4.7: Exponential Relationships (5 of 6)

Learning Objectives

- Use an exponential model (when appropriate) to describe the relationship between two quantitative variables. Interpret the model in context.

The exponential model used in the Chinook salmon example showed a decline over the years and a negative association between the variables; we call such a model an **exponential decay model**. (Compare this model to the exponential growth model we investigated earlier with the eagle pairing data.)

Now we investigate the meaning of the numbers in the exponential decay model.

Example

Understanding the Numbers in the Exponential Decay Model

Our goal in this example is to understand the meaning of the numbers 49,304 and 0.854 in the exponential model for predicting the Chinook population.

$$\text{Predicted Chinook population} = 49{,}304\ (0.854)^t$$

As before, the 49,304 is the initial value for the exponential model. It is the predicted value for y when $t = 0$.

- To see this, plug $t = 0$ into the exponential model:

$$\text{Predicted Chinook population} = 49{,}304\ (0.854)^t$$

- Interpretation in context: When $t = 0$, the year is 1970. In 1970, the predicted number of Chinook is 49,304.
- It is also the y-intercept where the exponential model crosses the y-axis.

Now let's investigate the meaning of 0.854 in the context of Chinook population.

For 1971, when $t = 1$, the model predicts $y = 49{,}304\ (0.854)^1 = 42{,}106$ Chinook salmon in the Sacramento River.

For 1972, when $t = 2$, the model predicts $y = 49{,}304\ (0.854)^2 = 35{,}958$ Chinook.

We can also view the calculation for $t = 2$ as repeated multiplication by 0.854:

$$\begin{array}{l}\hat{y}=49{,}304{(0.854)}^{2}\approx 35{,}985\\ \hat{y}=\underbrace{49{,}304\cdot(0.854)}_{42{,}106\text{ }\mathrm{Chinook\; when}\text{ }t=1}\cdot(0.854)\text{ }\approx 35{,}985\\ \text{ }35{,}985\text{ }\mathrm{Chinook\; when}\text{ }t=2\end{array}$$

Note: From this viewpoint, we find the Chinook population for 1972 by multiplying the 42,106 Chinook from the previous year by 0.854.

Here is another example: For 1973, when $t = 3$, we can rewrite $(0.854)^3$ as repeated multiplication: (0.854)(0.854)(0.854). The exponent 3 tells us to multiply the initial value 49,304 by 0.854 three times.

$$\begin{array}{l}\hat{y}=49{,}304{(0.854)}^{3}\approx \mathrm{30{,}708}\\ \hat{y}=\underbrace{49{,}304\cdot(0.854)\cdot(0.854)}_{\mathrm{35{,}985}\text{ }\mathrm{Chinook\; when}\text{ }t=\mathrm{2}}\cdot(0.854)\text{ }\approx \mathrm{30{,}708}\\ \text{ }\mathrm{30{,}708}\text{ }\mathrm{Chinook\; when}\text{ }t=3\end{array}$$

Note: We can also view this process as multiplying the estimated 35,985 Chinook from the previous year by 0.854.

In general, to find the estimated number of Chinook for the next year, we multiply the previous year's estimated population by 0.854. We call this the **decay factor**.

We view the decay factor as containing information about the **percentage decrease** in the population over the previous year. To see how this works, let's start with a hypothetical situation in which there is no change in the number of Chinook from one year to the next. Then we look at different percentages of decay for the first year to build to an understanding of the meaning of 0.854. This is the same type of thinking we performed to analyze the exponential growth model previously.

No change in the number of Chinook: If there is no change in a year, we have 100% of the fish from the previous year, so the decay factor is 1.00, which is 100% written in decimal form. As before, this is important to understand. A decay factor of 1.00 means no decline in the population. This makes sense because 49,304 (1.00) = 49,304; there is no change when we multiply by 1.00.

5% decay in the first year:

- 100% of the Chinook **– 5% decrease** in the Chinook = 95% remaining.
- Convert to decimal form to find the decay factor: 95% = 0.95.
- Now multiply the decay factor by 49,304 to find the number of Chinook for the next year: 49,304(0.95) = 46,839
- So, if we multiply by 0.95, this is a *5% decrease*.

6.8% decay in the first year:

- 100% of the Chinook **– 6.8% decrease** in the Chinook = 93.2% remaining.
- Convert to decimal form to find the decay factor: 93.2% = 0.932.
- Now multiply the decay factor by 49,304 to find the number of Chinook for the next year: 49,304 (0.932) = 45,951
- So, if we multiply by 0.932, this is a *6.8% decrease*.

What is the meaning of 0.854 in the model Predicted Chinook population = 49,304 $(0.854)^t$?

Answer: The 0.854 is the decay factor; as a percentage, it is 85.4%. This tells us that 85.4% of the previous year's Chinook population are here this year. To determine the percent decrease, calculate 100% – 85.4% = 14.6%. There is an estimated 14.6% decrease in the number of Chinook each year. (Remember the 100% represents no change in the population.)

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