

## 11.7: Test of Independence (2 of 3)

### Learning Objectives

- Conduct a chi-square test of independence. Interpret the conclusion in context.

Here we continue our chi-square test of independence for the variables *gender* and *body image* in the population of U.S. college students.

### Example

### Gender and Body Image Continued

#### Step 1: State the hypotheses.

Here are our hypotheses from the previous page:

$H_0$ : There is no relationship between gender and body image for U.S. college students. (The variables are independent.)

$H_a$ : There is a relationship between gender and body image for U.S. college students. (The variables are dependent.)

#### Step 2: Collect and analyze the data.

If the variables are independent, the percentage of males and females with a given response will be the same or at least close. Previously, we determined that in our sample, there are differences in the percentage of males and females who answer “about right,” “overweight,” or “underweight.”

Conditional Percents				
	About right	Overweight	Underweight	Row Totals
Female	560/760 73.68%	163/760 21.45%	37/760 4.87%	760/760 100%
Male	295/440 67.00%	72/440 16.36%	73/440 16.59%	440/440 100%
Column Totals	855/1200 71.25%	235/1200 19.58%	110/1200 9.17%	1200/1200 100%

↓ ↓ ↓  
If gender and body image are independent, then percents for a response category are the same.

We need to determine if these differences are typical in random samples from a population where gender and body image are independent. Perhaps the differences we see in this sample are just fluctuations expected in random sampling. Or perhaps these differences are too large to be explained by chance. We will not know until we complete the hypothesis test.

#### Step 3: Assess the evidence.

We need to determine the expected values and the chi-square test statistic so that we can find the P-value.

#### Calculating Expected Values for a Test of Independence

Expected counts always describe what we expect to see in a sample if the null hypothesis is true. In this situation, if gender and body image are independent, then we expect the probability that a student answers “about right” in the sample to be the same probability that a male (or a female) student answers “about right” (similarly for “overweight” or “underweight” responses).

Here are the calculations of expected counts for the response “about right”:

Probability that a student will answer “about right”:  $P(\text{about right}) = (855/1,200) = 0.7125$

Expected count of females in the sample who will answer “about right”:  $0.7125(760) = 541.5$

Expected count of males in the sample who will answer “about right”:  $0.7125(440) = 313.5$



Here are the calculations of expected counts for the response “overweight”:

- Probability that a student will answer “overweight”:  $P(\text{overweight}) = (235/1,200) = 0.1958$
- Expected count of females in the sample who will answer “overweight”:  $0.1958(760) = 148.8$
- Expected count of males in the sample who will answer “overweight”:  $0.1958(440) = 86.2$

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Example

## More on Gender and Body Image

### Checking Conditions

The conditions for use of the chi-square distribution are the same as we learned previously:

- The sample is random.
- All of the expected counts are 5 or greater.

Since the data meets the conditions, we can proceed with calculating  $\chi^2$  test statistic.

### Calculating the Chi-Square Test Statistic

We calculate the chi-square test statistic as we did in “Chi-Square Test for One-Way Tables.”

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

We will use technology to calculate the chi-square value. But for this sample, we will show the calculation.

$$\chi^2 = \frac{(560 - 541.5)^2}{541.5} + \frac{(295 - 313.5)^2}{313.5} + \frac{(163 - 148.8)^2}{148.8} + \frac{(72 - 86.17)^2}{86.17} + \frac{(37 - 69.67)^2}{69.67} + \frac{(73 - 40.33)^2}{40.33} \approx 47.18$$

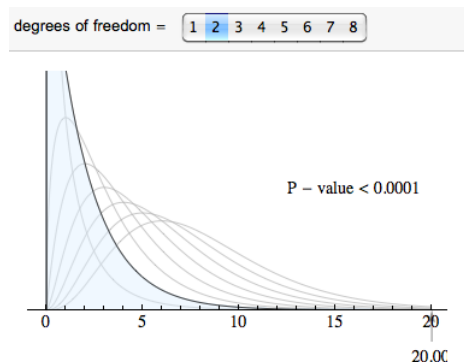
### Finding Degrees of Freedom and the P-Value

For a chi-square tests based on two-way tables, the degrees of freedom are

$$(\text{number of explanatory categories} - 1) \times (\text{number of response categories} - 1)$$

You will also see this written:  $(r - 1)(c - 1)$ , where  $r$  is the number of rows and  $c$  is the number of columns in the two-way table (when we write the table without row and column totals). In this case the degrees of freedom are  $(2 - 1)(3 - 1) = 2$ .

We use the chi-square distribution with  $df = 2$  to find the P-value. Note that the chi-square test statistic for this sample is so large that it is off the scale used in the simulation. So we conclude that the P-value is essentially zero.



### Step 4: Conclusion

The relationship between gender and body image is statistically significant in this sample. We reject the null hypothesis and accept the alternative hypothesis. Gender and body image are dependent variables in the population of U.S. college students. (P-value is

essentially 0.)

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### More on Gender and Body Piercing

A study was done on the relationship between gender and ear piercing among high-school students. A sample of 1,000 students was chosen, then classified according to both gender and whether or not they had either of their ears pierced. The following information is available:

Chi-Square Test: Pierced, No Pierced			
Observed Counts:			
	Pierced	No Pierced	Total
Female	576	64	640
Male	72	288	360
Total	648	352	1000
Expected Counts:			
	Pierced	No Pierced	Total
Female	414.72	225.28	640
Male	233.28	126.72	360
Total	648	352	1000
Chi-Square Contribution:			
	Pierced	No Pierced	
Female		115.462	
Male	111.502	205.265	

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