03. Analysis Tools 2

Ampere's Law

The long, hollow-core wire below has inner radius a, outer radius b and current i uniformly distributed across its area. The current flows in the +z-direction. Find the magnetic field at all points in space.

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For certain situations, typically ones with cylindrical symmetry, Ampere's Law allows you to calculate the magnetic field relatively easily. Ampere's Law, mathematically, states:

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Let's describe what this means in English. The left side of the equation involves the vector dot product between the magnetic field and an infinitesimally small length that is a piece of a larger closed path (termed the *amperian loop*). This dot product determines the amount of magnetic field that is parallel to this very small piece of a larger closed path. The integral simply tells us to add up all of these contributions around the entire closed path.

The gist of Ampere's Law is that this integral is exactly equal to the total amount of electric current flowing within the amperian loop, multiplied by the constant m0.

Somewhat counter intuitively, the key to applying Ampere's Law is to choose an amperian loop such that you never really have to do the integral on the left side of the equation! (Does this sound familiar?) To try to help you understand what I'm talking about, let's walk through the solution of the above problem. The following sequence of steps will help you understand the process of applying Ampere's law:

1. *Choose the appropriate amperian loop.*
2. *Carefully draw the hypthetical amperian loop at the location of interest.*
3. *Carefully draw the magnetic field at all points on the amperian loop.*
4. *Write an expression for the path length parallel to the magnetic field.*
5. *Write an expressioln for $i_{enclosed}$, the current inside the amperian loop.*
6. *Apply Ampere's Law and determine the magnetic field at all points on this hypothetical loop.*

There are thee distinct regions we will investigate:

- the region inside the "hole" in the wire($r < a$),
- the region within the actual material of the wire ($a < r < b$),
- and the region outside of the wire ($r > b$).

Let's start outside of the wire.

Outside of the wire: r > b

1. *Choose the appropriate amperian loop.*

The key to using Ampere's Law is to try to exploit the symmetry of the current-carrying wire. Since wires typically have circular cross-section, circular amperian loops are the norm.

2. *Carefully draw the hypothetical amperian loop at the location of interest.*

Since we are trying to determine the magnetic field for all points outside of the wire, draw a circular amperian surface with radius r. The value of r is *variable*, and can take on any value greater than b, the radius of the *real* wire. Remember, the amperian loop is *hypothetical*; it's a mathematical "object" that only exists to help you solve the problem. Try not to confuse it with the real wire of radius b.

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3. *Carefully draw the magnetic field at all points on the amperian loop.*

Although I have no idea what the *magnitude* of the magnetic field is at any point on my amperian loop, the symmetry of the situation tells me that the *direction* of the magnetic field must be either clockwise or counterclockwise. If the current is flowing out of the page, the field will be directed counterclockwise around the amperian loop.

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Moreover, even though I don't know the magnitude of the field, I *do* know that the magnitude is the same at every point on my loop.

4. *Write an expression for the path length parallel to the magnetic field.*

The left side of Ampere's law requires us to evaluate an integral around our amperian loop. The integral requires us to find the dot product between the magnetic field and the differential line element, and integrate this dot product around the entire loop. I mentioned earlier that you should never have to actually do this integral (assuming you chose the "correct" amperian loop). So why don't we have to do this integral?

The vector dot product can be re-written as:

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where is the angle between the magnetic field and the differential line element of the amperian loop. Traversing our amperian loop counterclockwise leads to the diagram at right

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Notice that the magnetic field vector and the vector representing the differential element are parallel at every point on the amperian loop. This simplifies the left-hand side of Ampere's law:

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Now note that the magnitude of the magnetic field is the same at every point on the amperian loop since every point is equal distance from the current distribution. Thus, magnetic field is constant and can be brought outside of the integral, leaving a pretty easy integral to evaluate.

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Notice that the entire left-hand side of Ampere's Law reduces to the product of the magnetic field magnitude and the length of the amperian loop, assuming the loop is parallel to this field. Thus, because of our wise choice of amperian loop, all we really need to calculate is the length of the loop parallel to the magnetic field. For a circular loop:

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5. *Apply Ampere's Law and determine the magnetic field at all points on this hypothetical loop.*

Since the amperian loop is outside of the real wire, *all* of the current flowing through the wire is enclosed by the amperian loop. Thus,

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6. *Apply Ampere's Law and determine the magnetic field at all points on this hypothetical loop.*

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Thus, the magnetic field outside of the hollow-core wire looks exactly the same as the magnetic field outside of a "normal" current-carrying wire.

Now we have to repeat this analysis for the other two regions.

Within the wire: a < r < b

1. *Choose the appropriate amperian loop.*

Again choose a circular amperian loop.

2. *Carefully draw the hypothetical amperian loop at the location of interest.*

Since we are trying to determine the magnetic field within the actual wire, the radius of our amperian loop is greater than a but less than b.

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3. *Carefully draw the magnetic field at all points on the amperian loop.*

As before, the magnitude of the magnetic field must be constant at all points on the amperian loop and directed counterclockwise.

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4. *Write an expression for the path length parallel to the magnetic field.*

The length parallel to the magnetic field is again the length of the circular amperian loop:

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5. *Write an expression for $i_{enclosed}$, the current inside the amperian loop.*

Since the amperian loop is within the wire, not all of the current in the wire is enclosed by the loop. The amount enclosed can be expressed as the product of a *current density* (J) and the cross-sectional area enclosed. First, since the current is uniformly distributed throughout the wire I can define the current density as:

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Then, the current enclosed by the amperian loop is the product of the current density and the area enclosed by the loop:

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6. *Apply Ampere's Law and determine the magnetic field at all points on this hypothetical loop.*

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Thus, the magnetic field inside the hollow-core wire actually increases with increasing r, since as r increases, more and more current is available to produce the magnetic field.

Inside the "hole": r < a

Since we must choose our amperian loop to have a radius less than a, it is located inside the hollow center of the wire. Since there is *no* current enclosed by this loop, the magnetic field in this region must be zero.

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