

8.4: Hypothesis Test Examples for Proportions

- In a hypothesis test problem, you may see words such as "the level of significance is 1%." The "1%" is the preconceived or preset α .
- The statistician setting up the hypothesis test selects the value of α to use before collecting the sample data.
- If no level of significance is given, a common standard to use is $\alpha = 0.05$.
- When you calculate the p -value and draw the picture, the p -value is the area in the left tail, the right tail, or split evenly between the two tails. For this reason, we call the hypothesis test left, right, or two tailed.
- The alternative hypothesis, H_a , tells you if the test is left, right, or two-tailed. It is the key to conducting the appropriate test.
- H_a never has a symbol that contains an equal sign.
- Thinking about the meaning of the p -value: A data analyst (and anyone else) should have more confidence that he made the correct decision to reject the null hypothesis with a smaller p -value (for example, 0.001 as opposed to 0.04) even if using the 0.05 level for alpha. Similarly, for a large p -value such as 0.4, as opposed to a p -value of 0.056 ($\alpha = 0.05$ is less than either number), a data analyst should have more confidence that she made the correct decision in not rejecting the null hypothesis. This makes the data analyst use judgment rather than mindlessly applying rules.

Full Hypothesis Test Examples

Example 8.4.7

Joon believes that 50% of first-time brides in the United States are younger than their grooms. She performs a hypothesis test to determine if the percentage is **the same or different from 50%**. Joon samples **100 first-time brides** and 53 reply that they are younger than their grooms. For the hypothesis test, she uses a 1% level of significance.

Answer

Set up the hypothesis test:

The 1% level of significance means that $\alpha = 0.01$. This is a **test of a single population proportion**.

$$H_0 : p = 0.50 \quad H_a : p \neq 0.50$$

The words "**is the same or different from**" tell you this is a two-tailed test.

Calculate the distribution needed:

Random variable: P' = the percent of first-time brides who are younger than their grooms.

Distribution for the test: The problem contains no mention of a mean. The information is given in terms of percentages. Use the distribution for P' , the estimated proportion.

$$P' - N \left(p, \sqrt{\frac{p-q}{n}} \right)$$

Therefore,

$$P' - N \left(0.5, \sqrt{\frac{0.5-0.5}{100}} \right)$$

where $p = 0.50$, $q = 1 - p = 0.50$, and $n = 100$

Calculate the p -value using the normal distribution for proportions:

$$p\text{-value} = P(p' < 0.47 \text{ or } p' > 0.53) = 0.5485$$

where

$$x = 53, p' = \frac{x}{n} = \frac{53}{100} = 0.53$$

Interpretation of the p -value: If the null hypothesis is true, there is 0.5485 probability (54.85%) that the sample (estimated) proportion p' is 0.53 or more OR 0.47 or less (see the graph in Figure).

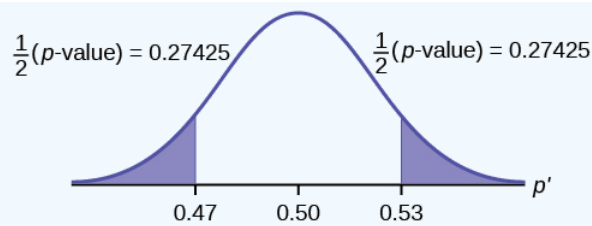


Figure 8.4.12

$\mu = p = 0.50$ comes from H_0 , the null hypothesis.

$p' = 0.53$. Since the curve is symmetrical and the test is two-tailed, the p' for the left tail is equal to $0.50 - 0.03 = 0.47$ where $\mu = p = 0.50$. (0.03 is the difference between 0.53 and 0.50.)

Compare α and the p -value:

Since $\alpha = 0.01$ and $p\text{-value} = 0.5485$, $\alpha < p\text{-value}$.

Make a decision: Since $\alpha < p\text{-value}$, you cannot reject H_0 .

Conclusion: At the 1% level of significance, the sample data do not show sufficient evidence that the percentage of first-time brides who are younger than their grooms is different from 50%.

The p -value can easily be calculated.

Press **STAT** and arrow over to **TESTS**. Press **5:1-PropZTest**. Enter .5 for p_0 , 53 for x and 100 for n . Arrow down to **Prop** and arrow to **not equals** p_0 . Press **ENTER**. Arrow down to **Calculate** and press **ENTER**. The calculator calculates the p -value ($p = 0.5485$) and the test statistic (z -score). **Prop not equals .5** is the alternate hypothesis. Do this set of instructions again except arrow to **Draw** (instead of **Calculate**). Press **ENTER**. A shaded graph appears with $(z = 0.6)$ (test statistic) and $p = 0.5485$ (p -value). Make sure when you use **Draw** that no other equations are highlighted in $Y =$ and the plots are turned off.

The Type I and Type II errors are as follows:

The Type I error is to conclude that the proportion of first-time brides who are younger than their grooms is different from 50% when, in fact, the proportion is actually 50%. (Reject the null hypothesis when the null hypothesis is true).

The Type II error is there is not enough evidence to conclude that the proportion of first time brides who are younger than their grooms differs from 50% when, in fact, the proportion does differ from 50%. (Do not reject the null hypothesis when the null hypothesis is false.)

Exercise 8.4.7

A teacher believes that 85% of students in the class will want to go on a field trip to the local zoo. She performs a hypothesis test to determine if the percentage is the same or different from 85%. The teacher samples 50 students and 39 reply that they would want to go to the zoo. For the hypothesis test, use a 1% level of significance.

First, determine what type of test this is, set up the hypothesis test, find the p -value, sketch the graph, and state your conclusion.

Answer

Since the problem is about percentages, this is a test of single population proportions.

- $H_0 : p = 0.85$
- $H_a : p \neq 0.85$
- $p = 0.7554$

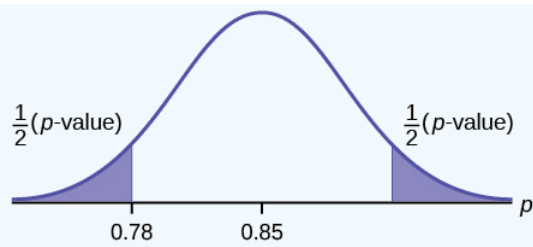


Figure 8.4.13

Because $p > \alpha$, we fail to reject the null hypothesis. There is not sufficient evidence to suggest that the proportion of students that want to go to the zoo is not 85%.

Example 8.4.8

Suppose a consumer group suspects that the proportion of households that have three cell phones is 30%. A cell phone company has reason to believe that the proportion is not 30%. Before they start a big advertising campaign, they conduct a hypothesis test. Their marketing people survey 150 households with the result that 43 of the households have three cell phones.

Answer

Set up the Hypothesis Test:

$$H_0 : p = 0.30, H_a : p \neq 0.30$$

Determine the distribution needed:

The **random variable** is $P' =$ proportion of households that have three cell phones.

The **distribution** for the hypothesis test is $P' \sim N\left(0.30, \sqrt{\frac{(0.30 \cdot 0.70)}{150}}\right)$

Exercise 9.6.8.2

- a. The value that helps determine the p -value is p' . Calculate p' .

Answer

- a. $p' = \frac{x}{n}$ where x is the number of successes and n is the total number in the sample.

$$x = 43, n = 150$$

$$p' = \frac{43}{150}$$

Exercise 9.6.8.3

- b. What is a **success** for this problem?

Answer

- b. A success is having three cell phones in a household.

Exercise 9.6.8.4

- c. What is the level of significance?

Answer

- c. The level of significance is the preset α . Since α is not given, assume that $\alpha = 0.05$.

Exercise 9.6.8.5

d. Draw the graph for this problem. Draw the horizontal axis. Label and shade appropriately.

Calculate the p -value.

Answer

d. p -value = 0.7216

Exercise 9.6.8.6

e. Make a decision. _____ (Reject/Do not reject) H_0 because _____.

Answer

e. Assuming that $\alpha = 0.05$, $\alpha < p$ -value. The decision is do not reject H_0 because there is not sufficient evidence to conclude that the proportion of households that have three cell phones is not 30%.

Exercise 8.4.8

Marketers believe that 92% of adults in the United States own a cell phone. A cell phone manufacturer believes that number is actually lower. 200 American adults are surveyed, of which, 174 report having cell phones. Use a 5% level of significance. State the null and alternative hypothesis, find the p -value, state your conclusion, and identify the Type I and Type II errors.

Answer

- $H_0 : p = 0.92$
- $H_a : p < 0.92$
- p -value = 0.0046

Because $p < 0.05$, we reject the null hypothesis. There is sufficient evidence to conclude that fewer than 92% of American adults own cell phones.

- Type I Error: To conclude that fewer than 92% of American adults own cell phones when, in fact, 92% of American adults do own cell phones (reject the null hypothesis when the null hypothesis is true).
- Type II Error: To conclude that 92% of American adults own cell phones when, in fact, fewer than 92% of American adults own cell phones (do not reject the null hypothesis when the null hypothesis is false).

The next example is a poem written by a statistics student named Nicole Hart. The solution to the problem follows the poem. Notice that the hypothesis test is for a single population proportion. This means that the null and alternate hypotheses use the parameter p . The distribution for the test is normal. The estimated proportion p' is the proportion of fleas killed to the total fleas found on Fido. This is sample information. The problem gives a preconceived $\alpha = 0.01$, for comparison, and a 95% confidence interval computation. The poem is clever and humorous, so please enjoy it!

Example 8.4.9

My dog has so many fleas,
They do not come off with ease.
As for shampoo, I have tried many types
Even one called Bubble Hype,
Which only killed 25% of the fleas,
Unfortunately I was not pleased.

I've used all kinds of soap,
Until I had given up hope
Until one day I saw
An ad that put me in awe.

A shampoo used for dogs
Called GOOD ENOUGH to Clean a Hog

Guaranteed to kill more fleas.

I gave Fido a bath
And after doing the math
His number of fleas
Started dropping by 3's!
Before his shampoo
I counted 42.

At the end of his bath,
I redid the math
And the new shampoo had killed 17 fleas.
So now I was pleased.

Now it is time for you to have some fun
With the level of significance being .01,
You must help me figure out

Use the new shampoo or go without?

Answer

Set up the hypothesis test:

$$H_0 : p \leq 0.25 \quad H_a : p > 0.25$$

Determine the distribution needed:

In words, CLEARLY state what your random variable \bar{X} or P' represents.

P' = The proportion of fleas that are killed by the new shampoo

State the distribution to use for the test.

Normal:

$$N \left(0.25, \sqrt{\frac{(0.25)(1 - 0.25)}{42}} \right)$$

Test Statistic: $z = 2.3163$

Calculate the p -value using the normal distribution for proportions:

$$p\text{-value} = 0.0103$$

In one to two complete sentences, explain what the p -value means for this problem.

If the null hypothesis is true (the proportion is 0.25), then there is a 0.0103 probability that the sample (estimated) proportion is $0.4048 \left(\frac{17}{42} \right)$ or more.

Use the previous information to sketch a picture of this situation. CLEARLY, label and scale the horizontal axis and shade the region(s) corresponding to the p -value.

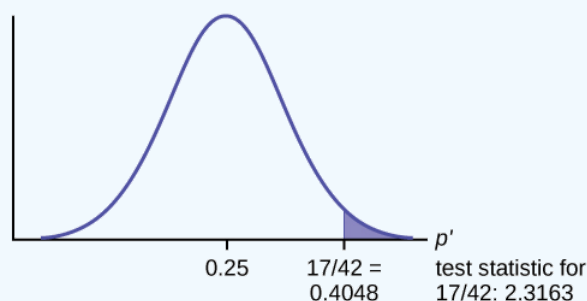


Figure 8.4.14

Compare α and the p -value:

Indicate the correct decision (“reject” or “do not reject” the null hypothesis), the reason for it, and write an appropriate conclusion, using complete sentences.

alpha	decision	reason for decision
0.01	Do not reject H_0	$\alpha < p$ -value

Conclusion: At the 1% level of significance, the sample data do not show sufficient evidence that the percentage of fleas that are killed by the new shampoo is more than 25%.

Construct a 95% confidence interval for the true mean or proportion. Include a sketch of the graph of the situation. Label the point estimate and the lower and upper bounds of the confidence interval.

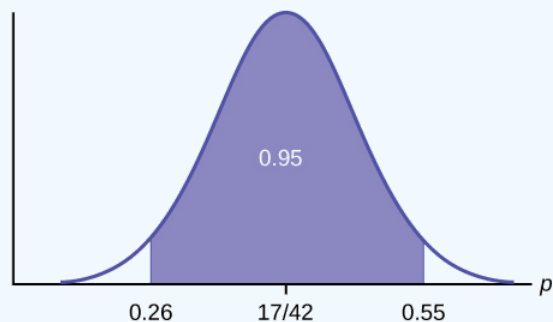


Figure 8.4.15

Confidence Interval: (0.26,0.55) We are 95% confident that the true population proportion p of fleas that are killed by the new shampoo is between 26% and 55%.

This test result is not very definitive since the p -value is very close to α . In reality, one would probably do more tests by giving the dog another bath after the fleas have had a chance to return.

Example 8.4.11

In a study of 420,019 cell phone users, 172 of the subjects developed brain cancer. Test the claim that cell phone users developed brain cancer at a greater rate than that for non-cell phone users (the rate of brain cancer for non-cell phone users is 0.0340%). Since this is a critical issue, use a 0.005 significance level. Explain why the significance level should be so low in terms of a Type I error.

Answer

We will follow the four-step process.

1. We need to conduct a hypothesis test on the claimed cancer rate. Our hypotheses will be

a. $H_0 : p \leq 0.00034$

b. $H_a : p > 0.00034$

If we commit a Type I error, we are essentially accepting a false claim. Since the claim describes cancer-causing environments, we want to minimize the chances of incorrectly identifying causes of cancer.

2. We will be testing a sample proportion with $x = 172$ and $n = 420,019$. The sample is sufficiently large because we have $np = 420,019(0.00034) = 142.8$ and $nq = 420,019(0.99966) = 419,876.2$ two independent outcomes, and a fixed probability of success $p = 0.00034$. Thus we will be able to generalize our results to the population.

3. The associated TI results are

Figure 9.6.11.

Figure 9.6.12.

4. Since the p -value = 0.0073 is greater than our alpha value = 0.005, we cannot reject the null. Therefore, we conclude that there is not enough evidence to support the claim of higher brain cancer rates for the cell phone users.

Example 8.4.12

According to the US Census there are approximately 268,608,618 residents aged 12 and older. Statistics from the Rape, Abuse, and Incest National Network indicate that, on average, 207,754 rapes occur each year (male and female) for persons aged 12 and older. This translates into a percentage of sexual assaults of 0.078%. In Daviess County, KY, there were reported 11 rapes for a population of 37,937. Conduct an appropriate hypothesis test to determine if there is a statistically significant difference between the local sexual assault percentage and the national sexual assault percentage. Use a significance level of 0.01.

Answer

We will follow the four-step plan.

1. We need to test whether the proportion of sexual assaults in Daviess County, KY is significantly different from the national average.
2. Since we are presented with proportions, we will use a one-proportion z -test. The hypotheses for the test will be
 - a. $H_0 : p = 0.00078$
 - b. $H_a : p \neq 0.00078$
3. The following screen shots display the summary statistics from the hypothesis test.

Figure 9.6.13.

Figure 9.6.14.

4. Since the p -value, $p = 0.00063$, is less than the alpha level of 0.01, the sample data indicates that we should reject the null hypothesis. In conclusion, the sample data support the claim that the proportion of sexual assaults in Daviess County, Kentucky is different from the national average proportion.

Review

The **hypothesis test** itself has an established process. This can be summarized as follows:

1. Determine H_0 and H_a . Remember, they are contradictory.
2. Determine the random variable.
3. Determine the distribution for the test.
4. Draw a graph, calculate the test statistic, and use the test statistic to calculate the p -value. (A z -score and a t -score are examples of test statistics.)
5. Compare the preconceived α with the p -value, make a decision (reject or do not reject H_0), and write a clear conclusion using English sentences.

Notice that in performing the hypothesis test, you use α and not β . β is needed to help determine the sample size of the data that is used in calculating the p -value. Remember that the quantity $1 - \beta$ is called the **Power of the Test**. A high power is desirable. If the power is too low, statisticians typically increase the sample size while keeping α the same. If the power is low, the null hypothesis might not be rejected when it should be.

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Glossary

Central Limit Theorem

Given a random variable (RV) with known mean μ and known standard deviation σ . We are sampling with size n and we are interested in two new RVs - the sample mean, \bar{X} , and the sample sum, $\sum X$. If the size n of the sample is sufficiently large, then $\bar{X} - N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ and $\sum X - N(n\mu, \sqrt{n}\sigma)$. If the size n of the sample is sufficiently large, then the distribution of the sample means and the distribution of the sample sums will approximate a normal distribution regardless of the shape of the population. The mean of the sample means will equal the population mean and the mean of the sample sums will equal n times the population mean. The standard deviation of the distribution of the sample means, $\frac{\sigma}{\sqrt{n}}$, is called the standard error of the mean.

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