

## 7.E: Confidence Intervals (Optional Exercises)

These are homework exercises to accompany the Textmap created for "Introductory Statistics" by OpenStax.

### 8.1: Introduction

### 8.2: A Single Population Mean using the Normal Distribution

#### Q 8.2.1

Among various ethnic groups, the standard deviation of heights is known to be approximately three inches. We wish to construct a 95% confidence interval for the mean height of male Swedes. Forty-eight male Swedes are surveyed. The sample mean is 71 inches. The sample standard deviation is 2.8 inches.

- a.
  - i.  $\bar{x}$  = \_\_\_\_\_
  - ii.  $\sigma$  = \_\_\_\_\_
  - iii.  $n$  = \_\_\_\_\_
- b. In words, define the random variables  $X$  and  $\bar{X}$ .
- c. Which distribution should you use for this problem? Explain your choice.
- d. Construct a 95% confidence interval for the population mean height of male Swedes.
  - i. State the confidence interval.
  - ii. Sketch the graph.
  - iii. Calculate the error bound.
- e. What will happen to the error bound obtained if 1,000 male Swedes are surveyed instead of 48? Why?

#### S 8.2.1

- a.
  - i. 71
  - ii. 3
  - iii. 48
- b.  $X$  is the height of a Swedish male, and  $\bar{X}$  is the mean height from a sample of 48 Swedish males.
- c. Normal. We know the standard deviation for the population, and the sample size is greater than 30.
- d.
  - i. CI: (70.151, 71.849)

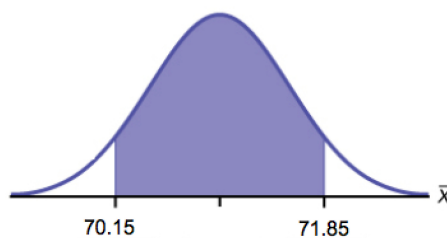


Figure 8.2.1.

- ii.  $EBM = 0.849$

e. The error bound will decrease in size, because the sample size increased. Recall, when all factors remain unchanged, an increase in sample size decreases variability. Thus, we do not need as large an interval to capture the true population mean.

#### Q 8.2.2

Announcements for 84 upcoming engineering conferences were randomly picked from a stack of IEEE Spectrum magazines. The mean length of the conferences was 3.94 days, with a standard deviation of 1.28 days. Assume the underlying population is normal.

1. In words, define the random variables  $X$  and  $\bar{X}$ .
2. Which distribution should you use for this problem? Explain your choice.
3. Construct a 95% confidence interval for the population mean length of engineering conferences.
  1. State the confidence interval.
  2. Sketch the graph.

3. Calculate the error bound.

### Q 8.2.3

Suppose that an accounting firm does a study to determine the time needed to complete one person's tax forms. It randomly surveys 100 people. The sample mean is 23.6 hours. There is a known standard deviation of 7.0 hours. The population distribution is assumed to be normal.

- a.
  - i.  $\bar{x}$  = \_\_\_\_\_
  - ii.  $\sigma$  = \_\_\_\_\_
  - iii.  $n$  = \_\_\_\_\_
- b. In words, define the random variables  $X$  and  $\bar{X}$ .
- c. Which distribution should you use for this problem? Explain your choice.
- d. Construct a 95% confidence interval for the population mean time to complete the tax forms.
  - i. State the confidence interval.
  - ii. Sketch the graph.
  - iii. Calculate the error bound.
- e. If the firm wished to increase its level of confidence and keep the error bound the same by taking another survey, what changes should it make?
- f. If the firm did another survey, kept the error bound the same, and only surveyed 49 people, what would happen to the level of confidence? Why?
- g. Suppose that the firm decided that it needed to be at least 96% confident of the population mean length of time to within one hour. How would the number of people the firm surveys change? Why?

### S 8.2.3

- a.
  - i.  $\bar{x}$  = 23.6
  - ii.  $\sigma$  = 7
  - iii.  $n$  = 100
- b.  $X$  is the time needed to complete an individual tax form.  $\bar{X}$  is the mean time to complete tax forms from a sample of 100 customers.
- c.  $N\left(23.6, \frac{7}{\sqrt{100}}\right)$  because we know sigma.
- d.
  - i. (22.228, 24.972)

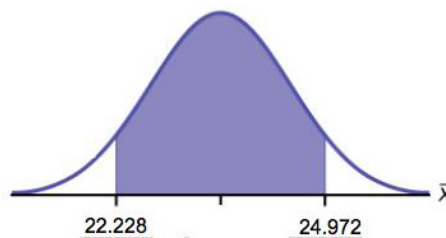


Figure 8.2.7.

- ii.
- iii.  $EBM = 1.372$
- e. It will need to change the sample size. The firm needs to determine what the confidence level should be, then apply the error bound formula to determine the necessary sample size.
- f. The confidence level would increase as a result of a larger interval. Smaller sample sizes result in more variability. To capture the true population mean, we need to have a larger interval.
- g. According to the error bound formula, the firm needs to survey 206 people. Since we increase the confidence level, we need to increase either our error bound or the sample size.

### Q 8.2.4

A sample of 16 small bags of the same brand of candies was selected. Assume that the population distribution of bag weights is normal. The weight of each bag was then recorded. The mean weight was two ounces with a standard deviation of 0.12 ounces. The population standard deviation is known to be 0.1 ounce.

- a.
  - i.  $\bar{x} =$  \_\_\_\_\_
  - ii.  $\sigma =$  \_\_\_\_\_
  - iii.  $s_x =$  \_\_\_\_\_
- b. In words, define the random variable  $X$ .
- c. In words, define the random variable  $\bar{X}$ .
- d. Which distribution should you use for this problem? Explain your choice.
- e. Construct a 90% confidence interval for the population mean weight of the candies.
  - i. State the confidence interval.
  - ii. Sketch the graph.
  - iii. Calculate the error bound.
- f. Construct a 98% confidence interval for the population mean weight of the candies.
  - i. State the confidence interval.
  - ii. Sketch the graph.
  - iii. Calculate the error bound.
- g. In complete sentences, explain why the confidence interval in part f is larger than the confidence interval in part e.
- h. In complete sentences, give an interpretation of what the interval in part f means.

### Q 8.2.5

A camp director is interested in the mean number of letters each child sends during his or her camp session. The population standard deviation is known to be 2.5. A survey of 20 campers is taken. The mean from the sample is 7.9 with a sample standard deviation of 2.8.

- a.
  - i.  $\bar{x} =$  \_\_\_\_\_
  - ii.  $\sigma =$  \_\_\_\_\_
  - iii.  $s_x =$  \_\_\_\_\_
- b. Define the random variables  $X$  and  $\bar{X}$  in words.
- c. Which distribution should you use for this problem? Explain your choice.
- d. Construct a 90% confidence interval for the population mean number of letters campers send home.
  - i. State the confidence interval.
  - ii. Sketch the graph.
  - iii. Calculate the error bound.
- e. What will happen to the error bound and confidence interval if 500 campers are surveyed? Why?

### S 8.2.5

- a.
  - i. 7.9
  - ii. 2.5
  - iii. 20
- b.  $X$  is the number of letters a single camper will send home.  $\bar{X}$  is the mean number of letters sent home from a sample of 20 campers.
- c.  $N7.9 \left( \frac{2.5}{\sqrt{20}} \right)$
- d.
  - i. CI: (6.98, 8.82)

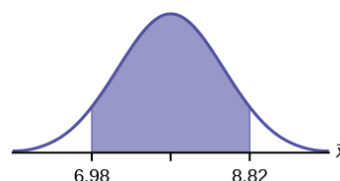


Figure 8..2.2: Copy and Paste Caption here. (Copyright; author via source)

- i.  $EBM : 0.92$

e. The error bound and confidence interval will decrease.

### Q 8.2.6

What is meant by the term “90% confident” when constructing a confidence interval for a mean?

1. If we took repeated samples, approximately 90% of the samples would produce the same confidence interval.
2. If we took repeated samples, approximately 90% of the confidence intervals calculated from those samples would contain the sample mean.
3. If we took repeated samples, approximately 90% of the confidence intervals calculated from those samples would contain the true value of the population mean.
4. If we took repeated samples, the sample mean would equal the population mean in approximately 90% of the samples.

### Q 8.2.7

The Federal Election Commission collects information about campaign contributions and disbursements for candidates and political committees each election cycle. During the 2012 campaign season, there were 1,619 candidates for the House of Representatives across the United States who received contributions from individuals. Table shows the total receipts from individuals for a random selection of 40 House candidates rounded to the nearest \$100. The standard deviation for this data to the nearest hundred is  $\sigma = \$909,200$ .

\$3,600	\$1,243,900	\$10,900	\$385,200	\$581,500
\$7,400	\$2,900	\$400	\$3,714,500	\$632,500
\$391,000	\$467,400	\$56,800	\$5,800	\$405,200
\$733,200	\$8,000	\$468,700	\$75,200	\$41,000
\$13,300	\$9,500	\$953,800	\$1,113,500	\$1,109,300
\$353,900	\$986,100	\$88,600	\$378,200	\$13,200
\$3,800	\$745,100	\$5,800	\$3,072,100	\$1,626,700
\$512,900	\$2,309,200	\$6,600	\$202,400	\$15,800

- a. Find the point estimate for the population mean.
- b. Using 95% confidence, calculate the error bound.
- c. Create a 95% confidence interval for the mean total individual contributions.
- d. Interpret the confidence interval in the context of the problem.

### S 8.2.7

- a.  $\bar{x} = \$568,873$
- b.  $CL = 0.95$   
 $\alpha = 1 - 0.95 = 0.05$   
 $z_{\frac{\alpha}{2}} = 1.96$   
 $EBM = z_{0.025} \frac{\sigma}{\sqrt{n}} = 1.96 \frac{909,200}{\sqrt{40}} = \$281,764$
- c.  $\bar{x} - EBM = 568,873 - 281,764 = 287,109$   
 $\bar{x} + EBM = 568,873 + 281,764 = 850,637$

Alternate solution:

1. Press **STAT** and arrow over to **TESTS**.
2. Arrow down to **7:ZInterval**.
3. Press **ENTER**.
4. Arrow to Stats and press **ENTER**.
5. Arrow down and enter the following values:
  - $\sigma : 909,200$
  - $\bar{x} : 568,873$
  - $n : 40$
  - $CL : 0.95$

6. Arrow down to Calculate and press `ENTER` .
  7. The confidence interval is (\$287,114, \$850,632).
  8. Notice the small difference between the two solutions—these differences are simply due to rounding error in the hand calculations.
- d. We estimate with 95% confidence that the mean amount of contributions received from all individuals by House candidates is between \$287,109 and \$850,637.

### Q 8.2.8

The American Community Survey (ACS), part of the United States Census Bureau, conducts a yearly census similar to the one taken every ten years, but with a smaller percentage of participants. The most recent survey estimates with 90% confidence that the mean household income in the U.S. falls between \$69,720 and \$69,922. Find the point estimate for mean U.S. household income and the error bound for mean U.S. household income.

### Q 8.2.9

The average height of young adult males has a normal distribution with standard deviation of 2.5 inches. You want to estimate the mean height of students at your college or university to within one inch with 93% confidence. How many male students must you measure?

### S 8.2.9

Use the formula for  $EBM$ , solved for  $n$ :

$$n = \frac{z^2 \sigma^2}{EBM^2}$$

From the statement of the problem, you know that  $\sigma = 2.5$ , and you need  $EBM = 1$  .

$$z = z_{0.035} = 1.812$$

(This is the value of  $z$  for which the area under the density curve to the **right** of  $z$  is 0.035.)

$$n = \frac{z^2 \sigma^2}{EBM^2} = \frac{1.812^2 2.5^2}{1^2} \approx 20.52$$

You need to measure at least 21 male students to achieve your goal.

## 8.3: A Single Population Mean using the Student t Distribution

### Q 8.3.1

In six packages of “The Flintstones® Real Fruit Snacks” there were five Bam-Bam snack pieces. The total number of snack pieces in the six bags was 68. We wish to calculate a 96% confidence interval for the population proportion of Bam-Bam snack pieces.

- a. Define the random variables  $X$  and  $P'$  in words.
- b. Which distribution should you use for this problem? Explain your choice
- c. Calculate  $p'$ .
- d. Construct a 96% confidence interval for the population proportion of Bam-Bam snack pieces per bag.
  - i. State the confidence interval.
  - ii. Sketch the graph.
  - iii. Calculate the error bound.
- e. Do you think that six packages of fruit snacks yield enough data to give accurate results? Why or why not?

### Q 8.3.2

A random survey of enrollment at 35 community colleges across the United States yielded the following figures: 6,414; 1,550; 2,109; 9,350; 21,828; 4,300; 5,944; 5,722; 2,825; 2,044; 5,481; 5,200; 5,853; 2,750; 10,012; 6,357; 27,000; 9,414; 7,681; 3,200; 17,500; 9,200; 7,380; 18,314; 6,557; 13,713; 17,768; 7,493; 2,771; 2,861; 1,263; 7,285; 28,165; 5,080; 11,622. Assume the underlying population is normal.

- a.
  - i.  $\bar{x} =$  \_\_\_\_\_
  - ii.  $s_x =$  \_\_\_\_\_
  - iii.  $n =$  \_\_\_\_\_

- iv.  $n - 1 =$  \_\_\_\_\_
- b. Define the random variables  $X$  and  $\bar{X}$  in words.
- c. Which distribution should you use for this problem? Explain your choice.
- d. Construct a 95% confidence interval for the population mean enrollment at community colleges in the United States.
  - i. State the confidence interval.
  - ii. Sketch the graph.
  - iii. Calculate the error bound.
- e. What will happen to the error bound and confidence interval if 500 community colleges were surveyed? Why?

### S 8.3.2

- a.
  - i. 8629
  - ii. 6944
  - iii. 35
  - iv. 34
- b.  $t_{34}$
- c.
  - i.  $CI : (6244, 11,014)$

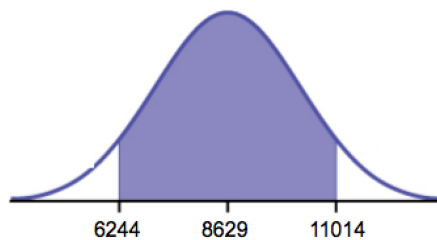


Figure 8.3.4.

- ii.
- iii.  $EB = 2385$
- d. It will become smaller

### Q 8.3.3

Suppose that a committee is studying whether or not there is waste of time in our judicial system. It is interested in the mean amount of time individuals waste at the courthouse waiting to be called for jury duty. The committee randomly surveyed 81 people who recently served as jurors. The sample mean wait time was eight hours with a sample standard deviation of four hours.

- a.
  - i.  $\bar{x} =$  \_\_\_\_\_
  - ii.  $s_x =$  \_\_\_\_\_
  - iii.  $n =$  \_\_\_\_\_
  - iv.  $n - 1 =$  \_\_\_\_\_
- b. Define the random variables  $X$  and  $\bar{X}$  in words.
- c. Which distribution should you use for this problem? Explain your choice.
- d. Construct a 95% confidence interval for the population mean time wasted.
  - i. State the confidence interval.
  - ii. Sketch the graph.
  - iii. Calculate the error bound.
- e. Explain in a complete sentence what the confidence interval means.

### Q 8.3.4

A pharmaceutical company makes tranquilizers. It is assumed that the distribution for the length of time they last is approximately normal. Researchers in a hospital used the drug on a random sample of nine patients. The effective period of the tranquilizer for each patient (in hours) was as follows: 2.7; 2.8; 3.0; 2.3; 2.3; 2.2; 2.8; 2.1; and 2.4.

- a.
  - i.  $\bar{x} =$  \_\_\_\_\_
  - ii.  $s_x =$  \_\_\_\_\_
  - iii.  $n =$  \_\_\_\_\_

- iv.  $n-1 =$  \_\_\_\_\_
- b. Define the random variable  $X$  in words.
- c. Define the random variable  $\bar{X}$  in words.
- d. Which distribution should you use for this problem? Explain your choice.
- e. Construct a 95% confidence interval for the population mean length of time.
  - i. State the confidence interval.
  - ii. Sketch the graph.
  - iii. Calculate the error bound.
- f. What does it mean to be “95% confident” in this problem?

#### S 8.3.4

- a.
  - i.  $\bar{x} = 2.51$
  - ii.  $s_x = 0.318$
  - iii.  $n = 9$
  - iv.  $n-1 = 8$
- b. the effective length of time for a tranquilizer
- c. the mean effective length of time of tranquilizers from a sample of nine patients
- d. We need to use a Student's-t distribution, because we do not know the population standard deviation.
- e.
  - i.  $CI : (2.27, 2.76)$
  - ii. Check student's solution.
  - iii.  $EBM : 0.25$
- f. If we were to sample many groups of nine patients, 95% of the samples would contain the true population mean length of time.

#### Q 8.3.5

Suppose that 14 children, who were learning to ride two-wheel bikes, were surveyed to determine how long they had to use training wheels. It was revealed that they used them an average of six months with a sample standard deviation of three months. Assume that the underlying population distribution is normal.

- a.
  - i.  $\bar{x} =$  \_\_\_\_\_
  - ii.  $s_x =$  \_\_\_\_\_
  - iii.  $n =$  \_\_\_\_\_
  - iv.  $n-1 =$  \_\_\_\_\_
- b. Define the random variable  $X$  in words.
- c. Define the random variable  $\bar{X}$  in words.
- d. Which distribution should you use for this problem? Explain your choice.
- e. Construct a 99% confidence interval for the population mean length of time using training wheels.
  - i. State the confidence interval.
  - ii. Sketch the graph.
  - iii. Calculate the error bound.
- f. Why would the error bound change if the confidence level were lowered to 90%?

#### Q 8.3.6

The Federal Election Commission (FEC) collects information about campaign contributions and disbursements for candidates and political committees each election cycle. A political action committee (PAC) is a committee formed to raise money for candidates and campaigns. A Leadership PAC is a PAC formed by a federal politician (senator or representative) to raise money to help other candidates' campaigns.

The FEC has reported financial information for 556 Leadership PACs that operating during the 2011–2012 election cycle. The following table shows the total receipts during this cycle for a random selection of 20 Leadership PACs.

\$46,500.00	\$0	\$40,966.50	\$105,887.20	\$5,175.00
\$29,050.00	\$19,500.00	\$181,557.20	\$31,500.00	\$149,970.80

\$2,555,363.20	\$12,025.00	\$409,000.00	\$60,521.70	\$18,000.00
\$61,810.20	\$76,530.80	\$119,459.20	\$0	\$63,520.00
\$6,500.00	\$502,578.00	\$705,061.10	\$708,258.90	\$135,810.00
\$2,000.00	\$2,000.00	\$0	\$1,287,933.80	\$219,148.30

$$\bar{x} = \$251,854.23$$

$$s = \$521,130.41$$

Use this sample data to construct a 96% confidence interval for the mean amount of money raised by all Leadership PACs during the 2011–2012 election cycle. Use the Student's  $t$ -distribution.

### S 8.3.6

$$\bar{x} = \$251,854.23$$

$$s = \$521,130.41$$

Note that we are not given the population standard deviation, only the standard deviation of the sample.

There are 30 measures in the sample, so  $n = 30$ , and  $df = 30 - 1 = 29$

$$CL = 0.96, \text{ so } \alpha = 1 - CL = 1 - 0.96 = 0.04$$

$$\frac{\alpha}{2} = 0.02, t_{0.02} = t_{0.02} = 2.150$$

$$EBM = t_{\frac{\alpha}{2}} \left( \frac{s}{\sqrt{n}} \right) = 2.150 \left( \frac{521,130.41}{\sqrt{30}} \right) = \$204,561.66$$

$$\bar{x} - EBM = \$251,854.23 - \$204,561.66 = \$47,292.57$$

$$\bar{x} + EBM = \$251,854.23 + \$204,561.66 = \$456,415.89$$

We estimate with 96% confidence that the mean amount of money raised by all Leadership PACs during the 2011–2012 election cycle lies between \$47,292.57 and \$456,415.89.

### Alternate Solution

Enter the data as a list.

Press **STAT** and arrow over to **TESTS** .

Arrow down to **8:TInterval** .

Press **ENTER** .

Arrow to **Data** and press **ENTER** .

Arrow down and enter the name of the list where the data is stored.

Enter **Freq : 1**

Enter **C-Level : 0.96**

Arrow down to **Calculate** and press **Enter** .

The 96% confidence interval is (\$47,262, \$456,447).

The difference between solutions arises from rounding differences.

### Q 8.3.7

*Forbes* magazine published data on the best small firms in 2012. These were firms that had been publicly traded for at least a year, have a stock price of at least \$5 per share, and have reported annual revenue between \$5 million and \$1 billion. The [Table](#) shows the ages of the corporate CEOs for a random sample of these firms.

48	58	51	61	56
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59	74	63	53	50
59	60	60	57	46
55	63	57	47	55
57	43	61	62	49
67	67	55	55	49

Use this sample data to construct a 90% confidence interval for the mean age of CEO's for these top small firms. Use the Student's  $t$ -distribution.

### Q 8.3.8

Unoccupied seats on flights cause airlines to lose revenue. Suppose a large airline wants to estimate its mean number of unoccupied seats per flight over the past year. To accomplish this, the records of 225 flights are randomly selected and the number of unoccupied seats is noted for each of the sampled flights. The sample mean is 11.6 seats and the sample standard deviation is 4.1 seats.

- $\bar{x} =$  \_\_\_\_\_
  - $s_x =$  \_\_\_\_\_
  - $n =$  \_\_\_\_\_
  - $n - 1 =$  \_\_\_\_\_
- Define the random variables  $X$  and  $\bar{X}$  in words.
- Which distribution should you use for this problem? Explain your choice.
- Construct a 92% confidence interval for the population mean number of unoccupied seats per flight.
  - State the confidence interval.
  - Sketch the graph.
  - Calculate the error bound.

### S 8.3.8

- $\bar{x} =$
  - $s_x =$
  - $n =$
  - $n - 1 =$
- $X$  is the number of unoccupied seats on a single flight.  $\bar{X}$  is the mean number of unoccupied seats from a sample of 225 flights.
- We will use a Student's  $t$ -distribution, because we do not know the population standard deviation.
- $CI : (11.12, 12.08)$
  - Check student's solution.
  - $EBM : 0.48$

### Q 8.3.9

In a recent sample of 84 used car sales costs, the sample mean was \$6,425 with a standard deviation of \$3,156. Assume the underlying distribution is approximately normal.

- Which distribution should you use for this problem? Explain your choice.
- Define the random variable  $\bar{X}$  in words.
- Construct a 95% confidence interval for the population mean cost of a used car.
  - State the confidence interval.
  - Sketch the graph.
  - Calculate the error bound.
- Explain what a "95% confidence interval" means for this study.

### Q 8.3.10

Six different national brands of chocolate chip cookies were randomly selected at the supermarket. The grams of fat per serving are as follows: 8; 8; 10; 7; 9; 9. Assume the underlying distribution is approximately normal.

- Construct a 90% confidence interval for the population mean grams of fat per serving of chocolate chip cookies sold in supermarkets.
  - State the confidence interval.
  - Sketch the graph.
  - Calculate the error bound.
- If you wanted a smaller error bound while keeping the same level of confidence, what should have been changed in the study before it was done?
- Go to the store and record the grams of fat per serving of six brands of chocolate chip cookies.
- Calculate the mean.
- Is the mean within the interval you calculated in part a? Did you expect it to be? Why or why not?

### S 8.3.10

- i. CI: (7.64, 9.36)

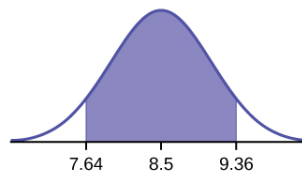


Figure 8.3.5.

ii.

iii.  $EBM : 0.86$

- The sample should have been increased.
- Answers will vary.
- Answers will vary.
- Answers will vary.

### Q 8.3.11

A survey of the mean number of cents off that coupons give was conducted by randomly surveying one coupon per page from the coupon sections of a recent San Jose Mercury News. The following data were collected: 20¢; 75¢; 50¢; 65¢; 30¢; 55¢; 40¢; 40¢; 30¢; 55¢; \$1.50; 40¢; 65¢; 40¢. Assume the underlying distribution is approximately normal.

- $\bar{x} =$  \_\_\_\_\_
  - $s_x =$  \_\_\_\_\_
  - $n =$  \_\_\_\_\_
  - $n - 1 =$  \_\_\_\_\_
- Define the random variables  $X$  and  $\bar{X}$  in words.
- Which distribution should you use for this problem? Explain your choice.
- Construct a 95% confidence interval for the population mean worth of coupons.
  - State the confidence interval.
  - Sketch the graph.
  - Calculate the error bound.
- If many random samples were taken of size 14, what percent of the confidence intervals constructed should contain the population mean worth of coupons? Explain why.

Use the following information to answer the next two exercises: A quality control specialist for a restaurant chain takes a random sample of size 12 to check the amount of soda served in the 16 oz. serving size. The sample mean is 13.30 with a sample standard deviation of 1.55. Assume the underlying population is normally distributed.

## Q 8.3.12

Find the 95% Confidence Interval for the true population mean for the amount of soda served.

- a. (12.42, 14.18)
- b. (12.32, 14.29)
- c. (12.50, 14.10)
- d. Impossible to determine

## S 8.3.12

b

## Q 8.3.13

What is the error bound?

- a. 0.87
- b. 1.98
- c. 0.99
- d. 1.74

## 8.4: A Population Proportion

## Q 8.4.1

Insurance companies are interested in knowing the population percent of drivers who always buckle up before riding in a car.

- a. When designing a study to determine this population proportion, what is the minimum number you would need to survey to be 95% confident that the population proportion is estimated to within 0.03?
- b. If it were later determined that it was important to be more than 95% confident and a new survey was commissioned, how would that affect the minimum number you would need to survey? Why?

## S 8.4.1

- a. 1,068
- b. The sample size would need to be increased since the critical value increases as the confidence level increases.

## Q 8.4.2

Suppose that the insurance companies did do a survey. They randomly surveyed 400 drivers and found that 320 claimed they always buckle up. We are interested in the population proportion of drivers who claim they always buckle up.

- a.
  - i.  $x =$  \_\_\_\_\_
  - ii.  $n =$  \_\_\_\_\_
  - iii.  $p' =$  \_\_\_\_\_
- b. Define the random variables  $X$  and  $P'$  in words.
- c. Which distribution should you use for this problem? Explain your choice.
- d. Construct a 95% confidence interval for the population proportion who claim they always buckle up.
  - i. State the confidence interval.
  - ii. Sketch the graph.
  - iii. Calculate the error bound.
- e. If this survey were done by telephone, list three difficulties the companies might have in obtaining random results.

## Q 8.4.3

According to a recent survey of 1,200 people, 61% feel that the president is doing an acceptable job. We are interested in the population proportion of people who feel the president is doing an acceptable job.

- a. Define the random variables  $X$  and  $P'$  in words.
- b. Which distribution should you use for this problem? Explain your choice.
- c. Construct a 90% confidence interval for the population proportion of people who feel the president is doing an acceptable job.
  - i. State the confidence interval.

- ii. Sketch the graph.
- iii. Calculate the error bound.

### S 8.4.3

- a.  $X$  = the number of people who feel that the president is doing an acceptable job;

$P'$  = the proportion of people in a sample who feel that the president is doing an acceptable job.

b.  $N \left( 0.61, \sqrt{\frac{(0.61)(0.39)}{1200}} \right)$

- c. i.  $CI : (0.59, 0.63)$
- ii. Check student's solution
- iii.  $EBM : 0.02$

### Q 8.4.4

An article regarding interracial dating and marriage recently appeared in the Washington Post. Of the 1,709 randomly selected adults, 315 identified themselves as Latinos, 323 identified themselves as blacks, 254 identified themselves as Asians, and 779 identified themselves as whites. In this survey, 86% of blacks said that they would welcome a white person into their families. Among Asians, 77% would welcome a white person into their families, 71% would welcome a Latino, and 66% would welcome a black person.

- a. We are interested in finding the 95% confidence interval for the percent of all black adults who would welcome a white person into their families. Define the random variables  $X$  and  $P'$ , in words.
- b. Which distribution should you use for this problem? Explain your choice.
- c. Construct a 95% confidence interval.
  - i. State the confidence interval.
  - ii. Sketch the graph.
  - iii. Calculate the error bound.

### Q 8.4.5

Refer to the information in Exercise.

- a. Construct three 95% confidence intervals.
  - i. percent of all Asians who would welcome a white person into their families.
  - ii. percent of all Asians who would welcome a Latino into their families.
  - iii. percent of all Asians who would welcome a black person into their families.
- b. Even though the three point estimates are different, do any of the confidence intervals overlap? Which?
- c. For any intervals that do overlap, in words, what does this imply about the significance of the differences in the true proportions?
- d. For any intervals that do not overlap, in words, what does this imply about the significance of the differences in the true proportions?

### S 8.4.5

- a. i.  $(0.72, 0.82)$
- ii.  $(0.65, 0.76)$
- iii.  $(0.60, 0.72)$
- b. Yes, the intervals  $(0.72, 0.82)$  and  $(0.65, 0.76)$  overlap, and the intervals  $(0.65, 0.76)$  and  $(0.60, 0.72)$  overlap.
- c. We can say that there does not appear to be a significant difference between the proportion of Asian adults who say that their families would welcome a white person into their families and the proportion of Asian adults who say that their families would welcome a Latino person into their families.
- d. We can say that there is a significant difference between the proportion of Asian adults who say that their families would welcome a white person into their families and the proportion of Asian adults who say that their families would welcome a black person into their families.

## Q 8.4.6

Stanford University conducted a study of whether running is healthy for men and women over age 50. During the first eight years of the study, 1.5% of the 451 members of the 50-Plus Fitness Association died. We are interested in the proportion of people over 50 who ran and died in the same eight-year period.

- Define the random variables  $X$  and  $P'$  in words.
- Which distribution should you use for this problem? Explain your choice.
- Construct a 97% confidence interval for the population proportion of people over 50 who ran and died in the same eight-year period.
  - State the confidence interval.
  - Sketch the graph.
  - Calculate the error bound.
- Explain what a “97% confidence interval” means for this study.

## Q 8.4.7

A telephone poll of 1,000 adult Americans was reported in an issue of Time Magazine. One of the questions asked was “What is the main problem facing the country?” Twenty percent answered “crime.” We are interested in the population proportion of adult Americans who feel that crime is the main problem.

- Define the random variables  $X$  and  $P'$  in words.
- Which distribution should you use for this problem? Explain your choice.
- Construct a 95% confidence interval for the population proportion of adult Americans who feel that crime is the main problem.
  - State the confidence interval.
  - Sketch the graph.
  - Calculate the error bound.
- Suppose we want to lower the sampling error. What is one way to accomplish that?
- The sampling error given by Yankelovich Partners, Inc. (which conducted the poll) is  $\pm 3$ . In one to three complete sentences, explain what the  $\pm 3\%$  represents.

## S 8.4.7

- $X$  = the number of adult Americans who feel that crime is the main problem;  $P'$  = the proportion of adult Americans who feel that crime is the main problem
- Since we are estimating a proportion, given  $P' = 0.2$  and  $n = 1000$ , the distribution we should use is  $N\left(0.61, \sqrt{\frac{(0.2)(0.8)}{1000}}\right)$ .
- $CI : (0.18, 0.22)$
  - Check student's solution.
  - $EBM : 0.02$
- One way to lower the sampling error is to increase the sample size.
- The stated “ $\pm 3$ ” represents the maximum error bound. This means that those doing the study are reporting a maximum error of 3%. Thus, they estimate the percentage of adult Americans who feel that crime is the main problem to be between 18% and 22%.

## Q 8.4.8

Refer to [Exercise](#). Another question in the poll was “[How much are] you worried about the quality of education in our schools?” Sixty-three percent responded “a lot”. We are interested in the population proportion of adult Americans who are worried a lot about the quality of education in our schools.

- Define the random variables  $X$  and  $P'$  in words.
- Which distribution should you use for this problem? Explain your choice.
- Construct a 95% confidence interval for the population proportion of adult Americans who are worried a lot about the quality of education in our schools.
  - State the confidence interval.
  - Sketch the graph.

iii. Calculate the error bound.

- d. The sampling error given by Yankelovich Partners, Inc. (which conducted the poll) is  $\pm 3$ . In one to three complete sentences, explain what the  $\pm 3\%$  represents.

*Use the following information to answer the next three exercises:* According to a Field Poll, 79% of California adults (actual results are 400 out of 506 surveyed) feel that “education and our schools” is one of the top issues facing California. We wish to construct a 90% confidence interval for the true proportion of California adults who feel that education and the schools is one of the top issues facing California.

#### Q 8.4.9

A point estimate for the true population proportion is:

- a. 0.90
- b. 1.27
- c. 0.79
- d. 400

#### S 8.4.9

c

#### Q 8.4.10

A 90% confidence interval for the population proportion is \_\_\_\_\_.

- a. (0.761, 0.820)
- b. (0.125, 0.188)
- c. (0.755, 0.826)
- d. (0.130, 0.183)

#### Q 8.4.11

The error bound is approximately \_\_\_\_\_.

- a. 1.581
- b. 0.791
- c. 0.059
- d. 0.030

#### S 8.4.11

d

*Use the following information to answer the next two exercises:* Five hundred and eleven (511) homes in a certain southern California community are randomly surveyed to determine if they meet minimal earthquake preparedness recommendations. One hundred seventy-three (173) of the homes surveyed met the minimum recommendations for earthquake preparedness, and 338 did not.

#### Q 8.4.12

Find the confidence interval at the 90% Confidence Level for the true population proportion of southern California community homes meeting at least the minimum recommendations for earthquake preparedness.

- a. (0.2975, 0.3796)
- b. (0.6270, 0.6959)
- c. (0.3041, 0.3730)
- d. (0.6204, 0.7025)

#### Q 8.4.13

The point estimate for the population proportion of homes that do not meet the minimum recommendations for earthquake preparedness is \_\_\_\_\_.

- a. 0.6614
- b. 0.3386
- c. 173
- d. 338

### S 8.4.13

a

### Q 8.4.14

On May 23, 2013, Gallup reported that of the 1,005 people surveyed, 76% of U.S. workers believe that they will continue working past retirement age. The confidence level for this study was reported at 95% with a  $\pm 3$  margin of error.

- a. Determine the estimated proportion from the sample.
- b. Determine the sample size.
- c. Identify  $CL$  and  $\alpha$ .
- d. Calculate the error bound based on the information provided.
- e. Compare the error bound in part d to the margin of error reported by Gallup. Explain any differences between the values.
- f. Create a confidence interval for the results of this study.
- g. A reporter is covering the release of this study for a local news station. How should she explain the confidence interval to her audience?

### Q 8.4.15

A national survey of 1,000 adults was conducted on May 13, 2013 by Rasmussen Reports. It concluded with 95% confidence that 49% to 55% of Americans believe that big-time college sports programs corrupt the process of higher education.

- a. Find the point estimate and the error bound for this confidence interval.
- b. Can we (with 95% confidence) conclude that more than half of all American adults believe this?
- c. Use the point estimate from part a and  $n = 1,000$  to calculate a 75% confidence interval for the proportion of American adults that believe that major college sports programs corrupt higher education.
- d. Can we (with 75% confidence) conclude that at least half of all American adults believe this?

### S 8.4.15

- a.  $p' = \frac{(0.55+0.49)}{2} = 0.52$ ;  $EBP = 0.55 - 0.52 = 0.03$
- b. No, the confidence interval includes values less than or equal to 0.50. It is possible that less than half of the population believe this.
- c.  $CL = 0.75$ , so  $\alpha = 1 - 0.75 = 0.25$  and  $\frac{\alpha}{2} = 0.125$   $z_{\frac{\alpha}{2}} = 1.150$ . (The area to the right of this  $z$  is 0.125, so the area to the left is  $1 - 0.125 = 0.875$ )  
 $EBP = (1.150)\sqrt{\frac{0.52(0.48)}{1,000}} \approx 0.018$   
 $(p' - EBP, p' + EBP) = (0.52 - 0.018, 0.52 + 0.018) = (0.502, 0.538)$

Alternate Solution

STAT TESTS A: 1-PropZInterval with  $x = (0.52)(1,000)$ ,  $n = 1,000$ ,  $CL = 0.75$

Answer is (0.502, 0.538)

- d. Yes – this interval does not fall less than 0.50 so we can conclude that at least half of all American adults believe that major sports programs corrupt education – but we do so with only 75% confidence.

### Q 8.4.16

Public Policy Polling recently conducted a survey asking adults across the U.S. about music preferences. When asked, 80 of the 571 participants admitted that they have illegally downloaded music.

- a. Create a 99% confidence interval for the true proportion of American adults who have illegally downloaded music.
- b. This survey was conducted through automated telephone interviews on May 6 and 7, 2013. The error bound of the survey compensates for sampling error, or natural variability among samples. List some factors that could affect the survey's outcome that are not covered by the margin of error.

- c. Without performing any calculations, describe how the confidence interval would change if the confidence level changed from 99% to 90%.

#### Q 8.4.17

You plan to conduct a survey on your college campus to learn about the political awareness of students. You want to estimate the true proportion of college students on your campus who voted in the 2012 presidential election with 95% confidence and a margin of error no greater than five percent. How many students must you interview?

#### S 8.4.17

$CL = 0.95$   
 $\alpha = 1 - 0.95 = 0.05$   
 $\frac{\alpha}{2} = 0.025$   
 $z_{\frac{\alpha}{2}} = 1.96$ . Use  $p' = q' = 0.5$ .

$$n = \frac{z_{\frac{\alpha}{2}}^2 p' q'}{EB^2} = \frac{1.96^2 (0.5)(0.5)}{0.05^2} = 384.16$$

You need to interview at least 385 students to estimate the proportion to within 5% at 95% confidence.

#### Q 8.4.18

In a recent Zogby International Poll, nine of 48 respondents rated the likelihood of a terrorist attack in their community as “likely” or “very likely.” Use the “plus four” method to create a 97% confidence interval for the proportion of American adults who believe that a terrorist attack in their community is likely or very likely. Explain what this confidence interval means in the context of the problem.

### 8.5: Confidence Interval (Home Costs)

### 8.6: Confidence Interval (Place of Birth)

### 8.7: Confidence Interval (Women's Heights)

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