17.3: Appendix C- Data on some common distributions

Discrete distributions

Indicator function $X = I_E \;\; P(X=1) = P(E) = p \;\; P(X=0) = q = 1-p$

$$E[X] = p \; \text{Var}[X] = pq \; M_X(s) = q + pe^s \;\; g_X(s) = q + ps$$

Simple random variable $X = \sum_{i=1}^n t_i I_{A_i}$ (a primitive form) $P(A_i) = p_i$

$$E[X] = \sum_{i=1}^n t_i p_i \; \text{Var}[X] = \sum_{i=1}^n t_i^2 p_i q_i - 2\sum_{i<j} t_i t_j p_i p_j \;\; M_X(s) = \sum_{i=1}^n p_i e^{st_i}$$

Binomial$(n,p) X = \sum_{i=1}^n I_{E_i}$ with $\{I_{E_i} : 1 \le i \le n\}$ iid $P(E_i) = p$

$$P(X=k) = C(n,k)p^k q^{n-k}$$

$$E[X] = np \; \text{Var}[X] = npq \; M_X(s) = (q+pe^s)^n \;\; g_X(s) = (q+ps)^n$$

MATLAB: $P(X=k) = \text{ibinom}(n,p,k) \;\; P(X \ge k) = \text{cbinom}(n,p,k)$

Geometric$(p) P(X=k) = pq^k \;\; \forall k \ge 0$

$$E[X] = q/p \; \text{Var}[X] = q/p^2 \; M_X(s) = dfracp1 - qe^s \;\; g_X(s) = \frac{p}{1-qs}$$

If $Y-1$ ~ geometric (p), so that $P(Y=k) = pq^{k-1} \;\; \forall k \ge 1$, then

$$E[Y] = 1/p \; \text{Var}[X] = q/p^2 \; M_Y(s) = \frac{pe^s}{1-qe^s} \;\; g_Y(s) = \frac{ps}{1-qs}$$

Negative binomial(m,p), X is the number of failures before the mth success.

$P(X=k) = C(m+k-1, m-1)p^m q^k \;\;\; \forall k \ge 0$

$$E[X] = mq/p \; \text{Var}[X] = mq/p^2 \; M_X(s) = (\frac{p}{1-qe^s})^m \;\; g_X(s) = (\frac{p}{1-qs})^m$$

For $Y_m = X_m + m$, the number of the trial on which mth success occurs. $P(Y=k) = C(k-1, m-1)p^m q^{k-m}$ $\forall k \ge m$.

$$E[Y] = m/p \; \text{Var}[Y] = mq/p^2 \; M_Y(s) = (\frac{pe^s}{1-qe^s})^m \;\; g_Y(s) = (\frac{ps}{1-qs})^m$$

MATLAB: $P(Y=k) = \text{nbinom}(m,p,k)$

Poisson(μ). $P(X=k) = e^{-\mu}\frac{\mu^k}{k!} \;\; \forall k \ge 0$

$$E[X] = \mu \; \text{Var}[X] = \mu \; M_X(s) = e^{\mu(e^s-1)} \;\; g_X(s) = e^{\mu(s-1)}$$

MATLAB: $P(X=k) = \text{ipoisson}(m,k) \;\; P(X \ge k) = \text{cpoisson}(m,k)$

Absolutely continuous distributions

Uniform$(a,b) \; f_x(t) = \frac{1}{b-a} \;\; a < t < b$ (zero elsewhere)

$$E[X] = \frac{b+a}{2} \;\; \text{Var}[X] = \frac{(b-a)^2}{12} \;\; M_X(s) = \frac{e^{sb} - e^{sa}}{s(b-a)}$$

Symmetric triangular $(-a,a) \;\; f_X(t) = \begin{cases} (a+t)/a^2 & -a \le t < 0 \\ (a-t)/a^2 & 0 \le t \le a \end{cases}$

$$E[X] = 0 \; \text{Var}[X] = \frac{a^2}{6} \;\; M_X(s) = \frac{e^{as} + e^{-as} - 2}{a^2 s^2} = \frac{e^{as}-1}{as} \cdot \frac{1-e^{-as}}{as}$$

Exponential$(\lambda) f_X(t) = \lambda e^{-\lambda t} \;\; t \ge 0$

$$E[X] = \frac{1}{\lambda} \;\; \text{Var}[X] = \frac{1}{\lambda^2} \;\; M_X(s) = \frac{\lambda}{\lambda - s}$$

Gamma$(\alpha, \lambda) f_X(t) = \dfrac{\lambda^\alpha t^{\alpha - 1} e^{-\lambda t}}{\Gamma(\alpha)} \ \ t \ge 0$

$$E[X] = \frac{\alpha}{\lambda} \ \text{Var}[X] = \frac{\alpha}{\lambda^2} \ M_X(s) = (\frac{\lambda}{\lambda - s})^\alpha$$

MATLAB: $P(X \le t) = \text{gammadbn}(\alpha, \lambda, t)$

Normal$N(\mu, \sigma^2) f_X(t) = \dfrac{1}{\sigma\sqrt{2\pi}} \exp(-\dfrac{1}{2}(\dfrac{t-\mu}{\sigma})^2)$

$$E[X] = \mu \ \text{Var}[X]\sigma^2 \ M_X(s) = \exp(\frac{\sigma^2 s^2}{2} + \mu s)$$

MATLAB: $P(X \le t) = \text{gaussian}(\mu, \sigma^2, t)$

Beta(r, s)

$$f_X(t) = \frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)} t^{r-1}(1-t)^{s-1} \ \ 0 < t < 1\,, r > 0,\, s > 0$$

$$E[X] = \frac{r}{r+s} \ \ \text{Var}[X] = \frac{rs}{(r+s)^2(r+s+1)}$$

MATLAB: $f_X(t) = \text{beta}(r, s, t) \ \ P(X \le t) = \text{betadbn}(r, s, t)$

Weibull(α, λ, ν)

$$F_X(t) = 1 - e^{-\lambda(t-\nu)^\alpha} \ , \alpha > 0, \lambda > 0, \nu \ge 0, t \ge \nu$$

$$E[X] = \frac{1}{\lambda^{1/\alpha}}\Gamma(1 + 1/\alpha) + \nu \ \ \text{Var}[X] = \frac{1}{\lambda^{2/\alpha}}[\Gamma(1 + 2/\lambda) - \Gamma^2(1 + 1/\lambda)]$$

MATLAB: ($\nu = 0$ only)

$$f_X(t) = \text{weibull}(a, l, t) \ P(X \le t) = \text{weibull}(a, l, t)$$

Relationship between gamma and Poisson distributions

- If X ~ gamma (n, λ), then $P(X \le t) = P(Y \ge n)$ where Y ~ Poisson (λt).
- If Y ~ Poisson (λt), then $P(Y \ge n) = P(X \le t)$ where X ~ gamma (n, λ).

This page titled 17.3: Appendix C- Data on some common distributions is shared under a CC BY 3.0 license and was authored, remixed, and/or curated by Paul Pfeiffer via source content that was edited to the style and standards of the LibreTexts platform.