

1.1: Analysis Tools

A Constant Pressure Process

An approximately spherical, 9.4 cm radius helium balloon is brought from a chilly room (15 °C) into a warm room (20 °C).

To analyze this situation, we should first carefully determine and define the sequence of events that take place. At each of these instants, let's tabulate what we know about the state of the helium gas. Then, we can analyze the process that leads from one state to another.

Event 1: The instant before the balloon is brought into the warmer room	Event 2: The instant the balloon stops expanding
$p_1 =$ $V_1 = \frac{4}{3}\pi(0.094 \text{ m})^3$ $T_1 = 15 \text{ }^\circ\text{C} = 288 \text{ K}$	$p_2 =$ $V_2 =$ $T_2 = 20 \text{ }^\circ\text{C} = 293 \text{ K}$

Process:

$$W_{12} =$$

$$Q_{12} =$$

The situation describes a gas that undergoes a process taking it from one state to another. Therefore, at each state I have tabulated the value (if know) of each of the state variables. Between the states I have indicated a location for the recording of the process undertaken as well as the energy transfer during the process, via work or heat.

Do I know anything else about the initial state of the gas in the balloon? The force that the helium gas exerts on the inner surface of the balloon must be equal in magnitude to the force that the outside air exerts on the outer surface of the balloon plus the additional force inward due to the elastic nature of the balloon. If I ignore this additional force due to the elastic balloon, the pressure of the helium must be equal in magnitude to the outside air pressure. Average air pressure on the surface of the earth, referred to as an atmosphere (atm), is

$$1 \text{ atm} = 1.01325 \times 10^5 \text{ Pa}$$

In addition, since the outside air pressure doesn't change, neither does the pressure of the helium gas. Therefore, the process is at constant pressure.

Event 1: The instant before the balloon is brought into the warmer room	Event 2: The instant the balloon stops expanding
$p_1 = 1.01 \times 10^5 \text{ Pa}$ $V_1 = 3.48 \times 10^{-3} \text{ m}^3$ $T_1 = 15 \text{ }^\circ\text{C} = 288 \text{ K}$	$p_2 = 1.01 \times 10^5 \text{ Pa}$ $V_2 =$ $T_2 = 20 \text{ }^\circ\text{C} = 293 \text{ K}$

Process: Constant Pressure

$$W_{12} =$$

$$Q_{12} =$$

The next step is to apply the equation of state to the two states of the gas:

$p_1 V_1 = nRT_1$ $(1.01 \times 10^5 \text{ Pa})(3.48 \times 10^{-3} \text{ m}^3) = n(8.31 \text{ J/K})(288 \text{ K})$ $n = 0.147 \text{ moles}$	$p_2 V_2 = nRT_2$ $(1.01 \times 10^5 \text{ Pa}) V_2 = (0.147)(8.31 \text{ J/K})(293 \text{ K})$ $V_2 = 3.54 \times 10^{-3} \text{ m}^3$ Thus, the final radius of the balloon is: $r_2 = 9.46 \text{ cm}$
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We have completely defined the initial and final states of the helium gas in the balloon, but what about the process that took the balloon from the first state to the second state? From the First Law of Thermodynamics,

$$U_{\text{internal, i}} + W + Q = U_{\text{internal, f}}$$

$$\frac{3}{2}nRT_1 + \left(-\int p dV\right) + Q = \frac{3}{2}nRT_2$$

Since the process is at constant pressure,

$$W = -p \int dV$$

$$W = -p(V_2 - V_1)$$

$$W = -(1.01 \times 10^5 \text{ Pa})(3.54 - 3.48) \times 10^{-3} \text{ m}^3$$

$$W = -6.06 \text{ J}$$

Thus,

$$\frac{3}{2}nRT_1 + \left(-\int p dV\right) + Q = \frac{3}{2}nRT_2$$

$$\frac{3}{2}(0.147)(8.31)(288) - 6.06 + Q = \frac{3}{2}(0.147)(8.31)(293)$$

$$Q = 15.2 \text{ J}$$

During the constant pressure process, 15.2 J of energy were transferred to the gas via heat (from the 20 °C room) and 6.06 J of energy were transferred out of the gas via the work done by the gas in expanding the balloon.

A Constant Volume Process

A miniature (1.1 m³), robotic submarine is loaded with scientific apparatus at the ocean's surface, sealed, and programmed to dive from the surface (15 °C) to a depth of 3000 m (3 °C).

When the submarine is open at the ocean's surface, the air inside the sub is at a pressure of 1 atm. The sub is then sealed so that no air can leak in or out. Assume the sub is made of strong material so that its volume does not change during its descent.

Event 1: The instant the submarine is sealed at the surface	Event 2: The instant the submarine achieves temperature equilibrium at a depth of 3000 m
$p_1 = 1.01 \times 10^5 \text{ Pa}$ $V_1 = 1.1 \text{ m}^3$ $T_1 = 288 \text{ K}$	$p_2 =$ $V_2 = 1.1 \text{ m}^3$ $T_2 = 276 \text{ K}$

Process: Constant Volume

$$W_{12} =$$

$$Q_{12} =$$

The next step is to apply the equation of state to the two states of the gas:

$p_1 V_1 = nRT_1$ $(1.01 \times 10^5 \text{ Pa})(1.1 \text{ m}^3) = n(8.31 \text{ J/K})(288 \text{ K})$ $n = 46.4 \text{ moles}$	$p_2 V_2 = nRT_2$ $p_2 (1.1 \text{ m}^3) = (46.4)(8.31 \text{ J/K})(276 \text{ K})$ $p_2 = 0.968 \times 10^5 \text{ Pa}$
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We have completely defined the initial and final states of the gas in the sub, but what about the process that took the gas from the first state to the second state? From the First Law of Thermodynamics,

$$U_{\text{internal},i} + W + Q = U_{\text{internal},f}$$

$$\frac{3}{2}nRT_1 + \left(-\int p dV\right) + Q = \frac{3}{2}nRT_2$$

Since the process is at constant volume, no energy transfer can take place through work. Thus,

$$\frac{3}{2}nRT_1 + 0 + Q = \frac{3}{2}nRT_2$$

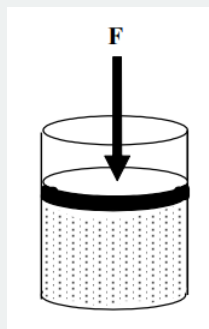
$$\frac{3}{2}(46.4)(8.31)(288) + Q = \frac{3}{2}(46.4)(8.31)(276)$$

$$Q = -6940 \text{ J}$$

During the sub's descent, 6940 J of energy were transferred out of the gas into the surrounding ocean water via heat.

A Constant Temperature Process

The piston at right is immersed in an ice-water bath. With a 100 N force applied to the circular, 2 cm radius piston head, the volume of gas in the piston is 200 cm³. The external force is slowly reduced to zero, leaving only atmospheric pressure pressing on the piston head.



Since the piston is immersed in ice water, the initial temperature of the gas is 0 °C. If the external force is removed slowly, the temperature of the gas should always remain in equilibrium with the ice water bath. Hence, the final temperature of the gas is also 0 °C. Note that the initial pressure of the gas is equal to the sum of atmospheric pressure plus the pressure due to the applied force.

Event 1: The instant before the force is decreased	Event 2: The instant the applied force reaches 0 N
$p_1 = 1.01 \times 10^5 \text{ Pa} + 100 \text{ N}/(\pi(0.02 \text{ m})^2)$ $p_1 = 1.81 \times 10^5 \text{ Pa}$ $V_1 = 200 \text{ cm}^3 = 2 \times 10^{-4} \text{ m}^3$ $T_1 = 273 \text{ K}$	$p_2 = 1.01 \times 10^5 \text{ Pa}$ $V_2 =$ $T_2 = 273 \text{ K}$

Process: Constant Temperature

$$W_{12} =$$

$$Q_{12} =$$

The next step is to apply the equation of state to the two states of the gas:

$p_1 V_1 = nRT_1$ $(1.81 \times 10^5 \text{ Pa})(2 \times 10^{-4} \text{ m}^3) = n(8.31 \text{ J/K})(273 \text{ K})$ $n = 0.0159 \text{ moles}$	$p_2 V_2 = nRT_2$ $(1.01 \times 10^5) V_2 = (0.0159)(8.31 \text{ J/K})(273 \text{ K})$ $V_2 = 3.57 \times 10^{-4} \text{ m}^3$
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We have completely defined the initial and final states of the gas in the piston, but what about the process that took the gas from the first state to the second state? From the First Law of Thermodynamics,

$$U_{\text{internal}, i} + W + Q = U_{\text{internal}, f}$$

$$\frac{3}{2}nRT_1 + \left(-\int p \, dV\right) + Q = \frac{3}{2}nRT_2$$

Let's examine the work done on the gas.

$$W = -\int p \, dV$$

Since the volume is the variable of integration, we must express the pressure in terms of the volume. From the equation of state,

$$pV = nRT$$

$$p = \frac{nRT}{V}$$

Therefore,

$$W = -\int \frac{nRT}{V} dV$$

Since the temperature is constant,

$$W = -nRT \int \frac{dV}{V}$$

$$W = -nRT (\ln(V_2) - \ln(V_1))$$

$$W = -nRT \ln\left(\frac{V_2}{V_1}\right)$$

$$W = -(0.0159)(8.31)(273) \ln\left(\frac{3.57 \times 10^{-4}}{2.0 \times 10^{-4}}\right)$$

$$W = -20.9 \, \text{J}$$

Since the temperature is constant, the internal energy is the same in both states. Thus,

$$\frac{3}{2}nRT_1 - 20.9 + Q = \frac{3}{2}nRT_2$$

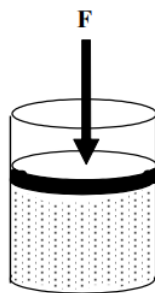
$$\frac{3}{2}nRT_1 - 20.9 + Q = \frac{3}{2}nRT_1$$

$$Q = 20.9 \, \text{J}$$

During the removal of the applied force, 20.9 J of energy were transferred to the gas via heat and 20.9 J of energy were transferred out of the gas via the work done by the expanding gas.

An Adiabatic Process

The piston at right is immersed in an ice-water bath. With a 100 N force applied to the circular, 2 cm radius piston head, the volume of gas in the piston is 200 cm³. The external force is quickly removed, leaving only atmospheric pressure pressing on the piston head.



Since the piston is immersed in ice water, the initial temperature of the gas is 0°C . If the external force is quickly removed, the gas will quickly expand, transferring energy out of the gas via work. In the previous problem, since the expansion was slow, energy was able to flow into the gas, via heat, as this process took place, thus maintaining a constant temperature. If the expansion is fast, there is not sufficient time for energy transfer to take place via heat. A process in which there is no transfer of energy via heat is termed adiabatic. Adiabatic processes occur either very quickly or in systems with thermal insulation.

Event 1: The instant before the force is removed	Event 2: The instant the gas reaches its new equilibrium state
$p_1 = 1.81 \times 10^5 \text{ Pa}$ $V_1 = 200 \text{ cm}^3 = 2 \times 10^{-4} \text{ m}^3$ $T_1 = 273 \text{ K}$	$p_2 = 1.01 \times 10^5 \text{ Pa}$ $V_2 =$ $T_2 =$

Process: Adiabatic

$$W_{12} =$$

$$Q_{12} =$$

The next step is to apply the equation of state to the two states of the gas:

$p_1 V_1 = nRT_1$ $(1.81 \times 10^5 \text{ Pa})(2 \times 10^{-4} \text{ m}^3) = n(8.31 \text{ J/K})(273 \text{ K})$ $n = 0.0159 \text{ moles}$	$p_2 V_2 = nRT_2$ $(1.01 \times 10^5) V_2 = (0.0159)(8.31 \text{ J/K}) T_2$ Uh-oh, two variables in one equation.
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Let's see if the First Law of Thermodynamics can help us out,

$$U_{\text{internal}, i} + W + Q = U_{\text{internal}, f}$$

$$\frac{3}{2}nRT_1 + W + 0 = \frac{3}{2}nRT_2$$

$$W = \frac{3}{2}nR(T_2 - T_1)$$

If we knew the final temperature, we could determine the work done by the gas. If we knew the work done by the gas, we could determine the final temperature. Unfortunately, we don't know either. In previous examples, since one variable was held constant (p , V or T), the equation of state was sufficient to determine the remaining state variables, and the First Law was sufficient to quantify the process. Somehow, from somewhere, we need more information.

Let's look again at the equation of state (pay attention, this gets tricky):

$$pV = nRT$$

If this relation is valid, so is its differential form,

$$d(pV) = d(nRT)$$

Using the product rule and noting that n and R are constants,

$$p \, dV + V \, dp = nR \, dT$$

Since

$$W = - \int p \, dV$$

$$dW = -p \, dV$$

and for an adiabatic process (illustrated above using the First Law),

$$dW = \frac{3}{2} nR \, dT$$

$$-p \, dV = \frac{3}{2} nR \, dT$$

Therefore,

$$p \, dV + V \, dp = nR \, dT$$

$$-\frac{3}{2} nR \, dT + V \, dp = nR \, dT$$

$$V \, dp = \frac{5}{2} nR \, dT$$

$$\frac{3}{5} V \, dp = \frac{3}{2} nR \, dT$$

It is reasonable for you to ask at this point, “What’s the point?”. The point is that we can now relate $p \, dV$ and $V \, dp$:

$$-p \, dV = \frac{3}{2} nR \, dT \qquad \frac{3}{5} V \, dp = \frac{3}{2} nR \, dT$$

$$-p \, dV = \frac{3}{5} V \, dp$$

rearranging

$$-\frac{5}{3} (1/V) \, dV = (1/p) \, dp$$

and integrating

$$-\frac{5}{3} \int (1/V) \, dV = \int (1/p) \, dp$$

$$-\frac{5}{3} \ln(V_2/V_1) = \ln(p_2/p_1)$$

Let’s define the factor $\frac{5}{3}$ to be γ , termed the *adiabatic constant*.

$$-\gamma \ln(V_2/V_1) = \ln(p_2/p_1)$$

$$\ln(V_2/V_1)^{-\gamma} = \ln(p_2/p_1)$$

“Applying” the exponential function to both sides

$$\exp[\ln(V_2/V_1)^{-\gamma}] = \exp[\ln(p_2/p_1)]$$

yields

$$(V_2/V_1)^{-\gamma} = p_2/p_1$$

$$(V_1/V_2)^{\gamma} = p_2/p_1$$

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

We now have a second relationship between the state variables of a gas, valid only for an adiabatic process. Using this relationship (with $\gamma = \frac{5}{3}$),

$$\begin{aligned}p_1 V_1^\gamma &= p_2 V_2^\gamma \\(1.81 \times 10^5 \text{ Pa}) (2 \times 10^{-4} \text{ m}^3)^\gamma &= (1.01 \times 10^5) V_2^\gamma \\0.124 &= (1.01 \times 10^5) V_2^\gamma \\1.23 \times 10^{-6} &= V_2^\gamma \\V_2 &= (1.23 \times 10^{-6})^{1/\gamma} \\V_2 &= 2.84 \times 10^{-4} \text{ m}^3\end{aligned}$$

Now, by re-examining the second state:

$$\begin{aligned}p_2 V_2 &= nRT_2 \\(1.01 \times 10^5) (2.84 \times 10^{-4} \text{ m}^3) &= (0.0159)(8.31 \text{ J/K})T_2 \\T_2 &= 217 \text{ K}\end{aligned}$$

and from the First Law,

$$\begin{aligned}W &= \frac{3}{2}(0.0159)(8.31 \text{ J/K})(217 \text{ K} - 273 \text{ K}) \\W &= -11.1 \text{ J}\end{aligned}$$

During the abrupt removal of the applied force, 11.1 J of energy was transferred out of the gas via the work done by the expanding gas.

Other Processes

The preceding examples are not meant to imply that all (or even most) thermodynamic processes that occur in nature fall into one of the 4 categories examined above. However, at this level of inquiry, only processes in which one of the thermodynamic state variables is held constant, or the process occurs adiabatically, will be investigated in numerical detail.

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