

## 11.3: Goodness-of-Fit Test

In this type of hypothesis test, you determine whether the data "fit" a particular distribution or not. For example, you may suspect your unknown data fit a binomial distribution. You use a chi-square test (meaning the distribution for the hypothesis test is chi-square) to determine if there is a fit or not. The null and the alternative hypotheses for this test may be written in sentences or may be stated as equations or inequalities.

The test statistic for a goodness-of-fit test is:

$$\sum_k \frac{(O - E)^2}{E} \quad (11.3.1)$$

where:

- $O$  = observed values (data)
- $E$  = expected values (from theory)
- $k$  = the number of different data cells or categories

The observed values are the data values and the expected values are the values you would expect to get if the null hypothesis were true. There are  $n$  terms of the form  $\frac{(O-E)^2}{E}$ .

The number of degrees of freedom is  $df = (\text{number of categories} - 1)$ .

The goodness-of-fit test is almost always right-tailed. If the observed values and the corresponding expected values are not close to each other, then the test statistic can get very large and will be way out in the right tail of the chi-square curve.

The expected value for each cell needs to be at least five in order for you to use this test.

### ✓ Example 11.3.1

Absenteeism of college students from math classes is a major concern to math instructors because missing class appears to increase the drop rate. Suppose that a study was done to determine if the actual student absenteeism rate follows faculty perception. The faculty expected that a group of 100 students would miss class according to the table below.

Number of absences per term	Expected number of students
0–2	50
3–5	30
6–8	12
9–11	6
12+	2

A random survey across all mathematics courses was then done to determine the actual number (**observed**) of absences in a course. The chart in the table below displays the results of that survey.

Number of absences per term	Actual number of students
0–2	35
3–5	40
6–8	20
9–11	1
12+	4

Determine the null and alternative hypotheses needed to conduct a goodness-of-fit test.

- $H_0$ : Student absenteeism **fits** faculty perception.

The alternative hypothesis is the opposite of the null hypothesis.

- $H_a$ : Student absenteeism **does not fit** faculty perception.

### ? Exercise 11.3.1.1

a. Can you use the information as it appears in the charts to conduct the goodness-of-fit test?

#### Answer

a. **No.** Notice that the expected number of absences for the "12+" entry is less than five (it is two). Combine that group with the "9–11" group to create new tables where the number of students for each entry are at least five. The new results are in the table below.

Number of absences per term	Expected number of students
0–2	50
3–5	30
6–8	12
9+	8

Number of absences per term	Actual number of students
0–2	35
3–5	40
6–8	20
9+	5

### ? Exercise 11.3.1.2

b. What is the number of degrees of freedom ( $df$ )?

#### Answer

b. There are four "cells" or categories in each of the new tables.

$$df = \text{number of cells} - 1 = 4 - 1 = 3$$

### ? Exercise 11.3.1

A factory manager needs to understand how many products are defective versus how many are produced. The number of expected defects is listed in the table below.

Number produced	Number defective
0–100	5
101–200	6
201–300	7
301–400	8
401–500	10

A random sample was taken to determine the actual number of defects. The table below shows the results of the survey.

Number produced	Number defective
0–100	5
101–200	7
201–300	8
301–400	9
401–500	11

State the null and alternative hypotheses needed to conduct a goodness-of-fit test, and state the degrees of freedom.

**Answer**

$H_0$ : The number of defects fits expectations.

$H_a$ : The number of defects does not fit expectations.

$df = 4$

### ✓ Example 11.3.2

Employers want to know which days of the week employees are absent in a five-day work week. Most employers would like to believe that employees are absent equally during the week. Suppose a random sample of 60 managers were asked on which day of the week they had the highest number of employee absences. The results were distributed as in the table below. For the population of employees, do the days for the highest number of absences occur with equal frequencies during a five-day work week? Test at a 5% significance level.

Day of the Week Employees were Most Absent

	Monday	Tuesday	Wednesday	Thursday	Friday
Number of Absences	15	12	9	9	15

**Answer**

The null and alternative hypotheses are:

- $H_0$ : The absent days occur with equal frequencies, that is, they fit a uniform distribution.
- $H_a$ : The absent days occur with unequal frequencies, that is, they do not fit a uniform distribution.

If the absent days occur with equal frequencies, then, out of 60 absent days (the total in the sample:  $15 + 12 + 9 + 9 + 15 = 60$ ), there would be 12 absences on Monday, 12 on Tuesday, 12 on Wednesday, 12 on Thursday, and 12 on Friday. These numbers are the **expected** ( $E$ ) values. The values in the table are the **observed** ( $O$ ) values or data.

This time, calculate the  $\chi^2$  test statistic by hand. Make a chart with the following headings and fill in the columns:

- Expected ( $E$ ) values (12, 12, 12, 12, 12)
- Observed ( $O$ ) values (15, 12, 9, 9, 15)
- $(O - E)$
- $(O - E)^2$
- $\frac{(O - E)^2}{E}$

Now add (sum) the last column. The sum is three. This is the  $\chi^2$  test statistic.

To find the  $p$ -value, calculate  $P(\chi^2 > 3)$ . This test is right-tailed. (Use a computer or calculator to find the  $p$ -value. You should get  $p\text{-value} = 0.5578$ .)

The  $dfs$  are the number of cells  $- 1 = 5 - 1 = 4$

Press **2nd DISTR** . Arrow down to  $\chi^2$ cdf. Press **ENTER** . Enter  $(3, 10^{99}, 4)$  . Rounded to four decimal places, you should see 0.5578, which is the  $p$ -value.

Next, complete a graph like the following one with the proper labeling and shading. (You should shade the right tail.)

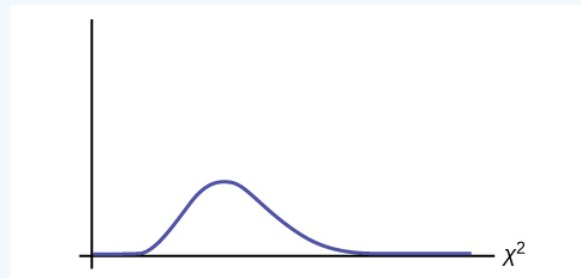


Figure 11.3.1.

The decision is not to reject the null hypothesis.

**Conclusion:** At a 5% level of significance, from the sample data, there is not sufficient evidence to conclude that the absent days do not occur with equal frequencies.

TI-83+ and some TI-84 calculators do not have a special program for the test statistic for the goodness-of-fit test. The next example [Example](#) has the calculator instructions. The newer TI-84 calculators have in **STAT TESTS** the test **Chi2 GOF** . To run the test, put the observed values (the data) into a first list and the expected values (the values you expect if the null hypothesis is true) into a second list. Press **STAT TESTS** and **Chi2 GOF** . Enter the list names for the Observed list and the Expected list. Enter the degrees of freedom and press **calculate** or **draw** . Make sure you clear any lists before you start. **To Clear Lists in the calculators:** Go into **STAT EDIT** and arrow up to the list name area of the particular list. Press **CLEAR** and then arrow down. The list will be cleared. Alternatively, you can press **STAT** and press 4 (for **ClrList** ). Enter the list name and press **ENTER** .

### ? Exercise 11.3.2

Teachers want to know which night each week their students are doing most of their homework. Most teachers think that students do homework equally throughout the week. Suppose a random sample of 49 students were asked on which night of the week they did the most homework. The results were distributed as in the table below.

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Number of Students	11	8	10	7	10	5	5

From the population of students, do the nights for the highest number of students doing the majority of their homework occur with equal frequencies during a week? What type of hypothesis test should you use?

**Answer**

$$df = 6$$

$$p\text{-value} = 0.6093$$

We decline to reject the null hypothesis. There is not enough evidence to support that students do not do the majority of their homework equally throughout the week.

### ✓ Example 11.3.3

One study indicates that the number of televisions that American families have is distributed (this is the **given** distribution for the American population) as in the table below.

Number of Televisions	Percent

Number of Televisions	Percent
0	10
1	16
2	55
3	11
4+	8

The table contains expected ( $E$ ) percents.

A random sample of 600 families in the far western United States resulted in the data in the table below.

Number of Televisions	Frequency
0	66
1	119
2	340
3	60
4+	15
	Total = 600

The table contains observed ( $O$ ) frequency values.

### ? Exercise 11.3.3.1

At the 1% significance level, does it appear that the distribution "number of televisions" of far western United States families is different from the distribution for the American population as a whole?

#### Answer

This problem asks you to test whether the far western United States families distribution fits the distribution of the American families. This test is always right-tailed.

The first table contains expected percentages. To get expected ( $E$ ) frequencies, multiply the percentage by 600. The expected frequencies are shown in the table below.

Number of Televisions	Percent	Expected Frequency
0	10	$(0.10)(600) = 60$
1	16	$(0.16)(600) = 96$
2	55	$(0.55)(600) = 330$
3	11	$(0.11)(600) = 66$
over 3	8	$(0.08)(600) = 48$

Therefore, the expected frequencies are 60, 96, 330, 66, and 48. In the TI calculators, you can let the calculator do the math. For example, instead of 60, enter  $0.10 * 600$ .

$H_0$ : The "number of televisions" distribution of far western United States families is the same as the "number of televisions" distribution of the American population.

$H_a$ : The "number of televisions" distribution of far western United States families is different from the "number of televisions" distribution of the American population.

Distribution for the test:  $\chi^2_4$  where  $df = (\text{the number of cells}) - 1 = 5 - 1 = 4$  .

**Note 11.3.3.1**

$df \neq 600 - 1$

**Calculate the test statistic:**  $\chi^2 = 29.65$

**Graph:**

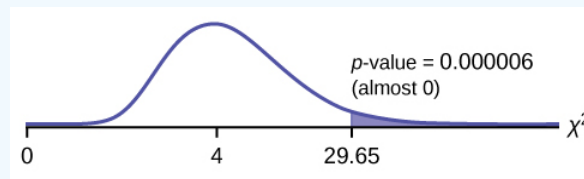


Figure 11.3.2.

**Probability statement:**  $p\text{-value} = P(\chi^2 > 29.65) = 0.000006$

**Compare  $\alpha$  and the  $p$ -value:**

$\alpha = 0.01$

$p\text{-value} = 0.000006$

So,  $\alpha > p\text{-value}$ .

**Make a decision:** Since  $\alpha > p\text{-value}$ , reject  $H_0$ .

This means you reject the belief that the distribution for the far western states is the same as that of the American population as a whole.

**Conclusion:** At the 1% significance level, from the data, there is sufficient evidence to conclude that the "number of televisions" distribution for the far western United States is different from the "number of televisions" distribution for the American population as a whole.

Press **STAT** and **ENTER** . Make sure to clear lists **L1** , **L2** , and **L3** if they have data in them (see the note at the end of [Example](#)). Into **L1** , put the observed frequencies 66 , 119 , 349 , 60 , 15 . Into **L2** , put the expected frequencies .10\*600, .16\*600 , .55\*600 , .11\*600 , .08\*600 . Arrow over to list **L3** and up to the name area "**L3**" . Enter  $(L1-L2)^2/L2$  and **ENTER** . Press **2nd QUIT** . Press **2nd LIST** and arrow over to **MATH** . Press **5** . You should see "sum" (Enter **L3**) . Rounded to 2 decimal places, you should see 29.65 . Press **2nd DISTR** . Press **7** or Arrow down to **7:χ2cdf** and press **ENTER** . Enter (29.65,1E99,4) . Rounded to four places, you should see 5.77E-6 = .000006 (rounded to six decimal places), which is the p-value.

The newer TI-84 calculators have in **STAT TESTS** the test **Chi2 GOF** . To run the test, put the observed values (the data) into a first list and the expected values (the values you expect if the null hypothesis is true) into a second list. Press **STAT TESTS** and **Chi2 GOF** . Enter the list names for the Observed list and the Expected list. Enter the degrees of freedom and press **calculate** or **draw** . Make sure you clear any lists before you start.

**? Exercise 11.3.3**

The expected percentage of the number of pets students have in their homes is distributed (this is the given distribution for the student population of the United States) as in the table below.

Number of Pets	Percent
0	18
1	25
2	30

Number of Pets	Percent
3	18
4+	9

A random sample of 1,000 students from the Eastern United States resulted in the data in the table below.

Number of Pets	Frequency
0	210
1	240
2	320
3	140
4+	90

At the 1% significance level, does it appear that the distribution “number of pets” of students in the Eastern United States is different from the distribution for the United States student population as a whole? What is the  $p$ -value?

**Answer**

$p\text{-value} = 0.0036$

We reject the null hypothesis that the distributions are the same. There is sufficient evidence to conclude that the distribution “number of pets” of students in the Eastern United States is different from the distribution for the United States student population as a whole.

#### ✓ Example 11.3.4

Suppose you flip two coins 100 times. The results are 20  $HH$ , 27  $HT$ , 30  $TH$ , and 23  $TT$ . Are the coins fair? Test at a 5% significance level.

**Answer**

This problem can be set up as a goodness-of-fit problem. The sample space for flipping two fair coins is  $HH, HT, TH, TT$ . Out of 100 flips, you would expect 25  $HH$ , 25  $HT$ , 25  $TH$ , and 25  $TT$ . This is the expected distribution. The question, “Are the coins fair?” is the same as saying, “Does the distribution of the coins (20 $HH$ , 27 $HT$ , 30 $TH$ , 23 $TT$ ) fit the expected distribution?”

**Random Variable:** Let  $X$  = the number of heads in one flip of the two coins.  $X$  takes on the values 0, 1, 2. (There are 0, 1, or 2 heads in the flip of two coins.) Therefore, the **number of cells is three**. Since  $X$  = the number of heads, the observed frequencies are 20 (for two heads), 57 (for one head), and 23 (for zero heads or both tails). The expected frequencies are 25 (for two heads), 50 (for one head), and 25 (for zero heads or both tails). This test is right-tailed.

$H_0$ : The coins are fair.

$H_a$ : The coins are not fair.

**Distribution for the test:**  $\chi^2_2$  where  $df = 3 - 1 = 2$ .

**Calculate the test statistic:**  $\chi^2 = 2.14$

**Graph:**

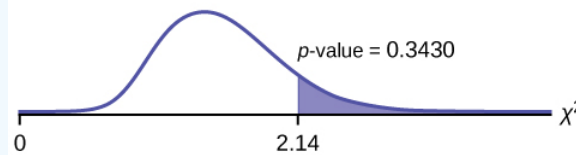


Figure 11.3.3.

**Probability statement:**  $p\text{-value} = P(\chi^2 > 2.14) = 0.3430$

**Compare  $\alpha$  and the  $p$ -value:**

$$\alpha = 0.05$$

$$p\text{-value} = 0.3430$$

$$\alpha < p\text{-value}.$$

**Make a decision:** Since  $\alpha < p\text{-value}$ , do not reject  $H_0$ .

**Conclusion:** There is insufficient evidence to conclude that the coins are not fair.

Press **STAT** and **ENTER**. Make sure you clear lists **L1**, **L2**, and **L3** if they have data in them. Into **L1**, put the observed frequencies 20, 57, 23. Into **L2**, put the expected frequencies 25, 50, 25. Arrow over to list **L3** and up to the name area "L3". Enter  $(L1-L2)^2/L2$  and **ENTER**. Press **2nd QUIT**. Press **2nd LIST** and arrow over to **MATH**. Press **5**. You should see "sum". Enter **L3**. Rounded to two decimal places, you should see 2.14. Press **2nd DISTR**. Arrow down to **7:χ<sup>2</sup>cdf** (or press **7**). Press **ENTER**. Enter 2.14, 1E99, 2). Rounded to four places, you should see .3430, which is the  $p$ -value.

The newer TI-84 calculators have in **STAT TESTS** the test **Chi2 GOF**. To run the test, put the observed values (the data) into a first list and the expected values (the values you expect if the null hypothesis is true) into a second list. Press **STAT TESTS** and **Chi2 GOF**. Enter the list names for the Observed list and the Expected list. Enter the degrees of freedom and press **calculate** or **draw**. Make sure you clear any lists before you start.

### ? Exercise 11.3.4

Students in a social studies class hypothesize that the literacy rates across the world for every region are 82%. The table below shows the actual literacy rates across the world broken down by region. What are the test statistic and the degrees of freedom?

MDG Region	Adult Literacy Rate (%)
Developed Regions	99.0
Commonwealth of Independent States	99.5
Northern Africa	67.3
Sub-Saharan Africa	62.5
Latin America and the Caribbean	91.0
Eastern Asia	93.8
Southern Asia	61.9
South-Eastern Asia	91.9
Western Asia	84.5
Oceania	66.4

**Answer**

$$df = 9$$

$$\chi^2 \text{ test statistic} = 26.38$$



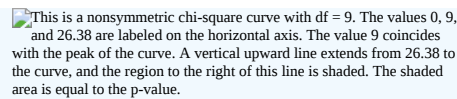


Figure 11.3.4.

Press `STAT` and `ENTER`. Make sure you clear lists `L1`, `L2`, and `L3` if they have data in them. Into `L1`, put the observed frequencies 99, 99.5, 67.3, 62.5, 91, 93.8, 61.9, 91.9, 84.5, 66.4. Into `L2`, put the expected frequencies 82, 82, 82, 82, 82, 82, 82, 82, 82, 82. Arrow over to list `L3` and up to the name area "`L3`". Enter  $(L1-L2)^2/L2$  and `ENTER`. Press `2nd QUIT`. Press `2nd LIST` and arrow over to `MATH`. Press `5`. You should see "`sum`". Enter `L3`. Rounded to two decimal places, you should see 26.38. Press `2nd DISTR`. Arrow down to `7:χ2cdf` (or press `7`). Press `ENTER`. Enter 26.38, 1E99, 9). Rounded to four places, you should see .0018, which is the  $p$ -value.

The newer TI-84 calculators have in `STAT TESTS` the test `Chi2 GOF`. To run the test, put the observed values (the data) into a first list and the expected values (the values you expect if the null hypothesis is true) into a second list. Press `STAT TESTS` and `Chi2 GOF`. Enter the list names for the Observed list and the Expected list. Enter the degrees of freedom and press `calculate` or `draw`. Make sure you clear any lists before you start.

## References

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2. Data from the College Board. Available online at <http://www.collegeboard.com>.
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5. Ogden, Cynthia L., Margaret D. Carroll, Brian K. Kit, Katherine M. Flegal, "Prevalence of Obesity in the United States, 2009–2010." NCHS Data Brief no. 82, January 2012. Available online at <http://www.cdc.gov/nchs/data/databriefs/db82.pdf> (accessed May 24, 2013).
6. Stevens, Barbara J., "Multi-family and Commercial Solid Waste and Recycling Survey." Arlington Count, VA. Available online at [www.arlingtonva.us/departments.../file84429.pdf](http://www.arlingtonva.us/departments.../file84429.pdf) (accessed May 24, 2013).

## Review

To assess whether a data set fits a specific distribution, you can apply the goodness-of-fit hypothesis test that uses the chi-square distribution. The null hypothesis for this test states that the data come from the assumed distribution. The test compares observed values against the values you would expect to have if your data followed the assumed distribution. The test is almost always right-tailed. Each observation or cell category must have an expected value of at least five.

## Formula Review

$\sum_k \frac{(O-E)^2}{E}$  goodness-of-fit test statistic where:

$O$ : observed values

$E$ : expected value

$k$ : number of different data cells or categories

$df = k - 1$  degrees of freedom

Determine the appropriate test to be used in the next three exercises.

### ? Exercise 11.3.5

An archeologist is calculating the distribution of the frequency of the number of artifacts she finds in a dig site. Based on previous digs, the archeologist creates an expected distribution broken down by grid sections in the dig site. Once the site has been fully excavated, she compares the actual number of artifacts found in each grid section to see if her expectation was accurate.

**? Exercise 11.3.6**

An economist is deriving a model to predict outcomes on the stock market. He creates a list of expected points on the stock market index for the next two weeks. At the close of each day's trading, he records the actual points on the index. He wants to see how well his model matched what actually happened.

**Answer**

a goodness-of-fit test

**? Exercise 11.3.7**

A personal trainer is putting together a weight-lifting program for her clients. For a 90-day program, she expects each client to lift a specific maximum weight each week. As she goes along, she records the actual maximum weights her clients lifted. She wants to know how well her expectations met with what was observed.

Use the following information to answer the next five exercises: A teacher predicts that the distribution of grades on the final exam will be and they are recorded in the table below.

Grade	Proportion
A	0.25
B	0.30
C	0.35
D	0.10

The actual distribution for a class of 20 is in the table below.

Grade	Frequency
A	7
B	7
C	5
D	1

**? Exercise 11.3.8**

$df =$  \_\_\_\_\_

**Answer**

3

**? Exercise 11.3.9**

State the null and alternative hypotheses.

**? Exercise 11.3.10**

$\chi^2$  test statistic = \_\_\_\_\_

**Answer**

2.04

### ? Exercise 11.3.11

$p$ -value = \_\_\_\_\_

### ? Exercise 11.3.12

At the 5% significance level, what can you conclude?

#### Answer

We decline to reject the null hypothesis. There is not enough evidence to suggest that the observed test scores are significantly different from the expected test scores.

Use the following information to answer the next nine exercises: The following data are real. The cumulative number of AIDS cases reported for Santa Clara County is broken down by ethnicity as in the table below.

Ethnicity	Number of Cases
White	2,229
Hispanic	1,157
Black/African-American	457
Asian, Pacific Islander	232
	Total = 4,075

The percentage of each ethnic group in Santa Clara County is as in the table below.

Ethnicity	Percentage of total county population	Number expected (round to two decimal places)
White	42.9%	1748.18
Hispanic	26.7%	
Black/African-American	2.6%	
Asian, Pacific Islander	27.8%	
	Total = 100%	

### ? Exercise 11.3.13

If the ethnicities of AIDS victims followed the ethnicities of the total county population, fill in the expected number of cases per ethnic group.

*Perform a goodness-of-fit test to determine whether the occurrence of AIDS cases follows the ethnicities of the general population of Santa Clara County.*

### ? Exercise 11.3.14

$H_0$ : \_\_\_\_\_

#### Answer

$H_0$ : the distribution of AIDS cases follows the ethnicities of the general population of Santa Clara County.

### ? Exercise 11.3.15

$H_a$ : \_\_\_\_\_

### ? Exercise 11.3.16

Is this a right-tailed, left-tailed, or two-tailed test?

**Answer**

right-tailed

### ? Exercise 11.3.17

degrees of freedom = \_\_\_\_\_

### ? Exercise 11.3.18

$\chi^2$  test statistic = \_\_\_\_\_

**Answer**

88,621

### ? Exercise 11.3.19

$p$ -value = \_\_\_\_\_

### ? Exercise 11.3.20

Graph the situation. Label and scale the horizontal axis. Mark the mean and test statistic. Shade in the region corresponding to the  $p$ -value.

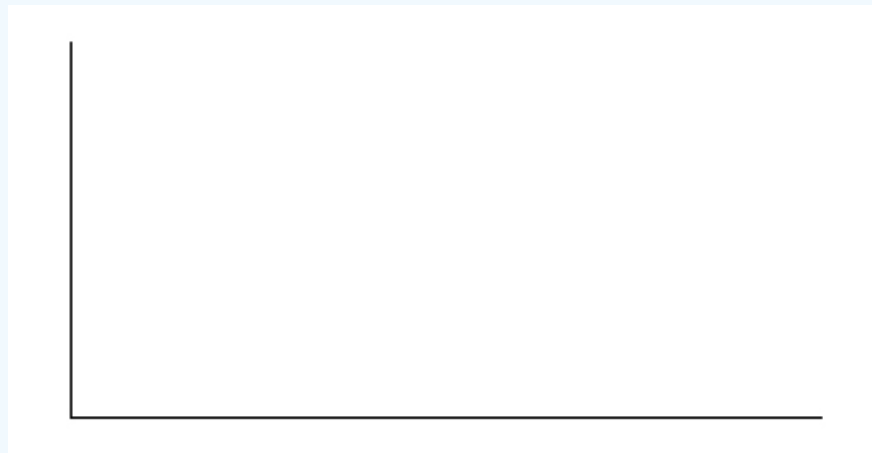


Figure 11.3.5.

Let  $\alpha = 0.05$

Decision: \_\_\_\_\_

Reason for the Decision: \_\_\_\_\_

Conclusion (write out in complete sentences): \_\_\_\_\_

**Answer**

Graph: Check student's solution.

Decision: Reject the null hypothesis.

Reason for the Decision:  $p\text{-value} < \alpha$

Conclusion (write out in complete sentences): The make-up of AIDS cases does not fit the ethnicities of the general population of Santa Clara County.

### ? Exercise 11.3.21

Does it appear that the pattern of AIDS cases in Santa Clara County corresponds to the distribution of ethnic groups in this county? Why or why not?

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