CHAPTER OVERVIEW

4: Expected Value

Expected value is one of the fundamental concepts in probability, in a sense more general than probability itself. The expected value of a real-valued random variable gives a measure of the center of the distribution of the variable. More importantly, by taking the expected value of various functions of a general random variable, we can measure many interesting features of its distribution, including spread, skewness, kurtosis, and correlation. Generating functions are certain types of expected value that completely determine the distribution of the variable. Conditional expected value, which incorporates known information in the computation, is one of the fundamental concepts in probability.

In the advanced topics, we define expected value as an integral with respect to the underlying probability measure. We also revisit conditional expected value from a measure-theoretic point of view. We study vector spaces of random variables with certain expected values as the norms of the spaces, which in turn leads to modes of convergence for random variables.