law is primarily useful when the objects under investigation have a high degree of symmetry. Guass' law basically exploits that symmetry to make the calculation of the electric field relatively painless. To exploit the symmetry of the situation, you should always choose a gaussian surface to mimic the symmetry of the object you are investigating. In this example, since the object is a cylinder, my gaussian surfaces will all be cylinders.

2. *Carefully draw the hypothetical gaussian surface at the location of interest.*

Since we are trying to determine the electric field for all points outside of the cylinder, draw a cylindrical gaussian surface with radius r. The value of r is *variable*, and can take on any value greater than b, the radius of the *real* cylinder. Remember, the gaussian surface is *hypothetical*; it's a mathematical "object" that only exists to help you solve the problem. Try not to confuse it with the real cylinder of radius b.

The gaussian surface is represented from two separate viewpoints below. (The gray cylinder is the actual, charged cylinder while the dashed cylinder is the gaussian surface.) The gaussian surface has radius r, where r can be any value greater than b, and length L. It is located near the midpoint of the actual cylinder. The gaussian surface also includes the circular "end caps" of the cylinder since a gaussian surface must be a *closed* surface.

pic

3. *Carefully draw the electric field at all points on the gaussian surface.*

Although I have no idea what the *magnitude* of the electric field is at any point on my gaussian surface, the symmetry of the situation tells me that the *direction* of the electric field must be either radially away from or radially toward the central axis of the cylinder. If the charge density is positive, the field will be directed radially outward from the cylinder axis.

Moreover, even though I don't know the magnitude of the field, I *do* know that the magnitude is the same at every point on my surface.

pic

4. *Write an expression for the surface area parallel to the electric field.*

The left side of Gauss' law requires us to evaluate an integral over the surface of our gaussian cylinder. The integral requires us to find the dot product between the electric field and the area, and the integrate this dot product ove the entire surface. I mentioned earlier that you should never have to actually do this integral (assuming you chose the "correct" gaussian surface). So why don't we have to do this integral?

The vector dot product can be re-written as:

pic

where is the angle between the electric field and the area of the gaussian surface. You nay recall from calculus that the "direction" associated with area is perpendicular to its surface. Thus, the direction of each infinitesimal area is indicated on the diagrams below.

pic

Notice that the electric field vector and the area vector are parallel at every point on the cylindrical portion of the gaussian surface and perpendicular at every point on the circular end caps. Thus, breaking the integral into two parts, one over the cylindrical portion of the gaussian surface and one over the end caps, yields:

pic

Now note that the magnitude of the electric field is the same at every point on the cylindrical portion of the gaussian surface since every point is equal distance from the charge distribution. Thus, the electric field is constant and can be brought outside of the integral, leaving a pretty easy integral to evaluate.

pic

Notice that the entire left-hand side of Gauss' law reduces to the product of the electric field magnitude and the magnitude of the surface area parallel to this field. Thus, because of our wise choice of gaussian surface, all we really need to calculate is the magnitude of the surface area parallel to the electric field. In this case, the parallel area is given by:

pic

5. *Write an expression for* $q_{enclosed}$ *the charge enclosed within the gaussian surface.*

Since the gaussian surface is outside of the real cylinder, *all* of the charge on the cylinder within the length L is enclosed. Since we know the volume charge density on the cylinder, p, the total charge on the cylinder within the length L is the product of the charge density and the volume of the cylinder.

pic

6. *Apply Gauss' Law and determine the electric field at all points on this hypothetical surface.*

pic

Thus, the electric field outside of the cylinder is inversely proportional to the distance from the cylinder.

Now we have to repeat this analysis for the other two regions.

Within the cylinder: a < r < b

1. *Choose the appropriate gaussian surface.*

We will again use a cylindrical surface.

2. *Carefully draw the hypothetical gaussian surface at the location of interest.*

Since we are interested in the electric field within the actual cylinder, the radius of our gaussian surface is larger than a but less than b.

pic

3. *Carefully draw the electric field at all points on the gaussian surface.*

As before, the magnitude of the electric field must be constant on the gaussian surface and directed radially outward due to the symmetry of the situation.

pic

4. *Write an expression for the surface area parallel to the electric field.*

The area parallel to the electric field is again the surface area of the cylindrical portion of the gaussian surface:

pic

5. *Write an expression for* $q_{enclosed}$ *the charge enclosed within the gaussian surface.*

Since the gaussian surface is inside the real cylinder, not all of the charge on the real cylinder is enclosed by the gaussian surface. All of the charge between a and r is enclosed, but the charge between r and b is not enclosed by the gaussian surface. Thus, the amount enclosed is the product of the volume charge density and the volume of the portion of the cylinder enclosed by the gaussian surface.

pic

6. *Apply Gauss' Law and determine the electric field at all points on this hypothetical surface.*

pic

Notice that this electric field increases with increasing r, since as r increases, more and more charge is availablre to produce the electric field.

The last region we have to investigate is inside of the "hole" in the cylinder.

Inside the "hole": r < a

Since we must choose our gaussian surface to have a radius less than a, it is located inside the hollow center of the cylinder. Since there is *no* charge enclosed by this gaussian surface, the electric field in this region must be zero.

pic

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1.7: Activities (The Electric Field)

Paul D’Alessandris (Monroe Community College)

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CHAPTER OVERVIEW

2: Electric Forces

Topic hierarchy

Thumbnail: A generator with a single rectangular coil rotated at constant angular velocity in a uniform magnetic field produces an emf that varies sinusoidally in time. Note the generator is similar to a motor, except the shaft is rotated to produce a current rather than the other way around. (CC BY; OpenStax).

Paul D’Alessandris (Monroe Community College)

2.1: Concepts and Principles - The Gravitational Analogy

The Gravitational Analogy

In introducing the concept of the electric field, I tried to illustrate it by drawing an analogy with the gravitational field, (pic). This analogy can be extended to electric force and gravitational force.

From mechanics, the relationship for the gravitational force on an object is:

pic

where

- m is the mass of the particle of interest,
- and (pic) is the net gravitational field at the location of the particle of interest. (This field was typically approximated as that of the earth, but should actually be the field created by all of the massive particles in the universe, other than the particle of interest.)

Mass is the property that allows particles to *create* gravitational fields and it is also the property that allow thems to *interact* with other particles' gravitational fields. This interaction is termed the gravitational force.

Wouldn't it be great if charge played the same role with regard to the electric field? Well, it is great! The electric force on a particle is given by the relation,

pic

where

- q is the charge on the particle of interest,
- and (pic) is the net electric field at the location of the particle of interest (created by all of the *other* charged particles in the universe).

All charged particles create electric fields, but this is only half of the story. All charged particles also interact with *other* particles' electric fields. This interaction is termed the electric force.

Paul D'Alessandris (Monroe Community College)

2.2: Analysis Tools - Point Charges

Point Charges

The three charges below are located as shown. Each grid square has width a. Find the net electric force on the positive charge.

pic

To find the electric force on the positive charge, you must first find the electric field at its location in space. Thus, this is really just more practice in calculating the electric field!

The electric field at the location of the positive charge will be the vector sum of the electric field from the -2q charge ($\mathbf{E}_{-2Q}$) and the electric field from the -2q charge ($\mathbf{E}_{-2q}$). Let's calculate these two fields separately and then add them together.

From the -2q charge:

pic

and from the +2q charge:

pic

Adding these two contributions together yields

pic

So the force on the positive charge is:

pic

The force on the positive charge is to the left, and slightly downward, attracted to the two negative charges.

Force and Motion

In many applications, oppositely charged parallel plates are used to "steer" beams of charged particles. In this example, a proton is injected at 2.0×10^6 m/s into the space between the plates. The plates are 2.0 cm long. What charge density is needed on the plates to give proton a y-velocity of 2.0×10^5 m/s as it exits the plates?

pic

Since this problem involves the motion of a particle between two distinct events, let's complete a motion table.

Event 1: The proton enters the device. Event 2: The proton exits the device.

$t_1 = 0$ s $t_2 =$

$r_{1x} = 0$ m $r_{2x} = 0.02$ m

$r_{1y} = 0$ m $r_{2y} =$

$v_{1x} = 2 \times 10^6$ m/s $v_{2x} =$

$v_{1y} = 0$ m/s $v_{2y} = 2 \times 10^5$ m/s

Between the plates, the proton will experience an electrical force, and hence acceleration, in the y-direction. Remember from mechanics that this y-acceleration will not affect thte kinematics of the proton in the x-direction. Thus, in the x-direction the acceleration of the proton is zero.

Applying the kinematic equations in the x-direction yields:

pic

Applying the same kinematics equations in the y-direction yields:

pic

Now, using Newton's Second Law and the relation for electric field from parellel conducting plates, find the necessary charge density.

pic

The bottom plate should be made positive and the top plate negative, both with this charge density. (Note that the force of gravity acting on the proton is completely insignificant compared to the electrical force. This is generally true and we will typically ignore the force of gravity acting on individual particles such as protons and electrons.)

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2.3: Analysis Tools - Force and Motion

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CHAPTER OVERVIEW

3: Magnetic Fields

Paul D’Alessandris (Monroe Community College)

01. Concepts and Principles

Moving Charges

All charged particles create electric fields, and these fields can be detected by other charged particles resulting in electric force. However, a completely *different* field, both qualitatively and quantitatively, is created when charged particles move. This is the magnetic field. All *moving* charged particles create magnetic fields, and all *moving* charged particles can detect magnetic fields resulting in magnetic force. *This is in addition to the electric field that is always present surrounding charged particles*.

This should strike you as rather strange. Whenever a charged particle begins to move a completely new field springs into existence (distributing business cards throughout the universe). Other charged particles, if they are at rest relative to this new field, *do not notice this new field* and do not feel a magnetic force. Only if they move relative to this new field can they sense its existence and feel a magnetic force. It's as if only while in motion can they read the business cards distributed by the original moving charge, and only while in motion does the original charge distribute these business cards in the first place! Does it sound strange yet?

Why the magnetic field exists, and its relationship to the electric field and relative motion will be explored later in the course. For now, we will concentrate on learning how to calculate the value of the magnetic field at various points surrounding moving charges. Next chapter, we will learn how to calculate the value of the magnetic force acting on *other* charges moving relative to a magnetic field.

Permanent Magnets

I claimed above that the magnetic field only exists when the source charges that create it are moving. But what about permanent magnets, like the ones holding your favorite physics assignments to your refrigerator? Where are the moving charges in those magnets?

The simplest answer is that the electrons in "orbit" in each of the atoms of the material create magnetic fields. In most materials, these microscopic magnetic fields are oriented in random directions and therefore cancel out when summed over all of the atoms in the material. In some materials, however, these microscopic magnetic field (large enough to interact with the microscopic magnetic fields present in your refrigerator door). Although this is a gross simplification of what actually takes place, it's good enough for now.

The magnetic properties of real materials are extremely complicated. In addition to the orbital contribution to magnetic field, individual electrons and protons have an intrinsic magnetic field associated with them due to a property called *spin*. Moreover, even neutrons, *with no net electric charge*, have an intrinsic magnetic field surrounding them. To learn more about the microscopic basis of magnetism, consider becoming a physics major....

Electric Current

Moving electric charges form an electric current. We will consider the source of all magnetic fields to be electric current, whether that current is macroscopic and flows through a wire or whether it is microscopic and flows "in orbit" around an atomic nucleus.

Thd simplest source of magnetic field is electric current flowing through a long, straight wire. In this case, the magnetic field at a particular point in space is given by the relation,

pic

where

- m_0 is the *permeability of free space*, a constant equal to 1.26 x 10^{-6} Tm/A.
- i is the source current, the electric current that creates the magnetic field, and is measured in amperes (A). One ampere is equal to one coulomb of charge flowign through the wire per second.(We will always enter the current as a positive value in this relation. The definition of (t hat) below will be used to specify the correct direction of the field.)
- r is the distance between the source current and the point of interest.
- (t hat) is the unit vector that is *tangent* to a circle centered on the source current, and located at the point of interest. To determine the sense of (t hat), place your thumb in the direction of current flow. The sense in which the fingers of your *right*

hand curl is the sense of (t hat). (For example, for current flowing out of the page, (t hat) is counterclockwise.)

If the source of the magnetic field is more complicated than long, straight current-carrying wires, then a more general relationship is needed. The magnetic field at a particular point in space, from any current distribution, is given by the relation,

pic

where

- (pic) is an infinitesimally small portion of current-carrying wire.
- (r hat) is the unit vector that points *from* the small portion of current-carrying wire *to* the point of interest.
- The integral is over the entire length of wire.

Paul D'Alessandris (Monroe Community College)

02. Analysis Tools

Long, Parallel Wires

Find the magnetic field at the indicated point. The long, parallel wires are separated by a distance 4a.

The magnetic field at this point will be the vector sum of the magnetic field from the left wire ($\mathbf{B}_L$) and the magnetic field from the right wire ($\mathbf{B}_R$).

For the left wire, I've indicated the direction of the tangent vector. Remember, with your thumb pointing in the direction of the current (out of the page), the direction in which the fingers of your right hand curl is the direction of the tangent vector (counterclockwise).

pic

By definition, the tangent vector is perpendicular to the radial vector. You should know how to specify the radial unit vector from the chapter on electric field; to specify the tangent unit vector, you have to construct a vector *perpendicular* to the radial vector.

To do this, simply flip the x- and y-components of the radial vector and add the appropriate algebraic signs. (Ask your math professor to prove that this always results in a vector perpendicular to the original vector.) Using this trick, the magnetic field from the left wire is:

pic

For the right wire your thumb should point into the page, making your right-hand fingers curl clockwise.

pic

pic

Adding these two contributions together yields

pic

More General Current Distribution

Find the magnetic field at the origin. The wire forms a circle of radius R.

PIC

Finding the magnetic field from a current-carrying wire involves several distinct steps. Until you become very comfortable setting up and evaluating magnetic field integrals, I would suggest you systematically walk through these steps.

1. *Carefully identify and label the location of the differential element on a diagram of the situation.*
2. *Carefully identify and label the location of the point of interest on a diagram of the situation.*
3. *Write an expression for (pic), the vector differential element.*
4. *Write an expression for r, the distance between the differential element and the point of interest.*
5. *Write an expression for (r hat), the unit vector representing the direction from the element to the point of interest.*
6. *Insert your expressions into the integral for the magnetic field.*
7. *Carefully choose the limits of integration.*
8. *Evaluate the integral.*

I'll demonstrate each of these steps for the scenario under investigation.

1. *Carefully identify and label the location of the differential element on a diagram of the situation.*

The differential element is a small (infinitesimal) piece of the current-carrying wire. The location of this differential element must be *arbitrary*, meaning it is not at a "special" location like the top or bottom of the loop. Its location must be represented by a variable, where this variable is the variable of integration and determines the limits of the integral.

For this example, select the differential element to be located at an angle "q" counter-clockwise from the x-axis. (Later, you will select the limits of integration to go from 0 to 2p to allow this arbitrary element to "cover" the entire loop.)

pic

2. *Carefully identify and label the location of the point of interest on a diagram of the situation.*

The point of interest is the center of the loop of wire.

pic

3. *Write an expression for (pic), the vector differential element.*

The differential element in the integral for magnetic field is a vector, meaning it has both a magnitude (its length) and a direction. The direction of the differential element is the direction in which the current is flowing through the element. Thus, we need an expression for the vector illustrated at right (and greatly magnified below).

The length of the differential element is Rdq, since the element forms an arc on a circle of radius R. Its direction can be determined by noting that it makes an angle of q with respect to the y-axis, and is directed in the -x-direction and +y-direction. This results in:

pic

4. *Write an expression for r, the distance between the differential element and the point of interest.*

The distance between the differential element and the point of interest is just the radius of the loop;

pic

5. *Write an expression for , the unit vector representing the direction from the element to the point of interest.*

Since this vector points *from* the element *to* the point of interest, it is directed in the -x-direction and -y-direction. This results in:

pic

pic

6. *Insert your expressions into the integral for the magnetic field.*

pic

7. *Carefully choose the limits of integration.*

The limits of integration are determined by the range over which the differential element must be "moved" to cover the entire object. In this case, the element must move all the way around the circular loop:

pic

8. *Evaluate the integral.*

pic

The magnetic field at the center of any current-carrying loop of radius R is given by the expression above.

Paul D’Alessandris (Monroe Community College)

03. Analysis Tools 2

1. Ampere's Law

Ampere's Law

The long, hollow-core wire below has inner radius a, outer radius b and current i uniformly distributed across its area. The current flows in the +z-direction. Find the magnetic field at all points in space.

pic

For certain situations, typically ones with cylindrical symmetry, Ampere's Law allows you to calculate the magnetic field relatively easily. Ampere's Law, mathematically, states:

pic

Let's describe what this means in English. The left side of the equation involves the vector dot product between the magnetic field and an infinitesimally small length that is a piece of a larger closed path (termed the *amperian loop*). This dot product determines the amount of magnetic field that is parallel to this very small piece of a larger closed path. The integral simply tells us to add up all of these contributions around the entire closed path.

The gist of Ampere's Law is that this integral is exactly equal to the total amount of electric current flowing within the amperian loop, multiplied by the constant m0.

Somewhat counter intuitively, the key to applying Ampere's Law is to choose an amperian loop such that you never really have to do the integral on the left side of the equation! (Does this sound familiar?) To try to help you understand what I'm talking about, let's walk through the solution of the above problem. The following sequence of steps will help you understand the process of applying Ampere's law:

1. *Choose the appropriate amperian loop.*
2. *Carefully draw the hypthetical amperian loop at the location of interest.*
3. *Carefully draw the magnetic field at all points on the amperian loop.*
4. *Write an expression for the path length parallel to the magnetic field.*
5. *Write an expressioln for $i_{enclosed}$, the current inside the amperian loop.*
6. *Apply Ampere's Law and determine the magnetic field at all points on this hypothetical loop.*

There are thee distinct regions we will investigate:

- the region inside the "hole" in the wire(r < a),
- the region within the actual material of the wire (a < r < b),
- and the region outside of the wire (r > b).

Let's start outside of the wire.

Outside of the wire: r > b

1. *Choose the appropriate amperian loop.*

The key to using Ampere's Law is to try to exploit the symmetry of the current-carrying wire. Since wires typically have circular cross-section, circular amperian loops are the norm.

2. *Carefully draw the hypothetical amperian loop at the location of interest.*

Since we are trying to determine the magnetic field for all points outside of the wire, draw a circular amperian surface with radius r. The value of r is *variable*, and can take on any value greater than b, the radius of the *real* wire. Remember, the amperian loop is *hypothetical*; it's a mathematical "object" that only exists to help you solve the problem. Try not to confuse it with the real wire of radius b.

pic

3. *Carefully draw the magnetic field at all points on the amperian loop.*

Although I have no idea what the *magnitude* of the magnetic field is at any point on my amperian loop, the symmetry of the situation tells me that the *direction* of the magnetic field must be either clockwise or counterclockwise. If the current is flowing out of the page, the field will be directed counterclockwise around the amperian loop.

pic

Moreover, even though I don't know the magnitude of the field, I *do* know that the magnitude is the same at every point on my loop.

4. *Write an expression for the path length parallel to the magnetic field.*

The left side of Ampere's law requires us to evaluate an integral around our amperian loop. The integral requires us to find the dot product between the magnetic field and the differential line element, and integrate this dot product around the entire loop. I mentioned earlier that you should never have to actually do this integral (assuming you chose the "correct" amperian loop). So why don't we have to do this integral?

The vector dot product can be re-written as:

pic

where is the angle between the magnetic field and the differential line element of the amperian loop. Traversing our amperian loop counterclockwise leads to the diagram at right

pic

Notice that the magnetic field vector and the vector representing the differential element are parallel at every point on the amperian loop. This simplifies the left-hand side of Ampere's law:

pic

Now note that the magnitude of the magnetic field is the same at every point on the amperian loop since every point is equal distance from the current distribution. Thus, magnetic field is constant and can be brought outside of the integral, leaving a pretty easy integral to evaluate.

pic

Notice that the entire left-hand side of Ampere's Law reduces to the product of the magnetic field magnitude and the length of the amperian loop, assuming the loop is parallel to this field. Thus, because of our wise choice of amperian loop, all we really need to calculate is the length of the loop parallel to the magnetic field. For a circular loop:

pic

5. *Apply Ampere's Law and determine the magnetic field at all points on this hypothetical loop.*

Since the amperian loop is outside of the real wire, *all* of the current flowing through the wire is enclosed by the amperian loop. Thus,

pic

6. *Apply Ampere's Law and determine the magnetic field at all points on this hypothetical loop.*

pic

Thus, the magnetic field outside of the hollow-core wire looks exactly the same as the magnetic field outside of a "normal" current-carrying wire.

Now we have to repeat this analysis for the other two regions.

Within the wire: a < r < b

1. *Choose the appropriate amperian loop.*

Again choose a circular amperian loop.

2. *Carefully draw the hypothetical amperian loop at the location of interest.*

Since we are trying to determine the magnetic field within the actual wire, the radius of our amperian loop is greater than a but less than b.

pic

3. *Carefully draw the magnetic field at all points on the amperian loop.*

As before, the magnitude of the magnetic field must be constant at all points on the amperian loop and directed counterclockwise.

pic

4. *Write an expression for the path length parallel to the magnetic field.*

The length parallel to the magnetic field is again the length of the circular amperian loop:

pic

5. *Write an expression for* $i_{enclosed}$*, the current inside the amperian loop.*

Since the amperian loop is within the wire, not all of the current in the wire is enclosed by the loop. The amount enclosed can be expressed as the product of a *current density* (J) and the cross-sectional area enclosed. First, since the current is uniformly distributed throughout the wire I can define the current density as:

pic

Then, the current enclosed by the amperian loop is the product of the current density and the area enclosed by the loop:

pic

6. *Apply Ampere's Law and determine the magnetic field at all points on this hypothetical loop.*

pic

Thus, the magnetic field inside the hollow-core wire actually increases with increasing r, since as r increases, more and more current is available to produce the magnetic field.

Inside the "hole": r < a

Since we must choose our amperian loop to have a radius less than a, it is located inside the hollow center of the wire. Since there is *no* current enclosed by this loop, the magnetic field in this region must be zero.

pic

Paul D’Alessandris (Monroe Community College)

04. Activities

CHAPTER OVERVIEW

4: Magnetic Forces

Topic hierarchy

Paul D’Alessandris (Monroe Community College)

01. Concepts and Principles

From Magnetic Field to Magnetic Force

As mentioned previously, only *moving* charged particles can interact with a magnetic field. Stationary electric charges are completely oblivious to the presence of magnetic fields. The magnetic force (also known as **Lorentz force**) on a moving electric charge is given by the relation,

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right) \tag{1}$$

where

- q is the charge on the particle of interest,
- $\vec{v}$ the velocity of the particle of interest,
- and $\vec{B}$ is the net magnetic field at the location of the particle of interest (created by all of the *other* moving charge particles in the universe).

Thus, the direction of the magnetic force on a moving charge is more complicated than in the analogous case of the electric force. In addition to determining the magnetic field at the location of the particle, you must know its velocity and perform a vector cross-product.

Magnetic Force on a Current-Carrying Wire

In many cases, instead of considering moving electric charges individually we will focus our attention on the collection of moving charges that make up an electric current. In an electric current, consider a small amount of charge, $\mathrm{d}q$, moving with velocity $\vec{v}$. This velocity can be represented as:

$$\vec{v} = \frac{\mathrm{d}\vec{l}}{\mathrm{d}t} \tag{2}$$

where $\vec{l}$ is the instantaneous displacement of this small collection of charge. Based on this observation, we can calculate the force acting on this small amount of charge by:

$$\mathrm{d}\vec{F} = (\mathrm{d}q)\,\vec{v} \times \vec{B} \tag{3}$$

$$\mathrm{d}\vec{F} = (\mathrm{d}q)\,\frac{\mathrm{d}\vec{l}}{\mathrm{d}t} \times \vec{B} \tag{4}$$

$$\mathrm{d}\vec{F} = \left(\frac{\mathrm{d}q}{\mathrm{d}t}\right)\mathrm{d}\vec{l} \times \vec{B} \tag{5}$$

Since the current in a wire is the amount of charge passing through any cross-section of the wire per second,

$$\mathrm{d}\vec{F} = i\left(\mathrm{d}\vec{l}\right) \times \vec{B} \tag{6}$$

Therefore the magnetic force on the entire current-carrying wire is given by the relation,

$$\vec{F} = \int i\left(\mathrm{d}\vec{l}\right) \times \vec{B} \tag{7}$$

where

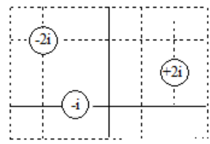
- i is the current in the wire of interest,
- $\mathrm{d}\vec{l}$ is an infinitesimal length of the wire of interest,
- $\mathrm{d}\vec{B}$ is the net magnetic field at the location of $\mathrm{d}\vec{l}$ (created by all of the *other* moving charged particles and currents in the universe),
- and the integral is over the entire length of the wire.

Paul D’Alessandris (Monroe Community College)

02. Analysis Tools

Long, Parallel Wires

Three long, parallel wires are located as shown. Each grid square has a width a. Find the net magnetic force per unit length on the rightmost wire.



The magnetic force on a current-carrying wire is given by the relation,

$$\vec{F} = \int i \left(\mathrm{d}\vec{l}\right) \times \vec{B} \tag{1}$$

However, for this scenario the relationship simplifies. First, the magnetic field from the two source wires is constant along the entire length of the wire of interest. Second, since the wire of interest is straight, $\mathrm{d}\vec{l}$ is also constant. Therefore, instead of an integral you have:

$$\vec{F} = i\vec{L} \times \vec{B}$$

where L is the total length of the wire of interest.

To solve this problem, first find the net magnetic field at the location of the wire of interest, and then perform the cross-product. The magnetic field will be the vector sum of the magnetic field for the leftmost wire ($\mathbf{B}_L$) and the magnetic field from the central wire ($\mathbf{B}_C$).

$$\vec{B}_L = \frac{\mu_0 i}{2\pi r}\hat{t} \qquad \vec{B}_C = \frac{\mu_0 i}{2\pi r}\hat{t}$$

$$\vec{B}_L = \frac{\mu_0 (2i)}{2\pi\sqrt{(4a)^2 + (a)^2}}\left(\frac{-a\hat{i} - 4a\hat{j}}{\sqrt{(4a)^2 + (a)^2}}\right) \qquad \vec{B}_C = \frac{\mu_0 (i)}{2\pi\sqrt{(3a)^2 + (a)^2}}\left(\frac{a\hat{i} - 3a\hat{j}}{\sqrt{(3a)^2 + (a)^2}}\right)$$

$$\vec{B}_L = \frac{2\mu_0 i}{2\pi(17a^2)}(-a\hat{i} - 4a\hat{j}) \qquad \vec{B}_C = \frac{\mu_0 i}{2\pi(10a^2)}(a\hat{i} - 3a\hat{j})$$

$$\vec{B}_L = \frac{\mu_0 i}{17\pi a}(-\hat{i} - 4\hat{j}) \qquad \vec{B}_C = \frac{\mu_0 i}{20\pi a}(\hat{i} - 3\hat{j})$$

$$\vec{B}_L = (-0.0588\hat{i} - 0.235\hat{j})\frac{\mu_0 i}{\pi a} \qquad \vec{B}_C = (0.05\hat{i} - 0.15\hat{j})\frac{\mu_0 i}{\pi a}$$

Adding these two contributions together yields

$$\vec{B} = (-0.0088\hat{i} - 0.385\hat{j})\frac{\mu_0 i}{\pi a}$$

So the magnetic force on the rightmost wire is:

$$\vec{F} = i\vec{L} \times \vec{B}$$

$$\vec{F} = (2i)(L\hat{k}) \times (-0.0088\hat{i} - 0.385\hat{j})\frac{\mu_0 i}{\pi a}$$

$$\vec{F} = (0.385\hat{i} - 0.0088\hat{j})\frac{2\mu_0 i^2 L}{\pi a}$$

$$\vec{F} = (0.245\hat{i} - 0.0056\hat{j})\frac{\mu_0 i^2 L}{a}$$

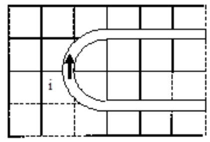
and the force per unit length is

$$\frac{\vec{F}}{L} = (0.245\hat{i} - 0.0056\hat{j})\frac{\mu_0 i^2}{a}$$

The force is to the right, and slightly downward. Notice that wires carrying currents in opposite directions tend to *repel* each other, while wires carrying currents in the same direction tend to *attract* each other.

Curved Wires

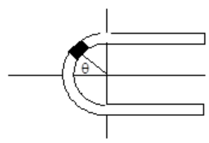
The bent wire below carries current i, consists of two straight segments of length 3l and a half-circle of radius L, and lies in a region of uniform magnetic field, B, in the +x-direction. Find the total force acting on the wire.

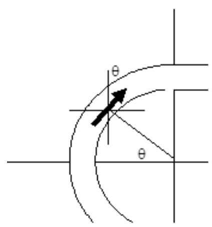


To find the total magnetic force on this bent wire, treat the wire as three separate wires. The force on the two straight sections is zero, because the current in these sections is parallel to the field, resulting in no magnetic force. Therefore, the force on the entire wire is equal to the force on the curved section, which will require setting up and evaluating an integral.

The differential element is located at an angle q, measured clockwise from the -y-axis. At this location, $d\vec{l}$ is directed in the +x-direction and +y-direction. This results in:

$$d\vec{l} = Ld\theta(\sin\theta\hat{i} + \cos\theta\hat{j})$$





Using this differential element, the force on the curved secton of wire is given by:

$$\vec{F} = \int i d\vec{l} \times \vec{B}$$

$$\vec{F} = \int_{-\pi/2}^{\pi/2} i(Ld\theta(\sin\theta\hat{i} + \cos\theta\hat{j})) \times (B\hat{i})$$

$$\vec{F} = -iLB\hat{k}\int_{-\pi/2}^{\pi/2} d\theta\cos\theta$$

$$\vec{F} = -iLB\hat{k}(\sin(\frac{\pi}{2}) - \sin(-\frac{\pi}{2}))$$

$$\vec{F} = -2iLB\hat{k}$$

The magnetic force will cause the curved end of the wire to sink into the plane of the page.

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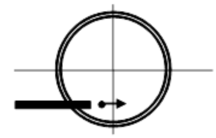
03. Analysis Tools 2

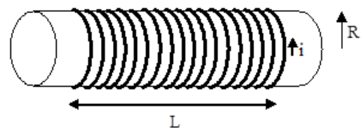
1. Force and Motion

Force and Motion

Protons are injected at 2.0 x 10^5 m/s into a 2000 turn-per-meter solenoid carrying 3.0 A clockwise in the diagram at right. Determine the protons' orbit radius.



A solenoid is a very useful device for generating a uniform magnetic field. A solenoid consists of a wire carrying current i wrapped N times around a hollow core of radius R and length L. Typically, L is substantially larger than R (much larger than illustrated below). Contrast this with a *coil* of wire, in which R is typically larger than L.



In a solenoid, the magnetic field inside the core is extremely uniform. In a sense, the small radius and long length "concentrate" the magnetic field within the core leading to an approximately constant value. Again, this contrasts with a coil of wire, in which the field varies at different locatoins inside and outside the coil.

By using **Ampere's Law**, and a few simplifying approximations, it can be shown that the field inside the solenoid is given by:

$$B = \mu_0 (\frac{N}{L}) i$$
$$B = \mu_0 n i$$

where n is the *turn density*, the number of loops of wire, or turns, per meter. Therefore, the solenoid described in the problem creates a magnetic field

$$B = \mu_0 n i$$
$$B = (1.26x10^{-6})(2000)(3)$$
$$B = 7.56x10^{-3} T$$

This field is directed into the page since the current flows clockwise.

This magnetic field will create a magnetic force on the proton. When the proton first enters the device the magnetic force will be directed upward, causing the path of the proton to bend upward. As the proton begins to move upward the direction of the magnetic force changes, and when the proton is moving directly upward the force will be to the left. This will cause the proton to bend toward the left. When the proton is moving directly leftward the magnetic force will be directed downward, causing the proton to begin to bend downward. And so on...

Since the magnetic force is always perpendicular to the direction of travel of the proton, the magnetic force causes the proton to make a non-stop left-hand turn! The proton will begin moving in circles due to this force, and since the force has no component along the direction of travel of the proton it does no work on the proton and the proton moves at constant speed. Basically, magnetic fields "steer" charged particles but don't make them speed up or slow down.

With this in mind, let's apply Newton's Second Law to the proton the instant it enters the solenoid:

$$\vec{F} = m\vec{a}$$
$$q\vec{v} \times \vec{B} = m\vec{a}$$

It enters the solenoid traveling in the +x-direction, and it will accelerate toward the center of its circular path, the +y-direction. (Remember that this radial acceleration can be expressed as v^2/r.)

$$q(v\hat{i}) \times (-\mu_0 ni\hat{k}) = m(\frac{v^2}{r}\hat{j})$$

$$qv\mu_0 ni\hat{j} = m\frac{v^2}{r}\hat{j}$$

$$q\mu_0 ni = m\frac{v}{r}$$

$$r = \frac{mv}{q\mu_0 ni}$$

$$r = \frac{(1.67x10^{-27})(2x10^5)}{(1.6x10^{-19})(7.56x10^{-3})}$$

$$r = 0.28m$$

The proton will circle at constant speed at this radius.

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04. Activities

No headers

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CHAPTER OVERVIEW

5: Electromagnetic Induction

Topic hierarchy

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01. Concepts and Principles

Creating Electrical Energy

When electric charges move, their electric fields vary. In the previous two chapters we considered moving electric charges as the source of magnetic fields, but we could just as easily have considered the variation in the electric field as the source of the magnetic field. This leads to an interesting question: If varying electric fields can create magnetic fields, can varying magnetic fields create electric fields? The answer is yes, and this process, termed *electromagnetic induction*, is at the heart of almost all electrical power generation worldwide. In addition to incredible technological importance, electromagnetic induction hints at a deep inter-relationship and symmetry between electric and magnetic fields that will be explored more fully in later chapters.

Imagine a region of space with a magnetic field. Surrounding a portion of this region is a hypothetical closed path.(Often, a real loop of wire will be the closed path of interest, but induction occurs whether or not a real wire loop is present.)

pic 1

First, let me define *magnetic flux*. In analogy ot electric flux, introduced in conjunction with Gauss' Law, magnetic flux is defined to be:

pic 2

This equation involves the vector dot product between the magnetic field and an infinitesimally small area within the area bounded by the closed path. This dot product between magnetic field and area often visualized as the amount of field that "passes through" the little piece of area. The integral simply tells us to add up all of these infinitesimal magnetic fluxes to get the total flux through the area enclosed by the path.

Second, let me define *emf*. (Actually, let me apologize. Emf *used* to stand for electromotive force, even though it is not a force. In light of this misleading name, emf now, officially, stands for emf. It's not short for anything. I'm not making this up.) Emf is the name for the electrical energy per unit charge created by changing magnetic flux. In general, any process that generates electrical energy "creates" emf. In addition to changing magnetic flux, chemical batteries and some solar cells create emf. Mathematically, emf is defined by integrating the electric field around the closed path described above:

pic 3

(Don't worry, you will never actually calculate this integral.) All you need to know is that emf is the energy created per unit charge. The units of emf, joules per coulomb, are given the name *volts*(V). In crude language, an emf is a "voltage".

Let's put this all together. The central relationship describing electromagnetic induction, termed *Faraday's Law*, claims that:

pic 4

where

- e is the emf induced (the voltage created) in the closed path,
- F is the magnetic flux that passes through the closed path,
- and the negative sign indicates that the emf's direction in the closed path is to *oppose the change in magnetic flux*. (If the closed path is a real loop of wire, the emf will drive an induced current whose direction is such that the magnetic field produced by this induced current is opposite to the change in magnetic field that produces the induced current. Crystal clear?)

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02. Analysis Tools

Abrupt Change in Flux

An N-turn loop of radius r is R from a very long straight wire carrying current i. If the current is reduced to zero in a time T, what is the average induced emf in the loop? What is the direction of the induced current? Assume r >> R.

pic 5

When the current is reduced, the magnetic flux through the small loop will change. By Faraday's Law, this change in flux will create an emf in the loop. The first step toward finding the emf involves finding the magnetic flux through the loop of interest. For this example, the direction of the magnetic field and the direction of the loop's area are parallel, so the dot product reduces to:

pic 6

Moreover, since the loop is far from the wire, and the loop's diameter is small, the magnetic field from the wire is approximately constant over the area of the loop, and the flux is given by:

pic 7

The N indicates that there are N loops of wire through which the flux passes.

This flux is reduced to zero over some finite time interval. Interpreting Faraday's Law over a finite time interval results in:

pic 8

This is the average emf induced during the time period under investigation.

Since initially the magnetic field through the loop was in the +z-direction, and was then removed, the induced emf will drive an induced current counterclockwise around the loop in an attempt to counter the reduction in flux. In other words, the induced current will try to maintain a constant magnetic flux through the loop.

Continuous Change in Flux

A 1000-turn secondary coil of radius 2.0 cm is concentric with a 400-turn primary coil of radius 20 cm carrying AC current at 60 Hz with peak current 2.0 A. (AC current can be modeled as $i(t) = i_0 \cos (2pft)$.) What is the induced emf in the secondary coil as a function of time?

pic 9

Again, the first step toward finding the emf involves finding the magnetic flux through the secondary loop. For this example, the direction of the magnetic field and the direction of the loop's area are parallel, flux reduces to:

pic 10

Moreover, since the magnetic field is approximately constant over the relatively small area of the secondary loop, the flux is given by:

pic 11

Plugging in values yields:

pic 12

Applying Faraday's Law:

pic 13

Thus, the secondary coil of wire has 60 Hz AC voltage 90° out of phase with the voltage in the primary coil. (Sine and cosine functions are 90° out of phase with each other.) The maximum emf in the secondary coil is 1.19 V.

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03. Activities

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CHAPTER OVERVIEW

6: Electric Potential

Topic hierarchy

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01. Concepts and Principles

An Alternative Approach

The electric field surrounding electric charges and the magnetic field surrounding moving electric charges can both be conceptualized as information embedded in space. In both cases, the information is embedded as *vectors*, detailing both the magnitude and direction of each field. Moreover, when this information is "read" by other moving electric charges, the result is a *force* acting on the charge. These forces can be calculated to allow us to determine the subsequent motion of the charge.

Just as in mechanics, there is an alternative to this force-based approach to analyzing the behavior of electric charges. In this chapter I will define a new field, the *electric potential*, which surrounds every electric charge. This field differs from the electric field because it is a *scalar* field, meaning that this field has only a magnitude at every point in space and no associated direction. Moreover, the information in this field, when read by other electric charges, does not result in a force on the charge but rather determines the *electric potential energy* the charge possesses at that point in space.

Understanding how to calculate this new field, how this field relates to energy, and how this field is related to the electric field will be the focus of this chapter.

A Gravitational Analogy

Rather than thinking in terms of the gravitional force and Newton's Second Law, an alternative way to examine mechanics scenarios is by using the concept of gravitational potential energy and the conservation of energy.

In the force approach, we envision a vector field surrounding the earth, regardless of whether a second mass is nearby to interact with this field. If a mass is present, the mass interacts with this field and feels a gravitational force. This idea is captured in the equation:

pic 1

In the energy approach, we can envision a scalar field, the *gravitational potential*, which is present regardless of whether a second mass is nearby to interact with this field. If a mass is present, the mass interacts with this field and has gravitational potential energy. Near the surface of the earth, the familiar expression for gravitational potential energy:

pic 2

can be thought of as the product of the mass of the object and this pre-existing gravitational potential, V_G:

pic 3

if we define the gravitational potential by:

pic 4

Although we didn't use the concept of gravitational potential while studying mechanics, it will prove to be a very useful concept in our study of electrical phenomenon. The general expression for gravitational potential, valid regardless of distance from a massive object, is:

pic 5

In summary, just as a mass will interact with the vector gravitational field as a force, a mass will interact with the scalar gravitational potential field as potential energy.

The situation is very similar for electrical phenomenon. We can envision a scalar field, the *electric potential*, which is present regardless of whether a second charge is nearby to interact with this field. If a charge is present, the charge interacts with this field and has *electric potential energy*. The electric potential, V_E, is defined by the relationship:

pic 6

where

- q is the source charge, the electric charge that creates the field,
- and r is the distance between the source charge and the point of interest.

This leads to an expression for the electric potential energy of:

pic 7

where

- q is the charge on the particle of interest, the charge that is interacting with the field,
- and V_E is the net electric potential at the location of the particle of interest (created by all of the *other* charged particles in the universe).

We will typically leave the subscript off the electric potential and electric potential energy unless the possibility of confusion with the gravitational potential and potential energy are present.

Relating the Electric Field and the Electric Potential

The electric field and the electric potential are not two, independent fields. They are two independent ways of conceptualizing the effect that an electric charge has on the space surrounding it. Just as problems in mechanics can be analyzed using a force-approach or an energy-approach, problems dealing with electrical phenomenon can be analyzed by focusing on the electric field or on the electric potential.

Additionally, just as it is sometimes necessary in mechanics to transfer between force and energy representations, it is sometimes necessary to transfer between the electric field and electric potential representations. The relationship between two fields can be understood by examining the expression for work, which relates force to transfer of energy.

Utilizing the dot product, the work donw in moving a particle from an initial point, i, to a final point, f, can be written as:

pic 8

where (pic 9) is an infinitesimal portion of the path along which the particle moves. You may also recall that the difference in potential energy between initial and final locations is defined as the opposite of the work needed to move the particles between the two points:

pic 10

Putting these two ideas together yields:

pic 11

We can now use this result to relate the electric potential and the electric field. Substituting in expressions for potential energy and force in terms of the fields that convey them leads to:

pic 12

In English, this final result states that the electric potential difference between any two points is defined as the negative of the integral of the electric field along a path connecting the two points. (I'm sure that doesn't seem like particularly *clear* English, but this idea will become more tangible once you get to work on some of the fun activities in this chapter.) The bottom line is that the electric potential can be determined by intergrating the electric field, and, conversely, the electric field can be determined by differentiating the electric potential.

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02. Analysis Tools

Point Charges

Find the electric potential at the indicated point. The charges are separated by a distance 4a.

Pic1

The electric potential at the point specified will be the sum of the electric potential from the left charge (V_L) and the electric potential from the right charge (V_R).

Pic 2

Notice that since the electric potential is a scalar, calculating the electric potential is often much easier than calculating the electric field.

Continuous Charge Distribution

The plastic rod of length L at the bottom has uniform charge density l. Find the electric potential at all points to the right of the rod on the x-axis.

Pic 3

Since electric charge is discrete, the electric potential can always be calculated by summing the electric potential from each of the electrons and protons that make up an object. However, macroscopic objects contain a *lot* of electrons and protons, so this summation has many, many terms:

pic 4

As described earlier, we will replace this summation over a very large number of discrete charges with an integral over a hypothetically continuous distribution of charge. This leads to a relationship for the electric potential at a particular point in space, from a continuous distribution of charge, of:

pic 5

where dq is the charge on a infinitesimally small portion of the object, and the integral is over the entire physical object.

The steps for finding the electric potential from a continuous distribution of charge are:

1. Carefully identify and label the location of the differential element on a diagram of the situation.

2. Carefully identify and label the location of the point of interest on a diagram of the situation.

3. Write an expression for dq, the charge on the differential element.

4. Write an expression for r, the distance between the differential element and the point of interest.

5. Insert your expressions into the integral for the electric potential.

6. Carefully choose the limits of integration.

7. Evaluate the integral.

I'll demonstrate below each of these steps for the scenario under investigation.

1. Carefully identify and label the location of the differential element on a diagram of the situation.

The differential element is a small (infinitesimal) piece of the object that we will treat like a point charge. The location of this differential element must be *arbitrary*, meaning it is not at a "special" location like the top, middle, or bottom of the rod. Its location must be represented by a variable, where this variable is the variable of integration and determines the limits of the integral.

For this example, select the differential element to be located a distance "y" above the center of the rod. The length of this element is "dy". (Later, you will select the limits of integration to go from -L/2 to +L/2 to allow this arbitrary element to "cover" the entire rod.)

Pic 6

2. Carefully identify and label the location of the point of interest on a diagram of the situation.

You are interested in the electric potential at all points along the x-axis to the right of the rod. Therefore, select an *arbitrary* location along the x-axis and label it with its location.

pic 7

3. Write an expression for dq, the charge on the differential element.

The rod has a uniform charge density l, meaning the amount of charge per unit length along the rod is constant. Since the differntial element has a length dy, the total charge on this element (dq) is the product of the density and the length:

pic 8

4. Write an expression for r, the distance between the differential element and the point of interest.

By Pythagoras' theorem, the distance between the differential element and the point of interest is:

pic 9

6. Insert your expressions into the integral for the electric potential.

pic 10

7. Carefully choose the limits of integration.

The limits of integration are determined by the range over which the differential element must be "moved" to cover the entire object. The location of the element must vary between the bottom of the rod (-L/2) and the top of the rod (+L/2) in order to include every part of the rod. The two ends of the rod form the two limits of integration.

pic 11

8. Evaluate the integral.

pic 12

This expression looks daunting, but its limiting behaviors (as x approaches zero and x approaches infinity) are correct. Consider the case where x gets smaller and smaller, approaching zero:

pic 13

This expression diverges (goes to infinity) as x becomes close to zero, as you should expect since the electric potential directly "on" an electric charge is infinite.

As x gets larger and larger, the L/2 terms become insignificant:

pic 14

This expression becomes zero as x becomes infinite, as you should expect since the electric potential extremely far from an electric charge is zero.

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03. Analysis Tools 2

Potential Difference

The long, hollow plastic cylinder at the bottom has inner radius a, outer radius 2a, and uniform charge density r. Find the electric potential difference between the inner and outer radius.

pic 1

An alternative method for calculating the electric potential at a point, or the electric potential difference between two points, is by using knowledge of the electric field. The following relation,

pic 2

states that the potential difference between two points can be determined by integrating the component of the electric field that lies along the path connecting the two points. This means that if you know the electric field in a region of space, you can easily (more or less) find the potential difference between any two points that lie in that region. This relation is particularly useful in conjunction with Gauss' Law for situations with cylindrical or spherical symmetry.

To find the potential difference between points a and b (i.e., what a voltmeter would read if connected across points a and b), we need the electric field in this region. Gauss' Law can be used to find that (review Gauss' Law if this step is a little fuzzy):

pic 3

This electric field points radially away from the center of the cylinder.

For simplicity, choose a path that directly connects a to 2a, i.e., a radial path.

pic 4

pic 5

Notice that this method directly calculates the *difference* in electric potential between two points, without actually determining the *value* of the electric potential at either point. Since electric potential is related to electric potential energy, this method allows to you to find the difference in energy between two points but not the actual value of the energy of an electric charge.

This should strike you as quite similar to the gravitational case. For gravitational potential energy, the choice of the zero-point is arbitrary and only energy differences lead to changes in kinetic energy. For electrical potential energy, the situation is identical. The zero-point of electric potential energy (and electric potential) is typically taken at infinity, although you can "zero" the potential at a more convenient point by "grounding" the system at that point. The physical act of grounding a point on an electrical device is mathematically equivalent to setting the potential equal to zero at that point.

Electric Potential Energy

In many applications, oppositely charged parallel plates (with small holes cut for the beam to pass through) are used to accelerate beams of charged particles. In this example, a proton is injected at v_1 into the space between the plates. The potential difference between the plates is DV(the left plate is at higher potential). What is the velocity of the proton as it exits the device?

pic 6

Since the electric potential changes as the proton moves from the left plate to the right plate, its potential energy changes. This change in potential energy results in a change in kinetic energy of the proton by energy conservation.

pic 7

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04. Activities

SECTION OVERVIEW

7: Electric Circuits

Paul D’Alessandris (Monroe Community College)

01. Concepts and Principles

1. Electric Circuits as Applied Physics
2. The Resistor
3. The Capacitor
4. The Inductor

Electric Circuits as Applied Physics

Electric circuits are one of the most practical applications of our understanding of electric and magnetic fields. In general, an electric circuit is any device that consists of a closed path for charges to move (a current), a source of energy to "drive" the motion of the charges (a potential difference, or voltage, often in the form of a battery), and various circuit elements that can either convert (resistors) or store (capacitors and inductors) the energy supplied by the energy source.

The study of circuits is incredibly broad, since there are limitless ways to combine these elements into an electric circuit. We will restrict ourselves to studying circuits with only a limited number of elements, and with a source that supplies a constant voltage[1].

The Resistor

In general, a *resistor* is any device that converts electrical energy into another form of energy, often heat. For example, a fluorescent light bulb converts electrical energy into light (with about a 20% efficiency, the remaining energy is converted into heat) and an incandescent light bulb converts electrical energy very efficiently into heat (with only about 5% of the incident energy converted to light). Since a conversion of electrical energy takes place in these devices, they are resistors.

In all resistors, the electric potential energy of the charges entering the device is larger than the electric potential energy of the charges exiting the device, because some of the potential energy has been converted to other forms. This decrease in potential energy is due to a decrease in electric potential between the two ends of the device and is directly proportional to the *resistance* of the device.

The definition of resistance for a device is:

pic 1

where

- DV is the potential difference between the two ends of the device, often termed the *voltage drop* across the device,
- and i is the current that flows through the device.

The unit of resistance, pic 2, is defined as the ohm (W).

The previous expression relates the resistance of a resistor to properties of the circuit it is part of. However, it is also sometimes useful to directly relate the resistance to the actual physical parameters of the device itself. For simple, passive resistors (basically blocks of material connected to a voltage source), resistance is defined as:

pic 3

where

- is the *resistivity* of the material from which the resistor is constructed,
- L is the length of the resistor in the direction of current flow,
- and A is the cross-sectional area of the resistor.

Resistivity can range from 0 for a perfect conductor to for a perfect insulator.

One final note on the properties of resistors concerns their rate of energy conversion. Since electric potential is the electric potential per unit of charge, and current is the charge flowing through the devicde per second, the product of change in electric potential and current is the change in electric potential energy per second. Thus, the rate of energy conversion, or *power*, in a resistor is given by:

pic 4

The Capacitor

A *capacitor* is a device that stores energy in the electric field between two closely spaced conducting surfaces. When connected to a voltage source, electric charge accumulates on the two surfaces but, since the conducting surfaces are separated by an insulator, the charges cannot travel from one surface to the other. The charges create an electric field in the space between the surfaces, and the two surfaces have a difference in electrical potential.

pic 5

Once "charged", if the capacitor is removed from the original circuit and connected to a second circuit it can act as a voltage source and "drive" its collected charge through the second circuit. When used in this way, the capacitor clearly acts as a temporary storehouse of energy.

To determine the energy stored in a capacitor, we first need to define the *capacitance* of the capacitor. The capacitance of a capacitor is defined as:

pic 6

where

- Q is the magnitude of the electric charge stored on either conducting surface,
- and DV is the potential difference between the surfaces.

The unit of capacitance,pic 7, is defined as the *farad* (F).

The amount of energy that can be stored on a capacitor is a function of both its capacitance and the potential difference between its surfaces. The relationship between stored energy and these parameters is:

pic 8

The Inductor

An *inductor* is a device that stores energy in the magnetic field created when current passes through a coil of wire. When connected to a voltage source, current will flow through the inductor, establishing a magnetic field.

pic 9

If the voltage source is suddenly removed, current will continue to flow in the coil because of electromagnetic induction. This induced current will act to replace the disappearing source current. The energy needed to drive this current comes from the energy stored in the magnetic field, so in this case the inductor acts as a temporary storehouse of energy.

To determine the energy stored in an inductor, we first need to define the *inductance* of the inductor. The inductance of the inductor is defined as:

pic 10

where

- F is the magnetic flux within the inductor,
- and i is the current flowing through the inductor.

The unit of capacitance, pic 11, is defined as the *henry* (H).

The amount of energy that can be stored in an inductor is a function of both its inductance and the current flowing through it. The relationship between stored energy and these parameters is:

pic 12

---

[1]Circuits with constant voltage sources are referred to as DC, or direct current, circuits.

Paul D'Alessandris (Monroe Community College)

02. Analysis Tools

Resistors in Circuits

The circuit at right represents a 12 V car battery and two mismatched headlights, R_1 = 1.9 W and R_2 = 2.1 W.

a. Determine the magnitude of the potential difference across and the current through each circuit component.

b. If the battery has a total stored energy of 800 W hr, and produce a constant potential difference until discharged, how long will the bulbs stay lit?

pic 1

The potential difference across the car battery is given as 12 V. This means that the electric potential in the wire coming out of the "top" of the battery is 12 V larger than the potential in the wire coming from the "bottom". Since each of the resistors are attached to these same two wires, the top of each resistor is 12 V higher in potential than the bottom. Therefore the potential difference across each resistor is 12 V. When circuit elements are connected such that the elements all have the same potential difference, the elements are said to be in *parallel*.

Since the potential difference across each resistor is known, we can use the definition of resistance to calculate the current through each branch of the circuit. Analyzing branch #1 yields

pic 2

and branch #2

pic 3

The current that flows through R_1 and the current that flows through R_2 must also flow through both the top and bottom wires connected to the battery. To complete the mental image of a closed circuit of current, we will say the current flows "through" the battery as well, although this is not technically true. Therefore, the current that flows through the battery (the total current flowing in the circuit) is:

pic 4

We can summarize this information in a simple table:

	V_{across} (V)	$i_{through}$ (A)
battery	12	12.0
R_1	12	6.32
R_2	12	5.71

To determine how long the headlights will stay lit, we must calculate the total power of the circuit (the total amount of electrical energy converted per second). We can do this separately for each headlight and then add the results:

pic 5

and

pic 6

so the total power of the circuit is:

pic 7

Therefore, the battery will last for

pic 8

Capacitor Properties

Imagine a pair of long, hollow nested cylinders of inner radius a and outer radius b. Calculate the capacitance, per meter, for these nested cylinders.

pic 9

Since capacitance is defined by the relation

pic 10

we need to determine the potential difference that would develop between these cylinders if charges Q (and -Q) where placed on the two surfaces.'To do this, imagine that a charge +Q (per meter) was placed on the inner cylinder. Using Gauss' Law, this leads to an electric field between the cylinders of:

pic 11

This field is directed radially away from the central axis of the cylinders.

Once the electric field between the cylinders is known, the magnitude of the potential difference between the cylinders can be calculated by:

pic 12

Substituting this result into definition of capacitance yields:

pic 13

Thus, the capacitance permeter of a set of nested cylinders depends on the natural logarithm of the ratio of the cylinder radii. Notice that is the cylinders are very close together (b is not much larger than a), the capacitance is very large. The capacitance of a capacitor is always enhanced by having the two charged surfaces very close together. However, as the surfaces get closer together, the possibility of electrical breakdown (charges "jumping" across the gap) becomes larger. For this reason, and several others, the space between the surfaces in a capacitor is typically filled with a type of material, called a *dielectric*, which both enhances the capacitance of the capacitor and inhibits electrical breakdown.

Capacitors in Circuits

The device at right represents a simplified camera flash circuit. With V = 3 V and R = 100 , find C such that the flash reaches 80% of its final voltage in 1.0 s.

pic 14

The circuit above, termed an *RC circuit*, can best be analyzed by considering the changes in electric potential experienced by a hypothetical charge "journeying" around the circuit:

- as it "passes through" the battery the potential increases by V,
- as it passes through the resistor the potential decreases by

pic 15

- and as it "passes through" the capacitor the potential decreases by

pic 16

Putting these changes in potential together results in:

pic 17

Note that the total change in potential (and potential energy) must be zero since the energy given to the charge by the battery is partially converted by the resistor and partially stored by the capacitor.

If we take a time derivative of the above equation (noting that V, R, and C are constants, but that Q, the charge on the capacitor, is changing) we are left with a differential equation for the current in the circuit:

pic 18

This equation says that the time derivative of the cureent is equal to the product of the current and the numerical factor pic 19. The only mathematical function that has the property that its derivative is proportional to itself is the exponential function. Therefore, the current must be given by the function:

pic 20

where i_0 is the current at t = 0 s.

If we assume that the capacitor is uncharged when the switch is first closed, then

pic 21

so the final expression for the current in the circuit as a function of time is:

pic 22

Using this expression we can determine the time-dependence of any other circuit parameter.

For example, the question asks about the voltage across the capacitor. Since the voltage across the resistor can be expressed as:

pic 23

the voltage across the capacitor is the amount of the source voltage that "remains":

pic 24

This function shows that after a long time ($t \rightarrow \infty$), the voltage across the capacitor will equal the voltage of the source.

Therefore,

pic 25

Thus, a 6.21 mF capacitor will reach 80% of its final voltage in 1.0 s.

Paul D'Alessandris (Monroe Community College)

03. Analysis Tools 2

Inductor Properties

Imagine a pair of long, hollow nested wires of inner radius a and outer radius b, designed to carry current into and out of the page. Calculate the inductance, per meter, for these nested wires.

pic 1

Since inductance is defined by the relation

pic 2

we need to determine the flux that would develop between these wires if current i (and -i) flowed along the two wires:

To do this, imagine that current i flowed out of the page along the inner wire. Using Ampere's Law, this leads to a magnetic field between the cylinders of:

pic 3

To help calculated the flux between the wires, the diagram at right is a top view of the nested wires. The dashed area is the area over which we will calculate the flux. (The current along the inner wire flows toward the top of the page, resulting in magnetic field pointing directly out of the page in the area of interest.)

The shaded sliver is the differential element, located a distance r from the center of the wires, with thickness dr and length l. The magnetic flux is then:

pic 4

Substituting this result into the definition of inductance yields:

pic 5

The inductance per meter is then:

pic 6

Thus, the inductance per meter of a set of nested wires depends on the natural lograithm of the ratio of the wire radii. Notice that if the wires are very far apart, the inductance is larger. However, as the wires get farther apart, the size of the device gets larger and may become impractical. For this reason, and several others, the space containing the magnetic flux in an inductor is typically filled with a material with a high magnetic permeability, like iron, in order to "concentrate" the magnetic flux into a smaller region of space.

Inductors in Circuits

The device below represents a simplified electromagnet. With V = 100 V and R = 15 W, find L such that the current reaches 5.0 A in 0.5 s.

pic 7

The circuit above, termed an *RL circuit*, can best be analyzed by considering the changes in electric potential experienced by a hypothetical charge "journeying" around the circuit:

- as it "passes through" the battery the potential increases by V,
- as it passes through the resistor the potential decreases by

pic 8

- and as it "passes through" the inductor's potential changes by

pic 9

Since by Faraday's Law of Induction,

pic 10

The emf induced by the inductor is the potential drop across it, so

pic 11

Putting these changes in potential together results in:

pic 12

Again, note that the total change in potential (and potential energy) must be zero since the energy given to the charge by the battery is partially converted by the resistor and partially stored by the inductor.

If we take a time derivative of the above equation (noting that V, R, and L are constant) we are left with a differential equation for the current in the circuit:

pic 13

This equation says that the time derivative of the *derivative* of the current is equal to the product of the *derivative* of the current and a numerical factor. This means that the derivative of the current must be exponential function. Therefore, the derivative of the current must be given by the function:

pic 14

where A is an arbitrary constant. Integrating this result leads to a current of the form:

pic 15

where B and D are arbitrary constants.

To determine these constants, consider the current in the circuit after a very long time ($t \rightarrow \infty$). After this amount of time the circuit will have reached an equilibrium value, so the change in the current will be zero. Thus,

pic 16

Therefore,

pic 17

Now consider the current in the circuit the instant you first close the switch ($t \rightarrow 0$). At this instant, no current can be flowing in the circuit. This is because if there was current flowing instantaneously after the switch was closed, this would be discontinuous change in current and the inductor would create an infinite emf to oppose this "infinite" increase in current. Therefore,

pic 18

Now that we know the values of the two constants, the final expression for the current in the circuit as a function of time is:

pic 19

Using this expression we can determine the time-dependence of any other circuit parameter.

Since the question asks about the current directly,

pic 20

Therefore, if the electromagnet has an inductance of 5.4 H, it will take 0.5 s for the current to rise to 5.0 A.

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04. Activities

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CHAPTER OVERVIEW

8: Electromagnetic Waves

Topic hierarchy

Paul D'Alessandris (Monroe Community College)

01. Concepts and Principles

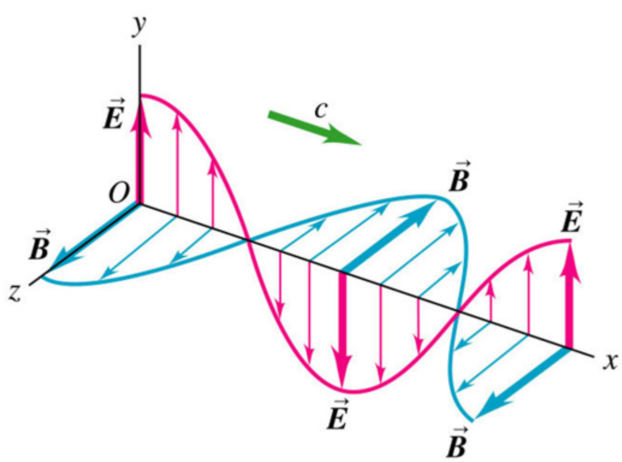
Putting it All Together

Electric charges are surrounded by electric fields. When these charges move, the electric field changes. Additionally, these moving charges (or, more importantly, these changing electric fields) create magnetic fields. Since before the motion no magnetic field existed, this is a change in the value of the magnetic field (from zero to non-zero). By electromagnetic induction, this change in magnetic field creates emf, which is a change in the value of the electric field in a region of space. But this change in electric field must cause further change in the magnetic field, which must cause further change in the electric field, which must...

Sorting through this interrelationship between electric and magnetic fields, which involves the simultaneously solution of a set of coupled, partial-differential equations, is the grand accomplish of James Maxwell. This work of Maxwell's, in the 1860s, is generally regarded as on par with the work of Isaac Newton and Albert Einstein. Although we won't actually solve this set of equations, we will study several of the consequences of Maxwell's work.

Electromagnetic Waves

By far the most important conclusion from Maxwell's work is that the changes in electric and magnetic fields, coupled together as described above, progagate through space as an electromagnetic wave. Near the moving charge that created the wave, the mathematical description of the wave is very complicated, but once the wave has moved a "reasonable" distance from the charge that created it, it can be visualized as below:



and mathematically represented as:

$$\vec{E} = E_{max}\cos(\frac{2\pi x}{\lambda} - 2\pi f t)\hat{j}$$
$$\vec{B} = B_{max}\cos(\frac{2\pi x}{\lambda} - 2\pi f t)\hat{k}$$

where

- l is the wavelength of the wave, the distance between maximum values of the field vectors,
- f is the frequency of the wave, the number of complete cycles of oscillation that wave passes through per second,
- the electric and magnetic field vectors are perfectly in phase, meaning they pass through their maximum and minimum values at the same points in space and time,
- the field vectors are oriented at 90° from each other,
- and the wave propagates in the directon, given by $\vec{E} \times \vec{B}$, which in this case is the +x-direction.

Of course, a similar mathematical description could be given for an electromagnetic wave propagating in any direction.

It's important to realize that the "picture" of the wave above is for a specific instant of time. This is a *traveling* wave, in that it moves through space, in this case in the +x-direction. Since it moves through space, an important characteristic of the wave is its speed. While solving the set of equations, Maxwell found that the speed of electromagnetic waves, denoted c, is given by the expression:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

where e_0 and m_0 are the familiar permittivity and permeability of free space.

What is somewhat odd about this result is that it states that the speed of the wave is a universal constant, since both e_0 and m_0 are constant. This means that regardless of the wavelength of the wave, or its frequency, or whether the charges that created the wave were wiggling in a stationary radio antenna on top of a mountain or in the headlight of a spaceship moving at a billion miles per hour (or if you viewed the wave from a second spaceship moving a billion mph in the other direction!) the wave always moves at exactly the same speed. *The wave speed is completely independent of how it was created.* Once the wave is created, its motion through space is completely determined by how its electric and magnetic fields interact with each other, and its speed has no relationship to the physical charge that created it. (This fact will have some curious repercussions about 40 years after Maxwell's discovery.)

Plugging in the known values for e_0 and m_0 yields:

$$c = 3.0x10^8 m/s$$

Maxwell realized that this was the known speed for visible light, and soon came to the conclusion that light was, in fact, an electromagnetic wave. Electromagnetic waves of other frequencies, such as radio waves, microwaves, x rays, etc, all propagate at this same speed and obey the same mathematical framework described above.

One final interesting conclusion from Maxwell's solution was that the ratio of the electric field to the magnetic field at any point in the wave has a constant value, equal to the wave speed:

$$\frac{E}{B} = c$$

Energy of Electromagnetic Waves

Energy can be stored in electric and magnetic fields, as you learned when you analyzed capacitors and inductors in electric circuits. Since electromagnetic waves involve the propagation of changes in these fields, they should involve the flow of energy in the direction of the wave's motion.

The *intensity*, or energy flow per unit area, of an electromagnetic wave is given by:

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

where pic 8 is termed the *Poynting vector*, and indicates both the direction of the energy flow and its magnitude per unit area. Since the electric and magnetic fields are oriented at 90° from each other, the magnitude of the intensity is given by:

$$S = \frac{EB}{\mu_0}$$

This expression gives you the intensity at any point in the electromagnetic wave at one specific instant of time. However, in many ways this isn't particularly useful. If you are dealingwith a visible light wave, for example, the frequency of the wave is a approximately 10^{14} cycles per second. This means that the intensity of the wave cycles through maximum and minimum values every 10^{-14} s!

A much more useful expression would be for the *average* intensity of the wave. To find this expression, rearrange

$$\frac{E}{B} = c$$

$$B = \frac{E}{c}$$

and substitute into the expression for the instantaneous intensity

$$S = \frac{EB}{\mu_0}$$
$$S = \frac{E^2}{c\mu_0}$$

Substituting our description of the electric field-portion of an electromagnetic wave

$$E = E_{max} \cos(\frac{2\pi x}{\lambda} - 2\pi f t)$$

into the above expression yields

$$S = \frac{E^2}{c\mu_0}$$
$$S = \frac{(E_{max} \cos(\frac{2\pi x}{\lambda} - 2\pi f t))^2}{c\mu_0}$$
$$S = \frac{E_{max}^2 \cos^2(\frac{2\pi x}{\lambda} - 2\pi f t)}{c\mu_0}$$

To find the average value of this function we need only to find the average value of the cosine-squared function, since all of the other terms are constant. It's common knowledge[1] that the average value of the cosine-squared function is ½. Therefore, the average intensity of an electromagnetic wave is:

$$\overline{S} = \frac{E_{max}^2}{2c\mu_0}$$

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02. Concepts and Principles 2

Momentum of Electromagnetic Waves

Just as electromagnetic waves involve the flow of energy, they also carry momentum in the direction of motion. Although it may seem confusing that mass-less, non-material fields, or at least changes in these fields, can have momentum, the momentum carried by electromagnetic waves can be directly measured. The long tail of a comet, pointing directly away from the sun, is a result of the momentum carried by the sun's electromagnetic waves.

The simplest phenomenon directly involving the momentum of electromagnetic waves occurs when the waves are absorbed or scattered from an object. Since the wave has momentum, some of this momentum is transferred to the object the wave "collides" with. This interaction can be directly studied by applying conservation of momentum, but an easier shortcut involves defining the concept of *radiation pressure*. Radiation pressure is the force per unit area that the wave exerts on the object during the collision. Radiation pressure is defined as:

pic 15

where a is a number between 1 and 2 that depends on the portion of the electromagnetic wave absorbed during the collision. If the wave if fully absorbed, a = 1. If the wave is perfectly reflected, a = 2. (We won't analyze situations between these two extremes.)

Polarization of Electromagnetic Waves

In many cases, the electric field component of an electromagnetic wave oscillates in a well-defined direction. When this is the case, the wave is said to be *polarized*. The polarization direction is the direction in which the electric field oscillates. For example, in the diagram below the wave is polarized in the y-direction.

pic 16

Certain materials allow only one specific directoin of electric field vector to progagate through them. These materials, called *polarizers*, typically absorb (or reflect) all waves with electric field vectors not aligned with their transmission axis. For example, a polarizer with transmission axis along the +x-direction would completely block the propagation of the wave illustrated above.

If the incident electric field vectors is not perfectly aligned with the transmission axis, only the component of the field vector along the transmission axis can pass through the polarizer. Thus the electric field magnitude that passes through the polarizer, E, is given by:

pic 17

where

- E_0 is the electric field magnitude incident on the polarizer,
- and q is the angle between the incident polarization direction and the transmission axis.

After passing through the polarizer, the electromagnetic wave is now polarized along the transmission axis.

Rather than concentrating on the electric field magnitude, in most cases its more useful to focus on the intensity of the wave. Since the average intensity of the wave is given by

pic 18

the intensity is proportional to the square of the electric field vector. Thus, a more useful version of the polarization equation is:

pic 19

where

- S_0 is the intensity incident on the polarizer,
- and q is the angle between the incident polarization direction and the transmission axis.

Interference of Electromagnetic Waves

Since electromagnetic waves consist of alternating electric and magnetic field vectors, if two or more waves pass through the same point in space (at the same time) their field vectors must add according to the basic principles of vector addition. This combination of waves through the vector addition of their field vectors is called *interference.*

In general, the interference of two electromagnetic waves can be incredibly difficult to analyze. We will restrict ourselves to a specific sub-class of interference phenomenon, in which there are only two sources of waves and the two sources produce waves of exactly the same frequency perfectly in phase. Typically, the easiest way to accomplish this is by having a single source of waves and then somehow dividing the waves and sending each portion of the wave along a separate path to a final common location. (This is a lot simpler than it sounds.)

The interference that results when the two waves are recombined depends only on the *path length difference*, Dd, the waves traveled to reach the recombination location. The path length difference is, as the name implies, simply the difference between the distance traveled by wave #1 and the distance traveled by wave #2 between separation and recombination,

pic 20

If this difference is exactly equal to zero, one wavelength, or any integer number of wavelengths, the waves will be perfectly in phase when they recombine and exhibit *constructive interference*, resulting in a locally maximum value for the electric field vector. Therefore, for constructive interference:

pic 21

where m is zero or any integer.

If the path length difference is exactly equal to one-half a wavelength, or any half-integer number of wavelengths, the waves will be perfectly out of phase when they recombine and exhibit *destructive interference*, resulting in a locally minimum value for the electric field vector. Therefore, for destructive interference:

pic 22

where m is zero or any integer.

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03. Analysis Tools

Energy, Power and Intensity

Future lunar colonists may want to watch their favority earthly TV shows. Suppose a television station on Earth has a power of 1.0 MW and broadcasts isotropically.

1. *a. What is the intensity of this signal on the moon?*
2. *b. What total power would be received by a 20 cm radius "satellite" dish on the moon?*
3. *c. What is the maximum electric field strength of this signal on the moon?*

If the signal is broadcast isotropically (the same in all directions), its intensity should decrease as the surface area of the sphere over which the signal spreads increases. Since the distance to the moon is 3.82 x 10^8,

pic 1

The power received by the dish would be

pic 2

This corresponds to a maximum electric field of

pic 3

Pressure

High-power lasers are used to compress a small hydrogen pellet in an attempt to initiate fusion. A laser generating pulses of radiation of power 1.5 GW is focused isotropically onto the 0.5 mm radius pellet. What is the pressure exerted on the pellet if the pellet absorbs the incident light?

When an electromagnetic wave is absorbed by a surface, it exerts a pressure on the surface given by

pic 4

The intensity of the light on the surface of the pellet is

pic 5

Thus, the pressure on the pellet is

pic 6

Polarization

Initially unpolarized light is sent through three polarizing sheets with transmission axes oriented at q_1 = 0°, q_2 = 45°, q_3 = 90° measured counterclockwise from the x-axis. What percentage of the initial intensity is transmitted by the system of the three sheets?

If polarized light of intensity S_0 is incident on a polarizing sheet, the intensity transmitted by the sheet depends on the angle between the polarization direction of the light and the transmission axis of the sheet, q, via

pic 7

But what if the incident light is *unpolarized?*

If the incident wave is unpolarized, we can imagine that it is a combination of waves of all possible polarizations. Then, to determine the intensity that passes through the polarizer we can average over all of these different hypothetical polarization directions. Since the intensity that passes through the polarizer depends on the square of the cosine function, this means we must find the average value of the cosine-squared function. Since it's common knowledge[1]

that the average value of the cosine-squared functions is ½, the intensity after the first sheet is one-half of the initial intensity:

pic 8

and the wave is now polarized along the x-axis.

After the second sheet,

pic 9

and the wave is now polarized at 45° from the x-axis.

After the third sheet,

pic 9

and the wave is now polarized along the y-axis. (Notice that qis ***not*** the angle of the transmission axis (90°), but rather the angle *between* the previous polarization direction and the transmission axis.) Thus, 12.5% of the initial light intensity passes through the three sheets and the resulting light is vertically polarized.

Two Source Interference

Two radar sources are separated by d = 10 m. During testing, the two sources broadcast a test frequency perfectly in phase. If destructive interference occurs at x = 40 m, what are the possible values for the source wavelength?

pic 10

Destructive interference occurs when the two waves have a relative phase difference of p (one wave is "flipped over" relative to the other). Since the two waves are emitted in phase, this phase difference must be due to the different distances the waves travel to reach the point of destructive interference. For their phase difference to equal p, their path length difference must be a half-integer multiple of their wavelength.

With the top source labeled #1 and the bottom source #2:

pic 11

For destructive interference,

pic 12,13,14,15

The source could have any of the above (or many other) wavelengths.

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04. Analysis Tools 2

Double Slit Interference

Monochromatic green light of wavelength 550 nm illuminates two parallel narrow slits d = 8.00 apart. If the interference pattern is projected on a screen D = 5.0 m from the slits, where are the bright fringes (constructive maximas) located on the screen?

pic 1

Constructive interference occurs when the two waves have a relative phase difference of 0°. Since the two waves are emitted in phase, any phase difference must be due to the different distances the waves travel to reach the various points on the screen. For their phase difference to be 0°, their path length difference must be an integer multiple of their wavelength.

We could proceed exactly as in the previous example and directly calculate the path length difference between the two waves that recombine on the screen. However, since the distance to the screen (D) is much, much larger than the separation between the slits (d) we can make use of a simplifying observation.

Let's imagine we are interested in the location on the screen indicated below.

pic 2

Although the paths from each slit toward the scren are not precisely parallel, if the drawing was to scale they would be *extremely* close to parallel. Below is a blow-up of the region near the slits:

pic 3

By adding a line perpendicular to the two parallel paths, and noting that this line makes the same angle q with the vertical that the paths make with the horizontal, the path length difference between these two paths is just the small distance indicated in the diagram below:

pic 4

so

pic 5

where

pic 6

from the diagram on the previous page. Although these results are only approximate, when D >> d they are very useful.

Since we are looking for constructive interference in this example,

pic 7

These angles correspond to

pic 8

Thin Film Interference

A camera lens with index of refraction 1.40 is coated with a thin transparent film of index of refraction 1.20 to eliminate the reflection of blue light (l = 480nm) normal to its surface. What is the minimum thickness of the film?

If blue light is not reflected from this coated camera lens, this means that waves that reflect from the *front* of the lens coating (the air-coating interface) and waves that reflect from the *rear* of the lens coating (the coating-lens interface) destructively interface for l = 481 nm light. These waves may have a difference in phase for two, independent reasons.

First, when a wave from reflects from a material having a larger index of refracton than the medium it is currently traveling through, it will reflect with a complete phase inversion (i.e., the wave will "flip over" when reflecting from something with a larger index).

Second, the wave reflecting from the rear of the coating will have traveled a greater distance than the wave reflecting from the front of the coating. This difference in path length will also cause a phase shift between the two waves.

For this particular example, both the front-reflecting wave and rear-reflecting wave reflect from surfaces having a larger index than they currently are traveling through, so both waves are "flipped" upon reflection. Since both waves are flipped, this effect will not influence the relative phase of the two waves.

Calling the thickness of the coating d, the rear-reflecting wave travels a distance 2d further than the front-reflecting wave. Thus,

pic 9

If this distance is equal to one-half (or three-halves, etc.) wavelength, the reflected interference will be destructive.

pic 10

However, there is one last complication. The wavelength of the light *in the coating* is not equal to the wavelength of the light in air, and 2d must equal one-half of the wavelength of the light in the coating for destructive reflection. The wavelength of light in a material other than vacuum is given by:

pic 11

Combining this with the previous relationship yields

pic 12

Thus, the coating must have a minimum thickness of 100 nm.

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05. Activities

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