15.3: Problems on Random Selection

Exercise 15.3.1

(See Exercise 3 from "Problems on Random Variables and Joint Distributions") A die is rolled. Let X be the number of spots that turn up. A coin is flipped X times. Let Y be the number of heads that turn up. Determine the distribution for Y.

Answer

```
PX = [0 (1/6)*ones(1,6)];
PY = [0.5 0.5];
gend
Do not forget zero coefficients for missing powers
Enter gen fn COEFFICIENTS for gN  PX
Enter gen fn COEFFICIENTS for gY  PY
Results are in N, PN, Y, PY, D, PD, P
May use jcalc or jcalcf on N, D, P
To view the distribution, call for gD.
disp(gD)             % Compare with P8-3
         0    0.1641
    1.0000    0.3125
    2.0000    0.2578
    3.0000    0.1667
    4.0000    0.0755
    5.0000    0.0208
    6.0000    0.0026
```

Exercise 15.3.2

(See Exercise 4 from "Problems on Random Variables and Joint Distributions") As a variation of Exercise 15.3.1, suppose a pair of dice is rolled instead of a single die. Determine the distribution for Y.

Answer

```
PN = (1/36)*[0 0 1 2 3 4 5 6 5 4 3 2 1];
PY = [0.5 0.5];
gend
Do not forget zero coefficients for missing powers
Enter gen fn COEFFICIENTS for gN  PN
Enter gen fn COEFFICIENTS for gY  PY
Results are in N, PN, Y, PY, D, PD, P
May use jcalc or jcalcf on N, D, P
To view the distribution, call for gD.
disp(gD)
         0    0.0269
    1.0000    0.1025
    2.0000    0.1823
    3.0000    0.2158
```

```
     4.0000    0.1954
     5.0000    0.1400
     6.0000    0.0806
     7.0000    0.0375
     8.0000    0.0140     % (Continued next page)
     9.0000    0.0040
    10.0000    0.0008
    11.0000    0.0001
    12.0000    0.0000
```

Exercise 15.3.3

(See Exercise 5 from "Problems on Random Variables and Joint Distributions") Suppose a pair of dice is rolled. Let X be the total number of spots which turn up. Roll the pair an additional X times. Let Y be the number of sevens that are thrown on the X rolls. Determine the distribution for Y. What is the probability of three or more sevens?

Answer

```
PX = (1/36)*[0 0 1 2 3 4 5 6 5 4 3 2 1];
PY = [5/6 1/6];
gend
Do not forget zero coefficients for missing powers
Enter gen fn COEFFICIENTS for gN  PX
Enter gen fn COEFFICIENTS for gY  PY
Results are in N, PN, Y, PY, D, PD, P
May use jcalc or jcalcf on N, D, P
To view the distribution, call for gD.
disp(gD)
          0    0.3072
     1.0000    0.3660
     2.0000    0.2152
     3.0000    0.0828
     4.0000    0.0230
     5.0000    0.0048
     6.0000    0.0008
     7.0000    0.0001
     8.0000    0.0000
     9.0000    0.0000
    10.0000    0.0000
    11.0000    0.0000
    12.0000    0.0000
   P = (D>=3)*PD'
P =  0.1116
```

Exercise 15.3.4

(See Example 7 from "Conditional Expectation, Regression") A number X is chosen by a random selection from the integers 1 through 20 (say by drawing a card from a box). A pair of dice is thrown X times. Let Y be the number of “matches” (i.e., both ones, both twos, etc.). Determine the distribution for Y.

Answer

```
gN = (1/20)*[0 ones(1,20)];
gY = [5/6 1/6];
gend
Do not forget zero coefficients for missing powers
Enter gen fn COEFFICIENTS for gN  gN
Enter gen fn COEFFICIENTS for gY  gY
Results are in N, PN, Y, PY, D, PD, P
May use jcalc or jcalcf on N, D, P
To view the distribution, call for gD.

disp(gD)
         0    0.2435
    1.0000    0.2661
    2.0000    0.2113
    3.0000    0.1419
    4.0000    0.0795
    5.0000    0.0370
    6.0000    0.0144
    7.0000    0.0047
    8.0000    0.0013
    9.0000    0.0003
   10.0000    0.0001
   11.0000    0.0000
   12.0000    0.0000
   13.0000    0.0000
   14.0000    0.0000
   15.0000    0.0000
   16.0000    0.0000
   17.0000    0.0000
   18.0000    0.0000
   19.0000    0.0000
   20.0000    0.0000
```

Exercise 15.3.5

(See Exercise 20 from "Problems on Conditional Expectation, Regression") A number X is selected randomly from the integers 1 through 100. A pair of dice is thrown X times. Let Y be the number of sevens thrown on the X tosses. Determine the distribution for Y. Determine $E[Y]$ and $P(Y \leq 20)$.

Answer

```
gN = 0.01*[0 ones(1,100)];
gY = [5/6 1/6];
gend
Do not forget zero coefficients for missing powers
Enter gen fn COEFFICIENTS for gN  gN
Enter gen fn COEFFICIENTS for gY  gY
Results are in N, PN, Y, PY, D, PD, P
May use jcalc or jcalcf on N, D, P
To view the distribution, call for gD.
EY = dot(D,PD)
EY =   8.4167
P20 = (D<=20)*PD'
P20 =  0.9837
```

Exercise 15.3.6

(See Exercise 21 from "Problems on Conditional Expectation, Regression") A number X is selected randomly from the integers 1 through 100. Each of two people draw X times independently and randomly a number from 1 to 10. Let Y be the number of matches (i.e., both draw ones, both draw twos, etc.). Determine the distribution for Y. Determine $E[Y]$ and $P(Y \leq 10)$.

Answer

```
gN = 0.01*[0 ones(1,100)];
gY = [0.9 0.1];
gend
Do not forget zero coefficients for missing powers
Enter gen fn COEFFICIENTS for gN  gN
Enter gen fn COEFFICIENTS for gY  gY
Results are in N, PN, Y, PY, D, PD, P
May use jcalc or jcalcf on N, D, P
To view the distribution, call for gD.
EY = dot(D,PD)
EY =  5.0500
P10 = (D<=10)*PD'
P10 = 0.9188
```

Exercise 15.3.7

Suppose the number of entries in a contest is N ~ binomial (20, 0.4). There are four questions. Let Y_i be the number of questions answered correctly by the ith contestant. Suppose the Y_i are iid, with common distribution

$$Y = [1\ 2\ 3\ 4]\ PY = [0.2\ 0.4\ 0.3\ 0.1]$$

Let D be the total number of correct answers. Determine $E[D]$, $\text{Var}[D]$, $P(15 \leq D \leq 25)$, and $P(10 \leq D \leq 30)$.

Answer

```
gN = ibinom(20,0.4,0:20);
gY = 0.1*[0 2 4 3 1];
gend
Do not forget zero coefficients for missing powers
Enter gen fn COEFFICIENTS for gN  gN
Enter gen fn COEFFICIENTS for gY  gY
Results are in N, PN, Y, PY, D, PD, P
May use jcalc or jcalcf on N, D, P
To view the distribution, call for gD.
ED = dot(D,PD)
ED =  18.4000
VD = (D.^2)*PD' - ED^2
VD =  31.8720
P1 = ((15<=D)&(D<=25))*PD'
P1 =   0.6386
P2 = ((10<=D)&(D<=30))*PD'
P2 =   0.9290
```

Exercise 15.3.8

Game wardens are making an aerial survey of the number of deer in a park. The number of herds to be sighted is assumed to be a random variable N ~ binomial (20, 0.5). Each herd is assumed to be from 1 to 10 in size, with probabilities

Value	1	2	3	4	5	6	7	8	9	10
Probability	0.05	0.10	0.15	0.20	0.15	0.10	0.10	0.05	0.05	0.05

Let D be the number of deer sighted under this model. Determine $P(D \le t)$ for $t = 25, 50, 75, 100$ and $P(D \ge 90)$.

Answer

```
gN = ibinom(20,0.5,0:20);
gY = 0.01*[0 5 10 15 20 15 10 10 5 5 5];
gend
Do not forget zero coefficients for missing powers
Enter gen fn COEFFICIENTS for gN  gN
Enter gen fn COEFFICIENTS for gY  gY
Results are in N, PN, Y, PY, D, PD, P
May use jcalc or jcalcf on N, D, P
To view the distribution, call for gD.
k = [25 50 75 100];
P = zeros(1,4);
for i = 1:4
    P(i) = (D<=k(i))*PD';
end
disp(P)
    0.0310    0.5578    0.9725    0.9998
```

Exercise 15.3.9

A supply house stocks seven popular items. The table below shows the values of the items and the probability of each being selected by a customer.

Value	12.50	25.00	30.50	40.00	42.50	50.00	60.00
Probability	0.10	0.15	0.20	0.20	0.15	0.10	0.10

Suppose the purchases of customers are iid, and the number of customers in a day is binomial (10,0.5). Determine the distribution for the total demand D.

a. How many different possible values are there? What is the maximum possible total sales?
b. Determine $E[D]$ and $P(D \le t)$ for $t = 100, 150, 200, 250, 300$
   Determine $P(100 < D \le 200)$.

Answer

```
gN = ibinom(10,0.5,0:10);
Y  = [12.5 25 30.5 40 42.5 50 60];
PY = 0.01*[10 15 20 20 15 10 10];
mgd
Enter gen fn COEFFICIENTS for gN  gN
Enter VALUES for Y  Y
Enter PROBABILITIES for Y  PY
Values are in row matrix D; probabilities are in PD.
To view the distribution, call for mD.
s = size(D)
s =    1   839
M = max(D)
M =    590
t = [100 150 200 250 300];
P = zeros(1,5);
for i = 1:5
    P(i) = (D<=t(i))*PD';
end
disp(P)
    0.1012    0.3184    0.6156    0.8497    0.9614
P1 = ((100<D)&(D<=200))*PD'
P1 =   0.5144
```

Exercise 15.3.10

A game is played as follows:

1. A wheel is spun, giving one of the integers 0 through 9 on an equally likely basis.
2. A single die is thrown the number of times indicated by the result of the spin of the wheel. The number of points made is the total of the numbers turned up on the sequence of throws of the die.
3. A player pays sixteen dollars to play; a dollar is returned for each point made.

Let Y represent the number of points made and $X = Y - 16$ be the net gain (possibly negative) of the player. Determine the maximum value of

$X, E[X], \text{Var}[X], P(X > 0), P(X \geq 10), P(X \geq 16)$

Answer

```
gn = 0.1*ones(1,10);
gy = (1/6)*[0 ones(1,6)];
[Y,PY] = gendf(gn,gy);
[X,PX] = csort(Y-16,PY);
M = max(X)
M =  38
EX = dot(X,PX)                % Check EX = En*Ey - 16 = 4.5*3.5
EX  =  -0.2500                % 4.5*3.5 - 16 = -0.25
VX = dot(X.^2,PX) - EX^2
VX =  114.1875
Ppos = (X>0)*PX'
Ppos =  0.4667
P10 = (X>=10)*PX'
P10 =   0.2147
P16 = (X>=16)*PX'
P16 =   0.0803
```

Exercise 15.3.11

Marvin calls on four customers. With probability $p_1 = 0.6$ he makes a sale in each case. Geraldine calls on five customers, with probability $p_2 = 0.5$ of a sale in each case. Customers who buy do so on an iid basis, and order an amount Y_i (in dollars) with common distribution:

$$Y = [200\ 220\ 240\ 260\ 280\ 300]\ PY = [0.10\ 0.15\ 0.25\ 0.25\ 0.15\ 0.10]$$

Let D_1 be the total sales for Marvin and D_2 the total sales for Geraldine. Let $D = D_1 + D_2$. Determine the distribution and mean and variance for D_1, D_2, and D. Determine $P(D_1 \geq D_2)$ and $P(D \geq 1500)$, $P(D \geq 1000)$, and $P(D \geq 750)$.

Answer

```
gnM = ibinom(4,0.6,0:4);
gnG = ibinom(5,0.5,0:5);
Y = 200:20:300;
PY = 0.01*[10 15 25 25 15 10];
[D1,PD1] = mgdf(gnM,Y,PY);
[D2,PD2] = mgdf(gnG,Y,PY);
ED1 = dot(D1,PD1)
ED1 =  600.0000               % Check: ED1 = EnM*EY = 2.4*250
VD1 = dot(D1.^2,PD1) - ED1^2
VD1 =    6.1968e+04
ED2 = dot(D2,PD2)
ED2 =  625.0000               % Check: ED2 = EnG*EY = 2.5*250
VD2 = dot(D2.^2,PD2) - ED2^2
VD2 =    8.0175e+04
```

```
[D1,D2,t,u,PD1,PD2,P] = icalcf(D1,D2,PD1,PD2);
Use array opertions on matrices X, Y, PX, PY, t, u, and P
[D,PD] = csort(t+u,P);
ED = dot(D,PD)
ED =   1.2250e+03
eD = ED1 + ED2              % Check: ED = ED1 + ED2
eD =   1.2250e+03           % (Continued next page)

VD = dot(D.^2,PD) - ED^2
VD =   1.4214e+05
vD = VD1 + VD2            % Check: VD = VD1 + VD2
vD =   1.4214e+05
P1g2 = total((t>u).*P)
P1g2 = 0.4612
k = [1500 1000 750];
PDk = zeros(1,3);
for i = 1:3
   PDk(i) = (D>=k(i))*PD';
end
disp(PDk)
    0.2556    0.7326    0.8872
```

Exercise 15.3.12

A questionnaire is sent to twenty persons. The number who reply is a random number N ~ binomial (20, 0.7). If each respondent has probability $p = 0.8$ of favoring a certain proposition, what is the probability of ten or more favorable replies? Of fifteen or more?

Answer

```
gN = ibinom(20,0.7,0:20);
gY = [0.2 0.8];
gend
Do not forget zero coefficients for missing powers
Enter gen fn COEFFICIENTS for gN  gN
Enter gen fn COEFFICIENTS for gY  gY
Results are in N, PN, Y, PY, D, PD, P
May use jcalc or jcalcf on N, D, P
To view the distribution, call for gD.
P10 = (D>=10)*PD'
P10 =   0.7788
P15 = (D>=15)*PD'
P15 =   0.0660
pD = ibinom(20,0.7*0.8,0:20);  % Alternate: use D binomial (pp0)
D = 0:20;
p10 = (D>=10)*pD'
p10 =  0.7788
```

```
p15 = (D>=15)*pD'
p15 =  0.0660
```

Exercise 15.3.13

A random number N of students take a qualifying exam. A grade of 70 or more earns a pass. Suppose N ~ binomial (20, 0.3). If each student has probability $p = 0.7$ of making 70 or more, what is the probability all will pass? Ten or more will pass?

Answer

```
gN = ibinom(20,0.3,0:20);
gY = [0.3 0.7];
gend
Do not forget zero coefficients for missing powers
Enter gen fn COEFFICIENTS for gN  gN
Enter gen fn COEFFICIENTS for gY  gY
Results are in N, PN, Y, PY, D, PD, P
May use jcalc or jcalcf on N, D, P
To view the distribution, call for gD.
Pall = (D==20)*PD'
Pall =  2.7822e-14
pall = (0.3*0.7)^20    % Alternate: use D binomial (pp0)
pall =  2.7822e-14
P10 = (D >= 10)*PD'
P10 = 0.0038
```

Exercise 15.3.14

Five hundred questionnaires are sent out. The probability of a reply is 0.6. The probability that a reply will be favorable is 0.75. What is the probability of at least 200, 225, 250 favorable replies?

Answer

```
n = 500;
p = 0.6;
p0 = 0.75;
D = 0:500;
PD = ibinom(500,p*p0,D);
k = [200 225 250];
P = zeros(1,3);
for i = 1:3
   P(i) = (D>=k(i))*PD';
end
disp(P)
    0.9893    0.5173    0.0140
```

Exercise 15.3.15

Suppose the number of Japanese visitors to Florida in a week is $N1$ ~ Poisson (500) and the number of German visitors is $N2$ ~ Poisson (300). If 25 percent of the Japanese and 20 percent of the Germans visit Disney World, what is the distribution for the total number D of German and Japanese visitors to the park? Determine $P(D \ge k)$ for $k = 150, 155, \cdots, 245, 250$

Answer

JD ~ Poisson (500*0.25 = 125); GD ~ Poisson (300*0.20 = 60); D ~ Poisson (185).

```
k = 150:5:250;
PD = cpoisson(185,k);
disp([k;PD]')
  150.0000    0.9964
  155.0000    0.9892
  160.0000    0.9718
  165.0000    0.9362
  170.0000    0.8736
  175.0000    0.7785
  180.0000    0.6532
  185.0000    0.5098
  190.0000    0.3663
  195.0000    0.2405
  200.0000    0.1435
  205.0000    0.0776
  210.0000    0.0379
  215.0000    0.0167
  220.0000    0.0067
  225.0000    0.0024
  230.0000    0.0008
  235.0000    0.0002
  240.0000    0.0001
  245.0000    0.0000
  250.0000    0.0000
```

Exercise 15.3.16

A junction point in a network has two incoming lines and two outgoing lines. The number of incoming messages N_1 on line one in one hour is Poisson (50); on line 2 the number is N_2 ~ Poisson (45). On incoming line 1 the messages have probability $P_{1a} = 0.33$ of leaving on outgoing line a and $1 - p_{1a}$ of leaving on line b. The messages coming in on line 2 have probability $p_{2a} = 0.47$ of leaving on line a. Under the usual independence assumptions, what is the distribution of outgoing messages on line a? What are the probabilities of at least 30, 35, 40 outgoing messages on line a?

Answer

```
m1a = 50*0.33;  m2a = 45*0.47; ma = m1a + m2a;
PNa = cpoisson(ma,[30 35 40])
PNa =   0.9119    0.6890    0.3722
```

Exercise 15.3.17

A computer store sells Macintosh, HP, and various other IBM compatible personal computers. It has two major sources of customers:

1. Students and faculty from a nearby university
2. General customers for home and business computing. Suppose the following assumptions are reasonable for monthly purchases.

- The number of university buyers $N1$ ~ Poisson (30). The probabilities for Mac, HP, others are 0.4, 0.2, 0.4, respectively.
- The number of non-university buyers $N2$ ~ Poisson (65). The respective probabilities for Mac, HP, others are 0.2, 0.3, 0.5.
- For each group, the composite demand assumptions are reasonable, and the two groups buy independently.

What is the distribution for the number of Mac sales? What is the distribution for the total number of Mac and Dell sales?

Answer

Mac sales Poisson (30*0.4 + 65*0.2 = 25); HP sales Poisson (30*0.2 + 65*0.3 = 25.5); total Mac plus HP sales Poisson(50.5).

Exercise 15.3.18

The number N of “hits” in a day on a Web site on the internet is Poisson (80). Suppose the probability is 0.10 that any hit results in a sale, is 0.30 that the result is a request for information, and is 0.60 that the inquirer just browses but does not identify an interest. What is the probability of 10 or more sales? What is the probability that the number of sales is at least half the number of information requests (use suitable simple approximations)?

Answer

```
X = 0:30;
Y = 0:80;
PX = ipoisson(80*0.1,X);
PY = ipoisson(80*0.3,Y);
icalc:  X  Y  PX  PY
- - - - - - - - - - - -
PX10 = (X>=10)*PX'    % Approximate calculation
PX10 =  0.2834
pX10 = cpoisson(8,10)   % Direct calculation
pX10 =  0.2834
M = t>=0.5*u;
PM = total(M.*P)
PM =    0.1572
```

Exercise 15.3.19

The number N of orders sent to the shipping department of a mail order house is Poisson (700). Orders require one of seven kinds of boxes, which with packing costs have distribution

Cost (dollars)	0.75	1.25	2.00	2.50	3.00	3.50	4.00
Probability	0.10	0.15	0.15	0.25	0.20	0.10	0.05

What is the probability the total cost of the $2.50 boxes is no greater than $475? What is the probability the cost of the $2.50 boxes is greater than the cost of the $3.00 boxes? What is the probability the cost of the $2.50 boxes is not more than $50.00

greater than the cost of the $3.00 boxes? *Suggestion*. Truncate the Poisson distributions at about twice the mean value.

Answer

```
X = 0:400;
Y = 0:300;
PX = ipoisson(700*0.25,X);
PY = ipoisson(700*0.20,Y);
icalc
Enter row matrix of X-values  X
Enter row matrix of Y-values  Y
Enter X probabilities  PX
Enter Y probabilities  PY
 Use array operations on matrices X, Y, PX, PY, t, u, and P
P1 = (2.5*X<=475)*PX'
P1 =   0.8785
M = 2.5*t<=(3*u + 50);
PM = total(M.*P)
PM =   0.7500
```

Exercise 15.3.20

One car in 5 in a certain community is a Volvo. If the number of cars passing a traffic check point in an hour is Poisson (130), what is the expected number of Volvos? What is the probability of at least 30 Volvos? What is the probability the number of Volvos is between 16 and 40 (inclusive)?

Answer

```
P1 = cpoisson(130*0.2,30) = 0.2407
P2 = cpoisson(26,16) - cpoisson(26,41) = 0.9819
```

Exercise 15.3.21

A service center on an interstate highway experiences customers in a one-hour period as follows:

- Northbound: Total vehicles: Poisson (200). Twenty percent are trucks.
- Southbound: Total vehicles: Poisson (180). Twenty five percent are trucks.

- Each truck has one or two persons, with respective probabilities 0.7 and 0.3.
- Each car has 1, 2, 3, 4, or 5 persons, with probabilities 0.3, 0.3, 0.2, 0.1, 0.1, respectively

Under the usual independence assumptions, let D be the number of persons to be served. Determine $E[D]$, $\text{Var}[D]$, and the generating function $g_D(s)$.

Answer

T ~ Poisson (200*0.2 = 180*0.25 = 85), P ~ Poisson (200*0.8 + 180*0.75 = 295).

```
a =   85
b = 200*0.8 + 180*0.75
b =  295
YT = [1 2];
```

```
PYT = [0.7 0.3];
EYT = dot(YT,PYT)
EYT =  1.3000
VYT = dot(YT.^2,PYT) - EYT^2
VYT =  0.2100
YP = 1:5;
PYP = 0.1*[3 3 2 1 1];
EYP = dot(YP,PYP)
EYP =  2.4000
VYP = dot(YP.^2,PYP) - EYP^2
VYP =   1.6400
EDT = 85*EYT
EDT =  110.5000
EDP = 295*EYP
EDP =  708.0000
ED = EDT + EDP
ED =  818.5000
VT = 85*(VYT + EYT^2)
VT =  161.5000
VP = 295*(VYP + EYP^2)
VP =    2183
VD = VT + VP
VD =   2.2705e+03

NT = 0:180;                   % Possible alternative
gNT = ipoisson(85,NT);
gYT = 0.1*[0 7 3];
[DT,PDT] = gendf(gNT,gYT);
EDT = dot(DT,PDT)
EDT =  110.5000
VDT = dot(DT.^2,PDT) - EDT^2
VDT =  161.5000
NP = 0:500;
gNP = ipoisson(295,NP);
gYP = 0.1*[0 3 2 2 1 1];
[DP,PDP] = gendf(gNP,gYP);     %  Requires too much memory
```

$$g_{DT}(s) = \exp(85(0.7s + 0.3s^2 - 1)) \;\; g_{DP}(s) = \exp(295(0.1(3s + 3s^2 2s^3 + s^4 + s^5) - 1))$$

$$g_D(s) = g_{DT}(s) g_{DP}(s)$$

Exercise 15.3.22

The number N of customers in a shop in a given day is Poisson (120). Customers pay with cash or by MasterCard or Visa charge cards, with respective probabilties 0.25, 0.40, 0.35. Make the usual independence assumptions. Let N_1, N_2, N_3 be the numbers of cash sales, MasterCard charges, Visa card charges, respectively. Determine $P(N_1 \ge 30)$, $P(N_2 \ge 60)$, $P(N_3 \ge 50$, and $P(N_2 > N_3)$.

Answer

```
X = 0:120;
PX = ipoisson(120*0.4,X);
Y = 0:120;
PY = ipoisson(120*0.35,Y);
icalc
Enter row matrix of X values  X
Enter row matrix of Y values  Y
Enter X probabilities  PX
Enter Y probabilities  PY
Use array opertions on matrices X, Y, PX, PY, t, u, and P
M = t > u;
PM = total(M.*P)
PM =    0.7190
```

Exercise 15.3.23

A discount retail store has two outlets in Houston, with a common warehouse. Customer requests are phoned to the warehouse for pickup. Two items, a and b, are featured in a special sale. The number of orders in a day from store A is N_A ~ Poisson (30); from store B, the nember of orders is N_B ~ Poisson (40).

For store A, the probability an order for a is 0.3, and for b is 0.7.

For store B, the probability an order for a is 0.4, and for b is 0.6. What is the probability the total order for item b in a day is 50 or more?

Answer

P = cpoisson(30*0.7+40*0.6,50) = 0.2468

Exercise 15.3.24

The number of bids on a job is a random variable N ~ binomial (7, 0.6). Bids (in thousands of dollars) are iid with Y uniform on [3, 5]. What is the probability of at least one bid of $3,500 or less? *Note* that “no bid” is not a bid of 0.

Answer

```
% First solution ---  FY(t) = 1 - gN[P(Y>t)]
P = 1-(0.4 + 0.6*0.75)^7
P  =    0.6794
% Second solution --- Positive number of satisfactory bids,
% i.e. the outcome is indicator for event E, with P(E) = 0.25
pN = ibinom(7,0.6,0:7);
gY = [3/4 1/4];         % Generator function for indicator
[D,PD] = gendf(pN,gY);  % D is number of successes
Pa = (D>0)*PD'          % D>0 means at least one successful bid
Pa =    0.6794
```

Exercise 15.3.25

The number of customers during the noon hour at a bank teller's station is a random number N with distribution

$$N = 1:10,\ PN = 0.01 * [5\ 7\ 10\ 11\ 12\ 13\ 12\ 11\ 10\ 9]$$

The amounts they want to withdraw can be represented by an iid class having the common distribution Y ~ exponential (0.01). Determine the probabilities that the maximum withdrawal is less than or equal to t for $t = 100, 200, 300, 400, 500$

Answer

Use $F_W(t) = g_N[P(Y \leq T)]$

```
gN = 0.01*[0 5 7 10 11 12 13 12 11 10 9];
t = 100:100:500;
PY = 1 - exp(-0.01*t);
FW = polyval(fliplr(gN),PY)  % fliplr puts coeficients in
                             % descending order of powers
FW =    0.1330    0.4598    0.7490    0.8989    0.9615
```

Exercise 15.3.26

A job is put out for bids. Experience indicates the number N of bids is a random variable having values 0 through 8, with respective probabilities

Value	0	1	2	3	4	5	6	7	8
Probability	0.05	0.10	0.15	0.20	0.20	0.10	0.10	0.07	0.03

The market is such that bids (in thousands of dollars) are iid, uniform [100, 200]. Determine the probability of at least one bid of $125,000 or less.

Answer

Probability of a successful bid $PY = (125 - 100)/100 = 0.25$

```
PY =0.25;
gN = 0.01*[5 10 15 20 20 10 10 7 3];
P = 1 - polyval(fliplr(gN),PY)
P =  0.9116
```

Exercise 15.3.27

A property is offered for sale. Experience indicates the number N of bids is a random variable having values 0 through 10, with respective probabilities

Value	0	1	2	3	4	5	6	7	8	9	10
Probability	0.05	0.15	0.15	0.20	0.10	0.10	0.05	0.05	0.05	0.05	0.05

The market is such that bids (in thousands of dollars) are iid, uniform [150, 200] Determine the probability of at least one bid of $180,000 or more.

Answer

Consider a sequence of N trials with probabilty $p = (180 - 150)/50 = 0.6$.

```
gN = 0.01*[5 15 15 20 10 10 5 5 5 5 5];
gY = [0.4 0.6];
[D,PD] = gendf(gN,gY);
P = (D>0)*PD'
P =   0.8493
```

Exercise 15.3.28

A property is offered for sale. Experience indicates the number N of bids is a random variable having values 0 through 8, with respective probabilities

Number	0	1	2	3	4	5	6	7	8
Probability	0.05	0.15	0.15	0.20	0.15	0.10	0.10	0.05	0.05

The market is such that bids (in thousands of dollars) are iid symmetric triangular on [150 250]. Determine the probability of at least one bid of $210,000 or more.

Answer

```
gN = 0.01*[5 15 15 20 15 10 10 5 5];
PY = 0.5 + 0.5*(1 - (4/5)^2)
PY = 0.6800
>> PW = 1 - polyval(fliplr(gN),PY)
PW = 0.6536
%alternate
gY = [0.68 0.32];
[D,PD] = gendf(gN,gY);
P = (D>0)*PD'
P = 0.6536
```

Exercise 15.3.29

Suppose N ~ binomial (10, 0.3) and the Y_i are iid, uniform on [10, 20]. Let V be the minimum of the N values of the Y_i. Determine $P(V > t)$ for integer values from 10 to 20.

Answer

```
gN = ibinom(10,0.3,0:10);
t = 10:20;
p = 0.1*(20 - t);
P = polyval(fliplr(gN),p) - 0.7^10
P =
  Columns 1 through 7
    0.9718    0.7092    0.5104    0.3612    0.2503    0.1686    0.1092
```

```
  Columns 8 through 11
    0.0664    0.0360    0.0147         0
Pa = (0.7 + 0.3*p).^10 - 0.7^10     % Alternate form of gN
Pa =
  Columns 1 through 7
    0.9718    0.7092    0.5104    0.3612    0.2503    0.1686    0.1092
  Columns 8 through 11
    0.0664    0.0360    0.0147         0
```

Exercise 15.3.30

Suppose a teacher is equally likely to have 0, 1, 2, 3 or 4 students come in during office hours on a given day. If the lengths of the individual visits, in minutes, are iid exponential (0.1), what is the probability that no visit will last more than 20 minutes.

Answer

```
gN = 0.2*ones(1,5);
p = 1 - exp(-2);
FW = polyval(fliplr(gN),p)
FW =    0.7635
gY = [p 1-p];                % Alternate
[D,PD] = gendf(gN,gY);
PW = (D==0)*PD'
PW =    0.7635
```

Exercise 15.3.31

Twelve solid-state modules are installed in a control system. If the modules are not defective, they have practically unlimited life. However, with probability $p = 0.05$ any unit could have a defect which results in a lifetime (in hours) exponential (0.0025). Under the usual independence assumptions, what is the probability the unit does not fail because of a defective module in the first 500 hours after installation?

Answer

```
p = 1 - exp(-0.0025*500);
FW = (0.95 + 0.05*p)^12
FW =   0.8410
gN = ibinom(12,0.05,0:12);
gY = [p 1-p];
[D,PD] = gendf(gN,gY);
PW = (D==0)*PD'
PW =   0.8410
```

Exercise 15.3.32

The number N of bids on a painting is binomial (10, 0.3). The bid amounts (in thousands of dollars) Y_i form an iid class, with common density function $f_Y(t) = 0.005(37 - 2t), 2 \le t \le 10$. What is the probability that the maximum amount bid is greater than $5,000?

Answer

$$P(Y \leq 5) = 0.005 \int_2^5 (37 - 2t)\, dt = 0.45$$

```
p = 0.45;
P = 1 - (0.7 + 0.3*p)^10
P =   0.8352
gN = ibinom(10,0.3,0:10);
gY = [p 1-p];
[D,PD] = gendf(gN,gY);  % D is number of "successes"
Pa = (D>0)*PD'
Pa =  0.8352
```

Exercise 15.3.33

A computer store offers each customer who makes a purchase of $500 or more a free chance at a drawing for a prize. The probability of winning on a draw is 0.05. Suppose the times, in hours, between sales qualifying for a drawing is exponential (4). Under the usual independence assumptions, what is the expected time between a winning draw? What is the probability of three or more winners in a ten hour day? Of five or more?

Answer

N_t ~ Poisson (λt), N_{Dt} ~ Poisson (λpt), W_{Dt} exponential (λp).

```
p = 0.05;
t = 10;
lambda = 4;
EW = 1/(lambda*p)
EW =    5
PND10 = cpoisson(lambda*p*t,[3 5])
PND10 =  0.3233    0.0527
```

Exercise 15.3.34

Noise pulses arrrive on a data phone line according to an arrival process such that for each $t > 0$ the number N_t of arrivals in time interval $(0, t]$, in hours, is Poisson $(7t)$. The ith pulse has an "intensity" Y_i such that the class $\{Y_i : 1 \leq i\}$ is iid, with the common distribution function $F_Y(u) = 1 - e^{-2u^2}$ for $u \geq 0$. Determine the probability that in an eight-hour day the intensity will not exceed two.

Answer

N_8 is Poisson (7*8 = 56) $g_N(s) = e^{56(s-1)}$.

```
t = 2;
FW2 = exp(56*(1 - exp(-t^2) - 1))
FW2 =   0.3586
```

Exercise 15.3.35

The number N of noise bursts on a data transmission line in a period $(0, t]$ is Poisson (μ). The number of digit errors caused by the ith burst is Y_i, with the class $\{Y_i : 1 \leq i\}$ iid, $Y_i - 1$ ~ geometric (p). An error correcting system is capable or correcting five or fewer errors in any burst. Suppose $\mu = 12$ and $p = 0.35$. What is the probability of no uncorrected error in two hours of operation?

Answer

$F_W(k) = g_N[P(Y \leq k)] P(Y \leq k) - 1 - q^{k-1}$ N_t ~ Poisson (12t)

```
q = 1 - 0.35;
k = 5;
t = 2;
mu = 12;
FW = exp(mu*t*(1 - q^(k-1) - 1))
FW =  0.0138
```