Ch 7.1 Central Limit Theorem for Sample Means

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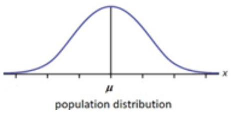
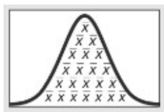
Sample distribution of sample mean:

When sample means $\bar{x}$ of same size n taken from the same population, the Sample means have the following behavior:

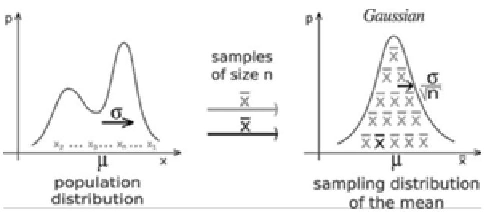
1) If the population distribution of X is normal, the distribution of $\bar{x}$ is always normal for all sample size n.





Sampling distribution of x-bar

2) When population distribution of X is not normal, The sampling distribution x-bar tends to be a normal distribution. The distribution become closer to normal when sample size increase.



Activity to discover the Central Limit Theorem:

https://stats.libretexts.org/Bookshelves/Ancillary_Materials/02%3A_Interactive_Statistics/15%3A_Discover_the_Central_Limit_Theorem_Activity

Central Limit Theorem for Sample Mean:

For all sample of the same size n with n > 30, the sampling distribution of $\bar{x}$ can be approximated by a normal distribution with mean μ and standard deviation $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Note: -This applies to all distribution of x. If X is normally distributed, n > 30 is not needed. Any n will work.

-The sample should be a Simple Random Sample.

Central Limit Theorem: $\mu_{\bar{x}} = \mu$,

\(\sigma _{\bar{x}} = \frac{\sigma}{\sqrt{n} \)}

Ex1 A standardized test with scores that are normally distributed with mean μ = 150 and standard deviation σ = 18. A class of 20 students take the test. The mean score $\bar{x}$ of the 20 students are calculated.

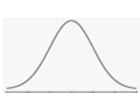
a) Is the distribution of mean score $\bar{x}$ of 20 students Normally distributed?

Ans: Yes because the original score is Normal.

a) What is the mean and standard deviation of ?

Use Central Limit Theorem: mean = 150, SD = $\frac{18}{\sqrt{20}} \approx 4.0249$

b) Find the probability that a student's score is greater than 160.



Use Online Normal Calculator, Mean = 150, SD = 4.0249

c) Find the probability that the mean score $\bar{x}$ of 20 students is greater than 160.

Click above, enter 160. Recalculate. P($\bar{x}$ > 160) = 0.0065

Ex2: Coke cans are filled so that the actual amounts have a mean of 12 oz and a standard deviation of 0.11 oz. The distribution of amount of coke is unknown.

a) Is the distribution of mean amount of coke in 36 cans normally distributed?

Yes, because n > 30, according to CLT, $\bar{x}$ will be normally distributed.

b) What is the mean and standard deviation of $\bar{x}$?

Ans: according to CLT: $\mu_{\bar{x}}$ = 12, $\sigma_{\bar{x}}$ = 0.11/√36 ≈ 0.01833

c) Find the percent of individual coke with amount between 11.9 to 12.1 oz.

Use online Normal Calculator: Mean = 12, SD = 0.11

Click between, enter 11.9 and 12.1, Recalculate. P(11.9 < x < 12.1) = 0.6367

63.67% of coke have amount between 11.9 oz to 12.1 oz.

d) Find the percent of mean amount of 36 coke with between 11.9 and 12.1 oz.

Use online Normal Calculator: Mean = 12, SD = 0.01833,

Click between, enter 11.9 and 12.1, Recalculate. P(11.9 < $\bar{x}$ < 12.1) = 1

100% of mean amount of 36 coke is between 11.9 and 12.1 oz.

Ex3. Annual incomes are known to have a distribution that is skewed to the right. Assume that 20 workers' mean incomes $\bar{x}$ are collected.

a) Will the distribution of mean income $\bar{x}$ be normally distributed?

Ans: No, since X is not normal and n < 30, CLT does not apply, $\bar{x}$ may not be normally distributed.
