4.E: Probability Topics (Optional Exercises)

These are homework exercises to accompany the Textmap created for "Introductory Statistics" by OpenStax.

3.1: Introduction

3.2: Terminology

Q 3.2.1

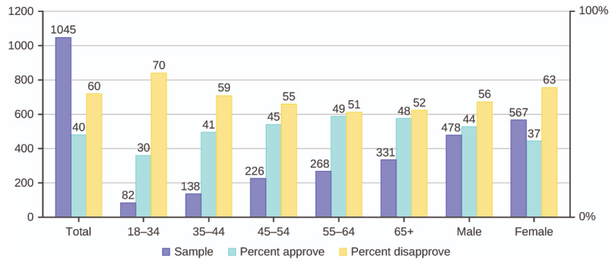


Figure 3.2.3.2.11.

The graph in Figure 3.2.1 displays the sample sizes and percentages of people in different age and gender groups who were polled concerning their approval of Mayor Ford's actions in office. The total number in the sample of all the age groups is 1,045.

a. Define three events in the graph.
b. Describe in words what the entry 40 means.
c. Describe in words the complement of the entry in question 2.
d. Describe in words what the entry 30 means.
e. Out of the males and females, what percent are males?
f. Out of the females, what percent disapprove of Mayor Ford?
g. Out of all the age groups, what percent approve of Mayor Ford?
h. Find $P(\text{Approve}|\text{Male})$.
i. Out of the age groups, what percent are more than 44 years old?
j. Find $P(\text{Approve}|\text{Age} < 35)$.

Q 3.2.2

Explain what is wrong with the following statements. Use complete sentences.

a. If there is a 60% chance of rain on Saturday and a 70% chance of rain on Sunday, then there is a 130% chance of rain over the weekend.
b. The probability that a baseball player hits a home run is greater than the probability that he gets a successful hit.

S 3.2.2

a. You can't calculate the joint probability knowing the probability of both events occurring, which is not in the information given; the probabilities should be multiplied, not added; and probability is never greater than 100%
b. A home run by definition is a successful hit, so he has to have at least as many successful hits as home runs.

3.3: Independent and Mutually Exclusive Events

Use the following information to answer the next 12 exercises. The graph shown is based on more than 170,000 interviews done by Gallup that took place from January through December 2012. The sample consists of employed Americans 18 years of age or older. The Emotional Health Index Scores are the sample space. We randomly sample one Emotional Health Index Score.

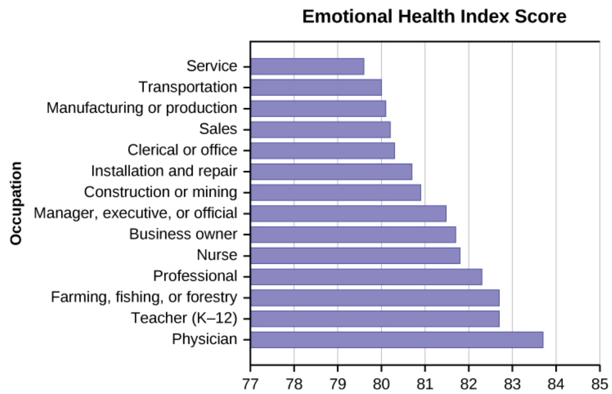


Figure 3.3.1.

Q 3.3.1

Find the probability that an Emotional Health Index Score is 82.7.

Q 3.3.2

Find the probability that an Emotional Health Index Score is 81.0.

S 3.3.2

0

Q 3.3.3

Find the probability that an Emotional Health Index Score is more than 81?

Q 3.3.4

Find the probability that an Emotional Health Index Score is between 80.5 and 82?

S 3.3.4

0.3571

Q 3.3.5

If we know an Emotional Health Index Score is 81.5 or more, what is the probability that it is 82.7?

Q 3.3.6

What is the probability that an Emotional Health Index Score is 80.7 or 82.7?

S 3.3.6

0.2142

Q 3.3.7

What is the probability that an Emotional Health Index Score is less than 80.2 given that it is already less than 81.

Q 3.3.8

What occupation has the highest emotional index score?

S 3.3.8

Physician (83.7)

Q 3.3.9

What occupation has the lowest emotional index score?

Q 3.3.10

What is the range of the data?

S 3.3.10

83.7 − 79.6 = 4.1

Q 3.3.11

Compute the average EHIS.

Q 3.3.12

If all occupations are equally likely for a certain individual, what is the probability that he or she will have an occupation with lower than average EHIS?

S 3.3.12

$P(\text{Occupation} < 81.3) = 0.5$

3.4: Two Basic Rules of Probability

Q 3.4.1

On February 28, 2013, a Field Poll Survey reported that 61% of California registered voters approved of allowing two people of the same gender to marry and have regular marriage laws apply to them. Among 18 to 39 year olds (California registered voters), the approval rating was 78%. Six in ten California registered voters said that the upcoming Supreme Court's ruling about the constitutionality of California's Proposition 8 was either very or somewhat important to them. Out of those CA registered voters who support same-sex marriage, 75% say the ruling is important to them.

In this problem, let:

- $\text{C} =$ California registered voters who support same-sex marriage.
- $\text{B} =$ California registered voters who say the Supreme Court's ruling about the constitutionality of California's Proposition 8 is very or somewhat important to them
- $\text{A} =$ California registered voters who are 18 to 39 years old.

a. Find $P(\text{C})$.
b. Find $P(\text{B})$.
c. Find $P(\text{C}|\text{A})$.
d. Find $P(\text{B}|\text{C})$.
e. In words, what is $\text{C}|\text{A}$?
f. In words, what is $\text{B}|\text{C}$?
g. Find $P(\text{C AND B})$.
h. In words, what is C AND B?
i. Find $P(\text{C OR B})$.
j. Are C and B mutually exclusive events? Show why or why not.

Q 3.4.2

After Rob Ford, the mayor of Toronto, announced his plans to cut budget costs in late 2011, the Forum Research polled 1,046 people to measure the mayor's popularity. Everyone polled expressed either approval or disapproval. These are the results their poll produced:

- In early 2011, 60 percent of the population approved of Mayor Ford's actions in office.
- In mid-2011, 57 percent of the population approved of his actions.
- In late 2011, the percentage of popular approval was measured at 42 percent.

a. What is the sample size for this study?
b. What proportion in the poll disapproved of Mayor Ford, according to the results from late 2011?
c. How many people polled responded that they approved of Mayor Ford in late 2011?
d. What is the probability that a person supported Mayor Ford, based on the data collected in mid-2011?

e. What is the probability that a person supported Mayor Ford, based on the data collected in early 2011?

S 3.4.2

a. The Forum Research surveyed 1,046 Torontonians.
b. 58%
c. 42% of 1,046 = 439 (rounding to the nearest integer)
d. 0.57
e. 0.60.

Use the following information to answer the next three exercises. The casino game, roulette, allows the gambler to bet on the probability of a ball, which spins in the roulette wheel, landing on a particular color, number, or range of numbers. The table used to place bets contains of 38 numbers, and each number is assigned to a color and a range.

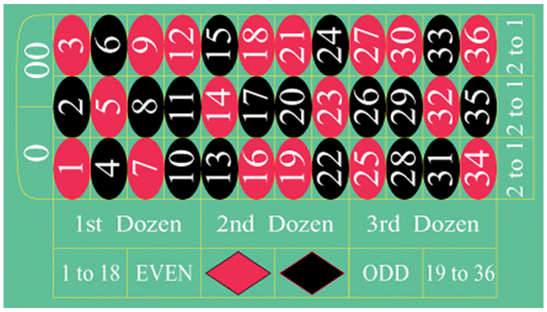


Figure 3.4.1

Q 3.4.3

a. List the sample space of the 38 possible outcomes in roulette.
b. You bet on red. Find $P(\text{red})$.
c. You bet on -1st 12- (1st Dozen). Find $P(\text{-1st 12-})$.
d. You bet on an even number. Find $P(\text{even number})$.
e. Is getting an odd number the complement of getting an even number? Why?
f. Find two mutually exclusive events.
g. Are the events Even and 1st Dozen independent?

Q 3.4.4

Compute the probability of winning the following types of bets:

a. Betting on two lines that touch each other on the table as in 1-2-3-4-5-6
b. Betting on three numbers in a line, as in 1-2-3
c. Betting on one number
d. Betting on four numbers that touch each other to form a square, as in 10-11-13-14
e. Betting on two numbers that touch each other on the table, as in 10-11 or 10-13
f. Betting on 0-00-1-2-3
g. Betting on 0-1-2; or 0-00-2; or 00-2-3

S 3.4.4

1. $P(\text{Betting on two line that touch each other on the table}) = \frac{6}{38}$
2. $P(\text{Betting on three numbers in a line}) = \frac{3}{38}$
3. $P(\text{Betting on one number}) = \frac{1}{38}$
4. $P(\text{Betting on four number that touch each other to form a square}) = \frac{4}{38}$
5. $P(\text{Betting on two number that touch each other on the table}) = \frac{2}{38}$
6. $P(\text{Betting on 0-00-1-2-3}) = \frac{5}{38}$
7. $P(\text{Betting on 0-1-2; or 0-00-2; or 00-2-3}) = \frac{3}{38}$

Q 3.4.5

Compute the probability of winning the following types of bets:

a. Betting on a color
b. Betting on one of the dozen groups
c. Betting on the range of numbers from 1 to 18
d. Betting on the range of numbers 19–36
e. Betting on one of the columns
f. Betting on an even or odd number (excluding zero)

Q 3.4.6

Suppose that you have eight cards. Five are green and three are yellow. The five green cards are numbered 1, 2, 3, 4, and 5. The three yellow cards are numbered 1, 2, and 3. The cards are well shuffled. You randomly draw one card.

- $\mathrm{G} =$ card drawn is green
- $\mathrm{E} =$ card drawn is even-numbered

  a. List the sample space.
  b. $P(\mathrm{G}) =$ _____
  c. $P(\mathrm{G}|\mathrm{E}) =$ _____
  d. $P(\mathrm{G\ AND\ E}) =$ _____
  e. $P(\mathrm{G\ OR\ E}) =$ _____
  f. Are G and E mutually exclusive? Justify your answer numerically.

S 3.4.6

1. $\{G1, G2, G3, G4, G5, Y1, Y2, Y3\}$
2. $\frac{5}{8}$
3. $\frac{2}{3}$
4. $\frac{2}{8}$
5. $\frac{}{6}8$
6. No, because $P(\mathrm{G\ AND\ E})$ does not equal 0.

Q 3.4.7

Roll two fair dice. Each die has six faces.

a. List the sample space.
b. Let A be the event that either a three or four is rolled first, followed by an even number. Find $P(\mathrm{A})$.
c. Let B be the event that the sum of the two rolls is at most seven. Find $P(\mathrm{B})$.
d. In words, explain what "$P(\mathrm{A}|\mathrm{B})$" represents. Find $P(\mathrm{A}|\mathrm{B})$.
e. Are A and B mutually exclusive events? Explain your answer in one to three complete sentences, including numerical justification.
f. Are A and B independent events? Explain your answer in one to three complete sentences, including numerical justification.

Q 3.4.8

A special deck of cards has ten cards. Four are green, three are blue, and three are red. When a card is picked, its color of it is recorded. An experiment consists of first picking a card and then tossing a coin.

a. List the sample space.
b. Let A be the event that a blue card is picked first, followed by landing a head on the coin toss. Find $P(\mathrm{A})$.
c. Let B be the event that a red or green is picked, followed by landing a head on the coin toss. Are the events A and B mutually exclusive? Explain your answer in one to three complete sentences, including numerical justification.
d. Let C be the event that a red or blue is picked, followed by landing a head on the coin toss. Are the events A and C mutually exclusive? Explain your answer in one to three complete sentences, including numerical justification.

S 3.4.9

The coin toss is independent of the card picked first.

a. $\{(G, H)(G, T)(B, H)(B, T)(R, H)(R, T)\}$
b. $P(\mathrm{A}) = P(\text{blue})P(\text{head}) = \left(\frac{3}{10}\right)\left(\frac{1}{2}\right) = \frac{3}{20}$
c. Yes, A and B are mutually exclusive because they cannot happen at the same time; you cannot pick a card that is both blue and also (red or green). $P(\mathrm{A\ AND\ B}) = 0$
d. No, A and C are not mutually exclusive because they can occur at the same time. In fact, C includes all of the outcomes of A; if the card chosen is blue it is also (red or blue). $P(\mathrm{A\ AND\ C}) = P(\mathrm{A}) = 320$

Q 3.4.10

An experiment consists of first rolling a die and then tossing a coin.

a. List the sample space.
b. Let A be the event that either a three or a four is rolled first, followed by landing a head on the coin toss. Find $P(\mathrm{A})$.
c. Let B be the event that the first and second tosses land on heads. Are the events A and B mutually exclusive? Explain your answer in one to three complete sentences, including numerical justification.

Q 3.4.11

An experiment consists of tossing a nickel, a dime, and a quarter. Of interest is the side the coin lands on.

a. List the sample space.
b. Let A be the event that there are at least two tails. Find $P(\mathrm{A})$.
c. Let B be the event that the first and second tosses land on heads. Are the events A and B mutually exclusive? Explain your answer in one to three complete sentences, including justification.

S 3.4.12

a. $S = (\mathrm{HHH}), (\mathrm{HHT}), (\mathrm{HTH}), (\mathrm{HTT}), (\mathrm{THH}), (\mathrm{THT}), (\mathrm{TTH}), (\mathrm{TTT})$
b. $\frac{4}{8}$
c. Yes, because if A has occurred, it is impossible to obtain two tails. In other words, $P(\mathrm{A\ AND\ B}) = 0$.

Q 3.4.13

Consider the following scenario:

Let $P(\mathrm{C}) = 0.4$.

Let $P(\mathrm{D}) = 0.5$.

Let $P(\mathrm{C}|\mathrm{D}) = 0.6$.

a. Find $P(CANDD)$.
b. Are C and D mutually exclusive? Why or why not?
c. Are C and D independent events? Why or why not?
d. Find $P(\mathrm{C\ OR\ D})$.
e. Find $P(\mathrm{D}|\mathrm{C})$.

Q 3.4.14

Y and Z are independent events.

a. Rewrite the basic Addition Rule $P(\mathrm{Y\ OR\ Z}) = P(\mathrm{Y}) + P(\mathrm{Z}) - P(\mathrm{Y\ AND\ Z})$ using the information that Y and Z are independent events.
b. Use the rewritten rule to find $P(\mathrm{Z})$ if $P(\mathrm{Y\ OR\ Z}) = 0.71$ and $P(\mathrm{Y}) = 0.42$.

S 3.4.14

a. If Y and Z are independent, then $P(\mathrm{Y\ AND\ Z}) = P(\mathrm{Y})P(\mathrm{Z})$, so $P(\mathrm{Y\ OR\ Z}) = P(\mathrm{Y}) + P(\mathrm{Z}) - P(\mathrm{Y})P(\mathrm{Z})$.
b. 0.5

Q 3.4.15

G and H are mutually exclusive events. $P(\mathrm{G})=0.5 P(\mathrm{H})=0.3$

a. Explain why the following statement MUST be false: $P(\mathrm{H} \mid \mathrm{G})=0.4$.
b. Find $P(\mathrm{H} \text{ OR } \mathrm{G})$.
c. Are G and H independent or dependent events? Explain in a complete sentence.

Q 3.4.16

Approximately 281,000,000 people over age five live in the United States. Of these people, 55,000,000 speak a language other than English at home. Of those who speak another language at home, 62.3% speak Spanish.

Let: $\mathrm{E}=$ speaks English at home; $\mathrm{E}'=$ speaks another language at home; $\mathrm{S}=$ speaks Spanish;

Finish each probability statement by matching the correct answer.

Probability Statements	Answers
a. $P(\mathrm{E}')=$	i. 0.8043
b. $P(\mathrm{E})=$	ii. 0.623
c. $P(\mathrm{S} \text{ and } \mathrm{E}')=$	iii. 0.1957
d. $P(\mathrm{S} \mid \mathrm{E}')=$	iv. 0.1219

S 3.4.16

a. iii
b. i
c. iv
d. ii

Q 3.4.17

1994, the U.S. government held a lottery to issue 55,000 Green Cards (permits for non-citizens to work legally in the U.S.). Renate Deutsch, from Germany, was one of approximately 6.5 million people who entered this lottery. Let $G=$ won green card.

a. What was Renate's chance of winning a Green Card? Write your answer as a probability statement.
b. In the summer of 1994, Renate received a letter stating she was one of 110,000 finalists chosen. Once the finalists were chosen, assuming that each finalist had an equal chance to win, what was Renate's chance of winning a Green Card? Write your answer as a conditional probability statement. Let $\mathrm{F}=$ was a finalist.
c. Are G and F independent or dependent events? Justify your answer numerically and also explain why.
d. Are G and F mutually exclusive events? Justify your answer numerically and explain why.

Q 3.4.18

Three professors at George Washington University did an experiment to determine if economists are more selfish than other people. They dropped 64 stamped, addressed envelopes with \$10 cash in different classrooms on the George Washington campus. 44% were returned overall. From the economics classes 56% of the envelopes were returned. From the business, psychology, and history classes 31% were returned.

Let: $\mathrm{R}=$ money returned; $\mathrm{E}=$ economics classes; $\mathrm{O}=$ other classes

a. Write a probability statement for the overall percent of money returned.
b. Write a probability statement for the percent of money returned out of the economics classes.
c. Write a probability statement for the percent of money returned out of the other classes.
d. Is money being returned independent of the class? Justify your answer numerically and explain it.
e. Based upon this study, do you think that economists are more selfish than other people? Explain why or why not. Include numbers to justify your answer.

S 3.4.18

a. $P(\mathrm{R}) = 0.44$
b. $P(\mathrm{R}|\mathrm{E}) = 0.56$
c. $P(\mathrm{R}|\mathrm{O}) = 0.31$
d. No, whether the money is returned is not independent of which class the money was placed in. There are several ways to justify this mathematically, but one is that the money placed in economics classes is not returned at the same overall rate; $P(\mathrm{R}|\mathrm{E}) \neq P(\mathrm{R})$.
e. No, this study definitely does not support that notion; *in fact*, it suggests the opposite. The money placed in the economics classrooms was returned at a higher rate than the money place in all classes collectively; $P(\mathrm{R}|\mathrm{E}) > P(\mathrm{R})$.

Q 3.4.19

The following table of data obtained from www.baseball-almanac.com shows hit information for four players. Suppose that one hit from the table is randomly selected.

Name	Single	Double	Triple	Home Run	Total Hits
Babe Ruth	1,517	506	136	714	2,873
Jackie Robinson	1,054	273	54	137	1,518
Ty Cobb	3,603	174	295	114	4,189
Hank Aaron	2,294	624	98	755	3,771
Total	8,471	1,577	583	1,720	12,351

Are "the hit being made by Hank Aaron" and "the hit being a double" independent events?

a. Yes, because $P(\text{hit by Hank Aaron}|\text{hit is a double}) = P(\text{hit by Hank Aaron})$
b. No, because $P(\text{hit by Hank Aaron}|\text{hit is a double}) \neq P(\text{hit is a double})$
c. No, because $P(\text{hit is by Hank Aaron}|\text{hit is a double}) \neq P(\text{hit by Hank Aaron})$
d. Yes, because $P(\text{hit is by Hank Aaron}|\text{hit is a double}) = P(\text{hit is a double})$

Q 3.4.29

United Blood Services is a blood bank that serves more than 500 hospitals in 18 states. According to their website, a person with type O blood and a negative Rh factor (Rh-) can donate blood to any person with any blood type. Their data show that 43% of people have type O blood and 15% of people have Rh- factor; 52% of people have type O or Rh- factor.

a. Find the probability that a person has both type O blood and the Rh- factor.
b. Find the probability that a person does NOT have both type O blood and the Rh- factor.

S 3.4.30

a. $P(\text{type O OR Rh-}) = P(\text{type O}) + P(\text{Rh-}) - P(\text{type O AND Rh-})$

$0.52 = 0.43 + 0.15 - P(\text{type O AND Rh-})$; solve to find $P(\text{type O AND Rh-}) = 0.06$

6% of people have type O, Rh- blood

b. $P(\text{NOT}(\text{type O AND Rh-})) = 1 - P(\text{type O AND Rh-}) = 1 - 0.06 = 0.94$

94% of people do not have type O, Rh- blood

Q 3.4.31

At a college, 72% of courses have final exams and 46% of courses require research papers. Suppose that 32% of courses have a research paper and a final exam. Let F be the event that a course has a final exam. Let R be the event that a course requires a research paper.

1. Find the probability that a course has a final exam or a research project.
2. Find the probability that a course has NEITHER of these two requirements.

Q 3.4.32

In a box of assorted cookies, 36% contain chocolate and 12% contain nuts. Of those, 8% contain both chocolate and nuts. Sean is allergic to both chocolate and nuts.

1. Find the probability that a cookie contains chocolate or nuts (he can't eat it).
2. Find the probability that a cookie does not contain chocolate or nuts (he can eat it).

S 3.4.32

a. Let $C =$ be the event that the cookie contains chocolate. Let $N =$ the event that the cookie contains nuts.
b. $P(\text{C OR N}) = P(\text{C}) + P(\text{N}) - P(\text{C AND N}) = 0.36 + 0.12 - 0.08 = 0.40$
c. $P(\text{NEITHER chocolate NOR nuts}) = 1 - P(\text{C OR N}) = 1 - 0.40 = 0.60$

Q 3.4.33

A college finds that 10% of students have taken a distance learning class and that 40% of students are part time students. Of the part time students, 20% have taken a distance learning class. Let $\text{D} =$ event that a student takes a distance learning class and $\text{E} =$ event that a student is a part time student

a. Find $P(\text{D AND E})$.
b. Find $P(\text{E}|\text{D})$.
c. Find $P(\text{D OR E})$.
d. Using an appropriate test, show whether D and E are independent.
e. Using an appropriate test, show whether D and E are mutually exclusive.

3.5: Contingency Tables

Use the information in the Table to answer the next eight exercises. The table shows the political party affiliation of each of 67 members of the US Senate in June 2012, and when they are up for reelection.

Up for reelection:	Democratic Party	Republican Party	Other	Total
November 2014	20	13	0	
November 2016	10	24	0	
Total				

Q 3.5.1

What is the probability that a randomly selected senator has an “Other” affiliation?

S 3.5.1

0

Q 3.5.2

What is the probability that a randomly selected senator is up for reelection in November 2016?

Q 3.5.3

What is the probability that a randomly selected senator is a Democrat and up for reelection in November 2016?

S 3.5.3

$\frac{10}{67}$

Q 3.5.4

What is the probability that a randomly selected senator is a Republican or is up for reelection in November 2014?

Q 3.5.5

Suppose that a member of the US Senate is randomly selected. Given that the randomly selected senator is up for reelection in November 2016, what is the probability that this senator is a Democrat?

S 3.5.5

$\frac{10}{34}$

Q 3.5.6

Suppose that a member of the US Senate is randomly selected. What is the probability that the senator is up for reelection in November 2014, knowing that this senator is a Republican?

Q 3.5.7

The events “Republican” and “Up for reelection in 2016” are ________

a. mutually exclusive.
b. independent.
c. both mutually exclusive and independent.
d. neither mutually exclusive nor independent.

S 3.5.7

d

Q 3.5.8

The events “Other” and “Up for reelection in November 2016” are ________

a. mutually exclusive.
b. independent.
c. both mutually exclusive and independent.
d. neither mutually exclusive nor independent.

Q 3.5.9

This table gives the number of participants in the recent National Health Interview Survey who had been treated for cancer in the previous 12 months. The results are sorted by age, race (black or white), and sex. We are interested in possible relationships between age, race, and sex.

Race and Sex	15-24	25-40	41-65	over 65	TOTALS
white, male	1,165	2,036	3,703		8,395
white, female	1,076	2,242	4,060		9,129
black, male	142	194	384		824
black, female	131	290	486		1,061
all others					
TOTALS	2,792	5,279	9,354		21,081

Do not include "all others" for parts f and g.

a. Fill in the column for cancer treatment for individuals over age 65.
b. Fill in the row for all other races.
c. Find the probability that a randomly selected individual was a white male.
d. Find the probability that a randomly selected individual was a black female.
e. Find the probability that a randomly selected individual was black
f. Find the probability that a randomly selected individual was a black or white male.
g. Out of the individuals over age 65, find the probability that a randomly selected individual was a black or white male.

S 3.5.9

a.

Race and Sex	1–14	15–24	25–64	over 64	TOTALS
white, male	210	3,360	13,610	4,870	22,050
white, female	80	580	3,380	890	4,930
black, male	10	460	1,060	140	1,670
black, female	0	40	270	20	330
all others				100	
TOTALS	310	4,650	18,780	6,020	29,760

b.

Race and Sex	1–14	15–24	25–64	over 64	TOTALS
white, male	210	3,360	13,610	4,870	22,050
white, female	80	580	3,380	890	4,930
black, male	10	460	1,060	140	1,670
black, female	0	40	270	20	330
all others	10	210	460	100	780
TOTALS	310	4,650	18,780	6,020	29,760

c. $\frac{22,050}{29,760}$
d. $\frac{330}{29,760}$
e. $\frac{2,000}{29,760}$
f. $\frac{23,720}{29,760}$
g. $\frac{5,010}{6,020}$

Use the following information to answer the next two exercises. The table of data obtained from*www.baseball-almanac.com* shows hit information for four well known baseball players. Suppose that one hit from the table is randomly selected.

NAME	Single	Double	Triple	Home Run	TOTAL HITS
Babe Ruth	1,517	506	136	714	2,873
Jackie Robinson	1,054	273	54	137	1,518
Ty Cobb	3,603	174	295	114	4,189
Hank Aaron	2,294	624	98	755	3,771
TOTAL	8,471	1,577	583	1,720	12,351

Q 3.5.10

Find $P(\text{hit was made by Babe Ruth})$.

a. $\frac{1518}{2873}$
b. $\frac{2873}{12351}$
c. $\frac{583}{12351}$
d. $\frac{4189}{12351}$

Q 3.5.11

Find $P(\text{hit was made by Ty Cobb}|\text{The hit was a Home Run})$.

a. $\frac{4189}{12351}$
b. $\frac{114}{1720}$
c. $\frac{1720}{4189}$
d. $\frac{114}{12351}$

S 3.5.11

b

Q 3.5.12

Table identifies a group of children by one of four hair colors, and by type of hair.

Hair Type	Brown	Blond	Black	Red	Totals
Wavy	20		15	3	43
Straight	80	15		12	
Totals		20			215

a. Complete the table.
b. What is the probability that a randomly selected child will have wavy hair?
c. What is the probability that a randomly selected child will have either brown or blond hair?
d. What is the probability that a randomly selected child will have wavy brown hair?
e. What is the probability that a randomly selected child will have red hair, given that he or she has straight hair?
f. If B is the event of a child having brown hair, find the probability of the complement of B.
g. In words, what does the complement of B represent?

Q 3.5.13

In a previous year, the weights of the members of the **San Francisco 49ers** and the **Dallas Cowboys** were published in the *San Jose Mercury News*. The factual data were compiled into the following table.

Shirt#	≤ 210	211–250	251–290	> 290
1–33	21	5	0	0
34–66	6	18	7	4
66–99	6	12	22	5

For the following, suppose that you randomly select one player from the 49ers or Cowboys.

a. Find the probability that his shirt number is from 1 to 33.
b. Find the probability that he weighs at most 210 pounds.
c. Find the probability that his shirt number is from 1 to 33 AND he weighs at most 210 pounds.
d. Find the probability that his shirt number is from 1 to 33 OR he weighs at most 210 pounds.
e. Find the probability that his shirt number is from 1 to 33 GIVEN that he weighs at most 210 pounds.

S 3.5.13

a. $\frac{26}{106}$
b. $\frac{33}{106}$
c. $\frac{21}{106}$
d. $\left(\frac{26}{106}\right)+\left(\frac{33}{106}\right)-\left(\frac{21}{106}\right)=\left(\frac{38}{106}\right)$
e. $\frac{21}{33}$

Exercise 3.6.8

The probability that a man develops some form of cancer in his lifetime is 0.4567. The probability that a man has at least one false positive test result (meaning the test comes back for cancer when the man does not have it) is 0.51. Let: $\mathrm{C}=$ a man develops cancer in his lifetime; $\mathrm{P}=$ man has at least one false positive. Construct a tree diagram of the situation.

Answer

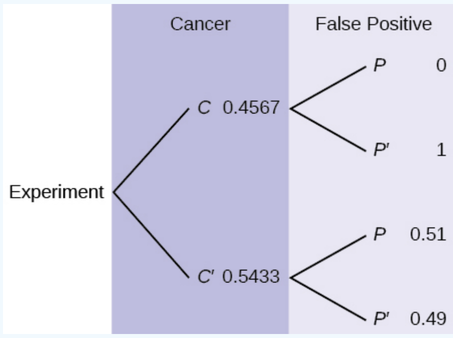


Figure 3.6.12

Bring It Together

Use the following information to answer the next two exercises. Suppose that you have eight cards. Five are green and three are yellow. The cards are well shuffled.

Exercise 3.6.9

Suppose that you randomly draw two cards, one at a time, **with replacement**.

Let $\mathrm{G}_1=$ first card is green
Let $\mathrm{G}_2=$ second card is green

a. Draw a tree diagram of the situation.
b. Find $P(\mathrm{G}_1 \mathrm{AND} \mathrm{G}_2)$.
c. Find $P(\text{at least one green})$.
d. Find $P(\mathrm{G}_2 | \mathrm{G}_1)$.
e. Are G_1 and G_2 independent events? Explain why or why not.

Answer

a.

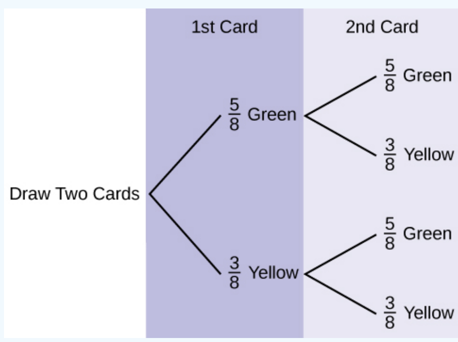


Figure 3.6.14

b. $P(\mathrm{GG})=\left(\frac{5}{8}\right)\left(\frac{5}{8}\right)=\frac{25}{64}$
c. $P(\text{at least one green})=P(\mathrm{GG})+P(\mathrm{GY})+P(\mathrm{YG})=\frac{25}{64}+\frac{15}{64}+\frac{15}{64}=\frac{55}{64}$
d. $P(\mathrm{G}|\mathrm{G})=\frac{5}{8}$
e. Yes, they are independent because the first card is placed back in the bag before the second card is drawn; the composition of cards in the bag remains the same from draw one to draw two.

Exercise 3.6.10

Suppose that you randomly draw two cards, one at a time, **without replacement**.

$G_1 =$ first card is green
$G_2 =$ second card is green

a. Draw a tree diagram of the situation.
b. Find $P(G_1 \text{ AND } G_2)$.
c. Find $P(\text{at least one green})$.
d. Find $P(G_2|G_1)$.
e. Are G_2 and G_1 independent events? Explain why or why not.

Use the following information to answer the next two exercises. The percent of licensed U.S. drivers (from a recent year) that are female is 48.60. Of the females, 5.03% are age 19 and under; 81.36% are age 20–64; 13.61% are age 65 or over. Of the licensed U.S. male drivers, 5.04% are age 19 and under; 81.43% are age 20–64; 13.53% are age 65 or over.

Exercise 3.6.11

Complete the following.

a. Construct a table or a tree diagram of the situation.
b. Find $P(\text{driver is female})$.
c. Find $P(\text{driver is age 65 or over}|\text{driver is female})$.
d. Find $P(\text{driver is age 65 or over AND female})$.
e. In words, explain the difference between the probabilities in part c and part d.
f. Find $P(\text{driver is age 65 or over})$.
g. Are being age 65 or over and being female mutually exclusive events? How do you know?

Answer

a.

	<20	**20–64**	**>64**	**Totals**
Female	0.0244	0.3954	0.0661	0.486
Male	0.0259	0.4186	0.0695	0.514
Totals	0.0503	0.8140	0.1356	1

b. $P(\text{F}) = 0.486$
c. $P(>64|\text{F}) = 0.1361$
d. $P(>64 \text{ and F}) = P(\text{F})P(>64|\text{F}) = (0.486)(0.1361) = 0.0661$
e. $P(>64|\text{F})$ is the percentage of female drivers who are 65 or older and $P(>64 \text{ and F})$ is the percentage of drivers who are female and 65 or older.
f. $P(>64) = P(>64 \text{ and F}) + P(>64 \text{ and M}) = 0.1356$
g. No, being female and 65 or older are not mutually exclusive because they can occur at the same time $P(>64 \text{ and F}) = 0.0661$.

Exercise 3.6.12

Suppose that 10,000 U.S. licensed drivers are randomly selected.

a. How many would you expect to be male?
b. Using the table or tree diagram, construct a contingency table of gender versus age group.
c. Using the contingency table, find the probability that out of the age 20–64 group, a randomly selected driver is female.

Exercise 3.6.13

Approximately 86.5% of Americans commute to work by car, truck, or van. Out of that group, 84.6% drive alone and 15.4% drive in a carpool. Approximately 3.9% walk to work and approximately 5.3% take public transportation.

a. Construct a table or a tree diagram of the situation. Include a branch for all other modes of transportation to work.
b. Assuming that the walkers walk alone, what percent of all commuters travel alone to work?
c. Suppose that 1,000 workers are randomly selected. How many would you expect to travel alone to work?
d. Suppose that 1,000 workers are randomly selected. How many would you expect to drive in a carpool?

Answer

a.

	Car, Truck or Van	**Walk**	**Public Transportation**	**Other**	**Totals**
Alone	0.7318				
Not Alone	0.1332				
Totals	0.8650	0.0390	0.0530	0.0430	1

b. If we assume that all walkers are alone and that none from the other two groups travel alone (which is a big assumption) we have: $P(\text{Alone}) = 0.7318 + 0.0390 = 0.7708$
c. Make the same assumptions as in (b) we have: (0.7708)(1,000) = 771
d. (0.1332)(1,000) = 133

Exercise 3.6.14

When the Euro coin was introduced in 2002, two math professors had their statistics students test whether the Belgian one Euro coin was a fair coin. They spun the coin rather than tossing it and found that out of 250 spins, 140 showed a head (event H) while 110 showed a tail (event T). On that basis, they claimed that it is not a fair coin.

a. Based on the given data, find $P(\text{H})$ and $P(\text{T})$.
b. Use a tree to find the probabilities of each possible outcome for the experiment of tossing the coin twice.
c. Use the tree to find the probability of obtaining exactly one head in two tosses of the coin.
d. Use the tree to find the probability of obtaining at least one head.

Exercise 3.6.15

Use the following information to answer the next two exercises. The following are real data from Santa Clara County, CA. As of a certain time, there had been a total of 3,059 documented cases of AIDS in the county. They were grouped into the following categories:

* includes homosexual/bisexual IV drug users

	Homosexual/Bisexual	**IV Drug User***	**Heterosexual Contact**	**Other**	**Totals**
Female	0	70	136	49	____
Male	2,146	463	60	135	____
Totals	____	____	____	____	____

Suppose a person with AIDS in Santa Clara County is randomly selected.

a. Find $P(\text{Person is female})$.
b. Find $P(\text{Person has a risk factor heterosexual contact})$.
c. Find $P(\text{Person is female OR has a risk factor of IV drug user})$.
d. Find $P(\text{Person is female AND has a risk factor of homosexual/bisexual})$.
e. Find $P(\text{Person is male AND has a risk factor of IV drug user})$.

f. Find $P(\text{Person is female GIVEN person got the disease from heterosexual contact})$.
g. Construct a Venn diagram. Make one group females and the other group heterosexual contact.

Answer

The completed contingency table is as follows:

* includes homosexual/bisexual IV drug users

	Homosexual/Bisexual	**IV Drug User***	**Heterosexual Contact**	**Other**	**Totals**
Female	0	70	136	49	**255**
Male	2,146	463	60	135	**2,804**
Totals	**2,146**	**533**	**196**	**184**	**3,059**

a. $\frac{255}{2059}$
b. $\frac{196}{3059}$
c. $\frac{718}{3059}$
d. 0
e. $\frac{463}{3059}$
f. $\frac{136}{196}$

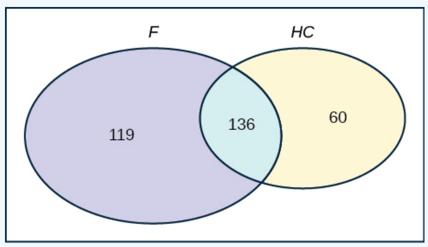
g. 

Figure 3.6.15

Exercise 3.6.16

Answer these questions using probability rules. Do NOT use the contingency table. Three thousand fifty-nine cases of AIDS had been reported in Santa Clara County, CA, through a certain date. Those cases will be our population. Of those cases, 6.4% obtained the disease through heterosexual contact and 7.4% are female. Out of the females with the disease, 53.3% got the disease from heterosexual contact.

a. Find $P(\text{Person is female})$.
b. Find $P(\text{Person obtained the disease through heterosexual contact})$.
c. Find $P(\text{Person is female GIVEN person got the disease from heterosexual contact})$
d. Construct a Venn diagram representing this situation. Make one group females and the other group heterosexual contact. Fill in all values as probabilities.

Use the following information to answer the next two exercises. This tree diagram shows the tossing of an unfair coin followed by drawing one bead from a cup containing three red (R), four yellow (Y) and five blue (B) beads. For the coin, $P(\text{H}) = \frac{2}{3}$ and $P(\text{T}) = \frac{1}{3}$ where H is heads and T is tails.

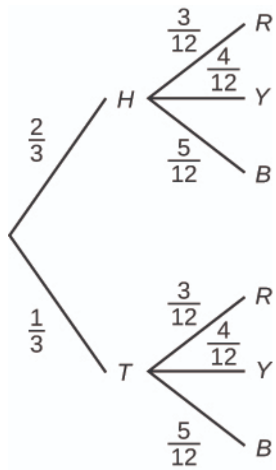


Figure 3.6.1.

Q 3.6.1

Find $P(\text{tossing a Head on the coin AND a Red bead})$

a. $\frac{2}{3}$
b. $\frac{5}{15}$
c. $\frac{6}{36}$
d. $\frac{5}{36}$

Q 3.6.2

Find $P(\text{Blue bead})$.

a. $\frac{15}{36}$
b. $\frac{10}{36}$
c. $\frac{10}{12}$
d. $\frac{6}{36}$

S 3.6.2

a

Q 3.6.3

A box of cookies contains three chocolate and seven butter cookies. Miguel randomly selects a cookie and eats it. Then he randomly selects another cookie and eats it. (How many cookies did he take?)

a. Draw the tree that represents the possibilities for the cookie selections. Write the probabilities along each branch of the tree.
b. Are the probabilities for the flavor of the SECOND cookie that Miguel selects independent of his first selection? Explain.
c. For each complete path through the tree, write the event it represents and find the probabilities.
d. Let S be the event that both cookies selected were the same flavor. Find $P(\mathrm{S})$.
e. Let T be the event that the cookies selected were different flavors. Find $P(\mathrm{T})$ by two different methods: by using the complement rule and by using the branches of the tree. Your answers should be the same with both methods.
f. Let U be the event that the second cookie selected is a butter cookie. Find $P(\mathrm{U})$.

3.7: Probability Topics