

8.3: Hypothesis Test Examples for Means with Unknown Standard Deviation

Full Hypothesis Test Examples

Example 8.3.6

Statistics students believe that the mean score on the first statistics test is 65. A statistics instructor thinks the mean score is higher than 65. He samples ten statistics students and obtains the scores 65 65 70 67 66 63 63 68 72 71. He performs a hypothesis test using a 5% level of significance. The data are assumed to be from a normal distribution.

Answer

Set up the hypothesis test:

A 5% level of significance means that $\alpha = 0.05$. This is a test of a **single population mean**.

$$H_0 : \mu = 65 \quad H_a : \mu > 65$$

Since the instructor thinks the average score is higher, use a ">". The ">" means the test is right-tailed.

Determine the distribution needed:

Random variable: \bar{X} = average score on the first statistics test.

Distribution for the test: If you read the problem carefully, you will notice that there is **no population standard deviation given**. You are only given $n = 10$ sample data values. Notice also that the data come from a normal distribution. This means that the distribution for the test is a student's t .

Use t_{df} . Therefore, the distribution for the test is t_9 where $n = 10$ and $df = 10 - 1 = 9$.

Calculate the p -value using the Student's t -distribution:

$p\text{-value} = P(\bar{x} > 67) = 0.0396$ where the sample mean and sample standard deviation are calculated as 67 and 3.1972 from the data.

Interpretation of the p -value: If the null hypothesis is true, then there is a 0.0396 probability (3.96%) that the sample mean is 65 or more.

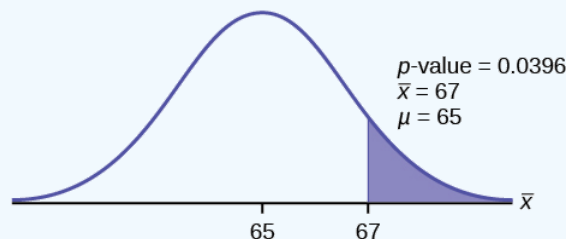


Figure 8.3.11

Compare α and the $p\text{-value}$:

Since $(\alpha = 0.05)$ and $p\text{-value} = 0.0396$, $\alpha > p\text{-value}$.

Make a decision: Since $\alpha > p\text{-value}$, reject H_0 .

This means you reject $\mu = 65$. In other words, you believe the average test score is more than 65.

Conclusion: At a 5% level of significance, the sample data show sufficient evidence that the mean (average) test score is more than 65, just as the math instructor thinks.

The p -value can easily be calculated.

Put the data into a list. Press **STAT** and arrow over to **TESTS**. Press **2:T-Test**. Arrow over to **Data** and press **ENTER**. Arrow down and enter 65 for μ_0 , the name of the list where you put the data, and 1 for **Freq**:. Arrow down to μ : and arrow over to $> \mu_0$. Press **ENTER**. Arrow down to **Calculate** and press **ENTER**. The calculator not only calculates the p -value ($p = 0.0396$) but it also calculates the test statistic (t -score) for the sample mean, the sample mean, and

the sample standard deviation. $\mu > 65$ is the alternative hypothesis. Do this set of instructions again except arrow to **Draw** (instead of **Calculate**). Press **ENTER**. A shaded graph appears with $t = 1.9781$ (test statistic) and $p = 0.0396$ (p -value). Make sure when you use **Draw** that no other equations are highlighted in $Y =$ and the plots are turned off.

Exercise 8.3.6

It is believed that a stock price for a particular company will grow at a rate of \$5 per week with a standard deviation of \$1. An investor believes the stock won't grow as quickly. The changes in stock price is recorded for ten weeks and are as follows: \$4, \$3, \$2, \$3, \$1, \$7, \$2, \$1, \$1, \$2. Perform a hypothesis test using a 5% level of significance. State the null and alternative hypotheses, find the p -value, state your conclusion, and identify the Type I and Type II errors.

Answer

- $H_0 : \mu = 5$
- $H_a : \mu < 5$
- $p = 0.0082$

Because $p < \alpha$, we reject the null hypothesis. There is sufficient evidence to suggest that the stock price of the company grows at a rate less than \$5 a week.

- Type I Error: To conclude that the stock price is growing slower than \$5 a week when, in fact, the stock price is growing at \$5 a week (reject the null hypothesis when the null hypothesis is true).
- Type II Error: To conclude that the stock price is growing at a rate of \$5 a week when, in fact, the stock price is growing slower than \$5 a week (do not reject the null hypothesis when the null hypothesis is false).

Example 8.3.10

The National Institute of Standards and Technology provides exact data on conductivity properties of materials. Following are conductivity measurements for 11 randomly selected pieces of a particular type of glass.

1.11; 1.07; 1.11; 1.07; 1.12; 1.08; .98; .98 1.02; .95; .95

Is there convincing evidence that the average conductivity of this type of glass is greater than one? Use a significance level of 0.05. Assume the population is normal.

Answer

Let's follow a four-step process to answer this statistical question.

1. **State the Question:** We need to determine if, at a 0.05 significance level, the average conductivity of the selected glass is greater than one. Our hypotheses will be
 - a. $H_0 : \mu \leq 1$
 - b. $H_a : \mu > 1$
2. **Plan:** We are testing a sample mean without a known population standard deviation. Therefore, we need to use a Student's-t distribution. Assume the underlying population is normal.
3. **Do the calculations:** We will input the sample data into the TI-83 as follows.

Figure 9.6.7.

Figure 9.6.8.

Figure 9.6.9.

Figure 9.6.10.

4. **State the Conclusions:** Since the p -value ($p = 0.036$) is less than our alpha value, we will reject the null hypothesis. It is reasonable to state that the data supports the claim that the average conductivity level is greater than one.

Review

The **hypothesis test** itself has an established process. This can be summarized as follows:

1. Determine H_0 and H_a . Remember, they are contradictory.

2. Determine the random variable.
3. Determine the distribution for the test.
4. Draw a graph, calculate the test statistic, and use the test statistic to calculate the p -value. (A z -score and a t -score are examples of test statistics.)
5. Compare the preconceived α with the p -value, make a decision (reject or do not reject H_0), and write a clear conclusion using English sentences.

Notice that in performing the hypothesis test, you use α and not β . β is needed to help determine the sample size of the data that is used in calculating the p -value. Remember that the quantity $1 - \beta$ is called the **Power of the Test**. A high power is desirable. If the power is too low, statisticians typically increase the sample size while keeping α the same. If the power is low, the null hypothesis might not be rejected when it should be.

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Glossary

Central Limit Theorem

Given a random variable (RV) with known mean μ and known standard deviation σ . We are sampling with size n and we are interested in two new RVs - the sample mean, \bar{X} , and the sample sum, $\sum X$. If the size n of the sample is sufficiently large, then $\bar{X} - N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ and $\sum X - N(n\mu, \sqrt{n}\sigma)$. If the size n of the sample is sufficiently large, then the distribution of the sample means and the distribution of the sample sums will approximate a normal distribution regardless of the shape of the population. The mean of the sample means will equal the population mean and the mean of the sample sums will equal n times the population mean. The standard deviation of the distribution of the sample means, $\frac{\sigma}{\sqrt{n}}$, is called the standard error of the mean.

Contributors and Attributions

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