

12.8: Rank Randomization for Association

Learning Objectives

- Compute Spearman's ρ
- Test Spearman's ρ for significance

The rank randomization test for association is equivalent to the randomization test for Pearson's r except that the numbers are converted to ranks before the analysis is done. Table 12.8.1 shows 5 values of X and Y . Table 12.8.2 shows these same data converted to ranks (separately for X and Y).

Table 12.8.1: Example data

X	Y
1.0	1.0
2.4	2.0
3.8	2.3
4.0	3.7
11.0	2.5

Table 12.8.2: Ranked data

X	Y
1	1
2	2
3	3
4	5
5	4

The approach is to consider the X variable fixed and compare the correlation obtained in the actual ranked data to the correlations that could be obtained by rearranging the Y variable ranks. For the ranked data shown in Table 12.8.2 the correlation between X and Y is 0.90. The correlation of ranks is called "Spearman's ρ ."

Table 12.8.3: Ranked data with correlation of 1.0

X	Y
1	1
2	2
3	3
4	4
5	5

There is only one arrangement of Y that produces a higher correlation than 0.90: A correlation of 1.0 results if the fourth and fifth observations' Y values are switched (see Table 12.8.3). There are also three other arrangements that produce an r of 0.90 (see Tables 12.8.4, 12.8.5, and 12.8.6). Therefore, there are five arrangements of Y that lead to correlations as high or higher than the actual ranked data (Tables 12.8.2 through 12.8.6).

Table 12.8.4: Ranked data with correlation of 0.90

X	Y
1	1
2	2
3	4
4	3
5	5

Table 12.8.5: Ranked data with correlation of 0.90

X	Y
1	1
2	3
3	2
4	4
5	5

Table 12.8.6: Ranked data with correlation of 0.90

X	Y
1	2
2	1
3	3
4	4
5	5

The next step is to calculate the number of possible arrangements of Y . The number is simply $N!$, where N is the number of pairs of scores. Here, the number of arrangements is $5! = 120$. Therefore, the probability value is $5/120 = 0.042$. Note that this is a one-tailed probability since it is the proportion of arrangements that give a correlation as large or larger. The two-tailed probability is 0.084.

Since it is hard to count up all the possibilities when the sample size is even moderately large, it is convenient to have a table of critical values.

Table of critical values for Spearman's ρ

From the table linked to above, you can see that the critical value for a one-tailed test with 5 observations at the 0.05 level is 0.90. Since the correlation for the sample data is 0.90, the association is significant at the 0.05 level (one-tailed). As shown above, the probability value is 0.042. Since the critical value for a two-tailed test is 1.0, Spearman's ρ is not significant in a two-tailed test.

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