

## 9.E: Hypothesis Testing with Two Samples (Optional Exercises)

These are homework exercises to accompany the Textmap created for "Introductory Statistics" by OpenStax.

### 10.1: Introduction

### 10.2: Two Population Means with Unknown Standard Deviations

Use the following information to answer the next 15 exercises: Indicate if the hypothesis test is for

- a. independent group means, population standard deviations, and/or variances known
- b. independent group means, population standard deviations, and/or variances unknown
- c. matched or paired samples
- d. single mean
- e. two proportions
- f. single proportion

#### Exercise 10.2.3

It is believed that 70% of males pass their drivers test in the first attempt, while 65% of females pass the test in the first attempt. Of interest is whether the proportions are in fact equal.

**Answer**

two proportions

#### Exercise 10.2.4

A new laundry detergent is tested on consumers. Of interest is the proportion of consumers who prefer the new brand over the leading competitor. A study is done to test this.

#### Exercise 10.2.5

A new windshield treatment claims to repel water more effectively. Ten windshields are tested by simulating rain without the new treatment. The same windshields are then treated, and the experiment is run again. A hypothesis test is conducted.

**Answer**

matched or paired samples

#### Exercise 10.2.6

The known standard deviation in salary for all mid-level professionals in the financial industry is \$11,000. Company A and Company B are in the financial industry. Suppose samples are taken of mid-level professionals from Company A and from Company B. The sample mean salary for mid-level professionals in Company A is \$80,000. The sample mean salary for mid-level professionals in Company B is \$96,000. Company A and Company B management want to know if their mid-level professionals are paid differently, on average.

#### Exercise 10.2.7

The average worker in Germany gets eight weeks of paid vacation.

**Answer**

single mean

#### Exercise 10.2.8

According to a television commercial, 80% of dentists agree that Ultrafresh toothpaste is the best on the market.

**Exercise 10.2.9**

It is believed that the average grade on an English essay in a particular school system for females is higher than for males. A random sample of 31 females had a mean score of 82 with a standard deviation of three, and a random sample of 25 males had a mean score of 76 with a standard deviation of four.

**Answer**

independent group means, population standard deviations and/or variances unknown

**Exercise 10.2.10**

The league mean batting average is 0.280 with a known standard deviation of 0.06. The Rattlers and the Vikings belong to the league. The mean batting average for a sample of eight Rattlers is 0.210, and the mean batting average for a sample of eight Vikings is 0.260. There are 24 players on the Rattlers and 19 players on the Vikings. Are the batting averages of the Rattlers and Vikings statistically different?

**Exercise 10.2.11**

In a random sample of 100 forests in the United States, 56 were coniferous or contained conifers. In a random sample of 80 forests in Mexico, 40 were coniferous or contained conifers. Is the proportion of conifers in the United States statistically more than the proportion of conifers in Mexico?

**Answer**

two proportions

**Exercise 10.2.12**

A new medicine is said to help improve sleep. Eight subjects are picked at random and given the medicine. The means hours slept for each person were recorded before starting the medication and after.

**Exercise 10.2.13**

It is thought that teenagers sleep more than adults on average. A study is done to verify this. A sample of 16 teenagers has a mean of 8.9 hours slept and a standard deviation of 1.2. A sample of 12 adults has a mean of 6.9 hours slept and a standard deviation of 0.6.

**Answer**

independent group means, population standard deviations and/or variances unknown

**Exercise 10.2.14**

Varsity athletes practice five times a week, on average.

**Exercise 10.2.15**

A sample of 12 in-state graduate school programs at school A has a mean tuition of \$64,000 with a standard deviation of \$8,000. At school B, a sample of 16 in-state graduate programs has a mean of \$80,000 with a standard deviation of \$6,000. On average, are the mean tuitions different?

**Answer**

independent group means, population standard deviations and/or variances unknown

**Exercise 10.2.16**

A new WiFi range booster is being offered to consumers. A researcher tests the native range of 12 different routers under the same conditions. The ranges are recorded. Then the researcher uses the new WiFi range booster and records the new ranges. Does the new WiFi range booster do a better job?

**Exercise 10.2.17**

A high school principal claims that 30% of student athletes drive themselves to school, while 4% of non-athletes drive themselves to school. In a sample of 20 student athletes, 45% drive themselves to school. In a sample of 35 non-athlete students, 6% drive themselves to school. Is the percent of student athletes who drive themselves to school more than the percent of nonathletes?

**Answer**

two proportions

*Use the following information to answer the next three exercises:* A study is done to determine which of two soft drinks has more sugar. There are 13 cans of Beverage A in a sample and six cans of Beverage B. The mean amount of sugar in Beverage A is 36 grams with a standard deviation of 0.6 grams. The mean amount of sugar in Beverage B is 38 grams with a standard deviation of 0.8 grams. The researchers believe that Beverage B has more sugar than Beverage A, on average. Both populations have normal distributions.

**Exercise 10.2.18**

Are standard deviations known or unknown?

**Exercise 10.2.19**

What is the random variable?

**Answer**

The random variable is the difference between the mean amounts of sugar in the two soft drinks.

**Exercise 10.2.20**

Is this a one-tailed or two-tailed test?

*Use the following information to answer the next 12 exercises:* The U.S. Center for Disease Control reports that the mean life expectancy was 47.6 years for whites born in 1900 and 33.0 years for nonwhites. Suppose that you randomly survey death records for people born in 1900 in a certain county. Of the 124 whites, the mean life span was 45.3 years with a standard deviation of 12.7 years. Of the 82 nonwhites, the mean life span was 34.1 years with a standard deviation of 15.6 years. Conduct a hypothesis test to see if the mean life spans in the county were the same for whites and nonwhites.

**Exercise 10.2.21**

Is this a test of means or proportions?

**Answer**

means

**Exercise 10.2.22**

State the null and alternative hypotheses.

a.  $H_0$ : \_\_\_\_\_

b.  $H_a$ : \_\_\_\_\_

**Exercise 10.2.23**

Is this a right-tailed, left-tailed, or two-tailed test?

**Answer**

two-tailed

**Exercise 10.2.24**

In symbols, what is the random variable of interest for this test?

**Exercise 10.2.25**

In words, define the random variable of interest for this test.

**Answer**

the difference between the mean life spans of whites and nonwhites

**Exercise 10.2.26**

Which distribution (normal or Student's  $t$ ) would you use for this hypothesis test?

**Exercise 10.2.27**

Explain why you chose the distribution you did for [Exercise](#).

**Answer**

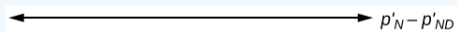
This is a comparison of two population means with unknown population standard deviations.

**Exercise 10.2.28**

Calculate the test statistic and  $p$ -value.

**Exercise 10.2.29**

Sketch a graph of the situation. Label the horizontal axis. Mark the hypothesized difference and the sample difference. Shade the area corresponding to the  $p$ -value.



**Answer**

Check student's solution.

**Exercise 10.2.30**

Find the  $p$ -value.

**Exercise 10.2.31**

At a pre-conceived  $\alpha = 0.05$ , what is your:

- Decision:
- Reason for the decision:
- Conclusion (write out in a complete sentence):

**Answer**

- Reject the null hypothesis
- $p\text{-value} < 0.05$
- There is not enough evidence at the 5% level of significance to support the claim that life expectancy in the 1900s is different between whites and nonwhites.

### Exercise 10.2.32

Does it appear that the means are the same? Why or why not?

**DIRECTIONS:** For each of the word problems, use a solution sheet to do the hypothesis test. The solution sheet is found in [Appendix E](#). Please feel free to make copies of the solution sheets. For the online version of the book, it is suggested that you copy the .doc or the .pdf files.

#### NOTE

If you are using a Student's  $t$ -distribution for a homework problem in what follows, including for paired data, you may assume that the underlying population is normally distributed. (When using these tests in a real situation, you must first prove that assumption, however.)

#### Q 10.2.1

The mean number of English courses taken in a two-year time period by male and female college students is believed to be about the same. An experiment is conducted and data are collected from 29 males and 16 females. The males took an average of three English courses with a standard deviation of 0.8. The females took an average of four English courses with a standard deviation of 1.0. Are the means statistically the same?

#### Q 10.2.2

A student at a four-year college claims that mean enrollment at four-year colleges is higher than at two-year colleges in the United States. Two surveys are conducted. Of the 35 two-year colleges surveyed, the mean enrollment was 5,068 with a standard deviation of 4,777. Of the 35 four-year colleges surveyed, the mean enrollment was 5,466 with a standard deviation of 8,191.

#### S 10.2.2

Subscripts: 1: two-year colleges; 2: four-year colleges

- $H_0 : \mu_1 \geq \mu_2$
- $H_a : \mu_1 < \mu_2$
- $\bar{X}_1 - \bar{X}_2$  is the difference between the mean enrollments of the two-year colleges and the four-year colleges.
- Student's- $t$
- test statistic: -0.2480
- $p$ -value : 0.4019
- Check student's solution.
- Alpha: 0.05
  - Decision: Do not reject
  - Reason for Decision:  $p$ -value  $> \alpha$
  - Conclusion: At the 5% significance level, there is sufficient evidence to conclude that the mean enrollment at four-year colleges is higher than at two-year colleges.

#### Q 10.2.3

At Rachel's 11<sup>th</sup> birthday party, eight girls were timed to see how long (in seconds) they could hold their breath in a relaxed position. After a two-minute rest, they timed themselves while jumping. The girls thought that the mean difference between their jumping and relaxed times would be zero. Test their hypothesis.

| Relaxed time (seconds) | Jumping time (seconds) |
|------------------------|------------------------|
| 26                     | 21                     |
| 47                     | 40                     |
| 30                     | 28                     |
| 22                     | 21                     |
| 23                     | 25                     |

| Relaxed time (seconds) | Jumping time (seconds) |
|------------------------|------------------------|
| 45                     | 43                     |
| 37                     | 35                     |
| 29                     | 32                     |

#### Q 10.2.4

Mean entry-level salaries for college graduates with mechanical engineering degrees and electrical engineering degrees are believed to be approximately the same. A recruiting office thinks that the mean mechanical engineering salary is actually lower than the mean electrical engineering salary. The recruiting office randomly surveys 50 entry level mechanical engineers and 60 entry level electrical engineers. Their mean salaries were \$46,100 and \$46,700, respectively. Their standard deviations were \$3,450 and \$4,210, respectively. Conduct a hypothesis test to determine if you agree that the mean entry-level mechanical engineering salary is lower than the mean entry-level electrical engineering salary.

#### S 10.2.4

Subscripts: 1: mechanical engineering; 2: electrical engineering

- $H_0 : \mu_1 \geq \mu_2$
- $H_a : \mu_1 < \mu_2$
- $\bar{X}_1 - \bar{X}_2$  is the difference between the mean entry level salaries of mechanical engineers and electrical engineers.
- $t_{108}$
- test statistic:  $t = -0.82$
- $p$ -value : 0.2061
- Check student's solution.
- $\alpha : 0.05$
  - Decision: Do not reject the null hypothesis.
  - Reason for Decision:  $p$ -value  $> \alpha$
  - Conclusion: At the 5% significance level, there is insufficient evidence to conclude that the mean entry-level salaries of mechanical engineers is lower than that of electrical engineers.

#### Q 10.2.5

Marketing companies have collected data implying that teenage girls use more ring tones on their cellular phones than teenage boys do. In one particular study of 40 randomly chosen teenage girls and boys (20 of each) with cellular phones, the mean number of ring tones for the girls was 3.2 with a standard deviation of 1.5. The mean for the boys was 1.7 with a standard deviation of 0.8. Conduct a hypothesis test to determine if the means are approximately the same or if the girls' mean is higher than the boys' mean.

Use the information from [\[link\]](#) to answer the next four exercises.

#### Q 10.2.6

Using the data from Lap 1 only, conduct a hypothesis test to determine if the mean time for completing a lap in races is the same as it is in practices.

#### S 10.2.6

- $H_0 : \mu_1 = \mu_2$
- $H_a : \mu_1 \neq \mu_2$
- $\bar{X}_1 - \bar{X}_2$  is the difference between the mean times for completing a lap in races and in practices.
- $t_{20.32}$
- test statistic:  $-4.70$
- $p$ -value : 0.0001
- Check student's solution.
- $\alpha : 0.05$
  - Decision: Reject the null hypothesis.
  - Reason for Decision:  $p$ -value  $> \alpha$

- iv. Conclusion: At the 5% significance level, there is sufficient evidence to conclude that the mean time for completing a lap in races is different from that in practices.

#### Q 10.2.7

Repeat the test in Exercise 10.83, but use Lap 5 data this time.

#### Q 10.2.8

Repeat the test in Exercise 10.83, but this time combine the data from Laps 1 and 5.

#### S 10.2.8

- $H_0 : \mu_1 = \mu_2$
- $H_a : \mu_1 \neq \mu_2$
- is the difference between the mean times for completing a lap in races and in practices.
- $t_{40.94}$
- test statistic:  $-5.08$
- $p$ -value : 0
- Check student's solution.
- $\alpha : 0.05$
  - Decision: Reject the null hypothesis.
  - Reason for Decision:  $p\text{-value} < \alpha$
  - Conclusion: At the 5% significance level, there is sufficient evidence to conclude that the mean time for completing a lap in races is different from that in practices.

#### Q 10.2.9

In two to three complete sentences, explain in detail how you might use Terri Vogel's data to answer the following question. "Does Terri Vogel drive faster in races than she does in practices?"

Use the following information to answer the next two exercises. The Eastern and Western Major League Soccer conferences have a new Reserve Division that allows new players to develop their skills. Data for a randomly picked date showed the following annual goals.

| Western          | Eastern       |
|------------------|---------------|
| Los Angeles 9    | D.C. United 9 |
| FC Dallas 3      | Chicago 8     |
| Chivas USA 4     | Columbus 7    |
| Real Salt Lake 3 | New England 6 |
| Colorado 4       | MetroStars 5  |
| San Jose 4       | Kansas City 3 |

Conduct a hypothesis test to answer the next two exercises.

#### Q 10.2.10

The **exact** distribution for the hypothesis test is:

- the normal distribution
- the Student's  $t$ -distribution
- the uniform distribution
- the exponential distribution

#### Q 10.2.11

If the level of significance is 0.05, the conclusion is:

- There is sufficient evidence to conclude that the **W** Division teams score fewer goals, on average, than the **E** teams
- There is insufficient evidence to conclude that the **W** Division teams score more goals, on average, than the **E** teams.
- There is insufficient evidence to conclude that the **W** teams score fewer goals, on average, than the **E** teams score.
- Unable to determine

#### Q 10.2.12

Suppose a statistics instructor believes that there is no significant difference between the mean class scores of statistics day students on Exam 2 and statistics night students on Exam 2. She takes random samples from each of the populations. The mean and standard deviation for 35 statistics day students were 75.86 and 16.91. The mean and standard deviation for 37 statistics night students were 75.41 and 19.73. The “day” subscript refers to the statistics day students. The “night” subscript refers to the statistics night students. A concluding statement is:

- There is sufficient evidence to conclude that statistics night students' mean on Exam 2 is better than the statistics day students' mean on Exam 2.
- There is insufficient evidence to conclude that the statistics day students' mean on Exam 2 is better than the statistics night students' mean on Exam 2.
- There is insufficient evidence to conclude that there is a significant difference between the means of the statistics day students and night students on Exam 2.
- There is sufficient evidence to conclude that there is a significant difference between the means of the statistics day students and night students on Exam 2.

#### Q 10.2.13

Researchers interviewed street prostitutes in Canada and the United States. The mean age of the 100 Canadian prostitutes upon entering prostitution was 18 with a standard deviation of six. The mean age of the 130 United States prostitutes upon entering prostitution was 20 with a standard deviation of eight. Is the mean age of entering prostitution in Canada lower than the mean age in the United States? Test at a 1% significance level.

#### S 10.2.13

Test: two independent sample means, population standard deviations unknown.

Random variable:

$$\bar{X}_1 - \bar{X}_2 \quad (9.E.1)$$

Distribution:  $H_0 : \mu_1 = \mu_2$   $H_a : \mu_1 < \mu_2$  The mean age of entering prostitution in Canada is lower than the mean age in the United States.

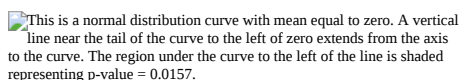
This is a normal distribution curve with mean equal to zero. A vertical line near the tail of the curve to the left of zero extends from the axis to the curve. The region under the curve to the left of the line is shaded representing p-value = 0.0157.

Figure 10.2.1.

Graph: left-tailed

p-value : 0.0151

Decision: Do not reject  $H_0$ .

Conclusion: At the 1% level of significance, from the sample data, there is not sufficient evidence to conclude that the mean age of entering prostitution in Canada is lower than the mean age in the United States.

#### Q 10.2.14

A powder diet is tested on 49 people, and a liquid diet is tested on 36 different people. Of interest is whether the liquid diet yields a higher mean weight loss than the powder diet. The powder diet group had a mean weight loss of 42 pounds with a standard deviation of 12 pounds. The liquid diet group had a mean weight loss of 45 pounds with a standard deviation of 14 pounds.

#### Q 10.2.15

Suppose a statistics instructor believes that there is no significant difference between the mean class scores of statistics day students on Exam 2 and statistics night students on Exam 2. She takes random samples from each of the populations. The mean and standard



deviation for 35 statistics day students were 75.86 and 16.91, respectively. The mean and standard deviation for 37 statistics night students were 75.41 and 19.73. The “day” subscript refers to the statistics day students. The “night” subscript refers to the statistics night students. An appropriate alternative hypothesis for the hypothesis test is:

- a.  $\mu_{\text{day}} > \mu_{\text{night}}$
- b.  $\mu_{\text{day}} < \mu_{\text{night}}$
- c.  $\mu_{\text{day}} = \mu_{\text{night}}$
- d.  $\mu_{\text{day}} \neq \mu_{\text{night}}$

### S 10.2.15

d

## 10.3: Two Population Means with Known Standard Deviations

Use the following information to answer the next five exercises. The mean speeds of fastball pitches from two different baseball pitchers are to be compared. A sample of 14 fastball pitches is measured from each pitcher. The populations have normal distributions. Table shows the result. Scouts believe that Rodriguez pitches a speedier fastball.

| Pitcher   | Sample Mean Speed of Pitches (mph) | Population Standard Deviation |
|-----------|------------------------------------|-------------------------------|
| Wesley    | 86                                 | 3                             |
| Rodriguez | 91                                 | 7                             |

### Exercise 10.3.2

What is the random variable?

**Answer**

The difference in mean speeds of the fastball pitches of the two pitchers

### Exercise 10.3.3

State the null and alternative hypotheses.

### Exercise 10.3.4

What is the test statistic?

**Answer**

-2.46

### Exercise 10.3.5

What is the  $p$ -value?

### Exercise 10.3.6

At the 1% significance level, we can reject the null hypothesis. There is sufficient data to conclude that the mean speed of Rodriguez’s fastball is faster than Wesley’s.

Use the following information to answer the next five exercises. A researcher is testing the effects of plant food on plant growth. Nine plants have been given the plant food. Another nine plants have not been given the plant food. The heights of the plants are recorded after eight weeks. The populations have normal distributions. The following table is the result. The researcher thinks the food makes the plants grow taller.

| Plant Group | Sample Mean Height of Plants (inches) | Population Standard Deviation |
|-------------|---------------------------------------|-------------------------------|
| Food        | 16                                    | 2.5                           |

| Plant Group | Sample Mean Height of Plants (inches) | Population Standard Deviation |
|-------------|---------------------------------------|-------------------------------|
| No food     | 14                                    | 1.5                           |

#### Exercise 10.3.7

Is the population standard deviation known or unknown?

#### Exercise 10.3.8

State the null and alternative hypotheses.

**Answer**

Subscripts: 1 = Food, 2 = No Food

$$H_0 : \mu_1 \leq \mu_2$$

$$H_a : \mu_1 > \mu_2$$

#### Exercise 10.3.9

What is the  $p$ -value?

#### Exercise 10.3.10

Draw the graph of the  $p$ -value.

**Answer**

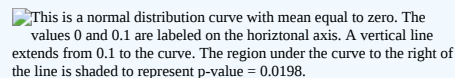
 This is a normal distribution curve with mean equal to zero. The values 0 and 0.1 are labeled on the horizontal axis. A vertical line extends from 0.1 to the curve. The region under the curve to the right of the line is shaded to represent  $p\text{-value} = 0.0198$ .

Figure 10.3.3.

#### Exercise 10.3.11

At the 1% significance level, what is your conclusion?

Use the following information to answer the next five exercises. Two metal alloys are being considered as material for ball bearings. The mean melting point of the two alloys is to be compared. 15 pieces of each metal are being tested. Both populations have normal distributions. The following table is the result. It is believed that Alloy Zeta has a different melting point.

|             | Sample Mean Melting Temperatures (°F) | Population Standard Deviation |
|-------------|---------------------------------------|-------------------------------|
| Alloy Gamma | 800                                   | 95                            |
| Alloy Zeta  | 900                                   | 105                           |

#### Exercise 10.3.12

State the null and alternative hypotheses.

**Answer**

Subscripts: 1 = Gamma, 2 = Zeta

$$H_0 : \mu_1 = \mu_2$$

$$H_a : \mu_1 \neq \mu_2$$

**Exercise 10.3.13**

Is this a right-, left-, or two-tailed test?

**Exercise 10.3.14**

What is the  $p$ -value?

**Answer**

0.0062

**Exercise 10.3.15**

Draw the graph of the  $p$ -value.

**Exercise 10.3.16**

At the 1% significance level, what is your conclusion?

**Answer**

There is sufficient evidence to reject the null hypothesis. The data support that the melting point for Alloy Zeta is different from the melting point of Alloy Gamma.

*DIRECTIONS: For each of the word problems, use a solution sheet to do the hypothesis test. The solution sheet is found in [\[link\]](#). Please feel free to make copies of the solution sheets. For the online version of the book, it is suggested that you copy the .doc or the .pdf files.*

**NOTE**

If you are using a Student's  $t$ -distribution for one of the following homework problems, including for paired data, you may assume that the underlying population is normally distributed. (When using these tests in a real situation, you must first prove that assumption, however.)

**Q 10.3.1**

A study is done to determine if students in the California state university system take longer to graduate, on average, than students enrolled in private universities. One hundred students from both the California state university system and private universities are surveyed. Suppose that from years of research, it is known that the population standard deviations are 1.5811 years and 1 year, respectively. The following data are collected. The California state university system students took on average 4.5 years with a standard deviation of 0.8. The private university students took on average 4.1 years with a standard deviation of 0.3.

**Q 10.3.2**

Parents of teenage boys often complain that auto insurance costs more, on average, for teenage boys than for teenage girls. A group of concerned parents examines a random sample of insurance bills. The mean annual cost for 36 teenage boys was \$679. For 23 teenage girls, it was \$559. From past years, it is known that the population standard deviation for each group is \$180. Determine whether or not you believe that the mean cost for auto insurance for teenage boys is greater than that for teenage girls.

**S 10.3.3**

Subscripts: 1 = boys, 2 = girls

- $H_0 : \mu_1 \leq \mu_2$
- $H_a : \mu_1 > \mu_2$
- The random variable is the difference in the mean auto insurance costs for boys and girls.
- normal
- test statistic:  $z = 2.50$
- $p$ -value : 0.0062
- Check student's solution.
- i.  $\alpha : 0.05$

- ii. Decision: Reject the null hypothesis.
- iii. Reason for Decision:  $p\text{-value} < \alpha$
- iv. Conclusion: At the 5% significance level, there is sufficient evidence to conclude that the mean cost of auto insurance for teenage boys is greater than that for girls.

#### Q 10.3.4

A group of transfer bound students wondered if they will spend the same mean amount on texts and supplies each year at their four-year university as they have at their community college. They conducted a random survey of 54 students at their community college and 66 students at their local four-year university. The sample means were \$947 and \$1,011, respectively. The population standard deviations are known to be \$254 and \$87, respectively. Conduct a hypothesis test to determine if the means are statistically the same.

#### Q 10.3.5

Some manufacturers claim that non-hybrid sedan cars have a lower mean miles-per-gallon (mpg) than hybrid ones. Suppose that consumers test 21 hybrid sedans and get a mean of 31 mpg with a standard deviation of seven mpg. Thirty-one non-hybrid sedans get a mean of 22 mpg with a standard deviation of four mpg. Suppose that the population standard deviations are known to be six and three, respectively. Conduct a hypothesis test to evaluate the manufacturers claim.

#### S 10.3.5

Subscripts: 1 = non-hybrid sedans, 2 = hybrid sedans

- a.  $H_0 : \mu_1 \geq \mu_2$
- b.  $H_a : \mu_1 < \mu_2$
- c. The random variable is the difference in the mean miles per gallon of non-hybrid sedans and hybrid sedans.
- d. normal
- e. test statistic: 6.36
- f.  $p\text{-value} : 0$
- g. Check student's solution.
- h. i.  $\alpha : 0.05$ 
  - ii. Decision: Reject the null hypothesis.
  - iii. Reason for decision:  $p\text{-value} < \alpha$
  - iv. Conclusion: At the 5% significance level, there is sufficient evidence to conclude that the mean miles per gallon of non-hybrid sedans is less than that of hybrid sedans.

#### Q 10.3.6

A baseball fan wanted to know if there is a difference between the number of games played in a World Series when the American League won the series versus when the National League won the series. From 1922 to 2012, the population standard deviation of games won by the American League was 1.14, and the population standard deviation of games won by the National League was 1.11. Of 19 randomly selected World Series games won by the American League, the mean number of games won was 5.76. The mean number of 17 randomly selected games won by the National League was 5.42. Conduct a hypothesis test.

#### Q 10.3.7

One of the questions in a study of marital satisfaction of dual-career couples was to rate the statement "I'm pleased with the way we divide the responsibilities for childcare." The ratings went from one (strongly agree) to five (strongly disagree). Table contains ten of the paired responses for husbands and wives. Conduct a hypothesis test to see if the mean difference in the husband's versus the wife's satisfaction level is negative (meaning that, within the partnership, the husband is happier than the wife).

|                        |   |   |   |   |   |   |   |   |   |   |
|------------------------|---|---|---|---|---|---|---|---|---|---|
| <b>Wife's Score</b>    | 2 | 2 | 3 | 3 | 4 | 2 | 1 | 1 | 2 | 4 |
| <b>Husband's Score</b> | 2 | 2 | 1 | 3 | 2 | 1 | 1 | 1 | 2 | 4 |

**S 10.3.7**

- a.  $H_0 : \mu_d = 0$
- b.  $H_a : \mu_d < 0$
- c. The random variable  $X_d$  is the average difference between husband's and wife's satisfaction level.
- d.  $t_9$
- e. test statistic:  $t = -1.86$
- f.  $p$ -value : 0.0479
- g. Check student's solution
- h.
  - i.  $\alpha : 0.05$
  - ii. Decision: Reject the null hypothesis, but run another test.
  - iii. Reason for Decision:  $p\text{-value} < \alpha$
  - iv. Conclusion: This is a weak test because alpha and the  $p$ -value are close. However, there is insufficient evidence to conclude that the mean difference is negative.

**10.4: Comparing Two Independent Population Proportions**

Use the following information for the next five exercises. Two types of phone operating system are being tested to determine if there is a difference in the proportions of system failures (crashes). Fifteen out of a random sample of 150 phones with OS<sub>1</sub> had system failures within the first eight hours of operation. Nine out of another random sample of 150 phones with OS<sub>2</sub> had system failures within the first eight hours of operation. OS<sub>2</sub> is believed to be more stable (have fewer crashes) than OS<sub>1</sub>.

**Exercise 10.4.2**

Is this a test of means or proportions?

**Exercise 10.4.3**

What is the random variable?

**Answer**

$P'_{OS_1} - P'_{OS_2}$  = difference in the proportions of phones that had system failures within the first eight hours of operation with OS<sub>1</sub> and OS<sub>2</sub>.

**Exercise 10.4.4**

State the null and alternative hypotheses.

**Exercise 10.4.5**

What is the  $p$ -value?

**Answer**

0.1018

**Exercise 10.4.6**

What can you conclude about the two operating systems?

Use the following information to answer the next twelve exercises. In the recent Census, three percent of the U.S. population reported being of two or more races. However, the percent varies tremendously from state to state. Suppose that two random surveys are conducted. In the first random survey, out of 1,000 North Dakotans, only nine people reported being of two or more races. In the second random survey, out of 500 Nevadans, 17 people reported being of two or more races. Conduct a hypothesis test to determine if the population percents are the same for the two states or if the percent for Nevada is statistically higher than for North Dakota.

**Exercise 10.4.7**

Is this a test of means or proportions?

**Answer**

proportions

**Exercise 10.4.8**

State the null and alternative hypotheses.

a.  $H_0$ : \_\_\_\_\_

b.  $H_a$ : \_\_\_\_\_

**Exercise 10.4.9**

Is this a right-tailed, left-tailed, or two-tailed test? How do you know?

**Answer**

right-tailed

**Exercise 10.4.10**

What is the random variable of interest for this test?

**Exercise 10.4.11**

In words, define the random variable for this test.

**Answer**

The random variable is the difference in proportions (percents) of the populations that are of two or more races in Nevada and North Dakota.

**Exercise 10.4.12**

Which distribution (normal or Student's  $t$ ) would you use for this hypothesis test?

**Exercise 10.4.13**

Explain why you chose the distribution you did for the [Exercise 10.56](#).

**Answer**

Our sample sizes are much greater than five each, so we use the normal for two proportions distribution for this hypothesis test.

**Exercise 10.4.14**

Calculate the test statistic.

**Exercise 10.4.15**

Sketch a graph of the situation. Mark the hypothesized difference and the sample difference. Shade the area corresponding to the  $p$ -value.

 This is a horizontal axis with arrows at each end. The axis is labeled  $p'N - p'ND$ .

Figure 10.4.5.

**Answer**

Check student's solution.

**Exercise 10.4.16**

Find the  $p$ -value.

**Exercise 10.4.17**

At a pre-conceived  $\alpha = 0.05$ , what is your:

- Decision:
- Reason for the decision:
- Conclusion (write out in a complete sentence):

**Answer**

- Reject the null hypothesis.
- $p\text{-value} < \alpha$
- At the 5% significance level, there is sufficient evidence to conclude that the proportion (percent) of the population that is of two or more races in Nevada is statistically higher than that in North Dakota.

**Exercise 10.4.18**

Does it appear that the proportion of Nevadans who are two or more races is higher than the proportion of North Dakotans? Why or why not?

*DIRECTIONS: For each of the word problems, use a solution sheet to do the hypothesis test. The solution sheet is found in [\[link\]](#). Please feel free to make copies of the solution sheets. For the online version of the book, it is suggested that you copy the .doc or the .pdf files.*

**NOTE**

If you are using a Student's  $t$ -distribution for one of the following homework problems, including for paired data, you may assume that the underlying population is normally distributed. (In general, you must first prove that assumption, however.)

**Q 10.4.1**

A recent drug survey showed an increase in the use of drugs and alcohol among local high school seniors as compared to the national percent. Suppose that a survey of 100 local seniors and 100 national seniors is conducted to see if the proportion of drug and alcohol use is higher locally than nationally. Locally, 65 seniors reported using drugs or alcohol within the past month, while 60 national seniors reported using them.

**Q 10.4.2**

We are interested in whether the proportions of female suicide victims for ages 15 to 24 are the same for the whites and the blacks races in the United States. We randomly pick one year, 1992, to compare the races. The number of suicides estimated in the United States in 1992 for white females is 4,930. Five hundred eighty were aged 15 to 24. The estimate for black females is 330. Forty were aged 15 to 24. We will let female suicide victims be our population.

**S 10.4.2**

- $H_0 : P_W = P_B$
- $H_a : P_W \neq P_B$
- The random variable is the difference in the proportions of white and black suicide victims, aged 15 to 24.
- normal for two proportions
- test statistic:  $-0.1944$
- $p\text{-value} : 0.8458$
- Check student's solution.
- $\alpha : 0.05$
  - Decision: Reject the null hypothesis.
  - Reason for decision:  $p\text{-value} > \alpha$

- iv. Conclusion: At the 5% significance level, there is insufficient evidence to conclude that the proportions of white and black female suicide victims, aged 15 to 24, are different.

#### Q 10.4.3

Elizabeth Mjelde, an art history professor, was interested in whether the value from the Golden Ratio formula,  $\left(\frac{\text{larger} + \text{smaller dimension}}{\text{larger dimension}}\right)$  was the same in the Whitney Exhibit for works from 1900 to 1919 as for works from 1920 to 1942. Thirty-seven early works were sampled, averaging 1.74 with a standard deviation of 0.11. Sixty-five of the later works were sampled, averaging 1.746 with a standard deviation of 0.1064. Do you think that there is a significant difference in the Golden Ratio calculation?

#### Q 10.4.4

A recent year was randomly picked from 1985 to the present. In that year, there were 2,051 Hispanic students at Cabrillo College out of a total of 12,328 students. At Lake Tahoe College, there were 321 Hispanic students out of a total of 2,441 students. In general, do you think that the percent of Hispanic students at the two colleges is basically the same or different?

#### S 10.4.4

Subscripts: 1 = Cabrillo College, 2 = Lake Tahoe College

- $H_0 : p_1 = p_2$
- $H_a : p_1 \neq p_2$
- The random variable is the difference between the proportions of Hispanic students at Cabrillo College and Lake Tahoe College.
- normal for two proportions
- test statistic: 4.29
- $p$ -value : 0.00002
- Check student's solution.
- $\alpha : 0.05$
  - Decision: Reject the null hypothesis.
  - Reason for decision:  $p$ -value < alpha
  - Conclusion: There is sufficient evidence to conclude that the proportions of Hispanic students at Cabrillo College and Lake Tahoe College are different.

Use the following information to answer the next three exercises. Neuroinvasive West Nile virus is a severe disease that affects a person's nervous system. It is spread by the Culex species of mosquito. In the United States in 2010 there were 629 reported cases of neuroinvasive West Nile virus out of a total of 1,021 reported cases and there were 486 neuroinvasive reported cases out of a total of 712 cases reported in 2011. Is the 2011 proportion of neuroinvasive West Nile virus cases more than the 2010 proportion of neuroinvasive West Nile virus cases? Using a 1% level of significance, conduct an appropriate hypothesis test.

- "2011" subscript: 2011 group.
- "2010" subscript: 2010 group

#### Q 10.4.5

This is:

- a test of two proportions
- a test of two independent means
- a test of a single mean
- a test of matched pairs.

#### Q 10.4.6

An appropriate null hypothesis is:

- $p_{2011} \leq p_{2010}$
- $p_{2011} \geq p_{2010}$
- $\mu_{2011} \leq \mu_{2010}$



d.  $p_{2011} > p_{2010}$

### S 10.4.6

a

### Q 10.4.7

The  $p$ -value is 0.0022. At a 1% level of significance, the appropriate conclusion is

- There is sufficient evidence to conclude that the proportion of people in the United States in 2011 who contracted neuroinvasive West Nile disease is less than the proportion of people in the United States in 2010 who contracted neuroinvasive West Nile disease.
- There is insufficient evidence to conclude that the proportion of people in the United States in 2011 who contracted neuroinvasive West Nile disease is more than the proportion of people in the United States in 2010 who contracted neuroinvasive West Nile disease.
- There is insufficient evidence to conclude that the proportion of people in the United States in 2011 who contracted neuroinvasive West Nile disease is less than the proportion of people in the United States in 2010 who contracted neuroinvasive West Nile disease.
- There is sufficient evidence to conclude that the proportion of people in the United States in 2011 who contracted neuroinvasive West Nile disease is more than the proportion of people in the United States in 2010 who contracted neuroinvasive West Nile disease.

### Q 10.4.8

Researchers conducted a study to find out if there is a difference in the use of eReaders by different age groups. Randomly selected participants were divided into two age groups. In the 16- to 29-year-old group, 7% of the 628 surveyed use eReaders, while 11% of the 2,309 participants 30 years old and older use eReaders.

### S 10.4.9

Test: two independent sample proportions.

Random variable:  $p'_1 - p'_2$

Distribution:

$$H_0 : p_1 = p_2$$

$$H_a : p_1 \neq p_2$$

The proportion of eReader users is different for the 16- to 29-year-old users from that of the 30 and older users.

Graph: two-tailed

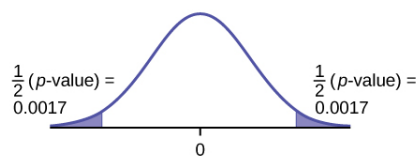


Figure 10.4.1.

$p$ -value : 0.0033

Decision: Reject the null hypothesis.

Conclusion: At the 5% level of significance, from the sample data, there is sufficient evidence to conclude that the proportion of eReader users 16 to 29 years old is different from the proportion of eReader users 30 and older.

### Q 10.4.10

are considered obese if their body mass index (BMI) is at least 30. The researchers wanted to determine if the proportion of women who are obese in the south is less than the proportion of southern men who are obese. The results are shown in Table. Test at the 1% level of significance.

|       | Number who are obese | Sample size |
|-------|----------------------|-------------|
| Men   | 42,769               | 155,525     |
| Women | 67,169               | 248,775     |

#### Q 10.4.11

Two computer users were discussing tablet computers. A higher proportion of people ages 16 to 29 use tablets than the proportion of people age 30 and older. Table details the number of tablet owners for each age group. Test at the 1% level of significance.

|              | 16–29 year olds | 30 years old and older |
|--------------|-----------------|------------------------|
| Own a Tablet | 69              | 231                    |
| Sample Size  | 628             | 2,309                  |

#### S 10.4.11

Test: two independent sample proportions

Random variable:  $p'_1 - p'_2$

Distribution:

$$H_0 : p_1 = p_2$$

$$H_a : p_1 > p_2$$

A higher proportion of tablet owners are aged 16 to 29 years old than are 30 years old and older.

Graph: right-tailed

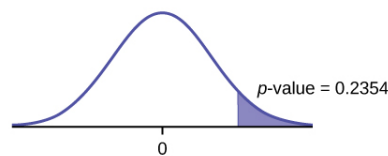


Figure 10.4.2.

$p$ -value : 0.2354

Decision: Do not reject the  $H_0$ .

Conclusion: At the 1% level of significance, from the sample data, there is not sufficient evidence to conclude that a higher proportion of tablet owners are aged 16 to 29 years old than are 30 years old and older.

#### Q 10.4.12

A group of friends debated whether more men use smartphones than women. They consulted a research study of smartphone use among adults. The results of the survey indicate that of the 973 men randomly sampled, 379 use smartphones. For women, 404 of the 1,304 who were randomly sampled use smartphones. Test at the 5% level of significance.

#### Q 10.4.13

While her husband spent 2½ hours picking out new speakers, a statistician decided to determine whether the percent of men who enjoy shopping for electronic equipment is higher than the percent of women who enjoy shopping for electronic equipment. The population was Saturday afternoon shoppers. Out of 67 men, 24 said they enjoyed the activity. Eight of the 24 women surveyed claimed to enjoy the activity. Interpret the results of the survey.

#### S 10.4.13

Subscripts: 1: men; 2: women

a.  $H_0 : p_1 \leq p_2$

b.  $H_a : p_1 > p_2$

- c.  $P'_1 - P_2$  is the difference between the proportions of men and women who enjoy shopping for electronic equipment.
- d. normal for two proportions
- e. test statistic: 0.22
- f.  $p$ -value : 0.4133
- g. Check student's solution.
- h.
  - i.  $\alpha : 0.05$
  - ii. Decision: Do not reject the null hypothesis.
  - iii. Reason for Decision:  $p$ -value  $> \alpha$
  - iv. Conclusion: At the 5% significance level, there is insufficient evidence to conclude that the proportion of men who enjoy shopping for electronic equipment is more than the proportion of women.

#### Q 10.4.14

We are interested in whether children's educational computer software costs less, on average, than children's entertainment software. Thirty-six educational software titles were randomly picked from a catalog. The mean cost was \$31.14 with a standard deviation of \$4.69. Thirty-five entertainment software titles were randomly picked from the same catalog. The mean cost was \$33.86 with a standard deviation of \$10.87. Decide whether children's educational software costs less, on average, than children's entertainment software.

#### Q 10.4.15

Joan Nguyen recently claimed that the proportion of college-age males with at least one pierced ear is as high as the proportion of college-age females. She conducted a survey in her classes. Out of 107 males, 20 had at least one pierced ear. Out of 92 females, 47 had at least one pierced ear. Do you believe that the proportion of males has reached the proportion of females?

#### S 10.4.15

- a.  $H_0 : p_1 = p_2$
- b.  $H_a : p_1 \neq p_2$
- c.  $P'_1 - P_2$  is the difference between the proportions of men and women that have at least one pierced ear.
- d. normal for two proportions
- e. test statistic: -4.82
- f.  $p$ -value : 0
- g. Check student's solution.
- h.
  - i.  $\alpha : 0.05$
  - ii. Decision: Reject the null hypothesis.
  - iii. Reason for Decision:  $p$ -value  $< \alpha$
  - iv. Conclusion: At the 5% significance level, there is sufficient evidence to conclude that the proportions of males and females with at least one pierced ear is different.

#### Q 10.4.16

Use the data sets found in [\[link\]](#) to answer this exercise. Is the proportion of race laps Terri completes slower than 130 seconds less than the proportion of practice laps she completes slower than 135 seconds?

#### Q 10.4.17

"To Breakfast or Not to Breakfast?" by Richard Ayore

In the American society, birthdays are one of those days that everyone looks forward to. People of different ages and peer groups gather to mark the 18th, 20th, ..., birthdays. During this time, one looks back to see what he or she has achieved for the past year and also focuses ahead for more to come.

If, by any chance, I am invited to one of these parties, my experience is always different. Instead of dancing around with my friends while the music is booming, I get carried away by memories of my family back home in Kenya. I remember the good times I had with my brothers and sister while we did our daily routine.

Every morning, I remember we went to the shamba (garden) to weed our crops. I remember one day arguing with my brother as to why he always remained behind just to join us an hour later. In his defense, he said that he preferred waiting for breakfast before he came to weed. He said, "This is why I always work more hours than you guys!"

And so, to prove him wrong or right, we decided to give it a try. One day we went to work as usual without breakfast, and recorded the time we could work before getting tired and stopping. On the next day, we all ate breakfast before going to work. We recorded how long we worked again before getting tired and stopping. Of interest was our mean increase in work time. Though not sure, my brother insisted that it was more than two hours. Using the data in [Table](#), solve our problem.

| Work hours with breakfast | Work hours without breakfast |
|---------------------------|------------------------------|
| 8                         | 6                            |
| 7                         | 5                            |
| 9                         | 5                            |
| 5                         | 4                            |
| 9                         | 7                            |
| 8                         | 7                            |
| 10                        | 7                            |
| 7                         | 5                            |
| 6                         | 6                            |
| 9                         | 5                            |

#### S 10.4.17

- $H_0 : \mu_d = 0$
- $H_a : \mu_d > 0$
- The random variable  $X_d$  is the mean difference in work times on days when eating breakfast and on days when not eating breakfast.
- $t_9$
- test statistic: 4.8963
- $p$ -value : 0.0004
- Check student's solution.
- $\alpha : 0.05$
  - Decision: Reject the null hypothesis.
  - Reason for Decision:  $p\text{-value} < \alpha$
  - Conclusion: At the 5% level of significance, there is sufficient evidence to conclude that the mean difference in work times on days when eating breakfast and on days when not eating breakfast has increased.

### 10.5: Matched or Paired Samples

Use the following information to answer the next five exercises. A study was conducted to test the effectiveness of a software patch in reducing system failures over a six-month period. Results for randomly selected installations are shown in [Table](#). The “before” value is matched to an “after” value, and the differences are calculated. The differences have a normal distribution. Test at the 1% significance level.

| Installation | A | B | C | D | E | F | G | H |
|--------------|---|---|---|---|---|---|---|---|
| Before       | 3 | 6 | 4 | 2 | 5 | 8 | 2 | 6 |
| After        | 1 | 5 | 2 | 0 | 1 | 0 | 2 | 2 |

#### Exercise 10.5.4

What is the random variable?

**Answer**

the mean difference of the system failures

#### Exercise 10.5.5

State the null and alternative hypotheses.

#### Exercise 10.5.6

What is the  $p$ -value?

**Answer**

0.0067

#### Exercise 10.5.7

Draw the graph of the  $p$ -value.

#### Exercise 10.5.8

What conclusion can you draw about the software patch?

**Answer**

With a  $p$ -value 0.0067, we can reject the null hypothesis. There is enough evidence to support that the software patch is effective in reducing the number of system failures.

Use the following information to answer next five exercises. A study was conducted to test the effectiveness of a juggling class. Before the class started, six subjects juggled as many balls as they could at once. After the class, the same six subjects juggled as many balls as they could. The differences in the number of balls are calculated. The differences have a normal distribution. Test at the 1% significance level.

| Subject | A | B | C | D | E | F |
|---------|---|---|---|---|---|---|
| Before  | 3 | 4 | 3 | 2 | 4 | 5 |
| After   | 4 | 5 | 6 | 4 | 5 | 7 |

#### Exercise 10.5.9

State the null and alternative hypotheses.

#### Exercise 10.5.10

What is the  $p$ -value?

**Answer**

0.0021

#### Exercise 10.5.11

What is the sample mean difference?

#### Exercise 10.5.12

Draw the graph of the  $p$ -value.

**Answer**


 This is a normal distribution curve with mean equal to zero. The values 0 and 1.67 are labeled on the horizontal axis. A vertical line extends from 1.67 to the curve. The region under the curve to the right of the line is shaded to represent  $p\text{-value} = 0.0021$ .

Figure 10.5.4.

### Exercise 10.5.13

What conclusion can you draw about the juggling class?

Use the following information to answer the next five exercises. A doctor wants to know if a blood pressure medication is effective. Six subjects have their blood pressures recorded. After twelve weeks on the medication, the same six subjects have their blood pressure recorded again. For this test, only systolic pressure is of concern. Test at the 1% significance level.

| Patient | A   | B   | C   | D   | E   | F   |
|---------|-----|-----|-----|-----|-----|-----|
| Before  | 161 | 162 | 165 | 162 | 166 | 171 |
| After   | 158 | 159 | 166 | 160 | 167 | 169 |

### Exercise 10.5.14

State the null and alternative hypotheses.

**Answer**

$$H_0 : \mu_d \geq 0$$

$$H_a : \mu_d < 0$$

### Exercise 10.5.15

What is the test statistic?

### Exercise 10.5.16

What is the  $p$ -value?

**Answer**

0.0699

### Exercise 10.5.17

What is the sample mean difference?

### Exercise 10.5.18

What is the conclusion?

**Answer**

We decline to reject the null hypothesis. There is not sufficient evidence to support that the medication is effective.

## Bringing It Together

Use the following information to answer the next ten exercises. indicate which of the following choices best identifies the hypothesis test.

- independent group means, population standard deviations and/or variances known
- independent group means, population standard deviations and/or variances unknown
- matched or paired samples
- single mean
- two proportions

f. single proportion

**Exercise 10.5.19**

A powder diet is tested on 49 people, and a liquid diet is tested on 36 different people. The population standard deviations are two pounds and three pounds, respectively. Of interest is whether the liquid diet yields a higher mean weight loss than the powder diet.

**Exercise 10.5.20**

A new chocolate bar is taste-tested on consumers. Of interest is whether the proportion of children who like the new chocolate bar is greater than the proportion of adults who like it.

**Answer**

e

**Exercise 10.5.21**

The mean number of English courses taken in a two-year time period by male and female college students is believed to be about the same. An experiment is conducted and data are collected from nine males and 16 females.

**Exercise 10.5.22**

A football league reported that the mean number of touchdowns per game was five. A study is done to determine if the mean number of touchdowns has decreased.

**Answer**

d

**Exercise 10.5.23**

A study is done to determine if students in the California state university system take longer to graduate than students enrolled in private universities. One hundred students from both the California state university system and private universities are surveyed. From years of research, it is known that the population standard deviations are 1.5811 years and one year, respectively.

**Exercise 10.5.24**

According to a YWCA Rape Crisis Center newsletter, 75% of rape victims know their attackers. A study is done to verify this.

**Answer**

f

**Exercise 10.5.25**

According to a recent study, U.S. companies have a mean maternity-leave of six weeks.

**Exercise 10.5.26**

A recent drug survey showed an increase in use of drugs and alcohol among local high school students as compared to the national percent. Suppose that a survey of 100 local youths and 100 national youths is conducted to see if the proportion of drug and alcohol use is higher locally than nationally.

**Answer**

e

### Exercise 10.5.27

A new SAT study course is tested on 12 individuals. Pre-course and post-course scores are recorded. Of interest is the mean increase in SAT scores. The following data are collected:

| Pre-course score | Post-course score |
|------------------|-------------------|
| 1                | 300               |
| 960              | 920               |
| 1010             | 1100              |
| 840              | 880               |
| 1100             | 1070              |
| 1250             | 1320              |
| 860              | 860               |
| 1330             | 1370              |
| 790              | 770               |
| 990              | 1040              |
| 1110             | 1200              |
| 740              | 850               |

### Exercise 10.5.28

University of Michigan researchers reported in the *Journal of the National Cancer Institute* that quitting smoking is especially beneficial for those under age 49. In this American Cancer Society study, the risk (probability) of dying of lung cancer was about the same as for those who had never smoked.

**Answer**

f

### Exercise 10.5.29

Lesley E. Tan investigated the relationship between left-handedness vs. right-handedness and motor competence in preschool children. Random samples of 41 left-handed preschool children and 41 right-handed preschool children were given several tests of motor skills to determine if there is evidence of a difference between the children based on this experiment. The experiment produced the means and standard deviations shown [Table](#). Determine the appropriate test and best distribution to use for that test.

|                           | Left-handed | Right-handed |
|---------------------------|-------------|--------------|
| Sample size               | 41          | 41           |
| Sample mean               | 97.5        | 98.1         |
| Sample standard deviation | 17.5        | 19.2         |

- Two independent means, normal distribution
- Two independent means, Student's-t distribution
- Matched or paired samples, Student's-t distribution
- Two population proportions, normal distribution



### Exercise 10.5.30

A golf instructor is interested in determining if her new technique for improving players' golf scores is effective. She takes four (4) new students. She records their 18-hole scores before learning the technique and then after having taken her class. She conducts a hypothesis test. The data are as [Table](#).

|                         | Player 1 | Player 2 | Player 3 | Player 4 |
|-------------------------|----------|----------|----------|----------|
| Mean score before class | 83       | 78       | 93       | 87       |
| Mean score after class  | 80       | 80       | 86       | 86       |

This is:

- a test of two independent means.
- a test of two proportions.
- a test of a single mean.
- a test of a single proportion.

**Answer**

a

*DIRECTIONS: For each of the word problems, use a solution sheet to do the hypothesis test. The solution sheet is found in [Appendix E](#). Please feel free to make copies of the solution sheets. For the online version of the book, it is suggested that you copy the .doc or the .pdf files.*

NOTE

If you are using a Student's *t*-distribution for the homework problems, including for paired data, you may assume that the underlying population is normally distributed. (When using these tests in a real situation, you must first prove that assumption, however.)

### Q 10.5.1

Ten individuals went on a low-fat diet for 12 weeks to lower their cholesterol. The data are recorded in [Table](#). Do you think that their cholesterol levels were significantly lowered?

| Starting cholesterol level | Ending cholesterol level |
|----------------------------|--------------------------|
| 140                        | 140                      |
| 220                        | 230                      |
| 110                        | 120                      |
| 240                        | 220                      |
| 200                        | 190                      |
| 180                        | 150                      |
| 190                        | 200                      |
| 360                        | 300                      |
| 280                        | 300                      |
| 260                        | 240                      |

### S 10.5.1

$p\text{-value} = 0.1494$

At the 5% significance level, there is insufficient evidence to conclude that the medication lowered cholesterol levels after 12 weeks.

Use the following information to answer the next two exercises. A new AIDS prevention drug was tried on a group of 224 HIV positive patients. Forty-five patients developed AIDS after four years. In a control group of 224 HIV positive patients, 68 developed AIDS after four years. We want to test whether the method of treatment reduces the proportion of patients that develop AIDS after four years or if the proportions of the treated group and the untreated group stay the same.

Let the subscript  $t$  = treated patient and  $ut$  = untreated patient.

### Q 10.5.2

The appropriate hypotheses are:

- a.  $H_0 : p_t < p_{ut}$  and  $H_a : p_t \geq p_{ut}$
- b.  $H_0 : p_t \leq p_{ut}$  and  $H_a : p_t > p_{ut}$
- c.  $H_0 : p_t = p_{ut}$  and  $H_a : p_t \neq p_{ut}$
- d.  $H_0 : p_t = p_{ut}$  and  $H_a : p_t < p_{ut}$

### Q 10.5.3

If the  $p$ -value is 0.0062 what is the conclusion (use  $\alpha = 0.05$ )?

- a. The method has no effect.
- b. There is sufficient evidence to conclude that the method reduces the proportion of HIV positive patients who develop AIDS after four years.
- c. There is sufficient evidence to conclude that the method increases the proportion of HIV positive patients who develop AIDS after four years.
- d. There is insufficient evidence to conclude that the method reduces the proportion of HIV positive patients who develop AIDS after four years.

### S 10.5.3

b

Use the following information to answer the next two exercises. An experiment is conducted to show that blood pressure can be consciously reduced in people trained in a “biofeedback exercise program.” Six subjects were randomly selected and blood pressure measurements were recorded before and after the training. The difference between blood pressures was calculated (after - before) producing the following results:  $\bar{x}_d = -10.2$   $s_d = 8.4$ . Using the data, test the hypothesis that the blood pressure has decreased after the training.

### Q 10.5.4

The distribution for the test is:

- a.  $t_5$
- b.  $t_6$
- c.  $N(-10.2, 8.4)$
- d.  $N\left(-10.2, \frac{8.4}{\sqrt{6}}\right)$

### Q 10.5.5

If  $\alpha = 0.05$ , the  $p$ -value and the conclusion are

- a. 0.0014; There is sufficient evidence to conclude that the blood pressure decreased after the training.
- b. 0.0014; There is sufficient evidence to conclude that the blood pressure increased after the training.
- c. 0.0155; There is sufficient evidence to conclude that the blood pressure decreased after the training.
- d. 0.0155; There is sufficient evidence to conclude that the blood pressure increased after the training.

### S 10.5.5

c

### Q 10.5.6

A golf instructor is interested in determining if her new technique for improving players' golf scores is effective. She takes four new students. She records their 18-hole scores before learning the technique and then after having taken her class. She conducts a hypothesis test. The data are as follows.

|                         | Player 1 | Player 2 | Player 3 | Player 4 |
|-------------------------|----------|----------|----------|----------|
| Mean score before class | 83       | 78       | 93       | 87       |
| Mean score after class  | 80       | 80       | 86       | 86       |

The correct decision is:

- Reject  $H_0$ .
- Do not reject the  $H_0$ .

### Q 10.5.7

A local cancer support group believes that the estimate for new female breast cancer cases in the south is higher in 2013 than in 2012. The group compared the estimates of new female breast cancer cases by southern state in 2012 and in 2013. The results are in Table.

| Southern States | 2012   | 2013   |
|-----------------|--------|--------|
| Alabama         | 3,450  | 3,720  |
| Arkansas        | 2,150  | 2,280  |
| Florida         | 15,540 | 15,710 |
| Georgia         | 6,970  | 7,310  |
| Kentucky        | 3,160  | 3,300  |
| Louisiana       | 3,320  | 3,630  |
| Mississippi     | 1,990  | 2,080  |
| North Carolina  | 7,090  | 7,430  |
| Oklahoma        | 2,630  | 2,690  |
| South Carolina  | 3,570  | 3,580  |
| Tennessee       | 4,680  | 5,070  |
| Texas           | 15,050 | 14,980 |
| Virginia        | 6,190  | 6,280  |

### S 10.5.7

Test: two matched pairs or paired samples ( $t$ -test)

Random variable:  $\bar{X}_d$

Distribution:  $t_{12}$

$H_0 : \mu_d = 0$   $H_a : \mu_d > 0$

The mean of the differences of new female breast cancer cases in the south between 2013 and 2012 is greater than zero. The estimate for new female breast cancer cases in the south is higher in 2013 than in 2012.

Graph: right-tailed

$p$ -value : 0.0004

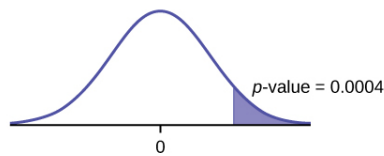


Figure 10.5.1.

Decision: Reject  $H_0$

Conclusion: At the 5% level of significance, from the sample data, there is sufficient evidence to conclude that there was a higher estimate of new female breast cancer cases in 2013 than in 2012.

#### Q 10.5.8

A traveler wanted to know if the prices of hotels are different in the ten cities that he visits the most often. The list of the cities with the corresponding hotel prices for his two favorite hotel chains is in Table. Test at the 1% level of significance.

| Cities         | Hyatt Regency prices in dollars | Hilton prices in dollars |
|----------------|---------------------------------|--------------------------|
| Atlanta        | 107                             | 169                      |
| Boston         | 358                             | 289                      |
| Chicago        | 209                             | 299                      |
| Dallas         | 209                             | 198                      |
| Denver         | 167                             | 169                      |
| Indianapolis   | 179                             | 214                      |
| Los Angeles    | 179                             | 169                      |
| New York City  | 625                             | 459                      |
| Philadelphia   | 179                             | 159                      |
| Washington, DC | 245                             | 239                      |

#### Q 10.5.9

A politician asked his staff to determine whether the underemployment rate in the northeast decreased from 2011 to 2012. The results are in Table.

| Northeastern States | 2011 | 2012 |
|---------------------|------|------|
| Connecticut         | 17.3 | 16.4 |
| Delaware            | 17.4 | 13.7 |
| Maine               | 19.3 | 16.1 |
| Maryland            | 16.0 | 15.5 |
| Massachusetts       | 17.6 | 18.2 |
| New Hampshire       | 15.4 | 13.5 |
| New Jersey          | 19.2 | 18.7 |
| New York            | 18.5 | 18.7 |
| Ohio                | 18.2 | 18.8 |
| Pennsylvania        | 16.5 | 16.9 |

| Northeastern States | 2011 | 2012 |
|---------------------|------|------|
| Rhode Island        | 20.7 | 22.4 |
| Vermont             | 14.7 | 12.3 |
| West Virginia       | 15.5 | 17.3 |

### S 10.5.9

Test: matched or paired samples ( $t$ -test)

Difference data:  $\{-0.9, -3.7, -3.2, -0.5, 0.6, -1.9, -0.5, 0.2, 0.6, 0.4, 1.7, -2.4, 1.8\}$

Random Variable:  $\bar{X}_d$

Distribution:  $H_0 : \mu_d = 0 H_a : \mu_d < 0$

The mean of the differences of the rate of underemployment in the northeastern states between 2012 and 2011 is less than zero. The underemployment rate went down from 2011 to 2012.

Graph: left-tailed.

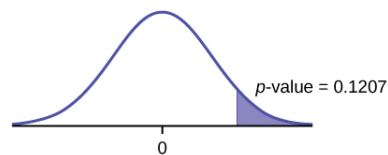


Figure 10.5.2.

$p$ -value : 0.1207

Decision: Do not reject  $H_0$ .

Conclusion: At the 5% level of significance, from the sample data, there is not sufficient evidence to conclude that there was a decrease in the underemployment rates of the northeastern states from 2011 to 2012.

## 10.6: Hypothesis Testing for Two Means and Two Proportions

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