Ch 8.3 Confidence Interval for Population Proportion

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Terms: Population proportion: p

Sample proportion: $\hat{p}$ = (x/n)

Number of success: x; sample size: n

Confidence Level: C-level

Significant Level: α = 1 – C-level (probability of unlikely)

EBP: Error Bound for proportion, margin of error

A) To Estimate p:

$\boxed{1) \text{ point estimate: } \hat{p} = x/n}$

$\boxed{2) \text{ Interval estimate: } \hat{p} - E \text{ to } \hat{p} + E}$

$E(EBP) = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

Note: E is the approximated value of sampling error of sampling distribution of $\hat{p}$, which has a normal distribution when x and n-x ≥ 5.

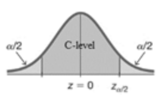
Use Statdisk online calculator to find the confidence interval.

- identify C-level, sample size n and success = x.

- https://www.statdisk.com/# , Analysis/ Confidence Intervals/Proportion one sample/

output: E (EBP) and   lower < p < upper

$z_{\alpha/2}$ is the critical value with C-level in the middle.



To find $z_{\alpha/2}$

Use online Inverse Normal calculator, set area = $\alpha/2$, mean = 0, sd =1,

click above, recalculate.

Explanation:

X = number of success is a binomial distribution with mean = np and sd = $\sqrt{npq}$.

When np and nq ≥ 5, distribution of X is normal with mean = np and sd = $\sqrt{npq}$.

So distribution of $\hat{p} = \frac{x}{n}$ is normal with mean = p and SD = $\sqrt{\frac{\hat{p}\hat{q}}{n}}$.

At a given C-level, the maximum error of $\hat{p}$ and p is E where $E(EBP) = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

Note: the requirement for this confidence interval is not n > 30 but np and nq ≥ 5.

Interpret a 95% Confidence Interval:

We are 95% confidence that the interval from ____ to ____ actual contain the true value of the population proportion of the category of interest.

The Confidence interval also shows that 95% of all confidence Intervals contain the true value of p.

Make conclusion from confidence interval:

1) Any value in the confidence Interval can be p.

2) If the whole interval > a, we can conclude p > a

3) If the whole interval < a, we can conclude p < a.

4) When two confidence interval overlap, we can conclude that the two p may be the same. We cannot conclude one of the p is higher.

Ex1. A research is conducted to determine how many household use Netflix to stream videos. A random sample of 500 households show that 442 households use Netflix.

a) Use a 90% confidence level to compute a confidence interval estimate of true proportion of households using Netflix.

use Statdisk Analysis/Confidence Intervals/Proportion one sample : Clevel = 0.9, n = 500, x = 442,

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Margin of Error, E = 0.02356

90% Confidence Interval (using normal approx):
0.86044 < p < 0.90756
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b) Find the critical value.

Use Inverse Normal calculator, since clevel = 90%, $\alpha = 0.1, \alpha/2 = 0.05$, set area = 0.05, mean = 0, sd = 1,

click above, recalculate.

$z_{\alpha/2}$ = 1.645

c) Interpret the confidence interval in non-technical term.

We estimate with 90% confidence that the true proportion of all households that use Netflix is between 86.0% to 90.8%

d) Can we conclude with 90% confidence that more than 80% of households use Netflix?

Since the interval contains 86.0% to 90.8%, all values are more than 80%, so we can conclude that.

Ex2: A Gallup poll of 1487 adults showed that 43% of the respondents have Facebook pages.

a) Find the number in the sample who have Facebook pages. $x = n(\hat{p})$ = 1487 (0.43) = 639

c) Find the interval estimate of p at 95% confidence level and margin of error E.

Use statdisk with Clevel = 0.95, n = 1487, x = 639

Interval estimate is 0.405 < p < 0.455, E = 0.025

d) Write a non-technical interpretation of the above

We are 95% confidence that the true percent of adults who have Facebook pages are between 40.5% to 45.5%.

e) Can we claim that less than 60% of all adults have Facebook pages?

Since the whole interval is less than 0.6, yes, we can conclude less than 60% of all adults have Facebook pages.

B) Determine sample size for a desired E

Since $E(EBP) = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$n = \frac{(z_{\alpha/2})^2 \hat{p}\hat{q}}{E^2}$ when an estimate $\hat{p}$ is known.

$n = \frac{(z_{\alpha/2})^2 \cdot 0.25}{E^2}$ when no estimate $\hat{p}$ is known.

Use statdisk / Analysis/Sample size Determination/ Estimate proportion to find sample size for a given error E.

Input C-Level, Desired E, Estimate of p = $\hat{p} = \frac{x}{n}$, evaluate.

Ex3. What sample size should be used if we want the keep the margin of error within 2.1% when estimating a proportion at a 90% confidence interval. use $\hat{p} = 0.43$ as an old estimation of p.

C-level = 0.9, E = 0.021, estimate of p = 0.43, Evaluate

Sample size p = 1504.

b) If no previous study has been done , what sample size will be needed. (do not use p-hat = 0.43)

Use statdisk / Analysis/Sample size Determination/ Estimate proportion

C-level = 0.9, E = 0.021, estimate of p = blank, evaluate

sample size p = 1534
