Ch 4.3 Binomial Distribution

Requirements for Binomial Distribution:

X can be modeled by binomial distribution if it satisfies four requirements:

1. The procedure has a fixed number of trials. (n)

2. The trials must be independent.

3. Each trial has exactly two outcomes, success and failure, where x = number of success in n trials.

4. The probability of a success remains the same in all trials. P(success in one trial) = p.

P(failure in one trial) = 1 – p = q

P(X) = x number of success in n trials.

Note: for sampling, use 5% guideline for independent.

Ex1: Determine if the following X is binomial or not

a. X = number of adults out of 5 who use iPhone.

b. X = number of times a student raises his/her hand in a class.

c. X = number of one after tossing a die 7 times.

d. X = number of tosses until the “one” shows up.

e. X = the way student commute to school.

a and c are binomial. a = B(5, p), c = B(7, 1/6)

b,d does not have a fixed number of trials.

e : X is not a count of success.

Find P(X) or P(range of X) when X is binomial:

n = number of trials, p= P(success in one trial)

q = P(failure in one trial) = 1 - p, X = number of success.

Method 1: use formula:

$P(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x}$

Method 2: Use Statdisk /Analysis/ Probability Distribution/Binomial distribution

Enter n, p, x.. output in sample editor under P(x), P(x or fewer) or P(x or greater).

Optional: use OnlineStatbook binomial calculator:

http://onlinestatbook.com/2/calculators/binomial_dist.html

input n and p (to N and ∏), select above, below or between.

Parameters of binomial distribution:

mean μ = np

variance: \(σ^2 = npq \)

standard deviation $\sigma = \sqrt{npq}$

Range rule of thumb:

Values not significant: Between $(\mu - 2\sigma)$ and $(\mu + 2\sigma)$

Find parameters of binomial distribution

Use Statdisk /Analysis/ Probability Distribution/ Binomial distribution, enter n, p, x, evaluate.

Mean, standard deviation and variances are under the sample editor.

Ex1. In a college, 35% of all students are full-time students. If 11 students are randomly chosen.

a) Can probability of X = number of full time students out of 11 be modeled by binomial distribution?

Ans: yes, since 11 students is less than 5% of all students, P(one student is full time) = 0.35 = constant, 11 is a constant number of trials.

there are two outcomes for each student, full-time or not full-time.

b) What is the probability that there are 4 full-time students out of 11?

Use statdisk/analysis/probability distribution/ binomial distribution n = 11, p = 0.35, x = 4, evaluate. Use P(x)

P(x) = 0.2428, P(4 out of 11 are full-time) = 0.2428.

c) What is the probability that there are less than 5 full-time students?

Use statdisk/analysis/probability distribution/ binomial distribution n = 11, p = 0.35, x = 4. use P(x or fewer)

P(x or fewer) = 0.6683. The chance of less than 5 full-time students out of 11 is 0.6683.

d) What is the probability of there are more than 3 full-time students?

P(more than 3) = P(4 or more)

Use Statdisk/analysis/probability distribution/ binomial distribution n= 11, p = 0.35, x = 4 use P(x or greater)

P(x or greater) = 0.5744

Ex2: A bookstore manager estimates that 9.5% of all customers coming in the store will buy a book or magazine. If 24 customers visit the store on a certain business hour,

a) Can x = number of customers out of 24 who buy a book or magazine be modeled by binomial distribution?

Yes X can be modeled by binomial distribution because there are 2 outcomes, buy book or magazine or not "buy book or magazine". 24 customers should be less than 5% of the population of all customers, so sample are independent. P(one customer buy) = 0.095 is constant.

So we can use binomial distribution n = 24, p =0.095,

b) Find the probability that exactly 3 customers will buy a book or magazine.

Use Statdisk/analysis/probability distribution/ binomial distribution n = 24, p =0.095, x = 3, evaluate, use P(x)

P(x) = 0.2133

c) Find the probability that at least 5 customers will buy a book or magazine.

P(at least 5) = P(5 or more)

Use Statdisk/analysis/probability distribution/ binomial distribution n = 24, p =0.095, x = 5, evaluate, use P(x or greater)

P(x or greater) = 0.0714

d) Find the probability that at most 2 customers will buy a book or magazine.

P(at most 2) = (2 or fewer)

Use Statdisk/analysis/probability distribution/ binomial distribution n = 24, p =0.095, x = 2, evaluate, use P(x or fewer)

P(x or fewer) = 0.5977

e) Find the non-significant range of customer who will buy a book or magazine out of 24 customers.

Find mean and standard deviation from statdisk/analysis/probability distribution/ binomial distribution, n = 24, p = 0.095, x = 0, evaluate

Evaluate, look at the bottom of the table.

Mean = 2.28, sd = 1.44,

Non-significant range = 2.28 – 2(1.44) = -0.60 to 2.28 + 2(1.44) = 5.16.

X values from -0.6 to 5.2 are non-significance.

Ex3. A small airline has a policy of booking as many as 60 persons on an airplane that can seat only 53. (Past studies have revealed that only 78% of the booked passengers actually arrive for the flight.)

a) Find the probability that if the airline books 60 persons, not enough seats will be available.

Use binomial distribution n= 60, p= 0.78, P(not enough seats) = P(54 or more)

Use Statdisk/analysis/probability distribution/ binomial distribution n = 60, p =0.78, x = 54, evaluate, use P(x or more)

P(x or more) = 0.013.

b) Find the non-significant range of passengers who will arrive out of 60 passengers.

Look at the bottom of the statdisk table, mean = 46.80, sd =3.21

non-significant range is from 46.8 - 2(3.21) to 46.8 + 2(3.21). From 40.38 to 53.22 or 40.4 to 53.2
