

8.2: Hypothesis Test Examples for Means

Full Hypothesis Test Examples

Example 8.2.4

Jeffrey, as an eight-year old, **established a mean time of 16.43 seconds** for swimming the 25-yard freestyle, with a **standard deviation of 0.8 seconds**. His dad, Frank, thought that Jeffrey could swim the 25-yard freestyle faster using goggles. Frank bought Jeffrey a new pair of expensive goggles and timed Jeffrey for **15 25-yard freestyle swims**. For the 15 swims, **Jeffrey's mean time was 16 seconds**. Frank thought that the goggles helped Jeffrey to swim faster than the 16.43 seconds. Conduct a hypothesis test using a preset $\alpha = 0.05$. Assume that the swim times for the 25-yard freestyle are normal.

Answer

Set up the Hypothesis Test:

Since the problem is about a mean, this is a **test of a single population mean**.

$$H_0 : \mu = 16.43, H_a : \mu < 16.43$$

For Jeffrey to swim faster, his time will be less than 16.43 seconds. The "<" tells you this is left-tailed.

Determine the distribution needed:

Random variable: \bar{X} = the mean time to swim the 25-yard freestyle.

Distribution for the test: \bar{X} is normal (population standard deviation is known: $\sigma = 0.8$)

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \text{ Therefore, } \bar{X} \sim N\left(16.43, \frac{0.8}{\sqrt{15}}\right)$$

$\mu = 16.43$ comes from H_0 and not the data. $\sigma = 0.8$, and $n = 15$.

Calculate the p - value using the normal distribution for a mean:

$p\text{-value} = P(\bar{x} < 16) = 0.0187$ where the sample mean in the problem is given as 16.

$p\text{-value} = 0.0187$ (This is called the **actual level of significance**.) The p - value is the area to the left of the sample mean is given as 16.

Graph:

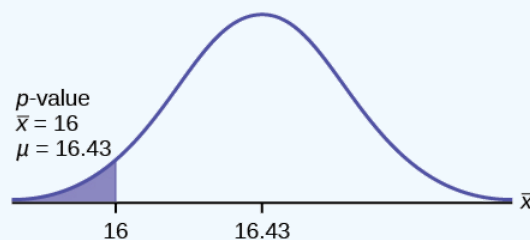


Figure 8.2.7

$\mu = 16.43$ comes from H_0 . Our assumption is $\mu = 16.43$.

Interpretation of the p - value: If H_0 is true, there is a 0.0187 probability (1.87%) that Jeffrey's mean time to swim the 25-yard freestyle is 16 seconds or less. Because a 1.87% chance is small, the mean time of 16 seconds or less is unlikely to have happened randomly. It is a rare event.

Compare α and the p - value:

$$\alpha = 0.05, p\text{-value} = 0.0187, \alpha > p\text{-value}$$

Make a decision: Since $\alpha > p\text{-value}$, reject H_0 .

This means that you reject $\mu = 16.43$. In other words, you do not think Jeffrey swims the 25-yard freestyle in 16.43 seconds but faster with the new goggles.

Conclusion: At the 5% significance level, we conclude that Jeffrey swims faster using the new goggles. The sample data show there is sufficient evidence that Jeffrey's mean time to swim the 25-yard freestyle is less than 16.43 seconds.

The p -value can easily be calculated.

Press **STAT** and arrow over to **TESTS**. Press **1:Z-Test**. Arrow over to **Stats** and press **ENTER**. Arrow down and enter 16.43 for μ_0 (null hypothesis), .8 for σ , 16 for the sample mean, and 15 for n . Arrow down to μ : (alternate hypothesis) and arrow over to $< \mu_0$. Press **ENTER**. Arrow down to **Calculate** and press **ENTER**. The calculator not only calculates the p -value ($p = 0.0187$) but it also calculates the test statistic (z-score) for the sample mean. $\mu < 16.43$ is the alternative hypothesis. Do this set of instructions again except arrow to **Draw** (instead of **Calculate**). Press **ENTER**. A shaded graph appears with $z = -2.08$ (test statistic) and $p = 0.0187$ (p -value). Make sure when you use **Draw** that no other equations are highlighted in $Y =$ and the plots are turned off.

When the calculator does a Z -Test, the Z -Test function finds the p -value by doing a normal probability calculation using the central limit theorem:

$$P(\bar{X} < 16) = 2^{\text{nd}} \text{ DISTR } \text{normcdf}((-10^{99}, 16, 16.43, \frac{0.8}{\sqrt{15}}))$$

The Type I and Type II errors for this problem are as follows:

The Type I error is to conclude that Jeffrey swims the 25-yard freestyle, on average, in less than 16.43 seconds when, in fact, he actually swims the 25-yard freestyle, on average, in 16.43 seconds. (Reject the null hypothesis when the null hypothesis is true.)

The Type II error is that there is not evidence to conclude that Jeffrey swims the 25-yard free-style, on average, in less than 16.43 seconds when, in fact, he actually does swim the 25-yard free-style, on average, in less than 16.43 seconds. (Do not reject the null hypothesis when the null hypothesis is false.)

Exercise 8.2.4

The mean throwing distance of a football for a Marco, a high school freshman quarterback, is 40 yards, with a standard deviation of two yards. The team coach tells Marco to adjust his grip to get more distance. The coach records the distances for 20 throws. For the 20 throws, Marco's mean distance was 45 yards. The coach thought the different grip helped Marco throw farther than 40 yards. Conduct a hypothesis test using a preset $\alpha = 0.05$. Assume the throw distances for footballs are normal.

First, determine what type of test this is, set up the hypothesis test, find the p -value, sketch the graph, and state your conclusion.

Press **STAT** and arrow over to **TESTS**. Press **1: Z-Test**. Arrow over to **Stats** and press **ENTER**. Arrow down and enter 40 for μ_0 (null hypothesis), 2 for σ , 45 for the sample mean, and 20 for n . Arrow down to μ : (alternative hypothesis) and set it either as $<$, \neq , or $>$. Press **ENTER**. Arrow down to **Calculate** and press **ENTER**. The calculator not only calculates the p -value but it also calculates the test statistic (z-score) for the sample mean. Select $<$, \neq , or $>$ for the alternative hypothesis. Do this set of instructions again except arrow to **Draw** (instead of **Calculate**). Press **ENTER**. A shaded graph appears with test statistic and p -value. Make sure when you use **Draw** that no other equations are highlighted in $Y =$ and the plots are turned off.

Answer

Since the problem is about a mean, this is a test of a single population mean.

- $H_0 : \mu = 40$
- $H_a : \mu > 40$
- $p = 0.0062$

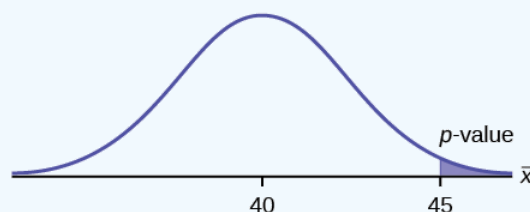


Figure 8.2.8

Because $p < \alpha$, we reject the null hypothesis. There is sufficient evidence to suggest that the change in grip improved Marco's throwing distance.

Historical Note

The traditional way to compare the two probabilities, α and the p -value, is to compare the critical value (z -score from α) to the test statistic (z -score from data). The calculated test statistic for the p -value is -2.08 . (From the Central Limit Theorem, the test statistic formula is $z = \frac{\bar{x} - \mu_x}{\left(\frac{\sigma_x}{\sqrt{n}}\right)}$. For this problem, $\bar{x} = 16$, $\mu_x = 16.43$ from the null hypotheses is, $\sigma_x = 0.8$, and $n = 15$.)

You can find the critical value for $\alpha = 0.05$ in the normal table (see **15.Tables** in the Table of Contents). The z -score for an area to the left equal to 0.05 is midway between -1.65 and -1.64 (0.05 is midway between 0.0505 and 0.0495). The z -score is -1.645 . Since $-1.645 > -2.08$ (which demonstrates that $\alpha > p$ -value), reject H_0 . Traditionally, the decision to reject or not reject was done in this way. Today, comparing the two probabilities α and the p -value is very common. For this problem, the p -value, 0.0187 is considerably smaller than $(\alpha = 0.05)$. You can be confident about your decision to reject. The graph shows α , the p -value, and the test statistics and the critical value.

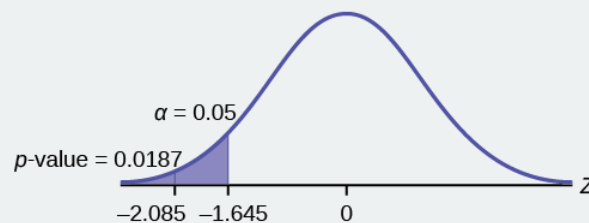


Figure 8.2.9

Example 8.2.5

A college football coach thought that his players could bench press a **mean weight of 275 pounds**. It is known that the **standard deviation is 55 pounds**. Three of his players thought that the mean weight was **more than** that amount. They asked **30** of their teammates for their estimated maximum lift on the bench press exercise. The data ranged from 205 pounds to 385 pounds. The actual different weights were (frequencies are in parentheses) 205(3); 215(3); 225(1); 241(2); 252(2); 265(2); 275(2); 313(2); 316(5); 338(2); 341(1); 345(2); 368(2); 385(1).

Conduct a hypothesis test using a 2.5% level of significance to determine if the bench press mean is more than 275 pounds.

Answer

Set up the Hypothesis Test:

Since the problem is about a mean weight, this is a test of a single population mean.

- $H_0 : \mu = 275$
- $H_a : \mu > 275$

This is a right-tailed test.

Calculating the distribution needed:

Random variable: \bar{X} = the mean weight, in pounds, lifted by the football players.

Distribution for the test: It is normal because σ is known.

- $\bar{X} \sim N\left(275, \frac{55}{\sqrt{30}}\right)$
- $\bar{x} = 286.2$ pounds (from the data).
- $\sigma = 55$ pounds (**Always use σ if you know it.**) We assume $\mu = 275$ pounds unless our data shows us otherwise.

Calculate the p -value using the normal distribution for a mean and using the sample mean as input (see [link](#)) for using the data as input):

$$p\text{-value} = P(\bar{x} > 286.2) = 0.1323.$$

Interpretation of the p -value: If H_0 is true, then there is a 0.1331 probability (13.23%) that the football players can lift a mean weight of 286.2 pounds or more. Because a 13.23% chance is large enough, a mean weight lift of 286.2 pounds or more is not a rare event.

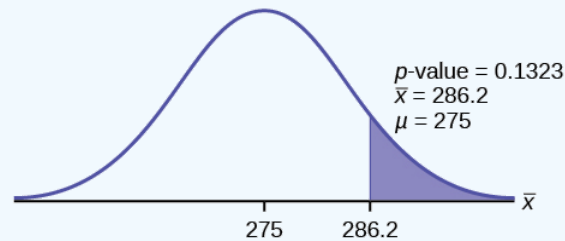


Figure 8.2.10

Compare α and the p - value:

$$\alpha = 0.025 p\text{-value} = 0.1323$$

Make a decision: Since $\alpha < p\text{-value}$, do not reject H_0 .

Conclusion: At the 2.5% level of significance, from the sample data, there is not sufficient evidence to conclude that the true mean weight lifted is more than 275 pounds.

The p - value can easily be calculated.

Put the data and frequencies into lists. Press **STAT** and arrow over to **TESTS**. Press **1:Z-Test**. Arrow over to **Data** and press **ENTER**. Arrow down and enter 275 for μ_0 , 55 for σ , the name of the list where you put the data, and the name of the list where you put the frequencies. Arrow down to μ : and arrow over to $> \mu_0$. Press **ENTER**. Arrow down to **Calculate** and press **ENTER**. The calculator not only calculates the p - value) ($p = 0.1331$), a little different from the previous calculation - in it we used the sample mean rounded to one decimal place instead of the data) but it also calculates the test statistic (z-score) for the sample mean, the sample mean, and the sample standard deviation. $\mu > 275$ is the alternative hypothesis. Do this set of instructions again except arrow to **Draw** (instead of **Calculate**). Press **ENTER**. A shaded graph appears with $z = 1.112$ (test statistic) and $p = 0.1331$ (p - value). Make sure when you use **Draw** that no other equations are highlighted in $Y =$ and the plots are turned off.

References

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Glossary

Central Limit Theorem

Given a random variable (RV) with known mean μ and known standard deviation σ . We are sampling with size n and we are interested in two new RVs - the sample mean, \bar{X} , and the sample sum, $\sum X$. If the size n of the sample is sufficiently large, then $\bar{X} - N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ and $\sum X - N(n\mu, \sqrt{n}\sigma)$. If the size n of the sample is sufficiently large, then the distribution of the sample means and the distribution of the sample sums will approximate a normal distribution regardless of the shape of the population. The mean of the sample means will equal the population mean and the mean of the sample sums will equal n times the population mean. The standard deviation of the distribution of the sample means, $\frac{\sigma}{\sqrt{n}}$, is called the standard error of the mean.

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