7.8: Distribution of Sample Proportions (5 of 6)

Learning Objectives

- Use a z-score and the standard normal model to estimate probabilities of specified events.

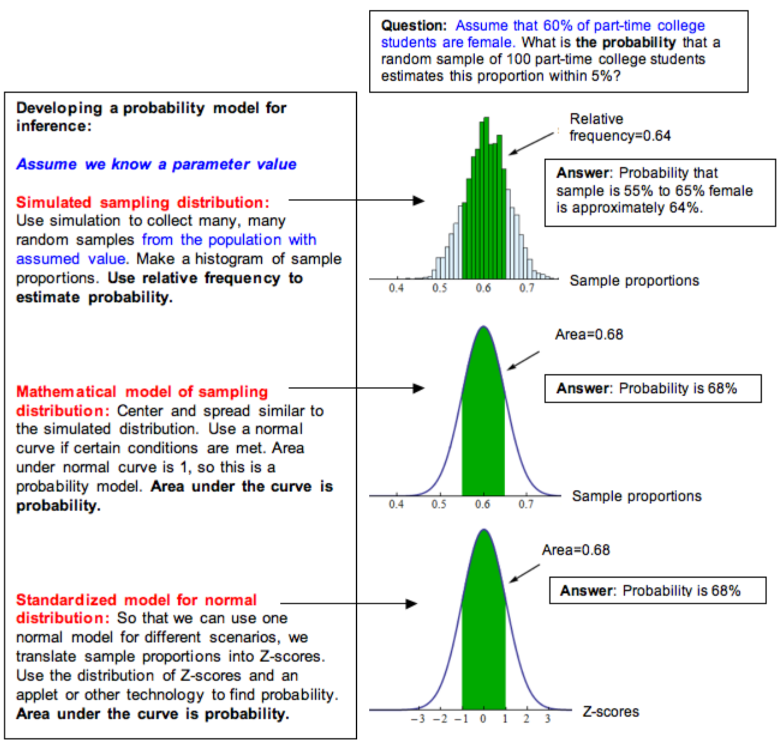
From our work on the previous page, we now have a mathematical model of the sampling distribution of sample proportions. This model describes how much variability we can expect in random samples from a population with a given parameter. If a normal model is a good fit for a sampling distribution, we can apply the empirical rule and use *z*-scores to determine probabilities. Here we link probability to the kind of thinking we do in inference.

Making Connections to Probability Models in *Probability and Probability Distribution*

Probability describes the chance that a random event occurs. Recall the concept of a random variable from the module *Probability and Probability Distribution*. When a variable is random, it varies unpredictably in the short run but has a predictable pattern in the long run. Sample proportions from random samples are a random variable. We cannot predict the proportion for any one random sample; they vary. But we can predict the pattern that occurs when we select a great many random samples from a population. The sampling distribution describes this pattern. When a normal model is a good fit for the sampling distribution, we can use what we learned in the previous module to find probabilities.

Recall probability models we saw in *Probability and Probability Distribution*. We saw examples of models with skewed curves, but we focused on normal curves because we use normal probability models to describe sampling distributions in Modules 7 to 10 when we make inferences about a population. As we now know, we can use a normal model only when certain conditions are met. Whenever we want to use a normal model, we must check the conditions to make sure a normal model is a good fit.

Here we summarize our general process for developing a probability model for inference. This is essentially the same process we used in the previous module for developing normal probability models from relative frequencies.



If a normal model is a good fit for the sampling distribution, we can standardize the values by calculating a *z*-score. Then we can use the standard normal model to find probabilities, as we did in *Probability and Probability Distribution*.

The *z*-score is the error in the statistic divided by the standard error. For sample proportions, we have the following formulas.

$$\begin{array}{l}\mathrm{standard}\text{ }\mathrm{error}=\sqrt{\frac{p(1-p)}{n}}\\ Z=\frac{\mathrm{statistic}-\mathrm{parameter}}{\mathrm{standard}\text{ }\mathrm{error}}=\frac{\stackrel{^}{p}-p}{\mathrm{standard}\text{ }\mathrm{error}}\end{array}$$

We can also write this as one formula:

$$Z=\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$$

Comment

This *z*-score formula is similar to the *z*-score formula we used in *Probability and Probability Distribution*. We described the *z*-score as the number of standard deviations a data value is from the mean. Here we can describe the *z*-score as the number of standard errors a sample proportion is from the mean. Because the mean is the parameter value, we can say that the *z*-score is the number of standard errors a sample proportion is from the parameter.

A positive *z*-score indicates that the sample proportion is larger than the parameter. A negative *z*-score indicates that the sample proportion is smaller than the parameter.

Example

Probability Calculations for Community College Enrollment

Let's return to the example of community college enrollment. Recall that a 2007 report by the Pew Research Center stated that about 10% of the 3.1 million 18- to 24-year-olds in the United States were enrolled in a community college. Let's again suppose we randomly selected 100 young adults in this age group and found that 15% of the sample was enrolled in a community college.

Previously, we determined that 15% is a surprising result. Now we want to be more precise. We ask this question: *What is the probability that a random sample of this size has 15% or more enrolled in a community college?*

To answer this question, we first determine if a normal model is a good fit for the sampling distribution.

Check normality conditions:

Yes, the conditions are met. The number of expected successes and failures in a sample of 100 are at least 10. We expect 10% of the 100 to be enrolled in a community college, $np=100(0.10)=10$. We expect 90% of the 100 to not be enrolled, $n(1-p)=100(0.90)=90$.

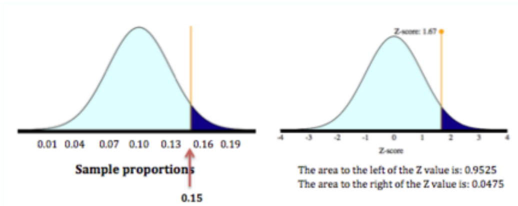
We therefore can use a normal model, which allows us to use a *z*-score to find the probability.

Find the *z*-score:

$$\mathrm{standard}\text{ }\mathrm{error}\text{ }=\text{ }\sqrt{\frac{p(1-p)}{n}}=\sqrt{\frac{0.10(0.90)}{100}}\approx 0.03$$

$$Z=\frac{\mathrm{statistic}-\mathrm{parameter}}{\mathrm{standard}\text{ }\mathrm{error}}=\frac{0.15-0.10}{0.03}\approx 1.67$$

Find the probability using the standard normal model:



We want the probability that the sample proportion is 15% or more. So we want the probability that the *z*-score is greater than or equal to 1.67. The probability is about 0.0475.

Conclusion: If it is true that 10% of the population of 18- to 24-year-olds are enrolled at a community college, then it is unusual to see a random sample of 100 with 15% or more enrolled. The probability is about 0.0475.

Note: This probability is a conditional probability. Recall from *Relationships in Categorical Data with Intro to Probability* that we write a conditional probability $P(A \text{ given } B)$ as $P(A \mid B)$. Here we write P(a sample proportion is 0.15 given that the population proportion is 0.10) as

$$P(\text{ }\hat{p}\text{ }\geq\text{ }0.15\text{ }|\text{ }p=0.10)\text{ }\approx \text{ }0.0475$$

Click here to open this simulation in its own window.

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