

2.2: Quantitative Data

The graph for quantitative data looks similar to a bar graph, except there are some major differences. First, in a bar graph the categories can be put in any order on the horizontal axis. There is no set order for these data values. You can't say how the data is distributed based on the shape, since the shape can change just by putting the categories in different orders. With quantitative data, the data are in specific orders, since you are dealing with numbers. With quantitative data, you can talk about a distribution, since the shape only changes a little bit depending on how many categories you set up. This is called a **frequency distribution**.

This leads to the second difference from bar graphs. In a bar graph, the categories that you made in the frequency table were determined by you. In quantitative data, the categories are numerical categories, and the numbers are determined by how many categories (or what are called classes) you choose. If two people have the same number of categories, then they will have the same frequency distribution. Whereas in qualitative data, there can be many different categories depending on the point of view of the author.

The third difference is that the categories touch with quantitative data, and there will be no gaps in the graph. The reason that bar graphs have gaps is to show that the categories do not continue on, like they do in quantitative data. Since the graph for quantitative data is different from qualitative data, it is given a new name. The name of the graph is a **histogram**. To create a histogram, you must first create the frequency distribution. The idea of a frequency distribution is to take the interval that the data spans and divide it up into equal subintervals called classes.

Summary of the Steps Involved in Making a Frequency Distribution

1. Find the range = largest value – smallest value
2. Pick the number of classes to use. Usually the number of classes is between five and twenty. Five classes are used if there are a small number of data points and twenty classes if there are a large number of data points (over 1000 data points). (Note: categories will now be called classes from now on.)
3. Class width = $\frac{\text{range}}{\# \text{ classes}}$ Always round up to the next integer (even if the answer is already a whole number go to the next integer). If you don't do this, your last class will not contain your largest data value, and you would have to add another class just for it. If you round up, then your largest data value will fall in the last class, and there are no issues.
4. Create the classes. Each class has limits that determine which values fall in each class. To find the class limits, set the smallest value as the lower class limit for the first class. Then add the class width to the lower class limit to get the next lower class limit. Repeat until you get all the classes. The upper class limit for a class is one less than the lower limit for the next class.
5. In order for the classes to actually touch, then one class needs to start where the previous one ends. This is known as the class boundary. To find the class boundaries, subtract 0.5 from the lower class limit and add 0.5 to the upper class limit.
6. Sometimes it is useful to find the class midpoint. The process is
$$\text{Midpoint} = \frac{\text{lower limit} + \text{upper limit}}{2}$$
7. To figure out the number of data points that fall in each class, go through each data value and see which class boundaries it is between. Utilizing tally marks may be helpful in counting the data values. The frequency for a class is the number of data values that fall in the class.

Note

The above description is for data values that are whole numbers. If your data value has decimal places, then your class width should be rounded up to the nearest value with the same number of decimal places as the original data. In addition, your class boundaries should have one more decimal place than the original data. As an example, if your data have one decimal place, then the class width would have one decimal place, and the class boundaries are formed by adding and subtracting 0.05 from each class limit.

Example 2.2.1 creating a frequency table

Example 2.2.1 contains the amount of rent paid every month for 24 students from a statistics course. Make a relative frequency distribution using 7 classes.

Table 2.2.1: Data of Monthly Rent

1500	1350	350	1200	850	900
1500	1150	1500	900	1400	1100
1250	600	610	960	890	1325
900	800	2550	495	1200	690

Solution

- Find the range:

largest value - smallest value = $2550 - 350 = 2200$

- Pick the number of classes:

The directions to say to use 7 classes.

- Find the class width:

$$\text{width} = \frac{\text{range}}{7} = \frac{2200}{7} \approx 314.286$$

Round up to 315.

Always round up to the next integer even if the width is already an integer.

- Find the class limits:

Start at the smallest value. This is the lower class limit for the first class. Add the width to get the lower limit of the next class. Keep adding the width to get all the lower limits.

$$350 + 315 = 665, 665 + 315 = 980, 980 + 315 = 1295 \Rightarrow$$

The upper limit is one less than the next lower limit: so for the first class the upper class limit would be $665 - 1 = 664$.

When you have all 7 classes, make sure the last number, in this case the 2550, is at least as large as the largest value in the data. If not, you made a mistake somewhere.

- Find the class boundaries:

Subtract 0.5 from the lower class limit to get the class boundaries. Add 0.5 to the upper class limit for the last class's boundary.

$$350 - 0.5 = 349.5, \quad 665 - 0.5 = 664.5, \quad 980 - 0.5 = 979.5, \quad 1295 - 0.5 = 1294.5 \Rightarrow$$

Every value in the data should fall into exactly one of the classes. No data values should fall right on the boundary of two classes.

- Find the class midpoints:

$$\text{midpoint} = \frac{\text{lower limit} + \text{upper limit}}{2}$$

$$\frac{350 + 664}{2} = 507, \quad \frac{665 + 979}{2} = 822, \Rightarrow$$

- Tally and find the frequency of the data:

Go through the data and put a tally mark in the appropriate class for each piece of data by looking to see which class boundaries the data value is between. Fill in the frequency by changing each of the tallies into a number.

Table 2.2.2: Frequency Distribution for Monthly Rent

Class Limits	Class Boundaries	Class Midpoint	Tally	Frequency
350-664	349.5-664.6	507		4
665-979	664.5-979.5	822		8
980-1294	979.5-1294.5	1137		5
1295-1609	1294.5-1609.5	1452		6
1610-1924	1609.5-1924.5	1767		0
1925-2239	1924.5-2239.5	2082		0
2240-2554	2239.5-2554.5	2397		1

Make sure the total of the frequencies is the same as the number of data points.

It is difficult to determine the basic shape of the distribution by looking at the frequency distribution. It would be easier to look at a graph. The graph of a frequency distribution for quantitative data is called a **frequency histogram** or just histogram for short.

Definition 2.2.1: Histogram

A Histogram is a graph of the frequencies on the vertical axis and the class boundaries on the horizontal axis. Rectangles where the height is the frequency and the width is the class width are drawn for each class.

Example 2.2.1: Drawing a Histogram

Draw a histogram for the distribution from Example 2.2.1.

Solution

The class boundaries are plotted on the horizontal axis and the frequencies are plotted on the vertical axis. You can plot the midpoints of the classes instead of the class boundaries. *Graph 2.2.1* was created using the midpoints.

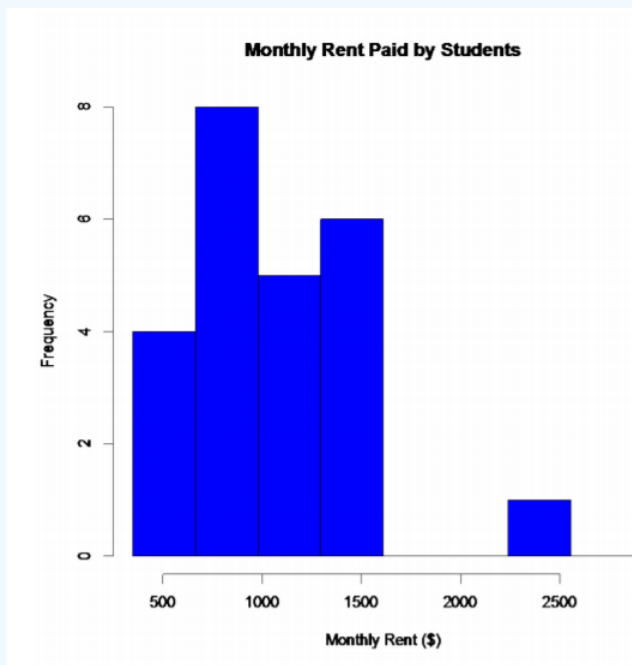


Figure 2.2.1: Histogram for Monthly Rent

Notice the graph has the axes labeled, the tick marks are labeled on each axis, and there is a title.

Reviewing the graph you can see that most of the students pay around \$750 per month for rent, with about \$1500 being the other common value. You can see from the graph, that most students pay between \$600 and \$1600 per month for rent. Of course, these values are just estimates from the graph. There is a large gap between the \$1500 class and the highest data value. This seems to say that one student is paying a great deal more than everyone else. This value could be considered an outlier. An **outlier** is a data value that is far from the rest of the values. It may be an unusual value or a mistake. It is a data value that should be investigated. In this case, the student lives in a very expensive part of town, thus the value is not a mistake, and is just very unusual. There are other aspects that can be discussed, but first some other concepts need to be introduced.

Frequencies are helpful, but understanding the relative size each class is to the total is also useful. To find this you can divide the frequency by the total to create a relative frequency. If you have the relative frequencies for all of the classes, then you have a relative frequency distribution.

Definition 2.2.2

Relative Frequency Distribution

A variation on a frequency distribution is a relative frequency distribution. Instead of giving the frequencies for each class, the relative frequencies are calculated.

$$\text{Relative frequency} = \frac{\text{frequency}}{\# \text{ of data points}}$$

This gives you percentages of data that fall in each class.

Example 2.2.3 creating a relative frequency table

Find the relative frequency for the grade data.

Solution

From Example 2.2.1, the frequency distribution is reproduced in Example 2.2.2.

Table 2.2.2: Frequency Distribution for Monthly Rent

Class Limits	Class Boundaries	Class Midpoint	Frequency
350-664	349.5-664.5	507	4
665-979	664.5-979.5	822	8
980-1294	979.5-1294.5	1127	5
1295-1609	1294.5-1609.5	1452	6
1610-1924	1609.5-1924.5	1767	0
1925-2239	1924.5-2239.5	2082	0
2240-2554	2239.5-2554.5	2397	1

Divide each frequency by the number of data points.

$$\frac{4}{24} = 0.17, \frac{8}{24} = 0.33, \frac{5}{24} = 0.21, \dots$$

Table 2.2.3: Relative Frequency Distribution for Monthly Rent

Class Limits	Class Boundaries	Class Midpoint	Frequency	Relative Frequency
350-664	349.5-664.5	507	4	0.17
665-979	664.5-979.5	822	8	0.33
980-1294	979.5-1294.5	1127	5	0.21
1295-1609	1294.5-1609.5	1452	6	0.25
1610-1924	1609.5-1924.5	1767	0	0
1925-2239	1924.5-2239.5	2082	0	0
2240-2554	2239.5-2554.5	2397	1	0.04
Total			24	1

The relative frequencies should add up to 1 or 100%. (This might be off a little due to rounding errors.)

The graph of the relative frequency is known as a relative frequency histogram. It looks identical to the frequency histogram, but the vertical axis is relative frequency instead of just frequencies.

Example 2.2.4 drawing a relative frequency histogram

Draw a relative frequency histogram for the grade distribution from Example 2.2.1.

Solution

The class boundaries are plotted on the horizontal axis and the relative frequencies are plotted on the vertical axis.

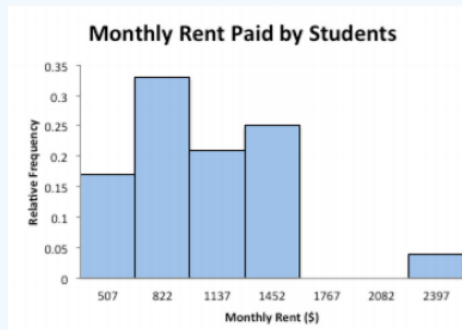


Figure 2.2.2: Relative Frequency Histogram for Monthly Rent

Notice the shape is the same as the frequency distribution.

Another useful piece of information is how many data points fall below a particular class boundary. As an example, a teacher may want to know how many students received below an 80%, a doctor may want to know how many adults have cholesterol below 160, or a manager may want to know how many stores gross less than \$2000 per day. This is known as a **cumulative frequency**. If you want to know what percent of the data falls below a certain class boundary, then this would be a **cumulative relative frequency**. For cumulative frequencies you are finding how many data values fall below the upper class limit.

To create a **cumulative frequency distribution**, count the number of data points that are below the upper class boundary, starting with the first class and working up to the top class. The last upper class boundary should have all of the data points below it. Also include the number of data points below the lowest class boundary, which is zero.

Example 2.2.5 creating a cumulative frequency distribution

Create a cumulative frequency distribution for the data in Example 2.2.1.

Solution

The frequency distribution for the data is in Example 2.2.2.

Table 2.2.2: Frequency Distribution for Monthly Rent

Class Limits	Class Boundaries	Class Midpoint	Frequency
350-664	349.5-664.5	507	4
665-979	664.5-979.5	822	8
980-1294	979.5-1294.5	1127	5
1295-1609	1294.5-1609.5	1452	6
1610-1924	1609.5-1924.5	1767	0
1925-2239	1924.5-2239.5	2082	0
2240-2554	2239.5-2554.5	2397	1

Now ask yourself how many data points fall below each class boundary. Below 349.5, there are 0 data points. Below 664.5 there are 4 data points, below 979.5, there are $4 + 8 = 12$ data points, below 1294.5 there are $4 + 8 + 5 = 17$ data points, and continue this process until you reach the upper class boundary. This is summarized in Example 2.2.4.

To put this into a cumulative frequency table we just add up the frequencies at or above each row.

Table 2.2.4: Cumulative Distribution for Monthly Rent

Class Limits	Class Boundaries	Class Midpoint	Frequency	Cumulative Frequency
350-664	349.5-664.5	507	4	4
665-979	664.5-979.5	822	8	12
980-1294	979.5-1294.5	1127	5	17
1295-1609	1294.5-1609.5	1452	6	23
1610-1924	1609.5-1924.5	1767	0	23
1925-2239	1924.5-2239.5	2082	0	23
2240-2554	2239.5-2554.5	2397	1	24

Again, it is hard to look at the data the way it is. A graph would be useful. The graph for cumulative frequency is called an **ogive** (o-jive). To create an ogive, first create a scale on both the horizontal and vertical axes that will fit the data. Then plot the points of the class upper class boundary versus the cumulative frequency. Make sure you include the point with the lowest class boundary and the 0 cumulative frequency. Then just connect the dots.

Example 2.2.6 drawing an ogive

Draw an ogive for the data in Example 2.2.1.

Solution

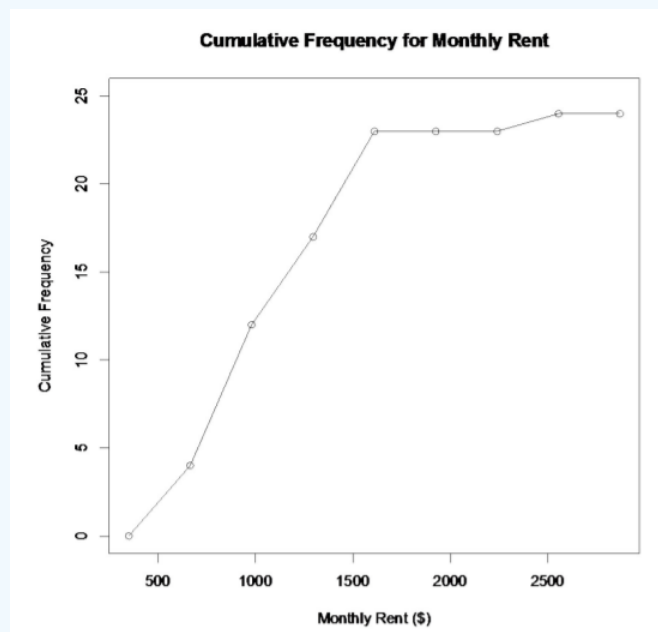


Figure 2.2.3: Ogive for Monthly Rent

The usefulness of a ogive is to allow the reader to find out how many students pay less than a certain value, and also what amount of monthly rent is paid by a certain number of students. As an example, suppose you want to know how many students pay less than \$1500 a month in rent, then you can go up from the \$1500 until you hit the graph and then you go over to the cumulative frequency axes to see what value corresponds to this value. It appears that around 20 students pay less than \$1500. (See *Graph 2.2.4*.)

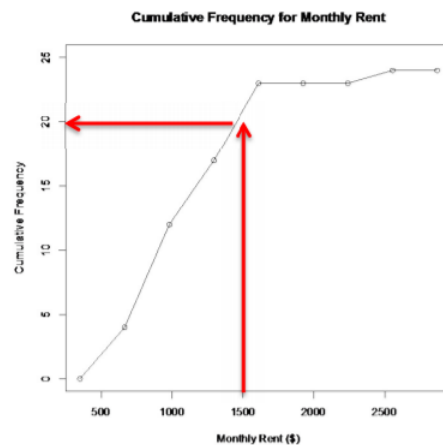


Figure 2.2.4: Ogive for Monthly Rent with Example

Also, if you want to know the amount that 15 students pay less than, then you start at 15 on the vertical axis and then go over to the graph and down to the horizontal axis where the line intersects the graph. You can see that 15 students pay less than about \$1200 a month. (See *Graph 2.2.5*.)

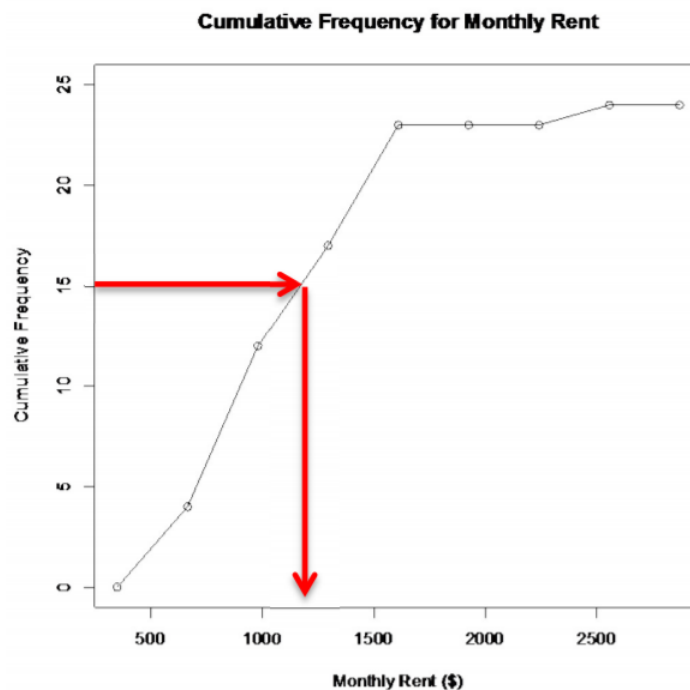


Figure 2.2.5: Ogive for Monthly Rent with Example

If you graph the cumulative relative frequency then you can find out what percentage is below a certain number instead of just the number of people below a certain value.

Shapes of the distribution:

When you look at a distribution, look at the basic shape. There are some basic shapes that are seen in histograms. Realize though that some distributions have no shape. The common shapes are symmetric, skewed, and uniform. Another interest is how many peaks a graph may have. This is known as modal.

Symmetric means that you can fold the graph in half down the middle and the two sides will line up. You can think of the two sides as being mirror images of each other. Skewed means one “tail” of the graph is longer than the other. The graph is skewed in the direction of the longer tail (backwards from what you would expect). A uniform graph has all the bars the same height.

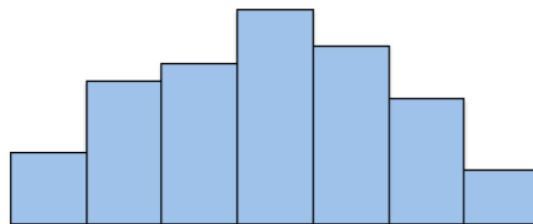
Modal refers to the number of peaks. Unimodal has one peak and bimodal has two peaks. Usually if a graph has more than two peaks, the modal information is not longer of interest.

Other important features to consider are gaps between bars, a repetitive pattern, how spread out is the data, and where the center of the graph is.

Examples of Graphs:

This graph is roughly symmetric and unimodal:

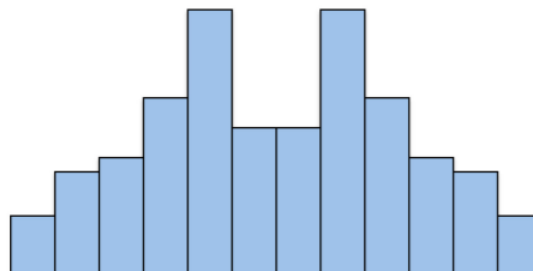
Roughly Symmetric Graph



Figure

This graph is symmetric and bimodal:

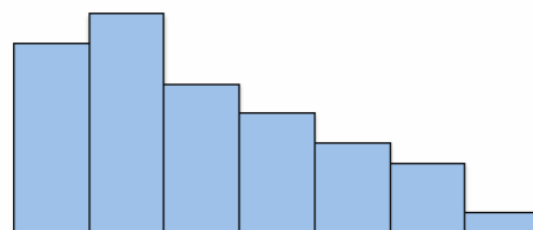
Bimodal and Symmetric Graph



Figure

This graph is skewed to the right:

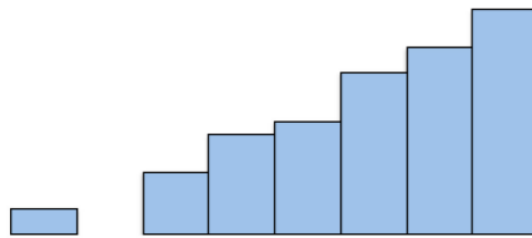
Skewed Right Graph



Figure

This graph is skewed to the left and has a gap:

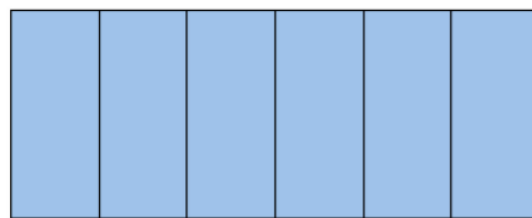
Skewed Left Graph



Figure

This graph is uniform since all the bars are the same height:

Uniform Graph



Figure

Example 2.2.7 creating a frequency distribution, histogram, and ogive

The following data represents the percent change in tuition levels at public, fouryear colleges (inflation adjusted) from 2008 to 2013 (Weissmann, 2013). Create a frequency distribution, histogram, and ogive for the data.

Table 2.2.5: Data of Tuition Levels at Public, Four-Year Colleges

19.5%	40.8%	57.0%	15.1%	17.4%	5.2%	13.0%
15.6%	51.5%	15.6%	14.5%	22.4%	19.5%	31.3%
21.7%	27.0%	13.1%	26.8%	24.3%	38.0%	21.1%
9.3%	46.7%	14.5%	78.4%	67.3%	21.1%	22.4%
5.3%	17.3%	17.5%	36.6%	72.0%	63.2%	15.1%
2.2%	17.5%	36.7%	2.8%	16.2%	20.5%	17.8%
30.1%	63.6%	17.8%	23.2%	25.3%	21.4%	28.5%
9.4%						

Solution

1. Find the range:

$$\text{largest value} - \text{smallest value} = 78.4\% - 2.2\% = 76.2\%$$

2. Pick the number of classes:

Since there are 50 data points, then around 6 to 8 classes should be used. Let's use 8.

3. Find the class width:

$$\text{width} = \frac{\text{range}}{8} = \frac{76.2\%}{8} \approx 9.525\%$$

Since the data has one decimal place, then the class width should round to one decimal place. Make sure you round up.

$$\text{width} = 9.6\%$$

4. Find the class limits:

$$2.2\% + 9.6\% = 11.8\%, 11.8\% + 9.6\% = 21.4\%, 21.4\% + 9.6\% = 31.0\%, \Leftarrow$$

5. Find the class boundaries:

Since the data has one decimal place, the class boundaries should have two decimal places, so subtract 0.05 from the lower class limit to get the class boundaries. Add 0.05 to the upper class limit for the last class's boundary.

$$2.2 - 0.05 = 2.15\%, 11.8 - 0.05 = 11.75\%, 21.4 - 0.05 = 21.35\% \Leftarrow$$

Every value in the data should fall into exactly one of the classes. No data values should fall right on the boundary of two classes.

6. Find the class midpoints:

$$\text{midpoint} = \frac{\text{lower limit} + \text{upper limit}}{2}$$

$$\frac{2.2 + 11.7}{2} = 6.95\%, \frac{11.8 + 21.3}{2} = 16.55\%, \Leftarrow$$

7. Tally and find the frequency of the data:

Table 2.2.6: Frequency Distribution for Tuition Levels at Public, Four-Year Colleges

Class Limits	Class Boundaries	Class Midpoint	Tally	Frequency	Relative Frequency	Cumulative Frequency
2.2-11.7	2.15-11.75	6.95		6	0.12	6
11.8-21.3	11.75-21.35	16.55		20	0.40	26
21.4-30.9	21.35-30.95	26.15		11	0.22	37
31.0-45.0	30.95-40.55	35.75		4	0.08	41
40.6-50.1	40.55-50.15	45.35		2	0.04	43
50.2-59.7	50.15-59.75	54.95		2	0.04	45
59.8-69.3	59.75-69.35	64.55		3	0.06	48
69.4-78.9	69.35-78.95	74.15		2	0.04	50

Make sure the total of the frequencies is the same as the number of data points.

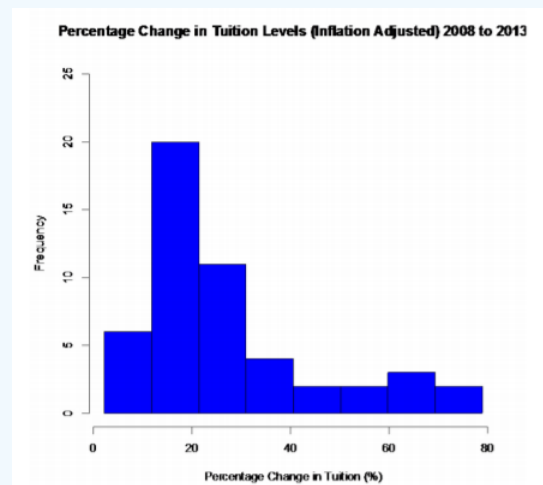


Figure 2.2.11: Histogram for Tuition Levels at Public, Four-Year Colleges

This graph is skewed right, with no gaps. This says that most percent increases in tuition were around 16.55%, with very few states having a percent increase greater than 45.35%.

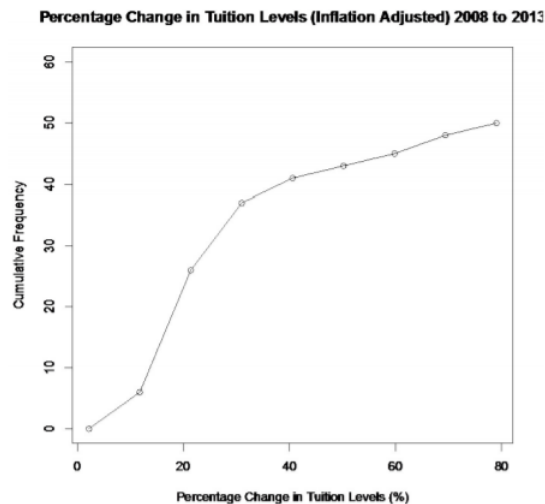


Figure 2.2.12: Ogive for Tuition Levels at Public, Four-Year Colleges

Looking at the ogive, you can see that 30 states had a percent change in tuition levels of about 25% or less.

There are occasions where the class limits in the frequency distribution are predetermined. Example 2.2.8 demonstrates this situation.

Example 2.2.8 creating a frequency distribution and histogram

The following are the percentage grades of 25 students from a statistics course. Make a frequency distribution and histogram.

Table 2.2.7: Data of Test Grades

62	87	81	69	87	62	45	95	76	76
62	71	65	67	72	80	40	77	87	58
84	73	93	64	89					

Solution

Since this data is percent grades, it makes more sense to make the classes in multiples of 10, since grades are usually 90 to 100%, 80 to 90%, and so forth. It is easier to not use the class boundaries, but instead use the class limits and think of the upper class limit being up to but not including the next classes lower limit. As an example the class 80 – 90 means a grade of 80% up to but not including a 90%. A student with an 89.9% would be in the 80-90 class.

Table 2.2.8: Frequency Distribution for Test Grades

Class Limit	Class Midpoint	Tally	Frequency
40-50	45		2
50-60	55		1
60-70	65		7
70-80	75		6
80-90	85		7
90-100	95		2

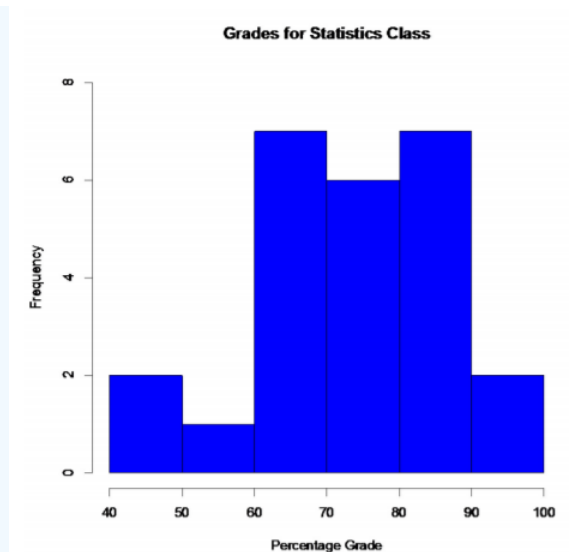


Figure 2.2.13: Histogram for Test Grades

It appears that most of the students had between 60 to 90%. This graph looks somewhat symmetric and also bimodal. The same number of students earned between 60 to 70% and 80 to 90%.

There are other types of graphs for quantitative data. They will be explored in the next section.

Homework

1. The median incomes of males in each state of the United States, including the District of Columbia and Puerto Rico, are given in Example 2.2.9 ("Median income of," 2013). Create a frequency distribution, relative frequency distribution, and cumulative frequency distribution using 7 classes.

Table 2.2.9: Data of Median Income for Males

\$42,951	\$52,379	\$42,544	\$37,488	\$49,281	\$50,987
\$60,705	\$50,411	\$66,760	\$40,951	\$43,902	\$45,494
\$41,528	\$50,746	\$45,183	\$43,624	\$43,993	\$41,612
\$46,313	\$43,944	\$56,708	\$60,264	\$50,053	\$50,580
\$40,202	\$43,146	\$41,635	\$42,182	\$41,803	\$53,033
\$60,568	\$41,037	\$50,388	\$41,950	\$44,660	\$46,176
\$41,420	\$45,976	\$47,956	\$22,529	\$48,842	\$41,464
\$40,285	\$41,309	\$43,160	\$47,573	\$44,057	\$52,805
\$53,046	\$42,125	\$46,214	\$51,630		

2. The median incomes of females in each state of the United States, including the District of Columbia and Puerto Rico, are given in Example 2.2.10 ("Median income of," 2013). Create a frequency distribution, relative frequency distribution, and cumulative frequency distribution using 7 classes.

Table 2.2.10: Data of Median Income for Females

\$31,862	\$40,550	\$36,048	\$30,752	\$41,817	\$40,236
\$47,476	\$40,500	\$60,332	\$33,823	\$35,438	\$37,242
\$31,238	\$39,150	\$34,023	\$33,745	\$33,269	\$32,684

\$31,844	\$34,599	\$48,748	\$46,185	\$36,931	\$40,416
\$29,548	\$33,865	\$31,067	\$33,424	\$35,484	\$41,021
\$47,155	\$32,316	\$42,113	\$33,459	\$32,462	\$35,746
\$31,274	\$36,027	\$37,089	\$22,117	\$41,412	\$31,330
\$31,329	\$33,184	\$35,301	\$32,843	\$38,177	\$40,969
\$40,993	\$29,688	\$35,890	\$34,381		

3. The density of people per square kilometer for African countries is in Example 2.2.11 ("Density of people," 2013). Create a frequency distribution, relative frequency distribution, and cumulative frequency distribution using 8 classes.

Table 2.2.11: Data of Density of People per Square Kilometer

15	16	81	3	62	367	42	123
8	9	337	12	29	70	39	83
26	51	79	6	157	105	42	45
72	72	37	4	36	134	12	3
630	563	72	29	3	13	176	341
415	187	65	194	75	16	41	18
69	49	103	65	143	2	18	31

4. The Affordable Care Act created a market place for individuals to purchase health care plans. In 2014, the premiums for a 27 year old for the bronze level health insurance are given in Example 2.2.12 ("Health insurance marketplace," 2013). Create a frequency distribution, relative frequency distribution, and cumulative frequency distribution using 5 classes.

Table 2.2.12: Data of Health Insurance Premiums

\$114	\$119	\$121	\$125	\$132	\$139
\$139	\$141	\$143	\$145	\$151	\$153
\$156	\$159	\$162	\$163	\$165	\$166
\$170	\$170	\$176	\$177	\$181	\$185
\$185	\$186	\$186	\$189	\$190	\$192
\$196	\$203	\$204	\$219	\$254	\$286

5. Create a histogram and relative frequency histogram for the data in Example 2.2.9. Describe the shape and any findings you can from the graph.
6. Create a histogram and relative frequency histogram for the data in Example 2.2.10. Describe the shape and any findings you can from the graph.
7. Create a histogram and relative frequency histogram for the data in Example 2.2.11. Describe the shape and any findings you can from the graph.

8. Create a histogram and relative frequency histogram for the data in Example 2.2.12. Describe the shape and any findings you can from the graph.
9. Create an ogive for the data in Example 2.2.9. Describe any findings you can from the graph.
10. Create an ogive for the data in Example 2.2.10. Describe any findings you can from the graph.
11. Create an ogive for the data in Example 2.2.11. Describe any findings you can from the graph.
12. Create an ogive for the data in Example 2.2.12. Describe any findings you can from the graph.
13. Students in a statistics class took their first test. The following are the scores they earned. Create a frequency distribution and histogram for the data using class limits that make sense for grade data. Describe the shape of the distribution.

Table 2.2.13: Data of Test 1 Grades

80	79	89	74	73	67	79
93	70	70	76	88	83	73
81	79	80	85	79	80	79
58	93	94	74			

14. Students in a statistics class took their first test. The following are the scores they earned. Create a frequency distribution and histogram for the data using class limits that make sense for grade data. Describe the shape of the distribution. Compare to the graph in question 13.

Table 2.2.14: Data of Test 1 Grades

67	67	76	47	85	70
87	76	80	72	84	98
84	64	65	82	81	81
88	74	87	83		

Answer

See solutions

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