

4.4: Problems on Independence of Events

Exercise 4.4.1

The minterms generated by the class $\{A, B, C\}$ have minterm probabilities

$$pm = [0.15 \ 0.05 \ 0.02 \ 0.18 \ 0.25 \ 0.05 \ 0.18 \ 0.12]$$

Show that the product rule holds for all three, but the class is not independent.

Answer

```
pm = [0.15 0.05 0.02 0.18 0.25 0.05 0.18 0.12];
y = imintest(pm)
The class is NOT independent
Minterms for which the product rule fails
y =
    1     1     1     0
    1     1     1     0    % The product rule hold for M7 = ABC
```

Exercise 4.4.2

The class $\{A, B, C, D\}$ is independent, with respective probabilities 0.65, 0.37, 0.48, 0.63. Use the m-function minprob to obtain the minterm probabilities. Use the m-function minmap to put them in a 4 by 4 table corresponding to the minterm map convention we use.

Answer

```
P = [0.65 0.37 0.48 0.63];
p = minmap(minprob(P))
p =
    0.0424    0.0249    0.0788    0.0463
    0.0722    0.0424    0.1342    0.0788
    0.0392    0.0230    0.0727    0.0427
    0.0667    0.0392    0.1238    0.0727
```

Exercise 4.4.3

The minterm probabilities for the software survey in [Example 2](#) from "Minterms" are

$$pm = [0 \ 0.05 \ 0.10 \ 0.05 \ 0.20 \ 0.10 \ 0.40 \ 0.10]$$

Show whether or not the class $\{A, B, C\}$ is independent: (1) by hand calculation, and (2) by use of the m-function imintest.

Answer

```
pm = [0 0.05 0.10 0.05 0.20 0.10 0.40 0.10];
y = imintest(pm)
The class is NOT independent
Minterms for which the product rule fails
y =
```

1	1	1	1	% By hand check product rule for any minterm
1	1	1	1	

Exercise 4.4.4

The minterm probabilities for the computer survey in [Example 3](#) from "Minterms" are

$$pm = [0.032 \ 0.016 \ 0.376 \ 0.011 \ 0.364 \ 0.073 \ 0.077 \ 0.051]$$

Show whether or not the class $\{A, B, C\}$ is independent: (1) by hand calculation, and (2) by use of the m-function imintest.

Answer

npr04_04

Minterm probabilities for Exercise 4.4.4. are in pm

y = imintest(pm)

The class is NOT independent

Minterms for which the product rule fails

y =

1	1	1	1
1	1	1	1

Exercise 4.4.5

Minterm probabilities $p(0)$ through $p(15)$ for the class $\{A, B, C, D\}$ are, in order,

$$pm = [0.084 \ 0.196 \ 0.036 \ 0.084 \ 0.085 \ 0.196 \ 0.035 \ 0.084 \ 0.021 \ 0.049 \ 0.009 \ 0.021 \ 0.020 \ 0.049 \ 0.010 \ 0.021]$$

Use the m-function imintest to show whether or not the class $\{A, B, C, D\}$ is independent.

Answer

npr04_05

Minterm probabilities for Exercise 4.4.5. are in pm

imintest(pm)

The class is NOT independent

Minterms for which the product rule fails

ans =

0	1	0	1
0	0	0	0
0	1	0	1
0	0	0	0

Exercise 4.4.6

Minterm probabilities $p(0)$ through $p(15)$ for the opinion survey in [Example 4](#) from "Minterms" are

$$pm = [0.085 \ 0.195 \ 0.035 \ 0.085 \ 0.080 \ 0.200 \ 0.035 \ 0.085 \ 0.020 \ 0.050 \ 0.010 \ 0.020 \ 0.020 \ 0.050 \ 0.015 \ 0.015]$$

show whether or not the class $\{A, B, C, D\}$ is independent.

Answer

npr04_06

Minterm probabilities for Exercise 4.4.6. are in pm

y = imintest(pm)

The class is NOT independent

Minterms for which the product rule fails

y =

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

Exercise 4.4.7

The class $\{A, B, C\}$ is independent, with $P(A) = 0.30$, $P(B^cC) = 0.32$, and $P(AC) = 0.12$. Determine the minterm probabilities.

Answer

$$P(C) = P(AC)/P(A) = 0.40 \text{ AND } P(B) = 1 - P(B^cC)/P(C) = 0.20 .$$

```
pm = minprob([0.3 0.2 0.4])
pm = 0.3360 0.2240 0.0840 0.0560 0.1440 0.0960 0.0360 0.0240
```

Exercise 4.4.8

The class $\{A, B, C\}$ is independent, with $P(A \cup B) = 0.6$, $P(A \cup C) = 0.7$, and $P(C) = 0.4$. Determine the probability of each minterm.

Answer

$$P(A^cC^c) = P(A^c)P(C^c) = 0.3 \text{ implies } P(A^c) = 0.3/0.6 = 0.5 = P(A).$$

$$P(A^cB^c) = P(A^c)P(B^c) = 0.4 \text{ implies } P(B^c) = 0.4/0.5 = 0.8 \text{ implies } P(B) = 0.2$$

```
P = [0.5 0.2 0.4];
pm = minprob(P)
pm = 0.2400 0.1600 0.0600 0.0400 0.2400 0.1600 0.0600 0.0400
```

Exercise 4.4.9

A pair of dice is rolled five times. What is the probability the first two results are “sevens” and the others are not?

Answer

$$P = (1/6)^2(5/6)^3 = 0.0161.$$

Exercise 4.4.10

David, Mary, Joan, Hal, Sharon, and Wayne take an exam in their probability course. Their probabilities of making 90 percent or more are

0.72 0.83 0.75 0.92 0.65 0.79

respectively. Assume these are independent events. What is the probability three or more, four or more, five or more make grades of at least 90 percent?

Answer

```
P = 0.01*[72 83 75 92 65 79];
y = ckn(P,[3 4 5])
y =    0.9780    0.8756    0.5967
```

Exercise 4.4.11

Two independent random numbers between 0 and 1 are selected (say by a random number generator on a calculator). What is the probability the first is no greater than 0.33 and the other is at least 57?

Answer

$$P = 0.33 \cdot (1 - 0.57) = 0.1419$$

Exercise 4.4.12

Helen is wondering how to plan for the weekend. She will get a letter from home (with money) with probability 0.05. There is a probability of 0.85 that she will get a call from Jim at SMU in Dallas. There is also a probability of 0.5 that William will ask for a date. What is the probability she will get money and Jim will not call or that both Jim will call and William will ask for a date?

Answer

$A \sim$ letter with money, $B \sim$ call from Jim, $C \sim$ William ask for date

```
P = 0.01*[5 85 50];
minvec3
Variables are A, B, C, Ac, Bc, Cc
They may be renamed, if desired.
pm = minprob(P);
p = ((A&Bc)|(B&C))*pm'
p =    0.4325
```

Exercise 4.4.13

A basketball player takes ten free throws in a contest. On her first shot she is nervous and has probability 0.3 of making the shot. She begins to settle down and probabilities on the next seven shots are 0.5, 0.6 0.7 0.8 0.8, 0.8 and 0.85, respectively. Then she realizes her opponent is doing well, and becomes tense as she takes the last two shots, with probabilities reduced to 0.75, 0.65. Assuming independence between the shots, what is the probability she will make k or more for $k = 2, 3, \dots, 10$?

Answer

```
P = 0.01*[30 50 60 70 80 80 80 85 75 65];
k = 2:10;
p = ckn(P,k)
p =
Columns 1 through 7
    0.9999    0.9984    0.9882    0.9441    0.8192    0.5859    0.3043
```

Columns 8 through 9
0.0966 0.0134

Exercise 4.4.14

In a group there are M men and W women; m of the men and w of the women are college graduates. An individual is picked at random. Let A be the event the individual is a woman and B be the event he or she is a college graduate. Under what condition is the pair $\{A, B\}$ independent?

Answer

$$P(A|B) = w/(m+w) = W/(W+M) = P(A)$$

Exercise 4.4.15

Consider the pair $\{A, B\}$ of events. Let $P(A) = p$, $P(A^c) = q = 1 - p$, $P(B|A) = p_1$, and $P(B|A^c) = p_2$. Under what condition is the pair $\{A, B\}$ independent?

Answer

$$p_1 = P(B|A) = P(B|A^c) = p_2 \quad (\text{see table of equivalent conditions}).$$

Exercise 4.4.16

Show that if event A is independent of itself, then $P(A) = 0$ or $P(A) = 1$. (This fact is key to an important "zero-one law".)

Answer

$$P(A) = P(A \cap A) = P(A)P(A) \quad . \quad x^2 = x \text{ iff } x = 0 \text{ or } x = 1.$$

Exercise 4.4.17

Does $\{A, B\}$ independent and $\{B, C\}$ independent imply $\{A, C\}$ is independent? Justify your answer.

Answer

```
% No. Consider for example the following minterm probabilities:
pm = [0.2 0.05 0.125 0.125 0.05 0.2 0.125 0.125];
minvec3
Variables are A, B, C, Ac, Bc, Cc
They may be renamed, if desired.
PA = A*pm'
PA = 0.5000
PB = B*pm'
PB = 0.5000
PC = C*pm'
PC = 0.5000
PAB = (A&B)*pm' % Product rule holds
PAB = 0.2500
PBC = (B&C)*pm' % Product rule holds
PBC = 0.2500
```

```
PAC = (A&C)*pm' % Product rule fails
PAC = 0.3250
```

Exercise 4.4.18

Suppose event A implies B (i.e. $A \subset B$). Show that if the pair $\{A, B\}$ is independent, then either $P(A) = 0$ or $P(B) = 1$.

Answer

$A \subset B$ implies $P(AB) = P(A)$; independence implies $P(AB) = P(A)P(B)$. $P(A) = P(A)P(B)$ only if $P(B) = 1$ or $P(A) = 0$.

Exercise 4.4.19

A company has three task forces trying to meet a deadline for a new device. The groups work independently, with respective probabilities 0.8, 0.9, 0.75 of completing on time. What is the probability at least one group completes on time? (Think. Then solve “by hand.”)

Answer

At least one completes iff not all fail. $P = 1 - 0.2 \cdot 0.1 \cdot 0.25 = 0.9950$

Exercise 4.4.20

Two salesmen work differently. Roland spends more time with his customers than does Betty, hence tends to see fewer customers. On a given day Roland sees five customers and Betty sees six. The customers make decisions independently. If the probabilities for success on Roland's customers are 0.7, 0.8, 0.8, 0.6, 0.7 and for Betty's customers are 0.6, 0.5, 0.4, 0.6, 0.6, 0.4, what is the probability Roland makes more sales than Betty? What is the probability that Roland will make three or more sales? What is the probability that Betty will make three or more sales?

Answer

```
PR = 0.1*[7 8 8 6 7];
PB = 0.1*[6 5 4 6 6 4];
PR3 = ckn(PR,3)
PR3 = 0.8662
PB3 = ckn(PB,3)
PB3 = 0.6906
PRgB = ikn(PB,0:4)*ckn(PR,1:5)'
PRgB = 0.5065
```

Exercise 4.4.21

Two teams of students take a probability exam. The entire group performs individually and independently. Team 1 has five members and Team 2 has six members. They have the following individual probabilities of making an “A” on the exam.

Team 1: 0.83 0.87 0.92 0.77 0.86 Team 2: 0.68 0.91 0.74 0.68 0.73 0.83

- What is the probability team 1 will make *at least as many* A's as team 2?
- What is the probability team 1 will make *more* A's than team 2?

Answer

```
P1 = 0.01*[83 87 92 77 86];  
P2 = 0.01*[68 91 74 68 73 83];  
P1geq = ikn(P2,0:5)*ckn(P1,0:5)'  
P1geq = 0.5527  
P1g = ikn(P2,0:4)*ckn(P1,1:5)'  
P1g = 0.2561
```

Exercise 4.4.22

A system has five components which fail independently. Their respective reliabilities are 0.93, 0.91, 0.78, 0.88, 0.92. Units 1 and 2 operate as a “series” combination. Units 3, 4, 5 operate as a two of three subsystem. The two subsystems operate as a parallel combination to make the complete system. What is reliability of the complete system?

Answer

```
R = 0.01*[93 91 78 88 92];  
Ra = prod(R(1:2))  
Ra = 0.8463  
Rb = ckn(R(3:5),2)  
Rb = 0.9506  
Rs = parallel([Ra Rb])  
Rs = 0.9924
```

Exercise 4.4.23

A system has eight components with respective probabilities

0.96 0.90 0.93 0.82 0.85 0.97 0.88 0.80

Units 1 and 2 form a parallel subsystem in series with unit 3 and a three of five combination of units 4 through 8. What is the reliability of the complete system?

Answer

```
R = 0.01*[96 90 93 82 85 97 88 80];  
Ra = parallel(R(1:2))  
Ra = 0.9960  
Rb = ckn(R(4:8),3)  
Rb = 0.9821  
Rs = prod([Ra R(3) Rb])  
Rs = 0.9097
```

Exercise 4.4.24

How would the reliability of the system in Exercise 4.4.23. change if units 1, 2, and 3 formed a parallel combination in series with the three of five combination?

Answer

```
Rc = parallel(R(1:3))
Rc = 0.9997
Rss = prod([Rb Rc])
Rss = 0.9818
```

Exercise 4.4.25

How would the reliability of the system in Exercise 4.4.23. change if the reliability of unit 3 were changed from 0.93 to 0.96? What change if the reliability of unit 2 were changed from 0.90 to 0.95 (with unit 3 unchanged)?

Answer

```
R1 = R;
R1(3) = 0.96;
Ra = parallel(R1(1:2))
Ra = 0.9960
Rb = ckn(R1(4:8), 3)
Rb = 0.9821
Rs3 = prod([Ra R1(3) Rb])
Rs3 = 0.9390
R2 = R;
R2(2) = 0.95;
Ra = parallel(R2(1:2))
Ra = 0.9980
Rb = ckn(R2(4:8), 3)
Rb = 0.9821
Rs4 = prod([Ra R2(3) Rb])
Rs4 = 0.9115
```

Exercise 4.4.26

Three fair dice are rolled. What is the probability at least one will show a six?

Answer

$$P = 1 - (5/6)^3 = 0.4213$$

Exercise 4.4.27

A hobby shop finds that 35 percent of its customers buy an electronic game. If customers buy independently, what is the probability that at least one of the next five customers will buy an electronic game?

Answer

$$P = 1 - 0.65^5 = 0.8840$$

Exercise 4.4.28

Under extreme noise conditions, the probability that a certain message will be transmitted correctly is 0.1. Successive messages are acted upon independently by the noise. Suppose the message is transmitted ten times. What is the probability it is transmitted correctly at least once?

Answer

$$P = 1 - 0.9^{10} = 0.6513$$

Exercise 4.4.29

Suppose the class $\{A_i : 1 \leq i \leq n\}$ is independent, with $P(A_i) = p_i, 1 \leq i \leq n$. What is the probability that at least one of the events occurs? What is the probability that none occurs?

Answer

$$P1 = 1 - P0, P0 = \prod_{i=1}^n (1 - p_i)$$

Exercise 4.4.30

In one hundred random digits, 0 through 9, with each possible digit equally likely on each choice, what is the probability 8 or more are sevens?

Answer

$$P = \text{cbinom}(100, 0.1, 8) = 0.7939$$

Exercise 4.4.31

Ten customers come into a store. If the probability is 0.15 that each customer will buy a television set, what is the probability the store will sell three or more?

Answer

$$P = \text{cbinom}(10, 0.15, 3) = 0.1798$$

Exercise 4.4.32

Seven similar units are put into service at time $t = 0$. The units fail independently. The probability of failure of any unit in the first 400 hours is 0.18. What is the probability that three or more units are still in operation at the end of 400 hours?

Answer

$$P = \text{cbinom}(7, 0.82, 3) = 0.9971$$

Exercise 4.4.33

A computer system has ten similar modules. The circuit has redundancy which ensures the system operates if any eight or more of the units are operative. Units fail independently, and the probability is 0.93 that any unit will survive between maintenance periods. What is the probability of no system failure due to these units?

Answer

$$P = \text{cbinom}(10, 0.93, 8) = 0.9717$$

Exercise 4.4.34

Only thirty percent of the items from a production line meet stringent requirements for a special job. Units from the line are tested in succession. Under the usual assumptions for Bernoulli trials, what is the probability that three satisfactory units will be found in eight or fewer trials?

Answer

$$P = \text{cbinom}(8, 0.3, 3) = 0.4482$$

Exercise 4.4.35

The probability is 0.02 that a virus will survive application of a certain vaccine. What is the probability that in a batch of 500 viruses, fifteen or more will survive treatment?

Answer

$$P = \text{cbinom}(500, 0.02, 15) = 0.0814$$

Exercise 4.4.36

In a shipment of 20,000 items, 400 are defective. These are scattered randomly throughout the entire lot. Assume the probability of a defective is the same on each choice. What is the probability that

1. Two or more will appear in a random sample of 35?
2. At most five will appear in a random sample of 50?

Answer

$$P1 = \text{cbinom}(35, 0.02, 2) = 0.1547.$$

$$P2 = 1 - \text{cbinom}(35, 0.02, 6) = 0.9999$$

Exercise 4.4.37

A device has probability p of operating successfully on any trial in a sequence. What probability p is necessary to ensure the probability of successes on all of the first four trials is 0.85? With that value of p , what is the probability of four or more successes in five trials?

Answer

$$p = 0.85^{1/4} \approx 0.954 \quad P(\text{cbinom}(5, p, 4) = 0.9854.$$

Exercise 4.4.38

A survey form is sent to 100 persons. If they decide independently whether or not to reply, and each has probability 1/4 of replying, what is the probability of k or more replies, where $k = 15, 20, 25, 30, 35, 40$?

Answer

$P = \text{cbinom}(100, 1/4, 15:40)$						
$P =$	0.9946	0.9005	0.5383	0.1495	0.0164	0.0007

Exercise 4.4.39

Ten numbers are produced by a random number generator. What is the probability four or more are less than or equal to 0.63?

Answer

$$P1 = \text{cbinom}(10, 0.63, 4) = 0.9644$$

Exercise 4.4.40

A player rolls a pair of dice five times. She scores a “hit” on any throw if she gets a 6 or 7. She wins iff she scores an *odd* number of hits in the five throws. What is the probability a player wins on any sequence of five throws? Suppose she plays the game 20 successive times. What is the probability she wins at least 10 times? What is the probability she wins more than half the time?

Answer

$$\text{Each roll yields a hit with probability } p = \frac{6}{36} + \frac{5}{36} = \frac{11}{36}.$$

```
PW = sum(ibinom(5, 11/36, [1 3 5]))
PW = 0.4956
P2 = cbinom(20, PW, 10)
P2 = 0.5724
P3 = cbinom(20, PW, 11)
P3 = 0.3963
```

Exercise 4.4.41

Erica and John spin a wheel which turns up the integers 0 through 9 with equal probability. Results on various trials are independent. Each spins the wheel 10 times. What is the probability Erica turns up a seven more times than does John?

Answer

$$P = \text{ibinom}(10, 0.1, 0:9) * \text{cbinom}(10, 0.1, 1:10)' = 0.3437$$

Exercise 4.4.42

Erica and John play a different game with the wheel, above. Erica scores a point each time she gets an integer 0, 2, 4, 6, or 8. John scores a point each time he turns up a 1, 2, 5, or 7. If Erica spins eight times; John spins 10 times. What is the probability John makes *more* points than Erica?

Answer

$$P = \text{ibinom}(8, 0.5, 0:8) * \text{cbinom}(10, 0.4, 1:9)' = 0.4030$$

Exercise 4.4.43

A box contains 100 balls; 30 are red, 40 are blue, and 30 are green. Martha and Alex select at random, with replacement and mixing after each selection. Alex has a success if he selects a red ball; Martha has a success if she selects a blue ball. Alex selects seven times and Martha selects five times. What is the probability Martha has more successes than Alex?

Answer

$$P = \text{ibinom}(7, 0.3, 0:4) * \text{cbinom}(5, 0.4, 1:5)' = 0.3613$$

Exercise 4.4.44

Two players roll a fair die 30 times each. What is the probability that each rolls the same number of sixes?

Answer

$$P = \text{sum}(\text{ibinom}(30, 1/6, 0:30).^2) = 0.1386$$

Exercise 4.4.45

A warehouse has a stock of n items of a certain kind, r of which are defective. Two of the items are chosen at random, without replacement. What is the probability that at least one is defective? Show that for large n the number is very close to that for selection with replacement, which corresponds to two Bernoulli trials with probability $p = r/n$ of success on any trial.

Answer

$$P1 = \frac{r}{n} \cdot \frac{r-1}{n-1} + \frac{r}{n} \cdot \frac{n-r}{n-1} + \frac{n-r}{n} \cdot \frac{r}{n-1} = \frac{(2n-1)r - r^2}{n(n-1)}$$

$$P2 = 1 - \left(\frac{r}{n}\right)^2 = \frac{2nr - r^2}{n^2}$$

Exercise 4.4.46

A coin is flipped repeatedly, until a head appears. Show that with probability one the game will terminate.

tip:

The probability of not terminating in n trials is q^n .

Answer

Let N = event never terminates and N_k = event does not terminate in k plays. Then $N \subset N_k$ for all k implies $0 \leq P(N) \leq P(N_k) = 1/2^k$ for all k , we conclude $P(N) = 0$.

Exercise 4.4.47

Two persons play a game consecutively until one of them is successful or there are ten unsuccessful plays. Let E_i be the event of a success on the i th play of the game. Suppose $\{E_i : 1 \leq i\}$ is an independent class with $P(E_i) = p_1$ for i odd and $P(E_i) = p_2$ for i even. Let A be the event the first player wins, B be the event the second player wins, and C be the event that neither wins.

- Express A , B , and C in terms of the E_i .
- Determine $P(A)$, $P(B)$, and $P(C)$ in terms of p_1 , p_2 , $q_1 = 1 - p_1$, and $q_2 = 1 - p_2$. Obtain numerical values for the case $p_1 = 1/4$ and $p_2 = 1/3$.
- Use appropriate facts about the geometric series to show that $P(A) = P(B)$ iff $p_1 = p_2/(1 + p_2)$.
- Suppose $p_2 = 0.5$. Use the result of part (c) to find the value of p_1 to make $P(A) = P(B)$ and then determine $P(A)$, $P(B)$, and $P(C)$.

Answer

$$a. C = \bigcap_{i=1}^{10} E_i^c$$

$$A = E_1 \vee E_1^c E_2^c E_3 \vee E_1^c E_2^c E_3^c E_4^c E_5 \vee E_1^c E_2^c E_3^c E_4^c E_5^c E_6^c E_7 \vee E_1^c E_2^c E_3^c E_4^c E_5^c E_6^c E_7^c E_8^c E_9$$

$$B = E_1^c E_2 \vee E_1^c E_2^c E_3^c E_4 \vee E_1^c E_2^c E_3^c E_4^c E_5^c E_6 \vee E_1^c E_2^c E_3^c E_4^c E_5^c E_6^c E_7^c E_8 \vee E_1^c E_2^c E_3^c E_4^c E_5^c E_6^c E_7^c E_8^c E_9^c E_{10}$$

$$P(A) = p_1[1 + q_1 q_2 + (q_1 q_2)^2 + (q_1 q_2)^3 + (q_1 q_2)^4] = p_1 \frac{1 - (q_1 q_2)^5}{1 - q_1 q_2}$$

$$P(B) = q_1 p_2 \frac{1 - (q_1 q_2)^5}{1 - q_1 q_2} \quad P(C) = (q_1 q_2)^5$$

For $p_1 = 1/4$, $p_2 = 1/3$, we have $q_1 q_2 = 1/2$ and $q_1 p_2 = 1/4$. In this case

$$P(A) = \frac{1}{4} \cdot \frac{31}{16} = 31/64 = 0.4844 = P(B), \quad P(C) = 1/32$$

Note that $P(A) + P(B) + P(C) = 1$.

c. $P(A) = P(B)$ iff $p_1 = q_1 p_2 = (1 - p_1) p_2$ iff $p_1 = p_2 / (1 + p_2)$.

d. $p_1 = 0.5 / 1.5 = 1/3$

Exercise 4.4.48

Three persons play a game consecutively until one achieves his objective. Let E_i be the event of a success on the i th trial, and suppose $\{E_i : 1 \leq i\}$ is an independent class, with $P(E_i) = p_1$ for $i = 1, 4, 7, \dots$, $P(E_i) = p_2$ for $i = 2, 5, 8, \dots$, and $P(E_i) = p_3$ for $i = 3, 6, 9, \dots$. Let A, B, C be the respective events the first, second, and third player wins.

a. Express A, B , and C in terms of the E_i .

b. Determine the probabilities in terms of p_1, p_2, p_3 , then obtain numerical values in the case $p_1 = 1/4$, $p_2 = 1/3$, and $p_3 = 1/2$.

Answer

$$a. A = E_1 \vee \bigvee_{k=1}^{\infty} \bigcap_{i=1}^{3k} E_i^c E_{3k+1}$$

$$B = E_1^c E_2 \vee \bigvee_{k=1}^{\infty} \bigcap_{i=1}^{3k+1} E_i^c E_{3k+2}$$

$$C = E_1^c E_2^c E_3 \vee \bigvee_{k=1}^{\infty} \bigcap_{i=1}^{3k+2} E_i^c E_{3k+3}$$

$$b. P(A) = p_1 \sum_{k=0}^{\infty} (q_1 q_2 q_3)^k = \frac{p_1}{1 - q_1 q_2 q_3}$$

$$P(B) = \frac{q_1 p_2}{1 - q_1 q_2 q_3}$$

$$P(C) = \frac{q_1 q_2 p_3}{1 - q_1 q_2 q_3}$$

For $p_1 = 1/4$, $p_2 = 1/3$, $p_3 = 1/2$, $P(A) = P(B) = P(C) = 1/3$.

Exercise 4.4.49

What is the probability of a success on the i th trial in a Bernoulli sequence of n component trials, given there are r successes?

Answer

$$P(A_{rn} E_i) = p C(n-1, r-1) p^{r-1} q^{n-r} \quad \text{and} \quad P(A_{rn}) = C(n, r) p^r q^{n-r}.$$

$$\text{Hence } P(E_i | A_{rn}) = C(n-1, r-1) / C(n, r) = r/n.$$

Exercise 4.4.50

A device has N similar components which may fail independently, with probability p of failure of any component. The device fails if one or more of the components fails. In the event of failure of the device, the components are tested sequentially.

- What is the probability the first defective unit tested is the n th, given one or more components have failed?
- What is the probability the defective unit is the n th, given that exactly one has failed?
- What is the probability that more than one unit has failed, given that the first defective unit is the n th?

Answer

Let A_1 = event one failure, B_1 = event of one or more failures, B_2 = event of two or more failures, and F_n = the event the first defective unit found is the n th.

a. $F_n \subset B_1$ implies $P(F_n|B_1) = P(F_n)/P(B_1) = \frac{q^{n-1}p}{1 - q^N}$

$$P(F_n|A_1) = \frac{P(F_n A_1)}{P(A_1)} = \frac{q^{n-1}pq^{N-n}}{Npq^{N-1}} = \frac{1}{N}$$

(see Exercise)

b. Since probability not all from n th are good is $1 - q^{N-n}$.

$$P(B_2|F_n) = \frac{P(B_2 F_n)}{P(F_n)} = \frac{q^{n-1}p(1 - q^{N-1})}{q^{n-1}p} = 1 - q^{N-n}$$

This page titled [4.4: Problems on Independence of Events](#) is shared under a [CC BY 3.0](#) license and was authored, remixed, and/or curated by [Paul Pfeiffer](#) via [source content](#) that was edited to the style and standards of the LibreTexts platform.