02. Analysis Tools

Long, Parallel Wires

Find the magnetic field at the indicated point. The long, parallel wires are separated by a distance 4a.

The magnetic field at this point will be the vector sum of the magnetic field from the left wire ($\mathbf{B}_L$) and the magnetic field from the right wire ($\mathbf{B}_R$).

For the left wire, I've indicated the direction of the tangent vector. Remember, with your thumb pointing in the direction of the current (out of the page), the direction in which the fingers of your right hand curl is the direction of the tangent vector (counterclockwise).

pic

By definition, the tangent vector is perpendicular to the radial vector. You should know how to specify the radial unit vector from the chapter on electric field; to specify the tangent unit vector, you have to construct a vector *perpendicular* to the radial vector.

To do this, simply flip the x- and y-components of the radial vector and add the appropriate algebraic signs. (Ask your math professor to prove that this always results in a vector perpendicular to the original vector.) Using this trick, the magnetic field from the left wire is:

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For the right wire your thumb should point into the page, making your right-hand fingers curl clockwise.

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Adding these two contributions together yields

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More General Current Distribution

Find the magnetic field at the origin. The wire forms a circle of radius R.

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Finding the magnetic field from a current-carrying wire involves several distinct steps. Until you become very comfortable setting up and evaluating magnetic field integrals, I would suggest you systematically walk through these steps.

1. *Carefully identify and label the location of the differential element on a diagram of the situation.*
2. *Carefully identify and label the location of the point of interest on a diagram of the situation.*
3. *Write an expression for (pic), the vector differential element.*
4. *Write an expression for r, the distance between the differential element and the point of interest.*
5. *Write an expression for (r hat), the unit vector representing the direction from the element to the point of interest.*
6. *Insert your expressions into the integral for the magnetic field.*
7. *Carefully choose the limits of integration.*
8. *Evaluate the integral.*

I'll demonstrate each of these steps for the scenario under investigation.

1. *Carefully identify and label the location of the differential element on a diagram of the situation.*

The differential element is a small (infinitesimal) piece of the current-carrying wire. The location of this differential element must be *arbitrary*, meaning it is not at a "special" location like the top or bottom of the loop. Its location must be represented by a variable, where this variable is the variable of integration and determines the limits of the integral.

For this example, select the differential element to be located at an angle "q" counter-clockwise from the x-axis. (Later, you will select the limits of integration to go from 0 to 2p to allow this arbitrary element to "cover" the entire loop.)

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2. *Carefully identify and label the location of the point of interest on a diagram of the situation.*

The point of interest is the center of the loop of wire.

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3. *Write an expression for (pic), the vector differential element.*

The differential element in the integral for magnetic field is a vector, meaning it has both a magnitude (its length) and a direction. The direction of the differential element is the direction in which the current is flowing through the element. Thus, we need an expression for the vector illustrated at right (and greatly magnified below).

The length of the differential element is Rdq, since the element forms an arc on a circle of radius R. Its direction can be determined by noting that it makes an angle of q with respect to the y-axis, and is directed in the -x-direction and +y-direction. This results in:

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4. *Write an expression for r, the distance between the differential element and the point of interest.*

The distance between the differential element and the point of interest is just the radius of the loop;

pic

5. *Write an expression for , the unit vector representing the direction from the element to the point of interest.*

Since this vector points *from* the element *to* the point of interest, it is directed in the -x-direction and -y-direction. This results in:

pic

pic

6. *Insert your expressions into the integral for the magnetic field.*

pic

7. *Carefully choose the limits of integration.*

The limits of integration are determined by the range over which the differential element must be "moved" to cover the entire object. In this case, the element must move all the way around the circular loop:

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8. *Evaluate the integral.*

pic

The magnetic field at the center of any current-carrying loop of radius R is given by the expression above.

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