04. Analysis Tools 2

Double Slit Interference

Monochromatic green light of wavelength 550 nm illuminates two parallel narrow slits d = 8.00 apart. If the interference pattern is projected on a screen D = 5.0 m from the slits, where are the bright fringes (constructive maximas) located on the screen?

pic 1

Constructive interference occurs when the two waves have a relative phase difference of 0°. Since the two waves are emitted in phase, any phase difference must be due to the different distances the waves travel to reach the various points on the screen. For their phase difference to be 0°, their path length difference must be an integer multiple of their wavelength.

We could proceed exactly as in the previous example and directly calculate the path length difference between the two waves that recombine on the screen. However, since the distance to the screen (D) is much, much larger than the separation between the slits (d) we can make use of a simplifying observation.

Let's imagine we are interested in the location on the screen indicated below.

pic 2

Although the paths from each slit toward the scren are not precisely parallel, if the drawing was to scale they would be *extremely* close to parallel. Below is a blow-up of the region near the slits:

pic 3

By adding a line perpendicular to the two parallel paths, and noting that this line makes the same angle q with the vertical that the paths make with the horizontal, the path length difference between these two paths is just the small distance indicated in the diagram below:

pic 4

so

pic 5

where

pic 6

from the diagram on the previous page. Although these results are only approximate, when D >> d they are very useful.

Since we are looking for constructive interference in this example,

pic 7

These angles correspond to

pic 8

Thin Film Interference

A camera lens with index of refraction 1.40 is coated with a thin transparent film of index of refraction 1.20 to eliminate the reflection of blue light (l = 480nm) normal to its surface. What is the minimum thickness of the film?

If blue light is not reflected from this coated camera lens, this means that waves that reflect from the *front* of the lens coating (the air-coating interface) and waves that reflect from the *rear* of the lens coating (the coating-lens interface) destructively interface for l = 481 nm light. These waves may have a difference in phase for two, independent reasons.

First, when a wave from reflects from a material having a larger index of refracton than the medium it is currently traveling through, it will reflect with a complete phase inversion (i.e., the wave will "flip over" when reflecting from something with a larger index).

Second, the wave reflecting from the rear of the coating will have traveled a greater distance than the wave reflecting from the front of the coating. This difference in path length will also cause a phase shift between the two waves.

For this particular example, both the front-reflecting wave and rear-reflecting wave reflect from surfaces having a larger index than they currently are traveling through, so both waves are "flipped" upon reflection. Since both waves are flipped, this effect will not influence the relative phase of the two waves.

Calling the thickness of the coating d, the rear-reflecting wave travels a distance 2d further than the front-reflecting wave. Thus,

pic 9

If this distance is equal to one-half (or three-halves, etc.) wavelength, the reflected interference will be destructive.

pic 10

However, there is one last complication. The wavelength of the light *in the coating* is not equal to the wavelength of the light in air, and 2d must equal one-half of the wavelength of the light in the coating for destructive reflection. The wavelength of light in a material other than vacuum is given by:

pic 11

Combining this with the previous relationship yields

pic 12

Thus, the coating must have a minimum thickness of 100 nm.

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