2.4: Conservation Laws

Concepts and Principles

What is a Conservation Law?

In general, a conservation law is a statement that a certain quantity does not change over time. If you know how much of this quantity you have today, you can be assured that the exact same amount of the quantity will be available tomorrow. A famous (at least to physicists) explanation of the nature of a conservation law was given by Richard Feynman.

> *Imagine your child has a set of 20 wooden blocks. Every day before bedtime you gather up your child's blocks to put them away. As you gather up the blocks, you keep count in your head. Once you reach 20, you know you have found all of the blocks and it is unnecessary for you to search any longer. This is because the number of blocks is conserved. It is the same today as it was yesterday.*
>
> *If one day you only find 18 blocks, you know to keep looking until you find the missing 2 blocks. Also, with experience, you discover the typical hiding places for the blocks. You know to check under the sofa, or behind the curtains.*
>
> *If your child is rambunctious, you may even have to look outside of the room. Perhaps he threw a block or two out of the window. Even though blocks can disappear from inside of the room, and appear out in the yard, if you search everywhere you will always find the 20 blocks.*

Physicists have discovered a number of quantities that behave exactly like the number of wooden blocks. We will examine two of these quantities, energy and momentum, below.

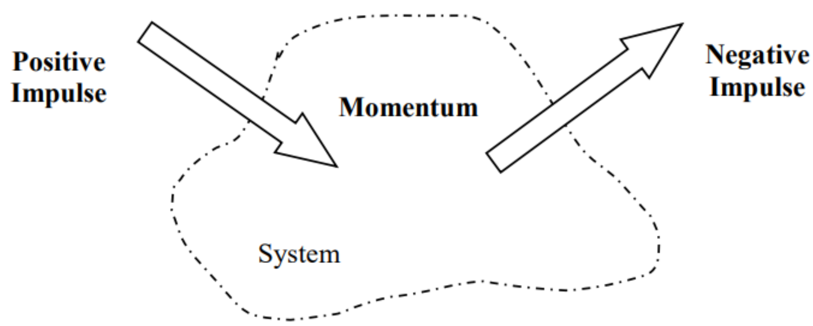
The Impulse-Momentum Relation

While Newton's Second Law directly relates the total force that acts on an object at a specific time to the object's acceleration at that exact same time, conservation laws relate the amount of a certain quantity present at one time to the amount present at a later time.

The first conserved quantity we will investigate is *momentum*. Of course, just because momentum is conserved doesn't mean that the momentum of any particular object or system of objects is always constant. The momentum of a single object, like the number of blocks in the playroom, can change. Just as blocks can be thrown out of the window of the playroom, the momentum of a single object can be changed by applying *impulse* to it. The relationship between impulse and momentum is, conceptually,

$$\text{initial momentum} + \text{impulse} = \text{final momentum}$$

Pictorially, we can visualize this as



In practice, we will identify an object or collection of objects (a *system*) and determine the amount of momentum the system contains at some initial time. This quantity cannot change unless impulse is done to the system. We call processes that bring momentum into the system as positive impulses, and processes that remove momentum from the system as negative impulses.

Mathematically this is written as

$$\text{initial momentum} + \text{impulse} = \text{final momentum}$$
$$P_i + J_{if} = P_f$$
$$\Sigma m v_i + \Sigma F(\Delta t) = \Sigma m v_f$$

where

- momentum (P) is the product of an object's mass and velocity,
- impulse (J) is the product of a force *external to the system* and the time interval over which it acts,
- and Σ indicates that you must sum the momentum of all of the objects in the system and all of the impulses acting on the system.

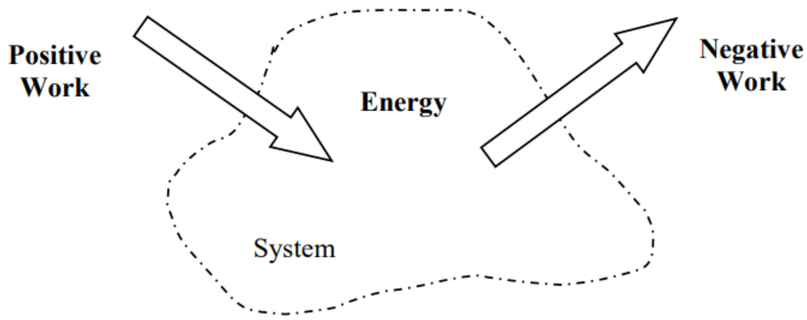
In short, if no impulse is applied to a system, its momentum will remain constant. However, if an impulse is applied to the system, its momentum will change by an amount exactly equal to the impulse applied. This momentum does not appear or disappear without a trace. It is simply transferred to the object *supplying* the impulse. In this sense, impulse is the transfer of momentum into or out of a system, analogous to tossing blocks into or out of a playroom.

The Work-Energy Relation

The second conserved quantity we will investigate is *energy*. Just like momentum, or wooden blocks, the conservation of energy doesn't mean that the energy of any particular object is always constant. The energy of a single object or system of objects can be changed by doing *work* to it. The relationship between work and energy is, conceptually,

$$\text{initial energy } + \text{ work } = \text{ final energy}$$

Pictorially, we can visualize this as



The similarity between momentum and energy is not complete, however. While there is only one form of momentum (i.e., one hiding place for momentum "blocks") there are several forms of energy. These different forms of energy will be introduced as you progress through more and more complicated models of the physical world. For now, the only "hiding place" I want to discuss is *kinetic energy*. In terms of kinetic energy, the above conceptual relationship between work and energy becomes, expressed mathematically,

$$\begin{gathered}\text{initial energy } + \text{ work } = \text{ final energy}\\ KE_i + W_{if} = KE_f\\ \Sigma\frac{1}{2}mv_i^2 + \Sigma|F||\Delta r|\cos\phi = \Sigma\frac{1}{2}mv_f^2\end{gathered}$$

where

- kinetic energy (KE) is the product of one-half an object's mass and squared velocity,
- work (W) is the product of a force *(even an internal force)* and the displacement over which it acts (with more subtle details discussed below),
- Σ indicates that you must sum the kinetic energy of all of the objects in the system and all of the work done to the system,
- and we define a new unit, Joule (J), as J = kg (m/s)^2 = N m

Unlike anything we've studied up to this point, the work-energy relation is a *scalar* equation. This will become especially important when we study objects moving in more than one dimension. For now, all this means is that in the expression for work, $|F||\Delta r|\cos\phi$, we should use the *magnitude* of the force and the *magnitude* of the change in position. This product is then multiplied by $\cos\phi$, where ϕ is defined to be the angle between the applied force and the displacement of the object. If the force and displacement are in the same direction $\phi = 0^\circ$, and the work is positive (the object gains energy). If the force and displacement are in the opposite direction $\phi = 180^\circ$, and the work is negative (the object loses energy). Note that the actual directions of the force and the displacement are unimportant, only their directions *relative to each other* affect the work.

In general, if no work is done to a system, its kinetic energy will remain constant. However, if work is done to system, its total energy will change by an amount exactly equal to the work done. Work is the transfer of energy from one system to another, again analogous to tossing blocks from the playroom into the yard.

Analysis Tools

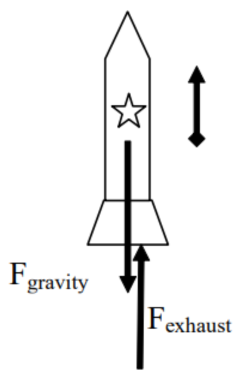
Applying the Impulse-Momentum Relation to a Single Object

Let's investigate the following scenario:

> *A 0.35 kg model rocket is fitted with an engine that produces a thrust of 11.8 N for 1.8 s. The rocket is launched vertically upward.*

To apply the impulse-momentum relation, you must clearly specify the initial and final events at which you will tabulate the momentum. For example:

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches maximum height.
$P_1 = 0$	$P_2 = (0.35)\ v_2$
$J_{12} = +F_{\text{exhaust}}\ (1.8) - F_{\text{gravity}}\ (1.8)$	



Note that each external force acting on the rocket is multiplied by the time interval over which it acts. (Also note that the rocket's engine does not produce a force on the rocket! The engine produces a downward force on the hot exhaust gases emitted from the engine and these hot gases exert an equal magnitude force back up on the rocket. That is why the force on the rocket is labeled as $F_{exhaust}$ rather than F_{engine}.)

Applying impulse-momentum to the rocket during this time interval yields:

$$\begin{aligned} &P_1 + J_{12} = P_2 \\ &0 + F_{\text{exhaust}}\ (1.8) - F_{\text{gravity}}\ (1.8) = 0.35 v_2 \\ &0 + (11.8)(1.8) - (0.35)(9.8)(1.8) = 0.35 v_2 \\ &v_2 = 43.0\ \mathrm{m/s} \end{aligned}$$

Thus, the rocket is traveling at 43.0 m/s at the instant the engine shuts off.

Of course, there is no reason why we had to analyze the rocket's motion between the two instants of time we selected above. We could have selected the events:

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches maximum height.
$P_1 =$	$P_2 = 0$
$J_{12} = +F_{\text{exhaust}}\ (1.8) - F_{\text{gravity}}\ (\Delta t)$	

During this time interval, the force of the exhaust gases only act on the rocket for a *portion* of the entire time interval. Noting that the rocket's velocity when it reaches its maximum height is zero, impulse-momentum would look like this:

$$
\begin{aligned}
&P_1 + J_{12} = P_2 \\
&0 + F_{\text{exhaust}}(1.8) - F_{\text{gravity}}(\Delta t) = 0 \\
&0 + (11.8)(1.8) - (0.35)(9.8)(\Delta t) = 0 \\
&\Delta t = 6.19 s
\end{aligned}
$$

Thus, the rocket is in the air for 6.19 s before reaching its maximum height.

Applying the Work-Energy Relation to a Single Object

The work-energy relation also has many uses for investigating physical scenarios. For example, let's look again at our model rocket:

> *A 0.35 kg model rocket is fitted with an engine that produces a thrust of 11.8 N for 1.8 s. The rocket is launched vertically upward.*

Assuming we've already analyzed this scenario using impulse-momentum, what additional information can we extract using work-energy?

Event 1: The instant the engine is ignited.	Event 2: The instant the engine shuts off.
$KE_1 = 0$	$KE_2 = 1/2\,(0.35)(43)^2$
$W_{12} = F_{\text{exhaust}}\,(\Delta r)\cos 0 + F_{\text{gravity}}\,(\Delta r)\cos 180$	

Therefore,

$$
\begin{aligned}
&KE_1 + W_{12} = KE_2 \\
&0 + |F_{\text{exhaust}}||\Delta r|\cos 0 + |F_{\text{gravity}}||\Delta r|\cos 180 = \tfrac{1}{2}(0.35)(43.2)^2 \\
&0 + 11.8(\Delta r)(1) + (0.35)(9.8)(\Delta r)(-1) = 327 \\
&11.8\Delta r - 3.43\Delta r = 327 \\
&\Delta r = 39.1 \text{ m}
\end{aligned}
$$

Thus, the rocket rises to a height of 39.1 m before the engines shuts off.

What if we apply work-energy between the following two events:

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches its maximum height
$KE_1 = 0$	$KE_2 = 0$
$W_{12} = F_{\text{exhaust}}\,(39)\cos 0 + F_{\text{gravity}}\,(\Delta r)\cos 180$	

During this time interval, the force of the exhaust gases only act on the rocket for a *portion* of the entire displacement, namely 39 m, while the force of gravity acts over the entire displacement.

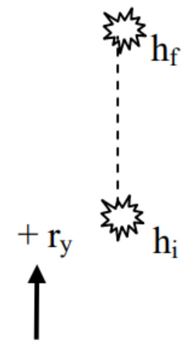
$$
\begin{aligned}
&KE_1 + W_{12} = KE_2 \\
&0 + (11.8)(39)\cos 0 + (0.35)(9.8)(\Delta r)\cos 180 = 0 \\
&0 + 460 - 3.43\Delta r = 0 \\
&\Delta r = 134 m
\end{aligned}
$$

Thus, the maximum height reached by the rocket is 134 m.

Gravitational Potential Energy

In any situation in which an object changes its height above the surface of the earth, the force of gravity does work on the object. It is possible to calculate this work in general, and to rewrite the work-energy relation in such a way as to incorporate the effects of this work. This is referred to as constructing a *potential energy function* for the work done by gravity.

Let's imagine an object of mass, m, located an initial height, h_i, above the zero of a vertical coordinate system, with the upward direction designated positive. It moves to a final height of h_f.



To calculate the work done by gravity on this object:

$$\begin{aligned}W_{\text{gravity}} &= |F||\Delta r|\cos\phi \\ W_{\text{gravity}} &= (mg)\,(h_f - h_i)\cos 180 \\ W_{\text{gravity}} &= -mgh_f + mgh_i\end{aligned}$$

The "mgh" terms are referred to as *gravitational potential energy*. Thus, the work done by gravity can be thought of as changing the gravitational potential energy of the object. Let's plug the above result into the work-energy relation:

$$\begin{aligned}&\tfrac{1}{2}mv_i^2 + \Sigma|F||\Delta r|\cos\phi = \tfrac{1}{2}mv_f^2 \\ &\tfrac{1}{2}mv_i^2 + \Sigma|F||\Delta r|\cos\phi + W_{\text{gravity}} = \tfrac{1}{2}mv_f^2 \\ &\tfrac{1}{2}mv_i^2 + \Sigma|F||\Delta r|\cos\phi - mgh_f + mgh_i = \tfrac{1}{2}m{v_f}^2 \\ &\tfrac{1}{2}mv_i^2 + mgh_i + \Sigma|F||\Delta r|\cos\phi = \tfrac{1}{2}mv_f^2 + mgh_f\end{aligned}$$

Therefore, this final relation:

$$\begin{aligned}&KE_i + GE_i + W_{if} = KE_f + GE_f \\ &\tfrac{1}{2}mv_i^2 + mgh_i + \Sigma|F||\Delta r|\cos\phi = \tfrac{1}{2}mv_f^2 + mgh_f\end{aligned}$$

can (and will) be used in place of the standard work-energy relation provided:

1. You do not include the force of gravity a second time by calculating the work done by gravity. Basically, in this relationship gravity is no longer thought of as a force that does work on objects but rather as a source of potential energy.
2. You calculate the initial and final heights, h_i and h_f, using a coordinate system in which the upward direction is positive.

Applying Work-Energy with Gravitational Potential Energy

Let's use the work-energy relation, with gravitational potential energy terms, to re-analyze the previous scenario:

> *A 0.35 kg model rocket is fitted with an engine that produces a thrust of 11.8 N for 1.8 s. The rocket is launched vertically upward.*

Let's apply work-energy between the following two events, setting the initial elevation of the rocket equal to zero:

Event 1: The instant the engine is ignited.	Event 2: The instant the rocket reaches its maximum height
$KE_1 = 0$	$KE_2 = 0$
$GE_1 = 0$	$GE_2 = (0.35)(9.8)\,h_2$
$W_{12} = F_{\text{exhaust}}\,(39)\cos 0$	

During this time interval, the force of the exhaust gases only act on the rocket for a *portion* of the entire displacement, namely 39 m. Remember, the force of gravity *does not do work* in this way of modeling nature, rather the gravitational energy of the rocket changes as it changes its elevation.

$$KE_i + GE_i + W_{if} = KE_f + GE_f$$
$$0 + 0 + (11.8)(39)\cos 0 = 0 + (0.35)(9.8)h_f$$
$$0 + 0 + 460 = 0 + 3.43h_f$$
$$h_f = 134m$$

results in, of course, the same maximum height reached by the rocket.

Applying the Impulse-Momentum Relation to a Collision

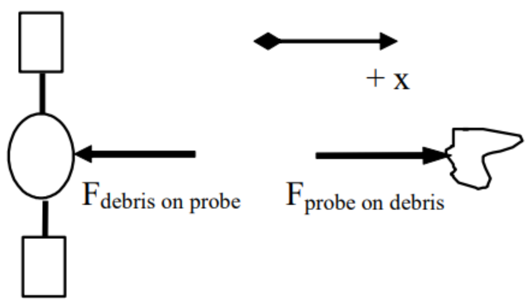
Probably the most useful application of the impulse-momentum relation is in the study of collisions. For example:

> *Far from the earth, a 250 kg space probe, moving at 5 km/s, collides head-on with a 60 kg piece of space debris initially at rest. The debris becomes entangled in the probe's solar collectors.*

Let's choose:

Event 1: The instant before the collision.	Event 2: The instant the debris and probe reach a common velocity.
Object: Space Probe	
P_1 = (250)(5000)	P_2 = 250 v_2
$J_{12} = -F_{\text{debrisonpobe}}\ (\Delta t)$	
Object: Debris	
P_1 = 0	P_2 = 60 v_2
$J_{12} = +F_{\text{probeondebris}}\ (\Delta t)$	

The free-body diagrams for the two objects during this time interval are shown below.



Applying the impulse-momentum relation to each object separately yields:

Probe	*Debris*
$P_1 + J_{12} = P_2$	$P_1 + J_{12} = P_2$
$250(5000) - F_{\text{debrisonprobe}}\ (\Delta t) = 250v_2$	$0 + F_{\text{probeondebris}}\ (\Delta t) = 60v_2$
$1250000 - F_{\text{debrisonprobe}}\ (\Delta t) = 250v_2$	$F_{\text{probeondebris}}\ (\Delta t) = 60v_2$

Notice that the final velocities of the two objects are the same, because they remain joined together following the collision. Also, the Δt's are the same because the time interval over which the force of the debris acts on the probe must be the same as the time interval over which the force of the probe acts on the debris. In fact, these two forces must be equal to each other in magnitude by Newton's Third Law.

Thus, the impulses must cancel if the two equations are added together:

$$1250000 - F_{\text{debrisonprobe}}(\Delta t) = 250 v_2$$
$$\underline{F_{\text{probeondebris}}(\Delta t) = 60 v_2}$$
$$1250000 = 310 v_2$$
$$v_2 = 4032 \text{ m/s}$$

The probe slows to a speed of 4032 m/s (and the debris changes direction and accelerates to a speed of 4032 m/s) via the collision. Thus, even though we do not know the magnitude of the force involved, or the duration of the collision, we can calculate the final velocities of the two objects colliding. This is because the forces involved comprise an interaction, and by Newton's Third Law forces that comprise an interaction are always equal in magnitude and opposite in direction.

In fact, in problems involving collisions (or explosions, which to physicists are merely collisions played backward in time!), you should almost always apply the impulse-momentum relation to the interacting objects because the forces involved comprise an interaction. Thus, by adding your equations together, these terms will always add to zero. This will often allow you to determine the final velocities of the colliding objects.

In conclusion, I should point out that the probe loses momentum during the collision and that the debris gains the exact same amount of momentum. (Check the numbers to verify this statement.) The momentum is transferred from the probe to the debris through the action of the impulse the probe and debris exert on each other. The momentum transfer from the probe to the debris is analogous to throwing a wooden block from the playroom into the yard: The playroom now has one less block and the yard has one more!

Applying the Work-Energy Relation to the Same Collision

Let's return to the collision scenario discussed above and attempt to investigate it using workenergy.

> *Far from the earth, a 250 kg space probe, moving at 5 km/s, collides head-on with a 60 kg piece of space debris initially at rest. The debris becomes entangled in the probe's solar collectors.*

Event 1: The instant before the collision.	Event 2: The instant the debris and probe reach a common velocity.
Object: Space Probe	
$\mathrm{KE}_1 = 1/2\ (250) 5000^2$	$\mathrm{KE}_2 = 1/2\ (250) \mathrm{v}_2{}^2$
$GE_1 = 0$	$GE_2 = 0$
$\mathrm{W}_{12} = F_{\text{on P}}\ (\Delta r_P) \cos 180$	
Object: Debris	
$KE_1 = 0$	$\mathrm{KE}_2 = 1/2\ (60) \mathrm{v}_2{}^2$
$GE_1 = 0$	$GE_2 = 0$
$\mathrm{W}_{12} = F_{\text{on D}}\ (\Delta r_D) \cos 0$	

Applying the work-energy relation to each object separately yields:

Probe	*Debris*
$KE_i + GE_i + W_{if} = KE_f + GE_f$	$KE_i + GE_i + W_{if} = KE_f + GE_f$
$\frac{1}{2}(250)(5000)^2 + (F_{onP})(\Delta r_{\text{probe}}) \cos 180 = \frac{1}{2}(250)(v_{2\text{ probe}})^2$	$0 + (F_{onD})(\Delta r_{\text{debris}}) \cos 0 = \frac{1}{2}(60)(v_{2\text{ debris}})^2$
$3.13 \times 10^9 - F_{\text{onP}}\ (\Delta r_{\text{probe}}) = 125 v_{2\text{ probe}}{}^2$	$F_{\text{onD}}\ (\Delta r_{\text{debris}}) = 30 v_{2\text{ debris}}{}^2$

The final velocities of the two objects are the same, because they remain joined together following the collision, and the two forces are the same by Newton's Third Law. **However, these two equations cannot be added together and solved because the two distances over which the forces act, Δr_{probe} and $\Delta \mathbf{r}_{\text{debris}}$, are not necessarily equal.** During the collision, the center of the probe will move a different distance that the center of the debris[3]. Since these two distances are different, the works will *not* cancel as the impulses did, and the equations are *not* solvable!

> 3 If the two objects were *actually* particles, rather than being *approximated* as particles, then the two distances would have to be the same and the two works would cancel when the equation were added together.

In fact, since we know v_2 = 4032 m/s from our momentum analysis,

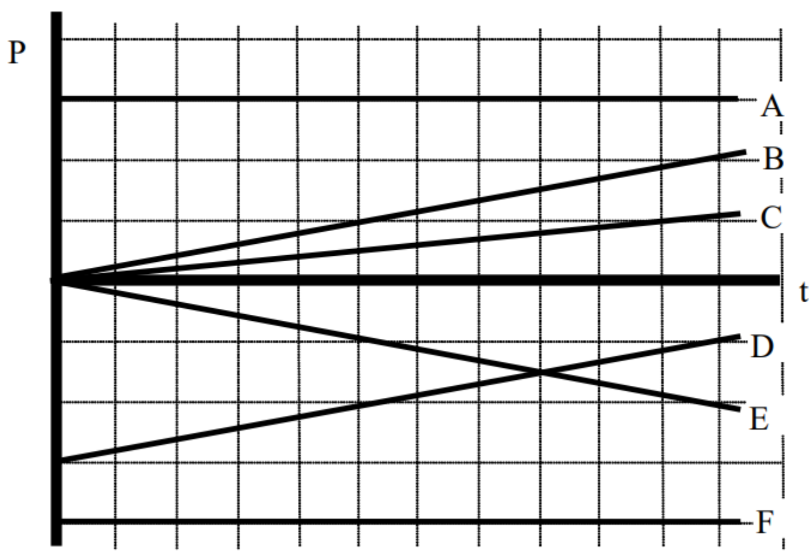
Probe	*Debris*
$3.13 \times 10^9 - F_{\text{onP}}\ (\Delta r_{\text{probe}}) = 125(4032)^2$	$F_{\text{onD}}\ (\Delta r_{\text{debris}}) = 30(4032)^2$
$3.13 \times 10^9 - F_{\text{onP}}\ (\Delta r_{\text{probe}}) = 2.03 \times 10^9$	$W_{\text{ondebris}} = F_{\text{onD}}\ (\Delta r_{\text{debris}}) = 0.49 \times 10^9 J$
$W_{\text{onprobe}} = -F_{\text{onP}}\ (\Delta r_{\text{probe}}) = -1.1 \times 10^9$ J	

Obviously, the two works do not cancel. In fact, the *internal work*, or work done by the objects on each other, totals -0.61 x 10^9 J. This means that there is 0.61 x 10^9 J *less* kinetic energy in the system of the probe and the debris after the collision than before the collision. This is sometimes referred to as the energy lost in the collision, although the energy is not lost but rather converted into other forms of energy (i.e., other hiding places for the wooden blocks that have yet to be discussed), such as thermal energy.

In short, the work-energy relation (as it now stands) cannot be used to effectively analyze collisions unless additional information regarding the internal energy is available. Occasionally, an approximation is made in which the total internal work is zero. When this approximation is made, the collision is referred to as an *elastic* collision. Realistic collisions, in which the total internal energy is not zero and kinetic energy is "lost", are referred to as *inelastic* collisions.

Activities

Below are momentum vs. time graphs for six different objects.



a. Rank these graphs on the basis of the change in momentum of the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

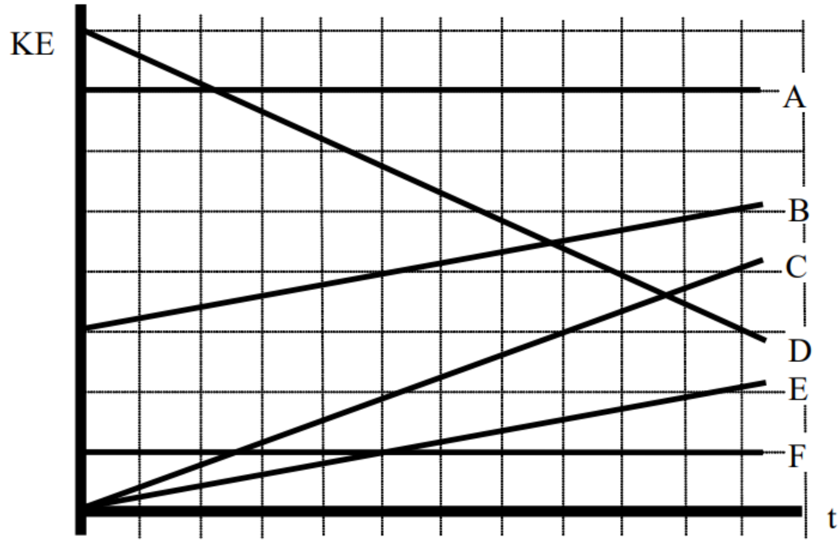
b. Rank these graphs on the basis of the total impulse on the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are kinetic energy vs. time graphs for six different objects. All of the objects move horizontally.



a. Rank these graphs on the basis of the change in kinetic energy of the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

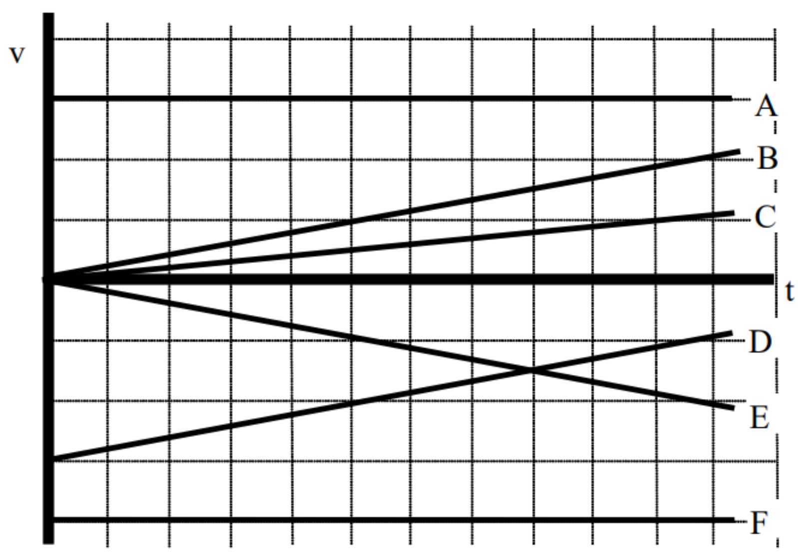
b. Rank these graphs on the basis of the total work on the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are velocity vs. time graphs for six equal-mass objects.



a. Rank these graphs on the basis of the change in momentum of the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

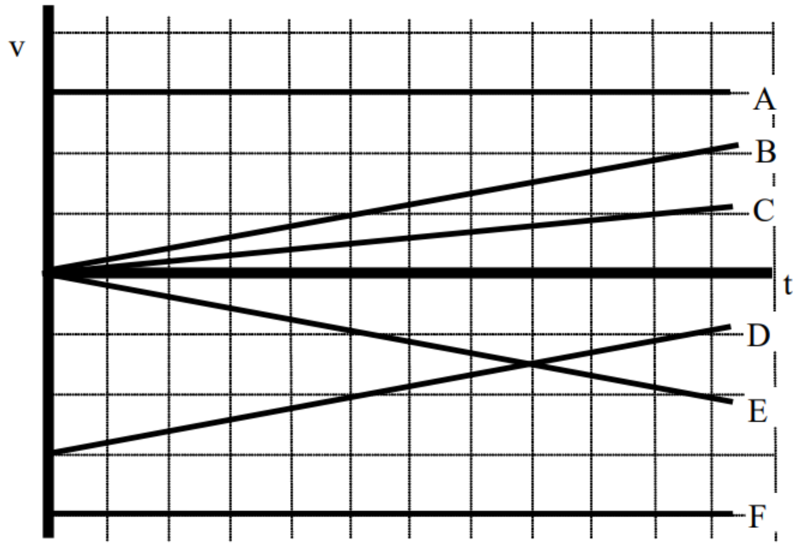
b. Rank these graphs on the basis of the change in kinetic energy of the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are velocity vs. time graphs for six equal-mass objects. All of the objects move horizontally.



a. Rank these graphs on the basis of the total impulse on the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

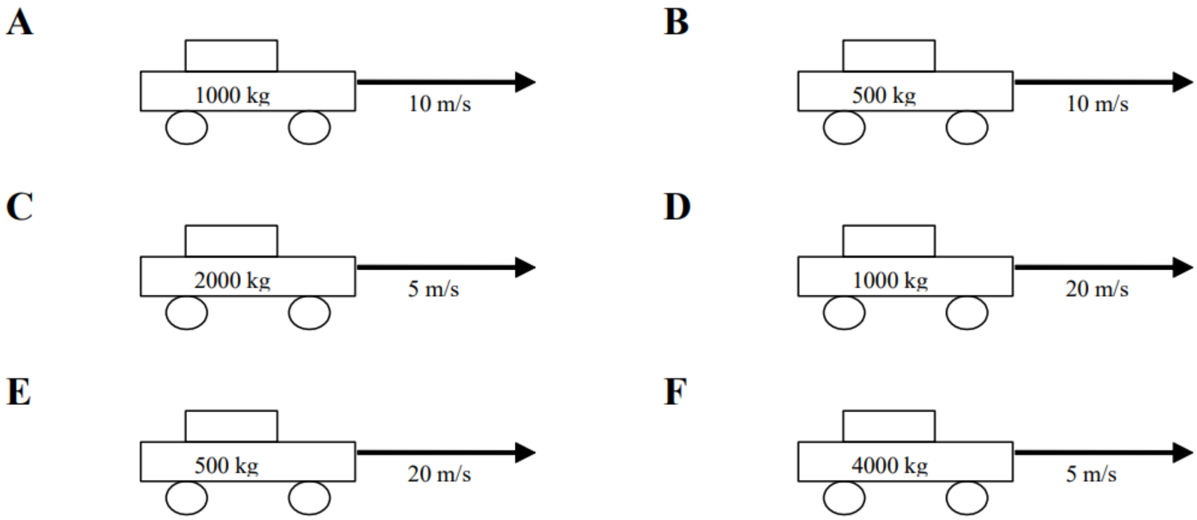
b. Rank these graphs on the basis of the total work on the object over the time interval indicated.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Largest Negative

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six automobiles initially traveling at the indicated velocity. The automobiles have different masses and velocities.



a. All automobiles will be stopped in the same amount of time. Rank these automobiles on the basis of the magnitude of the force needed to stop them.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

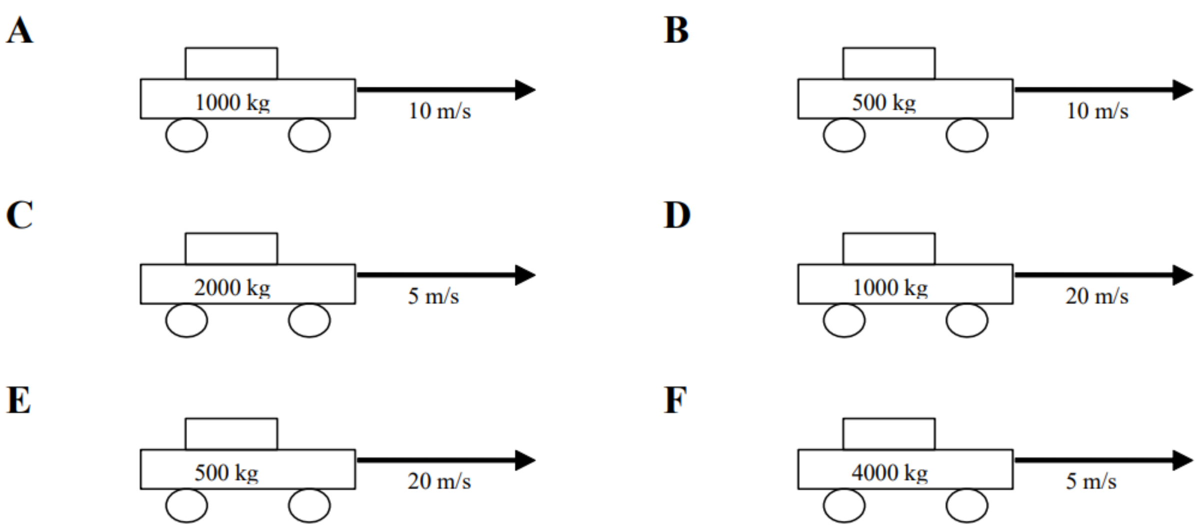
b. All automobiles will be stopped in the same amount of distance. Rank these automobiles on the basis of the magnitude of the force needed to stop them.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

Below are six automobiles initially traveling at the indicated velocity. The automobiles have different masses and velocities.



a. Rank these automobiles on the basis of the magnitude of the force needed to stop them.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

b. Rank these automobiles on the basis of the magnitude of the work needed to stop them.

Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

Explain the reason for your ranking:

c. Rank these automobiles on the basis of the magnitude of the impulse needed to stop them.

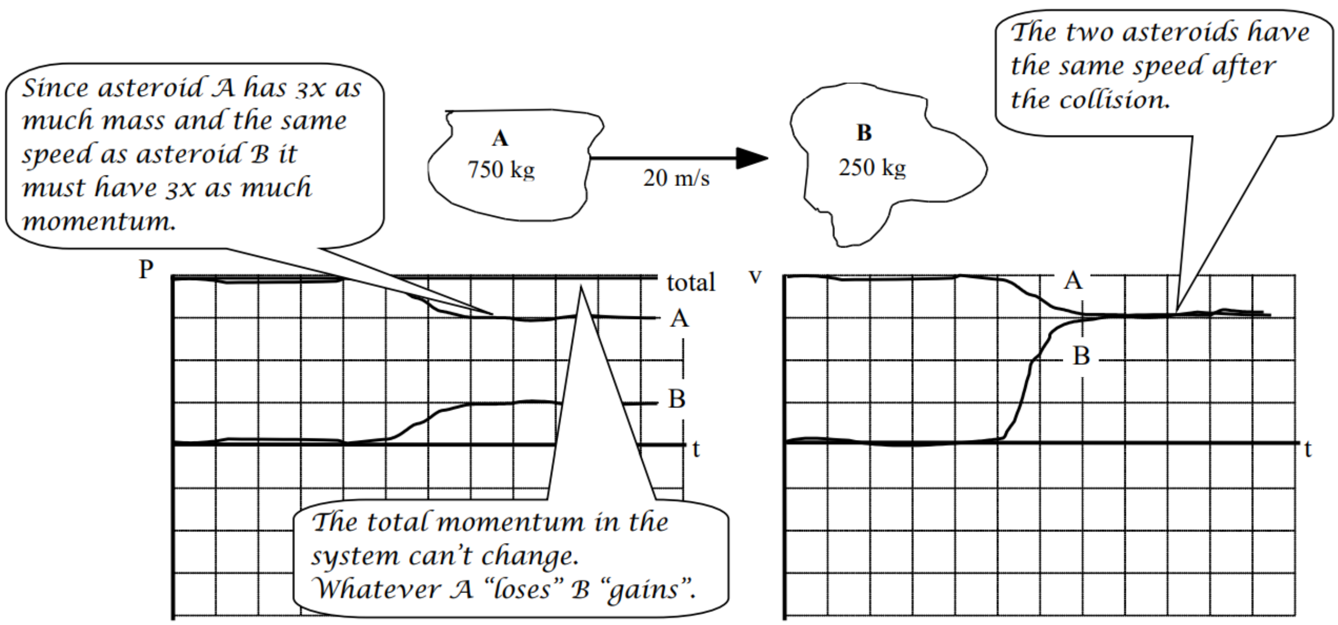
Largest Positive 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking cannot be determined based on the information provided.

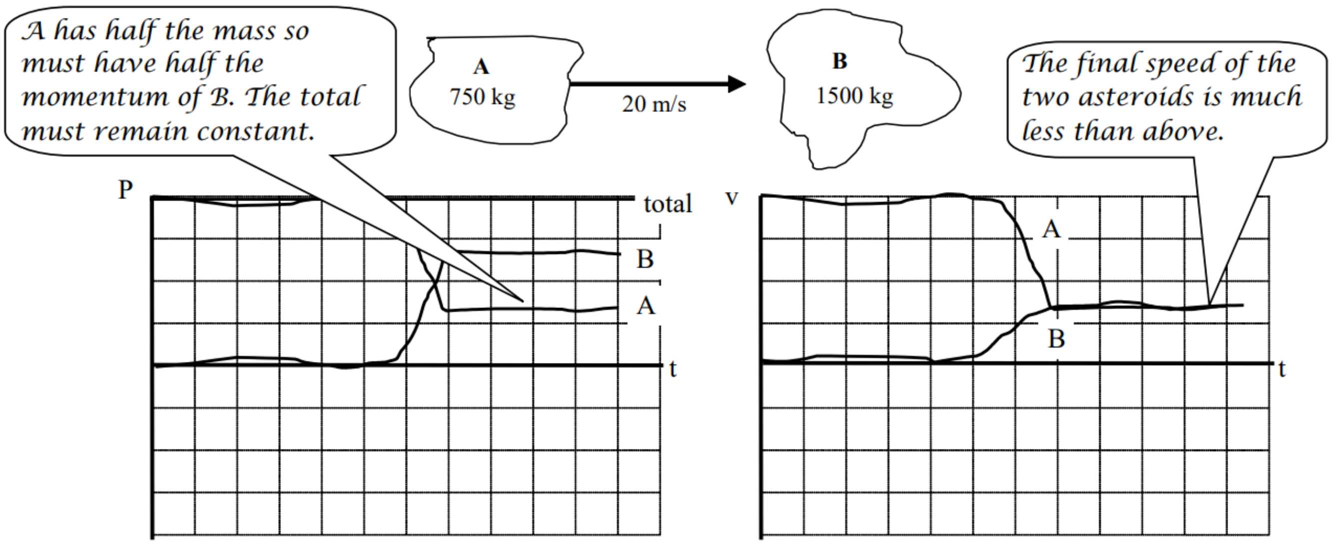
Explain the reason for your ranking:

For each of the collisions illustrated below, sketch a graph of the momentum and velocity of asteroid A, the momentum and velocity of asteroid B, and the total momentum in the system of the two asteroids. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

a. The two asteroids remain joined together after the collision.

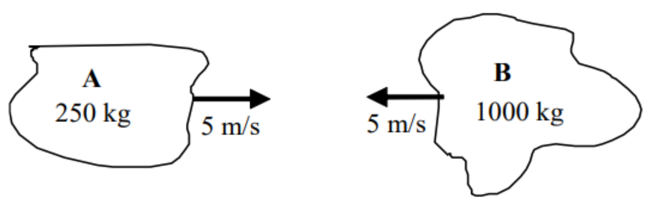


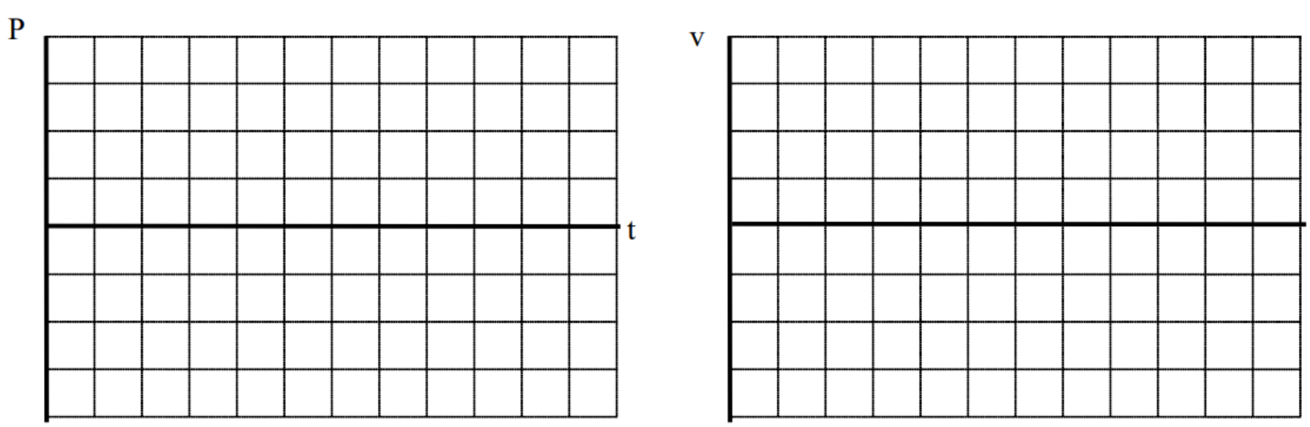
b. The two asteroids remain joined together after the collision.



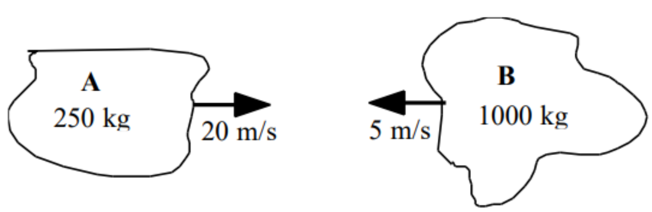
For each of the collisions illustrated below, sketch a graph of the momentum and velocity of asteroid A, the momentum and velocity of asteroid B, and the total momentum in the system of the two asteroids. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

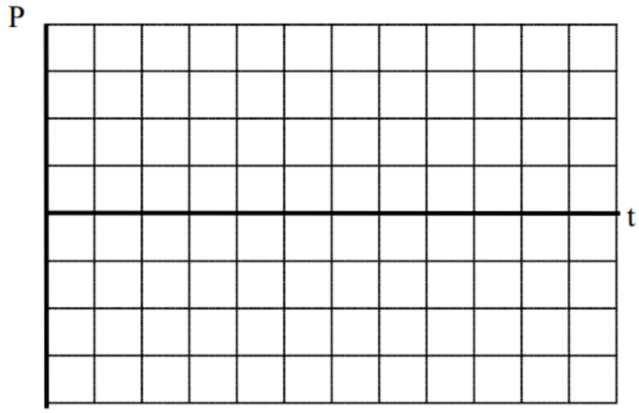
a. The two asteroids remain joined together after the collision.

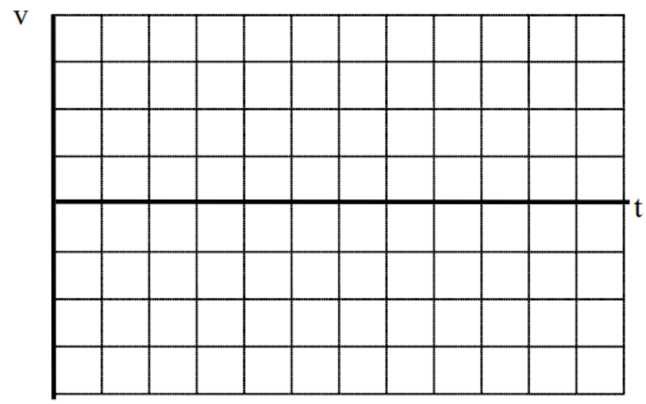




b. The two asteroids remain joined together after the collision.

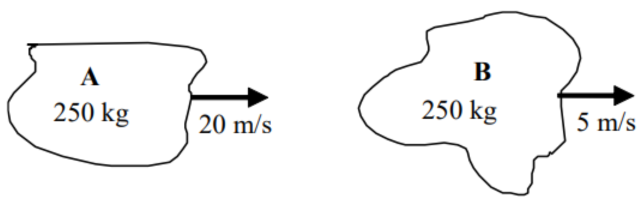


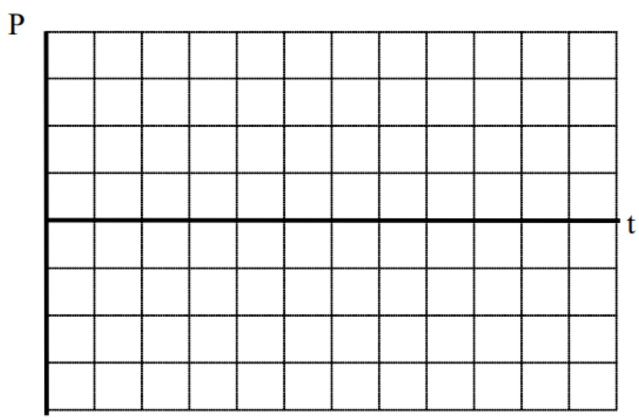


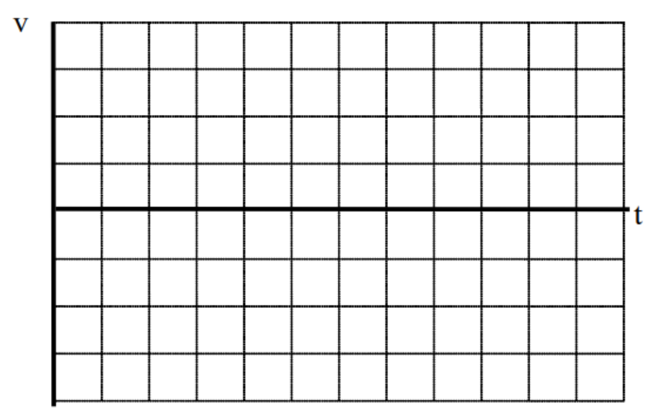


For each of the collisions illustrated below, sketch a graph of the momentum and velocity of asteroid A, the momentum and velocity of asteroid B, and the total momentum in the system of the two asteroids. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

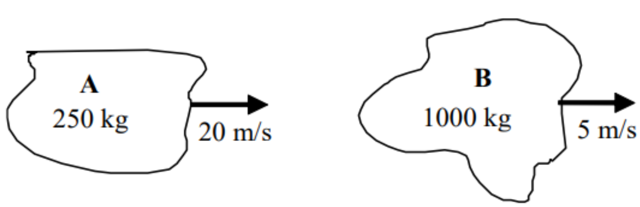
a. The two asteroids remain joined together after the collision.

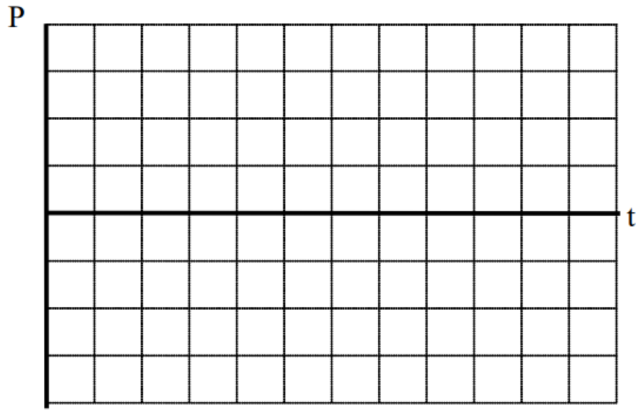


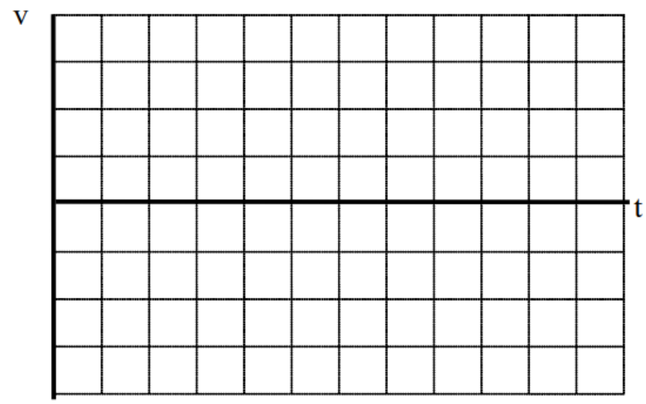




b. The two asteroids remain joined together after the collision.

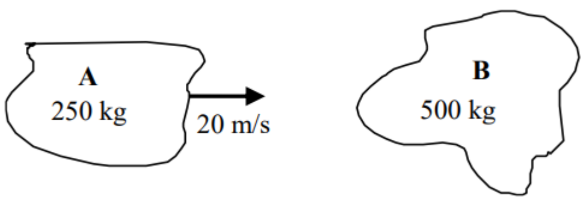


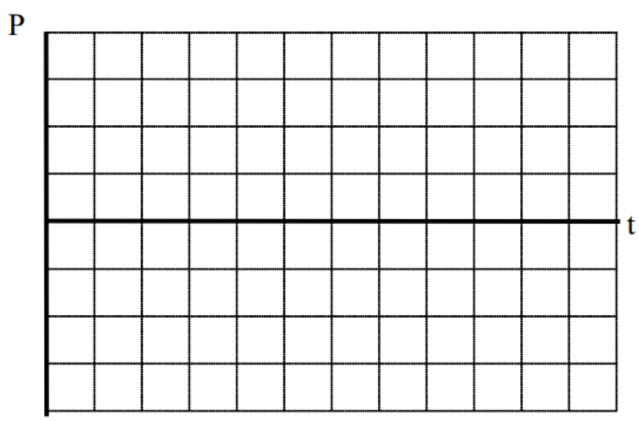


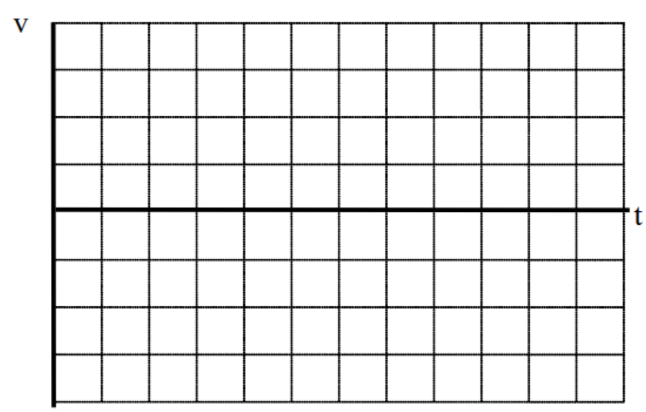


For each of the collisions illustrated below, sketch a graph of the momentum and velocity of asteroid A, the momentum and velocity of asteroid B, and the total momentum in the system of the two asteroids. Begin your graph before the collision takes place and continue it after the collision is over. Use a consistent scale on all graphs.

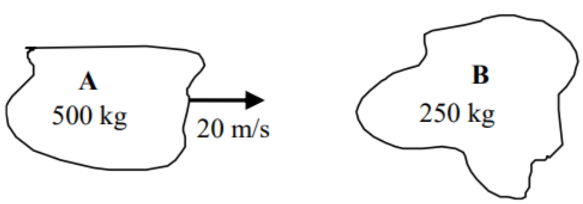
a. Asteroid A rebounds at 5 m/s after the collision.

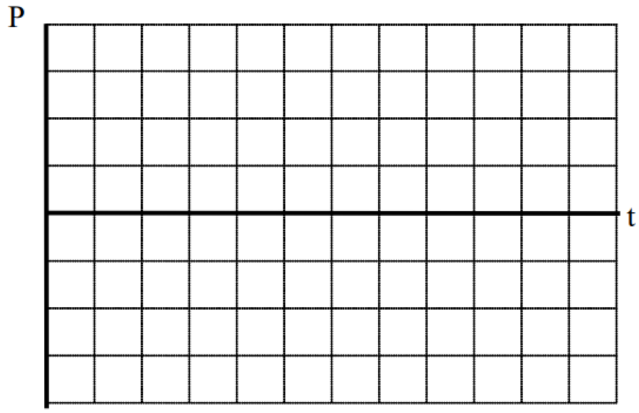


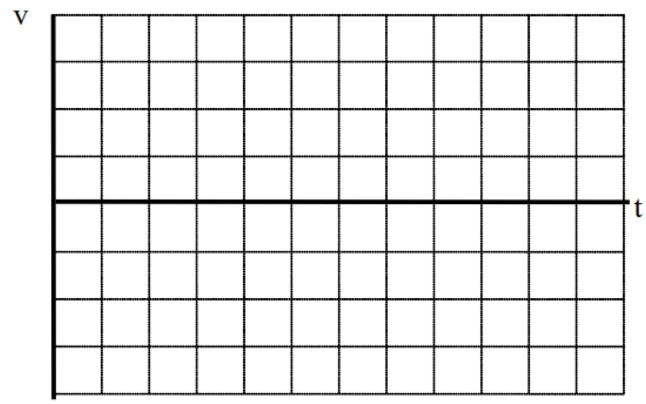




b. Asteroid B moves at 20 m/s after the collision.

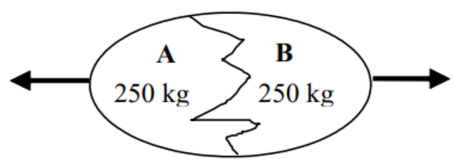


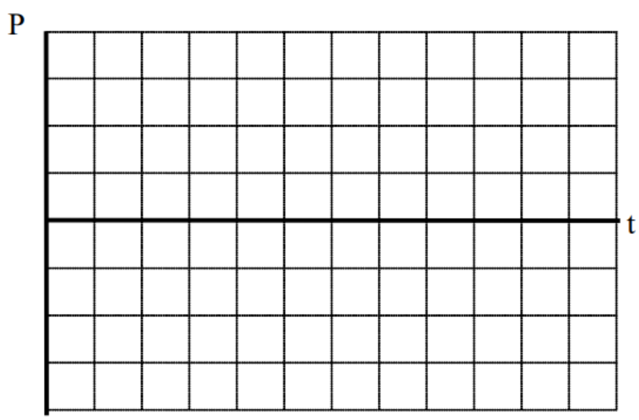


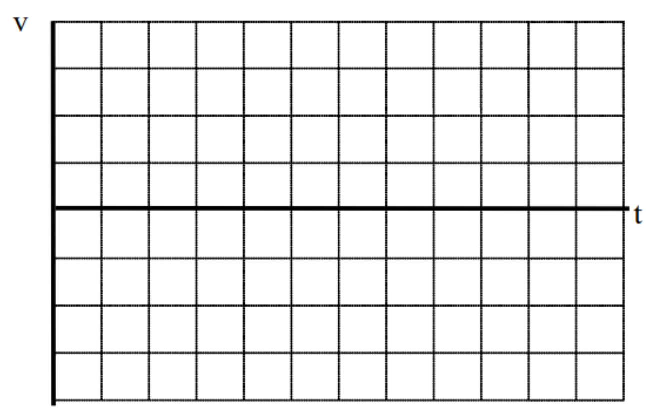


For each of the explosions illustrated below, sketch a graph of the momentum and velocity of fragment A, the momentum and velocity of fragment B, and the total momentum in the system of the two fragments. Begin your graph before the explosion takes place and continue it while the fragments travel away from the sight of the explosion. Use a consistent coordinate system and scale on all graphs.

a. The exploding egg is initially at rest.

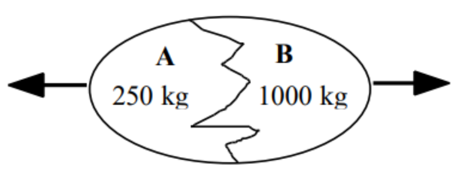


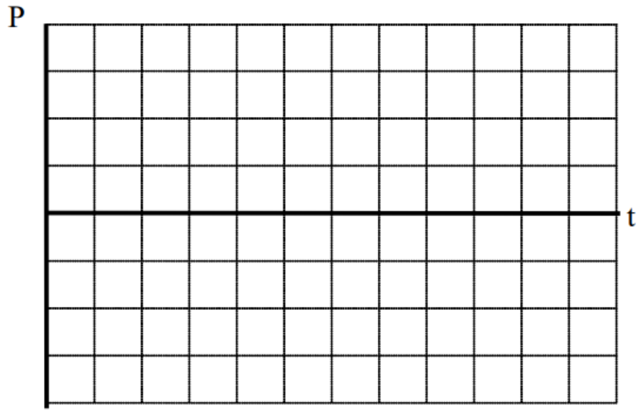


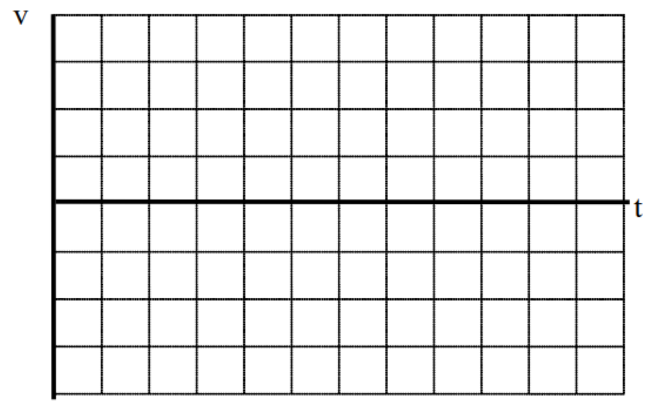


b. The exploding egg is initially at rest.

The exploding egg is initially at rest.

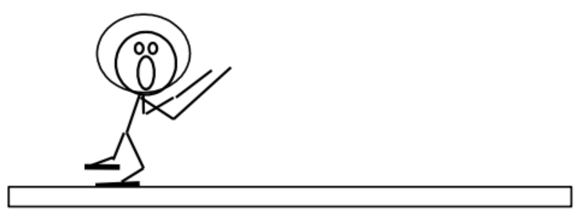


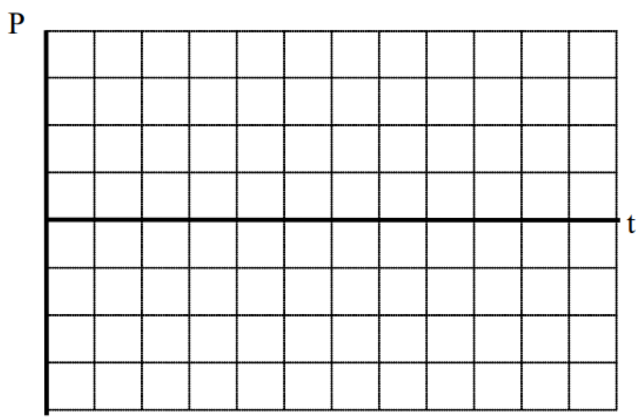


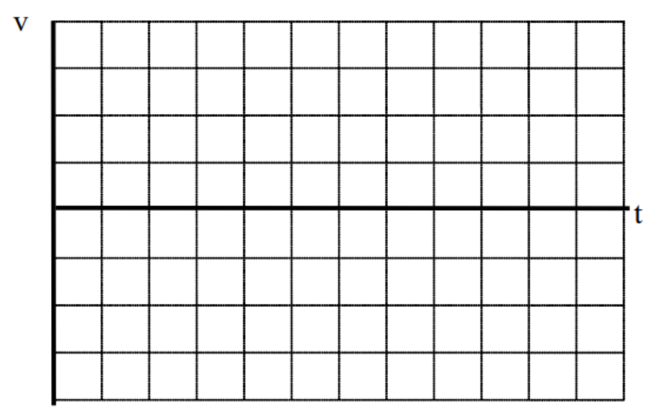


A 200 kg astronaut is initially at rest on the extreme edge of a 1000 kg space platform. She wears special magnetic shoes that allow her to walk along the metal platform. For each of the situations illustrated below, sketch a graph of the momentum and velocity of the astronaut, the momentum and velocity of the platform, and the total momentum in the system of the two objects. Begin your graph before the astronaut begins to walk and continue it while she walks along the platform. Use a consistent coordinate system and scale on all graphs.

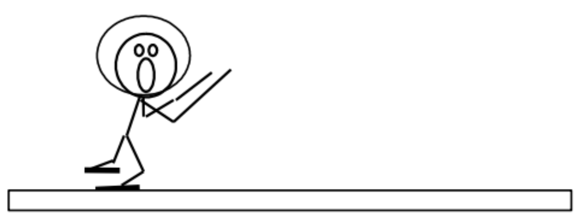
a. The astronaut and platform are initially at rest.

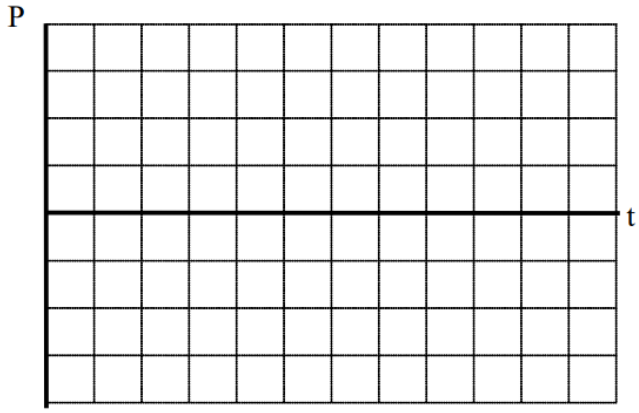


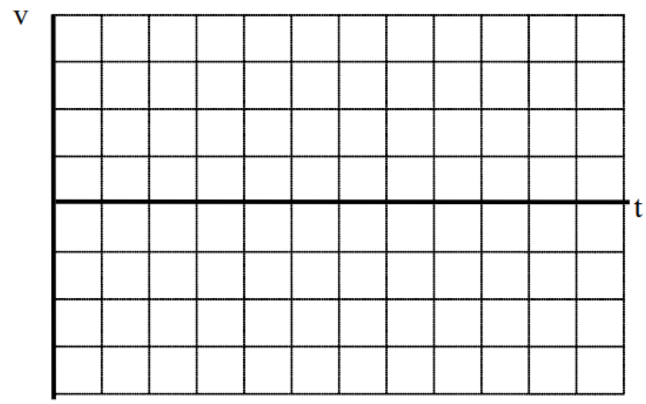




b. The astronaut and platform are initially drifting to the right.

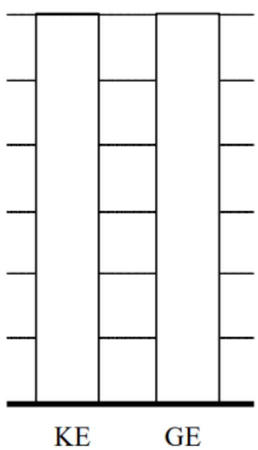




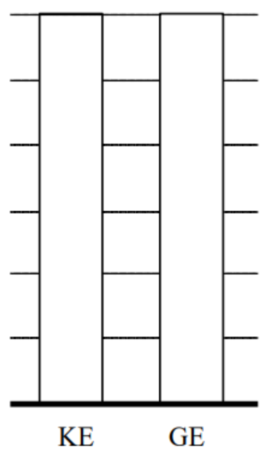


For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of the object at each of the events listed. Use a consistent scale throughout each motion. Set the lowest point of the motion as the zero-point of gravitational potential energy.

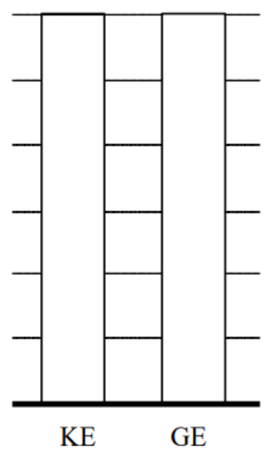
a. *A 4000 kg rocket's engine produces a thrust of 70,000 N for 15 s. The rocket is fired vertically upward.*



When the engine is first turned one.

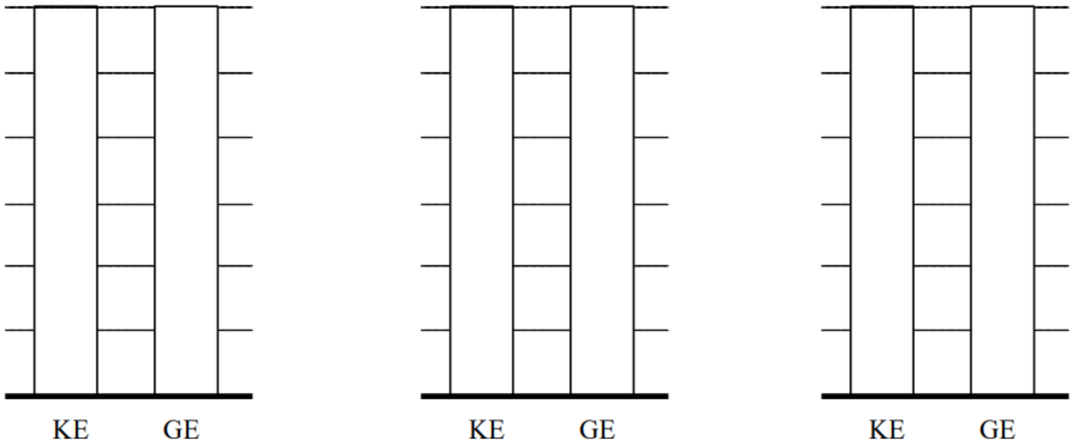


When the engine turns off.



When the rocket reaches its maximum height.

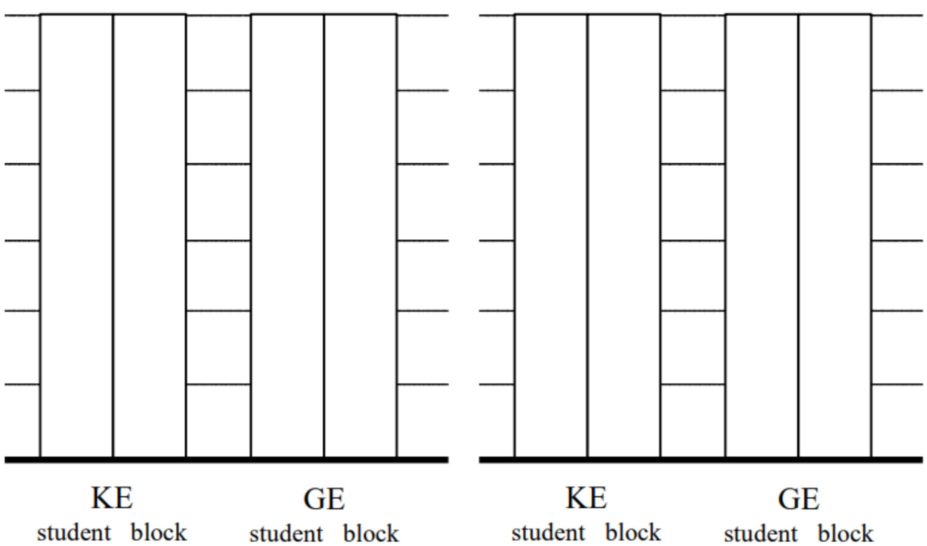
b. *To practice falling, a 55 kg pole-vaulter falls off of a wall 6.0 m above a 2.0 m thick foam cushion resting on the ground. However, he misses the cushion. The pole-vaulter sinks about 0.10 m into the ground before stopping.*



When the person first begins to fall. *When the person hits the ground.* *When the person finally stops.*

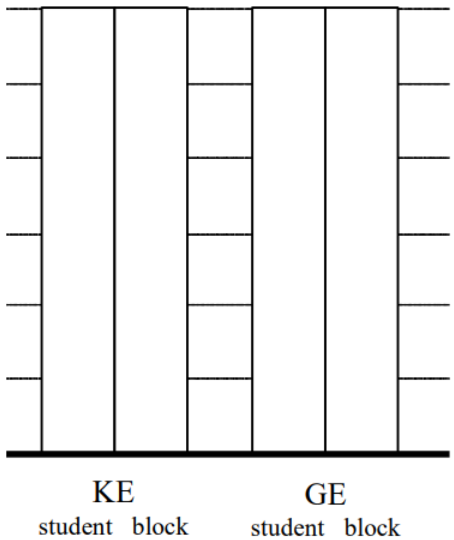
For each of the scenarios described below, indicate the amount of kinetic energy and gravitational potential energy of each object at each of the events listed. Use a consistent scale throughout each motion. Set the lowest point of the motion as the zero-point of gravitational potential energy

a. *Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. An 84 kg block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the 84 kg block is released, the student is pulled up to his dorm room, 8.0 m off the ground.*

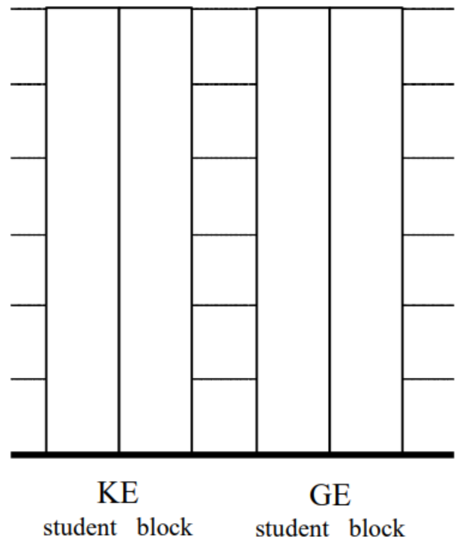


When the block is released. *When the student reaches his room.*

b. Tired of walking down the stairs, a 75 kg engineering student designs an ingenious device for reaching the ground from his third floor dorm room. A 60 kg block at rest on the ground is attached to a rope that passes over a pulley. The student grabs the other end of the rope and steps out of his window.



When the student steps out of the window.



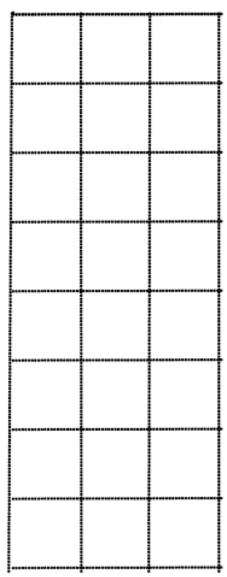
When the student reaches the ground.

A 100 kg man concerned about his weight decides to weigh himself in an elevator. He stands on a bathroom scale in an elevator that is moving upward at 3.0 m/s. As the elevator reaches his floor, it slows to a stop.

a. If the elevator slows to a stop over a distance of 2.0 m, what is the reading on the bathroom scale?

b. If the elevator slows to a stop in 1.5 s, what is the reading on the bathroom scale?

Motion Diagram



a. Motion Information

Event 1:	Event 2:
$KE_1 =$	$KE_2 =$
$GE_1 =$	$GE_2 =$
$W_{12} =$	

Mathematical Analysis[26]

Free-Body Diagram

b. Motion Information

Event 1:	Event 2:
P1 =	P2 =
J12 =	

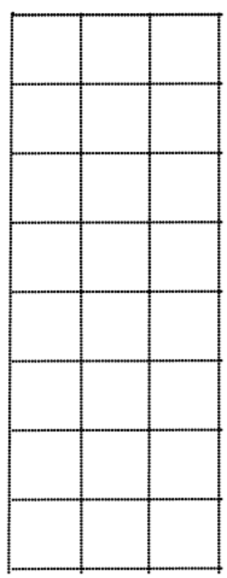
Mathematical Analysis

A 70 kg student is 120 m above the ground, moving upward at 3.5 m/s, while hanging from a rope hanging from a 280 kg helium balloon. The lift on the balloon due to the buoyant force is 3000 N.

a. With what speed does the student hit the ground?

b. How long does it take the student to reach the ground?

Motion Diagram

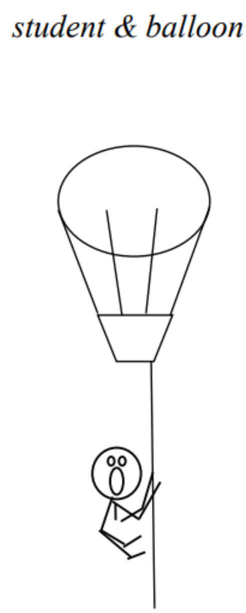


a. Motion Information

Event 1:	Event 2:
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Mathematical Analysis[27]

Free-Body Diagram



b. Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	

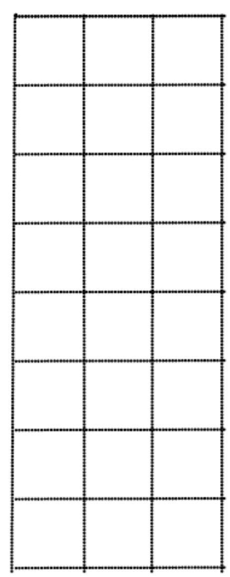
Mathematical Analysis

A 4000 kg rocket's engine produces a thrust of 70,000 N for 15 s. The rocket is fired vertically upward.

a. What is the speed of the rocket when its engine turns off?

b. How long does it take the rocket to reach its maximum height?

Motion Diagram



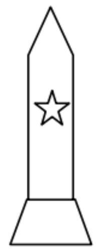
a. Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	

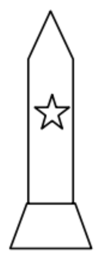
Mathematical Analysis[28]

Free-Body Diagram

while engine fires



after engine turns off



b. Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	

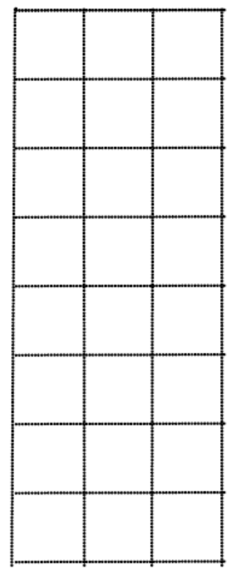
Mathematical Analysis

To practice falling, a 55 kg pole-vaulter falls off of a wall 6.0 m above a 2.0 m thick foam cushion resting on the ground. However, he misses the cushion. The pole-vaulter sinks about 0.10 m into the ground before stopping.

a. What is the speed of the pole-vaulter when he hits the ground?

b. What is the force exerted on the pole-vaulter by the ground as he comes to rest?

Motion Diagram

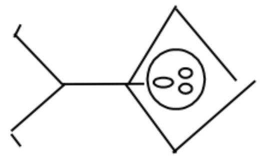


a. Motion Information

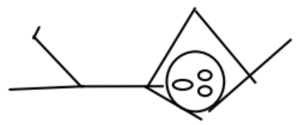
Event 1:	Event 2:
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Mathematical Analysis[29]

Free-Body Diagram



while dying



b. Motion Information

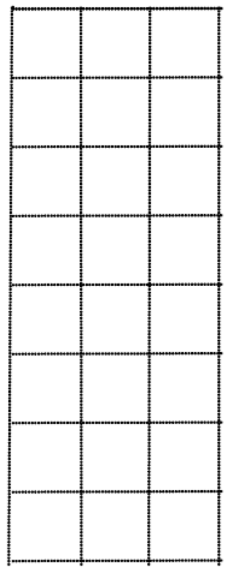
Event 1:	Event 2:
KE_1 =	KE_2 =
GE_1 =	GE_2 =

Event 1:	Event 2:
W_{12} =	

Mathematical Analysis

A decorative light fixture in an elevator consists of a 2.0 kg light suspended by a cable from the ceiling of the elevator. From this light, a separate cable suspends a second 0.80 kg light. The elevator is moving downward at 4.0 m/s when someone presses the emergency stop button. During the stop, the upper cable snaps. The elevator engineer says that the cable could withstand a force of 40 N without breaking. Find the maximum time and distance over which the elevator stopped.

Motion Diagram



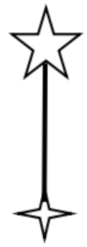
Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Mathematical Analysis[30]

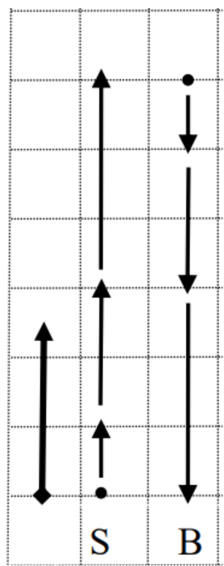
Free-Body Diagram

the two lights



Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. An 84 kg block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the 84 kg block is released, the student is pulled up to his dorm room, 8.0 m off the ground.

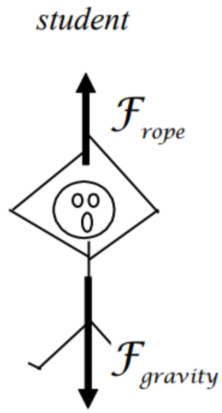
Motion Diagram

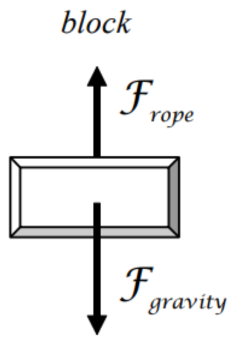


Motion Information

Event 1: The block is released	Event 2: The student reaches the room.
Object: Student	
$KE_1 = 0$	$KE_2 = 1/2(80)\mathcal{V}_f^2$
$GE_1 = 0$	$GE_2 = 80(9.8)(8)$
$W_{12} = \mathcal{F}_R(8)\cos 0^\circ$	
Object: Block	
$KE_1 = 0$	$KE_2 = 1/2(84)\mathcal{V}_f^2$
$GE_1 = 84(9.8)(8)$	$GE_2 = 0$
$W_{12} = \mathcal{F}_R(8)\cos 180^\circ$	

Free-Body Diagram





Since the distance the student and block travel is known, applying work-energy should allow us to solve the problem. I'll apply it separately to each object.

student	block
$0+F_{\text{rope}}\ (8)=\frac{1}{2}80v_f^2+80(9.8)(8)$	$84(9.8)(8)-F_{\text{rope}}\ (8)=\frac{1}{2}84v_f^2$
$8F_{\text{rope}}\ =40v_f^2+6272$	$6586-8F_{\text{rope}}\ =42v_f^2$

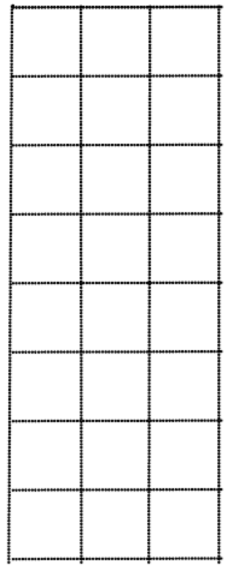
$\mathcal{F}_{\text{rope}}$ *is the same in both equations, as is the final speed. Thus the two equations can be added together to yield:*

$$6586=40v_f^2+42v_f^2+6272$$
$$314=82v_f^2$$
$$v_f=1.96\ \text{m/s}$$

Notice that if you applied work-energy to the entire system you would have generated this same equation. Initially, the only form of energy present is the gravitational energy of the block (mgh = 6586 J). At the second event, both objects have kinetic energy plus the student has gravitational potential energy (mgh = 6272 J).

Tired of walking up the stairs, an engineering student designs an ingenious device for reaching his third floor dorm room. A 100 kg block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room, 8.0 m off the ground. He is traveling at 2.2 m/s when he reaches his room.

Motion Diagram

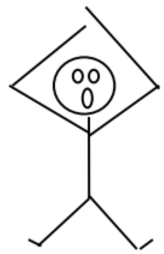


Motion Information

Event 1:	Event 2:
Object:	
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	
Object:	
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Free-Body Diagram

student

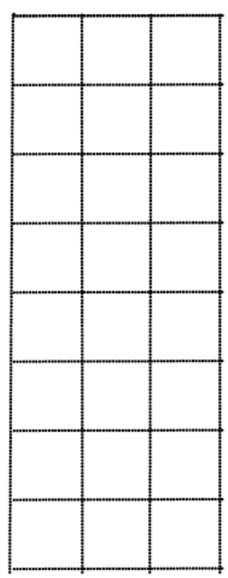


block

Mathematical Analysis[31]

Tired of walking down the stairs, a 75 kg engineering student designs an ingenious device for reaching the ground from his third floor dorm room. A 60 kg block at rest on the ground is attached to a rope that passes over a pulley. The student grabs the other end of the rope and steps out of his window. He falls for 5.5 s before reaching the ground.

Motion Diagram



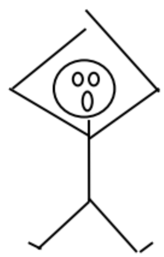
Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =

Event 1:	Event 2:
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Free-Body Diagram

student

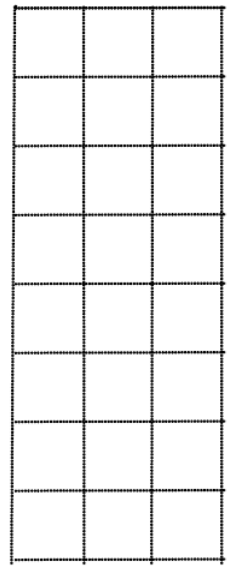


block

Mathematical Analysis[32]

Tired of walking down the stairs, a 75 kg engineering student designs an ingenious device for reaching the ground from his dorm room. A 60 kg block at rest on the ground is attached to a rope that passes over a pulley. The student grabs the other end of the rope and steps out of his window. He hits the ground at 3.3 m/s.

Motion Diagram

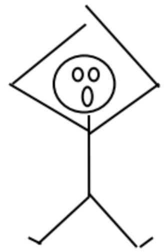


Motion Information

Event 1:		Event 2:	
Object:		Object:	
P_1 =	P_2 =	P_1 =	P_2 =
J_{12} =		J_{12} =	
P_1 =	P_2 =	P_1 =	P_2 =
KE_1 =	KE2 =	KE_1 =	KE2 =
GE_1 =	GE2 =	GE_1 =	GE2 =
W_{12} =		W_{12} =	

Free-Body Diagram

student

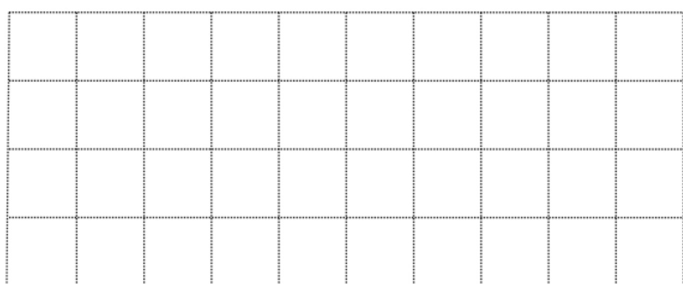


block

Mathematical Analysis[33]

Far from any other masses, a 2000 kg asteroid traveling at 12 m/s collides with a 1200 kg asteroid traveling in the other direction at 16 m/s. After the collision they remain joined together and move with a common velocity.

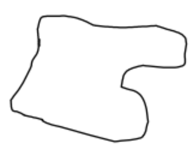
Motion Diagram



Free-Body Diagrams

2000 kg asteroid

1200 kg asteroid



Motion Information

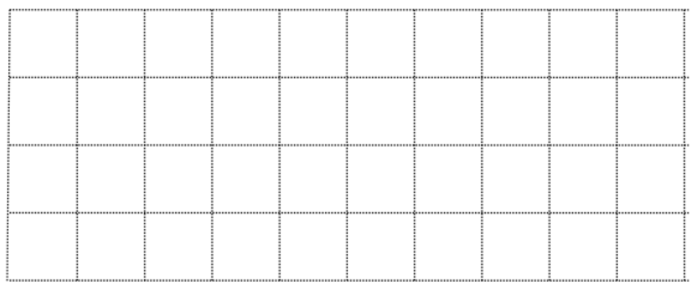
Event 1:	Event 2:
Object:	
P_1 =	P_2 =

Event 1:	Event 2:
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

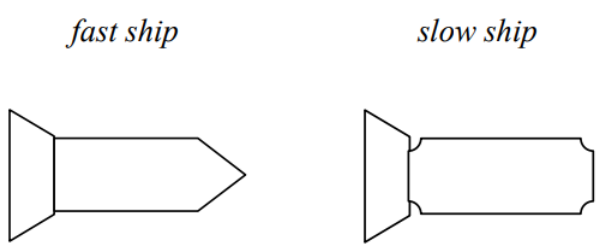
Mathematical Analysis[34]

On a remote stretch of intergalactic highway, a 7.5 x 10^6 kg spaceship traveling at 10 percent the speed of light (0.10c = 3.0 x 10^7 m/s) doesn't notice the slower spaceship ahead clogging the lane. The fast-moving ship rear-ends the slower ship, an older 5.5 x 10^6 kg model, and the two ships get entangled and drift forward at 0.07c.

Motion Diagram



Free-Body Diagrams



Motion Information

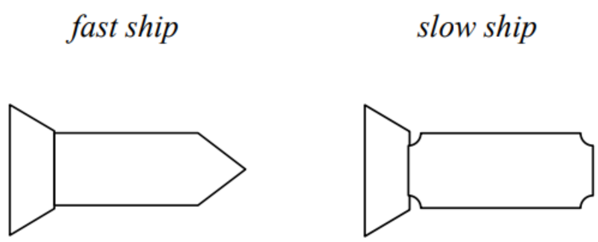
Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Mathematical Analysis[35]

On a remote stretch of intergalactic highway, a 7.5 x 10^6 kg spaceship traveling at 10 percent the speed of light (0.10c = 3.0 x 10^7 m/s) doesn't notice the slower spaceship ahead, moving at 0.05c, clogging the lane. The fast-moving ship rear-ends the slower ship, an older 4.5 x 10^6 kg model, and the slower ship gets propelled forward at 0.13c.

Motion Diagram

Free-Body Diagrams



Motion Information

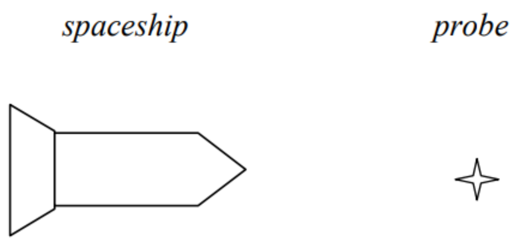
Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Mathematical Analysis[36]

In the farthest reaches of deep space, an 8000 kg spaceship, including contents, is at rest relative to a space station. The spaceship recoils after it launches a 600 kg scientific probe with a speed of 300 m/s relative to the space station.

Motion Diagram

Free-Body Diagrams



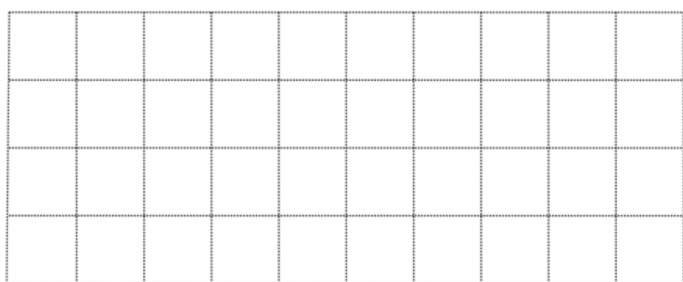
Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

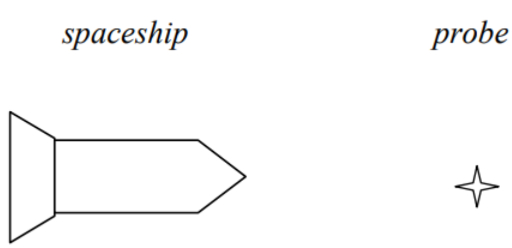
Mathematical Analysis[37]

In the farthest reaches of deep space, an 8000 kg spaceship, including contents, is drifting at 50 m/s relative to a space station. The spaceship is brought to rest, relative to the space station, by the recoil from launching a 600 kg scientific probe.

Motion Diagram



Free-Body Diagrams



Motion Information

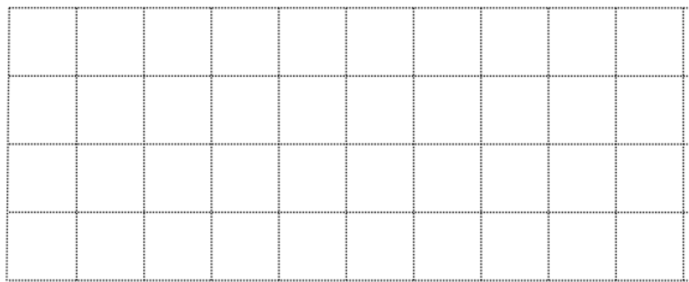
Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	

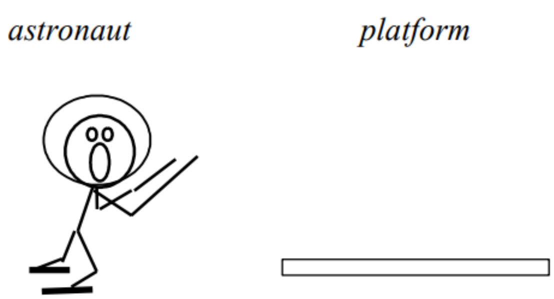
Mathematical Analysis[38]

A 140 kg astronaut is standing on the extreme edge of a 1000 kg space platform, at rest relative to the mother ship. She begins to walk toward the other edge of the platform, reaching a speed of 2.0 m/s relative to the mother ship. (She wears special magnetic shoes that allow her to walk along the metal platform.)

Motion Diagram



Free-Body Diagrams



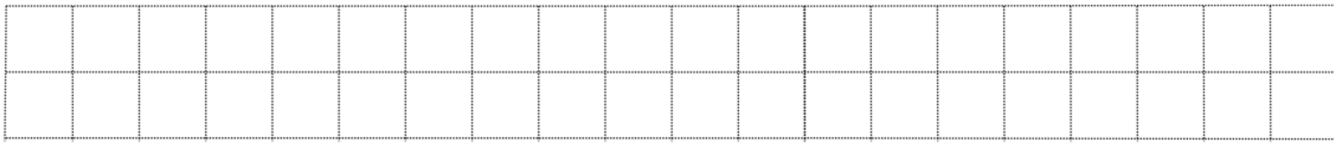
Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

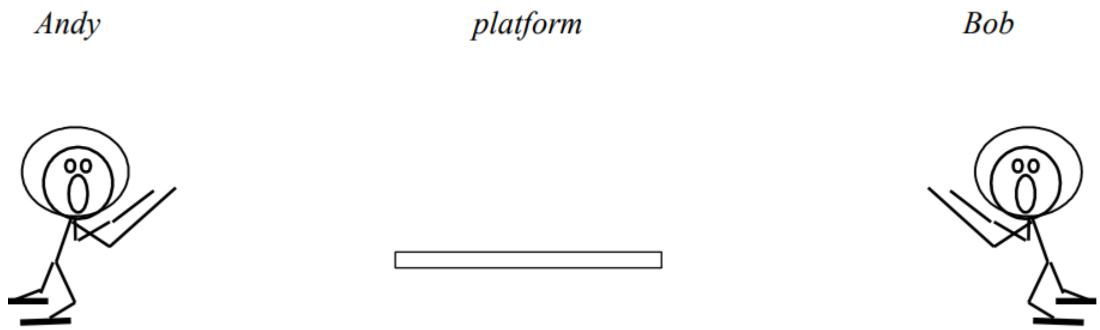
Mathematical Analysis[39]

Two astronauts, 140 kg Andy and 170 kg Bob, are standing on opposite edges of a 1000 kg space platform, at rest relative to the mother ship. They each begin to walk toward the opposite ends of the platform, Andy reaching a speed of 2.0 m/s and Bob 1.5 m/s, both relative to the mother ship. (They wear special magnetic shoes that allow them to walk along the metal platform.)

Motion Diagram



Free-Body Diagrams



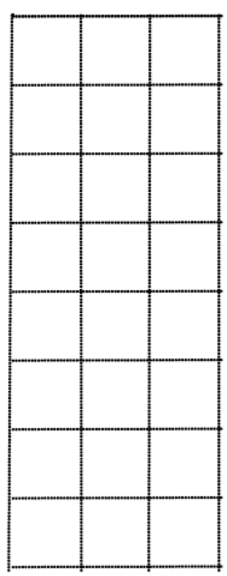
Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Mathematical Analysis[40]

A 70 kg student is hanging from a 280 kg helium balloon. The balloon is rising at a constant speed of 8.0 m/s relative to the ground. The lift on the balloon due to the buoyant force is constant. The student begins to climb up the rope at a speed of 15 m/s relative to the ground. The balloon's upward speed is decreased as the student climbs.

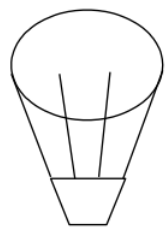
Motion Diagram



Free-Body Diagrams

student

balloon



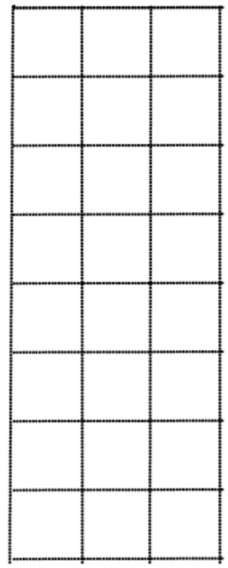
Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Mathematical Analysis[41]

A man of mass m, concerned about his weight, decides to weigh himself in an elevator. He stands on a bathroom scale in an elevator which is moving upward at v. As the elevator reaches his floor, it slows to a stop over a time interval, T. Determine the reading on the bathroom scale (F_{scale}) as a function of m, v, T, and g.

Motion Diagram



Free-Body Diagrams

Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	

Questions

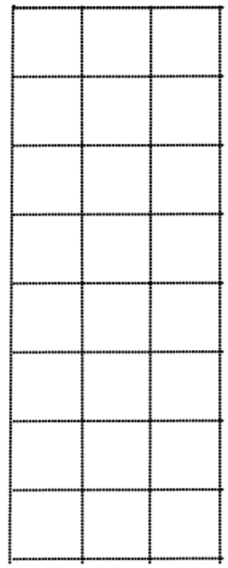
If $T = \infty$, what should F_{scale} equal? Does your function agree with this observation?

For what combination of v and T would the bathroom scale read 0 N?

If the elevator were initially going down, would the above combination of v and T also lead to a scale reading of 0 N?

A rocket of mass m is fired vertically upward from rest. The rocket's engine produces a thrust of constant magnitude F for t_{thrust} seconds. Determine the time it takes the rocket to reach its apex (t_{apex}) as a function of F, t_{thrust}, m, and g.

Motion Diagram



Free-Body Diagrams

Motion Information

Event 1:	Event 2:
P_1 =	P_2 =
J_{12} =	

Questions

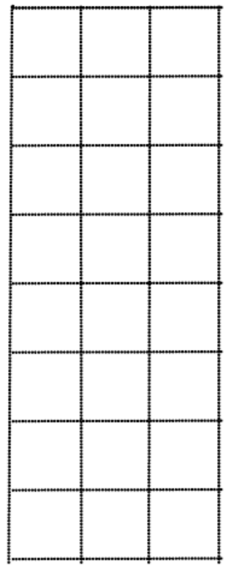
If $g = 0\ m/s^2$, what should t_{apex} equal? Does your function agree with this observation?

If $F = mg$, what should t_{apex} equal? Does your function agree with this observation?

For what value of F would $t_{apex} = 2t_{thrust}$?

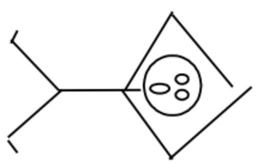
To practice falling, a pole-vaulter of mass m falls off of a wall a distance D above a thick foam cushion. The pole-vaulter sinks a distance d into the cushion before stopping. Determine the force exerted on the pole-vaulter due to the cushion ($F_{cushion}$) as a function of m, D, d, and g.

Motion Diagram

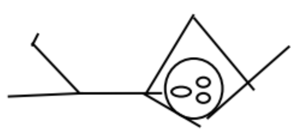


Free-Body Diagrams

while falling



while dying



Motion Information

Event 1:	Event 2:
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Questions

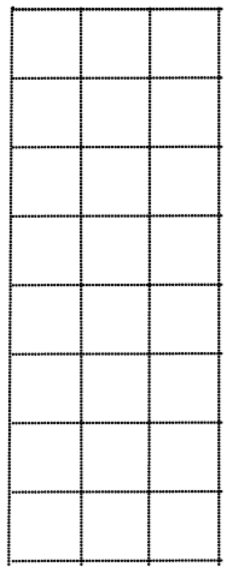
If $D = \infty$, what should $F_{cushion}$ equal? Does your function agree with this observation?

If d = 0 m, what should $F_{cushion}$ equal? Does your function agree with this observation?

What would be worse for the pole-vaulter, starting at twice the initial distance above the cushion or sinking half of the original distance into the cushion?

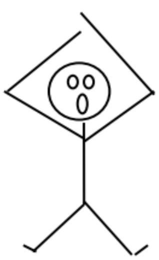
Tired of walking up the stairs, an engineering student of mass m designs an ingenious device for reaching his third floor dorm room. A block of mass M is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room in a time T. Determine the velocity of the student (v) when he reaches his room as a function of m, M, T and g.

Motion Diagram



Free-Body Diagrams

student



block

Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Questions

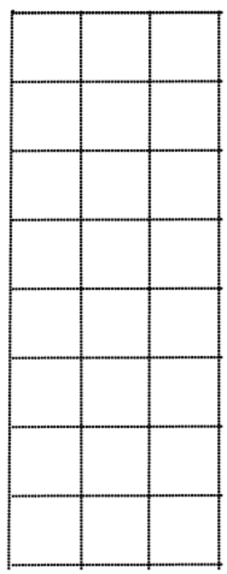
If g = 0 m/s^2, what should v equal? Does your function agree with this observation?

If m = M, what should v equal? Does your function agree with this observation?

If M = ∞, what should v equal? Does your function agree with this observation?

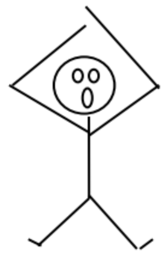
Tired of walking down the stairs, an engineering student of mass m designs an ingenious device for reaching the ground from her dorm room. A block of mass M at rest on the ground is attached to a rope that passes over a pulley. The student grabs the other end of the rope and steps out of her window a distance D above the ground. Determine the velocity of the student (v) when she reaches the ground as a function of m, M, D and g.

Motion Diagram

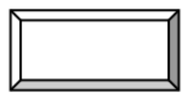


Free-Body Diagrams

student



block



Motion Information

Event 1:	Event 2:
Object:	
KE_1 =	KE2 =
GE_1 =	GE_2 =
W_{12} =	
Object:	
KE_1 =	KE_2 =
GE_1 =	GE_2 =
W_{12} =	

Questions

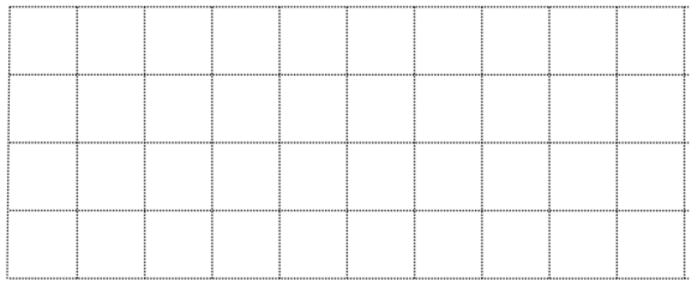
If $g = 0\ m/s^2$, what should v equal? Does your function agree with this observation?

If m = M, what should v equal? Does your function agree with this observation?

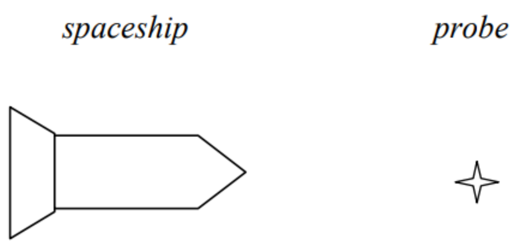
If M = ∞, what should v equal? Does your function agree with this observation?

In the farthest reaches of deep space, a spaceship of mass M, including contents, is at rest relative to a space station. The spaceship recoils after it launches a scientific probe of mass m at a speed v relative to the space station. Determine the recoil speed of the spaceship (V) as a function of M, m, and v.

Motion Diagram



Free-Body Diagrams



Motion Information

Event 1:	Event 2:
Object:	
P_1 =	P_2 =
J_{12} =	
Object:	
P_1 =	P_2 =
J_{12} =	

Questions

If M = 2m, what should V equal? Does your function agree with this observation?

If M = ∞, what should V equal? Does your function agree with this observation?

Model One

Summary Problems

1. A sprinter in a 100 meter dash starts from rest and accelerates at 2.5 m/s^2 for 3.9 s. She then runs at a constant speed, until she either tires or finishes the race. She can run at a constant velocity for at most 7.0 s without tiring. Once tired, she begins to slow, accelerating at 1.0 m/s^2. How long does it take her to finish the race?
2. The *Schindler Mobile* is a self-propelled elevator, powered by a small motor attached to the bottom of the elevator cab that drives *the cab up and* down the tracks of two high-strength aluminum columns. Assume the Schindler Mobile can travel from the 1st floor to the 10th floor (approximately 40 m) in 15 s. Assume the elevator both speeds up and slows
3. *The Thrust SSC*, a 7 ton jet mobile powered by two Royal Air Force Phantom jets providing 110,000 hp, was designed to break the speed of sound. The "car" was tested at a 15 mile long track in the Black Rock desert of Nevada. The car accelerated for nearly five miles, then moved through a measured mile at top speed. The car slowed by cutting power and releasing parachutes for five additional miles before applying brakes at speeds below 300 mph. Complete a kinematic description for the car's motion, assuming it reaches a maximum speed of Mach 1 (750 mph at the temperatures encountered at the Black Rock desert raceway).
4. Often the reason for the formation of congested traffic is obvious; accidents, lane closings, or other bottlenecks. However, you have probably also experienced "phantom" traffic jams, which emerge without any obvious reason, seemingly *out of nothing*.

This phenomenon can be understood by the collective behavior of many drivers. If one vehicle drives more slowly than others, the vehicle behind has to brake to maintain the desired *safety time*. (The safety time is the elapsed time between the two objects passing the same point. Thus, the distance associated with this "time cushion" varies with the speed of the traffic.) Consequently, the next vehicle behind has to brake, and so on. If traffic flow is unstable, each following vehicle has to brake stronger than its predecessor. Thus, a small initial perturbation triggers a backward propagating "wave" of slower vehicles with increasing amplitude. Finally, the vehicles come nearly to a stop; a full-fledged traffic jam has evolved. The driver having caused the small perturbation by driving unusually slowly escapes without even realizing what he has triggered!

To get a better feel for the kinematics involved in instigating phantom jams, imagine a car approaching a slow-moving truck on a one-lane road. The car is initially traveling at 120 km/hr, while the truck moves at 70 km/hr. The car is 100 m behind the truck when the driver first notices the truck. Find the minimum acceleration necessary for the car to come to equilibrium behind the truck and achieve a safety time of 2 s. Assume the truck does not accelerate.

5. When modeling traffic flow, various psychological factors must be incorporated. One is the politeness factor. The *politeness factor* quantifies how much one weights disadvantages imposed on other drivers against one's own advantage when considering a lane change. Lane changes are more common when the politeness factor is low. Different regions of the country have, on average, different politeness factors. In addition, urban vs. rural drivers differ in politeness factor. At high values of the politeness factor, drivers run the risk of getting stuck permanently behind slow-moving vehicles or other obstacles.

   To get a better feel for how "politeness" affects traffic flow, imagine a car stuck behind a 5 m long, stationary obstacle blocking its lane. 30 m ahead of the car is a turn in the road. Therefore, the driver of the car cannot see an approaching car or truck until it is 30 m from the car.

   a. If the car pulls out to go around the obstacle (with acceleration 4 m/s^2) just as a truck moving at 55 mph rounds the bend, is this an “extremely polite" maneuver? (A maneuver is extremely polite when the truck does not need to slow down in order to avoid an accident.)
   b. What acceleration is necessary for an extremely polite driver to pull out from behind the barrier? (If the car cannot generate this acceleration, an extremely polite driver must spend the rest of their life stuck behind the barrier!)
6. Traffic engineers are concerned with selecting the proper "yellow time" to ensure safe passage through stoplights. To understand this scenario, imagine yourself driving down a relatively empty road. Up ahead, the traffic light turns yellow. If you are close enough to the traffic light you can pass through the intersection before the light turns red. If you are far from the traffic light you can safely slow down and stop before the intersection. But what if you are in-between, in what is termed the "no-win" zone, and are too far to make it and too close to stop? Traffic engineers design the duration of the yellow signal to eliminate this no-win zone.
   a. You are driving at the speed limit (45 mph) on a straight, empty road with perfect visibility. Your maximum acceleration while braking is 7.0 m/s^2. The yellow time is 1.0 s. Determine the location of the no-win zone (i.e., the range of positions from which you cannot safely traverse the intersection). Assume you don't speed up to "run" the yellow, since this is an illegal activity.
   b. If you do want to "run" the yellow from anywhere in the no-win zone, what minimum acceleration is needed? Is this feasible? How fast would you be traveling as you go through the intersection?
7. Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching his third floor dorm room. A heavy block is attached to a rope that passes over a pulley. The student holds the other end of the rope. When the block is released, the student is pulled up to his dorm room, 8.0 m off the ground. However, the student has a weak stomach and will get nauseous if he accelerates at greater than 4.0 m/s^2. Also, the rope he used can transmit a force of only 1100 N before breaking. If possible, what mass ballast block should he use to avoid breaking the rope and avoid getting nauseous?
8. Tired of walking up the stairs, an 80 kg engineering student designs an ingenious device for reaching her third floor dorm room. A heavy block is attached to a rope that passes over a pulley. The student holds the other end of the rope. However, the dorm room window is 12 m off the ground and the block is initially only 10 m off the ground. The student wants to choose a mass for the block such that when the block hits the ground, the student is “launched” upward, and reaches her window at the apex of her motion. (That will make it easiest to climb in the window.) What mass block should she use?

Selected Answers

[1] $r2 = 1.4$ m

[2] t2 = 1.45 m

[3] t3 = 3.4 s

[4] t3 = 4.55 s

[5] r3 = 36.6 m

[6] r3 = 3520 m

[7] r2 = 14.9 m

[8] r4 = 2000 m

[9] t4 = 3.7 x 105 s

[10] t2 = 14.9 s

[11] t2 = 7.8 s

[12] t3 = 15.7 s

[13] t3 = 2.87 s

[14] F rope = 420 N

[15] F cushion = 1910 N

[16] a ≥ 4.49 m/s2

[17] t2 = 17.1 s to reach ground

[18] F cushion = 2830 N

[19] F ground = 43700 N

[20] r3 = 63.5 m

[21] r3 = 1550 m

[22] m block = 240 kg

[23] m block = 26 kg

[24] F rope = 500 N

[25] F rope = 466 N

[26] a. F scale = 755 N b. F scale = 780 N

[27] a. v = 17.5 m/s b. t = 17.1 s

[28] a. v = 116 m/s b. t = 26.8 s

[29] a. v = 12.5 m/s b. F ground = 43700 N

[30] t2 = 0.89 s r2 = 1.78 m

[31] m student = 94 kg

[32] v2 = 6.0 m/s

[33] Student falls 5.0 m in 3.03 s

[34] v2 = 1.5 m/s

[35] v1 slowship = 0.029c = 8.73 x 10^6 m/s

[36] v2 fastship = 0.052c = 1.56 x 10^7 m/s

[37] v2 ship = 24.3 m/s

[38] v probe = 667 m/s

[39] v2 platform = 0.28 m/s

[40] v2 platform = 0.025 m/s

[41] v2 balloon = 6.3 m/s