11.3: Hypotheses and Decision Criteria

The process of testing hypotheses using an independent samples t-test is the same as it was in the last three chapters, and it starts with stating our hypotheses and laying out the criteria we will use to test them.

Our null hypothesis for an independent samples t-test follows a similar format as all others. For a two tailed test, the means of the two groups are the same under the null hypothesis, no matter how those groups were formed. Mathematically, this takes on two equivalent forms:

$$H_0 : \mu_1 = \mu_2$$

or

$$H_0 : \mu_1 - \mu_2 = 0$$

Both of these formulations of the null hypothesis tell us exactly the same thing: that the numerical value of the means is the same in both groups. This is more clear in the first formulation, but the second formulation also makes sense (any number minus itself is always zero) and helps us out a little when we get to the math of the test statistic. Either one is acceptable and you only need to report one. The English interpretation of both of them is also the same:

$$H_0 : \text{ There is no difference between the means of the two groups}$$

Note that you should make sure that you make the hypothesis specific to your experiment/situation when you write it out.

Our alternative hypotheses also follow a similar format.

For two tailed tests:

$$H_A : \mu_1 \neq \mu_2$$

Or

$$H_A : \mu_1 - \mu_2 \neq 0$$

In words:

$$H_A : \text{There is a difference between the means of the two groups}$$

For one tailed tests, the alternative hypothesis will include predictions about one group being higher or lower than the other group:

$$H_A : \mu_1 > \mu_2 \; H_A : \mu_1 < \mu_2$$

And the appropriate null hypothesis:

$$H_0 : \mu_1 \leq \mu_2 \; H_0 : \mu_1 \geq \mu_2$$

In words, these would be:

$$H_A : \text{Group 1 is higher than group 2}$$

OR

$$H_A : \text{Group 1 is lower than group 2}$$

$$H_0 : \text{Group 1 is not higher than group 2}$$

OR

$$H_0 : \text{Group 1 is not lower than group 2}$$

Notice that we are now dealing with two means instead of just one, so it will be very important to keep track of which mean goes with which population and, by extension, which dataset and sample data. We use subscripts to differentiate between the populations, so make sure to keep track of which is which. If it is helpful, you can also use more descriptive subscripts. To use the experimental medication example:

H_0: There is no difference between the scores of the those who received the treatment and the control groups

$$H_0 : \mu_{\text{treatment}} = \mu_{\text{control}}$$

H_A: There is a difference between the scores of those who received the treatment and the control groups

$$H_A : \mu_{\text{treatment}} \neq \mu_{\text{control}}$$

Once we have our hypotheses laid out, we can set our criteria to test them using the same three pieces of information as before: significance level (α), directionality (left, right, or two-tailed), and degrees of freedom, which for an independent samples t-test are:

$$df = n_1 + n_2 - 2$$

This looks different than before, but it is just adding the individual degrees of freedom from each group (n– 1) together. Notice that the sample sizes, n, also get subscripts so we can tell them apart.

For an independent samples t-test, it is often the case that our two groups will have slightly different sample sizes, either due to chance or some characteristic of the groups themselves. Generally, this is not as issue, so long as one group is not massively larger than the other group. What is of greater concern is keeping track of which is which using the subscripts.

Contributors and Attributions

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