Preface

Preface to Pfeiffer Applied Probability

The course

This is a "first course" in the sense that it presumes no previous course in probability. The units are modules taken from the unpublished text: Paul E. Pfeiffer, ELEMENTS OF APPLIED PROBABILITY, USING MATLAB. The units are numbered as they appear in the text, although of course they may be used in any desired order. For those who wish to use the order of the text, an outline is provided, with indication of which modules contain the material.

The mathematical prerequisites are ordinary calculus and the elements of matrix algebra. A few standard series and integrals are used, and double integrals are evaluated as iterated integrals. The reader who can evaluate simple integrals can learn quickly from the examples how to deal with the iterated integrals used in the theory of expectation and conditional expectation. Appendix B provides a convenient compendium of mathematical facts used frequently in this work. And the symbolic toolbox, implementing MAPLE, may be used to evaluate integrals, if desired.

In addition to an introduction to the essential features of basic probability in terms of a precise mathematical model, the work describes and employs user defined MATLAB procedures and functions (which we refer to as *m-programs*, or simply *programs*) to solve many important problems in basic probability. This should make the work useful as a stand alone exposition as well as a supplement to any of several current textbooks.

Most of the programs developed here were written in earlier versions of MATLAB, but have been revised slightly to make them quite compatible with MATLAB 7. In a few cases, alternate implementations are available in the Statistics Toolbox, but are implemented here directly from the basic MATLAB program, so that students need only that program (and the symbolic mathematics toolbox, if they desire its aid in evaluating integrals).

Since machine methods require precise formulation of problems in appropriate mathematical form, it is necessary to provide some supplementary analytical material, principally the so-called *minterm analysis*. This material is not only important for computational purposes, but is also useful in displaying some of the structure of the relationships among events.

A probability model

Much of "real world" probabilistic thinking is an amalgam of intuitive, plausible reasoning and of statistical knowledge and insight. Mathematical probability attempts to to lend precision to such probability analysis by employing a suitable *mathematical model*, which embodies the central underlying principles and structure. A successful model serves as an aid (and sometimes corrective) to this type of thinking.

Certain concepts and patterns have emerged from experience and intuition. The mathematical formulation (the mathematical model) which has most successfully captured these essential ideas is rooted in measure theory, and is known as the *Kolmogorov model*, after the brilliant Russian mathematician A.N. Kolmogorov (1903-1987).

One cannot prove that a model is *correct*. Only experience may show whether it is *useful* (and not incorrect). The usefulness of the Kolmogorov model is established by examining its structure and showing that patterns of uncertainty and likelihood in any practical situation can be represented adequately. Developments, such as in this course, have given ample evidence of such usefulness.

The most fruitful approach is characterized by an interplay of

- A formulation of the problem in precise terms of the model and careful mathematical analysis of the problem so formulated.
- A grasp of the problem based on experience and insight. This underlies both problem formulation and interpretation of analytical results of the model. Often such insight suggests approaches to the analytical solution process.

MATLAB: A tool for learning

In this work, we make extensive use of MATLAB as an aid to analysis. I have tried to write the MATLAB programs in such a way that they constitute useful, ready-made tools for problem solving. Once the user understands the problems they are designed to solve, the solution strategies used, and the manner in which these strategies are implemented, the collection of programs should provide a useful resource.

However, my primary aim in exposition and illustration is to *aid the learning process* and to deepen insight into the structure of the problems considered and the strategies employed in their solution. Several features contribute to that end.

1. Application of machine methods of solution requires precise formulation. The data available and the fundamental assumptions must be organized in an appropriate fashion. The requisite *discipline* for such formulation often contributes to enhanced understanding of the problem.
2. The development of a MATLAB program for solution requires careful attention to possible solution strategies. One cannot instruct the machine without a clear grasp of what is to be done.
3. I give attention to the tasks performed by a program, with a general description of how MATLAB carries out the tasks. The reader is not required to trace out all the programming details. However, it is often the case that available MATLAB resources suggest alternative solution strategies. Hence, for those so inclined, attention to the details may be fruitful. I have included, as a separate collection, the m-files written for this work. These may be used as patterns for extensions as well as programs in MATLAB for computations. Appendix A provides a directory of these m-files.
4. Some of the details in the MATLAB script are presentation details. These are refinements which are not essential to the solution of the problem. But they make the programs more readily usable. And they provide illustrations of MATLAB techniques for those who may wish to write their own programs. I hope many will be inclined to go beyond this work, modifying current programs or writing new ones.

An Invitation to Experiment and Explore

Because the programs provide considerable freedom from the burden of computation and the tyranny of tables (with their limited ranges and parameter values), standard problems may be approached with a new spirit of experiment and discovery. When a program is selected (or written), it embodies one method of solution. There may be others which are readily implemented. The reader is invited, even urged, to explore! The user may experiment to whatever degree he or she finds useful and interesting. The possibilities are endless.

Acknowledgments

After many years of teaching probability, I have long since lost track of all those authors and books which have contributed to the treatment of probability in this work. I am aware of those contributions and am most eager to acknowledge my indebtedness, although necessarily without specific attribution.

The power and utility of MATLAB must be attributed to to the long-time commitment of Cleve Moler, who made the package available in the public domain for several years. The appearance of the professional versions, with extended power and improved documentation, led to further appreciation and utilization of its potential in applied probability.

The Mathworks continues to develop MATLAB and many powerful "tool boxes," and to provide leadership in many phases of modern computation. They have generously made available MATLAB 7 to aid in checking for compatibility the programs written with earlier versions. I have not utilized the full potential of this version for developing professional quality user interfaces, since I believe the simpler implementations used herein bring the student closer to the formulation and solution of the problems studied.

CONNEXIONS

The development and organization of the CONNEXIONS modules has been achieved principally by two people: C.S.(Sid) Burrus a former student and later a faculty colleague, then Dean of Engineering, and most importantly a long time friend; and Daniel Williamson, a music major whose keyboard skills have enabled him to set up the text (especially the mathematical expressions) with great accuracy, and whose dedication to the task has led to improvements in presentation. I thank them and others of the CONNEXIONS team who have contributed to the publication of this work.

Paul E. Pfeiffer

Rice University