1.4: Analysis Tools - Continuous Charge Distribution

Continuous Charge Distribution

The plastic rod of length L below has uniform charge density l. Find the electric field at all points on the x-axis to the right of the rod.

pic

Electric charge is discrete, meaning it comes in integer multiples of electron and proton charge. Therefore, the electric field can always be calculated by summing the electric field from each of the electrons and protons that make up an object. However, macroscopic objects contain a *lot* of electrons and protons, so this summation has many, many terms:

pic

As described earlier, we will assume that the charge on macroscopic objects is continuous, and distributed throughout the object. Mathematically, this means we will replace a summation over a very large number of discrete charges with an integral over a hypothetically continuous distribution of charge. This leads to a relationship for the electric field at a particular point in space, from a continuous distribution of charge, of:

pic

where dq is the charge on a infinitesimally small portion of the object, and the integral is over the entire physical object.

Finding the electric field from a continuous distribution of charge involves several distinct steps. Until you become very comfortable setting up and evaluating electric field integrals, I would suggest you systematically walk through these steps.

1. *Carefully identify and label the location of the differential element on a diagram of the situation.*
2. *Carefully identify and label the location of the point of interest on a diagram of the situation.*
3. *Write an expression for dq, the charge on the differential element.*
4. *Write an expression for r, the distance between the differential element and the point of interest.*
5. *Write an expression for (r hat), the unit vector representing the direction from the element to the point of interest.*
6. *Insert your expressions into the integral for the electric field.*
7. *Carefully choose the limits of integration.*
8. *Evaluate the integral.*

I'll demonstrate below each of these steps for the scenario under investigation.

1. *Carefully identify and label the location of the differential element on a diagram of the situation.*

The differential element is a small (infinitesimal) piece of the object that we will treat like a point charge. The location of this differential element must be *arbitrary*, meaning it is not at a "special" location like the top, middle, or bottom of the rod. Its location must be represented by a variable, where this variable is the variable of integration and determines the limits of the integral.

For this example, select the differential element to be located a distance "y" above the center of the rod. The length of this element is "dy". (Later, you will select the limits of integration ot go from-L/2 to +L/2 to allow this arbitrary element to "cover" the entire rod.)

pic

2. *Carefully identify and label the location of the point of interest on a diagram of the situation.*

You are interested in the electric field at all points along the x-axis to the right of the rod. Therefore, select an *arbitrary* location along the x-axis and label it with its location.

pic

3. *Write an expression for dq, the charge on the differential element*.

The rod has a uniform charge density l, meaning the amount of charge per unit length along the rod is constant. Since the differential element has a length dy, the total charge on this elment (dq) is the product of the density and the length:

pic

4. *Write an expression for r, the distance between the differential element and the point of interest.*

By Pythagoras' theorem, the distance between the differential element and the point of interest is:

pic

5. *Write an expression for (r hat), the unit vector representing the direction from the element to the point of interest.*

The vector going *from* the element *to* the point of interest heads "down" a distance y and then to the right a distance x. This leads to a unit vector:

pic

6. *Insert your expressions into the integral for the electric field.*

pic

7. *Carefully choose the limits of integration.*

The limits of integration are determined by the range over which the differential element must be "moved" to cover the entire object. The location of the element must vary between the bottom of the rod (-L/2) and the top of the rod (+L/2) in order to include every part of the rod. The two ends of the rod form the two limits of integration.

pic

8. *Evaluate the integral.*

pic

It's typically easier to think of a vector integral as two, separate scalar integrals, one "in the x-direction" and one "in the y-direction", as above.

Examining the x-integral (and using an integral table):

pic

I'll leave it to you to evaluate the y-integral, but you should find that it equals zero. (Without doing calculation whatsoever, you should *know* that the y-component of the electric field along the x-axis *must* be zero because of the symmetry of the situation. If it's not clear why the y-component of the field must be zero, talk to you instructor!)

Thus, the electric field along the x-axis is given by:

pic

Does this function make sense? Two ways to check whether this field is reasonable are to determine how the funciton behaves for very small and very large values of x.

As x gets very small, you are getting closer and closer to the charged rod. This *should* lead to an electric field that grows larger and larger (without bound). Notice that the limit of the function as x approaches zero does "go to infinity", so the function does have the proper behavior for small x.

As x gets very large, you are getting farther and farther to the charged rod. Not only should the field decrease, but as you get very far from the rod should begin to look like a point charge. Notice that as x gets large, the term (L^2/4) becomes negligible compared to x^2. This leads to a field

pic

Since the total charge on the rod (Q) is simply the product of the charge density and the total length of the rod, this reduces to

pic

This is exactly the expression for the electric field from a point charge. Thus, as you move farther and farther from the rod, the rod does indeed begin to look like a point charge.

Paul D’Alessandris (Monroe Community College)